Adaptive Control of Uncertain Nonlinear Systems with Quantized Input Signal

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Abstract

This paper proposes new adaptive controllers for uncertain nonlinear systems in the presence of input quantization. The control signal is quantized by a class of sector-bounded quantizers including the uniform quantizer, the logarithmic quantizer and the hysteresis quantizer. To clearly illustrate our approaches, we will start with a class of single-loop nonlinear systems and then extend the results to multi-loop interconnected nonlinear systems. By using backstepping technique, a new adaptive control algorithm is developed by constructing a new compensation method for the effects of the input quantization. A hyperbolic tangent function is introduced in the controller with a new transformation of the control signal. When considering multi-loop interconnected systems with interactions, a totally decentralized adaptive control scheme is developed with a new compensation method incorporated for the unknown nonlinear interactions and quantization error. Each local controller, designed simply based on the model of each subsystem by using the adaptive backstepping technique, only employs local information to generate control signals. Unlike some existing control schemes for systems with input quantization, the developed controllers do not require the global Lipschitz condition for the nonlinear functions and also the quantization parameters can be unknown. Besides showing global stability, tracking error performance is also established and can be adjusted by tuning certain design parameters. Simulation results illustrate the effectiveness of our proposed schemes.

Key words: Adaptive control; backstepping; input quantization; nonlinear systems; decentralized control.

1 Introduction

In quantized control systems, the control signal to the system is a piece-wise constant function of time and the system is interacted with information quantization. Due to its theoretical and practical importance in the study of digital control systems and networked control systems, there has been a great deal of interest in the development of quantized control systems. The main motivation for considering quantization in control systems comes from the observation that for many control systems, quantization is not only inevitable, but also useful. An important aspect is to use quantization schemes that have sufficient precision and require low communication rate. Much attention has been paid to quantized feedback control, in order to understand the required quantization density or information rate in stability analysis. The stabilization problem of linear or nonlinear systems with quantized control signals has been studied, see for examples [1–4], where the systems considered are completely known.

Uncertainties and nonlinearities always exist in many practical systems. Thus it is more reasonable to consider controller design for uncertain nonlinear systems. Quantized control of uncertain systems with known quantization parameters has been studied by using robust approaches in [5,6] and adaptive approaches in [7–13]. Adaptive control schemes for uncertain systems with logarithmic or hysteresis input quantization have been reported in [7] and [8], where the hysteresis type of quantization was originally introduced. However the stability condition in [7] and [8] depends on the control signal, which is hard to be checked in advance as the control signal is only available after the controller is put in opera-

tion. Since backstepping technique was proposed in [14], it has been widely used to design adaptive controllers for uncertain systems [15,16]. Adaptive backstepping control of uncertain nonlinear systems with quantized input have been studied in [9–12]. In [9,10], a backstepping-based adaptive control scheme is presented for a class of uncertain strict-feedback nonlinear systems with hysteresis quantized input. Although the proposed method can avoid stability conditions depending on the control input, it requires the nonlinear functions to satisfy global Lipschitz conditions with known Lipschitz constants. This strict condition has been relaxed recently in [11] by using am implicit adaptive controller. However in [11], the unknown parameters are only contained in the last differential equation of the system and the control signal is implicitly involved in the proposed control law. That is, the control signal needs to satisfy the equation resulted from the control law. It is nontrivial to solve the equation to obtain the control signal explicitly. In [12], a new quantizer is proposed based on logarithmic and uniform quantizers. By using such a quantizer, the resulting quantization error is bounded. However, in our paper, the quantization error depends on the input control signal and cannot be assumed bounded. Clearly, how to handle such a unbounded quantization error is difficult and challenging.

Due to difficulties in considering the effects of uncertain interconnections, extension of single-loop results to multi-loop interconnected systems is challenging, especially both input quantization and unknown interconnections are considered. In the control of uncertain interconnected systems, decentralized adaptive control strategy, designed independently for local subsystems and using locally available signals for feedback propose, is an efficient and practical strategy, see for examples [17,18]. Research on decentralized adaptive control using backstepping technique has also received great attention, see for examples [19–22]. In the presence of input quantization in unknown interconnected systems, the number of available decentralized control is still limited. Only paper [13] has addressed the issue of decentralized quantized control via output-feedback for interconnected systems. In [13], the original system needs to be transformed to a form including only the output signal and the signals from filters. So interactions only exist in the equation for the output in the final control systems and the rest equations related to the filter signals do not involve interactions. In this paper, more general class of interconnected systems is considered in the sense that interactions exist in all the differential equations of the subsystems. Thus for such systems, it is more challenging to design appropriate controllers to account for the effects of unknown interactions.

In this paper, we propose new adaptive backstepping approaches to solve the tracking control problems of both single-loop uncertain nonlinear systems and multi-loop uncertain interconnected nonlinear systems, which are preceded by quantized input signal. The control signal is quantized by a class of sector-bounded quantizers in-

cluding the uniform quantizer, the logarithmic quantizer and the hysteresis quantizer. Unknown parameters are contained in each differential equation of the system and their bounds are not required to be known. Based on backstepping approach, a new adaptive controller is developed by introducing a hyperbolic tangent function. By proposing a new transformation of the final control signal and using the property of the quantizer, the effects from the quantization input are effectively compensated so that the global Lipschitz conditions required for the nonlinearities in [9,10] are relaxed. To handle unknown quantized parameters, new parameter updating laws are developed which do not require the knowledge on the bounds of such unknown parameters. When considering multi-loop interconnected systems with interactions allowed in every state equation, a totally decentralized adaptive controller design approach is developed together with a new compensation method constructed for the unknown nonlinear interactions and quantization error. A well defined smooth function is introduced in the decentralized adaptive controllers to compensate for the effects of unknown nonlinear interactions. Besides showing global stability of the systems, the tracking error can asymptotically converge to a residual, which can be made arbitrarily small by choosing suitable design parameters and thus adjustable. The main contributions of this paper are summarized as follows.

- The global Lipschitz condition for the nonlinear functions considered in a class of strict-feedback uncertain systems with input quantization is removed.
- A new adaptive control scheme is developed to achieve desired tracking performance for a larger class of nonlinear systems by constructing a new compensation method for the effects of the input quantization.
- Extension of single-loop results to multi-loop interconnected systems with both input quantization and unknown interconnections.
- A decentralized adaptive control scheme is developed together with constructing a new compensation method for the unknown nonlinear interactions.

To clearly illustrate our approach, we will start with single-loop uncertain nonlinear systems with input quantization. Then the obtained results are extended to a class of uncertain interconnected nonlinear systems with both input quantization and unknown interconnections.

