

Corporate Hedging under a Resource Rent Tax Regime

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Abstract

In addition to the ordinary corporate income tax, special purpose taxes are sometimes levied to extract abnormal profits arising from the use of natural resources. Such dual tax regimes exist in Norway for oil and hydropower, where the corresponding special purpose tax bases are unaffected by any derivatives payments. Dual tax firms with hedging programs therefore face the risk of potentially large discrepancies between the tax bases for corporate income tax and special purpose taxes. I investigate how this tax base asymmetry influences the extent of hedging of value-maximizing firms facing hedgeable as well as unhedgeable risk. Dual tax firms facing deadweight costs in low-profit events generally demand less hedging than ordinary firms, but otherwise respond similarly to characteristics of the underlying risk exposures. The special purpose tax does not influence firms' hedge portfolios in the absence of deadweight cost.

Keywords: Resource tax, special purpose tax, hedging, nonfinancial firms, rent taxation, tax base asymmetry, dual taxes, neutral tax

1. Introduction

Companies involved in upstream activities on the Norwegian continental shelf are liable for a special purpose petroleum tax at a rate of 50% on top of corporate income tax at a rate of 28%. Because crude oil sales are taxed at norm prices set by an official board instead of actual prices, derivative payoffs are excluded from the special purpose tax base. Norwegian hydroelectricity producers similarly face tax provisions that preclude derivative payoffs from the tax base of a special purpose hydropower tax at a rate of 30%. These tax provisions form a wedge between the tax bases for corporate income and special purpose taxes for firms with hedging programs involving derivatives. How does this tax base asymmetry influence the extent of hedging demanded by firms? Alternatively, are these special purpose taxes, allegedly designed to extract a portion of abnormal profits arising from the use of natural resources like oil and waterfalls, neutral in terms of the hedge ratios chosen by firms? While the conditions under which such taxes are neutral in terms of firms' investment decisions have been extensively studied¹, few have studied the potential impact of dual taxes on firms' risk management strategies. With cross-border market integration, firms that operate in essentially the same market sometimes face special purpose taxes evaded by their competitors across the border. Norwegian and Swedish hydroelectricity producers constitute one prominent example: the former set of firms is subject to special purpose taxation, while the latter is liable to tax on corporate income at a rate of 26.3% (28% before 2009).² Both sets of firms operate at Nord Pool, the Nordic electricity exchange, facing basically the same prices except for occasional divergence between price areas (Marckhoff & Wimschulte, 2009). My research identifies

¹ See Lund (vedlegg 1) and Hagen and Åvitsland (vedlegg 2) in NOU 2000:18 for an extensive discussion of neutral resource rent extraction (in Norwegian). Interested readers may also consult the references therein, e.g., Bulow & Summers (1984) and Fane (1987). More recent discussions are provided by Lund (2009) and Boadway and Keen (2009). Updated information about the Norwegian hydro power tax regime may be found in Sections 9 and 21 of Ot.prp. nr. 1, versions 2007-2008 and 2009, respectively (also in Norwegian). For a brief description in English, see "Tax Facts Norway 2009, A Survey of the Norwegian Tax System" by KPMG LAW ADVOKATFIRMA DA. Information about the petroleum tax regime in Norway may be found in the publication "Facts – The Norwegian Petroleum Sector" by the Norwegian Ministry of Petroleum and Energy (downloadable at www.petrofacts.no).

² See PWC Worldwide tax summaries or the publication "Taxes in Sweden - An English Summary of Tax Statistical Yearbook of Sweden" published by the Swedish Tax Agency (www.skatteverket.se).

under what conditions and to what extent special purpose taxes can be expected to influence the hedging strategies of firms operating in incomplete markets.

The influence of tax base asymmetry on the extent of hedging has not been extensively analyzed in the literature, but Lien (2004) attributes the low trading volumes of U.S. corn yield (quantity) futures in the late 1990s to the inability of these contracts to offset tax gains or losses from the spot positions. Lien arrives at this conclusion by analyzing a utility-maximizing constant-returns-to-scale firm that faces uncertain quantity and price to be realized the next period ($t = 1$). To reduce the risk, the firm may trade both price and quantity forward contracts at $t = 0$. In terms of taxation, any loss from trading in the price futures contract can offset production profits; this is not the case for quantity futures. A no-hedging result follows for the tax-disadvantaged quantity futures contract under the restrictive assumptions that the value of any negative tax income is zero and that a zero correlation exists between price and quantity innovations. Lien (2004) concludes that "quantity futures contracts do not provide any hedging function" (p. 32), so the failure of these contracts should come as no surprise.

Although firms can sometimes trade their quantity exposure, this is generally not the case for energy companies. Hydroelectricity producers face unpredictable variations in yearly inflows that cannot be hedged, at least not at reasonable terms. Oil producers also face unpredictable variations in output. The research question now posed therefore differs from that of Lien (2004); quantity risk is presumed unhedgeable, while price risk may be transferred in organized derivatives markets. How does tax base asymmetry influence how firms manage their price risk exposure under these circumstances? In order to disentangle this influence under less restrictive tax assumptions than those employed by Lien, the possibility that the absolute value of the tax on an arbitrary positive profit may be larger than the absolute value of the tax savings arising from a loss of similar magnitude is considered (Altshuler &

Auerbach, 1990; Eldor & Zilcha, 2002). Only in a few countries are firms actually compensated for negative tax incomes (tax loss carryback), and the ability to carry losses forward is time-constrained in most countries. Besides, interest is usually foregone when only tax loss carryforward applies. This type of tax asymmetry is characterized by nondecreasing marginal tax rates or convex tax functions. My analysis shows that firms subject to tax base asymmetry are expected to hedge less than firms facing corporate taxes only under certain conditions, but the reduction in hedging demand is far from the 'no hedge' result of Lien. However, Lien analyzes a tax disadvantaged derivative contract with quantity-dependent payoff; I address the influence of a similar tax disadvantage on a hedge portfolio with price-dependent payoffs under less restrictive tax and correlation assumptions. Quantity risk is presumed unhedgeable in this setting.³

The remainder of the paper is organized as follows. Section 2 outlines a general model of firms' hedging activities with dual taxes and, accordingly, tax base asymmetry. The conditions under which special purpose taxes distort firms' hedging choices are outlined, namely when firms face deadweight costs, that is, direct and indirect costs of financial distress or costly external financing in low-profit events. Section 3 analyzes how tax base asymmetry influences firms' hedging choices under such conditions when the representative firm adheres to linear hedging instruments. An analytical result applicable to firms facing tax base asymmetry and linear tax functions is presented together with an extensive numerical analysis addressing hedge portfolio distortions under more general tax exposure. Section 4 concludes the paper by arguing that the findings are expected to extend beyond the two Norwegian special purpose tax regimes referred to above. After all, these are nothing but variants of tax

³ The problems addressed are related to previous research on the effect of price uncertainty on the operation of competitive firms (Broll, Kit Pong, & Zilcha, 1999; Domar & Musgrave, 1944; Moschini & Lapan, 1992; Sandmo, 1971; Stiglitz, 1969). Zilcha and Eldor (2004) integrate different strands of this literature and demonstrate that the optimal production of a utility-maximizing producer is unaffected by convex tax exposures in the presence of markets for forward contracts. Nevertheless, convex tax exposures will affect the optimal forward sales of a utility-maximizing producer. This paper takes a different starting point by focusing on how tax asymmetries in general influence the hedging policies of value-maximizing firms facing stochastic production and price and, possibly, deadweight costs in low after-tax profit events.

regimes for rent extraction found in many other countries (see Table 3 in Baunsgaard, (2001)).

2. A general model of hedging under a dual tax regime

Special purpose taxes may be analyzed in the economic setting of Brown and Toft (2002), that is, in terms of short-run hedging strategies conditional on fixed capital structure, dividend policy, production technology, investments, etc. Firms face two sources of uncertainty at $t = 0$ in this economic environment. First, the revenue from selling one unit of production is uncertain because market clearing prices are under the influence of exogenous processes, such as weather conditions or business cycles. Second, firms' production may vary as a result of unpredictable demand variations or stochastic production technology. These risk exposures may be partially hedged at $t = 0$ by entering into a set of derivatives contracts paying $\mathbf{a}'\mathbf{h}_g(p)$ at $t = 1$, where p is the realized spot price, \mathbf{h}_g a vector of real valued functions representing gross contract payments for long positions in the different contracts, and \mathbf{a} a vector representing the number of long contracts for different derivatives. Under the assumptions of no arbitrage and zero risk-free interest rates, any derivative contract settled at $t = 1$ must satisfy the condition

$$\int_P h(p)g(p)dp = 0 \quad (1)$$

where h represents net contract payoff and g is the risk neutral marginal density of P . Thus, any derivative contract must have zero risk neutral expectation after the price has been deducted from the gross contract payoff to avoid potential arbitrage profits inconsistent with economic equilibrium. Firms are liable for corporate taxes on net profits and possibly also a special purpose tax on the net spot value of production. The net profit subject to corporate taxation is defined by subtracting variable and fixed costs from sales revenue and adding the net payoffs from derivatives contracts. Distinct from the corporate tax, the special purpose tax

is unaffected by derivatives transactions. Subtracting taxes from net profits defines net profit after taxes, the determinant of deadweight costs for all firms. Net economic profit at $t = 1$ is given by net profit after taxes minus deadweight costs.

