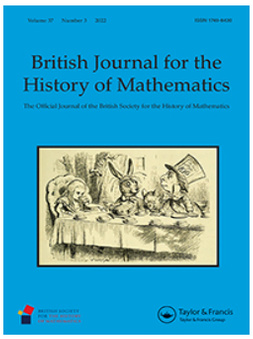

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The sine anecdote in Kovalevskaya's memoirs

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In Sofya Vasilyevna Kovalevskaya's memoirs there is a rather ambiguous story about how she came to understand trigonometric functions on her own as a teenager by reading the chapter on optics in Tyrtov's elementary physics textbook. Furthermore, she claims that in so doing, she happened to follow 'the same road that had been taken historically: that is, instead of a sine I used a chord'. We examine Tyrtov's textbook in search of sources for such inspiration and quote hitherto unknown critical reactions to her autobiographical reflections by Kovalevskaya's teacher I I Malevich. We conclude that Kovalevskaya's memoirs may well be marred by personal interests and/or faltering memory. By adding new sources about Kovalevskaya's early mathematical education and by critiquing some previously published reactions to the sine anecdote, we hope to contribute some nuances to the biographical literature on this world-famous pioneering female professor of mathematics.

Резюме

В воспоминаниях Софьи Васильевны Ковалевской мы можем найти довольно двусмысленный рассказ о том, как она в подростковом возрасте самостоятельно пришла к пониманию тригонометрических функций, прочитав раздел оптики в учебнике начальной физики Тыртова. Более того, она утверждала, что при этом она случайно 'пошла тем же путём, который употреблялся исторически, т. е. вместо синуса брала хорду'. Мы исследуем учебник Тыртова в поисках вдохновляющего начала таких утверждений и приводим доселе неизвестные критические отклики на автобиографические размышления Ковалевской, сделанные её учителем И И Малевичем. Мы пришли к выводу, что некоторые факты в мемуарах Ковалевской также могут быть искажены в связи с личными интересами или неточностью воспоминаний. Добавляя новые источники о раннем математическом образовании Ковалевской и критикуя некоторые ранее опубликованные реакции на рассказ о синусе, мы надеемся внести некоторые нюансы в биографическую литературу о первой женщине-профессоре математики, известной на весь мир.

1. An anecdote about sines and chords

The Russian mathematician Sofya Vasilyevna Kovalevskaya (1850–1891), née Korvin-Krukovskaya, was the first woman ever to work as a professor of mathematics, from 1884 onwards, at the then newly established Stockholms Högskola. Kovalevskaya's memoirs about her childhood appeared in *Vestnik Evropy* (Kovalevskaya 1890, 19). A shorter autobiographical story, focusing on her mathematical education and career, was published posthumously in *Russkaya Starina* (Kovalevskaya 1891). The copious biographical literature on Kovalevskaya relies heavily on these two autobiographical publications.

In the latter, Kovalevskaya reports the following episode from her adolescence, which we quote in its entirety from Stillman's English translation[1]:

> One of our neighboring landowners, Professor Tyrtov, brought us the textbook of elementary physics he had written. I made an attempt to read it, but in the section on optics, to my chagrin, I encountered trigonometric formulas, sines, cosines and tangents.
>
> What was a sine? I was nonplussed by this question and turned to Malevich for help in solving it. But since the matter was not a part of his curriculum, he replied that he did not know what a sine was[2].
>
> Then, trying to cope with the formulas contained in the book, I tried to explain it for myself. By strange coincidence I took the same road that had been taken historically: that is, instead of a sine I used a chord. In the case of small angles these quantities almost coincided with one another. And since the formulas in Tyrtov's book dealt with infinitely small angles exclusively they tallied excellently with the basic definition I had adopted. And with this I contented myself.
>
> Some time later I was having a conversation with Professor Tyrtov about his book, and he expressed doubt at first that I could have understood it. To my declaration that I had read it with great interest he said, "Come, now – aren't you bragging?" But when I told him the means I had used to explain the trigonometric formulas he completely changed his tone. He went straight to my father, heatedly arguing the necessity of providing me with the most serious kind of instruction, and even comparing me to Pascal.
>
> After some hesitation my father agreed to have me taught by Professor Strannolyubsky, with whom I then settled down to work successfully. In the course of that winter, we went through analytic geometry, differential and integral calculus.
>
> The next year I married V. O. Kovalevsky … …
>
> (Kovalevskaya 1891, 453–4), translation in Stillman et al. (1978, 217–218)

Professor Nikolay Nikanorovich Tyrtov (1822–1888), protagonist of the sine anecdote, taught physics at the Naval Cadet School in Kronstadt outside St. Petersburg, where he had also received his education. It has developed into the present N G Kuznetsov Naval Academy. The summer visits that Professor Tyrtov paid to his neighbour in the countryside, Sofya's father, retired army general Vasily Vasilyevich Korvin-Krukovsky (1801–1879), brought together two gentlemen of the privileged classes, both of whom were landowners and high-ranking military men. As a faculty member of the Naval Academy, Tyrtov was on a military career path which would eventually lead to his promotion to Lieutenant General, the same rank that Korvin-Krukovsky had when he retired.

[1]We rely on Stillman's translation for the quotes in this article. We have checked it carefully against the Russian original in *Russkaya Starina* and find it impeccable.