2 Problem Statement

2.1 Modeling of uncertain nonlinear systems

A class of uncertain nonlinear systems is considered in the following parametric strict-feedback form as in [14,23].

$$\dot{x}_i = x_{i+1} + \psi_i(\bar{x}_i) + \phi_i^T(\bar{x}_i)\theta$$

$$\dot{x}_n = q(u(t)) + \psi_n(\bar{x}_n) + \phi_n^T(\bar{x}_n)\theta \tag{1}$$
$$y = x_1(t), \quad i = 1, \ldots, n-1,$$

where $x_i(t) \in \Re^1, i = 1, \ldots, n$, $u(t) \in \Re^1$ and $y(t) \in \Re^1$ are the states, input and output of the system respectively, $\bar{x}_i(t) = [x_1(t), \ldots, x_i(t)]^T \in \Re^i$, the vector $\theta \in \Re^r$ is constant and unknown, $\psi_i(\bar{x}_i) \in \Re^1$ and $\phi_i(\bar{x}_i) \in \Re^r$ are known nonlinear functions and differentiable, $q(u(t))$ represents a quantizer and takes the quantized values. The control objective is to design a feedback control law for $u(t)$ ensure that the output $y(t)$ can track a reference signal $y_r(t)$ and all closed-loop signals are bounded.

Assumption 1 *The reference signal $y_r(t)$ and its nth order derivatives are known and bounded.*

Remark 1 *The proposed scheme in [9,10] requires the nonlinearities in the system to be globally Lipschitz continuous with known Lipschitz constants. Compared with [9,10], this condition is now relaxed. Also in contrast to [11], unknown parameters are contained in each differential equation of the system and their bounds are not required to be known. Thus the system considered in this paper is more general.*

2.2 Quantizer

The quantizer $q(u)$ has the following property.

$$|q(u) - u| \le \delta|u| + u_{min}, \tag{2}$$

where $0 < \delta < 1$ and $u_{min} > 0$ are quantization parameters. It can be shown that most practical quantizers, such as uniform, logarithmic, and hysteresis quantizers illustrated below, satisfy the property in (2).

2.2.1 Uniform quantizer

The uniform quantizer is modeled as

$$q(u) = \begin{cases} u_i sgn(u), & u_i - \frac{l}{2} < |u| \le u_i + \frac{l}{2} \\ 0, & |u| \le u_0 \end{cases}, \tag{3}$$

where $u_0 > 0$ and $u_1 = u_0 + \frac{l}{2}$, $u_i = u_{i-1} + l$ with $i = 2, \ldots,$ and l is the length of the quantization interval. $q(u)$ is in the set $U = \{0, \pm u_i\}$. Clearly the property (2) is satisfied with $|q(u) - u| \le u_{min} = \max\{u_0, l\}$. The map of the uniform quantizer (3) is shown in Figure 1.

2.2.2 Logarithmic quantizer

The logarithmic quantizer in [5] is modeled as

$$q(u) = \begin{cases} u_i sgn(u), & \frac{u_i}{1+\delta} < |u| \le \frac{u_i}{1-\delta} \\ 0, & |u| \le \frac{u_{min}}{1+\delta} \end{cases}, \tag{4}$$

where $u_i = \rho^{(1-i)}u_{min}$ with $i = 1, 2, \ldots$ and parameter $\rho = \frac{1-\delta}{1+\delta}$, $0 < \delta < 1$. $q(u)$ is in the set $U = \{0, \pm u_i\}$, $u_{min} > 0$ determines the size of the dead-zone for $q(u)$. The property (2) is satisfied in [10]. The map of the logarithmic quantizer (4) is shown in Figure 2.

2.2.3 Hysteresis quantizer

The hysteresis quantizer in [10,24] is modeled as

$$q(u) = \begin{cases} u_i sgn(u), & \frac{u_i}{1+\delta} < |u| \le u_i, \dot{u} < 0, \text{or} \\ & u_i < |u| \le \frac{u_i}{1-\delta}, \dot{u} > 0 \\ u_i(1+\delta)sgn(u), & u_i < |u| \le \frac{u_i}{1-\delta}, \dot{u} < 0, \text{or} \\ & \frac{u_i}{1-\delta} < |u| \le \frac{u_i(1+\delta)}{(1-\delta)}, \dot{u} > 0 \\ 0, & 0 \le |u| < \frac{u_{min}}{1+\delta}, \dot{u} < 0, \text{or} \\ & \frac{u_{min}}{1+\delta} \le u \le u_{min}, \dot{u} > 0, \\ q(u(t^-)), & \dot{u} = 0 \end{cases} \tag{5}$$

where $u_i = \rho^{(1-i)}u_{min}$ with integer $i = 1, 2, \ldots$ and parameters $u_{min} > 0$ and $0 < \rho < 1$, $\rho = \frac{1-\delta}{1+\delta}$. $q(u)$ is in the set $U = \{0, \pm u_i, \pm u_i(1+\delta)\}$. The property (2) is satisfied in [10]. The map of the hysteresis quantizer (5) is shown in Figure 3.

Remark 2 *The uniform quantizer has the uniformly spaced quantization levels. The logarithmic and hysteresis quantizers belong to non-uniform quantizaters with unequal quantization levels. Such kind of quantizers are the coarsest quantizers which minimize the average rate of communication instances and are easy to implement. Compared with the logarithmic quantizer, the hysteresis quantizer has additional quantization levels, which are used to avoid chattering. Whenever $q(u)$ in Figure 3 makes a transition from one value to another, some dwell time will elapse before a new transition can occur. Detailed discussions can be found in [10].*

3 Adaptive control design

3.1 Known quantization parameters

In this section, we will design adaptive backstepping feedback control laws for the nonlinear uncertain system (2) where the parameters of the quantizers are known. We begin by introducing the change of coordinates

$$z_1 = y - y_r \tag{6}$$
$$z_i = x_i - \alpha_{i-1}, \ i = 2, 3, \ldots, n, \tag{7}$$

where α_i are virtual controllers. The design procedure is outlined in the following steps.
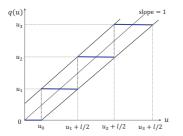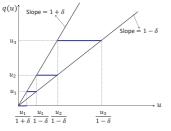
Fig. 1. The map of uniform quantizer $q(u)$

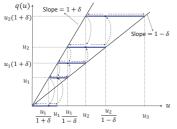Fig. 2. The map of logarithmic quantizer $q(u)$

Fig. 3. The map of hysteresis quantizer $q(u)$

Step i, $(i = 1, \ldots, n-1)$. The design for the first $n-1$ subsystems follows the backstepping design procedure in [14]. Design the stabilizing function α_i as

$$\alpha_i = -c_i z_i - z_{i-1} - \psi_i - \omega_i^T \hat{\theta} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (x_{j+1} + \psi_j)$$
$$+ \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \left(\Gamma \tau_i - \Gamma l_\theta (\hat{\theta} - \theta_0) \right)$$
$$+ \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \omega_i z_j + y_r^{(i)} \tag{8}$$

$$\tau_i = \tau_{i-1} + \omega_i z_i \tag{9}$$

$$\omega_i = \phi_i - \sum_{j=1}^{i} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j, \tag{10}$$

where c_i and l_θ are positive constants, Γ is a positive definite matrix, $\hat{\theta}$ is an estimate of θ, and θ_0 is a constant vector.

