UNIVERSITY OF AGDER

Dynamic Asset Allocation Strategies Based on Volatility, Unexpected Volatility and Financial Turbulence

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This master's thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

Preface

This Master's thesis is the last contribution in my master's degree in Economic and Business Administration with specializing in financial economics, at School of Business and Law in University of Agder. This education has given me a deeper understanding of financial theory, especially how financial models can describe real-world financial problems and how to implement created and theoretical models empirically. The latter was motivation for this thesis, along with lectures and published papers from my supervisor Professor Valeriy Zakamulin. This thesis has also given me useful skills in computation and programing with the free statistical software R. The process of writing this thesis has been extensive and challenging, however, I have learn a lot, and it has also been both fun and interesting.

I would like to thank my supervisor Professor Valeriy Zakamulin for constructive feedback and exceptional guidance whenever needed, I am also grateful for support with functions used in R-programming. My follow classmates have given me an enjoyable companionship along with constructive discussions and comparisons throughout the Master's programme, for this I am thankful. Last but not least, I would like to thank my closest family for being supportive during the countless amount of hours I was away when studying.

David Borkner Grimsrud

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Abstract

This master thesis looks at unexpected volatility- and financial turbulence's predictive ability, and exploit these measures of financial risk, together with volatility, to create three dynamic asset allocation strategies, and test if they can outperform a passive and naively diversified buy-and-hold strategy. The idea with the dynamic strategies is to increase the portfolio return by keeping the portfolio risk at a low and stable level over time. This is be done by changing the allocation between risky asset and risk-free asset, as the market environment changes. Three dynamic asset allocation strategies are implemented: a turbulence-responsive strategy, an unexpected volatility-responsive strategy, and a volatility-responsive strategy. The empirical results show that all three dynamic asset allocation strategies strongly outperform a passive equally-weighted benchmark in the out-of-sample period with respect to risk-adjusted return.

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1. Introduction

A rational investor will try to maximize his/hers portfolio return adjusted for its risk. Risk is usually measured as the volatility of the return, that is the chance that an investment's actual return will be different from expected. However, there is alternative measures of risk such as unexpected volatility and financial turbulence[1].

Figure 1: Development of 10 Industry Portfolios from 01.1975 to 07.2014. We can see several bear markets categorized by persistent periods of negative returns, high volatility and high turbulence, while bull markets are considered with persistent periods of positive returns, low volatility and low turbulence.

Development in time
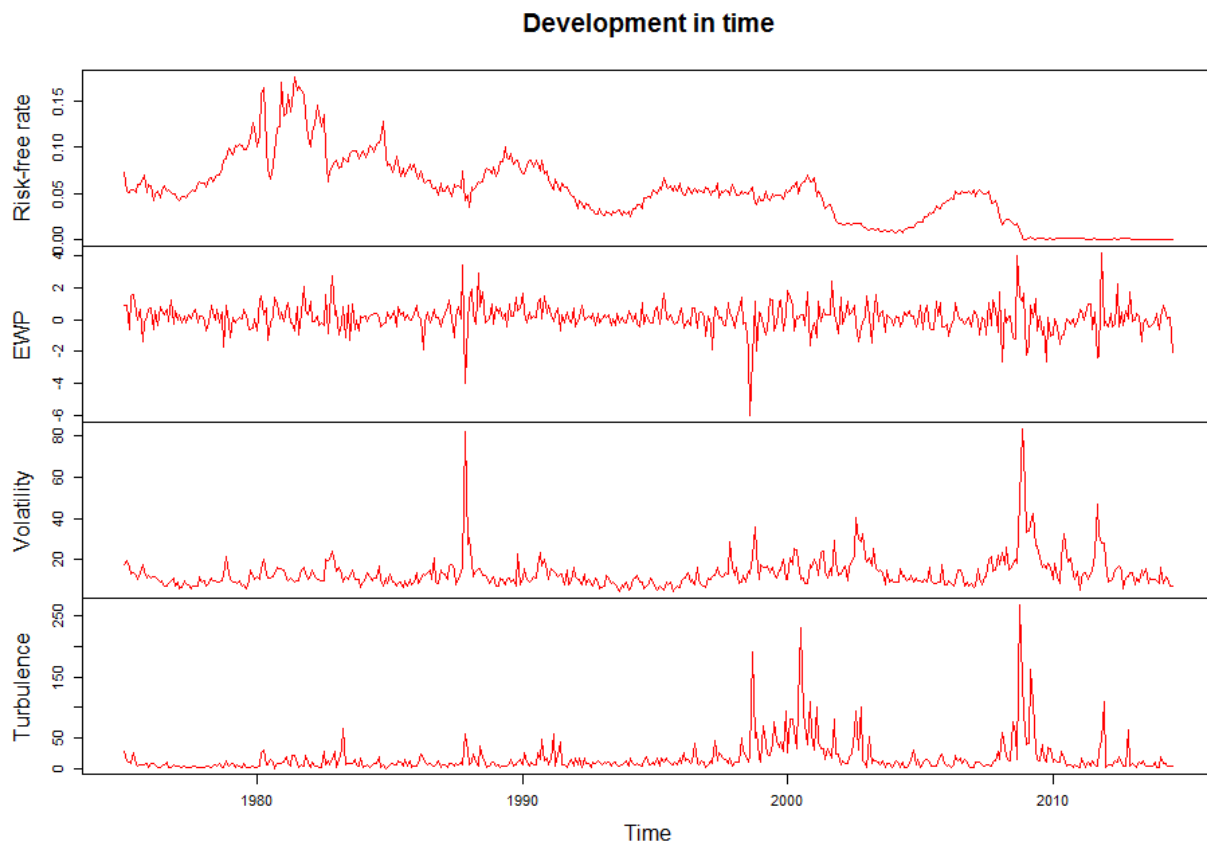
Asset allocation is generally very fixed at institutional investors, because the decision makers review the strategic plan for asset allocation with respect to the market conditions too infrequently. Although a portfolio of 50% risky assets and 50% risk-free asset (50/50 portfolio) may deliver a wanted level of risk on average in the long-run, it is rarely an optimal portfolio in bear markets, where the market volatility is high. Fixed-weighted asset allocation strategies can have very unpredictable performance in high-volatile periods. Baker and Haugen (2012) found

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[1]Financial turbulence is in the following of this thesis just denoted as turbulence.

that low-volatility and low-beta portfolios produced exceptional high returns and small drawdowns in the period 1968-2008, in contrast to the traditional assumption that higher risk is rewarded with higher expected return. This low-volatility anomaly from Baker and Haugen (2012) supports that it rarely optimal be invested in a fixed-weighted (passive) portfolio in volatile periods.

A dynamic volatility-responsive asset allocation strategy takes care of that problem, by adjusting the weights between a risky asset and a risk-free asset according to the market volatility, such that the portfolio volatility is managed. That is, in periods of high volatility, the dynamic strategy will reduce the weight in the risky asset, and hence increase the weight in the risk-free asset. In periods with low volatility, the strategy will keep a high weight in the risky asset and a low weight in the risk-free asset.

Figure 2: Monthly volatility of a dynamic volatility-responsive strategy (blue) and a passive 50/50 portfolio (red). Both strategies uses an equally weighted portfolio of U.S. 10 Industry Portfolio returns as the risky asset, and U.S. Treasury bill return as the risk-free asset. The 50/50 portfolio are all-time fixed 50 percent in the risky asset and fixed 50 percent in the risk-free asset, whereas the volatility-responsive strategy allocate the weighs in the two assets after changes in the economic environment.
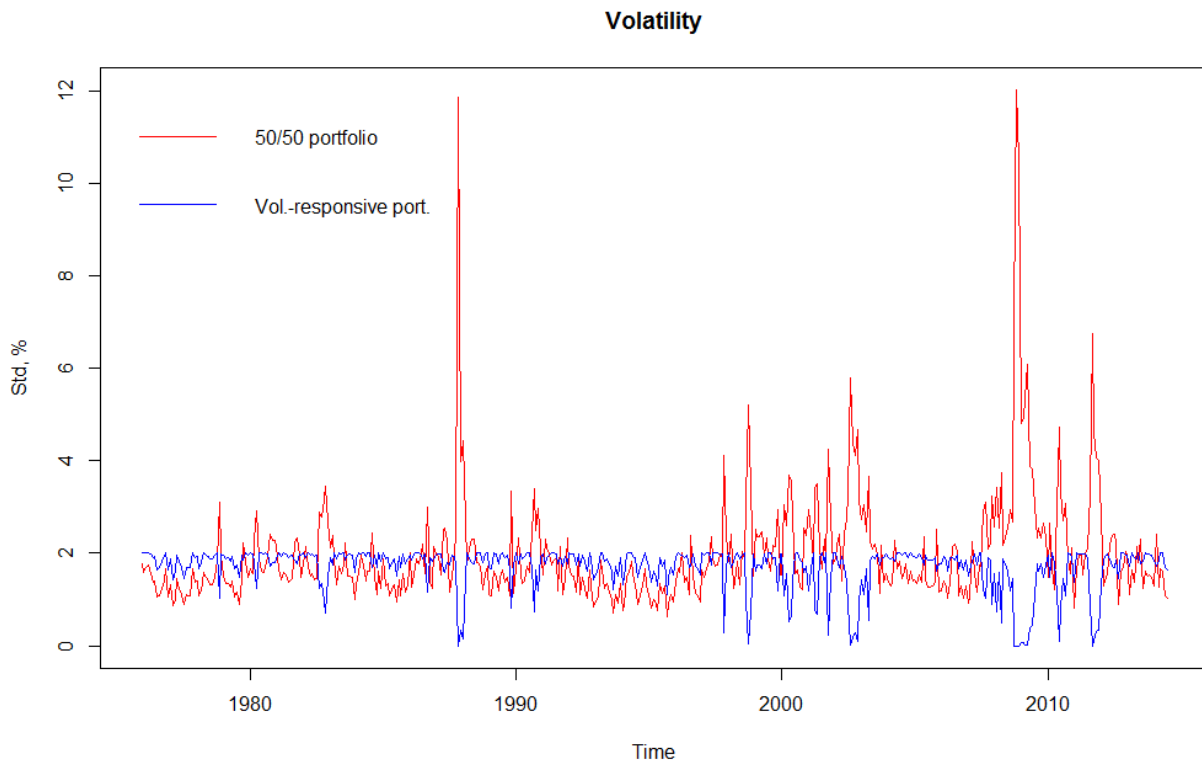
In Figure 2, you see an example of the difference in volatility between a passive 50/50 portfolio

and a dynamic volatility-responsive strategy. The volatility in a dynamic volatility-responsive portfolio rarely get monthly volatility above 2 percent. Moreover, in the volatile periods, e.g. 1987, '98, '03, '08 and 2012, the volatility-responsive portfolio weight down in risky asset to approximately zero monthly portfolio volatility. Whereas the passive 50/50 portfolio reaches high levels of monthly volatility in distressful periods.

Merton (1980) argue that excess return and market volatility should be positively related, yet, many researchers find a negative relation between excess return and market volatility over time. French, Schwert and Stambaugh (1987) explain that volatility consist of expected- and unexpected volatility, while expected volatility is positively related to excess return, unexpected volatility is negatively related to excess return. In fact, many empirical studies find that active strategies that seek to keep the volatility at a target level outperform passive buy-and-hold strategies, but few have explored how unexpected volatility predicts future volatility and can be used as a determining factor in a dynamic asset allocation strategy.

Zakamulin (2014) covers a gap in the literature with his paper "Dynamic Asset Allocation Strategies Based on Unexpected Volatility". However, there are still lots of room for textbook in finance to write about unexpected volatility. Likewise, turbulence is only mentioned by a handful of researchers in finance, Kritzman and Li (2010) and Harman (2014) is three of them, they define financial turbulence and use turbulence as a risk measure in asset allocation strategies to improve portfolio risk-adjusted return.

There is an argument among researchers on whether an optimized portfolio, that have a specific strategy to diversify across assets and time, can significantly outperform a naively diversified portfolio. DeMiguel, Garlappi and Uppal (2009) and Duchin and Levy (2009) claim that none of the tested optimized portfolios preformed significantly better than the naively diversified portfolio, in terms of Sharpe ratio. Kritzman, Page and Turkington (2010) answered by presenting a minimum-variance portfolio and a mean-variance portfolio where both of them delivered higher Sharpe ratios out-of- sample than a naively diversified portfolio and the market portfolio. Kirby and Ostdiek (2012) also replied by representing an optimized portfolio with time diversification.

This thesis investigates three different dynamic asset allocation strategies based on volatility, unexpected volatility and turbulence. These three dynamic strategies monthly allocate the weight in a risky asset and a risk-free asset according to last month measured risk, in order to keep a high reward and at the same time keep a low level of risk in the portfolio. They are compared against a benchmark, which is a naively diversified equally weighted market portfolio.

Before I go further with these asset allocation strategies, I present in Section 5.1 unexpected volatility and its ability to predict future volatility and excess return. I find that unexpected volatility can predict future volatility and future excess return, this result is in line with French et al. (1987) and Zakamulin (2014), and is a key element for making a dynamic unexpected volatility-responsive strategy.

Similarly, I present historical turbulence's ability to predict future turbulence and future excess return in Section 5.2. I find that turbulence is like volatility very persistent, turbulence can be used to predict next month turbulence, equivalent to Kritzman and Li (2010). This result give an opportunity to use turbulence in a dynamic asset allocation strategy. However, I find no significant relation between turbulence and future excess return.

The three dynamic asset allocation strategies implemented in this thesis are: a volatility-responsive strategy, an unexpected volatility-responsive strategy and a turbulence-responsive strategy. They diversify the portfolios across assets and over time, and they are evaluated on risk-adjusted returns. To estimate the input in these dynamic asset allocation strategies, volatility, unexpected volatility and turbulence, I apply a five years in-sample period and a five years rolling estimation window that runs throughout the out-of-sample period. I find that all three dynamic asset allocation strategies deliver a considerable higher risk-adjusted return than the passive benchmark in the out-of-sample period.

The rest of the thesis is organized as follows: Section 2 is relevant theory and review on literature, Section 3 informs about the dataset, Section 4 explains the methodology, Section 5 presents the

empirical result, Section 6 discusses the findings, and Section 7 concludes. The R programs used to compute the empirical results are presented in the Appendix.

2. Relevant theory –and literature review

2.1 Modern portfolio theory

The pioneer of modern portfolio theory (MPT), Harry Markowitz, introduced MPT in Markowitz (1952) and Markowitz (1959). It is a financial theory where the investor is assumed to be rational, and attempts to optimize a portfolio by allocate weights of specific various assets according to investor's risk aversion. Such that it maximizes portfolio expected return for a given level portfolio risk, or minimize portfolio risk for a given level of expected return. This means that you should not select individual assets based on its own qualities, but rather based on how its returns will affect the portfolio qualities, in that how the individual asset returns correlates to other assets returns in the portfolio, in order to diversify portfolio risk.

In more mathematical terms and matrix notation, we can find the minimum portfolio variance, σ_p^2, for any particular portfolio return, μ_p. The weights, w_i, invested in each asset, assuming N different assets exist, is limited to 1.

$$\sum_{i=1}^{N} w_i = 1. \qquad (2.1)$$

The weights is a (N x 1) vector, $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix}$ [2].

The portfolio return, r_p, is the weighed sum of the individual asset returns, $\boldsymbol{r}$, where r_p is a (1 x 1) scalar and $\boldsymbol{r} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{pmatrix}$ is a (N x 1) vector of returns.

$$r_p = \boldsymbol{w}'\boldsymbol{r} = (w_1, w_2, \cdots, w_N) \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{pmatrix} = w_1 r_1 + w_2 r_2 + \cdots + w_N r_N. \qquad (2.2)$$

This gives us the expected portfolio return,

$$\mu_p = E[r_p] = \boldsymbol{w}'E[\boldsymbol{r}] = \boldsymbol{w}'\boldsymbol{\mu}, \qquad \text{where } \boldsymbol{\mu} = E[\boldsymbol{r}]. \qquad (2.3)$$

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[2] Vectors and matrices are expressed in bold typography.

The portfolio variance is given by,

$$\sigma_p^2 = \boldsymbol{w'\Sigma w} =$$

$$(w_1, w_2, \cdots, w_N) \begin{pmatrix} Var(r_1) & cov(r_1, r_2) & \cdots & cov(r_1, r_N) \\ cov(r_2, r_1) & Var(r_2) & \cdots & cov(r_2, r_N) \\ \vdots & \vdots & \ddots & \vdots \\ cov(r_N, r_1) & cov(r_N, r_2) & \cdots & Var(r_N) \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix}. (2.4)$$

Where $\boldsymbol{\Sigma}$ is a (N x N) covariance matrix containing the variance of all N assets returns and their pair wise covariance between the N assets returns.

The minimum variance for a target portfolio return, μ^*, can be found by solving this quadratic function,

$$min 0.5\sigma_p^2, with\ respect\ to\ \mu_p = \mu^* \ and\ \sum_{i=1}^{N} w_i = 1. \qquad (2.5)$$

By solving this problem, you get the optimal asset allocation weights that will minimize the risk for a given level of return. This optimal solution lies on the efficient frontier (described by Markowitz (1952)). The efficient frontier is a set of risky assets that offers a minimum amount of risk for a given feasible target return, this forms as an upper part of a hyperbola in a $(\mu_p,\ \sigma_p)$-space. The exact allocation in the efficient frontier depends on the investor's risk tolerance.

