Analytical Expressions for Radiative Losses in Solar Cells

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Abstract

Analytical expressions for the fundamental losses in single junction solar cells are revised and improved. The losses are, as far as possible, described using parameters with clear physical interpretations. One important improvement compared to earlier work is the use of Lambert's W function, which allows for analytical expressions for the voltage and current at the maximum power point. Other improvements include the use of Stefan Boltzmann's law to describe the incoming energy flux as well as taking into account the fermionic nature of the electrons when calculating the thermalization loss. A new expression, which combines emission, Boltzmann and Carnot losses, is presented. Finally, an expression which combines all energy losses derived in this work is presented.

1 Introduction

In 1961, *Shockley* and *Queisser* published an article where the theoretical efficiency limits of single-junction solar cells (SC) were studied. They presented a model based on a detailed particle balance and found a theoretical upper limit of 40.8% [1]. This *Shockley-Queisser* (SQ) limit shows that a single-junction solar cell is unable to convert almost 60% of the incoming solar radiation into useful energy. In order to understand what happens to this 60% of the total incoming radiation, five different mechanisms of energy loss were identified and studied in Ref. [2]. There, the authors mathematically modeled the energy losses that are theoretically unavoidable. These fundamental energy losses are due to: (i) unabsorbed photons, (ii) carriers thermalizing to the bandgap, (iii) radiative emission from the cell and, voltage losses caused by (iv) the cell having a temperature higher than 0 K and, (v) a mismatch of the solid angles of absorption and emission of radiation. These five mechanisms of energy loss are further studied and discussed in section 4.

The expressions derived in Ref. [2] are based on approximations which may lead to some inaccuracies. In this work, we intend to improve those expressions.

In Ref. [3], exact expressions for both the optimal voltage and current, and consequently the efficiency, were derived by making use of *Lambert's W function*. We will follow this approach and combine it with Refs. [1] and [2] to find compact expressions for the fundamental losses in solar cells.

Before starting our discussion, let us summarize our strategies to improve the model presented in Ref. [2]. These will later on be explained in detail:

- Stefan Boltzmann's law is used to describe the incoming solar radiation.
- Lambert's W function is used to obtain exact expressions for the maximum power point voltage and current. Consequently, some of the fundamental losses are expressed in terms of Lambert's W function.
- The fermionic nature of electrons and holes is considered in the expression for the thermalization loss.
- A new expression that combines Carnot, Boltzmann and emission losses is derived. The derivation consists in computing the difference in output power when the cell is at 0 K and at a nonzero temperature T_c.
- The new expressions together with the output power account for 100% of the total incident solar radiation.

2 Conventions and notation

We will follow *Shockley* and *Queisser*'s detailed balance approach [1] and study a single junction solar cell operating at temperature $T_c = 300K$. The Sun is assumed to be a blackbody radiating at temperature $T_s = 6000K$. The flux of photons with energy in the interval $[E, E + dE]$ is given by Planck's law

$$n(E, T, \mu) = \frac{2F_s}{c^2h^3} \frac{E^2 \, dE}{\exp\left(\frac{E-\mu}{kT}\right) - 1}, \tag{1}$$

where μ is the chemical potential, or the splitting of *quasi-Fermi levels*, of the material and F_s is a geometrical factor associated with the solid angle in which the cell absorbs or emits radiation. Thermal energy emission, such as solar radiation, has a 0 chemical potential, while in the case of luminescent emission from a solar cell, the chemical potential is $\mu = qV$, where q is the electron charge and V is the voltage across the device.

The total electrical current produced by the cell is calculated as the difference between the absorbed and emitted photons times the electric charge

$$J = q \int_{E_g}^{\infty} \left[n(E, T_s, 0, F_{abs}) - n(E, T_c, qV, F_{emi}) \right] \, dE, \tag{2}$$

where F_{abs} and F_{emi} are the geometrical factors associated with absorption and emission of radiation, respectively. The first term on the right hand-side of Eq. (2) is known as *generation current*, J_G, while the second is known as *recombination current*, J_R.

