

Analysis-forecast of expected return and risk on an equity-linked note

----- Indeksobligasjon Nordea Norske Aksjer II 2009/2013

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This Master's Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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Abstract

The main topic for the article is analyzing return and risk of an equity-linked note. The analysis will concentrate on a specific product whose return is compounded by zero-coupon bond and return on a risky portfolio with 7 underlying stocks. First I represent the relative and necessary theories and models for taking Monte Carlo simulation to simulate underlying stock price paths and then compute the expected return on structured products; and then I will analyze the return-risk by taking different risk measures analysis for return on the specific product. Meanwhile the article will give the inspiration to the investors on how to choosing an optimal portfolio by presenting the performance evaluating with different portfolio constructions.

I appreciate that I got much help from Professor Valeri Zakamouline, for blazing the way in constructing this article for me. Although the research work is intrinsically to be a progress from unknown to known, I wish it can give an effective summery to the knowledge I got and proper end to my master degree studying.

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1. Introduction

1.1 Research problems and purpose

In this article I will analyze the return and risk on a specific equity-lined note which is chosen from Norwegian financial market. I estimate the return on the product by simulating the underlying equities' price paths. Then I analyze the performance evaluation of the product with different risk measures.

The analysis will mainly demonstrate those 4 problems about the return and risk on equity-lined notes:

- i. How to simulate the equity-lined note's expected return that is depending on return of correlated underlying equities?
- ii. Is the expected return of the prospect really meaningful for investors to estimate whether they should invest the product? Or they should evaluate it with considering risk together with return?
- iii. Has the assumption of theory that to simulate the return distribution and risk give any deviation from the practical simulation?
- iv. How is the performance of the specific product? Should

investors choose it?

With my analysis and comparison, it shows that my estimation has a considerable discrepancy from the issuer's forecast. Issuer's high expected return forecast may be too optimistic, and the mean of the expected return may not well represent the product's prospect. Because the distribution of the return shows that the mean expected return is allocated with low probability, while the probability of expected return that lower than the mean expected return, and even the probability of loss is overwhelming high.

The performance evaluation analysis shows that the "principal guaranteed" product is only a narrow meaning that when the costs do not considered. If we let "the same risk, the highest return" to be the criteria to evaluate the performance of financial product, the product I analyze here is no advantage at all can be weighted, especially when it's invested by taking loan.

1.2 Development of structured products

Structured product is using the financial engineering method to compound one or more basic financial assets and derivatives into a pack, and construct them into a new financial product.

The most accepted earliest structured product is said to be the

Standard&Poor's 500 indexed note (SPIN) issued by Salomon Brothers in 1986. So the history of structured product is not very long, but develops very fast. Along the development, it plays a very important role in the financial derivative market for personal investment in large part of the world now. And it can be traded in American Stock Exchange (Amex), London Stock Exchange (LSE) and Hong Kong Exchanges and Clearing Limited, etc.

An advantage of structured product is so called principal guarantee, it satisfies the different demands of investors with different degree of risk preference. Most standard derivatives normally require huge transaction volume which leads to a high cost, and return and risk of standard derivatives are not that explicit which need investors have some financial knowledge to manage them. Therefore, structured products meet the need of investors who are not professional, with low capital and afraid of losing money, but willing to participate in stock market and investors who want to increase the participation rate on specific stocks, but can't directly. Additionally, they are also widely applied for hedging risk.

Structured products are very popular in Europe. Now in Asia financial market, for example China, Hongkong, and Japan, also has potential market with huge turnover of structured products. But in USA, it seems that it's not that widely traded although the market is very potential, a

reason for this might be the government financial regulating.¹

Structured products can be designed to link with many different financial assets, for example interest rate, equity, exchange rate, commodities and even credit. But products that are linked with first three kinds of assets I listed are most popular and widely issued. Hereinto, products that are linked with equity are called equity-linked notes (ELN), and these kind of structured products are the product I'm going to discuss.

¹ The point of view is from Thorsten Hens and Marc Oliver Rieger(2008)

2. Product description

2.1 brief introduction of equity-linked notes

In general, a combination of a bond and one or more equity options is called ELN. The underlying equities of ELN can be stock prices, index prices, and the return of the stock or index, and the return of ELN are decided by the change of return on underlying equities.

In international markets, ELNs can be divided into Principal Guaranteed Notes (PGN) and High Yield Notes (HYN) basing on the different structures. Using bond and option as example, PGN is constructed as buying in bond and option, while HYN is constructed as buying in bond, but selling the option. HYNs are not principal guaranteed, but investors can get option price as profit when value of option exercising is too low or the option price at maturity is out of the exercise price; meanwhile, this gain a intrinsic limitation of the excess return.

So according the structure of the ELNs, the return of ELNs comes from two parts: one is fixed return on bonds or deposits which gives capital protection, and can be represented as principal plus interest; another part is the uncertain return affected by the return on underlying assets. In other words, investors gain from price or return increase of underlying assets, but losses are very limited. That is what those individual investors, for

example, the customers in the news we mentioned before are expecting and looking for. But actually, some times and somehow, issuers and even investors are willing to put more interest and focus on how good the loss control and principal guarantee is, but are blind to the possible fact of risk the products have. The reason for the interest departure might be the asymmetric information between the issuers and investors: issuers always have deeper, faster and more covered information than individual investors.

In Norway financial market, the most common and familiar structured products, or ELNs are known with names aksjeindeksobligasjoner(AIO), banksparing med aksjeavkastning(BMA) and warrants. They are compounded by two parts: one part invested for gain, the other part is saving for principal guarantee. We can find the detail information and prospect analysis of these products in all the main big banks' websites.²

2.2 Necessary information of Indeksobligasjon Nordea Norske Aksjer II 2009/2013

With investing in Indeksobligasjon Nordea Norske Aksjer II 2009/2013, the return is depend on the return of a basket of 7 biggest companies which are listed on Oslo stock market. The return of the product is

² See https://www.dnbnor.no/markets/investeringsprodukter/bma_io/
<http://www.nordea.no/Privat/Sparing+og+investering/Aksjer+og+andre+verdipapirer/Mer+fakta/401564.html>

measured by compare the price of every stock in the basket at the maturity with the starting price. All stocks have the same weight in the basket, which is 14.285%. The 7 underlying stocks is as table below:

<i>number</i>	<i>weight</i>	<i>Stock name</i>	<i>title</i>
1	1/7	StatoilHydro ASA	STL:NO
2	1/7	Telenor ASA	TEL:NO
3	1/7	Orkla ASA	ORK:NO
4	1/7	DnB NOR ASA	DNBNOR:NO
5	1/7	Yara International ASA	YAR:NO
6	1/7	Norsk Hydro ASA	NHY:NO
7	1/7	Renewable Energy Corp ASA	REC:NO

Table1: product information

But there is a limitation of the return on each underlying stocks. If one or more stocks' prices increase more than 80% at the end of the exercise date(mature date), the price increase will be set limited as 80%, this can be seen as a strike price in the option. And if the price of the stock basket at the end of the holding period is not higher than the starting price, investors are still be guaranteed to get 100% of net invest amount being pay back at the mature date, but drawing costs are exclusive.

The formula to compute the additional payout to the principal of the

product, which is presented by the issuer and can be found in the prospect brochure of the product on Nordea's website:

$$T = GL \times AF \times \max \left\{ 0; \sum_{i=1}^7 w_i \times \min \left\{ 80\%; \left(\frac{Aksje_i^{forfall} - Aksje_i^{start}}{Aksje_i^{start}} \right) \right\} \right\} \quad (2.1)$$

Where GL is the face value of the investment amount without the commission and other management costs, or we can say the principal. If the average-weighted sum of the returns of the underlying stocks is negative, the product owner can get back 100% of GL.

The product has "Asian tale" on underlying stocks at maturity. Therefore prices at maturity of stocks in the basket of the product INNA II are computed as an average of the last 7 months' observations.

The return participation rate of the product is set as 92%. It's determined by many factors, such as interest level, volatility and the life of the note. And the return of the product can be computed by participation rate multiplying with potential value increase from the underlying assets. For example, if return participation rate is 100% and value increase rate is 1%, the return of the product will be 1%.

And taking a good look at the underlying part of the equation (3.1), we'll find that it has the same structure of the payout as options, so it's the option element of the product return.

The cost of drawing the product is as below:

Tegningsomkostninger:

NOK 10.000 - 990.000 = 3,00 %

NOK 1.000.000 - 4.990.000 = 2,00 %

NOK 5.000.000 eller høyere = 0,50 %

3. Theory present

From the equation (2.1) we can see that to estimate the future price of the underlying stocks is the key point to estimate the future return on the product. So we need to see how stock price is developing, in this section, I'll discuss the theory of stock price development.

To model the stock price, first we need to assume that in efficient market, the stock price is generated by all the official available information, include all the historical price information; if there is any new information, the stock price has equally possibility to go up or down, as we model the random walk. Then, the movement of the stock price today is independent with historic movements.

3.1 From Black-scholes assumption to geometric Brownian motion

According the Black-scholes assumption, the price of the asset (stock) follows a process which can be write as:

$$\frac{dS(t)}{S(t)} = \alpha dt + \sigma dZ(t) \quad (3.1)$$

Where $S(t)$ is the stock price,

$dS(t)$ is the change in the stock price,

α is the continuously compounded expected return on the stock,

σ is the continuously compounded standard deviation,

$Z(t)$ is a normally distributed random variable that follows

Brownian motion.

$dZ(t)$ is the change in $Z(t)$ over a short period of time.

This equation is the differential form equation of a geometric Brownian motion and it is surprisingly useful to demonstrate that the stock price is lognormally distributed, based on the assumption foundation that the stock price follows geometric Brownian is also lognormally distributed.

Brownian motion is a random walk occurring in continuous time with continuous movements. The random walk was demonstrated properly in Samuelson in 1965, and it is modeled by flipping a coin n time repeatedly to see what is displayed when it lands, and the outcome of the flip is a random variable Y . Then the value of $Y(t)$ at each flip is either 1 or -1, corresponding the outcome head or tail. Let $Z(n)$ denote the cumulative outcome at the n flip, and we write

$$Z(n) = \sum_{i=1}^n Y(i)$$

Also easy to know that

$$Z(n) - Z(n-1) = Y(n) \tag{3.2}$$

To solve the problems that the random walk model is not adequate to present the practical stock price movements, the binomial model was introduced in, and it assumes that continuously compounded returns are random walk.

