Level-Crossing Rate and Average Duration of Fades of Non-Stationary Multipath Fading Channels

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***Abstract*—The level-crossing rate (LCR) and average duration of fades (ADF) are important statistical quantities describing the fading behaviour of mobile radio channels. To date, these quantities have only been analysed under the assumption that the mobile radio channel is wide-sense stationary, which is generally not the case in practice. In this paper, we propose a concept for the analysis of the LCR and ADF of non-stationary channels. Rice's standard formula for the derivation of the LCR of wide-sense stationary processes is extended to a more general formula enabling the computation of the instantaneous LCR of non-stationary processes. The application of the new concept results in closed-form expressions for the instantaneous LCR and ADF of non-stationary multipath flat fading channels. The contribution of this paper is of central importance for the statistical characterization of non-stationary mobile radio channels.**

I. Introduction

Time-varying multipath fading channels are commonly characterized by the level-crossing rate (LCR) and the average duration of fades (ADF). The LCR is a measure for the average number of times a radio signal drops below a given threshold level, while the ADF provides the average time interval during which the signal remains below the threshold [1], [2]. These statistical quantities are often referred to as the second-order statistics. They are important for the channel characterization as well as for the performance evaluation of wireless communication systems. In addition, the LCR is useful for estimating the velocity of mobile units [3]. Furthermore, the knowledge of the ADF is helpful for analyzing the error burst statistics [4], [5], choosing proper channel coding schemes, and optimizing the interleaver size. Moreover, for deep fading thresholds, the LCR and ADF are useful for computing the outage rate and average outage duration of wireless communication systems [6], respectively.

As is well known, the level-crossing properties depend on both the distribution and the spectral characteristics of the random process. In his original work [7], Rice presented in the 1940s a standard formula that is useful for the derivation of the LCR of any wide-sense stationary continuous random process. Based on this standard formula, to date, there exists a large body of research characterizing the LCR and ADF of many types of mobile fading channels, including Rayleigh [1], Rice [8], Nakagami [9], Hoyt [10], and Weibull [11] channels. Because of their importance, the LCR and ADF have also been studied intensively in the context of cooperative communications [12], and multiple antenna systems [13], [14]. All the above LCR- and ADF-related research works have in common that they have been carried out under the assumption of stationary fading channels.

However, the stationarity assumption is not realistic in practice, and has been introduced only for mathematical tractability of the analysis. In fact, several studies of measurement data collected in diverse propagation environments have shown that multipath fading channels exhibit non-stationarity features [15], [16]. In conjunction with this, the modeling and analysis of non-stationary fading channels has recently been a subject of intensive research [17]–[19].

In this paper, we study the LCR of non-stationary fading channels, providing an insight into the impact of various non-stationarity effects on the behavior of the crossing statistics. To this end, we consider a non-stationary multipath fading channel model for the complex channel gain described by a sum of chirps (SOC$_h$), which captures the non-stationarity effects caused by the variation of the speed of the mobile station (MS). For this scenario, we first derive an expression for the joint probability density function (PDF) of the fading envelope and its time derivative. In contrast to the case of stationary processes, this joint PDF is found to be time dependent. Using this new joint PDF, we then determine an expression for the instantaneous LCR. Particularly, it is shown that the derived expression obeys the same formalism as the LCR of stationary processes, except that the variance parameters are time dependent. In addition, a new approach is provided to determine the instantaneous LCR of measured and simulated non-stationary processes. Finally, this approach is applied to verify the accuracy of the derived analytical expression for the instantaneous LCR.

The paper is organized as follows. Section II starts from Rice's standard LCR formula for stationary processes and extends his fundamental expression to the instantaneous LCR of non-stationary processes. Section III describes briefly an SOC$_h$ process as an appropriate model for the complex channel gain of a non-stationary multipath fading channel under speed variations. Section IV is devoted to the derivation of the instantaneous LCR and ADF of SOC$_h$ processes. The numerical results are then presented in Section V. Finally, the conclusions are drawn in Section VI.

II. LCR of Stationary and Non-Stationary Processes

The LCR $N_\zeta(r)$ is defined as the expected number of times (per second) that a stochastic process $\zeta(t)$ passes through a given level r with positive (or negative) slope. If the stochastic process $\zeta(t)$ is wide-sense stationary, then the LCR $N_\zeta(r)$ of $\zeta(t)$ can be computed analytically by means of Rice's standard formula [7]

$$N_\zeta(r) = \int_0^\infty \dot{x}\, p_{\zeta\dot{\zeta}}(r,\dot{x})\, d\dot{x} \tag{1}$$

where $p_{\zeta\dot{\zeta}}(x,\dot{x})$ denotes the joint PDF of the process $\zeta(t)$ and its time derivative $\dot{\zeta}(t)$ at the same point in time t. Note that the LCR $N_\zeta(r)$ is independent of time for stationary processes.