[2]This passage is somewhat ambiguous, and the allegation of ignorance is hardly credible. After this story had been published, Malevich stated that he had intended to teach Kovalevskaya trigonometry from 1867. (Birkeland and Nossum 2022).

The sine anecdote is Kovalevskaya's only remark about Tyrtov and about her first encounter with trigonometry and physics anywhere in her memoirs. The crucial passage is in the centre of the quote:

> By strange coincidence I took the same road that had been taken historically: that is, instead of a sine I used a chord. In the case of small angles these quantities almost coincided with one another. And since the formulas in Tyrtov's book dealt with infinitely small angles exclusively they tallied excellently with the basic definition I had adopted.

We shall substantiate our treatment of the sine anecdote by examining Tyrtov's book, which has not been done so far in the literature. This will serve to relativize or even disprove some of Kovalevskaya's claims. We shall also dispute some explanations of her claims by other historians (Cooke, Audin) which we find somewhat premature and unconvincing, and we shall have occasion to refer to two contemporary reactions to the anecdote; one by Kovalevskaya's brother Fyodor Vasilyevich Korvin-Krukovsky (1855–1920), and another by her teacher Iosif Ignatyevich Malevich (1813–1898), who is mentioned in the anecdote.

Iosif Ignatyevich Malevich was the resident house teacher of the Korvin-Krukovsky family. From age 8 to 17, Kovalevskaya was home schooled in mathematics and other subjects by him. He taught mathematics according to a method very similar to that devised by August Wilhelm Grube (1816–1884). Grube's method is time-consuming and attracted strong criticism, among which a sarcastic flourish from the pen of none other than Leo Tolstoy (1860–63, 300). As a matter of fact, unnoticed by most of Kovalevskaya's biographers, Malevich angrily distanced himself from the sine anecdote and several other parts of her memoirs (Birkeland and Nossum 2022).

Kovalevskaya mentions her father's hesitation to let her study mathematics soon after remarking that he 'harbored a strong prejudice against learned women' (Stillman et al. 1978, 216). The episode with Tyrtov is portrayed as a turning point, after which her father's misgivings with her studies in mathematics were overcome, and this view is widely echoed in the biographical literature on Kovalevskaya.

However, there is some evidence to suggest that her father's attitude to studies in mathematics was, on the contrary, always quite positive. Kovalevskaya's in-house teacher Malevich reported about her father that 'at the beginning of my service at Palibino[3] he asked me for one thing only: 'more mathematics and less philology'[4] (Birkeland and Nossum 2022).

2. The historical road

To say it right at the beginning: we shall not be able to ultimately determine what Kovalevskaya meant by the 'road that had been taken historically'. The information she gives is too sparse. We must also be aware of the gap of twenty-odd

[3]The Korvin-Krukovsky family estate, in Pskov Oblast, western Russia.
[4]побольше математики и поменьше словесности

years between the incident described and the time when Kovalevskaya dictated this part of her memoirs. The most trivial explanation would of course be that, historically, applied mathematicians, optical physicists, and astronomers were originally interested in calculating chords belonging to angles while the sine was introduced later by the Indians as an auxiliary notion for reasons of mathematical convenience.

The actual historical paths along which trigonometry developed, so diffusely alluded to in Kovalevskaya's memoirs, have been well charted through the efforts of historians of mathematics, most notably perhaps Anton von Braunmühl (1900/1903), Glen van Brummelen (2009, 2021), and Galina Pavlovna Matvievskaya (2012). Quite sophisticated trigonometry, in particular elaborate tabulations of chords, existed long before the sine function (cf. Figure 1) was introduced. Aristarchus's inequality, the sine subtraction law, and other trigonometric laws were known already in antiquity, albeit formulated in terms of chords, not trigonometric notions like the sine. Any talk about approximating sines with chords or arcs must be measured against these historical accomplishments.

3. Using chords instead of sines

Unlike the remark about 'the historical road', the middle part of Kovalevskaya's central quote is quite understandable. Young Sofya did not know what 'sine' meant and interpreted it as the chord belonging to a given angle (arc). Since the chord of an angle depends on the radius of the circle, we must further assume that she understood something about mathematical scaling, relating the chord to a standard circle, for instance the unit circle. Assuming this reduction, the chord is an immediately given and intuitive geometric magnitude in isosceles triangles while there is no *a priori* reason to consider the sine, that is the ratio between the perpendicular dropped from one radius vector onto another, and the radius itself, like in Figure 1.

Visualizations like Figure 1—easily understandable to young Sofya—may illustrate this point. Here the slanted line is the chord belonging to θ and the vertical line is equal to the sine in the case of a unit circle with radius $R = 1$.

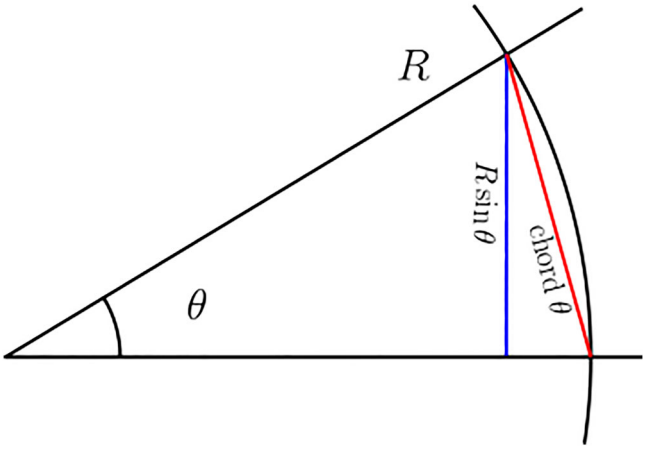


Figure 1. Visualization of sine and chord.