When a risk-free asset is included, the efficient frontier will no longer be a set of portfolios, but one specific portfolio of risky assets, called the tangency portfolio (sometimes also called the optimal risky portfolio). That is the portfolio that tangents the efficient frontier when you draw a line from the risk-free asset to the efficient frontier in a $(\mu_p,\ \sigma_p)$-space. The tangency portfolio together with the risk-free asset, will be the best fit for each investor's individual risk tolerance. And the portfolio return, r_p is,

$$r_p = wr_t^* + (1 - w)rf. \qquad (2.6)$$

Where r_t^* denote return from the tangency portfolio, w is the weight invested in the tangency portfolio and rf denotes the return on the risk-free asset. r_p is called the best possible capital allocation line (CAL). Due to the fact that the variance and the risk of a risk-free asset is zero, the variance of this portfolio will be,

$$\sigma_p^2 = w^2\sigma_t^2 + (1-w)^2\sigma_{rf}^2 + 2w^2(1-w)^2 cov(r_t, rf)$$

$$= w^2\sigma_t^2 + (1-w)^2 0 + 2w^2(1-w)^2 0 = w^2\sigma_t^2. \qquad (2.7)$$

Now that we know the portfolio return and we know that the standard deviation is the square root of the variance, we can compute the CAL as,

$$r_p = rf + \left(\frac{r_t - rf}{\sigma_t}\right)\sigma_p. \qquad (2.8)$$

Figure 3: Illustration of Efficient frontier, CAL and tangency portfolio, with and without short sale restriction

10 US Industry portfolios
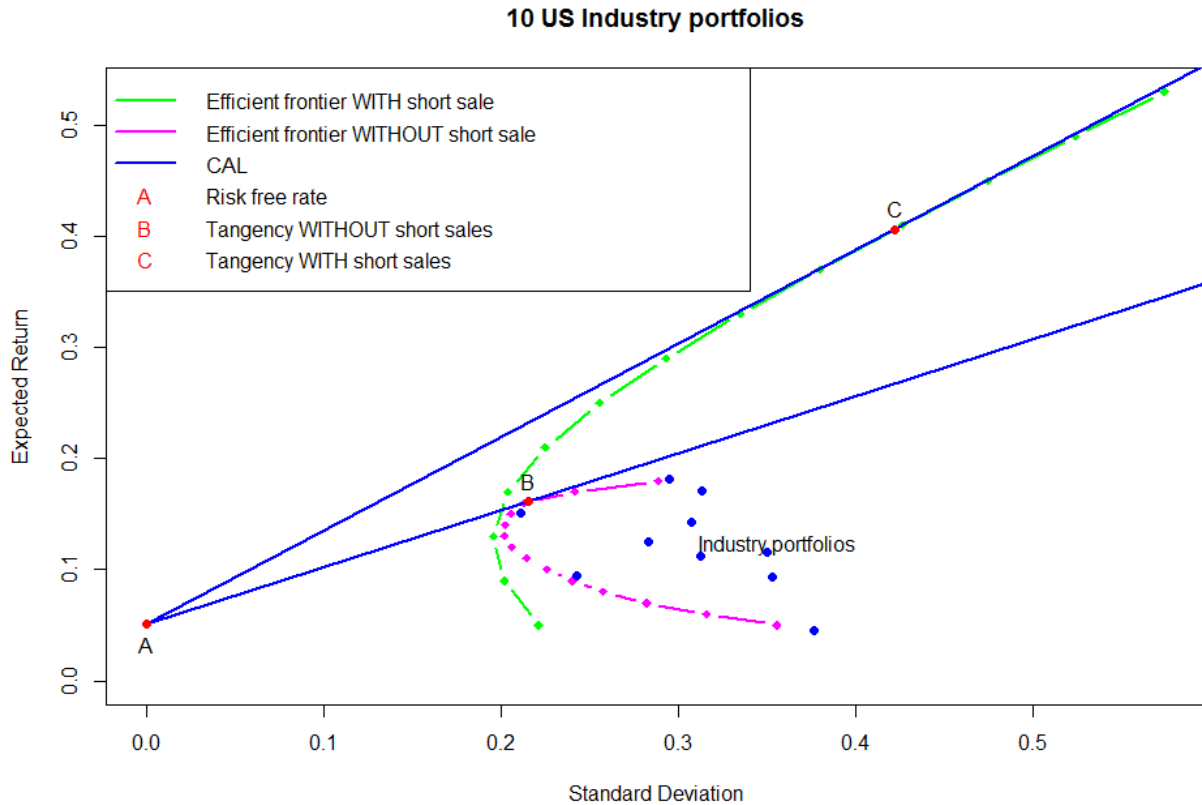
Figure 3 illustrates the efficient frontier with and without short sale restrictions, and the corresponding CAL and tangency portfolios. As you see, the short sale restriction make a difference. With short sale allowed, the CAL is steeper, and the tangency portfolio have a higher expected return and higher risk. According to modern portfolio theory, a risk averse investor will set his/hers portfolio somewhere on the CAL. When short sale is restricted, the investor's portfolio will be on the CAL, either in point A, in point B, or between point A and B, depending on investor's attitudes to risk. In the state where short selling is allowed, a rational investor will allocate his/hers investments on the CAL, from point A to point C or further out on the line. If the investor are risk tolerant and adapt the portfolio on the CAL to the right of point C, then the

investor buys risky asset on margin and borrows money at a risk-free rate to finance the tangency portfolio. This implies that the investor have weights larger than one in the tangency portfolio, $w > 1$, and negative weights in the risk-free assets, $(1 - w) < 0$.

2.2 Capital Asset Pricing Model

In early 1960s, Jack Treynor (1961,1962), William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) independently extended the work on MPT and derived the Capital Asset Pricing Model (CAPM). CAPM claim that portfolio returns is given by this equation in an equilibrium state,

$$E[r_p] = r_f + \beta_p(E[r_m] - r_f), \qquad (2.9)$$

Where portfolios expected return, $E[r_p]$, is a sum of risk-free rate of return, r_f, in addition to the systematic risk coefficient, β_p, multiplied with expected market excess return, $E[r_m] - r_f$.

The CAPM has a long list of assumptions, that are unrealistic, but CAPM is still popular due to its simplicity and variety. According to CAPM, all investors will hold a part of their wealth in the tangency portfolio, because investors are assumed to be rational, be utility-maximizes and that information asymmetries does not exist. When all investors hold the tangency portfolio and a risk-free asset, then this becomes the market portfolio, which will have the highest possible excess return given the risk.

Jensen (1968) developed an extension of the CAPM to estimate how much a manager's forecasting ability contributes to the fund's returns, this is now known as the Single Index Model, which is the CAPM regressed on market portfolio's excess return,

$$r_p - r_f = \alpha_p + \beta_p(r_m - r_f) + e_p \qquad (2.10)$$

where α_p is a constant and a risk-adjusted measure that indicate additional portfolio return compared to market return, e_p is an error term.

Taking expectations,

$$E[r_p] - r_f = \alpha_p + \beta_p(E[r_m] - r_f) \qquad (2.11)$$

When the market is efficient, and CAPM is true, then $E[\alpha_p] = 0$. If we take the variance of excess return in (2.10), we get the portfolio variance = systematic risk + non-systematic risk often called firm-specific risk,

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sigma^2(e_p) \qquad (2.12)$$

2.3 Volatility

Volatility is a measure of the spread in returns for a given security or market index. In general, the higher the volatility, the higher the risk. Therefore, volatility is a main measurement of risk. Rational expectation model states that investors should get a risk premium for taking on risk, i.e. the higher the volatility the higher excess return demanded. So, according to rational expectation model, market excess return and market volatility is positively correlated over a long-run in the cross-section of assets.

Although the relationship between market excess return and volatility should be positive in the long-run, this is not always the empirical result, often in long.run time-series it occurs a negative relationship between market excess return and market volatility. Whaley (2000) describes the Chicago Board Option Exchange's Market volatility index (VIX), how it is constructed and how it behaves during the period 1986 to 2000, with respect to how market volatility predict the market returns. This is also examined by Giot (2005) and Banerjee, Doran and Peterson (2007) among others.

This negative relationship was also found in my data: $r_t - rf_t = 0.161513 - 0.00614\sigma_t$, but the explanation degree was zero and the volatility coefficient was insignificant.

To get an rational explanation of this empirical phenomenon, French et al. (1987) suggested to decompose market volatility into two parts, expected volatility and unexpected volatility, $\sigma_t = \sigma_t^e + \sigma_t^u$, where expected volatility σ_t^e is predicted using GARCH(1,1) and unexpected volatility is given by the difference between volatility and expected volatility $\sigma_t^u = \sigma_t - \sigma_t^e$. French et al. (1987) argued that the negative relationship between market excess return and market volatility exist because excess return is positively correlated to expected volatility, but volatility is highly persistent, so an increase in unexpected volatility would increase the future expected risk premium, hence, decrease the current stock price.

In my data, I have computed the realized monthly variance of index returns as the sum of squared daily returns by using this equation:

$$\sigma_t^2 = \sum_{i=1}^{N_t} r_{it}^2 \qquad (2.13)$$

where *Nt* is the number of trading days in month t and r_{it} is the return on day i of month t. The return is assumed to have the properties:

$$r_t = \mu + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma_t^2).$$ (2.14)

2.4 Sharpe ratio

The Sharpe ratio aim to measure risk-adjusted performance by subtracting the risk-free interest rate from the portfolio rate of return, such that we get excess return of the portfolio, and then divide excess return by the standard deviation of the portfolio returns.

$$SR = \frac{r_p - rf}{\sigma_p}$$ (2.15)

The Sharpe ratio is the slope of the CAL, and was developed by William F. Sharpe (1966). It has two main versions from Sharpe (1994), ex-ante Sharpe ratio, which uses expected portfolio return in the calculations, and ex-post Sharpe ratio, which uses realized portfolio return. Since my objective is backward looking, I use the ex-post Sharpe ratio.

The Sharpe ratio is popular in finance due to its simplicity and its ability to measure the tradeoff between risk and return. The Sharpe ratio follows the ideology of the rational expectation model in that an investor should be properly compensated for taking on additional risk. If the excess return on the investment is relatively low with respect to the risk, then the Sharpe ratio would be low. We want as high value as possible in the Sharpe ratio, as we want as high α-value as possible in the Single Index Model.

A drawback with the Sharpe ratio is that it includes standard deviation of excess return, which assumes that the excess return in the portfolio follows a normal distribution, therefore, kurtosis and skewness can decrease the accuracy of the Sharpe ratio. The standard deviation are measured by the distance each return has from the mean, so a large observed return, positive or negative, in a series of relatively small returns will penalize the Sharpe ratio. An example from Harding (2002) is that a suddenly large positive return in a series of small, consistent and positive returns will generate a lower Sharpe ratio, due to the increased standard deviation. One solution to this problem is to use the Sortino rate, which produce a semi-standard deviation based on only negative returns to use in the denominator instead of standard deviation. Another drawback in

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an ex-ante Sharpe ratio is the estimation, if the estimates are spurious then the Sharpe ratio will be spurious.

2.5 Turbulence

Turbulence is a substitute for volatility as a measurement of risk. Turbulence is defined by Kritzman and Li (2010) as uncharacteristic behavior of asset prices with respect to their historical behavior pattern, this includes extreme price moves, diverging of correlated assets, and convergence of uncorrelated assets. Periods with turbulence is often characterized with excessive risk aversion, illiquidity, and falling prices of risky assets. This thesis uses the mathematical measure for turbulence presented by Chow, Jacquier, Kritzman and Lowry (1999) which is equivalent to the "Mahalanobis distance" introduced by Mahalanobis (1927) used to analyze distances and resemblances in the human skull. Turbulence is defined as:

$$d_t = \sqrt{(r_t - \mu)\Sigma^{-1}(r_t - \mu)'} \qquad (2.16)$$

Where d_t is a scalar of turbulence for a particular time period t, r_t is a vector (N x 1) of assets returns for time period t, μ is the sample average vector (N x 1) of historical returns and Σ is the sample covariance matrix (N x N) of historical returns.

Four steps to interpret d_t:

Step 1. We want to capture the magnitude to which of the returns that was unusually high or low, we do this by taking each asset's return, r_t, and subtract by the historical average, μ.

Step 2. To make d_t scale independent and to capture the interaction of the assets, we multiplying these differences by the inverse of covariance matrix of returns.

Step 3. To convert d_t from a vector to a scalar, we post multiplying by the transpose of the differences between the asset returns and their averages.

Step 4: Square the sum.

Turbulence presented in this form can be estimated for any set of asset, not only for asset with liquid option markets, it also has the advantage that it secure interactions between combinations of assets in addition to the degree of the assets' returns. Like volatility, turbulence is highly persistent, illustrated in Figure 7 (page 37), and return to risk are substantially lower in turbulent periods.

These features gives an incentive to lower the amount of risky assets in presence of turbulence, in order to maintain risk and hope to improve the tradeoff between return and risk. This is done by Kritzman and Li (2010), where they build an optimal turbulence-resistant portfolio and an unconditioned optimal portfolio. The turbulence-resistant portfolio substantially outperformed the unconditioned portfolio in the out-of sample turbulent periods, but slightly underperformed the unconditioned portfolio, on average, in all market conditions. Kritzman and Li (2010) also show how to use turbulence as a filter for scaling exposure to risk in risky strategies, the result compared to an unfiltered strategy was greater return and information ratio, lower standard deviation and lower negative skewness.

Note that turbulence is not made to recognize cheap or expensive asset, it is rather a measure to determine how fragile the market is, and to see how far we are from 'normal' market conditions. Turbulence is the degree of uncharacteristic behavior, capturing extreme price movements and changing intra-relationships.

2.6 Efficient market hypothesis

Efficient market hypothesis (EMH) state that no stock price history can be used in a strategy and still be superior to the market portfolio. This statement is studied in Section 5.3. The EMH was developed by Eugene Fama in the early 1960s and has been very popular until the behavioural finance started to grow in the 1990s. EMH states that the stock market is efficient, meaning all stocks are fairly priced and fully reflect available information. In EMH there does not exist overvalued or undervalued assets, so it is impossible to get a higher excess return than the market given equal risk. To get a higher excess return than the market, you have to take on more risky assets.

The EMH is closely related to the random walk hypothesis which states that the stock market prices is independently and identically distributed (i.i.d), this means that the stock prices evolve according to a random walk, and past trends or movements cannot predict the future stock price. Evidence that confirms the random walk hypothesis was found by Cootner (1962), Fama (1965a) and Granger and Morgenstern (1963) among others, all though Alexander (1961), Steiger (1964)

and Lo and Mackinlay (2001) dismiss the random walk hypothesis. Fama (1965b) describes the relationship between the random walk hypothesis and EMH as "the action of the many market participants should cause the actual price of a security to wander randomly about its intrinsic value".

There is three categories of EMH, Bodie, Kane and Marcus (2011) describes these:

Weak-form EMH: Assets prices reflect all information of relevant historical data, such as past prices, trading volume and short interest.

Semistrong-form EMH: Assets prices reflect all publicly available information. This includes past prices, the firm's fundamental data, quality on management and products, balance sheet composition, patents held, earnings forecast, and accounting practice.

Strong-form EMH: Asset prices reflect all relevant information including insider information.

For a strong-form efficient market to be possible, Fama (1970) argue that these assumptions must be fulfilled:

- No transaction costs.
- All information is costless available to all participants.
- Investors are homogeneous in the expectations about implications of current information and distribution of future prices.

These three forms of market efficiency implies according to EMH that technical analysis would be pointless, and fundamental analysis will only be beneficial in a week-form efficient market. Strong-form EMH is generally not supported, but Chan, Gup and Pan (1997) proofs by a unit root test market efficiency in weak-form EMH. Basu (1977), Rosenberg, Reid and Lanstein (1985), and many other studies have shown that the market is inefficient, which is why the EMH is a controversial hypothesis.

2.7 Low-volatility anomaly

Over the years, it is been written many scientific about the dynamic relationship and behaviour between excess return and market risk. The traditional assumption of the risk-reward relationship, where you expect a higher excess return when taking on more risk, have many anomalies. Many studies have discovered that portfolios with low-risk stocks produce higher risk-

adjusted returns than portfolios with high-risk stocks. That is why it exist many low-risk strategies, which aim to lower the risk in a portfolio in order to get a higher risk-adjusted return. One of these strategies are low-beta strategies. Under the assumptions of CAPM, CAPM predicts a positive relation between the systematic risk coefficient beta and excess return, so high-beta portfolios have greater expected return than low-beta portfolios. Jensen, Black and Scholes (1972), Roll and Ross (1994) and Baker and Haugen (2012) among others finds empirical evidence that there exist a flat and/or negative relationship between beta and excess return. This rises the popularity of low-beta portfolios, which aim to keep a low systematic risk in the portfolio in order to get a high excess return. Frazzini and Pedersen (2014) present a "betting against beta"-strategy where they construct a portfolio that go long in leveraged low-beta assets and short high-beta assets, their portfolio produces significant positive risk-adjusted returns.

Low-volatility strategies use volatility as a key determinant to allocate assets in a portfolio. This has become a very popular strategy, Baker and Haugen (1991) constructs a low-volatility portfolio consisting of 1000 US stocks that are weighted such that they minimizes the portfolio volatility. Every quarter the portfolio weights are re-balanced and optimized over the trailing twenty-four months. Their low-volatility portfolio significantly outperformed the market portfolio with both lower volatility and higher return. Jagannathan and Ma (2003) found that their low-volatility portfolio with no short-sale constraints generated higher returns and had lower realized volatility than the value-weighted market portfolio. Kuo and Li (2013) explains that a traditional minimum variance portfolio is guaranteed to be free of noisy return expectations, therefore, the minimum variance portfolio avoids dealing with "garbage-assets" with high risk, and thus preforms better than the mean-variance optimal portfolio. Clarke, De Silva and Thorley (2010) study a minimum variance portfolio with long-only constraint, and concludes that the analytical and empirical results show that the optimal portfolio weights is determined by the beta coefficient, and that low-beta stocks have relatively high mean returns. Baker and Haugen (2012) found evidence that support the low-volatility anomaly over the time period 1991 to 2011 in 33 different markets, and in each country, the low-risk stocks is superior with respect to Sharpe ratio.

In Blitz and Van Vliet (2007) they try to explain why low-volatility stocks earn higher risk-adjusted returns that the market portfolio. In a factor model based on size, value and momentum, they

found that there was significant higher positive alpha in low-volatility portfolios versus high-volatility portfolios, and that low-volatility stocks had low beta. They remark that their ordinary risk factors cannot explain all of the volatility effect. Baker, Bradley and Wurgler (2011) claim that the low-risk portfolio is superior because institutional investors focus too much on their alpha and the information ratio, instead their benchmark-free Sharpe ratio.

Risk-parity strategy is a special asset diversification strategy that is constructed such as each asset are weighted after their respectively risk. That gives high weights on low-risk assets and low weights on high-risk assets. According to a white paper by Allen (2010), a levered risk party portfolio would have significantly outperformed an average institutional portfolio in the time period 2000 to 2010, but the risk parity portfolio would have significantly underperformed against the average institutional investor in the time period 1990 to 2000. Asness, Frazzini and Pedersen (2012) found that in the time period 1926 to 2010, the risk parity portfolio has superior Sharpe ratio compared to a 40/60 stock/bond portfolio and the value weighted market portfolio, but the risk parity portfolio generates lower average returns.

Over to timing diversification, Busse (1999) studied how well mutual fund managers succeed to time market volatility, and he found that volatility timing has huge effect on mutual funds returns, and the better the mutual fund manager is to time market volatility, the higher risk-adjusted return was yield. (Collie, Sylvanus and Thomas (2011), Albeverio, Steblovskaya, and Wallbaum (2013), Perchet, de Carvalho, Heckel, and Moulin (2015)) among others have used volatility's persistence as motivation to make a volatility-targeting strategy that aims to target a constant level of risk over time in a portfolio by rebalancing between risky assets and risk-free asset as the market volatility changes. They found by using historical data that this active volatility-targeting strategy significantly outperformed the passive benchmark. Hallerbach (2012) found that volatility targeting generally increases the risk-adjusted return, but the risk-adjusted return is sensitive to the accuracy of the volatility forecast.

French et al. (1987) study the relation between stock return and stock market volatility. They found evidence that the expected excess return is positively related to the expected volatility of stock returns. In addition, that unexpected stock market returns are negatively related to the

unexpected volatility of stock returns. French et al. (1987) lead us to the Unexpected volatility strategy, that break volatility up into two pieces, expected volatility and unexpected volatility. Unexpected volatility strategy uses unexpected volatility to predict future volatility, and then allocate weights between stocks and bonds based on the predicted volatility. Zakamulin (2014) write how unexpected volatility can predict future volatility and future excess return using a GARCH(1,1) model. After that, he creates different dynamic asset allocation strategies based on unexpected volatility, and describe how they preform compared to a passive strategy.

Turbulence strategies are a relatively new concept based on the paper of Chow et al. (1999). Turbulence strategies aim to lower the amount of risky assets in presence of turbulence, in order to maintain risk-adjusted return. This is done by Kritzman and Li (2010), where they build an optimal turbulence-resistant portfolio and an unconditioned optimal portfolio. The turbulence-resistant portfolio substantially outperformed the unconditioned portfolio in the out-of sample turbulent periods, but slightly underperformed the unconditioned portfolio, on average, in all market conditions. Kritzman and Li (2010) also show how to use turbulence as a filter for scaling exposure to risk in risky strategies, the result compared to an unfiltered strategy was greater return and information ratio, lower standard deviation and lower negative skewness. The white paper of Harman (2014) suggest using turbulence in a regime shift strategy, that weights risky assets to 0 and risk-free assets to 1 in turbulent periods, and in non-turbulent periods it weights risky assets to 1 and risk-free assets to 0.

