HOW ARE PROOF AND PROVING CONCEPTUALIZED IN MATHEMATICS CURRICULUM DOCUMENTS IN THE USA AND JAPAN?

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Only a few international comparative studies have reported on proof and proving in curriculum documents. This report proposes a method of comparing the meaning of proof-related words in two specific countries' curriculum documents (the USA and Japan) through quantitative and interpretative analyses. Using a text mining approach to explore text data, we found that the co-occurrence network of the words "proof" and "prove" in curriculum documents from the two countries is quite different. In the USA, the word "proof" is concerned with justification and "prove" is used as a general process, while in Japan "proof" is more related to discovery and "prove" is more associated with specific theorems.

INTRODUCTION

Although the universality of mathematics is widely recognized, mathematics educators also acknowledge that it is situated differently in different countries' educational systems. This is also the case for proof and proving in a mathematics curriculum. However, only a few international research studies have reported on the role of proof in curricula (Reid, Jones, & Even, 2019). An international comparative study on proof and proving is promising but challenging because educational, linguistic, and cultural conditions vary according to country (Reid, 2015).

Currently, proof and proving are mentioned in the official curriculum documents of many countries. However, there are still debates among researchers over what constitutes proof, even after repeated discussions over the last three decades (e.g., Mariotti, 2006; Stylianides, Bieda, & Morselli, 2016). How are proof and proving conceptualized in curriculum documents? What are the specificities of their meanings in documents from different countries? How can we compare and analyze them? To address these questions, this study proposes a method of comparison based on the text mining approach, which allows us to analyze the co-occurrence of words in documents, both quantitatively and qualitatively.

The present study is a part of an ongoing international research project for comparative studies on argumentation and proof. In this report, we present a case study by focusing on documents from two specific countries: *Principles and Standards for School Mathematics* (NCTM, 2000; hereafter called *Standards*) in the USA and *Teaching Guide of the Course of Study: Mathematics* (MEXT, 2008; hereafter called *CoS*) in Japan. *Standards* is one of the most well-known and influential curricular documents

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in the world. Though it has strongly influenced curricula in the USA and elsewhere, it is not the national curriculum in the USA. *CoS* is less known outside Japan, but it is an elaboration of the national curriculum. Therefore, while the two documents are different, they play similar roles in their contexts, and so are comparable.

THEORETICAL BACKGROUND AND LITERATURE REVIEW

Although there are limited international comparative studies on curriculum regarding proof and proving (except for Hemmi, Lepik, & Viholainen, 2013), several comparative studies have been conducted on mathematics textbooks (e.g., Miyakawa, 2017) and classrooms (e.g., Knipping, 2004). To gain a better understanding of how proof and proving are conceptualized in different countries' curricula, it is important for researchers to develop a methodological approach to compare and analyze them. To do so, it is reasonable to pay more attention to linguistic aspects related to proof and proving, although some other aspects such as "structure" and "function" can also be considered (Miyakawa & Shinno, 2021). While the approach developed in the Lexicon project (e.g., Clarke, Mesiti, Cao, & Novotná, 2017) in which the methodology focuses on general pedagogical vocabulary used by teachers is promising, we require a more particular approach to investigate how proof-related words are used in curricula.

Some previous studies have considered linguistic issues involving cultural elements, which may affect the nature of proof (e.g., Balacheff, 1987; Sekiguchi, 2002). For example, according to Sekiguchi (2002), "argumentation" is a culturally dependent notion and its meaning in Japanese is not equivalent to that in English or any other Western languages. However, even the term "culture" is often ambiguous, which sometimes represents an obstacle to international communications in our research field.

How then to compare the meanings of words across different languages and cultures? As Wittgenstein reminds us, "the meaning of a word is its use in the language" (Philosophical Investigations, §43). Hence, our study adopts a text mining approach that allows us to analyze proof-related words through quantitative comparisons of their *use* between different countries. Although it pays little direct attention to cultural issues, the results may create an opportunity for discussion among researchers, which may bring new insights into proof and proving from a cultural perspective.