Step n. In the last step n, the actual control input u appears and is at our disposal.

$$u = -\tanh(z_n u_n / \lambda) u_n \tag{11}$$

$$u_n = \frac{1}{1 - \delta} \left(-\alpha_n + \mu \tanh(\mu z_n / \lambda) \right) \tag{12}$$

$$\alpha_n = -c_n z_n - z_{n-1} - \psi_n + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (x_{j+1} + \psi_j)$$
$$- \omega_n^T \hat{\theta} + y_r^{(n)} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \Gamma \left(\tau_n - l_\theta (\hat{\theta} - \theta_0) \right)$$
$$+ \sum_{j=2}^{n-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \omega_n z_j \tag{13}$$

$$\tau_n = \tau_{n-1} + \omega_n z_n \tag{14}$$

$$\omega_n = \phi_n - \sum_{j=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_j} \phi_j \tag{15}$$

$$\dot{\hat{\theta}} = \Gamma \tau_n - \Gamma l_\theta (\hat{\theta} - \theta_0), \tag{16}$$

where c_n, λ and μ are positive constants with $\mu \geq u_{min}$.

Theorem 1 *Consider the closed-loop adaptive system consisting of plant (2) with an input quantization, the adaptive backstepping controller (11) with virtual control laws (8)-(10) and (12)-(15), parameter estimator with updating law (16). The global boundedness of all the signals in the system is ensured. Furthermore, the tracking error $e(t) = y(t) - y_r(t)$ is ultimately bounded as follows:*

$$|e(t)| \leq B_1, \text{ where } B_1 = \sqrt{\max \left\{ 2U_n(0), \frac{2M_1}{F_1} \right\}}, \tag{17}$$

where $U_n(0) = \sum_{i=1}^{n} \frac{1}{2} z_i^2(0) + \frac{1}{2} \tilde{\theta}(0)^T \Gamma^{-1} \tilde{\theta}(0)$, $M_1 = 0.557\lambda + \frac{l_\theta}{2} \parallel \theta - \theta_0 \parallel^2$, $F_1 = \min\{2c_1, 2c_2, ..., 2c_n, l_\theta \lambda_{min}(\Gamma)\}$, and $\lambda_{min}(\Gamma)$ is the minimum eigenvalue of Γ.

Proof 1 *Consider the Lyapunov function as follows*

$$U_i = \sum_{j=1}^{i} \frac{1}{2} z_j^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}, \ i = 1, 2, ..., n. \tag{18}$$

where $\tilde{\theta} = \theta - \hat{\theta}$. The derivative of U_n satisfies

$$\dot{U}_n = \dot{U}_{n-1} + z_n \Big(q(u) + \psi_n + \theta^T \phi_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - y_r^{(n)}$$
$$- \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (x_{j+1} + \psi_j + \theta^T \phi_j) \Big). \tag{19}$$

The following inequality is derived by multiplying $|z_n|$ on both sides of (2) and using (11)

$$z_n q(u) \leq z_n u + \delta |z_n u| + u_{min} |z_n|$$
$$\leq -z_n u_n \tanh(z_n u_n / \lambda) + \delta |z_n u_n \tanh(z_n u_n / \lambda)|$$
$$+ u_{min} |z_n|$$
$$\leq -(1 - \delta) z_n u_n \tanh(z_n u_n / \lambda) + u_{min} |z_n|$$

$$\leq -(1-\delta)|z_n u_n| + u_{min}|z_n| + \epsilon_1$$
$$\leq -(1-\delta)z_n u_n + \mu|z_n| + \epsilon_1, \tag{20}$$

where $\epsilon_1 = 0.2785\lambda(1-\delta)$ and we have used the property that $|x| - x\tanh(x/\lambda) \leq 0.2785\lambda$ in [25,26]. Using (12), (13), (16), (19), and (20), the derivative of U_n satisfies

$$\dot{U}_n \leq \dot{U}_{n-1} + z_n\Big(\alpha_n + \psi_n + \theta^T\phi_n - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - y_r^{(n)}$$
$$- \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(x_{j+1} + \psi_j + \theta^T\phi_j)\Big)$$
$$- \mu z_n\tanh(\mu z_n/\lambda) + \mu|z_n| + \epsilon_1$$
$$\leq -\sum_{j=1}^n c_j z_j^2 + \tilde{\theta}^T\Big(\tau_n - \Gamma^{-1}\dot{\hat{\theta}}\Big)$$
$$+ \Big(\sum_{j=1}^{n-1}\frac{\partial\alpha_j}{\partial\hat{\theta}}z_{j+1}\Big)\Big(\Gamma\tau_n - \Gamma l_\theta(\hat{\theta} - \theta_0) - \dot{\hat{\theta}}\Big) + \epsilon_2$$
$$\leq -\sum_{i=1}^n c_i z_i^2 - \frac{1}{2}l_\theta\parallel\tilde{\theta}\parallel^2 + \epsilon_2 + \frac{1}{2}l_\theta\parallel\theta - \theta_0\parallel^2$$
$$\leq -F_1 U_n + M_1, \tag{21}$$

where $\epsilon_2 = 0.2785\lambda(2-\delta) \leq 0.557\lambda$ and the following property is used.

$$l_\theta\tilde{\theta}(\hat{\theta} - \theta_0) \leq -\frac{1}{2}l_\theta\parallel\tilde{\theta}\parallel^2 + \frac{1}{2}l_\theta\parallel(\theta - \theta_0)\parallel^2. \tag{22}$$

By direct integration of the differential inequality (21), we have

$$U_n \leq U_n(0)e^{-F_1 t} + \frac{M_1}{F_1}(1 - e^{-F_1 t}), \tag{23}$$

which shows that U_n is uniformly bounded, yielding that $z_1, z_2, \ldots, z_n$ and $\hat{\theta}$ are all bounded. The boundedness of x_i ($i = 1, \ldots, n$) can be ensured from the boundedness of α_i in (8) and the nth order derivatives of y_r, and the fact that $x_i = z_i + \alpha_{i-1}$ and $x_1 = z_1 + y_r$. Combining this with (11) and (12), $u(t)$ is bounded. Thus all the signals of the overall closed-loop system are globally uniformly bounded. Particularly, the bound of z_i is bounded in the set $\Big\{z \mid \parallel z\parallel \leq \sqrt{\max\Big\{2U_n(0), \frac{2M_1}{F_1}\Big\}}\Big\}$, which is adjustable by tuning the design parameters c_i, l_θ θ_0, and $\lambda_{min}(\Gamma)$.