The economic environment described above can be represented by a recursive equation system governed by the two random variables (RVs) P and Q . Net profit (NP) and special purpose income (SPI), the bases of corporate (CT) and special purpose taxes (SPT), are both given by the realizations p and q of the RVs P and Q at $t = 1$. However, the two tax incomes differ in terms of treatment of derivatives payoff and, possibly, fixed costs. While net profit is the income from the spot market minus variable and fixed costs plus the net derivatives payoff $\mathbf{a}'\mathbf{h}(p)$, special purpose income ignores derivatives payments. Furthermore, the tax code for special purpose income may limit the amount of interest deductions and prescribe different rules for depreciation of assets. These differences are represented by the two functions $u: \mathbb{R}^2 \rightarrow \mathbb{R}$ and $v: \mathbb{R}^2 \rightarrow \mathbb{R}$. Both taxes are assumed to be increasing functions of their respective tax incomes. Deadweight costs (DWC) incurred at year end in low after-tax profit events are given as a function of net profit after taxes ($NPAT$). Finally, the economic profit (Π) of a firm is defined as net profit after taxes minus deadweight costs.⁴

$$NP = u(p, q) + \mathbf{a}'\mathbf{h}(p) \quad (2)$$

$$CT(NP) \equiv \int_0^{NP} CT'(s)ds, \quad 0 \leq CT' < 1 \quad (3)$$

$$SPI = v(p, q) \quad (4)$$

⁴ Altschuler and Auerbach (1990) make a distinction between the current and the effective marginal tax rates. The former is defined as the derivative of current taxes with respect to current income and the latter as the current marginal tax rate adjusted for the influence on future and previous taxes. These authors argue that the two potential corrections needed to derive a firm's effective marginal tax rate are (1) the reduction in future taxes due to increased carryback potential and (2) the increase in future taxes due to a reduction in unused tax shields. In the non-dynamic setup of Brown and Toft, the derivatives of the tax functions $CT(NP)$ and $SPT(SPI)$ are best interpreted as effective marginal tax rates.

$$SPT(SPI) \equiv \int_0^{SPI} SPT'(s)ds, \quad 0 \leq SPT' < 1 \quad (5)$$

$$0 \leq CT' + NRI' < 1 \quad (6)$$

$$NPAT = NP - CT - SPT \quad (7)$$

$$DWC = DWC(NPAT) \quad DWC' \leq 0 \quad (8)$$

$$\Pi = NPAT - DWC \quad (9)$$

The general model is defined by equations (1)-(9), a behavioral assumption, two exogenously specified marginal tax rate functions, a technical restriction and two RVs, P and Q , in a risk neutral probability space $(\Omega, \mathfrak{F}, P_m)$. The behavioral assumption states that firms maximize the expected value of net economic profit at $t = 1$, that is, for a given set of tradable contracts \mathbf{h} , firms solve the maximization problem:

$$\max_{\mathbf{a} \in S} E[\Pi(\mathbf{a})] = \max_{\mathbf{a} \in S} \int_P \int_Q \{ NP(p, q; \mathbf{a}) - CT(NP(p, q; \mathbf{a})) - SPT(SPI(p, q)) - DWC(NPAT(p, q; \mathbf{a})) \} f(p, q) dq dp \quad (10)$$

under a risk neutral measure given by the density f . Each choice of \mathbf{a} generally entails a different marginal density for the RV Π , so the maximization problem is equivalent to choosing from among alternative RVs Π solely on the basis of expectation. Under the technical restriction that $E[\Pi(\mathbf{a})]$ is concave in an open set of contract numbers S , value-maximization is equivalent to solving the first order conditions (one for each derivative contract):

$$\frac{\partial E[\Pi(\mathbf{a})]}{\partial \mathbf{a}} = \int_P \int_Q \left\{ \frac{\partial NP(p, q; \mathbf{a})}{\partial \mathbf{a}} (1 - CT'(NP(p, q; \mathbf{a}))) (1 - DWC'(NPAT(p, q; \mathbf{a}))) \right\} f(p, q) dq dp = \mathbf{0} \quad (11)$$

Thus, value-maximizing firms choose a vector \mathbf{a} that equates the expected marginal change in net profit after taxes with the expected marginal change in deadweight costs for all

derivatives. In the special case of constant marginal corporate tax rates, this condition simplifies to

$$\frac{\partial E[\Pi(\mathbf{a})]}{\partial \mathbf{a}} = \int_P \int_Q \left\{ \frac{\partial NP(p, q; \mathbf{a})}{\partial \mathbf{a}} DWC'(NPAT(p, q; \mathbf{a})) \right\} f(p, q) dq dp = \mathbf{0} \quad (12)$$

This is true as expected net profit after taxes is unaffected by the choice of hedge portfolio for a linear tax function.

The model above generalizes Brown and Toft's (2002) model to an economy where firms could face dual taxes and, accordingly, tax base asymmetry. In this setup, hedge portfolio gains and losses are taxed only once, while every other item is liable to corporate and special purpose taxes. As the period of interest is one calendar year (in conformity with the tax code), the cost and capital structure of the firm are considered predetermined. Predictions are therefore best understood as optimal, short-run hedging strategies conditional on some factors that are presumed to be fixed during a calendar year (capital structure, dividend policy, production technology, etc.). Thus, the analysis effectively controls for the potential influence of the debt capacity hedging incentive (Stulz, 1996). Even on this general level, two important results follow:

Proposition 1: *Consider firms that do not face deadweight costs in low after-tax profit events, i.e., $DWC'(npat) = 0 \forall npat$. Under these conditions, firms subject to special purpose tax liabilities will choose the same hedge portfolio as otherwise identical firms facing corporate taxes only. Because these firms cannot change their expected special purpose tax liabilities by entering into derivatives contracts, their only motivation for hedging would be to reduce expected corporate tax liabilities facing convex corporate tax exposures (Smith & Stulz, 1985). This motivation is the same for otherwise identical firms; therefore, the special purpose tax is neutral in terms of firms' hedging choices. **Proof.** Follows directly from (11) and the concavity assumption. Note that this result is not conditional on constant-returns-to-scale.*

Proposition 2: *Consider firms that face deadweight costs in low after-tax profit events. In this case, firms facing dual taxes may choose different hedge portfolios than otherwise identical firms liable for corporate taxes only. The special purpose tax is not neutral in terms of firms'*

hedging choices under these circumstances. **Proof.** Follows directly from (11) and the fact that the choice of hedge portfolio (the vector \mathbf{a}) influences after-tax profits and consequently, deadweight costs.

The current specification of deadweight costs makes sense when these costs are primarily driven by firm value (Hahnenstein & Röder, 2007; Merton, 1973, 1974). However, if costly cash shortfall is the key constituent of deadweight costs in the spirit of Froot and Stein (1993), a more cash-flow oriented approach is appropriate. Such an economic environment may be modeled by adding one restriction to the general model, that is, by calculating after-tax profits in equation (8) using zero marginal tax rates for all negative profits. In that case, any future tax benefits associated with negative profits are irrelevant for deadweight costs because these benefits do not mitigate concurrent costs of external financing. In conclusion, special purpose taxation will only distort the hedging choices of firms that face prospects of deadweight costs in low after-tax profit events. The significance of these adjustments is analyzed in the next section.

3. Forward contract demand with special purpose taxes and deadweight costs

The following analysis is confined to hedging with forward contracts, consistent with the tradition of the literature preceding Eldor and Zilcha (2004). Just as forward and futures contracts (swaps) have been the main vehicles for risk transferral in the Nordic electricity market since the market's inception in 1995, there is ample evidence in the literature documenting the widespread use of linear hedging strategies. In a study of U.S. nonfinancial firms, Gay, Nam, and Turac (2002) report that 69% of commodity risk exposures, 75% of currency exposures, and 70% of interest exposures are managed with linear derivatives. Huang, Ryan, and Wiggins (2007) find that 73% of firms that use derivatives manage interest and currency risk exposures entirely in terms of linear hedging instruments. Benson and

Oliver (2004) and Bodnar and Gebhardt (1999) report qualitatively similar results. From a theoretical point of view, Brown and Toft's (2002) perfect exotic hedge, a quadratic function of price, features the same sensitivity as the optimal forward hedge at the expected price (pp. 1294-1295). Thus, linear hedging instruments play an important part in firms' hedging programs, in theory as well as in practice. However, as shown by Brown and Toft (2002) and by Gay, Nam, and Turac (2003), the significance of nonlinear payoffs increases in importance when quantity risk increases. We must therefore keep in mind that some information *is* lost for firms exposed to high levels of unhedgeable relative to hedgeable risk when addressing linear hedging strategies (first-order effects) only.

3.1 Assumptions

To study the hedging behavior of a value-maximizing nonfinancial price taker using the general model of Section 2, the functions u , v , DWC , and the risk neutral probability measure must be specified. Following Brown and Toft (2002) closely, I chose the bivariate normal risk neutral probability density f with the parameters $\mu_P = \mu_Q = 1$. Thus, expected revenues are normalized to approximately one, depending on the assumed price-quantity correlation coefficient ρ and the volatilities σ_P and σ_Q . The cost structure of the representative firm is composed of a variable and a fixed part, both accepted by the corporate tax code, amounting to cq and C . Revenue is simply the product of the realized quantity and the realized price, so $u(p,q) = pq - cq - C$. $NRI = v(p,q) = pq - cq - C_{SPT}$, where C_{SPT} is set equal to C for convenience. The statutory special purpose tax rate is assumed to be 30%, while the corporate statutory tax rate is set to 28%. Like Brown and Toft, I assume exponentially decreasing deadweight costs, $DWC(NPAT) = c_1 e^{-c_2 NPAT}$ for $c_1, c_2 > 0$, where c_1 is a scale or location parameter controlling the horizontal alignment of DWC , while c_2 controls slope and curvature. When deadweight costs are not set to zero, the base case assumptions $c_1 = 0.0075$ and $c_2 = 9$

are used. c_2 is set high—BT employed $c_2 \in \{2,5,8\}$ —in order to really test the potential for hedge portfolio distortions resulting from tax base asymmetry.

To account for convex tax functions, marginal tax rates are represented by the following functions:⁵

$$CT'(NP) \equiv \frac{t_{CT}}{1 + \theta(e^{-\beta_{CT} NP} - 1)} \quad 0 \leq \theta, \beta_{CT} \quad (13)$$

$$SPT'(SPI) \equiv \frac{t_{SPT}}{1 + \theta(e^{-\beta_{SPT} SPI} - 1)} \quad 0 \leq \theta, \beta_{SPT} \quad (14)$$

For sufficiently small θ , these marginal tax rate functions can approximate statutory tax rates applicable to positive tax incomes and allow for non-decreasing marginal tax rates for negative tax incomes. This case study sets θ to 0.001, in which case the approximation errors of the marginal tax rates applicable to positive tax incomes are negligible. The parameters β_{CT} and β_{SPT} govern the rate of decline in marginal tax rates as income becomes increasingly negative and the marginal tax rates eventually become zero. Higher β s mean higher tax function convexity or increased potential hedging gains in the form of lower expected taxes. The partial influence of varying β on a firm's tax exposure is illustrated in Figure 1.

