An activity theory analysis of group interactions in mathematical modelling with the aid of digital technologies

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This paper reports on a study in progress of group interactions in mathematical modelling activities at the secondary school level. The study focuses on students' use of digital technologies in solving a mathematical modelling task. This study analyses how group interactions are influenced by digital technologies using Cultural-Historical Activity Theory as an analytical framework. The results from the data analysed indicate that a key factor to the modelling outcome resides in the interactions within group activities generated by digital technologies, such as the digital technology serving as a reference tool to visually demonstrate one's ideas. Furthermore, the digital technology provides a platform for observing and improving strategies within group activities.

Keywords: Activity theory, group interactions, mathematical modelling, digital technologies.

Introduction

Digital technologies (DT) play an important role in mathematics teaching and learning (Greefrath et al., 2018), and an area where such technologies are used in complex problem solving is mathematical modelling (MM), a process that maps real-world situations in mathematical terms with the goal of finding a real-world solution (Niss & Blum, 2020). Research supports the assertion that DT as mediating artefacts have an impact on students' modelling processes (Molina-Toro et al., 2019). Interactions generated within group activities in MM affect and are affected by DT. More specifically, GeoGebra and other Dynamic Geometry Software as mediating artefacts have an impact on sense-making in group interactions as students solve mathematical tasks (Zengin, 2021; Granberg & Olsson, 2015). Granberg and Olsson (2015) argues that students use GeoGebra as a reference tool to visually demonstrate their reasoning to one another in group interactions. Questioning and challenging of ideas is seen as one of the factors in group interaction success (Goos et al., 2002) and high-achieving students often dominate in group interactions (Esmonde, 2009). Time constraints might affect strategy selection by a group as they work on a problem (Caviola et al., 2017).

Research in MM is often done from a cognitive perspective with the focus on its heuristics and modelling process (often schematized in a cyclic diagram), and this applies to studies on group interactions in MM activities. In one example, Jankvist and Niss (2020) reports on difficulties of upper secondary students in MM and found "understanding of the tasks", the first step in the modelling process, as a major challenge. Vos and Frejd (2022) argues that with only an emphasis on cognitive aspects, a research study might not capture other important aspects (e.g., a dimension for tool use, social norms, etc.) that play a role in MM. On another hand, there are significant studies that take a socio-cultural perspective to MM. For instance, Williams and Goos (2012) reports that MM should be understood in relation to the developmental needs and hence the subjectivity and personalities of the learners, and the wider institutional and professional context. English et al (2016) claim that literature on MM and DT has not yet been sufficiently explored and as such, this paper focuses on group interactions of students solving a MM task with the aid of DT, taking a socio-cultural stance. To narrow the focus of this paper I look at how DT affect interactions generated by group activities in MM (although group interactions also affect DT in a dialectical manner). A 26 minutes episode in which a group of students solved a MM task with the aid of GeoGebra (a dynamic mathematics software) and calculator is selected and the analysis of this episode provides evidence of how DT shapes group interactions in MM activities. The research question guiding the analysis was: How do DT as mediating artefacts influence students' group interactions as they solve a MM task?

Analytical framework

To study how DT influences group interactions in a MM activity, I consider Cultural-Historical Activity Theory (CHAT) (Engeström, 1987). CHAT draws from the idea that all social actions are mediated by discourse and other cultural means. Mediated actions (multiple mediations) are considered within the analytical components of activity in CHAT (Figure 1).

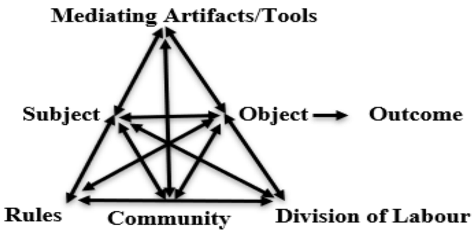

Figure 1: The structure of a human activity system (Engeström, 1987, p. 78)

Students solving MM tasks with mediational means could be explored by analysing interactions between the components of the activity system. The mediational means for the various interactions could be tools for the subject-object interaction, rules for the subject-community interaction, and division of labour for the community-object interaction. Engeström presents the activity system as an indivisible "whole" and in this study the unit of analysis is a group of students interacting with DT in solving a MM task. The subjects engage in object-oriented (objective of the activity system) activity with the objective of obtaining an outcome (develop a technology-based model/solution). An example in mathematics education of using CHAT as an analytical tool to analyze group work in mathematical modelling is Hernandez-Martinez and Harth (2015).