3. Data

In this thesis, the data consist of three time-series, risk-free interest rate with 475 monthly observations, 10 Industry Portfolio with 11250 daily observations divided on 10 variables, and Portfolios Formed on Size where the decile portfolios are used, the three datasets start at December 31, 1969, and end in July 31, 2014. I have chosen this period because it is the last forty-five years, which includes several periods with large volatility and turbulence, especially the stock market crash in November 1987, the dot-com bubble in 2000 and the financial crisis in 2008.

Both 10 Industry Portfolios and Portfolios Formed on Size is split in equally -and value-weighted returns, and are available at Kenneth French's online data library[3], in monthly or daily data. I use value-weighted daily data in my computations, and then convert the data into monthly data when needed. 10 Industry Portfolios are data originating from Center for Research in Security Prices[4] (CRSP), where the data are stock returns from each stock listed at NYSE, AMEX and NASDAQ, all U.S. stocks, these stocks are sorted into one of ten categories based on their current four digit SIC code. The categories are[5]: (Variable-names in brackets)

1. Consumer NonDurables (NoDur)
2. Consumer Durables (Durbl)
3. Manufacturing (Manuf)
4. Oil, Gas, and Coal Extraction and Products (Enrgy)
5. Business Equipment (HiTec)
6. Telephone and Television Transmission (Telcm)
7. Wholesale, Retail, and some Services (Shops)
8. Healthcare, Medical Equipment, and Drugs (Hlth)
9. Utilities (Utils)
10. Other (Other).

The risk-free interest rate is also distributed by Kenneth French's data library, I use the risk-free rate from the dataset 'Fama/French 3 Factors', which are available in daily, weekly or monthly data. The risk-free rate is based on one month's U.S. Treasury Bill return, which originating from Ibbotson and Associates, Inc. Portfolios Formed on Size[6] consists of 20 columns with returns where I only use the last ten columns, that is the decile portfolios formed on size. 10 Size is only included as a secondary dataset to confirm that the risk-adjusted performance of the created portfolios from dataset 10 Industry Portfolios is analogous in dataset 10 Size. Hence, the risk-free interest rate and 10 Industry Portfolios is the primary datasets used in this thesis, and 10 Industry Portfolios is the source in the following, unless 10 Size is specified. All three datasets are collected

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[3] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[4] http://www.crsp.com
[5] A more extensive definition at: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/Siccodes10.zip
[6] The dataset Portfolios Formed on Size is in the following denoted 10 Size.

from Kenneth French's data library and is limited to U.S. assets, this make them comparable and consistent.

I primarily use monthly data, except in the estimation of volatility and turbulence, which demands daily data. The argument for using data with horizon of one month is that most institutional investors and other active investors usually only re-balance their portfolio each months, this is due to institutional investor's investment committee that cannot meet every day to discus and re-balance the their portfolio. In addition, rebalancing an active portfolio every day would generate a significant transaction cost. On the contrary, rebalancing the portfolio every quarter or more infrequently would make the portfolio too static and the portfolio would yield a lower average return and Sharpe ratio. The analysis of the data is computed with help from the statistical free software R, which can be downloaded from http://www.r-project.org/.

4. Methodology

For the reader to understand how the how I got the empirical results and the meaning of these data, I present the methodology of this study. I will describe how four different portfolios are constructed in this thesis, along with their estimated parameters. I explain the performance measurements and how statistical tests are applied. The computations of my empirical results from the statistical software R is given in appendix. This thesis exclude transaction cost and tax from the portfolios, because transaction cost is negligible, especially for institutional investors. Tax is constantly changing and can be different from location to location, tax often depend on the level of capital, and dividend and capital yield can have different tax rates.

4.1 Models implemented and estimation process.

This thesis is built on the perspective of a utility maximizing investor, whose main focus is to maximize the portfolio risk-adjusted return. Which in this thesis are measured and ranked by the Sharpe ratio, although, I will also use other measurement as secondary options. Note that according to the CAPM, the value weighted market portfolio is the optimal portfolio with respect to Sharpe ratio. Although, the empirical result in Section 5.3 and Table 5.10 tell the contrary. No matter how different assets in a portfolio are weighted, the portfolio performance can always be measured by the Sharpe ratio.

Of the four portfolios build in this thesis for a utility maximizing investor, three of them are dynamic asset allocation strategies responsive to volatility, unexpected volatility and turbulence respectively. The last one is a passive equally weighted portfolio (EWP) that contain an equal amount of all ten Industry portfolios, all risky assets. By constructing this EWP, I get a large naively diversified portfolio with value-weighted returns from ten different industries sectors that is simple to construct compared to building a portfolio of specific companies and/or industries. Building a selected portfolio of risky assets will also be time-consuming and will have a greater risk in volatile and turbulent periods.

DeMiguel et al. (2009) stated that "Of the 14 models we evaluate across seven empirical datasets, none is consistently better than the 1/N rule in terms of Sharpe ratio, certainty-equivalent return, or turnover..." They concluded that the EWP would be a natural benchmark due to its simplicity and low implementation cost. Therefore, it would be natural in this thesis to use the Sharpe ratio of the EWP as the benchmark.

The three dynamic asset allocation strategies can be considered as "optimized" portfolios, in the sense that they aim to have the same characteristics as Markowitz's Minimum Variance Portfolio. That is, their goal is to earn a high Sharpe ratio compared to the benchmark, by minimizing the portfolio risk. They do this by rebalancing the portfolio every month, such that they respond by reducing the weights in the EWP when the risk increases, and respond by increasing the weights in the EWP when the risk declines. The portfolio risk is defined differently in the three dynamic asset allocation strategies; they allocate the weighting of the assets according to volatility, unexpected volatility and turbulence respectively. The four portfolios made in this thesis are made on the constraint of "long only", meaning borrowing and short sale is restricted. Their portfolio return is equivalent to Equation (2.6) subject to budget constraint (2.1).

It is assumed that risky-asset returns at time t follows a random walk which are given by:

$$\boldsymbol{r_t} = \boldsymbol{\mu} + \boldsymbol{\varepsilon_t} \qquad\qquad (4.1)$$

Where r_t is a (N x 1) vector of returns at time t, μ is a (N x 1) vector of mean returns, and ε_t is a (N x 1) vector of random shock at time t, that is an i.i.d random variable with zero mean and constant variance.

The rolling window covariance matrices are computed using the following method, demonstrated with two assets, where historical covariance between daily returns from asset i and daily returns from asset j is applied:

$$\Sigma = \sigma_{i,j} = \frac{\sum_{t=1}^{T} r_{i,t} r_{j,t}}{T} \qquad (4.2)$$

Where T is the length of the rolling window.

The constant mean return estimate, μ, is given as:

$$\hat{\mu}_t = \frac{1}{T} \sum_{i=t-T}^{t-1} r_i \qquad (4.3)$$

Where T will be the length of the in-sample period, which is the distance from 31.12.1969 to 31.12.1974, 1260 days. The computation of the portfolios are done in matrix -and vector-form, and is equivalent to the notation in Equation (2.1) to (2.4).

Table 4.1 exhibit the four portfolios in this study and their requirement of input estimates. As you can see underneath, the turbulence-responsive portfolio is the most complex portfolio with respect to input estimates.

Table 4.1: The four portfolios constructed for the empirical study.

Portfolio model	Abbreviation	Input estimates
Equally weighted portfolio	EWP	None
Dynamic unexpected volatility portfolio	unex.port	Σ
Volatility-responsive portfolio	vol.port	Σ
Turbulence-responsive portfolio	turb.port	Σ, μ

The three dynamic portfolios all depend on the covariance matrix in the computation of volatility, unexpected volatility and turbulence, in addition, the turbulence also need the mean return in the building process. These input factors must be estimated, since we do not know their true realizations, in order to construct the three optimized portfolios. These parameters will be

estimated by using a specific time period, called in-sample or lookback periods, to estimate the parameters for the following out-of-sample period, which is the period I will study. In-sample periods or lookback period is the basis of our estimates. So, a short or irrelevant in-sample period will give a bad estimate, it is therefore important that the in-sample period reflect the timeframe you want estimate.

I implement a rolling-window of 60 months to my in-sample period. These 60 months should be more than enough to give a good estimate of the covariance matrix of daily returns for my out-of-sample period, computed after Equation (4.2). This means that my in-sample period that estimate the covariance matrix used to generate the portfolio weights, goes from 31.12.1969 to 31.12.1974. After one month, it is time to re-balance the portfolio. A new covariance matrix is estimated, this time from 31.01.1970 to 31.01.1975, this new covariance matrix is used to give new weight in the portfolio construction, while the old covariance matrix is ignored. This re-balancing process is used in all three dynamic strategies, and continues every month by always generating a new sample covariance matrix based on the last 60 months to make a new updated portfolio weights.

In the computation of turbulence, the mean returns need to be estimated from the in-sample period, but unlike the covariance matrix, the mean returns used in the computations are fixed through the whole out-of-sample period. According to Chopra and Ziemba (1993), the estimation error from covariance matrix is significantly less than the estimation error from mean returns. This implies that the volatility-responsive portfolio and the unexpected volatility-responsive portfolio should be more accurate than the turbulence-responsive portfolio.

4.1.1 Equally weighted portfolio

The equally weighted portfolio is in general a portfolio consisting of assets that are weighted equally and summed to one, that is, $w_i^{EWP} = \frac{1}{N}$ with respect to $\sum_{i=1}^{N} w_i^{EWP} = 1$, no factors of the assets are taken into consideration. In this thesis, the equally weighted portfolio consist of all risky assets, which is the value weighted returns from the 10 Industry Portfolios, where each of the ten industries are weighted as $\frac{1}{10}$ to construct one whole portfolio with equally weighted

returns from risky assets. The EWP is a passive, naïve diversified buy-and-hold portfolio that do not require any estimation, hence no estimation error will occur. Many economist, DeMiguel et al. (2009) and Duchin and Levy (2009) among others, argue that the EWP is an effective and better alternative to advanced asset allocation strategies because it is so simple, easy to implement and is a cheap asset allocation strategy.

This EWP is the closest we come to the value weighted market portfolio in the sense that the 10 Industry Portfolio returns are value weighted in their individual portfolios and then the ten portfolios are equally weighted. Based on strong assumptions the CAPM tell us that the optimal strategy for the investor is to hold the market portfolio of risky assets in terms of Sharpe ratio. The investor should be best off by allocate his/hers wealth between the market portfolio of risky assets and risk-free asset based on the individual risk aversion. This support my usage of the EWP's Sharpe ratio as a benchmark in this thesis, and this is also why EWP is chosen as the risky asset in the three dynamical asset allocation strategies.

4.1.2   Volatility-responsive strategy

The volatility-responsive portfolio aim to improve the risk-adjusted return by dynamically allocate the weighting between risky asset and a risk-free asset in response to the market volatility such that the portfolio volatility is minimized. This portfolio will be the portfolio on the efficient frontier that offers the lowest volatility, as you see illustrated in Figure 9 (page 42). The volatility-responsive portfolio is equivalent to Markowitz's Minimum variance portfolio, except the volatility-responsive portfolio allocate the weights between a risky asset and a risk-free asset, while the minimum variance portfolio only consider risky assets.

The key ingredient in this portfolio, naively forecasted volatility, $\hat{\sigma}_{t+1} = \sigma_t$, that is realized volatility in months t used as forecast for month $t + 1$. The volatility vector is computed after the following equation:

$$\boldsymbol{\sigma} = \sqrt{\boldsymbol{w'\Sigma w}} \qquad\qquad (4.4)$$

Where the first 21 daily returns of the EWP is used to make a variance covariance matrix, Σ, which is multiplied with the inverse of the vector $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix} = \begin{pmatrix} 0.1_1 \\ 0.1_2 \\ \vdots \\ 0.1_{10} \end{pmatrix}$ and $\boldsymbol{w}$ and then squared rooted to make it volatility, this procedure is used in a rolling window of 535 months. The output is, $\boldsymbol{\sigma}$, a vector of monthly portfolio volatility of EWP from the start of the in-sample period January 1970 to the end of the out-of-sample period July 2014.

The next step is to compute the volatility-responsive portfolio, which actively allocate the position in the risky EWP, and the position in the risk-free interest rate in response to changes in the volatility environment of U.S. 10 Industries Portfolios. This volatility-responsive portfolio in periods of high predicted volatility will be weighted close too, or equal one in the risk-free asset and will be weighted close too, or equal zero in the risky EWP. And vica versa, in periods of predicted low volatility, the volatility-responsive portfolio will be weighted close too, or equal one in the EWP and will be weighted close too, or equal zero in the risk-free asset.

In the volatility-responsive strategy, the weight at time $t + 1$ invested in the EWP is based on the volatility at time t. This means that volatility-responsive strategy use a naïve model that uses realized volatility as a forecast for the volatility in the upcoming month. The weight at time $t + 1$ is computed after the following equation:

$$w_{t+1}^{EWP} = N\left(\frac{E_t[\sigma] - \sigma_t}{std_t[\sigma]}\right) \qquad (4.5)$$

Where N is the Normal cumulative distribution function, $E_t[\sigma]$ is the mean volatility until time t, σ_t is month t's realized volatility, used as forecast for volatility in month $t + 1$. and $std_t[\sigma]$ is the standard deviation of volatility until time t. By subtracting the last month realized volatility from the mean volatility, I get the deviation from the mean. Then I divide by the standard deviation of volatility and by taking the normal distribution of the product, I get the weight in percent of the wealth invested in the EWP portfolio at time $t + 1$.

The weight at time $t + 1$ invested in the risk-free asset is equivalent to (2.6):

$$w_{t+1}^{rf} = 1 - w_{t+1}^{EWP} \qquad (4.6)$$

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In order to make the volatility-responsive portfolio, the weights is re-balanced every month by re-estimate the parameters in (4.7) in a rolling-window of 60 months to find new "optimal" weights. This procedure is done every month from the start one month ahead of the out-of-sample period to the end of the out-of-sample period. These weights is then multiplied with their corresponding asset in order to make the volatility-responsive portfolio.

4.1.3   Unexpected volatility-responsive strategy

The unexpected volatility-responsive strategy aims to improve the risk-adjusted return by actively allocate the weights invested in the risky asset and the risk-free asset in response to changes in unexpected volatility.

This strategy also depend on input estimate of the covariance matrix since it depend on the estimation of realized monthly volatility. The computation of expected volatility, which is a forecast/prediction of future volatility only needs daily returns and the in-sample period as input. In the forecasting of expected volatility, I use the Generalized Autoregressive Conditionally Hetroscedastic (GARCH) model by Bollerslev (1986) due to its mean reversion and its symmetric degree of past returns. The GARCH model let the conditional variance σ_t^2 depend on its own lags, so the simples GARCH model, GARCH(1,1) can be written as:

$$\sigma_t^2 = a + b\sigma_{t-1}^2 + c\varepsilon_t^2 \qquad\qquad (4.7)$$

Where σ_t^2 is a one period ahead estimate for the variance computed on any past information considered as relevant. $b\sigma_{t-1}^2$ is the information about volatility from the previous period. $c\varepsilon_t^2$ tell us the degree of how much volatility changes due to lagged shocks. Since (4.4) only hold three parameters, it is very parsimonious, and allow an infinite number of past squared errors to influence volatility at time t.

The GARCH model can be extended to a GARCH(p,q) model, where p is the lags of the conditional variance and q is the lags of squared error:

$$\sigma_t^2 = a + \sum_{i=1}^{p} b_i\sigma_{t-i}^2 + \sum_{j=1}^{q} c_j\varepsilon_{t-j}^2 \qquad\qquad (4.8)$$

However, I have chosen to use a GARCH(1,1) model in my calculations, because it is very simple, robust and sufficient to capture the volatility clustering in my dataset. For stationarity in the GARCH model, one need $b + c < 1$, such that $var(\varepsilon_t) = \frac{a}{1-(b+c)} > 0$, this is called long-run

mean. If $b + c \geq 1$, then you have an "integrated GARCH" process, which is non-stationarity in variance.

The GARCH(1,1) method produce $\hat{\sigma}_t^e$, a vector of monthly expected (forecasted) volatility from daily returns of EWP based on the rolling estimation window of five years, that ranges through the hole out-of-sample period. As described in Section 2.3, unexpected volatility is the difference between realized volatility and expected volatility, which is used as input in the unexpected volatility-responsive strategy.

The unexpected volatility-responsive portfolio is equivalent to the volatility-responsive portfolio, but the unexpected volatility-responsive portfolio allocate the weights in response to changes in unexpected volatility based on Equation (4.9). In periods with high unexpected volatility, the unexpected volatility-responsive portfolio will allocate the weight of the risky EWP close too or equal zero, and allocate the weight of the risk-free interest rate close too or equal one. In periods of low unexpected volatility will the unexpected volatility-responsive portfolio allocate the weights in opposite direction.

The dynamic unexpected volatility portfolio weight at time $t + 1$ invested in the EWP is computed after this equation:

$$w_{U,t+1}^{EWP} = N \left(\frac{E[\sigma^u] - \sigma_t^u}{std[\sigma^u]} \right) \tag{4.9}$$

The weight invested in the risk-free asset is:

$$w_{U,t+1}^{rf} = 1 - w_{U,t+1}^{EWP} \tag{4.10}$$

The notation and explanation of (4.9) and (4.10) is analogous to (4.5) and (4.6), but unexpected volatility is substituted with volatility in the Equation (4.9). The weights is re-balanced every month by re-estimate the parameters in (4.9) in a rolling-window of 60 months to find new updated weights. This procedure is done every month from the start one month ahead of the out-of-sample period to the end of the out-of-sample period. These weights is then multiplied with their corresponding asset in order to make the unexpected volatility-responsive portfolio.

4.1.4   Turbulence-responsive strategy

The purpose of the turbulence-responsive strategy is to improve the risk-adjusted return by dynamically allocate the weighting between risky asset and a risk-free assets such that the portfolio turbulence is minimized. Turbulence demands input estimate of both covariance matrix and mean return, which make more room for estimation error in the computation compared to the volatility-responsive portfolio.

The turbulence vector, $\boldsymbol{d}$, is computed based on this equation:

$$\boldsymbol{d} = \sqrt{(\boldsymbol{r} - \boldsymbol{\mu})\Sigma^{-1}(\boldsymbol{r} - \boldsymbol{\mu})'} \qquad (4.11)$$

Where $\boldsymbol{r}$ is a vector of 1260 daily returns of the EWP in the in-sample period used to construct a variance covariance matrix, Σ, and a vector of mean return from each portfolio in 10 Industry Portfolios, $\boldsymbol{\mu}$. This is applied in a rolling estimation window of five years that returns realized daily turbulence from the start of the out-of-sample period to the end of the out-of-sample period. The daily turbulence is then converted to monthly turbulence in order to make fitted weights to the turbulence-responsive portfolio.