METHODOLOGICAL CONSIDERATIONS

Text mining approach

The methodological approach adopted in our study employs text mining, specifically co-occurrence network analysis. This approach interprets the meaning of a word from its occurrence with other words, that is, co-occurrence relations. Since the meaning of a given word may vary from country to country, we cannot determine the "true" meaning of the word. However, a word's use in any given text can be interpreted quantitatively by its co-occurrence network in that text. The advantage here is that we

can avoid possible ambiguities due to the linguistic nuance in each country's language, since it makes use of linguistic networks to characterize the usage of the word within the document.

Using this approach, we can compare and analyze the commonalities and specificities in the co-occurrences of proof-related words in curricular documents of both countries. In short, if the co-occurrence of a particular word in different languages is similar, then we can interpret that the word has a similar meaning. If not, the word can be interpreted as having a different meaning.

Data set

For the USA, *Standards* (NCTM, 2000) is used for the analysis. Although the actual mathematics curriculum varies from state to state, *Standards* has influenced the curriculum in most states. The recently published *Common Core State Standards for Mathematics* (CCSSI, 2010) is also influential nationally and internationally, but it contains fewer explanations about mathematical contents and processes than those in *Standards*. Therefore, we chose *Standards* for our quantitative text analysis due to the abundance of data in *Standards* for our analysis.

The words "proof" and "prove" often appear in the content standard "Geometry" and the process standard "Reasoning and Proof". The section analyzed in this paper is all text in the overview (Chapter 3) and standards for grades 6-8 part (Chapter 6) (See Table 1).

Standards	*CoS*
Overview of the Standards for mathematics education - Geometry - Reasoning and Proof	Section 1.1: Objectives of Mathematics Section 2: Content
Standard for Grades 6-8 - Geometry - Reasoning and Proof	Section 1.2: Objectives for Each Grade Section 3: Contents of Each Grade - Geometrical Figures - Mathematical Activities
Total 1,090 sentences (in English)	Total 930 sentences (in Japanese)

Table 1: Contrast of the data.

For Japan, the national curriculum provided by the Ministry of Education (MEXT) consists of a small number of pages for mathematics and has no additional explanations about the objectives and contents. The document we analysed, *CoS*, is the teaching guide to the curriculum, which contains a greater number of pages with a detailed description of the objectives and contents. In practice, Japanese teachers use both publications as curriculum sources. We used the *Teaching Guide of the Course of*

Study: Mathematics (Grade 7-9), published in 2008 (MEXT, 2008) for our analysis. Although the latest *CoS* was published in 2017, we used the 2008 version because this version has an English translation (Isoda, 2010), which made identifying corresponding words easier. The Sections corresponding to the parts of *Standards* that were analyzed are shown in Table 1.

Text analysis software

In this report, we utilized *KH Coder* (Higuchi, 2016, 2017), which can be applied to both Japanese and English. One of its advantages is that it allows easy visualization of the results, thus helping us to perform an exploratory study. The procedure with KH Coder can be summarized into the following four steps: data preparation, pre-processing, visualizing, and exploring the co-occurrence network chart. KH Coder performs pre-processing and visualizing steps automatically, and exploring the displayed chart is an important step for us to understand and re-interpret the meaning of the words. Because the latter process takes place qualitatively, this is considered a mixed method study. The four steps are as follows.

Data Preparation. KH Coder can analyze text format data using sentences as the unit of analysis. Text files were prepared from *Standards* and *CoS* and anything that could not be identified as a sentence in the text was not included in the data. For example, section headings with no periods or words within the figures were not included.

Pre-processing. Pre-processing consists of morphological analysis and word counting of the text files. For English, the *Stanford POS Tagger* software was used to tokenize sentences into words and identify the part of speech. The stop words function in KH Coder identified common words that could occur in any text, such as articles and forms of the verb "to be," and these were omitted. For Japanese, *Chasen* software was used for morphological analysis. *Chasen* could not distinguish between a noun (e.g., 証明; *shōmei*, proof) and a nominal verb (e.g., 証明スル; *shōmei-suru*, prove), so the latter was manually specified as one word so that it could be counted separately.

Visualizing. The "Word Association" command was used to determine which words were closely associated with specific words. The command, under the condition "a specific word (e.g., proof, prove) must appear," searched for sentences satisfying the condition, and listed the words that occur with a particularly high probability. The results were displayed in the co-occurrence network chart and analyzed visually.

