## FINDING THE BEST PORTFOLIO FOR THE LONG TERM INVESTOR

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This Master Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

## Preface

This thesis is written as the final part of the master's programme in business administration at the University of Agder.

Today it is common for financial advisors to recommend long-term investors to allocate most of their investments in stocks, as one assumes that stocks are less risky in the long-term.

However, some research shows that bonds are likely to perform better than stocks in the long run.

With this thesis I want to find out which portfolio that is best for the long-term investor. I think it is an interesting problem to discuss, because long-term investments are relevant for almost everyone, either they are saving for their retirement or in college funds for their kids etc, and therefore also my results are relevant for many.

Earlier in my master's degree I have taken the course BE-411 Derivatives and Risk Management which gave me an introduction to the program Matlab. Therefore it was natural for me to use Matlab in my analysis.

I would like to thank my supervisor Valeri Zakamouline for useful guidance and for helping me develop the Matlab programs.


#### Abstract

This paper examines different portfolios of U.S financial assets, trying to find the best portfolio for the long-term investor. I compare bond portfolios with stock portfolios based on statistics as book-to-market ratio, market capitalization, earnings yield, dividend yield and cash flow yield, stock portfolios based on industry and the market portfolio. The ranking devices I am using are Sharpe ratio and Sortino ratio. Dependent on the assumption on whether the returns are independent and identically distributed or not, the probability distribution of the return are simulated by using two different methods, the standard bootstrap method and the block-bootstrap method. Since I am calculating both the Sharpe ratio and the Sortino ratio with both methods, I have four different outcomes of my analysis.

The results depend on my assumptions, when I calculate Sharpe ratios by using the bootstrap method, bonds tend to outperform stocks. In the three other scenarios however, my analysis shows that stocks tend to outperform bonds, and stock portfolios with high book-to-market ratios, high dividend yields, high earnings yield or high cash flow yield tend to perform better than other portfolios.


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## 1 Introduction

It is a common knowledge, that for a long-term investor, the best choice is to allocate most of the investment portfolio in stocks. The concept behind this thinking is called the "time diversification". The concept suggests that as the time horizon increases, the volatility of stocks will decrease as high returns will tend to offset the low returns. Some researchers support the concept, but the concept is also often criticized.

Some studies show that the best long-term investment is investing in bonds, while others show that stocks are the best investment. Some results show that large capitalization stocks outperform small capitalization stocks, while others get the exact opposite result.

There are several reasons to why researchers' results are so conflicted, even though most of them are using the same data. There are differences in what performance measure they are using, Sharpe ratio is the most commonly used measure, but also Treynor ratio and Sortino ratio is used. There are also differences in the assumptions researchers are taking whether returns are independent and identically distributed or not, and this reflect on the way of sampling. And there are also differences in what frequency of the considered returns that are used.

### 1.1 Topic

In this paper I am trying to find the best portfolio for the long-term investor. There are several complications with solving this problem. To evaluate the performance of a portfolio I have to use a performance measure. One of the complications is which performance measure I should use to determine which portfolio that is the best portfolios for a long-term investor. Different performance measures have different advantages and disadvantages. Which performance measure that should be used, also depends on the investors preferences. Sharpe ratio is the most widely used performance measure, but it is also criticized for assuming that the distributions of returns are normally distributed and that investors have quadratic preferences.

A lot of new performance measures have been developed lately; however, there is still no agreement on which measure that is most accurate and reliable. Sortino ratio is one of these new measures. These two ratios are the performance measures I use to compare the portfolios, and I base my suggestion on which portfolio that is the best portfolio for the longterm investor on these measures.

To be able to compute Sharpe ratio and Sortino ratio I need to know the probability distribution. This leads to the second problem; the data of the annual returns of the bonds and stocks only go from 1926/1927 to 2008/2009. That means that there are just above 80 years of data, and when my holding period is 20 years there is only 4 periods that do not overlap each other. This will make the precision of the estimation of the probability distribution very low, and it will not be good enough to make a reliable suggestion on which portfolios that will perform best over a 20 year period. As a result of the lack of annual returns, I need to use simulations. To solve this problem I am using bootstrap methods to simulate the probability distribution.

The third complication is whether the distribution of the returns is independent and identically distributed or not independent and identically distributed. It is not possible to reject either of the possibilities, so my analysis includes both assumptions. There are two different bootstrap methods; the standard bootstrap method which assumes that returns are independent and identically distributed and the block-bootstrap method which assumes that returns are not independent and identically distributed. Since neither hypothesis can be rejected, I have to use both methods.

### 1.2 Structure

I am starting this thesis by looking at earlier studies. Earlier studies might give me an idea of what I can expect in my analysis, but it also shows the spectre of different opinions and results.

Chapter 3 is a theoretical chapter where I first explain the concept of time diversification. Afterwards I present the portfolios and their characteristics. I am also explaining how the
long-term risk for stocks and bonds are affected by different aspects in the society. Afterwards I am explaining the basic key measures that are needed to be able to calculate and understand the performance measures. In the next section I explain why we need performance measures to be able to rank the different portfolios and different categories for the measures, and in the end of this chapter I present the two performance measures I have chosen to use.

Chapter 4 is the methodology chapter, where I first explain why we need to use simulations methods. Afterwards I am presenting the two different methods; the standard bootstrap method and the block-bootstrap method, with a focus on how they work, which assumptions they are relevant for, and complications that may occur.

Chapter 5 describes the data. This chapter describes where the returns are collected, and it shows how the stocks are divided into portfolios. Also the annual means, standard deviations, skewness and kurtosis of the portfolios are presented here.

Chapter 6 is the analysis. It starts with a comparison of bonds and stocks, divided in portfolios by both their capitalization size and their book to market ratio. This analysis is followed by an analysis where the bonds are compared to stock portfolios based on the same parameters again, except that there are now 3 portfolios based on the capitalization size and 3 portfolios based on the book-to-market-ratio. In these first two analyses I am mainly checking whether stock portfolios or bond portfolio perform best, and if small cap stocks and high book-tomarket ratio stocks perform better than the other stocks, which some earlier studies has shown.

In the third analysis I am using several different parameters to divide the stocks into their portfolios; cash flow yield, earnings yield and dividend yield, and I am comparing the portfolios based on these parameters to the portfolios based on cap size and book to market ratio. All the stock portfolios are compared to the bond portfolios to see which perform best. In the last analysis I am not dividing the stocks into portfolios based on different properties, but after 30 different industries. I am looking at which industry that performs best, and if the industry portfolios perform better than the bonds.

In every section I am looking at both Sharpe ratios and Sortino ratios, and since I calculate the ratios by using both the bootstrap method and the block-bootstrap method, I have four different results for each section.

In each of these sections I first look at the figures and/or tables and describe each of them, and then I discuss and compare the different results I get in every table.

Chapter 7 is the discussion of the results and the conclusion.

## 2 Review of literature

There are a lot of studies on whether bonds or stocks perform best in the long run. Some studies show that a long-term investor should prefer stocks, while other studies show the exact opposite. There are several reasons for these conflicted results, the two most important may be the choice of performance measure and the assumption on whether stocks are independent and identically distributed. There might also be differences in the empirical data, some researchers have chosen to use monthly returns, while others are using annual.

Levy is perhaps the pioneer in this type of research. Levy (1972) analyzed the performance of assets using Sharpe ratio and showed that as the time horizon increases, Sharpe ratio tend to first increase, and then start to decrease. He found this pattern especially with assets with high volatilities, while assets with lower volatilities might have increasing Sharpe ratios as the time horizon increases. This means that defensive assets as bonds would over longer time periods outperform all stocks. Several studies have in the years after supported that the time horizon has an important effect on the performance measures, among others; Chen and Lee (1981), Levy (1981), Levy (1984), Chen and Lee (1986) and Levy and Samuelson (1992). Most of these studies have assumed that returns are independent and identically distributed.

Hodges, Taylor and Yoder (1997) saw the same effect, by comparing the Sharpe ratios of common stocks with small stocks and long-term corporate bonds, they showed that initially the common stocks outperform the small stocks and the bonds, but for longer holding periods the bonds outperform both stock portfolios. Also in this study it is assumed that returns are independent and identically distributed, and they use the standard bootstrap method for their simulations.

Lin and Chou (2003) compare different stock portfolios based on the stocks market capitalization. They show that when one uses a standard bootstrap method the big cap stocks outperform both mid cap stocks and small stocks according to Sharpe ratio. In their study the time horizon does not affect the ranking, big cap stocks perform best both in short and long horizons. They are also using the block-bootstrap method and they show that mid cap stocks perform best in the long run.

Mukherji (2002) use the Sortino ratio to compare small stocks and big stocks to T-bills and bonds. The results show that for short horizons T-bills perform best, intermediate-term bonds are preferable for medium-term investments, large stocks for long-term and small stocks for very long-term investments. However, the author suggest investing in both stocks, bonds and bills, even when the holding periods are expected to be long, in case the holding period may turn out shorter than expected. Sinha and Sun (2005) also find that small stocks perform best over long horizons when one is using the Sortino ratio to compare the portfolios. They used the bootstrap method for the simulations.

There has also been some research on if different statistics as book-to-market ratio, market capitalization etc, have an impact on the performance of the portfolio. Basu (1977 and 1983) discovered that portfolios with low price-to-earnings ratios performed better than portfolios with high price-to-earnings ratios. Banz (1981) was the first to document the effect of the size of firms. He divided the NYSE stocks into 10 portfolios based on the firm size, and showed that the smaller the firm, the bigger excess returns it would generate. The difference in the annual returns that he had calculated was $10,3 \%$ between the portfolio with the smallest firms and the portfolio with the largest firms. Smaller firms tend to be more risky than larger ones, but even when the returns are adjusted for risk by using the CAPM the small firms would outperform the larger firms.

Fama and French (1992) showed that also the book-to-market ratio has an impact on the returns of the portfolios. Fama and French divided the stocks into 10 different portfolios based on their book-to-market values, and showed that the portfolio with the highest book-to-market ratios outperform the portfolios with lower book-to-market ratios.

Looking at earlier studies would normally give one an insight in what results one could expect in the analysis to be performed, and this is also the case in my thesis of course. But at the same time since the earlier results are so conflicted, it makes my analysis even more interesting as the results are not given in advance.

## 3 Theory

In this chapter I first explain the concept of time diversification. Then I look at the different portfolios I am using in my analysis, and I explain what some of the statistics I use mean. This is followed by a section that shows which factors in society that affect the long-term risk of bonds and which factors that affect the long-term risk of stocks. I need to calculate a performance measure to compare the portfolios. I have chosen to use two "reward-tovariability" ratios; Sharpe ratio and Sortino ratio. To compute the measures and understand why they might rank the portfolios differently I need to know some key measures as mean, standard deviation, skewness and kurtosis. I explain how these key measures are calculated and what they mean before I focus on the performance measures.

### 3.1 Time diversification

It is generally accepted that the investment horizon plays a crucial role in determining the optimal investment portfolio composition. Financial investors commonly believe that as the investment horizon increases, so should the part of the portfolio consisting of stocks do. The concept of time diversification suggests that the volatility in the returns of stocks will decline as the investment horizon increases. The theory behind this is that over time returns above average will tend to offset returns below average, and therefore decline the probability of loss.

If the concept of time diversification holds, then when the investment horizon gets sufficiently long, the optimal investment portfolio would consist only of stocks. The only problem would be to find out how long is a long-term investment, and to predict when the time diversification benefits start to kick in.

However, the issue of time diversification has been controversial. Some studies support time diversification (as for example Mukherji (2002)), and some studies do not (as for example Hodges, Taylor and Yoder (1997)).

### 3.2 Portfolios

In this paper I compare different stock portfolios and bond portfolios. I have three different bond portfolios; long-term corporate bonds, long-term government bonds and intermediateterm government bonds. The stock portfolios are different stock portfolios based on different measures or different industries and the market portfolio

### 3.2.1 Bonds

The portfolios of bonds are consisting of long-term corporate bonds, long-term government bonds or intermediate-term government bonds. The long-term government bonds portfolio and the intermediate-term government bonds portfolio are both a one-bond portfolio, while the long term corporate bonds portfolio consists of bonds from Citigroup, Long-term high grade and Corporate bond index.

### 3.2.2 Stocks

In addition to the market portfolio and the industry portfolios, I also use stock portfolios based on different properties of the stocks; the capitalization size, book-to-market ratio, dividend yield (dividends-to-price ratio), cash flow yield (cash flow-to-price ratio) and earnings yield ( earnings-to-price ratio).

### 3.2.2.1 Size

When I use portfolios of stocks based on size, the size (small, medium, big) refers to the size of the market capitalization. The market capitalization is calculated by multiplying the number of a company's outstanding shares by its stock price. The definition of a small cap stock can vary among brokers, but in general it is a company with a market capitalization
from $\$ 300$ million to $\$ 2$ billion. ${ }^{1}$ Large or big cap stocks are normally referring to stocks with a market capitalization value above $\$ 10$ billion. ${ }^{2}$ The mid cap stocks will of course be between the small cap stocks and the big cap stocks, ergo between $\$ 2$ billion and $\$ 10$ billion.

### 3.2.2.2 Book-to-market-ratio

The book-to-market ratio is used to find the value of a company by comparing the book value of a firm to its market value. Book value is computed by looking at the historical cost or accounting value of the firm and the market value is determined in the stock market by its market capitalization. ${ }^{3}$ It is calculated as follows:

Baok - to - market ratio $=\frac{\text { Book value of firm }}{\text { Market value of firm }}$

If the book-to-market value is more than 1 , one would usually suggest that the stock is undervalued. Many investors will think of this as a good investment. This is because obtaining a ratio greater than one requires the book value to exceed the market value, which may indicate that investors have not given the company the credit it deserves. Similarly, if the book-to-market ratio is below 1 , one would usually suggest that the stock is overvalued. However, companies that do not have a lot of physical asset will also often have low book-tomarket ratios. ${ }^{4}$

The book-to-market ratio will usually be more than 1 for most companies. As a result of this, low book-to-market ratios do not necessarily mean below 1 and high book-to-market ratios do not necessarily mean above 1 . The relationship between low, medium and high is relative. Low book-to-market ratio could for instance mean a book-to-market ratio below 3, and high book-to-market ratio could for instance mean a book-to-market ratio above 10 .

[^0]Stocks with high book-to-market ratios are often referred to as value stocks, while stocks with low book-to-market ratio are called growth or glamour stocks.

### 3.2.2.3 Dividend yield

Investopedia defines the dividend yield as follows: "The dividend yield is a financial ratio that shows how much a company pays out in dividends each year relative to its share price. In the absence of any capital gains, the dividend yield is the return on investment for a stock". 5 Dividend yield is calculated as follows:

Dividend yield $=\frac{\text { Annual dividends per share }}{\text { Price per share }}$

Well-established companies tend to have higher dividend yields than companies that are less established. Newly established firms may not pay any dividends at all, because they are keeping all the money in their companies to maximize their growth.

### 3.2.2.4 Cash flow yield

The cash flow yield or cash flow-to-price ratio is calculated as follows:

$$
\text { Cash flow yield }=\frac{\text { Cash flow per share }}{\text { Price pershare }}
$$

[^1]
### 3.2.2.5 Earnings yield

The earnings yield shows the percentage of each dollar invested in the stock that was earned by the company. The measure is used by investor to determine the optimal asset allocation. It is the opposite of the price-to-earnings ratio. The earnings yield or earnings-to-price ratio is calculated as follows:

## Earnings yield $=\frac{\text { Earnings per share }}{\text { Price per share }}$

In general, a low ratio indicates that investors are expecting higher earnings growth in the future compared to companies with a higher rate. However, it is more useful to compare the earnings-to-price ratio with other companies in the same industry, the whole market or earlier earnings-to-price rates for the company it selves, than comparison between industries.

The earnings yield of a broad market index can be compared to for instance the 10-year Treasury yield. If the earnings yield to the market index is less the 10-year Treasury yield, the market may be overvalued as a whole. ${ }^{6}$

### 3.2.3 Difference in risk

In the long run stocks and bonds do not face the same type of risk. The most important longterm risk for stocks is how the trend growth rate of the overall economy will develop. Corporate earnings are the fundamental determinants of stock prices. Changes in inflation rates and the government budget are likely to affect stocks in the short run, but the long-term growth rate for corporate earnings may not be affected. In the long run businesses can increase the prices of their product so that the customers are paying for the increase in costs

[^2]due to a higher inflation rate, so that corporate earnings are not affected. However, inflation rates and the government budget do affect stock prices indirectly, through their impact on trend productivity growth. If this growth rate increases due to lower inflation, then the stock returns will also increase.

The long-term risk concerning government bonds are changes in the long-term inflation rate and government budget. Since bonds have fixed nominal coupon rates, changes in the inflation will affect the real return. An increase in the inflation rate will reduce the real return for investors. If the increase leads to a higher expected inflation rate in the future, investors will demand higher nominal coupon rates on new bonds. This will depress the prices on currently held bonds.

An increase in government budget deficits also affects the real returns from government bonds. Increasing government spending without increasing income will mean a higher future borrowing for the government. In theory this leads to higher interest rates on new government bonds, reducing the price on existing bonds.

### 3.3 Key measures

We need to use one or several performance measures to be able to give an estimation of which one of the portfolios that performs best in the long term. Most performance measures are calculated by dividing the excess return over a measure of risk. The risk may be market risk, standard deviation, downside deviation etc. The excess return is calculated by subtracting a benchmark as the risk free rate from the expected return. We therefore need to know some key measures to able to calculate and understand the performance measures. Four key measures are especially important; the mean, variance, skewness and kurtosis.

### 3.3.1 Mean

The mean is the average of two or more observations. It can be calculated as follows:

$$
\mu=\frac{1}{N} \times \sum_{i=1}^{N} x_{i}
$$

The mean or expected value of the returns can be calculated by taking the average of earlier historical returns.

