



UNIVERSITETET I AGDER

Interacting heterogeneous algo-traders

An extension of the Day and Huang model

Håvard Kroglund

Lauritz Fevang

Supervisor

Jochen Jungeilges

This Master's Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

University of Agder, 2013

Faculty of Economics and Social Sciences

Department of Economics and Business Administration

Abstract

We develop a model with heterogeneous and socially interacting investors applying different technical trading rules (algorithms), by extending the seminal model of Day and Huang (1990). The original model consists of (sophisticated) α -investors, (unsophisticated) β -investors and a market maker. We have studied the nonlinearity features and described the dynamic behavior of the market. In the extended model, β -investors are replaced by heterogeneous and socially integrated algo-traders. Through the communication process, each investor is able to obtain information about certain other investors and his characteristics (wealth, stress indicator and trading rule). If he finds a superior investor, he will adapt his or hers algorithm. Based on ten dissimilar technical trading rules we constructed some numerical experiments, and simulated the model. Then we evaluated the mean wealth and the long run price behavior. The combination of algo-traders and the sophisticated investors resulted in price fluctuates of different types. The volatility was typically highest at the beginning of the different price series, and in one of the series a stable 10-cycle appeared. This cycle seems consistent for some levels of the flocking coefficient in the bifurcation diagram that was generated for the original Day and Huang model. The main conclusion is that unsophisticated investors does not destabilize the market. Our extended model provides several starting points for future work.

Acknowledgments

This master's thesis, consisting of 30 ECTS credits, is written as a part of our Master of Science degree in business and administration at the University of Agder (Handelshøyskolen i Kristiansand)

During last fall we contacted our supervisor and presented some ideas for the thesis, and after some meetings the topic became clear. We are greatly thankful to our supervisor Professor Jochen Jungeilges, for his guidance and many late meetings. We would never be able to accomplish this topic if it wasn't for his knowledge and programming skills.

Kristiansand, June 3, 2013

Lauritz Fevang

Håvard Kroglund

Contents

1	Introduction	1
1.1	Outline	2
2	Literature review	2
2.1	Technical analysis	2
2.2	Automated trading	4
2.3	Market impact	6
2.4	Summary	7
3	Investment selections and market mechanism	8
3.1	Fundamental analysis	9
3.2	Technical analysis	10
3.3	Market maker's mechanism	14
4	Day and Huang model	14
4.1	The participants	15
4.1.1	α -Investors	15
4.1.2	β -Investors	17
4.1.3	Market maker	18
4.2	Summary	19
5	Analyzing the model	20
5.1	Deriving the equation of the model	20
5.2	Switching market	21
5.3	α -Investors behavior	22
5.4	β -investors behavior	22
5.4.1	Bifurcation diagram of the flocking coefficient.	22
5.5	Model behavior when $u \neq v$	28
5.6	Market maker behavior	28
5.7	Summary	30
5.8	Empirical relevance	31
6	The extended model	31
6.1	The parameters	32
6.1.1	Risky and risk free assets.	32
6.1.2	Investor wealth.	33
6.1.3	Algorithm rules.	33
6.1.4	The last signal	34

6.1.5	Stress indicator.	34
6.1.6	Excess demand.	34
6.2	Accumulating the excess demand (supply)	35
6.3	The price adjustment	36
6.4	Details on the communication process	37
6.5	Description of period t	38
6.5.1	Choosing the algorithm	38
6.5.2	Implementing the rule and receiving the signal	41
6.5.3	Updating the investor characteristics	41
6.6	Summary	43
7	Simulations	43
7.1	Initial conditions	44
7.2	Technical trading rules (algorithms)	44
7.2.1	Simple MA:	45
7.2.2	Filtered MA:	45
7.2.3	Double MA:	46
7.3	Numerical experiment based on simple MA	46
7.3.1	Low initial price	47
7.3.2	High initial price	47
7.3.3	Random initial price history	48
7.3.4	The social interaction process	48
7.4	Numerical experiment based on generalized MA	50
7.4.1	Experiment 1	50
7.4.2	Experiment 2	53
8	Conclusion and future research	54
8.1	Future research	55
A	Appendix of Day and Huang model	57
A.1	Price adjustment function in the Day and Huang model	57
A.2	Properties of the Day and Huang model	57
A.3	R codes	59
B	Appendix of the extended model	60

List of Figures

3.1	Technical trading rules; golden gross, MACD and Envelope	12
3.2	Technical trading rules; TTR, RSI and PSY	13
4.1	α -investors	17
4.2	β -investors and aggregated excess demand	18
5.1	Model is out of range: explode.	21
5.2	Bear and bull markets	23
5.3	Market dominating by α -investors	24
5.4	Bifurcation diagram of the b	25
5.5	Windows in the bifurcation diagram	26
5.6	Periodic cycles, orbits	27
5.7	Bearish market and bullish market	29
6.1	The scaling function Ψ	37
6.2	Investor preferences over (ω, s) -space	39
6.3	Indifference curves on subset of (ω, s) -space	41
6.4	Tree representation of the updating process	42
7.1	Numerical experiment of simple MA in different condition	49
7.2	Price series of the tree experiments generated by the extended model . . .	51
7.3	Mean wealth of the tree experiments generated by the extended model . .	52

List of Tables

4.1	Description of the Day and Huang model parameters	15
6.1	Notation in the extended model	32
6.2	Investor's action space	34
6.3	Effects of investor's actions	35
7.1	Number and their characteristics to algorithm rules	47
7.2	Experiment long run behavior $t = 500$	53
7.3	Experiment 2b long run behavior $t = 500$	53
7.4	Experiment 3 long run behavior $t = 500$	53

1 Introduction

Recently there has been a rapid growth in the amount of automated trading in stock markets and other financial markets. Automated trading is a general terminology used to characterize computerized trading. The computer technology has revolutionized financial markets, and nowadays these markets are highly dependent on artificial intelligence. Automated trading is also known as algorithmic trading or robot trading, where different securities are traded automatically by computers, generating an output signal, based on a data set. This signal might be generated by an algorithm, often referred to as a technical trading rule.

An investor trading in a financial market, cannot know for sure whether other participants operating in the same market, are computers or usual investors.

Financial markets are said to have nonlinearity and chaotic dynamics. In Day and Huang they develop a deterministic model that generates stochastic fluctuating prices. We use this model as a groundwork in our study.

The objective of this thesis is to develop an extended version of the Day and Huang (1990) model, to see how heterogeneous and socially integrated investors affect the market.

The investors use different algorithmic trading rules to operate in this stylized nonlinear model. The original model focus on a given population, but we are emphasizing on the individual investor in the extended model. β -investors are substituted by algo-traders in the extended model. While Day and Huang explain the stock volatility, we expand this view by looking beyond the “hidden surface” and focusing on the microstructure of the investors. A new dimension is given to the extended model, in the sense of a social integration process.

We generate some sub-results of the original model, such as the bifurcation diagram of the flocking coefficient, and we made some small corrections. The extended model is tested based different numerical experiment, applying a subset of 10 dissimilar algorithmic rules.

It is a common intuition saying that the more unsophisticated investors going into the financial market the more destabilization. In contrary, Suhadolnik et al. (2010) find that if one introduce more of these unsophisticated investors (robot traders) that are social integrated, then it causes more stabilized stock markets.

The motive for using the seminal model by Day and Huang is a combination of its simplicity, and that it is able to provide several stylized facts about the stock market (more on the advantages of the model in section 5.8).

Our main focus is to develop the extended model and to provide a functional environment for communication. The combination of the communication process and the

algorithmic trading is what differs this thesis from other papers.

1.1 Outline

The outline of this thesis is as follows. In Section 2, we presents a review of the literature on technical analysis, automated trading and market impact. Then, in Section 3 different investment strategies, fundamental and technical, are described and the market mechanism is explained briefly. The fundamentals of the Day and Huang model are carefully described in Section 4, and in Section 5 we analyze the same model.. The extended model is revealed in Section 6. In Section 7 we simulate the model, and finally, in Section 8 the conclusions and our proposals for future research are outlined.

2 Literature review

This master's thesis is related to different fields of research, and we have divided the literature review into three different topics: technical analysis, automated trading and market impact. The first part focuses on the behavior of price changes in financial markets, and if technical analysis provides any valuable information. Automated trading, often referred to as algorithmic trading, is mainly based on technical analysis and is discussed in the second part. The third and last part is about how different markets are affected by the growing amount of automatic trading.

2.1 Technical analysis

Technical analysis is often said to be old as the market itself. Early in the 20th century Charles Dow develops a series of principles designed to describe and forecast the behavior of financial markets. The “Dow theory” is often regarded as the groundwork of technical analysis.

In Bachelier (1900) the famous model “random walk” is developed, which describes the behavior of speculative prices in financial markets. The model implies that technical analysis is useless. One important assumption for Bachelier's random walk is that prices in financial markets follow a Gaussian (normal) distribution. This assumption is later rejected by Mandelbrot (1963) and Fama (1965). In other papers random walk often refers to price changes that may have other distributions than the Gaussian. The other main assumption related to random walk is that price changes are independent.

The forecasting skills of professional agencies are tested and evaluated in Cowles (1933). At first 36 financial and insurance companies are trying to foretell which securities that will provide the highest return. Then 25 financial publications and their ability to forecast the stock market are tested. In the conclusion it is stated that no

significant evidence of forecasting skills is found. This indicates that the forecasters are unable to predict price changes in the market, or at least not able to profit from it.

Kendall (1953) tests whether price changes in the past provide any valuable information about price changes in the future. The data set consists of 19 industry indices, two commodity price series (wheat and cotton) and one monthly average of the wheat price series. It is stated that random changes in the price series are large enough to “swamp” any systematic effect, but aggregative index numbers seem to behave more systematically compared to their simple components. Alexander (1961, 1964) is partly based on data and results from Kendall’s work. The author concludes that price changes in speculative markets appear to follow a random walk over time, but at the same time a move in the price changes seems to exist.

In Fama (1965) the theory underlying the random walk model is discussed, and the empirical validity of the model is tested. The data set consists of daily prices from the Dow-Jones Industrial Average in the period 1957 to 1962. Both the price distribution and the independence of the price series are tested. He finds that the distribution of the daily price changes contains too many centered observations and the tails are too “fat” to be consistent with the Gaussian distribution. Fama also finds that the amount of dependence in the price series is approximately zero. It is concluded that stock prices seem to follow a random walk model. This finding is consistent with the result from Mandelbrot (1963). Samuelson (1965) shows that stock prices are efficient¹, and follows a martingale process.

Later Fama (1970) finds evidence of positive dependencies between changes in day-to-day prices on common stocks. This finding may be used by technicians² to make profitable trading rules, but according to the paper even with low transaction costs this profit will be eliminated. The famous theory “Efficient Market Hypothesis”, developed by the economist Eugene Fama in Fama (1965, 1970), is based on Bachelier’s thinking.

At this time it seems to be an agreement among many academics that technical analysis is useless, even though some weak trends in different price series have been discovered. This view is about to change later in the 20th century, at least among some researchers.

Brock et al. (1992) find strong support for technical analysis. The objective of the paper is to test two of the most popular technical trading rules, moving average and trading-range break, using an empirical approach. At first the strategies are tested in an artificial market based on Dow Jones Industrial Average (DJIA). The data set contains 90 years of daily observations. Then the returns, generated by the two different rules in the artificial market, are compared to the return from the actual Dow Jones. It is concluded that buy signals consistently generate higher return than sell signals, which is not consistent with the random walk model.

¹With all available information; efficient in the sense that prices are fairly valued.

²Users of technical analysis.

Sullivan et al. (1999) use the same DJIA data set as Brock et al. (1992) for an in-sample test. They robust the in-sample period for data-snooping, and they found even better performance of the best technical trading rule. However, they studied a 10-years period, 1987-1996, testing the best trading rule for an out-of-sample test. The result is that the best trading rule in the in-sample does not repeat its performance in the out-of-sample period.

2.2 Automated trading

Recently financial markets have been heavily influenced by algorithmic trading, but algorithms have existed in the literature for a long time. Hartl (1989) has the objective to solve an optimal control problem, and to generalize this trading model using “forward algorithms” on every wheat price path. He finds a solution for selected specific price movements, but no closed solution for every price fluctuation.

To solve optimization problems, evolutionary algorithm system can be exploited. Under the creation of artificial intelligence³, in the beginning of the computers, one of the goals was to develop computer programs that had the capability of adapting and learning in their environments. This idea is the foundation of genetic algorithms.

Genetic algorithms, invented by Holland (1962, 1975), is a technique based on a class of research, adaptation, and optimization. According to Goldberg (1989) a genetic algorithm is a stochastic searching technique based on the mechanism of genetics and natural selection. Holland’s original goal was to “... *study the phenomenon of adaptation as it occurs in nature and to develop ways in which the mechanisms of natural adaption might be imported into computer systems*” Michell (1996, p. 2-3). In the same book Mitchell builds the understanding of genetic algorithms to students and researchers, and provides insight about complex adaptive systems. She explains how genetic algorithms have been used to model processes of innovation, bidding strategies and the emergence of economic markets.

Machine learning algorithms, such as genetic algorithms, have contributed improvements to trading of stocks, bonds and securities. Bauer and Liepins (1992) are regarded as the first to connect investment strategies and genetic algorithms together in an understandable way. In Bauer (1994) a practical assistance, using genetic algorithms to develop attractive strategies of trading based on fundamental information, is created. “*These techniques can be easily extended to include other types of information such as technical and macroeconomic data as well as past prices*” Papadamou and Stephanides (2007, p.190).

Allen and Karjalainen (1999) use genetic algorithms to find technical trading rules based on historical data from the S&P 500. The data set consists of daily prices from

³The intelligence of a computer (software).

1928 to 1995. According to this paper, technical rules do not provide any excess return when transaction costs are included, but the rules are able to identify periods when the daily return is positive and the risk is low. This implies that genetic algorithms are an appropriate method to discover technical trading rules.

Neural networks are used to generate and to test the performance of trading rules in artificial markets. Fernandez-Rodriguez et al. (2000) develop a simple technical trading rule based on Artificial Neural Networks. This rule is applied to the General Index of the Madrid Stock Market. In their result it is suggested that the technical trading rule performs better than a simple buy-and-hold strategy for both bear and stable markets, but not for bull markets. Transaction costs are not included in these tests.

Echo State Network (ESN) is a special case of neural networks. Lin et al. (2011) use genetic algorithms and genetic programming to improve the technical trading rules used on stock in the trading system ESN. These rules perform significantly better than a simple buy-and-hold strategy on the S&P 500.

Fernandez-Rodriguez et al. (2001) investigate the profitability of technical trading rules using genetic algorithms to optimize the parameters. They suggest that such rules are always superior to a risk-adjusted buy-and-hold strategy for reasonable trading costs.

According to Hsu et al. (2009), the central idea of successful stock market forecasting is to achieve best possible results using historical data and the least complex stock market model.

While Allen and Karjalainen focused on applying genetic algorithms models to generate profitable trading rules, Shangkun et al. (2012) test the robustness of such algorithms on currency trading. Their results depend on the similarity of the trend between training and testing period. “. . . we conclude that a trading rule trained in a period is profitable with relatively high probability in periods which are similar in trends but is mixed in periods which are different in trends” Shangkun et al. (2012, p. 93).

Nowadays the research in the context of automated trading is emphasized on high-frequency trading. Strassburg et al. (2012) test the possibility to speed up the genetic algorithm process by introducing parallelization⁴. They find that the average result time used to find one set of technical trading rules is shorter when using more CPU⁵ cores. This means that the genetic algorithm process can be improved by introducing parallelization.

In Gerber et al. (2004) they simulate and test the “Optimark’s electronic matching algorithm”⁶. The authors develop a “new” algorithm to improve the system and to make it easier to use. Papadamou and Stephanides (2007) use genetic algorithms to improve

⁴A large problem is divided into smaller ones and solved simultaneously.

⁵Short for “central processor unit” and represent a part of the computer’s hardware.

⁶Optimark Inc. “*Provides exchange solutions to electronic marketplaces and communities*”, URL: <http://www.optimark.com/>

parameters in the technical analysis by generating a software program in MATLAB⁷. This is a tool to optimize the trading rule parameters and the prediction of trend for technical analysis, which results in better profit and saving of time compared to other relevant software.

Schmidt et al. (2010) use the support of different online algorithms for trading decisions and evaluate their empirical performance. They also analyze the algorithm by using historical prices of DAX-30 index from 1998 to 2007. The threat-based algorithm dominates the other algorithms.

Fong et al. (2012) evaluate the performance of different “trend-following” algorithms. A simulation of the Hang Sang Index Futures is developed to test the algorithms. The algorithm, “trend recalling”, achieves the best performance, and it is a huge gap between the performance of the best and the second best “trend-following” algorithm.

Financial funds or investors executing large stock positions may cause a shortfall in the market. Pemy (2012) develops one algorithm to minimize the execution shortfall and one to maximize the volume-weighted average price. The idea behind both strategies (algorithms) is to divide large orders into several small orders, and to execute them into the market during a predetermined period of time.

2.3 Market impact

Furbush (1989) is an empirical study of the marked crash that occurred the 19th of October 1987 - known as “Black Monday”. The paper focuses on the relationship between program trading, and stock and futures price changes the days before and after Black Monday. He finds that program trading is associated with these extreme price fluctuations in the cash market, and that the pattern of the program trading during the 19th and 20th of October is different compared to the days before. After Black Monday it has been discussed who was to blame for the crash. According to Shiller (1988) the explanation of the market crash may be a sociological or psychological phenomenon as well an economic one. *“It is difficult to place the blame for the crash of 1987 on the program trading since stock quotes were changing so rapidly on Black Monday that program trading could not have occurred because the market information needed to make transactions was continuously being updated”* Kim (2007, p. 10).

Lux (1995) attempts to formalize the “herd behavior” of investors, and describes the rise of bubbles as a self-organizing process. In which degree speculators behave similar to other speculators depends on the economic variable called actual return according to Lux. He also states that investors’ beliefs with respect to the fundamental values are what cause the phenomena of bullish and bearish beliefs and bubbles. In Lux (1998)

⁷A numerical computing software developed by MathWorks, URL: <http://www.mathworks.se/>

he develops a socio-economic model of the interaction of speculators in a financial market. “*The key result of the paper is that the distribution of returns that follows from the (quasi-deterministic mean-value) dynamics conforms in important aspects with empirical regularities*” Lux (1998, p. 162).

In Izumi et al. (2009) the authors want to extend the artificial-market simulation to the practical financial markets, and to find out how automated trading affects the stability of the stock market. Eight different trading rules (strategies) are tested and evaluated in an artificial market and by using a back test. They conclude that the artificial market evaluation provides better information than the back test evaluation. It is also concluded that the market impact depends on the trading rules and how they are combined together.

On the 6th of May 2010, also known as the “flash crash”, one of the largest point swing and the biggest one-day point decline in the history of the Dow Jones Industrial Average took place. CFTC and SEC (2010) concludes that the crash was complex and triggered by an automated execution of a large sell order. Automated programs and algorithmic trading may destabilize the market according to the report. A Combination of the large automatic selling order, high frequency trading and algorithmic trading was one of the reasons for these extreme price movements and increased volatility.

Suhadolnik et al. (2010) suggest to use automated trading as an alternative to monetary policy and financial regulations to make the economy more stable. The motivation of this suggestion is that if financial markets are complex, monetary policy and financial regulations may be unable to protect these markets against bubbles and crashes. “*Under such circumstances, why not use the robot traders as an anti-bubble decoy?*” Suhadolnik et al. (2010, p. 5183). A new stochastic-cellular automata model, where traders are socially integrated in and interacting in the neighborhood of each other, is introduced in the paper. The robot traders are implemented in a simulated model of the Boverspa⁸ index. It is concluded that the introduction of such robot traders results in a more Gaussian market, and that the use of intelligent robots to prevent bubbles and lower the volatility can be justified.

2.4 Summary

The value of technical analysis has been discussed among academics for a long time. Burton G. Malkiel describes the situation like this: “*Obviously, I’m biased. This is not only a personal bias but a professional one as well. Technical analysis is anathema to much of the academic world. We love to pick on it*” Malkiel (2011, p. 139).

Lately, the literature has been emphasized on automated trading which is often based on technical analysis. The objective of various papers is to use algorithms to find optimal

⁸Sao Paulo Stock Exchange Index.

trading strategies, and some of them conclude that it is possible to “beat” the market using technical trading rules developed by genetic algorithms. Fast and sophisticated computers are able to exploit mismatches in the market in no time, but some argues that this excess return will be eliminated by the transaction costs. Both historical data and simulation has been used to test different strategies, but the latter often provides performance estimates with higher precision.

In the light of financial bubbles and crashes, e.g Black Monday, some economists have blamed automated trading for these extreme price fluctuations. This is only partly supported in the academic world, but the literature available focusing on how automated trading affects the market is scarce. The conclusion from one paper, using an artificial simulation approach, is that the market impact will depend on the trading rules and how they are combined together.

Robot trading is quite unpopular among many investors, and they claim that these robots manipulate and destabilize the market. The view is totally different in a paper written a few years ago. In this paper robot trading is suggested as an alternative to monetary policy and financial regulations to make the markets more stable. It is concluded that the use of intelligent robots to prevent bubbles and lower the volatility can be justified.

3 Investment selections and market mechanism

Many different techniques are applied by investors to try to earn money in financial markets, and it is often distinguished between investors using fundamental analysis (fundamentalists) and they using technical analysis (technicians or chartists). Even though they have the same objective, namely to make a profit in the market, the methods used are totally different. Fundamentalists calculate the intrinsic value and compare it to the market price and try to find underpriced companies. Technicians focus on nothing else but the stock charts. A third strategy, used by many investors, is to combine these two techniques.