To understand the continuously compounded return, we'd better see how

continuously compounded return is defined. Suppose we invest A for n years at annual compounded return R. If the compounded return is encashed m times annual, the terminal value of the investment will be:

$$A_t = A \left(1 + \frac{R}{m} \right)^{mn}$$

Notice that we say an investment invests in continuously compounded return when $m \rightarrow \infty$. In this case, the terminal value becomes:

$$\lim_{m \rightarrow \infty} A \left(1 + \frac{R}{m} \right)^{mn} = A \cdot \lim_{m \rightarrow \infty} \left(1 + \frac{R}{m} \right)^{mn} = A \cdot \lim_{m \rightarrow \infty} \left(1 + \frac{1}{\frac{m}{R}} \right)^{\frac{m}{R} \cdot R \cdot n} \quad (3.3)$$

Here when $m \rightarrow \infty$, we have $\frac{m}{R} \rightarrow \infty$, then employ a mathematical formula:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x = e$$

If we let $x = \frac{m}{R}$, and substitute the formula into the equation (3.3), we get

$$A_t = \lim_{m \rightarrow \infty} A \left(1 + \frac{R}{m} \right)^{mn} = A \cdot e^{R \cdot n} \quad (3.4)$$

In this equation, R is the continuously compounded return. We can see that the final value of investment is independent to the frequency of taking continuously compounded return, m.

But for Brownian motion, by “continuous”, the flip should be infinitely fast and taken with infinitesimally small steps at each time, thus movements of the process are continuous. For definition and more properties and characteristics of Brownian motion can refer the material

from Merton (1990).

As Brownian motion is “continuous”, the time scale of every movement should be infinitely small, so we introduce in a scale factor \sqrt{h} . Note that $Z(t)$ is the value at time t , and let $h = \frac{T}{n}$. As the change in Brownian motion is assumed to be normally distributed, we can therefore assume to take random draw $Y(t)$ from either a binomial or a normal distribution, and have

$$E[Y(t)] = 0 \quad \text{and} \quad V[Y(t)] = 1$$

In addition, the characteristic that increments of Brownian motion are independent distributed is fulfilled. This is very important, because the assumption of stock price movements exhibits this tendency that today’s price movement isn’t affected or determined by historic movements.

From equation (3.2), now we can write the change in Z as:

$$Z(t+h) - Z(t) = Y(t+h)\sqrt{h} \tag{3.5}$$

Note that, when $n \rightarrow \infty$, we get $h = \frac{T}{n} \rightarrow 0$, and consequently $Z(t+h) - Z(t) \rightarrow 0$, which means the change in Brownian motion is infinitesimal. This could be a well reflect for that the Brownian motion is a continuous stochastic process and the scale factor \sqrt{h} is being multiplied appropriate here.

Since $n = \frac{T}{h}$, we can solve the Brownian motion by the limit of the cumulative increments as following

$$Z(T) - Z(0) = \sum_{i=1}^n (Z[ih] - Z[(i-1)h]) = \sum_{i=1}^n Y(ih)\sqrt{h}$$

$$= \sqrt{T} \left[\frac{1}{\sqrt{n}} \sum_{i=1}^n Y(ih) \right] \quad (3.6)$$

But 0 mean means that the expectation of stock price movements, or expected return of the stock is 0, and then the stock is valueless to invest. For arithmetic Brownian motion, we have to make it allow an arbitrary variance and a nonzero mean, so we further assume that

$$E[Y(t)] = \alpha \quad \text{and} \quad V[Y(t)] = \sigma$$

And we can write

$$X(t+h) - X(t) = \alpha h + \sigma Y(t+h) \sqrt{h}$$

With $h = \frac{T}{n}$, in the same way, we solve

$$\begin{aligned} X(T) - X(0) &= \sum_{i=1}^n \left[\alpha \frac{T}{n} + \sigma Y(ih) \sqrt{\frac{T}{n}} \right] \\ &= \alpha T + \sigma \left(\sqrt{T} \sum_{i=1}^n \frac{Y(ih)}{\sqrt{n}} \right) \end{aligned} \quad (3.7)$$

Recall the equation (3.6), and compare it with the right side of the equation (3.7). Easily we get

$$X(T) - X(0) = \alpha T + \sigma Z \quad (3.8)$$

We can rewrite the equation as:

$$X(T) - X(0) = \alpha(T-0) + \sigma[Z(t) - Z(0)]$$

So the differential form of the equation is:

$$dX(t) = \alpha dt + \sigma dZ(t) \quad (3.9)$$

Here, α is the instantaneous mean, and is called drift rate of change in $X(t)$, and σ is instantaneous standard deviation. Since $Z(t)$ follows Brownian motion, with the compare, we know that $X(t)$ follows a

generalized Brownian motion, so it should have most characteristics that the Brownian motion has.

However, this model can still not represent the stock price movements very well. In practice, stock price is not definable to be negative, since stockholders have limited liability. But here the random variable X is free to be negative, to modify this, we use geometric Brownian motion which assumes that the drift and volatility are functions of a random variable W :

$$dX(t) = \alpha[W(t)]dt + \sigma[W(t)]dZ(t)$$

Let the random variable W be the stock price X , moreover that the function could be that the drift and volatility is proportional to stock price X , which the model still follows basic requirements of the geometric Brownian motion. Then we can write:

$$dX(t) = \alpha X(t)dt + \sigma X(t)dZ(t) \quad (3.10)$$

Divided both sides of equation above by $X(t)$, we get:

$$\frac{dX(t)}{X(t)} = \alpha dt + \sigma dZ(t) \quad (3.11)$$

Comparing with equation (3.9), we can see $\frac{dX(t)}{X(t)}$ is normally distributed.

$\frac{dX(t)}{X(t)}$ represents the percentage change in the stock price.

3.2. Lognormal stock price

By definition, if random variable x follows normal distribution, $y = e^x$ follows lognormal distribution. Taking logs at both sides, the equation

can be written as $\ln(y) = x$. And again by definition, the continuously compounded return from time t to $t+h$ is:

$$r_h = \ln\left(\frac{S_{t+h}}{S_t}\right) \quad (3.12)$$

If r_h is normally distributed, according to the definition of lognormal distribution, the stock price S is lognormally distributed. Exponentiating both sides and rearranging the equation (3.12), denote stock price at time t and $t+h$ as S_t and S_{t+h} , the continuously compounded return in per unit time change $\frac{1}{h}$ as r and it is additive. Then the lognormal model for stock price simply can be written as:

$$S_{t+h} = S_t e^{r_h} \quad (3.13)$$

The continuously compounded return is assumed to be constructed by two parts: certain part and uncertain part which comes from the uncertain stock price movements. We can estimate the uncertain part with variance of the continuously compounded return. Suppose the variance of return from time t to $t+h$ is σ^2 , and returns in per unit time change are uncorrelated, meaning that variances are additive. Then the variance of return can be written in terms of variance in per unit time change as $\sigma^2 = h\sigma_p^2$. We can rewrite it as $\sigma_p^2 = \frac{1}{h}\sigma^2$, then take the square root of both sides and get the standard deviation of return in per unit time change:

$\sigma_p = \sqrt{\frac{1}{h}}\sigma$, where $\frac{1}{h}$ is per unit time change. If we denote per unit time change as Δt , it becomes:

$$\sigma_p = \sqrt{\Delta t} \sigma$$

Add this uncertain part to equation (3.4), the adjusted form of the binomial model for stock price is:

$$S_{t+h} = S_t e^{rh \pm \sigma \sqrt{h}} \quad (3.14)$$

Moving S_t to the left side and then taking logs both side, we get:

$$\ln\left(\frac{S_{t+h}}{S_t}\right) = rh \pm \sigma \sqrt{h}$$

An explanation to the equation is that from time t to time $t+h$, stock price moves h times, if the expected continuously compounded return in every movement is r , and the variance of h moves is σ , the continuously compounded return from time t to time $t+h$ can be estimated as h moves times expected continuously compounded return, adding the uncertain part of return caused by uncertain up and down stock price moves.

Investors who choose to invest in stocks must tolerate more risk of losing than risk-free product, so they will request a risk premium as compensation. Therefore, if the risk-free return on government bonds is r_f , the expected return on stock is:

$$r = r_f + \text{risk pre} \quad (3.15)$$

When we exam the total return on a stock, r , normally it can be seen in two aspects, or it comes from two forms: the stock price change and dividend payout. Denote dividends yield be δ , we have:

$$S_{t+h} = S_t e^{(r-\delta)h \pm \sigma \sqrt{h}}$$

Since the binomial model assumes that continuously compounded returns

on stock price follow random walk, we add the standard normally distributed random variable z to the uncertain part, and get:

$$S_{t+h} = S_t e^{(r-\delta)h \pm \sigma \sqrt{h} z}$$

But now, we are at a position that the model still can't interpret the relationship between stock prices and the expected continuously return very well. Properties of normal distribution tell us that if $z: N(m, v^2)$, expect of e^z is $E(e^z) = e^{m + \frac{1}{2}v^2}$. And also if $z: N(0,1)$, easy to know $\sigma \sqrt{h} z: N(0, (\sigma \sqrt{h})^2)$. Therefore, the mean of random variable $e^{\sigma \sqrt{h} z}$ can be computed as:

$$E(e^{\sigma \sqrt{h} z}) = e^{0 + \frac{1}{2}(\sigma \sqrt{h})^2} = e^{\frac{1}{2}\sigma^2 h} \quad (3.16)$$

Expect of stock price at time t+h is:

$$E(S_{t+h}) = E(S_t e^{(r-\delta)h + \sigma \sqrt{h} z}) = E(S_t e^{(r-\delta)h} e^{\sigma \sqrt{h} z}) = S_t e^{(r-\delta)h} E(e^{\sigma \sqrt{h} z})$$

Substitute equation (2.15) into it, the result is:

$$E(S_{t+h}) = S_t e^{(r-\delta)h} e^{\frac{1}{2}\sigma^2 h} = S_t e^{(r-\delta + \frac{1}{2}\sigma^2)h}$$

An interpretation to the equation can be that the expected stock price at time t+h equals the expected forward value at time t+h of stock price at time t. In this case, the expression $r - \delta + \frac{1}{2}\sigma^2$ must be the expected continuously compounded return on stock price change from time t to time t+h. Here comes the problem: how to interpret the expression? We analyzed earlier that $r - \delta$ is the expected continuously compounded return on stock price change, but there is a $\frac{1}{2}\sigma^2$ which we can't interpret it properly and it's meaningless for expressing the expected return. Therefore, we subtract a $\frac{1}{2}\sigma^2$ in the certain part of the expectation of the

continuously return, to make the expected continuously compounded return easy interpreted as $r - \delta$. After the modification, we finally model the lognormal stock price properly:

$$S_{t+h} = S_t e^{(r-\delta-\frac{1}{2}\sigma^2)h \pm \sigma\sqrt{h}z} \quad (3.17)$$

Where z is a standard normal distributed random variable follows random walk.

3.3 From lognormal stock price to geometric Brownian motion model stock price

We know that the price of derivative product is function of underlying assets price and time, and write the function as

$$C(S, K, \sigma, r, T-t, \delta)$$

Where S is underlying asset price, K is strike price,

Delta –gamma approximation is using delta and gamma to approximate the derivative product price by adjusting approximation error.