The weights in the turbulence-responsive portfolio is determined by the forecast of turbulence. To forecasting turbulence I use the naïve forecasting model, that is the realized turbulence in month t is used as forecast for turbulence in months $t + 1$, $\hat{d}_{t+1} = d_t$. Using this approach to forecast turbulence works fine because turbulence is highly persistent, as displayed in Table 5.5 in Section 5.2. This means that the weight of the turbulence-responsive portfolio is set by the past month realized turbulence. The weight at time $t + 1$ invested in the EWP is computed after the following equation:

$$w_{T,t+1}^{EWP} = N\left(\frac{E[d]-d_t}{std[d]}\right) \qquad (4.12)$$

The weight invested in the risk-free asset is:

$$w_{T,t+1}^{rf} = 1 - w_{T,t+1}^{EWP} \qquad (4.13)$$

The notation and explanation of (4.12) and (4.13) is equivalent to Equation (4.5) and (4.6), but turbulence is substituted with volatility in the Equation (4.12). These weights are re-balanced every month in the out-of-sample period due to a rolling window, which re-estimate the

parameters in (4.12) based on turbulence from the last five years. These weights is then multiplied with their corresponding asset in order to make the turbulence-responsive portfolio.

Similar to the volatility-responsive portfolio and the unexpected volatility-responsive portfolio, the turbulence-responsive portfolio weight in the EWP (risk-free asset) is low (high) in turbulent periods and high (low) in non-turbulent periods.

4.2 Portfolio performance measurement

The different portfolios will be evaluated after mean returns, standard deviation, CAPM alpha, skewness, capital accumulation and Sharpe ratio, where the latter is the main measurement. These measurements are given numerically in chapter 5, where the portfolio performance is revealed.

In the evaluation of the different portfolios, I check the robustness of my results and data by splitting the out-of-sample period into four sub-periods. All sub-periods is between nine and ten years. This way, it will be easier to see which portfolio who performs best and worst in the given sub-periods, and it will be interesting to see if the portfolio who performs best in the whole out-of-sample period also is superior in the four sub-periods.

Table 4.2: Time periods used to determine the performance of the portfolios.

Categorization of time period	Date of time period
Out-of-sample period	31.12.1975-31.07.2014
Sub-period #1	31.12.1975-31.12.1984
Sub-period #2	31.01.1985-30.12.1994
Sub-period #3	31.01.1995-31.12.2004
Sub-period #4	31.01.2005-31.07.2014

These four sub-periods all cover periods with some financial distress, although the two last sub-periods include the periods with most risk, in form of the dot-com bubble burst and the financial crisis. The second sub-period include the 19'Th of October's Black Monday (1987) that caused a lot of financial frustration. While the first sub-period was relatively calm without any huge burst.

4.2.1   Mean return and standard deviation

Because the mean return is so simple, it is useful to get an overall view of the empirical portfolios. The specific portfolio mean return is computed from Equation (2.6), however, the portfolio mean return is annualized in this thesis by using this equation for simple interest:

$$\bar{\mu}_p = \hat{\mu}_p 12 \qquad\qquad (4.14)$$

Where $\bar{\mu}_p$ is the annual portfolio mean return, and $\hat{\mu}_p$ is realized monthly portfolio mean return. The variance is the spread of the observations, and is computed after Equation (2.7). The standard deviation is the square root of variance. To annualize the portfolios monthly standard deviation, the monthly standard deviation of a given portfolio, $\hat{\sigma}_p$, is multiplied with the square root of 12.

$$\bar{\sigma}_p = \hat{\sigma}_p \sqrt{12} \qquad\qquad (4.15)$$

Where $\bar{\sigma}_p$ is the annualized standard deviation of a given portfolio return.

All result in this thesis are reported in annual terms, to simplify and avoid confusion.

4.2.2   Skewness

Skewness measures the deviation of symmetry in a dataset, it measures if the dataset deviate to the left or the right of the center point. A perfectly symmetric dataset, like the normal distribution, looks exactly the same on the right hand side of the mean, as on the left hand side of the mean. A dataset is symmetric if it has a skewness value of zero. The dataset has more values on the left hand side of the mean if the skewness value is negative, meaning that the data are skewed to the left of the mean, and the left tail is longer than the right tail. Vica versa, if the skewness value is positive, then the right hand side of the mean has a longer tail than the left hand side of the mean, and the dataset is skewed to the right of the mean.

In my empirical portfolios, a negative skewness will indicate that the mass of the returns is concentrated to the right of the mean, the portfolio has a tail of returns that are lower than the mean; investors do generally not prefer this. A positive skewness indicate that the mass of the returns is concentrated to the left of the mean, the portfolio has a tail of returns that are higher than the mean; investors generally prefer positive skewness above and beyond their preference for a higher mean and lower volatility. Note that portfolio skewness unequal zero implies that the portfolios are not normally distributed. Skewness has this formula:

$$S = \frac{\sum_{i=1}^{N}(r_i - \hat{\mu}_p)^3 / N}{\hat{\sigma}_p{}^3} \qquad (4.16)$$

Where N is the number of returns in the portfolio. r_i, $\hat{\mu}_p$ and $\hat{\sigma}_p$ are monthly portfolio returns, monthly portfolio mean return and monthly portfolio standard deviation respectively, equivalent to previous notations.

4.2.3   CAPM alpha

I include CAPM alpha, also known as Jensen's alpha, as a measure because it is much used in practice by traders and investors who manage an active portfolio to measure their additional portfolio return compared to market return. The CAPM alpha is given in Equation (2.10) in Section 2.2, but can also be written as:

$$\alpha_p = E[r_p] - r_f - \beta_p(E[r_m] - r_f) \qquad (4.17)$$

Where $E[r_p]$ is the expected portfolio return, and $r_f - \beta_p(E[r_m] - r_f)$ is fair compensation for systematic risk. α_p can be defined as the excess return generated by the portfolio over its benchmark

When Jensen developed the alpha in 1968, he gathered annual returns from 115 mutual fund from the period 1945-1964 to test for positive alpha, while using the S&P500 as benchmark. . Jensen (1968) found that the majority of the funds had a negative estimated alpha with a mean alpha of 0.4%. He found only three funds that had a significant positive alpha at the 5% level of the 115 funds tested.

4.2.4   Sharpe ratio

The out-of-sample Sharpe ratio is the main performance measurement in this thesis, because it can compare portfolios with different exposure to risk. A rational investor will prefer the portfolio with the highest Sharpe ratio, therefore it serves as the main performance measurement in this thesis, regardless of its limitations.

In the evaluation process, I will use this version of the Sharpe ratio,

$$\widehat{SR}_p = \frac{\hat{\mu}_p - rf}{\hat{\sigma}_p} \qquad (4.18)$$

The difference from Equation (2.15) is that I use monthly out-of-sample mean return, mean risk-free rate of return and out-of-sample standard deviation of monthly portfolio's returns. The out-of-sample Sharpe ratio is annualized by multiplying by the square root of 12.

$$\overline{SR}_p = \widehat{SR}_p 12^{0.5} \tag{4.19}$$

To test if the three dynamical asset allocation portfolio's Sharpe ratio is statistical distinguishable compared to the passive benchmark portfolio, I implement Jobson and Korkie (1981)'s test with the modification by Memmel (2003). This test take two Sharpe ratios and test the null hypothesis: $H_0: SR_1 - SR_2 = 0$, where the test statistics is given by the following equation:

$$\bar{z} = \frac{\overline{SR}_1 - \overline{SR}_2}{\sqrt{\frac{1}{T}\left[2(1-\bar{\rho}^2) + 0.5\left(\overline{SR}_1^2 + \overline{SR}_2^2 - 2\overline{SR}_1\overline{SR}_2\bar{\rho}^2\right)\right]}} \tag{4.20}$$

$\bar{z}$ is standard normally distributed test statistics. $\overline{SR}_1$, $\overline{SR}_2$ and $\bar{\rho}$ is the annual out-of-sample Sharpe ratio of portfolio 1, annual out-of-sample Sharpe ratio of portfolio 2 and their correlation coefficient respectively. T is the sample size. The null hypothesis is rejected if the test's p-value is less then significance level α=0.05.

5. Empirical results

In this Section, I will report my empirical results for the four portfolios in this study, and compare the four portfolios, especially with weight on the three dynamic asset allocation strategies described in Section 4.1. The empirical results are given in annual terms. Section 5.1 to 5.3 exhibit the empirical results based on the primary dataset 10 Industry Portfolios and risk-free interest rate, while Section 5.4 display the empirical results for the four portfolios based on dataset 10 Size and risk-free interest rate. An extensional discussion on the empirical results are given in Section 6.

I will start this chapter by looking at the summary statistic of my data to get a rough impression of the underlying factors. Where Table 5.1 shows: minimum value, first quartile of the observations, median, mean, third quartile of the observations and maximum value. The notation in Table 5.1 is equal to what I used in R software.

*Table 5.1: Summary statistic for the time period 01.1975-31.07.2014, based on dataset 10 Industry Portfolios and risk-free interest rate. *The impossible number is not valid, it is a result of a monthly return multiplied by 12 to annualize.*

Summary statistics in percent						
	Min.	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**
EWP	-603.50*	-25.80	11.80	12.83	53.75	420.30
rf	0.0000	2.13	4.92	4.873	6.72	16.20
ex.return	-608.70*	-32.06	8.5	7.937	47.81	420.30
std.port	4.422	9.336	11.500	13.68	15.87	83.30
std.pred	7.333	11.08	13.13	14.20	15.59	62.08
std.unex	-28.75	-3.261	-1.286	-0.5213	1.41	67.92
Mturb	0.6418	4.8448	9.1621	17.3816	16.6056	267.2369

EWP is the equally weighted portfolio described in Section 4.1.2, The summary statistics tell us that the EWP has a positive annual mean return of 12.23 percent. The first quartile, that is the middle return between the smallest and the median return, show a negative return of 25.92 percent. On the other hand, the third quartile, that is the middle return between the median and the highest return, display a positive return of 53.75 percent. If you look at Figure 1, you see that the second half has much more spread in the returns than the first half.

rf is the risk-free interest rate, it has a mean return of 4.873 percent. From the quartiles, we can state that 50 percent of the observations is between 2.13 percent and 6.72 percent. As we can see from Figure 1 (page 1), rf was as high as 16.2 percent in June 1981, but rf has since then evolved downwards in the out-of-sample period, and after the financial crisis rf has been approximately zero.

ex.return is the excess return which has a positive mean return of 7.937 percent.

std.port is the estimated realized equally weighted volatility, that has a mean volatility of 13.68 percent, 75 percent of std.port lies below 15.87 percent which is good, but std.port has a maximum at 83.3 percent.

std.pred is the predicted volatility generated by the GARCH(1,1) model, std.pred has a mean of 14.20 percent which is higher than std.port, also the maximum volatility is a bit lower. Figure 4 illustrates how std.pred follow std.port, the difference between them is std.unex.

std.unex is the unexpected volatility, that is std.port subtracted by std.pred as described in Section 2.3. std.unex has a mean of -0.5213 percent, this tell us that on average, historical volatility is less than forecasted volatility.

Mturb is turbulence which has a mean of 17.38 percent, that is over the third quantile of 16.6 percent, this is due to extreme levels of turbulence that happens rarely but is drives the mean upwards. The three most extreme observations of turbulence happened at September 1998, July 2000 and October 2009 as illustrated in Figure 1 (page 1).

Figure 4: Historical vs. predicted monthly volatility. The GARCH(1,1) model applied to a 60 months rolling window predicts volatility.
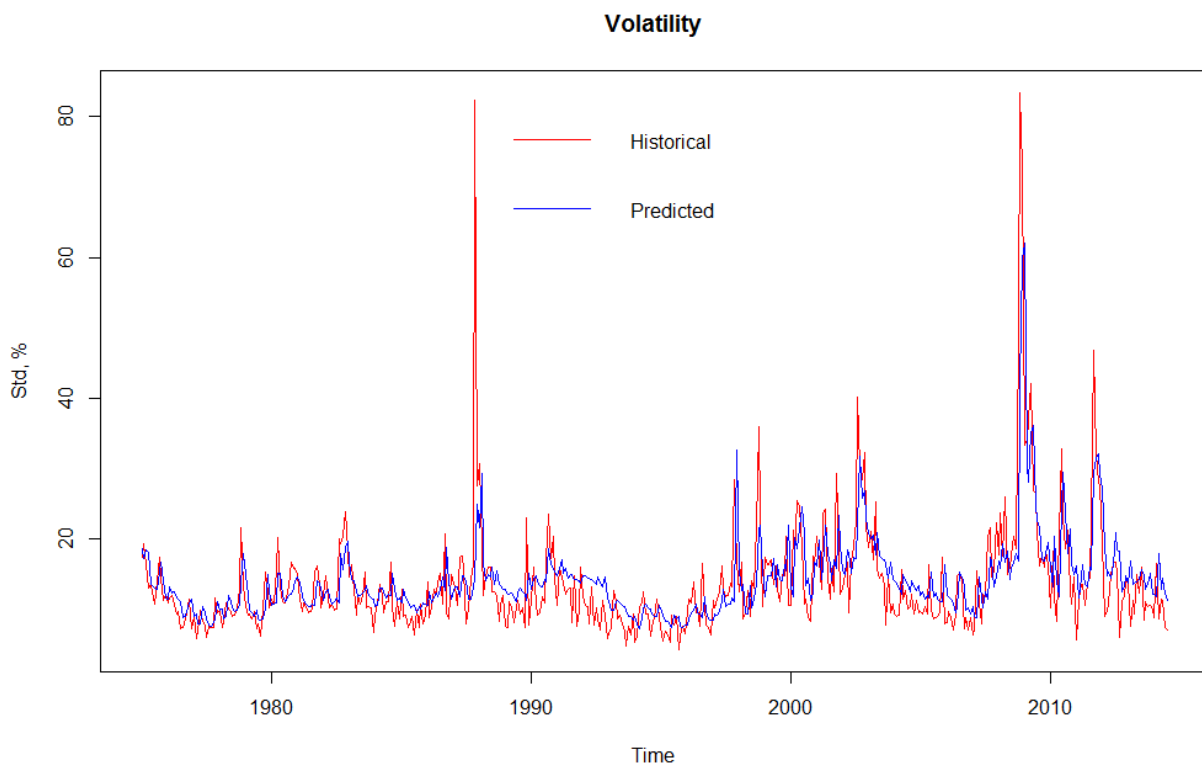
5.1 Predictive abilities of unexpected volatility

To check if unexpected volatility can predict monthly volatility of the returns on EWP and the monthly excess return, as suggested by French et al. (1987) and Zakamulin (2014). I check if lagged unexpected volatility can predict the volatility of the returns on EWP and the excess return by running the following regression:

$$\sigma_t = \alpha + \beta \sigma^u_{t-1} + \varepsilon_t \qquad\qquad (5.1)$$
$$\text{and } r_t - rf_t = \alpha + \beta \sigma^u_{t-1} + \varepsilon_t \qquad\qquad (5.2).$$

The sign on the β coefficient tell us how the future dependent variable is predicted to respond to a change in unexpected volatility. If β < 0, then the future dependent variable is expected to move

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in the opposite direction of unexpected volatility. If β = 0, then the future dependent variable is unrelated to unexpected volatility. If β > 0, then the future dependent variable is expected to move in the same direction as unexpected volatility.

The R software output is given below:

Table 5.2: Regression of volatility on its lagged unexpected volatility.

Regression	Time period	β-coefficient	P-value	R-square adj.
	01.1975 – 12.1984	0.95573	<2e-16	0.4669
	01.1985 – 12.1994	0.9838	<2e-16	0.8466
(5.1)	01.1995 – 12.2004	0.83651	8.22e-16	0.419
	01.2005 – 07.2014	1.0674	<2e-16	0.4714
	01.1975 – 07.2014	0.98803	<2e-16	0.5223

Figure 5: Volatility regresses on lagged unexpected volatility yields a positive correlation in the out-of-sample period. $\sigma_t = \alpha + \beta\sigma_{t-1}^u + \varepsilon_t$, results: $\alpha = 14.19$, $\beta = 0.988$, $p-value = 0.5223$ => Significant positive relationship between σ_t and σ_{t-1}^u.

Volatility regressed on lagged unexpected volatility
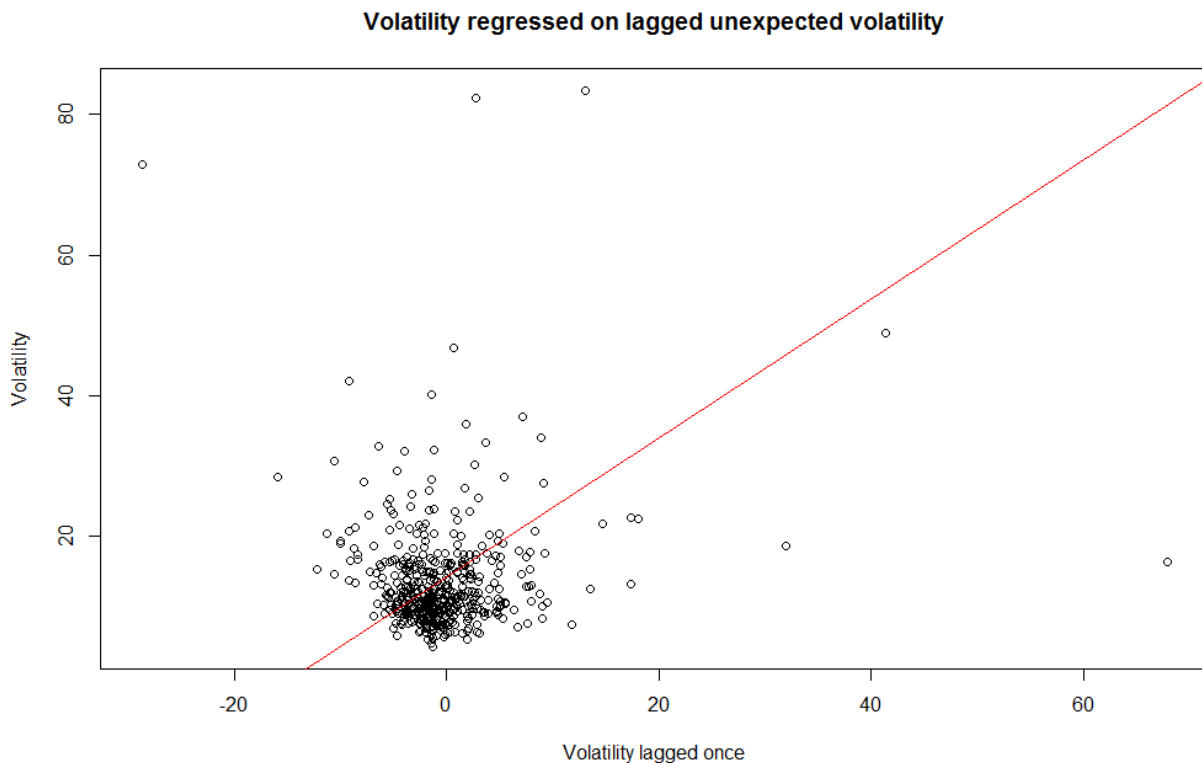
Regression of (5.1) shows a strong positive β coefficient in the out-of-sample period and in all four sub-periods. This indicates that unexpected volatility move in the same direction as future volatility. To be more specific, β=0.988 means that a one percentage increase in unexpected

volatility would increase the future value of volatility with 0.988 percent. The p- value is highly significant, and all periods are significant on α = 1% significance level. This means that the coefficient contributes significantly such that there is a linear relationship between volatility and unexpected volatility. The out-of-sample period have an acceptable explanation degree of 0.5223, unexpected volatility predicts future volatility in an excellent way. The sub-period results is consistent with the results for the out-of-sample period regression of volatility on past values of unexpected volatility, hence these results are robust.

Table 5.3: Regression of excess return on lagged unexpected volatility.

