# State dependence in sequential equity judgements 

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#### Abstract

We report the results from a questionnaire-type experiment designed to elicit whether individuals decide in accordance with the equity axiom constituent for Rawls's second principle. The experiment is sequential in nature. Hence it generates panel data. We use recently developed panel data methods for studying the role that state dependence and unobservable individual-specific effects play for the observed equity judgements. The results indicate that a dominant share of our probants initially adhere to Hammond's equity axiom, but that many of these leave the Rawlsian position at later stages of the experiment. Although state dependence plays a significant role it cannot alone explain the observed decision behavior. Individual-specific effects are also important.


## 1 Introduction

The Rawlsian school claims that welfare judgements should be based on how policies affect the utility of the worst-off individual in society. More specifically, welfare judgements should be guided by the Rawlsian difference principle, which underlies the maximin principle of Rawls (1971). Rather than contemplating this normative statement, we turn in the present paper to the positive issue of whether individual decisions are consistent with a specific version of the difference principle. Since economic theory cannot provide an answer to this question, our examination will be empirical.

[^0]Empirical examinations are inflicted with their own problems. In particular, actual choices usually will be determined by a mixture of ethical and selfish considerations, the constraints under which choices are made, and strategic considerations. Thus it may be difficult to recover the underlying ethical principles from observed choices. A number of economists have therefore turned to experiments as a method for eliciting the principles that guide individuals when prioritizing on behalf of society.

We use an experimental setup developed by Gaertner (1992). Since probands are sequentially exposed to different levels of a treatment variable, one should ask to what degree choices made by probands at the later stages of our experiment are affected by their earlier choices. This phenomenon has long been recognized as a basic human response. In the (social) psychology literature this tendency to weight past choices heavily in present decisions is referred to as "preference for consistency". As argued in Cialdini et al. (1995) it exists as a measurable personality trait. ${ }^{1}$ The trait manifests itself in experimental data as what is called (positive) state dependence in econometrics. To capture the phenomenon we apply panel data methods described by Honoré and Kyriazidou (2000) and Magnac (2000), who in turn build on the work of Chamberlain (1985), and Heckman (1981). An important feature of the estimation technique proposed by Chamberlain (1985) is that it allows state dependence to be examined independently of individual-specific effects.

The present paper is related to the empirical literature that uses experiments in the examination of equity judgements. Yaari and Bar-Hillel (1984) carried out one of the first experiments aimed at revealing the principles that guide individuals who are given the hypothetical task of allocating goods to others. After having examined nine different principles, including the Rawlsian maximin principle and Utilitarianism, they concluded that people in experiments tended to act in accordance with the Rawlsian maximin rule when taking decisions in situations involving "need". In other situations, however, they found that the maximin rule did not apply. Frohlich et al. (1987a, b) found that the vast majority of participants preferred a compromise between the Rawlsian maximin principle and Utilitarianism, rather than one of these "extreme" principles. In a third experiment, reported by Gaertner (1994), the results are again mixed and difficult to understand as the outcome of one of the pure principles set out in the literature.

Gaertner et al. (2001) summarize the findings in the line of research to which the present paper belongs. They conclude that whether people base their welfare assessments on the Rawlsian difference principle is both context-dependent and dependent on the political and cultural environment. Gaertner (1992), Jungeilges and Theisen (2005), and Jungeilges and Theisen (2008) arrive at similar conclusions. The contextdependence is emphasized also in the review paper of Konow (2003).

Our current research effort makes use of the established experimental design due to Gaertner (1992). We contribute, however, through addressing new research questions, and by exploiting data from an additional country. The examination of the question "Do people act in accordance with the Rawlsian difference principle?" motivated the strand of the empirical social choice literature cited above. It also stimulated

[^1]our current effort. The results for Norway by and large confirm the findings of the previous research. In the present paper, however, we investigate whether individual decision behavior depends on the size of the group of individuals who are better off no matter which social state is realized. While this question was only touched upon in previous papers, we provide an in depth treatment of the issue. Finally, and most important, we contribute through examining whether choices in the sequential experiment are conditioned on previous choices, i.e., whether there is state dependence. We do this by applying modern econometric methods. By considering explicitly the dynamics involved in sequential experimental design, we carry the work of Jungeilges and Theisen (2005) a substantial step forwards.

In Sect. 2, we provide a brief theoretical account of the social choice context motivating the experiment. A discussion of the experimental design, and the instrument used in the empirical examination is given in Sect. 3. The data collection procedure and the sample are described in Sect. 4. The subsequent section contains results on individuals' propensity to act in accordance with Rawls' second principle, and an analysis where decision trees are used for describing the sequential nature of our experiment. In Sect. 6, we specify the econometric model used for examining and testing for the presence of state dependence. The maximum likelihood estimator for the state dependence parameter of a dynamic binary choice model in which the dependent variable is lagged once is determined explicitly. Next, in Sect. 7, we present and discuss the estimation results. In addition, we assess how well state dependence alone can replicate our observations, and we discuss the extent to which the results are driven by individ-ual-specific effects. Section 8 summarizes the main results and outlines some ideas for further research.

## 2 Theoretical background

Let $X=\{x, y, \ldots\}$ represent a finite or infinite set of social states. The set $N=$ $\{1,2, \ldots, i, j, \ldots, n\}$ refers to a finite group of individuals. Suppose that $\sharp X \geq 3$ and $\sharp N \geq 3$. Next, define $\mathcal{R}$ as the set of orderings on the set of social states $X$. Then for $R \in \mathcal{R} \forall x, y \in X$, we write $x R y$ to indicate that a social state (or policy) $x$ is at least as good as the state $y$ from a collective point of view.