Delta measures the change in the derivative product price caused by per unit change in underlying asset price; Gamma measures the change in delta when the underlying asset price changes; theta measures the change in the derivative product price for change in observation time. By definition, these measures are partial derivatives of the derivative product, for example option price, we get:

$$\Delta \equiv C_S \quad \Gamma \equiv C_{SS} \quad \theta = C_t$$

Back to the delta-gamma approximation, we have the formula:

$$C(S_{t+h}, t+h) = C(S_t, t) + \varepsilon \Delta(S_t, t) + \frac{1}{2} \varepsilon^2 \Gamma(S_t, t) + h \theta(S_t, t) \quad (3.18)$$

Notice that h is the time interval from time t to $t+h$, and $C(S_{t+h}, t+h)$ is derivative product price at time $t+h$, ε is the change in asset price S caused by per unit time change h .

When regarding the stock price $S(t)$ as a function of Brownian motion $Z(t)$ and time t with the expression:

$$S(t) = S(0) e^{(\alpha - \delta - \frac{1}{2}\sigma^2)t + \sigma Z(t)} \quad (3.19)$$

Here the variable $Z(t)$ is a random variable follows a random walk from time 0 to time t . If denote the length from time 0 to time t as T ; let $n = \frac{T}{h}$ be the number of time intervals during T , the length of each stock price movement be h , the stock price at each movement can be writ as equation (3.17), and the relationship between variable Z and z is $Z = \sqrt{T}z$. In addition, if $n \rightarrow \infty$, $Z(t)$ can be seen as following a random walk in continuous time, or say Brownian motion. We may discuss a little bit about parameter α and r . In lognormal stock price model, r is the estimated total continuously compounded return; in geometric Brownian motion model, $\alpha X(t)dt$ in equation (3.10) is drift term, and $\alpha X(t)$ is defined as mean of stock price change. Dividing both sides $X(t)$, $\frac{dX(t)}{X(t)}$ is compounded, and α in equation (3.11) is mean of compounded return on stock price change. Therefore the relationship between r and α can be

represented by equation $E(r) = \alpha$, and consequently r can be replaced by α in equation (3.19).

According to the formula (2.25), easily can get:

$$S(Z_{t+h}, t+h) = S(Z_t, t) + \Delta(Z_t, t) + \frac{1}{2} \varepsilon^2 \Gamma(Z_t, t) + \theta(Z_t, t) h$$

Rewrite it as:

$$S(Z_{t+h}, t+h) - S(Z_t, t) = \varepsilon \Delta(Z_t, t) + \frac{1}{2} \varepsilon^2 \Gamma(Z_t, t) + h \theta(Z_t, t) \quad (3.20)$$

Make the time interval h be an equally time change unit, then per unit change in stock price is $\Delta S = S(Z_{t+h}, t+h) - S(Z_t, t)$. Moreover, when $h \rightarrow dt$, we have $\Delta S \rightarrow dS$ and $\varepsilon = \Delta Z \rightarrow dZ$, substitute them into equation (3.20), get

$$dS(t) = dZ(t) \Delta(Z_t, t) + \frac{1}{2} [dZ(t)]^2 \Gamma(Z_t, t) + dt \theta(Z_t, t) \quad (3.21)$$

Now we need to solve Δ , Γ and θ from $S(t) = S(0) e^{(\alpha - \delta - \frac{1}{2} \sigma^2)t + \sigma Z(t)}$:

$$\Delta(Z_t, t) = \frac{\partial S(t)}{\partial Z(t)} = \sigma S(t)$$

$$\Gamma(Z_t, t) = \frac{\partial^2 S(t)}{\partial Z^2(t)} = \sigma^2 S(t)$$

$$\theta(Z_t, t) = \frac{\partial S(t)}{\partial t} = \left(\alpha - \delta - \frac{1}{2} \sigma^2 \right) S(t)$$

Substitute them into the equation (3.21) and get:

$$dS(t) = \sigma S(t) dZ(t) + \frac{1}{2} \sigma^2 S(t) [dZ(t)]^2 + \left(\alpha - \delta - \frac{1}{2} \sigma^2 \right) S(t) dt \quad (3.22)$$

Recall that the change in Brownian motion $dZ(t)$ is modeled as binomial times scale factor \sqrt{h} as equation (3.5). When time interval h is infinitesimal and $h \rightarrow 0$, we know that per unit change in $Z(t)$ is:

$$dZ(t) = Z(t) - Z(t-h) = Z(t+h) - Z(t) = Y(t)\sqrt{h} = Y(t+h)\sqrt{h}$$

Since $Y(t) = \pm 1$ and $Y(t+h) = \pm 1$, we get:

$$[dZ(t)]^2 = h \rightarrow dt$$

With it, equation (3.22) can be written as:

$$\begin{aligned} dS(t) &= \sigma S(t)dZ(t) + \frac{1}{2}\sigma^2 S(t)dt + \left(\alpha - \delta - \frac{1}{2}\sigma^2\right)S(t)dt \\ &= \sigma S(t)dZ(t) + (\alpha - \delta)S(t)dt \end{aligned}$$

Compare it with the expression for geometric Brownian motion, it follows the definition which the mean and variance are proportional to the variable follows the process, and this means that the change in stock price is proportional to stock price which generates compounding and, vice versa, the lognormal stock price with the normal distributed compounded return fulfills the geometric Brownian movement of stock price.

4. Methodology-- Monte Carlo simulation

Monte Carlo simulation is a popular and widely used method to analyze the probability distribution of the movement or development of object. It takes by researching, assuming and setting the random process of object's development, estimating the parameters by historical stochastic data, generating the time series and computing the future value of object.

Monte Carlo simulation is basing on that the object has random process characteristic, then we can use computer to simulate the sample result and estimate the stochastic value of the object. When the times of simulation is more enough, we can get stable result by averaging all the simulations.

4.1 Simulate the correlated stock price path

In practice of analyzing the return on ELNs which may be constructed with several correlated underlying assets, as we use uncorrelated random variables to simulate uncorrelated assets prices, consistently, to analyze the price of correlated assets prices, we first need to generate correlated random variables.

Now we need to find out the correlations between correlated random variables. If there are n correlated underlying assets, we denote $\rho_{i,j}$ as the correlated coefficient between underlying assets i and j . With knowing that $\rho_{i,i}$ is 1, and $\rho_{i,j}$ is equal to $\rho_{j,i}$, we can write the correlation matrix as:

$$\boldsymbol{\rho} = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \dots & \rho_{1n} \\ \rho_{21} & 1 & \rho_{23} & \dots & \rho_{2n} \\ \rho_{31} & \rho_{32} & 1 & \dots & \rho_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ \rho_{n1} & \rho_{n2} & \rho_{n3} & \dots & 1 \end{pmatrix}$$

Our mathematic focus is on $\rho_{i,j}$. Correlation is defined as $\rho_{ij} = \frac{\text{COV}_{ij}}{\sigma_i \sigma_j}$, therefore we first need to figure out σ_i , volatilities of continuously compounded stock return for all underlying stocks, which are also important and necessary parameters for the simulation. Approximately we can use historical volatilities which are computed from historical data of underlying assets prices and compound returns on them.

Denote one observation of one underlying asset price, for example stock price, at time t_k as $S_i(t_k)$. Assume that the time intervals, Δt between every two sequential observations are equal length, meaning that $t_{k+1} = t_k + \Delta t$, where $k = 0, 1, 2, \dots, n$, and if there are $n+1$ observations, at time t_0 is the first and at time t_n is the last.

According to the lognormal model, equation (3.17), we can write:

$$S_i(t_{k+1}) = S_i(t_k) e^{(r_i - \delta_i - \frac{1}{2}\sigma_i^2)\Delta t + \sigma_i \sqrt{\Delta t} z_i}$$

Taking log at both sides and rearranging the equation above, we get the compounded return on underlying stock i from time t_k to time t_{k+1} . And if denote it as $\nu_i(t_k)$, we get:

$$\nu_i(t_{k+1}) = \ln\left(\frac{S_i(t_{k+1})}{S_i(t_k)}\right) = (r_i - \delta_i - \frac{1}{2}\sigma_i^2)\Delta t + \sigma_i \sqrt{\Delta t} z_i \quad (4.1)$$

First two terms beside the first equal sign are very useful for estimating

parameters σ and ρ . If we can obtain historical observation data about stock prices sequences, we can solve $v_i(t_{k+1})$, and n observations of stock prices S_i compute $n-1$ compounded returns; $n+1$ observations gives n compounded returns. Then if we know over $n+1$ observations, we can compute the expect of the returns by equation:

$$\bar{v}_i = \frac{1}{n+1} \sum_{k=0}^n v_i(t_{k+1})$$

The parameter interpretation of the equation as follow: t_0 is the time point for first price observation of stock i , and at t_{n+1} is the last observation. So there are $n+2$ observations which gives $n+1$ compounded returns on price of stock i .

Standard deviation σ is equal to the square root of variance. For stock i , by definition, the short period standard deviation which is computed from return at each historical stock price observation is:

$$\sqrt{\frac{1}{n+1} \sum_{k=0}^n (v_i(t_{k+1}) - \bar{v}_i)^2}$$

Then we can compute the standard deviation from the return during the length of all observations by multiplying the short period standard deviation by the square root of number of short period time intervals. Additionally, if the time interval of the short period is smaller enough as $\Delta t \rightarrow 0$, we can obtain the standard deviation of continuous return. The number of short period time intervals are unlimited big when Δt is unlimited small, therefore, the number of time intervals can be presented

as $\frac{1}{\Delta t}$. The standard deviation of continuous return during the stock price observations is:

$$\sigma_i = \sqrt{\frac{1}{\Delta t(n+1)} \sum_{k=0}^n (v_i(t_{k+1}) - \bar{v}_i)^2} \quad (4.2)$$

In the same way, the covariance of returns on stock i and j is:

$$c o v_{i,j} = \frac{1}{\Delta t} E \left[(v_i(t_{k+1}) - \bar{v}_i) (v_j(t_{k+1}) - \bar{v}_j) \right]$$

Now we finally can compute the correlation coefficient between returns on stock i and j by equation:

$$\rho_{ij} = \frac{\sum_{k=0}^n (v_i(t_{k+1}) - \bar{v}_i)(v_j(t_{k+1}) - \bar{v}_j)}{\sqrt{\sum_{k=0}^n (v_i(t_{k+1}) - \bar{v}_i)^2 \sum_{k=0}^n (v_j(t_{k+1}) - \bar{v}_j)^2}} \quad (4.3)$$

But this is a closed form set of correlations. To obtain correct form of correlations, let it be the matrix \mathbf{A} , we can solve it from the matrixes equation $\mathbf{A} * \mathbf{A}^T = \boldsymbol{\rho}$, where \mathbf{A}^T is the transpose of \mathbf{A} . To use this method proper, we need to mention that the matrix $\boldsymbol{\rho}$ must be positive-definite. Otherwise, the estimated correlations $\boldsymbol{\rho}$ is not valid to give the required correlated random variables. But in practice of simulating, if observations of asset prices or compounded returns on asset are frequent enough, for example, m assets, n observations and $n > m$, and it's easy to imagine that the more frequent observations are.