The value of technical and fundamental analysis are heavily discussed among academics and others. The random walk hypothesis, which is said to be consistent with the efficient-market hypothesis (EMH), implies that both techniques are useless. But there are also many papers suggesting that financial markets are inefficient, meaning that it might be possible to predict future prices. For instant, the January effect, weekend effect, sunshine effect, and sports effect (see Thaler (1987), Huberman and Regev (2001), Hirshleifer and Shumway (2003), and Edmans et al. (2007) respectively), all indicate some kind of inefficiency in the market.

A more detailed description of fundamental and technical analysis is given below. In-

dependent of strategy, all investors need a market environment in order to trade. Different market platforms and mechanisms are discussed briefly at the end of the section.

3.1 Fundamental analysis

Fundamental analysis is used to find the intrinsic value of securities such as stocks. The intrinsic value is often called the “true value” and is based on real data. The idea is to find the true value and compare it to the market price, to determine whether the stock is overpriced or underpriced. Fundamentalists consider a stock as mispriced if the intrinsic value differs from the market price. This is because they expect the intrinsic value to be equal to the market price in the long run. If the intrinsic value is higher than the stock price, then analysts expect the price to increase and recommend to buy. Similarly, it is considered as a sell signal if the stock price is higher than the intrinsic value, because fundamentalists expects the price decrease in that case.

Fundamental analysis is often divided into a study of factors that is specific to the firm and macroeconomic variables. The idea is to include and consider all factors and variables that may affect the company and the stock value. Firm-specific factors like earnings, debt-equity and dividends are obtained from the company’s financial statement. A fundamentalist may also be looking at qualitative information, such as the quality of the company’s management. Gross domestic product (GDP), inflation rates and interest rates are examples of macroeconomic variables that are evaluated.

Let’s construct an example of how fundamental analysis may be used: At first the firm’s paid dividends are obtained from the financial statement. Then the yearly expected future growth of the dividend is calculated. Further a discount rate is calculated based on the interest rate and the company’s total risk. A dividend discount model is used to find the intrinsic value of the equity and this value is then compared to the market price. The analyst recommends to buy shares if intrinsic value exceeds market price, and to sell in the opposite case.

The relatively wide time horizon is probably one the reasons why fundamental analysis is rarely used in automated trading. Many of the factors that fundamental analysis is based on are reported one or a few times each year only, and consequently the intrinsic value is not changed rapidly. Because of that the benefit of using automated trading may be small for fundamentalists. Another reason is that qualitative information often is a part of the estimation intrinsic value. Such information is very hard or at most impossible for computers to handle.

3.2 Technical analysis

In technical analysis historical prices and its qualities, such as momentum and volume, are used to forecast future price movements. Unlike fundamentalists chartists do not calculate a firm's true value, nor do they care about financial statements or any economic variables. The idea is that the market repeats itself, and therefore they try to find patterns and trends in the chart of the stocks. Because of their use of charts to predict future prices, technical analysts are often referred to as chartists.

Automated trading is primarily based on technical analysis and there are at least two good reasons for that. The first reason is that chartists do not use any qualitative information, which makes the technical rules suitable for *algo-trading*. The second reason is that the time horizon for such rules is relatively short, and the conditions change fast and the use computers may give considerable advantages.

It is said to exist hundreds of different technical trading rules, and some of the most used rules are explained in words and graphically below. The descriptions are based this on different literature such as Brock et al. (1992); Sullivan et al. (1999); Bodie et al. (2011); Malkiel (2011); Izumi et al. (2009).

Some of the rules may be implemented with a band (filter). If the band is one percent, then the price must exceed the resistance level by one percent to generate a buy signal. Similarly, the price must fall below the support level by one percent to produce a sell signal.

Golden Cross. This rule is based on two simple moving averages - one long-term and one short-term moving average. The long-term line captures the main trend, while the short-term line captures the shorter price movements. A buy (sell) signal is generated if the short-term line breaks above (below) the long-term line (see Figure 3.1a). Every intersection between the short-term line and the long-term line generates a buy or sell signal.

MACD. Moving Average Convergence/Divergence (MACD) is a technical indicator. The MACD-rule, is based on a MACD line and a signal line. If the MACD line breaks above the signal line, then chartists consider this as a buy signal because they expect the price to increase. Similarly, it is a sell signal if the MACD line breaks below the signal line, because the price is expected to decrease (see Figure 3.1b).

Envelope. The rule, Envelope, is defined by upper and lower price range levels, which are based one simple moving average (MA) of historical prices. A percentage is added to and subtracted from the MA to generate the upper and lower levels respectively. Envelope is used to identify conditions in which the stock is overbought or oversold in the market.

If the price break above the upper level, the stock is considered as overbought which is a sell signal. Similarly, a buy signal is generated if the price breaks below the lower level, because the stock is oversold (see Figure 3.1c). Analysts may apply and interpret the rule differently, but the overall strategy is to identify when the stock price breaks above the upper level and below the lower level.

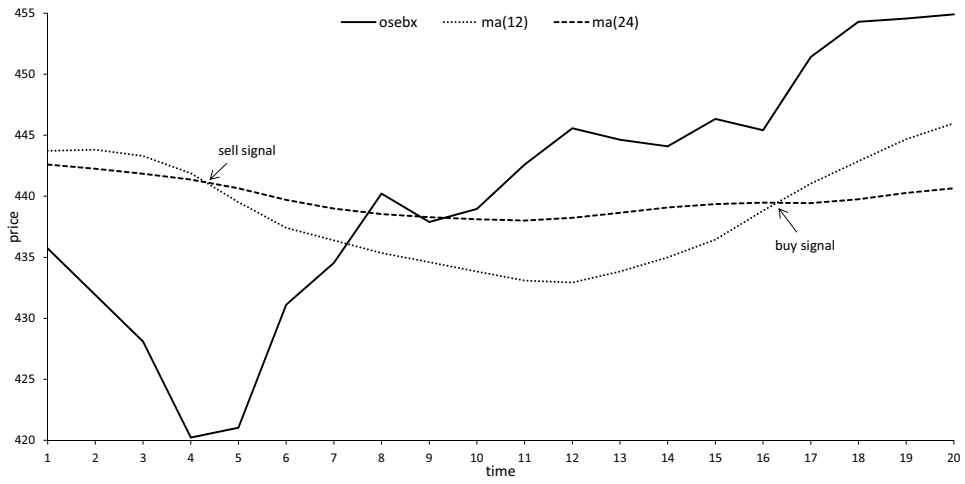
Trading Range Break. Trading Range Break or High/Low Band is a simple trading rule based on local maximum and minimum prices. A local maximum (minimum) price is the highest (lowest) price during a determined period of time. The length of the periods varies between different strategies. Local maximum and minimum prices are often called resistance levels and support levels respectively.

According to chartists, resistance (support) levels exist because many investors are willing to sell (buy) at this price level, and when the current price equals the resistance (support) level, the price drops because of many selling (buying) orders. If the price still breaks above the resistance level, then it is a buy signal because the price is expected to increase even more. In the opposite case, when the price breaks below the support level, a sell signal is generated because the price is expected to decrease further (see Figure 3.2a). In some variants of the strategy the buy or sell position is held for a predetermined period of time and during the period all other buy or sell signals are ignored.

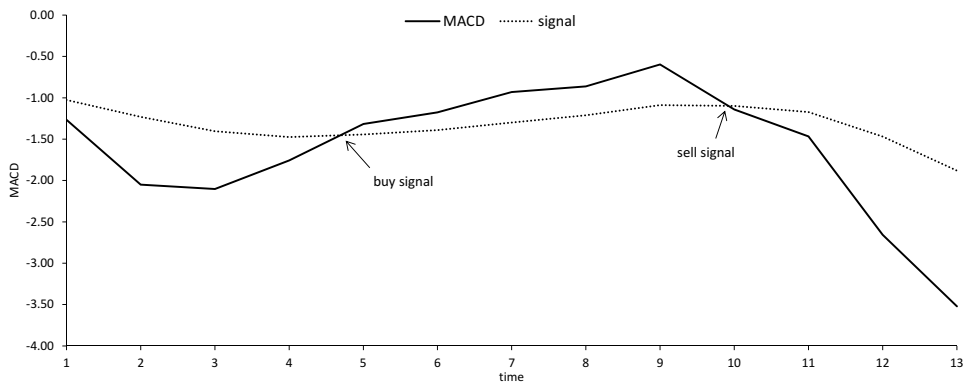
Relative Strength Index. Relative strength index (RSI) is a momentum indicator and is based on recent gains and losses. The idea is to compare average losses and gains during the last 14 days⁹ to determine conditions in which the stock is overbought or oversold. The index ranges from 0 to 100. An increase in the RSI indicates a “strength”, while a decrease is a sign of “weakness”. If the RSI breaks below (above) a predetermined support (resistance) level, then the stock is oversold (overbought). The support and resistance level are normally set equal to 30 and 70 respectively. It is considered as a sell signal if the stock is overbought, and similarly it is a buy signal if the stock is oversold (see Figure 3.2b).

Psychological Line. Psychological (PSY) line is another momentum indicator, and may be considered as a simple version of the RSI. The PSY-line ranges from 0 to 1. To calculate the PSY line, during a period of time the sum of all days in which the price increases is divided by the total number of days. The strategy also consists of predetermined resistance and support levels. If the PSY line exceeds the resistance level, then it is a sell signal, and a buy signal is generated if the PSY line falls below support level (see Figure 3.2c).

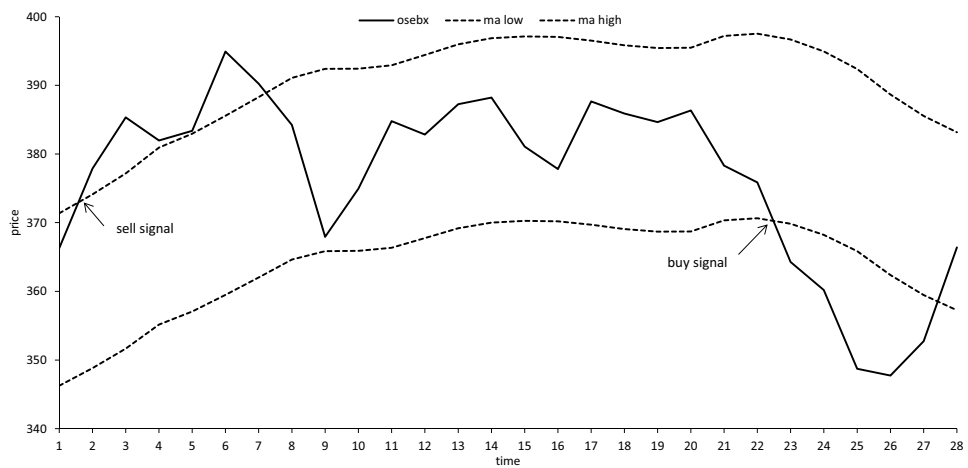
⁹This of course depends among chartists.



(a) Golden Cross rule

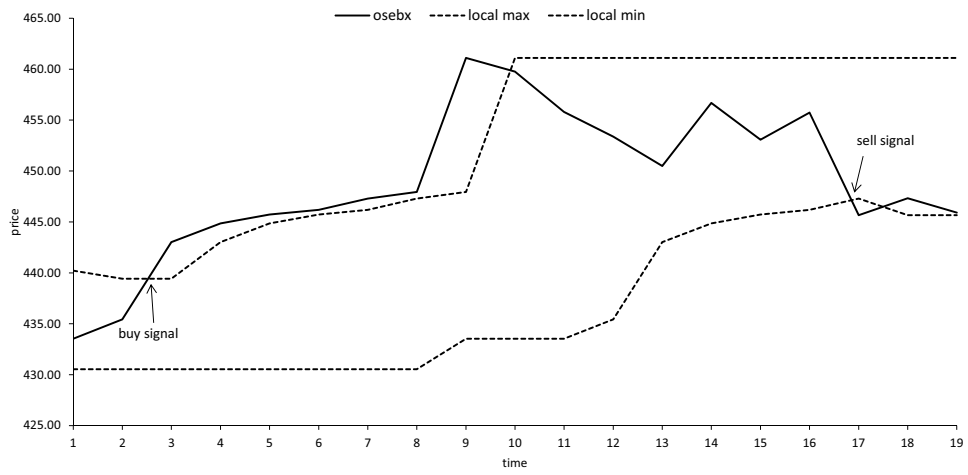


(b) MACD rule

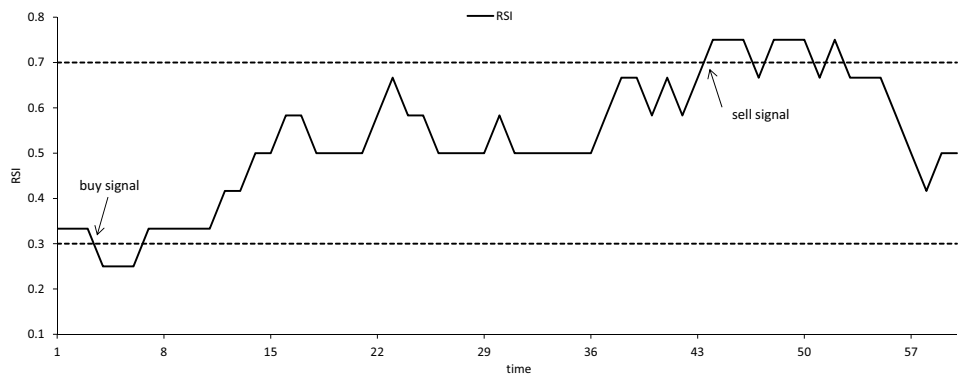


(c) Envelope rule

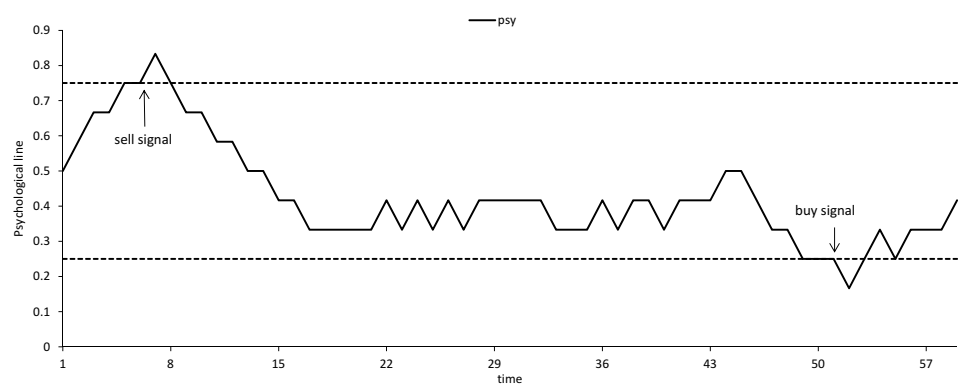
Figure 3.1: Technical trading rules; golden gross, MACD and Envelope



(a) Trading Range Break rule



(b) RSI rule.



(c) Psychological Line rule.

Figure 3.2: Technical trading rules; TTR, RSI and PSY

3.3 Market maker's mechanism

Financial markets or security exchanges are established to meet the needs of different traders, and organizing markets of trading. Back in time, direct negotiations was needed to trade securities, but in modern finance the market has emerged from meeting places to more efficient electronic market platforms. Broadly speaking one might distinguish between three trading systems (applied in the United States): over-the-counter (OTC), electronic communication network (ECN) and formal exchanges, according to Bodie et al. (2011, p. 62).

NASDAQ is an over-the-counter quotation system for securities not listed on regular stock exchanges. The system was developed to link brokers and dealers in computer networks (electronic trading) to median quotes. Today NASDAQ is a trading system, handling the majority of trades with sophisticated electronic trading platforms, and typically the standard for exchange markets worldwide. It is a computer-based market, with a system of market makers. NASDAQ was one of the major developers of ECN, which is a computer-operated trading network offering financial products on the outside of stock exchanges.

Formal exchanges are managed through a specialist, and New York Stock Exchange (NYSE) is an example of such an exchange. Specialists may act either as a broker or a dealer, and each security is assigned to one specialist.

If no trades are carried out, meaning that the market lacks either demand or supply, then the specialist may buy or sell shares (of stocks) for his own account (inventory), *to make the other side of a trade*. The function of the specialist is to make and maintain a “fair and orderly market” by buying and selling from his inventory. The specialist earns commission on managing the orders.

4 Day and Huang model

In the seminal paper by Day and Huang (1990) they develop a financial market model with heterogeneous interacting agents. The object of the paper is to model market behavior in the stock market. Day and Huang describe it as: “*A deterministic excess demand model of stock market behavior is presented that generates stochastically fluctuating prices and randomly switching bear and bull markets*” Day and Huang (1990, p. 299).

The model is based on stylized institutional realities and behavior specifications. To make the results more rigorous, the model has some simplifications. A number of key variables are treated as parameters, and should be thought of as changing endogenously variables.

Assumption: The market consists of only one type of shares (one company).

The model and its participants are described in the following.

symbol	description
p	current market price
p_0	initial value
u	estimate of investment value
M	estimated topping price
m	estimated bottoming price
$f(\cdot)$	chance function
ε	parameters in the chance function
$d_{1,2}$	parameters in the chance function
α	magnitude of α -investor's strength
v	current fundamental value
b	flocking coefficient
c	price adjustment coefficient
$E(\cdot)$	aggregate excess demand

Table 4.1: Description of the Day and Huang model parameters

4.1 The participants

There are three different participants operating in this market; α -investors, β -investors and the market maker.

Assumption: All α -investors are homogenous and all β -investors are homogenous.

In some papers¹⁰ α -investors and β -investors are referred to as fundamentalists and chartists respectively. This is not consistent with the definitions of fundamentalists and chartists used in this paper, but there are of course different ways to describe such investors.

Assumption: The results are derived from the behavior of the participants, and no exogenous inputs influence the system.

4.1.1 α -Investors

α -investors base their trading strategy on the spread between market price p and a sophisticated estimate of investment value u . These investors might also be referred to as information traders or sophisticated investors. The investment value is a complicated statistical analysis of the future value of fundamental value v . An information trader wants to buy when the price is well below the investment value ($p < u$), and to sell when the price is well above the investment value ($p > u$).

¹⁰e.g. Gu (1993); Wieland and Westerhoff (2005)

α -investors try to include all the most recent information into their estimation of u . This information is based on macroeconomic variables, industry information, company performance and others. I contains quantitative and qualitative information used in estimating u . To obtain all necessary data is costly and requires a lot a resources. If ϕ reflects the cost of I , then $u = \phi(I)$.

In addition to estimate investment value, information traders also consider the chance for capital gains and losses. The market is regarded as dynamical, where prices are in constant motion. If p is less then u , then investors expect a capital gain. Subsequently, if p decreases, then the chance of an increase in the future is higher. Therefore, investors might wait to buy the share, expecting the price to drop even lower. If instead the price rises, then investors miss the opportunity of a higher gain, and has to settle for a lower gain. These conditions are reversed when p is larger than u . The chance for capital gain or loss is small or equal to zero when p is close to u . This hypothesis is described in Keynes (1936). His work within the aspect of speculation, resulted in the function $f(p)$, which represents the chance function. $f(p)$ is bounded to the topping price M and the bottoming price m . If p is close to M , then the chance of losing a capital gain is great. If p is close to m , then the chance of missing a possible capital gain is great. This is illustrated in figure 4.1a

The chance function is specified as

$$f(p) := \begin{cases} 0 & p < m \\ (p - m + \varepsilon)^{-d_1} (M + \varepsilon - p)^{-d_2} & m \leq p \leq M \\ 0 & p > M \end{cases} \quad (4.1)$$

for $0 < d_1 < 1$ and $0 < d_2 < 1$. $f(\cdot)$ is assumed to be non-negative and differentiable. Properties of the function: $f'(p) < 0$ when $m < p < u$, $f'(p) = 0$ when $p = u$ and $f'(p) > 0$ when $u < p < M$.

α -investors' excess demand is represented by the function

$$\alpha(p) = \begin{cases} 0 & p < m, \\ a(u - p)f(p) & p \in [m, M] \\ 0 & p > M \end{cases} \quad (4.2)$$

where a is a measure of the strength of α -investors' demand. As described in Black (1986), information traders become more and more aggressive, as the difference between current price and investment value increases. Figure 4.1b illustrates the non-linearity and the complexity of α -investors' excess demand.

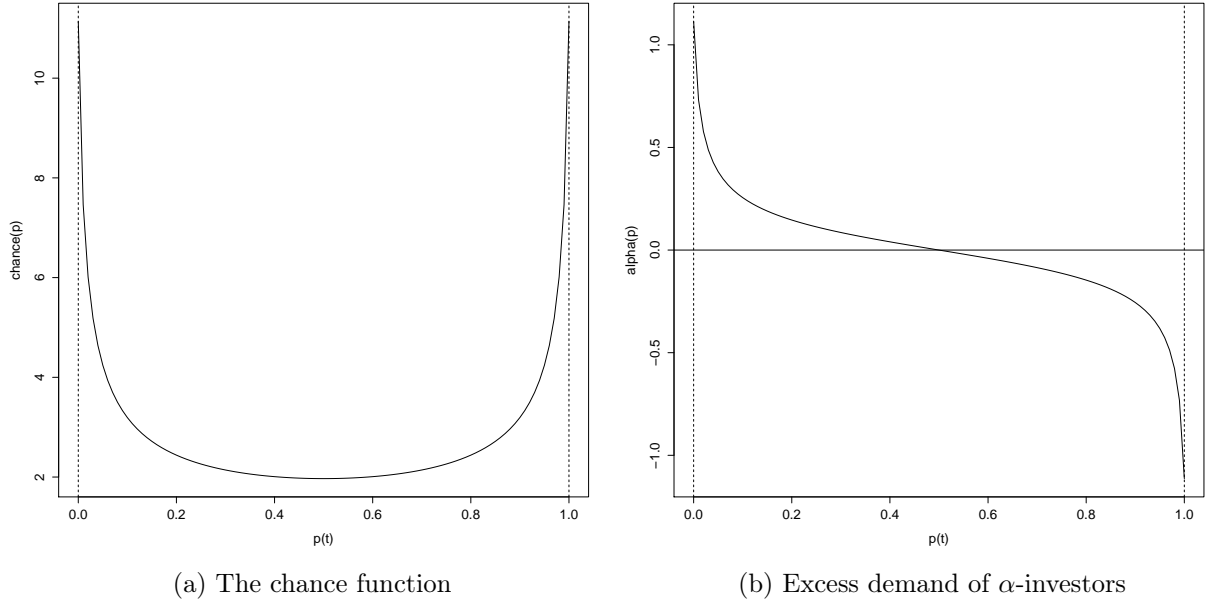


Figure 4.1: α -investors

4.1.2 β -Investors

In contrast to α -investors, the majority of investors can not afford sophisticated and expensive analyzes. Instead β -investors base their trading strategy on simple rules, which are less costly. These investors are regarded as unsophisticated and behave like “noise traders” (introduced in Black (1986)).