To reflect the evaluations of social states by individuals, consider the Cartesian product of the set of individuals and the set of social states: $X \times N$. Elements of this set are of the form $(x, i)$. Such pairs are interpreted as referring to person $i$ under social state $x$. Let $\mathcal{U}$ denote the set of bounded functions defined on $X \times N$. Functions from this set are used for welfare comparisons between individuals under a given social state as well as between different individuals across alternative social states. Given $u \in \mathcal{U} \forall i, j \in N$, and $\forall x, y \in X$, the statement $u(x, i) \geq u(y, i)$ says that social state $x$ is at least as good as social state $y$ from the point of view of individual $i$. To express that under social state $x$ individual $i$ is at least as well off as individual $j$ under social state $y$, we write $u(x, i) \geq u(y, j)$. Social choice theory is concerned with finding characterizations of social welfare functionals. That is, one tries to characterize the type of functions which can be defined from the set $\mathcal{U}$ to the set of orderings $\mathcal{R}$, given some reasonable restrictions. Among the typical requirements such as independence
of irrelevant alternatives, Pareto-type principles and the anonymity principle one finds Hammond's equity axiom

Axiom 1 For some $u \in \mathcal{U}$ and any $x, y \in X$, if for some pair of individuals $i, j \in N$

$$
u(y, i)<u(x, i)<u(x, j)<u(y, j)
$$

and for all $k \in N \backslash\{i, j\}: u(y, k)=u(x, k)$, then $x R y$.
This axiom says that the individual who is better off anyway should not determine the social ordering. The relationship to Rawls's second principle is apparent. Deschamps and Gevers (1978) and Gaertner (1992) discuss the technical significance as well as extentions of this important static axiom. The assumption of a stable utility function made in this line of research is acceptable also when sequential choices are made within a short span of time, and in a controlled experimental setting. We now turn to the experimental design used to examine whether human decisions are consistent with Axiom 1.

## 3 Experimental design

We address three research questions related to Hammond's equity axiom.

1. Do individuals facing decision contexts as described in Sect. 2 decide in accordance with Hammond's equity axiom?
2. Does the decision behavior depend on the size $(S)$ of the group of individuals who are better off no matter which social state is realized?
3. Is the propensity to make a specific choice conditioned on previous choices, i.e., do we observe state dependence?

The first research question links this paper to the previous literature. Results concerning items 2 and 3 may shed light on whether the axiom has enough structure to be valid under widely different circumstances. The question is now what structure an experiment should have to be informative about our three research questions. First of all, we have to subject a proband to a situation which mimics the type of decision context described in Sect. 2. Second, we consider a situational aspect which is not made explicit in Hammond's axiom: the size $(S)$ of the group of those who are better off irrespective of the social alternative. In the axiom the size $S$ is fixed, but to address the second research question the size $(S)$ should be made an integral part of the decision context, so that probands have to make decisions under different levels of $S$. Finally, to answer the third research question we observe probands' responses (decisions) to a sequence of increasing levels of the $S$.

In response to these requirements we chose the following structure for the hypothetical decision questions: In each situation, person $i$ is better off under $x$ than under $y$, while person $j$ (or a group) is better off under $y$ than under $x$. The utility of person $j$ is always higher than the utility of person $i$ irrespective of the social state chosen (cf. Axiom 1 of Sect. 2). The proband has to choose between the alternatives $x$ and $y$. Each individual is sequentially subjected to the same choice situation with four
increasing levels of the treatment variable $S$ (size of the group of those who are better off regardless of the alternative chosen): $s_{0}<s_{1}<s_{2}<s_{3}$.

Our interest lies in the sequential decision pattern evolving under the systematic variation of the factor $S$ only. We use six different frames designed by Gaertner (1992), cf. Table 1. The richness of situational detail was chosen on purpose. The resulting real world flavor of the choice contexts was supposed to trigger the involvement of probands. The wide variation of detail between situations allows us to assess the robustness of the results.

Let us use Situation 1 to give a concrete description of the format. In the baseline question of Situation 1, probands are asked to decide whether a certain amount of money should be allocated exclusively to the assistance of a handicapped person (alternative $x$ ), or to alternative $y$, the education of one intelligent child ( $s_{0}=1$.) In the second step, alternative $x$ still is to allocate the money to the assistance of the handicapped person, but the number of intelligent children that could get education in alternative $y$ is increased to two $\left(s_{1}=2\right)$. In the third and the fourth step, the number of intelligent children who would benefit from alternative $y$ is increased to three $\left(s_{2}=3\right)$ and four $\left(s_{3}=4\right)$, respectively. In other words, more and more individuals who unanimously would prefer alternative $y$ to $x$ are gradually introduced, while the number of individuals who would benefit from $x$ is kept constant at one. The crucial question is whether the number of individuals who would benefit from alternative $y$ will affect the allocation of money, when money either must be allocated exclusively to $x$ or exclusively to $y$.

In Situations 2, 3, 4, 5, and 6, the alternatives among which probands are asked to prioritize differ from Situation 1. In Situation 2, probands are asked whether they would allocate money to an aid program against hunger in Sub-Saharan Africa ( $x$ ) or to environmental protection programs in or close to the probands' home country (alternative y). In Situation 3, the issue at stake is whether a poor country should allocate its limited reserves of foreign currency to the purchase of dialysis machines $(x)$ or to the purchase of fruit and vitamin pills $(y)$ to selected parts of the population.

In Situation 4, the issue is whether a poor country should allocate foreign currency to the purchase of dialysis machines $(x)$ or to the import of Bordeaux wine $(y)$. In Situation 5, the question is whether a poor country should allocate foreign currency to the purchase of clothing for a group of needy people $(x)$ or to the import of Bordeaux wine ( $y$ ). Finally, in Situation 6 the issue is whether a run-down country should emphasize workers' rights to strike and to choose occupation freely and pull itself up by its bootstraps $(x)$ or accept a condition to set aside workers' basic rights in order to obtain a favorable loan that would benefit significant groups of the population. A $p d f$-file containing the questionnaire due to Gaertner (1992) can be found at http:// www.vwl-theorie.uni-osnabrueck.de/Basic.pdf. For a hard copy, see Jungeilges and Theisen (2008).