Regression	Time period	β-coefficient	P-value	R-square adj.
(5.2)	01.1975 – 12.1984	0.05874	0.0215	0.03588
	01.1985 – 12.1994	-0.04206	6.83e-06	0.1511
	01.1995 – 12.2004	-0.0249	0.165	0.007931
	01.2005 – 07.2014	0.01587	0.224	0.004308
	01.1975 – 07.2014	-0.011943	0.0838	0.004206

On the contrary, regression of (5.2) displays a more tangled result. The β coefficient is marginally negative in the out-of-sample period, while it is positive first- and last sub-period, and the β coefficient is negative in the two middle sub-periods. In the two first sub-periods, the p-value is significant and the explanation degree is low but useful, however, it is troubling that the β coefficient has opposite signs. In the two last sub-periods, the p-values are insignificant and the explanation degree is approximately zero. Overall, in the out-of-sample period excess return regressed on lagged values of unexpected volatility is insignificant, and past values of unexpected volatility fails to explain present values of excess return.

Excess return regressed on lagged unexpected volatility
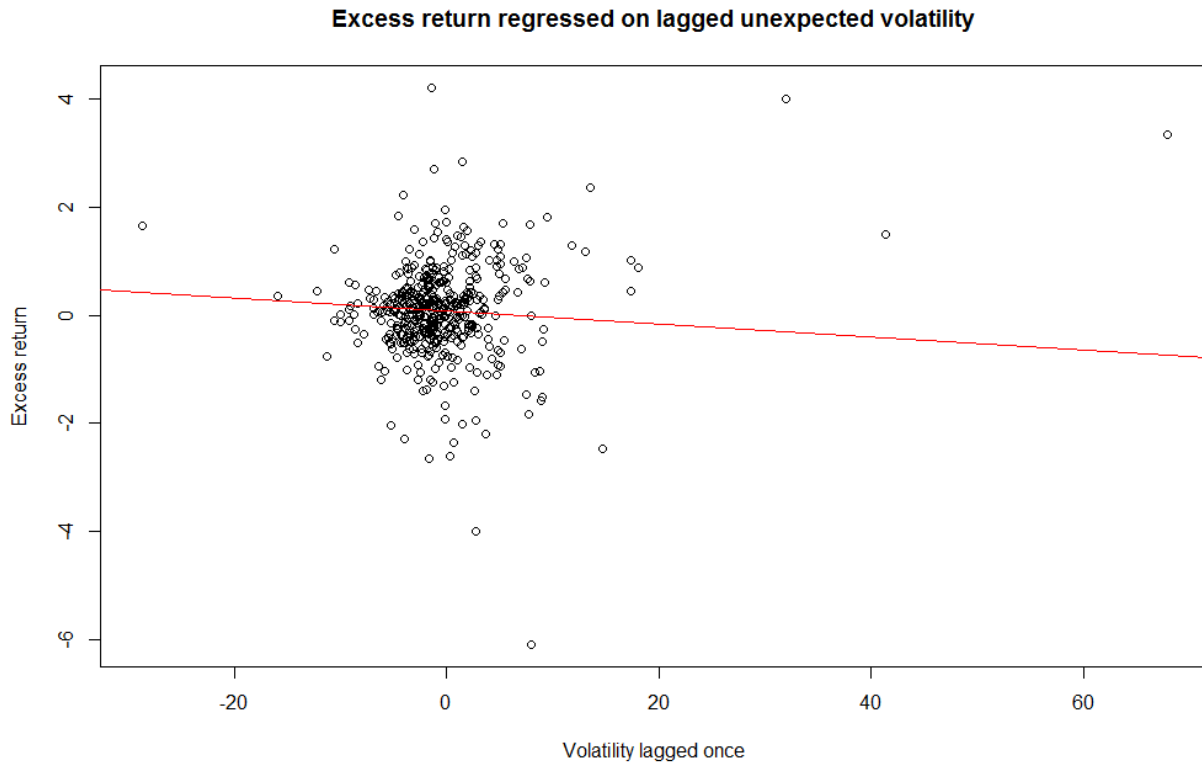
To get a clearer point of view of how it is all connected, I run a regression where the dependent variable is excess return, and the independent variables are unexpected- and expected volatility.

$$r_t - rf_t = \alpha + \beta_1\sigma^u_{t-1} + \beta_2\sigma^e_{t-1} + \varepsilon_t \qquad (5.3)$$

Table 5.4: Excess return regressed on unexpected- and expected volatility lagged once.

Regression	Time period	β_1-coefficient (unexpected)	P-value of β_1	β_2-coefficient (expected)	P-value of β_2	R-square adj.
	01.1975 – 12.1984	0.06002	0.0188	0.02902	0.2457	0.03882
	01.1985 – 12.1994	-0.041326	7.36e-06	0.044975	0.0353	0.1758
(5.3)	01.1995 – 12.2004	-0.025782	0.158	-0.005413	0.769	0.0001953
	01.2005 – 07.2014	0.0159102	0.226	-0.0006658	0.955	-0.004552
	01.1975 – 07.2014	-0.0119397	0.0843	0.0003062	0.9666	0.0021

The result of (5.3) is negative unexpected volatility coefficient β_1 and marginally positive expected volatility coefficient β_2 in the out-of-sample period. The β_1 coefficient is significant on

an α=0.1 significance level, while the β_2 coefficient is insignificant. The adjusted R^2 is approximately zero. Similar to French et al. (1987) and Zakamulin (2014) there is a positive relationship between excess return and expected volatility lagged once and a negative relationship between excess return and unexpected volatility lagged once in the same period. The β_1 coefficient is significant in the two first sub-periods, but the sign is different in the two first sub-periods equivalent to regression (5.2). The β_2 coefficient is only significant in the second sub-period, where it is positive. The second sub-period is the only period with a decent explanation degree of 17.6 percent, the other periods have adjusted R^2 near zero (analogous to (5.2)).

Out of the results from regression (5.1), (5.2) and (5.3), I can argue that unexpected volatility can predict future volatility and there is a positive linear relationship between them, this is also illustrated in Figure 5, hence unexpected volatility can be used in as input in a dynamic asset allocation strategy. Unexpected volatility is able to predict future excess return in the out-of-sample period, where it is a negative linear relationship between them, but the explanation degree is low. Only in the first and second sub-period, unexpected volatility significantly predict future excess return. Expected volatility is not able to predict future excess return in the out-of-sample period. Only in the second sub-period where there is a positive linear relationship between expected volatility and future excess return is significant on α=0.05 level.

5.2 Predictive abilities of turbulence.

Turbulence is as volatility very persistent, this is proofed by regressing turbulence on last month turbulence, given by this equation:

$$d_t = \alpha + \beta d_{t-1} + \varepsilon_t \qquad (5.4)$$

 Table 5.5 display the output of regression (5.4), where a one percentage change in turbulence is estimated to change future turbulence with 0.4907 percent in the same direction in the out-of-sample period. In the out-of-sample period and all sub-periods, the β-coefficient is positive, they are all significant except in the second sub-period where also the explanation degree is almost zero. From Table 5.5 we can state that turbulence can be used to predict future values of turbulence, and there exist a positive linear relationship between them.

Table 5.5: Persistence of turbulence: Turbulence regressed on turbulence lagged once, that is $d_t = \alpha + \beta d_{t-1} + \varepsilon_t$

Regression	Time period	β-coefficient	P-value	R-square adj.
	01.1975 – 12.1984	0.28155	0.00149	0.07515
	01.1985 – 12.1994	0.10761	0.241	0.003264
(5.4)	01.1995 – 12.2004	0.4129	2.75e-06	0.1636
	01.2005 – 07.2014	0.48855	3.13e-08	0.2316
	01.1975 – 07.2014	0.4907	< 2e-16	0.2392

Figure 7: Persistence of turbulence: Turbulence regressed on its own lag in the out-of-sample period, that is $d_t = \alpha + \beta d_{t-1} + \varepsilon_t$. Results:α=8.81, β=0.49, p-value=0.00 => Significant positive relationship between d_t and d_{t-1}.
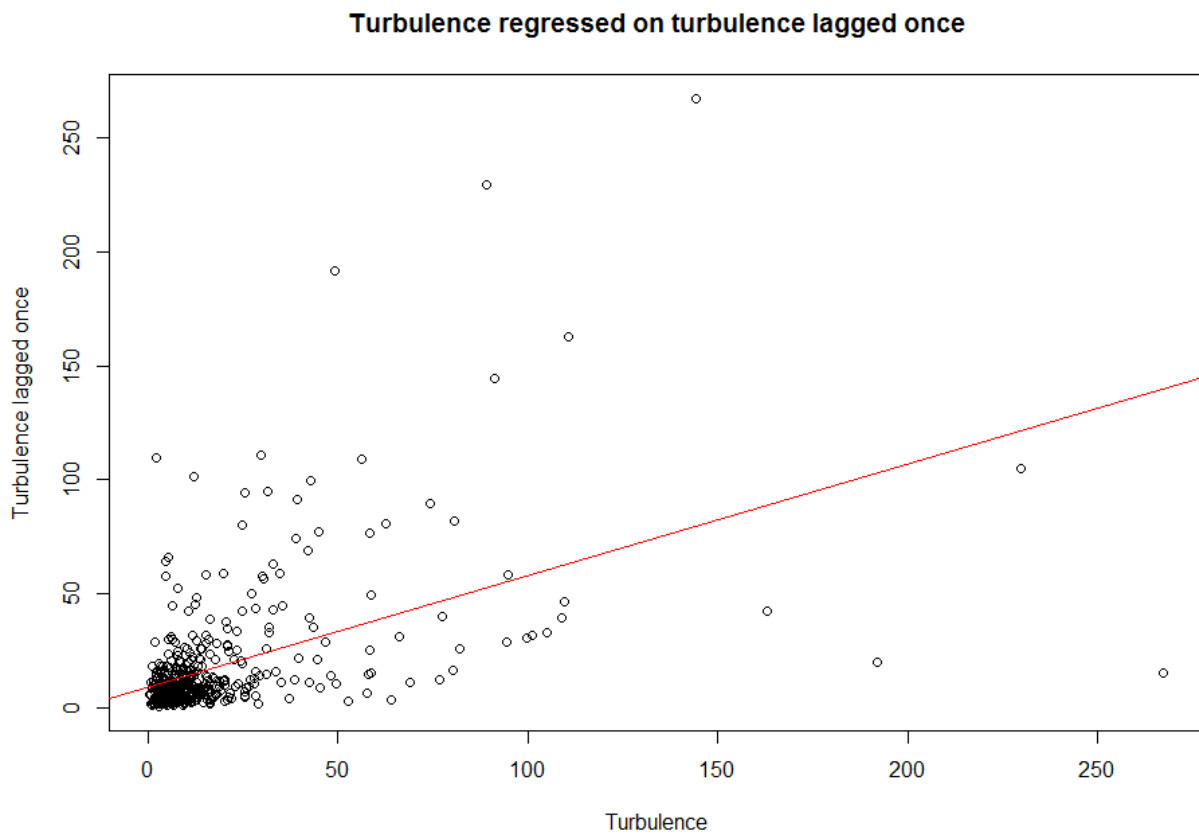
Now that it is proved that turbulence is persistent and can be used to predict itself, it is time to test how turbulence can be used to predict future excess return. As displayed in Table 5.6 underneath, where excess return is regressed on last month turbulence, given by this equation:

$$r_t - rf_t = \alpha + \beta d_{t-1} + \varepsilon_t \qquad (5.5)$$

The result of regression (5.5) is that all the β-coefficients are positive which indicated the direction, however none of the periods is significant on a α=5% significance level, the last sub-period has a p-value of 5.79 percent and is significant on an α=10% significance level. The adjusted R^2 is very low in all periods, implying that what the current turbulence provides essentially zero information about upcoming excess returns. Based on these data, I will not state that turbulence can predict future excess return.

Table 5.6: Regression of excess return on turbulence lagged once.

Regression	Time period	β-coefficient	P-value	R-square adj.
(5.5)	01.1975 – 12.1984	0.004158	0.576	-0.005836
	01.1985 – 12.1994	0.005749	0.419	-0.002894
	01.1995 – 12.2004	0.0005263	0.839	-0.008121
	01.2005 – 07.2014	0.005241	0.0579	0.02288
	01.1975 – 07.2014	0.002028	0.18	0.001695

Figure 8: Excess return regressed on turbulence lagged once, that is $r_t - rf_t = \alpha + \beta d_{t-1} + \varepsilon_t$. Results: $\alpha = 0.043,\ \beta = 0.002,\ p-value = 0.18$ => Insignificant unbiased relation between $r_t - rf_t$ and d_{t-1}.

Excess return regressed on turbulence lagged once
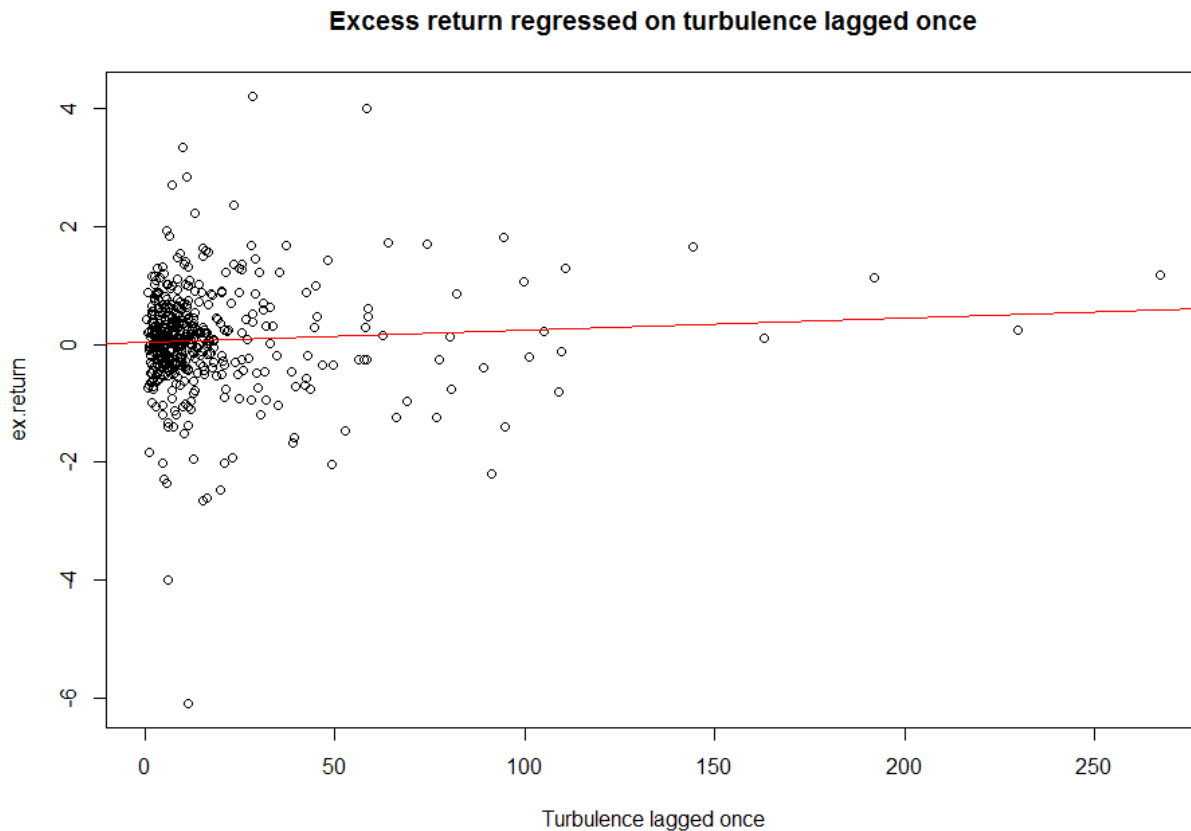
The persistence of volatility is well known documented in many scientific papers, this thesis will therefore not bother to replicate the volatility's persistence before making a volatility-responsive strategy. Now that features of unexpected volatility and turbulence are identified, it is time to test how well they works as input in a dynamic asset allocation strategy.

5.3 Performance of constructed portfolios.

We saw in the part 5.1 and 5.2 that unexpected volatility can be used to predict volatility, which also was positively related. Only unexpected volatility could predict excess returns to some degree, but not very convincingly. Turbulence is persistent and can be used to predict future turbulence. However, this part aim to evaluate four constructed portfolios that consist of different allocations between of market returns of EWP and risk-free interest rate.

Table 5.7: Mean returns

Time period	EWP	vol.port	unex.port	turb.port
01.1976 – 12.1984	0.16911	0.11235	0.10220	0.13373
01.1985 – 12.1994	0.26554	0.17150	0.17816	0.20669
01.1995 – 12.2004	0.07137	0.08217	0.09744	0.06317
01.2005 – 07.2014	-0.01841	0.00002	-0.00357	-0.00717
01.1976 – 07.2014	0.12230	0.09200	0.09440	0.09942

The portfolios mean returns is presented in Table 5.7. In the out-of-sample period, the EWP has the highest mean return with 12.23 percent, followed by turb.port with 9.94 percent mean return. Out of the four portfolios, vol.port has the lowest mean return with 9.2 percent. The EWP has highest mean return in the two first sub-periods, and is worst of all four portfolios in the last sub-period. Only vol.port managed to get a positive mean return in the last sub-period. Among the three dynamic portfolios, turb.port and vol.port both have two sub-periods each where they performs best of the three, turb.port is best in the two first decades, unex.port has highest mean return in the third sub-period, while vol.port is best in the last decade.

Table 5.8: Standard deviation.

Time period	EWP	vol.port	unex.port	turb.port
01.1976 – 12.1984	0.20041	0.10587	0.08699	0.12706
01.1985 – 12.1994	0.22270	0.09575	0.09888	0.12060
01.1995 – 12.2004	0.28594	0.14519	0.13716	0.12669
01.2005 – 07.2014	0.30285	0.11387	0.16783	0.14332
01.1976 – 07.2014	0.25820	0.11791	0.12806	0.13123

As anticipated, the EWP has the highest standard deviation in all periods and vol.port has the lowest standard deviation in the out-of-sample period. Unex.port has the lowest standard deviation in first sub-period, vol.port is superior in the second and fourth sub-period, turb.port has the lowest standard deviation in the third sub-period. Notice the large spread in portfolio standard deviation in the fourth sub-period, ranging from 0.11387 to 0.30285.

Table 5.9: Skewness.

Time period	EWP	vol.port	unex.port	turb.port
01.1976 – 12.1984	0.35118	-0.30812	-0.36388	0.16812
01.1985 – 12.1994	-0.43048	0.64641	1.53532	1.75402
01.1995 – 12.2004	-2.02904	-1.93319	-0.30471	-2.25571
01.2005 – 07.2014	0.67494	-0.34394	0.80325	0.31765
01.1976 – 07.2014	-0.57120	-1.02104	0.32508	-0.09490

In terms of skewness, the closer value to zero, the better. In the out-of-sample period, turb.port has the skewness closest to zero, which means that the turb.port follows the normal distribution best and indicating that the standard deviation and hence the Sharpe ratio is most accurate in the turbulence-responsive strategy. vol.port has the worst skewness in the out-of-sample period, the skewness is negative which means that vol.port is skewed to the left of a normal distribution.