Methodology

This paper is a qualitative study that focuses on a group of three second year upper secondary school students, aged 16-17, solving a MM task with DT. It is part of a study in progress that involves four secondary schools in southern Norway selected based on geographical accessibility, digital tool use, and a mathematics curriculum that supports MM. In each school participants were randomly selected (forming the focus group) from amongst the students who volunteered. The study was performed within lesson hours, and written informed consent was obtained from each student and all ethical requirements outlined by the Norwegian Centre for Research Data (NSD) were followed. Prior to the students solving the two MM tasks without any help (main activity), they solved similar MM tasks with the help of the teacher and researcher (introductory activity). The students solved the tasks in groups of 3-4 sharing one computer throughout the activities. Data was collected during the main activity for the whole study but for this paper I focus on the analysis of one group of students solving one of the two tasks in the main activity. These students had already covered topics on functions and equations (linear, quadratic, and exponential), and had prior experience of using DT in doing mathematics. The MM task given to the students is presented below.

Solar power car task: A car making company is launching a new solar powered car. Recent market research showed that one hundred people would buy the car for a selling price of €5000. Further, the market research showed that for every €100 price increase, people's interest in buying the car would decrease by one person. Find the best-selling price for the car, as to maximize the company's sales revenue. Send a letter explaining how you solved the problem to the company's sales manager (Mousoulides, 2011).

Data sources for the analysis consisted of 26 minutes of recorded conversations (in both English and Norwegian; video recordings) and computer activities (screen capture software). The video recordings provide evidence of the students' interactions during the activities; such evidence may reveal the working process of the students and their interactions with each other and the DT. The screen capture software complements the video recordings as it provides information about how the students solve the task on the computer. Dialogue between the students was transcribed verbatim and interaction with GeoGebra, calculator, and gestures (such as pointing to the screen) are described below using square brackets (e.g., [Rolf draws the graph $f(x) = -x + 100$]).

Data analysis

To address the research question, the episode was structured and analyzed according to the elements of the activity system. The analysis focused on students' language (suggestions, questions, answers, arguments, etc.) and actions (gestures and interaction with GeoGebra and calculator). The students are the subject of the activity, and the object of the activity is to solve a MM task. GeoGebra and calculator are the mediating artifacts/tools of the activity, and under these tools the concepts 'creating a shared goal' and 'observing and repairing divergencies' (Roschelle & Teasley, 1995; Granberg & Olsson, 2015) were given a special focus in looking into the influence of DT on subject-object interaction. To create a shared goal, the students have the facility (DT) to look at the same thing as they negotiate and agree on the appearance of the mathematical representation generated by the DT. They also might use the DT as a reference tool to visually demonstrate their individual ideas to one another. For instance, a student might suggest a function/equation to the peers and use GeoGebra to graphically represent this function. To observe and repair divergences, the DT is used to maintain shared knowledge and ideas through the verification of ideas or settling disagreement by performing tests, referencing, among others. Under the division of labour element, the concept 'personalizing of problems in group situations' (Lowrie, 2011) was given a special focus to look into the influence of DT at the division of labour mediating the community-object interaction. Personalizing of problems is based on an individual's interest, and this interest could be the problem-solving strategies (Yerushalmy, 2000) adopted or the choice of mathematical representation and representational types offered by the DT. The transcript was coded separately by the author and two other researchers (using the constructs above) and the definitions for the codes were negotiated leading to shared interpretations. I will, in the results section, present each of the elements of the activity system.

Results

The activity system (hereafter referred to as "activity") analysed in this paper is a group of students solving a MM task with DT. The elements of the activity are interrelated, but I will present them in a linear manner.

Subject of the activity

The group of three students (all pseudonyms) that was followed were: Thea, who was mainly in charge of the computer and the writings with paper-and-pencil; Rolf, who sometimes took charge of the computer; and Kåre, who was in charge of the calculator (on a mobile phone) and sometimes recording results with paper-and-pencil (these roles the students adopted where not assigned by the teacher). The teacher describes these three students as average in terms of performance (although there are hierarchies within bands of performance). These students shared a common problem space or object, and they used approximately 26 minutes to solve the task.

Community

The community of the activity was formed of students (the teacher and researcher were only visible in the introductory activity). The community was formed spontaneously and for the purpose of solving the MM task, and then dissolved.

Object of the activity

The object is the immediate goal of the activity, and it is assumed by the researcher that the object of the activity is to solve the MM task with DT and write a report. The students ratified this objective at one point in their engagement with the task. For instance, Thea ratified the main object of the activity as she pointed out the need to find the maximum revenue of the company. This was done after the students had classified their variables (the people and the price of the car) in the problem text and agreed on the function $f(x) = -x + 100$ representing the number of people that buys the car. Below is part of the transcription, where participants ratify their objective:

Thea: Yeah. Good, now we are going to find out when the price is erm… when the maximum company sell is, so….