The figure suggests that sine, chord, and arc are nearly the same for small angles θ. In fact we have

$$(*)\ (R \sin \theta / \text{chord } \theta) \to 1$$

with vanishing angle (arc) θ. This asymptotic equality (which numerically goes beyond assuming the vanishing of the *difference* between the two quantities) follows rigorously via calculus from (**) and the Taylor expansion of the sine although such an argument was of course out of reach for young Sofya at the time:

$$(**)\ \sin \theta = \frac{\text{chord}(2\theta)}{2R}$$

Because (**) is the definition of sine for any angle and because Kovalevskaya admits that her interpretation of sine as chord was only approximately true for small angles, we can rule out that she alluded to this equation when talking about 'using chord for sine'. In other words, she must have meant the exchange of chord and sine for the *same*, but small angle θ.

From the Taylor expansion of sin θ also follows for $arc\ \theta = \theta\ R$

$$(***)\ (arc\ \theta / R \sin \theta) \to 1$$

As we shall see below, Tyrtov formulated it like this:

> The angles i and r will be so small that their sines can be considered proportional to the angles themselves. (Tyrtov 1861/1862 vol. II, 31)

Kovalevskaya apparently assumed that Tyrtov meant 'chord' when he said 'sine', which means:

$$(****)\ (arc\ \theta / \text{chord } \theta) \to 1$$

Both statements are of course correct, and Kovalevskaya accepted (****) either on Tyrtov's authority or supported by a picture. While Kovalevskaya recognized formulas (*), (**), and (***) in 1890 in hindsight, her ignorance of the sine notion at the time of the sine anecdote indicates that she was only aware of (****) when reading Tyrtov's book.

4. Reconstructing Kovalevskaya's reading of Tyrtov

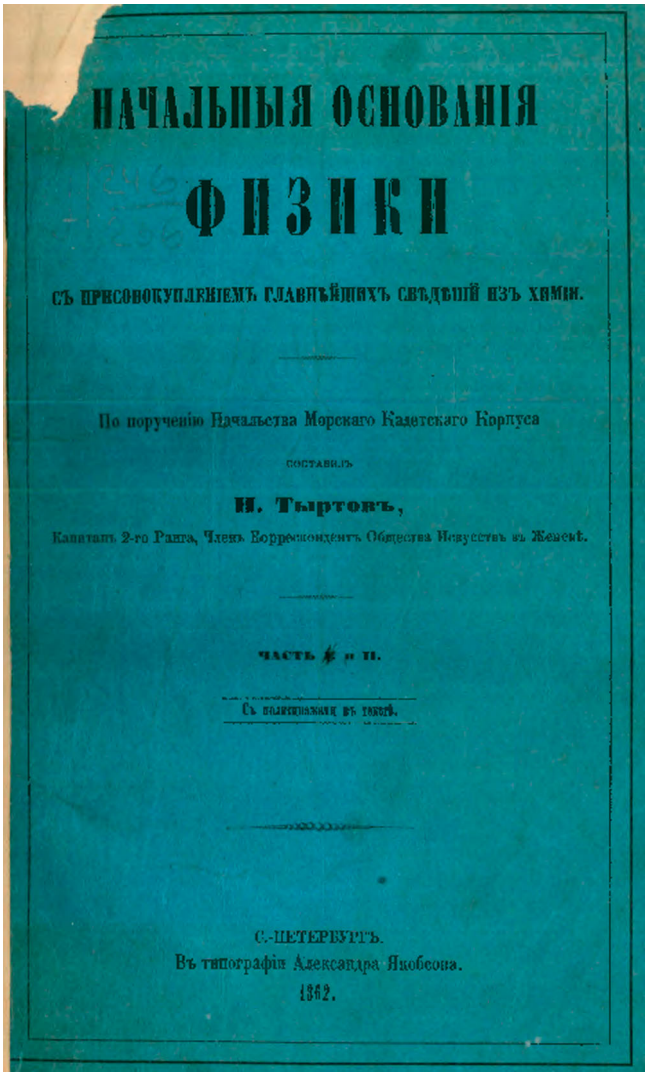
The family anecdote about sines and chords is promulgated with minor modifications as an indication of young Sofya's talent by many biographers. We are, however, not aware of any previous comparisons with the actual text of Tyrtov (1861/1862), which we shall now turn to. We proceed to comment on the chapter on optics in Tyrtov's book, finding several discrepancies with Kovalevskaya's story.

Tyrtov wrote or contributed to at least three physics textbooks: (Pisarevsky and Tyrtov 1859), (Tyrtov 1861/1862) in two volumes, and (Tyrtov 1875). His contribution to the first is clarified on the title page and in Nikolay Grigoryevich

Pisarevsky's (1821–1895) preface; Tyrtov wrote a chapter on galvanism, and Pisarevsky wrote the rest. In contrast (Tyrtov 1875), which was published long after the time of the anecdote we are discussing, was his own work from beginning to end: the library reference indicates that it is a lithographed handwritten manuscript.

