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# Implied Volatility

A theoretical study on explaining the stylized facts of implied volatility using the utility indifference model.

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# Abstract

The breakthrough Black-Scholes (BS) model predicts a horizontal line when plotting the implied volatility (IV) against the strike price. However, empirical studies uncovered that the implied volatility derived from option market prices in the BS model varies with strike prices and time to maturity, leading to the identification of three stylized facts that the BS model fails to explain. First, the IV curves exhibit a smile/smirk pattern, with an upward-sloping term structure for at-the-money options. Second, option prices tend to reflect higher implied volatility compared to the realized volatility of asset returns. Third, the negative skewness implied by options prices is greater in absolute terms compared with the realized skewness. Consequently, numerous sophisticated models have been developed to address these stylized facts. Nevertheless, traditional models often fall short of fully explaining all aspects of these phenomena. This study introduces a novel approach to the utility indifference model by incorporating behavioral utility functions to provide a more accurate representation of these anomalies. To evaluate the model's performance, the standard function used in expected utility theory and behavioral utility functions are tested under both normal and Normal Inverse Gaussian (NIG) distributions. The findings indicate that the conventional utility function fails to capture the observed smirk patterns. In contrast, the behavioral utility function generates the IV smirks that closely align with empirical shapes, even under the normal distribution. These results highlight the effectiveness of the utility indifference model with behavioral utility functions in explaining these stylized facts that standard models struggle to reproduce.

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# Chapter 1

# Introduction

The seminal work of Black & Scholes (1973) (BS) represented a significant advancement in the development of option pricing models. This framework continues to be one of the most successful and widely adopted applications to date, with market option prices frequently quoted in terms of its implied volatility (IV). The implied volatility of an option contract is defined as the volatility of the underlying instrument which, when entered into e.g., the BS formula, returns a theoretical value equal to the current market price of the option (McDonald, 2014). According to the BS model, plotting the IV as a function of the strike price results in a horizontal line. However, empirical observations have revealed significant deviations from this expectation, leading to the identification of stylized facts that the BS model cannot explain.

The most prominent stylized facts are as follows: firstly, market prices for out-of-the-money put options and in-the-money call options tend to be higher than the prices predicted by the BS model, known as the IV smirk (Rubinstein, 1985). Graphically, the BS implied volatility values exhibit a U-shaped pattern prior to the 1987 stock market crash and a smirk pattern post-crash when plotted against the strike price (Bates, 2000). Furthermore, the implied volatility term structure for at-the-money options is upward-sloping (Zhang & Xiang, 2008). Secondly, implied volatility in option prices typically exceeds realized volatility of the underlying asset returns. This disparity between realized and implied volatility is termed the volatility risk premium (Jackwerth & Rubinstein, 1996). Thirdly, the negative skewness implied by options prices is greater in absolute terms compared with the realized skewness, and the spread in skewness between the two measures is referred to as the skewness risk premium (Bakshi et al., 2003).

The primary theoretical explanations for those anomalies revolve around the simplifying assumptions that underlie the BS model. Specifically, the model assumes that the probability distribution of the underlying asset log returns is normal. However, it is well known that the stock return distributions exhibit a left skew and higher kurtosis compared to normally distributed returns. In order to reconcile the inconsistency between the BS model and empirical observations, numerous more realistic alternative models have been introduced in the option pricing literature.

Merton (1976) introduced the jump-diffusion model, which enhances the underlying price dynamics of the BS model with a "Poisson-driven" jump process that accounts for rare events. By considering volatility as a stochastic process, Heston (1993) proposed a stochastic volatility model that captures the negative correlation between the instantaneous volatility and stock price, as well as the leverage effect. However, the jump-diffusion model falls short in accurately fitting long-maturity data, while the stochastic volatility model struggles to generate accurate IV smirk for short maturities and produces an incorrect term structure. Additionally, neither of these models can adequately account for volatility and skewness risk premiums. Eberlein & Prause (2002) utilized the four-parameter NIG class distribution to derive an option pricing formula under the risk-neutral measure that performs better in fitting the observed IV smirk, nevertheless, it may not account for other stylized facts. Inspired by behavioral finance, Suzuki et al. (2009) developed an agent-based equilibrium model with exponential-type utility functions to explain those empirical puzzles using loss aversion of heterogeneous agents. However, the term structure of their model remains unclear, and it is unknown whether the estimated magnitude of the risk premia aligns with the market data. This observation indicates that the current body of literature does not offer a comprehensive option pricing model that can incorporate all of the observed stylized facts.

To mitigate the limitations of existing option pricing models, this study introduces a novel approach to the utility indifference model by incorporating behavioral utility functions to more accurately capture the three stylized facts of IV. The novelty of the study lies in its application of behavioral utility functions for utility-based option pricing, which is an unexplored area of research in the literature.

The utility indifference model is founded on the equilibrium principle that option prices can be determined by identifying the price at which the representative investor is indifferent between investing exclusively in stocks or in both stocks and put options to maximize expected utility. At the equilibrium option price, the representative investor neither buys nor sells options, resulting in an optimal quantity of options of zero and thereby maintaining a zero net supply of options. Given the absence of analytical solutions for the model, even with the standard utility function within the expected utility theory, numerical methods are employed to obtain option prices. Furthermore, the measures of volatility and skewness in the physical and risk-neutral (estimated) contexts are compared to determine the risk premiums for volatility and skewness. This thesis evaluates the performance of the model by testing it employing both the standard function used in expected utility theory and various behavioral utility functions, considering both normal and NIG distributions.

The findings reveal that the conventional utility function falls short in capturing the observed smirk patterns. In contrast, the behavioral utility function successfully generates IV smirks that closely resemble the empirical shapes, even when applied to the normal distribution. Moreover, the model reveals its ability to account for upward IV term structure, as well as the NIG distribution forms more pronounced smirk patterns across all utility functions. The results obtained from the evaluation of volatility and skewness risk premia provide support for the model's capability to generate negative risk premiums for both volatility and skewness which is in line with the existing literature. In addition, the results suggest that the $n$ degree lower partial moment (LPM) behavioral utility function, where $n$ ranges from 1 to 2, generates risk premia that are in close accordance with the empirically observed values. These findings emphasize the effectiveness of the utility indifference model with behavioral utility functions in successfully explaining these stylized facts, while standard models fall short in reproducing them. By overcoming the limitations of standard models, this research offers valuable insights into understanding the dynamics of implied volatility. The findings are expected to have important implications for option pricing, risk management, and investment decision-making.

The remaining thesis is structured as follows. Chapter 2 reviews the previous literature. The model is presented in Chapter 3. Chapter 4 provides the numerical results of the model and the concluding remarks is given in Chapter 5.

# Chapter 2

# Literature review

Relevant literature explaining the stylized facts of implied volatility, namely, implied volatility surface, volatility risk premium, and skewness risk premium is reviewed in this chapter. The most common theoretical explanation lies in the fact that the probability distribution of the underlying asset log returns deviates significantly from the normality assumed in the BS formula, rather is skewed to the left and additionally exhibits higher kurtosis. As a result, several more sophisticated option pricing models have been developed in the literature, primarily involving generalizations of the BS framework, to overcome these empirical anomalies. The organization of this chapter follows these empirical statistical regularities accordingly.

## 2.1 Implied volatility surface

The estimates of implied volatility typically depend on the option's strike price and time to maturity, suggesting that the implied volatility surface as a collection of implied volatilities can be expressed as a two-dimensional function of strike price (IV curve) and expiration time (term structure) (Cont & da Fonseca, 2002). Mathematically, this implied volatility surface can be obtained by inverting option market prices for the volatility parameter in the BS model (Rubinstein, 1994; Dumas et al., 1998; Das & Sundaram, 1999), which reflects the return probability distribution of the underlying asset in different time horizons in a risk-neutral world (Zhang & Xiang, 2008). Quoting options directly in terms of implied volatilities is a common practice since the implied volatility surface contains all the knowledge about market option prices, enabling direct comparisons and relative valuation.

Under the BS framework, a strong assumption states that the underlying price process follows geometric Brownian motion with constant volatility and a Gaussian return distribution. Therefore, the implied volatility of all options on the same underlying asset should be identical regardless of strike price and time to maturity. Graphically, the shape of the implied volatility surface for an option should be the plane at the height of the constant volatility value, that is, implied volatility as a function of the strike price (moneyness) of an option ought to be flat and maintained a constant slope over different expiration time, with no term structure.

In stark contrast, it is widely observed that the implied volatilities derived by the BS model from option market prices for the same underlying asset at the same point in time fluctuate with both strike price and time to expiration. In other words, a U-shaped (smile) IV curve (before the 1987 stock market crash), where deep in-the-money and deep out-of-the-money options have higher implied volatilities than at-the-money options, or the smirk pattern (following the market crash), in which implied volatility decreases monotonically as strike price increases, emerges in empirical data. Moreover, these IV curves extend to nonflat surfaces as time-to-maturity changes, i.e., the shape of the IV curve is more pronounced in short maturities and flattens out in long maturities (e.g., Rubinstein, 1985; Das & Sundaram, 1999; Bates, 2000; Chen et al., 2016). The stylized fact of the smile/smirk pattern is generally interpreted as the presence of high excess kurtosis and negative skewness in the risk-neutral distribution of the underlying return (e.g., Jackwerth & Rubinstein, 1996; Bakshi et al., 2003; Carr & Wu, 2003; Bates, 2022). The term structure of implied volatility, on the other hand, reflects investors' market fluctuation expectations on the distribution of asset returns over different time horizons (Ap Gwilym & Buckle, 1997; Zhang & Xiang, 2008). The invalidation of the BS framework, especially after the 1987 market crash, has attracted widespread attention in the financial literature in an effort to construct alternative option pricing models that could account for stylized facts.

In order to reconcile the inconsistency between the BS model and empirical observations, a considerable number of more realistic models have been proposed in the option pricing literature. These models most often model the implicit distribution of underlying asset returns using jump-diffusion, stochastic volatility, and other non-Gaussian distributions, such as the Normal Inverse Gaussian (NIG) distribution.

Acknowledging the leptokurtic character of asset returns in the short run, Merton (1976) first presented the jump-diffusion model by augmenting the underlying price dynamics of the BS model with a "Poisson-driven" jump process that captures rare events. This model was later extended by Kou (2002) and Kou & Wang (2004) to a double exponential jump-diffusion model through a compound Poisson process to achieve a more pronounced asymmetric leptokurtic character, along with analytical tractability with path-dependent options. However, these exponential Lévy class models assume a finite number of shocks, and independent increments lead to zero autocorrelation in the return series, suggesting the absence of volatility clustering (as in the BS model), even though they are able to produce excess kurtosis and skewness in compliance with market option data (Bates, 2000; Carr & Wu, 2003; Broadie & Detemple, 2004). Consequently, the considerable kurtosis/skewness generated by the jump component does not always persist over the medium- or long-term expirations, that is, the shape of the implied volatility surface levels off with time to maturity faster than empirical evidence would suggest (Das & Sundaram, 1999; Lorig & Lozano-Carbassé, 2015). This result prompts researchers to view the dynamics of underlying volatility as a stochastic rather than a constant process.

The stochastic volatility option pricing model generalizes the BS pricing formula in a way that models the joint process of stock returns and instantaneous volatility allowing for randomness. Early studies examining random variance option pricing include Hull & White (1987), Scott (1987), and Wiggins (1987). While the previous two papers assumed that stochastic volatility was independent of stock returns, the work done by Wiggins (1987) allowed for an imperfect correlation between volatility and stock returns. However, these studies can only provide numerical solutions. The Heston (1993) model compensates for the shortcomings of previous models by providing a quasi-closed-form solution and is capable of capturing the nature of the negative correlation between the changes in volatility and returns, as well as the leverage effect, making it the most well-known and popular of all stochastic volatility models. Nonetheless, a sizeable amount of literature provides empirical evidence indicating that the Heston model is misspecified (lack of jumps in asset returns) due to its affine square-root structure when modeling stochastic volatility (e.g., Bakshi et al., 1997; Aït-Sahalia & Kimmel, 2007; Broadie et al., 2007; Christoffersen et al., 2010; Zhang et al., 2017). Thus, the IV curve created by the Heston model is too shallow at short-term maturities compared with the empirically expressed implied volatility surface, even though the smile/smirk pattern persists at long-term maturities (Bakshi et al., 1997; Das & Sundaram, 1999; Jones, 2003; Gatheral, 2006). Furthermore, it fails to match the term structure of implied volatilities generated by market data (Pan, 2002). What this reveals is the need to build a more realistic model that goes beyond stochastic volatility models.