### 3.3.2 Variance

The variance is a measure of the dispersion of a set of data points around their mean value. Variance is a mathematical expectation of the average squared deviations from the mean. It is commonly used in the world of finance to describe the risk of a security. The standard deviation is the square root of the variance. The variance can be calculated as follows:

$$
\sigma^{2}=\frac{\sum(x-\mu)^{2}}{N-1}
$$

### 3.3.3 Skewness

Skewness describes asymmetry from the normal distribution.

Skewness $=\frac{E\left[x_{i}-\mu\right]^{3}}{a^{3}}$

A normal distribution has zero skewness. If the distribution is skewed to the left, the skewness is negative. This means that the left tail is greater than the right tail. If the distribution is skewed to the right of the normal distribution, the skewness will be positive. This means that the right tail is greater than the left tail. Most investors prefer positive skewness. It is very important to know the value of a security's skewness to be able to better understand performance measures.

### 3.3.4 Kurtosis

Kurtosis refers to the flatness of a distribution. It is defined as:

Kurtosis $=\frac{E\left[x_{i}-\mu\right]^{4}}{\sigma^{4}}$

A normal distribution has a kurtosis of 3. It is normal to calculate the excess kurtosis, which is kurtosis minus 3, so that a normal distribution has excess kurtosis of 0 . Positive excess kurtosis means that the distribution is recognized with a pointy tip around the mean and fat tails on both sides.

### 3.4 Performance measurements

Everyone wants to get the highest return possible. However, investments that are likely to generate a high return will often have a higher risk. Without knowing anything about a investors risk-preference, it is not possible to say which investment is the best, if one of the alternatives has a low expected return and low risk, and the other has a higher expected return and higher risk. We cannot compare to alternatives if we only know expected return and risk. But to be able to compare different portfolios we can calculate different ratios which are comparable.

There are many different measures. These can be divided into categories. First they can be categorized after the type of skills reflected in the measures: Asset selection and market timing. The asset selection category can be divided into the standardized risk-adjusted performance measures and those that explicitly depend on investors' preferences. And the standardized risk-adjusted performance measures can be classified after the measure of value creation, whether it is an excess return or gain potential, and after the type of performance translation, in relative or absolute terms. There are also sub-categories in these categories.

### 3.4.1 Sharpe ratio

Sharpe ratio is a ratio developed by William Sharpe (1966) to measure risk-adjusted performance. The ratio is without doubt the most commonly used performance measure. It is an absolute measure. The ratio is called "reward-to-variability".

Sharpe ratio is defined as the ratio of the mean return in excess of the risk free rate over its standard deviation.

Sharpe ratio $=\frac{\mu-r_{f}}{\sigma}$

Where $\mu$ is the mean return,
$r_{f}$ is the risk free rate, $\sigma$ is the standard deviation

Sharpe ratio assumes that the returns are normally distributed or quadratic preferences.

Sharpe can only be used as a ranking device. One can calculate Sharpe ratio for different assets and compare their performance to each other. If we are comparing different portfolios based on their Sharpe ratio, the best portfolio is the one with the highest ratio. A negative ratio suggests that the investor would be better off placing his money in the risk-free alternative, instead of the stock.

When using Sharpe ratio, both positive and negative risk from the average on the same level, most investors however only fear the negative risk, and if the returns are not normally distributed, and there are differences in the size of the tails, then Sharpe ratio may undervalue or overvalue the risk.

In the last decade several other performance measures have been developed, trying to fix the problems with Sharpe ratio. However, one cannot be sure that they all will behave rationally, due to lack of a solid theoretical underpinning. Some of these new measures are according to Ingersoll, Spiegel and Goetzmann (2007), prone to manipulations. It is possible to manipulate the measure by borrowing and lending. The Sharpe ratio cannot be manipulated by leverage. Many of the newer performance measures are also very complicated, and this may be a part of the explanation of why Sharpe ratio still is so widely used, when there have been developed new measures that are more accurate.

### 3.4.2 Sortino ratio

We have discussed the problem with using standard deviation, and there have been different suggestions how to make a better risk measure. Ang and Chua (1979) introduced the reward to half-variance index, where they had replaced the standard deviation with the half-variance, which considers only the returns lower than the mean. Ziemba introduced in 2005 the
downside-risk Sharpe ratio that has replaced the standard deviation with pure downside risk, which considers only pure losses with a return lower than zero.

Sortino and van deer Meer (1991) suggested that downside risk deals with the risk of not reaching a specific level of return, and according to Sortino and van deer Meer the returns below this level where the only returns including any risk for the investor. This specific level is different for each investor.

One measurement in this sub-category is the Sortino ratio. It is the most widely used measure within its category. This can be calculated by dividing the mean return in excess of the reserve return that specific for each investor over its downside risk below the reserve return.

Sortino ratio $=\frac{\mu-M A R}{D D}$

Where $\mu$ is the mean return,
MAR is the Minimum Average Return,
DD is downside deviation

The MAR is an investor specific minimum level of return. When I am using Sortino ratio in my analysis, I have set the MAR equal to the risk free rate of return.

The downside deviation is calculated as follows:
$D D=\sqrt{\frac{\sum_{i=1}^{N} L_{i}^{2}}{N}}$

Where N is the number of observations,

$$
\begin{array}{ll}
L_{i} \text { is } & r_{i}-\text { MAR if } r_{i}-\text { MAR }>0 \\
& \text { and } \\
& 0 \text { if } r_{i}-\text { MAR } \geq 0
\end{array}
$$

This means that in this paper Sortino ratio is calculated as follows:

Sortinoratio $=\frac{\mu-r_{f}}{\sqrt{\frac{\sum_{i=1}^{N} L_{i}^{2}}{N}}}$

When using the Sortino ratio, we assume that investors do not have quadratic preferences. Instead we assume that investors only fear the downside risk. The ratio is a solution to the problem with using standard deviation, when the distribution of returns is skewed to the right or left. However, it does not solve problems due to kurtosis and autocorrelation.

## 4 Methodology

One of the complications with finding the best portfolio for the long term investor is that there is not enough data available. To compute the Sharpe ratio or Sortino ratio, we need to know the probability distributions of the returns. The data for the returns of the market portfolio is from 1927 to 2009, while the data for the returns of the bonds portfolios are from 1926 to 2008, and when I compare the portfolios I need to use annual return from the same period. The annual returns I am using are therefore from 1927-2008. When we look at portfolios based on dividend yield, cash flow yield, earnings yield etc, the available data is even less.

The long-term investment horizon is 20 years. I need to compute the means, standard deviation and the downside deviation for the portfolio to be able to calculate the Sharpe ratio and the Sortino ratio. With only 82 years of data, there is only 4 non-overlapping periods of 20 years. 4 periods is too few periods to give a good estimation of the probability distribution of the returns, and if the precision of the probability distribution is low, so will the precision of the Sharpe ratio and the Sortino ratio be. Then there will be no point in suggesting the portfolio a long-term investor should invest in, as this suggestion will not be reliable.

To overcome the lack of sufficient data, researchers rely on statistical bootstrap methods, as they assume that these bootstrap methods will improve the quality of the estimation of the probability distribution. The bootstrap methods are computer-intensive methods of estimation of parameters and distributions by resampling the original data. If we should use a method called the standard bootstrap method or a method called the block-bootstrap method depends on our assumption on whether the returns are serial dependent or not. If we assume that the returns are independent and identically distributed we should use the standard bootstrap method, and if we assume that the returns are not independent and identically distributed we should use the block-bootstrap method.

### 4.1 The standard bootstrap method

The standard bootstrap method was first introduced by Efron (1979). The standard bootstrap method is used if we assume that the returns are independent and identically distributed. This means that we assume that the returns one year are not dependent on what the returns were the year before. If we instead assume that the returns are serial dependent this method cannot be used, because it will destroy any serial dependency in the observed returns.

The reason for using this method is to get an approximate probability distribution of the observations, so that the probability distribution can be used to calculate a specific parameter. The method consists of drawing random samples from a set of observations, with replacement. For example in this thesis, when I am comparing stock portfolios based on cap size and book-to-market ratios to the market portfolio and the bond portfolios I have returns from 1927 to2008, and I want to estimate the probability distribution for a 20 year horizon. To simulate this probability distribution the bootstrap method could draw random returns as the returns from 1929, 1952, 1954, and 1969 and so on until it had drawn 20 individual returns. Then it compounds the returns to get a 20 year period return, and by doing this several times we can obtain the approximate probability distribution. Since the returns are only drawn individually, any dependency between the returns one year, and the returns the year before or after would be destroyed, as the method would most likely only draw one of these years. This is why the assumption on whether returns are independent and identically distributed is so important for the choice of method.

I am using this method in my thesis to get the probability distribution for a 20 year horizon for the different portfolios, based on annual returns from 1927-2008. I am calculating Sharpe ratio and Sortino ratio based on these probability distributions. The number of resamples should be as big as possible, but it is limited by time and available computing power. In my thesis the number of resamples is 50000 .

The standard bootstrap method is a commonly used method, and it has for instance been used by Lloyd and Modani (1983), Lloyd and Haney (1985), Leibowitz and Langetieg (1989),

Butler and Domain (1991), Hodges, Taylor and Yoder (1997), Mukherji (2003) and Sinha and Sun (2005). During the analysis I will refer to this method by only calling it the bootstrap method.

Levy (1972) showed that the Sharpe ratio calculated over a T period holding horizon will first rise and then fall as T increases. The explanation for this is that if the returns are independent and identically distributed over time, both expected returns and standard deviation increases with the holding period, however, the rate of increase is greater for the standard deviation than for the expected returns. Also the rate of the increase in the standard deviation is larger for assets with high mean and high volatility, than for assets with low mean and low volatility. This means that when the investment horizon increases, so will the performance of assets with low mean and volatility tend to do, compared with assets with high mean and volatility. Normally one would expect stocks to have higher mean and volatility than bonds. As a result of this difference in mean and volatility, bonds will perform better than stocks as the investment horizon increases.

And also, the longer the horizon, the greater the skewness. It seems that when the time horizon increases, so will the right tail. The right tail will increase with a higher rate for assets with high volatilities, and this is the explanation for the higher rate of increase in the standard deviation of assets with high volatilities. Since Sharpe ratio does not take into account the difference between the right and the left tail, an increase in the right tail will lead to an increase in the standard deviation. Higher standard deviation means lower Sharpe ratio.

### 4.2 The block-bootstrap method

In the 1980's several studies claimed that returns are serial dependent, among others; Schiller (1981), Summers (1986) and Fama and French (1988). If we assume that the returns are serial dependent the bootstrap method cannot be used, as it by its random resampling of individual returns destroys the dependency between the returns.

Another bootstrap method is the block-bootstrap method. Hall (1985) was the first to introduce this method by suggesting drawing random blocks of data instead of individual
observations as it is done in the bootstrap method. This method can be used if we assume that the observations are not independent and identically distributed, instead of the bootstrap method, since the bootstrap method destroys the dependency. But when we replace the individual observations with blocks of observations the dependence is to a larger degree preserved.

If we set the size of the blocks to be 5 , instead of drawing individual returns as with the bootstrap method, the block-bootstrap method would draw blocks of 5 and 5 returns. It could for example draw returns from 1929, 1928,..., 1933, and from1952, 1953,..., 1956, and continue until it had drawn 20 returns. Then it compounds the returns to get a 20 year period return, and by doing this several times we obtain the approximate probability distribution.

The length of the blocks differs almost with each study that has used the block-bootstrap method. Some use only a fraction of the holding period while others have chosen to set the block length equal to the holding period. There is no exact answer to how long the block length should be, but in this thesis it is set to $0,75 \times T$, and $T$ is the time horizon.

The block-bootstrap method can be performed either by using overlapping blocks or nonoverlapping blocks. These approaches are called the moving block bootstrap method and the non-overlapping bootstrap method. Both methods were introduced by Hall (1985), but Carlstein (1986) proposed non-overlapping blocks for univariate time series data, while Künch (1989) suggested overlapping blocks for the same setting.

The non-overlapping method is drawing blocks of data from the observations, but the blocks are not overlapping each other. This means that is the length of the block is 10 , and we have 82 observations, and the blocks cannot overlap each other, we get maximum 8 blocks. This is too few to make a good estimation of the probability distribution. Therefore the nonoverlapping method is not fitted for simulations where the amount of observations is small.

The annual returns for the different portfolios in my thesis are 82. Since I have relative few observations, it would be better to use the moving block-bootstrap method than the nonoverlapping method. This method is also drawing blocks of observations from the data set, instead of individual observation. However, this method allows the blocks to be overlapping. If we are using the moving block-bootstrap method and the non-overlapping block-bootstrap
method on the same number of observations and with the same block length, there will be a lot more potential blocks with the moving block-bootstrap method than with the nonoverlapping method. When we have relative few observations, we will get a better estimation on the probability distribution with the method that provides the largest amount of potential blocks. This means that in this thesis the preferable method, together with the bootstrap method, is the moving block-bootstrap method.

## 5 The data description

### 5.1 Collection of data

In this paper I have chosen to calculate Sharpe ratio and Sortino ratio for the portfolios. The measures are calculated in Matlab. I use both the bootstrap method and the block-bootstrap method to calculate the measures. The data for the stocks are collected from Kenneth R. French's data library ${ }^{7}$ and the data for the bonds are from Ibbotson ${ }^{8}$. For some reason the data on the T-bills return in French's data library are not exactly the same as the data from Ibbotson. I have chosen to use the data from French. Most of the historical returns are from 1927-2008. When I'm using portfolios based on cash flow yield and earnings yield the data is from 1952-2008, and therefore when I am comparing these portfolios to others I have to use the same period for all portfolios.

Kenneth French is using percentage-blocks to determine which stocks go into which group. The $30 \%$ smallest capitalization stocks go into low, the $30 \%$ largest capitalization stocks go into high, and the $40 \%$ in the middle is called medium.

The book-to-market ratio is determined as follows: The $30 \%$ lowest book-to-market ratio stocks go into low, the $30 \%$ highest book-to-market ratio stocks go into high, and the $40 \%$ in the middle is called medium.

The cash flow yield: the $30 \%$ lowest cash flow yield stocks go into low, the $30 \%$ highest cash flow yield stocks go into high, and the $40 \%$ in the middle is called medium.

The 3 portfolios based on the earnings yield are: low (the $30 \%$ lowest earnings yield stocks), high (the $30 \%$ highest earnings yield stocks) and medium (the $40 \%$ in the middle).

And dividend yield: The $30 \%$ lowest dividend yield stocks go into low, the $30 \%$ highest dividend yield stocks go into high, and the $40 \%$ in the middle is called medium.

[^3]
### 5.2 Statistics

| Portfolios | Mean | Standard deviation | Skewness | Kurtosis |
| :--- | ---: | ---: | ---: | ---: |
| Long term corporate bonds | 6,177 | 8,4214 | 1,3997 | 6,5748 |
| Long term government bonds | 6,059 | 9,4140 | 1,1811 | 4,7013 |
| Intermediate term government bonds | 5,577 | 5,7185 | 1,2806 | 5,4152 |
| Low size and low BM stocks | 12,972 | 33,6086 | 1,1189 | 6,9313 |
| Low size and medium BM stocks | 16,724 | 29,0802 | 0,3028 | 3,9160 |
| Low size and high BM stocks | 19,136 | 32,1159 | 0,3236 | 3,6844 |
| Big size and low BM stocks | 10,989 | 20,7018 | $-0,2846$ | 2,4801 |
| Big size and medium BM stocks | 12,040 | 21,8601 | $-0,0905$ | 5,0407 |
| Big size and high BM stocks | 15,115 | 27,3880 | 0,2904 | 4,6865 |
| Market | 11,394 | 20,7462 | $-0,4066$ | 2,9165 |

Table 5.1: The annual mean, standard deviation, skewness and kurtosis bond portfolios, stock portfolios based on capitalization size and book-to-market ratio and the market portfolio.

In table 5.1 we can see that the relationship between the returns on the bonds and the returns on the stocks are perhaps what one would expect, the returns on the stocks are about 2-3 times as high as the returns on the bonds. The standard deviations vary more, both for bonds and stocks, but the lowest standard deviation for the stocks are about twice as big as the highest standard deviation for bonds. We see that among the portfolios of stocks the ones with the lowest means are the market portfolio and the big cap stocks with low or medium book-tomarket ratios. These are also the portfolios of stocks with the lowest standard deviation. The portfolios with the highest means are low cap stocks with high or medium book-to-market ratios and the big cap stocks with high book-to-market ratio. The portfolios with the highest standard deviation are the portfolios of low cap stocks.

We can see that all portfolios have positive skewness except for the market portfolio and the big cap stocks with medium or high book-to-market values, which have negative values. The distribution of the returns of portfolios with positive values is skewed to the right compared to the normal distribution. The distribution of the returns of portfolios with negative values is skewed to the left of the normal distribution.

The kurtosis of most of the portfolios is above 3 , which means that excess kurtosis is positive. Positive excess kurtosis describes a curve that is peakier than the normal distribution, and the curve has fat tails. Two of the portfolios have kurtosis below 3, which means that excess kurtosis is negative. The distribution of the returns of the portfolios that have negative excess is more flat than the normal distribution, and the curve has thin tails. The portfolios with negative excess kurtosis are the market portfolio and the portfolio of big stocks with low book-to-market values.

| Portfolios | Mean | Standard deviation | Skewness | Kurtosis |
| :--- | ---: | ---: | ---: | ---: |
| Long term corporate bonds | 6,177 | 8,4214 | 1,3997 | 6,5748 |
| Long term government bonds | 6,059 | 9,4140 | 1,1811 | 4,7013 |
| Intermediate term government bonds | 5,577 | 5,7185 | 1,2806 | 5,4152 |
| Small size stocks | 16,790 | 34,9030 | 0,6989 | 4,7630 |
| Medium size stocks | 14,563 | 26,6462 | 0,1691 | 3,7466 |
| Big size stocks | 11,153 | 19,9837 | $-0,4175$ | 2,9324 |
| Low BM stocks | 10,887 | 20,8188 | $-0,2845$ | 2,4529 |
| Medium BM stocks | 12,383 | 22,1044 | $-0,0732$ | 4,9312 |
| High BM stocks | 16,041 | 27,8368 | 0,3250 | 4,6846 |
| Market | 11,394 | 20,7462 | $-0,4066$ | 2,9165 |

Table 5.2: The annual mean, standard deviation, skewness and kurtosis bond portfolios, stock portfolios based on capitalization size, portfolios based book-to-market ratio and the market portfolio.