Concerning the authorship of Tyrtov (1861/1862), the Pskov Encyclopedia (Pskov 2003) maintains that it was published 'under Tyrtov's editorship'.[5] The title page indicates that Tyrtov composed or compiled[6] it by assignment of the Naval Cadet Corps.

Both (Pisarevsky and Tyrtov 1859) and (Tyrtov 1861/1862) are aimed at secondary institutions of education, the latter specifically for the Naval Cadet Schools, where academic subjects were taught alongside a practical education of mariners.


НАЧАЛЬНЫЯ ОСНОВАНІЯ

ФИЗИКИ

СЪ ПРИСОВОКУПЛЕНІЕМЪ ГЛАВНѢЙШИХЪ СВѢДѢНІЙ ИЗЪ ХИМІИ.

По порученію Начальства Морскаго Кадетскаго Корпуса

составилъ

Н. Тыртовъ,

Капитанъ 2-го Ранга, Членъ Корреспондентъ Общества Искусствъ въ Женевѣ.

ЧАСТЬ I и II.

С.-ПЕТЕРБУРГЪ.
Въ типографіи Александра Якобсона.
1862.

Figure 2. Title page of Tyrtov's book.

Volume II of Tyrtov (1861/1862) starts with a chapter on optics, and we therefore think this was the book that was presented to the Korvin-Krukovsky family one summer day, most likely in 1867, and read by Sofya (Figure 2). This is where she struggled with trigonometric formulas, was refused help from the resident house teacher Malevich, and proceeded to understand trigonometric functions on her own by replacing sines with chords, all according to the anecdote from her autobiographical story.

We have found few references to Tyrtov's books in the literature. Only (Pisarevsky and Tyrtov 1859) is mentioned in the recent compendious overview 'Two Centuries of Physics Textbooks in Russia' (Pushyreva and Brashnikov 2021), and not very prominently at that. The ubiquitous book (Lenz 1856) which the Baltic-German immigrant, Professor and Academician, Rector of the Imperial University in St. Petersburg, Heinrich Friedrich Emil Lenz (1804–1865), famous for Lenz's rule in electromagnetism, wrote by assignment of the Ministry of Popular Enlightenment, and which was reprinted many times, must have been a formidable contender for first place among textbooks in nineteenth-century Russian physics classrooms. Nevertheless, Tyrtov (1861/1862) is not entirely forgotten, and it earned a comparison with Lenz (1856) in Teleshov and Teleshova (2018, 109).

The relationship between the textbooks (Tyrtov 1861/1862) and (Lenz 1856) is in fact quite close. In his preface, Tyrtov acknowledges his debt to the highly regarded Academician Lenz, for allowing him to copy a large number of figures as well as, word for word, many accompanying explanations. In fact, much of the material we touch on when discussing Kovalevskaya's reading of Tyrtov's book originates with Lenz. We have, however, no information about personal or professional connections between Tyrtov and Lenz, beyond this permission to reuse material from Lenz (1856).

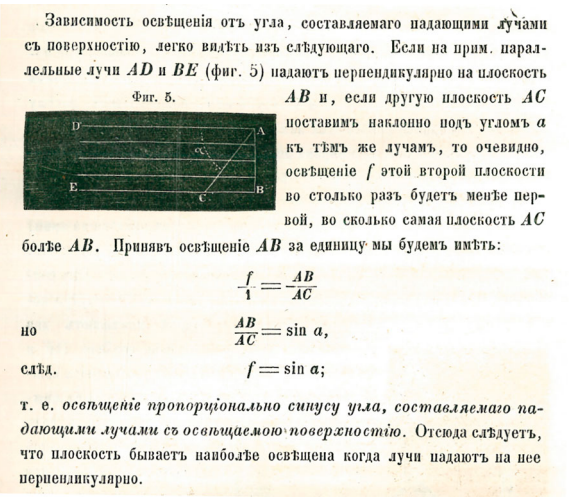
We now attempt a reconstruction of Kovalevskaya's reading of Tyrtov (1861/1862) assuming, as per the anecdote, that she started without any idea of what the word 'sine' means. Chapter 1 on optics covers the first 83 pages of part II of the book. The optics chapter is divided into 77 sections, of which 8 contain trigonometric formulas with sines and tangents.[7]

It is not the case, as Kovalevskaya maintains, that every angle in Tyrtov (1861/1862) is infinitesimally small. A great variety of angles of substantial size are discussed, and small angles are in fact only mentioned in 3 of the 8 sections containing trigonometric formulas.

Surprisingly enough, the reader finds the answer to Kovalevskaya's question 'What is a sine?' already in the first pages of the chapter on optics, in section 4 on pages 4–5: 'Intensity of light' (Figure 3).

In Figure 3[8] the surfaces AB and AC are presented edgewise, and are assumed rectangular with identical widths. To paraphrase the text, light shines in from the left, parallel to rays AD and BE, and at right angles to the surface AB. Taking the intensity

[5]под его редакцией
[6]составил
[7]But we are unable to find a single mention of cosines as reminisced by Kovalevskaya, and we cannot find any explanation of the relation between sine and tangent either.
[8]Cf. Figure 5 from Tyrtov (1861/1862, vol. II, 5), which is a copy of Figure 7 from Lenz (1856, chapter VII, 7), and the accompanying text and formulas are taken almost verbatim from the same source.