One strand in the literature accounts for those empirical anomalies by reconstructing the risk-neutral density of the underlying asset using non-Gaussian distributions. Among them, a semiparametric option pricing model was introduced by Jarrow & Rudd (1982), in which they approximated the lognormal distribution of the underlying asset prices according to a generalized Edgeworth series expansion. Later, Corrado & Su (1996) derived an analogous model, with the major difference that they modeled the distribution of underlying returns using an A-type Gram-Charlier series expansion of the normal density function for the convenience of illustration. Both models extend the BS option pricing formula to address the biases invoked by non-normal skewness and kurtosis in the underlying security distribution. The additional terms added to the BS formula improve pricing accuracy and align the models more closely with market option data. However, negative probability values may be returned when the skewness and kurtosis coefficient values fed into their model are high, which is often observed in practice due to extreme events.

On the other hand, Eberlein & Prause (2002) exploit attractive properties of the four-parameter NIG class distribution, such as computational simplicity, flexibility, estimability of parameters, and sufficiency as a density, to reproduce the risk-neutral probability density of the underlying log returns. The authors proposed a generalized hyperbolic model as an asset pricing model, which is capable of generating distributions that more accurately approximate observed empirical distributions. In addition, they developed an option pricing formula under the risk-neutral measure that effectively accounts for the "smile effect" produced by the BS model.

## 2.2 Volatility risk premium

The at-the-money volatility implied in option prices (risk-neutral volatility) is widely acknowledged to exhibit an upward bias compared to the ex-post realized volatility of asset returns (physical volatility) across different time horizons, excluding the period that includes the 1987 stock market crash (Jackwerth & Rubinstein, 1996; Poon & Granger, 2003; Jiang & Tian, 2005). This disparity between realized and implied volatilities is commonly referred to as the volatility risk premium. Notably, researchers have identified the negative volatility risk premium as the prominent explanation for this observed bias (e.g., Bakshi & Kapadia, 2003; Engle, 2004; Zhao et al., 2013). Intuitively, all else being equal, when the market price of volatility risk is negative, the drift of the risk-neutral volatility process increases, thereby inflating option prices. Economically, the incentive to hedge against market depreciation makes investors willing to pay more to hold options (bear negative expected returns), which in turn makes options more expensive than they would be if volatility were unpriced (Jackwerth & Rubinstein, 1996; Bakshi & Kapadia, 2003).

This stylized fact suggests that, in addition to the fundamental price risk of the underlying, there is a risk premium component in option prices associated with other risk factors. The two most dominant risk factors contributing to this risk premium are stochastic volatility and jumps (Buraschi & Jackwerth, 2001). According to Pan (2002), this implied volatility bias arises when neither stochastic volatility risk nor jump risk is priced in an option pricing model. This is the case for the BS model, where, by construction, no risks related to higher moments of the underlying asset returns are priced since the asset return dynamics follow a two-parameter geometric Brownian motion (Coval & Shumway, 2001).

Numerous scholars have extensively investigated the role of stochastic volatility risk in explaining the discrepancy between realized and implied volatilities using various methods, including parameter estimation (Chernov & Ghysels, 2000; Andersen et al., 2002; Ederington & Guan, 2013), hedging performance (Bakshi & Kapadia, 2003), replicating strategies (Carr & Wu, 2009), and Monte Carlo simulations (Rambharat & Brockwell, 2011). Despite employing different methodologies, these studies consistently demonstrate that stochastic volatility risk is a key driver of the negative market prices of volatility risk premiums embedded in equity index options, providing the explanation for the observed discrepancy.

In contrast to the emphasis on volatility risk, an alternative perspective posits the significance of the jump risk factor in explaining the observed stylized fact, with a negligible impact from volatility risk. Notable studies in this area include Guo (1998), Bates (2000), Pan (2002), Eraker (2004), Broadie et al. (2007), and Christoffersen et al. (2012), among others. This conclusion is attributed to the model specification error and discretization error associated with those stochastic volatility risk studies. Pan (2002) argues that the research on stochastic volatility risk through the parametric method may be affected by model specification errors, implying that the presence of a significant volatility risk premium relies substantially on the choice of a model. Therefore, improperly specified models can result in misleading conclusions. The same argument is also drawn by Broadie et al. (2007). Furthermore, Branger & Schlag (2008) point out that, in those surveys using model-free methods, discretization errors may appear on top of misspecification errors, which would lead to unreliable inferences.

Earlier literature, such as the jump-diffusion model of Merton (1976), as well as the stochastic volatility models of Hull & White (1987), Scott (1987), and Wiggins (1987), usually assumes a zero risk premium, according to Bates (2003). In contrast, Heston's (1993) model allows for a volatility risk premium, but it is the negative correlation between its asset volatility and returns rather than the stochastic property itself that shapes skewness (Dumas et al., 1998). Furthermore, the Heston model suffers from model specification issues, leading to unclear risk premium results, and empirical evidence shows that even with a non-zero volatility risk premium, it is still unable to properly fit real data (Bates, 2000; Pan, 2002). This suggests that the negative volatility risk premium cannot be explained by the jump-diffusion model of Merton (1976) or the stochastic volatility model of Heston (1993). Non-Gaussian models do not have an explicit risk premium parameter in their model, as they focus on considering the effect of higher-order moments under the risk-neutral measure. In fact, theoretical research has shown that the higher moments of the physical return (especially negative skewness) are the primary attributes of the volatility risk premium (Bakshi & Madan, 2006; Chabi-Yo, 2012). Overall, the conventional models are unable to successfully explain this stylized fact.

## 2.3 Skewness risk premium

Another consistently observed empirical anomaly is the greater absolute value of negative skewness implied by options prices (risk-neutral skewness) compared to the realized skewness (physical skewness), particularly more pronounced in indices following the 1987 stock market crash (Bates, 1991; Rubinstein, 1994; Bakshi et al., 2003). Graphically, risk-neutral skewness is associated with the slope of the IV curve (Zhang & Xiang, 2008), indicating that the IV curve (to the left) has a higher slope than that implied by the realized skewness. This study adopts the term skewness risk premium, following the terminology of the volatility risk premium, to describe the spread between risk-neutral skewness and physical skewness. It implies that the tail fears perceived by investors are generally not reflected in the historically estimated skewness and the need to hedge against them (Finta & Aboura, 2020). This observed stylized fact has attracted considerable research attention, leading to numerous studies aimed at providing an explanation.

Analytical and empirical examining of models performed in a model-free manner is a general approach among researchers. Building on Bakshi & Madan's (2000) fundamental results showing that it is feasible to span and price the risk-neutral payoff by option positioning across different strike prices, Bakshi et al. (2003) theoretically demonstrated how higher moments of risk-neutral return density can be retrieved from option portfolios. An explicit expression of risk-neutral skewness of index returns related to its physical counterpart was given as well. Moreover, they empirically confirmed a negative skewness risk premium rooted in leptokurtic physical index density and risk aversion. Zhao et al. (2013) exploited Zhang & Xiang's (2008) method by contrasting higher-order cumulants under the risk-neutral measure (inferred from index options) with those obtained from the physical measure. Their analysis attributed the skewness risk premium to jump risk, supported by empirical evidence. However, this model-free approach is not free from limitations. When converting risk-neutral moments into physical moments, assumptions need to be made about the relationship between them (Langlois, 2020). In particular, variations in physical moments and risk premiums are both affected by variations in risk-neutral moments (Chang et al., 2013).

Previous studies have established that the skewness premium is attributable to higher moments of return density and risk aversion (Bakshi et al., 2003; Bakshi & Madan, 2006) and jump risk (Zhao et al., 2013). Further, it is well noted that modeling with symmetric stochastic volatility and jumps are only able to produce fat tails, while leverage and asymmetric jumps can introduce skewness (Bates, 2022). Thereby, the ability of Merton's (1976) jump-diffusion model and Heston's (1993) stochastic volatility model to explain this stylized fact can be ruled out. Specifically, Merton (1976) assumes a diversifiable symmetric jump process that leaves the same jump parameters in risk-neutral and physical densities (Zhang et al., 2012). Meanwhile, Heston's (1993) model lacks the richness to generate an asymmetric distribution (Bakshi & Kapadia, 2003). Non-Gaussian models may fail to reproduce skewness premium as well. First, from a parametric perspective, there is no wedge between risk-neutral and physical density in the Gram-Charlier-based models, making it impossible to estimate the skewness under the two distributions. Second, when a nonparametric approach requires integral operations on strike prices to recover skewness contract prices, non-Gaussian models will not be able to handle it since the estimation accuracy for non-Gaussian settings is unknown (Feunou et al., 2017). The preceding discussion highlights the inability of traditional models to explain this observed stylized fact.

## 2.4 Utility-based models

A frequently observed limitation of stochastic volatility and jump-diffusion models is the exogenous nature of their asset price and volatility process. Consequently, the factors that shape the price process and their corresponding effects on option pricing remain uncertain. In contrast, utility-based equilibrium models exhibit better explanatory power for stylized facts where the model's price process is given endogenously by the dynamics of supply and demand. Additionally, these models take into account the heterogeneity of trader types and their respective risk preferences.

Within this strand, one approach attempts to account for those empirical regularities by exploiting a pricing kernel as a linkage between risk-neutral and physical probability distributions (converting physical density to its risk-neutral counterpart). Since the market under their assumed simple economy is incomplete, such an approach usually employs a general equilibrium setting that involves a utility function. Using a representative investor with a constant relative risk averse (CRRA) utility, Fu & Yang

(2012) explained implied volatility smirk, negative volatility risk premium, and negative skewness risk premium (with a negative sign and greater absolute value of risk-neutral skewness) in the general case of a jump-diffusion model with a Lévy process. Their option pricing model can be regarded as a modification of Merton's (1976) model by including a jump process in the risk-neutral density. Furthermore, their pricing kernel is proven to be identical to the option pricing formula provided by Bakshi & Madan (2000) in a model-free manner (discussed above). The volatility smirk and volatility and skewness risk premiums are attributed to risk aversion. However, the term structure of the implied volatility in their model is unspecified and it is uncertain whether the estimated level of risk premia corresponds to the actual market data. In the same spirit, Li et al. (2017) subsequently generalized Fu & Yang's (2012) model by incorporating stochastic volatility to the jump component and obtains a similar result.

Inspired by behavioral finance theory, an alternative approach involves using an agent-based simulation method. The equilibrium option pricing model used in the early study of Benninga & Mayshar (2000) and Ziegler (2002) incorporated the heterogeneous beliefs of investors and indicate that such heterogeneity could account for the observed volatility smiles implied by options. In similar efforts, Suzuki et al. (2009) proposed a prospect theory-based equilibrium model with exponential-type utility functions for different types of investors and illustrate that the loss aversion behavior of the heterogeneous agent can explain the implied volatility smile and the negative skewness risk premium. Furthermore, they implicitly account for the negative volatility risk premium. Liu et al. (2014) extended the micro-individual aspect of the previous models by taking into account the collective behavior of traders. The proposed model by the authors effectively replicates the observed empirical pattern of an asymmetric implied volatility curve, whereby an increase in the degree of collective behavior corresponds with a steeper slope of the curve.

## 2.5 Summary

The discrepancies between the BS model and empirical observations have prompted numerous studies attempting to construct more realistic models that can explain or capture stylized facts, such as implied volatility smirk and term structure, volatility risk premium, and skewness risk premium. Typical approaches for modeling the returns of underlying assets involve the use of jump-diffusion, stochastic volatility, and non-Gaussian distribution models, which aim to incorporate stochastic volatility and jump risks. However, jump-diffusion models fail to fit long-maturity data, and stochastic volatility models cannot generate correct IV curves at short maturities. Nor can they account for volatility and skewness risk premiums. Although non-Gaussian models can better fit the observed IV surface, they are incapable of explaining the other two stylized facts. It is worth emphasizing that while certain models can generate IV curves with the correct shape, their predicted level is only half of the empirical shape (Ghysels et al., 1996). Utility-based models, which take into account the microscopic view of investor behavior with various types of investors, offer a better explanation for those stylized facts. However, the IV term structure of these models remains unclear, and it is uncertain whether the estimated magnitude of the risk premia is in accordance with market estimates. Evidently, the existing literature lacks a comprehensive model that can successfully explain all aspects of these stylized facts. This study aims to bridge this gap by proposing a novel approach based on the utility indifference argument, in which a representative investor is endowed with a loss aversion utility function. To the best of the authors' knowledge, this approach has not been explored previously in the literature and offers a promising framework for capturing these empirical regularities.