Experimentally, the LCR $N_\zeta(r)$ is usually computed by invoking the ergodicity concept. If the process $\zeta(t)$ is ergodic, then the LCR $N_\zeta(r)$ can be obtained from the measurement (or simulation) of a single sample function of the stochastic process $\zeta(t)$. Let $\zeta^{(k)}(t)$ ($k = 1, 2, \ldots$) be the kth sample function of $\zeta(t)$, then the measured (simulated) LCR $N_{\zeta^{(k)}}(r)$ equals the number of up-crossings (or down-crossings) $L_{\zeta^{(k)}}(r)$ of $\zeta^{(k)}(t)$ through the level r observed within a time interval T, and then dividing $L_{\zeta^{(k)}}(r)$ by T, i.e., $N_{\zeta^{(k)}}(r) = L_{\zeta^{(k)}}(r)/T$. In the limit, as T tends to infinity, we obtain

$$\lim_{T\to\infty} N_{\zeta^{(k)}}(r) = \lim_{T\to\infty} \frac{L_{\zeta^{(k)}}(r)}{T} = N_\zeta(r) \tag{2}$$

for all $k = 1, 2, \ldots$ In practice, however, T is limited and we have to write $N_{\zeta^{(k)}}(r) \approx N_\zeta(r)$, where the approximation error is small if T is sufficiently large. This procedure is illustrated in Fig. 1.

Now, if the stochastic process $\zeta(t)$ is non-stationary, then the joint PDF $p_{\zeta\dot{\zeta}}(x,\dot{x};t)$ depends on time t. By regarding t as a parameter, it is straightforward to show that the LCR of a non-stationary process $\zeta(t)$ is also a function of time t and can be computed analytically by

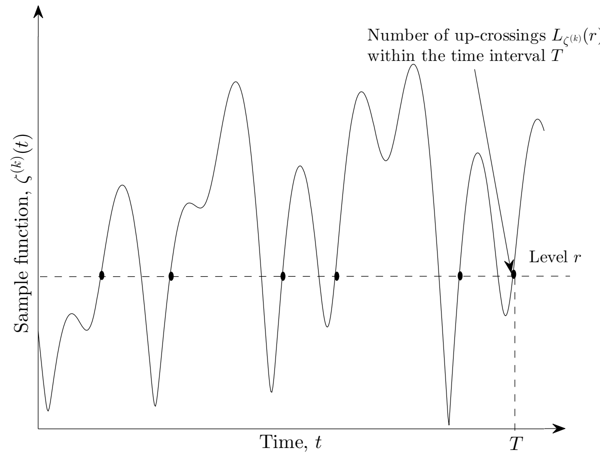

Fig. 1. Illustration of the conceptual procedure for the experimental computation of the LCR $N_\zeta(r)$ of ergodic processes $\zeta(t)$.

$$N_\zeta(r,t) = \int_0^\infty \dot{x}\, p_{\zeta\dot{\zeta}}(r,\dot{x};t)\, d\dot{x}\,. \tag{3}$$

In the following, we call $N_\zeta(r,t)$ the *instantaneous LCR*.

The experimental computation of the instantaneous LCR $N_\zeta(r,t)$ of non-stationary processes $\zeta(t)$ differs completely from the stationary case. The reason is that the concept of ergodicity cannot be invoked for non-stationary processes, which prevents computing the instantaneous LCR $N_\zeta(r,t)$ from the measurements (simulations) of a single sample function of $\zeta(t)$. However, $N_\zeta(r,t)$ can be computed from a large set of K sample functions $\zeta^{(1)}(t), \zeta^{(2)}(t), \ldots, \zeta^{(K)}(t)$. This can be shown by defining A_t as the elementary event that the non-stationary stochastic process $\zeta(t)$ crosses the signal level r within an infinitesimal interval $(t, t+\Delta t)$ of duration Δt either from down to up or from up to down. According to the relative frequency definition of probability, the probability of the event A_t, denoted as $P\{A_t\}$, equals the limit

$$P\{A_t\} = \lim_{K\to\infty} \frac{K_{A_t}}{K} \tag{4}$$

where K_{A_t} denotes the number of occurrences of A_t at time t, and K is the number of sample functions (trials). The instantaneous LCR $N_\zeta(r,t)$ can then be obtained as

$$N_\zeta(r,t) = \lim_{\Delta t\to 0} \frac{P\{A_t\}}{\Delta t} = \lim_{\substack{\Delta t\to 0\\ K\to\infty}} \frac{K_{A_t}}{\Delta t K}\,. \tag{5}$$

In a physical experiment, the time interval Δt is larger than zero and the number of sample functions K is finite. However, provided that Δt is sufficiently small and K is large, we may write $N_\zeta(r,t) \approx K_{A_t}/(\Delta t K)$. Fig. 2 illustrates the conceptual procedure for computing the instantaneous LCR $N_\zeta(r,t)$ of non-stationary processes.