Remark 3 *The controller designed in this section achieves the goals of stabilization and tracking with quantized input signal. The difficulty to achieve the control objective is to handle the quantization error because its bound depends on the control input $u(t)$. In [9,10], global Lipschitz condition for the nonlinear functions is required to guarantee the stability and compensate for the effects of the quantization error. In this paper, a new*

controller is developed in (11) which is a function of the virtual controller u_n and includes a hyperbolic tangent function $\tanh(z_n u_n/\lambda)$. Together with the property of the quantizer, this new control strategy enables the effects from the quantization error $|\delta z_n u|$ to be compensated by taking out $z_n u$ from the absolute function and transforming it to a term including only the virtual control signal $u_n(t)$ as shown in (20). As a result, the global Lipschitz continuous restriction in [9,10] for the nonlinear functions is removed, which largely broadens the class of systems to be controlled. In [11], the control signal is implicitly involved in the proposed control law to compensate for the effects of quantization input. Compared to [11], the proposed new control signal is an explicit function of the states and estimated parameters, and thus easy for implementation in practice. In addition, unknown parameters are contained in each differential equation of the system considered in our paper and their bounds are not required to be known.

Remark 4 *The inequality (20) is a key step. It transforms the quantized input term $z_n q(u)$ to $-(1-\delta)z_n u_n$ which is an explicit function of the virtual control signal u_n and can be directly designed based on Lyapunov stability.*

Remark 5 *The ultimate tracking error is adjustable and can be made smaller by increasing the design parameter c_i and $\lambda_{min}(\Gamma)$. Note that λ used in the hyperbolic tangent function in (11) should be chosen as a suitable positive constant. While the tracking error becomes theoretically small for sufficiently small λ, the tangent hyperbolic function approaches the sign function. Thus, there is a trade-off between the tracking performance and the realization of controller.*

3.2 Unknown quantized parameters

In this section, we consider the case that the parameters δ and u_{min} of the quantizer are unknown. So far there is no result available to address this issue due to the challenge of the problem involved. It is also found difficult to find a feasible solution to the adaptive control problem formulated in Section 2, if we design estimators to directly identify these parameters. After extensive research, an innovative solution is arrived by proposing suitable estimators to identify the bounds of certain parameters related to these unknown quantized parameters. As the first $n-1$ steps in the recursive adaptive backstepping process are the same as the design procedure in Section 3.1, so we only focus on the last step, which gives the control input u and the estimators to identify the unknown parameters summarized below.

$$u = -\tanh(z_n u_n/\lambda)u_n \tag{24}$$
$$u_n = \hat{\beta}\left(-\alpha_n + \hat{\mu}\tanh(z_n/\lambda)\right) \tag{25}$$
$$\dot{\hat{\theta}} = \Gamma\tau_n - \Gamma l_\theta(\hat{\theta} - \theta_0) \tag{26}$$

$$\dot{\hat{\mu}} = \gamma_1 z_n \tanh(z_n/\lambda) - \gamma_1 l_1 (\hat{\mu} - \mu_0) \quad (27)$$

$$\dot{\hat{\beta}} = \gamma_2 z_n \left(-\alpha_n + \hat{\mu} \tanh(z_n/\lambda) \right) - \gamma_2 l_2 (\hat{\beta} - \beta_0), \quad (28)$$

where γ_1, l_1, γ_2, l_2, μ_0 and β_0 are positive constants, $\hat{\mu}$ and $\hat{\beta}$ are estimates of the bound $\mu \geq u_{min}$ and $\beta = \frac{1}{1-\delta}$.

Theorem 2 *Consider the closed-loop adaptive system consisting of plant (2) with an input quantization, the adaptive backstepping controller (24) with virtual control laws (8)-(10), (13),(15) and (25), the parameter estimators with updating laws (26), (27), and (28). The global boundedness of all the signals in the system is ensured. Furthermore, the tracking error $e(t) = y(t) - y_r(t)$ is ultimately bounded as follows:*

$$|e(t)| \leq B_2, \text{ where } B_2 = \sqrt{\max \left\{ 2U_n(0), \frac{2M_2}{F_2} \right\}}, \quad (29)$$

where $F_2 = \min\{2c_1, 2c_2, ..., 2c_n, l_\theta \lambda_{min}(\Gamma), l_1\gamma_1, l_2\gamma_2\}$, $M_2 = 0.557\lambda + \frac{l_\theta}{2} \parallel \theta - \theta_0 \parallel^2 + \frac{l_1}{2}(\mu - \mu_0)^2 + \frac{l_2}{2}(1 - \delta)(\beta - \beta_0)^2$, $U_n(0) = \sum_{i=1}^{n} \frac{1}{2} z_i(0)^2 + \frac{1}{2} \tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{2\gamma_1}\tilde{\mu}^2(0) + \frac{(1-\delta)}{2\gamma_2}\tilde{\beta}^2(0)$.

Proof 2 *We choose the final Lyapunov function as follows*

$$U_n = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2\gamma_1} \tilde{\mu}^2 + \frac{(1-\delta)}{2\gamma_2} \tilde{\beta}^2. \quad (30)$$

Now substituting (24) and (25) into (20), the following inequality is obtained.

$$
\begin{aligned}
z_n q(u) &\leq -(1-\delta)z_n u_n + \mu|z_n| + \epsilon_1 \\
&= -(1-\delta)(\beta - \tilde{\beta})z_n \bar{u}_n + \mu|z_n| + \epsilon_1 \\
&= -z_n \bar{u}_n + (1-\delta)\tilde{\beta}z_n \bar{u}_n + \mu|z_n| + \epsilon_1 \\
&\leq z_n \alpha_n + (1-\delta)\tilde{\beta}z_n \bar{u}_n \\
&\quad -\hat{\mu}z_n \tanh(z_n/\lambda) + \mu z_n \tanh(z_n/\lambda) + \epsilon_2 \\
&\leq z_n \alpha_n + (1-\delta)\tilde{\beta}z_n \bar{u}_n + \tilde{\mu}z_n \tanh(\frac{z_n}{\lambda}) + \epsilon_2 (31)
\end{aligned}
$$

where $\epsilon_2 = 0.2785\lambda(1 - \delta + \mu)$ and $\bar{u}_n = -\alpha_n + \hat{\mu}\tanh(z_n/\lambda)$. Using (27), (28) and (31), the derivative of U_n satisfies

$$
\begin{aligned}
\dot{U}_n &\leq -\sum_{i=1}^{n} c_i z_i^2 - \frac{1}{2} l_\theta \parallel \tilde{\theta} \parallel^2 + \frac{1}{2} l_\theta \parallel \theta - \theta_0 \parallel^2 + \epsilon_2 \\
&\quad + \frac{1}{\gamma_1} \tilde{\mu} \left(\gamma_1 z_n \tanh(z_n/\lambda) - \dot{\hat{\mu}} \right) \\
&\quad + \frac{(1-\delta)}{\gamma_2} \tilde{\beta} \left(\gamma_2 z_n \bar{u}_n - \dot{\hat{\beta}} \right) \\
&\leq -\sum_{i=1}^{n} c_i z_i^2 - \frac{l_\theta}{2} \parallel \tilde{\theta} \parallel^2 - \frac{l_1}{2}\tilde{\mu}^2 - \frac{l_2(1-\delta)}{2}\tilde{\beta}^2 + M_2 \\
&\leq -F_2 U_n + M_2. \quad (32)
\end{aligned}
$$

By direct integration of the differential inequality (32), we have

$$U_n \leq U_n(0)e^{-F_2 t} + \frac{M_2}{F_2}(1 - e^{-F_2 t}). \quad (33)$$

Based on (33), all signals of the overall closed-loop system are globally uniformly bounded and $z_i(t)$ approaches to a compact set $\{z_i \mid |z_i(t)| \leq B_2\}$ where B_2 is given in (29).