# Chapter 3

# Option pricing models

To facilitate the assessment of models' ability to account for stylized facts, this chapter presents several option pricing models for evaluating European-style option contracts, which can in turn provide a direct comparison with the utility indifference model. The BS option pricing formula is given primary focus as it lays the foundation for the other models. However, the model's assumption of normally distributed log returns fails to generate the observed implied volatility in real markets, highlighting the need for improved models. To address this issue, this thesis also covers Merton's (1976) jump-diffusion, Heston's (1993) stochastic volatility, and Eberlein & Prause's (2002) NIG model, given their significant role in the literature. Finally, the chapter concludes by shifting its focus to the utility indifference model.

## 3.1 Theoretical models

### 3.1.1 The BS model

The Black & Scholes (1973) option pricing formula is utilized to ascertain the theoretical value of derivative securities with regard to the price of other investment vehicles. This formula is grounded in the fundamental principle of risk elimination (through delta-hedging options) via the creation of a self-financing, risk-free portfolio that involves purchasing one option and short-selling a portion of the underlying asset. The no-arbitrage argument underpins the BS model, forming its very foundation.

Under the model assumption, the process for the stock price $S$ follows a geometric

Brownian motion:

$$\frac{dS}{S} = \mu dt + \sigma dZ, \tag{3.1}$$

where $\mu$ is the mean logarithmic return on the stock, $\sigma$ is constant stock volatility, and $Z$ is the Brownian motion that captures the potential uncertainty in the market.

The strategy is to create a self-financing risk-free portfolio with a return equal to the continuously compounded risk-free rate of return, $r$, in order to prevent arbitrage opportunities (risk-neutral). Denote by $V(S(t), t)$ the current value of the security with underlying $S$ which pays $V(S(T), T)$ at maturity. By applying Itô's Lemma, one can derive the famous Black-Scholes partial differential equation (PDE) that the value process of any derivative security must satisfy:

$$V_t + rSV_S + \frac{1}{2}\sigma^2 S^2 V_{SS} = rV, \tag{3.2}$$

along with the boundary conditions of a European put option $V(S(T), T) = P(T) = (K - S(T))^+$, where $K$ is the strike price. The value of the put $P$ options on a non-dividend paying asset can be shown as:

$$P(S, K, \sigma, r, T) = Ke^{-rT} \times N(-d_2) - S \times N(-d_1), \tag{3.3}$$

where $N(\cdot)$ is the cumulative normal distribution function, and

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}},$$
$$d_2 = d_1 - \sigma\sqrt{T}.$$

Alternatively, a straightforward application of the put-call parity for European options on non-dividend paying assets can also yield the put option equation. The call $(C)$ and put options with the same expiration date and strike prices are linked by the non-arbitrage condition that states:

$$C + Ke^{-rT} = P + S. \tag{3.4}$$

Merton (1973b) further demonstrated the validity of the BS model under relaxed assumptions, including stochastic interest rates and the presence of dividend-paying stocks. The BS pricing framework continues to be the predominant option pricing model, as evidenced by the widespread use of implicit volatility as the standard method for quoting options prices. The BS formula can also be interpreted as pricing the option in a manner that achieves its equilibrium expected return. Furthermore, the BS option price strictly increases relative to volatility, which establishes a unique correspondence between price and volatility (Chen et al., 2016).

### 3.1.2 The Merton model

To account for jumps in stock prices, the Merton (1976) jump-diffusion model expands upon the BS stock price dynamics by incorporating a Poisson jump process. As a result, the model generates stock price returns through a combination of these two processes, which can be expressed by a stochastic differential equation:

$$\frac{dS}{S} = (\mu - \lambda k)dt + \sigma dZ + dq, \tag{3.5}$$

where $q(t)$ is the Poisson process with a constant jump intensity $\lambda$ and a random jump magnitude $Y$, independent of $dZ$, $dq$ is 0 when no jump occurs, and $Y - 1$ otherwise. In addition, the occurrence of a jump has a probability of $\lambda dt$. $k = \mathrm{E}(Y-1)$ is the expected percentage jump of $S$.

The inclusion of discrete jumps in the continuous stock process creates an incomplete market, as these jumps cannot be hedged using derivatives. Consequently, the option payoff structure can not be replicated, and pricing becomes impossible. To overcome this challenge, Merton made the assumption that the jump risk is non-systematic and therefore diversifiable. The Black-Scholes PDE then becomes:

$$V_t + (r - \lambda k)SV_S + \frac{1}{2}\sigma^2 S^2 V_{SS} + \lambda \mathrm{E}_Y[V(SY,t) - V(S,t)] = rV. \tag{3.6}$$

Assuming $i$ independent and identically distributed (i.i.d.) jumps follow a common lognormal distribution with mean $\mu_J$ and variance $\sigma_J^2$, that is, $\ln(Y) \sim \mathcal{N}(\mu_J, \sigma_J^2)$, and incorporating boundary conditions, Merton demonstrated that the price of a European call option can be written as:

$$C(S, K, \sigma, r, T; \mu_J, \sigma_J, \lambda) = \sum_{i=0}^{\infty} \frac{e^{-\lambda' T}(\lambda' T)^i}{i!} \text{BSCall}(S, K, \sigma_i, r_i, T), \tag{3.7}$$

where

$$\lambda' \equiv \lambda e^{\mu_J},$$
$$\sigma_i \equiv \sqrt{\sigma^2 + \frac{i\sigma_J^2}{T}},$$
$$r_i \equiv r - \lambda k + \frac{i\mu_J}{T}.$$

Equation (3.7) expresses the option value as the expected value of European option prices, which takes into account the likelihood of a certain number of jumps occurring (McDonald, 2014). Assigning a negative value to the expected jump size parameter $k$ creates a thicker left tail in comparison to the lognormal distribution. However, this in turn leads to a decline in implied volatility (Broadie & Detemple, 2004). Furthermore, the European put option pricing formula can be obtained using put-call parity.

### 3.1.3 The Heston model

The Heston (1993) model allows the volatility to vary stochastically while remaining correlated with the stock returns. Assume the spot price dynamics follows the process:

$$\frac{dS}{S} = \mu dt + \sqrt{v(t)} dZ_1, \tag{3.8}$$

where $v(t)$ is the stochastic return variance (instantaneous) that follows a mean-reverting square-root process:

$$dv(t) = k[\theta - v(t)]dt + \sigma_v \sqrt{v(t)} dZ_2, \tag{3.9}$$

where $v(t)$ approaches its long-run mean leval $\theta$ at a rate of $k$. $\sigma_v$ is the volatility of the variance, and $Z_1$ and $Z_2$ are correlated with a coeffient $\rho$. The above equation also produces the volatility clustering effect (Broadie & Detemple, 2004).

Given the assumed stock price and volatility process, the Black-Scholes PDE for $V(S(t), v(t), t)$ is then as follows:

$$\frac{1}{2}v(t)S^2V_{SS} + \frac{1}{2}\sigma_v^2 v(t)V_{vv} + \rho v(t)\sigma_v S V_{Sv}$$
$$+ rSV_S + \{k[\theta - v(t)] - \lambda(S, v, t)\}V_v + V_t = rV, \tag{3.10}$$

where $\lambda(S, v, t)$ denotes the variance risk premium assumed to be proportional to $v(t)$, that is, $\lambda(S, v, t) = \lambda v(t)$, for analytical convenience. As there are two sources of risk, namely stock price risk ($dZ_1$) and volatility risk ($dZ_2$), and no asset exists to hedge volatility, this model resorts to an equilibrium approach to price the option contract. Consequently, the variance process under the risk-neutral measure can be expressed as:

$$\begin{aligned} dv(t) &= \{k[\theta - v(t)] - \lambda v(t)\}dt + \sigma_v\sqrt{v(t)}dZ_2^* \\ &= k^*[\theta^* - v(t)]dt + \sigma_v\sqrt{v(t)}dZ_2^*, \end{aligned} \tag{3.11}$$

where $k^* = k + \lambda$ and $\theta^* = k\theta/(k + \lambda)$ provide the mapping between the physical parameters and their risk-neutral counterparts. The Heston model is the stochastic variance model of the Equation (3.11). Note that in order to implement or test this model, it is necessary to estimate four parameters: $\rho, \sigma_v, k$, and $\theta$, as well as to filter the spot variance $v(t)$.

Heston (1993) demonstrated that the square-root stochastic volatility model permits an analytical option pricing formula, where the necessary probabilities are obtained by numerically integrating the conditional characteristic function of the underlying asset price using the Fourier inversion technique. Additionally, he observes that the presence of a correlation between volatility and the spot price is crucial to produce skewness. In the absence of such correlation, stochastic volatility only affects the kurtosis.

### 3.1.4 The NIG model

Eberlein & Prause (2002) developed an option pricing model based on the premise that in a risk-neutral world, the stock return should be equivalent to the risk-free rate of return. To accomplish this, they employed the NIG distribution, which is the unconditional distribution of a random variable $X$ under the assumption that $X$ is normally distributed with mean $a + bY$ and conditional on the variance $Y$ following an Inverse Gaussian distribution.

The NIG density can be expressed by the scale-invariant parameters as follows:

$$f(x; \alpha, \beta, \eta t, \delta t) = \frac{\delta \alpha t e^{\delta t \varphi + \beta(x - \eta t)}}{\pi \sqrt{(\delta t)^2 + (x - \eta t)^2}} K_1\Big(\alpha \sqrt{(\delta t)^2 + (x - \eta t)^2}\Big), \tag{3.12}$$

where

$$\varphi = \sqrt{\alpha^2 - \beta^2},$$

and $\alpha > 0$, $0 < |\beta| < \alpha$, are the shape parameters that determine the steepness and asymmetry, respectively; $\eta \in \mathbb{R}$, is the location parameter; $\delta > 0$, is the scale parameter; and $K_1(\cdot)$ represents the modified Bessel function of the third kind of index 1. This distribution can be fully characterized by its first four moments and has the ability to produce heavier tails and more pronounced skewness, which are consistent with real-world data.

The first four moments of the NIG distribution, the mean ($\mu_x$), variance ($\sigma_x^2$), skewness ($\mathcal{S}$), and excess kurtosis ($\mathcal{K}$) of $x$ are given by:

$$\begin{aligned} \mu_x &= \eta t + \delta t \frac{\beta}{\varphi}, \quad \sigma_x^2 = \delta t \frac{\alpha^2}{\varphi^3}, \\ \mathcal{S} &= 3 \frac{\beta}{\alpha \sqrt{\delta t \varphi}}, \quad \mathcal{K} = \frac{3}{\delta t \varphi}\Big(1 + 4\Big(\frac{\beta}{\alpha}\Big)^2\Big). \end{aligned} \tag{3.13}$$

The NIG parameters can be solved explicitly according to Equation (3.13). Assuming $t = 1$, one can obtain:

$$\beta = \frac{\sigma_x \mathcal{S} \theta^2}{3}, \quad \alpha = \sqrt{\beta^2 + \theta^2}, \quad \delta = \frac{\sigma_x^2 \theta^2}{\theta^2 + \beta^2}, \quad \eta = \mu_x - \frac{\beta \delta}{\theta}, \tag{3.14}$$

where

$$\theta = \frac{3}{\sigma_x \sqrt{3\mathcal{K} - 5\mathcal{S}^2}},$$

and $3\mathcal{K} - 5\mathcal{S}^2 > 0$ to acquire meaningful parameters for the NIG distribution.

The put option price $P$ can be written as:

$$P = e^{-rT} K \Phi(\gamma) - e^{-rT - \omega T} S \int_{-\infty}^{\gamma} e^x f(x) dx, \tag{3.15}$$

where

$$\gamma = \ln(\frac{K}{S}) + \omega T,$$
$$\omega = \eta + \delta\Big(\varphi - \sqrt{\alpha^2 - (1+\beta)^2}\Big) - r.$$

The compensation term $\omega$ arises in the context of equivalent martingale measures, under which the arbitrage-free prices can be calculated as expectations with respect to these measures. $\Phi(\cdot)$ is the cumulative probability distribution function of $x$.