Basically, the strategy of these investors is based on the current price p and current fundamental value v . Their excess demand function is

$$\beta(p) := b(p - v). \quad (4.3)$$

where b is the flocking coefficient and “...reflects the relative importance of β -investors and the strength of their response to price signals” Day and Huang (1990, p. 305). Contrary to α -investors, β -investors do not take into account the chance function. More precisely, the chance function is assumed to be constant, and doesn’t affect the excess demand.

β -investors do not try to sell if the prices are increasing, nor do they try to buy if the prices are decreasing. They will enter the market when the price is high, expecting the price to increase further, and exit when the price is low, expecting a further decrease. Because of their characteristics, they “chase” the stock prices up and down and causes bull and bear markets.

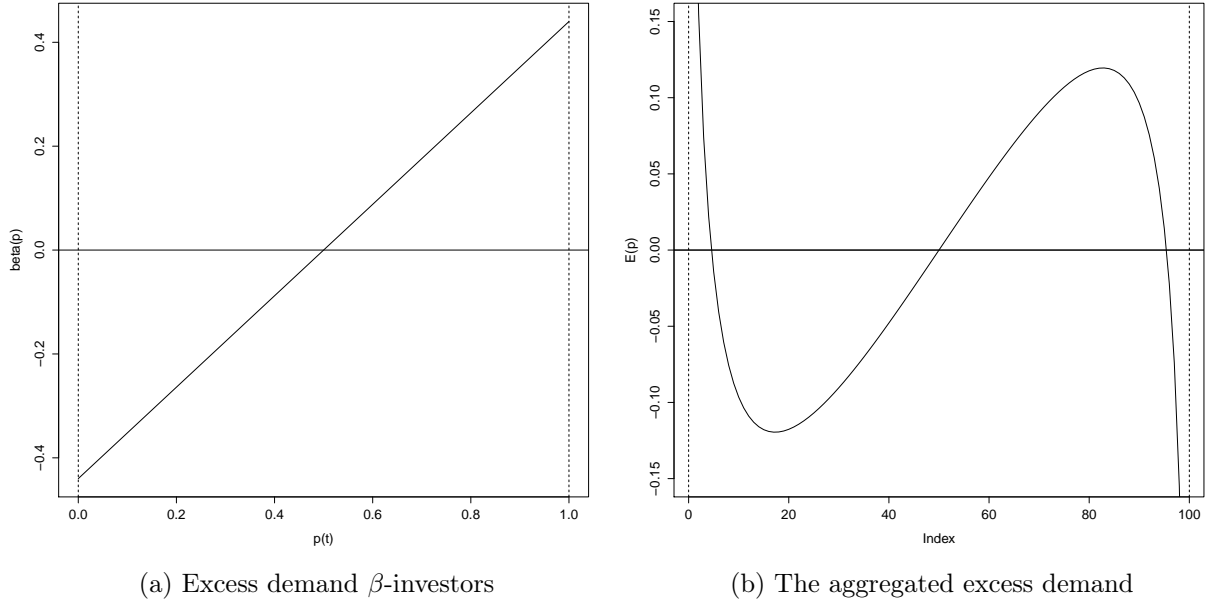


Figure 4.2: β -investors and aggregated excess demand

4.1.3 Market maker

The market maker (specialist) is an intermediary, and might be compared to a matching platform. Its main function is to set the price when the market is out of equilibrium, which is true if the demand exceeds the supply or vice versa. If demand equals supply, then there will be no change in the price. To be able to provide all buyers when the demand exceeds the supply, the market maker has an inventory of stocks. In such a situation his inventory of stocks decreases. When supply exceeds demand, the market maker uses his financial resources to buy shares, and his inventory increases.

Assumption: Prices are announced at discrete intervals of time, and orders are executed at the announced price.

The aggregated excess demand $E(p)$ is calculated by adding the excess demand for all investors, which is

$$E(p) := \alpha(p) + \beta(p). \quad (4.4)$$

The (aggregated) excess demand is negative if the supply exceeds the demand. The change in the inventory is represented by the function

$$V_t - V_{t+1} = E(p_t) \quad (4.5)$$

where V_t is the inventory of stocks at time t . Note that equation 4.5 is dissimilar from the corresponding one in Day and Huang model. To our best knowledge there has been

some kind of a typing error in their equation, because the equation inconsistent with the written description. We tried to investigate this further by looking it up in a more recent version of the model (Huang and Day, 1993), but the equation is not reproduced in this article.

Anyway, it is important to keep equation (4.5) in balance because the market maker's financial resources is limited. To achieve a balanced formula, the market maker adjusts the price from one period to another. To avoid a destabilization of the market, the price adjustments have to be as "moderate" as possible. This is the standard Walrasian market mechanism or the *tatonnement process* operating - setting the price out of equilibrium.¹¹

Because of its function in the market, a market maker must buy from investors when the price is high and sell when it's low. But these losses can be offset by investing on their own account. Similar to what we described in Section 3.3, the specialist also earn a commission on the trade between α and β investors.

The model assumes "...that the change in price $p_{t+1} - p_t$ is determined by a continuous, monotonically increasing function $c\gamma[E(p)]$ where $\gamma(0) = 0$ and where c is an adjustment coefficient..." Day and Huang (1990, p. 306). It is also assumed that

$$p_{t+1} = f(p_t) := p_t + c\gamma[E(p_t)].$$

To make the model more simple it is assumed that $\gamma[E(p)] \equiv E(p)$ for all p , which leads to

$$p_{t+1} = f(p_t) := p_t + cE(p_t). \tag{4.6}$$

$f(p)$ is called the price adjustment function. More on price adjustment functions in different market situations will be described in the next sections.

4.2 Summary

There are two types of investors (α -investors and β -investors) and a market maker operating in the model. α -investors buy (sell) when the price is below (above) their estimated investment value, while β -investors buy (sell) when the price is above (below) the current fundamental value. The market maker is an intermediary, and all buy and sell orders are executed through this specialist. For each period the market maker quotes a price based on last period's price and the aggregated excess demand (supply).

¹¹For more see Walras and Jaffé (1954); Friedman (1955); Kaldor and Tobin (1985); Cowell (2006, p. 164)

5 Analyzing the model

To understand the model it does not hold to just write down the equation of the system, because there is a nonlinearity in the equation. In this section we will try to capture the essential features of the model, and describe the market behavior. At first, the complete price adjustment function is derived, and then we have analyzed the market situation based on the numerical experiment in Day and Huang. This situation is used as a basis to analyze how changes in a single parameter affects the whole market when the other parameters are treated as constants. In the real market, parameters like v and u would change within business markets as well as reactions to macro-economical features and other news, and therefore it is interesting to see how such changes affects the model. We have described and evaluated changes in the following parameters: a , b , c , u and v . Lastly, we sum up and briefly discuss the model's empirical relevance.

5.1 Deriving the equation of the model

Based on the description of the participants from earlier, we can now complete the equation of the model. The price in the next period ($t + 1$) is given by combining the equations (4.1a)-(4.4):

$$p_{t+1} = f(p) = \begin{cases} m & p_t < m \\ p_t + c \left[\frac{a(u-p)}{(p_t - m + \varepsilon)^{d_1} (M + \varepsilon - p_t)^{d_2}} + b(p_t - v) \right] & m \leq p_t \leq M \\ M & p_t > M \end{cases} \quad (5.1)$$

where $f(p)$ is the price adjustment function. This map $f(p)$ represents the equation in the system set up by the parameters described in Table 4.1.

The model is said to “explode” if the price is below 0 or above 1 at any point. Negative share prices ($p < 0$) make of course no sense, because then the company is already bankrupt. If $p > 1$, then the α -investors' chance function and their excess demand is both equal to 0 (see equation (4.1)). β -investors' excess demand increases as the price goes upwards, regardless of whether $p > 1$ or not. These characteristics of α and β investors imply that the aggregate excess demand $E(p)$ is always positive if $p > 1$, and from earlier we know that the market maker adjusts the price upwards as long as $E(p) > 0$. It follows from this reasoning that if $p > 1$, then the price increases infinitely in the long run.

This “explosion” occurs if one or more parameters are set too high or low with respect to what the model is able to deal handle. An example of a situation where $p < 0$ is given in Figure 5.1. The model explodes because the price adjustment function is below 0 (and above 1) as shown in Figure 5.1a.

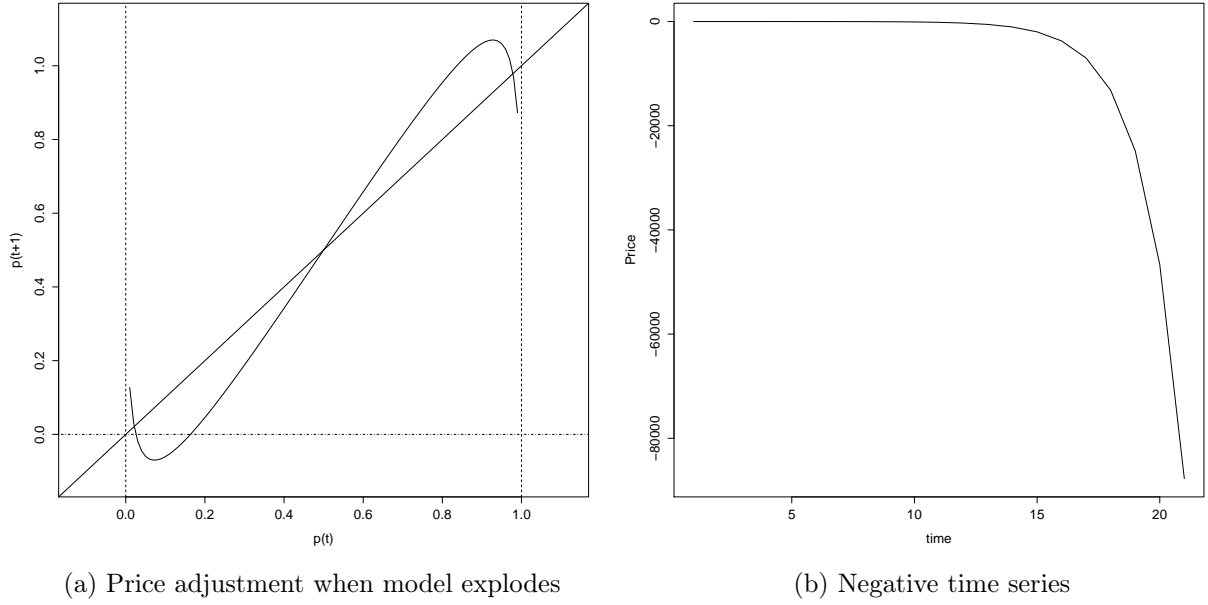


Figure 5.1: Model is out of range: explode.

5.2 Switching market

This market situations is based on the numerical experiment in Day and Huang, and the parameter values are

$$\begin{aligned}
 a = 0.2 \quad b = 0.88 \quad c = 1 \quad m = 0 \quad M = 1 \\
 u = 0.5 \quad v = 0.5 \quad \varepsilon = 0.01 \quad d_1 = 0.5 \quad d_2 = 0.5
 \end{aligned}
 \tag{5.2}$$

Figure 5.2b shows the time series and how the price decreases in the beginning. α -investors' chance function is small and their excess demand is low at this point, because the initial price is set just below 0.5 ($p_0 = 0.49$). β -investors' excess supply exceeds α -investors' excess demand, and the aggregated excess demand is negative. The market maker uses his financial resources to buy shares and adjusts the price downwards. The price decreases until it is very close to the bottoming price, and the α -investors' chance function and demand increases sufficiently to cause a positive aggregated excess demand. Then, shares are sold by the market maker and the price is adjusted upwards. This situation, in which aggregated excess demand alternates in sign and prices go up and down within the bear zone, continues for an unpredictable number of periods.

At some point, when the price reaches too close to the bottoming price, the aggregated excess demand is sufficiently high to pull the price into the bull zone. The price fluctuates within the bull zone for an unpredictable number of periods. When the price reaches too close to the topping price, the price is pulled back into the bear zone and as before it goes up and down within this zone. The price fluctuations switch between bull and bear at

random intervals in the long run.

Unsophisticated investors dominate the market in this situation and cause this erratic and unpredictable behavior. They buy high and sell low, and in this way they “make a market” for the information traders. The behavior of the fluctuations cannot be distinguished from a stochastic process or a random walk.

Figure 5.2a shows three fixed points \bar{p} .¹² Prices are flocking around these points as shown in the price distribution Figure 5.2c. The return in Figure 5.2d is generated by the function

$$rm_t = \ln(p_{t+1}) - \ln(p_t).$$

5.3 α -Investors behavior

If the magnitude of α -investor’s strength a is above 0.2, then the market becomes more stable and less volatile. This is because an increase in a makes the information traders more dominant. The market is in full equilibrium if a is sufficiently high (0.45 approximately), and the price is stable at 0.5 in the long run (see Figure 5.3b). In this situation there will be no trading between the participants because both α - and β -investors’ excess demand are equal to zero. The properties of full and temporary equilibrium is derived in (A.2). α -investors dominates the market, and under such circumstances the volatility will be more-or-less zero (particularly in the long run) and price fluctuations can only be explained by exogenous shocks. This “ α -dominated” market situation is shown in Figure 5.3.

The model explodes if a is smaller than 0.2 approximately. Unsophisticated investors dominate the market in this situation and cause “extreme” prices as exemplified in Figure 5.1b. Dependent on initial price, current price will then decrease or increase infinitely in the long run (see Section (5.1) for more details).

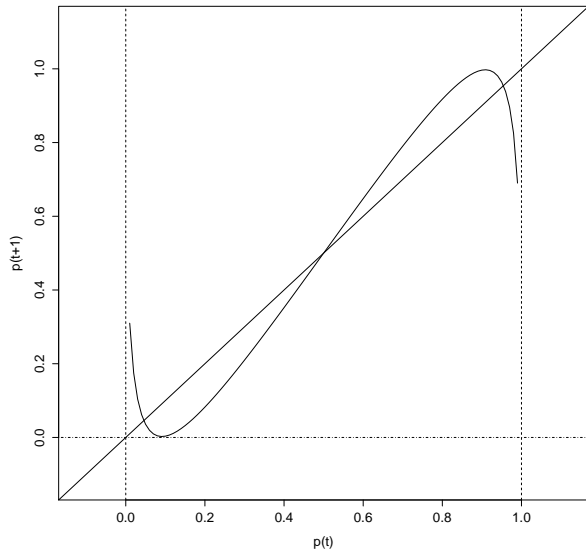
5.4 β -investors behavior

As mentioned earlier, there will be no trading between the participants in an α -dominating market. Such a market situation is not very interesting from an analytical point of view, and in this section we focus on the more interesting case in which β -investors are more dominant, as the flocking coefficient b rises.

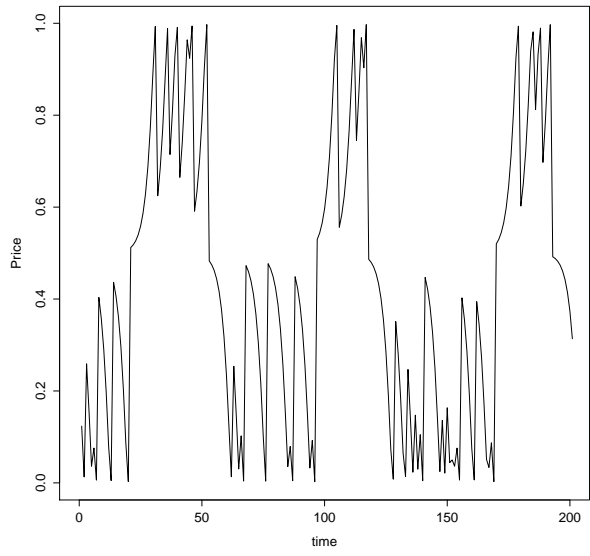
5.4.1 Bifurcation diagram of the flocking coefficient.

To capture the essential features of the model and to provide a better understanding of the relation between the market dynamics and the flocking coefficient b , we have generated

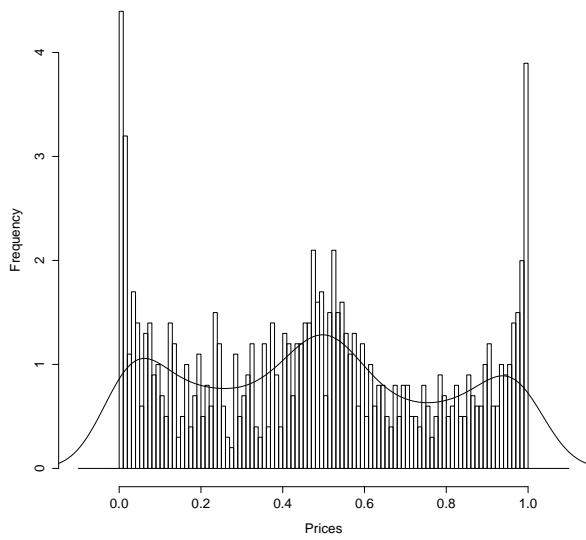
¹²Fixed points are given by the intersection between the price adjustment function and the 45°-line (see (A.2)), or by solving equation (5.1) for $p_{t+1} = p_t = \bar{p}$. According to Wieland and Westerhoff (2005, p. 120), the value of these points are approximately equal to 0.04, 0.50 and 0.96.



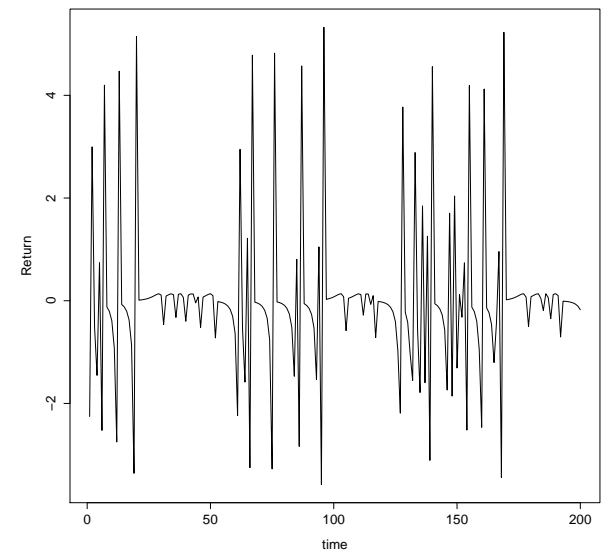
(a) The price adjustment function



(b) Stock price series



(c) The distribution of stock price



(d) The return series

Figure 5.2: Bear and bull markets

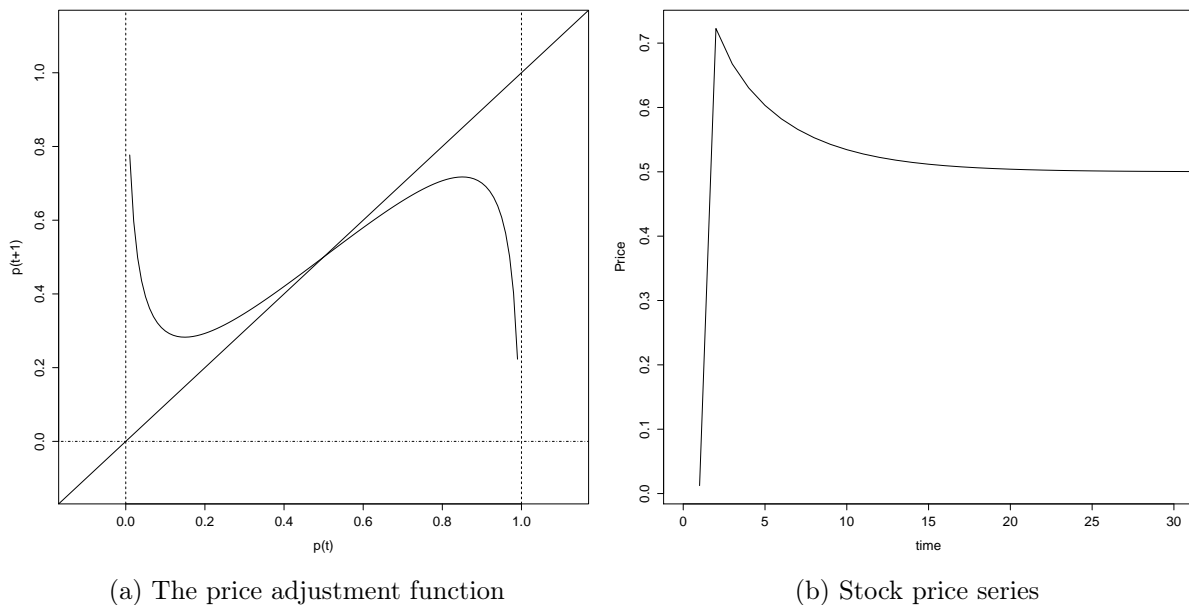


Figure 5.3: Market dominating by α -investors

a bifurcation diagram as an alternative of individual diagrams. In general, bifurcations of higher-order cycles are difficult to detect, but by generating a bifurcation diagram this will show the complexity in a simple, systematic way.