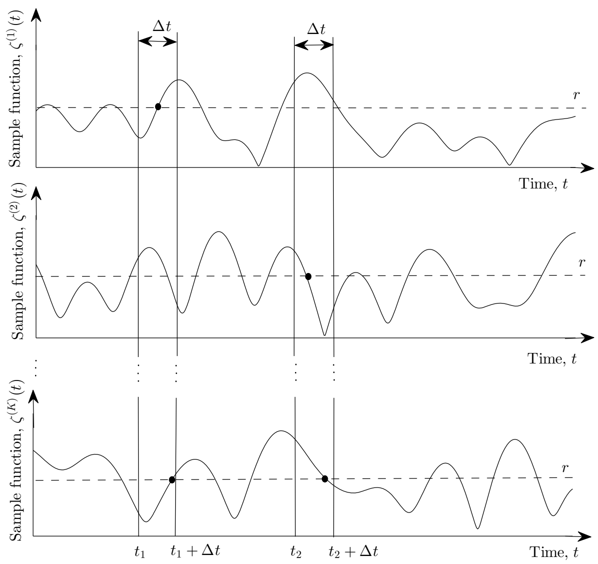

Fig. 2. Illustration of the conceptual procedure for the experimental computation of the instantaneous LCR $N_\zeta(r,t)$ of non-stationary processes $\zeta(t)$, where the bullets (•) denote elementary level-crossing events A_t.

In analogy to the stationary case, we mention (without proof) that the instantaneous ADF $T_\zeta(r,t)$ of $\zeta(t)$ can approximately be computed with a high degree of accuracy at low levels r as

$$T_\zeta(r,t) \approx \frac{F_\zeta(r,t)}{N_\zeta(r,t)} \tag{6}$$

where $F_\zeta(r,t)$ denotes the time-dependent CDF of $\zeta(t)$, which is defined as

$$F_\zeta(r,t) = P\{\zeta(t) \le r\} = \int_0^r p_\zeta(z;t)\,dz \tag{7}$$

with $p_\zeta(z;t)$ being the time-dependent PDF of $\zeta(t)$.

As an application, we derive in the following the instantaneous LCR of a non-stationary Rayleigh process. Let $\mu(t) = \mu_1(t) + j\mu_2(t)$ be a zero-mean non-wide-sense stationary complex Gaussian process. The underlying zero-mean real-valued Gaussian processes $\mu_1(t)$ and $\mu_2(t)$ are supposed to be uncorrelated and have identical time-variant variances $\sigma_0^2(t)$. The non-stationary Rayleigh process $\zeta(t)$ is defined by the absolute value of $\mu(t)$, i.e., $\zeta(t) = |\mu(t)|$. In the Appendix, it is shown that the instantaneous LCR $N_\zeta(r,t)$ of $\zeta(t)$ can be expressed in close form as

$$N_\zeta(r,t) = \sqrt{\frac{\beta(t)}{2\pi}}\, p_\zeta(r;t)\,. \tag{8}$$

where $p_\zeta(r;t)$ denotes the time-dependent Rayleigh distribution with parameter $\sigma_0^2(t)$. In (8), $\beta(t)$ represents the negative curvature of the time-dependent autocorrelation function (ACF)

$$R_{\mu_i\mu_i}(\tau,t) = E\left\{\mu_i\left(t+\frac{\tau}{2}\right)\mu_i\left(t-\frac{\tau}{2}\right)\right\} \tag{9}$$

at the origin $\tau = 0$, i.e.,

$$\beta(t) = -\frac{d^2}{d\tau^2} R_{\mu_i\mu_i}(\tau,t)\Big|_{\tau=0} = -\ddot{R}_{\mu_i\mu_i}(0,t) \tag{10}$$

for $i = 1,2$, where $E\{\cdot\}$ stands for the expectation operator.

III. Review of Non-Stationary Multipath Fading Channel Models

A non-stationary flat fading multipath channel model for the downlink (base station to MS link) has been proposed in [18]. There, it has been shown that the complex channel gain $\mu(t)$ of a flat fading channel can be modelled under linear speed variations as an SOC$_h$ process

$$\mu(t) = \sum_{n=1}^{N} c_n e^{\,j\left[2\pi\left(f_n t + \frac{k_n}{2}t^2\right)+\theta_n\right]}\,. \tag{11}$$

In (11), N denotes the number of multipath components, c_n is the path gain of the nth multipath component, f_n is the associated initial Doppler frequency, k_n describes the Doppler frequency change in Hertz per second, and θ_n is the path phase, which is modelled as a random variable with uniform distribution over 0 to 2π, i.e., $\theta_n \sim \mathcal{U}(0,2\pi]$. The path gains c_n in (11) can either be random variables with $E\{c_n^2\} = 2\sigma_0^2/N$ or constants $c_n = \sigma_0\sqrt{2/N}$, and f_n and k_n are given by [18]

$$f_n = f_{\max}\cos(\alpha_n) \tag{12}$$

$$k_n = f_{\max}\frac{a_0}{\mathrm{v}_0}\cos(\alpha_n) \tag{13}$$

where $f_{\max}$ denotes the maximum Doppler frequency at $t = 0$, v_0 represents the initial speed of the MS, α_n refers to the angle of arrival (AOA), and a_0 accounts for the acceleration ($a_0 > 0$) or deceleration ($a_0 < 0$) of the MS. The non-stationary channel model described by (11) captures the effect of a linear speed change of the MS according to

$$\mathrm{v}(t) = \mathrm{v}_0 + a_0 t\,. \tag{14}$$

With reference to (11), it can be observed that a linear speed change of the MS results in a frequency modulation of the received multipath components.