In table 5.2 we can see that the relationship between the returns on the bonds and the returns on the stocks are about 2-3 times as high as the returns on the bonds. The standard deviations vary more, both for bonds and stocks, but the lowest standard deviation for the stocks are about twice as big as the highest standard deviation for bonds. The assets with the highest means are small cap stocks, stocks with high book-to-market values and mid cap stocks. These portfolios also have the highest standard deviations. The portfolios with the lowest means and standard deviations are the bonds.

We can see that all portfolios have positive skewness except for the market portfolio, the big cap stocks and stocks with low and medium book-to-market ratios which have negative values. The distribution of the returns of portfolios with positive values is skewed to the right compared to the normal distribution. The distribution of the returns of portfolios with negative values is skewed to the left of the normal distribution.

The kurtosis of most of the portfolios is above 3 , which means that excess kurtosis is positive. Assets with positive excess kurtosis have probability distributions that are peakier than the normal distribution, and the probability distribution has fat tails. Three of the portfolios have kurtosis below 3, which means that excess kurtosis is negative. The assets that have negative excess returns have probability distributions that are more flat than the normal distribution, and the distributions have thin tails. The portfolios with negative excess kurtosis are the market portfolio, the portfolio of big stocks and the portfolio with low book-to-market values.

## 6 Analysis

### 6.1 Portfolios based on both cap size and book-to-market ratios

The first portfolios of stocks I am comparing to the long-term corporate bonds, long-term government bonds and intermediate-term government bonds are 6 portfolios where the stocks are first divided in to two parts after size (small or big), and then the two parts are both divided in three after book-to-market ratio (low, medium, high), and also the market portfolio. This gives me 10 portfolios.

Since I am simulating the probability distribution, the results can differ every time I perform the simulation procedure. This means that there might be minor differences between the results of the figures and the tables for the same methods and measures.

### 6.1.1 Figures



Figure 6.1: Sharpe ratio for the bond portfolios, stock portfolios based on capitalization size and book-to-market ratio and the market portfolio, calculated by using the bootstrap method.

In figure 6.1 we see that up to about 6 years the Sharpe ratios develop in almost the same way for all the portfolios, but after 6 years the path of the Sharpe ratios of the stocks and the Sharpe ratios of the bonds are going in different directions. While the Sharpe ratios of the bonds continues to increase as the time horizon increases, the Sharpe ratios of the stocks starts to fall.


Figure 6.2: Sharpe ratio for the bond portfolios, stock portfolios based on capitalization size and book-to-market ratio and the market portfolio, calculated by using the block-bootstrap method.

In 6.2 we see that when the time horizon exceeds 8 years the path of the Sharpe ratios of the bonds seems to flat out and actually decrease a bit. The Sharpe ratios of the stocks just continue to rise.


Figure 6.3: Sortino ratio for the bond portfolios, stock portfolios based on capitalization size and book-to-market ratio and the market portfolio, calculated by using the bootstrap method.

In 6.3 we see that the development of all the Sortino ratios follows similar curves, but the stocks are superior to the bonds. We see that the portfolios that rank the highest are those with a high book-to-market ratio, especially those with small stocks.


Figure 6.4: Sortino ratio for the bond portfolios, stock portfolios based on capitalization size and book-to-market ratio and the market portfolio, calculated by using the block-bootstrap method.

In 6.4 we see the same development, and even the ranking of the portfolios performance are the same as in 6.3. The difference between the two figures lies in the spread of the Sortino ratios. We can see that in 6.3 the values after 20 years are closer to each other then in 6.4

### 6.1.2 Tables

| Horizon | Longterm <br> Corporate | Longterm <br> Government | Intermed.term <br> Goverment |  | Small size |  |  | Bigsize |  | Market |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Bonds | Bonds | Bonds | Low BM | Medium BM | High BM | Low BM | Medium BM | High BM |  |
|  | 0,29 (8) | 0,25 (10) | 0,32 71 | 0,28 (9) | 0,45 (2) | 0,48 (1) | 0,35 (6) | 0,38 (4) | 0,42 (3) | 0,37(5) |
|  | 0,40 88 | 0,34 (10) | 0,45 71 | 0,36(9) | 0,58(2) | 0,62 (1) | 0,47(6) | 0,51 (4) | 0,55 (3) | 0,49 (5) |
|  | 0,49(8) | 0,42 (10) | 0,55 77 | 0,42 (9) | 0,68 (2) | 0,71 (1) | 0,56(6) | 0,61 (4) | 0,64 (3) | 0,59 (5) |
|  | 0,56 (8) | 0,4791 | 0,62 77 | 0,46(10) | 0,73 (2) | 0,76 (1) | 0,63 (6) | 0,67 (4) | 0,70 (3) | 0,66 (5) |
|  | 0,61 (8) | 0,52 (9) | 0,69 6) | 0,48(10) | 0,75 (2) | 0,77(1) | 0,67 (7) | 0,71 (4) | 0,72 (3) | 0,70 (5) |
| 6 | 0,67/8) | 0,57 (9) | 0,75 (4) | 0,50 (10) | 0,78(2) | 0,79 (1) | 0,71 (7) | 0,75 (5) | 0,75 (3) | 0,74(6) |
|  | 0,71(8) | 0,60 (9) | 0,80 (1) | 0,50 (10) | 0,78 (3) | 0,78(2) | 0,73 77 | 0,76(4) | 0,75 (6) | 0,76 (5) |
| 8 | 0,75 77 | 0,64 97 | 0,85 (1) | 0,50 (10) | 0,78 (3) | 0,77(5) | 0,74(8) | 0,78(2) | 0,76 (6) | $0,77(4)$ |
|  | 0,79 (4) | 0,67 (9) | 0,90 (1) | 0,51 (10) | 0,78(5) | 0,77(7) | 0,76(8) | 0,80 (2) | 0,77(6) | 0,80 (3) |
| 10 | 0,82 (2) | 0,709) | 0,93 [1] | 0,50 (10) | 0,76 (6) | 0,75 (8) | 0,78(5) | 0,80 (4) | 0,76 (7) | 0,81 (3) |
| 11 | 0,84 (2) | 0,7299 | 0,96 (1) | 0,50 (10) | 0,76(6) | 0,74 (8) | 0,78(5) | 0,81 (4) | 0,76 (7) | 0,81 (3) |
| 12 | 0,88(2) | 0,75 66 | 1,01 (1) | 0,49 (10) | 0,74 (7) | 0,72(9) | 0,78(5) | 0,81 (4) | 0,74 481 | 0,81 (3) |
| 13 | 0,89 (2) | 0,76 (6) | 1,03 11 ) | 0,47(10) | 0,72 77 | 0,69(9) | 0,79 (5) | 0,80(4) | 0,72(8) | 0,81 (3) |
| 14 | 0,92 (2) | 0,79 (5) | 1,06 (11) | 0,46 (10) | 0,70 (8) | 0,66(9) | 0,78(6) | 0,79 (4) | 0,71 71 | 0,80 (3) |
| 15 | 0,94 (2) | 0,81 (4) | 1,10(1) | 0,46 (10) | 0,70 (8) | 0,65 (9) | 0,78(6) | 0,80(5) | 0,70 77 | 0,81 31 |
| 16 | 0,96(2) | 0,81 (3) | 1,11 11 | 0,44 (10) | 0,67 78 | 0,63(9) | 0,78(6) | 0,80(5) | 0,69 (7) | 0,81 (4) |
| 17 | 0,99 (2) | 0,84 (3) | 1,15 11 ) | 0,42 (10) | 0,63 (8) | 0,59(9) | 0,77(5) | 0,7716 | 0,66 (7) | 0,79 (4) |
| 18 | 0,99(2) | 0,84 (3) | 1,16(11) | 0,43 100 | 0,64 (8) | 0,59 (9) | 0,78(5) | 0,78(6) | 0,66 (7) | 0,80 (4) |
| 19 | 1,00 (2) | 0,85 (3) | 1,18(1) | 0,38(10) | 0,60 (8) | 0,55 (9) | 0,75 (5) | 0,75 (6) | 0,63 77 | 0,77(4) |
| 20 | 1,02 (2) | 0,87(3) | 1,20(1) | 0,40 (10) | 0,60 (8) | 0,53(9) | 0,76(5) | 0,75 (6) | 0,61 77 | 0,78(4) |

Table 6.1: Sharpe ratios for bond portfolios, stock portfolios based on size and book-tomarket ratio and the market portfolio for different horizons, calculated by using the bootstrap method.

In table 6.1 we have made the assumption that the returns are independent and identically distributed over time, and therefore the bootstrap method is chosen to simulate the probability distribution. According to the Sharpe ratio the small cap stocks with high book-to-market
values perform best for rather small holding periods. The small cap stocks with medium book-to-market values are ranked as the second best portfolio in the short run followed by the big cap stocks with high book-to-market ratios. The bonds and the small cap stocks with low book-to-market ratios perform the worst in the short run, but as the time horizon increases we see that the ranking of the bonds are improving. In year 7 the best portfolio is the intermediate-term government bond, and this is the best portfolio also for the long-term investor. In year 10 the long-term corporate bonds are ranked second after the intermediateterm government bonds and it stays as the second best through the rest of the period. After 16 years the third best portfolio is the long-term government bonds. This means that after 16 years all bonds are outperforming the stocks, so for a long-term investor the best option would be to invest in bonds, and preferable the intermediate-term government bonds. The portfolios that performed best for rather small holding periods are in the long horizon performing worse than the other stocks. The exception is the small cap stocks with low book-to-market ratios, as this portfolio performs worse than all the other stock both for short and long horizons.

| Horizon | $\begin{array}{\|l} \hline \text { Long.term } \\ \text { Corporate } \\ \text { Bonds } \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Long:term } \\ & \text { Government } \\ & \text { Bonds } \\ & \hline \end{aligned}$ | Intermed.term <br> Goverment <br> Bonds | Small size |  |  | Bigsize |  |  | Narket |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Low BM | Medium BM | High BM | Low BM | Medium BM | High BM |  |
|  | 0,28(8) | 0,24 (10) | 0,32 (7) | 0,28 9) | 0,45 (2) | 0,48 (1) | 0,35 6) | 0,38 (4) | 0,42 (3) | 0,37 (5) |
|  | 0,38 (9) | 0,32 (10) | 0,40 77 | 0,38 88) | 0,62 (2) | 0,65 (1) | 0,49 (6) | 0,58(4) | 0,62 (3) | 0,54 (5) |
|  | 0,47(8) | 0,40 (10) | 0,50 (7) | 0,44 9) | 0,70 (2) | 0,73 (1) | 0,57 (6) | 0,66 (4) | 0,69 (3) | 0,62 (5) |
|  | 0,50 (8) | 0,42 (10) | 0,51(7) | 0,489) | 0,76(3) | 0,79 (2) | 0,65 (6) | 0,75 (4) | 0,79 (1) | 0,71 (5) |
|  | 0,54 (7) | 0,46 (10) | 0,54(8) | 0,51 91 | 0,80(4) | 0,82 (3) | 0,71 (6) | 0,82 (2) | 0,88 (1) | 0,79 (5) |
|  | 0,56(7) | $0,77(10)$ | 0,55 (9) | 0,55 (8) | 0,86(4) | 0,87 (3) | 0,77 (6) | 0,89 (2) | 0,95 (1) | 0,85 (5) |
|  | 0,59 (7) | 0,50 (10) | 0,58(8) | 0,58 9) | 0,90 (4) | 0,89 (5) | 0,81 60 | 0,94(2) | 1,00 (1) | 0,90 (3) |
| 8 | 0,60 (8) | 0,50 (10) | 0,58(9) | 0,62 (7) | 0,97(4) | 0,97(5) | 0,88(6) | 1,03(2) | 1,08 (1) | 0,98(3) |
|  | 0,61 81 | 0,51 (10) | 0,59(9) | 0,67 (7) | 1,05 (4) | 1,03 55 | 0,94 (6) | 1,12(2) | 1,16 (1) | 1,05 (3) |
| 10 | 0,60 (8) | 0,50 (10) | 0,58(9) | 0,68(7) | 1,08(4) | 1,06 (5) | 0,97 (6) | 1,17(2) | 1,20 (1) | 1,10 (3) |
| 11 | 0,62 87 | 0,52 (10) | 0,60 (9) | 0,71 (7) | 1,11 (4) | 1,07 (5) | 1,00 (6) | 1,20 (2) | 1,22 (1) | 1,12(3) |
| 12 | 0,61 (8) | 0,51 (10) | 0,58(9) | 0,76 (7) | 1,16(4) | 1,11 (5) | 1,05 (6) | 1,28 (2) | 1,311 1 ) | 1,20 (3) |
| 13 | 0,59 (8) | 0,50 (10) | 0,57(9) | 0,78 71 | 1,15 (4) | 1,10(5) | 1,06(6) | 1,29 (2) | 1,30 (1) | 1,21 (3) |
| 14 | 0,59 (8) | 0,50 (10) | 0,56(9) | 0,81 71 | 1,16(4) | 1,10 (5) | 1,08(6) | 1,31 (2) | 1,32 (1) | 1,23 33) |
| 15 | 0,60 (8) | 0,51 (10) | 0,57(9) | 0,83 71 | 1,18(4) | 1,10(5) | 1,07 (6) | 1,31 (1) | 1,31 (2) | 1,23(3) |
| 16 | 0,57 78 | 0,49 (10) | 0,55 99 | 0,83 71 | 1,17(4) | 1,12 (5) | 1,09 (6) | 1,34 (1) | 1,34 (2) | 1,26(3) |
| 17 | 0,57 (8) | 0,49 (10) | 0,55 99 | 0,84 (7) | 1,19(4) | 1,14(5) | 1,10 (6) | 1,35 (1) | 1,344 21 | 1,27 37 |
| 18 | 0,56 (8) | 0,47 (10) | 0,54(9) | 0,86(7) | 1,22 (4) | 1,16(5) | 1,10 (6) | 1,37 (1) | 1,34 (2) | 1,29 (3) |
| 19 | 0,57 (8) | 0,49 (10) | 0,55 (9) | 0,84 (7) | 1,21 (4) | 1,15 (5) | 1,09 (6) | 1,36 (1) | 1,33(2) | 1,28(3) |
| 20 | 0,55 (8) | 0,47 (10) | 0,53(9) | 0,92 77 | 1,26(4) | 1,17(5) | 1,10 (6) | 1,38(1) | 1,35 21 | 1,30 (3) |

Table 6.2: Sharpe ratios for bond portfolios, stock portfolios based on size and book-tomarket ratio and the market portfolio for different horizons, calculated by using the blockbootstrap method.

In table 6.2 we are assuming that the returns are serial dependent. When we are assuming that returns are not independent and identically distributed, we cannot use the bootstrap method since this method would destroy the serial dependency. Therefore we need to use the blockbootstrap method.

By looking at the table we see that for rather short holding periods the stocks that perform best are small cap stocks with high book-to-market values, or medium book-to-market values
and big cap stocks with high book-to-market values. The portfolios that perform the worst in the short-run are the bonds and the small cap stocks with low book-to-market ratios.

For the longer horizons bonds are performing worse than the stocks, even the small cap stocks with low book-to-market ratios. However the best portfolio is not the small cap stocks with high book-to-market ratios as in the short run. The portfolios that were ranked 1,2 and 3 for rather short horizons are now ranked 5, 4 and2. The best portfolio in the long run is the big stocks with medium book-to-market ratios. This portfolio is ranked as the best portfolio from year 15 . Also the market portfolio performs well according to Sharpe ratio when we assume that the returns are not independent and identically distributed, the market portfolio is ranked as the third best portfolio from holding periods of 7 years and throughout the rest of the horizon.