## 3.2 The utility indifference model

This section presents the utility indifference model, an alternative approach to price options contracts, which was proposed by Samuelson & Merton (1969) prior to the celebrated BS model. In this study, the utility indifference model expands upon the original model by incorporating a behavioral utility function for the representative investor. To obtain option prices, numerical methods are employed. Additionally, the section provides methodologies for estimating volatility and skewness risk premiums.

### 3.2.1 Model setup

Suppose a financial market consists of three assets: a stock index, a put option on the stock index, and a risk-free asset. The risk-free asset yields a continuously compounded risk-free rate of return $r$, and the stock pays no dividends. Given that the stock price at time 0 is $S_0$, the stock price at future time $T$ is:

$$S_T = S_0 e^x, \tag{3.16}$$

where $x$ is the stock log return, which is a random variable. Let the probability density function of $x$ be denoted as $f(x)$. The model considers two densities: the first is the normal distribution, which is given by

$$f(x, \mu T, \sigma^2 T) = \frac{1}{\sigma\sqrt{2\pi T}} e^{-\frac{(\mu T - x)^2}{2\sigma^2 T}}, \tag{3.17}$$

where $\mu T$ and $\sigma^2 T$ are the mean and variance of $x$, and the second is the NIG distribution, which is given by the probability density function in Equation (3.12).

To determine the utility indifference option price, consider two portfolio choices in which the investor optimally distributes wealth among the three assets (i.e., the stock, the put option, and the risk-free asset). Let $W_0$ and $P_0$ represent wealth and put option price at time 0, respectively. The objective is to ascertain the value of the unknown variable, $P_0$. The first portfolio contains stocks and risk-free assets, in which the investor purchases $a$ shares of stocks, and the rest $B$ is deposited in the bank. The bank account balance is then:

$$B = W_0 - aS_0,$$

the future wealth of the investor at time $T$ becomes:

$$W_T = aS_0(e^x - e^{rT}) + W_0 e^{rT}.$$

Let $U$ denote the investor's utility, which is defined in terms of ultimate wealth $W_T$, that is, $U(W_T)$, the expected utility is then $E[U(W_T)]$. The goal of the investor is to choose the optimal value of $a$ that maximizes the expected utility:

$$\begin{aligned} J(W_0, S_0) &= \max_a E[U(W_T)] \\ &= \max_a E[U(aS_0(e^x - e^{rT}) + W_0 e^{rT})]. \end{aligned} \tag{3.18}$$

The second portfolio includes all three assets, that is, the investor allocates his/her wealth to $a$ shares of stocks, $b$ put options on the stock, and the remaining wealth $B$ is placed in the bank. The sum of money in the bank is then:

$$B = W_0 - aS_0 - bP_0,$$

the investor's wealth at future time $T$ is equal to:

$$W_T = aS_T + b\max(K - S_T, 0) + (W_0 - aS_0 - bP_0)e^{rT}.$$

The objective of the investor is to select $a$ and $b$ so as to maximize the expected utility:

$$V(W_0, S_0, P_0) = \max_{a,b} E[U(W_T)]$$
$$= \max_{a,b} E[U(aS_T + b\max(K - S_T, 0) + (W_0 - aS_0 - bP_0)e^{rT})]. \quad (3.19)$$

The utility indifference option price can be then derived from the following relationship:

$$V(W_0, S_0, P_0) = J(W_0, S_0). \quad (3.20)$$

This equation defines the indifference option price $P_0$, which represents the price where the investor is indifferent between investing solely in stocks and investing in both stocks and put options. It is worth noting that at the equilibrium option price, the optimal number of options in the portfolio is required to be zero, $b = 0$.

### 3.2.2 Utility functions

This thesis considers various types of utility functions, including the standard function used in expected utility theory, as well as behavioral utility functions. The axiomatic expected utility theory, developed by Von Neumann & Morgenstern (1944) serves as a normative framework for rational decision-making. The theory posits that expected utility is a weighted average of the utilities of all possible outcomes, where the weights are the probabilities of each outcome. Moreover, the utility function in this model is concave, implying that individuals' preferences are risk-averse, and their choices are consistent with maximizing expected utility.

The CRRA utility function, the standard function used in the expected utility theory, defined over the investor's final wealth $W_T$, can be represented using Arrow (1971) relative risk aversion measure as follows:

$$U(W_T) = \begin{cases} \frac{W_T^{1-\gamma}}{1-\gamma} & \gamma > 0, \gamma \neq 1, \\ \ln(W_T) & \gamma = 1, \end{cases} \quad (3.21)$$

where $\gamma$ is the "relative risk aversion coefficient". A higher value of $\gamma$ indicates a greater degree of risk aversion. The CRRA utility function is defined for outcomes that are expressed as a "proportion of wealth", and it maintains a constant rate of relative risk aversion. However, the expected utility theory has a limited ability in capturing decision-making patterns, prompting the development of behavioral utility functions that can account for observed departures from expected utility maximization.

Behavioral utility functions are widely recognized as a more realistic approach to decision-making since they incorporate the irrational aspect of human behavior. Unlike the expected utility theory, which defines the utility function solely over an investor's final wealth, behavioral utility functions introduce a reference point that separates outcomes into domains of losses and gains. By doing so, these functions account for loss aversion and provide a more accurate representation of an investor's risk preferences (Barberis & Thaler, 2003).

The conventional method of measuring risk by means of the variance (or standard deviation) of returns is inadequate when the return distribution deviates from normality (Jarrow & Zhao, 2006). To address this shortcoming, Bawa (1975) and Fishburn (1977) proposed the lower partial moment (LPM) as a substitute risk measure for variance. One notable advantage of this approach is that it concentrates on downside risk by disregarding the right tail of the probability distribution, making it more suitable for investors who are primarily concerned with minimizing their losses. The Mean-LPMn utility function of Fishburn (1977) expressed as expected utility as follows:

$$E[U(W_T)] = E[W_T] - \lambda \times LPMn(W_T, R), \tag{3.22}$$

where

$$LPM_n(W_T, R) = \int_{R>W_T} (R - W_T)^n dF$$

indicates the lower partial moment of order $n$ of the distribution of $W_T$. Additionally, by varying $n$, the risk measure can capture investor utility towards both risk and losses. In this thesis $n \in \{1, 2\}$. $\lambda$ is the loss aversion coefficient, $R = W_0 e^{\kappa T}$ is the reference level. Typically, $\kappa$ is assumed to be zero, thereby establishing the reference point as the "status quo".

Finally, the Prospect Theory (PT) utility function of Kahneman & Tversky (1979,

1992) is defined as:

$$U(W_T) = \begin{cases} (W_T - R)^\alpha & \text{if } W_T > R, \\ -\lambda(R - W_T)^\alpha & \text{otherwise,} \end{cases} \tag{3.23}$$

where $R$ and $\lambda$ are defined the same as in Equation (3.22). The downside aversion is reflected in the curvature of the utility function at reference point $R$, where a kink is typically observed.

The standard definition of the PT utility in Equation (3.23) can be presented as the Mean-LPMn utility:

$$E[U(W_T)] = UPM_\alpha(W_T, R) - \lambda \times LPM_\alpha(W_T, R), \tag{3.24}$$

where $UPM_\alpha(W_T, R)$ is the upper partial moment of $W_T$ with respect to $R$. Compared to the expected utility theory, investors with PT utility functions based on Prospect Theory exhibit a stronger tendency towards loss aversion, which is characterized by a steeper decline in utility for losses than for gains.

Unfortunately, there is no analytical solution available for maximizing the expected utility problem of Equations of (3.18) and (3.19), therefore, numerical methods need to be relied upon. In addition, one needs to search for the value of $P_0$ that satisfies Equation (3.20).

### 3.2.3 Numerical optimization

The R package Rsolnp is employed to address the non-linear optimization problems in Equations (3.18) and (3.19). This package is designed to solve general nonlinear inequality constraints problems:

$$\min_{\mathbf{x}} \quad f(\mathbf{x})$$

$$s.t.$$

$$g(\mathbf{x}) = \mathbf{B}$$

$$\mathbf{L} \leq h(\mathbf{x}) \leq \mathbf{U}$$

$$\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$$

where $\mathbf{x}$ is the vector of optimization parameters, vector $\mathbf{B}$ defines the equality constraints, and vectors $\mathbf{L}, \mathbf{U}, \mathbf{l}$, and $\mathbf{u}$ define the lower and the upper bounds on the inequality function $h(\mathbf{x})$ and vector $\mathbf{x}$, respectively. To tackle the optimization problems, the following constraints are imposed: $a \geq 0, b \geq 0$, and $B \geq 0$. That is, selling short the stock, selling put options, and borrowing money at the risk-free rate are not permitted.

To illustrate the implementation of numerical optimization, consider the Mean-LPM$_1$ (the bilinear mean-expected loss) utility. Its expected utility function can be written as:

$$E[U(W_T)] = E[W_T] - \lambda \times LPM_1(W_T, R). \tag{3.25}$$

When dealing with only stocks and risk-free assets, the objective function that needs to be maximized is:

$$\max_{a} \quad E[W_T] - \lambda \times LPM_1(W_T, R). \tag{3.26}$$

The solutions for $E[W_T]$ is represented by the following equations:

$$E[W_T] = aS_0 \int_{-\infty}^{\infty} e^x f(x)dx + (W_0 - aS_0)e^{rT}. \tag{3.27}$$

The solution for $LPM_1(W_T, R)$ is contingent upon the condition:

$$W_0(e^{rT} - e^{\kappa T}) - aS_0 e^{rT} > 0, \tag{3.28}$$

assuming $\kappa \leq r$. If the condition (3.28) is met, then $LPM_1(W_T, R) = 0$, otherwise:

$$LPM_1(W_T, R) = \int_{-\infty}^{\delta} (W_0(e^{\kappa T} - e^{rT}) - aS_0(e^x - e^{rT}))f(x)dx, \tag{3.29}$$

where

$$\delta = \ln\Big(\frac{W_0}{aS_0}(e^{\kappa T} - e^{rT}) + e^{rT}\Big).$$

Numerical integration can be used to evaluate the integral equations described in (3.27) and (3.29). The optimization problem presented in (3.26) can be solved using Rsolnp, taking into account the aforementioned constraints.

In a similar vein, when a put option is included in addition to stocks and risk-free assets, the objective function to maximize becomes:

$$\max_{a,b} \quad E[W_T] - \lambda \times LPM_1(W_T, R), \tag{3.30}$$

solutions to $E[W_T]$ and $LPM_1(W_T, R)$ in this scenario are expressed as:

$$E[W_T] = aS_0 \int_{-\infty}^{\infty} e^x f(x)dx + (W_0 - aS_0 - bP_0)e^{rT} + b\int_{-\infty}^{d} (K - S_0e^x)f(x)dx, \tag{3.31}$$

$$\begin{aligned} LPM_1(W_T, R) = I_b &\int_{d_1^b}^{d_2^b} (A - bK - (a-b)S_0e^x)f(x)dx \\ + I_a &\int_{d_1^a}^{d_2^a} (A - aS_0e^x)f(x)dx, \end{aligned} \tag{3.32}$$

where

$$d = \ln(\frac{K}{S_0}),$$

$$A = W_0(e^{\kappa T} - e^{rT}) + (aS_0 + bP_0)e^{rT}.$$

The indicator function $I_b$ takes the value of 1 if $R - W_T$ can be positive when $x < d$, and 0 otherwise. In this case, $d_1^b$ and $d_2^b$ represent the lower and upper limits for $x$. Similarly, $I_a$ is an indicator function that evaluates to 1 when $R - W_T$ can take positive values for $x > d$, and 0 otherwise. Here, $d_1^a$ and $d_2^a$ denote the lower and upper limits for $x$. The price of the put option is determined by finding the value of $P_0$ in Equation (3.30) that produces the same maximum value as Equation (3.26).