It should be mentioned that the non-stationary model described by (11) has recently been refined in [19] by taking additionally into account that the angle of motion $\alpha_\mathrm{v}(t)$ and the AOA $\alpha_n(t)$ might change with time t along the route of the MS. Under these conditions, it has been shown in [19] that the non-stationary channel model described by (11) remains valid in the sense of a first-order approximation, but the model parameters f_n

and k_n in (12) and (13), respectively, have to be replaced by more complicated expressions (see [19, Eqs. (13) and (14)]).

IV. Derivation of the Instantaneous LCR and ADF of a Sum-of-Chirps

In the previous section, we have seen that the complex channel gain $\mu(t)$ of a non-stationary flat fading multipath channel can be modelled by an SOC$_h$ process. In the following, we want to derive the instantaneous LCR and ADF of the envelope of this class of non-stationary processes. Therefore, we consider a propagation scenario with fixed scatterers $S_n\,(n = 1, 2, \ldots, N)$ at known positions (x_n, y_n) in the xy-plane. In this case, the parameters f_n [see (12)] and k_n [see (13)] are constant. In such a deterministic propagation environment, the path gains c_n are also constants, leaving only the phases θ_n as random variables, which are supposed to be independent and identically distributed (i.i.d.). For fixed values of time $t = t_0$, the SOC$_h$ process $\mu(t)$ in (11) reduces to a random variable $\mu(t_0)$ that can be analyzed like any other random variable. By invoking the central limit theorem [20, pp. 278], it is obvious that $\mu(t_0) = \mu_1(t_0)+j\mu_2(t_0)$ approaches a complex Gaussian random variable if N tends to infinity. For limited values of N, the distribution of $\mu_1(t_0)$ and $\mu_2(t_0)$ is close to a Gaussian distribution even if N is in the order of 10 provided that the gains c_n are equal or at least well balanced without any dominant multipath component.

The time derivative $\dot{\mu}(t)$ of the SOC$_h$ process $\mu(t)$ in (11) is obtained as

$$\dot{\mu}(t) = j2\pi \sum_{n=1}^{N} c_n f_n(t)\, e^{j\left[2\pi\left(f_n t + \frac{k_n}{2} t^2\right) + \theta_n\right]} \qquad (15)$$

where

$$f_n(t) = f_n + k_n t \qquad (16)$$

represents the time-variant Doppler frequency.

In the following, we analyse the time-dependent ACF and time-dependent cross-correlation function (CCF) of the SOC$_h$ process $\mu(t)$ and its time derivative $\dot{\mu}(t)$.

A. Time-Dependent ACF $\mathcal{R}_{\mu\mu}(\tau, t)$ of $\mu(t)$

The time-dependent ACF $\mathcal{R}_{\mu\mu}(\tau, t)$ of the complex SOC$_h$ process $\mu(t)$ is defined as

$$\mathcal{R}_{\mu\mu}(\tau, t) = E\left\{\mu\left(t + \frac{\tau}{2}\right)\mu^*\left(t - \frac{\tau}{2}\right)\right\} \qquad (17)$$

where the superscript asterisk $(\cdot)^*$ denotes the complex conjugate operator. Substituting (11) in (17) and averaging over the i.i.d. random phases $\theta_n \sim \mathcal{U}(0, 2\pi]$ results in

$$\mathcal{R}_{\mu\mu}(\tau, t) = \sum_{n=1}^{N} c_n^2\, e^{j2\pi f_n(t)\tau}\,. \qquad (18)$$

By using equal gains c_n, defined as $c_n = \sigma_0\sqrt{2/N}$, the variance of the SOC$_h$ process $\mu(t)$ can be obtained as

$$\mathcal{R}_{\mu\mu}(0, t) = \sum_{n=1}^{N} c_n^2 = \sum_{n=1}^{N} \sigma_0^2 \frac{2}{N} = 2\sigma_0^2\,. \qquad (19)$$

For the complex SOC$_h$ process $\mu(t) = \mu_1(t) + j\mu_2(t)$, the following relations hold:

$$\mathcal{R}_{\mu_1\mu_1}(\tau, t) = \mathcal{R}_{\mu_2\mu_2}(\tau, t) = \frac{1}{2}\mathrm{Re}\{\mathcal{R}_{\mu\mu}(\tau, t)\} = \sum_{n=1}^{N} \frac{c_n^2}{2}\cos(2\pi f_n(t)\tau) \qquad (20)$$

$$\mathcal{R}_{\mu_1\mu_2}(\tau, t) = \mathcal{R}_{\mu_2\mu_1}(-\tau, t) = \frac{1}{2}\mathrm{Im}\{\mathcal{R}_{\mu\mu}(\tau, t)\} = \sum_{n=1}^{N} \frac{c_n^2}{2}\sin(2\pi f_n(t)\tau)\,. \qquad (21)$$

The last equation states that $\mu_1(t_1)$ and $\mu_2(t_2)$ are in general correlated, but they are uncorrelated at the same point in time $t = t_1 = t_2$, because if $\tau = t_1 - t_2 = 0$, then we obtain $\mathcal{R}_{\mu_1\mu_2}(0, t) = 0$.