Зависимость освѣщенія отъ угла, составляемаго падающими лучами съ поверхностію, легко видѣть изъ слѣдующаго. Если на прим. параллельные лучи AD и BE (фиг. 5) падаютъ перпендикулярно на плоскость AB и, если другую плоскость AC поставимъ наклонно подъ угломъ a къ тѣмъ же лучамъ, то очевидно, освѣщеніе f этой второй плоскости во столько разъ будетъ менѣе первой, во сколько самая плоскость AC болѣе AB. Принявъ освѣщеніе AB за единицу мы будемъ имѣть:

Фиг. 5.

$$\frac{f}{1} = \frac{AB}{AC}$$

но $$\frac{AB}{AC} = \sin a,$$

слѣд. $$f = \sin a;$$

т. е. *освѣщеніе пропорціонально синусу угла, составляемаго падающими лучами съ освѣщаемою поверхностію.* Отсюда слѣдуетъ, что плоскость бываетъ наиболѣе освѣщена когда лучи падаютъ на нее перпендикулярно.

Figure 3. An explanation of the sine can be found in Tyrtov's treatment of the intensity of light striking a slanted surface.

of light striking the surface AB (in the absence of AC) to be 1, we seek the intensity f of light striking the surface AC, placed in front of AB at an angle a. The intensity f diminishes by the same rate as the area of AC exceeds that of AB. Now, the ratio $\frac{AB}{AC}$ is the sine of the angle a, so in conclusion f is just sin a:

$$\frac{f}{1} = \frac{AB}{AC}$$

$$\frac{AB}{AC} = \sin a$$

$$f = \sin a.$$

Here the angle a is not necessarily small, and the text gives a clear definition of the sine of an acute angle. This could mean that Kovalevskaya did not read this part of Tyrtov's book very attentively, or that her memory of it had faded during the long interlude between the events of the anecdote and the time when it was written down.

But let us assume that Kovalevskaya had simply overlooked the definition of the sine on page 5 and was still wondering what the sine was when reading section 18, pp 19–21 'Refraction of light in a medium bounded by a plane surface', of Tyrtov's book, where he introduces the law of refraction. Here is a translation of the first few paragraphs, referring to our Figure 4 (Tyrtov's Figure 21):

Фиг. 21.

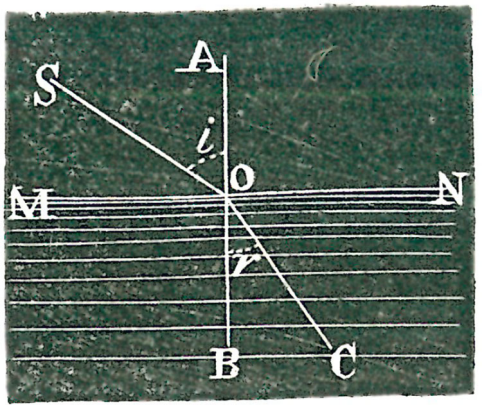


Figure 4. Refraction in a translucent medium bounded by a plane surface.

18. Refraction of light in a medium bounded by a plane surface. We already said that a ray of light falling on a translucent body penetrates inside it; however inside the body the light does not retain its original direction, but deviates sideways; this deviation is called *refraction of light* and is the topic of *Dioptrics*.

Let *SO* be a ray (fig. 21) falling from empty space onto a plane *MN*, which bounds a translucent medium; the angle *SOA* formed by the *incident ray SO* and the *perpendicular AO*, is called the *angle of incidence*. Experience shows that within the medium the ray is declined towards the perpendicular *OB*, hence the angle *BOC*, formed by the *refracted ray CO* and the perpendicular *OB*, is called the *angle of refraction*. Refraction of light takes place according to the following laws:

(1) The incident and refracted rays lie in a plane with the perpendicular.
(2) For one and the same refracting medium, the ratio of the sine of the angle of incidence and the sine of the angle of refraction remains constant.

And therefore, if i denotes the angle of incidence, r the angle of refraction, and μ the constant ratio, then we get:

$$\frac{\sin\ i}{\sin\ r} = \mu \quad \text{or} \quad \sin\ r = \frac{1}{\mu} \cdot \sin\ i.$$

Let us point out that the law of light refraction, formulated in the last line, is an empirical law and thus independent from mathematical identities strictly connected to the geometry of a circle such as formulas (*) through (****) above. Therefore, the law of light refraction required an entirely different kind of understanding on Kovalevskaya's part. We are inclined to think that at this point she realized something of the difference between mathematics, which is here only a tool to express a physical

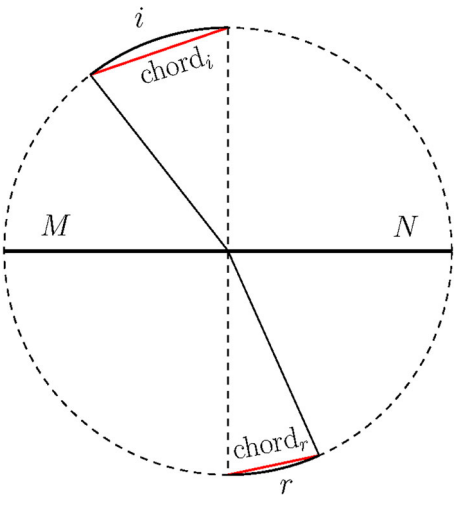


Figure 5. Chords of incident and refracted rays of light.

law, and physics itself. This would explain the lavish praise she reported receiving from the physicist Tyrtov.