All situations are designed so that there will be unanimous support for the claim that alternative $x$ is a social goal worth pursuing. In Situations 1, 2, 3, and 6, the same is likely to hold for alternative $y$. Situations 4 and 5 , however, were deliberately designed so that some may consider alternative $y$ not to be warranted as a social goal. This was done in order to test the logic and consistency of probands' answers across situations and to test for context dependence of choices.

Table 1 Summary of situations


## 4 Data collection

The data were collected in March 2001, using two different student groups at the University of Agder located on the Southern coast of Norway. Group 1 consists of first-year students in a two-year study program in basic business administration. The second group consists of third-year students in an advanced two-year program in business administration.

Descriptions of the six situations together with answering sheets were administered during normal lecture hours. It was pointed out to the students that there was no such thing as a single right answer to a question. The first-year students had not been exposed to welfare theory or social choice theory prior to the experiment. The third-year students, by contrast, had been introduced to the concepts of utilitarian and Rawlsian welfare functions. Discussions with the students after the answering sheets had been collected revealed that at least some of the third-year students had figured out that the experiment was related to welfare economic issues and the concepts of Rawlsianism and Utilitarianism.

In total, the sample consists of 130 probands of which 66 were first-year students and 64 were third-year students. There was an equal split between males and females. Probands' age ranged from 19 to 40 , but $95 \%$ of them were between 19 and 25 years old. As pointed out in Jungeilges and Theisen (2005) the sample is not too far from being representative of the Norwegian population when it comes to social background.

## 5 Data analysis

For each situation, a sequence of four decisions, indexed by $t \in\{0, \ldots, 3\}$, constitute the response of a proband. At each step in the sequence there is a binary choice: The decision can be in favor of the individual(s) who is (are) worst off under both policies, in which case it is in accordance with the equity axiom. Alternatively the decision can be in favor of those who are best off under both policies, which means that the decision follows a non-Rawlsian logic. The results of the experiment can be displayed by means of decision trees, one for each situation, as shown in Figs. 1 and 2. Each node in these trees represents a step in the sequence of four decisions-indicated by (0), (1), (2), (3) at the top of each subfigure. At each node, the upper branch represents a decision in accordance with the equity axiom, and the lower branch represents a decision that is not in accordance with the Rawlsian logic.

Let us use Fig. 1a for explaining the sequential nature of the experiment. The upper branch of the tree (branch zero from the top) represents a sequence of decisions that follows the Rawlsian logic at each and every step of the sequence. The number 0.8692 on the first leg of this branch is the relative frequency of making a Rawlsian decision at the first step. Similarly, the number 0.7611 on the second leg of the branch is the relative frequency of a Rawlsian decision at the second step, conditional on a Rawlsian decision in the baseline question. In the same manner, the numbers 0.8023 and 0.9275 on the third and fourth leg of the upper branch are the relative frequencies of making Rawlsian decisions at steps number three and four, conditional on having made a Rawlsian decision at the previous steps. The leaf frequency 0.4923 at the end


Fig. 1 Empirical decision trees. a Decision tree for Situation 1. b Decision tree for Situation 2. c Decision tree for Situation 3. d Decision tree for Situation 4
of the branch is the frequency of deciding in accordance with the Rawlsian logic at all steps. Hence, the leaf frequency is the product of the frequencies on all the legs that make up the branch. Notice that the leaf frequencies of branch zero are large in all the six situations, indicating a strong tendency to decide in accordance with Hammond's axiom throughout the experiment.


Fig. 2 Empirical decision trees. a Decision tree for Situation 5. b Decision tree for Situation 6

Branch number 1 in Fig. 1a represents a sequence where the proband in the baseline question $(t=0)$ decides in accordance with the equity axiom, sticks to the Rawlsian position at steps 1 and 2 as the group of individuals that would benefit from policy $y$ is increased, but with a conditional frequency of 0.0725 deviates from the Rawlsian position at step 3. Similarly, branch 3 represents a sequence where the proband at steps 0 and 1 follows the Rawlsian logic, but with a frequency of 0.1977 leaves the Rawlsian position at step 2 , and with a frequency of 0.9412 sticks to the non-Rawlsian position also at step 3.

Let us now summarize the main features of the choices in all six situations (cf. Figs. 1, 2). Starting at the root of the trees, observe first the high frequencies with which the individual in the baseline question chooses in accordance with the equity axiom. Second, having initially made a Rawlsian decision, the frequencies are high that the decision is replicated at later steps. The frequencies of deviating from the initial Rawlsian decision are, however, in Situations $1,2,3$, and 6 , also sizable, while this is not the case in Situations 4 and 5. Third, the frequencies of initially making a non-Rawlsian decision are moderate in Situations 1, 2, 3, and 6, but having made such a decision at the first step there is a strong tendency to stick to the same decision at the later steps.

The observations summarized so far indicate that the results for Situations 4 and 5 differ from the others. In both situations, the frequency of initially choosing in accordance with the equity axiom is close to 1 , and once having made a Rawlsian choice at the initial step, very few deviate from this decision at later steps. Hence, in Situations 4 and 5, we find a high relative frequency of support for the worst-off individual(s), for any level of the treatment variable. In Situations 1, 2, 3, and 6, by contrast, the
fraction of probands that at all steps showed support for those who are worst-off is substantially lower and fairly stable between 40 and $50 \%$, cf. the leaf frequencies of the very upper branches.