⁵ The marginal tax rate function (13) implies a corporate tax function on the form

$$CT(NP) = \frac{t_{CT}}{\beta_{CT}(1-\theta)} \ln \left[\theta - e^{\beta_{CT} NP} (\theta - 1) \right], \beta \neq 0, \theta \neq 1 \dots \text{The case of special purpose tax follows analogously.}$$

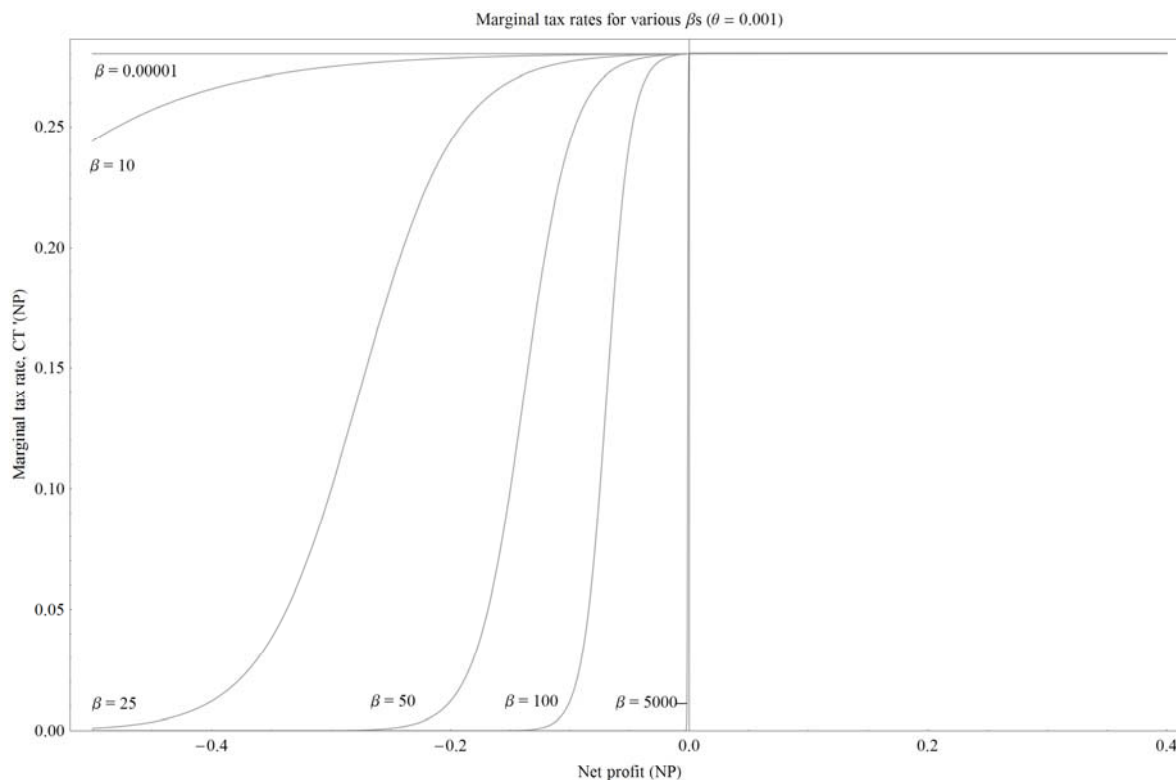


Figure 1. Marginal tax rates for various β s representing different degrees of tax function convexity. The limiting case $\beta \rightarrow \infty$ is the tax function employed in Lien's (2004) analysis of tax base asymmetry. The other extreme ($\beta \rightarrow 0$) represents the special case of linear tax functions, i.e., when the absolute value of the tax on an arbitrary positive profit equals the absolute value of the tax savings arising from a loss of a similar magnitude.

3.2 Forward contract demand: the special case of linear tax functions

The number of contracts, or, equivalently when assuming $\mu_Q = 1$, the hedge ratios of value-maximizing firms that account for deadweight costs under conditions with constant marginal tax rates, are given as⁶

$$a^* = a_{\min \sigma_{NPAT}^2} + a_{DWC}^{CORR} \quad (15)$$

where

⁶ Proof is provided in appendices A and B. a^* is consistent with the optimal hedge ratio derived by Brown and Toft for the case of linear tax functions and no special purpose taxes. This is easily seen by reformulating deadweight costs in terms of net profit before taxes. By replacing c_2 with $c_3 = c_2(1-t_{CT})$, we find that $c_1 e^{-c_2 NPAT} = c_1 e^{-c_2 NP(1-t_{CT})} = c_1 e^{-c_3 NP}$. Nevertheless, the result on the minimum variance hedge differs slightly from Brown and Toft's equation (14) in that the contribution margin ($\mu_p - c$) affects the minimum variance hedge ratio. Clearly, variance minimization is neither a necessary nor a sufficient strategy for value maximization (a related result is derived by Hahnenstein and Röder (2003)). Note that these results hold for arbitrarily small c_2 and a positive c_1 ; any hedge ratio is optimal (not only the minimum variance solution) for $c_2 = 0$, including the no-hedge solution $a^* = 0$. Also note that under linear tax exposure, the minimum variance solution applies both before and after taxes, that is, $a_{\min \sigma_{NPAT}^2} = a_{\min \sigma_{NP}^2}$.

$$a_{\min \sigma_{NPAT}^2} = \frac{1-t_{CT}-t_{SPT}}{1-t_{CT}} (-\mu_Q - (\mu_P - c) \rho \frac{\sigma_Q}{\sigma_P}) \quad (16)$$

$$a_{DWC}^{CORR} = \frac{(1-t_{CT}-t_{SPT})^2}{1-t_{CT}} (\mu_P - c)(1-\rho^2) c_2 \sigma_Q^2 \quad (17)$$

In general, value-maximizing firms that face deadweight costs deviate from the minimum variance hedge ratio by a_{DWC}^{CORR} . Some interesting implications follow from (15)-(17):

Corollary 1: Consider two otherwise identical value-maximizing firms A and B, differing only in that firm A is liable to pay a special purpose tax not faced by firm B. The ratio of firm A's to firm B's minimum variance forward hedge ratio equals $(1-t_{CT}-t_{SPT})/(1-t_{CT})$, while the ratio of the two firms' deadweight cost correction terms equals $((1-t_{CT}-t_{SPT})/(1-t_{CT}))^2$. Thus, special purpose taxes will always reduce the minimum variance hedge ratio compared to otherwise identical firms liable to corporate taxes only. On the other hand, the deadweight cost correction term, which is positive for firms expecting positive contribution margins, will always be smaller for firms facing special purpose taxes. Consequently, the possibility that $|a_{CT}^*| < |a_{CT+SPT}^*|$ cannot be ruled out, but in most economically interesting cases $|a_{CT}^*| > |a_{CT+SPT}^*|$.

Corollary 2: Firms that face dual taxes generally choose hedge ratios closer to the minimum variance forward hedge than firms facing corporate taxes only, provided that $\mu_P > c$.

Corollary 3: Ruling out some rare cases when the minimum hedge ratios are close to zero, the smaller the speed with which deadweight costs increase as profits drop (smaller c_2), the closer is the relative hedge ratio a_A^*/a_B^* to the minimum variance relative hedge ratio $(1-t_{CT}-t_{SPT})/(1-t_{CT})$ when $\mu_P > c$. The same argument applies to a smaller σ_Q and a $|\rho|$ closer to one.

The main result is illustrated in Figure 2. While it is possible to consider cases where $|a_{CT}^*| < |a_{CT+SPT}^*|$, the vast majority of economically interesting risk exposures entail less hedging with special purpose taxes. This finding applies to the Nordic electricity market, where Norwegian and Swedish hydroelectricity producers are subject to different tax regimes while operating in essentially the same market. Norwegian and Swedish hydroelectricity producers

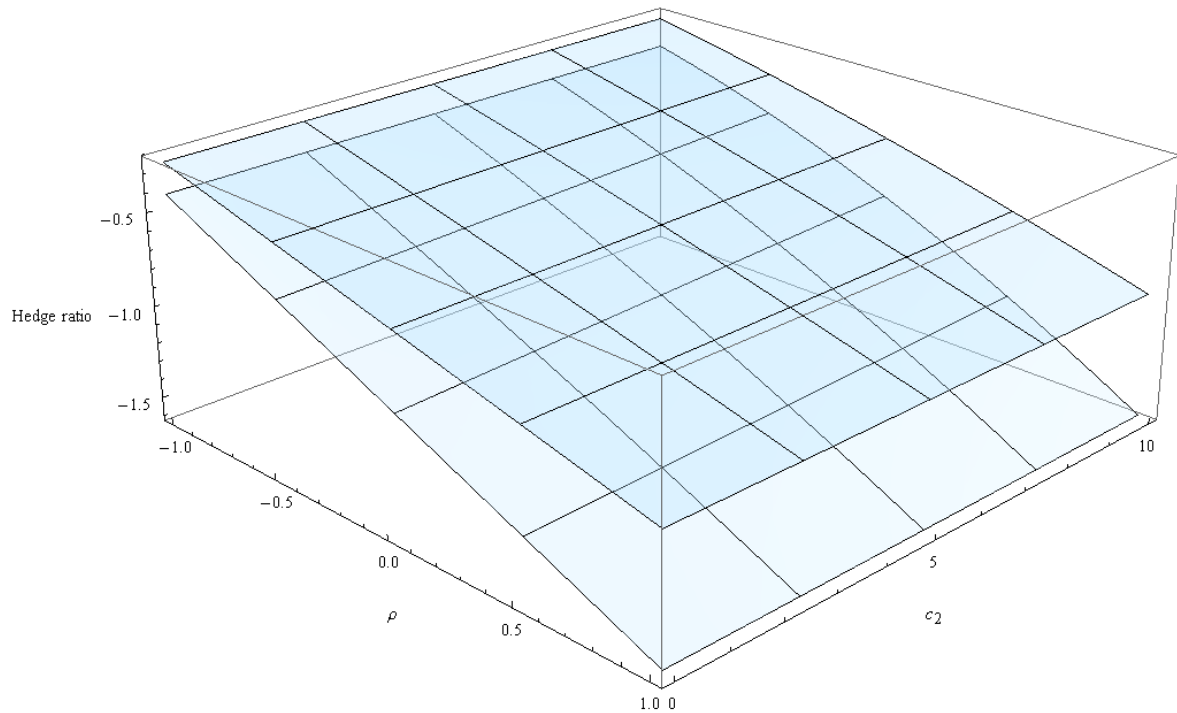


Figure 2. The impact of tax income basis asymmetry on forward hedge ratios under conditions of linear tax functions across various price-quantity correlation (ρ) and deadweight cost (c_2) assumptions for $\sigma_P = 0.2$, $\sigma_Q = 0.15$, $\mu_P = \mu_Q = 1$, and $c = 0.2$. The upper surface, which represents the case with tax base asymmetry, generally prescribes less hedging relative to the case with corporate taxes only (lower surface), cf. Corollary 1.

are subject to similar corporate tax rates (28% in both countries until 2009, 26.3% in Sweden beginning in 2009), and Norwegian producers are currently liable to special purpose taxes at a rate of 30% in addition to corporate taxes. The analysis predicts that Norwegian hydroelectricity producers would generally hedge less than their Swedish counterparts under conditions with linear tax exposures, everything else being equal.