The incident and refracted rays form angles i and r, respectively, with the perpendicular. Let us suppose that, reading section 18, Kovalevskaya was still entertaining the idea that 'sine' actually means chord, and that she initially thought the physical phenomenon of refraction might be such that the ratio $\dfrac{\text{chord } i}{\text{chord } r}$ remains constant for varying angles i in Figure 5. That is, she assumed $\sin\theta = \text{chord } \theta$. This is in fact not implausible if one considers small angles i and r.

On page 21, Tyrtov explicitly mentions the limiting cases $i = 0^\circ$ and $i = 90^\circ$, and the latter case is illuminating: Tyrtov shows that when a ray of light passes from air into glass of refractive index $\mu = \dfrac{3}{2}$, $r = 41^\circ\ 50'$ is the greatest refracted angle which can be obtained as $i \to 90^\circ$. Now, if the ratio $\dfrac{\text{chord } i}{\text{chord } r}$ were constant and equal to the refractive index $\dfrac{3}{2}$, this would correspond to chord $r = 0.9428$ in a circle of radius $R = 1$. This should have raised Kovalevskaya's suspicion, because a chord so close to $R = 1$ is clearly the chord of an angle much closer to 60° than to $41^\circ\ 50'$, since a triangle with all sides equal to or close to 1 must have all angles close to 60°. In fact, it is the chord subtended by an angle of $56^\circ\ 15'$.

To us the most likely explanation is that Kovalevskaya took the law of light refraction from section 18 (because she would need it further below in her reading), interpreted it as being valid for chords but did not check the numerical examples given by Tyrtov. It seems likely that she read the book only partially and non-linearly, in any case not paying careful attention to section 18 and perhaps skipping section 4 altogether, thus not noticing the general definition of sine. Furthermore, her inaccurate report that only small angles were considered in Tyrtov seems to indicate that the only sections she paid careful attention to were those dealing with small angles.

The first time Tyrtov mentions small angles is in section 26, pp 29–32: 'Refraction of light in spherical lenses'.

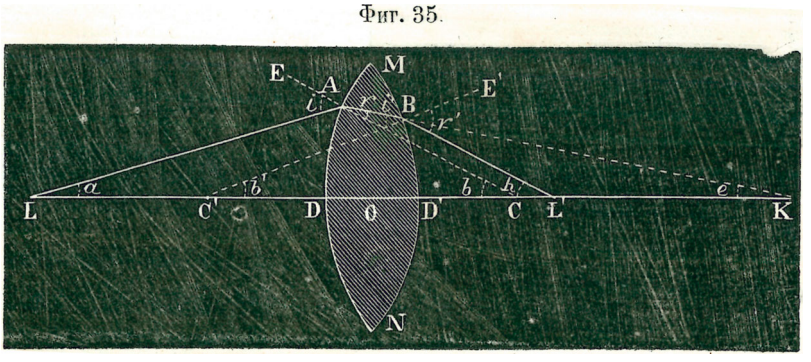


Figure 6. Refraction in a biconvex spherical lens

We translate a few paragraphs from page 31–32 in Tyrtov's book, referring to our Figure 6 (Tyrtov's Figure 35, page 30):

> Suppose that LA is one of the central rays, i.e., incident very close to the axis; then *the arc AD will coincide with a/the straight line touching it; then also the angles i and r will be so small that their sines can be considered proportional to the angles themselves* (our emphasis). For brevity let us denote angle $ALD = a$, angle $ACD = b$, angle $AKD = e$, $LD = d$, $AD = n$, $KD = f'$, and finally radius $CD = R$.

For the first surface MDN we have:

$$\sin i = \mu \cdot \sin r, \textit{ or } i = \mu \cdot r.$$

But from the triangle ALC we have:

$$i = a + b$$

and in the triangle ACK

$$r = b - e.$$

Consequently, we get:

$$a + b = \mu \cdot (b - e).$$

Note that Tyrtov approximates ratios of sines with ratios of the corresponding angles in accordance with (***)[10] on the premise that the angles i and r in question are small. Reading the emphasized phrase to mean that small angles coincide with their corresponding chords, Kovalevskaya may have settled on the idea that

[10]Further below in his text Tyrtov does it also with tan x, as indicated in the quote from Kovalevskaya's autobiographical story, but we do not go into this here.

$\sin x = \text{chord } x$ for small angles x after reading section 26, interpreting Tyrtov's approximation as meaning (****).

5. Commentaries to the sine anecdote in the literature

We now proceed to discuss four reactions to Kovalevskaya's sine anecdote: two by contemporaries (her brother and her teacher), and two by recent historians.

It is of course more than a simple exaggeration to say that she recreated a whole branch of mathematics, as her brother F V Korvin-Krukovsky says in his obituary for his sister:

> Thus she, quite independently, rediscovered some facts which at that time, of course, had long since been known and entered the elementary gymnasium teaching. In other words, it was as if she, without even realizing it, had recreated an entire branch of science – trigonometry. If she had lived a few hundred years earlier and done the same thing, it would have been enough for posterity to put her on a par with the greatest minds of mankind. (Korvin-Krukovsky 1891, 631: our translation from the Russian)

In fact, we have no evidence at all that Kovalevskaya developed detailed relations involving chords, such as the ones mentioned above which existed already in antiquity (certainly more than 'a few hundred years' earlier) and were analogous to later equations with sines. After all, Tyrtov's textbook focused on physics, not trigonometry, and it did not contain any trigonometrical formulas of complexity comparable to the ones already used by the ancients.