Remark 6 *The virtual control laws $\alpha_i(i = 1, \ldots, n)$ are the same for both cases of known quantization parameters and unknown quantization parameters. When the parameters δ and u_{min} of the quantizer are unknown, two on-line estimators (27) and (28) are developed and the estimates $\hat{\mu}$ and $\hat{\beta}$ are used in the adaptive controller (25). Note that, instead of directly estimating the unknown quantization parameters u_{min} and δ, estimators (27) and (28) are designed to identify two parameters related to them.*

4 Decentralized adaptive control of interconnected systems with input quantization

In this section, we extend our approach to control a class of nonlinear interconnected systems. Due to difficulties in considering both effects of uncertain interconnections and quantization input, extension of single-loop results to multi-loop interconnected systems is challenging.

4.1 Modeling of nonlinear interconnected systems

A class of interconnected systems consisting N single-input single-output subsystems is considered in the following.

$$
\begin{aligned}
\dot{x}_{i,j} &= x_{i,j+1} + \psi_{i,j}(\bar{x}_{i,j}) + \phi_{i,j}^T(\bar{x}_{i,j})\theta_i \\
&\quad + h_{i,j}(y_1, \cdots, y_N), \quad j = 1, \ldots, n_i - 1 \\
\dot{x}_{i,n_i} &= q_i(u_i) + \psi_{i,n_i}(\bar{x}_{i,n_i}) + \phi_{i,n_i}^T(\bar{x}_{i,n_i})\theta_i \\
&\quad + h_{i,n_i}(y_1, \cdots, y_N) \\
y_i(t) &= x_{i,1}(t), \quad i = 1, \ldots, N, \quad (34)
\end{aligned}
$$

where $x_{i,j}(t) \in \Re^1$, $u_i(t) \in \Re^1$ and $y_i(t) \in \Re^1$ $i = 1, \ldots, N$, $j = 1, \ldots, n_i$, are the state, input and output of the subsystem respectively, $\bar{x}_{i,j}(t) = [x_{i,1}(t), \ldots, x_{i,j}(t)]^T \in \Re^j$ the vector $\theta_i \in \Re^{r_i}$ is constant and unknown, $\psi_{i,j} \in \Re^1$ and $\phi_{i,j}(.) \in \Re^{r_i}$ are known smooth nonlinear functions, $h_{i,j}(.)$ denotes the nonlinear interaction from the jth subsystem to the ith subsystem for $j \neq i$ or a nonlinear un-modeled part of the ith subsystem for $j = i$, the input $q_i(u_i)$ represents a quantizer satisfying the property in (2). For such a class of systems, we need the following assumptions.

Assumption 2 *The nonlinear interactions satisfy*

$$\left(h_{i,j}(y_1, \cdots, y_N, t)\right)^2 \le \sum_{k=1}^{N} r_{i,j,k} \bar{h}_{i,j,k}(y_k), \tag{35}$$

where $\bar{h}_{i,j,k}(.)$ are known smooth functions and $r_{i,j,k}$ are positive constants denoting the strengths of the uncertain subsystem interactions.

The control objective is to design a totally decentralized adaptive controller for system (34) such that the closed-loop system is stable and the output $y_i(t)$ can track a given reference signal $y_{ri}(t)$ as close as possible.

Assumption 3 *The reference signal $y_{ri}(t)$ and its n_ith order derivatives are known and bounded.*

4.2 Design of decentralized adaptive controller

The local adaptive controllers are summarized in (36)-(45) below.
Coordinate transformation:

$$z_{i,1} = y_i - y_{ri} \tag{36}$$
$$z_{i,j} = x_{i,j} - \alpha_{i,j-1}, \ j = 2, 3, \ldots, n_i \tag{37}$$

Control laws:

$$u_i = -\tanh(z_{i,n_i} u_{n_i}/\lambda_i) u_{n_i} \tag{38}$$

$$u_{n_i} = \frac{1}{1-\delta_i}\left(-\alpha_{i,n_i} + \mu_i \tanh(\mu_i z_{i,n_i}/\lambda_i)\right) \tag{39}$$

$$\alpha_{i,1} = -c_{i,1} z_{i,1} - \frac{1}{4} z_{i,1} - \psi_{i,1} - \omega_{i,1}^T \hat{\theta}_i$$
$$\qquad -sg_i(z_{i,1}) \sum_{j=1}^{n_i} (n_i - j + 1) \sum_{k=1}^{N} r_{i,j,k} \bar{h}_{k,j,i}(y_i) \tag{40}$$

$$\alpha_{i,j} = -c_{i,j} z_{i,j} - \frac{1}{4} z_{i,j} - \frac{1}{4} \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}}\right)^2 z_{i,j} - z_{i,j-1}$$
$$\qquad -\psi_{i,j} - \omega_{i,j}^T \hat{\theta}_i + \sum_{k=1}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}}(x_{i,k+1} + \psi_{i,k})$$
$$\qquad + \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_i}\left(\Gamma_i \tau_{i,j} - \Gamma_i l_{\theta i}(\hat{\theta}_i - \theta_{i0})\right)$$
$$\qquad + \sum_{k=2}^{j-1} \frac{\partial \alpha_{i,k-1}}{\partial \hat{\theta}_i} \Gamma_i \omega_{i,j} z_k + y_{ri}^{(j)} \tag{41}$$

with

$$\tau_{i,j} = \tau_{i,j-1} + \omega_{i,j} z_{i,j} \tag{42}$$

$$\omega_{i,j} = \phi_{i,j} - \sum_{k=1}^{j} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} \phi_{i,k}, \ j = 1, \ldots, n_i \tag{43}$$

$$sg_i(z_{i,1}) = \begin{cases} \frac{1}{z_{i,1}} & |z_{i,1}| \ge \sigma_i \\ \frac{z_{i,1}}{(z_{i,1}^2 - \sigma_i^2)^{n_i} + \sigma_i} & |z_{i,1}| < \sigma_i \end{cases} \tag{44}$$

Parameter update law:

$$\dot{\hat{\theta}}_i = \Gamma_i \tau_{i,n_i} - \Gamma_i l_{\theta i}(\hat{\theta}_i - \theta_{i0}) \tag{45}$$

where $c_{i,j}$, $l_{\theta i}$, σ_i, λ_i, and μ_i are positive constants, $\mu_i \ge u_{min,i}$, Γ_i is a positive definite matrix, and $\hat{\theta}_i$ is an estimate of θ_i, and θ_{i0} is a constant, $i = 1, \ldots, N$, $j = 1, \ldots, n_i$.