The method to solve the matrixes equation is Cholesky decomposition, we can employ the Matlab function chol(x), write $\mathbf{A} = \text{chol}(\boldsymbol{\rho})$ in Matlab, the output is a upper triangle matrix \mathbf{A} , and fulfils $\mathbf{A} * \mathbf{A}^T = \boldsymbol{\rho}$; if $\boldsymbol{\rho}$ is not

positive-definite, the output is a fail message.

We simulate a unitary matrix ξ of uncorrelated standard normal distributed variables at every observation, and then the matrix of correlated random variables z can be calculated by solving $z=A*\xi$.

5. Estimation of parameters

In this chapter I will estimate parameters for simulation in detail. We may also take a look at the difference on parameters when estimation is basing on different historical data and by different method, then how to choose the most relevant parameters.

5.1 selection of risk-free interest rate

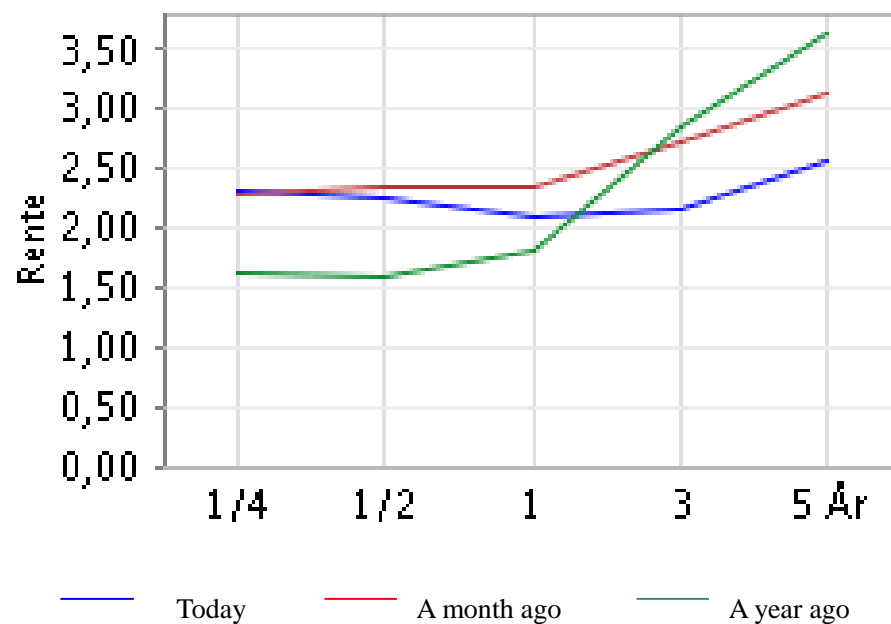


Figure 1: Rate of return on government bond for different running period and predict time

Figure 2³ below is the rate of return on Norwegian government bond with different expire year and predict time, which was estimated at May, 2010. In the same website, I found another figure of interest rate which is

the same curves. This means it's used to let the rate of return on government bond be instead of the risk-free rate, since the government bonds are comparatively safe and the returns are very stable.

If we make a middle point between 3-year and 5-year and draw a vertical from the point, the intersection with the green line indicates the effective rate of return on 4-year government bond, which is approximately be 3.25%. We can also see the figure of curve of Consumer Price Index (CPI) from year 1988-2006⁴ can be seen on the website of Statistics Norway, it shows that the CPI went down stably. But since the financial crises 2008, the CPI rate was changed more frequently than before.

However, Norges Bank comes out with estimated data series of effective return on government bond every certain period directly. On the website, there are annual averaged synthetic 3-, 5-, 10-year effective yields from 1985-2009⁵. Instead to find the risk-free interest rate around the date that the product started, 06.02.09, I can use the averaged 2009 synthetic 4-year risk-free return estimated from 3-year and 5-year government bonds' effective yields rates by the linear equations, which is 3.02%. And if we estimate from 5-year and 10-year rates, the result will be 2.89%. If we make an average on two estimations, it will be 2.96%. About the fundamental calculation method and database which used to estimate and the all historical data, can see appendix 1.

³ From the website <http://www.osloabm.no/>, accessed at 28.05.10

⁴ See figure 7 in <http://www.ssb.no/vis/samfunnsspeilet/utg/200504/14/art-2005-09-27-01.html> accessed 20.5.10

Compare this effective yields rate on 4-year government bond, 2.96%, with the green curve in figure 2. The rate of return on 4-year government bond one year ago, approximately in May, 2009, was around 3.25%. Since the product start date, February, 2009 are pretty near to May, 2009, we can let estimated return at May, 2009 be an approximation of the rate of return at February, 2009. The difference between to estimates is only 0.29%.

Although we can estimated the risk-free return in all ways above, for this product, this parameter is given out as 3.25% in the product prospect brochure since it's an important parameter for investor to evaluate the product. And it's just as same as the approximation we got from figure 2, we can say that the return on 4-year government bond around date 06.02.09 and 20.05.09 nearly didn't change.

5.2 Selection of Risk premium

In Section 3.2, I discussed how to estimate total stock return by equation (3.15). And for each stock i , the risk premium is equal to the beta of the stock i in the market multiply the market risk premium. So we need to know the Norwegian equity market risk premium first.

By definition, risk premium is the return in excess of the risk-free rate of

⁵ See data on http://www.norges-bank.no/templates/article_57357.aspx accessed at 20.05.10

return that an investment is expected to yield. So it can be estimated by observing difference of return between stock market and money market. Here I'll use the risk premium citing from the estimation of Espen Sirnes's⁶ blog note: Risikopremien for Oslo Børs 1915-2009⁷, at 07.05.2010. He estimated and represented the Norwegian equity market risk premiums using databases in different length of historical periods. The table 1 of his estimated result is as below, and I'm citing full version of his historical statistic database selection and estimating analysis in appendix 2. In addition, he took the detailed database of Oslo Børs Market and excel estimated for risk premium estimating, and it can be seen on the website: <http://ansatte.uit.no/esi000/risikopremieOB.xls>.

	<i>1915-2009</i>	<i>1970-2009</i>	<i>1990-2009</i>
<i>Aksjeavkastning</i>	10.7 %	18.0 %	12.4 %
<i>Rente</i>	5.3 %	10.3 %	6.3 %
<i>Risikopremie</i>	5.4 %	7.7 %	6.1 %

Table 2: estimated Norwegian equity market risk premiums form historical data

With analysis and suggestion in his article, the risk premium which computed with database from 1915-2009 is most proper, and I'll also use

⁶ Associated professor of Finance, in University in Tromsø.

⁷ See <http://espensirnes.blogspot.com/2010/05/risikopremien-for-oslo-brs-1915-2009.html> accessed at 20.05.10

this historical estimated risk premium as 5.4%.

5.3 Selection of beta β and dividend yield δ

Beta is an important historical statistic value of the stock market, it's used to measure correlation of risk between individual stock and market portfolio. Dividend yield and volatility is changing in a wide range, using the historical data estimated value may not give accurate or proper prediction of the future value. Different selection of dividend yield and volatility can give variable simulation results, so it's important to select the practicable ones.

For dividend rate, I'll use the value estimated by historical observations in trailing twelve months (ttm.) to instead the value estimated by last 4 years' observations until the product start day. Because since the financial crisis in 2008, the stock price, dividend payout and stock return was changing strongly, using the data 1 year ago from now which affected by financial crisis should better predict the trend of the future development than the data before financial crisis, unless the financial crisis straightens up rapidly and market trend changes over strongly in following 4 years.

On the website of Bloomberg⁸, with searching code of each stock, we can find the historical stochastic computed beta value to the Oslo Børs

⁸ See <http://preview.bloomberg.com/apps/quote?T=en10/quote.wm&ticker=STL:NO>

Market for each stock.

I collect the beta values as table 3 below. In the same table, I also collect the dividend rates. With the efficient risk-free return equity market risk premium and beta values, I compute the total return by equation:

$$r_{total} = r_f + \beta_i \times riskpremium_{market}$$

For stock STL, it's computed as:

$$3.25\% + 0.884 \times 5.4\% = 8.024\%$$

Then the expected return on change in stock price is equal to the expected total return on stock subtract from estimated dividend rate. All results of computations are collected in the table 3.

	<i>BETA</i>	<i>Risk</i>	<i>Total expected</i>	<i>Dividend</i>	<i>Expected return on</i>
	β	<i>Premium</i>	<i>return</i>	<i>yield δ</i>	<i>Stock price</i>
<i>STL</i>	0.884	4.774%	8.024%	4.098%	3.926%
<i>TEL</i>	0.748	4.039%	7.289%	2.749%	4.54%
<i>ORK</i>	0.885	4.779%	8.029%	4.395%	3.634%
<i>DNB</i>	1.192	6.437%	9.687%	2.333%	7.354%
<i>YAR</i>	1.171	6.323%	9.573%	2.061%	7.512%
<i>NHY</i>	1.157	6.248%	9.498%	1.128%	8.37%
<i>REC</i>	1.180	6.372%	9.622%	None ⁹	9.622%

Table 3: Estimated parameters.

⁹ According to the annual reports of REC group, it pays no dividend. Annual report can see at

5.4 Estimate volatilities and correlations

Volatilities are estimated by standard deviations of stock return. As we discussed in Section (4.1), we can use historical stock price observations to compute the return by equation (4.1).

Historical stock price observations for last 4 years can be collected by searching the code of each stock on Bloomberg website. For the purpose to mitigate the computation and database workload, I'll use monthly stock price observations to calculate compounded returns.

Let STL be an example: the 61 stock price observations on date 15th every month in every last 5 years were sequentially been collected and uploaded as price 5 in the Matlab program. Then use Matlab program to estimate the compounded returns and the monthly volatility. In the same way, we get the volatilities of all 7 underlying stocks and correlations between 7 underlying stocks. The monthly volatilities and correlations are been computed as table 3 and the matrix following. The mathematic interpretation to explain the computation is in Section 4.1. For the MATLAB program to take this estimation can see Appendix 3.