Let us now compute the geometrical factors. Although this is found in textbooks, we include their derivation here since they are important for the Boltzmann loss. The geometrical factors arise from integrating over the solid angles of emission, Ω_{emi}, and absorption, Ω_{abs} with respect to the normal of the cell. We will assume that the cell emits through and angle θ_{emi} and absorbs energy through an angle θ_X. Defining θ' as the polar angle with respect to the surface normal, we have

$$F_{emi} = \int_{\Omega_{emi}} \cos\theta' d\Omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos\theta' \sin\theta' d\theta' d\phi = \pi \sin^2 \theta_{emi}, \tag{3}$$

$$F_{abs} = \int_{\Omega_{abs}} \cos\theta' d\Omega = \int_0^{2\pi} \int_0^{\theta_X} \cos\theta' \sin\theta' d\theta' d\phi = \pi \sin^2 \theta_X. \tag{4}$$

θ_X can be expressed in terms of the Sun concentration factor, X, by making use of the $\sin\theta_X = \sqrt{X} \sin\theta_{sun}$, with $\theta_{sun} = 0.267°$ being the angle which the Sun subtends without any concentrators. Defining $X_{max} = 1/\sin^2\theta_{sun}$, as the maximum concentration factor, we can express F_{abs} as $F_{abs} = \pi(X/X_{max})$. With this in mind, J_G can be expressed as

$$J_G(E_g) = \frac{2\pi q}{c^2 h^3} \frac{X}{X_{max}} \int_{E_g}^{\infty} \frac{E^2}{\exp\left(\frac{E}{kT_s}\right) - 1} \, dE. \tag{5}$$

It is possible to simplify Eq. (1). When $E - \mu >> kT$, the exponential term in the denominator becomes dominant. Hence, we can neglect the -1. This is called *Boltzmann's approximation* and is valid in the regime of useful bandgaps ($E_g \geq 0.5\ eV$ for the normal conditions experienced by solar cells). We will use this approximation to express J_R as

$$J_R(E_g) \approx \frac{2q}{c^2h^3} F_{emi} \int_{E_g}^{\infty} E^2 \exp\left(-\frac{E}{kT_c}\right) \exp\left(\frac{\mu}{kT_c}\right) \mathrm{d}E = J_0(E_g) \exp\left(\frac{qV}{kT_c}\right), \quad (6)$$

where J_0 is known as the *dark saturation current.* Unless otherwise stated, we will assume in the following that the cell emits radiation in a hemisphere, i.e., $F_{emi} = \pi$. Finally in this section, we want to point out that Boltzmann's approximation should not be used when calculating J_G since kT_s is large compared to the photon energies in question.

3 Optimal power out efficiency

The efficiency of the solar cell is given by:

$$\eta = \frac{VJ}{P_{\text{in}}}, \quad (7)$$

where $P_{\text{in}} = \sigma T_S^4$, with σ being the Stefan-Boltzmann constant. The optimal efficiency is found by differentiating $\eta(V)$ with respect to the voltage, equating to zero and solving for V. The achieved limiting efficiency receives the name of Shockley-Queisser (SQ) limit and is 40.8%. In Fig. 1, the optimal efficiency as a function of the bandgap, E_g, is presented for Sun concentration factors, $X = X_{max}$ and $X = 1$. The limiting efficiencies are 40.8% and 30.9%, respectively.