4.3 Stability Analysis

The main results are formally stated in the following theorem.

Theorem 3 *Consider the interconnected systems (34) with input quantization and nonlinear interconnections under Assumptions 2-3, the decentralized adaptive backstepping controller (38) with virtual control laws (39)-(41) and the parameter estimator with updating law (45), the following results can be guaranteed.*

(1) *All the closed-loop signals are globally uniformly bounded.*
(2) *The tracking error signals $e(t) = [e_1, e_2, \ldots e_N]^T$, where $e_i = y_i - y_{ri}$ for $i = 1, 2, \ldots, N$ will converge to a compact set.*

Proof 3 *Define the estimation error $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. For subsystem i, we choose the local Lyapunov function as*

$$U_{i,n_i} = \sum_{j=1}^{n_i} \frac{1}{2} z_{i,j}^2 + \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i. \tag{46}$$

The derivative of U_{i,n_i} is given by

$$\dot{U}_{i,n_i} \le -\sum_{j=1}^{n_i} c_{i,j} z_{i,j}^2 - \frac{1}{2} l_{\theta i} \parallel \tilde{\theta}_i \parallel^2 + \frac{1}{2} l_{\theta i} \parallel \theta_i - \theta_{i0} \parallel^2$$
$$\qquad + \epsilon_{i,2} + \sum_{j=1}^{n_i} z_{i,j}\left(h_{i,j} + \sum_{k=1}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} h_{i,k}\right)$$
$$\qquad - \sum_{j=1}^{n_i}\left(\frac{1}{4} z_{i,j}^2 + \frac{1}{4} \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}}\right)^2 z_{i,j}^2\right)$$
$$\qquad - sg_i(z_{i,1}) z_{i,1} \sum_{j=1}^{n_i}(n_i - j + 1)\sum_{k=1}^{N} r_{i,j,k}\bar{h}_{k,j,i}(y_i)$$
$$\le -F_i U_{i,n_i} + \frac{1}{2} l_{\theta i} \parallel \theta_i - \theta_{i0} \parallel^2 + \epsilon_{i,2}$$
$$\qquad - sg_i(z_{i,1}) z_{i,1} \sum_{j=1}^{n_i}(n_i - j + 1)\sum_{k=1}^{N} r_{k,j,i}\bar{h}_{k,j,i}(y_i)$$

$$+ \sum_{j=1}^{n_i} (n_i - j + 1) \sum_{k=1}^{N} r_{i,j,k} \bar{h}_{i,j,k}(y_k), \qquad (47)$$

where $F_i = \min\{2c_{i,1}, 2c_{i,2}, ..., 2c_{i,n_i}, l_{\theta_i}\lambda_{min}(\Gamma_i)\}$, $\epsilon_{i2} = 0.2785\lambda_i(2-\delta_i)$, Young's inequality and (35) are applied, by noting that.

$$\sum_{j=1}^{n_i} z_{i,j} \left(h_{i,j} + \sum_{k=1}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} h_{i,k} \right)$$

$$- \sum_{j=1}^{n_i} \left(\frac{1}{4} z_{i,j}^2 + \frac{1}{4} \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} \right)^2 z_{i,j}^2 \right)$$

$$\leq \sum_{j=1}^{n_i} (n_i - j + 1) h_{i,j}^2$$

$$\leq \sum_{j=1}^{n_i} (n_i - j + 1) \sum_{k=1}^{N} r_{i,j,k} \bar{h}_{i,j,k}(y_k). \qquad (48)$$

Choose a Lyapunov function for the entire group of subsystems as $U = \sum_{i=1}^{N} U_{i,n_i}$. From (38)-(47), we obtain

$$\dot{U} \leq - \sum_{i=1}^{N} F_i U_{i,n_i} + \sum_{i=1}^{N} \left(\frac{1}{2} l_{\theta_i} \parallel \theta_i - \theta_{i0} \parallel^2 + \epsilon_{i,2} \right) + H \qquad (49)$$

where

$$H = - \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{N} sg_i(z_{i,1}) z_{i,1}(n_i - j + 1) r_{k,j,i} \bar{h}_{k,j,i}(y_i)$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{N} (n_i - j + 1) r_{i,j,k} |\bar{h}_{i,j,k}(y_k)|. \qquad (50)$$

From the definition of sg_i in (44), it is clear that $H = 0$ for $|z_{i,1}| \geq \sigma_i$. For $|z_{i,1}| < \sigma_i$,

$$H \leq \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{N} (n_i - j + 1) r_{i,j,k} \bar{h}_{i,j,k}(z_{i,1} + y_{ri}). \qquad (51)$$

Clearly H has an upper bound $\bar{H} \geq 0$ from the boundedness of y_{ri} and $|z_{i,1}| < \sigma_i$. It follows that

$$\dot{U} \leq -FU + M, \qquad (52)$$

where $F = \min\{F_i\}, i = 1, \ldots, N$, $M = \sum_{i=1}^{N} \left(\frac{1}{2} l_{\theta_i} \parallel \theta_i - \theta_{i0} \parallel^2 + \epsilon_{i,2} \right) + \bar{H}$. By direct integration of the differential inequality (52), we have

$$U \leq U(0)e^{-Ft} + \frac{M}{F}(1 - e^{-Ft}), \qquad (53)$$

where $U(0) = \sum_{i=1}^{N} \sum_{j=1}^{n_i} \frac{1}{2} z_{i,j}^2(0) + \frac{1}{2}\tilde{\theta}_i^T(0)\Gamma_i^{-1}\tilde{\theta}_i(0)$. It shows that U is uniformly bounded, yielding that $x_{i,j}$, $\hat{\theta}_i$, $\alpha_{i,j}$, u_i for $i = 1, \ldots, N$ and $j = 1, \ldots, n_i$ are all bounded. From the definitions of $e(t)$, U and (53), we obtain that

$$\parallel e(t) \parallel^2 \leq \max \left\{ 2U(0), \frac{2M}{F} \right\}. \qquad (54)$$

It implies that the tracking errors will converge to a compact set.