Notice that the leaf frequencies that a proband will follow branches 1,3 , or 7 in Figs. 1 and 2 are much smaller than the leaf frequency of branch zero. Relatively small frequencies are also associated with the very lower branch (15), which represents a complete sequence of non-Rawlsian decisions. Finally, observe the very small leaf frequencies of branches $2,4,5,6$, and 8 to 14 . This large collection of branches contains two subsets. The first category consists of the eight branches $2,4,5,6,9,10$, 11 , and 13, where the individual switches back and forth between deciding in accordance with the equity axiom and not adhering to the axiom. Such decision patterns are difficult to rationalize, and may indicate that the individual has not understood the logic of the experiment. In the second category, consisting of the three branches 8,12 , and 14 , the individual at an early step does not decide in accordance with the equity axiom, but at a later step does adhere to that axiom. Such decision patterns seem peculiar. Throughout the rest of the paper we will refer to the decisions corresponding to all the branches $2,4,5,6$, and 8 through 14 as inconsistent. The fact that all the branches corresponding to sequences that are classified as inconsistent carry very small leaf frequencies indicates that probands largely must have understood the logic of the experiment. Moreover, one may speculate that the low frequency of inconsistent decisions simply may be due to errors in decision making. When it comes to fulfillment of the equity axiom, the branches representing inconsistent decisions are not of primary interest. On the other hand, when testing for state dependence, we shall see that the some inconsistent branches carry decisive information. If an inconsistent decision is made at an early stage and the individual continues to stick to that decision, it may be indicative of state dependence.

The descriptive analysis so far shows that decision behavior is quite similar across contexts, in particular for Situations 1,2,3, and 6 . For all situations, our results confirm earlier findings that a considerable fraction of probands decide in line with Axiom 1 (cf. research question 1). In the sequel of this section, we will compare the responses in the six situations under two different perspectives. First, we address the second research question by focussing on the change in the probability of a decision adhering to the equity axiom as the level of the treatment variable increases, i.e., as the group of individuals who are better off under both alternatives is extended. Second, the distribution of three types of decisions will be scrutinized: decisions that are consistent and in line with the equity axiom, those that are consistent but do not follow the axiom, and finally inconsistent decisions. In each case, we aggregate information contained in the decision trees to reduce the complexity of the evidence. We seek a clearer view of whether there is a common decision pattern across the different contexts. Apart from that, both views are in some way related to state dependence. The graphs associated with the first perspective can be thought of as an informal way to assess the possibility that state dependence may play a significant role. The second perspective produces an overview of the results for all situations. In addition, it provides prior information concerning the existence of the state dependence estimator derived in Sect. 6.

The evidence on changes in the frequency of adherence to the equity axiom is for Situations 1, 2, 3, and 6 summarized in Fig. 3. Each subfigure is associated with one


Fig. 3 Fulfillment of the equity axiom $[N=130]$
of these decision contexts, and contains a plot of the relative frequency of a decision fulfilling the equity axiom against the level of our treatment variable, i.e., the size of the group of those who are better off under both alternatives. For the baseline question ( $t=0$ ) the (unconditional) relative frequency can be read off the first leg of branch 0 of the decision tree. The frequency at step $t \in\{1,2,3\}$ is computed as the product of the first $t+1$ (conditional) frequencies found on branch 0 of the decision tree for the respective situation. Each observed frequency is indicated by $\bullet$. In addition, we superimpose a plot of hypothetical probabilities based on the assumption that a proband, once (s)he decided in line with the axiom at step 0 , lets a random process (independent identically distributed Bernoulli trials $B(p=0.5)$ ) determine her or his subsequent choices. Each hypothetical probability is indicated by the symbol o. Finally, we add a horizontal line extending to the right of the baseline frequency of choosing according to the Rawlsian logic.


Fig. 4 Situations 1-6 in $\left(\hat{P}_{r}, \hat{P}_{u}\right)$-space, $[N=130]$
The graph of the relative frequencies indicated by $\bullet$ shows the effect of a variation in the treatment variable on the judgements of probands. The hypothetical probabilities indicated by o show how the evidence should look like if probands did not take the information on group size into consideration (or they considered it but it did not influence their choice at all) and there was no state dependence. The horizontal line also relates to a hypothetical scenario: If probands who adhere to the equity axiom in the baseline decision continue to do so on each and every later step, despite the information on the increase in the magnitude of the treatment variable (group size), then the observed frequencies for steps 1 to 3 would lie on this line. Such a pattern would indicate a "preference for consistency".

As indicated by Fig. 3, the fraction of the respondents adhering to the equity axiom is a decreasing function of the magnitude of the treatment variable. The fraction adhering to the axiom drops from about $90 \%$ at the lowest level of the treatment variable to slightly less than $50 \%$ for the highest level of the treatment variable. In all four situations the graph of observed frequencies is distinctly different from the graph for the hypothetical probabilities. Although there are some differences between the four graphs, we conclude that proband's behavior seems to be driven by systematic factors that operate in a similar way in Situations 1, 2, 3, and 6. At this stage of the analysis it is unclear whether state dependence and/or individual specific effects can explain our observations.

Next, let us consider the distribution of three types of decisions previously introduced: consistent decisions in line with the equity axiom, consistent decisions not following the axiom, and inconsistent decisions. Figure 4 gives this distribution for all situations. The frequency of deciding in accordance with the equity axiom is measured on the horizontal axis in Fig. 4. The probability of either initially or eventually deciding in a non-Rawlsian way is measured on the vertical axis. Finally, the estimated probability of not deciding according to any of these patterns is measured by the vertical or horizontal distance from the point representing a situation to the hypotenuse
of the equilateral triangle. This distance reflects the frequency for the occurrence of what we have characterized as inconsistent decisions. Notice that our probands were most likely to decide inconsistently in Situations 2 and 3. In the other situations very few decisions were inconsistent. The crucial role of the inconsistent decisions will become apparent when we test for state dependence in Sect. 7.

The descriptive analysis of this section indicates strong similarities in the decision pattern across contexts, in particular this is the case for Situations 1, 2, 3, and 6. In these situations we have seen that the majority of the respondents reveal themselves as either Rawlsians ( $40-50 \%$ ) or non-Rawlsians ( $10-20 \%$ ). The majority of the remaining probands eventually respond to an increase of the treatment variable by leaving the Rawlsian position.

Our experiment generates panel data. For that data structure Heckman (1981) provides an extensive discussion of the different kinds of forces that may be at work. Two phenomena are particularly relevant in our context: Individual heterogeneity, and true state dependence. On the basis of the analysis in the present section it is not possible to say which of these two forces are driving the observed outcomes. ${ }^{2}$ In the sequel, we will therefore turn to research question 3 by making use of a model of sequential decisions that allows for state dependence as well as for individual effects.