In Ref. [3], it was shown that an analytical expression for the optimal voltage, V_{opt}, can be obtained by making use of Lambert's W function, defined as $z = \mathrm{W}(ze^z)$, to solve $\frac{\partial(JV)}{\partial V} = 0$. The obtained optimal voltage then is

$$V_{opt} = \frac{kT_c}{q} \left(\mathrm{W}\left(e\frac{J_G}{J_0} \right) - 1 \right). \quad (8)$$

An expression for the optimal current, J_{opt}, was found by plugging Eq. (8) into Eq. (2). The obtained expression is

$$J_{opt} = J_G(E_g) \left(1 - \frac{1}{\mathrm{W}\left(e\frac{J_G}{J_0} \right)} \right), \quad (9)$$

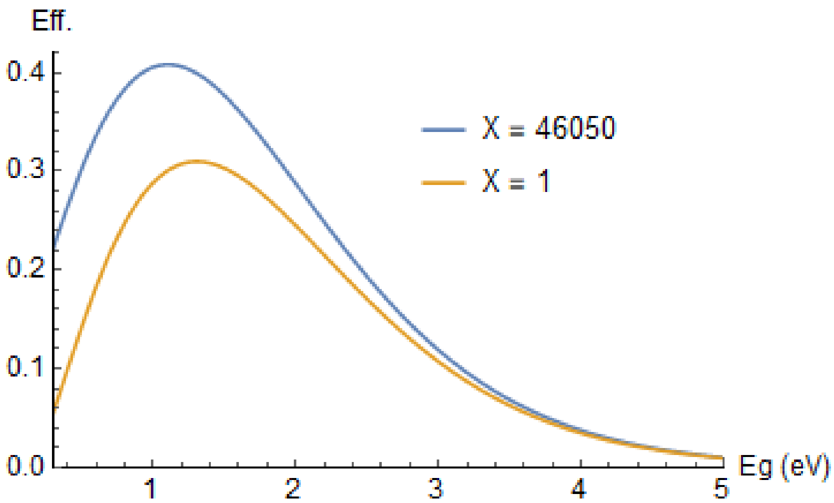

Figure 1: Power out efficiency as a function of the bandgap for different Sun concentration factors.

where they made use of $\mathrm{W}(z)\exp[\mathrm{W}(z)] = z$ to simplify the expression. The optimal power out efficiency then is found by plugging Eqs. (8) and (9) into Eq. (7). We then have

$$\eta(E_g) = \frac{kT_c}{q}\left(\mathrm{W}\left(e\frac{J_G}{J_0}\right) - 2 + \frac{1}{\mathrm{W}\left(e\frac{J_G}{J_0}\right)}\right) J_G. \tag{10}$$

A comparison between the efficiencies obtained in Ref. [3] and in Ref. [2] is presented in Fig 2. We notice a difference between the maxima of both efficiencies. The efficiency calculated with the approximations used in Ref. [2] has its maximum at 39.2%, while Ref. [3] yields a maximum of 40.8%, as expected from the *Shockley-Queisser limit* [1]. As we mentioned in section 2, Boltzmann approximation should not be used to calculate J_G. In Ref. [2], it was done in order to find a compact expression for the optimal voltage. A large drawback of using Boltzmann's approximation to calculate J_G was that the achieved power out efficiency and, later on, the intrinsic losses were slightly lower than they should be.

4 Intrinsic losses

From Fig 2, we clearly see that more than half of the incoming solar radiation is not converted into useful energy for the cell. Intrinsic losses for idealized single junction

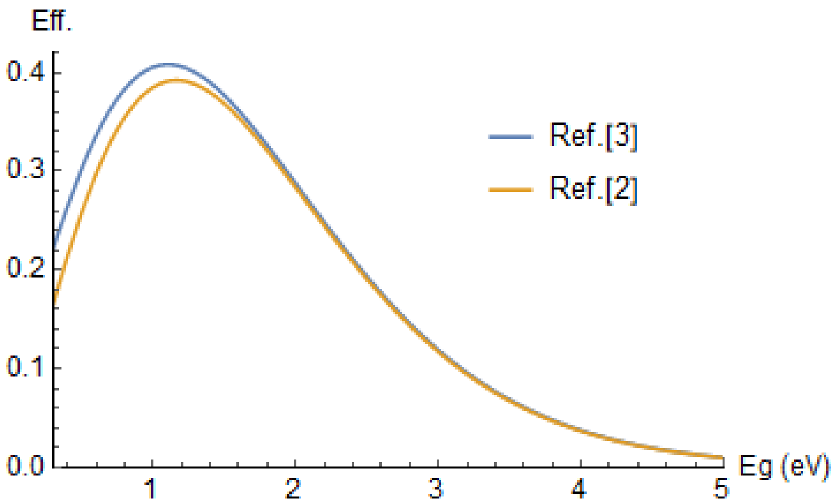

Figure 2: Power out efficiency as a function of the energy gap at maximum Sun concentration factor. Ref. [3] makes use of an expression for the efficiency that relies in fewer approximations than the one derived in Ref. [2]. This results into a higher efficiency.