*Table 5.10: **Sharpe ratio**. The p-value of Sharpe ratio test between the dynamical portfolios and EWP in parentheses.*

Time period	EWP	vol.port	unex.port	turb.port
01.1976 – 12.1984	0.40534	0.23115 (0.11)	0.16467 (0.02)	0.36088 (0.00)
01.1985 – 12.1994	0.93998	1.20410 (0.00)	1.23336 (0.00)	1.24778 (0.00)
01.1995 – 12.2004	0.11560	0.30204 (0.01)	0.43104 (0.00)	0.19614 (0.11)
01.2005 – 07.2014	-0.10933	-0.12894 (0.80)	-0.10889 (0.81)	-0.15264 (0.95)
01.1976 – 07.2014	0.28491	0.36696 (0.00)	0.35658 (0.00)	0.38628 (0.00)

Finally, it is time to look at the key measurement. In the first sub-period, the passive EWP is superior with a Sharpe ratio of 0.40534. Out of the three dynamic portfolios, turb.port has without doubt the greatest Sharpe ratio and unex.port has the worst Sharpe ratio. In the second sub-period, turb.port is superior with a Sharpe ratio of 1.24778, followed by unex.port. EWP has the lowest Sharpe ratio. In the third sub-period, unex.port is clearly superior with a Sharpe ratio of 0.431, followed by vol.port. EWP has again the weakest Sharpe ratio. In the fourth and last sub-period, all the four portfolios has a negative Sharpe ratio and the distance between their Sharpe ratios is small. However, unex.port has the least negative Sharpe ratio and is superior, followed by EWP, turb.port has the lowest Sharpe ratio.

In the out-of-sample period, there is a close race between vol.port and turb.port, but the turbulence-responsive strategy is the optimal strategy out of these four portfolios with a Sharpe ratio of 0.38628. It is tight between the two portfolios in the middle, but vol.port take the second place in the ranking with a Sharpe ratio of 0.36696, followed by unex.port with a Sharpe ratio of 0.35658. EWP has clearly the lowest Sharpe ratio with 0.28491 in the out/of/sample period. These findings indicate that it is not rational to hold the passive EWP in terms of risk-adjusted return; one should invest in the optimized portfolio turb.port, which has superior Sharpe ratio in the out-of-sample period. The out-of-sample period performance in a (mean return, standard deviation)-space is illustrated in Figure 9.

Figure 9: Illustration of risk/reward performance. Note that rf represents "all-in" in risk-free interest rate (0/100 portfolio). The blue line is the Capital allocation line.
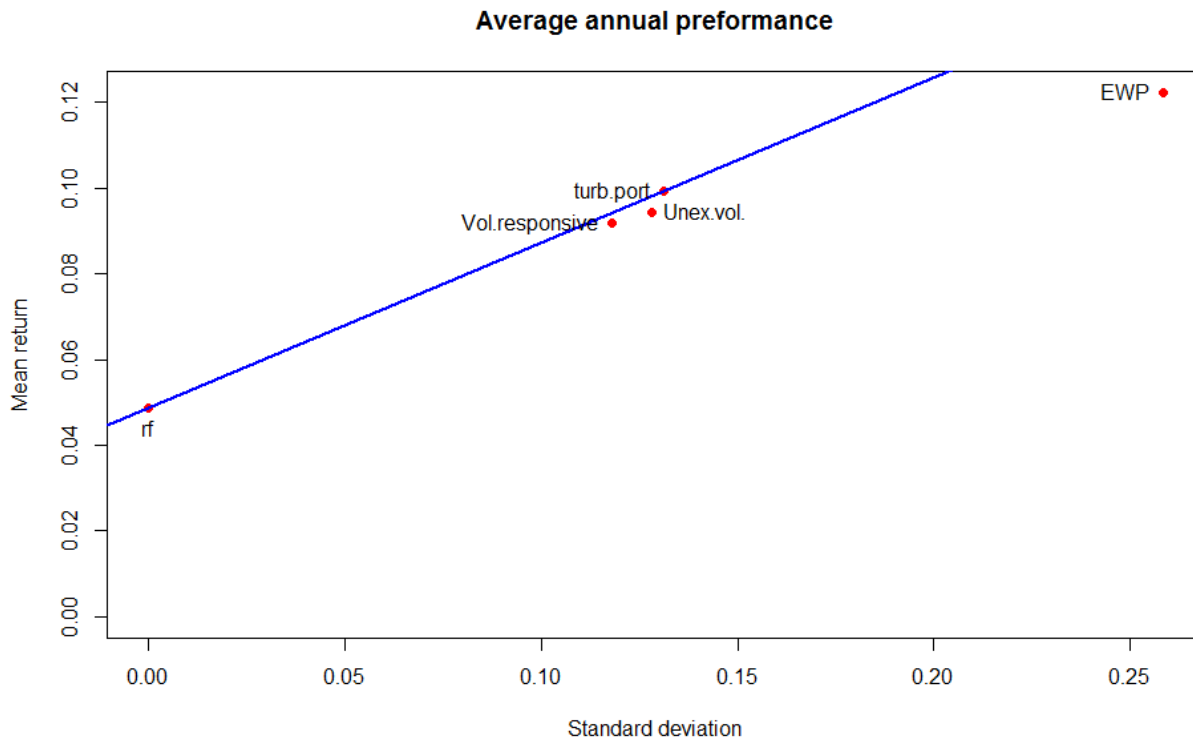
Since EWP is the benchmark, EWP is not included in the computation of CAPM alpha. In the out-of-sample period turb.port has a superior CAPM alpha with an α of 1.74 percent out of the three portfolios. vol.port follow with an α of 1.5 percent in the out-of-sample period and unex.port has an α of 1.405 percent in the out-of-sample period. In the sub-periods, turb.port is superior in the first. unex.port has the best CAPM alpha in the second, third and fourth sub-period. unex.port performs worst in the first, second and fourth sub-period in terms of CAPM alpha.

Table 5.11: CAPM alpha. The p-value of CAPM alpha in parentheses.

Time period	vol.port	unex.port	turb.port
01.1976 – 12.1984	-0.01505 (0.28)	-0.01795 (0.13)	-0.00490 (0.50)
01.1985 – 12.1994	0.04237 (0.02)	0.04960 (0.01)	0.04895 (0.00)
01.1995 – 12.2004	0.02832 (0.10)	0.04479 (0.02)	0.01204 (0.54)
01.2005 – 07.2014	-0.00558 (0.82)	-0.00234 (0.93)	-0.00833 (0.72)
01.1976 – 07.2014	0.01500 (0.14)	0.01405 (0.17)	0.01744 (0.07)

For a graphical view of the development of weights in the three dynamic portfolios, see Figure 10, 11 and 12. unex.port seems to have a more rapid response in the allocation between stocks

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and risk-free interest. While the vol.port moves a bit smoother, and turb.port has the smoothest response in the allocation between EWP and risk-free interest. As displayed in Table 5.12 underneath, there is 2.65 percent difference in the mean weights of unex.port and turb.port, and the median weight in EWP of turb.port is at 61.93 percent, while unex.port has a median weight in EWP of 55.2 percent. It is also worth mentioning that while unex.port ranges from zero to hundred percent weight in EWP, turb.port has only a maximum weight in EWP of 73.17 percent.

Table 5.12: Summary statistics of portfolio weights.

Summary statistics of portfolio weights in percent						
	Min.	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**
Unex.port	0.00	37.09	55.20	51.98	68.26	100.00
Vol.port	0.00	39.39	60.66	53.92	70.52	87.36
Turb.port	0.00	51.02	61.93	54.63	67.94	73.17

Figure 10: Weights of stocks in unexpected volatility-responsive portfolio.

Weight of EWP in Unexpected Volatility-Responsive portfolio

Figure 11: Weights of stocks in volatility-responsive portfolio.
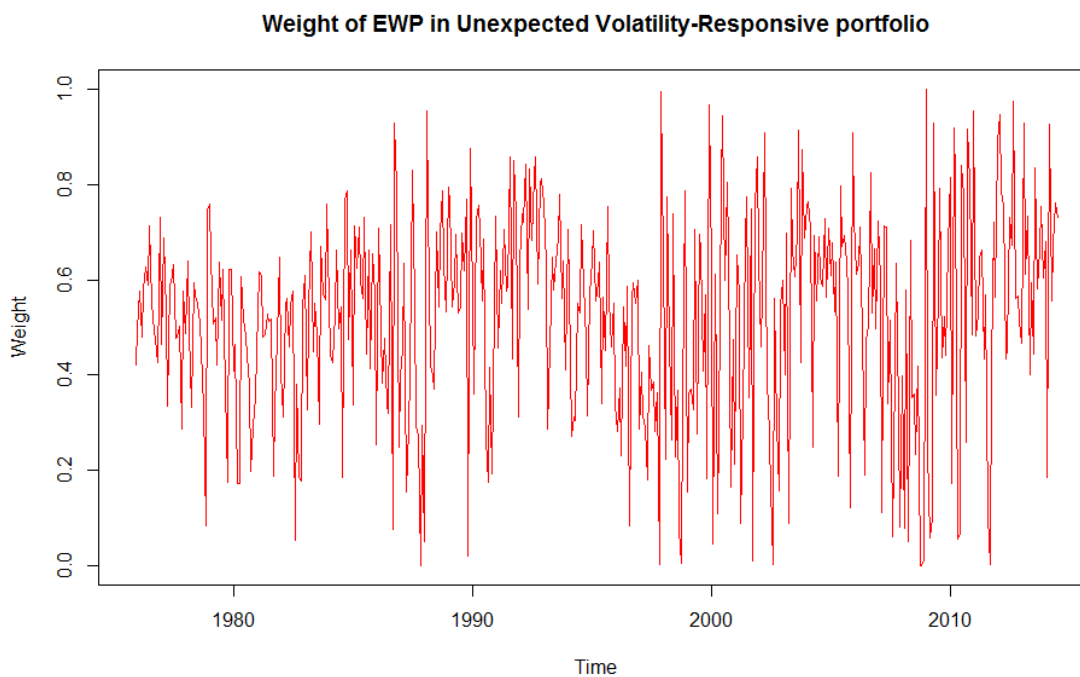

Weight of stocks in Volatility-Responsive portfolio

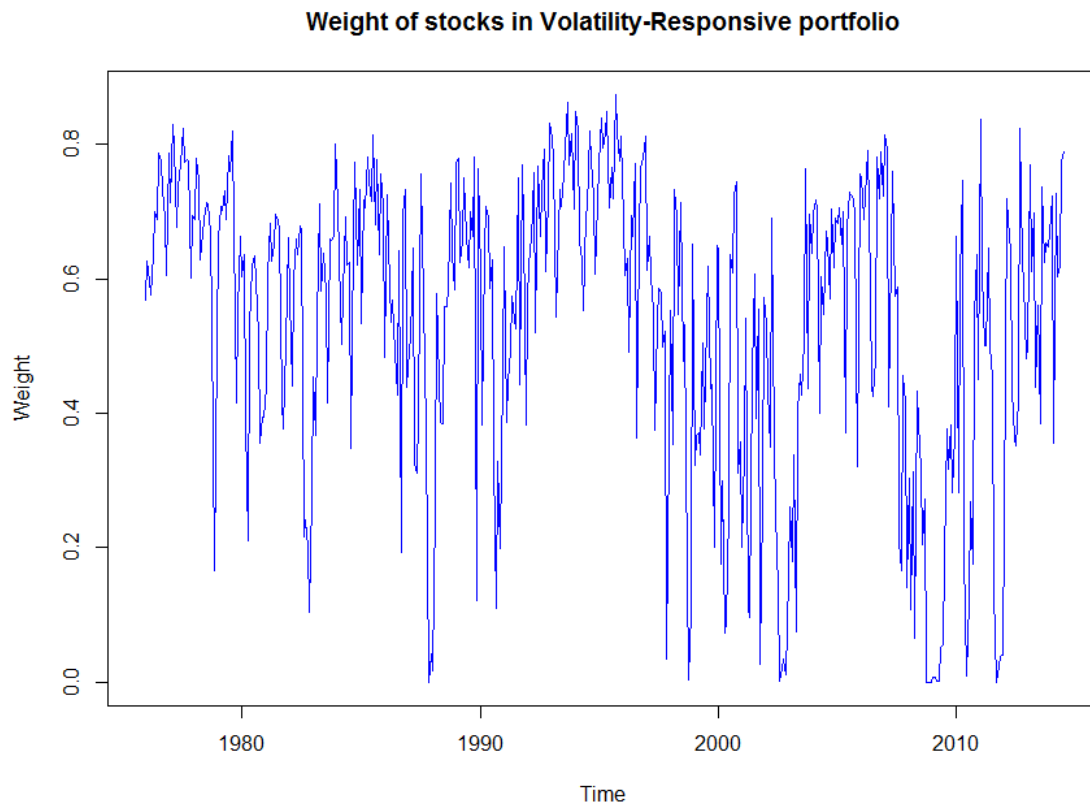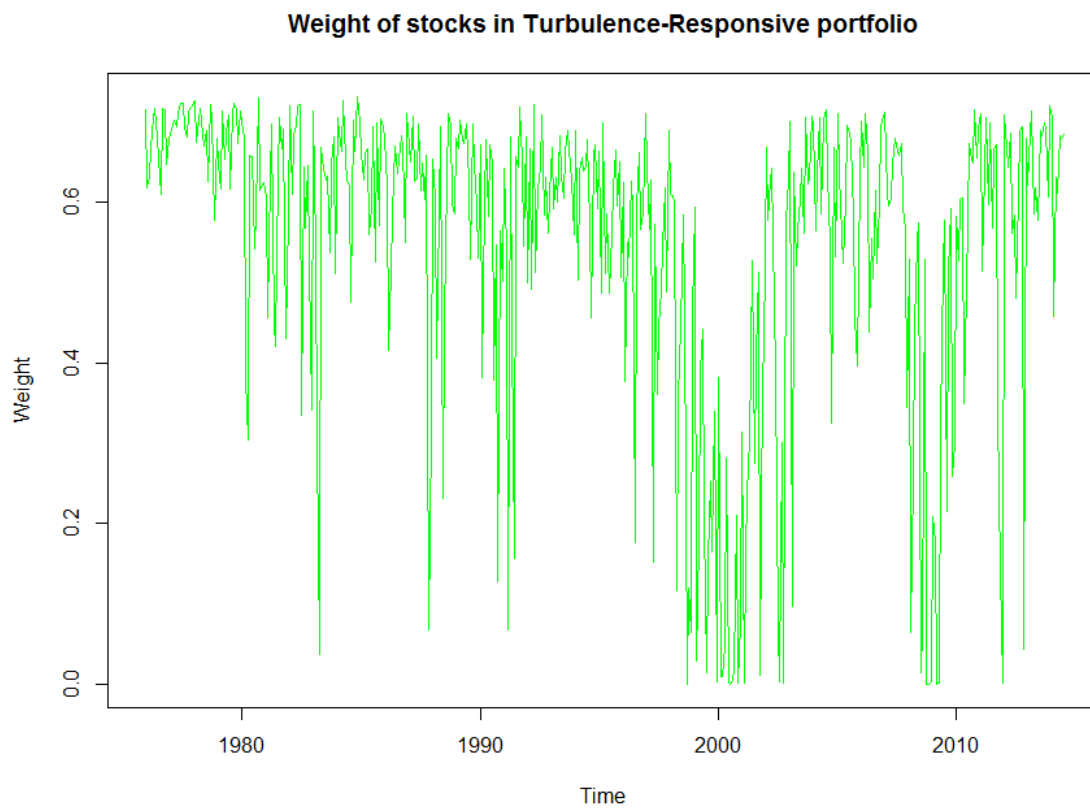*Figure 12: Weights of stocks in Turbulence-responsive portfolio.*

Weight of stocks in Turbulence-Responsive portfolio

5.4 Performance of constructed portfolios based on dataset 10 Size.

My primary goal of this thesis is to evaluate the dynamic asset allocation strategies based on 10 Industry Portfolios. However, to proof that the dynamic strategies also hold on other dataset and that my conclusion is right, I run the constructed portfolios on the dataset 10 Size. Table 5.13 display the performance from out-of-sample period with dataset 10 Size.

Table 5.13: Statistics of the performance of the four constructed portfolios in the out-of-sample period 01.1976 – 07.2014 based on 10 Size dataset. P-values in parentheses.

Measure	EWP	vol.port	unex.port	turb.port
Mean returns	0.2269	0.1546	0.1470	0.1410
Standard deviation	0.2626	0.1231	0.1380	0.1266
Skewness	-0.2681	-0.8402	1.0087	0.3779
Sharpe ratio	0.6786	0.8604 (0.00)	0.7123 (0.02)	0.7285 (0.00)
CAPM alpha		0.0382 (0.00)	0.0185 (0.11)	0.0171 (0.07)

The completely different dataset 10 Size present EWP as superior in terms of mean returns out of the four portfolios, this is in line with the mean returns based on 10 Industry Portfolios, however, the order has changed in Table 5.13, vol.port has highest mean returns out of the three dynamic portfolios, followed by unex.port. In terms of standard deviation unex.port and turb.port has changed places compared to the ranking in Table 5.8, although the standard deviation to unex.port and turb.port is very close to each other in both dataset. All the portfolios is skewed a bit more to the right in the 10 Size dataset, which changes the skewness from an average of -0.33 in dataset 10 Industry Portfolios to a better average of 0.07 in dataset 10 Size. In terms of the primary measurement, Sharpe ratio, the three dynamic portfolios have still a significant higher Sharpe ratio than EWP in the 10 Size dataset, but vol.port is superior followed by turb.port and unex.port. All three dynamic portfolios have a higher CAPM alpha in Table 5.13 than in Table 5.11. vol.port yields the highest CAPM alpha in dataset 10 Size and turb.port yields the lowest CAPM alpha out of the three portfolios.

6. Discussion

As far as I know, there exist no other study scientific paper that explore how dynamic asset allocation strategies based on unexpected volatility or turbulence perform on Kenneth French's 10 Industry Portfolio or 10 Size. So, I have no exact study to compare with. The study closest to this thesis, is Zakamulin (2014) and Kritzman & Li (2010). Zakamulin does a study on the S&P500- and DJIA index from 1970 to 2012. He finds a significant negative relation between excess return and unexpected volatility lagged once, however, his explanation degree is low. Zakamulin (2014) also found a significant positive relation between volatility and unexpected volatility lagged once. When he made a dynamic unexpected volatility-responsive portfolio, it clearly outperformed a passive 50/50 portfolio and a dynamic volatility-responsive portfolio.

When I run the regression excess return on lagged unexpected volatility in the out-of-sample period, I got an weak negative relationship between expected return and unexpected volatility lagged once, significant on an α=0.1 significance level, but the explanation degree was approximately zero. In the second sub-period, it was a clear significant negative relationship between unexpected volatility and future excess return. When unexpected volatility increased in the second sub-period, next month's excess return where predicted to decrease. When I run a regression between volatility and unexpected volatility lagged once, I got a significant strong positive relationship, which states that unexpected volatility is able to explain about 52 percent of future volatility in the out-of-sample period, and up to 85 percent in second sub-period. Explicitly, When unexpected volatility increases, next month's volatility is predicted to increase. Regression (5.3) display an insignificant weak positive relationship with expected volatility and future excess return. All these data from unexpected volatility is consistent with Zakamulin (2014) and French et al. (1987), even though they used a different dataset then what is applied in this thesis.

When testing the predictive abilities of turbulence on future excess return, I got an insignificant weak positive relation between excess return and turbulence lagged once, in out-of-sample period and all sub-periods at α=5% significance level. In addition, the explanation degree was approximately zero. However, turbulence can be used to predict next month's turbulence. Turbulence lagged once was able to explain about 24 percent of future turbulence and there exist

a significant positive relation between them in the out-of-sample period, and all four sub-periods. Kritzman and Li (2010) showed that Mahalanobis distance is mathematically equivalent to turbulence, which is replicated in this thesis. Kritzman and Li (2010) found that turbulence substantially lower risk-adjusted returns, and that turbulence is continues when is arrives. Even though Kritzman and Li (2010) uses a different dataset then what is applied in this thesis, the conclusions with respect to turbulence is equivalent in this thesis.