The tone of the letter that Kovalevskaya's teacher I I Malevich sent to the editor of *Russkaya Starina* after her autobiographical, and by then posthumous, story had been published, sharply contrasts with her brother's eulogy:

> Who among people is without weakness? – Who, having achieved something, does not wish for more? … And so the unusual scientist, not content with world fame, wishes to also be known as a self-taught genius, first of all in Sweden, where nobody knew about her course of education, and then, maybe not noticing it herself, she got carried away with ripened, deceptive fantasies and set in motion an unprecedented tale in a small circle of interlocutors, believing that her story, going from one to the other, even aside from printing, would be considered a true, undisputable fact. (Malevich 1892), translated and commented in Birkeland and Nossum (2022)

Malevich's letter is quite long, it attacks many parts of her autobiographical story, not only the sine anecdote, and in particular it fervently laments Kovalevskaya's perceived self-glorification and her depreciation of his, her teacher's, significance in her initial preparation for world-class mathematical achievements. He puts into perspective Kovalevskaya's ambiguous allegation that he had said he did not know what the sine was:

> After returning to Russia,[11] I had intended to begin teaching Trigonometry. (Malevich 1892), translated in Birkeland and Nossum (2022, 103)

[11]Malevich had joined the family on their trip to Switzerland and Germany 1866–1867. His tutelage of Kovalevskaya ended in the fall of 1867.

Her brother and her teacher being contemporaries of Kovalevskaya and closely connected to her, we now discuss two more recent attempts to locate her historical path, namely Roger L Cooke's fine treatise (Cooke, 1984) and Michèle Audin's fond and personal accolade (Audin, 2011).

Cooke says that Kovalevskaya followed the path of one of the first pioneers, Ptolemy, and that her:

> … guess was that the sine of a central angle in a circle is proportional to the chord subtended by the angle. In fact it is proportional to half the subtended chord of twice the angle, but for small angles the difference is negligible. Indeed the earliest trigonometric tables – in Ptolemy's *Almagest*, for example – were tables of chords rather than half-chords. Thus Kovalevskaya was following the route of the pioneers in the area. (Cooke 1984, 10)

Cooke alludes to the definition (**) of sine which is valid for any angle and to the approximation (****) for small angles. Formula (**) was introduced by the Indians around 500 CE and simplified the calculations used for instance in Ptolemy. If one should compare this 'route of the pioneers' to Kovalevskaya's, one could only say that she saw or felt something about the relation between sines (unknown to her) and chords for small angles and traced the historical development backwards. Historically it was chords (the primarily interest of astronomers) that were replaced by sines, not the other way around as done by Kovalevskaya. From this perspective her interpretation of sine as chord appears retrograde rather than progressive.

Cooke's remark about the sine episode should not be overvalued because it is—like our remarks—not based on any scientific publication by Kovalevskaya but only on her rather vague remark in her autobiographical story. Cooke appears not to have used Tyrtov's book,[12] and his commentary on the sine episode appears only in the introductory biographical chapter of his excellent book under '1.6 Early Mathematical Training.'

Audin, however, seems to feel differently and drives the misinterpretation of Kovalevskaya's memoirs to an extreme. Apparently misreading Cooke, she even joins Kovalevskaya's brother in ascribing to her the reinvention of the sine:

> According to Roger Cooke (1984, p. 10), the early trigonometric tables, those of Ptolemaios for example, gave the chords rather than the sines: Sofya invented the sine as it had appeared historically. (Audin 2011, 35)

In her popular book, she embellishes the sine anecdote by inventing fictitious dialogues between Kovalevskaya and Tyrtov and making unwarranted remarks about Malevich:

> Her good tutor Malevich didn't know a whole lot, so Sofya had to figure it out for herself. (Audin 2011, 34)

[12]He does not refer directly to Tyrtov's book in his bibliography but refers to biographies by Koblitz (1983) and Kochina (1983). These in turn do list Tyrtov, but without analysing his book.

As remarked above, Malevich wrote an angry rebuttal of Kovalevskaya's autobiographical story, in a letter which was still unpublished at the time when Audin wrote her book: cf. (Birkeland and Nossum 2022).

6. Reliability of the transmission of the story

The literary salon in St. Petersburg, hosted by Mikhail Ivanovich Semevsky (1837–1892), editor of *Russkaya Starina*, at which Kovalevskaya's autobiographical story was told and stenographed, took place on May 29, 1890. Semevsky was an acquaintance of the Korvin-Krukovsky family and had himself been home-schooled by Malevich, with whom he maintained a life-long correspondence. After failed career attempts in the military and civil services, he had established himself as a publicist and socialite in St. Petersburg. The stenographic notes of Kovalevskaya's story were kept in Semevsky's archive until after her death the following year, when they were written out and prepared for publication by Kovalevskaya's brother F V Korvin-Krukovsky.