“By trying different values of this nonlinear parameter, May found that he could dramatically change the system’s character. Rising the parameter meant rising the degree of nonlinearity, and that changed not just the quantity of the outcome, but also its quality.” Gleick (1988, p. 70)

May et al. (1976) was able to connect all the information and plot it in a single picture or diagram. A bifurcation diagram is a plot that shows possible long run values, in the system, of a bifurcation parameter (or a changing parameter). Bifurcation diagram may also be called Feigenbaum structure. Such diagrams make it possible to analyze the entire changes in the properties of the system of equilibria, and get a better view of the critical boundaries between steadiness and oscillation (see Gleick (1988); Schroeder (2009)). The bifurcation diagram shows the price fluctuations (time series) for increasing values of b - from left to right. It can be used to distinguish between steady states (equilibria), states of instability and chaotic behavior in the market.

The time series consist 1000 observations, but to show the long-run behavior only the last 100 are plotted in our bifurcation diagram. Every time series is based on a given value of b . Figure 5.4 shows the diagram when the initial price is set both high (within the bull zone) and low (within the bear zone), and how the initial price influences the time series. The initial price is set below 0.5 in all the diagrams in Figure 5.5. If $p_0 = 0.5$,

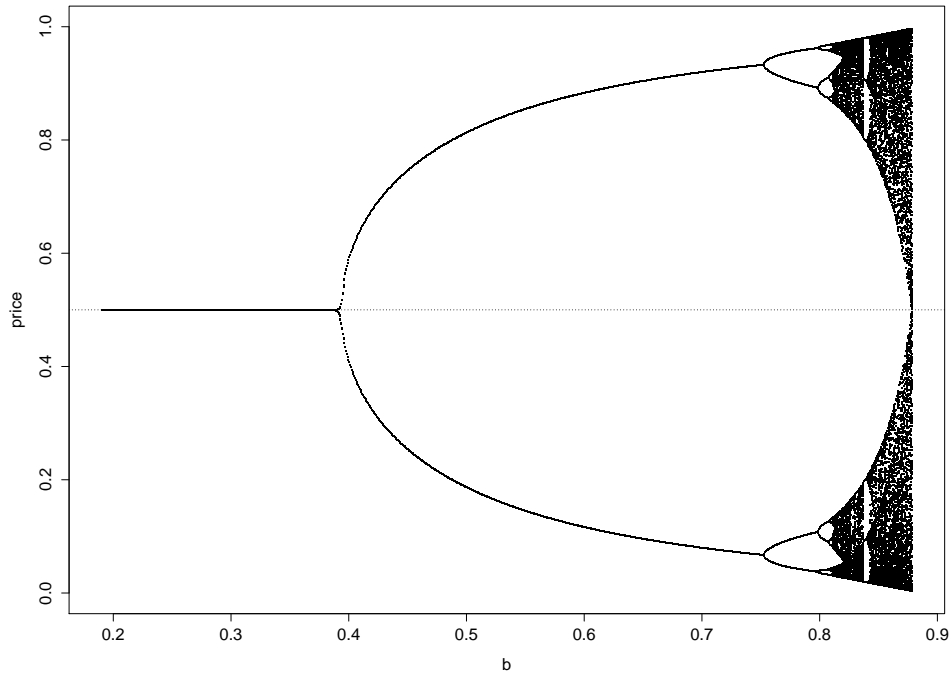


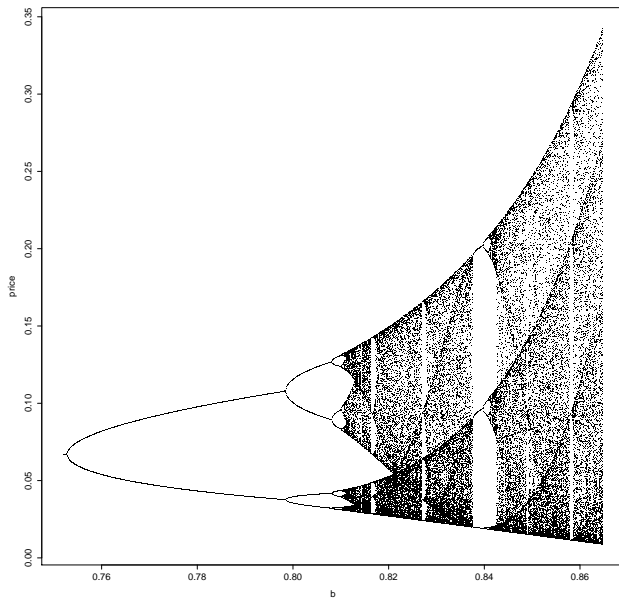
Figure 5.4: Bifurcation diagram of the b

then equation (A.8) is fulfilled, and the price is always stable at 0.5 independent of the flocking coefficient. This situation is illustrated by dotted line in Figure 5.4.

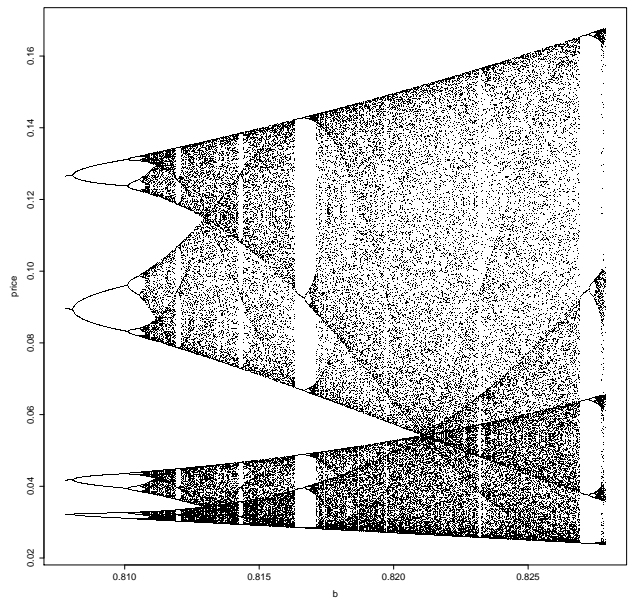
In Figure 5.4 it is easily seen that the level of b has a significant influence on the model and the price behavior. When the flocking coefficient increases, the market changes from being in equilibrium (steady state), to a situation where the fixed price is splitted into two prices. This process in which the number of fixed prices is doubled is called a bifurcation, and the prices fluctuate in a 2-cycle. As b increases additionally, the bifurcations occur more and more frequently, until some point in which the price fluctuations become erratic and chaotic.

Low level. The market is in temporary or full equilibrium when b is low. If b is less than 0.4, then the market is in full equilibrium and the price is stable and unique at 0.5 (see Section 5.3). The aggregate excess demand is equal to zero, and will be no transactions between the investors in this situation. When b is between 0.4 and 0.75, the β -investors become more dominant and the market price drops below 0.5. The market is still stable, but the equilibrium is only temporary.

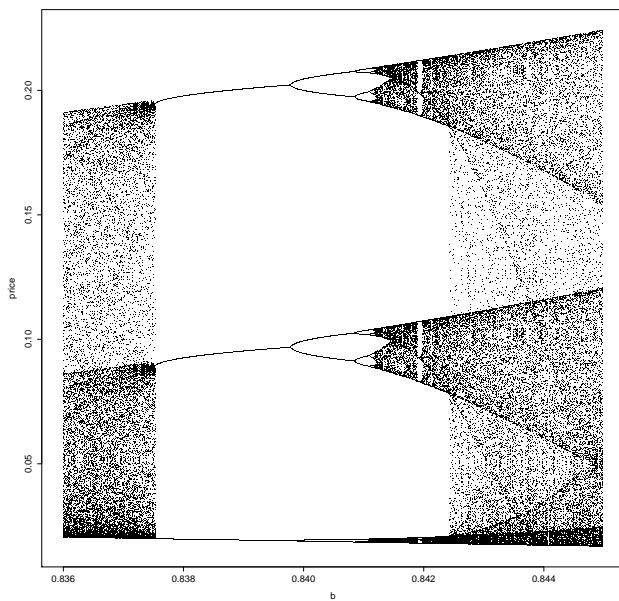
Mid-high level. When the level of b increases additionally and above 0.75 (moving to the right in the diagram), the prices is no longer stable at one equilibrium price. A bifurcation of the price occurs when $b \approx 0.76$, and the price fluctuates in a periode-2



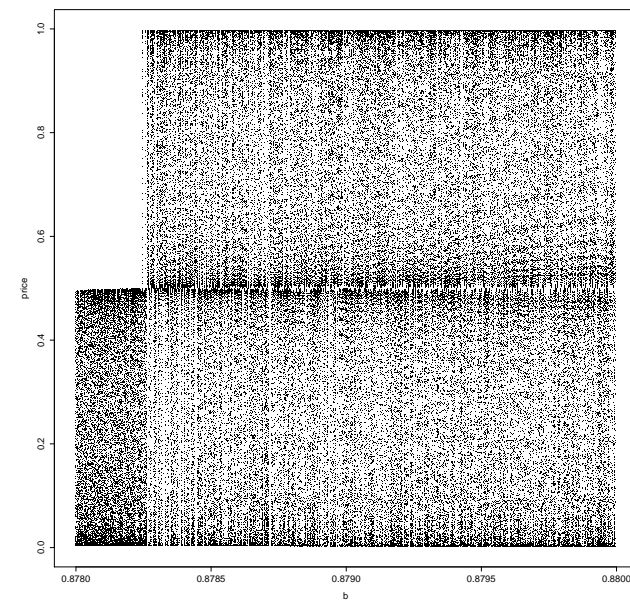
(a) First produced bifurcation



(b) Windows of cycles and regional chaos.



(c) Period tree occurs



(d) Complete chaos

Windows of the bifurcation diagram. (a) Beginning in full equilibrium, a bifurcation (cycle) occurs and the equilibrium is no longer unique. The cycles grow at the power of 2, meaning that the 2-cycle turns into 4-, 8-, and 16-cycles and so on. (b) “Black regions” and “white windows” appear, meaning that the market switches between predictable cycles and more-or-less chaotic fluctuations. (c) A stable 3-cycle appears. (d) Eventually the number of period-doubles is “infinitely” and the market becomes chaotic (stochastic).

Figure 5.5: Windows in the bifurcation diagram

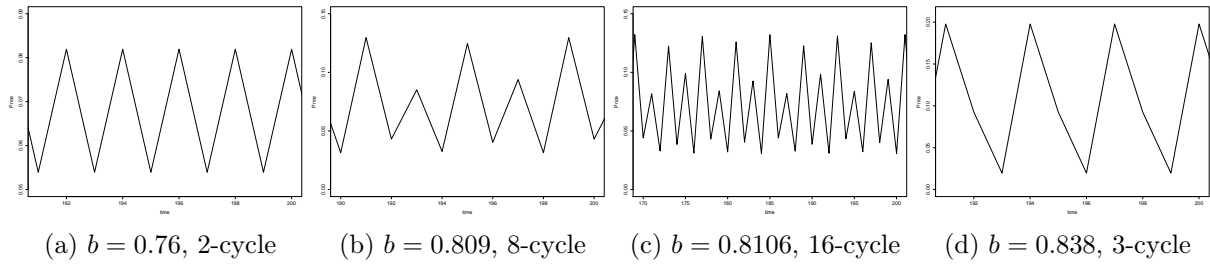


Figure 5.6: Periodic cycles, orbits

orbit¹³ (2-cycle). The price series is unstable in the short run, but stabilized in the long run and switches back and forth between two prices (see Figure 5.6a). The fixed point has lost its steady state and is changed from an attractor to a repeller (see A.2 for more details).

As we are moving more to the right in the diagram, the bifurcations occur more and more frequently. The 2-cycle grows to a 4-, 8-, 16-cycle and so on (see Figure 5.5b). Eventually, the period-doubling becomes so vast that “black regions” appear in the diagram, as the points spread within the upper and lower bands. At some b -level above 0.81 the environment is difficult to distinguish from n -cycles and regional randomness; fluctuations being more-or-less chaotic. One might expect more chaotic behavior as the flocking coefficient increases further, but suddenly, beyond a certain point, “white windows” appear. If the windows are wide enough, then it is possible to count the number of fixed points.

The market is switching back and forth between predictable cycles and chaotic fluctuations ($0.81 < b < 0.88$), even though a rise of the parameter b is said to rise the degree of nonlinearity (Gleick (1988)). This is illustrated in the different plots in Figure (5.5).

A “period three” appears. In Figure 5.5a it is easily seen that there exist “white windows” when $b \approx 0.84$. This is even easier to see in Figure 5.5c, which shows that a 3-cycle occurs when $b = 0.838$. This is a very important case and is described in Li and Yorke (1975). They proved that for any one-dimensional system, that has a three-periodic cycle (period-3 orbit), cycles of infinite orders will eventually occur.

Strong level ($b = 0.88$) Until now, the bifurcation process has repeated itself over and over again and more-or-less chaotic behavior has occurred as b has been increased. The prices have fluctuated in 8-, 16-, 32-cycles (see Figure 7.1e and 5.6c) and so on *ad infinitum*, ending up in chaos as b goes to 0.88 (the model explodes if $b > 0.88$). When $b = 0.88$, then the price behavior is chaotic and the market is said to be dominated by β -investors, see Figure 5.5d. A more detailed description of this situation is given in

¹³Orbit is the technical term for a succession of iterates p_t .

Section 5.2.

5.5 Model behavior when $u \neq v$

In the switching market we assumed for simplicity that the investment value u is equal to the fundamental value v . The model gets far more complicated (as if the model wasn't complex enough) if this doesn't hold ($u \neq v$), because the market is heavily affected by changes in investment value and fundamental value.

If the investment value is higher than the current fundamental value ($u > v$) and the market is dominated by β -investors ($b = 0.88$), then the majority of investors are said to be bearish. The prices fluctuate within the bear zone in the long run. In the opposite case, where $u < v$, the majority of investors are said to be bullish and the price fluctuates within the bull zone. The price behavior depends on the initial price in both situations - particularly in the short run. Price fluctuations are still more-or-less chaotic when $u \neq v$, but the prices do not switch between bull and bear at random intervals. There may be one, two or three temporary fixed points, but the market is never in full equilibrium if $v \neq u$ (cf. (A.8)).

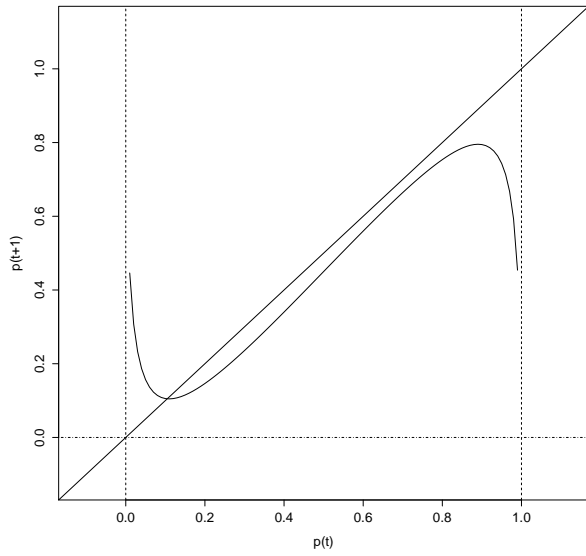
It should be stated that the model is particularly sensitive to changes in v or u when $b = 0.88$, and the model is able to handle small changes only before the model explodes, see Figure 5.1. If the flocking coefficient is set lower, say $b = 0.5$, then it is possible to see how relatively large changes in u or v affects the market. Figure 5.7a shows that a bear market is generated if fundamental value is set higher than investment value ($v = 0.6$ and $u = 0.5$). Contrary, a bull market is generated if $v = 0.5$ and $u = 0.6$, see Figure 5.7c. In both situations there will be one steady state only, which is temporary and locally stable.

5.6 Market maker behavior

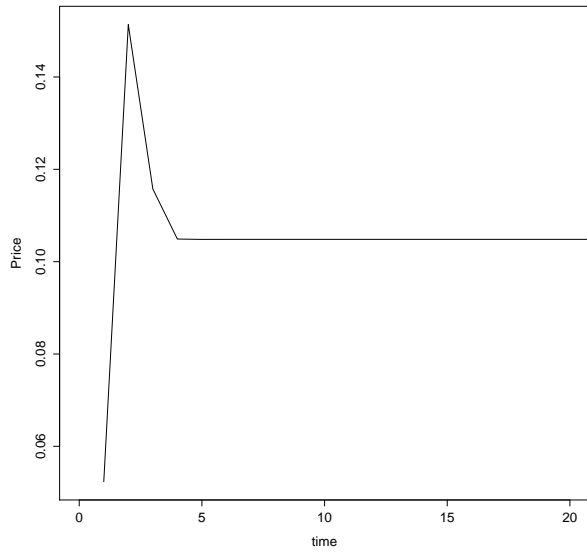
The coefficient c is a parameter connected to the price adjustment function.

“According to Propositions 2 and 3, changes in the price adjustment coefficient can profoundly influence the qualitative features of market dynamics. A precise impression of this relationship can be obtained by computing the bifurcation diagram for the parameter c .” Day and Huang (1990, p. 321)

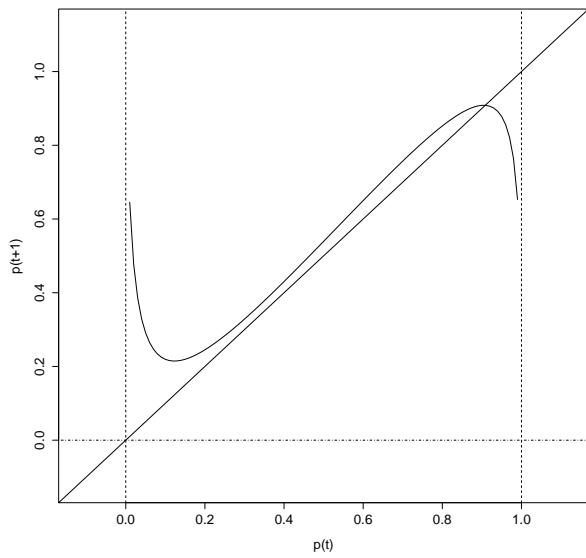
Day and Huang consider the market maker's role in the market and how their rational self-interest may influence their decisions. In this thesis we aren't not investigating the market maker's role in detail, as we are more interested in how the unsophisticated investors influence the market. But we have considered the basic elements of the market makers.



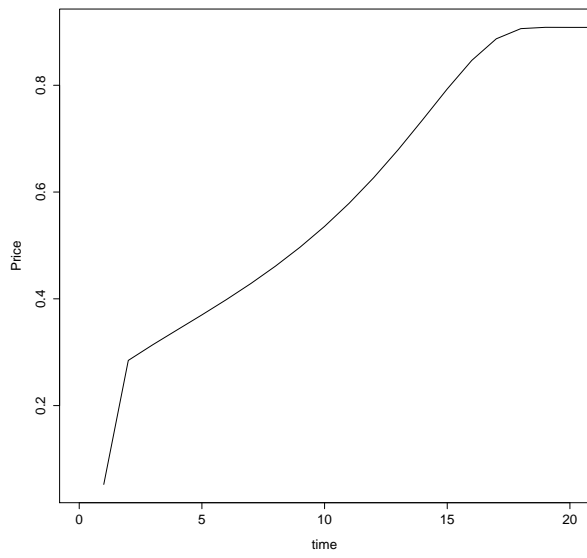
(a) The price adjustment function - bearish market



(b) Stock price series - bearish market



(c) The price adjustment function - bullish market



(d) Stock price series - bullish market

Figure 5.7: Bearish market and bullish market

A more detailed description is given in Gu (1995), which had the purpose to carry out an economic analysis of the specialist's simple price adjustment rules.

One can say that the price adjustment coefficient adjusts or scales the aggregate excess demand's effect on tomorrow's price p_{t+1} (cf. equation (4.6)). If c is zero, then $p_{t+1} = p_t$, which means the aggregated excess demand has no effect on tomorrow's price. If c is high (low), then the aggregated excess demand has large (small) influence on p_{t+1} .

Given a switching market situation (see Section 5.2), the market is no longer switching between bull and bear if c is set below 1. Price fluctuations might still be more-or-less chaotic, but the prices are never pulled from bull to bear zone or vice versa. Whether the price fluctuates within the bull or the bear zone depends on the initial price. When c is sufficiently low, the market is stable at one price and converge to m or M (dependent of p_0).

In the case in which $c > 1$, the prices switch more frequently between bull and bear compared to the switching market. The model explodes if c is set higher than 1.02 approximately.

5.7 Summary

The model is highly sensitive to changes in the different parameters, and even small modifications might affect the characteristics of the model dramatically. Our main concern has been how the β -investors and different values of the flocking coefficient b influences the price behavior, but changes in other parameters have been evaluated too.