	<i>STL-</i>	<i>TEL-</i>	<i>ORK-</i>	<i>DNB-</i>	<i>YAR-</i>	<i>NHY-</i>	<i>REC-</i>
	<i>Price5</i>	<i>Price6</i>	<i>Price3</i>	<i>Price1</i>	<i>Price7</i>	<i>Price2</i>	<i>Price4</i>
<i>Volatility</i>	31.60	41.01	36.17	39.30	50.50	46.50	71.07
(%)							

Table 4: Volatility of return on each underlying stock, price 1 to price 7 is denoted in the Matlab syntax.

$$\rho = \begin{pmatrix} 1 & 0.6074 & 0.5450 & 0.0298 & 0.4023 & 0.4951 & 0.4707 \\ 0.6074 & 1 & 0.5457 & 0.1013 & 0.7378 & 0.3840 & 0.5556 \\ 0.5450 & 0.5457 & 1 & 0.0190 & 0.5881 & 0.4964 & 0.4367 \\ 0.0298 & 0.1013 & 0.0190 & 1 & 0.1168 & 0.2433 & 0.0726 \\ 0.4023 & 0.7378 & 0.5881 & 0.1168 & 1 & 0.3247 & 0.4730 \\ 0.4951 & 0.3840 & 0.4964 & 0.2433 & 0.3247 & 1 & 0.6561 \\ 0.4707 & 0.5556 & 0.4367 & 0.0726 & 0.4730 & 0.6561 & 1 \end{pmatrix}$$

On website of Bloomberg, we can collect volatilities computed from 90-day observations until today, and list as below:

	<i>STL</i>	<i>TEL</i>	<i>ORK</i>	<i>DNB</i>	<i>YAR</i>	<i>NHY</i>	<i>REC</i>
<i>Volatility</i>	27.720	33.573	30.140	38.269	46.675	39.597	65.448
¹⁰ (%)							

Table 5: volatility of return on each underlying stock, estimated from 90-day daily observation until today

Comparing the volatilities in table 5 with table 4, we find that the annualized volatilities which estimated from 90 days' daily observation

¹⁰ <http://preview.bloomberg.com/apps/quote?T=en10/quote.wm&ticker=STL:NO> Accessed at 26.07.2010

hold similar level as the annualized volatilities estimated from 4 years' monthly observation, only a bit lower.

6. Simulation result

At the position now, I have almost already estimated, selected and prepared all the parameters which I need, and get to simulate the future stock price paths. As you might have attention on that I put a quite weight on trying to find out the available, right and accurate parameter values in many different ways, because I realize that the perfect parameter values are crucial important to get a satisfying simulating result.

Not only for professional worker, but also for the normal individual investors, learning to recognize and grand the useful, valid, right and accurate information is the key to understand the market and product development and lead to success.

The last but same important step is taking the Monte Calro Simulation in MATLAB. With the simulated future stock price path for each underlying stock, and the return calculation formula which be given out in product prospect brochette, we can finally compute the expected estimated return on the product. About the Matlab program which I partly cited from professor Valeri Zakamouline and made adjust in some syntax for this specific product, I will not explain more here, but give out them directly in Appendix 4.

The only one point I need mention here is “Asian tale” of computing the maturity underlying stock prices. The return of the derivative part of the

product is highly connected with the price development of the 7 underlying stocks through the product's holding period. Using "Asian tale" to compute the maturity price can reduce the affection from the extreme price including both positive changed extreme price and negative moved extreme price. Therefore the time value of the product must be lower than the time value of the underlying stocks.

Here the maturity price of each stock is required to compute as arithmetic average of the stock prices of last 7monthes from 24.07.2012 to 24.01.2013. So in MATLAB, I simulate each stock price path monthly during the coming 4years. And for reducing workload and saving the MATLAB running time, I simulate the last 7 price broadcast points with 6 monthly intervals, and the period between the beginning price and the last 7 prices is only one interval with rest time of 4years. Then compute the mean of the last 7price simulations for each underlying stock.

6.1 Expected return estimation of the product when it's 100% self-financed

When running the program, a dialog box will automatically be presented and ask parameters about the proportional percent of loan in the investing amount. For computing the annual rate of return without considering the interest rate of taking loan in the investing, as well as the cost of investing in the product, type 0 in both first and second blank. The result of return

and probabilities of return I got under the 100000 times simulation of underlying stock price paths presents in the table 6. With running program several times, we can specify different parameter value, which is 0.5%, 2% and 3% in second blank, and get look at the change on annual product return caused by different product investing cost levels in 100% self-financed investment.

	<i>Management costs level</i>			
	<i>0</i>	<i>0.5%</i>	<i>2%</i>	<i>3%</i>
<i>Expected annual return</i>	2.21 %	2.09%	1.70%	1.48%
<i>Prob. < risk free rate</i>	68.78%	69.30%	70.92%	71.65%
<i>Prob. Of loss</i>	56.21%	56.74%	58.38%	58.94%

Table 6: Expected annual return of the product and the probabilities of expected return smaller than risk free rate and loss for different cost level, under 100% self-financed situation

We can see that with the 100% self-financed investment, the product may give the positive average expected annual return in all given investment costs levels. The lower cost, the higher is the expected return, the higher is the probability of getting the return that less than the risk-free rate, and the higher probability of loss. But even do not consider the investment

cost, the estimated annual return is still only 2.21% on average of 100000 simulations. This return level is even lower than risk-free rate of return, which means that if investing in the government bond, investors may get higher return than the product with nearly 40% possibility.

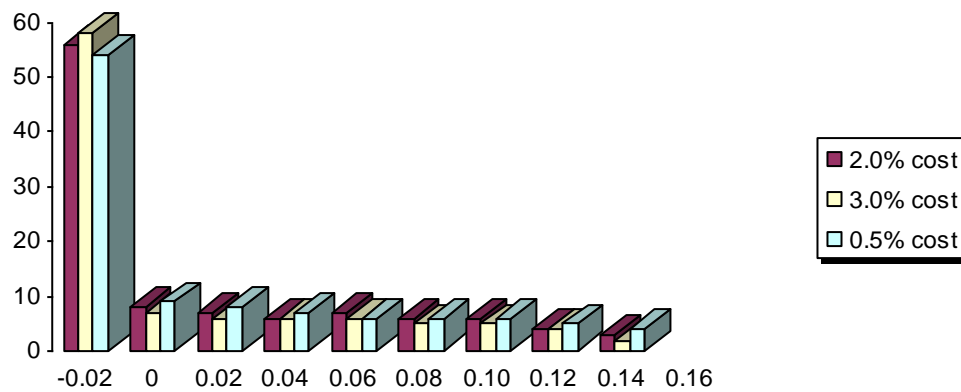


Figure 2: the probability distribution of the expected return on the 100% self-financed product, and it's presented in different costs levels.

Figure 2 shows the probability distribution of the expected return on product, it gives more intuitive view of the performance of the product. We find that although the mean of the expected return lies between 1.48% and 2.21%, the probability of getting an expected return at the mean level is approximately only 5% to 7%. Meanwhile, the probabilities to get negative expected returns are higher than 50%. Which means when investors invest a product which be forecasted with positive return, but actually they are taking the risk that there is more than 50% percent possibility that they will get negative return.

Since the return level is even lower than risk-free rate, we can say that the product gives no expected reward, and at same time it has relatively high risk to lose even more, therefore no rational investors will invest it.

The possible reason for such a poor expected return may be the structure of the product. Remember that the product has limitation of the return on each single stock to be max 80%. To test the affection on the product return by the return limitation on each underlying stocks, we can simulate the product return by taking away the 80% limitation on each underlying stock.

I replace this line in the Matlab program

$$AP(n) = AP(n) + w(i) * \min(0.8, (SF(i) - S0(i)) / S0(i));$$

by this line

$$AP(n) = AP(n) + w(i) * (SF(i) - S0(i)) / S0(i);$$

The product returns when it's 100% self-financed investment and do not consider the limitation on each single underlying stock presents in figure 7.

We can see that the adjusted mean of expected return will be between 5.54% and 6.34% with different costs levels. This level of expected return is higher than the risk-free rate of return, although it's still lower than the market index return.

	<i>Management costs level</i>			
	<i>0</i>	<i>0.5%</i>	<i>2%</i>	<i>3%</i>
<i>Expected annual return</i>	<i>6.34 %</i>	<i>6.25%</i>	<i>5.83%</i>	<i>5.54%</i>
<i>Prob. < risk free rate</i>	<i>57.99%</i>	<i>57.90%</i>	<i>59.12%</i>	<i>60.01%</i>
<i>Prob. Of loss</i>	<i>49.07%</i>	<i>49.12%</i>	<i>50.26%</i>	<i>51.05%</i>

Table 7: Expected annual return of the product and the probabilities of expected return smaller than risk free rate and loss for different cost level, under 100% self-financed situation, no 80% limitation on each underlying stock's return.

Comparing the table 7 with table 6, we know that the limitation on the return of each single underlying stock has considerable affection on the expected return on the product, it restricts the product's expected return; meanwhile, when losing the limitation, the probability of loss is reducing with 7% and probability of the return less than risk-free rate return is reducing 10%.

From this comparison we understand that the return can be constructed flexibly of the structure the product, it's a characteristic point of the structure product.

6.2 Expected return of the product when invest by taking loan

To compute the expected product annual return rate under loan financed investment, there is still one more parameter which is crucial: the interest rate of loan.

In the product prospect brochure it didn't come out with a determined value of this interest rate. It actually increases the risk to invest the product by taking loan. Because investors could never know the effective interest rate for loan, and couldn't estimate the precise return in the leverage position, and if the interest rate of borrowing is excessively higher than the risk-free rate, it's a poor leverage position which investors may not pay enough attention on when invest in the product. The market fact is that it's nearly impossible for investors taking financial loan with risk-free rate, so the high borrowing interest rate affects the return significantly in negative movements.

Therefore an appropriate estimated borrowing loan interest rate is necessary; here I use the risk-free rate plus the estimated annual market risk premium for financial loan. The risk premium of financial loan I use 2.07% which is taken from professor Valeri Zakamoline's data estimation.

The estimated interest rate for financial loan is:

$$3.25\% + 2.07\% = 5.32\%$$

Then the 4 year interest rate for loan is:

$$(1 + 5.32\%)^4 - 1 = 23.04\%$$

When invest the product by taking loan, the percentage of the loan

(PerLoan) in the investment amount affect the return of the product by paying back the loan with interest. Considering those factors, the return of the product is written as:

$$AP = (1 + \max(AP, 0) - (1 + \text{cost}) * (1 + \text{PerLoan} * 23.04\%)) / (1 + \text{cost})$$

As checking the express above: when it's 100% self-financed investment, the PerLoan is equal to 0, the return becomes $AP = (\max(AP, 0) - \text{cost}) / (1 + \text{cost})$. And when costs level is equal to 0, the return becomes $AP = \max(AP, 0) - \text{PerLoan} * 23.04\%$. Those make sense.

Management costs 0.5%

	<i>100% loan</i>	<i>80% loan</i>	<i>60% loan</i>
<i>Expected annual return</i>	-3.88%	-2.60%	-1.37%
<i>Prob. < risk free rate</i>	86.80%	83.79%	80.55%
<i>Prob. Of loss</i>	77.11%	73.38%	69.53%

Table 8: Expected annual return of the product and the probabilities of expected return smaller than risk free rate and loss for different cost level, under 100% loan-financed investment

The simulation result under the 100% loan financed investment as table 8 below. With the result we know the affection that the loan percent makes

to the expected return. The higher the loan percent, the lower the expected return.

Comparing with the 100% self-financed investment; we can see that the expected returns under different loan levels all become negative. When the product is 100% loan-financed with 0.5% cost level, the probability of loss increases to 77.11%, and the probability of getting an expected return that less than risk free rate is even 86.80%. It means that if invest in alternative product, for example risk free product like government bonds, it may get a higher return very possibly. Meanwhile notice that the 0.5% investment costs require a huge amount invest, individual investors rarely will take such a risk to take huge loan. If investors invest fewer amounts with cost level 2% or 3%, the mean of expected return will be even lower and in even high possibilities they will get loss or return less than risk-free rate.