B. Time-Dependent ACF $\mathcal{R}_{\dot{\mu}\dot{\mu}}(\tau, t)$ of $\dot{\mu}(t)$

The time-dependent ACF $\mathcal{R}_{\dot{\mu}\dot{\mu}}(\tau, t)$ of $\dot{\mu}(t)$ is obtained by substituting (15) in $\mathcal{R}_{\dot{\mu}\dot{\mu}}(\tau, t) = E\{\dot{\mu}(t + \tau/2)\dot{\mu}^*(t - \tau/2)\}$ and computing the expected (mean) value w.r.t. the i.i.d. phases $\theta_n \sim \mathcal{U}(0, 2\pi]$. The final result equals

$$\mathcal{R}_{\dot{\mu}\dot{\mu}}(\tau, t) = (2\pi)^2 \sum_{n=1}^{N} (c_n f_n(t))^2\, e^{j2\pi f_n(t)\tau}\,. \qquad (22)$$

From the equation above and by using $\mathcal{R}_{\dot{\mu}_i\dot{\mu}_i}(\tau, t) = 0.5\,\mathrm{Re}\{\mathcal{R}_{\dot{\mu}\dot{\mu}}(\tau, t)\}$, we can compute $\beta(t)$ [see (A.2)], which is equal to the variance of $\dot{\mu}_i(t)$, i.e.,

$$\beta(t) = \mathrm{Var}\{\dot{\mu}_i(t)\} = \mathcal{R}_{\dot{\mu}_i\dot{\mu}_i}(0, t) = 2\pi^2 \sum_{n=1}^{N} (c_n f_n(t))^2\,. \qquad (23)$$

It is important here to note that the variance $\beta(t)$ of $\dot{\mu}_i(t)$ depends on time t. This stands in contrast to the variance σ_0^2 of $\mu_i(t)$, which is constant. For the correlation properties of $\dot{\mu}_1(t)$ and $\dot{\mu}_2(t)$, similar statements hold as pointed out in the previous subsection.

C. Time-Dependent CCF $R_{\mu\dot{\mu}}(\tau, t)$ of $\mu(t)$ and $\dot{\mu}(t)$

The time-dependent CCF $R_{\mu\dot{\mu}}(\tau, t)$ of $\mu(t)$ and $\dot{\mu}(t)$ is obtained by substituting (11) and (15) in $\mathcal{R}_{\mu\dot{\mu}}(\tau, t) = E\{\mu(t + \tau/2)\dot{\mu}^*(t - \tau/2)\}$ and computing the expected value w.r.t. the i.i.d. phases $\theta_n \sim \mathcal{U}(0, 2\pi]$. Thus,

$$\mathcal{R}_{\mu\dot{\mu}}(\tau, t) = -j2\pi \sum_{n=1}^{N} c_n^2\, f_n(t)\, e^{j2\pi f_n(t)\tau}\,. \qquad (24)$$

It can be shown that the time-dependent CCF $\mathcal{R}_{\mu\dot{\mu}}(\tau,t)$ at $\tau=0$ can be expressed in terms of the time-variant mean Doppler shift $B^{(1)}_{\mu\mu}(t)$ as

$$\mathcal{R}_{\mu\dot{\mu}}(0,t) = -j4\pi\sigma_0^2\, B^{(1)}_{\mu\mu}(t)\,. \tag{25}$$

where $B^{(1)}_{\mu\mu}(t)$ is given by

$$B^{(1)}_{\mu\mu}(t) = \frac{\sum\limits_{n=1}^{N} c_n^2\, f_n(t)}{\sum\limits_{n=1}^{N} c_n^2} = \frac{\sum\limits_{n=1}^{N} c_n^2\, f_n(t)}{2\sigma_0^2}\,. \tag{26}$$

As the time-variant mean Doppler shift $B^{(1)}_{\mu\mu}(t)$ is in general unequal to zero, it can be concluded from (25) that $\mu_1(t)$ $(\mu_2(t))$ and $\dot{\mu}_2(t)$ $(\dot{\mu}_1(t))$ are generally correlated at the same point in time, while $\mu_i(t)$ and $\dot{\mu}_i(t)$ are uncorrelated for $i=1,2$.

D. Instantaneous LCR $N_\zeta(r,t)$ and ADF $T_\zeta(r,t)$

For convenience and to simplify the mathematics, we neglect the cross-correlation between $\mu_1(t)$ $(\mu_2(t))$ and $\dot{\mu}_2(t)$ $(\dot{\mu}_1(t))$. In this case, the processes $\mu_1(t), \mu_2(t), \dot{\mu}_1(t)$, and $\dot{\mu}_2(t)$ can be considered as mutually uncorrelated. In the limit $N\to\infty$, they approach mutually independent non-wide-sense stationary Gaussian processes, which are described by the joint PDF $p_{\mu_1\mu_2\dot{\mu}_1\dot{\mu}_2}(x_1,x_2,\dot{x}_1,\dot{x}_2)$ as given in (A.3) if we replace there $\sigma_0^2(t)$ by σ_0^2 and notice that $\beta(t)$ is given by (23). If the number of multipath components N is limited (but sufficiently large), then the joint PDF $p_{\mu_1\mu_2\dot{\mu}_1\dot{\mu}_2}(x_1,x_2,\dot{x}_1,\dot{x}_2)$ in (A.3) and thus the instantaneous LCR $N_\zeta(r,t)$ in (8) are approximately valid. This motivates the closed-form approximations of the instantaneous LCR

$$N_\zeta(r,t) \approx \sqrt{\frac{\beta(t)}{2\pi}}\, p_\zeta(r) \tag{27}$$

and the instantaneous ADF

$$T_\zeta(r,t) \approx \frac{F_\zeta(r)}{N_\zeta(r,t)} \tag{28}$$

of the envelope of SOC$_h$ processes. In (27), the symbol $p_\zeta(r)$ denotes the Rayleigh distribution with parameter σ_0^2, and the characteristic quantity $\beta(t)$ is given by (23).