The market is in full equilibrium and there will be no trading between the participants if α -investors dominate and the flocking coefficient is low. As b increases and the β -investors become more dominant, the market is no longer in full equilibrium, but still stable at one fixed price. The bifurcation process begins when the flocking coefficient is 0.75 approximately, and the prices fluctuate in 2-, 4-, and 8-cycles and so on. Prices are predictable as long as such cycles exist, but cycles of high orders might be very hard to observe. At some point, when $b = 0.88$ approximately, price fluctuations of chaotic type occur and the prices switch between bull and bear markets and random intervals. This behavior cannot be distinguished from a stochastic process or a random walk. β -investors dominates the market in this situation, and they are said to make a market for the α -investors. This situation ($b = 0.88$), which is referred to as a switching market in our thesis, is based on the numerical experiment in Day and Huang's paper.

For simplicity it is assumed that $u = v$ in the switching market. If the market is dominated by β -investors ($b = 0.88$) and estimated investment value is higher than current fundamental value ($u > v$), then a bear market occurs. A bull market occurs in the opposite case where $v > u$.

The price adjustment coefficient c scales the aggregated excess demand's effect on

tomorrow's price. If c is set lower than 1, then the price volatility decreases, and the model explodes if $c > 1.02$ approximately.

5.8 Empirical relevance

Is the model empirically relevant? Gu (1993) among others has the objective to answer this question. The purpose of his paper is to test whether the type of volatility generated in the Day and Huang model, is anything similar to that observed in real stock markets. To do that, he compares the distribution of price changes in the Day and Huang model to that of the S&P composite stock price index. The data set contains 62 years of monthly average prices, which are adjusted for inflation and economic growth. He concludes that the two distributions are *strikingly* similar, or more precisely, the tests used in the paper fail to reject that the two distributions are not statistically different.

Day and Huang shows that their model can generate price series that have stochastic characters like that tails are fatter for the short averages and get less and less leptokurtic as the length of the average increases. Gu (1995) investigated the arguments of the market makers ability to “churn the market”.

It is exceedingly debated that financial markets have chaotic dynamics, but it is certified that conditions of nonlinearity is an underlying property in the financial market. To cause chaos, conditions for nonlinearity has to be fulfilled.

6 The extended model

In this section we introduce a new and extended version of the Day and Huang model, derived in the previous section. As the *original*, the new model consists of α -investors, β -investors, and a market maker, but the unsophisticated investors are no longer homogenous (at least in the initial state). Investor i represents each individual β -investor, where $i \in \{1, 2, 3, \dots, I\}$ denotes the investor index. The time index is defined by $t \in \{1, 2, 3, \dots, T\}$.

The characteristics of the information traders and the specialist or not changed in the new version. In the original model all β -investors based their trading strategy on the relation between current price and current fundamental value, but the latter parameter is removed in this extended version. Instead, β -investors base their investment decisions on different technical trading rules or algorithms, and therefore they may be referred to as *algo-traders*.

The *algo-traders are socially integrated*, in the sense that they are able to obtain and adapt information from other algo-traders to improve their own strategy. This motivation of interacting, is an outcome reflected by a *stress* indicator.

The original Day and Huang model doesn't say anything about the situation in which an investor that does not hold any shares or has sold his position. As in Shangkun et al. (2012), we consider a safe, interest bearing account as an alternative to holding shares. In other words, investors either hold risky assets (shares) or risk free assets (interest bearing account) at all times. In this model we do not consider any transaction cost.

Assumption: Short selling is prohibited in the model.

All assumptions and simplifications made in the original model still hold unless something else is pointed out.

6.1 The parameters

To make the model functioning some new parameters have to be introduced. These parameters are listed in the Table 6.1 and subsequently the most important ones are explained.

symbol	meaning
i	investor index
t	time index
γ_{it}	units of the risky asset held by investor i at time t
ω_{it}	investor i 's wealth at time t
s_{it}	stress indicator for investor i at time t
b_{it}	investor i 's excess demand for the risky asset at time t
a_{it}	algorithm used by investor i at time t
p_t	price per unit of the risky asset at time t
r	interest rate paid by the safe asset
Ψ	function used to scale the excess demand
ipm_t	investor population matrix at time t
nn	number of neighbors

Table 6.1: Notation in the extended model

6.1.1 Risky and risk free assets.

Investor i holds γ_{it} units of the risky asset at time t , where $\gamma_{it} \in \mathbb{R}$ and $\gamma_{it} \geq 0$. To indicate that an investor holds his funds in a risk free account, that has bearing interest of r , the γ_{it} is set equal to zero. An investor is said to be "in" the market if he holds risky assets, and "out" of the market if he holds his funds in a risk free account.

$$\gamma_{it} \begin{cases} = 0, & \text{all wealt invested in the safe asset at time } t; \\ > 0, & \text{units of risky asset held at time } t. \end{cases} \quad (6.1)$$

Assumption: γ_{it} is not defined as an integer so it is possible to buy fractions of shares.

6.1.2 Investor wealth.

ω_{it} denotes the investor i 's wealth at time t , where $\omega_{it} \in \mathbb{R}$ and $\omega_{it} \geq 0$. If the investor holds risky assets, i.e. $\gamma_{it} > 0$, then his wealth at time t is determined simply by valuing his stock of risky assets by their current value, that is $\omega_{it} = \gamma_{it}p_t$. If the investor holds his wealth in a safe, i.e. $\gamma_{it} = 0$, then the wealth ω_{it} includes the interest paid for period t : $\omega_{it} = \omega_{it-1}(1+r)$. Since variable ω_{it} does not carry the entire information about the nature of investor i 's wealth, only the numerical value, one will usually consider the pair $(\gamma_{it}, \omega_{it})$ to fully characterize the wealth position of an investor.

6.1.3 Algorithm rules.

We consider a set of alternative trading algorithms. Each investor might use different technical trading rule (but only one at the same time) and each rules are labeled using integers. So the set of potential algorithm is given by $\{1, 2, 3, \dots, A\}$. If a trading rule is applied at time t , it generates a binary signal S on the basis of a subset of past prices $p_{t-1}, p_{t-2}, p_{t-3}, \dots, p_{t-\tau}$. Therefore each algorithm can be written as $F : \mathbb{R}^\tau \rightarrow \{0, 1\}$ or $S \leftarrow F(p_t, p_{t-1}, p_{t-2}, p_{t-3}, \dots, p_{t-\tau})$, where

$$S = \begin{cases} 1, & \text{buy;} \\ 0, & \text{sell.} \end{cases} \quad (6.2)$$

Assumption: Each algo-trader chooses one trading algorithm. The choice of investor i at time t is recorded on the variable a_{it} . Consequently, the variable takes integer values: $a_{it} \in \{1, 2, 3, \dots, A\}$.

To see what type of action is implied by the binary signal indicated by rule F , one has to consider whether investor i currently is in or out of the market. Let SI denote the state indicator for investor i , assuming values from the set $\{\text{in out}\}$. Table 6.2 specifies the action suggested by rule F conditional on SI . In each cell in Table 6.2 we identify the action triggered by the signal S conditional on whether or not the investor currently holds the risky asset.

	S	
SI	1 (buy)	0 (sell)
in	hold risky assets	sell risky assets
out	buy risky assets	hold safe assets

Table 6.2: Investor's action space

Assumption: A algo-trader always implements the action (strategy) suggested by the trading rule a_{it} he has decided to use at time t .

6.1.4 The last signal

The signal generated from the last period is recorded and is denoted sl_{it} , where $sl_{it} = S_{it-1}$. We need this book keeping, of the last signal, because some of the algorithm requires this information in their calculations. sl is connected to investor i , note that if an investor substitute his algorithm in a communication process he will not adapt this sl signal.

6.1.5 Stress indicator.

The stress indicator for investor i at time t s_{it} , reflects the number of consecutive losses the investor has experienced. Whenever the investor's wealth position improves, the variable is zeroed out. s_{it} is a discrete variable and take values from the set $\{0, 1, 2, 3, \dots, N\}$. It is the level of s_{it} that might trigger a change in the trading rule currently used by investor i . Given that s_{it} exceeds a *tolerance level*, which is individual for each investor, he starts to look for alternatives. In that case the investor uses the stress levels and wealth positions of selected neighbors to identify a rule that is superior to his own.

Technically speaking, the investor has a preference order over (ω_{jt}, s_{jt}) which allows for a mathematical representation by means of a utility function u_i . Consequently, u_i is defined on the tuple (ω_{jt}, s_{jt}) for some $j \neq i$. Investor j represents each individual investor that investor i communicates with in this case. Meaning that, the stress levels and wealth positions of each investor j is known for investor i . Investor j 's algorithm a_{jt} is also known, but not a part of investor i 's utility function. Further detailed are discussed in Section 6.5.1.

6.1.6 Excess demand.

Analyzing Table 6.2 one notices that in the two "hold" positions, on the diagonal, no transactions will be carried out. The investor's demand for the risky asset will be zero in

each case. Someone that is holding units of the risky asset will sell (supply) those units if the trading algorithm generates a sell signal, and consequently the excess demand is reduced. Alternatively, an investor who currently holds his wealth in form of the safe asset will demand units of the risky asset if the algorithm produces a buy signal. In that way, excess demand will be increased. Let b_{it} represent investor i 's demand for the risky asset (in units of the asset), and we can distinguish between three cases:

$$b_{it} \begin{cases} > 0, & \text{investor } i \text{ is a } \textit{net demander} \text{ of the risky asset;} \\ = 0, & \text{investor } i \text{ does not affect the price;} \\ < 0, & \text{investor } i \text{ is a } \textit{net supplier} \text{ of the risky asset.} \end{cases} \quad (6.3)$$

Then table 6.2 can be transformed into Table 6.3 that gives the effect of alternative outputs of a trading algorithm applied by investor i at time t .

	S	
SI	1 (buy)	0 (sell)
in	$b_{it} = 0$	$b_{it} < 0$
out	$b_{it} > 0$	$b_{it} = 0$

Table 6.3: Effects of investor's actions

6.2 Accumulating the excess demand (supply)

At each point in time the following is known about investor i : the number of units of risky assets in his portfolio (γ_{it}), his wealth position (ω_{it}), the excess demand (supply) of the risky asset (b_{it}), his stress level s_{it} , an indicator of the trading rule applied (a_{it}), and signal generated by a_{it} in $t - 1$ is recorded. That is, the 1 by 6 row vector

$$\left(\gamma_{it} \quad \omega_{it} \quad b_{it} \quad s_{it} \quad a_{it} \quad sl_{it} \right) \quad (6.4)$$

provides a complete characterization of investor i at time t . To represent the population of algo-traders we arrange the rows characterizing all I investor in an I by 6 matrix. This

matrix is referred to as the *investor population matrix*:

$$ipm_t = \begin{pmatrix} \gamma_{1t} & \omega_{1t} & b_{1t} & s_{1t} & a_{1t} & sl_{1t} \\ \gamma_{2t} & \omega_{2t} & b_{2t} & s_{2t} & a_{2t} & sl_{2t} \\ \gamma_{3t} & \omega_{3t} & b_{3t} & s_{3t} & a_{3t} & sl_{3t} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_{it} & \omega_{it} & b_{it} & s_{it} & a_{it} & sl_{it} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_{It} & \omega_{It} & b_{It} & s_{It} & a_{It} & sl_{It} \end{pmatrix} \quad (6.5)$$

The excess demand of the algo-traders is computed at the end of period t as

$$\beta(p_t) = \Psi\left(\sum_{i=1}^I b_{it}\right) \quad (6.6)$$

where $\Psi : \mathbb{R} \rightarrow \mathbb{R}$ denotes the function used to scale the excess demand to be compatible with the β -component in the original Day and Huang model. Ψ is defined as

$$\Psi := \frac{1}{1 + e^{-\lambda(\sum_{i=1}^I b_{it})}} - \frac{\psi}{2} \quad (6.7)$$

where ψ maps the excess demand in the range $(\frac{-\psi}{2}, \frac{\psi}{2})$, and λ adjust the steepness. The scaling function Ψ prevents the model from exploding if the excess demand or supply of the algo-traders is high. If ψ , which is an exogenous parameter, is set low, then the amplitude of the prices is low. In the opposite case, where ψ is high, the amplitude is (of course) high. The explanation is that the price changes made by the market maker (when there is excess demand (supply)) is larger when ψ is high, compared to a situation in which it is low. While the ψ cover the height, the λ plays an intricate role conduction the steepness of the scale function. Figure 6.1 illustrates the characteristic of the equation (6.7) with a fixed, but with different, values of λ . The smaller λ gets (going to zero in the limit), the curving of the Ψ is less converge to a straight horizontal line if $\lambda = 0$. If the λ is set relatively high this will make the function curving vertically. At present, the λ is set to a level that coincide with the wealth and I .

6.3 The price adjustment

The price at time $t+1$ is determined in the course of the *tatonnement process* established in the reference model:

$$p_{t+1} = p_t + c[\alpha(p_t) + \beta(p_t)].$$

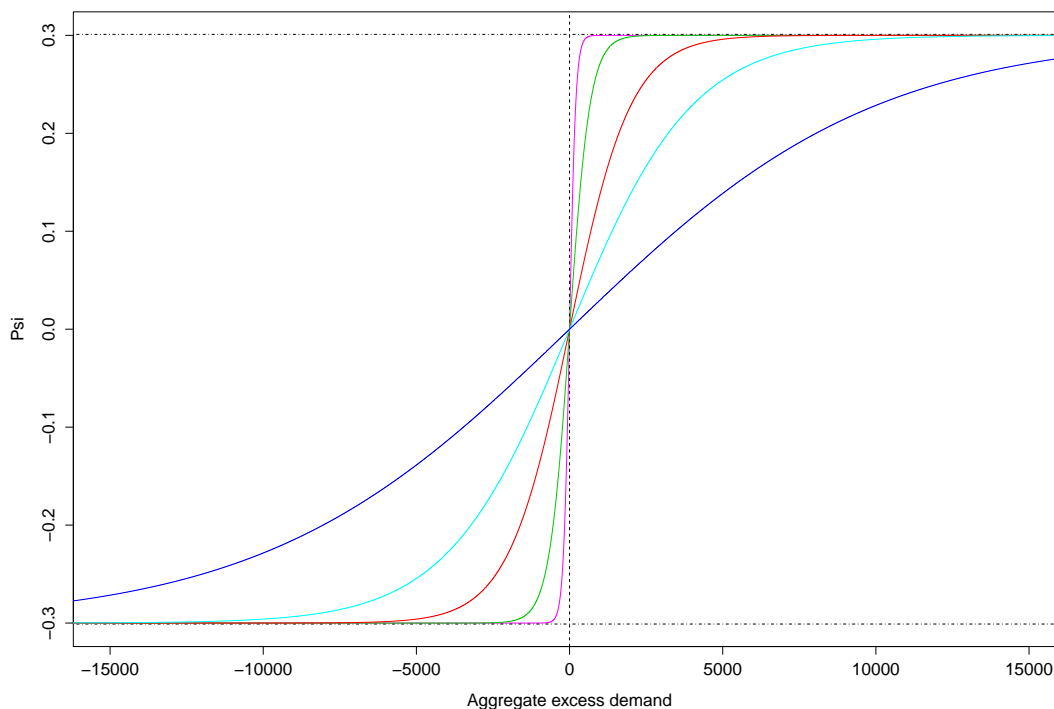


Figure 6.1: The scaling function Ψ

For simplicity the price adjustment coefficient is assumed to be 1, and therefore the latter equation can be written as

$$\begin{aligned}
 p_{t+1} &= p_t + \alpha(p_t) + \beta(p_t) \\
 &= p_t + \alpha(p_t) + \Psi\left(\sum_{i=1}^I b_{it}\right)
 \end{aligned} \tag{6.8}$$

The event of fixing the price for the risky asset defines the end of a period. Once the price has been fixed and *made public* the subsequent periods begins.

6.4 Details on the communication process

In this prevailing version of the model we have implemented a very stylized form of “communication process”. An investor who sees the need to communicate will make a number of random contacts in the population of algo-traders. The chatter with those individuals will always be successful in the sense that the true value of the relevant characteristics will be revealed by the contacts in the course of the communication with investor i . We let the nn be the “number of neighbors” parameter that investor i may be in contact with.

Assumption: When investor i gets information about investor j 's wealth, stress and algorithm, he will always be given the truth in the communication process.

6.5 Description of period t

At the beginning of period t the path of past prices up to and including the current price is *common knowledge*: $P_t = \begin{pmatrix} p_0 & p_1 & p_2 & p_3 & \dots & p_{t-2} & p_{t-1} & p_t \end{pmatrix}$. Each algo-trader has access to this price series and a subset of rows of the investor population matrix from the previous period ipm_{t-1} . That is, each investor has a chance to inform himself about the characteristics of other algo-traders through a communication process to be described below.

Given this information basis, the i 'th investor is able to perform a sequence of decisions leading to actions, i.e. performing transactions, which eventually will lead to a new vector of investor characteristics. In a first approach, we give a coarse description of this sequence. Given P_t and ipm_{t-1} the i 'th investor

1. chooses an algorithm; F (rule) from the set $\{1, 2, 3, \dots, A\}$,
2. implements the rule and receives the signal,
3. carries out the transactions suggested by the rule at the current price p_t .

At the beginning of period t the situation of investor i is described by the elements of the i 'th row of the ipm_{t-1} matrix. Once the sequence given above is terminated, this row vector of investor characteristics is updated. A more detailed description of this updating process is given in Section 6.5.3.

As soon as the sequence 1-3 has been performed by each of the I investors and the updating of their characteristics is complete the new investor population matrix ipm_t is available. Finally, marking the end of the period t , the information on population comprehensive excess demand (3rd column of ipm_t matrix) is accumulated and used to determine the price of the risky asset prevailing in the period $t + 1$.

6.5.1 Choosing the algorithm

Assumption: An investor who realizes increases in wealth does not see the need to change the trading rule he is using, i.e. the algorithm is never substituted if $s_{it} = 0$.

It is the scenario where he experiences one or more periods of loss in a row, i.e. $s_{it} > 0$, that provides a motivation for substituting the rule used so far, by an alternative one. The choice of the new rule is based on the outcome of a communication process.

At the beginning of this process investor i contacts investor j . It is assumed that as a result of the "conversation" with investor j , investor i manages to get information about a

subset of investor j 's characteristics for the previous period. To be more specific, investor i will have information about investor j 's wealth and stress level at time $t - 1$, and the trading algorithm used during the last period. If investor i "talks" to investors $l, k, m,$ and n , then he has the following *data base* at his disposal:

$$\begin{pmatrix} \omega_{lt-1} & s_{lt-1} & a_{lt-1} \\ \omega_{kt-1} & s_{kt-1} & a_{kt-1} \\ \omega_{mt-1} & s_{mt-1} & a_{mt-1} \\ \omega_{nt-1} & s_{nt-1} & a_{nt-1} \end{pmatrix}.$$

Since the wealth level is a real number and the stress level is an integer variable, the space (ω, s) lacks convexity. The (wealth, stress)-situation experienced by investor i in period $t - 1$ is represented by the point having the coordinates $(\omega_{it-1}, s_{it-1})$. These coordinates are plotted in the (wealth, stress)-space shown in Figure 6.2, and each (wealth, stress)-situation will be represented as a point on one of the gray lines.

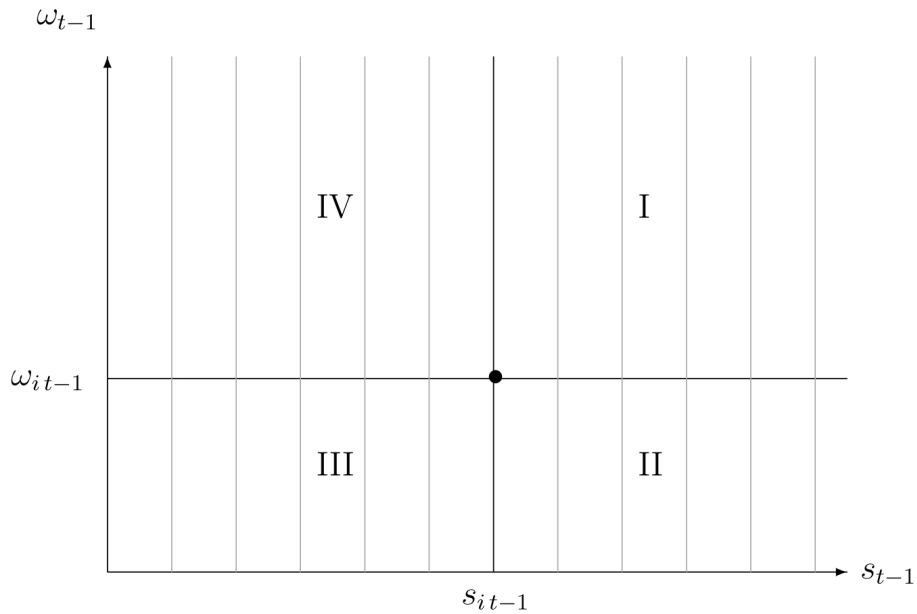


Figure 6.2: Investor preferences over (ω, s) -space

Investor i 's (wealth, stress)-situation partitions the (ω, s) -space into four regions: I, II, III, and IV. An investor (that has decided to substitute his rule) will try to "learn" from other investors by comparing their own (wealth, stress)-position to that of other investors. Investor i selects those contacts who have currently reached a higher wealth position at a lower stress level. Suppose we were to plot the (wealth, stress)-situations of those investors who happen to outperform investor i in Figure 6.2, then the resulting points would lie on the gray lines in region IV.

If no such cases exist, i.e the region IV is empty, then the investor interprets the fact that none of his contacts does better both in terms of wealth and level, as an indication that the circumstances are just “bad”. Then the investor will not change the rule he currently applies, meaning that he will stick to his own rule. It should be pointed out that even though the marked situation is “bad”, it is never an option to ignore buy (or sell) signals generated by the algorithm. These investors are assumed to unsophisticated and they carry out the transactions suggested by the rule no matter what, see assumption on page 34. An investor will not copy rules used by contacts positioned in region I, II, or III. Contacts in region I are better in terms of wealth and in III they are better in terms of stress, but none of them are superior in both. In region II the contacts are worse of in both wealth and stress.