| Horion | $\begin{array}{\|l} \mid \text { Longiterm } \\ \text { Corporate } \end{array}$ | Long.term <br> Government | Intermed.term <br> Goverrment |  | Small size |  |  | Big size |  | Narket |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Bonds | Bonds | Bonds | Low BM | Medium BM | High BM | Low BM | Medium BM | High BM |  |
|  | 0,53 9) | 0,48 (10) | 0,79 (4) | 0,56(8) | 0,97(2) | 1,09 (1) | 0,63 77 | 0,7(5) | 0,86 (3) | 0,66 (6) |
|  | 0,87 9 9) | 0,76 (10) | 1,27 (4) | 0,96(8) | 1,78(2) | 2,05 (1) | 1,10 77 | 1,2(5) | 1,53 (3) | 1,14 (6) |
|  | 1,18 (9) | 1,01 (10) | 1,75 (4) | 1,34(8) | 2,68(2) | 3,16 (1) | 1,54 (7) | 1,7(5) | 2,26 (3) | 1,63(6) |
|  | 1,49 (9) | 1,25 (10) | 2,20 (5) | 1,75 (8) | 3,72(2) | 4,46 (1) | 2,04 (7) | 2,2 244 | 3,08(3) | 2,16(6) |
|  | 1,79 99) | 1,48(10) | 2,72 (6) | 2,19(8) | 4,93(2) | 6,05 (1) | 2,55 (7) | 2,9 (4) | 4,02 (3) | 2,73 (5) |
|  | 2,07 (9) | 1,70 (10) | 3,24 (6) | 2,67(8) | 6,34(2) | 7,88 (1) | 3,14 (7) | 3,5 (4) | 5,03 33) | 3,35 (5) |
|  | 2,47 9 9 | 2,02 (10) | 3,93(5) | 3,10(8) | 7,80 (2) | 9,9111) | 3,66 (7) | 4,1 (4) | 6,07 (3) | 3,92(6) |
|  | 2,77 (9) | 2,23 (10) | 4,41 (6) | 3,73(8) | 9,84(2) | $12,711(1)$ | 4,35 77 | 4, (4) | 7,48(3) | 4,71 (5) |
|  | 3,20 (9) | 2,56 (10) | 5,18(6) | 4,32(8) | 11,96(2) | 15,78 (1) | 5,15 77) | 5,8(4) | 9,10 (3) | 5,56 (5) |
| 10 | 3,47 9 9) | 2,78(10) | 5,70 (7) | 4,94(8) | 14,48(2) | 19,62 (1) | 5,83 (6) | 6,7(4) | 10,79 (3) | 6,36 (5) |
| 11 | 3,92 9 ) | 3,06(10) | 6,50 (7) | 5,56(8) | 17,10 (2) | 23,44 11) | 6,60 (6) | 7,7(4) | 12,53 3) | 7,21 (5) |
| 12 | 4,29 99) | 3,35 (10) | 7,30 (7) | 6,50(8) | 20,92 (2) | 29,4711 | 7,66 (6) | 9,0 (4) | 15,15 (3) | 8,43(5) |
| 13 | 4,65 9) | 3,63 (10) | 7,98(7) | 7,24(8) | 24,66 (2) | 35,35 (1) | 8,46 (6) | 10,1 (4) | 17,30 (3) | 9,37 (5) |
| 14 | 5,20 (9) | 3,96(10) | 9,02 (7) | 8,12 81 | 29,01 (2) | 42,49 (1) | 9,56 (6) | $11,4(4)$ | 20,33 (3) | 10,61 (5) |
| 15 | 5,61 (9) | 4,29 (10) | 10,08 (7) | 9,04(8) | $34,04(2)$ | 51,35 (1) | 10,65 (6) | 12,9 (4) | 23,47 (3) | 11,86(5) |
| 16 | 6,04(9) | 4,60 (10) | 11,1117 | 10,25 (8) | 40,29 (2) | 61,00 (1) | 12,04(6) | 14,6(4) | 27,00 (3) | 13,43 (5) |
| 17 | 6,61 19 ) | 5,00(10) | 12,15 (7) | 11,26(8) | 46,64 (2) | 73,12 (1) | 13,34(6) | 16,3 44 | 31,29 (3) | 14,33 (5) |
| 18 | 7,31 19) | 5,44(10) | 13,677 71 | 12,54 (8) | 56,04(2) | 89,49 (1) | 15,12 (6) | 18,5 (4) | 36,99 (3) | 17,04 (5) |
| 19 | 7,62 (9) | 5,70 (10) | 14,16 (7) | 14,04(8) | 64,66 (2) | 100,39 (1) | 16,88(6) | 20,7 (4) | 42,10 (3) | 19,07 (5) |
| 20 | 8,42 (9) | 6,17(10) | 15,84 (8) | 15,94 (7) | 77,43 (2) | 128,15 (1) | 18,79 (6) | $23,7(4)$ | 49,36 (3) | 21,41 (5) |

Table 6.3: Sortino ratios for bond portfolios, stock portfolios based on size and book-tomarket ratio and the market portfolio for different horizons, calculated by using the bootstrap method.

In table 6.3 we are assuming that the returns are independent and identically distributed.
When we make this assumption we have to use the bootstrap method. In this table the portfolios are ranked by the Sortino ratios. The higher Sortino ratio, the higher ranking.

For rather short holding periods the best portfolios are small cap stocks with high or medium book-to-market ratios, followed by the big stocks with high book-to-market ratios. We see
that for short horizons also the intermediate-term bonds perform well, however, as the time increases the portfolio performs worse relative to the other portfolios.

With a time horizon of 20 years, all the bonds are outperformed by the stocks. The stocks that perform the best are the same throughout the whole period; small cap stocks with high book-to-market ratios, small cap stocks with medium book-to-market ratios and big cap stocks with high book-to-market ratios.

|  | $\begin{aligned} & \text { Long:term } \\ & \text { Corporate } \\ & \text { Bonds } \\ & \hline \end{aligned}$ | Long.term <br> Goverrment <br> Bonds | Intermed.term <br> Goverment <br> Bonds | Small size |  |  | Bigsize |  |  | Market |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Low BM | Medium BM | High BM | Low BM | Medium BM | High BM |  |
|  | 0,54 (9) | 0,49 (10) | 0,79 (4) | 0,55 (8) | 0,96 (2) | 1,09 (1) | 0,61 77 | 0,69 (5) | 0,84 (3) | 0,64 (6) |
|  | 0,70 (9) | 0,60 (10) | 1,14(5) | 0,97(8) | 1,71 (2) | 2,00 (1) | 1,06 77 | 1,22 (4) | 1,57(3) | 1,111 (6) |
|  | 1,00 (9) | 0,85 (10) | 1,58(6) | 1,39 (8) | 2,64 (2) | 3,10 (1) | 1,53 77 | 1,75 (4) | 2,30 (3) | 1,63(5) |
|  | 1,16 (9) | 0,95 (10) | 1,84(7) | 1,80(8) | 3,68(2) | 4,37 (1) | 1,94 (6) | 2,30 (4) | 3,11 (3) | 2,12 (5) |
|  | 1,32 (9) | 1,06 (10) | 2,09(8) | 2,29 (7) | 4,97(2) | 5,99 (1) | 2,48(6) | 2,91 (4) | 4,05 (3) | 2,73(5) |
|  | 1,54 (9) | 1,21 (10) | 2,48(8) | 2,85 (7) | 6,49(2) | 8,09 (1) | 3,15 66) | 3,70 (4) | 5,28(3) | 3,50 (5) |
| 7 | 1,68(9) | 1,31(10) | 2,74(8) | 3,36 (7) | 8,03(2) | 10,13 (1) | 3,75 (6) | 4,32 (4) | 6,31 (3) | 4,16(5) |
|  | 1,96 (9) | 1,48(10) | 3,44(8) | 4,18(7) | $11,26(2)$ | 14,83 (1) | 4,911 (6) | 5,96(4) | 9,15 (3) | 5,66 (5) |
|  | 2,18 (9) | 1,60 (10) | 3,89 (8) | 4,98(7) | 15,79 (2) | 21,94 (1) | 6,28(6) | 8,33 (4) | 13,18(3) | 7,60 (5) |
| 10 | 2,31 9 9 | 1,69 (10) | 4,30 (8) | 5,68(7) | 21,28 (2) | 31,07 (1) | 7,72 (6) | 10,73(4) | 17,40 (3) | 9,73 (5) |
| 11 | 2,51 9 ) | 1,79 (10) | 4,72 (8) | 6,57 (7) | 27,04 (2) | 41,16 (1) | 8,97 (6) | 12,89 (4) | 21,27 (3) | 11,55 (5) |
| 12 | 2,61 (9) | 1,89 (10) | 5,07(8) | 7,21 (7) | 34,74 (2) | 54,48 (1) | 10,04(6) | 16,47 (4) | 29,48 (3) | 13,56 (5) |
| 13 | 2,87 (9) | 2,04 (10) | 5,88(8) | 8,38(7) | 46,92 (2) | 75,78 (1) | 12,30 (6) | 20,00 (4) | 36,59 (3) | 17,13 (5) |
| 14 | 2,93(9) | 2,10 (10) | 6,19(8) | 9,51 71 | 60,50 (2) | 98,36 (1) | 13,90(6) | 24,08 (4) | 44,58(3) | 20,26 (5) |
| 15 | 3,14 (9) | 2,22 (10) | 6,89(8) | 10,48 (7) | 74,05 (2) | 127,31 11) | 15,70 (6) | 28,11 (4) | 55,87(3) | 23,36 (5) |
| 16 | 3,27 (9) | $2,27(10)$ | 7,77(8) | 12,56 (7) | 105,82 (2) | 160,71 (1) | 18,04(6) | 35,90(4) | 67,74 (3) | 30,22 (5) |
| 17 | 3,44 (9) | 2,37 (10) | 8,51 81 | 13,76 (7) | 126,11 (2) | 227,12 (1) | 19,58(6) | 41,55 (4) | 90,36 (3) | 332,78 (5) |
| 18 | 3,50 (9) | 2,40 (10) | 9,14(8) | 15,36 (7) | 155,59 (2) | 298,50 (1) | 21,89 (6) | 47,62 (4) | 114,78 (3) | 37,64 (5) |
| 19 | 3,83 (9) | 2,60 (10) | 10,01 (8) | 17,25 (7) | 220,84 (2) | 407,92 (1) | 24,26(6) | 59,97 (4) | 148,06 (3) | 44,51 (5) |
| 20 | 3,61 9 9 | 2,45 (10) | 9,65 (8) | 17,97 (7) | 282,81 (2) | 605,21 (1) | 26,12 (6) | 66,21 (4) | 197,43 (3) | 49,21 (5) |

Table 6.4: Sortino ratios for bond portfolios, stock portfolios based on size and book-tomarket ratio and the market portfolio for different horizons, calculated by using the blockbootstrap method.

Table 6.4 presents the Sortino ratios for the portfolios with the assumption that the returns are not independent and identically distributed. This means that the appropriate method to use is the block-bootstrap method, which does not destroy any serial dependencies.

In the short run we see the same pattern as in 6.3; small cap stocks with high book-to-market ratios perform best, followed by small cap stocks with medium book-to-market ratios, and big cap stocks with high book-to-market ratios. These portfolios are ranked as 1,2 , and 3 throughout the whole period. Already at a holding period of 5 years all stocks outperforms the bonds, and the bonds are still ranked as the bottom 3 if the investor is looking at a 20 year holding period.

### 6.1.3 Discussion and comparison

By looking at the four tables and the four figures, we can see three completely different rankings, depending on which of the measures and methods that has been used. When we are calculating the Sharpe ratio by using the bootstrap method we see a similar ranking as in the other tables in the short run, but for longer horizons the portfolios consisting of different bonds perform best, while all the portfolios of small cap stocks perform worse than all of the other portfolios. When the Sharpe ratio is computed by using the block-bootstrap method, the rankings change. In this scenario the best portfolios consists of big cap stocks with medium or large book-to-market ratio, while the bonds perform worst. When the portfolios are ranked based on the Sortino ratio the portfolio that performs best is the small cap stocks with large book-to-market ratios, and the bonds perform worse than all the other portfolios.

When we calculate Sharpe ratio and use the bootstrap method, we are assuming that the returns on the assets are independent and identically distributed. Levy (1972) showed that when one makes this assumption, the assets with high means and high volatilities will perform worse than assets with low means and volatilities as the time horizon increases, due to the
more rapid increase in the standard deviation for assets with high volatility than assets with low volatility.

If we look at table 5.1 we see that the assets with low means and volatilities mainly are the bonds. If we only compare the stock portfolios we see that the stocks with the lowest means and volatilities are the market portfolio and the portfolios of big cap stocks with low or medium book-to-market values. The ones with the highest standard deviations are the portfolios of low cap stocks. If we take a look at table 6.1, which shows the Sharpe ratio for the portfolios by using the bootstrap method, we see that the portfolios that perform best are the bonds followed by the market, and then the big cap stocks with low or medium book-tomarket values, while the portfolios that perform the worst are the small cap stocks. The explanation for this ranking is that the small cap stocks that have high volatilities will have a higher rate of increase in their standard deviation than the bonds that have low volatilities. Also the big cap stocks with low or medium book-to-market ratios and the market portfolio have low volatility, which causes them to have a lower rate of increase in the standard deviation than the other stock portfolios.

As a result of this higher rate of increase in the standard deviation for the assets with high volatilities, the Sharpe ratio for assets with low standard deviation as the bonds will eventually as the time horizon increases become greater than the assets with high standard deviations. This higher rate of increase in the standard deviations for assets with high volatilities only occur when we assume that returns are independent and identically distributed, and it explains why there are differences between table 6.1 and table 6.2.

We see this effect in table 6.1 where the portfolios are ranked by their Sharpe ratios, however, when the portfolios are ranked after the Sortino ratio, still by using the block-bootstrap method, as in table 6.3, the stocks outperform the bonds. Also in this table the assumption is that returns are independent and identically distributed, but we do not see the same effect of the higher rate of increase in the standard deviation for assets with high volatilities compared to assets with low volatilities.

The reason for the higher rate of increase in the standard deviation for assets with high volatilities is due to an higher rate of increase in the upside variability in the returns, this means that the distribution is skewed more to the right. The higher volatility, the higher rate
of increase in skewness. When the right-tail reward increases so will the standard deviation as the standard deviation does not appreciate positive skewness, and when the standard deviation increases the Sharpe ratio will decrease. Sortino ratio however, is not affected by an increase in the upside variability, it is only affected by changes in the downside deviation. Therefore we do not see the same pattern in table 6.3 as in table 6.1 , because Sortino ratio is not influenced by the higher rate of increase in the standard deviation of assets with high volatilities, since the increase is caused by an increase in the upside variability.

As discussed earlier, the difference between the Sharpe ratio and the Sortino ratio is that Sharpe uses the standard deviation and Sortino uses the downside deviation. This means that if the portfolios have skewness different from zero, there might be differences in the ranking order from Sharpe and Sortino. If the skewness is positive, most of the portfolios risk would be upward variability. This is not a risk that investors fear, and therefore it is not included as risk in Sortino ratio, but it is a part of the risk when standard deviation is used, as in Sharpe ratio. Therefore, if a portfolio has positive skewness, the Sharpe ratio will underestimate the performance of the portfolio. And if the portfolio has negative skewness, which means that most of the standard deviation is downside deviation, then the Sharpe ratio will overestimate the portfolios performance.

When we assume that the returns are not independent and identically distributed, as in table 6.2 and 6.4 there are still differences in the ranking with Sharpe ratio and the ranking with Sortino ratio. This is not due to a higher rate increase in assets with high volatilities as this effect only occurs when we assume that the returns are independent an identically distributed. However, the skewness will still play an important role in explaining why the ratios rank the portfolio differently.

Table 5.1 shows the skewness of the portfolios, though these are only annual skewness. The difference between the ranking that Shape ratio gives and the ranking that Sortino ratio gives should be possible to explain by looking at the skewness, but the annual skewness is not the same as the skewness with a time horizon of 15 or 20 years. But if we look at table 5.1 we can still see a tendency that portfolios with a probability distribution which is skewed to the right tend to be ranked higher with Sortino ratio than with Sharpe ratio, and portfolio with probability distributions skewed to the left tend to be ranked higher with Sharpe ratio than with Sortino ratio.

The portfolio consisting of big cap stocks with medium book-to-market ratios is the best according to Sharpe, while Sortino rank it as the fourth best. If we look in table 5.1 we see that big cap stocks with medium book-to-market ratio have a negative skewness, which means that Sharpe ratio will overvalue and that the risk that most investors fear is actually greater than what Sharpe is taking account for.

The big cap stocks with high book-to-market ratios rank high with both measures. The market portfolio however, is ranked a lot higher with Sharpe than with Sortino. Looking at table 5.1 again we see that the market has negative skewness which means that Sharpe overestimates the value of the portfolio.

Small cap stocks with medium or high book-to-market values are the two best portfolios according to Sortino ratio, while Sharpe ratio prefers other portfolios. Table 5.1 tells us that both these portfolios have positive skewness. This means that a larger part of the standard deviation is upside variability, which investors do not fear. This means that Sharpe ratio undervalues these portfolios.

### 6.2 Portfolios by cap size and portfolios by book-to-market ratios

Here I compare the same portfolios as earlier of bonds and the market portfolio to 6 stock portfolios. The stocks are divided in three after size (low, medium, high) and in three after book-to-market ratio (low, medium, high). This gives me ten different portfolios.

I use the same portfolios in the next section, where I compare them to several other portfolios. The reason why I have chosen to compare them separately is that in the next section I have to exclude the empirical data from 1927 to 1951, because returns for the other portfolios are only documented from 1952. Since most of the earlier studies have used data from 1926 or 1927 I wanted to do the same, and then see if it affects the ranking of the portfolios whether I am using the returns from 1927 or from 1952.

Since I simulate the probability distribution, the results can differ every time I perform the simulation procedure. This means that there may be minor differences between the results of the figures and tables for the same methods and measures.

### 6.2.1 Figures



Figure 6.5: Sharpe ratio for the bond portfolios, stock portfolios based on capitalization size, stock portfolios based on book-to-market ratio and the market portfolio, calculated by using the bootstrap method.

In 6.5 we see that in the first years the stocks outperform the bonds, but all Sharpe ratios are still following the same pattern. After year 6 the paths of the Sharpe ratios of the stocks and the Sharpe ratios of the bonds go in different directions. The Sharpe ratios of the stocks are slowly decreasing, while the Sharpe ratios of the bond are still rising. After 16 years all bonds outperform all stocks.


Figure 6.6: Sharpe ratio for the bond portfolios, stock portfolios based on capitalization size, stock portfolios based on book-to-market ratio and the market portfolio, calculated by using the block-bootstrap method.

In 6.6 we can see that the first years look similar to the pattern in 6.5 , but around year 10 the Sharpe ratios of the bonds start to fall. In this figure the bonds never perform better than any of the stocks.


Figure 6.7: Sortino ratio for the bond portfolios stock portfolios based on capitalization size, stock portfolios based on book-to-market ratio and the market portfolio, calculated by using the bootstrap method.

In 6.7 we see that the development of all the Sortino ratios follows similar curves, after 20 years the stocks perform better than the bonds. We see that the portfolios that rank the highest are those with small and medium sized stocks, or high book-to-market ratio.


Figure 6.8: Sortino ratio for the bond portfolios, stock portfolios based on capitalization size, stock portfolios based on book-to-market ratio and the market portfolio, calculated by using the block-bootstrap method.

In 6.8 we see the same development, but the order of the portfolios is not the same. However, the main difference between the two figures lies in the spread of the Sortino ratios. We can see that in 6.7 the values after 20 years are closer to each other then in 6.8.