solar cells are unavoidable and presented in this section. Five mechanisms of energy loss have previously been discussed in Ref. [2]. In this section, we will revise them and propose new analytical expressions with the purpose of describing them more accurately.

4.1 Thermalization

Let us start by considering the energy loss due to thermalization. If an electron absorbs a high-energetic photon, it will acquire an energy way higher than the bandgap. Through strong interactions with lattice phonons, the overexcited electrons will thermalize to the edge of the conduction band, i.e., they will emit their energy excess as heat and relax to an energy of $E = E_g$. This process of energy loss is described in Ref. [2] by

$$L_{Therm} = \int_{E_g}^{\infty} (E - E_g) \cdot n(E, T_s, 0, F_{abs})\, dE. \tag{11}$$

After relaxing to the bandgap, electrons should distribute according to Fermi-Dirac statistics, which means that their mean energy will be a little above E_g. According to the literature (e.g. in Ref. [4]), the internal energy of the electrons is given by

$$\frac{\int_0^\infty EF(E)D(E)\;dE}{\int_0^\infty F(E)D(E)\;dE} = \frac{3}{2}kT_c, \tag{12}$$

where $F(E)$ is the Fermi-Dirac distribution and $D(E)$ is the density of states for the electrons. It should be noted that the internal energy of the electrons is equal to $\frac{3}{2}kT_c$ only when their dispersion relation is parabolic [4]. The same argument should apply to holes in the VB and, therefore, the mean energy of an electron-hole pair is $E_g + 3kT_c$.

The average energy of the exciting photons is given by

$$\overline{E}_G = \frac{q}{J_G}\int_{E_g}^{\infty} E \cdot n(E, T_s, 0, F_{abs})\;dE, \tag{13}$$

This quantity can be introduced together with the extra $3kT_c$ into Eq. (11) to obtained a more accurate expression for the energy loss due to thermalization of carriers. We finally have

$$L_{therm} = \frac{1}{q}(\overline{E}_G - E_g - 3kT_c)J_G. \tag{14}$$

4.2 Further Thermalization

We saw in the previous section that after thermalizing to the bandgap, carriers will carry extra energy due to their fermionic nature. As a consequence, an extra thermalization must occur in the extraction of carriers to the metal contact. This second thermalization has to be equal to the internal energy of the carriers, that is, $3kT_cJ_G$. Therefore, this second thermalization will cancel the energy gain that we discussed in section 4.1.

4.3 Emission and Voltage Losses

In this section, we will consider three mechanisms of energy loss that constitute a smaller fraction of the lost solar radiation in comparison to the loss due to thermalization (around 7%). We will first introduce each mechanism as well as the expressions used in Ref. [2] to describe them and, after that, we will present an approach to obtain an expression which will combine all three losses.

Let us first consider the energy loss due to emitted photons. According to Kirchoff's law, since the cell absorbs radiation, it should also emit. The energy loss associated with the emission of photons produced by the cell is given in Ref. [2] by

$$L_{Em} = E_g\int_{E_g}^{\infty} n(E, T_c, qV, F_{emi})\;dE. \tag{15}$$

Second, we have two fundamental mechanisms of energy loss that directly affect the maximum achievable voltage. This is easily seen in Ref [2], where the optimal voltage is given by

$$qV_{opt} = E_g \left(1 - \frac{T_c}{T_s}\right) - kT_c \ln\left(\frac{\Omega_{emi}}{\Omega_{abs}}\right). \tag{16}$$

The first term multiplying the energy gap on the right-hand side of Eq. (16) is called *Carnot Factor* because its mathematical form resembles of the expression for Carnot's efficiency [2, 5]. Since a solar cell may be considered a heat engine, the maximum achievable efficiency needs to be limited by Carnot's efficiency, which manifests as a voltage drop.