If, on the other hand, one of his contacts use a rule that is superior to his own (i.e. there is exactly one point in region IV), then this *contact is identified and investor i copies investor j's rule*. What will happen if the communication process leads to several points lying in region IV? In this instance the investor compares all alternatives and copies the rule used by the investor who had the most attractive position in the (wealth, stress)-space. In a special case in which more than one point lie on the most attractive (farthest out in the North-Western direction) indifference curve, the first occurrence is chosen.

Assumption: The investor has a preference order on (ω, s) which can be represented by a *utility function* $u(\omega, s)$ such that $(\omega, s) \succeq (\omega, s)' \Leftrightarrow u(\omega, s) \geq u((\omega, s)')$ where $u(\omega, s) = \omega - \gamma s$ and $\gamma > 0$.

This utility function is increasing in wealth (positive marginal product of wealth) and decreasing in stress. Moreover, it reflects the fact that investor i is willed to accept higher stress levels if he is compensated by higher wealth. Some representatives of the set of linear indifference curves implied by the utility function are plotted in region IV in Figure 6.3. Level curves lying further out in the North-Western direction are associated with higher utility levels. For example, we have $\bar{u}' > \bar{u}$ in Figure 6.3.

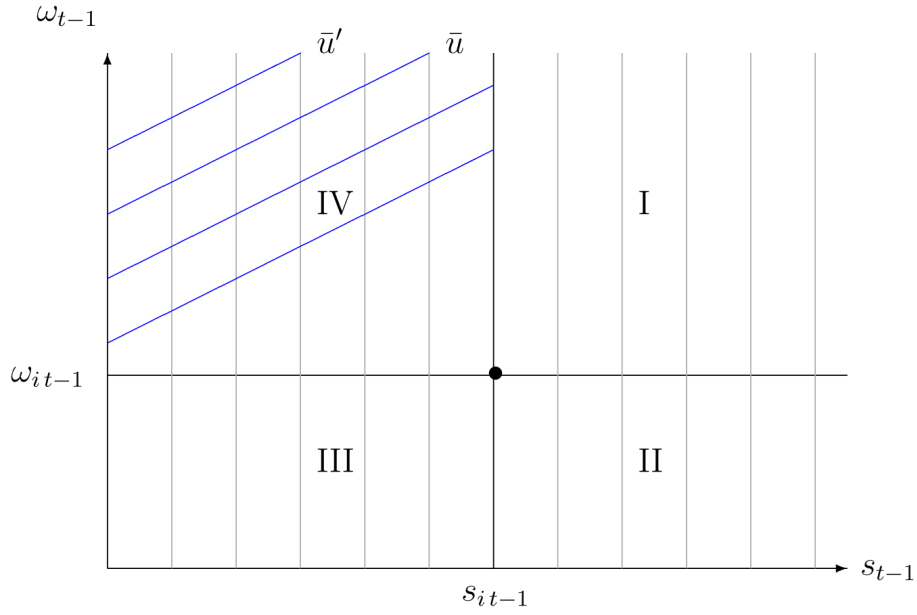


Figure 6.3: Indifference curves on subset of (ω, s) -space

To sum up, investor i will copy the trading rule of the contact whose (wealth, stress)-position maximizes his own utility function, and consequently, his vector of investor characteristics will be updated:

$$a_{it} \leftarrow a_{jt-1}.$$

6.5.2 Implementing the rule and receiving the signal

Once the decision concerning algorithm has been reached investor i applies the rule. Given his access to the price history, the algorithm operation on a subset of the price series is run. Investor i receives the binary signal S generated by the rule, and follows the rule by instigating the appropriate actions (c.f. Table 6.2 and Table 6.3). The possible changes in investor characteristics are discussed in the following section.

6.5.3 Updating the investor characteristics

The updating process is the third and last part of the sequence of decisions discussed in Section 6.5. As considered earlier, each algorithm (indicating by F) generates a binary signal S , 0 or 1, based on past prices. The different rules (algorithms) might generate different signals based on the same price series, but they all produce either a buy signal ($S = 1$) or sell signal ($S = 0$). Every signal leads to an updating process where investor i 's portfolio of risky assets, wealth and excess demand is updated, which means the investor population matrix, equation (6.5), is updated from ipm_{t-1} to ipm_t . Figure 6.4 gives the

updating rules for each and every possible reaction to the binary signal S .

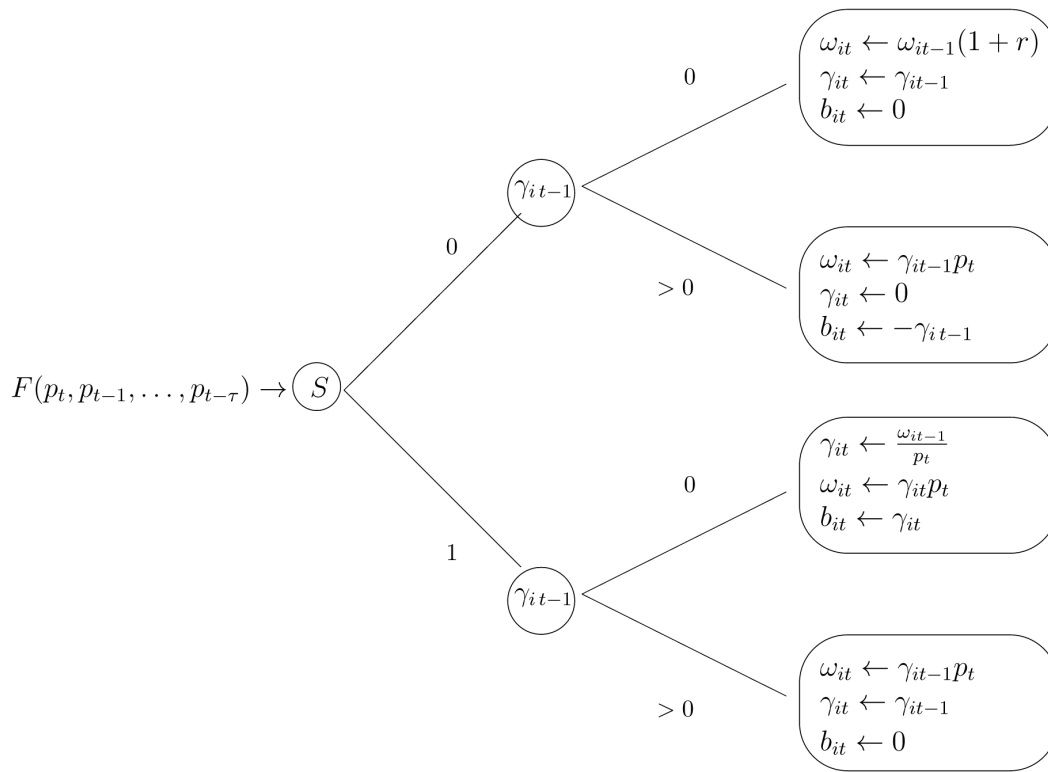


Figure 6.4: Tree representation of the updating process

Since the process depends on whether investor i is *in* or *out* of the market at time $t - 1$ and if the binary signal S is a buy or sell signal, there are four possible scenarios (see figure 6.4). Each scenario (situation) is described below.

In the first scenario investor i is out of the market and the algorithm (trading rule) generates a sell signal ($S = 0$). Since the investor by now is holding risk free assets ($\gamma_{it-1} = 0$) no transactions will be carried out. However, the elements γ_{it} , ω_{it} and b_{it} are updated at the end of the sequence, but only ω_{it} is changed. Investor i 's wealth at time t is equal to his wealth last period plus the risk-free interest rate earned during the period ($\omega_{it} = \omega_{it-1}(1+r)$). Obviously, since no transactions are carried out, investor i 's position in risky assets is unchanged, implying that $\gamma_{it} = \gamma_{it-1} = 0$. The excess demand at time t is equal to zero for the same reason.

Investor i is in the market and a sell signal is produced by the trading rule in the second situation. On the basis of this signal, investors i sells his position in risky assets and instead deposits his wealth in a safe account ($\gamma_{it} = 0$). Because of this selling order, the excess demand is negative (i.e. positive excess supply) at time t ($b_{it} = -\gamma_{it-1}$). Investor i 's updated wealth is equal to his number of shares at time $t - 1$ multiplied by

the current share price ($\omega_{it} = \gamma_{it-1}p_t$).

In the third scenario investor i is out of the market and a buy signal ($S = 1$) is generated. Because of the buy signal investor i withdraws his money from the risk free account and buys risky assets. The number of shares (units) bought is equal to his wealth at time $t - 1$ divided by the current share price ($\gamma_{it} = \frac{\omega_{it-1}}{p_t}$). Investor i 's wealth at time t is equal to his number of shares multiplied by the current price ($\omega_{it} = \gamma_{it}p_t$). Excess demand is positive and equal to investor i 's number of shares at time t , ($b_{it} = \gamma_{it}$).

Investor i is in the market and the algorithm generates a buy signal in the fourth and last situation. Since he is already in the market, no transactions will be carried out, and consequently there will be no excess demand at time t ($b_{it} = 0$). Investor i 's number of units of the risky asset is not changed during the process ($\gamma_{it} = \gamma_{it-1}$), and his updated wealth is equal to number of shares at time t multiplied by the current share price ($\gamma_{it-1}p_t$).

6.6 Summary

In this section we introduce and derive an extended version of the Day and Huang model. Our attention is directed against the β -investors (algo-traders), which are the only participants changed from the original model. The algo-traders are heterogeneous, and their investment strategies are based on different technical trading rules. Moreover, these investors are socially integrated and investor i has the opportunity to obtain information about investor j 's wealth, stress indicator and trading rule. The stress indicator reflects the number of losses the investor has experienced, and it is the level of stress that might trigger a change in his trading rule. A risk free and interest bearing account is introduced as an alternative to holding shares.

If investor i exceeds his *tolerance level* with respect to stress, then he uses the stress levels and wealth positions of selected neighbors to identify an algorithm that is superior to his own. The algorithm is superior only if the investor using the algorithm does better both in terms of wealth and stress. On the other hand, if no such cases exist (or he is below his tolerance level), then investor i will not change the algorithm he currently applies.

7 Simulations

In this section we run simulations of the extended model and derive the results. But before we are able to do simulations, we have to outline the initial conditions and the different properties of technical trading rules (algorithms) applied by the investors. The algorithms are basically based on moving averages of dissimilar lengths, which are divided

into three types: simple MA, filtered MA and double MA, similar to the collective term of generalized moving average (GMA) in Strassburg et al. (2012, p. 1308).

The simulations are based on two numerical experiments. In the first one, the investors use dissimilar versions of the simple MA only, which makes it easier to describe the dynamics of the market. It also give us a chance to control for possible defects of the model, or errors in the programming (before the model becomes too complicated). In the second experiment, dissimilar versions of the GMA are applied by the investors. In addition to describe the dynamics of the market, we also evaluate some wealth condition of the population.

7.1 Initial conditions

Starting values Let's consider the starting values for each element of the investor population matrix at the initial state ipm_0 . Each individual investor holds amount ω in an interest bearing account, i.e. $\gamma = 0$, and ω is set equal to 100 in the numerical experiment below. Understandably, the excess demand b and the stress level s both equals zero at the initial point. A technical trading rule is randomly drawn from the set of rules $(1, 2, 3, \dots, A)$ with replacement, and assigned to each investor in the ipm_0 . The last element of the matrix, which is the signal from the last period sl , is set equal to zero.

The initial price history Technical trading rules are based on past prices. For instant, to be able to calculate a moving average of five days, at least five historical prices are required. In other words we need an *initial price history* in order to implement the different rules. Such price histories might be constructed differently and we use two dissimilar methods below. In method (1) the initial price is chosen from the range $(0, 1)$, and during the first n periods α -investors are the only participants trading in the market. This means that prices during the first n periods, except from the initial price, are determined by the sophisticated investors. At the end of this initial price history, the algo-traders are no longer excluded from the market. In method (2) the initial price history consists of randomly chosen prices only. To make this alternative more realistic, the prices might be chosen from a relatively small range.

7.2 Technical trading rules (algorithms)

The three different GMA described here is consistent with Strassburg et al., and they are based on the same ideas as the golden cross described in Section Investment selections and market mechanism. Let's first formulate the moving average, which is equal to

$$MA_t(\theta) = \frac{1}{\theta} \sum_{i=0}^{\theta-1} p_{t-i}, \quad t = \theta, \theta + 1, \dots, N \quad (7.1)$$

where θ defines the order over the MA. p_t is the price that is known (made public) at time t . The binary signal S is given by

$$S_t(\theta_1, \theta_2, \theta_3) = MA_t(\theta_1) - (1 + (1 - 2S_{t-1})\theta_3)MA_t(\theta_2) \quad (7.2)$$

where θ_1 is defined as the short period, θ_2 is the long period, and θ_3 is the filter parameter. S_{t-1} represents the binary signal generated by the algorithm last period, which is denoted by sl in the population matrix.

7.2.1 Simple MA:

In the simple moving average case, $\theta_1 = 1$, $\theta_2 > 1$, $\theta_3 = 0$, which gives

$$S_t = p_t - MA_t(\theta_2). \quad (7.3)$$

where $p_t = MA_t(1)$ since $\theta_1 = 1$. The rule generates a buy signal (1) if $p_t > MA_t(\theta_2)$, and a sell signal (0) if $p_t \leq MA_t(\theta_2)$.

7.2.2 Filtered MA:

To avoid “false” or weak signals a filter is added to the rule, making the rule more conservative. Basically, this means that the rule only generates a signal if the short and the long moving MA differs significantly. In this case, $\theta_1 \geq 1$, $\theta_2 > \theta_1$, $\theta_3 > 0$, and for example if $\theta_1 = 1$, and $\theta_2 = 2$, then

$$S_t = p_t - (1 + (1 - 2S_{t-1})\theta_3)MA_t(2). \quad (7.4)$$

If the last signal applied by the investor indicated sell ($S_{t-1} = 0$), meaning that investor i 's is out of the market, then

$$S_t = p_t - (1 + \theta_3)MA_t(2).$$

In the opposite case, in which $S_{t-1} = 1$ and the investor is in the market, then

$$S_t = p_t - (1 - \theta_3)MA_t(2).$$

7.2.3 Double MA:

In contrast to the simple MA, θ_1 is always larger than 1 in the case of double MA. Further it is required that the order of the long MA is higher than the order the short MA, $\theta_2 > \theta_1$, and if $\theta_3 = 0$, then

$$S_t = MA_t(\theta_1) - MA_t(\theta_2). \quad (7.5)$$

It should be pointed out that both the simple and the double MA might be implemented with the filter, which is the case in some of the rules applied in the numerical experiment below.

7.3 Numerical experiment based on simple MA

In the first numerical experiment we simulate a market in which algo-traders only use simple moving averages of different length of, i.e. $\theta_2 = [2, \dots, 11]$. Consistent with the initial conditions described earlier, each algo-trader is assigned to one algorithm, arbitrarily selected from the set of different 10 rules. Since the rules are arbitrarily chosen, situations in which not all of the 10 rules are represented in the initial state might occur. The rules differs in the sense that the long MA varies, but the short MA equals to p_t for each algorithm. Based on the parameter values;

$$a = 0.2 \quad \psi = 0.6 \quad \lambda = 1 \quad u = 0.5 \quad I = 50 \quad nn = 2 \quad r = 0.001 \quad \omega_0 = 100, \quad (7.6)$$

we are able to generate different price series.

At first, in section 7.3.1 and 7.3.2, the *initial price history* is constructed using method (1), and it consists of 15 periods ($n = 15$). The initial price p_0 is set within the range (0.05, 0.5) in Figure 7.1a and within (0.5, 0.95) in Figure 7.1b. If $p_0 < 0.05$ or $p_0 > 0.95$, then the model might explode, meaning that the price is below 0 or above 1 at some point. As in the original Day and Huang model, the price is stable and in full equilibrium at 0.5 in the long run, if $p_0 = 0.5$. Then, in Section 7.3.3, we use method (2) to construct the initial price history, and the interval is set to (0.4, 0.6)

The communication process taking place in the different price series is described and discussed in a separate section at the end.

Since there is random elements (generally two in the communication process and assign algorithms) in the model, we run it for a number of times, and the price series presented in this section are typically in the sense that these graphs are similar to what were generated in most of the cases. This doesn't mean that price series of another features can not appear.

a_{it}	θ_1	θ_2	θ_3
1	5	10	0
2	2	7	0.1
3	1	2	0.15
4	2	3	0.1
5	2	4	0.1
6	2	4	0.15
7	2	8	0
8	3	5	0.5
9	5	10	0.1
10	2	6	0.1

Table 7.1: Number and their characteristics to algorithm rules

7.3.1 Low initial price

During the initial price history in Figure 7.1a, the aggregated excess demand is positive and the price converges to α -investors' investment value u . At the end of period 15, the algo-traders are no longer excluded from model, and right away the price rises dramatically. This increase occurs because all the simple MAs generate a buy signal at this point, which again can be explained by the price pattern during the initial price history. Based on the characteristics of the these rules, it is quite obvious that all of them produce buy signals since the price has increased for 15 periods in a row. At the end of period 16, when the price is relatively high, α -investors' excess supply exceeds algo-traders' excess demand and the market maker adjusts the price downwards for the next period. The price decreases for a couple of periods, and investors being in the market suffer losses. These losses trigger the algo-traders to start *looking* for alternatives, and if one (or more) superior alternative is found, then the rule he currently applies is substituted.

In the long run price behavior, the prices typically fluctuate in a 4- or 6-cycle, dependent on the communication process and on how many rules that still exist. In most of the cases only the first and the second rule, $MA(\theta_2 = 2)$ and $MA(\theta_2 = 3)$, survive in the long run.

7.3.2 High initial price

At first in Figure 7.1b, when α -investors are operating only, the (aggregated) excess demand is negative and the price declines. Subsequent to the initial price history, the algo-traders starts to participate in the model, but they stay out of the market (i.e. hold interest bearing account). Since the price decreased monotonically during the initial price

history, all the algorithms generate sell signals, and therefore the algo-traders stay out of the market.

The price drops until it converges to 0.5 in the long run. Since all of the algo-traders are out of the market, their stress levels equal zero, and none of them are motivated to substitute the rule currently used. From this it follows that each algo-trader applies the same rule in the long run as in initial state.

7.3.3 Random initial price history

Figure 7.1 illustrates dissimilar price series that might be generated when we use random initial price history. In other words, the model doesn't provide one typical price series, but there are some characteristic similarities between them.

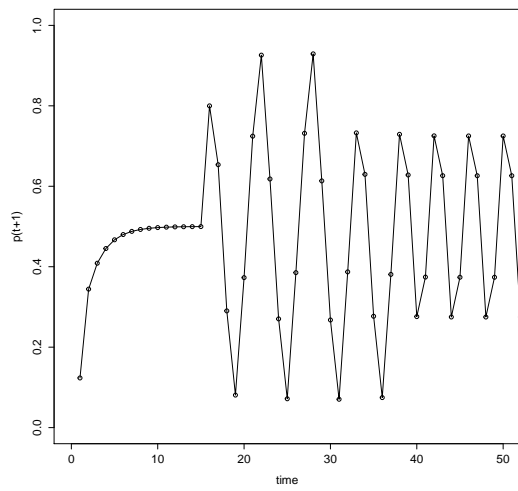
In Figure 7.1c - 7.1e, the price rises dramatically after the initial price history. This increase occurs because the demand is high relative to the supply, which is zero at this point. In fact, the supply will always be equal to zero at the end of period 15 since no investors are in the market (they are left out from the model during the initial price history). This means that even the slightest demand will cause an excess demand and an increase in the price.

In the long run, the prices fluctuate in 4-, 6- and 8-cycles in Figure 7.1c, 7.1d and 7.1e. The numbers of rules applied by the investors decreases during these time series, and typically in the long run only one, two or three different rules are used. In the general, rules with low θ_2 , meaning they are based on relatively short MAs, perform better than the other rules and are therefore frequently adopted by the other investors.

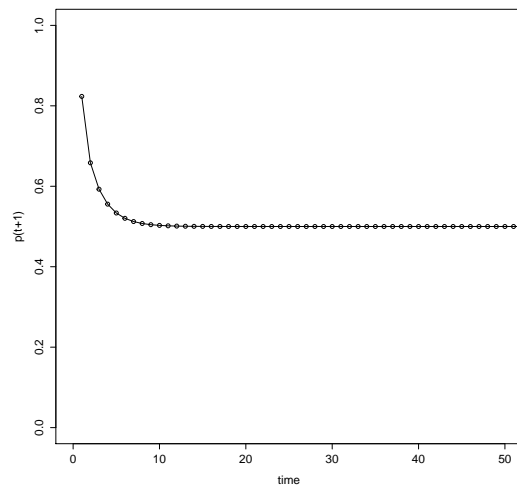
In figure 7.1f the price converges to 0.5 in the long run. The algo-traders are never in the market in this situation, and they never substitute the rule they are currently applying.

7.3.4 The social interaction process

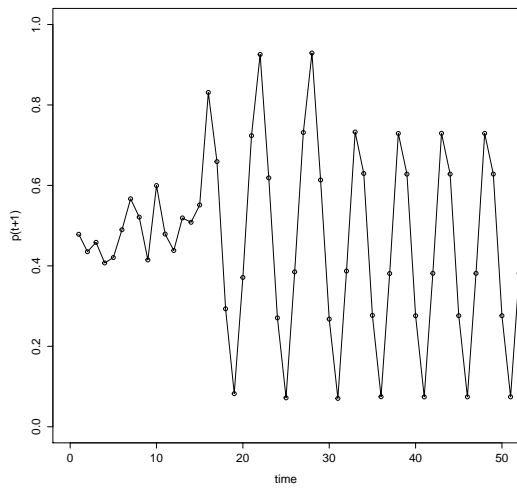
We have up til now describe the dynamics of how the price change, but left out the interaction between the algo-traders. The quantity that investor i can communicate with is denoted by the parameter nn . If the number of neighbors nn is set relatively high to the total number of investors, I , this will in general increase the "speed" of all these related processes. The outline of which rule the population end up using goes relatively faster than previously.