## 6 The econometric model

We now turn to a dynamic binary response model which relates the conditional probability for a success in period $t$ to state dependence ( $\gamma$ ) and unobservable individual effects $\left(\delta_{i}\right)$. In our context, the choice of individual $i$ at time $t, y_{i t}$, is defined as a success if it is in line with the Rawlsian logic $\left(y_{i t}=1\right)$. The dynamic binary response model with the dependent variable lagged once takes the form

$$
\begin{equation*}
P\left(y_{i t}=1 \mid \delta_{i}, y_{i}, \ldots, y_{i t-1}\right)=\frac{e^{\gamma y_{i t-1}+\delta_{i}}}{1+e^{\gamma y_{i t-1}+\delta_{i}}} \tag{1}
\end{equation*}
$$

where $i=1, \ldots, N$ indexes individuals and $t=1, \ldots, T$ serves as a time index. In our case $T$ equals 3 . We assume $\gamma \in \mathbb{R}$. For individual $i$ the probability for a success occurring in the initial period is assumed to depend on the unobservable individualspecific effect $\delta_{i} \in \mathbb{R}$ alone

$$
P\left(y_{i 0}=1 \mid \delta_{i}\right)=p_{0}\left(\delta_{i}\right)
$$

This model reflects how data are generated in the experiments discussed in the previous sections. Following an approach due to Chamberlain (1985) which is out-

[^2]Table 2 Outcomes relevant for estimation of $\hat{\gamma} . A=A_{1} \cup A_{2}$ and $B=B_{1} \cup B_{2}$

| set | outcome | \# state changes | $n_{i}$ |
| :---: | :---: | :---: | :---: |
|  | 0000 | 0 |  |
|  | 0001 | 1 |  |
| $A_{1}$ | 0010 | 2 | $n_{1}$ |
|  | $\begin{array}{lllll}0 & 0 & 1\end{array}$ | 1 | $n_{2}$ |
| $B_{1}$ | 0100 | 2 | $n_{4}$ |
|  | $\begin{array}{llll}0 & 1 & 0 & 1\end{array}$ | 3 | $n_{5}$ |
|  | 0110 | 2 |  |
|  | 0111 | 1 |  |
|  | 1000 | 1 |  |
|  | 1001 | 2 |  |
| $A_{2}$ | 1010 | 3 | $n_{3}$ |
|  | 1011 | 2 | $n_{1}$ |
| $B_{2}$ | 1100 | 1 | $n_{6}$ |
|  | 1101 | 2 | $n_{4}$ |
|  | 1110 | 1 |  |
|  | 1111 | 0 |  |

lined in Honoré and Kyriazidou (2000) and Jungeilges and Theisen (2006), it can be shown that inference about the state dependence parameter, $\gamma$, in the case where the data generating process is observed at exactly four points in time, can be based on the log-likelihood function

$$
\begin{equation*}
L(\gamma)=\sum_{i=1}^{N} \chi_{y_{i 1}+y_{i 2}=1}\left(y_{i 1}, y_{i 2}\right) \ln \left(\frac{e^{\gamma\left(y_{i 0}-y_{i 3}\right)^{y_{i 1}}}}{1+e^{\gamma\left(y_{i 0}-y_{i 3}\right)}}\right) \tag{2}
\end{equation*}
$$

The argument of the $\ln$ function is the probability that an individual will follow a particular branch in our decision trees, conditional on the choice made at the first step of the experiment, while $\chi$ is an indicator function taking the value 1 if the condition $y_{i 1}+y_{i 2}=1$ holds, otherwise it is equal to zero. The $\chi$ function selects the subset of individuals who either at time 1 or time 2, but not at both times, have made a Rawlsian choice. The log likelihood function (2) is independent of individual-specific effects $\delta_{i}$.

Since the maximum likelihood estimator of $\gamma$ does not depend on the unobservable individual-specific effects, we can obtain an estimate of $\gamma$ without having to make any distributional assumptions about the unobservable $\delta_{i}$. This desirable property is a consequence of the selection made by the indicator function $\chi$ in (2). For the rationale behind the selection rule see Chamberlain (1985). In our case the indicator function selects the subsets of outcomes $\left(y_{i 0} y_{i 1} y_{i 2} y_{i 3}\right)$ from $A=A_{1} \cup A_{2}$ and $B=B_{1} \cup B_{2}$ given in Table 2.

Once the likelihood function for the appropriate subsample has been established, the maximum likelihood estimator of the state dependence parameter is found in the standard way. From the first order condition it follows that

$$
\begin{equation*}
\hat{\gamma}=\ln \left(\frac{n_{2}+n_{6}}{n_{3}+n_{5}}\right) . \tag{3}
\end{equation*}
$$

Involving the second order derivative of (2) w.r.t. $\gamma$, Jungeilges and Theisen (2006) show that (3) indeed maximizes the likelihood function.

Notice that four of the events which are contained in the set $A \cup B$, and therefore were included in the log likelihood function, have dropped out in the course of deriving the maximum likelihood estimator. This is due to the fact that the probabilities of these events are independent of the state dependence parameter. Hence they are constants that vanish when finding the first order derivative of the likelihood function.

Next, notice that the estimator of the state dependence parameter is obtained by applying the natural logarithm to a ratio of counts based on a subsample of individuals. The numerator of this ratio is the count of cases in which just one state change occurs, conditional on the initial choice. The denominator is the count of cases in which the maximum number of (three) state changes are made, conditional on the initial choice. Consequently, the value of the estimator is positive (negative) if the number of sample outcomes in which one state change occurs is larger (smaller) than the number of incidences with three state changes. Positive values of $\hat{\gamma}$ reflect state dependence in the sense that the probability of realizing a success in period $t$ is high if a success has been observed in period $t-1$. If, on the other hand, $\hat{\gamma}$ assumes a negative value, it indicates state dependence of the type where there is a tendency that individuals switch back and forth between following the Rawlsian logic and not adhering to that logic. Finally, if the ratio of the absolute frequencies of cases with only one state change and cases with three changes is approximately equal to one, the $\gamma$ estimator will take a value close to zero. In the context of model (1) the probability for a success after a success had been observed before would then just be determined by the individual specific effect $\delta_{i}$.