### 3.2.4 Risk premia measurement

The risk premiums for volatility and skewness are determined by comparing the measures of volatility and skewness in the physical and risk-neutral (estimated) contexts. Specifically, the physical volatility ($\sigma_x$), skewness ($\mathcal{S}$), and excess kurtosis ($\mathcal{K}$) values are set to 18.5% (annual), -0.5 (monthly), and 1 (monthly), respectively, to facilitate comparisons with prior empirical findings. Additionally, these parameters represent the real-world characteristics of the S&P 500 index. To derive corresponding risk-neutral measures of volatility and skewness, this study employs a parametric approach by fitting the implied volatility of the model to the NIG option pricing model.

The estimation of the risk-neutral measure in the literature is typically conducted using market option prices and a model-free approach. However, in this study, the option prices are derived from a specific assumption about the physical probability distibution, the NIG distribution. Therefore, it is natural to employ a parametric approach.

The choice of the NIG option pricing model as the benchmark is motivated by two factors. Firstly, the model is based on the NIG distribution, which possesses a significant advantage over other non-Gaussian distributions in that the parameters can be specified by the distribution's first four moments. Secondly, the NIG option pricing model computes option prices using the physical volatility and skewness of the underlying asset, without requiring any additional adjustments. The only modification made to the stock price process is to adjust the drift parameter to the risk-free rate of return, to generate arbitrage-free option prices.

Assume that the underlying asset log return process adheres to a NIG distribution in accordance with Equation (3.12), and the option prices are computed utilizing the NIG option pricing model as given by Equation (3.15). This approach to estimating the risk-neutral volatility and skewness involves fitting the implied volatility of the utility indifference model for a fixed time to maturity to the implied volatility generated by the NIG option pricing model for the same period. As a result, the estimated values of the volatility and skewness under the risk-neutral measure can be derived as the parameters of the NIG option pricing model that produce the same implied volatility as the utility indifference model.

Denoting the implied volatility obtained by the utility indifference model and the NIG option pricing model as $IV$ and $IV^*$ respectively. Let $\sigma_{rn}$, $\mathcal{S}_{rn}$, and $\mathcal{K}_{rn}$ represent risk-neutral volatility, skewness, and excess kurtosis. The corresponding risk-neutral measures can be derived by minimizing the sum of the squared distance between the implied volatility generated by the two models with respect to these parameters. This can be expressed as follows:

$$\min_{\sigma_{rn},\mathcal{S}_{rn},\mathcal{K}_{rn}} \quad \sum_{i=0}^{N} \left( IV_i - IV_i^* \right)^2, \tag{3.33}$$

where $N$ is the moneyness. $IV$ and $IV^*$, as functions of $\sigma$, S, and K, are obtained by solving for the volatility parameter in the BS formula through inverse calculations of the option prices derived from the utility indifference model and NIG option pricing model, respectively.

The intertemporal asset pricing model of Merton (1973a) provides theoretical guidance on the sign of the volatility risk premium. According to this model, an increase in market volatility leads to a decrease in available investment opportunities. As a result, investors seek to hedge this deterioration by accepting lower expected returns on assets that are positively correlated with market volatility innovations. This results in a negative price of market volatility risk. Given that physical volatility is typically observed to be smaller than risk-neutral volatility, in this thesis, the volatility risk premium is defined as the difference between physical and risk-neutral volatility to be consistent with empirical results. Therefore, the volatility risk premium (VRP) can be expressed as follows:

$$VRP = \sigma_x - \sigma_{rn}. \tag{3.34}$$

Currently, there is no theoretical guidance available to determine the sign of the skewness risk premium, making its determination reliant on empirical analysis (Chang et al., 2013). However, empirical evidence has consistently shown that risk-neutral skewness tends to exceed physical skewness. To ensure the skewness risk premium retains a negative sign, which has been widely documented in the literature, this thesis defines it as the difference between risk-neutral and physical skewness. Therefore, the skewness risk premium (SRP) is given by:

$$SRP = \mathcal{S}_{rn} - \mathcal{S}. \tag{3.35}$$

It is worth noting that, in this approach, the risk-neutral volatility and skewness are jointly measured by calibrating the entire IV curve of the model to the NIG parameters, rather than just at the at-the-money point. However, defining the risk premia as the difference between at-the-money physical and risk-neutral measures is not a standard practice in the literature.

# Chapter 4

# Numerical results

This chapter presents the numerical results obtained from implementing the methodology introduced in the previous chapter, which includes the IV surface plots of the utility indifference model using various utility functions under both normal and NIG distributions, along with a table that contains the estimated values of volatility and skewness risk premiums. In addition, a detailed discussion of the results is provided, highlighting the key findings and implications for this research field. To offer context for the numerical results, the chapter begins by illustrating the numerical results of the theoretical models outlined in the preceding chapter.

## 4.1 Illustration of theoretical model results

### 4.1.1 Merton model results

Figure 4.1 exhibits the implied volatility of Merton's jump-diffusion model as a function of moneyness for four different times to expiration, $T \in \{0.25, 0.5, 0.75, 1\}$ years, with the given parameters $\sigma = 0.135, r = 0.03, \mu_J = -0.093, \sigma_J = 0.115$, and $\lambda = 1$. When the jump intensity parameter $\lambda$ equals 0, this option pricing model reduces to the BS model. The presence of a non-zero jump mean $\mu_J$ in the model results in the skewed distribution of the underlying log returns. Moreover, the stock uncertainty in this model is composed of a continuous Brownian motion and an independent Poisson jump process. As such, the total volatility is the sum of the volatilities from these two parts. To ensure consistency in the numerical computations, the parameters for the model are chosen such that the total volatility is equal to the standard volatility parameter, $\sigma_x = 0.2$, used throughout this thesis. The computation of the total volatility is executed in compliance with Navas (2003).

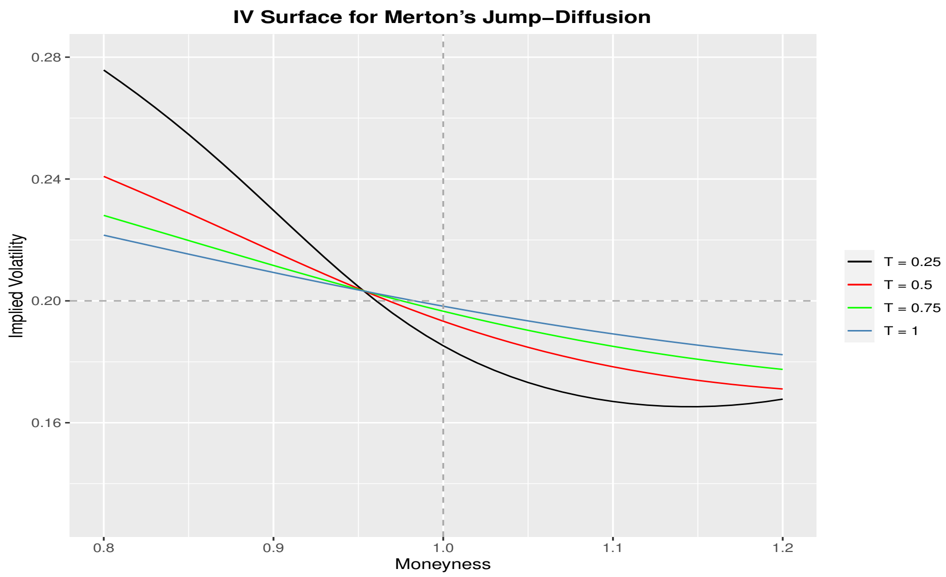


Figure 4.1: Implied volatility surface for Merton's jump-diffusion model. The dashed horizontal line represents a stock volatility of 20%. Moneyness is computed as the ratio of the option strike price (K) to the underlying stock price (S).

Graphically, Merton's jump-diffusion model is seen to generate the correct smirk pattern for a short expiration time, which is attributed to the presence of negative jumps (with non-zero jump mean) and resulting negative skewness in the implied stock return density. However, as the time to maturity increases, the IV curve flattens out more rapidly than empirical evidence would suggest. For instance, at a 0.75-year maturity, the IV curve practically reduces to a straight line. This feature aligns with prior research done by e.g., Carr & Wu (2003), Broadie & Detemple (2004), and Lorig & Lozano-Carbassé (2015). Furthermore, the at-the-money IV as a risk-neutral measure is less than the stock volatility of 20%, the physical measure, which is inconsistent with market data as reported in previous studies such as Jackwerth & Rubinstein (1996), Poon & Granger (2003), and Jiang & Tian (2005), among others. The implied volatility of Merton's jump-diffusion model is observed to be smaller than the realized volatility, implying a positive volatility risk premium. This result contrasts with the widely accepted negative volatility risk premium observed in the literature, which suggests that investors are willing to pay more for options to hedge against market depreciation, leading to expensive option prices and negative expected returns. Additionally, the term structure of implied volatility in Merton's model is consistent with empirical observations, where the implied volatility for at-the-money options increases as the time to expiration date rise (Heston & Nandi, 2000; Zhang & Xiang, 2008). Intuitively, the longer the time to expiration, the higher the option implied volatility (option price).

### 4.1.2 Heston model results

Following the parameters estimated in Bakshi et al. (1997) and Rompolis & Tzavalis (2007), that is, $r = 0.03, v = 0.04, \theta = 0.02, k = 1.15, \sigma_v = 0.39$, and $\rho = -0.64$, Figure 4.2 depicts the IV surface for the Heston stochastic volatility model at 3-month, 6-month, 9-month, and 12-month maturities. This model incorporates a negative correlation ($\rho$) between the changes in volatility and the spot price, which is necessary to generate skewness in the return distribution (Heston, 1993; Dumas et al., 1998). The kurtosis of the return distribution is largely determined by the magnitude of the volatility of variance ($\sigma_v$) relative to the mean-reverting speed ($k$), resulting in a heavier-tailed density compared to the lognormal distribution (Jones, 2003). However, the Heston model has been found to have limitations in fitting market data for short-term expiration due to its inability to generate sufficiently large skewness and kurtosis in the implied return distribution (Das & Sundaram, 1999; Zhang et al., 2017).

In contrast to Merton's jump-diffusion model, the IV curves of the Heston model do not level off at medium- or long-term expirations. The smirk shape is persistent at long-term maturities, as Figure 4.2 demonstrates. However, the short-term IV curves generated by the Heston model are often criticized for being too shallow compared to the empirically expressed IV surface. This shortcoming is attributed to the affine square-root structure used to model stochastic volatility, as pointed out by e.g., Bakshi et al. (1997), Jones (2003), and Gatheral (2006). In addition, similar to Merton's model, the at-the-money stock volatility ($\sigma_x = 0.2$) is greater than implied volatility for $T = 0.25$, which suggests a positive volatility risk premium contradicts the widely accepted notion of a negative one. Moreover, Pan (2002) noted that the Heston model also falls short in capturing the term structure of implied volatilities observed in the market, with the

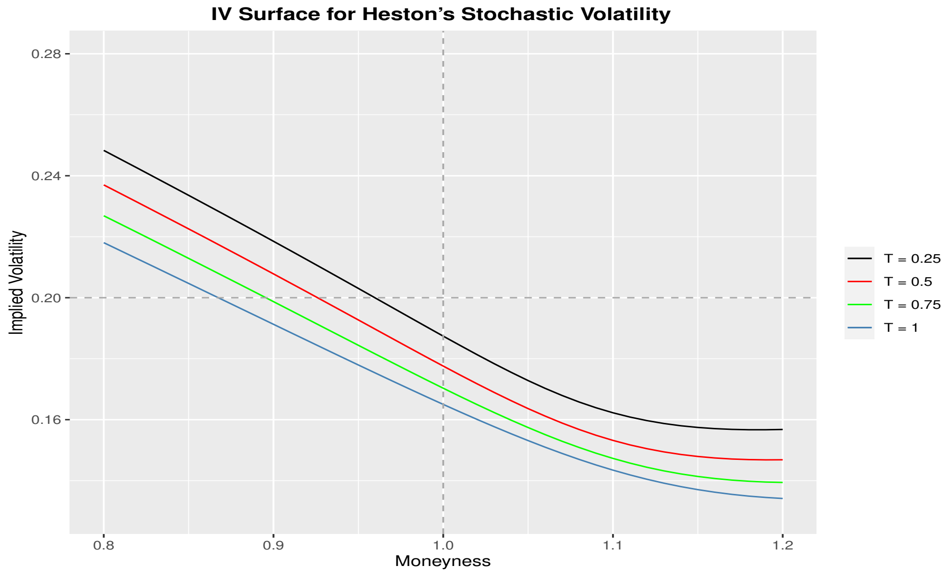


Figure 4.2: Implied volatility surface for Heston's stochastic volatility model. The dashed horizontal line represents a stock volatility of 20%. Moneyness is computed as the ratio of the option strike price (K) to the underlying stock price (S).

at-the-money implied volatility exhibiting a decreasing function of the time to maturity, as demonstrated in the figure. This implies that the option price (implied volatility) becomes cheaper as the time to maturity increases, which contradicts empirical evidence.