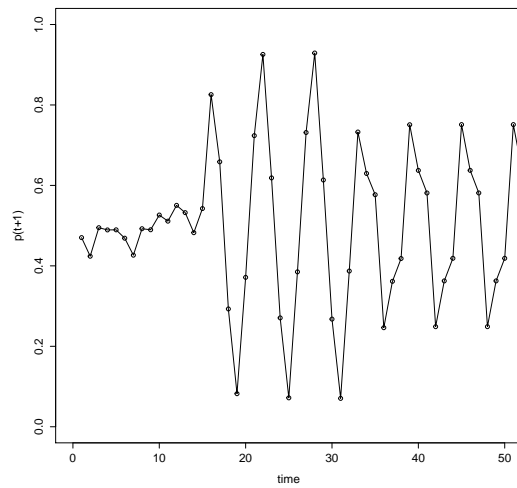
(a) Price series where the initial is low



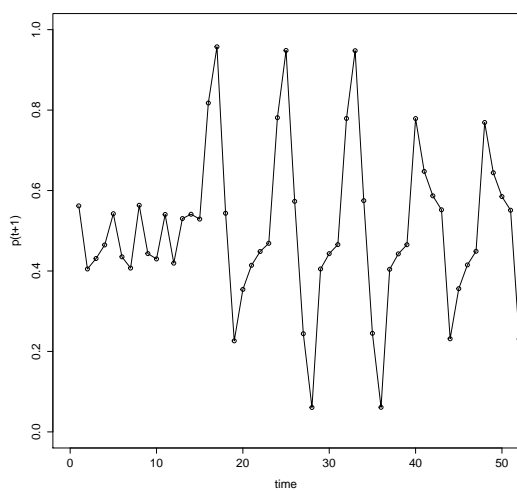
(b) Price series where initial is high



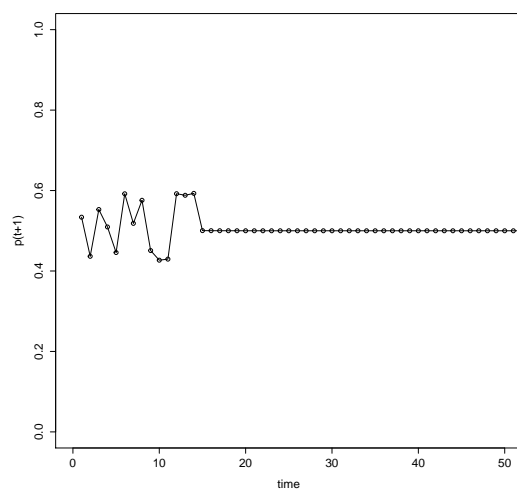
(c) $\theta_2 = 2$, 4-cycle



(d) $\theta \in \{3, 4\}$, 6-cycle



(e) $\theta = 5$, 8-cycle



(f) $\theta_2 \in \{2, \dots, 8\}$, one price

Figure 7.1: Numerical experiment of simple MA in different condition

7.4 Numerical experiment based on generalized MA

In this section we implement the remaining algorithms, meaning that all three types of GMA rules that might be used by the algo-traders. We set up two numerical experiments with different parameter values and 10 dissimilar rules. In each case, method (2) with a range of (0.4 0.6), is used to generate the initial price history. Based on these initial conditions, the simulation of each experiment is carried out. We run the simulations for several times, and the results seems to be quite different from time to time.

For each experiment we plot at least one price series, and make a corresponding plot of the (arithmetic) mean wealth at time t . In a sense, this mean-wealth parameter is a performance measure, which focuses on the population instead of each algorithm.

7.4.1 Experiment 1

In this experiment the parameter values are set equal to

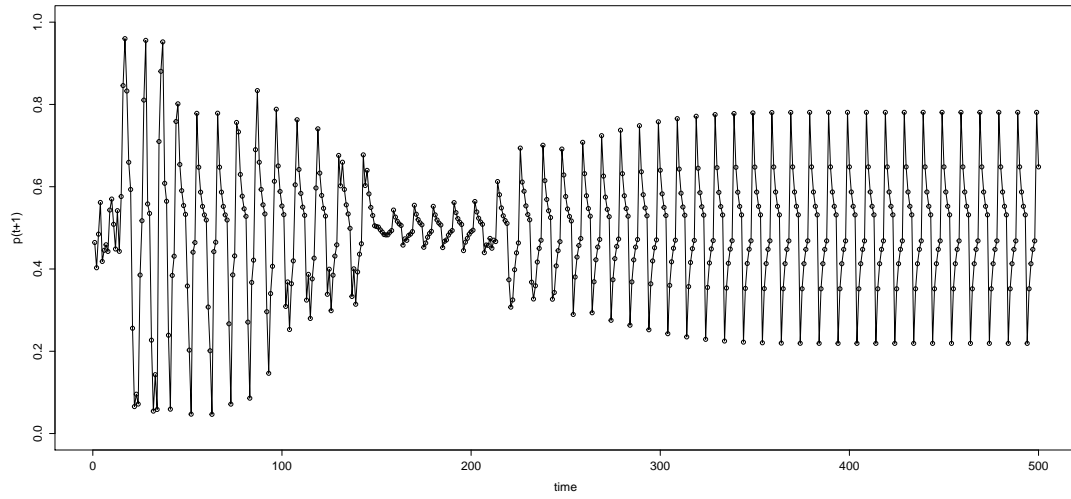
$$a = 0.2 \quad \psi = 0.6 \quad \lambda = 0.008 \quad u = 0.5 \quad I = 50 \quad nn = 2 \quad r = 0.001 \quad \omega_0 = 100, \quad (7.7)$$

and Table 7.1 describes the characteristics of the different algorithms.

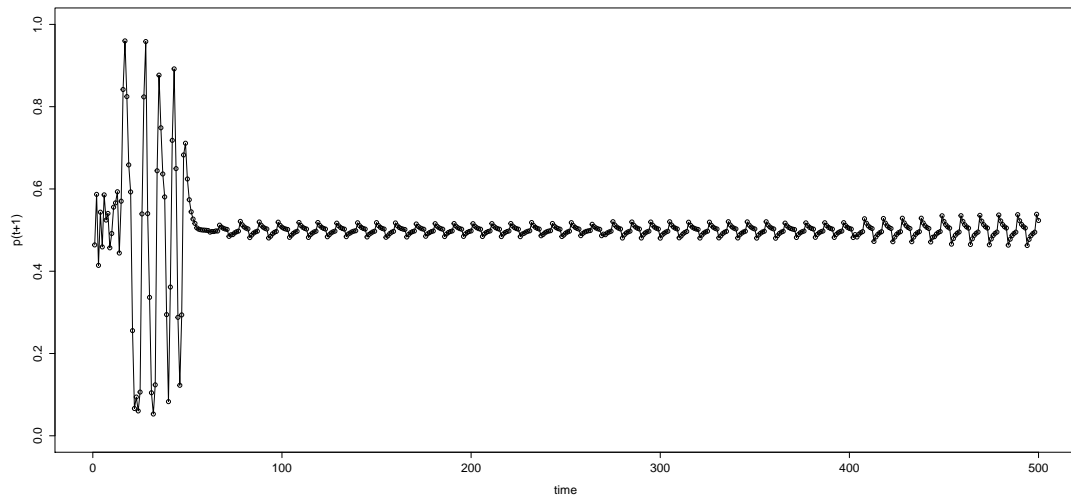
If one compares Figure 7.2a and 7.2b, then it is easy to see how different simulations based on the same initial conditions might generate totally dissimilar price series. Let's first focus on the price dynamics in Figure 7.2a. The plot shows large changes in the price volatility during the time horizon. At the beginning (beyond the initial price history), the prices are highly volatile and they fluctuate close to 0 and 1 at some points. Then, the volatility decreases, and the price fluctuates close to 0.5. The price behavior looks similar to cycles in this situation, but they are not stable. At some point, the price volatility increases again, and in the long run the prices fluctuate in a stable 10 cycle¹⁴.

The corresponding wealth-plot in Figure 7.3a, illustrates that the mean wealth decreases for a long time. At some point, the mean wealth converts close to zero, and subsequently it starts to increase. Comparing the wealth-plot to the price series, one can see that the wealth starts to increase approximately at same time as the price fluctuations turn into a stable cycle ($t \approx 300$).

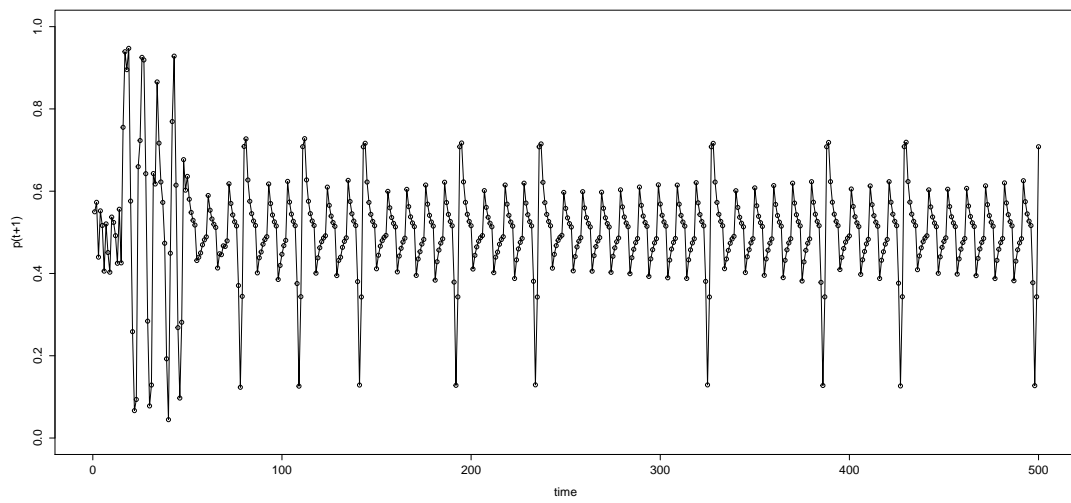
¹⁴See Appendix of the extended model and one can see a clear 10 cycle in the prices e.g. in period 489 and 499 consisting of the highest price in the amplitude.



(a) Prices series generated of experiment 1a

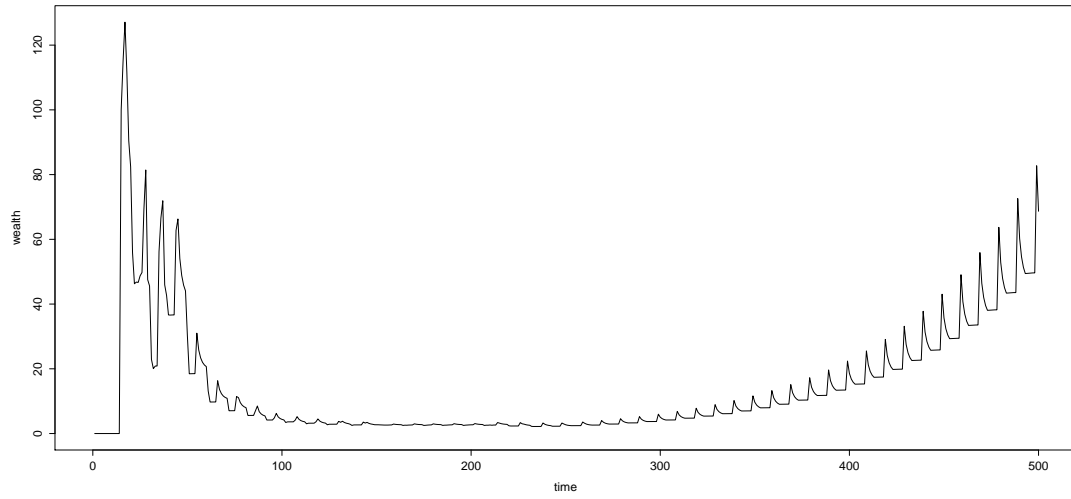


(b) Prices series generated of experiment 1b

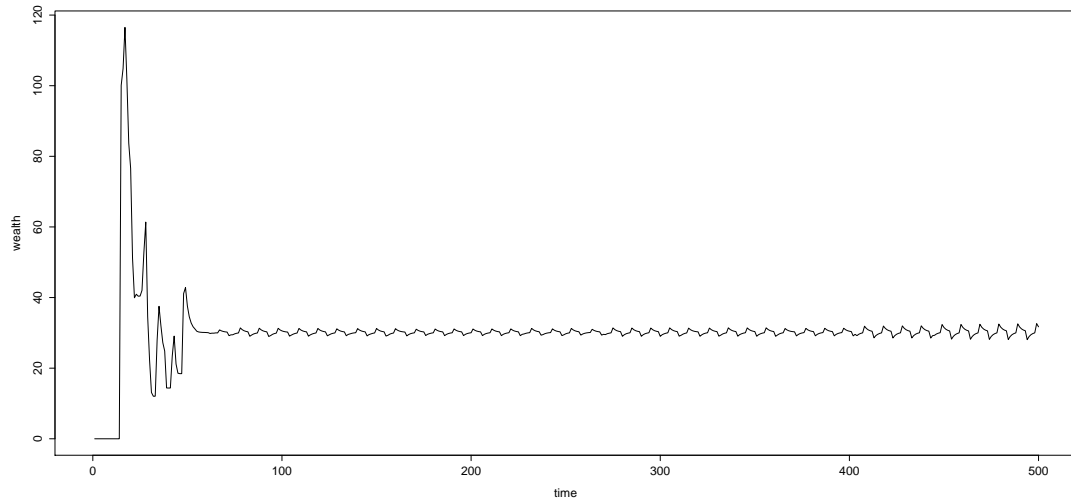


(c) Prices series generated of experiment 2

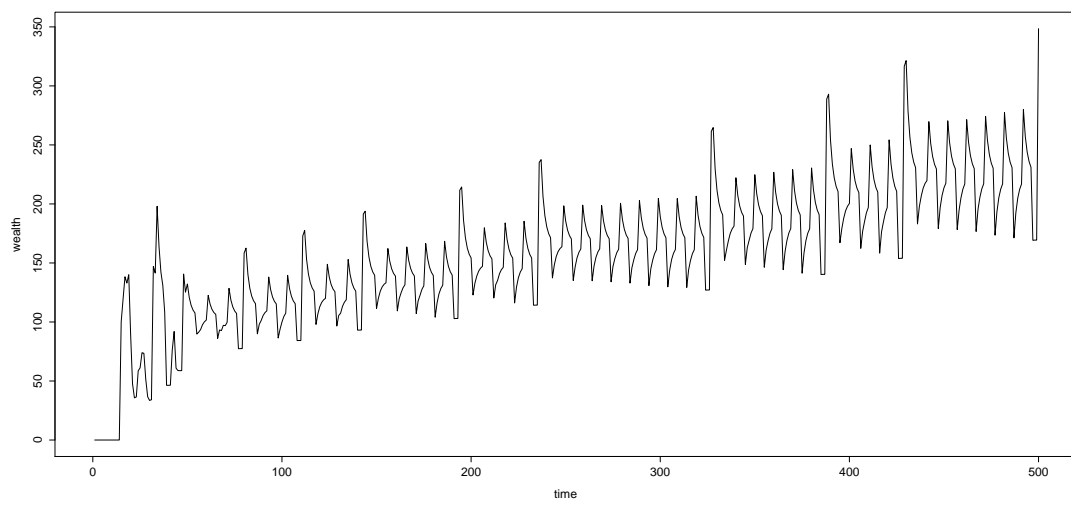
Figure 7.2: Price series of the tree experiments generated by the extended model



(a) The mean wealth of ipm_t generated by experiment 1a



(b) The mean wealth of ipm_t generated by experiment 1b



(c) The mean wealth of ipm_t generated by experiment 2

Figure 7.3: Mean wealth of the tree experiments generated by the extended model

Table 7.2 illustrates that every algo-trader use the same rule (number 7). Since the mean wealth increases in the long run, and rule number 7 is the only one used, it is implied that this algorithm has to generate positive return.

Rule	θ_1	θ_2	θ_3	Number of investors
7	2	8	0	50

Table 7.2: Experiment long run behavior $t = 500$

Looking at the other price series (Figure 7.2b), which is based on the same numerical experiment, the price dynamics is totally different. Except from the beginning, the volatility is very low, and the prices fluctuates close to 0.5. The prices seem to fluctuate in a stable cycle for a long time, but studying the price series more carefully, one could see that this is the not case. The price series contains 500 observations only, and stable cycles might occur in an extended time horizon.

In the wealth-plot in Figure 7.3b, a similar trend in the long run behavior as in the associated time series can be seen. At the beginning, the wealth decreases dramatically, but in the long run it is almost stable and only small changes occurs.

Table 7.3 shows the five different algorithms that “survive” in the long run. Note that four out of five rules are implemented with filter, which might partly explain the low volatility in the price series (Figure 7.2b).

Rule	θ_1	θ_2	θ_3	Number of investors
2	2	7	0.1	1
3	1	2	0.15	34
5	2	4	0.1	4
6	2	4	0.15	4
7	2	8	0	7

Table 7.3: Experiment 2b long run behavior $t = 500$

7.4.2 Experiment 2

Rule	θ_1	θ_2	θ_3	Number of investors
3	1	2	0.15	61
7	2	8	0	39

Table 7.4: Experiment 3 long run behavior $t = 500$

In this experiment the number of investors are set equal to 100, and one of the algorithms is marginally changed to:

a_{it}	θ_1	θ_2	θ_3
5	3	4	0.1

In the price series in Figure 7.2c it seems to be some kind of a pattern in the sense that similar spikes appear over and over again. These spikes occur at random intervals, and looking more carefully, one can see that they are unequal. At the beginning of the series, the prices are highly volatile, which is similar to what we observed in the two simulations in Experiment 1. After 50 periods approximately, the volatility seems to decrease, and except from the characteristic spikes the price fluctuates close to 0.5.

Figure 7.3c shows that the mean wealth increases considerably during the time horizon. Comparing this plot to the price series, similar spikes in the wealth-plot can be observed, and they occur at the same time interval. This indicates that at the algorithms applied by the investors are able to profit from the spikes moving upwards. Two different algorithms are used in the long run (see Table 7.4).

8 Conclusion and future research

The objective of this thesis was to develop an extended version of the Day and Huang model with heterogeneous and socially integrated investors. The investors use different algorithmic trading rules to operate in this stylized nonlinear model.

In the first part we investigated carefully the original Day and Huang model and its dynamics. Our main concern was the β -investors, and their relative importance in the market, which is denoted by the flocking coefficient b . A bifurcation diagram was made to see how different values of the flocking coefficient generate different price series. We found that the prices were predictable for some values of b , but for other values, fluctuations of more-or-less chaotic type occur. From the bifurcation we concluded that a *higher flocking coefficient not always is consistent with a more complex market*, and in some cases a *higher coefficient led to less complexity*.

In the next part, we developed an extended version of the Day and Huang model. The β -investors from the original model was replaced by algo-traders, but the α -investors and the market maker were not changed. The algo-traders are heterogeneous and their investment strategies are based on different technical trading rules, as we revealed in section 3.2. Further, these investors are socially integrated, and investor i is able to obtain information about investor j 's wealth, stress indicator and trading rule. If investor i , exceeds his tolerance level with respect to stress and he identifies an algorithm that is superior to his own, then the algorithm he currently applies is substituted.

When the fundamentals of the model were programmed, we started to simulate the model based on a numerical experiment. At first, the trading rules only consisted of simple moving averages. Keeping the rules simple, we were able to check the programming for errors and understanding the basics of the model. Then based on dissimilar initial conditions and the numerical experiment, we plotted three different price series to test the logic of the social integration process and to outline the price behavior. For each simulation, the long-run price was either in full equilibrium ($p = 0$), or it fluctuated in a stable cycle, typically a 2- 4- or 6-cycle. The number of different algorithms applied in the long run, was typically one, two or three. Rules with a long MA of a low order (few periods) tended to perform better than the other rules, in the sense that they survived in the long run. Looking at the price series, one could see frequently switches between bull and bear. Based on this price behavior it is quite logical that the long MA of low order, performs well, which indicates that the social integration process seems to work.

Further, dissimilar versions of the generalized MA rule were implemented. We generated different price series and plots of the mean wealth based on two numerical experiments.

Our main conclusion is that *the unsophisticated investors does not destabilize the market*. This is in line with the result from Suhadolnik et al. (2010). The intuition that more unsophisticated investors increase complexity to the market can not say to hold. Although, in our price series the short run price behavior shows more complexity, but in the long run, cycles occur and it is less complex.

Figure 7.2c, illustrates the price series fluctuates in a 10-cycle in the long run as the algo-traders and the sophisticated α -investors are trading in the market. This seems consistent for some levels of the flocking coefficient in the bifurcation diagram generated previously.

8.1 Future research

The current version of the extended model provides several starting points for future improvements, such as: introducing a more complex communication process, including more algorithmic rules, reconstructing the investor population and introducing a high-frequency trading perspective.

In the communication process in our model, it is assumed that the true information always will be revealed. This may not always be the case, and someone can for example lie or misunderstand. In In future versions of the model one might consider to encompass a more complex communication process.