### 6.2.2 Tables

| Horizon | $\begin{aligned} & \text { Longterm } \\ & \text { Corporate } \\ & \text { Bonds } \end{aligned}$ | $\begin{aligned} & \text { Longterm } \\ & \text { Goverenment } \\ & \text { Bonds } \end{aligned}$ | $\begin{aligned} & \text { Intermed: term } \\ & \text { Govermment } \\ & \text { Boonds } \end{aligned}$ | Size |  |  | $B / M$ |  |  | Market |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Low | Medium | High | Low | Medium | High |  |
|  | 0,29 (9) | 0,25 100 | 0,32 81 | 0,37 (4) | 0,40 (2) | 0,37(5) | 0,3477 | 0,39 (3) | 0,44 (1) | 0,37/6) |
|  | 0,41 9 | 0,35 100 | 0,46 (8) | 0,50 6) | 0,55 22 | 0.5144 | 04777 | 0.54 (3) | 0,5911 | 0,51/5 |
|  | 0,49 9) | 0,42 (10) | 0,55 77) | 0,55 6) | 0,62 (2) | 0,59(4) | 0,55 81 | 0,62 (3) | 0,67 (1) | 0,59(5) |
|  | 0,56 69 | 0,48 100 | 0,63 66 | 0,59 (8) | 0,68 (2) | 0,66(4) | 0,61 171 | 0,68(3) | 0,72 [11 | 0,65 (5) |
| 5 | 0,61 91 | 0,52 (10) | 0,69 (6) | 0,62 (8) | 0,72 (3) | 0,714 | 0,65 71 | 0,7321 | 0,76(1) | 0,70 (5) |
|  | 0,66 81 | 0,56(10) | 0,74 (3) | 0,63 (9) | 0,74 (4) | 0,74 (5) | 0,6877 | 0,75 (2) | 0,78 (1) | 0,7316 |
|  | 0,72 28 | 0,61 (10) | 0,81 (1) | 0,63 9) | 0,75 66 | 0,77(4) | 0,7277 | 0,78 (3) | 0,79 (2) | 0,76 (5) |
| 8 | 0,74 (7) | 0,63(9) | 0,85 (1) | 0,62 (10) | 0,76 (6) | 0,79 (3) | 0,7488 | 0,80 (2) | 0,79 (4) | 0,78(5) |
| 9 | 0,78(6) | 0,66(9) | 0,89 (1) | 0,61 (10) | 0,76(7) | 0,81 (2) | 0,75 81 | 0,80 (3) | 0,79(5) | 0,79(4) |
| 10 | 0,81 15 | 0,69 (9) | 0,93 (1) | 0,62 (10) | 0,77(7) | 0,83 22 | 0,7718) | 0,83 33 | 0,80(6) | 0,81 (4) |
| 11 | 0,84 (2) | 0,72(9) | 0,96 (1) | 0,60 (10) | 0,76(8) | 0,82 [3) | 0,7717) | 0,81 4 4 | 0,78(6) | 0,81 (5) |
| 12 | 0,86(2) | 0,74 (9) | 1,00 (1) | 0,57 (10) | 0,75 (8) | 0,83 [3) | 0,7716 | 0,82 (4) | 0,7677 | 0,81 (5) |
| 13 | 0,89 (2) | 0,76 (7) | 1,03 11 ) | 0,56 (10) | 0,73 9 ) | 0,83 [3) | 0,776 | 0,81 41 | 0,75 88 | 0,81 (5) |
| 14 | 0,9112) | 0,78(6) | 1,06 (1) | 0,52 (10) | 0,72 29 | 0,83 (3) | 0,7717) | 0,81 15 | 0,72 88 | 0,81 (4) |
| 15 | 0,94 (2) | 0,80 (6) | 1,10 (1) | 0,52 (10) | 0,71 19 | 0,83 (3) | 0,7717) | 0,80(5) | 0,72 281 | 0,81 (4) |
| 16 | 0,95 (2) | 0,81 (4) | 1,11111 | 0,51 (10) | 0,70 (8) | 0,83 33) | 0,77171 | 0,7916) | 0,69(9) | 0,80(5) |
| 17 | 0,97(2) | 0,83 (3) | 1,14 (1) | 0,50 (10) | 0,69 (8) | 0,82 (4) | 0,75 77 | 0,78(6) | 0,69(9) | 0,79(5) |
| 18 | 0,99 (2) | 0,84(3) | 1,1614 | 0,45 (10) | 0,66(8) | 0,81 (4) | 0,75 77 | 0,7716) | 0,65 99 | 0,79(5) |
| 19 | 1,01(2) | 0,86(3) | 1,18 (1) | 0,42 (10) | 0,65 (8) | 0,81 (4) | 0,75 77 | 0,76(6) | 0,64(9) | 0,78(5) |
| 20 | 1,02 (2) | 0,86(3) | 1,20 (1) | 0,41 (10) | 0,63 81 | 0,80 (4) | 0,7477 | 0,75 66 | 0,63 919 | 0,78(5) |

Table 6.5: Sharpe ratios for bond portfolios, stock portfolios based on size, stock portfolios based on book-to-market ratio and the market portfolio for different horizons, calculated by using the bootstrap method.

In table 6.5 we are assuming that returns are independent and identically distributed, and as when we make this assumption the appropriate method to use is the bootstrap method. For
rather short holding periods we see that stocks with high book-to-market values performs best followed by mid cap stocks, and third best are stocks with medium book-to-market values. The bonds perform the worst.

But as the time horizon increases, the order of the portfolios is dramatically changed. The bonds that performed the worst in the short run are the best portfolios in the long run. The best portfolio is the one consisting of intermediate-term government bond, followed by longterm corporate bonds and long-term government bonds. From a time horizon of 7 years the intermediate-term bonds are the best portfolio, from 11 years the long-term corporate bonds are ranked second best, and from year 17 all bonds outperform the stocks.

The best of the stocks are big cap stocks and the market portfolio. Small cap stocks perform the worst.

| Horizon | $\begin{array}{\|l\|l\|l\|l\|l\|l\|l\|l\|} \hline \text { Corporate } \end{array}$ | Long.term <br> Government | Intermed.term <br> Goverment |  | Size |  |  | B/M |  | Market |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Bonds | Bonds | Bonds | Low | Medium | High | Low | Medium | High |  |
|  | 0,29 9) | 0,25 (10) | 0,32 81 | 0,38(5) | 0,41 (2) | 0,38 (4) | 0,35 (7) | 0,40 (3) | 0,45 (1) | 0,38(6) |
|  | 0,38 99) | 0,33 (10) | 0,40 (8) | 0,50 (6) | 0,57 (3) | 0,53 (5) | 0,487) | 0,58(2) | 0,64 (1) | 0,53 (4) |
|  | 0,46 (9) | 0,39 (10) | 0,49 (8) | 0,58(6) | 0,67 (3) | 0,63 (5) | 0,58 77) | 0,68(2) | 0,73 (1) | 0,63 (4) |
|  | 0,50 (9) | 0,42 (10) | 0,52 (8) | 0,60 (7) | 0,74 (3) | 0,71 (5) | 0,65 (6) | 0,78(2) | 0,83 (1) | 0,72 (4) |
|  | 0,54 (9) | 0,46 (10) | 0,54 (8) | 0,60 (7) | 0,78 (3) | 0,77(5) | 0,71 (6) | 0,83 (2) | 0,90 (1) | 0,78(4) |
| 6 | 0,56 (8) | 0,47 (10) | 0,55 (9) | 0,65 (7) | 0,86(3) | 0,84 (5) | 0,77(6) | 0,92 (2) | 0,98 (1) | 0,86(4) |
|  | 0,59 (8) | 0,50 (10) | 0,58 (9) | 0,67 (7) | 0,91 (3) | 0,88(5) | 0,82 (6) | 0,97(2) | 1,03 (1) | 0,91 (4) |
| 8 | 0,60 (8) | 0,51 (10) | 0,58(9) | 0,71 (7) | 0,99 (3) | 0,95 (5) | 0,886) | 1,06(2) | 1,12 (1) | 0,99 (4) |
|  | 0,61 181 | 0,51 (10) | 0,59 (9) | 0,77(7) | 1,06(3) | 1,00(5) | 0,93 (6) | 1,13(2) | 1,19 (1) | 1,04(4) |
| 10 | 0,60 (8) | 0,50 (10) | 0,58 (9) | 0,81 71 | 1,11 (3) | 1,05 (5) | 0,98 (6) | 1,20(2) | 1,25 (1) | 1,10 (4) |
| 11 | 0,62 (8) | 0,52 (10) | 0,60 (9) | 0,84 (7) | 1,15 (3) | 1,08(5) | 1,01 (6) | 1,23(2) | 1,27 (1) | 1,13 (4) |
| 12 | 0,61 81 | 0,51 (10) | 0,58(9) | 0,90 (7) | 1,23(3) | 1,13(5) | 1,06 (6) | 1,30(2) | 1,34 (1) | 1,19 (4) |
| 13 | 0,59 81 | 0,50 (10) | 0,57 (9) | 0,91 71 | 1,24 (3) | 1,14 (5) | 1,07(6) | 1,31(2) | 1,34 (1) | 1,20 (4) |
| 14 | 0,58(8) | 0,49(10) | 0,56 (9) | 0,947) | 1,28(3) | 1,17(5) | 1,10 (6) | 1,35 (2) | 1,36 (1) | 1,24(4) |
| 15 | 0,59 81 | 0,50 (10) | 0,57 (9) | 0,96 (7) | 1,28(3) | 1,17(5) | 1,10 (6) | 1,34(2) | 1,37 (1) | 1,24(4) |
| 16 | 0,57 81 | 0,49 (10) | 0,55 (9) | 0,94(7) | 1,30 (3) | 1,19(5) | 1,12(6) | 1,38(2) | 1,41 (1) | 1,27 (4) |
| 17 | 0,56 88 | $0,48(10)$ | 0,55 (9) | 0,93(7) | 1,30 (3) | 1,20 (5) | 1,126) | 1,37(2) | 1,39 (1) | 1,27(4) |
| 18 | 0,56 81 | $0,48(10)$ | 0,54(9) | 0,94(7) | 1,32 (3) | 1,20 (5) | 1,13(6) | 1,39(1) | 1,38(2) | 1,28(4) |
| 19 | 0,56(8) | 0,48(10) | 0,55 (9) | 0,94(7) | 1,32 (3) | 1,20 (5) | 1,1316) | 1,39(1) | 1,38(2) | 1,28(4) |
| 20 | 0,55 81 | 0,47(10) | 0,54(9) | 0,99 71 | 1,38(3) | 1,20 (5) | 1,14(6) | 1,41 (2) | 1,42 (1) | 1,30(4) |

Table 6.6: Sharpe ratios for bond portfolios, stock portfolios based on size and book-tomarket ratio and the market portfolio for different horizons, calculated by using the blockbootstrap method.

In table 6.6 we are assuming that the returns are not independent and identically distributed, and we are therefore using the block-bootstrap method. For rather short holding periods, stocks with high and medium book-to-market values and the mid cap stocks perform best, while the bonds perform the worst.

There are not a lot of changes from the short run to the long run; in the long run we can see that the ones that perform best are stocks with high book-to-market ratios, followed closely by stocks with medium book-to-market ratio, and third the mid cap stocks. The market portfolio improves its ranking, from $6^{\text {th }}$ with a holding period of 1 year to $4^{\text {th }}$ with a holding period of 20 years. All the stocks perform better than the bonds no matter what time horizon.

| Horizon | $\begin{aligned} & \text { Long:term } \\ & \text { Corporate } \\ & \text { Bonds } \\ & \hline \end{aligned}$ | Long.term <br> Goverment <br> Bonds | Intermed.term <br> Goverment <br> Bonds | Size |  |  | $B / M$ |  |  | Narket |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Low | Medium | High | Low | Medium | High |  |
|  | 0,54 (9) | 0,49 (10) | 0,81 (4) | 0,83 3) | 0,83 (2) | 0,68 (6) | 0,62 88 | 0,75 (5) | 0,97 (1) | 0,677 71 |
| 2 | 0,86 (9) | 0,75 (10) | 1,25 (4) | 1,46(3) | 1,47 (2) | 1,14 (6) | 1,05 (8) | 1,25 (5) | 1,71 (1) | 1,13 717 |
|  | 1,18(9) | 1,01 100 | 1,75 (5) | 2,13 3) | 2,15 (2) | 1,64 (6) | 1,50(8) | 1,79 (4) | 2,55 (1) | 1,63 71 |
| 4 | $4 \quad 1,51$ (9) | $1,27(10)$ | 2,27(5) | 2,91 (3) | 2,93(2) | 2,16 (6) | 1,97(8) | 2,36(4) | 3,50 (1) | 2,15 77 |
|  | 1,80 (9) | 1.51 (10) | 2,76(5) | 3,75 (2) | 3,75 (3) | 2,69 (6) | 2,42 (8) | 3,00 (4) | 4,59 (1) | 2,68 77 |
| 6 | 6 2,10 9) | 1,75 100 | 3,26 77 | 4,73(3) | 4,74 (2) | 3,34 (5) | 2,98(8) | 3,72 (4) | 5,89 (1) | 3,32 (6) |
| 7 | 2,45 99 | 1,98(10) | 3,84 (7) | 5,86(2) | 5,80 (3) | 3,97(5) | 3,55 (8) | 4,46(4) | 7,32 (1) | 3,96(6) |
|  | 8 2,83 91 | $2,27(10)$ | 4,47(7) | 7,24 (2) | 7,13(3) | 4,75 (5) | 4,21 (8) | 5,37 (4) | 9,14 (1) | 4,74 (6) |
| 9 | 9) 3,18(9) | $2,53(10)$ | 5,16 77 | 8,55 (2) | 8,38(3) | 5,47 (5) | 4,81 81 | 6,27 (4) | 10,89 (1) | 5,46(6) |
| 10 | 3,49 99) | 2,79 (10) | 5,74(7) | 10,32 (2) | 10,10 (3) | 6,40 (5) | 5,60 (8) | 7,34 (4) | 13,30 (1) | 6,39 (6) |
| 11 | 3,86 (9) | 3,02 (10) | 6,45 (8) | 12,16 (2) | 11,83(3) | 7,41 (5) | 6,477) | 8,52 (4) | 15,72 (1) | 7,38(6) |
| 12 | 4,27 919 | 3,36 (10) | 7,34 (7) | 14,39 (2) | 13,86 (3) | 8,41 (5) | 7,24 (8) | 9,85 (4) | 18,88 (1) | 8,39 (6) |
| 13 | 4,76 (9) | 3,69 (10) | 8,15 (8) | 16,74 (2) | $16,24(3)$ | 9,67 (5) | 8,27(7) | 11.42 (4) | 22,32 (1) | 9,64 (6) |
| 14 | 5,10 (9) | 3,92 (10) | 8,84 (8) | 19,80 (2) | 18,75 (3) | 10,90 (5) | 9,32 77 | 12,85 (4) | 26,311 [1] | 10,87(6) |
| 15 | 5,63 9) | 4,31 100 | 10,07 (7) | 22,50 (2) | $21,03(3)$ | 11.71 (6) | 9,99(8) | 14,13 (4) | 30,03 (1) | 11,73(5) |
| 16 | 6,24 (9) | 4,71 (10) | 11,10 (8) | $26,55(2)$ | $24,64(3)$ | 13,48(5) | 11,37 77 | 16,38 (4) | 35,77 (1) | 13,48 (6) |
| 17 | 6,55 (9) | 4,911(10) | 11,766 8 | 30,54 (2) | 28,16 (3) | 14,91 (6) | 12,56 77 | 18,24 (4) | 41,18 (1) | 14,96(5) |
| 18 | 7,29 (9) | 5,44 (10) | 13,40 (8) | 36,06 (2) | $33,15(3)$ | 17,18(6) | 14,347) | 21,134 | 49,29 (1) | 17,20(5) |
| 19 | 7,66 (9) | 5,76(10) | 14,57 (8) | 39,74 (2) | 35,82 (3) | 18,17(6) | 15,18 77) | 22,58 (4) | 53,91 (1) | 18,211 (5) |
| 20 | 8,43919 | 6,18(10) | 16,01 (8) | 47,70 (2) | 43,22 (3) | 20,92 (6) | 17,25 77) | 26,46 (4) | 66,80 (1) | 21,01 (5) |

Table 6.7: Sortino ratios for bond portfolios, stock portfolios based on size and book-to-market ratio and the market portfolio for different horizons, calculated by using the bootstrap method.