From the figure 3 we can even observe that although the mean of expected loss is between -1.37% and -3.88% with different level of loan percent, the most possible loss is actually less than -4% with probability higher than 57%. So the mean of expected return actually is too narrow to present the product's future performance.

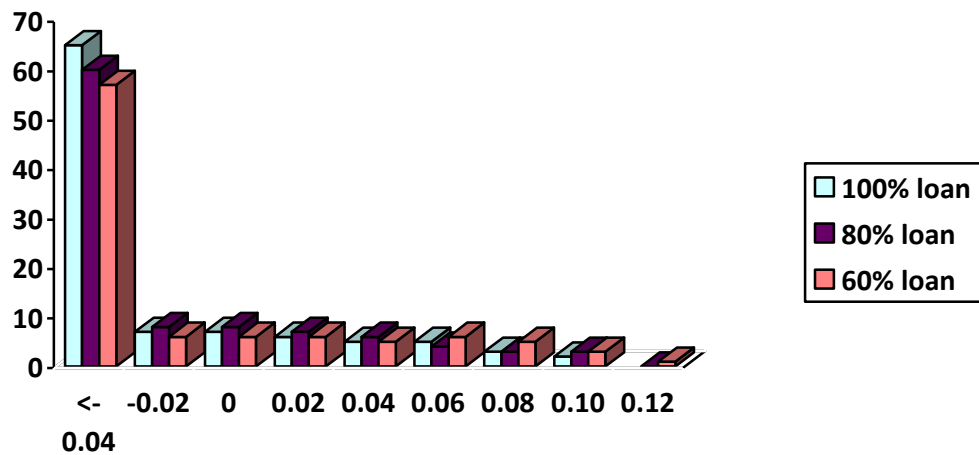


Figure 3: the probability distribution of the expected return on the product when it's invested with different loan levels. Management costs are 0.5%.

When I type “structured product” or “AIO and BMA” as keywords into the internet searching engine of main Norwegian websites, the most results be found are news as talking about that the normal individual buyers of these products were feeling cheated when they didn't get the benefit that products had showed, and even got loss that they didn't expect. With a view to the return analysis here for investing the product by taking loan, we will easy to understand that, if the normal investors invest with taking loan, such kind of dilemma will surely widely exist. Consider the analysis above, individual investors should think twice before invest in the product by taking loan.

6.3 Comparing the estimated result of return with issuer's forecast

As presented in the product prospect brochure, the expected annual future return of the product is broadcasted as 11.09% without the costs or 10.27% considering costs!

The discrepancy between my simulation result and the issuing bank's forecast is significantly striking.

The reason may be in two aspects. One aspect is on the deviation of the parameters selection between my simulation and Nordea's predict.

For example, I use the historical stock price observation from 15.06.2005 to 15.06.2009, but not as it should be, from 06.02.2005 to 06.02.2009.

And slao the dividend yield that has the significant affect on the final expected return on stock price. The estimated expected returns from historical data on underlying stocks are crucial parameters to determine the result of the simulating. Dividend rate of stock for each listed company can differ remarkably year to year, therefore estimating the mean of the historical dividend rate is necessary to be accurate in predictable future. But here for limited resource I could get access, I used only the trailing twelve month estimation which may be hard to represent the average annual dividend level in coming 4 years estimated period. Here we once again can see how important to choose the proper historical database and select accurate parameters.

Another caution may be that during 2004-2008 there was a strong bull market in the Norwegian stocks market with returns much higher than historical average. Nordea bank use this period to estimate mean returns and just extrapolated these high returns into the future. A too optimistic expected return can increase the confidence of investors' investment; it fulfills the benefit of the issuer bank.

7. Risk analysis

From the analysis of last section we know that the mean of the expected return is not adequate to present the future performance of the product. In this chapter, I will analyze the risk and comparing it between the product and the potential alternative product strategies which are constructed with relevant assets by employing several different risk measures and performance evaluation measures. And we will see which product strategies may be the most optimal one.

Most researchers, experts, officers, professors and media take critical attitude on such kind financial products as personal investment instruments. Problem is that investors should not only keep eyes on the expected return, but also need to pay attention on the possibility of risk exposures. When we are talking about any guaranteed produce, or "a low risk product", we must think about how we measure the risk.

7.1 Introduction of risk measures and performance measures

7.1.1 Risk measures for normal distributed return

In general, the risk of investments comes from market uncertainty and asymmetric information. And the financial risk can be measured respectively in systematic risk and non-systematic risk, the sum of them

is called total risk.

Usually we measure risk by two indicators in traditional performance evaluation measures: standard deviation and beta (β). Beta value of each financial asset comes from comparing the historical data of the asset's risk with market risk and measures the coefficient with the market risk changing. Therefore Beta value is appropriate to measure the market system risk. Standard deviation of the historical asset return is estimated by historical price observations. Since the asset price reflects complete market information and macro political information, the standard deviation of return should also reflect full size of risk factors, include market system risk and the non-system risk, therefore standard deviation can represent for the total risk.

Conventional Sharpe ratio measures total risk by standard deviation, and Treynor's measure and Jensen's measure uses the systematic risk measured by Beta; while information ratio uses the product's nonsystematic risk which may be reduced by holding a market index portfolio in theoretical level. These measures are well presented in the book Investments written by Bodie and other two.

7.1.2 Deviation from normal distribution

In the Monte Carlo simulation model, expected return of financial assets

as stocks is assumed to follow the normal distribution, which means that it assumes the standard deviation of the product return could measure the risk. But to assess that if the normal distribution is adequate for predict the assets' return, we can examine that if the probability distribution of the expected return is as symmetric as normal distribution.

7.1.2.1 Skewness of distribution of return

Here we can use **skew** ratio to measure the asymmetry or “skewness” of a distribution. The measure **skew** can be computed as:

$$\text{Skew} = E[r(s) - E(r)]^3 / \sigma^3$$

Where: $r(s)$ is the expected holding period return in each scenario, here is setting as in each simulation

$E(r)$ is the expected mean return of all simulations

σ is standard deviation of $E(r)$

With knowing that the cube of negative is negative, so the cubed extreme values of positive or negative will determine the positive or negative of skew. We can say that when the skew value of the distribution is positive, the standard deviation overestimates the risk, and the probability distribution of expected return is called positive skewed distribution. Same story that if the distribution is positively skewed, we know that the standard deviation will overestimates the risk. The probability of an annual return greater than mean of the expected return is higher for the

positively skewed distribution than the normal distribution with the same average return and standard deviation. This is significantly meaningful to select the risk measure for the product in principal.

Here we can compute the skew ratio of the distribution of the product return in Matlab. When considering a 100% self-financed investment in the product without management costs, the value of Skew is computed as:

$$\text{Skew of the product} \approx 1.17$$

This is a remarkable positive Skew value. From the comparison of distributions and Skew values of portfolios: world large stocks, U.S. large stocks and small stocks, world bonds and long-term U.S. treasury bonds with historical data from 1926-2005(see Investment, Bodie, Kane, Marcus; page 145-146), we can observe that the negative skew could be a sign of the risky portfolios like world large stocks, U.S. large stocks and U.S. small stocks while the positive skew could be a sign of the portfolio with relatively lower risk in theoretically, like the world bonds portfolio, long-term treasury bonds and T-Bills.

Therefore we can say that the product distribution should be more similar as the low risk-risk free portfolio. The level of average return and standard deviation should be close to the return and standard deviation of lower risk-risk free portfolios. And this is just in consistent with my simulation result. See the table 9, return of U.A. T-Bills is seen as risk-free rate, and assuming to follow normal distribution. U.S. small

stocks portfolio is risky. The expected return and standard deviation of the product is at similar level as the T-bills. The skew of the product is highly positive while the skew of small stocks portfolio is relatively negative, we can see that the risky portfolio has higher level of return and standard deviation than the product.

7.1.2.2 Kurtosis value of distribution of the expected return

In addition, we can compute the Kurtosis ratio that measures the “fat-tailed” distribution from normality. It’s computed as:

$$\text{Kurtosis} = E[r(s) - E(r)]^4 / \sigma^4 - 3$$

Since the Kurtosis ratio of normal distribution would be 3. The positive Kurtosis ratio of a distribution means the extreme values on both side of the mean have more probability in the tails than normal distribution, or say, the standard deviation underestimates the probability of extreme values, although it may be as symmetric distributed as normal distribution.

The Kurtosis value of the product when considering a 100% self-financed investment in the product without management costs is:

$$\text{Kurtosis of the product} \approx 3.10$$

This is an extreme value of positive kurtosis; it implies that the probability of the extreme return on the product is underestimated by standard deviation. Look at the figure 2, the probability of return between

-2% and 0% is around 50%, and between 4% and 14% is around 25%.

The observation is in consistent with the positive kurtosis.

	<i>The product</i>	<i>U.S. T-Bills</i>	<i>U.S. small stocks</i>
<i>Average return</i>	2.21%	3.75%	17.95%
<i>Standard deviation</i>	4.16%	3.15%	38.71%
<i>Sharpe ratio</i>	-0.25	0	0.36
<i>Skew</i>	1.17	0	-0.22
<i>Kurtosis</i>	3.10	0	0.86

Table9: Skew and kurtosis, return-standard deviation comparison between the product, and risk-free, risky portfolios. Data of T-bills and small stocks portfolio is historical estimated from 1926- 2005

From the figure 2 and 3 we can also easily see that the probability of product return actually follows a positive skewed distribution. Therefore, what we can do here is to adjust the standard deviation to measure the risk. And the downside risk adjust is useful for the product with a positively skewed distribution of return.

I'm going to demonstrate some details in later sections.

7.2 Sharpe ratio

Sharpe ratio is developed for choosing the optimal return-risk portfolio.

Since we estimate the product risk as standard deviation, we use Sharpe ratio to evaluate performance of the product.

7.2.1 Brief introduction of Sharpe ratio

There are two important premise for that Sharpe ration can be suitable used and measure the portfolio performance well. First, Sharpe ration is based on the mean-variance theory, which means that using arithmetic average return and the standard deviation to measure the variance, or say risk. Then, Sharpe ratio can only measure the portfolio with normally distributed returns or quadratic preferences.