It is said to exist hundreds of different technical trading rules, and by implementing additional rules, one can increase the complexity of this stylized model. In the current version of the model it assumed that if a rule doesn't "survive" the communication process,

then it will never come back. From some point of view this assumption might look unrealistic, and it would be interesting to develop a system in which such rules can back and into the market again. This system could be linked to the communication process, and it would be a way of bringing in rules endogenously.

It seems reasonable that not every investor are equipped with the same level of wealth, and that they can tolerate different level of stress. For example one could use some kind of distribution, giving investors unlike wealth at the initial state.

The perspective of high frequency trading could be encountered by providing α -traders to only trade in intervals of each hundred or thousand period and let the algo-traders trade in between. Lastly, in a smaller perspective one could remove the ban from short selling, and also investigate the scaling function more carefully.

A Appendix of Day and Huang model

A.1 Price adjustment function in the Day and Huang model

Derivation of the $\theta(\cdot)$

$$\begin{aligned} p_{t+1} = \theta(p) &= p + cE(p) \\ &= p + c[\alpha(p) + \beta(p)] \\ &= p + c[a(u-p)f(p) + b(p-v)], \quad p \in [m, M] \end{aligned} \tag{A.1}$$

First order condition w.r.t. p

$$\begin{aligned} \theta'(p) &= 1 + c[a(u-p)f'(p) + af(p) + b] \\ &= 1 + c[\alpha'(p) + b] \end{aligned} \tag{A.2}$$

where $\alpha'(p) = a(u-p)f'(p) + af(p)$, then from equation (4.4) we can write $E'(p) = \alpha'(p) + b$ when $\bar{p} = v = u$

$$\theta'(\bar{p}) = 1 + c[af(\bar{p}) + b] \tag{A.3}$$

if $c = 1$, then $\theta'(\bar{p}) = 1 + af(\bar{p}) + b$

A.2 Properties of the Day and Huang model

To derive some characterization properties of the model, some basic concept of dynamics systems theory.

Fixed points

The graphical map of the price adjustment function θ is said to have fixed points (steady states) \bar{p} , and are located in the intersection between the 45°-line and θ .

Hyperbolicity

Let \bar{p} denote a fixed point of θ . Then \bar{p} is said to be hyperbolic if

$$|\theta'(\bar{p})| \neq 1. \tag{A.4}$$

Attractor

If \bar{p} denotes a hyperbolic fixed point of θ and

$$|\theta'(\bar{p})| < 1 \tag{A.5}$$

is fulfilled, then \bar{p} is said to be an attracting fixed point (attractor) or asymptotically stable.

Repellor

If $\theta'(p)$ is not in the unit circle, meaning that

$$|\theta'(\bar{p})| > 1, \tag{A.6}$$

then \bar{p} is called a repelling fixed point (repellor).

Full and temporary equilibrium

Let current price be stationary at level p . Then the aggregated excess demand is

$$E(p) = \alpha(p) + \beta(p) = 0, \tag{A.7}$$

which implies that α -investors' excess demand equals β -investors' excess supply or vice versa [$\alpha(p) = \beta(p)$]. α -investors are only in full equilibrium if $\alpha(p) = 0$, which is true if $p = u$. Similarly, β -investors are only in full equilibrium if $\beta(p) = 0$, which is true if $p = v$. From this it follows that the market is in full equilibrium (and unique) if

$$\bar{p} = u = v. \tag{A.8}$$

If only equation (A.7) is fulfilled and equation (A.8) doesn't hold, then the equilibrium price is only temporary.

Stability of full equilibrium

To obtain local stability of full equilibrium it is required that equation (A.5) holds, which implies that $\theta'(\bar{p})$ lies within the unit circle $-1 < \theta'(\bar{p}) < 1$. This can be rewritten as

$$-2 < c[\alpha'(\bar{p}) - b] < 0. \tag{A.9}$$

If equation A.9 is fulfilled, then α -investors dominate the market and β -investors are more-or-less absent. There will be no fluctuations of chaotic type at this point.

Instability of full equilibrium

If equation (A.5) is violated and the \bar{p} is a repeller (see (A.2)), then the conditions of local stability of full equilibrium are violated and

$$c[\alpha'(\bar{p}) + b] < -2 \quad \text{or} \quad \alpha'(\bar{p}) + b > 0. \quad (\text{A.10})$$

For simplicity it is assumed that $c = 1$, and that a , b and $\alpha'(\bar{p})$ are all positive. If $\theta'(p) = 1 + \alpha'(\bar{p}) + b > 1$ holds, then one can be sure that full equilibrium is unstable.

In the non-hyperbolic situation

A fixed point is said to be non-hyperbolic if

$$|\theta'(\bar{p})| = 1.$$

In this situation $|1 + c[\alpha'(\bar{p}) + b]| = 1$. We can rewrite into

$$c[\alpha'(\bar{p}) + b] = -2 \quad \text{or} \quad \alpha'(\bar{p}) + b = 0,$$

if $c > 0$ then $\alpha'(\bar{p} = v = u) + b = 0$.

A.3 R codes

We revile the basic code that we constructed with the Day and Huang model.

```
chance <- function(p){ m<-0; d1<-0.5; d2<-0.5; M<-1; eps<-0.008; c<-1; out<-0
if (prod(p<m-0.0001|p>M+0.0001)) {out<-0}
else {out<-(p-m+eps)^(-d1)*(M+eps-p)^(-d2)}
  out}
```

```
alpha <- function(p){a<-0.2; u<-0.50; m<-0; M<-1; z<-0
      if (prod(p<m-0.01|p>M+0.01)) {z<-0}
      else {z<-a*(u-p)*chance(p)}
      z}
```

```
beta <- function(p){b<-0.88; v<-0.50; z<-b*(p-v)}
```

```
dom<-0.01*1:99 ;c<-1.0
y<-alpha(dom); yy<-alpha(dom)+beta(dom)
p1<-dom+c*(alpha(dom)+beta(dom))
```

```

plot(dom,p1,type="l")

price <- function(init, len, c){p<-c(1:(len+1)); p[1]<-init;
  for (i in 1:len) {p[i+1]<-p[i]+c*(alpha(p[i])+beta(p[i]))}
  p}
z<-price(0.1234654,200,1.0); plot(z,type="l")

hist(z,seq(-0.1,1.1,0.01),prob=TRUE);lines(density(z))

```

B Appendix of the extended model

We have constructed a matrix of the experiment with correlated Figure 7.2c and 7.3c in order to see the evolution in the model and the population. ¹⁵ In order the columns are: period t , price p_t , mean wealth $\bar{\omega}_{it}$, the fraction of risky asset $1 - \frac{1}{I} \sum_{i=1}^I \gamma_{it}$, mean stress level \bar{s}_{it} , and aggregated excess demand (supply) $\beta(p_{it})$.

	[,1]	[,2]	[,3]	[,4]	[,5]
...
[15,]	0.57615014	100.068000	0.32	0.00	2777.0538958
[16,]	0.84582715	115.072178	0.74	0.00	2485.2595513
[17,]	0.95995470	127.093696	0.90	0.00	835.0402392
[18,]	0.83263069	111.576936	0.90	0.90	0.0000000
[19,]	0.65936487	90.457717	0.90	1.80	0.0000000
[20,]	0.59328711	82.409777	0.60	2.70	-2161.7266228
[21,]	0.25592454	55.890822	0.38	2.40	-1398.2773102
[22,]	0.06550080	46.270293	0.12	1.90	-1744.7665707
[23,]	0.09566739	46.793110	0.02	0.00	-475.6742065
[24,]	0.07167580	46.687233	0.02	0.02	0.0000000
[25,]	0.38531206	48.718341	0.10	0.00	73.4499216
[26,]	0.51734992	49.764857	0.64	0.00	2903.0014175
[27,]	0.81051525	69.074976	0.98	0.00	959.3388676
[28,]	0.95577078	81.429537	1.00	0.00	7.1892020
[29,]	0.55810855	47.549603	1.00	1.00	0.0000000
[30,]	0.53508002	45.587623	0.86	2.00	-578.5973857
[31,]	0.22703927	22.914062	0.32	2.58	-2846.8191484
[32,]	0.05446154	20.052962	0.12	1.28	-399.7634552

¹⁵Parts of the matrix have been taken out.

[33,]	0.14312720	20.843415	0.00	0.00	-434.7083954
[34,]	0.05852204	20.861087	0.16	0.00	2709.9615448
[35,]	0.70985087	56.175547	0.28	0.00	334.6004610
[36,]	0.88051642	66.568325	0.94	0.00	693.4141341
[37,]	0.95206437	71.917216	1.00	0.00	38.9331597
[38,]	0.60825630	45.946578	1.00	1.00	0.0000000
[39,]	0.56463369	42.651404	0.82	2.00	-2848.4230347
[40,]	0.23897886	36.636250	0.04	2.46	-908.3811757
...
[120,]	0.63312209	3.858014	1.00	1.00	0.0000000
[121,]	0.57881396	3.527080	1.00	2.00	0.0000000
[122,]	0.54740453	3.335683	1.00	3.00	0.0000000
[123,]	0.52865953	3.221458	0.10	4.00	-171.8202727
[124,]	0.33848851	2.717946	0.08	0.50	-5.0498708
[125,]	0.39949666	2.875750	0.00	0.00	-127.8115444
[126,]	0.29858810	2.878626	0.00	0.00	0.0000000
[127,]	0.38496314	2.881504	0.00	0.00	0.0000000
[128,]	0.43146113	2.884386	0.00	0.00	0.0000000
[129,]	0.45869394	2.885410	0.92	0.00	202.7619497
[130,]	0.67606056	3.767909	0.92	0.00	0.0000000
[131,]	0.60216550	3.468247	1.00	0.92	85.2192579
[132,]	0.65958194	3.798944	1.00	0.00	0.0000000
[133,]	0.59340432	3.417786	1.00	1.00	0.0000000
[134,]	0.55599315	3.202312	1.00	2.00	0.0000000
[135,]	0.53381346	3.074565	0.90	3.00	-18.0667177
[136,]	0.49882909	2.885902	0.24	3.60	-155.9524010
[137,]	0.33315760	2.510046	0.20	1.20	-1.8916491
[138,]	0.40043135	2.662597	0.00	0.00	-112.0704399
[139,]	0.31426131	2.665259	0.00	0.00	0.0000000
[140,]	0.39282653	2.667925	0.00	0.00	0.0000000
...
[206,]	0.50858548	2.787490	0.52	3.20	-55.4758392
[207,]	0.43970545	2.502921	0.50	2.60	-4.3920516
[208,]	0.45834253	2.579186	0.36	0.00	-14.4266621
[209,]	0.45750573	2.576889	0.36	0.36	0.0000000
[210,]	0.47429460	2.640951	0.28	0.00	-28.4679791
[211,]	0.45041314	2.565763	0.28	0.28	0.0000000
[212,]	0.47002920	2.629573	0.22	0.00	-12.6561117

[213,]	0.46667494	2.619841	0.76	0.22	118.8391521
[214,]	0.61257852	3.395929	1.00	0.00	11.2239505
[215,]	0.58058580	3.218572	1.00	1.00	0.0000000
[216,]	0.54845221	3.040434	1.00	2.00	0.0000000
[217,]	0.52928918	2.934201	1.00	3.00	0.0000000
[218,]	0.51773879	2.870169	1.00	4.00	0.0000000
[219,]	0.51075075	2.831430	0.46	5.00	-118.8391521
[220,]	0.37376207	2.398817	0.08	2.76	-103.7630914
[221,]	0.30724528	2.328197	0.00	0.56	-54.5808974
...
[485,]	0.35184334	43.461730	0.00	0.00	0.0000000
[486,]	0.41282380	43.505192	0.00	0.00	0.0000000
[487,]	0.44766194	43.548697	0.00	0.00	0.0000000
[488,]	0.46837772	43.548697	1.00	0.00	4648.8865058
[489,]	0.78085163	72.601812	1.00	0.00	0.0000000
[490,]	0.64815666	60.264135	1.00	1.00	0.0000000
[491,]	0.58717620	54.594310	1.00	2.00	0.0000000
[492,]	0.55233806	51.355139	1.00	3.00	0.0000000
[493,]	0.53162228	49.429033	0.00	4.00	-4648.8865058
[494,]	0.21914837	49.478462	0.00	0.00	0.0000000
[495,]	0.35184334	49.527941	0.00	0.00	0.0000000
[496,]	0.41282380	49.577469	0.00	0.00	0.0000000
[497,]	0.44766194	49.627046	0.00	0.00	0.0000000
[498,]	0.46837772	49.627046	1.00	0.00	5297.7590957
[499,]	0.78085163	82.735276	1.00	0.00	0.0000000
[500,]	0.64815666	68.675556	1.00	1.00	0.0000000

References

- Alexander, S. S. (1961). Price Movements in speculative Markets: Trends or Random Walks. *Industrial Management Review*, 2(2):7–26.
- Alexander, S. S. (1964). Price Movements in speculative Markets: Trends or Random Walks, number 2. *Industrial Management Review*, 5(2):25–46.
- Allen, F. and Karjalainen, R. (1999). Using genetic algorithms to find technical trading rules. *Journal of Financial Economics*, 51(2):245 – 271.
- Bachelier, L. (1900). Theorie de la speculation. doctoral dissertation. *Annales Scientifiques de l'Econle Normale Superieure*, 17(iii):21–86. Translation in: Cootner, P. (Ed.), *The Random Character of Stock Market Prices*, vol. 2, MIT Press, Cambridge, pp. 338-372.
- Bauer, R. J. (1994). *Genetic Algorithms and Investment Strategies*. Wiley Finance Editions. Wiley.
- Bauer, R. J. and Liepins, G. (1992). Genetic algorithms and computerized trading strategies. *Expert Systems in Finance*.
- Black, F. (1986). Noise. *The Journal of Finance*, 41(3):pp. 529–543.
- Bodie, Z., Kane, A., and Marcus, A. J. (2011). *Investments and Portfolio Management*. The McGraw-Hill/Irwin series in finance, insurance and real estate. McGraw-Hill Education.
- Brock, W., Lakonishok, J., and LeBaron, B. (1992). Simple technical trading rules and the stochastic properties of stock returns. *The Journal of Finance*, 47(5):pp. 1731–1764.
- CFTC and SEC (2010). Findings regarding the market events of may 6, 2010. Report by the U.S. Commodity Futures Trading Commission and U.S. Securities and Exchange Commission, Washington, DC.
- Cowell, F. A. (2006). *Microeconomics: Principles and Analysis*. Number 9780199267774 in OUP Catalogue. Oxford University Press.
- Cowles, A. (1933). Can stock market forecasters forecast? *Econometrica*, 1(3):pp. 309–324.
- Day, R. H. and Huang, W. (1990). Bulls, bears and market sheep. *Journal of Economic Behavior & Organization*, 14(3):299 – 329.

- Edmans, A., Garc a, D., and Norli, O. (2007). Sports sentiment and stock returns. *The Journal of Finance*, 62(4):1967–1998.
- Fama, E. F. (1965). Portfolio analysis in a stable paretian market. *Management Science*, 11(3):pp. 404–419.
- Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance*, 25(2):pp. 383–417.
- Fernandez-Rodriguez, F., Gonzalez-Martel, C., and Sosvilla-Rivero, S. (2000). On the profitability of technical trading rules based on artificial neural networks:: Evidence from the madrid stock market. *Economics Letters*, 69(1):89 – 94.
- Fernandez-Rodriguez, F., Gonzalez-Martel, C., and Sosvilla-Rivero, S. (2001). Optimisation of technical rules by genetic algorithms: Evidence from the madrid stock market. (2001-14).
- Fong, S., Si, Y.-W., and Tai, J. (2012). Trend following algorithms in automated derivatives market trading. *Expert Systems with Applications*, 39(13):11378 – 11390.
- Friedman, M. (1955). Leon walras and his economic system. *The American Economic Review*, pages 900–909.
- Furbush, D. (1989). Program trading and price movement: Evidence from the october 1987 market crash. *Financial Management*, 18(3):pp. 68–83.
- Gerber, C., Teich, J., Wallenius, H., and Wallenius, J. (2004). A simulation and test of optimark’s electronic matching algorithm and its simple variations for institutional block trading. *Decision Support Systems*, 36(3):235 – 245.
- Gleick, J. (1988). *Chaos: making a new science*. A Penguin book. Penguin.
- Goldberg, D. E. (1989). *Genetic algorithms in search, optimization, and machine learning*. Artificial Intelligence. Addison-Wesley.
- Gu, M. (1993). An empirical examination of the deterministic component in stock price volatility. *Journal of Economic Behavior & Organization*, 22(2):239 – 252.
- Gu, M. (1995). Market mediating behavior: An economic analysis of the security exchange specialists. *Journal of Economic Behavior & Organization*, 27(2):237 – 256.
- Hartl, R. F. (1989). On forward algorithms for a generalized wheat trading model. *Engineering Costs and Production Economics*, 15(0):367 – 370.

- Hirshleifer, D. and Shumway, T. (2003). Good day sunshine: Stock returns and the weather. *The Journal of Finance*, 58(3):1009–1032.
- Holland, J. H. (1962). Outline for a logical theory of adaptive systems. *J. ACM*, 9(3):297–314.
- Holland, J. H. (1975). *Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence*. University of Michigan Press.
- Hsu, Y.-T., Hung, H.-F., Yeh, J., and Liu, M.-C. (2009). Profit refiner of futures trading using clustering algorithm. *Expert Systems with Applications*, 36(3, Part 2):6192 – 6198.
- Huang, W. and Day, R. H. (1993). Chaotically switching bear and bull markets: The derivation of stock price distributions from behavioral rules. In *Nonlinear Dynamics and Evolutionary Economics (R. Day and P. Chen, eds.)*, pages 169–182.
- Huberman, G. and Regev, T. (2001). Contagious speculation and a cure for cancer: A nonevent that made stock prices soar. *The Journal of Finance*, 56(1):387–396.
- Izumi, K., Toriumi, F., and Matsui, H. (2009). Evaluation of automated-trading strategies using an artificial market. *Neurocomputing*, 72:3469 – 3476.
- Kaldor, N. and Tobin, J. (1985). *Economics without equilibrium*. University College Cardiff Press Cardiff.
- Kendall, M. G. (1953). The analysis of economic time-series-part i: Prices. *Journal of the Royal Statistical Society. Series A (General)*, 116(1):pp. 11–34.
- Keynes, J. M. (1936). *The general theory of employment, interest and money*. Harcourt Brace, New York.
- Kim, K. (2007). *Electronic and Algorithmic Trading Technology: The Complete Guide*. Complete Technology Guides for Financial Services. Elsevier.
- Li, T.-Y. and Yorke, J. A. (1975). Period three implies chaos. *The American Mathematical Monthly*, 82(10):pp. 985–992.
- Lin, X., Yang, Z., and Song, Y. (2011). Intelligent stock trading system based on improved technical analysis and echo state network. *Expert Systems with Applications*, 38(9):11347 – 11354.
- Lux, T. (1995). Herd behaviour, bubbles and crashes. *The Economic Journal*, 105(431):pp. 881–896.

- Lux, T. (1998). The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions. *Journal of Economic Behavior & Organization*, 33(2):143 – 165.
- Malkiel, B. (2011). *A Random Walk Down Wall Street: The Time-Tested Strategy for Successful Investing (Tenth Edition)*. W. W. Norton.
- Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4):pp. 394–419.
- May, R. M. et al. (1976). Simple mathematical models with very complicated dynamics. *Nature*, 261(5560):459–467.
- Michell, M. (1996). An introduction to genetic algorithms.
- Papadamou, S. and Stephanides, G. (2007). Improving technical trading systems by using a new matlab-based genetic algorithm procedure. *Mathematical and Computer Modelling*, 46:189 – 197.
- Pemy, M. (2012). Optimal algorithms for trading large positions. *Automatica*, 48(7):1353 – 1358.
- Samuelson, P. A. (1965). Proof that properly anticipated prices fluctuate randomly. *Industrial management review*, 6(2).
- Schmidt, G., Mohr, E., and Kersch, M. (2010). Experimental analysis of an online trading algorithm. *Electronic Notes in Discrete Mathematics*, 36(0):519 – 526.
- Schroeder, M. (2009). *Fractals, chaos, power laws: minutes from an infinite paradise*. Dover Books on Physics Series. Dover Publications, Incorporated.
- Shangkun, D., Yizhou, S., and Akito, S. (2012). Robustness test of genetic algorithm on generating rules for currency trading. *Procedia Computer Science*, 13(0):86 – 98.
- Shiller, R. J. (1988). Portfolio insurance and other investor fashions as factors in the 1987 stock market crash. *NBER Macroeconomics Annual*, 3:pp. 287–297.
- Strassburg, J., Gonzalez-Martel, C., and Alexandrov, V. (2012). Parallel genetic algorithms for stock market trading rules. *Procedia Computer Science*, 9(0):1306 – 1313.
- Suhadolnik, N., Galimberti, J., and Silva, S. D. (2010). Robot traders can prevent extreme events in complex stock markets. *Physica A: Statistical Mechanics and its Applications*, 389(22):5182 – 5192.

- Sullivan, R., Timmermann, A., and White, H. (1999). Data-snooping, technical trading rule performance, and the bootstrap. *The Journal of Finance*, 54(5):pp. 1647–1691.
- Thaler, R. H. (1987). Anomalies: The January effect. *The Journal of Economic Perspectives*, 1(1):pp. 197–201.
- Walras, L. and Jaffé, W. (1954). *Elements of Pure Economics, Or, The Theory of Social Wealth*. Elements of Pure Economics, Or the Theory of Social Wealth. Taylor & Francis. Translation by Jaffé, William.
- Wieland, C. and Westerhoff, F. H. (2005). Exchange rate dynamics, central bank interventions and chaos control methods. *Journal of Economic Behavior & Organization*, 58(1):117 – 132.