According to table 6.7, where we are assuming that return are independent and identically distributes, the best portfolio throughout the whole period is the portfolio of stocks with high book-to-market ratios. The second best is the one consisting of small stocks, and the third is the medium size stocks, for short holding periods, and these two portfolios switch places in the long run. The long-term bonds are outperformed both with short and long holding periods, while the inter-mediate term government bonds are the $4^{\text {th }}-5^{\text {th }}$ best for short holding periods, while from year 16 all the bonds are outperformed by the stocks. The worst of the stocks are the stocks with low book-to-market ratios and the big cap stocks.

| Horizon | $\begin{aligned} & \text { Long:term } \\ & \text { Corporate } \\ & \text { Bonds } \\ & \hline \end{aligned}$ |  | $\begin{array}{\|l\|} \hline \text { Inermed.term } \\ \text { Government } \\ \text { Bonds } \\ \hline \end{array}$ | Size |  |  | B/M |  |  | Market |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Low | Medium | High | Low | Medium | High |  |
|  | 0,54 (9) | 0,49 (10) | 0,80 (4) | 0,83 (2) | 0,83 3) | 0,67 66 | 0,62 81 | 0,73 (5) | 0,95 (1) | 0,66 (7) |
|  | 0,70 (9) | 0,61 (10) | 1,13 (5) | 1,50(2) | 1,44 (3) | 1,13 (6) | 1,04(8) | 1,28(4) | 1,72 (1) | 1,12 71 |
|  | 1,02 (9) | 0,87 (10) | 1,61 (5) | 2,14 (2) | 2,10(3) | 1,611 61 | 1,46(8) | 1,81(4) | 2,54 (1) | 1,60(7) |
|  | 1,15 (9) | 0,95 (10) | 1,83(8) | 3,00 (2) | 2,85 3) | 2,14 (5) | 1,88(7) | 2,46(4) | 3,59 (1) | 2,1316) |
| 5 | 1,33 99) | 1,06 (10) | 2,111 (8) | 4,02 (2) | 3,80(3) | 2,78 (5) | 2,43 77 | 3,12 24 ) | 4,68 (1) | 2,771 (6) |
|  | 1,54 (9) | 1,20 (10) | 2,46(8) | 5,37(2) | 4,99(3) | 3,56 (5) | 3,09 71 | 3,97(4) | 6,17 (1) | 3,55 (6) |
|  | 1,71 (9) | 1,32 (10) | 2,79 (8) | 6,47(2) | 6,04(3) | 4,29 (5) | 3,69 (7) | 4,71 (4) | 7,51 (1) | 4,26(6) |
| 8 | 1,92(9) | 1,45 (10) | 3,33 88 | 8,28(2) | 7,97(3) | 5,60 (5) | 4,68(7) | 6,23(4) | 10,62 (1) | 5,55(6) |
| 9 | 2,19(9) | 1,61 100 | 3,96(8) | 10,72 (3) | 10,92 (2) | 7,60 (5) | 6,02 71 | 8,85 (4) | 15,79 (1) | 7,54(6) |
| 10 | 2,27(9) | 1,65 (10) | 4,23 (8) | 13,77 (3) | 14,40 (2) | 9,62 (6) | 7,37 77) | 11,53(4) | 21.81 (1) | 9,68(5) |
| 11 | 2,53(9) | 1,81 (10) | 4,75 (8) | 16,38 3) | 17,57(2) | $11,40(6)$ | 8,64 (7) | 14,04 (4) | 27,111 (1) | 11,53(5) |
| 12 | 2,67 (9) | 1,93 100 | 5,28(8) | 19,98(3) | 21,25 (2) | 13,10 (6) | 9,66 (7) | 17,49 (4) | 35,81 (1) | 13,39(5) |
| 13 | 2,75 (9) | 1,96(10) | 5,69 (8) | 23,93(3) | 27,24 (2) | 16,17 (6) | 11.71 | $21.511^{4} 4$ | 47,54 (1) | 16,73/5) |
| 14 | 2,96 (9) | 2,10 (10) | 6,45(8) | 29,46 (3) | 34,70 (2) | 19,75 (6) | 13,80 (7) | 27,30 (4) | 62,97 (1) | 20,59 (5) |
| 15 | 3,24(9) | $2,88(10)$ | 7,15 (8) | 35,00 (3) | 43,22 (2) | 22,94(6) | 15,78(7) | 32,54 (4) | 79,4 (1) | 24,12 (5) |
| 16 | 3,35 (9) | 2,30 (10) | 8,10 (8) | 47,36 (3) | 58,83 (2) | 29,18(6) | 18,62 (7) | 41,87 (4) | 96,01 (1) | 31,50(5) |
| 17 | 3,46(9) | 2,35 100 | 8,73(8) | 51,25 3) | 63,69 (2) | 30,19 (6) | 19,25 77 | 46,55 (4) | 126,52 (1) | 32,75 (5) |
| 18 | 3,41 (9) | 2,34 (10) | 8,95 (8) | 63,68(3) | 77,93(2) | 34,54 (6) | 21,66 77) | 55,72 (4) | 171,12 (1) | 38,44 (5) |
| 19 | 3,76 (9) | 2,55 (10) | 10,20 (8) | 74,86 (3) | 93,61 (2) | 39,06 (6) | 23,94 (7) | 65,10 (4) | 217,29 (1) | 43,48 (5) |
| 20 | 3,65 (9) | $2,47(10)$ | 9,58(8) | 93,14 (3) | 127,05 (2) | 43,36 (6) | 25,61 (7) | 81,67 (4) | 340,73 (1) | 50,16 (5) |

Table 6.8: Sortino ratios for bond portfolios, stock portfolios based on size and book-tomarket ratio and the market portfolio for different horizons, calculated by using the blockbootstrap method.

In table 6.8 we are assuming that returns are not independent and identically distributed, and we must use the block-bootstrap method to avoid destroying the serial dependency. The portfolio that performs best in table 6.8 is the one with high book-to-market ratio stocks. Second best is mid cap stocks, followed by the small cap stocks. These three are the three best portfolios both in the short run and the long run, but in the short run the small cap stocks are better than the mid cap stocks. We observe that from year 4 all portfolios of stocks are superior to all portfolios of bonds.

### 6.2.3 Discussion and comparison

By using different performance measures and methods in order to compute them, we get different predictions on which portfolio that will be the best investment in the long run. If we were only looking at the Sharpe ratio calculated by using the bootstrap method, the best option for a long-term is investing in bonds, and the intermediate-term government bonds perform the best of the bonds. By calculating the Sharpe ratio by using the block-bootstrap method we get a different order. The best portfolios are those with stocks with medium or big book-to-market values, the mid cap stocks and the market. This ranking is close to the ranking we get by using Sortino ratio. With Sortino ratio the best portfolios are medium and big book-to-market ratios and small and medium cap stocks.

When we assume that returns are independent and identically correlated, the standard deviation will increase more than the mean over time, and assets with high volatilities will have a higher rate of increase than assets with low volatilities. The increase in the standard deviation is mainly due to an increase in right-tail potential, which means the skewness is increasing and the probability distribution of the returns are skewed more to the left. The rate of increase is higher for assets with high volatilities than for assets with low volatilities.

From table 5.2 we see that small cap stocks, high book-to-market ratio stocks and mid cap stocks are the portfolio with highest standard deviation, while bonds are the portfolios with the lowest standard deviation. Therefore the rate of increase of the standard deviation will be higher for all stocks than for the bonds. This means that eventually as time increases all bonds will outperform the stocks. In table 6.5 we see that this is the case with holding periods of 17 years or longer.

In table 6.5 we also saw that the portfolios that performed the worst were the small and mid cap stocks and stocks with high book-to-market ratios. Since these are the portfolios with the highest volatilities, these are the portfolios that will have the highest rate of increase in the standard deviation, and they will therefore eventually be outperformed by all the other portfolios. This occurs with holding periods of 13 years or longer.

Since the main reason for the higher rate of increase in the standard deviation for assets with high volatilities is due to a higher rate of increase in the upside variability, we will not see the same pattern in 6.7 as the Sortino ratio only uses the downside deviation and not the whole standard deviation. So even though one assumes that the returns are independent and identically distributed also in table 6.7, it does not influence the rankings since we are using the Sortino ratio to rank the portfolios in this table.

So why do the market perform better when using Sharpe, and why do low stocks perform worse compared to Sortino ratio? The answer should be found in the skewness of the portfolios. Sharpe ratio overvalues the performance of stocks with negative skewness, because Sharpe does not take into account that the risk that the investors fear is actually greater than what one would expect just by looking at the standard deviation.

Table 5.2 shows the skewness of the portfolios, however only annual skewness. We can still see a tendency that assets with positive skewness are relative ranked higher with Sortino ratio than with Sharpe ratio.

As we can see the market portfolio has negative skewness, which means that Sharpe ratio overestimates the performance of the market portfolio, and this can explain why the portfolio performs better when using Sharpe ratio than using Sortino ratio.

The portfolio with small cap stocks has positive skewness, which means probability distribution is skewed to the right of the normal distribution. And as discussed this means that Sharpe ratio will underestimate the performance of the portfolio. If we compare with the other portfolios of stocks, we see that the small cap stocks have the largest skewness, which means that this is the portfolio Sharpe ratio will undervalue the most.

### 6.3 Portfolios based on size, B/M, D/P, CF/P, and E/P

In this section I am using portfolios based on size, book-to-market ratio, dividend yield, cash flow yield and earnings yield. These portfolios are compared to the portfolios of bonds and the market portfolio. I have 19 different portfolios. At this point the tables are getting so big,
that I have chosen not to include the whole time horizon, but show the results for the $1^{\text {st }}, 5^{\text {th }}$, $10^{\text {th }}, 15^{\text {th }}$ and $20^{\text {th }}$ year.

### 6.3.1 Tables

|  |  |  | 1 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Long-term | Corporate | Bonds | 0,20 (18) | $0,44(18)$ | 0,59 (18) | 0,68 (17) | 0,74 (16) |
| Long-term | Government | Bonds | 0,20 (19) | $\begin{aligned} & 0,42(19) \\ & 0,58(16) \end{aligned}$ | 0,54 (19) | 0,66 (19) | 0,71 (18) |
| Intermed.term | Government | Bonds | $0,27(17)$ |  | 0,74 (13) | 0,92 (9) | 1,02 (1) |
|  | Size | Low | $0,38(10)$ | 0,67 (12) | 0,72 (15) | 0,67 (18) | 0,60 (19) |
|  |  | Medium | 0,42 (9) | 0,77 (9) | 0,87 (9) | 0,85 (12) | 0,80 (12) |
|  |  | High | 0,37 (12) | 0,71 (10) | 0,84 (10) | 0,89 (10) | 0,87 (8) |
|  |  | Low | 0,32 (14) | 0,63 (14) | 0,74 (14) | 0,78 (13) | 0,77 (13) |
|  | B/M | Medium | 0,43 (6) | 0,83 (6) | 0,96 (6) | 0,99 (5) | 0,95 (5) |
|  |  | High | 0,51 (3) | 0,91 (3) | 0,99 (5) | 0,95 (8) | 0,87 (9) |
|  |  | Low | 0,33 (13) | 0,64 (13) | 0,75 (12) | 0,78 (14) | 0,77 (14) |
|  | D/P | Medium | 0,42 (8) | 0,81 (8) | 0,95 (7) | 1,00 (3) | 0,97 (3) |
|  |  | High | 0,47 (4) | 0,88 (4) | 0,99 (4) | 1,00 (4) | 0,94 (7) |
|  |  | Low | 0,30 (15) | 0,58 (15) | 0,70 (16) | 0,75 (15) | 0,75 (15) |
|  | CF/P | Medium | 0,43 (7) | 0,81 (7) | 0,95 (8) | 0,98(6) | 0,94 (6) |
|  |  | High | 0,55 (1) | 0,99 (1) | 1,09 (1) | 1,05 (1) | 0,96 (4) |
|  |  | Low | 0,29 (16) | 0,56 (17) | 0,68 (17) | 0,73 (16) | 0,73 (17) |
|  | E/P | Medium | 0,46 (5) | 0,87 (5) | 1,01 (3) | 1,04 (2) | 1,00 (2) |
|  |  | High | 0,55 (2) | 0,96 (2) | $\begin{aligned} & 1,03(2) \\ & 0,84(11) \end{aligned}$ | $\begin{aligned} & 0,96(7) \\ & 0,88(11) \end{aligned}$ | $\begin{aligned} & 0,85(11) \\ & 0,85(10) \end{aligned}$ |
| Market |  |  | 0,37 (11) | 0,71 (11) |  |  |  |

Table 6.9: Sharpe ratios for bond portfolios, stock portfolios based on either cap size, book-to-market ratio, dividend yield, cash flow yield, earnings yield and the market portfolio for 1, $5,10,15$ and 20 years, calculated by using the bootstrap method.

We can see than when we are using the bootstrap method to calculate the Sharpe ratio, the portfolio that performs best for rather short holding periods is the stock portfolio with high cash flow yield. The second best are stocks with high earnings yield, followed by stocks with
high book-to-market ratios and stocks with high dividend yield. All the stock portfolios performs better than the bonds with a 1 year holding period, but for longer holding periods we see that the portfolio that performs best is the intermediate-term government bonds. The second best portfolio consists of stocks with medium earnings yield, followed by the portfolio consisting of stocks with medium dividend yields. The differences between the Sharpe ratios of the portfolios are relative small, so if I was to run the program several times, I might not get the exact same ranking, some of the portfolios might switch places.

Table 6.1 and table 6.5, are also showing the Sharpe ratios for different portfolios, and using the bootstrap method. If we compare these tables to table 6.9 , we can see that the difference between the best and the worst portfolio is larger in the other tables than in this. The reason for this is that the data for portfolios based on the cash flow yields and the dividend yields starts in 1952, instead of 1927 as in the other tables.

Table 6.1 and 6.5 also shows that for rather long investment horizon the bond portfolios outperform all the stock portfolios. In table 6.9 we see that even though the intermediate-term bonds perform better than the other portfolios, the long-term bonds have almost the lowest Sharpe ratio no matter what holding period. For longer periods the long-term government bonds only perform better than the small stocks, while the long-term corporate bonds also beat the stocks with low earnings yield.

|  |  |  | 1 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Long-term | Corporate | Bonds | 0,20 (18) | 0,40 (18) | 0,49 (18) | 0,54 (18) | 0,56 (18) |
| Long-term | Government |  | 0,19 (19) | 0,36 (19) | 0,43 (19) | 0,48 (19) | 0,51 (19) |
| Intermed.term | Government | Bonds | 0,26 (17) | 0,47 (17) | 0,57 (17) | 0,63 (17) | 0,67 (17) |
|  |  | Low | 0,39 (10) | 0,82 (12) | 0,92 (13) | 0,94 (14) | 1,03 (13) |
|  | Size | Medium | 0,43 (9) | 1,00(8) | 1,17 (8) | 1,23 (9) | 1,29 (8) |
|  |  | High | 0,37 (12) | 0,83 (11) | 0,99 (11) | 1,10 (12) | 1,06 (12) |
|  |  | Low | 0,33 (14) | 0,73 (14) | 0,88 (14) | 0,99 (13) | 0,97 (14) |
|  | B/M | Medium | 0,44 (6) | 1,07 (5) | 1,30 (5) | 1,39 (5) | 1,37 (5) |
|  |  | High | 0,52 (3) | 1,27 (1) | 1,60 (1) | 1,68 (2) | 1,73 (1) |
|  |  | Low | 0,34 (13) | 0,77 (13) | 0,97 (12) | 1,11 (11) | 1,12 (11) |
|  | D/P | Medium | 0,43 (8) | 0,96 (9) | 1,14 (9) | 1,23 (8) | 1,19 (10) |
|  |  | High | 0,48 (4) | 1,17 (4) | 1,48 (4) | 1,59 (3) | 1,56 (3) |
|  |  | Low | 0,30 (15) | 0,69 (15) | 0,83 (15) | 0,91 (16) | 0,88 (16) |
|  | CF/P | Medium | 0,44 (7) | 1,01 (7) | 1,21 (7) | 1,30 (7) | 1,29 (7) |
|  |  | High | 0,57 (1) | 1,25 (2) | 1,58 (2) | 1,72 (1) | 1,70 (2) |
|  |  | Low | 0,29 (16) | 0,68 (16) | 0,82 (16) | 0,92 (15) | 0,90 (15) |
|  | E/P | Medium | 0,47 (5) | 1,06 (6) | 1,26 (6) | 1,36(6) | 1,33 (6) |
|  |  | High | 0,56 (2) | 1,22 (3) | 1,49 (3) | 1,56 (4) | 1,55 (4) |
|  | Market |  | 0,37 (11) | 0,88 (10) | 1,07 (10) | 1,20 (10) | 1,19 (9) |

Table 6.10: Sharpe ratios for bond portfolios, stock portfolios based on either cap size, book-to-market ratio, dividend yield, cash flow yield, earnings yield and the market portfolio for 1, $5,10,15$ and 20 years, calculated by using the block-bootstrap method.

In table 6.10 we assume that the returns are not independent and identically distributed, and we are therefore using the block-bootstrap method. The best long-term investment according to table 6.10 is the portfolio with high book-to-market ratio stocks, closely followed by the stocks with high cash flow yields. After these come the stocks with high dividend yields and the stocks with high earnings yields. All though the ranking of the portfolios changes with different holding periods, these 4 portfolios are in the top four for all holding period.

The portfolio that performs worst is the long-term government bonds, followed by the other bonds. And we see that the ranking of the bonds are the same for all horizons.

|  |  |  | 1 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Long.term | Corporate | Bonds | 0,38 (19) | 1,11 (19) | 1,93 (18) | 2,82 (19) | 3,86 (19) |
| Long-term | Government | Bonds | 0,40 (18) | 1,11 (18) | 1,92 (19) | 2,84 (18) | 3,90 (18) |
| Intermed.term | Government | Bonds | 0,64 (12) | 1,93 (15) | 3,70 (16) | 5,82 (16) | 8,76 (16) |
|  |  | Low | 0,80 (10) | 3,50 (10) | 8,87 (10) | 18,14 (10) | 34,78 (7) |
|  | Size | Medium | 0,83 (6) | 3,80 (6) | 9,71 (6) | 20,14 (6) | 38,24 (6) |
|  |  | High | 0,64 (11) | 2,68 (11) | 5,99 (12) | 11,10 (12) | 18,56 (12) |
|  |  | Low | 0,56 (15) | 2,20 (14) | 4,68 (14) | 8,25 (14) | 13,26 (14) |
|  | B/M | Medium | 0,82 (9) | 3,66 (9) | 9,14 (8) | 18,51 (8) | 34,16 (8) |
|  |  | High | 1,13 (3) | 6,17 (3) | 18,89 (3) | 45,64 (3) | 104,04 (3) |
|  |  | Low | 0,56 (14) | 2,22 (13) | 4,82 (13) | 8,54 (13) | 13,96 (13) |
|  | D/P | Medium | 0,82 (7) | 3,76 (7) | 9,17 (7) | 18,61 (7) | 33,78 (9) |
|  |  | High | 1,03 (4) | 5,13 (4) | 14,13 (4) | 31,48 (4) | 66,02 (4) |
|  |  | Low | 0,49 (16) | 1,84 (16) | 3,77 (15) | 6,34 (15) | 9,82 (15) |
|  | CF/P | Medium | 0,82 (8) | 3,68(8) | 9,09 (9) | 18,39 (9) | 33,74 (10) |
|  |  | High | 1,22 (2) | 7,15 (2) | 22,73 (2) | 58,80 (2) | 139,86 (2) |
|  |  | Low | 0,45 (17) | 1,70 (17) | 3,43 (17) | 5,67 (17) | 8,65 (17) |
|  | E/P | Medium | 0,91 (5) | 4,35 (5) | 11,32 (5) | 23,98(5) | 46,32 (5) |
|  |  | High | 1,27 (1) | 7,43 (1) | 24,33 (1) | 65,52 (1) | 159,57 (1) |
|  | Market |  | 0,64 (13) | 2,67 (12) | 6,00 (11) | 11,14 (11) | 18,74 (11) |

Table 6.11: Sortino ratios for bond portfolios, stock portfolios based on either cap size, book-to-market ratio, dividend yield, cash flow yield, earnings yield and the market portfolio for 1, $5,10,15$ and 20 years, calculated by using the bootstrap method.