### 4.1.3 NIG model results

Figure 4.3 illustrates IV surface for Eberlein & Prause's (2002) NIG option pricing model using parameters $\mu_x = 0.08, \sigma_x = 0.2, \mathcal{S} = -0.5, \mathcal{K} = 1, r = 0.03$, and $T \in \{0.25, 0.5, 0.75, 1\}$ years. The appropriate values for the skewness ($\mathcal{S}$) and excess kurtosis ($\mathcal{K}$) parameters are crucial for modeling stock log returns accurately. In this study, these parameters were determined using a rigorous historical estimation process with long-term time series of daily return data from the S&P 500 stock market index. The resulting values of -0.5 for the annual skewness and 1 for excess kurtosis provide a reliable basis for parameterization and align with observed market dynamics, indicating a heavy-tailed and left-skewed distribution of the underlying stock log returns. Furthermore, if both the NIG skewness and excess kurtosis parameters are set to zero, the NIG option pricing model simplifies to the BS model. Additionally, Equation (3.12) reveals a direct proportionality between the mean and variance of the NIG distribution with respect to time $t$. As a result, the location parameter $\eta$ and the scale parameter $\delta$ are adjusted accordingly for different expiration times when computing numerical results.

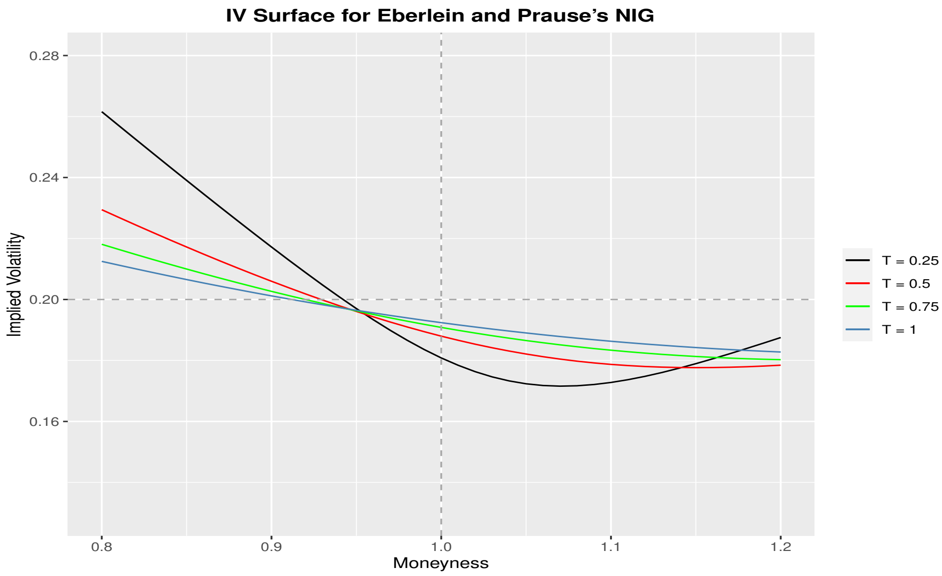


Figure 4.3: Implied volatility surface for Eberlein and Prause's NIG model with $\mathcal{S} = -0.5$ and $\mathcal{K} = 1$. The dashed horizontal line represents a stock volatility of 20%. Moneyness is computed as the ratio of the option strike price (K) to the underlying stock price (S).

Figure 4.3 shows a qualitative resemblance to Figure 4.1 of the Merton model, as both models exhibit similar features in their pricing processes. Specifically, the NIG model's pricing process is characterized by "purely discontinuous paths", which is considered more suitable for modeling the microstructure of asset prices compared to continuous processes (Eberlein & Prause, 2002). Additionally, similar to the Merton model, the NIG option pricing model displays a more pronounced smirk pattern for short expiration times, while the IV curves for longer expiration times tend to level off quickly. Furthermore, the at-the-money implied volatility of the NIG option pricing model is lower than the physical volatility for a 3-month maturity, as indicated by the black line that lies below the dashed line in the figure. This could lead to an incorrect positive volatility risk premium. However, despite this limitation, the term structure of at-the-money implied volatility of the NIG model is in line with the observed real-world data, suggesting that the model captures the underlying asset's volatility behavior over time.

## 4.2 Numerical results of the model

Table 4.1 itemizes the set of parameters used to obtain the numerical results for the utility indifference model with various utility functions. The values for the mean returns of $x$, volatility of the returns, skewness, and excess kurtosis are expressed in annual terms and are selected to be roughly in line with the long-term parameters of the S&P 500 stock market index. The time to maturity is set to $T \in \{0.25, 0.5, 0.75, 1\}$ years. In addition, the expected return for the lognormal distribution is computed as $\mu_x + \frac{1}{2}\sigma_x^2$, which corresponds to 10% based on the values of $\mu_x$ and $\sigma_x$ listed in Table 4.1. To maintain simplicity, a constant continuously compounded interest rate of 3% per annum is assumed and dividends are not considered.

Table 4.1: The standard set of parameters for the numerical computation of the model.

| Parameter | Value |
|---|---|
| $\sigma_x$ | 0.2 |
| $\mu_x$ | 0.08 |
| $\mathcal{S}$ | -0.5 |
| $\mathcal{K}$ | 1 |
| $\gamma$ | 2 |
| $\kappa$ | 0 |
| $\lambda$ | 3 |
| $\alpha$ | 0.88 |

### 4.2.1 Graphical illustration of the results

This subsection presents the numerical results of the utility indifference model, which is implemented using the methodology described in Section 3.2 and the parameters reported in Table 4.1. The utility functions are evaluated under both normal and NIG distributions, and the results are illustrated graphically through IV surface plots, as shown in Figure 4.4. These plots provide an intuitive representation of the implied volatility as a function of the option strike price and time to maturity and offer valuable insights into the model's performance.

Figure 4.4 demonstrates the IV surfaces generated by the utility indifference model, using four different utility functions, the CRRA, PT, Mean-LPM$_1$, and Mean-LPM$_2$, under both normal and NIG distributions. The plots are presented in a 4 by 2 matrix format, with each row corresponding to a specific utility function, and columns representing the normal and NIG distributions, respectively. The range of maturities covered in the plots includes 3-month intervals from 0.25 to 1 year. Furthermore, the y-axis has been set to a scale of 0.17 to 0.40 to facilitate easy comparison between the plots.

Taken together, the numerical results presented in the figures indicate that the model using the Mean-LPM$_2$ utility function creates the observed smirk curves even under the normal distribution. In general, the smirk shapes produced by the normal distribution are relatively shallow except for the Mean-LPM$_2$ utility, whereas the NIG distribution forms more pronounced smirk patterns across all utility functions. Furthermore, for both distributions, the CRRA utility function results in the flattest IV curves, followed by the PT and Mean-LPM$_1$ utilities, while the Mean-LPM$_2$ utility constructs the steepest surface. Excluding the CRRA utility under the normal distribution, all other plots exhibit an upward term structure of the at-the-money implied volatilities. Moreover, with the exception of the CRRA utility function under both normal and NIG distributions, the at-the-money implied volatilities consistently exceed the 20% physical volatility level for all maturities. This suggests that the utility indifference model with the mentioned utilities is capable of crafting a negative volatility risk premium.

The CRRA utility function is commonly used in expected utility theory to model investor preferences. In the implemented model, the relative risk aversion coefficient ($\gamma$) is set to 2, which is in accordance with the estimate reported by Fu & Yang (2012) and Lin et al. (2019). Under the normal distribution, the IV curves show four horizontal lines representing a scenario where the BS model with constant volatility is true. However, the implied volatility remains constant across different strike prices, indicating that this model specification is inadequate in capturing the empirically observed IV smirk in the options market. Conversely, the NIG distribution yields a satisfactory smirk pattern for options with a short-term expiration ($T = 0.25$) for a representative investor with a relatively low level of risk aversion. However, its at-the-money implied volatility is lower than the corresponding physical volatility, suggesting that this setting may not be sufficient to generate a negative volatility risk premium for short-dated options.

In contrast, the following three utility functions (PT, Mean-LPM$_1$, and Mean-LPM$_2$) fall under the behavior utility category, which underscores the originality of this thesis

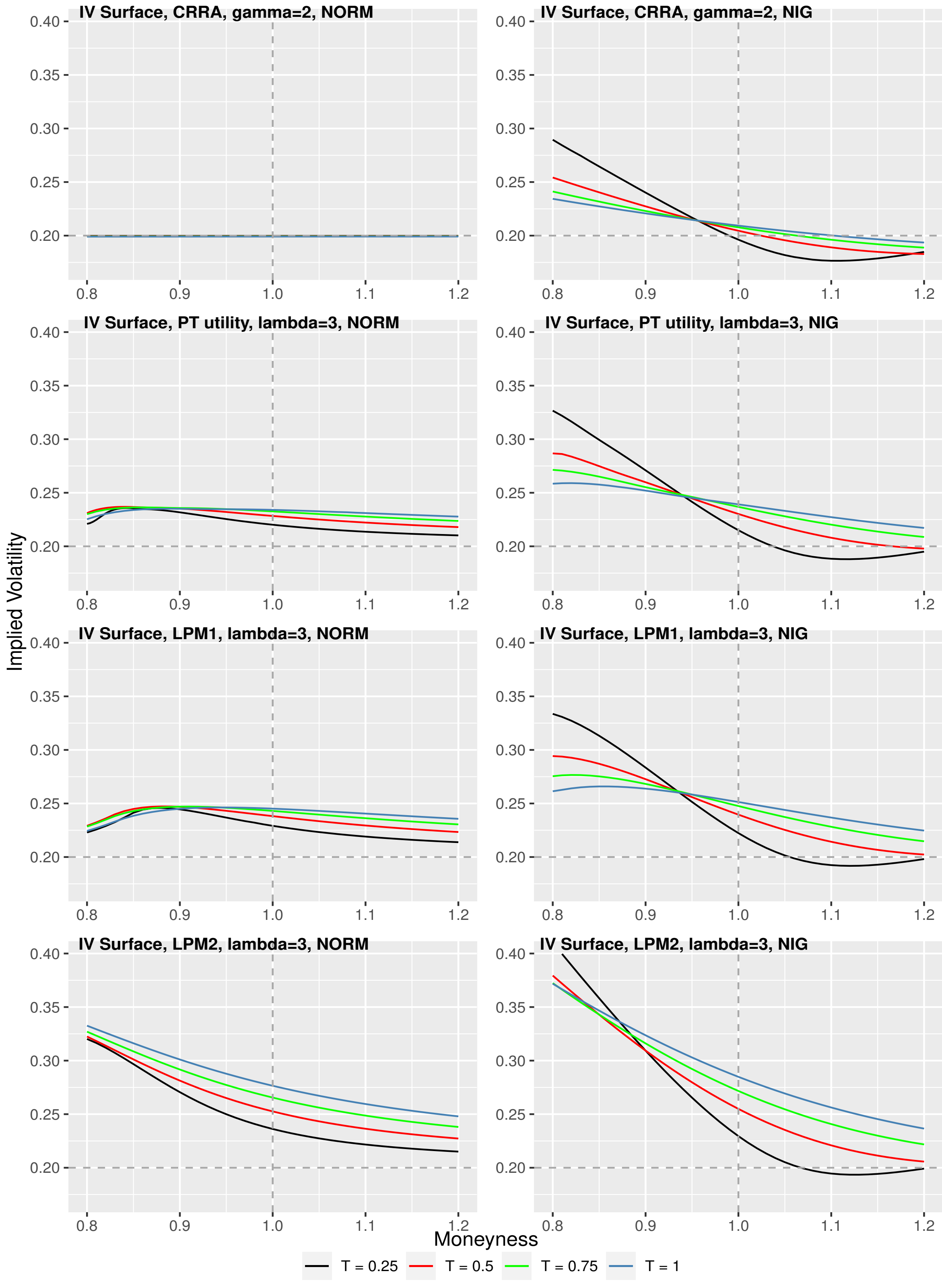


Figure 4.4: Implied volatility surfaces for the utility indifference model with various utility functions under both normal and NIG distributions. The dashed horizontal line represents a stock volatility of 20%. Moneyness is computed as the ratio of the option strike price (K) to the underlying stock price (S).

in using behavior utility functions for utility-based option pricing. The loss aversion coefficient ($\lambda$) for all three utility functions is assigned a value of 3, following the estimation of Abdellaoui et al. (2008), as a higher loss aversion coefficient leads to more robust numerical computations. For the PT utility function, the degree of the upper and lower partial moment ($n$) is set at 0.88, as used in the empirical study of Kahneman & Tversky (1992), which generates an IV surface resembling that of Mean-LPM$_1$, as illustrated in Figure 4.4. It is noteworthy that the PT utility function displays concavity for gains and convexity for losses relative to a reference level ($R$), whereas the Mean-LPM$_1$ utility function is characterized by linearity for both gains and losses with respect to the reference level. When considering the normal distribution, both the PT and Mean-LPM$_1$ utilities are capable of constructing negative volatility risk premiums and correct term structures for at-the-money implied volatilities. However, they are not adequate for producing significant IV smirk, even for short-term maturities. Nevertheless, compared to the PT utility, the Mean-LPM$_1$ function formulates steeper curves and higher implied volatility levels. When the NIG distribution is employed, both the PT and Mean-LPM$_1$ utility functions demonstrate satisfactory steepness in the IV smirk curve for the shortest maturity of 3 months, as their slope of the IV curve, quantified as the difference between the implied volatility at the maximum moneyness between 0.8 to 1 and that at moneyness 1, is greater than that of Merton's model (9.05%), which is widely acknowledged as capable of generating sufficient skewness for short-term maturities. However, the curvature decays rapidly for longer maturities in both cases.