V. Numerical Results

To confirm the correctness of the derived solutions for the instantaneous LCR (ADF), we compare our main theoretical findings by experimental Monte Carlo simulations.

In our experimental study, we have modelled a non-stationary multipath channel by an SOC$_h$ process $\mu(t)$ [see (11)] consisting of $N=10$ components. The extended method of exact Doppler spread (EMEDS) [21] has been applied to compute the path gains c_n and AOAs α_n according to

$$c_n = \sigma_0\sqrt{\frac{2}{N}} \quad \text{and} \quad \alpha_n = \frac{2\pi}{N}\left(n-\frac{1}{4}\right) \tag{29}$$

where σ_0 was chosen to be unity. The phases θ_n have been computed by considering them as the outcomes (realizations) of a random generator with a uniform distribution over $(0,2\pi]$. Each sample function $\mu^{(k)}(t)$ of $\mu(t)$ is characterized by the same path gains c_n and AOAs α_n, but different phases $\theta_n^{(k)}$ for all $k=1,2,\ldots,K$. The number K of generated waveforms was equal to 10^3. The remaining parameters have been chosen as follows: $f_{\max}=16.4\,\text{Hz}$, $\text{v}_0=3\,\text{km/h}$, $a_0=1.5\,\text{m/s}^2$, and $\Delta t=0.01\,\text{s}$.

Fig. 3 presents the instantaneous LCR $N_\zeta(r,t)$ by using the approximation in (27). In addition, Fig. 3 shows the experimental results obtained for $N_\zeta(r,t)\approx K_{A_t}/(\Delta t K)$ by applying the conceptual procedure described in Section II. From the fact that the experimental results match the numerical predictions of the theory, we can gain confidence that our analytical results are correct.

Furthermore, the corresponding instantaneous ADF $T_\zeta(r,t)$, computed by means of the approximation in (6), is depicted in Fig. 4. The corresponding simulation results of the instantaneous ADF $T_\zeta(r,t)$ have been obtained by generating K sample functions $\zeta^{(1)}(t),\zeta^{(2)}(t),\ldots,\zeta^{(K)}(t)$, where $\zeta^{(k)}(t)=|\mu^{(k)}(t)|$, and then computing the average value of the time intervals during which the sample functions $\zeta^{(k)}(t)$, $k=1,2,\ldots,K$, remain below a given signal level r on condition that the down-crossings, marking the beginning of a fading interval, occurred between t and $t+\Delta t$. For sufficiently small values of Δt and a large number of sample functions K, the instantaneous ADF $T_\zeta(r,t)$ can be approximated as

$$T_\zeta(r,t) \approx \frac{1}{M}\sum_{m=1}^{M} \Lambda^{(m)}(t) \tag{30}$$

where $M\le K$ represents the number of sample functions $\zeta^{(1)}(t),\zeta^{(2)}(t),\ldots,\zeta^{(M)}(t)$ with down-crossings in $(t,t+\triangle t)$, and $\Lambda^{(m)}(t)$ stands for the time interval between the down-crossing detected in the interval $(t,t+\triangle t)$ and the next up-crossing of the sample function $\zeta^{(m)}(t)$. The results in Fig. 4 provide further confidence on the correctness of the theory. Finally, the excellent match between theoretical and simulation results allow us to conclude that the approximation in (6) is remarkably accurate over a wide range of signal levels up to $r=3$.

VI. Conclusion

In this paper, Rice's standard formula for the derivation of the LCR of wide-sense stationary processes has been extended to find the LCR of non-stationary

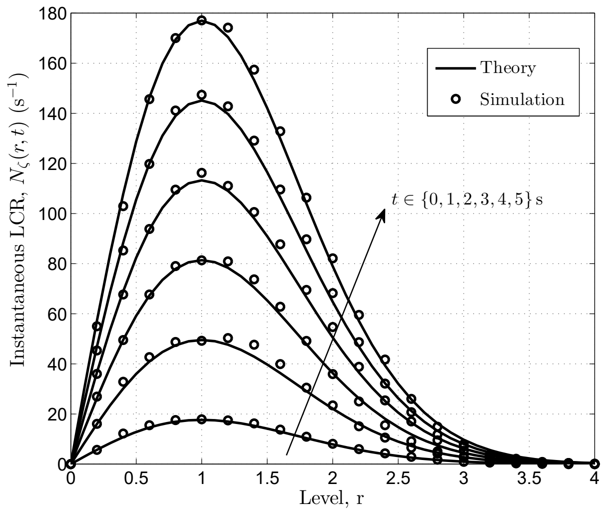

Fig. 3. Instantaneous LCR $N_\zeta(r,t)$ of a non-stationary multipath fading channel.