Few firms face linear corporate tax exposure (Eldor & Zilcha, 2002). However, because analytical solutions are available for linear taxes only, this special case proves crucial for the numerical procedures used to identify how firms facing special purpose taxes generally respond to increased corporate tax function convexity. Furthermore, the Norwegian special purpose tax regime was changed in 2008 in a way that essentially created linear exposure, at least if one considers the Norwegian government as a credible counterpart (no political risk).

The new tax provisions promise a tax refund in case of an unused negative accumulated special purpose tax income in the event of corporate restructuring events like mergers, acquisitions, business foreclosures, etc. Negative tax balances may be carried forward with the risk-free rate of interest to reflect the "sure thing" nature of the negative tax balance (Ot.prp. nr. 1, 2007-2008, Norwegian Ministry of Finance). In this case, the special purpose tax exposure is linear, or at least very close to linear.

3.3 The influence of corporate tax function convexity on relative forward contract demand

The hedging implications of dual tax exposure under different corporate tax function convexity assumptions are now analyzed by solving N first order conditions of value-maximizing firms' optimization problem, with and without special purpose taxes.

Specifically, I first solve N root conditions for N values of β_{CT}^i for ordinary firms and firms

$$\frac{\partial E[\Pi(a; \beta_i)]}{\partial a} = \int_P \int_Q \left\{ (p - \mu_p) \left(1 - \frac{t_{CT}}{1 + \theta \left(e^{-\beta_{CT}^i NP(p, q; a)} - 1 \right)} \right) \right. \\ \left. (1 + c_1 c_2 e^{-c_2 NPAT(p, q; a, \beta_{CT}^i, \beta_{SPT}^j)}) \right\} f(p, q) dq dp = 0 \quad i = 1, \dots, N, j = 1, 2 \quad (18)$$

liable to dual taxes when $\beta_{CT}^{i,1} = \beta_{CT}^i \forall i$ and $\beta_{SPT}^{i,1} = 0.00001 \forall i$ ($j = 1$). In this case, the special purpose tax exposure is approximately linear, as in the Norwegian hydropower tax regime, and value is the determinant of deadweight costs. Next, I analyze the case when cash shortfall or costly external financing is the determinant of deadweight costs ($j = 2$). In the latter case, both betas in the deadweight cost function are set very high to disregard any future tax benefits associated with negative profits; $\beta_{CT}^{i,2} = \beta_{SPT}^{i,2} = 5000 \forall i$. The relative magnitudes

of the hedge ratios of firms liable for dual and ordinary tax exposure, respectively, are calculated numerically using the following steps (setting $\beta_{CT} = \beta$ for notational convenience):⁷

1. Set the value of the corporate tax convexity parameter β_1 to 0.00001, i.e., the case approximating linear tax functions for which the known hedge ratios (15) apply.
2. Using the hedge ratio calculated from (15) as the starting value, solve the first order condition within given error tolerances to arrive at $a_{CT}^*(\beta_1)$.
3. Reset the tax convexity parameter to β_2 and solve for $a_{CT}^*(\beta_2)$ using $a_{CT}^*(\beta_1)$ as the starting value.
4. Repeat the procedure until $a_{CT}^*(\beta_N)$ has been calculated using $a_{CT}^*(\beta_{N-1})$ as the starting value. At this point, the set $\mathbf{a}_{CT}^*(\beta) \equiv \{a_{CT}^*(\beta_1), a_{CT}^*(\beta_2), \dots, a_{CT}^*(\beta_N)\}$ has been backed out from the N first order conditions associated with the N values for β .
5. Repeat steps 1-4 with dual taxes to arrive at the set $\mathbf{a}_{CT+SPT}^*(\beta)$. Calculate the set of

relative hedge ratios $\mathbf{a}_{RELATIVE}^* \equiv \left\{ \frac{a_{CT+SPT}^*(\beta_1)}{a_{CT}^*(\beta_1)}, \frac{a_{CT+SPT}^*(\beta_2)}{a_{CT}^*(\beta_2)}, \dots, \frac{a_{CT+SPT}^*(\beta_N)}{a_{CT}^*(\beta_N)} \right\}$.

Deviations from one represent distortions imposed on firms' hedge ratios by special purpose taxes, everything else equal.

Figure 3 illustrates relative forward hedge ratios for cases 1-4 under conditions with very high deadweight costs, while Figure 4 shows these values when cash shortfall is the key determinant of deadweight cost in the spirit of Froot and Stein (1993). Both figures show that special purpose taxes can distort the hedging strategies of nonfinancial firms. Defining any

⁷ Optimal hedge ratios were calculated for β up to 5000. However, since the values leveled out, only results for $\beta \leq 1500$ are presented. All calculations were performed using global adaptive integration methods in Mathematica 6.0.0 with PrecisionGoal = 7, AccuracyGoal = 12, MinRecursion = 5, MaxRecursion = 50, and WorkingPrecision = 40 at maximum, integrating over $z_P \times z_Q = \{(z_P \times z_Q) : z_P \in (-10, 10), z_Q \in (-10, 10)\}$. Several robustness checks were performed to validate the results, including reducing the integration region to include fewer standard deviations.

deviation of a relative hedge ratio from one as a distortion imposed by special purpose taxes on firm hedging, the significance of the distortions seems to depend on the underlying exposure as well on whether value or cash shortfall is the key determinant of deadweight costs. Experiments with $c_2 = 2$ and $c_2 = 5$ confirmed the a priori expectation that the distortions become less significant as the curvature parameter c_2 approaches zero. Note that Figures 3 and 4 can represent the case of Norwegian hydroelectricity producers; the distortions will be larger with the special purpose petroleum tax rate of 50% applicable to companies operating on the Norwegian continental shelf.

Overall, the distortions imposed by special purpose taxes seem moderate, even under conditions with very high deadweight costs. Remember that Brown and Toft (2002) assumed $c_2 \in \{2, 5, 8\}$, the equivalent of $c_2 \in \{2(1-t_{CT}), 5(1-t_{CT}), 8(1-t_{CT})\}$ under conditions with linear tax exposure when deadweight costs are specified in terms of after-tax profits, which are more moderate assumptions about the speed with which deadweight costs increase as profits drop. As corporate tax exposure become more convex, value-maximizing firms tend to increase the relative amount of hedging, eventually leveling out at some relative hedge ratio

$$\frac{1-t_{CT}-t_{SPT}}{1-t_{CT}} < \frac{a_{CT+SPT}^*}{a_{CT}^*} < 1 \text{ for economically interesting parameters. This pattern parallels the}$$

influence of an increasing deadweight cost parameter c_2 under conditions with linear tax exposure, in which case the relative hedging demand increases to reduce the value of deadweight costs (Corollary 1). In that case, the distortions become larger as deadweight costs turn less extreme, approaching the case of the relative minimum variance forward hedging demand $(1-t_{CT}-t_{SPT})/(1-t_{CT})$ (Corollary 3). Note that the special purpose tax does not distort the hedging demand of a value-maximizing dual tax firm when c_2 equals zero (Proposition 1), while a similar firm minimizing the profit variance would chose the relative

hedging demand $(1-t_{CT} - t_{SPT})/(1-t_{CT})$ in the case of linear tax exposure because the special purpose tax serves as a partial hedge.

All things considered, the argument put forward by the Norwegian tax authorities that special purpose taxes are essentially neutral is not entirely correct for firms operating in the Brown and Toft setting. This is particularly the case for value-maximizing firms facing potentially low or moderate deadweight costs and linear corporate tax exposure. The distortions imposed on firms facing convex corporate tax exposure as well as deadweight costs seem to be smaller the higher the degree of corporate tax function convexity. With less hedging initially, the downward adjustment necessary to restore optimality in response to higher tax function convexity is generally smaller in magnitude for dual tax firms than for ordinary firms. This result parallels the influence of the parameter c_2 on the extent of hedging described in Corollary 1.

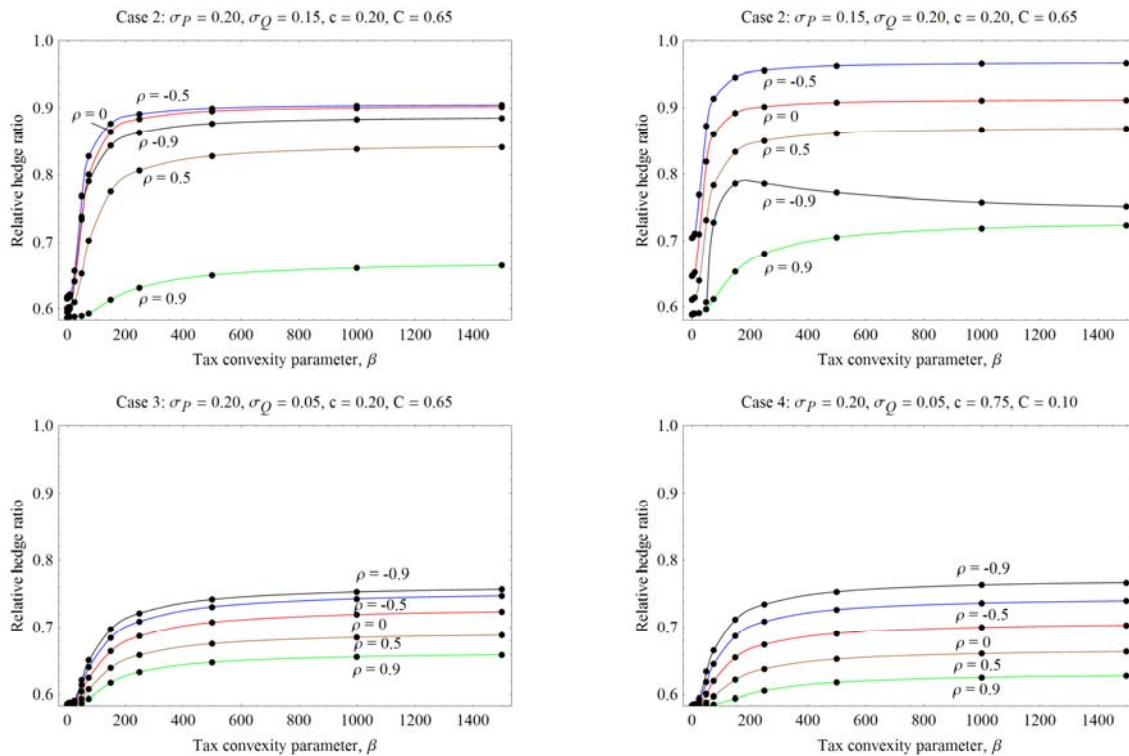


Figure 3. Relative hedge ratios $a_{CT+SPT}^*(\beta_{CT}, \beta_{SPT})/a_{CT}^*(\beta_{CT}, \beta_{SPT})$ for $\beta_{CT} = \beta$ and $\beta_{SPT} = 0.0001$ when value is the key determinant of deadweight costs, that is, numerical solutions of equation (18) given linear special purpose tax exposure. The leftmost markers correspond to the relative hedge ratios given by (16). The relative hedge ratios corresponding to the beta values $\{0.00001, 1, 5, 10, 25\}$ left out from the plot for case 2, $\rho = -0.9$, are $\{16.5, 15.8, 16.8, 321.5, -21\}$. Such results may arise when the hedge ratios are close to zero, cf.