Questions about the reliability of the transmission of the story may reasonably be raised here. We have been unable to locate the shorthand notes, and cannot now ascertain who took them down, but it is at least possible that it was Semevsky himself. His biographer Olga Borisovna Kokh (1952–) reports that he was an adept stenographer, and frequently contributed material to his own journal based on interviews he took down in a Russian shorthand (Kokh 2005). However, she also states that his shorthand was idiosyncratic, so much so that some surviving notes are unintelligible to historians, and this somewhat contrasts with the fact that Kovalevskaya's brother was able to write out these notes for publication. Her brother published his own eulogy for her (Korvin-Krukovsky 1891) in *Russkaya Starina* vol. 71, and it contains some of the same anecdotal material, including the sine anecdote, as Kovalevskaya's autobiographical story (Kovalevskaya 1891), which he had written out for publication and which appeared in vol. 72 of the same journal.

7. Dating the sine anecdote

The exact year of the encounter with Tyrtov is missing from Kovalevskaya's memoirs, although her lessons with Strannolyubsky are firmly placed in the winter of 1867/1868 by the reference to her marriage to V O Kovalevsky, which took place in 1868. Let us try to find out more exactly when Tyrtov visited the Korvin-Krukovskys and presented them with his book.

As noticed already by Stillman, the dating of the sine anecdote is ambiguous in the source material (Stillman, et al. 1978, 204). Kovalevskaya's younger brother maintains that she was fourteen years old at the time, i.e. that the year was 1864; cf. his eulogy for her, in which he describes the sine episode and mentions his sister's age:

> … and particularly knowledge of linear trigonometry (*sic*), which he in no way expected to find in a 14-year-old village girl. (Korvin-Krukovsky 1891, 630: our translation from Russian)

Many of her biographers have taken this at face value. However, there is some evidence which points to 1867 as the real point of time. Kovalevskaya says that her

lessons with Strannolyubsky took place 'in the course of that winter', and her brother says they started 'soon after' the incident described.

Anyway, the story about Tyrtov and Kovalevskaya would lose much of its relevance and interest if one were to assume that she was already seventeen at the time. We cannot rule out that slips of memory, and/or a conscious or unconscious desire for effective presentation of her mathematical prowess influenced her own report and her brother's eulogy and led to these events being predated.

In an earlier section, we discussed her father's alleged hesitation to let her study mathematics, but to bridge the apparent gap in time between Tyrtov's visit and Strannolyubsky's lessons, it would be necessary to construe her father's hesitation to have lasted three years.

8. The wallpaper and bedlamp anecdotes

The sine anecdote is preceded by two other anecdotes in Kovalevskaya (1891), which we shall call the wallpaper and bedlamp anecdotes, respectively. In the wallpaper anecdote, the walls of Kovalevskaya's childhood nursery were covered in scrap paper which happened to contain lecture notes on calculus by Ostrogradsky. The knowledge she had gleaned from the walls of her bedroom was later lauded by Strannolyubsky, who became her teacher in St. Petersburg. In the bedlamp anecdote, her father called for her mathematics lessons with Malevich to stop, whereupon she took an algebra book with her to bed and continued to study mathematics furtively at night. These three anecdotes together depict Kovalevskaya as a child prodigy, whose talents in mathematics flourished spontaneously and fortuitously, despite the reluctance of her father and the ignorance of her teacher. But in his letter to the editor of *Russkaya Starina* after (Kovalevskaya, 1891) had been published, her teacher I I Malevich angrily contradicted the sine anecdote and repudiated each part of the bedlamp anecdote: cf. (Birkeland and Nossum 2022).

9. Concluding remarks

Much of the sine episode in Kovalevskaya's memoirs remains shrouded in mystery. We believe that Tyrtov may well have been impressed by some of Kovalevskaya's remarks, but as we have shown by comparing the sine anecdote with the contents of Tyrtov's book, she cannot have studied it, or even the chapter on optics, linearly and completely. She missed the part which clarifies the meaning of the sine function, and she must have concentrated only on the few sections which deal with small angles.

Kovalevskaya's statement that by replacing the sine of a small angle with a chord, she happened to follow the same path that had been taken historically, is a coarse approximation at best. Historically, chords of large and small angles were central objects of study long before the sine was introduced as a convenient auxiliary concept.

Caution is obviously indicated when discussing a family anecdote which appeared in Kovalevskaya's own autobiographical material as well as in an obituary written by her brother and in numerous laudatory biographies since then. In recent work (Birkeland 2020; Birkeland and Nossum 2022), we have shown that Malevich distanced himself from Kovalevskaya's description of him in the sine anecdote, and contradicted many other parts of her memoirs, a fact no previous biographer seems to have taken full account of.

As pointed out by Andrew Baruch Wachtel (1959–) in his book 'Battle for Childhood. Creation of a Russian Myth', late-nineteenth century Russian autobiographers allowed themselves a great deal of leeway in describing their childhood (Wachtel 1990). We think Kovalevskaya's autobiographical material is a case in point. More generally, autobiographical claims of having been self-taught, which are no less common in mathematics than elsewhere, are notoriously hard to verify.

It cannot be ruled out that having read Professor Tyrtov's book only partially, Kovalevskaya exaggerated when she reported about it to her father and Tyrtov, with the goal of impressing them and depicting herself and her achievements in the most flattering light possible. Unless new documents surface, such as letters written by Tyrtov or others, we shall probably never know for sure. But by placing the sine anecdote under scrutiny, critiquing previous attempts at interpretation, and utilizing a hitherto unexplored source (Tyrtov), we hope to have shed some new light on the personal scientific development of this pioneering woman.

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