Exploring. Based on the co-occurrence of words centered around "proof" and "prove," their meanings were interpreted. In the co-occurrence network chart, words with similar appearance patterns (i.e., with high degrees of co-occurrence) are connected by edges. Thicker edges correspond to stronger co-occurrence. If words are not connected with edges, there is no strong co-occurrence. The number of edges drawn on the chart can be increased to the number at which a focused word can be interpreted by its co-occurrence. The color of each node represents sub-graphs, which means that the same color belongs to the same group. Edges between words belonging to different sub-

graphs are represented by dotted lines. By considering words with strong co-occurrences and sub-graphs, the meaning of a particular word in the text can be interpreted.

RESULTS

The co-occurrence network of frequently occurring words in the *Standards* is shown in Figure 1.

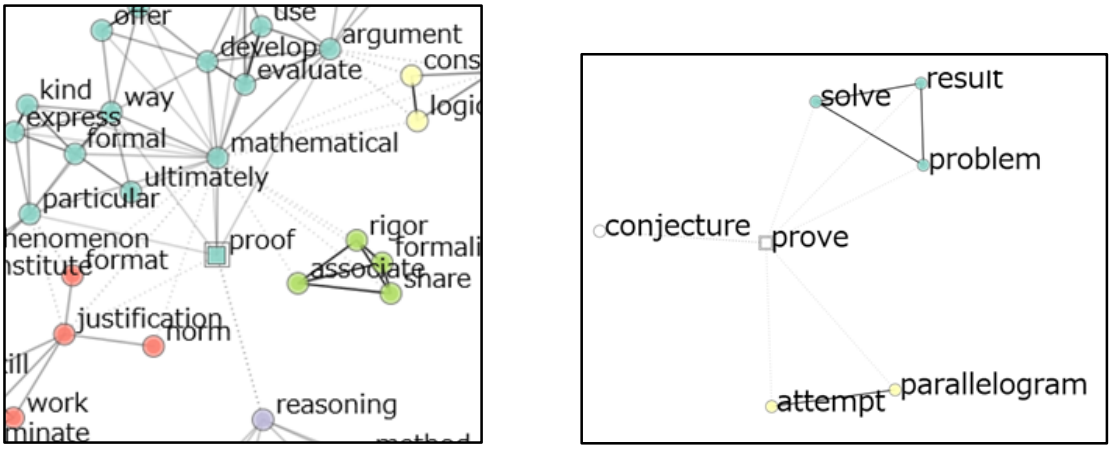

Figure 1: Extract of the co-occurrence network chart for "proof" (left; 164 edges) and "prove" (right; 10 edges) in *Standards.*

The word "proof" occurs 30 times and is strongly associated with the words "mathematical," "develop," "argument," "way," "particular," "justification," and "reasoning." The proof in the text can be re-interpreted in three ways using connected words: (1) developing mathematical arguments in a particular way, (2) justification, and (3) reasoning. While "proof" is associated with a rich set of words, the word "prove" appears only 4 times. It is surprising that the noun form occurs more often than verb form. This may suggest that the process standard "proof" is not always understood as a process. "Prove" is used in the phrases such as "to prove conjectures" (NCTM, 2000, p.42), or "to solve problems and to prove their results" (ibid., p.43). "Prove" can be re-interpreted as a process that targets the conjectures about a figure and the results of problem-solving. Of course, due to the small amount of data, the interpretation may be biased.

In the *CoS*, the noun 証明 (proof) appears 40 times and the nominal verb 証明スル (prove) 12 times. The numerical tendency is the same as that of *Standards*. The co-occurrence network for these words in *CoS* is shown in Figure 2. The Japanese in the figure is translated into the corresponding English word, with reference to Isoda (2010).