Corollary 1. Any deviation from one is interpreted as a distortion imposed by special purpose taxes on firm hedging. $\theta = 0.001$, $c_1 = 0.0075$, $c_2 = 9$, $t_{CT} = 0.28$, and $t_{SPT} = 0.3$.

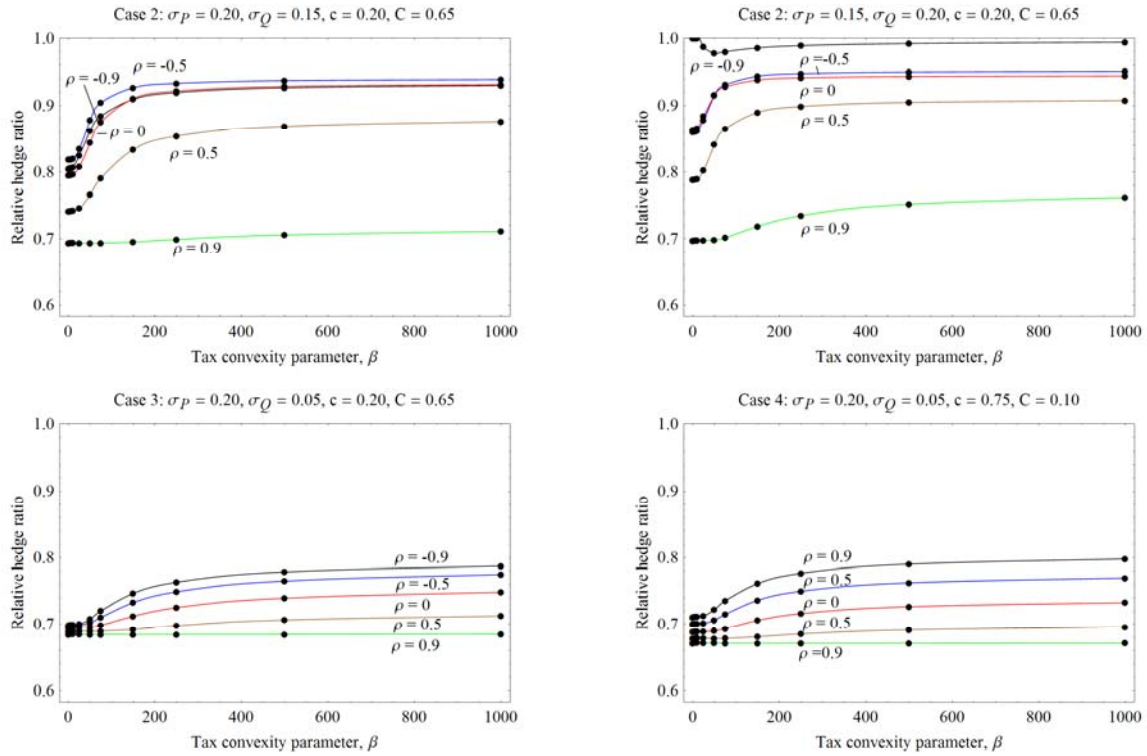


Figure 4. Relative hedge ratios $a_{CT+SPT}^*(\beta_{CT}, \beta_{SPT}) / a_{CT}^*(\beta_{CT}, \beta_{SPT})$ when cash shortfall is the key determinant of deadweight costs in the spirit of Froot & Stein (1993). Approximated by setting β_{SPT} and β_{SPT} equal to 5000 in the exponential deadweight cost function (approximates $\beta \rightarrow \infty$) while letting $\beta_{CT} = \beta$ in the second product term of the first order condition (18) vary. The leftmost markers correspond to the relative hedge ratios given by (16). Any deviation from one is interpreted as a distortion imposed by special purpose taxes on firm hedging. $\theta = 0.001$, $c_1 = 0.0075$, $c_2 = 9$, $t_{CT} = 0.28$, and $t_{SPT} = 0.3$.

4. Concluding remarks

While a large strand of literature studies how rent taxes may distort firms' operating and investment decisions (Baunsgaard, 2001; Lund, 2009; McPhail, Daniel, King, Moran, & Otto, 2009; Otto et al., 2006), to the best of my knowledge, this study is the first to analyze how firms' financial strategies may be influenced by rent taxation. I analyze the influence of dual tax exposure in the form of a special purpose tax on top of corporate income tax on the hedging demand of value-maximizing nonfinancial firms. Using an extended version of the model of Brown and Toft (2002), I demonstrate that special purpose taxes do not influence

the extent of firm hedging when corporate tax function convexity is the only theoretical motivation for hedging. This holds under general conditions about production technology, tax provisions, and tax function convexity. With the possibility of deadweight costs that are high when profits are low and low when profits are high, such as direct and indirect costs of financial distress and costly external financing, moderate distortions are introduced for constant-returns-to-scale firms committed to hedging with forward contracts. The relative hedging demand of firms liable to dual taxes lies between the ratio $(1 - t_{CT} - t_{SPT}) / (1 - t_{CT})$ and one in the vast majority of cases. This suggests that the type of special purpose taxes analyzed in this paper will not critically impede the development of a market for the transfer of price risk, like in the case of tax-disadvantaged corn yield futures on Chicago Board of Trade reported by Lien (2004). The success of Nord Pool ASA, the Nordic electricity exchange, in establishing and sustaining a market for the transferral of price risk even with special purpose taxation imposed on Norwegian hydroelectricity producers bears some evidence in support of this claim. However, because my analysis is predicated on a single-period incomplete market model, these results do not necessarily apply beyond the case when producers face identically and independently distributed random variables representing hedgeable and unhedgeable risk. In order to analyze multiperiod hedging with statistically dependent realizations of the hedgeable and unhedgeable random variables, a dynamic model is called for. This is left for future research.

Although this study has been motivated by the Norwegian hydropower and petroleum tax regimes, its implications are probably not confined to Norway. Various oil, gas and mineral tax regimes designed to extract rent are applied all over the world; see Table 3 in Baunsgaard (2001) for an overview. Despite considerable variations in the setup for rent taxation across countries, the global trend is to move away from production-based taxes to profit-based taxes for rent extraction, that is, to design rent tax regimes more like the

Norwegian model (McPhail et al., 2009, p. 37). The predictions of this paper are therefore expected to be relevant for most governments seeking to extract economic rent on behalf of their constituents, i.e., for understanding how dual taxes may influence firms' financial strategies. Because it is a possible that financial and real decisions sometimes interact, this paper could also prove important for the ongoing research into the optimal design of rent taxes on firms' investment and operating decisions reviewed by Lund (2009).

Appendix A. The optimal number of forward contracts for constant marginal tax rates

The optimal choice of forward sales is found by solving the problem

$$\max_{a \in \mathbb{R}} E[\Pi(a)] = \max_{a \in \mathbb{R}} \int_P \int_Q \left\{ NP(p, q; a)(1 - t_{CT}) - t_{SPT} SPI(p, q) - DWC(NPAT(p, q; a)) \right\} f(p, q) dq dp \quad (\text{A.1})$$

given $h(p) = (p - \mu_p)$. Because the derivative of the deadweight cost function is given by $DWC'(NPAT(p, q; a)) = -c_2 c_1 e^{-c_2 NPAT(p, q; a)}$ and the analytical expression for the bivariate normal distribution is

$$f(p, q) = \frac{1}{2\pi\sigma_p\sigma_q\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)} \left[\frac{(p-\mu_p)^2}{\sigma_p^2} + \frac{(q-\mu_q)^2}{\sigma_q^2} - \frac{2\rho(p-\mu_p)(q-\mu_q)}{\sigma_p\sigma_q} \right]}$$

the first order condition defined in equation (12) now becomes

$$\frac{\partial E[\Pi(a)]}{\partial a} = A \int_P (p - \mu_p) \int_Q e^{-c_2 NPAT(p, q; a) - B \left[\frac{(p-\mu_p)^2}{\sigma_p^2} + \frac{(q-\mu_q)^2}{\sigma_q^2} - \frac{2\rho(p-\mu_p)(q-\mu_q)}{\sigma_p\sigma_q} \right]} dq dp = 0 \quad (\text{A.2})$$

where $A = \frac{c_1 c_2}{2\pi\sigma_p\sigma_q\sqrt{1-\rho^2}}$ and $B = \frac{1}{2(1-\rho^2)}$ ($|\rho| \neq 1$). Expanding and collecting terms in the exponent with similar powers of p , q , and pq reduces the expression above to

$$\frac{\partial E[\Pi(a)]}{\partial a} = A e^{d_6(a)} \int_P (p - \mu_p) e^{-d_1 p^2 + d_2(a)p} \int_Q e^{-d_4 q^2 + d_3 pq + d_5 q} dq dp = 0 \quad (\text{A.3})$$

where

$$\begin{aligned}
d_1 &= \frac{B}{\sigma_P^2} > 0 \quad (\sigma_P \neq 0) \\
d_2(a) &= \frac{2B(\mu_P \sigma_Q - \rho \mu_Q \sigma_P)}{\sigma_P^2 \sigma_Q} - a c_2 (1 - t_{CT}) \\
d_3 &= \frac{2B\rho}{\sigma_P \sigma_Q} - c_2 (1 - t_{CT} - t_{SPT}) \\
d_4 &= \frac{B}{\sigma_Q^2} > 0 \quad (\sigma_Q \neq 0) \\
d_5 &= c c_2 (1 - t_{CT} - t_{SPT}) + \frac{2B(\mu_Q \sigma_P - \rho \mu_P \sigma_Q)}{\sigma_P \sigma_Q^2} \\
d_6(a) &= c_2 [(C + a \mu_P)(1 - t_{CT}) - C_{SPT} t_{SPT}] + B \left(\frac{2\rho \mu_P \mu_Q}{\sigma_P \sigma_Q} - \frac{\mu_P^2}{\sigma_P^2} - \frac{\mu_Q^2}{\sigma_Q^2} \right)
\end{aligned}$$