The second term affecting the voltage appears due to the possible mismatch between the solid angles of emission and absorption of radiation. As we saw in section 2, while we assume that the cell emits in a hemisphere, it typically absorbs solar radiation through a smaller solid angle. The mismatch between the solid angles of absorption and emission results in part of the incoming energy being lost in entropy generation [6]. As for the Carnot loss, because its mathematical expression resembles *Boltzmann's entropy equation*, it is referred to in Ref. [2] as the *Boltzmann factor*. The corresponding losses are calculated in Ref. [2] by multiplying each factor by the optimal current J_{opt}.

In the following section, we will present an expression that combines the three losses that have been explained in this section.

4.4 The CBE Loss. Derivation

From Eqs. (15) and (16), we notice that both Carnot and Boltzmann factors as well as the emission loss cancel when the temperature of the cell is 0 K. We hence should be able to find an expression that combines all three losses by calculating the difference in output power at $T_c = 0$ K and at a non-zero cell temperature, T_c. We can then define the combined loss, L_{CBE}, (Carnot, Boltzmann and Emission) as

$$L_{CBE} = V_{opt}(0)J_{opt}(0) - V_{opt}(T_c)J_{opt}(T_c). \tag{17}$$

Starting with the voltage, we first take the limit $T_c \to 0$ in Eq. (8). By doing it so, we obtain an undefined result since $\mathrm{W}(T_c^{-1} \to 0) = \mathrm{W}(\infty) \to \infty$. In order to walk this problem around, we make use of the asymptotic expansion of Lambert's W function [7], given by

$$\mathrm{W}(x) \approx \ln x - \ln\ln x + \frac{\ln\ln x}{\ln x}. \tag{18}$$

In Eq. (8), all the terms in this expansion are multiplied by the temperature of the cell, T_c. Since $T_c \to 0$ and the last two terms in the expansion grow very slowly in comparison, it is reasonable to cancel them out. This leads to

$$\begin{aligned} V_{opt}(T_c \to 0) &= \lim_{T_c \to 0} \frac{kT_c}{q} \left[\ln \left(e \frac{J_g}{J_0(T_c)} \right) - 1 \right] \\ &= \lim_{T_c \to 0} \frac{kT_c}{q} \ln \left(\frac{J_g}{J_0(T_c)} \right) \\ &= \lim_{T_c \to 0} -\frac{kT_c}{q} \ln J_0(T_c). \end{aligned} \tag{19}$$

In order to proceed now, we need to compute the integral for J_0 in Eq. (6). This is easily doable thanks to having taken Boltzmann's approximation. The integral gives

$$J_0(E_g) = \exp \left(-\frac{E_g}{kT_c} \right) kT_c \left(E_g^2 + 2E_g kT_c + 2k^2 T_c^2 \right). \tag{20}$$

Now, besides the exponential term, we have terms like $T_c \ln T_c$, $T_c \ln T_c^2$ and $T_c \ln T_c^3$, which will cancel in the limit $T_c \to 0$. We finally have

$$V_{opt}(T_c \to 0) = -\frac{kT_c}{q} \ln \left[\exp \left(-\frac{E_g}{kT_c} \right) \right] = \frac{E_g}{q}. \tag{21}$$

We now continue with the current and take the limit $T_c \to 0$ in Eq. (9). Again, we have a divergent $\mathrm{W}(T_c^{-1})$ but this time, in a denominator. This implies that the last term in Eq. (9) will cancel and, in the limit of 0 K temperature, the maximum power point current is just $J_G(E_g)$. Putting all together, we find L_{CBE} to be

$$L_{CBE} = \left(\frac{E_g}{q} - V_{opt}(T_c) \left[1 - \frac{1}{\mathrm{W}\left(e \frac{J_G}{J_0} \right)} \right] \right) J_G. \tag{22}$$

It should be noted that the asymptotic expansion for Lambert's W in Eq. (18) only holds for $x > e$. In our case, this implies that Eq. (22) only is true for $J_G > J_0$, but this is true in all interesting cases where Boltzmann's approximation can be used.