Sharpe ratio can be simply calculated by the formula:

$$S_C = (E(r_C) - r_f) / \sigma_C \quad (7.1)$$

Where:

S_C is the Sharpe ratio to the overall product in combination of risky portfolio with proportion y in investment amount, and with risk-free asset with the proportion as $(1-y)$;

$E(r_C)$ is the estimated return on combined product;

r_f is estimated rate of return on risk-free asset;

σ_C is the standard deviation of the combined product

Sharpe ration on complete product can be rewritten as:

$$S_C = \{r_f + y[E(r_{\text{risky portfolio}}) - r_f] - r_f\} / [(1-y)\sigma_f + y\sigma_{\text{risky portfolio}}]$$

$$\begin{aligned}
&= y[E(r_{\text{risky portfolio}}) - r_f] / y\sigma_{\text{risky portfolio}} \\
&= [E(r_{\text{risky portfolio}}) - r_f] / \sigma_{\text{risky portfolio}} \qquad (7.2)
\end{aligned}$$

So Sharpe ratio actually measures the excess return of the risky portfolio beyond the risk-free rate when increasing a unit of total risk. Investors can construct optimal portfolio by adjusting the return-risk. We can get a comprehensive impression from the following scenario:

For example, there are two portfolios A and B, the expected annual returns and standard deviations of portfolio A and B are present respectively in the second and third rows of the table 10 below, the risk-free rate of return is 5%. With the equation (7.1), we can easily calculate the Sharpe ratio for portfolio A and B, as $S_A = 1.5$ and $S_B = 2$. It means that if an investor has to bear one more unit of risk exposure of both two portfolios, the increasing of expected return he may grand from portfolio B is higher than portfolio A. In this case, he can borrow in additional same amount of the investing budget of portfolio B with risk-free rate of interest and double the investment. This is a leveraged position in adjusted portfolio B. The proportion that invested in the risky portfolio is

$$\begin{aligned}
Y &= \text{budget amount} * 2 / \text{budget amount} \\
&= 2
\end{aligned}$$

Which means $(1-y) = -1$, indicating a short position in the risk-free asset.

For calculate Sharpe ratio of the adjusted portfolio B:

$$E(r_{\text{adjusted B}}) = 2 * 15\% + (-1) * 5\% = 25\%$$

$$\sigma_{\text{adjusted B}} = 2 * 5\% + (-1) * 0 = 10\%$$

$$S_{\text{adjusted B}} = (25\% - 5\%) / 10\% = 2$$

	$E(r_{\text{portfolio}})$	$\sigma_{\text{portfolio}}$	r_f	Sharpe ratio S_P
<i>Portfolio A</i>	20%	10%	5%	1.5
<i>Portfolio B</i>	15%	5%	5%	2
<i>Adjusted portfolio B</i>	25%	10%	5%	2

Table 10: Example of using Sharpe ratio to optimize the portfolio.

We can see the results in the last row and compare them with original portfolio A and B. we find that the loan-taking adjusted portfolio B has same standard deviation as original portfolio A, but higher expected return than original portfolio A; at same time the Sharpe ratio of the adjusted portfolio B is still as same as original portfolio B. So the investor can get higher return without increasing the self-owned capital through borrowing and investing in adjusted portfolio B with higher Sharpe ratio. Notice again that the premise is investor can borrow in at risk-free interest rate. However, this proves that the portfolio with higher Sharpe ratio always can have the better return than the portfolio with lower Sharpe ratio when the risk exposure is at same level.

In another words, along the capital allocation line, investor can always

construct optimal portfolio by adjusted the proportion between of risk-free and risky. That's why we should consider the return conditionally with risk. The principal is simple: the highest return with lowest risk.

7.2.2 Sharpe ratio analysis of the product

Same story that we can try to figure out the best investment alternatives between portfolio alternatives in this product investment by comparing and ranking the Sharpe ratio of the different portfolio constructions.

We can see this product is constructed with risk-free asset and risky portfolio with 7 equal weighted underlying assets as stock prices. Among all involved financial assets, we can reconstruct the investment into alternative strategies with different assets construction.

We analyzed the expected return on the product, but what is the return on other involved assets? Should investors prefer the portfolio just like the product, or other alternative assets? First we consider a portfolio with only risky asset that is constructed with the 7 underlying stock prices. Then we can also invest in the portfolio with only some of the 7 underlying stock prices. At last we can even consider the portfolio with risk-free asset and risky asset that is constructed with part of the 7 underlying assets. Some of the estimated Sharpe ratios of these alternative portfolios show in the table 11.

If invest as 100% loan financed, it's a leverage position in the risky portfolio with 7 underlying assets. It got a negative Sharpe ratio as in the row 3 which means that when increasing per unit risk, the expected return doesn't increase but reduce. I believe that the reason is risk exposure of taking loan to invest, and the rate of return on financial loan is much higher than risk-free rate of return.

<i>product</i>	<i>Expected</i>	<i>volatility of exp.</i>	<i>Sharpe</i>	<i>Ranking</i>
<i>construction</i>	<i>return</i>	<i>return</i>	<i>ratio</i>	
<i>Issued product</i>	2.09%	4.17%	-0.28	10
<i>100% loan product</i>	-3.88%	4.90%	-1.46	11
<i>Only underlying p.</i>	6.42%	45.16%	0.07	6
<i>Market index</i>	8.65%	30.20% ¹¹	0.18	1
<i>STL</i>	3.926%	31.6%	0.02	8
<i>TEL</i>	4.54%	41.01%	0.03	7
<i>ORK</i>	3.634%	36.17%	0.01	9
<i>DNB</i>	7.354%	39.30%	0.10	3
<i>YAR</i>	7.512%	50.50%	0.08	5
<i>NHY</i>	8.37%	46.50%	0.11	2
<i>REC</i>	9.622%	71.07%	0.09	4

Table 11: Sharpe ratio and ranking of alternative product strategies, mean and standard deviation of the product expect return are computed assuming 100%

self-financed and no management costs

Usually, when we measure the risk as standard deviation, the Sharpe ratio of a guaranteed product in portfolio is lower than the Sharpe ratio of the underlying assets. The result of Sharpe ratios in table 10 is just in consistent with this, although the product expected return is even lower than the risk-free rate and market index return. Sharpe ratios of each underlying stock and the portfolio of all 7 underlying stocks all have higher ranking than the product.

I use Espen Sirnes's estimate of the market index standard deviation and Sharpe ratio, it can be seen in Appendix 2. Although the period of his estimate is not 4 years as the product's running period, it shows that the Sharpe ratio results are almost same when they are estimated from historical data in different lengths of period.

From the ranking comparison, we find that all the alternative strategies with the relevant assets of the product have worse performance than market index. With my simulation result, the optimal investment here is market index; the worst investment is 100% loan financed invest of the product. So investors should invest in any other alternative strategies rather than the product.

7.3 Sortino ratio

¹¹ The standard deviation is citing from Espen Sirnes's estimation with data from 1915-2009. See Appendix 2.

Sortino ratio was developed by Frank A. Sortino about 20 years ago to differentiate between good and bad volatility in the Sharpe ratio. Sortino ratio is pretty like the Sharpe ratio, the different is Sortino ratio uses downside deviation, rather than standard deviation to measure risk. The basic thinking point for using downside risk is that those positive movements on return are consistent with investors investing aspiration; therefore they shouldn't be counted in for measuring risk. So we adjust risk by the standard deviation of returns which only below the expected return, here we denote that the investors expect a risk-free rate return at least.

It is calculated as follows:

$$\text{Sortino ratio} = (E(r_C) - r_f) / \sigma_d \quad (7.3)$$

Here σ_d represents downside risk, and we use the lower partial standard deviation (LPSD) to measure the downside risk. Usually we use the risk-free rate of return to stand for investors expected return, and then the lower partial standard deviation is the standard deviation of the returns which are higher than the risk-free rate of return.

But for this product, the mean of expected return is even lower than the risk-free rate, to compute the downside risk becomes just not meaningless. So I won't analyze here, but for many other products, it will be more realistic to predict the risk then using standard deviation.

8. Conclusion

8.1 The situation of ELN

I have seen such news on Norwegian financial and economic news website E24.no with the title: Prohibit structured products¹². Main context was that there was going to come on a new rule for restricting or even prohibiting banks' market operation of loan financed compound structured product or selling such kind product to normal customers. The reason was that the Banking Complaints Board have received considerable quantities of complain from structured product customers who thought they knew the structured product well and it was a nice choose to control the loss with certain gain, but at last suffered in loss.

My analysis result seems to go to the same direction to be a critical exemplification, although a flaw may not represent the entire.

The problem we should ask first is that if the common investors know the risk and return on the investment of structured products as well as common knowledge? Is the common investor obtain the all the relevant information and data which could affect the return and risk on structured product just as the professional investor (banks and companies) obtained? Beside, issuing institutions should enhance the level of effective information disclosure. It shouldn't only appear the positive information

¹² See <http://e24.no/boers-og-finans/article2246709.ece>

that corresponds with issuers benefit on the prospect brochure of product, but also the potential negative point should be released. As this product in the article, the bank using the strong market points to give the optimistic forecast without a cautious consideration of the volatility of the return. It doesn't mean at we should take conservative attitude to the product, but give the complete product information to investors, both the expectation of return and the potentiation of risk.

The most important point investors should pay attention on is the product structure, for the product in the article, the structure is the main reason that investors may suffer in loss.

8.2. Investment with the product

Even do not consider the high commissions and costs; the expected return of the product is lower than the risk-free return. Loan-financed product is even worse, it's like investors are surely to pay the high interest when they didn't even know that if they will get a return that could cover it. And because investors will never get risk-free interest rate on loan, so at least they have to get a return beyond the margin between loan interest rate and risk-free rate to make sure that they won't lose.

The product is not as flexible as stock and other portfolios. Investor can decide to hold stocks in short-term or long-term and just need to wait a position when stock price is high and sell it to benefit. But the

equity-linked product has fixed holding period, if underlying stock price is extremely low, investors get loss; if the underlying stock price is extremely high, we still need to see if there is “Asian tale” to average the extreme price which means the return is adjustedly reduced.

8.3 Variance of Monte Carlo

Monte Carlo simulation I’m taking here is a developed method that it uses the positive defined matrix of correlation coefficient to consider the correlation between the underlying assets. But according to a report of Doctor Filonberg in Georgia University, there are 5 most popular and widely used program for generating random variables which gives wrong predict when they were used in a simple simulation of atoms behavior in the magnetic crystal. Therefore, to control the clustering of generated random variables by more proper random variable generator and to improve the Matlab program is very necessary to reduce the variance of Monte Carlo simulation result.

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Appendix 1

Explanation of the fundamental and database for estimating the efficient rate of return on government bonds and estimated historical data

Government bonds

Norwegian synthetic 3-, 5- or 10-year effective yields. Synthetic yields are calculated by weighting two Treasury bills with short and long remaining terms to maturity respectively. Prices used are those most recently traded if the most recently traded price is within the spread. Otherwise, the price is equal to the bid or offer price closest to the most recently traded price. If no trading has taken place, the price is equal to the middle price.

Before 1993: The series contains annual effective yields payable in arrears on government bonds for 0-3 years, 3-6 years or more than 6 years. The yield was calculated as a weighted average of volume outstanding for government bonds with remaining terms to maturity of 0-3 years, 3-6 years and more than 6 years quoted on the Oslo Stock Exchange. Daily closing quotes (middle prices or traded prices).

NB: The period 6 March 1987 – 31 December 1989 contains weekly closing quotes.