In table 6.11 we are assuming that returns are independent and identically distributed, and therefore using the bootstrap method to calculate the Sortino ratio. We see that for rather short terms the best portfolios are the stocks with high earnings yields, stocks with high cash flow yields, stocks with high book-to-market ratios and stocks with high dividend yields. The longterm bonds perform the worst in the short run together with stocks with low earnings yield.

Looking at table 6.11 we see that the best option for a long-term investment would be a portfolio consisting of stocks with high earnings yield. The next best portfolio consists of stocks with high cash flow yields, and third is the portfolio of stocks with high book-tomarket ratio, so we see that the time horizon does not affect the ranking of the best portfolios. The worst portfolios are the portfolios of bonds, closely followed by the portfolio with low earnings yield stocks. Also the bottom 3 portfolios stay the same independent of the time horizon.

|  |  |  | 1 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Long-term | Corporate | Bonds | 0,37 (19) | 0,87 (18) | 1,55 (18) | 2,41 (18) | 3,13 (18) |
| Long-term | Government | Bonds | 0,39 (18) | 0,77 (19) | 1,24 (19) | 1,86 (19) | 2,31 (19) |
| Intermed.-term | Government | Bonds | 0,63 (13) | 1,53 (17) | 3,24 (17) | 5,98 (17) | 9,00 (17) |
|  |  | Low | 0,80 (10) | 4,02 (10) | 10,23 (10) | 24,24 (9) | 67,26 (8) |
|  | Size | Medium | 0,85 (9) | 4,52 (9) | 12,22 (7) | 32,61 (7) | 102,93 (5) |
|  |  | High | 0,66 (11) | 3,36 (12) | 8,47 (12) | 16,99 (12) | 31,35 (12) |
|  |  | Low | 0,58 (15) | 2,57 (14) | 5,64 (14) | 10,49 (14) | 16,25 (14) |
|  | B/M | Medium | 0,85 (8) | 5,39 (6) | 14,65 (6) | 35,19 (6) | 93,12 (6) |
|  |  | High | 1,15 (3) | 11,25 (1) | 57,90 (1) | 335,45 (1) | $\infty$ (1) |
|  |  | Low | 0,58 (14) | 2,94 (13) | 7,33 (13) | 14,86 (13) | 27,83 (13) |
|  | D/P | Medium | 0,85 (6) | 4,75 (8) | 11,36 (9) | 22,82 (10) | 45,15 (10) |
|  |  | High | 1,05 (4) | 7,91 (4) | 32,15 (4) | 93,37 (4) | 366,13 (4) |
|  |  | Low | 0,50 (16) | 2,23 (15) | 4,55 (15) | 7,69 (15) | 10,95 (15) |
|  | CF/P | Medium | 0,85 (7) | 4,89 (7) | 12,13 (8) | 26,05 (8) | 56,58 (9) |
|  |  | High | 1,24 (2) | 11,15 (2) | 42,34 (2) | 200,47 (2) | 4018,22 (2) |
|  |  | Low | 0,47 (17) | 2,08 (16) | 4,28 (16) | 7,46 (16) | 10,70 (16) |
|  | E/P | Medium | 0,94 (5) | 5,71 (5) | 15,21 (5) | 35,30 (5) | 84,92 (7) |
|  |  | High | 1,29 (1) | 10,32 (3) | 41,25 (3) | 167,53 (3) | 1949,90 (3) |
|  | Market |  | 0,66 (12) | 3,37 (11) | 8,67 (11) | 18,17 (11) | 37,71 (11) |

Table 6.12: Sortino ratios for bond portfolios, stock portfolios based on either cap size, book-to-market ratio, dividend yield, cash flow yield, earnings yield and the market portfolio for 1 , $5,10,15$ and 20 years, calculated by using the block-bootstrap method.

In table 6.12 we assume that the returns are serial dependent and we must use the blockbootstrap method. The four best portfolios in the short run are stocks with high earnings yield, stocks with high cash flow yield, high book-to-market ratio and high dividend yield. The worst portfolios are the bonds and the stocks with low earnings yield.

The best portfolio in table 6.12 for longer holding periods is the one of stocks with high book-to-market ratios. We see that the Sortino ratio for this portfolio, when we are using the blockbootstrap method, goes toward infinity. The second and third best options are the portfolios of stocks with high cash flow yields and stocks with high earnings yield, followed by stocks with high dividend yields. We see that though the ranking of the portfolios have changed, the four portfolios are the top 4 through the whole horizon I am looking at. The worst portfolios are the bonds.

### 6.3.2 Discussion and comparison

The four different tables give different answers to which portfolio that would be the best choice for an investor that wants to make a long term investment. As earlier when we are calculating the Sharpe ratio and assuming that returns are independent and identically distributed we see that the intermediate-term government bond is the best bond for the longterm investor. In the other section the long-term bonds has also outperformed the stocks under this assumption. But in table 6.9 we see that the long-term bonds do not perform better than the stocks. One explanation is of course that we are comparing the bonds with several more portfolios than earlier as we in this section also have included portfolios based on earnings yield, dividend yield and cash flow yield. But also if we only compare the long-term bonds with the portfolios based on the cap size and the book-to-market ratio we can see that all the stock portfolios except for the small cap stocks perform better than the long-term bonds. The difference between table 6.9 and 6.5 beside the extra portfolios we are analyzing, are that the returns are not from the exact same period. As mentioned the returns for portfolios based on cash flow yields and earnings yield are only available from 1952, so in the whole section 6.3 I
am using returns from 1952-2008, while in section 6.2 I used returns from 1927-2008. The reason for the difference in rankings must be that stocks either relative better or that the bonds perform relative worse in the shorter period.

In table 6.10, 611 and 6.12 we see that the same portfolios are in the top 4 in every section, though the ranking between them are not the same. These 4 portfolios are the best both for short term-investors and long-term investors. If look at the worst portfolios for the long run for these three table we see that the bonds and the stock portfolio with low earnings yield perform among the 5 worst portfolios in all three tables.

We know that there is an explanation for the difference in results in table 6.9 and the three others. As explained in the earlier sections, when we assume that returns are independent and identically distributed the upside variability will increase with time, and therefore also the standard deviation increases. This effect is largest for assets with high volatilities. The reason that the long-term bond perform relatively worse in this section, than in section 6.2, may of course also be due to this effect. If the long-term bond have both higher mean returns and standard deviation in this period(or if the stocks have lower standard deviations in this period), they might have a more similar rate of increase in the standard deviation over time as the stocks, and therefore not perform better than the stocks in the holding period we look at. However, if we were to increase the holding period, the bonds would eventually perform better than the stocks if their standard deviations are smaller than the stocks'. We know that this effect do not influence the ranking when we use the Sortino ratio as the downside ratio as Sortino ratio uses, do not include the upside variability.

The difference between the rankings from Sharpe ratio and Sortino when we assume that returns are serial dependent and therefore uses the block-bootstrap method are small, and they are mainly due to the difference in skewness for the portfolios. The Sharpe ratios for portfolios with positive skewness are undervalued, while the Sharpe ratios for portfolios with negative skewness are overvalued. I would therefore assume that portfolios that are ranked relative better according to Sharpe ratio have probability distributions that are skewed more to the left than for the rest of the assets, while portfolios that perform relative better according to Sortino ratio will probably have a probability distribution that is skewed more to the right than the average for the portfolios.

### 6.4 Industry

In this section I am comparing different industries to the bonds and market portfolio. I have 30 different industry portfolios, and this gives me 34 portfolios in total that I am comparing. The tables are getting so big and complex that I am choosing to only show the ratios for holding periods of $1,5,10,15$ and 20years, instead of each year.

### 6.4.1 Tables

|  | 1 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Long-term Corporate Bonds | 0,28 (26) | 0,61 (16) | 0,81 (7) | 0,94 (2) | 1,01 (2) |
| Long-term Government Bonds | 0,24 (32) | 0,52 (25) | 0,69 (16) | 0,80 (9) | 0,87 (4) |
| Intermed.-term Government Bonds | 0,32 (21) | 0,69 (9) | 0,93 (2) | 1,09 (1) | 1,20 (1) |
| Food Products | 0,44 (2) | 0,83 (1) | 0,93 (1) | 0,94 (3) | 0,87 (3) |
| Beer \& Liquor | 0,36 (11) | 0,59 (18) | 0,58 (23) | 0,45 (31) | 0,45 (26) |
| Tobacco Products | 0,46 (1) | 0,82 (2) | 0,88 (4) | 0,83 (6) | 0,75 (10) |
| Recreation | 0,29 (24) | 0,51 (28) | 0,52 (28) | 0,49 (27) | 0,40 (28) |
| Printing and Publishing | 0,28 (25) | 0,51 (26) | 0,55 (25) | 0,52 (25) | 0,46 (22) |
| Consumer Goods | 0,37 (7) | 0,69 (8) | 0,78 (9) | 0,78 (11) | 0,73 (11) |
| Apparel | 0,28 (28) | 0,50 (29) | 0,54 (27) | 0,53 (23) | 0,46 (23) |
| Healthcare | 0,44 (3) | 0,80 (3) | 0,89 (3) | 0,87 (4) | 0,80 (5) |
| Chemicals | 0,35 (13) | 0,64 (14) | 0,71 (13) | 0,69 (14) | 0,65 (13) |
| Textiles | 0,26 (30) | 0,46 (32) | 0,48 (33) | 0,45 (32) | 0,39 (30) |
| Construction and Construction Materials | 0,29 (22) | 0,55 (21) | 0,61 (19) | 0,59 (19) | 0,54 (20) |
| Steel Works Etc | 0,26 (31) | 0,46 (31) | 0,49 (31) | 0,46 (28) | 0,40 (29) |
| Fabricated Products and Machinery | 0,32 (18) | 0,60 (17) | 0,65 (18) | 0,63 (18) | 0,57 (17) |
| Electrical Equipment | 0,40 (5) | 0,70 (7) | 0,74 (11) | 0,68 (16) | 0,62 (16) |
| Automobiles and Trucks | 0,29 (23) | 0,50 (30) | 0,50 (30) | 0,44 (34) | 0,38 (31) |
| Aircraft, ships, and railroad equipment | 0,33 (16) | 0,58 (20) | 0,59 (22) | 0,54 (22) | 0,46 (24) |
| Metal industry | 0,28 (29) | 0,51 (27) | 0,56 (24) | 0,55 (21) | 0,49 (21) |
| Coal | 0,35 (14) | 0,59 (19) | 0,60 (21) | 0,53 (24) | 0,45 (25) |
| Petroleum and Natural Gas | 0,43 (4) | 0,79 (4) | 0,86 (5) | 0,84 (5) | 0,77 (6) |
| Utilities | 0,33 (17) | 0,62 (15) | 0,71 (14) | 0,71 (12) | 0,67 (12) |
| Communication | 0,34 (15) | 0,66 (10) | 0,77 (10) | 0,79 (10) | 0,77 (7) |
| Personal and Business Services | 0,32 (19) | 0,53 (23) | 0,51 (29) | 0,46 (30) | 0,35 (34) |
| Business Equipment | 0,36 (10) | 0,64 (13) | 0,69 (17) | 0,64 (17) | 0,57 (18) |
| Business Supplies and Shipping Containers | 0,40 (6) | 0,74 (5) | 0,82 (6) | 0,81 (8) | 0,76 (9) |
| Transportation | 0,28 (27) | 0,53 (24) | 0,60 (20) | 0,59 (20) | 0,56 (19) |
| Wholesale | 0,24 (33) | 0,43 (33) | 0,47 (34) | 0,45 (33) | 0,37 (33) |
| Retail | 0,37 (9) | 0,66 (11) | 0,72 (12) | 0,69 (13) | 0,62 (15) |
| Restaraunts, Hotels, Motels | 0,32 (20) | 0,55 (22) | 0,54 (26) | 0,51 (26) | 0,38 (32) |
| Banking, Insurance, Real Estate, Trading | 0,36 (12) | 0,65 (12) | 0,71 (15) | 0,69 (15) | 0,62 (14) |
| Everything else | 0,23 (34) | 0,43 (34) | 0,49 (32) | 0,46 (29) | 0,43 (27) |
| Market | 0,37 (8) | 0,70 (6) | 0,80 (8) | 0,81 (7) | 0,76 (8) |

Table 6.13: Sharpe ratios for bond portfolios, stock portfolios based on the industries and the market portfolio for $1,5,10,15$ and 20 years, calculated by using the bootstrap method.

Table 6.13 shows that for rather short holding periods the portfolios are food products and tobacco products together with healthcare. The bonds do not perform well in the short run, though some stocks perform worse, as "everything else" and wholesale. For longer horizons we see that the bonds are the portfolios that perform best together with food products.
Tobacco products are ranked as the $10^{\text {th }}$ best portfolio with a holding period of 20 years, while healthcare is ranked as the $5^{\text {th }}$ best. Personal and business service and wholesale performs worst for the long holding periods.

|  | 1 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Long-term Corporate Bonds | 0,29 (25) | 0,54 (25) | 0,60 (32) | 0,59 (32) | 0,55 (32) |
| Long-term Government Bonds | 0,25 (32) | 0,46 (34) | 0,50 (34) | 0,50 (34) | 0,47 (34) |
| Intermed.-term Government Bonds | 0,32 (21) | 0,54 (27) | 0,58 (33) | 0,57 (33) | 0,54 (33) |
| Food Products | 0,45 (2) | 0,81 (3) | 0,91 (11) | 0,86 (21) | 0,84 (22) |
| Beer \& Liquor | 0,36 (12) | 0,63 (16) | 0,77 (20) | 0,76 (24) | 0,73 (28) |
| Tobacco Products | 0,46 (1) | 0,82 (2) | 0,85 (14) | 0,77 (23) | 0,74 (27) |
| Recreation | 0,30 (23) | 0,56 (23) | 0,77 (21) | 0,88 (18) | 0,96 (18) |
| Printing and Publishing | 0,29 (26) | 0,50 (30) | 0,68 (28) | 0,74 (25) | 0,81 (23) |
| Consumer Goods | 0,37 (7) | 0,75 (7) | 1,02 (3) | 1,11 (4) | 1,14 (6) |
| Apparel | 0,28 (28) | 0,59 (21) | 0,73 (24) | 0,73 (27) | 0,75 (26) |
| Healthcare | 0,44 (3) | 0,81 (4) | 1,01 (4) | 1,09 (5) | 1,12 (8) |
| Chemicals | 0,35 (13) | 0,69 (11) | 0,97 (8) | 1,08 (7) | 1,18 (4) |
| Textiles | 0,26 (30) | 0,51 (29) | 0,67 (29) | 0,71 (29) | 0,75 (25) |
| Construction and Construction Materials | 0,30 (22) | 0,61 (18) | 0,91 (10) | 1,05 (9) | 1,13 (7) |
| Steel Works Etc | 0,26 (31) | 0,49 (32) | 0,64 (31) | 0,69 (30) | 0,71 (31) |
| Fabricated Products and Machinery | 0,33 (18) | 0,59 (20) | 0,91 (12) | 1,15 (3) | 1,27 (3) |
| Electrical Equipment | 0,40 (6) | 0,73 (8) | 0,91 (9) | 0,96 (13) | 1,02 (12) |
| Automobiles and Trucks | 0,29 (24) | 0,56 (24) | 0,79 (18) | 0,89 (17) | 0,95 (19) |
| Aircraft, ships, and railroad equipment | 0,33 (17) | 0,65 (15) | 0,82 (17) | 0,89 (16) | 0,99 (15) |
| Metal industry | 0,28 (29) | 0,54 (26) | 0,73 (23) | 0,86 (20) | 0,99 (16) |
| Coal | 0,35 (14) | 0,63 (17) | 0,68 (27) | 0,66 (31) | 0,72 (30) |
| Petroleum and Natural Gas | 0,43 (4) | 0,87 (1) | 1,18 (1) | 1,28 (1) | 1,30 (1) |
| Utilities | 0,33 (16) | 0,66 (14) | 0,89 (13) | 0,99 (10) | 1,06 (11) |
| Communication | 0,34 (15) | 0,67 (12) | 0,84 (15) | 0,90 (15) | 0,86 (21) |
| Personal and Business Services | 0,32 (20) | 0,50 (31) | 0,65 (30) | 0,72 (28) | 0,79 (24) |
| Business Equipment | 0,36 (10) | 0,66 (13) | 0,72 (26) | 0,74 (26) | 0,73 (29) |
| Business Supplies and Shipping Containers | 0,40 (5) | 0,77 (6) | 0,99 (7) | 0,99 (11) | 1,01 (13) |
| Transportation | 0,28 (27) | 0,59 (19) | 0,84 (16) | 0,97 (12) | 1,07 (10) |
| Wholesale | 0,24 (33) | 0,53 (28) | 0,74 (22) | 0,86 (19) | 0,97 (17) |
| Retail | 0,37 (9) | 0,71 (10) | 1,00 (6) | 1,06 (8) | 1,11 (9) |
| Restaraunts, Hotels, Motels | 0,32 (19) | 0,58 (22) | 0,78 (19) | 0,92 (14) | 1,00 (14) |
| Banking, Insurance, Real Estate, Trading | 0,36 (11) | 0,73 (9) | 1,00 (5) | 1,09 (6) | 1,15 (5) |
| Everything else | 0,23 (34) | 0,49 (33) | 0,72 (25) | 0,84 (22) | 0,92 (20) |
| Market | 0,37 (8) | 0,78 (5) | 1,10 (2) | 1,24 (2) | 1,30 (2) |

Table 6.14: Sharpe ratios for bond portfolios, stock portfolios based on the industries and the market portfolio for $1,5,10,15$ and 20 years, calculated by using the block-bootstrap method.

In this table we assume that there exits serial dependency between the annual returns. We see that for rather short holding periods the petroleum industry, tobacco products and food
products perform best. Tobacco products do not perform well in the long run, neither do the food products.