Note that the appearance of the first order condition (A.3) is identical to Brown's and Toft's (2002) equation (A.3), except for the definitions of the "d-constants". Given that p is a constant when integrating over the space of the RV Q , we may define $d_7(p) = d_3 p + d_5$ and reformulate the innermost integral into the form $\int_Q e^{-d_4 q^2 + d_7(p)q} dq$. This definite integral has

the solution $\sqrt{\frac{\pi}{d_4}} e^{\frac{d_7^2}{4d_4}}$. Substituting for d_7 and rearranging yields the first order condition

$$\frac{\partial E[\Pi(a)]}{\partial a} = A \sqrt{\frac{\pi}{d_4}} e^{d_6(a) + \frac{d_5^2}{4d_4}} \int_P (p - \mu_P) e^{-\frac{1}{2} \left(\left(2d_1 - \frac{d_3^2}{2d_4} \right) p^2 - \left(\frac{d_3 d_5}{d_4} + 2d_2(a) \right) p \right)} dp = 0 \quad (\text{A.4})$$

The constant in front of the integral will never be zero, so the problem reduces to finding the a satisfying

$$\frac{\partial E[\Pi(a)]}{\partial a} = \int_P (p - \mu_P) e^{-\frac{1}{2}(C p^2 - D p)} dp = 0 \quad (\text{A.5})$$

where $C = 2d_1 - \frac{d_3^2}{2d_4}$ and $D = \frac{d_3 d_5}{d_4} + 2d_2(a)$. The finite solution of the lhs of (A.5) is

$$\frac{\partial E[\Pi(a)]}{\partial a} = e^{\frac{D(a)^2}{8C}} \frac{\sqrt{\frac{\pi}{2}} (2C\mu_P - D(a))}{C^{\frac{3}{2}}} \quad (\text{A.6})$$

for $C > 0$ (sufficient condition for keeping the integrand from exploding). This condition will always be satisfied for $|\rho| \neq 1$.

$$\mathbf{Proof.} \quad C > 0 \Leftrightarrow 2d_1 - \frac{d_3^2}{2d_4} > 0 \Leftrightarrow \frac{1 - (\rho - c_2(1 - \rho^2))(1 - t_{CT} - t_{SPT})\sigma_P\sigma_Q}{(1 - \rho^2)\sigma_P^2} > 0$$

Because the denominator is positive,

$$\begin{aligned} C > 0 &\Leftrightarrow 1 - (\rho - c_2(1 - \rho^2))(1 - t_{CT} - t_{SPT})\sigma_P\sigma_Q > 0 \Leftrightarrow \\ &1 > (\rho - c_2(1 - \rho^2))(1 - t_{CT} - t_{SPT})\sigma_P\sigma_Q \Leftrightarrow \\ &1 > \rho - c_2(1 - \rho^2)(1 - t_{CT} - t_{SPT})\sigma_P\sigma_Q \Leftrightarrow \\ &1 - \rho > -c_2(1 - \rho^2)(1 - t_{CT} - t_{SPT})\sigma_P\sigma_Q \end{aligned}$$

The fact that the lhs (rhs) is always positive (negative) proves the claim $C > 0$. ■

Thus, the first order condition (A.5) reduces to $2C\mu_P - D(a) = 0$. Inserting for C and $D(a)$ yields

$$\begin{aligned} \frac{\partial E[\Pi(a)]}{\partial a} &= 2c_2 \left(a(1 - t_{CT}) + (1 - t_{CT} - t_{SPT})\mu_Q + (\mu_P - c)\rho(1 - t_{CT} - t_{SPT})\frac{\sigma_Q}{\sigma_P} \right. \\ &\quad \left. - (\mu_P - c)(1 - \rho^2)c_2(1 - t_{CT} - t_{SPT})^2\sigma_Q^2 \right) = 0 \end{aligned} \quad (\text{A.7})$$

Solving for a , we get

$$\begin{aligned} a_{Tax}^* &= \frac{1 - t_{CT} - t_{SPT}}{1 - t_{CT}} \left(-\mu_Q - (\mu_P - c)\rho\frac{\sigma_Q}{\sigma_P} \right. \\ &\quad \left. + (\mu_P - c)(1 - \rho^2)c_2(1 - t_{CT} - t_{SPT})\sigma_Q^2 \right) \end{aligned} \quad (\text{A.8})$$

The expected economic profit is indeed a strictly concave function of a , so this is a unique solution provided $c_2 > 0$ (any hedge ratio will be optimal if $c_2 = 0$). This follows directly from the second derivative of equation (A.2):

$$\begin{aligned} \frac{\partial^2 E[\Pi(a)]}{\partial a^2} &= -c_2(1 - t_{CT})A \int_P (p - \mu_P)^2 \\ &\quad \int_Q e^{-c_2 NPAT(p,q;a) - B \left[\frac{(p - \mu_P)^2}{\sigma_P^2} + \frac{(q - \mu_Q)^2}{\sigma_Q^2} - \frac{2\rho(p - \mu_P)(q - \mu_Q)}{\sigma_P\sigma_Q} \right]} dq dp < 0 \quad \forall a \end{aligned}$$

Appendix B. The after-tax minimum variance forward contract demand for constant marginal tax rates

The variance of the net profit after taxes is defined as $E[NPAT^2] - (E[NPAT])^2$ or

$$\begin{aligned} \sigma_{NPAT}^2(a) = & \int_P \int_Q \left((pq - cq - C + a(p - \mu_P))(1 - t_{CT}) - t_{SPT} \right. \\ & \left. (pq - cq - C_{SPT}) \right)^2 f(p, q) dpdq - \\ & \left(\left(\mu_P \mu_Q + \rho \sigma_P \sigma_Q - c \mu_Q - C \right) (1 - t_{CT}) \right)^2 \\ & \left(-t_{SPT} (\mu_P \mu_Q + \rho \sigma_P \sigma_Q - c \mu_Q - C_{SPT}) \right)^2 \end{aligned} \quad (B.1)$$

The first order condition for the after-tax variance minimizing forward hedge ratio is

$$\begin{aligned} 2(1 - t_{CT}) \int_P \int_Q \left\{ (pq - cq - C + a(p - \mu_P))(1 - t_{CT}) - t_{SPT} \right. \\ \left. (pq - cq - C_{SPT}) \right\} (p - \mu_P) f(p, q) dpdq = 0 \end{aligned} \quad (B.2)$$

Inserting for the bivariate normal density, the first order condition may be reformulated as

$$\begin{aligned} d_6(a) + \frac{1}{2\pi \sigma_P \sigma_Q \sqrt{1 - \rho^2}} \int_P \int_Q \left\{ (d_1 p^2 q + d_2(a) p^2 + d_3 pq + d_4 q + d_5(a) p) \right. \\ \left. e^{\frac{-1}{2(1 - \rho^2)} \left[\frac{(p - \mu_P)^2}{\sigma_P^2} + \frac{(q - \mu_Q)^2}{\sigma_Q^2} - \frac{2\rho(p - \mu_P)(q - \mu_Q)}{\sigma_P \sigma_Q} \right]} \right\} dqdp = 0 \end{aligned} \quad (B.3)$$

where

$$\begin{aligned} d_1 &= 1 - t_{CT} - t_{SPT} & d_2(a) &= a(1 - t_{CT}) \\ d_3 &= -(c + \mu_P)(1 - t_{CT} - t_{SPT}) & d_4 &= c \mu_P (1 - t_{CT} - t_{SPT}) \\ d_5(a) &= -(1 - t_{CT})(C + 2a \mu_P) + t_{SPT} C_{SPT} \\ d_6(a) &= \mu_P ((1 - t_{CT})(C + a \mu_P) - t_{SPT} C_{SPT}) \end{aligned}$$

The solutions of the five integrals corresponding to the terms involving

$d_1, d_2(a), d_3, d_4$ and $d_5(a)$ are $d_1(\mu_Q(\mu_P^2 + \sigma_P^2) + 2\mu_P \rho \sigma_P \sigma_Q), d_2(\mu_P^2 + \sigma_P^2),$

$d_3(\mu_P \mu_Q + \rho \sigma_P \sigma_Q), d_4 \mu_Q,$ and $d_5(a) \mu_P,$ respectively. Thus, the first order condition is

$$d_6(a) + d_1(\mu_Q(\mu_P^2 + \sigma_P^2) + 2\mu_P \rho \sigma_P \sigma_Q) + d_2(a)(\mu_P^2 + \sigma_P^2) + d_3(\mu_P \mu_Q + \rho \sigma_P \sigma_Q) + d_4 \mu_Q + d_5(a) \mu_P = 0.$$

Inserting for the constants, expanding and collecting terms yields the first order condition

$$\frac{\partial \sigma_{NPAT}^2}{\partial a} = \sigma_P \left[(a + \mu_Q) \sigma_P (1 - t_{CT}) + (\mu_P - c) \rho \sigma_Q (1 - t_{CT} - t_{SPT}) - t_{SPT} \mu_Q \sigma_P \right] = 0 \quad (\text{B.4})$$

Solving for a yields the after-linear-tax minimum variance forward contract demand because σ_{NPAT}^2 is strictly convex in a .

$$\frac{\partial^2 \sigma_{NPAT}^2}{\partial a^2} = \sigma_P^2 (1 - t_{CT}) > 0 \quad (\text{B.5})$$

Thus, the unique after-linear-tax minimum variance contract demand is

$$a_{\min \sigma_{NPAT}^2}^{Tax} = \frac{1 - t_{CT} - t_{SPT}}{1 - t_{CT}} \left(-\mu_Q - (\mu_P - c) \rho \frac{\sigma_Q}{\sigma_P} \right) \quad (\text{B.6})$$

Given constant marginal tax rates, this is also the number of forward contracts that minimizes the variance of before-tax profits.