All three dynamical asset allocation portfolios deliver a statistically higher Sharpe ratio then EWP in the out-of-sample period in both dataset 10 Industry Portfolios and dataset 10 Size. However, there is differences with respect to Sharpe ratio in the four sub-periods. It is worth noticing that in the last sub-period, none of the dynamic portfolios is statistically different from the EWP with respect to Sharpe ratio, and all portfolios have negative Sharpe ratio. When considering a small time period, let's say five or ten years, it is not given that the dynamic asset allocation strategies would deliver consistently better Sharpe ratio than the EWP of risky assets. Even though the empirical results also display higher Sharpe ratios in some sub-periods and lower in other sub-periods, the four portfolios is always in a certain relation to each other. Compared to the dynamic portfolios, the passive EWP does not generate a stable risk/reward pattern in the long-run, the passive portfolio is too risky in volatile periods, and have too little risk in low volatile periods, see Figure 2. All three dynamic asset allocation strategies take care of this problem, and delivers a good and stable risk/reward pattern over time.

The superiority of the dynamic portfolios is interesting considering that they are the least volatile portfolios, this supports findings on low-volatility anomaly and reject the traditional risk-reward relationship of CAPM where risky asset get compensated with higher excess return. To some degree, these findings also rejects the EMH since the three active strategies all get a positive CAPM Alpha, In the primary dataset, only turb.port is significant on an α=0.1 significance level in the out-of-sample period. While all three dynamic portfolios is significant in the second sub-period. In dataset 10Size, vol.port yield a significant positive CAPM alpha on a five percent significance level. The EMH state that a CAPM Alpha ≤ 0 should be yield because the "market is efficient".

Transaction cost is not included in this thesis, for small investors who typically pay a much higher brokerage fee/transaction cost than institutional investors, the result of the three portfolios could be quite different. Consider a "small-amount" private investor who pays 0.05% brokerage fee on the transaction amount per transaction and/or a minimum brokerage fee of $ 10. The "small-amount" investor would allocate the portfolio rapidly if (s)he applied one of the three dynamic asset allocation strategies, and hence make many transactions to minimize the portfolio risk. This could result in a negative excess return because the transaction cost "eat up" the profit.

7. Conclusion

This thesis analysed the predictive abilities of unexpected volatility and turbulence and use these measures of financial risk as input, along with volatility, to create a dynamical asset allocation strategy that can beat a passive and naively diversified buy-and-hold strategy. The performance is measured in the outlook of a utility maximizing investor, whose main focus is to maximize the portfolio risk-adjusted return, hence, out-of-sample Sharpe ratio is applied as key measurement of portfolio performance. Mean return, standard deviation, skewness and CAPM alpha is also applied to distinguish the portfolios.

I created four portfolios, one passive buy-and-hold portfolio, a naively diversified EWP of risky assets. And three active asset allocation portfolios that aim to avoid unnecessary risk, one dynamic unexpected volatility-responsive portfolio, one dynamic volatility-responsive portfolio, and one dynamic turbulence-responsive portfolio. These three active portfolios allocate the weights between the risky asset EWP and the risk-free asset, one month's U.S. Treasury bill return, in the portfolios according to last month's unexpected volatility, last month's volatility and last month's turbulence respectively. The weights are re-balanced every month.

The empirical result showed that the GARCH(1,1) model was able to predict volatility pretty well. It exist a significant positive relation between unexpected volatility and future volatility. It was also found a negative relation between unexpected volatility and future excess return, significant on an $\alpha=0.1$ significance level. Turbulence was found to be persistent and it exist a positive relation between turbulence and future turbulence. However, turbulence is not able to predict future excess return.

In terms of the four portfolios, the empirical result in the out-of-sample period revealed that the standard deviation decreased in the three portfolios that included risk-free interest rate with over 51 percent on average, while the returns only decreased with 22 percent on average, and therefore improved the risk-to-reward returns. All the constructed dynamic asset allocation portfolios outperformed the passive portfolio in terms of Sharpe ratio, where the turbulence-responsive portfolio had highest Sharpe ratio. These founds support low-volatility anomaly and reject the traditional risk-reward relationship.

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A. Applied R code for accomplished empirical results.

To make this thesis's data completely reliable and possible to replicate, I present the R code used to achieve the empirical result. The package "xts" by Jeffrey A. Ryan and Joshua M. Ulrich was applied in order to handle extensive time-series matrices. The package "PerformanceAnalytics" by Brian G. Peterson, Peter Carl, Kris Boudt, Ross Bennett, Joshua Ulrich, Eric Zivot, Matthieu Lestel, Kyle Balkissoon and Diethelm Wuertz is a collection of econometric functions for performance and risk analysis, this was used to attain skewness and CAPM alpha. The package "quadprog" by Berwin A. Turlach and Andreas Weingessel is made for solving quadratic programing problems, I use "quadprog" to compute the illustration of the efficient frontier.

A1. R code: Program

This code show the computations from line 1 to line 120, the performance measures from line 121 to line 218, and line 221 to line 325 displays the code for graphical plots.

```
rm(list=ls(all=TRUE))
```

```r
library(xts)

library(PerformanceAnalytics)

library(fGarch)

source("functions.r")

Return <- read.csv("10IPd-Value.csv", header=TRUE, sep=";") #10 Industry Portfolios

rownames(Return) <- Return$Date


####################

# STOP HERE IF YOU WANT TO USE DATASET: 10 INDUSTRY PORTFOLIO, AND CONTINUE FROM
LINE 16,

# IF YOU WANT TO USE DATASET: 10 SIZE, THEN RUN THE FOLLOWING LINES.

SReturn<- read.table("size.txt", header=TRUE)  #Returns sorted on Size

rownames(SReturn) <- Return$Date

Return=as.xts(SReturn[,-(2:10)])

####################


Return <- as.xts(Return[,-1])

# assuming that 1 Month = 21 Days

# and 1 Year = 252 Days

nDays <- 21

nMonths <- floor(nrow(Return)/nDays)

years.lookback <- 5

lookback.period <- years.lookback*12

n <- nMonths - lookback.period # number of monthly portfolio returns in out-of-sample period

nact=ncol(Return) # number of activa.


############## estimating Volatility and 1/N portfolio ###############

std.hist <- matrix(data=NA, nrow=nMonths, ncol=nact) #allocate the space.

std.port <- rep(0,nMonths) #allocate the space.

for(i in 1:nMonths) {

  start <- (i-1)*nDays + 1
```

```
  end <- i*nDays
  ret <- Return[start:end,] #daily returns for the last month in the lookback period.
  covmat=cov(ret)*nDays #Variance covariance matrix from last month in the lookback period.
  std.hist[i,] <- sqrt(apply(ret^2, 2, mean))*sqrt(21)#Overall monthly volatility in %.


  w <- rep(1/nact, nact)
  std.port[i] <- sqrt(w %*% covmat %*% w) #vector of 1/N portfolio volatility, based on daily data
}


MReturn <- Return[ endpoints(Return, on="months", k=1), ] #Monthly returns!
MReturn <- MReturn[-1,]
ewp <- apply(MReturn,1,mean) #Realized monthly returns in equally-weighted portfolio
std.hist=ts(std.hist[(lookback.period + 1):nMonths,],start=c(1975,1),frequency = 12) #Overall
monthly volatility from 01.1975-31.07.2014
std.port=ts(std.port[(lookback.period + 1):nMonths],start=c(1975,1),frequency = 12) #Monthly
1/N portfolio volatility from 01.1975-31.07.2014
ewp=ts(ewp[(lookback.period + 1):nMonths],start=c(1975,1),frequency = 12) #as time serie
############### Turbulence ###################
r <- Return[1:(lookback.period*21), ] #daily returns in the lookback period, 31.12.69-31.12.74.
covmat = cov(r)                #Variance covariance matrix of lookback period.


turbulence = (Return[,1]*NA)
for( t in ((lookback.period*nDays)+1) : end ) {
  # measures turbulence for the current observation
  turbulence[t] = mahalanobis(Return[t,], colMeans(r), covmat) #Daily turbulence
  if( t %% 200 == 0) cat(t, 'out of', end, '\n')
}
Mturb <- turbulence[ endpoints(turbulence, on="months", k=1), ] #Monthly turbulence!
Mturb <- Mturb[(lookback.period + 1):nMonths,]
Fturb <- lag(Mturb,1) #Naively forcasted turbulence=Turbulence lagged once
turb <- ts(cbind(Mturb[-1],Fturb[-1]), start=c(1975,1), frequency=12)
```

```r
############### UNEXPECTED VOLATILITY and Excess return ########################

ewp.d=apply(Return, 1,mean)  #daily data in equally weighted portfolio.

std.pred  <-  ts((garch.forecast(ewp.d,  lookback.period))/100,start=c(1975,1),frequency  =  12)
#Predicted GARCH Monthly 1/N portfolio volatility from 01.1975-31.07.2014, based on daily data

std.unex=std.port-std.pred   #Monthly Unexpected volatility

std.unex.lag=lag(std.unex,1) #Monthly Unexpected volatility lagged once


RF<- read.csv("RF.csv", header=TRUE, sep=";")

RF <- ts(RF$RF[(lookback.period + 1):nMonths], start=c(1975,1), frequency=12)/100 #Risk free
rate from 01.1975-31.07.2014

rf <- RF*12  #This is Annual Risk free rates pr. months

ex.return <- ewp-rf #Excess return

ewp <- ewp[12:475]  #01.1976-31.07.2014 to use in the dynamic portfolios

rf <- rf[12:475]    #01.1976-31.07.2014 to use in the dynamic portfolios

###############################################################################

######## DYNAMIC UNEXPECTED VOLATILITY-RESPONSIVE STRATEGY #########

year.start <- 1971

MReturn <- ts(MReturn,start=c(1970,1), frequency=12)

ind.start <- which(time(MReturn) == year.start)

std.mean.u = mean(std.unex) # the mean of unexpected volatility

std.std.u  = sd(std.unex)   # standard deviation of unexpected volatility

ws.u <- rep(0,n)         # weight of ewp in the portfolio

for (j in 1:n) {
  # find the index of the past month
  period.end <- ind.start + j - 2
  # use the past month unexpected volatility as the forecast and compute the weight of ewp
  ws.u[j] <- pnorm((std.mean.u-std.unex[period.end])/std.std.u)
}

ws.u <- ws.u[1:464]
```

```r
unex.port= ts(ws.u*ewp+(1-ws.u)*rf, start=c(1975,12), frequency=12) #Return from dynamic
unex-vol-responsive strategy, 01.1976-31.07.2014


#DYNAMIC VOLATILITY-RESPONSIVE STRATEGY
std.mean.v <- mean(std.port)
std.std.v <- sd(std.port)
ws.v <- rep(0,n)
for (j in 1:n) {
  # find the index of the past month
  period.end <- ind.start + j - 2
  # use the past month volatility as the forecast and compute the weight of ewp
  ws.v[j] <- pnorm((std.mean.v-std.port[period.end])/std.std.v)
}
ws.v <- ws.v[1:464]
vol.port= ts(ws.v*ewp+(1-ws.v)*rf, start=c(1975,12), frequency=12) #Return from dynamic vol-
responsive strategy, 01.1976-31.07.2014


#DYNAMIC TURBULENCE-RESPONSIVE STRATEGY
Mturb <- ts(Mturb,start=c(1975,1),frequency = 12)
std.mean.t <- mean(Mturb)
std.std.t <- sd(Mturb)
ws.t <- rep(0,n)
for (j in 1:n) {
  # find the index of the past month
  period.end <- ind.start + j - 2
  # use the past month turbulence as the forecast and compute the weight of ewp
  ws.t[j] <- pnorm((std.mean.t-Mturb[period.end])/std.std.t)
}
ws.t <- ws.t[1:464]
turb.port= ts(ws.t*ewp+(1-ws.t)*rf, start=c(1975,12), frequency=12) #Return from dynamic turb-
responsive strategy, 01.1976-31.07.2014
```

####Perfomance measures in Sub-periods and OOS period####

#Periods

Sub1=cbind(ewp[1:109],vol.port[1:109], unex.port[1:109], turb.port[1:109], rf[1:109])

Sub2=cbind(ewp[110:229],vol.port[110:229],   unex.port[110:229],   turb.port[110:229],
rf[110:229])

Sub3=cbind(ewp[230:349],vol.port[230:349],   unex.port[230:349],   turb.port[230:349],
rf[230:349])

Sub4=cbind(ewp[350:464],vol.port[350:464],   unex.port[350:464],   turb.port[350:464],
rf[350:464])

OOSP=cbind(ewp,vol.port, unex.port, turb.port, rf)

#Sharpe ratios

Sharpe=matrix(c(SR(Sub1[,1]-mean(Sub1[,5])),SR(Sub1[,2]-mean(Sub1[,5])),SR(Sub1[,3]-
mean(Sub1[,5])), SR(Sub1[,4]-mean(Sub1[,5])),

        SR(Sub2[,1]-mean(Sub2[,5])),SR(Sub2[,2]-mean(Sub2[,5])),SR(Sub2[,3]-mean(Sub2[,5])),
SR(Sub2[,4]-mean(Sub2[,5])),

        SR(Sub3[,1]-mean(Sub3[,5])),SR(Sub3[,2]-mean(Sub3[,5])),SR(Sub3[,3]-mean(Sub3[,5])),
SR(Sub3[,4]-mean(Sub3[,5])),

        SR(Sub4[,1]-mean(Sub4[,5])),SR(Sub4[,2]-mean(Sub4[,5])),SR(Sub4[,3]-mean(Sub4[,5])),
SR(Sub4[,4]-mean(Sub4[,5])),

        SR(ewp-mean(rf)),SR(vol.port-mean(rf)),SR(unex.port-mean(rf)),   SR(turb.port-
mean(rf))),

        nrow=5, ncol=4, byrow=T,

        dimnames=list(c("1976-1985", "1985-1995","1995-2005","2005-2015","1976-2015"),

                c("ewp", "vol.port", "unex.port", "turb.port")))

#Sharpe ratio test

SharpeT=matrix(c(ST(Sub1[,2],Sub1[,1]),ST(Sub1[,3],Sub1[,1]),ST(Sub1[,4],Sub1[,1]),

        ST(Sub2[,2],Sub2[,1]),ST(Sub2[,3],Sub2[,1]),ST(Sub2[,4],Sub2[,1]),

        ST(Sub3[,2],Sub3[,1]),ST(Sub3[,3],Sub3[,1]),ST(Sub3[,4],Sub3[,1]),

```
            ST(Sub4[,2],Sub4[,1]),ST(Sub4[,3],Sub4[,1]),ST(Sub4[,4],Sub4[,1]),

            ST(vol.port,ewp),ST(unex.port,ewp),ST(turb.port,ewp)),

          nrow=5, ncol=3, byrow=T,

          dimnames=list(c("1976-1985", "1985-1995","1995-2005","2005-2015","1976-2015"),
                  c("vol.port", "unex.port", "turb.port")))
```

#Mean returns

```
Mean=matrix(c(mean(Sub1[,5]),      mean(Sub1[,1]),      mean(Sub1[,2]),      mean(Sub1[,3]),
mean(Sub1[,4]),
        mean(Sub2[,5]), mean(Sub2[,1]), mean(Sub2[,2]), mean(Sub2[,3]), mean(Sub2[,4]),
        mean(Sub3[,5]), mean(Sub3[,1]), mean(Sub3[,2]), mean(Sub3[,3]), mean(Sub3[,4]),
        mean(Sub4[,5]), mean(Sub4[,1]), mean(Sub4[,2]), mean(Sub4[,3]), mean(Sub4[,4]),
        mean(rf), mean(ewp), mean(vol.port), mean(unex.port), mean(turb.port)),
      nrow=5, ncol=5, byrow=T,
      dimnames=list(c("1976-1985", "1985-1995","1995-2005","2005-2015","1976-2015"),
              c("rf","ewp", "vol.port", "unex.port", "turb.port")))
```

#Standard deviation

```
Std=matrix(c(sd(Sub1[,1])/sqrt(12),      sd(Sub1[,2])/sqrt(12),      sd(Sub1[,3])/sqrt(12),
sd(Sub1[,4])/sqrt(12),
        sd(Sub2[,1])/sqrt(12), sd(Sub2[,2])/sqrt(12), sd(Sub2[,3])/sqrt(12), sd(Sub2[,4])/sqrt(12),
        sd(Sub3[,1])/sqrt(12), sd(Sub3[,2])/sqrt(12), sd(Sub3[,3])/sqrt(12), sd(Sub3[,4])/sqrt(12),
        sd(Sub4[,1])/sqrt(12), sd(Sub4[,2])/sqrt(12), sd(Sub4[,3])/sqrt(12), sd(Sub4[,4])/sqrt(12),
        sd(ewp)/sqrt(12), sd(vol.port)/sqrt(12), sd(unex.port)/sqrt(12), sd(turb.port)/sqrt(12)),
      nrow=5, ncol=4, byrow=T,
      dimnames=list(c("1976-1985", "1985-1995","1995-2005","2005-2015","1976-2015"),
              c("ewp", "vol.port", "unex.port", "turb.port")))
```

#Skewness

```
Skewness=matrix(c(skewness(Sub1[,1]),      skewness(Sub1[,2]),      skewness(Sub1[,3]),
skewness(Sub1[,4]),
```

skewness(Sub2[,1]), skewness(Sub2[,2]), skewness(Sub2[,3]), skewness(Sub2[,4]),

skewness(Sub3[,1]), skewness(Sub3[,2]), skewness(Sub3[,3]), skewness(Sub3[,4]),

skewness(Sub4[,1]), skewness(Sub4[,2]), skewness(Sub4[,3]), skewness(Sub4[,4]),

skewness(ewp), skewness(vol.port), skewness(unex.port), skewness(turb.port)),

nrow=5, ncol=4, byrow=T,

dimnames=list(c("1976-1985", "1985-1995","1995-2005","2005-2015","1976-2015"),

c("ewp", "vol.port", "unex.port", "turb.port")))

#CAPM Alpha

Alpha=matrix(c(CAPM.alpha(Sub1[,2],Sub1[,1],Rf=mean(Sub1[,5])),CAPM.alpha(Sub1[,3],Sub1[,1],Rf=mean(Sub1[,5])),CAPM.alpha(Sub1[,4],Sub1[,1],Rf=mean(Sub1[,5])),

CAPM.alpha(Sub2[,2],Sub2[,1],Rf=mean(Sub2[,5])),CAPM.alpha(Sub2[,3],Sub2[,1],Rf=mean(Sub2[,5])),CAPM.alpha(Sub2[,4],Sub2[,1],Rf=mean(Sub2[,5])),

CAPM.alpha(Sub3[,2],Sub3[,1],Rf=mean(Sub3[,5])),CAPM.alpha(Sub3[,3],Sub3[,1],Rf=mean(Sub3[,5])),CAPM.alpha(Sub3[,4],Sub3[,1],Rf=mean(Sub3[,5])),

CAPM.alpha(Sub4[,2],Sub4[,1],Rf=mean(Sub4[,5])),CAPM.alpha(Sub4[,3],Sub4[,1],Rf=mean(Sub4[,5])),CAPM.alpha(Sub4[,4],Sub4[,1],Rf=mean(Sub4[,5])),