From (3) it is immediately seen that the maximum likelihood estimator of $\gamma$ is not defined if a sample does not contain admissible cases with exactly one state change ( $n_{2}+n_{6}=0$ ) or one does not observe any binary sequences indicating three state changes $\left(n_{3}+n_{5}=0\right)$. From a formal point of view the estimator then does not exist. In cases with many elementary events and small samples this problem may often arise, since outcomes that occur with a probability close to zero may not be observed in the sample. Through increasing the sample size sufficiently, the problem may be solved. Although increasing the sample size certainly will be a sound strategy, there is no guarantee that the problem will always vanish with a sample that can realistically be collected. Moreover, to obtain a larger sample may be costly. The obstacles and costs of obtaining a sample of sufficient size may in particular be high for samples generated through experiments. Consequently, we need a procedure to come around the problem that $\hat{\gamma}$ does not exist, without needing to collect ever-larger real-world samples.

Such a procedure can be designed once it is recognized that the outcomes needed to estimate $\hat{\gamma}$ occur with a zero frequency in the observed sample. In the true underlying population distribution that the sample can be thought as drawn from, however, it is reasonable to assume that these events have strictly positive, albeit small, probabilities for occurring. On this assumption we can model the underlying true population distribution by means of the following mixed density

$$
\phi(\alpha)=\alpha \phi(\omega)+(1-\alpha) \hat{f}(\omega)
$$

where $\hat{f}(\omega)$ is the density estimate obtained from our sample, $\phi(\omega)$ represents a density of our choice where $\omega$ is a random variable defined over the set of 16 outcomes, and $\alpha \in(0,1)$. By assigning a sufficiently small value to the mixing constant $\alpha$, the density $\phi$ represents a marginally distorted summary of our sample. We choose $\alpha$ such that $\phi(\alpha)$ is contained in a "confidence region" around the true but unknown underlying discrete density function $f(\omega)$. In what follows, we set $\alpha=0.05$. The key criterion for the choice of $\phi$ is given by the need to put some positive probability mass over the outcomes which are associated with 0 probability in the empirical density $\hat{f}$. In our case, we specify $\phi(\omega)$ as a uniform density. Each outcome is assumed to occur with equal probability of $\frac{1}{16}$.

We generate data from the mixed cumulative distribution function $\Phi(\alpha)$ on the basis of $\Phi^{-1}$, the inverse of the distribution function. Once we have generated such a pseudo sample-with the number of observations identical to the number of observations in the original sample-we test the hypothesis $H_{0}: \phi(\alpha)=f(\omega)$ versus $H_{1}: \phi(\alpha) \neq f(\omega)$ using a $\chi^{2}$ test for the equality of to multinomial distributions described by Mood et al. (1974, pp. 448-452). The $\chi^{2}$ statistic and the associated $p$ value can be interpreted as a measure of the distance between the empirical densities $\hat{\phi}$ and $\hat{f}$. If we fail to reject the $H_{0}$, we can view the pseudo sample as a set of realizations drawn from a density being equivalent to the population density $f$. Such an equivalent sample is then used to estimate the state dependence parameter $\gamma$. If the estimator does not exist, a new pseudo sample is drawn. The process terminates once, we have obtained $\hat{\gamma}$. The likelihood of at least one repetition of the sampling process is inversely related to the size of $\alpha$. In our implementation of the procedure with $\alpha=0.05$, we observed the necessity of very few repetitions.

## 7 Inference

We estimated the state dependence parameter $\gamma$ for Situations 1, 2, 3, and 6. The estimates for Situations 2 and 3 were obtained from the collected sample of size $N=130$, while for the remaining situations the resampling procedure described in the last section had to be implemented. ${ }^{3}$ The estimation results are shown in Table 3, along with the count of observations in the numerator and denominator of the ratio under the ln function in the $\gamma$-estimator (3), the likelihood ratio (LR) for testing the hypotheses $H_{0}: \gamma=0$ versus $H_{1}: \gamma \neq 0$ and the associated $p$ values.

The hypothesis $H_{0}: \gamma=0$ is clearly rejected at the $5 \%$ level on the basis of the likelihood ratio statistic for all situations. Hence, we conclude that the results provide clear evidence for state dependence of the type where the likelihood of a success in

[^3]Table 3 Inference about $\gamma(N=130)$

| Situation | $n_{2}+n_{6}$ | $n_{3}+n_{5}$ | $\hat{\gamma}$ | LR | $p$ Value |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 11 | 1 | 2.40 | 9.75 | 0.0018 |
| 2 | 14 | 3 | 1.54 | 7.72 | 0.0055 |
| 3 | 21 | 1 | 3.04 | 22.36 | 0.0000 |
| 6 | 20 | 2 | 2.03 | 17.10 | 0.0000 |



Fig. 5 Kernel density estimates for $\hat{\gamma}$. a Situation 1 (mean $=2.44$, median $=2.56$ ). b Situation 2 $($ mean $=1.42$, median $=1.39) . \mathbf{c}$ Situation $3($ mean $=2.39$, median $=2.35)$. $\mathbf{d}$ Situation $6($ mean $=2.78$, median $=2.89$ )
period $t$ is high if a success was realized also in period $t-1$. Finally, notice that the estimates of the state dependence parameter vary moderately between situations, and that the estimates based on resampling do not differ strongly from the estimates for Situations 2 and 3.