Mediating artefacts/tools for the subject-object interaction

The students used the following artefacts in solving the task: GeoGebra and calculator. Below is an example showing how the students worked on the task with GeoGebra:

Kåre: Like this [points to the x and y axis in GeoGebra, draws a graph with paper-and-pencil and writes $f(x) = 100x$ representing the graph].

Thea: Erm no, then you say that erm… its going down with a 1000… If you understand.

Kåre: So, it will be naturally in there, right? 'Konstantled' [constant term] or something?

…

Thea: Yeah, it's going to be on the x-axis, it's not a constant. Do you have any idea? [Thea asks Rolf if he has any idea] … If we try… I just try something [draw the graph of the function $f(x) = -x + 100$ in GeoGebra]. Erm, it goes down by one person, if we just try, I don't think this is the right...

Rolf: It could be true.

Thea: Yeah, if we think that 5000 is zero then when [writes $x = 1$ in the algebra section in GeoGebra].

Rolf: Should be 99 or something.

Kåre: So, what you are showing here is erm… you lose one person per 100.

Thea: So, if we write [labels the x-axis as 'Start at €5000, and 1 is equal to 100£ in price' and the y-axis as 'People']. So, if we start at €5000 we have 100 people, and if we go down with the price at €100 then its 99, and that's what it says here [points at the point A = (1,99) with the cursor on the graph].

Kåre: So basically, the graph is showing that it will decline, how much it will decline based on prices and how many people.

The students negotiated and agreed on the function representing the number of people that buy the car. The students agreed on a shared goal through a flow of turn taking, dialogue, and action [in brackets]. They made a linear graph with the number of people on the y-axis and the price of car on the x-axis. They started from 0 which is equivalent to 5000 euros on the x-axis and every 1 point on the x-axis is equivalent to 100 euros (so, $x = 3$ is 5300). They also made a function $f(x) = -x + 100$ to represent the number of people, and so when $y = 70$ (people) it intersects the function f at $x = 30$. That is 70 people will buy the car at 8000 (30 times 100 plus 5000) euros, and later used the calculator to find the total revenue when the figures are large (e.g., the product of 70 and 8000). In the dialogue above, GeoGebra was used as a reference tool to visualize one's reasoning during the mathematical discourse. To visually demonstrate their individual reasoning to one another, they used GeoGebra as a reference tool by pointing to the coordinate axis and sketching with paper-and-pencil ($f(x) = 100x$) in relation to the coordinate axis. Thea responding to the initial function proposed by Kåre, used GeoGebra to visually demonstrate her suggested function $f(x) = -x + 100$. At a point, the students found themselves in a situation where they wanted an efficient way to find the company's maximum revenue. The except below describes the group interaction:

Rolf:	But isn't it like a faster way to find that out. I feel like there is, but I don't have any idea how to do it.
Thea:	We can make sliders, I think… We can try.
Kåre:	I don't know.
Thea:	Erm [makes a slider $a = 100$, but the slider has no effect on the function $f(x) = -x + 100$] … We just try something else [writes $y = 60$ on the graph and found the point of intersection with the line $f(x) = -x + 100$]. Here 40 multiplied with 100, 4000 so it's not more. So, I think we should try ...
Rolf:	Try 100.

GeoGebra could be used to maintain and improve shared ideas in group interactions. In the dialogue above, Rolf observed the solution strategy and felt there is a fastest way to find out the maximum revenue but could not visually demonstrate his ideas. This could be that Rolf wasn't confident enough to put forward his thought, however this triggers Thea to come up with the idea of making sliders. Thea made a slider, but it did not have an effect on the graph as the slider ($a = 100$) has no link with the function ($f(x) = -x + 100$), and as such they returned to the strategy they begin with. There wasn't any divergence in the strategy they began with, as the strategy adopted only needed an improvement to be more efficient. That is, GeoGebra might offer the possibility of observing and repairing divergences or improving solution strategies in group interactions if the function and the slider under construction mathematically linked (e.g., $f(x) = -x + 100$, $x = a$, eq1: $x = a$ and an intersection point $A = \text{Intersect}\ (f, \text{eq1})$).