Source: Oslo Stock Exchange 4 p.m. Calculations by Norges Bank (Department for Market Operations and Analysis)

Annual average of daily quotes

	3 year	5 year	10 year
2009	2.71	3.33	4.00
2008	4.53	4.43	4.47
2007	4.79	4.77	4.78
2006	3.74	3.90	4.07
2005	2.90	3.27	3.74
2004	2.95	3.61	4.36
2003	4.24	4.58	5.04
2002	6.39	6.36	6.38
2001	6.44	6.31	6.24

2000	6.61	6.38	6.22
1999	5.39	5.39	5.52
1998	5.32	5.34	5.40
1997	4.62	5.12	5.89
1996	5.46	5.98	6.78
1995	6.36	6.89	7.43
1994	6.59	7.04	7.46
1993	6.54	6.62	6.86
1992	10.54	9.78	9.62
1991	10.05	9.91	9.99
1990	10.97	10.71	10.68
1989	10.93	10.81	10.86
1988	13.13	13.07	12.88
1987	ND	13.58	13.31
1986	ND	13.56	13.30
1985	ND	ND	12.91

Appendix 2

The article is Cited directly from Espen Sirnes blog. For the computation can see <http://ansatte.uit.no/esi000/risikopremieOB.xls>.

FREDAG 7. MAI 2010

Risikopremien for Oslo Børs 1915-2009

Risikopremien i det norske aksjemarkedet er på 5,4%. Beregningen er gjort på grunnlag av data for perioden 1915-2009. Beregningen finner du [her](#). Om du har forslag til forbedringer av metoden, så tar jeg gjerne i mot. Det bør spesielt nevnes at beregningen baserer seg på en modellmessig estimering av utbytter for perioden 1915-1969.

Å kjenne til markedets risikopremie er helt avgjørende for å kunne beregne lønnsomheten til potensielle prosjekter. Markedets risikopremie er den avkastningen som børsen gir utover risikofri rente på sikt. I Norge vil det si avkastningen som du kan forvente på Oslo Børs over tid utover den risikofrie bankrenten.

Alternativet for enhver investering er å investere på børsen. Av den grunn er det et generelt krav til investeringer at den gir bedre avkastning enn markedet over tid, justert for risiko. Dette er hva vi kaller et "avkastningskrav", og er selve kjernen i det som kalles "kapitalverdimodellen".

For å kunne gjøre en slik vurdering er det imidlertid helt nødvendig at man vet hvilken avkastning børsen gir over tid. Overraskende nok er det ikke gjort noen grundige empiriske studier av dette for det norske aksjemarkedet de siste årene, etter det jeg vet. Det eneste jeg kunne finne var en referanse til Johnsen (1996), men den er ikke tilgjengelig på nett og uansett gammel.

	<i>1915-2009</i>	<i>1970-2009</i>	<i>1990-2009</i>
<i>Aksjeavkastning</i>	10.7 %	18.0 %	12.4 %
<i>Rente</i>	5.3 %	10.3 %	6.3 %
<i>Risikopremie</i>	5.4 %	7.7 %	6.1 %

Mangelen på empiriske studier kan kanskje forklares av at det er svært

vanskelig å finne data tilstrekkelig langt tilbake for Oslo Børs. MSCI Barra, som samarbeider med børsen om indeksene, har bare tall tilbake til 1970. Vi ser fra tabellen over at om en legger disse tallene til grunn blir aksjemarkedets risikopremie (dvs. avkastning over risikofri rente) på hele 7,7 %, hvilket er nokså høyt. I denne perioden har man faktisk tjent i gjennomsnitt 18 % hvert år. Årsaken er sannsynligvis en meget sterk økonomisk vekst på 70 og 80-tallet, som vi ikke kan regne med å se igjen i fremtiden.

Man kan imidlertid få bedre estimater på risikopremien dersom en går lenger tilbake, ettersom ekstreme avkastninger i spesielle perioder da blir av mindre betydning for gjennomsnittet. Heldigvis finnes det derfor tall både fra SSB og i kapittel Eithrem (2004) lenger tilbake. Ulempen er at disse ikke er justert for utbytter. Utbytter utgjør en betydelig del av markedets avkastning, så å ignorere dem vil gi helt feil resultat.

Nivået på utbyttene varierer imidlertid systematisk med avkastning fra gjeldende og forrige periode (høy avkastning denne perioden gir høye utbytter, og høy avkastning forrige periode gir lavere utbytte). Jeg har derfor estimert utbyttene i perioden 1915-1970 med en enkel modell der utbyttene er avhengig av årets og fjorårets ujusterte avkastning. Parametrene er estimert med dataene fra perioden 1970-2009 hvor både utbytte og avkastning er kjent. Modellen forklarer hele 48,5% av variasjonen i utbyttene, noe som tyder på at dette er en hensiktsmessig måte å løse problemet på.

Det kan også være interessant å ta med standardavvikene og Sharpe Ratioen:

	<i>1915-2009</i>	<i>1970-2009</i>	<i>1990-2009</i>
<i>SD</i>	30,2 %	43,1 %	32,3 %
<i>Sharpe Ratio (SR)</i>	0,18%	0,18	0,19

Vi ser at en overraskende stabil SR. I perioden 1970-2009 er det imidlertid betydelig større usikkerhet (SD). Dette tas imidlertid igjen i form av større avkastning slik at SR er relativt uforandret.

Litteratur:

Johnsen, T. (1996): "Avkastningskrav ved vurdering av lønnsomheten i statlig eiet forretningsdrift" *SNF-rapport 90/96*

Øyvind Eitrheim, Jan T. Klovland and Jan F. Qvigstad (eds.) (2004): "Historical monetary statistics for Norway 1819–2003", *Norges Banks skriftserie / Occasional Papers nr. 35, kap. 8*

Appendix 3

The Matlab program to estimate volatilities and correlation coefficients by historical prices.

```
Load "price1"  
Load "price2"  
Load "price3"  
Load "price4"  
Load "price5"  
Load "price6"  
Load "price7"  
price = [price1 price2 price3 price4 price5 price6 price7]  
return1 = price2ret(price)  
dt = 1/12  
rf = 0.0325  
[amean, astd, lowbnd, upbnd] = meanstd(return1, dt)  
ELPM = elpm(amean, astd, rf)  
cor = corcoef(return1)  
C = chol(cor)
```

Appendix 4

The Matlab program to simulate the underlying stock prices and estimate the product expected return.

```
clear, clc, close all  
  
minRet = -0.075  
maxRat = 0.15  
step = 0.015  
RB = minRet:step:maxRat  
P = length(RB)
```

```

ProbP = zeros(1,P);

% NEED TO DEFINE THE PARTICIPATION RATE
AF=0.92

rf = 0.0325
rloanpre = 0.0207
rloan = rf+rloanpre
rloan4 =(1+rloan)^4-1

prompt = {'Percentage of loan', 'tegningsomkostninger, in percent'}
dlg_title = 'input required data'
lines= 1
def = {'100', '3.0'}
res = inputdlg(prompt,dlg_title,lines,def)

if isempty(res)
    return
end

a = str2num(res{1})/100
kost = str2num(res{2})/100

ExpReturn = [0.0735 0.0837 0.0363 0.0962 0.0393 0.0454 0.0751];
sigma = [0.3930    0.4650    0.3617    0.7107    0.3160    0.4101    0.5050];
cor = [1.0000    0.6074    0.5450    0.0298    0.4023    0.4951    0.4707
       0.6074    1.0000    0.5457    0.1013    0.7378    0.3840    0.5556
       0.5450    0.5457    1.0000    0.0190    0.5881    0.4964    0.4367
       0.0298    0.1013    0.0190    1.0000    0.1168    0.2433    0.0726
       0.4023    0.7378    0.5881    0.1168    1.0000    0.3247    0.4730
       0.4951    0.3840    0.4964    0.2433    0.3247    1.0000    0.6561
       0.4707    0.5556    0.4367    0.0726    0.4730    0.6561    1.0000];

C = [1.0000    0.6074    0.5450    0.0298    0.4023    0.4951    0.4707
     0    0.7944    0.2702    0.1048    0.6211    0.1049    0.3396
     0    0    0.7937    -0.0322    0.2533    0.2497    0.1115
     0    0    0    0.9935    0.0482    0.2270    0.0267
     0    0    0    0    0.6212    -0.0222    0.0696
     0    0    0    0    0    0.7934    0.4475
     0    0    0    0    0    0    0.6670];

```

```

S0 = [100 100 100 100 100 100 100];
w = [1/7 1/7 1/7 1/7 1/7 1/7 1/7];

k = 7; % simulate only 7 prices
dt = 1/12*ones(1,k); % time interval half a year
dt(1) = 3.5; % first interval 3 and a half years

N = 100000; % Number of simulations

SP = zeros(7,k+1);
AP = zeros(1,N);
arp = zeros(1,N);
AZero = 0

h = waitbar(0,'Please wait...');
for n=1:N
    for i=1:7
        SP(i,1) = S0(i);
    end
    dZ = randn(7,k);
    dW = C'*dZ;
    for j=1:k
        for i=1:7
            SP(i,j+1)=SP(i,j)*exp((ExpReturn(i)-0.5*sigma(i)^2)*dt(j)+sigma(i)*sqrt(dt(j))*dW(i,j));
        end
    end
    % compute the mean from the second observation
    SF = [mean(SP(1,2:k+1)) mean(SP(2,2:k+1)) mean(SP(3,2:k+1)) mean(SP(4,2:k+1))
mean(SP(5,2:k+1)) mean(SP(6,2:k+1)) mean(SP(7,2:k+1))];
    for i=1:7 %
        AP(n) = AP(n)+ w(i)* min(0.8,(SF(i)-S0(i))/S0(i));
    end

    if AP(n) < 0
        AZero = AZero + 1;
    end
    % Note that there are management costs of 3%
    AP(n) = (0.97+AF*max(AP(n), 0)-(1+kost)*(1+a*rloan4))/(1+kost);
    arp(n) = (1+AP(n))^(1/4)-1;

    % compute the probabilities
    for i = 1:P

```



```

        if((arp(n) > RB(i)-step/2) & (arp(n) < RB(i)+step/2))
            ProbP(i) = ProbP(i)+1;
            break
        end
    end
end

if(rem(n,1000)==0)
    waitbar(n/N)
end
end

close(h)

% normalize the probabilities
ProbP=ProbP/N

p = 0;
for i=1:N
    if(AP(i)<=0)
        p = p+1;
    end
end
end
probofloss = p/N

p = 0;
for i=1:N
    if(arp(i)<=0.0325)
        p = p+1;
    end
end
end
proboflessthanrf = p/N

fprintf('Mean annual return %f\n',mean(arp))
fprintf('Std annual return %f\n',std(arp))
Skew = skewness(arp)
Kurtosis = kurtosis(arp)

figure
bar(RB,ProbP)
title('probability distribution')

```

```
xlabel('annual return')  
ylabel('Probability')  
axis([minRet maxRat 0 0.6])
```