4.5 The CBE Loss. Separation

Eq. (22) has the advantage with respect to Ref. [2] that the model now is more compact, reducing the amount of equations from five to three. A disadvantage to point out is that it now becomes a bit problematic to see each contribution (emission, Carnot and Boltzmann) individually.

In the following, we show alternative ways to find each contribution separately.

Emission loss Eq. (15) assumes that the emitted photons have an average energy of E_g. Let us instead compute the total energy flux $\int E \cdot n(E, T_c, qV, F_{emi})\, dE$ as

$$\begin{aligned} L_{Em} &= \int_{E_g}^{\infty} E \cdot n(E, T_c, qV, F_{emi})\, dE \\ &= \exp\left(\frac{qV}{kT_c}\right) \int_{E_g}^{\infty} E \cdot n(E, T_c, 0, F_{emi})\, dE. \end{aligned} \tag{23}$$

The last integral in Eq. (23) equals the energy flux emitted by the cell in thermal equilibrium. We denote it E_0 and use the expression for V_{opt} in Eq. (8) to get

$$L_{Em} = \frac{E_0}{\mathrm{W}\left(e\frac{J_G}{J_0}\right)} \frac{J_G}{J_0}. \tag{24}$$

Voltage Losses Both Carnot and Boltzmann losses reduce the maximum achievable voltage of a solar cell. Since this is the case, an alternative way of computing these losses may be the product of the correspondent voltage drop times the optimal current, J_{opt}, given in Eq. (9). This approach was already introduced in Ref. [2].

In the context of voltage losses, it is useful to define the ratio between the geometrical factors of emission and absorption. Let $\gamma = F_{abs}/F_{emi}$. The advantage of using γ with respect to X/X_{max} is that now, we can increase the efficiency by restricting the emission angle instead of having a high number of Suns. In terms of efficiency, it is equivalent to have a maximum Sun concentration factor and restricting the cell to emit in the same angle as it absorbs [6].

In order to incorporate γ to the notation presented in section 2, we need to make use of the general form of F_{emi} in Eq.(3). By doing this, γ appears in all the expressions where Lambert's W is involved. We have

$$\mathrm{W}\left(e\frac{J_G}{J_0}\right) = \mathrm{W}\left(\frac{\sin^2\theta_X}{\sin^2\theta_{emi}} \frac{eJ_{G,max}}{J_{0,max}}\right) = \mathrm{W}\left(\gamma \frac{eJ_{G,max}}{J_{0,max}}\right), \tag{25}$$

where $J_{G,max}$ and $J_{0,max}$ are given by Eqs. (5) and (6) evaluated at $F_{abs} = F_{emi} = \pi$. The dependence with the absorption and emission angles is now incorporated in γ. When $\gamma = 1$, the emission and the absorption angle are equal and therefore we will have maximum efficiency without having maximum Sun concentration.

In order to have a compact notation, let us introduce γ as an index in the expressions for the optimal voltage and current, defined in Eqs. (8) and (9). We make $V_{opt} \rightarrow V_{opt}^{\gamma}$ and $J_{opt} \rightarrow J_{opt}^{\gamma}$. We can now proceed to calculate the Carnot and Boltzmann losses.