The Mean-LPM$_2$ utility function presents a linear pattern for gains and a non-linear configuration for losses relative to a reference level ($R$). Compared to other utility functions, it generates the highest implied volatility levels under both the normal distribution and the NIG distribution, leading to a greater volatility risk premium at the at-the-money. The resulting smirk pattern persists as the time to maturity increases, in line with the Heston model. However, the IV slope of the Mean-LPM$_2$ under the normal distribution is lower than that of the Merton model. Meanwhile, under the NIG distribution, this utility function exhibits the strongest curvature for short-term maturities at 18.06%. This suggests that the utility indifference model with the Mean-LPM$_2$ utility under the NIG distribution can account for the empirical evidence by generating sufficient smirk for the short-term that persists over longer maturities.

### 4.2.2 Comparative statics analysis

In this subsection, a comparative statics analysis is conducted to examine the effect of changes in model parameters on implied volatility. Comparative statics analysis is a method used to assess how changes in model parameters affect the results by comparing the outcomes before and after the parameter changes. The analysis primarily focuses on the Mean-LPM$_2$ utility function under the NIG distribution with a time to maturity of 3 months. Furthermore, the dependence of implied volatility on the lower partial moment order ($n$) of the Mean-LPMn utility function is explored as well.

The comparative statics analysis is presented graphically in Figure 4.5. The plot in the top left shows IV curves of the Mean-LPM$_2$ utility function across different values of loss aversion coefficients ($\lambda$) ranging from 2 to 5. Interestingly, the results indicate that the Mean-LPM$_2$ utility is minimally impacted by variations in the loss aversion coefficient ($\lambda$). This can be attributed to the fact that the lower partial moment of order 2 already incorporates the investor's risk aversion and accounts for potential losses (Nawrocki, 1992). This point is further illustrated by the plot in the top right corner, which depicts the relationship between implied volatility and the degree of the lower partial moment, $n$, in the Mean-LPMn utility function. The plot includes specific values of $n$ for 1, 1.5, 2, and 2.5. As demonstrated by Fishburn (1977), the $n$ degree LPM effectively captures the investor utility concerning risk and loss, with higher values reflecting greater risk aversion. This is evident in the plot, where higher values of $n$ result in steeper IV curves, indicating increased risk/loss aversion. However, beyond a degree of 2, further raises in $n$ have little effect on implied volatility.

The bottom two plots display the IV curves of the Mean-LPM$_2$ utility function as they vary with different values of kurtosis (ranging from 3 to 6) and skewness (0, -0.25, -0.5, and -0.75). The plots reveal interesting patterns: when the underlying distribution is normal (with kurtosis of 3 and skewness of 0), the resulting IV curves exhibit a relatively flat shape, as indicated by the black curves in the plots. However, as the kurtosis and negative skewness increase, the IV curves become more pronounced with a steeper slope in both cases. Furthermore, the plots illustrate that kurtosis generates a symmetric IV smile, while negative skewness produces a smirk shape with a downward slope, which is consistent with the broad literature.

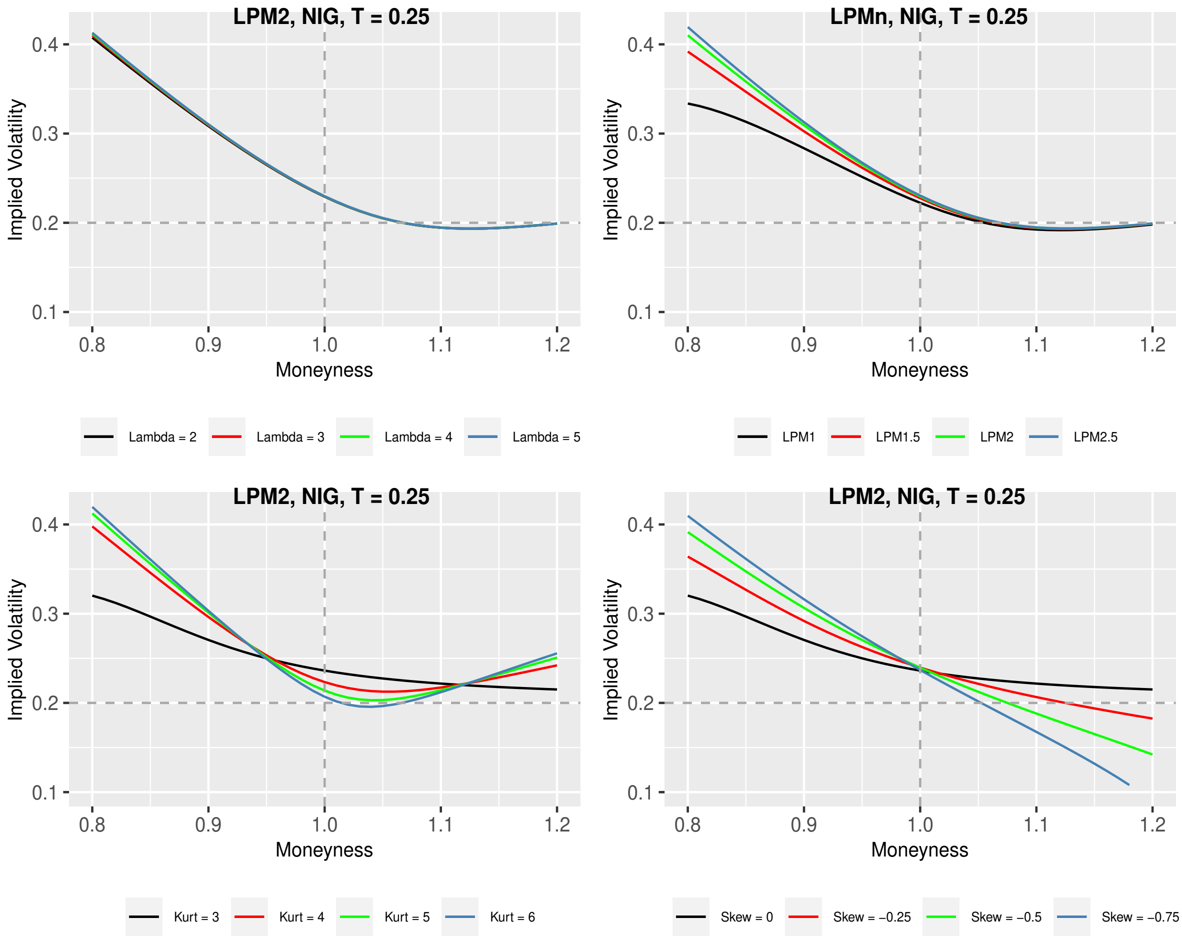


Figure 4.5: The comparative statics analysis plots. The upper-left panel depicts the IV curves of Mean-$LPM_2$ utility function as they vary with different loss aversion coefficients ($\lambda$). In the upper-right panel, the IV curves of the Mean-LPMn utility function are displayed, with various degrees ($n$) of the lower partial moment. The lower-left and lower-right panels illustrate the IV curves for Mean-$LPM_2$ corresponding to different kurtosis and skewness values, respectively. All the IV curves are plotted under the NIG distribution for T = 0.25. The dashed horizontal line represents a stock volatility of 20%. Moneyness is computed as the ratio of the option strike price (K) to the underlying stock price (S).

The results of this comparative statics analysis demonstrate the effectiveness of the utility indifference model with the Mean-LPMn behavioral utility function in generating IV curves that closely align with empirical observations. Moreover, these findings offer valuable insights into the integration of risk aversion and loss aversion through the Mean-LPMn utility function, which enhances the model's capacity to capture the dynamic nature of implied volatility.

### 4.2.3 Risk premia estimation of the model

In this subsection, the estimation results of volatility and skewness risk premia using the methodology outlined in Subsection 3.2.4 are reported in Table 4.3. These estimates will be discussed and compared with those from prior studies. To begin, empirical estimates of volatility and skewness risk premia documented in the literature will be provided.

Researchers commonly adopt a model-free approach to gauge the relationship between options and the underlying risk-neutral density. For instance, Zhao et al. (2013) utilized the methodology developed by Zhang & Xiang (2008) to measure the risk-neutral cumulants and estimated the volatility and skewness risk premia by comparing the risk-neutral and physical cumulants based on S&P 500 options data from 1996 to 2005, producing values of -5.2% (annually) and -0.67 (monthly), respectively. Similarly, Lin et al. (2019) estimated the risk-neutral moments using the Bakshi et al. (2003) method, with the physical moments derived from the jump-diffusion model with CRRA utility. Their estimates, using long-term S&P 500 options data spanning from 1990 to 2011, were -3.4% for an annual volatility risk premium and -3.39 for a monthly skewness risk premium. It should be noted that the estimation of volatility and skewness risk premia can be significantly influenced by the choice of estimation techniques and datasets, in addition to other factors. Table 4.2 provides a summary of these results. As this thesis focuses on the volatility risk premium rather than the variance risk premium addressed in previous literature, the VRP and SRP values reported in Table 4.2 are converted from variance to volatility measures accordingly.

Table 4.2: Previous estimation of the volatility and skewness risk premia.

| Paper | Method | Data | VRP | SRP |
|---|---|---|---|---|
| Zhao et al. (2013) | Zhang & Xiang (2008) | S&P 500 1996-2005 | -5.2 | -0.67 |
| Lin et al. (2019) | Bakshi et al. (2003) | S&P 500 1990-2011 | -3.4 | -3.39 |

Note: This table shows the estimated volatility and skewness risk premia previously documented in the literature, based on empirical data from the S&P 500 options. Both papers utilize a model-free approach to estimate risk-neutral measures. The volatility risk premium is expressed as an annualized percentage, while the skewness risk premium is reported in monthly terms.

Table 4.3 presents the estimated volatility and skewness risk premia of the utility indifference model for four utility functions, under both normal and NIG distributions, implementing the method introduced in Subsection 3.2.4. It is important to note that the risk-neutral volatility and skewness are not measured at the at-the-money point. Instead, these quantities are jointly estimated by fitting the implied volatility for a fixed time to maturity ($T = 0.25$) to the NIG model parameters, relying on the model's underlying assumption about the physical probability distribution. The volatility and skewness risk premia are then computed as the difference between the corresponding risk-neutral and physical measures, as expressed in Equations (3.34) and (3.35).