Appendix C. The marginal density and distribution function of the absolutely continuous RV NPAT for the class of price derivatives

To see why NPAT is an absolutely continuous RV, observe in Figure 5 that $NPAT^{-1}(p; npat, \mathbf{a})$ defines an infinitely large set of curves in \mathbb{R}^2 .⁸ There are two curves for every value of $npat$, one on each side of the vertical asymptote at $p = c$, covering mutually exclusive parts of the realization range of the RV Q . A small change in the ordered pair (p, q) will always cause a small change in $npat$; for no such change will a discontinuous change in $npat$ be observed. $NPAT^{-1}(p; npat, \mathbf{a})$ is discontinuous in an immediate region around the vertical asymptote, but NPAT is still an absolutely continuous RV. Thus, a marginal density of the RV NPAT exists and may be found using a well-known theorem from the theory of functions of RVs. Having found the marginal density, the distribution function of NPAT is easily found. However, an alternative, more intuitive derivation of the distribution function is provided by integrating over the space of P and Q using p -dependent integration limits.

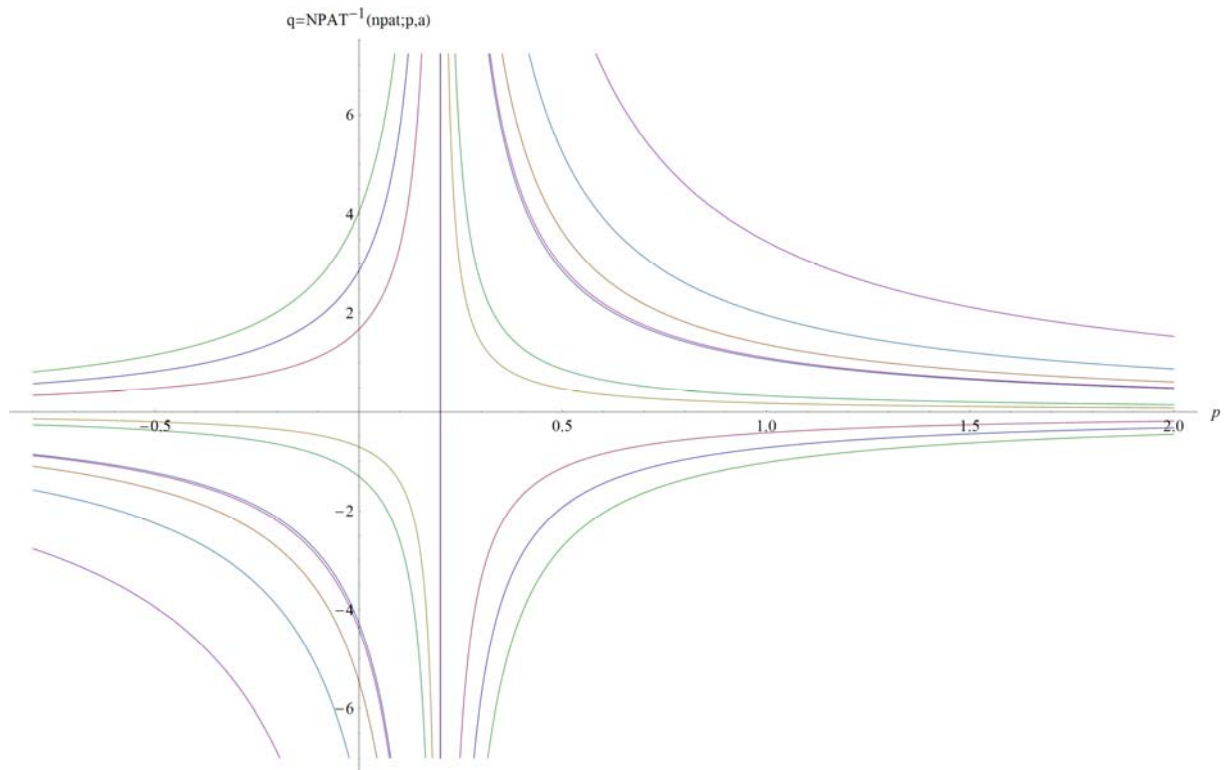


Figure 5. The inverse function $q = NPAT^{-1}(p; npat, a)$ for some arbitrarily chosen values of $npat$, under the assumption of no derivatives contracts, linear tax functions, $\mu_P = \mu_Q$, $t_{SPT} = 0.3$, $t_{CT} = 0.28$, $C = 0.45$, $C_{SPT} = 0.55$, and $c = 0.2$. For $p > c$, increasing q implies higher $npat$ for any given p . For $p = c$, $npat = npat_{p=c}^a$ and $NPAT^{-1}(c; npat_{p=c}^a, a)$ is arbitrary. For $p < c$, increasing q implies lower $npat$ for any given p .

⁸ The parameters β and θ are now incorporated into the general expressions for corporate taxes, purely for notational ease.

For the studied model,

$$\frac{\partial NPAT(p, q; \mathbf{a})}{\partial q} = (p - c)(1 - CT'(NP(p, q; \mathbf{a})) - SPT'(SPI(p, q))) \quad (C.1)$$

It follows from the model assumptions that

$$\begin{aligned} \frac{\partial NPAT(p, q; \mathbf{a})}{\partial q} &< 0, \quad p < c \\ \frac{\partial NPAT(c, q; \mathbf{a})}{\partial q} &= 0, \quad p = c \\ \frac{\partial NPAT(p, q; \mathbf{a})}{\partial q} &> 0, \quad p > c \end{aligned}$$

which proves the existence of the inverse functions $NPAT_{p < c}^{-1}(npat; p, \mathbf{a})$, $NPAT_{p > c}^{-1}(npat; p, \mathbf{a})$ and the non-existence of $NPAT_{p=c}^{-1}(npat_{p=c}^{\mathbf{a}}; c, \mathbf{a})$.⁹ Given that the choice of q is arbitrary for $p = c$ and $\int_c^c \int_Q f(p, q) dq dp = 0$, we may define the inverse function

$$NPAT^{-1}(npat; p, \mathbf{a}) = \begin{cases} NPAT_{p < c}^{-1}(npat; p, \mathbf{a}), & p < c \\ NPAT_{p > c}^{-1}(npat; p, \mathbf{a}), & p > c \end{cases} \quad (C.2)$$

Thus, $NPAT^{-1}(npat; p, \mathbf{a})$ yields the unique value q that corresponds to any $npat$ within the range of $NPAT(p, q; \mathbf{a})$, possibly excluding $npat_{p=c}^{\mathbf{a}}$ when p and the number of contracts are fixed. $NPAT^{-1}(npat; p, \mathbf{a})$ is generally implicitly defined by the equation

$$q - \frac{npat + C - \mathbf{a}'\mathbf{h}(p) + CT(NP(p, q; \mathbf{a})) + SPT(SPI(p, q))}{(p - c)} = 0 \quad (p \neq c) \quad (C.3)$$

but for the special case of linear tax functions, the inverse function is explicitly defined as

$$NPAT^{-1}(npat; p, \mathbf{a}) = \frac{npat + C(1 - t_{CT}) - t_{SPT} C_{SPT} - \mathbf{a}'\mathbf{h}(p)(1 - t_{CT})}{(p - c)(1 - t_{CT} - t_{SPT})} \quad (C.4)$$

Following Dudewicz's and Mishra's (1988) Theorem 4.6.4/4.6.18, the joint density of the RVs P and $NPAT$, $\psi(p, npat; \mathbf{a})$, may be derived as

⁹ For a given hedge ratio vector \mathbf{a} and payoff vector $\mathbf{h}(p)$, there is one unique after-tax profit, $npat_{p=c}^{\mathbf{a}}$, for $p = c$, irrespective of q .

$$\psi(p, npat; \mathbf{a}) = \frac{f(p, NPAT^{-1}(npat; p, \mathbf{a}))}{\left| \frac{\partial}{\partial q} NPAT(p, NPAT^{-1}(npat; p, \mathbf{a}); \mathbf{a}) \right|} = \begin{cases} \frac{f(p, NPAT^{-1}(npat; p, \mathbf{a}))}{\frac{\partial}{\partial q} NPAT(p, NPAT^{-1}(npat; p, \mathbf{a}); \mathbf{a})}, & p < c \\ \frac{f(p, NPAT^{-1}(npat; p, \mathbf{a}))}{\frac{\partial}{\partial q} NPAT(p, NPAT^{-1}(npat; p, \mathbf{a}); \mathbf{a})}, & p > c \end{cases} \quad (C.5)$$

where $\psi(p, npat; \mathbf{a}) \geq 0$ and $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(p, npat; \mathbf{a}) dp dnpat = 1$. Inserting for

$f(p, NPAT^{-1}(npat; p, \mathbf{a}))$ and setting $z_p = \frac{p - \mu_p}{\sigma_p}$, $z_Q[z_p, npat; \mathbf{a}] =$

$\frac{NPAT^{-1}(npat; z_p, \mathbf{a}) - \mu_Q}{\sigma_Q}$, $B = \frac{1}{2(1 - \rho^2)}$, the bivariate density of the RVs P and $NPAT$ may

be compactly reformulated as a function of the RVs Z_p and $NPAT$.

$$\psi(z_p, npat; \mathbf{a}) = \begin{cases} \frac{e^{-B(z_p^2 + z_Q^2[z_p, npat; \mathbf{a}] - 2\rho z_p z_Q[z_p, npat; \mathbf{a}])}}{2\pi\sigma_p\sigma_Q\sqrt{1-\rho^2}(z_p\sigma_p + \mu_p - c)(1-CT'-SPT')}, & z_p < \frac{c - \mu_p}{\sigma_p} \\ \frac{e^{-B(z_p^2 + z_Q^2[z_p, npat; \mathbf{a}] - 2\rho z_p z_Q[z_p, npat; \mathbf{a}])}}{2\pi\sigma_p\sigma_Q\sqrt{1-\rho^2}(z_p\sigma_p + \mu_p - c)(1-CT'-SPT')}, & z_p > \frac{c - \mu_p}{\sigma_p} \end{cases} \quad (C.6)$$