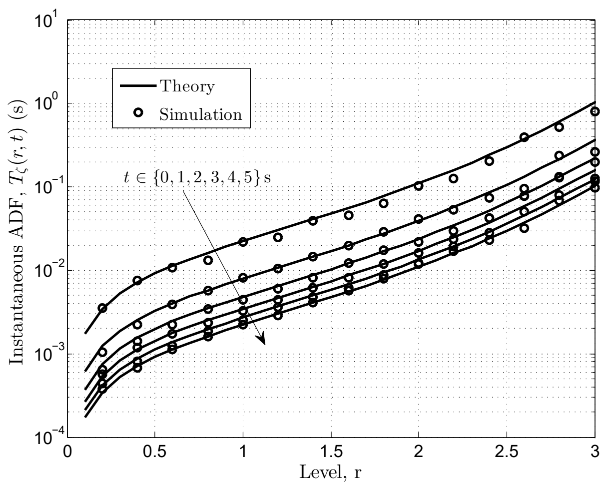

Fig. 4. Instantaneous ADF $T_\zeta(r,t)$ of a non-stationary multipath fading channel.

processes. The proposed concept has been applied to compute the LCR and ADF of non-stationary multipath flat fading channels. It turned out that the LCR (ADF) of non-stationary processes is a function of time, which motivated us to coin the term instantaneous LCR (ADF). It has been shown that the instantaneous LCR cannot be determined experimentally through simulations (or measurements) by determining the number of level down-crossings within a given time interval from a single sample function of the non-stationary process. Instead, the experimental computation of the instantaneous LCR (ADF) requires a large number of sample functions, which can easily be generated in computer simulations by using different realizations of the channel phases. However, in a real-world propagation environment, a large number of sample functions can only be obtained by employing huge multiple-input multiple-output (MIMO) antenna systems. In other words, massive MIMO techniques open new horizons for measuring the instantaneous LCR (ADF) of real-world channels in non-stationary propagation environments.

APPENDIX

A. Derivation of the Instantaneous LCR of Non-Stationary Rayleigh Processes

The starting point for deriving the instantaneous LCR $N_\zeta(r,t)$ of a non-stationary Rayleigh process $\zeta(t)$ is a complex non-wide-sense stationary Gaussian process $\mu(t) = \mu_1(t) + j\mu_2(t)$. Its time derivative will be denoted by $\dot{\mu}(t) = \dot{\mu}_1(t) + j\dot{\mu}_2(t)$. For the mean values and variances of $\mu_i(t)$ and $\dot{\mu}_i(t)$, we impose the following conditions:

$$E\{\mu_i(t)\} = 0 \qquad E\{\dot{\mu}_i(t)\} = 0 \tag{A.1}$$
$$\mathrm{Var}\{\mu_i(t)\} = \sigma_0^2(t) \qquad \mathrm{Var}\{\dot{\mu}_i(t)\} = \beta(t) \tag{A.2}$$

for $i = 1, 2$. Furthermore, we assume that the real-valued non-wide-sense stationary Gaussian processes $\mu_1(t), \mu_2(t), \dot{\mu}_1(t)$, and $\dot{\mu}_2(t)$ are mutually uncorrelated. Hence, the processes $\mu_1(t)$, $\mu_2(t)$, $\dot{\mu}_1(t)$, and $\dot{\mu}_2(t)$ are also mutually independent, implying that their joint PDF can be expressed as the product of the marginal densities of $\mu_1(t), \mu_2(t), \dot{\mu}_1(t)$, and $\dot{\mu}_2(t)$, i.e.,

$$\begin{aligned} p_{\mu_1\mu_2\dot{\mu}_1\dot{\mu}_2}&(x_1, x_2, \dot{x}_1, \dot{x}_2; t) \\ &= p_{\mu_1}(x_1;t)\, p_{\mu_2}(x_2;t)\, p_{\dot{\mu}_1}(\dot{x}_1;t)\, p_{\dot{\mu}_2}(\dot{x}_2;t) \\ &= \frac{e^{-\frac{x_1^2}{2\sigma_0^2(t)}}}{\sqrt{2\pi}\sigma_0(t)} \frac{e^{-\frac{x_2^2}{2\sigma_0^2(t)}}}{\sqrt{2\pi}\sigma_0(t)} \frac{e^{-\frac{\dot{x}_1^2}{2\beta(t)}}}{\sqrt{2\pi\beta(t)}} \frac{e^{-\frac{\dot{x}_2^2}{2\beta(t)}}}{\sqrt{2\pi\beta(t)}} \end{aligned} \tag{A.3}$$

for $|x_i| < \infty$ and $|\dot{x}_i| < \infty$ $(i = 1, 2)$. Transforming the Cartesian coordinates $(x_1, x_2, \dot{x}_1, \dot{x}_2)$ into polar coordinates $(z, \theta, \dot{z}, \dot{\theta})$ by means of