CAPM.alpha(OOSP[,2],OOSP[,1],   Rf=mean(rf)),CAPM.alpha(OOSP[,3],OOSP[,1],Rf=mean(rf)),CAPM.alpha(OOSP[,4],OOSP[,1], Rf=mean(rf))),

nrow=5, ncol=3, byrow=T,

dimnames=list(c("1976-1985", "1985-1995","1995-2005","2005-2015","1976-2015"),

c("vol.port", "unex.port", "turb.port")))

###########################

mean((std.port-std.pred)^2) #MSFE.pred

A.std.port=std.port*sqrt(12) #Annual Volatility

A.std.pred=std.pred*sqrt(12) #Annual predicted Volatility

A.std.unex=std.unex*sqrt(12) #Annual unexpected Volatility

A.std.unex.lag=std.unex.lag*sqrt(12) #Annual unexpected volatility lagged once

############# Estimate the AR(1) model, regressions #############

summary(lm(A.std.port ~ A.std.unex.lag))#regression of std on its lagd unexpected std.

summary(lm(A.std.port[1:120] ~ A.std.unex.lag[1:120]))

summary(lm(A.std.port[121:240] ~ A.std.unex.lag[121:240]))

summary(lm(A.std.port[241:360] ~ A.std.unex.lag[241:360]))

summary(lm(A.std.port[361:475] ~ A.std.unex.lag[361:475]))

summary(lm(ex.return~A.std.unex.lag)) #regression of ex.return on lagged unexpected std.

summary(lm(ex.return[1:120] ~ A.std.unex.lag[1:120]))

summary(lm(ex.return[121:240] ~ A.std.unex.lag[121:240]))

summary(lm(ex.return[241:360] ~ A.std.unex.lag[241:360]))

summary(lm(ex.return[361:475] ~ A.std.unex.lag[361:475]))

summary(lm(ex.return~A.std.port)) #Neg relationship between market excess return and market volatility, in contrast to rational expectation model

summary(lm(ex.return[1:120] ~ A.std.port[1:120]))

summary(lm(ex.return[121:240] ~ A.std.port[121:240]))

summary(lm(ex.return[241:360] ~ A.std.port[241:360]))

summary(lm(ex.return[361:475] ~ A.std.port[361:475]))

#Relationship between between the excess market return and both the expected and unexpected volatility

A.std.pred.lag <- lag(A.std.pred,1)

#regression of excess return on its expected and unexpected volatility.

summary(lm(ex.return~ A.std.pred.lag+A.std.unex.lag))

summary(lm(ex.return[1:120] ~ A.std.pred.lag[1:120]+A.std.unex.lag[1:120]))

summary(lm(ex.return[121:240] ~ A.std.pred.lag[121:240]+A.std.unex.lag[121:240]))

summary(lm(ex.return[241:360] ~ A.std.pred.lag[241:360]+A.std.unex.lag[241:360]))

summary(lm(ex.return[361:475] ~ A.std.pred.lag[361:475]+A.std.unex.lag[361:475]))

```
#Presisitance of Turbulence

summary(lm(Mturb~Fturb))#regression of turbulence on its own lag.

summary(lm(Mturb[1:120] ~ Fturb[1:120]))

summary(lm(Mturb[121:240] ~ Fturb[121:240]))

summary(lm(Mturb[241:360] ~ Fturb[241:360]))

summary(lm(Mturb[361:475] ~ Fturb[361:475]))


summary(lm(ex.return~Fturb)) #regression of ex.return on its lagged turbulence.

summary(lm(ex.return[1:120] ~ Fturb[1:120]))

summary(lm(ex.return[121:240] ~ Fturb[121:240]))

summary(lm(ex.return[241:360] ~ Fturb[241:360]))

summary(lm(ex.return[361:475] ~ Fturb[361:475]))

#############################################################

#################### Plots ####################

devel=cbind(exp(RF*12)-1            ,ts(apply(MReturn,1,mean)[(lookback.period        +
1):nMonths],start=c(1975,1),frequency = 12), A.std.port, Mturb)

colnames(devel)=c("Risk-free rate","EWP", "Volatility", "Turbulence")

plot(devel, main="Development in time", col="red")

rm(devel)

plot(cbind(A.std.port, A.std.pred),plot.type = "single", col=c("red", "blue"), main= "Volatility",
ylab="Std, %")

legend("top", legend=c("Historical", "Predicted"), col=c("red", "blue"), lwd=1, bty = "n")

plot(Mturb, main= "Financial Turbulence")

plot(turb, plot.type = "single", col=c("red", "blue"),main= "Financial Turbulence", xlab="",
ylab="Turb., %")

legend("top", legend=c("Historical", "Predicted"), col=c("red", "blue"), lwd=1, bty = "n")

plot(cbind(std.port,std.unex),  xlab="",  ylab="Std,  %",  main="Historical-  and  unexpected
volatility")

plot(cbind(std.port,std.unex),plot.type = "single", col=c("red", "green"), xlab="", ylab="Std, %",
main="Historical- and unexpected volatility")

legend("top", legend=c("Historical", "Unexpected"), col=c("red", "green"), lwd=1, bty = "n")
```

```r
plot(ex.return, ylab="Excess return, %", main="Excess return")
abline(h = mean(ex.return), col="red")


#Weights
plot(ts(ws.u,start=c(1975,12),frequency = 12), ylab="Weight",col="red", main="Weight of EWP in
Unexpected Volatility-Responsive portfolio")
plot(ts(ws.v,start=c(1975,12),frequency = 12), ylab="Weight", col="blue",main="Weight of EWP
in Volatility-Responsive portfolio")
plot(ts(ws.t,start=c(1975,12),frequency = 12), ylab="Weight", col="green",main="Weight of EWP
in Turbulence-Responsive portfolio")


#AR(1)
plot(A.std.unex.lag,    A.std.port,    main="Volatility    regressed    on    lagged    unexpected
volatility",xlab="Volatility lagged once", ylab="Volatility")
abline(lm(A.std.port ~ A.std.unex.lag), col="red")
plot(A.std.unex.lag, ex.return, main="Excess return regressed on lagged unexpected volatility",
xlab="Volatility lagged once", ylab="Excess return")
abline(lm(ex.return~A.std.unex.lag), col="red") #show the regression line, slighthly negative.
plot(ts(Mturb,    start=c(1975,1),    frequency=12),    ts(Fturb,    start=c(1975,1),    frequency=12),
main="Turbulence regressed on turbulence lagged once",xlab="Turbulence", ylab="Turbulence
lagged once")
abline(res.turb, col="red") #show the positive regression line.
plot(ts(Fturb,  start=c(1975,1),  frequency=12),  ex.return,  main="Excess  return  regressed  on
turbulence lagged once",xlab="Turbulence lagged once")
abline(res.exturb, col="red") #show the regression line, slighthly positive.


# Constructed portfolios
plot(c(0,Std[5,]),  Mean[5,],xlab="Standard  deviation",pch=19,  col="red",  ylab="Mean  return",
main="Average annual preformance", ylim=c(0,max(Mean[5,])), xlim=c(0, max(Std[5,])))
abline(a=mean(rf), b=max(Sharpe[5,2:4]), lwd=2, col="blue")
text(x=0, y=mean(rf), labels="rf", pos=1)
```

```r
text(x=sd(ewp)/sqrt(12), y=mean(ewp), labels="EWP", pos=2)

text(x=sd(vol.port)/sqrt(12), y=mean(vol.port), labels="Vol.responsive", pos=2)

text(x=sd(unex.port)/sqrt(12), y=mean(unex.port), labels="Unex.vol.", pos=4)

text(x=sd(turb.port)/sqrt(12), y=mean(turb.port), labels="turb.port", pos=2)


#10 Industry portfolios and efficient frontier.

source("functions.r")

library("quadprog")

er=apply(MReturn, 2, mean)

covmat=cov(MReturn)

r=mean(rf)

# EFFICIENT PORTFOLIOS

# compute the efficient frontier with short sales

mup.ws <- seq(from=0.05, to=0.54, by=0.04)

n <- length(mup.ws)

sigp.ws <- rep(0, n)

for (i in 1:n) {

  w <- effportfolio(er, covmat, mup.ws[i])

  sigp.ws[i] <- sqrt(w %*% covmat %*% w)

}

# compute the efficient frontier no short sales

mup.ns <- seq(from=0.05, to=max(er), by=0.01)

n <- length(mup.ns)

sigp.ns <- rep(0, n)

for (i in 1:n) {

  w <- effportfolio(er, covmat, mup.ns[i], FALSE)

  sigp.ns[i] <- sqrt(w %*% covmat %*% w)

}

#TANGENCY PORTFOLIO with short sales

w.tan.ws <- tanportfolio(er, covmat, r)

mup.tan.ws <- w.tan.ws %*% er
```

```r
sigp.tan.ws <- sqrt(w.tan.ws %*% covmat %*% w.tan.ws)

#TANGENCY PORTFOLIO no short sales

w.tan.ns <- tanportfolio(er, covmat, r, FALSE)

mup.tan.ns <- w.tan.ns %*% er

sigp.tan.ns <- sqrt(w.tan.ns %*% covmat %*% w.tan.ns)

# compute the maximum Sharpe ratio

sr.ws <- maxsharpe(er, covmat, r)

sr.ns <- maxsharpe(er, covmat, r, FALSE)

# T-bills plus tangency (capital allocation line)

sigp.tan.tbill = 0:13

mup.tan.tbill.ns = r + sr.ns*sigp.tan.tbill

mup.tan.tbill.ws = r + sr.ws*sigp.tan.tbill

# create portfolio plot

# mean-variance frontier with short sales

plot(sigp.ws/sqrt(12), mup.ws, type="b", pch=18, col="green", lwd=2,
    ylim=c(0, max(mup.ws)), xlim=c(0, max(sigp.ws/sqrt(12))),
    xlab="Standard Deviation", ylab="Expected Return",
    main="10 US Industry portfolios")

# mean-variance frontier no short sales

points(sigp.ns/sqrt(12), mup.ns, type="b",pch=18, col="magenta", lwd=2)

# original assets

sig <- sqrt(diag(covmat))/sqrt(12)

points(sig, er, pch=19, col="blue")

# Capital Allocation Line through tangency portfolios

lines(sigp.tan.tbill/sqrt(12), mup.tan.tbill.ns, lwd=2, col="blue")

lines(sigp.tan.tbill/sqrt(12), mup.tan.tbill.ws, lwd=2, col="blue")

# tangency portfolio with short sales

points(sigp.tan.ws/sqrt(12), mup.tan.ws, pch=19, col="red")

# tangency portfolio no short sales

points(sigp.tan.ns/sqrt(12), mup.tan.ns, pch=19, col="red")

# risk-free
```

```
points(0, r, pch=19, col="red")
# text labels
text(x=0, y=r, labels="A", pos=1)
text(x=sigp.tan.ws/sqrt(12), y=mup.tan.ws, labels="C", pos=3)
text(x=sigp.tan.ns/sqrt(12), y=mup.tan.ns, labels="B", pos=3)
text(x=mean(sig), y=mean(er), labels="Industry portfolios", pos=4)
legend("topleft", legend=c("Efficient frontier WITH short sale", "Efficient frontier WITHOUT short
sale", "CAL","Risk free rate","Tangency WITHOUT short sales","Tangency WITH short sales"),
col=c("green", "magenta", "blue","red","red","red"), lwd=c(2,2,2,NA, NA, NA), bty = "1",
pch=c(NA,NA,NA,"A","B", "C"))


std.vol.port=ts(ws.v*std.port[12:475]+(1-ws.v)*0, start=c(1975,12), frequency=12)
std.fifftyfiffty=ts(0.5*std.port[12:475]+0.5*0, start=c(1975,12), frequency=12)
plot(cbind(std.fifftyfiffty, std.vol.port),plot.type = "single", col=c("red", "blue"), main=
"Volatility", ylab="Std, %")
legend("topleft", legend=c("50/50 portfolio", "Vol.-responsive port."), col=c("red", "blue"),
lwd=1, bty = "n")
####################################################
```

A2. R code: Functions.

The R codes underneath present the different functions applied in the computation of the
program.

```
garch.forecast <- function(r, lookback.period) {
  # this function predicts the volatility using the GARCH(1,1) model
  nDays <- 21
  nMonths <- floor(length(r)/nDays)
  n <- nMonths - lookback.period # number of months in the OOS period
  std <- rep(0,n)
  for(i in 1:n) {
    start <- (i-1)*nDays + 1        # start of rolling estimation window
    end <- (i+lookback.period-1)*nDays # end of rolling estimation window
```

```
    ret <- r[start:end]

    coeff = garchFit(~ garch(1, 1), data = ret, trace=FALSE) # estimate GARCH(1,1)

    res = predict(coeff, n.ahead = nDays) # nDays ahead prediction

    std[i] = sqrt(sum(res$standardDeviation^2))*100

  }

  return(std)

}
```

exp.forecast returns the predicted monthly volatility over the OOS period using the exponentially-weighed moving average
r - is the vector of daily returns
lookback.period - is the number of months in the lookback period

```
exp.forecast <- function(r, lookback.period, lambda=0.88) {

  nDays <- 21

  nMonths <- floor(length(r)/nDays)

  n <- nMonths - lookback.period     # number of months in the OOS period

  nLookback <- lookback.period*nDays # number of days in the lookback period

  w <- rep(lambda,nLookback)^rev(0:(nLookback-1)) # weights of observations in the moving window

  bottom <- sum(w)  # sum of weights


  std <- rep(0,n)
  for(i in 1:n) {
    start <- (i-1)*nDays + 1        # start of rolling estimation window
    end <- (i+lookback.period-1)*nDays # end of rolling estimation window
    ret <- r[start:end]
    std[i] <-  sqrt(sum(w*ret^2)/bottom)*sqrt(21)*100 # to monthly vol
  }
  return(std)
}
```

Annual Sharpe ratio

```r
SR <- function(er) {
  return(mean(er)/sd(er)*sqrt(12))
}
#Sharpe ratio test
ST <- function(ex1, ex2) {
  # test for equality of two Sharpe ratios
  # ex1 - excess returns to the first portfolio
  # ex2 - excess returns to the second portfolio
  # returns the p-value of the test
  # the null hypothesis is rejected when p-value is small
  if (length(ex1) != length(ex2))
    stop("Different lengths of two returns!")
  SR1 <- (mean(ex1)/sd(ex1))
  SR2 <- (mean(ex2)/sd(ex2))
  ro <- cor(ex1,ex2)
  n <- length(ex1)
  z <- (SR2-SR1)/sqrt( (2*(1-ro)+0.5*(SR1^2+SR2^2-2*SR1*SR2*ro^2))/n )
  pval <- 2*pnorm(-abs(z))
  return(pval)
}


# Efficient portfolio subject to target return
effportfolio <- function(er, covmat, mu, shorts=TRUE) {
  # inputs:
  # er                    N x 1 vector of expected returns
  # covmat                    N x N covariance matrix of returns
  # mu            scalar, target expected return
  #
  # output is a vector of portfolio weights

  # check for valid inputs
```

```r
er <- as.vector(er)
covmat <- as.matrix(covmat)
if (!is.numeric(mu) || length(mu) !=1)
  stop("Target return is not a scalar")
if(length(er) != nrow(covmat))
  stop("Mismatch in number of rows")
if(any(diag(chol(covmat)) <= 0))
  stop("Covariance matrix is not positive definite")


# compute the weights of the efficient portfolio
if(shorts==TRUE){
  # closed-form solutions when short sales are allowed
  ones <- rep(1, length(er))     # vector of ones
  covmat.inv <- solve(covmat)     # inverse of covariance matrix
  A <- er %*% covmat.inv %*% er
  B <- er %*% covmat.inv %*% ones
  C <- ones %*% covmat.inv %*% ones
  lambda <- (C*mu - B)/(A*C - B^2)
  gamma <- (A - B*mu)/(A*C - B^2)
  w <- covmat.inv %*% (er*lambda + ones*gamma)
  w <- as.vector(w)
} else if(shorts==FALSE){
  # numerical solution with no short sales
  n = nrow(covmat)
  Dmat <- covmat
  dvec <- rep.int(0, n)
  Amat <- cbind(rep(1,n), er, diag(1,n))
  bvec <- c(1, mu, rep(0,n))
  result <- solve.QP(Dmat=Dmat,dvec=dvec,Amat=Amat,bvec=bvec,meq=2)
  w <- round(result$solution, 6)
} else {
```

```r
    stop("shorts needs to be logical. For no-shorts, shorts=FALSE.")
  }
  return(w)
}


# Tangency portfolio
tanportfolio <- function(er, covmat, r, shorts=TRUE) {
  # inputs:
  # er                        N x 1 vector of expected returns
  # covmat                    N x N covariance matrix of returns
  # r                scalar, risk-free rate return
  #
  # output is a vector of portfolio weights

  # check for valid inputs
  er <- as.vector(er)
  covmat <- as.matrix(covmat)
  if (!is.numeric(r) || length(r) !=1)
    stop("Risk-free rate is not a scalar")
  if(length(er) != nrow(covmat))
    stop("Mismatch in number of rows")
  if(any(diag(chol(covmat)) <= 0))
    stop("Covariance matrix is not positive definite")

  # Global minimum variance portfolio
  w <- gmvportfolio(covmat, shorts=shorts)
  if(w %*% er < r)
    stop("Risk-free rate greater than mean return on global minimum variance portfolio")
  #
  # compute the weights of the tangency portfolio
  if(shorts==TRUE){
```

```r
  # closed-form solutions when short sales are allowed
  ones <- rep(1, nrow(covmat))  # vector of ones
  covmat.inv <- solve(covmat)   # inverse of covariance matrix
  w <- (covmat.inv %*% (er-r))/as.numeric(ones %*% covmat.inv %*% (er-r))
  w <- as.vector(w)
} else if(shorts==FALSE){
  # numerical solution with no short sales
  n = nrow(covmat)
  Dmat <- covmat
  dvec <- rep.int(0, n)
  Amat <- cbind(er-r, diag(1,n))
  bvec <- c(1, rep(0,n))
  result <- solve.QP(Dmat=Dmat,dvec=dvec,Amat=Amat,bvec=bvec,meq=1)
  w <- round(result$solution/sum(result$solution), 6)
} else {
  stop("shorts needs to be logical. For no-shorts, shorts=FALSE.")
}
return(w)
}
# in-sample Sharpe ratio
maxsharpe <- function(er, covmat, r, shorts=TRUE) {
  # inputs:
  # er            N x 1 vector of expected returns
  # covmat                N x N covariance matrix of returns
  # r             scalar, risk-free rate return
  #
  # output is the maximum Sharpe ratio

  #
  # check for valid inputs
  er <- as.vector(er)
```

```r
covmat <- as.matrix(covmat)
if (!is.numeric(r) || length(r) !=1)
  stop("Risk-free rate is not a scalar")
if(length(er) != nrow(covmat))
  stop("Mismatch in number of rows")
if(any(diag(chol(covmat)) <= 0))
  stop("Covariance matrix is not positive definite")


# computations
if(shorts==TRUE){
  s <- sqrt((er-r) %*% solve(covmat) %*% (er-r))
} else if(shorts==FALSE){
  w <- tanportfolio(er, covmat, r, FALSE)
  mup <- w %*% er
  sigp <- sqrt(w %*% covmat %*% w)
  s <- (mup-r)/sigp
} else {
  stop("shorts needs to be logical. For no-shorts, shorts=FALSE.")
}
return(as.numeric(s))
}
```