Remark 7 *The main difficulties are to handle the unknown interactions in all the differential equations of the subsystems. At each control step j, in addition to compensate for the interaction $h_{i,j}$, we also need to compensate for the interactions $h_{i,k}$, $k = 1, .., j - 1$ from the previous differential equation of $x_{i,k}$ by using backstepping technique. In order to handle these effects, a new compensation scheme is constructed by introducing a well defined smooth function in (44) and new terms in the controller (40)and (41). Compared with the adaptive controller designed for single-loop nonlinear systems with input quantization, the new term $-sg_i(z_{i,1}) \sum_{j=1}^{n_i}(n_i - j + 1) \sum_{k=1}^{N} r_{i,j,k} \bar{h}_{k,j,i}(y_i)$ is introduced in the local control law $\alpha_{i,1}$ in (40) to compensate for the effects of interactions $h_{i,j}$ among other subsystems $j \neq i$. The other new terms $-\frac{1}{4} z_{i,j}$ and $-\frac{1}{4} \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} \right)^2 z_{i,j}$ in the local virtual control laws (40) and (41) are used to compensate for the effects from the un-modeled part $h_{i,j}$ of its own subsystem $j = i$. Note that a well defined function $sg_i(z_{i,1})$ in (44) is continuous and n_i th-order differentiable.*

Remark 8 *Note that the interaction $h_{i,j}$ at each equations of the interconnected systems (34) is transformed to a term $\sum_{j=1}^{n_i}(n_i - j + 1) \sum_{k=1}^{N} r_{i,j,k} \bar{h}_{i,j,k}(y_k)$ in (47). The key steps are (48) and (50) in the stability analysis, which results in the cancellation of the interaction effects from other subsystems.*

Remark 9 *In [13], the original system was transformed to a system including only the output signal and the signals from filters. Only interactions $h_{i,1}$ exist in the equation for the output in the final control systems and the rest equations related to the filter signals do not involve interactions. So the designed controller only need to compensate for the effect from the interaction $h_{i,1}$. Compared to [13], the class of interconnected systems given in (34) of this paper is more general, as interactions exist in all the differential equations of subsystems. From the proposed scheme, it can be noted that at step j of the backstepping design, we need not only to compensate for the effects of interaction $h_{i,j}$ from the jth equation, but also the effects of interactions $h_{i,k}$ $(k = 1, \ldots, j - 1)$ from the previous $j - 1$ equations. Thus for such systems, it is more challenging to design appropriate controllers to compensate for the effects of unknown interactions.*

5 Simulation results

5.1 Uncertain nonlinear systems

In this section we consider an uncertain nonlinear system with quantization input as follows.

$$\ddot{x} + \theta \sin(\dot{x}) + x^2 = q(u), \tag{55}$$

where θ is an unknown parameter and $q(u)$ is a quantized input. The objective is to design a quantized control input for u to make the output $y = x$ track the reference signal $y_r(t) = sin(t)$. In the simulation, we consider three quantizers: uniform in (3), logarithmic in (4) and hysteresis in (5). The actual parameter value is chosen as $\theta = 1$ for simulation.

Case 1: Known quantization parameters.

The quantization parameters are chosen as $l = 0.2$, $\delta = 0.2$ and $u_{min} = 0.1$. The initial states and parameter are set as $x(0) = 0.5, \dot{x}(0) = 0.2$ and $\hat{\theta}(0) = 0.8$. The control design parameters are chosen as $c_1 = c_2 = 6$, $\Gamma = 1, l_\theta = 0.01, \theta_0 = 0, \lambda = 0.2, \mu = 0.1$. The trajectory output, tracking error, the control signal and quantized control are shown in Figure 4 for a uniform quantizer, Figure 5 for a logarithmic quantizer and Figure 6 for a hysteresis quantizer, respectively. For three input quantizers, the simulation results show that the output tracks the desired reference signal and the tracking error is bounded. The simulation results in Figures 7-9 show that the magnitudes of tracking errors with control parameters $c_1 = c_2 = 1$ are larger than those with parameters $c_1 = c_2 = 6$ in Figures 4-6, respectively. This also verifies our theoretical findings in Theorem 1 that the tracking error can be made smaller by increasing c_i.

Case 2: Unknown quantization parameter.

When the quantized parameters are unknown, the adaptive backstepping controller (24)-(28) are employed. The initial states and parameter are set as $x(0) = 0, \dot{x}(0) = 0.9$ and the control parameters are chosen as $c_1 = 8, c_2 = 5, \Gamma = 1, \gamma_2 = 0.01$. Figures 10, 11 and 12 respectively show the trajectories of output , tracking error and the control signal for the system (55) preceded by three quantizers with unknown quantization parameters. The simulation results for three quantizers verify our theoretical findings in Theorem 2 that the output tracks the desired reference signal and the tracking errors are bounded.
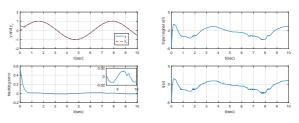
5.2 Interconnected systems

An interconnected system is considered with two subsystems and hysteresis quantization inputs as follows:

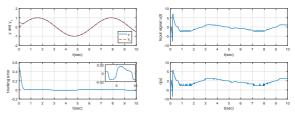$$\ddot{y}_i + \theta_i \phi_i + h_i = q_i(u_i), \quad i = 1, 2, \tag{56}$$

where $\phi_1 = y_1^2$, $\phi_2 = y_2 + y_2^2$, the interconnection terms $h_1 \le y_2 + y_1$, $h_2 \le y_1^2 + y_2$ $q_i(u_i)$ represents a hysteresis

quantizer in (5), the parameters θ_i are unknown. It is assumed that the tracking trajectories are $y_{r1} = sin(t)$ and $y_{r2} = 1 - cos(t)$, the quantization parameters are chosen as $\delta_i = 0.2$ and $u_{min,i} = 0.1$, the initial conditions are $[x_1(0), \ \dot{x}_1(0)] = [0.5, \ 0]^T, [x_2(0), \ \dot{x}_2(0)] = [0.5, \ 0]^T$ and $\hat{\theta}_i(0) = 0.8$. The design parameters are chosen as $c_{i1} = 6, c_{i2} = 4, \Gamma_i = 1, l_{\theta i} = 0.01, \theta_{i0} = 0.01, \lambda_i = 0.2, \mu_i = 0.1$. The responses of all subsystem outputs $y_i(t)$ and control inputs u_i are shown in Figures 13. Clearly, all these signals are bounded which is in accordance with the theoretical findings in Theorem 3.