$$x_1 = z\cos\theta, \quad \dot{x}_1 = \dot{z}\cos\theta - z\dot{\theta}\sin\theta \tag{A.4}$$
$$x_2 = z\sin\theta, \quad \dot{x}_2 = \dot{z}\sin\theta + z\dot{\theta}\cos\theta \tag{A.5}$$

allows us to derive the time-dependent joint PDF $p_{\zeta\vartheta\dot{\zeta}\dot{\vartheta}}(z, \theta, \dot{z}, \dot{\theta}; t)$ of the envelope $\zeta(t)$, phase $\vartheta(t)$, and their time derivatives $\dot{\zeta}(t)$ and $\dot{\vartheta}(t)$ by using

$$\begin{aligned} p_{\zeta\vartheta\dot{\zeta}\dot{\vartheta}}(z, \theta, \dot{z}, \dot{\theta}; t) = &\frac{1}{|J|}\, p_{\mu_1\mu_2\dot{\mu}_1\dot{\mu}_2}\big(z\cos\theta, z\sin\theta, \dot{z}\cos\theta \\ &-z\dot{\theta}\sin\theta, \dot{z}\sin\theta + z\dot{\theta}\cos\theta; t\big) \end{aligned} \tag{A.6}$$

where J denotes the Jacobian determinant

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial z} & \frac{\partial x_1}{\partial \theta} & \frac{\partial x_1}{\partial \dot{z}} & \frac{\partial x_1}{\partial \dot{\theta}} \\ \frac{\partial x_2}{\partial z} & \frac{\partial x_2}{\partial \theta} & \frac{\partial x_2}{\partial \dot{z}} & \frac{\partial x_2}{\partial \dot{\theta}} \\ \frac{\partial \dot{x}_1}{\partial z} & \frac{\partial \dot{x}_1}{\partial \theta} & \frac{\partial \dot{x}_1}{\partial \dot{z}} & \frac{\partial \dot{x}_1}{\partial \dot{\theta}} \\ \frac{\partial \dot{x}_2}{\partial z} & \frac{\partial \dot{x}_2}{\partial \theta} & \frac{\partial \dot{x}_2}{\partial \dot{z}} & \frac{\partial \dot{x}_2}{\partial \dot{\theta}} \end{vmatrix} = -\frac{1}{z^2}\,. \qquad \text{(A.7)}$$

Substituting (A.3) and (A.7) in (A.6) results in

$$p_{\zeta\vartheta\dot{\zeta}\dot{\vartheta}}(z,\theta,\dot{z},\dot{\theta};t) = \frac{z^2}{(2\pi\sigma_0(t))^2\beta(t)}\, e^{-\frac{z^2}{2\sigma_0^2(t)}}\, e^{-\frac{\dot{z}^2+z^2\dot{\theta}^2}{2\beta(t)}} \qquad \text{(A.8)}$$

for $z \geq 0, |\theta| \leq \pi, |\dot{z}| < \infty$, and $|\dot{\theta}| < \infty$. Integrating $p_{\zeta\vartheta\dot{\zeta}\dot{\vartheta}}(z,\theta,\dot{z},\dot{\theta};t)$ over the variables θ and $\dot{\theta}$ gives the time-dependent joint PDF $p_{\zeta\dot{\zeta}}(z,\dot{z};t)$ of the envelope $\zeta(t)$ and its time derivative $\dot{\zeta}(t)$ in product form

$$p_{\zeta\dot{\zeta}}(z,\dot{z};t) = p_{\zeta}(z;t)\cdot p_{\dot{\zeta}}(\dot{z};t) \qquad \text{(A.9)}$$

where

$$p_{\zeta}(z;t) = \frac{z}{\sigma_0^2(t)}\, e^{-\frac{z^2}{2\sigma_0^2(t)}} \qquad \text{(A.10)}$$

$$p_{\dot{\zeta}}(\dot{z};t) = \frac{1}{\sqrt{2\pi\beta(t)}}\, e^{-\frac{\dot{z}^2}{2\beta(t)}}\,. \qquad \text{(A.11)}$$

From the product form in (A.9), it can be concluded that the non-stationary processes $\zeta(t)$ and $\dot{\zeta}(t)$ are statistically independent. Obviously, the envelope $\zeta(t)$ follows the Rayleigh distribution with time-variant parameter $\sigma_0^2(t)$; and its time derivative $\dot{\zeta}(t)$ is Gaussian distributed with zero mean and time-variant variance $\beta(t)$.

Finally, after substituting (A.9) in (3) and solving the integral by using [22, Eq. (3.458-3)], we find the following expression for the instantaneous LCR $N_{\zeta}(r,t)$ of non-stationary Rayleigh processes

$$N_{\zeta}(r,t) = \sqrt{\frac{\beta(t)}{2\pi}}\, p_{\zeta}(r;t)\,. \qquad \text{(A.12)}$$

In the case that the underlying zero-mean Gaussian processes $\mu_i(t)$ and $\dot{\mu}_i(t)$ have constant variances $\sigma_0^2(t) = \sigma_0^2$ and $\beta(t) = \beta$, respectively, then the result in (A.12) reduces to the well-known formula for the LCR of wide-sense stationary Rayleigh processes that can be found, e.g., in [1, Eq. (1.3-35)].

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