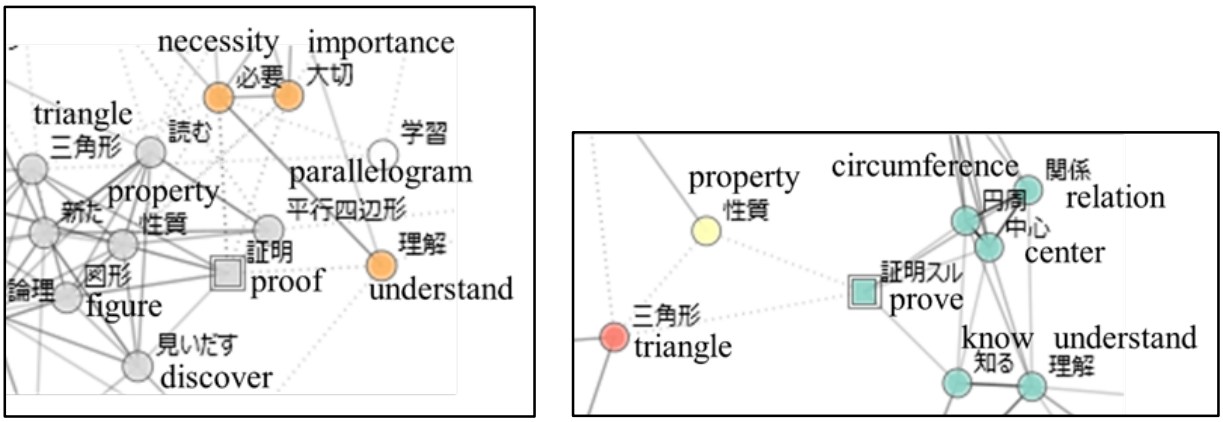

Figure 2: Extract of the co-occurrence network chart for 証明 (proof) (left; 150 edges) and 証明スル (prove) (right; 323 edges) in the *CoS*.

The word "proof" is relatively strongly associated with the words "property," "figure," "triangle," "parallelogram," "discover," "understand," "necessity," and "importance." The proof in the text can be re-interpreted in two ways: (1) to discover the properties of figures such as triangles and parallelograms and (2) something whose necessity and importance are supposed to be understood. Moreover, it is interesting to note that proof in the text is strongly associated with "discovery," not "justification." The description of the *CoS* emphasizes discovering new properties through reading proofs. On the contrary, it is found that the word "prove" is associated with words about the "inscribed angle theorem," such as "center," "circumference," "relation," and to "properties of triangles." Given that it did not co-occur with "discover," it is thus a different conceptualization from "proof." From the associated words, "prove" can be re-interpreted as a process that targets the properties of specific geometrical figures.

DISCUSSION AND CONCLUSION

The results show that the co-occurrence of "proof" and "prove" in curriculum documents in the USA and Japan is quite different. In *Standards*, the word "proof" is strongly associated with the development of a mathematical argument. This conceptualization is close to the definition by Stylianides (2007), who describes proof as a mathematical argument. In the *CoS*, it is associated with the understanding of the properties of specific geometrical figures. The former is more concerned with justification, whereas the latter is more concerned with discovery. Additionally, the word "prove" in *Standards* can be re-interpreted as a general reasoning process, and in the *CoS* as a process that associates with specific theorems. In this way, the text mining approach to the comparison allowed us to better understand the conceptualization of proof and proving in each document, since we could not get such an insight from a superficial comparison of the original texts, which describe the meaning of "proof" in each document as follows.

- "A formal way of expressing particular kinds of reasoning and justification […] arguments consisting of logically rigorous deductions of conclusions from hypotheses" (NCTM, 2000, p.56).
- "A proof is a series of statements starting with the 'hypothesis' and leading to the 'conclusion,' supported by the ideas that have already been accepted as true" (MEXT, 2008, p.115; translation by Isoda, 2010, p.181).

This suggests that the conceptualization of "proof" and "prove" in the texts is different and that curriculum developers in both countries may use the terms in different ways. To be sure, what we have articulated in this paper is only one reasonable interpretation of the meaning in the specific texts, not the "true" meaning. However, it is very important to consider the possible influence of cultural differences when conducting and utilizing international comparative studies. Since the intended curriculum influences the implemented and attained curriculum, it is necessary to examine whether these differences are also found at other curriculum levels (textbooks or classrooms).

The text analysis approach to the usages of words in curriculum documents can be applied to other related words, such as "reasoning" or "argumentation," in other countries, although a certain amount of text is required. Furthermore, it allows us to understand how certain words are conceptualized in documents based on the linguistic culture of the country. Articulating how proof and proving are conceptualized in different curricula using the same methodology is important for further development of international comparative research.

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