(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 4. Uniform quantizer with $c_1 = c_2 = 6$

(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 5. Logarithmic quantizer with $c_1 = c_2 = 6$

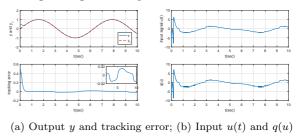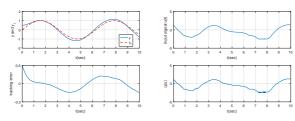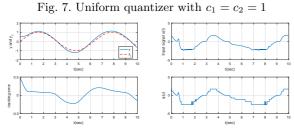(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 6. Hysteresis quantizer with $c_1 = c_2 = 6$
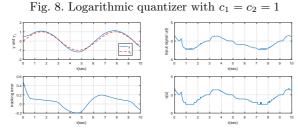
6 Conclusion

In this paper, we propose new adaptive backstepping approaches for single-loop uncertain nonlinear systems and multi-loop uncertain nonlinear interconnected systems with input quantization. By introducing a hyperbolic tangent function, proposing a new transformation of the final control signal and using the property of the quantizer, the effects from the quantization input are effectively compensated and the global Lipschitz conditions required for the nonlinearities are relaxed. When quantized parameters are not known, new parameter updating laws are developed which do not require the knowl-

(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 7. Uniform quantizer with $c_1 = c_2 = 1$

(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 8. Logarithmic quantizer with $c_1 = c_2 = 1$

(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 9. Hysteresis quantizer with $c_1 = c_2 = 1$

(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 10. Uniform quantizer with unknown parameters

(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 11. Logarithmic quantizer with unknown parameters

(a) Output y and tracking error; (b) Input $u(t)$ and $q(u)$

Fig. 12. Hysteresis quantizer with unknown parameters

(a) Outputs y_i and y_{ri};   (b) Inputs $q_i(u_i)$

Fig. 13. Interconnected system with hysteresis quantizer

edge on the bounds of such unknown parameters. Besides showing global stability of the system, the tracking error can asymptotically converge to a residual, which can be made smaller by choosing suitable design parameters and thus adjustable. When extension to interconnected systems, a decentralized adaptive control scheme is developed together with a new compensation method constructed for the unknown nonlinear interactions. It is established that the proposed decentralized controllers can ensure the global stability of the overall system and the transient performance of the tracking errors can be improved by appropriately tuning design parameters. Simulation results illustrate the effectiveness of our proposed schemes.
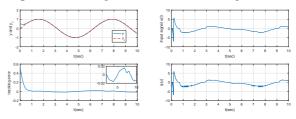
References

[1] H. Ishii and B. Francis, *Limited Data Rate in Control Systems with Network*. Berlin, Germany: Springer, 2002.

[2] S. Tatikonda and S. Mitter, "Control under communication constraints," *IEEE Transactions on Automatic Control*, vol. 49, pp. 1056–1068, 2004.

[3] N. Elia and S. Mitter, "Stabilization of linear systems with limited information," *IEEE Transactions on Automatic Control*, vol. 46, pp. 1384–1400, 2001.
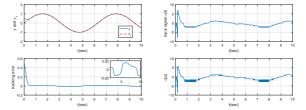
[4] D. Liberzon and J. Hespanha, "Stabilization of nonlinear systems with limited information feedback," *IEEE Transactions on Automatic Control*, vol. 50, pp. 910–915, 2005.

[5] C. D. Persis, "Robust stabilization of nonlinear systems by quantized and ternary control," *Systems & Control Letters*, vol. 58, pp. 602–609, 2009.

[6] T. Liu, Z. P. Jiang, and D. J. Hill, "A sector bound approach to feedback control of nonlinear systems with state quantization," *Automatica*, vol. 48, no. 1, pp. 145–152, 2012.

[7] T. Hayakawaa, H. Ishii, and K. Tsumurac, "Adaptive quantized control for linear uncertain discrete-time systems," *Automatica*, vol. 45, pp. 692–700, 2009.

[8] ——, "Adaptive quantized control for nonlinear uncertain systems," *Systems and Control Letters*, vol. 58, pp. 625–632, 2009.

[9] J. Zhou and C. Wen, "Adaptive backstepping control of uncertain nonlinear systems with input quantization," in *52nd IEEE Conference on Decision and Control*, Florence, Italy, 2013, pp. 5571–5576.

[10] J. Zhou, C. Wen, and G. Yang, "Adaptive backstepping stabilization of nonlinear uncertain systems with quantized input signal," *IEEE Transactions on Automatic Control*, vol. 59, pp. 460–464, 2014.

[11] L. Xing, C. Wen, H. Su, J. Cai, and L. Wang, "A new adaptive control scheme for uncertain nonlinear systems with quantized input signal," *Journal of the Franklin Institute*, vol. 352, pp. 5599–5610, 2015.

[12] L. Xing, C. Wen, Y. Zhu, H. Su, and Z. Liu, "Output feedback control for uncertain nonlinear systems with input quantization," *Automatica*, vol. 65, pp. 191–202, 2016.

[13] C. Wang, C. Wen, Y. Lin, and W. Wang, "Decentralized adaptive tracking control for a class of interconnected nonlinear systems with input quantization," *Automatica*, vol. 81, p. 359368, 2017.

[14] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York: Wiley, 1995.

[15] J. Zhou, C. Wen, and Y. Zhang, "Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis," *IEEE Transactions on Automatic Control*, vol. 49, pp. 1751–1757, 2004.

[16] J. Zhou, C. Zhang, and C. Wen, "Robust adaptive output control of uncertain nonlinear plants with unknown backlash nonlinearity," *IEEE Transactions on Automatic Control*, vol. 52, no. 3, pp. 503–509, 2007.

[17] P. Ioannou, "Decentralized adaptive control of interconnected systems," *IEEE Transactions on Automatic Control*, vol. 31, pp. 291 – 298, 1986.

[18] C. Wen, "Indirect robust totally decentralized adaptive control of continuous-time interconnected systems," *IEEE Transactions on Automatic Control*, pp. 1122–1126, 1995.

[19] ——, "Decentralized adaptive regulation," *IEEE Transactions on Automatic Control*, pp. 2163–2166, 1994.

[20] Z. P. Jiang, "Decentralized and adaptive nonlinear tracking of large-scale systems via output feedback," *IEEE Transactions on Automatic Control*, vol. 45, pp. 2122–2128, 2000.

[21] C. Wen and J. Zhou, "Decentralized adaptive stabilization in the presence of unknown backlash-like hysteresis," *Automatica*, vol. 43, pp. 426–440, 2007.

[22] C. Wen, J. Zhou, and W. Wang, "Decentralized adaptive backstepping stabilization of interconnected systems with dynamic input and output interactions," *Automatica*, vol. 45, pp. 55–67, 2009.

[23] R. Marino and P. Tomei, *Nonlinear Control Design: Geometric, Adaptive and Robust*. New York: Prentice Hall, 1995.

[24] C. De Persis and F. Mazenc, "Stability of quantized time-delay nonlinear systems: A Lyapunov-Krasowskii functional approach," *Mathematics of Control, Signals, and Systems*, vol. 21, pp. 4337–370, 2010.

[25] M. M. Polycarpou, "Stable adaptive neural control scheme for nonlinear system," *IEEE Transactions on Automatic Control*, vol. 41, no. 3, pp. 447–451, 1996.

[26] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Transactions on Automatic Control*, vol. 56, pp. 1672–1678, 2011.