In order to obtain an even better basis for assessing the $\gamma$-estimates, we have for each of the Situations $1,2,3$, and 6 generated 500 new samples, using the resampling procedure described at the end of Sect. 6. All these generated data sets fulfill the restrictions for the existence of the $\gamma$-estimator, and provide us with estimates of the state dependence parameter. Kernel density estimates based on the realizations $\hat{\gamma}_{1} \ldots \hat{\gamma}_{500}$ for each situation are given in Fig. 5. For each situation the average and the median of the $500 \gamma$ estimates are close to the point estimates given in column 4
of Table $3 .{ }^{4}$ Moreover, the generated $\gamma$-estimates are all positive. This lends support to our previous statements concerning state dependence.

For each Situation 1, 2, 3, and 6 we construct a hypothetical decision tree based on the values of $\hat{\gamma}$ given in Table 3. It is assumed that no individual effects are operating $\left(\delta_{i}=0\right)$. According to (1), the probability for arriving at a choice in line with the equity axiom at step $t$, conditional on the choice made at $t-1$, can then be stated as

$$
\begin{equation*}
\hat{P}\left(y_{i t}=1 \mid \delta_{i}=0, y_{i 0}, \ldots, y_{i t-1}\right)=\frac{e^{\hat{\gamma} y_{i t-1}}}{1+e^{\hat{\gamma} y_{i t-1}}} \tag{4}
\end{equation*}
$$

where we take the relative frequency for the initial choice as it is observed in the experiment. The individual specific effect cannot be removed from the probability estimate of the initial choice. The conditional probabilities computed from (4) are put on the branches of the decision tree. Apart from the dependence on the initial choice, the resulting tree depends on the state dependence estimator only. Hence, we call it a $\hat{\gamma}$-tree. The $\hat{\gamma}$-trees are shown in Fig. 6. Each tree provides (i) an estimate for the distribution of the logically possible outcomes of the experiment (leaf probabilities) and (ii) it gives an insight into the process by which this distribution is generated, i.e., via the probabilities on the branches for the hypothetical case with no individual specific effects operating once the initial decision has been made.

Let us contrast the distribution obtained under the hypothesis of no individual specific effects with the estimate of the distribution based on the entire sample evidence. For this purpose we compute the differences between the conditional branch frequencies based on all observations (shown on the decision trees in Figs. 1 and 2) and the probabilities found in the corresponding $\hat{\gamma}$-trees

$$
\begin{align*}
& \Delta_{t}=\hat{P}\left(y_{i t}=\xi \mid \delta_{i} \in \mathbb{R}, y_{i 0}, \ldots, y_{i t-1}\right) \\
& \quad-\hat{P}\left(y_{i t}=\xi \mid \delta_{i}=0, y_{i 0}, \ldots, y_{i t-1}\right), \quad \xi \in\{0,1\} \tag{5}
\end{align*}
$$

The tree constructed by positioning the differences between the conditional probabilities on the associated branches and attaching the differences in leaf probabilities to the leafs is referred to as a $\Delta$-tree. Notice that the $\Delta$-tree is only an expositional device, not a proper decision tree. The $\Delta$-trees for Situations $1,2,3$, and 6 are shown in Fig. 7. A positive (negative) value in the $\Delta$-tree suggests that the probability is underestimated (overestimated) by the model that ignores individual specific effects. The larger the absolute values of the observed differences, the stronger the role of individual specific effects.

If probands' choices conditional on the choice made at step 1 are determined only by state dependence, the $\hat{\gamma}$-trees should be identical to the observed decision trees in

[^4]


(c)

Fig. $6 \hat{\gamma}$-trees. a Situation 1. b Situation 2. c Situation 3. d Situation 6

Sect. 5, and the $\Delta$-trees should have zeros on all legs and leaves. Figure 7 clearly shows that this is not the case. Hence, it seems that the unobservable individual effects must play a significant role. The sum of the squared leaf-numbers in each subfigure a-d provides an aggregate measure of how well the leaf-probabilities in the $\hat{\gamma}$-trees can be predicted from the estimated state dependence parameters, on the assumption that


Fig. $7 \Delta$-trees. a Situation 1. b Situation 2. c Situation 3. d Situation 6
unobserved individual-specific effects play a role only at the first step of the experiment. This measure shows that the predictive properties of the estimated $\gamma$ values are much better for Situation 2 than for Situations 1, 3, and 6. Focusing on the differences shown for the leafs of the $\Delta$-tree we find a pronounced tendency to underestimate the prevalence of outcomes in which individuals reconsider their initial Rawlsian choice.

For several of these branches, the numbers on the legs and leaves of the $\Delta$-trees are of substantial magnitude, in particular for Situations 1, 3, and 6. Strong individual effects seem to be operating here, working in the direction of revision of initial choices.

From the $\Delta$-trees we observe that a model based on state dependence alone will tend to overestimate the probability for events which were classified as inconsistent. Individual factors seem to operate in the direction of keeping probands away from responding to the experiment by choice sequences that are hard to explain by a Rawlsian or a utilitarian logic. Moreover, for individuals who at the initial step made a non-Rawlsian decision, the model relying on state dependence only predicts choices in subsequent decisions fairly well. Those probands are not likely to revise their initial choice.

To summarize, we obtained statistically significant positive estimates of the state dependence parameters for all four situations considered. State dependence clearly plays a role. On the other hand, individual-specific effects are also important. The substantial role played by individual-specific effects warrants the conclusion that individuals do not seem to stick to more or less "automatic" patterns of responding to our experiment. The fact that the decision trees for Situations 4 and 5 in Sect. 5 differ strongly from the trees representing the other situations points in the same direction.

## 8 Conclusion

We have investigated three research issues. To begin with, we asked whether individuals act in accordance with Hammond's equity axiom-irrespective of situational detail. The results show that, in decision contexts that come close to real-world Situations ( $1,2,3$, and 6 ), between 40 and $50 \%$ of the probands always decide in line with the axiom, while between 10 and $20 \%$ never decide to the benefit of the worst-off individuals. In accordance with earlier research findings there is considerable, but not unconditional, support for Hammond's equity axiom.