Division of labour for the community-object interaction

Forms of distribution of actions and operations among the students were identified. The students had roles that were constant throughout the activities (described under the subject of the activity section). Other roles in the form of leading, questioning and challenging, suggesting, among others changed at different times in the course of the activity. Thea mostly took the leading role (dominating in the communications), and the group discourse was centered around her input on the computer, whilst Kåre mostly agreed with her ideas and sometimes questioned them if they were not clearly understood. Rolf mostly observed and came in at times to give a suggestion. Thea started with a problem-solving strategy that she was comfortable with, that is starting with graphical representation and later analyzing patterns of numbers and observing the increment in revenue. Rolf comes in with an idea to efficiently generate the data, but Thea had personalized the problem-solving strategy, see the example below:

Rolf:	Oh! we could have done all of it with the 'regneark'(spreadsheet).
Thea:	Yeah, that's right.
Rolf:	And then just try with the
Thea:	We didn't think about it.
Rolf:	Or we just… I mean we can do it now; it might take a shorter time.
Thea:	Do you think?
Rolf:	I think so.
Thea:	But we are already done, though.

Thea dismissed Rolf's comments and went back to the existing idea thinking they are already close to the answer (which might also be time factor). Subscribing to Rolf's suggestions might have helped the group in generating their data with the spreadsheet and find a function that represent the data. The features of GeoGebra allows multiple problem-solving strategies, however the approach used by the group depends on the representational choice of the student taking the leading role, especially when they think they are close to finding the answer.

Rules for the subject-community interaction

The rules of the activity are sets of conditions that influence how/why the participants might act within the activities. The average time of 20 minutes used in solving a single task during the introduction section, had influence on the students as they solve the MM task. Another rule is the dismissing of comments or suggestions when they don't fit into the current strategy. Looking at the second dialogue under the subsection 'mediating artefacts/tools for the subject-object interaction', the students first accepted the new idea of introducing a slider ($a = 100$) and rejected it after trying it out and not yielding any better results.

Discussion and conclusion

The research question in this paper was: How do DT as mediating artefacts influence students' group interactions as they solve a MM task? This question is addressed from a CHAT perspective. Even though this is a work in progress, and that it is limited to one episode of 26 minutes, the findings of this study might help to advance current knowledge in this field both from a theoretical and empirical point of view. CHAT helps in interpreting interactions that occur in group activity. The CHAT elements, that is the community (a group of students), the rules (time constraint, working in a group, accepting or rejecting an idea, among others), the division of labour (students' roles determined by their involvement with the mediating artefacts), mediating artefacts (GeoGebra and calculator), subject (three average upper secondary students) and object (solving a MM task) are seen as a whole, or as collective system interacting with each other, in contrast to cognitive approaches focusing on heuristics and modelling processes. From a CHAT perspective, the student-student interactions are directed by the individual's engagement with the mediating artefacts, which influence the outcome of the activity. From an empirical point of view, the study revealed important findings in comparison to previous research, which can be summarized as follows. The role taken by a group member shows the commitment of the participants displayed in generating a solution during the modelling activities. The student taking the leading role in the group interaction is mostly taking charge of what happens on the computer. Although the students are described as average performing students, it is possible that hierarchies within the bands of performance exist as Esmonde (2009) highlights the trend that high-achieving students often tend to dominate in group interactions. Lowrie (2011) argues that in such circumstances, the responses of other students are generally influenced by the idea and strategies that have been formulated by the dominant student. The DT became a shared working space in which students' actions were situated. The DT served as a reference tool as the students visually demonstrated their reasoning during the group interactions (Granberg & Olsson, 2015). DT provides the opportunity of observing and improving solution strategies (Granberg & Olsson, 2015), but in our case the students observed and had an idea, yet they weren't able to improve the solution strategy resulting from the inability to link the functions constructed (that is, the slider having no effect on the function). The student taking the leading role (dominant student) had a problem-solving strategy similar to what Yerushalmy (2000) reports as starting with graphical representation and analyzing patterns of numbers. Students taking the leading role might reject new ideas if they think they are close to finding an answer by their strategy. This echoes previous research that highlights personalizing of problems hindering the potential for sophisticated sense-making that could lead to a better outcome of the activity (Lowrie, 2011). Drawing students' attention on not personalizing the problem-solving strategy and taking input of peers into consideration might benefit students' learning and achievement in MM with DT. Hernandez-Martinez and Harth (2015) argues that new ideas introduced in group interaction are of little use if they are not specific and/or have no connection with the groups' understanding of the problem; one of the reasons of rejecting a new idea could be that the new ideas are either communicated without confidence or not clear enough to connect with the group's current thinking. This draw back could also be the results of time constraint. Caviola et al. (2017, p. 7) argues that time constraint might "interfere with decision making by altering strategy selection" in problem solving. Studying the influence of DT in group interactions requires further study (e.g., from Affordance theory perspective), and is the scope of my ongoing PhD study.

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