For longer horizons table 6.14 shows that all the portfolios of stocks perform better than any of the portfolios of bonds. Especially the portfolio of oil related stocks, the market portfolio and fabricated products and machinery perform well, so these portfolios are the best choice for an investor making a long-term investment. The portfolios that are the worst investments, besides the bonds, are steel works, coal and business equipment.

|  | 1 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Long-term Corporate Bonds | 0,53 (26) | 1,81 (31) | 3,51 (31) | 5,73 (31) | 8,31 (33) |
| Long-term Government Bonds | 0,48(32) | 1,51 (34) | 2,79 (34) | 4,36 (34) | 6,12 (34) |
| Intermed.-term Government Bonds | 0,79 (8) | 2,75 (15) | 5,75 (20) | 10,26 (22) | 15,58 (22) |
| Food Products | 0,94 (4) | 4,33 (3) | 12,10 (3) | 25,91 (4) | 49,49 (4) |
| Beer \& Liquor | 0,97 (3) | 4,28 (4) | 11,73 (4) | 27,21 (3) | 55,63 (3) |
| Tobacco Products | 1,04 (1) | 4,83 (1) | 13,96 (1) | 31,67 (1) | 65,91 (1) |
| Recreation | 0,52 (27) | 1,96 (26) | 4,43 (24) | 8,17 (24) | 14,27 (23) |
| Printing and Publishing | 0,51 (28) | 1,89 (27) | 4,21 (27) | 7,50 (27) | 12,82 (27) |
| Consumer Goods | 0,66 (17) | 2,60 (18) | 6,09 (17) | 11,64 (17) | 20,23 (17) |
| Apparel | 0,55 (23) | 2,02 (25) | 4,37 (25) | 7,85 (26) | 12,88 (26) |
| Healthcare | 1,00 (2) | 4,71 (2) | 13,14 (2) | 29,20 (2) | 59,13 (2) |
| Chemicals | 0,74 (11) | 3,03 (11) | 7,31 (12) | 14,43 (12) | 25,97 (13) |
| Textiles | 0,49 (31) | 1,81 (30) | 3,91 (28) | 6,99 (28) | 11,61 (28) |
| Construction and Construction Materials | 0,55 (25) | 2,08 (23) | 4,52 (23) | 8,21 (23) | 13,71 (24) |
| Steel Works Etc | 0,50 (30) | 1,82 (29) | 3,80 (30) | 6,82 (29) | 11,22 (29) |
| Fabricated Products and Machinery | 0,61 (21) | 2,40 (21) | 5,42 (21) | 10,26 (21) | 17,74 (21) |
| Electrical Equipment | 0,81 (7) | 3,60 (6) | 9,32 (6) | 19,93 (6) | 39,69 (6) |
| Automobiles and Trucks | 0,60 (22) | 2,31 (22) | 5,36 (22) | 10,41 (20) | 18,70 (18) |
| Aircraft, ships, and railroad equipment | 0,65 (18) | 2,83 (14) | 6,67 (14) | 13,57 (14) | 24,79 (14) |
| Metal industry | 0,55 (24) | 2,03 (24) | 4,36 (26) | 7,95 (25) | 12,92 (25) |
| Coal | 0,74 (10) | 3,09 (8) | 7,66 (8) | 15,59 (9) | 30,50 (9) |
| Petroleum and Natural Gas | 0,84 (5) | 3,97 (5) | 10,47 (5) | 22,87 (5) | 44,55 (5) |
| Utilities | 0,63 (20) | 2,54 (19) | 5,82 (18) | 10,73 (18) | 18,15 (19) |
| Communication | 0,66 (15) | 2,51 (20) | 5,81 (19) | 10,46 (19) | 17,94 (20) |
| Personal and Business Services | 0,72 (12) | 2,98 (12) | 7,62 (9) | 15,84 (8) | 31,85 (8) |
| Business Equipment | 0,71 (13) | 3,08 (10) | 7,56 (10) | 15,35 (10) | 29,26 (10) |
| Business Supplies and Shipping Containers | 0,82 (6) | 3,50 (7) | 8,89 (7) | 18,09 (7) | 33,50 (7) |
| Transportation | 0,50 (29) | 1,82 (28) | 3,80 (29) | 6,60 (30) | 10,56 (30) |
| Wholesale | 0,42 (34) | 1,55 (33) | 3,18 (33) | 5,52 (33) | 8,75 (31) |
| Retail | 0,71 (14) | 2,96 (13) | 7,23 (13) | 14,21 (13) | 25,99 (12) |
| Restaraunts, Hotels, Motels | 0,75 (9) | 3,08 (9) | 7,52 (11) | 15,25 (11) | 28,79 (11) |
| Banking, Insurance, Real Estate, Trading | 0,64 (19) | 2,63 (17) | 6,31 (16) | 12,07 (15) | 21,50 (15) |
| Everything else | 0,45 (33) | 1,58 (32) | 3,22 (32) | 5,56 (32) | 8,58 (32) |
| Market | 0,66 (16) | 2,71 (16) | 6,33 (15) | 12,01 (16) | 20,92 (16) |

Table 6.15: Sortino ratios for bond portfolios, stock portfolios based on the industries and the market portfolio for $1,5,10,15$ and 20 years, calculated by using the bootstrap method.

According to table 6.15 where we are comparing the portfolios based on their Sortino ratios calculated by using the bootstrap method, the best portfolios for rather short holding periods are tobacco products and healthcare. Also the food products and beer and liquor perform well in the short run. For longer horizons the best portfolio are still the tobacco products. And the
second best is healthcare. This means that the time horizon do not affect the choice of which industry an investor should invest in.

The worst portfolios for short horizons are the long term government bonds, wholesale and "everything else". For long horizons the long-term bonds are the worst portfolios followed by "everything else" stocks, while the intermediate-term bonds are ranked as the $22^{\text {nd }}$.

|  | 1 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Long-term Corporate Bonds | 0,53 (23) | 1,34 (33) | 2,28 (33) | 3,20 (33) | 3,70 (33) |
| Long-term Government Bonds | 0,49 (29) | 1,07 (34) | 1,66 (34) | 2,26 (34) | 2,52 (34) |
| Intermed.-term Government Bonds | 0,79 (7) | 2,10 (25) | 4,30 (30) | 7,14 (30) | 9,78 (31) |
| Food Products | 0,92 (4) | 4,44 (3) | 18,60 (2) | 48,93 (3) | 147,24 (3) |
| Beer \& Liquor | 0,95 (3) | 3,86 (6) | 12,15 (6) | 30,03 (7) | 70,02 (7) |
| Tobacco Products | 1,00 (1) | 6,29 (1) | 31,41 (1) | 135,50 (1) | 981,33 (1) |
| Recreation | 0,50 (27) | 2,30 (22) | 6,13 (22) | 12,65 (23) | 25,25 (21) |
| Printing and Publishing | 0,49 (28) | 2,17 (24) | 5,99 (23) | 12,37 (24) | 27,44 (18) |
| Consumer Goods | 0,63 (17) | 2,33 (21) | 6,33 (21) | 12,99 (21) | 23,82 (25) |
| Apparel | 0,53 (24) | 2,04 (28) | 4,96 (27) | 9,74 (27) | 19,41 (27) |
| Healthcare | 0,98 (2) | 4,70 (2) | 16,12 (4) | 40,84 (4) | 97,31 (5) |
| Chemicals | 0,71 (9) | 3,32 (8) | 11,20 (8) | 22,24 (10) | 43,31 (14) |
| Textiles | 0,47 (32) | 2,10 (26) | 5,99 (24) | 12,07 (26) | 25,21 (22) |
| Construction and Construction Materials | 0,53 (25) | 2,28 (23) | 6,82 (20) | 14,25 (17) | 28,00 (17) |
| Steel Works Etc | 0,48 (31) | 1,81 (32) | 4,02 (32) | 7,06 (31) | 10,87 (30) |
| Fabricated Products and Machinery | 0,58 (22) | 2,52 (18) | 7,36 (17) | 13,91 (19) | 25,99 (19) |
| Electrical Equipment | 0,77 (8) | 3,37 (7) | 11,75 (7) | 31,34 (6) | 75,97 (6) |
| Automobiles and Trucks | 0,58 (21) | 2,46 (19) | 7,07 (18) | 13,54 (20) | 24,92 (24) |
| Aircraft, ships, and railroad equipment | 0,63 (18) | 2,86 (14) | 8,07 (16) | 18,10 (16) | 39,11 (16) |
| Metal industry | 0,52 (26) | 1,87 (29) | 4,06 (31) | 5,81 (32) | 8,13 (32) |
| Coal | 0,70 (11) | 3,26 (9) | 8,14 (15) | 14,07 (18) | 19,92 (26) |
| Petroleum and Natural Gas | 0,82 (5) | 4,13 (5) | 18,33 (3) | 54,00 (2) | 206,86 (2) |
| Utilities | 0,62 (20) | 2,41 (20) | 5,93 (25) | 12,75 (22) | 25,91 (20) |
| Communication | 0,65 (15) | 2,67 (17) | 7,00 (19) | 18,81 (14) | 44,95 (10) |
| Personal and Business Services | 0,70 (12) | 3,11 (10) | 8,63 (13) | 20,17 (13) | 43,47 (11) |
| Business Equipment | 0,69 (13) | 3,10 (11) | 9,13 (12) | 21,22 (11) | 43,47 (12) |
| Business Supplies and Shipping Containers | 0,80 (6) | 4,17 (4) | 16,08 (5) | 37,57 (5) | 105,94 (4) |
| Transportation | 0,49 (30) | 1,82 (31) | 4,60 (28) | 8,83 (28) | 18,33 (28) |
| Wholesale | 0,41 (34) | 2,04 (27) | 5,81 (26) | 12,35 (25) | 25,08 (23) |
| Retail | 0,69 (14) | 2,95 (13) | 9,22 (11) | 20,28 (12) | 40,90 (15) |
| Restaraunts, Hotels, Motels | 0,70 (10) | 2,96 (12) | 8,18 (14) | 18,65 (15) | 43,45 (13) |
| Banking, Insurance, Real Estate, Trading | 0,62 (19) | 2,73 (16) | 9,40 (10) | 22,53 (9) | 53,53 (8) |
| Everything else | 0,43 (33) | 1,85 (30) | 4,53 (29) | 8,31 (29) | 16,20 (29) |
| Market | 0,64 (16) | 2,77 (15) | 9,59 (9) | 22,86 (8) | 51,70 (9) |

Table 6.16: Sortino ratios for bond portfolios, stock portfolios based on the industries and the market portfolio for 1, 5, 10, 15 and 20 years, calculated by using the block-bootstrap method.

The final table, in this section, shows the Sortino ratio for the portfolios calculated by using the block-bootstrap method. We see that for short horizons the worst portfolios are the longterm bonds and the steel works, while for long holding periods all bonds and also the metal industry are outperformed by the other portfolios. The tobacco stocks perform best for all time horizons. Also food and petroleum and natural gas perform well for all horizons.

### 6.4.2 Discussion and comparison

The four tables in this section rank the portfolios in different ways. If we assume that investors have quadratic preferences and that returns are independent and identically distributed then the best portfolio for rather short periods would be food products or tobacco products, while for the long term investor the best portfolio is intermediate-term bonds.

If we assume that the investors have quadratic preference, but that the returns are not independent and identically distributed the tobacco products or the petroleum and natural gas would be the best choice for short horizons, and the petroleum and natural gas is also the best portfolio for the long term investor.

If we assume that investors only do not have quadratic preferences, and that returns are independent and identically distributed the best tobacco products is the best choice for all horizons.

And finally, if we assume that investors do not have quadratic preferences and that returns are not independent and identically distributed, the best investment is the tobacco products for all horizons. This means that if we assume that investors do not have quadratic preferences, but that they instead only fear the downside risk, the time horizon does not affect which portfolio that performs the best.

## 7 Conclusion

During the analysis we have seen that it is not possible to give just one straight answer to which portfolio that is best for the long-term investor. The answer depends on which assumptions we make regarding the investor's preferences and on whether we assume that returns are independent and identically distributed or not. Also, the analysis has shown that the results depend on the period the empirical returns are collected from.

If we assume that investors have quadratic preferences and that returns are independent and identically distributed, we have seen that at least when we use the whole period from 19272008, the bonds tend to outperform the stocks for longer horizons. The exception is the food products in table 6.13 which were ranked as the $3^{\text {rd }}$ best portfolio for long-term investors, in front of the long-term government bonds. The bonds that tend to perform the best both in the short and the long run under these assumptions are the intermediate-term government bonds. The second best are the corporate bonds, while the long-term government bonds tend to perform the worst of the bonds. When the annual returns only are collected from 1952-2008 as in table 6.9 , we see a dramatic change in the ratings. Though the intermediate-term bonds still outperform the stocks in the long run, the long-term bond are ranked as the $16^{\text {th }}$ and $18^{\text {th }}$ of a total of 19 portfolios. Only under these assumption the ranking seems to be influenced by the change in the number of annual returns. If we only compare the stocks, we see that for long horizons big cap stocks tend to outperform mid and small cap stocks, medium book-tomarket ratios seems to outperform low and high book-to-market ratios, medium dividend yields tend to outperform low and high dividend yields, high cash flow yields seems to outperform medium and low cash flow yields and medium earnings yields tend to outperform low and high earnings yields. We also have seen that with these assumptions the market portfolio performs relative well.

When we assume that investors have quadratic preferences, and that returns are not independent and identically distributed, all the stocks outperforms the bonds for longer horizons. It does not matter if the annual returns are collected from 1952-2008 or 1927-2008. Under these assumptions mid cap stocks tend to outperform big and small cap stocks, and for the book-to-market ratios, the dividend yields, the cash flow yields and the earnings yields high values outperform the medium values, and the medium values outperform the low
values. The portfolio that performs best for the long-term investor when we are comparing portfolios based on statistics is the stocks with high book-to-market ratios. When we are comparing the industries, the petroleum and natural gas is the best. We also have seen that with these assumptions the market portfolio performs relative well.

If we assume that investors do not have quadratic preferences and that returns are independent and identically distributed, all the stocks outperforms the long-term bonds for longer horizons, the intermediate-term government bonds however, perform better than stocks with low earnings yields, and when we are comparing bonds and the industry portfolios, intermediateterm bond are ranked as the $22^{\text {nd }}$. Mid cap stocks tend to outperform big and small cap stocks, and for the book-to-market ratios, the dividend yields, the cash flow yields and the earnings yield high values outperform medium values, and the medium values outperform low values. The portfolio that performs best for the long-term investor when we are comparing portfolios based on statistics is the stocks with high earnings yields. When we are comparing the industries, the tobacco products perform the best. These portfolios perform best for all horizons. Under these assumptions the expected holding period does influence which portfolio one should invest in.

If we assume that investors do not have quadratic preferences and that returns are not independent and identically distributed, all the stocks outperforms the bonds for longer horizons, except for the metal industry that is outperformed by the intermediate-term government bonds. With these assumptions the mid cap stocks tend to outperform big and small cap stocks, and for the book-to-market ratios, the dividend yields, the cash flow yields and the earnings yield high values outperform medium values, and the medium values outperform low values. The portfolio that performs best for the long-term investor when we are comparing portfolios based on statistics is the stocks with high book-to-market ratios. When we are comparing the industries the tobacco products perform the best. Both these portfolios perform well for all horizons.

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## Appendix

## Matlab program:

```
clear; clc; close all
filename = '....txt'
%filename = '....txt'
data = load(filename)/100 % load data and divide by 100 to correct for
percents
nSim = 50000 % define the number of simulations
T = 1:20 % define investment horizons
numT = length(T) % compute the number of horizons
[nRows, numPort] = size(data); % find the number of risky portfolios
numPort = numPort - 1 % as one column contains TBill return
% allocate the space for performance measures
PM = zeros(numT, numPort);
% LOOP: BOOTSTRAP SIMULATION AND COMPUTATION OF PERFORMANCE MEASURES
% compute the performance measures for each T
for i=1:numT
    % perform bootstrap or block-bootstrap
    y = bootstrap(T(i),data,nSim);
    B = round(0.75*T(i));
    %y = blockbootstrap(B,T(i),data,nSim);
    r = y(:,end); % risk-free rate of return in the last column
    for j=1:numPort
        x = y(:, j);
            % compute the Sortino ratio
            %PM(i,j) = Sortino(x, r);
            % compute the Sharpe ratio
            PM(i,j) = SR(x, r);
    end % loop wrt i
end % loop wrt j
% PLOT THE RESULS
%plot(PM)
semilogy(T,PM(:, 1),'k',T,PM(:, 2),'--k',T,PM(:, 3),':k')
hold all
semilogy(T,PM(:, 4:end-1))
hold all
semilogy(T,PM(:, end),':r','LineWidth', 2)
xlabel('Investment horizon')
ylabel('Performance measure')
```

```
legend('Long term corporate bonds','Long term goverment
bonds','Intermediate term goverment bonds',...
    'Low size', 'Med size', 'High size', 'Low bm', 'Med bm', 'High
bm', . . 
    'Market', -1)
for i=1:numT
    for j=1:numPort
        fprintf('%5.2f ', PM(i,j))
    end % loop wrt i
    fprintf('\n')
end % loop wrt j
```


[^0]:    ${ }^{1} \mathrm{http}: / /$ www.investopedia.com/terms/s/small-cap.asp
    ${ }^{2}$ http://www.investopedia.com/terms/l/large-cap.asp
    ${ }^{3}$ http://www.investopedia.com/terms/b/booktomarketratio.asp
    ${ }^{4}$ http://www.investorwords.com/6749/book to market ratio.html

[^1]:    ${ }^{5}$ http://www.investopedia.com/terms/d/dividendyield.asp

[^2]:    ${ }^{6}$ http://www.investopedia.com/terms/e/earningsyield.asp

[^3]:    ${ }^{7} \mathrm{http}: / /$ mba.tuck.dartmouth.edu/pages/faculty/ken.french/data library.html
    ${ }^{8}$ Ibbotson yearbook 2009