Thus, the marginal density of the RV $NPAT$ is

$$\Phi(npat; \mathbf{a}) = -K \int_{-\infty}^{\frac{c - \mu_p}{\sigma_p}} \frac{e^{-B(z_p^2 + z_Q^2[z_p, npat; \mathbf{a}] - 2\rho z_p z_Q[z_p, npat; \mathbf{a}])}}{(z_p\sigma_p + \mu_p - c)(1-CT'-SPT')} dz_p + K \int_{\frac{c - \mu_p}{\sigma_p}}^{\infty} \frac{e^{-B(z_p^2 + z_Q^2[z_p, npat; \mathbf{a}] - 2\rho z_p z_Q[z_p, npat; \mathbf{a}])}}{(z_p\sigma_p + \mu_p - c)(1-CT'-SPT')} dz_p \quad (C.7)$$

where

$$K = \frac{1}{2\pi\sigma_Q\sqrt{1-\rho^2}}, CT' = CT'(NP(z_p, z_Q[z_p, npat; \mathbf{a}], \mathbf{a})), SPT' = SPT'(SPI(z_p, z_Q[z_p, npat; \mathbf{a}], \mathbf{a}))$$

There is no analytical solution of this integral, but the marginal distribution may be obtained using numerical methods because the integrand is well behaved. It is easy to see that

$\lim_{z_p \rightarrow \pm\infty} \frac{e^{-B(z_p^2 + z_Q^2[z_p, npat] - 2\rho z_p z_Q[z_p, npat])}}{(z_p \sigma_p + \mu_p - c)(1 - CT' - SPT')} = 0$, so the critical question is what happens to the integrand as $z_p \rightarrow \frac{c - \mu_p}{\sigma_p}$ from either side. This is a "0/0"-type limit, but the integrand is well behaved as the rate of convergence to zero is faster in the numerator than in the denominator.

The distribution function may be numerically obtained from $\Phi(npat; \mathbf{a})$, of course. However, it may also be derived by integrating over the space of the RVs P and Q using $NPAT^{-1}(npat; p, \mathbf{a})$ as p -dependent integration limits of Q for any given $npat$ within the range of $NPAT$. Inserting for $f(p, q)$, defining $B = \frac{1}{2(1 - \rho^2)}$ and rewriting the integrals in terms of the standardized RVs z_Q and z_p , the distribution function becomes

$$\begin{aligned}
 P[NPAT \leq npat; \mathbf{a}] &= F(npat; \mathbf{a}) \\
 &= \frac{1}{2\pi\sqrt{1 - \rho^2}} \int_{-\infty}^{\frac{c - \mu_p}{\sigma_p}} e^{-Bz_p^2} \int_{NPAT^{-1}(npat, z_p; \mathbf{a})}^{\infty} e^{-B(z_Q^2 - 2\rho z_p z_Q)} dz_Q dz_p + \\
 &\quad \frac{1}{2\pi\sqrt{1 - \rho^2}} \int_{\frac{c - \mu_p}{\sigma_p}}^{\infty} e^{-Bz_p^2} \int_{-\infty}^{NPAT^{-1}(npat, z_p; \mathbf{a})} e^{-B(z_Q^2 - 2\rho z_p z_Q)} dz_Q dz_p
 \end{aligned} \tag{C.8}$$

This distribution function may be more compactly formulated as

$$\begin{aligned}
 F(npat; \mathbf{a}) &= \frac{1}{2} - \frac{1}{2\sqrt{2}\pi} \int_{-\infty}^{\frac{c - \mu_p}{\sigma_p}} e^{-\frac{z_p^2}{2}} \text{Erf}(F(z_p, npat; \mathbf{a})) dz_p \\
 &\quad + \frac{1}{2\sqrt{2}\pi} \int_{\frac{c - \mu_p}{\sigma_p}}^{\infty} e^{-\frac{z_p^2}{2}} \text{Erf}(F(z_p, npat; \mathbf{a})) dz_p
 \end{aligned} \tag{C.9}$$

where

$$\text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad \text{and} \quad F(z_p, npat; \mathbf{a}) = \frac{1}{\sqrt{2(1 - \rho^2)}} \left(\frac{NPAT^{-1}[npat, z_p; \mathbf{a}] - \mu_Q}{\sigma_Q} - \rho z_p \right).$$

It is easily shown that $F'[npat; \mathbf{a}] = \Phi(npat; \mathbf{a})$, as required.

Appendix D. Proof of concavity for zero deadweight costs and $h(p) = p - \mu_P$

The maximand (10) is a concave function of the hedge ratio a for the case of nondecreasing marginal tax rates, zero deadweight costs, and $h(p) = (p - \mu_P)$.

Proof.

The first order condition (11) may be rewritten as follows:¹⁰

$$\frac{\partial E[\Pi(a)]}{\partial a} = \int_{z_P} \int_{z_Q} w(z_P, z_Q; a) f^*(z_P, z_Q) dz_Q dz_P$$

The derivative of $w(z_P, z_Q; a)$ becomes:

$$\begin{aligned} \frac{\partial w(a)}{\partial a} &= \frac{\partial}{\partial a} \left(\frac{\partial NP(z_P, z_Q; a)}{\partial a} (1 - CT'(NP(z_P, z_Q; a))) \right) \\ &= \sigma_P z_P (1 - CT''(NP(z_P, z_Q; a)) \sigma_P z_P) \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\partial^2 E[\Pi(a)]}{\partial a^2} &= \int_{z_P} \int_{z_Q} \sigma_P z_P (1 - CT''(NP(z_P, z_Q; a)) \sigma_P z_P) f^*(z_P, z_Q) dz_Q dz_P \\ &= \int_{z_P \geq 0} \sigma_P z_P \int_{z_Q} (1 - CT''(NP(z_P, z_Q; a)) \sigma_P z_P) f^*(z_P, z_Q) dz_Q dz_P \\ &\quad + \int_{z_P < 0} \sigma_P z_P \int_{z_Q} (1 - CT''(NP(z_P, z_Q; a)) \sigma_P z_P) f^*(z_P, z_Q) dz_Q dz_P \end{aligned}$$

Equation (13) implies that

$$CT''(NP) = \frac{\beta \theta e^{-\beta NP} t_{CT}}{(1 + \theta(e^{-\beta NP} - 1))^2}$$

Inserting for CT'' yields

$$\begin{aligned} \frac{\partial^2 E[\Pi(a)]}{\partial a^2} &= \int_{z_P \geq 0} \sigma_P z_P \int_{z_Q} \left(1 - \frac{\beta \theta e^{-\beta NP(z_P, z_Q; a)} t_{CT} \sigma_P z_P}{(1 + \theta(e^{-\beta NP(z_P, z_Q; a)} - 1))^2} \right) f^*(z_P, z_Q) dz_Q dz_P \\ &\quad + \int_{z_P < 0} \sigma_P z_P \int_{z_Q} \left(1 - \frac{\beta \theta e^{-\beta NP(z_P, z_Q; a)} t_{CT} \sigma_P z_P}{(1 + \theta(e^{-\beta NP(z_P, z_Q; a)} - 1))^2} \right) f^*(z_P, z_Q) dz_Q dz_P \end{aligned} \tag{D.1}$$

¹⁰ f^* is the bivariate density of the standardized price and quantity variables P and Q ; $f^* = \sigma_P f$.

Let's rewrite the first line of (D.1) as follows, denoting the marginal density of the standardized price variable as g^* :

$$\begin{aligned}
& \sigma_P \int_{z_P \geq 0} z_P \left[\int_{z_Q} f^*(z_P, z_Q) dz_Q - \sigma_P \int_{z_Q} \frac{\beta \theta e^{-\beta NP(z_P, z_Q; a)} t_{CT} z_P}{\left(1 + \theta \left(e^{-\beta NP(z_P, z_Q; a)} - 1\right)\right)^2} f^*(z_P, z_Q) dz_Q \right] dz_P \\
&= \sigma_P \int_{z_P \geq 0} z_P \left[g^*(z_P) - \sigma_P \int_{z_Q} \frac{\beta \theta e^{-\beta NP(z_P, z_Q; a)} t_{CT} z_P}{\left(1 + \theta \left(e^{-\beta NP(z_P, z_Q; a)} - 1\right)\right)^2} f^*(z_P, z_Q) dz_Q \right] dz_P \\
&= \sigma_P \int_{z_P \geq 0} z_P g^*(z_P) dz_P - \sigma_P^2 \int_{z_P \geq 0} z_P \int_{z_Q} \frac{\beta \theta e^{-\beta NP(z_P, z_Q; a)} t_{CT} z_P}{\left(1 + \theta \left(e^{-\beta NP(z_P, z_Q; a)} - 1\right)\right)^2} f^*(z_P, z_Q) dz_Q dz_P
\end{aligned}$$

A similar result is achieved for the second line of (D.1). Adding together, we find that $\frac{\partial^2 E[\Pi(a)]}{\partial a^2}$ equals

$$\begin{aligned}
& \sigma_P \mu_{z_P} \\
& - \sigma_P^2 t_{CT} \int_{z_P \geq 0} z_P \int_{z_Q} \frac{\beta \theta e^{-\beta((z_P \sigma_P + \mu_P - c)(z_Q \sigma_Q + \mu_Q) - C + a z_P \sigma_P)}}{\left(1 + \theta \left(e^{-\beta((z_P \sigma_P + \mu_P - c)(z_Q \sigma_Q + \mu_Q) - C + a z_P \sigma_P)} - 1\right)\right)^2} f^*(z_P, z_Q) dz_Q dz_P \\
& - \sigma_P^2 t_{CT} \int_{z_P < 0} z_P \int_{z_Q} \frac{\beta \theta e^{-\beta((z_P \sigma_P + \mu_P - c)(z_Q \sigma_Q + \mu_Q) - C + a z_P \sigma_P)}}{\left(1 + \theta \left(e^{-\beta((z_P \sigma_P + \mu_P - c)(z_Q \sigma_Q + \mu_Q) - C + a z_P \sigma_P)} - 1\right)\right)^2} f^*(z_P, z_Q) dz_Q dz_P
\end{aligned}$$

The first line above is zero because $\mu_{z_P} = 0$. The second and third lines are both

negative. Thus, $\frac{\partial^2 E[\Pi(a)]}{\partial a^2}$ has been shown to be negative, i.e., the expected economic profit is a concave function of a for the special case with no deadweight costs. ■

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