Table 4.3: The estimation of volatility and skewness risk premia of the model.

| | **Normal distribution** | | **NIG distribution** | |
|---|---|---|---|---|
| Utility | VRP | SRP | VRP | SRP |
| CRRA | 0.0 | 0.00 | -0.5 | -0.12 |
| PT | -0.4 | -0.32 | -2.8 | -0.64 |
| Mean-LPM1 | -1.1 | -0.44 | -3.6 | -0.72 |
| Mean-LPM2 | -4.5 | -2.36 | -7.9 | -3.16 |

Note: This table displays the estimated risk premia obtained by fitting to the NIG distribution, as described in Subsection 3.2.4, for a fixed time to maturity of $T = 0.25$. The volatility premium is presented as an annualized percentage, while the skewness risk premium is reported in monthly terms.

The results demonstrated in Table 4.3 indicate that, with the exception of the CRRA utility function under the normal distribution, the utility indifference model is capable of capturing negative volatility risk premiums and negative skewness premiums. This observation implies that the risk-neutral volatility exceeds the physical volatility. Moreover, both the risk-neutral and physical skewness exhibit negative values, with the risk-neutral skewness being greater than the physical skewness in absolute terms. These findings are consistent with the broader literature on option pricing.

Consistent with the pattern depicted in the graphical illustration, the CRRA utility function exhibits the lowest risk-neutral volatility and skewness under both the normal and NIG distributions, followed by the PT and Mean-$\text{LPM}_1$ utility functions. On the other hand, the Mean-$\text{LPM}_2$ utility function generates the highest risk-neutral volatility and skewness values under both distributions. Applying the CRRA utility function in the model under the NIG distribution produces negative values for both VRP and SRP, a finding that is consistent with prior research by Fu & Yang (2012), Li et al. (2017), and Lin et al. (2019). However, the magnitude of these risk premia is relatively lower than that found by Lin et al. (2019). Similarly, the PT and Mean-$LPM_1$ utility functions under the normal distribution fail to accurately capture the volatility and skewness risk premia that correspond to empirical data. Nonetheless, under the NIG distribution, the Mean-$LPM_1$ function effectively explains the real market volatility risk premium, as supported by the measurements provided by Lin et al. (2019). Furthermore, the magnitudes of the SRP for both the PT and Mean-$LPM_1$ utility functions are in close proximity to the estimations reported by Zhao et al. (2013). Interestingly, the model incorporating the Mean-$LPM_2$ utility function produces realistic values for VRP and SRP, even under the normal distribution. The quantified VRP of -4.5% aligns with the previous findings by Christoffersen et al. (2021). In contrast, the estimated VRP of -7.9% under the NIG distribution is significantly larger than prior literature estimates, entailing that the model with the Mean-$LPM_2$ utility function under the NIG distribution may overprice out-of-the-money put options. Despite that, the model is able to adequately account for the higher probability of losses, as indicated by the SRP. The above discussions may shed some light on the potential validity of the Mean-LPMn utility function, with the lower partial moment ($n$) ranging between 1 and 2, as a plausible choice for the true utility function.

In summary, the results indicate that the utility indifference model, with the exception of the CRRA utility function under the normal distribution, can account for the negative volatility and skewness risk premia. Specifically, Table 4.3 illustrates that the conventional CRRA utility function fails to generate sufficient premia under the NIG distribution compared to other behavioral utility functions. Furthermore, increasing the order of lower partial moments in the behavioral utility function leads to higher magnitudes of volatility and skewness risk premiums under both the normal and NIG distributions. Overall, comparing the model performance in accordance with empirical data is a challenging task due to the lack of consistency in the reported results across different datasets and methods. Nevertheless, the results of this study suggest that the Mean-LPMn ($1 < n < 2$) utility function may be the most appropriate utility function for the representative investor, given its ability to capture risk premia. Additionally, the utility indifference model, employing the Mean-LPMn utility function with specified parameters, exhibits the capacity to accurately reproduce the volatility and skewness risk premia that conform to the empirical evidence.

# Chapter 5

# Conclusion

The Black-Scholes model, despite its groundbreaking contribution to option pricing theory, exhibits deficiencies when compared to empirical evidence. Empirical studies frequently reveal anomalies, such as the smirk shape and term structure of implied volatility, as well as the presence of volatility and skewness risk premiums. These stylized facts have motivated researchers to develop more sophisticated models that can account for them. However, the current literature has limitations in comprehensively explaining all observed empirical anomalies of implied volatility. This study aims to address this gap by employing the utility indifference model with behavioral utility functions for utility-based option pricing. This novel approach offers a more effective means of explaining all three stylized facts.

The model's performance is evaluated under four utility functions (CRRA, PT, Mean-$LPM_1$, and Mean-$LPM_2$) and both normal and NIG distributions. Graphical illustrations show that the standard CRRA utility function falls short in capturing the observed smirk patterns, in stark contrast, the Mean-$LPM_2$ behavioral utility function produces IV curves that closely match the empirical shape, even under the normal distribution. The NIG distribution exhibits more prominent smirk patterns across all utility functions compared to the normal distribution. Furthermore, the Mean-$LPM_2$ utility function replicates the observed empirical evidence under the NIG distribution, successfully producing pronounced IV curves for short-term options that do not flatten out rapidly as the maturity extends. Assessing the volatility and skewness risk premia poses challenges due to inconsistencies in reported results across different datasets and estimation methods. Nevertheless, the estimation results confirm the graphical illustration and reveal negative risk premiums for both volatility and skewness, consistent with existing literature. In particular, the results suggest that the Mean-LPMn utility function (with $n$ values ranging from 1 to 2) yields risk premia that closely correspond to the values observed empirically. Overall, these findings highlight the efficacy of the utility indifference model incorporating behavioral utility functions in explaining these stylized facts that conventional models find challenging to replicate. Furthermore, these findings are expected to have broad implications for the existing literature in option pricing, as well as to provide valuable insights into areas such as risk management and investment decision-making.

While the results of the study are convincing, it is important to acknowledge that due to the limited timeframe of the study, the model parameters have not been calibrated using current market prices of highly liquid options, as such, it may not fully reflect future information from stock returns. These factors serve as potential areas for future research, as the model could be further improved to more accurately reflect market conditions.

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# Appendix A

# Discussion paper

### Discussion paper: Responsibility - Sofie Jahnsen

The concept of responsibility has gained significant attention within the field of finance, especially following the 2008 global financial crisis. This crisis highlighted the disparity between legally permissible actions and ethical standards, emphasizing the growing recognition of greater accountability and transparency in financial markets. This discussion paper delves into the intricate relationship between responsibility and my master's thesis, which investigates the issue of implied volatility. Specifically, it focuses on the empirically observed stylized facts associated with implied volatility and proposes the application of the utility indifference model with behavioral utility functions to effectively explain these facts. In this discussion paper, I will begin by providing a concise overview of my master's thesis and subsequently explore the intricate connection it shares with the broad concept of responsibility.

### The thesis

The Black & Scholes (1973) (BS) model, a breakthrough in option pricing, yields a horizontal line when plotting the implied volatility (IV) against the strike price. However, empirical studies have identified three stylized facts that the BS model fails to explain. Firstly, the IV curves exhibit a smile/smirk pattern (Rubinstein, 1985), with an upward-sloping term structure for at-the-money options (Zhang & Xiang, 2008). Secondly, option prices tend to imply higher volatility than realized asset returns, known as the volatility risk premium (Jackwerth & Rubinstein, 1996). Thirdly, the negative skewness implied by options prices is greater in absolute terms compared with the realized skewness, termed the skewness risk premium (Bakshi et al., 2003).

In response to these stylized facts, numerous sophisticated models have been developed. However, the standard models often fall short of fully explaining all aspects of these phenomena. This study presents a novel approach to the utility indifference model by incorporating behavioral utility functions to provide a more accurate representation of these anomalies. The model's performance is evaluated by testing the standard function used in expected utility theory and behavioral utility functions under both normal and Normal Inverse Gaussian (NIG) distributions.

The findings reveal that the conventional utility function inadequately captures the observed smirk patterns, while the behavioral utility function successfully generates volatility smirks that closely align with empirical shapes, even under normal distribution assumptions. These results underscore the effectiveness of the utility indifference model with behavioral utility functions in explaining these stylized facts. By overcoming the limitations of traditional models, this research offers valuable insights into understanding implied volatility dynamics. The findings are expected to shed light on risk management and investment decision-making.

## Responsibility

In the field of finance, the concept of responsibility refers to the ethical obligation of individuals and organizations to act in a manner that promotes the well-being of society and the environment (Siltaoja et al., 2015), has gained attention due to the recognition that legally permissible behavior may not always align with ethical standards. Implied volatility, reflecting market expectations of future volatility, carries implications for investors, financial institutions, and the broader market. The stylized facts of implied volatility, such as the smile/smirk pattern, volatility risk premium, and skewness risk premium, emphasize the need for responsible assessment and risk management. This research contributes to understanding responsibility in implied volatility by incorporating the impact of behavioral biases on investor behavior. By recognizing and addressing these biases, stakeholders can navigate the complexities of implied volatility responsibly, promoting stability and mitigating systemic risks in the financial industry.

Horrigan (1987) argues that the application of the option pricing model to analyze the relationships between equity holders within a firm gives rise to ethical challenges. The option pricing model assumes that all equities in a firm can be viewed as options for buying or selling the firm's assets. In firms with debt in their capital structure, common stocks can be seen as call options, while the limited liability provided to stockholders can be considered a form of put options that protects against the consequences of debt default. This perspective suggests that stockholders may increase the riskiness of a firm's operations to maximize the value of their call options, potentially at the expense of bondholders. The widespread adoption of the options paradigm by managers and owners would inevitably lead to a significant erosion of trust among creditors toward firms. Moreover, it significantly increases the risk of corporate insolvency, as stockholders can simply abandon their obligations. Overall, any strategy pursued by stockholders that relies on profiting from the direct losses of bondholders is highly unethical.

To effectively manage the ethical challenges posed by the option pricing model and promote responsible behavior, several strategies can be employed. First, it is crucial to establish robust corporate governance practices that prioritize transparency, accountability, and ethical decision-making. Such practices ensure that the interests of all stakeholders are considered and protected. Regulatory authorities play a key role in enforcing regulations that discourage unethical practices and ensure compliance with financial reporting standards, thereby fostering responsible behavior in the industry. Educating investors about the implications of the option pricing model and the ethical challenges it presents is essential in empowering them to make informed investment decisions. Open communication and collaboration among stakeholders, including stockholders, bondholders, managers, and regulators, enhance understanding and facilitate the development of responsible solutions. Furthermore, the development and promotion of ethical guidelines and best practices provide a comprehensive framework for responsible decision-making (Boatright, 2013). By adopting these strategies, stakeholders can effectively address ethical challenges, foster trust, and promote responsible behavior in the financial industry.

To further advance responsibility in implied volatility research, it is essential to address the potential biases that may arise in the research process. Biases can stem from various sources, including sample selection, variable choice, and statistical methods. For instance, a study focusing solely on a specific asset or market may lack generalizability to the broader market, while small sample sizes can limit the applicability of findings. Researchers must conscientiously consider these biases and take measures to mitigate their impact, ensuring the validity and reliability of their research. Additionally, the dissemination of research findings carries ethical implications. Researchers must ensure that their findings are communicated in a clear and unbiased manner and that they are not used to mislead or manipulate investors or other market participants.

In the master's thesis, I have taken several measures to ensure the robustness and transparency of the research. I have provided comprehensive details of the research approach, facilitating the replication of the study by fellow researchers. The chosen risk premia estimation method aligns with the underlying assumptions of the model, enhancing the accuracy of the results. However, it is important to acknowledge that certain limitations exist. Due to time constraints, I was unable to calibrate the model parameters using the current market prices of highly liquid options. While this limitation does not compromise the findings of the research, it presents an opportunity for further improvement to better reflect real-time market conditions. In future research, I will address these limitations and be aware of potential biases when collecting the dataset, ensuring a comprehensive and unbiased representation. Transparency will remain a key principle in the research, and I will strive to enhance the accuracy and reliability of the findings through improved calibration processes. Hopefully, this could foster a responsible and accountable research environment.

## Conclusion

In conclusion, responsibility is a crucial concept in the field of finance, and it is particularly relevant to research on implied volatility. As this paper has shown, research on implied volatility has important implications for market participants, regulators, and investors, and it is essential to consider the ethical and social implications of market mechanisms. By emphasizing responsibility and promoting fairness, transparency, and accountability, we can create a more stable and sustainable financial system that benefits not only market participants but also society at large.

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