The second research question focuses on the individual's decision behavior as the level of the treatment variable is increased. We observe a clear pattern in our results for the real-world Situations 1, 2, 3, and 6 (cf. Fig. 3). Although 75-90\% initially decided in accordance with the equity axiom, between 25 and $35 \%$ left their initial Rawlsian position as the the size of the group of those who are better off was increased. To conclude, although the adherence to Hammond's equity axiom is initially substantial, the larger the group of those who are better off irrespective of the policy chosen, the smaller is the propensity to act in accordance with the equity axiom.

The third research problem asked whether our results are driven by state dependence. The fact that a large share of the probands stick to their initial decisions throughout the experiment may be due to state dependence, but it may also be due to individual specific effects. To clarify the question, we first obtain an estimate of the state dependence parameter in a dynamic binary response model. The estimator is independent of the unobservable individual-specific effects. Then the null hypothesis of no state dependence is tested against the simple alternative of (any type of) state dependence using a likelihood ratio test. The null hypothesis is clearly rejected in Situations 1, 2, 3 , and 6 . All the estimated state dependence parameters are positive, indicating that
the results are-at least to some degree-driven by state dependence. In other words: Once an individual has made a choice (Rawlsian or non-Rawlsian) there is a tendency that it will stick to that decision.

The part of the systematic choice behavior in our experiment that cannot be explained by state dependence, is due to unobserved individual-specific effects. In the $\Delta$-trees in Fig. 7, large leaf numbers indicate that the individual effects play a substantial role for some choices. It seems that unobservable individual-specific effects are important for explaining why a large group of individuals adhere to the equity axiom throughout the experiment, in particular in Situations 1, 3, and 6. Moreover, the individual effects seem to explain much of the propensity to leave the initial Rawlsian position as the level of the treatment variable is increased.

To make inference about state dependence that is not distorted by the presence of unobservable individual-specific effects, it is important that our data contain also some "inconsistent" choices. Nevertheless, the large majority of probands decided consistently throughout the experiment. Notice also that when presented with peculiar decision contexts (Situations 4 and 5), decisions differ markedly from what we found in realistic situations. These results strongly indicate that probands understood the logic of the experiment and responded in a rational manner. Furthermore, the striking difference between the decision behavior in the real-world Situations (1, 2, 3, and 6), and the more peculiar situations ( 4 and 5), indicate that decision behavior is context dependent. At the same time, the similarity of the results for Situations 1, 2, 3, and 6 suggests that the main results are robust with respect to changes in the decision context. Jungeilges and Theisen (2005) provide a more extensive analysis of context dependence.

At the present state we have not tested whether our results are robust to changes in the starting point. In the context of empirical distributive justice Lars Schwettmann has recently confronted probands with an alternative sequence of levels of the treatment variable in the case we refer to as Situation 3. Relative to the natural increasing sequence of levels of the treatment variable, he reports small and statistically insignificant differences in responses. ${ }^{5}$ One of the areas where researchers, for many years, have payed attention to starting point bias (anchoring) is contingent valuation. Herriges and Shogren (1996) find that starting point bias may occur in sequential preference elicitation processes, but also that this is not necessarily the case. Moreover, Aprahamian et al. (2007) find that individuals are heterogenous. For some they find evidence of starting point bias, for others they fail to establish it. It is unclear whether and under what conditions starting-point bias plays an important role. An empirical investigation of the seriousness of the starting point bias in our context suggests itself as a subject for future research. We believe that our approach to estimating the state dependence parameter might be useful in clarifying also this question. To implement such an approach we will have to set up a new experiment to generate new data.

Another interesting issue for future research would be to exploit data from different countries to examine whether our results on state dependence are robust towards changes in culture. Finally, alternative designs of the experiment, e.g., systematic var-

[^5]iation of selected aspects of our situations, could be developed to isolate factors which influence individual decision behavior in distributive dilemmata.

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[^0]:    J. A. Jungeilges $(\boxtimes) \cdot T$. Theisen

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[^1]:    ${ }^{1}$ We are grateful to an anonymous referee for drawing our attention to this literature.

[^2]:    ${ }^{2}$ Using a standard logit model, Jungeilges and Theisen (2005) find that there is a form of time dependence also for the choice made at the first step. Specifically, the probability that this choice is in accordance with the Rawlsian logic depends on the following covariates: age, gender, parental background, job experience, and educational level reached. These covariates can be interpreted as capturing a proband's "history" up to the point in time when the experiment was carried out. This finding is very much in line with the pattern observed for the decision trees suggesting that the choice in the initial stage and the decisions taken at later stages are related.

[^3]:    3 As discussed above, to obtain a numerical measure of state dependence that is independent of the individual specific effects, inconsistent responses which do not seem to follow the logic of the equity principle or any other principle of distributive justice are essential. Such outcomes are not observed for Situations 1 and 6, a fact that can already be seen from Fig. 4. There, the distance of Situations 2 and 3 from the hypothenuse is not negligible since the fraction of inconsistent observations is positive. For Situations 1 and 6 the small distance from the hypothenuse indicates the lack of inconsistent observations.

[^4]:    ${ }^{4}$ Let $\hat{\gamma}_{l}$ denote the ML estimate for situation $l \in\{1,2,3,6\}$ and $\bar{\gamma}_{l}^{(500)}$ and $\sigma_{l}^{(500)}$ represent the arithmetic average and the standard deviation of the 500 estimates generated for situation $l$. To reflect the closeness between the point estimator and the estimate based on resampling we define the distance measure $d_{l}=\frac{\left|\hat{\gamma}_{l}-\bar{\gamma}_{l}^{(500)}\right|}{\sigma_{l}^{(500)}}$. We find the values $d_{1}=0.0861, d_{2}=0.1946, d_{3}=1.1931$, and $d_{6}=1.6516$ for our four situations.

[^5]:    5 These results have not been published. They were communicated verbally to one of the authors by Lars Schwettmann (Martin-Luther-University, Halle-Wittenberg).