Starting with Boltzmann loss, we have previously discussed that it is consequence of the possible mismatch between solid angles of emission and absorption. In terms of γ, Boltzmann loss will be zero at $\gamma = 1$ and nonzero otherwise. Hence, the Boltzmann energy loss can be computed as the difference in voltage $V_{opt}^{1} - V_{opt}^{\gamma}$ times the optimal current, that is

$$L_B = \frac{kT_c}{q}\left[\mathrm{W}\left(\frac{eJ_{G,max}}{J_{0,max}}\right) - \mathrm{W}\left(\gamma\frac{eJ_{G,max}}{J_{0,max}}\right)\right] J_{opt}^{\gamma}. \tag{26}$$

It should be noted that expanding the W functions to first order, i.e., $\mathrm{W}(x) \approx \ln x$ in Eq. (26) will result in the expression proposed in Ref. [2] for the Boltzmann loss.

Continuing with Carnot loss, the corresponding voltage drop is due to the cell having a nonzero temperature. We can hence calculate this loss as $(V_{opt}^{1}(T_c = 0) - V_{opt}^{1}(T_c))J_{opt}^{\gamma}$, where we have to set $\gamma = 1$ in the voltage drop to ensure that there is no Boltzmann loss. From Eq. (21), we have that $V_{opt}^{1}(T_c = 0) = E_g/q$. Hence, the Carnot energy loss is

$$L_C = \left(\frac{E_g}{q} - V_{opt}^{1}\right) J_{opt}^{\gamma}. \tag{27}$$

We want to point out that by introducing Eqs. (8) and (9) and further expanding the W functions to first order, the proposed expression for the Carnot loss in Ref. [2] is obtained if we also make use of Boltzmann's approximation in calculating J_G.

4.6 Unabsorbed photons

Finally, the last mechanism of energy loss is produced by the photons with energy lower than the bandgap of the material. These photons will not be absorbed. The energy lost due to unabsorbed photons is given by

$$L_{Below} = \int_0^{E_g} E \cdot n(E, T_s, 0, F_{abs})\, dE. \tag{28}$$

As for the optimal efficiency, the expression derived in Ref. [2] makes use of Boltzmann's approximation when calculating J_G. As we discussed in section 2, this approach is inaccurate. Even though the expression may not be as compact as desirable, not making use of Boltzmann's approximation in Eq. (28) gives a more accurate energy loss.

Eq. (28) can be expressed in terms of the incident solar radiation by making use of $\overline{E}_G$. Adding the energy flux of the exciting photons, $\int E \cdot n(E, T_s, 0, F_{abs})$, to

Eq. (28) gives

$$\left(\int_0^{E_g} + \int_{E_g}^{\infty}\right) E \cdot n(E, T_s, 0, F_{abs})\ dE = \int_0^{\infty} E \cdot n(E, T_s, 0, F_{abs})\ dE, \tag{29}$$

which is just $P_{\text{in}} = \sigma T_s^4$. We can hence write the energy loss due to unabsorbed photons as

$$L_{Below} = \left(\frac{\sigma T_s^4}{J_G} - \frac{\overline{E}_G}{q}\right) J_G. \tag{30}$$

4.7 The Total Loss

Finally, Eqs. (14) and (22) together with the second thermalization can be summed into a compact expression which accounts for all mechanisms of energy loss occurring in the cell. We denote it as L_T. The total sum gives

$$L_T = \left(\frac{\overline{E}_G}{q} - V_{opt}(T_c)\left[1 - \frac{1}{\mathrm{W}\left(e\frac{J_G}{J_0}\right)}\right]\right) J_G. \tag{31}$$

Note that by also adding the energy loss due to unabsorbed photons given by Eq. (30), we obtain a very obvious result which reads as: the total energy loss equals the difference between the input and the output power.

5 Numerical Results

The intrinsic losses and power out efficiency, all as a function of the energy gap, are plotted in Fig. 3 for a Sun concentration factor of $X = 1000$. The incident solar radiation is described by Stefan-Boltzmann law. A comparison between the results obtained in this work, (left), and the ones derived in Ref. [2], (right), is shown.

Tab 1 shows a comparison between the fraction of solar energy attributed to the different expressions derived both in this work and in Ref. [2]. All expressions in Tab. 1 are evaluated at the optimal bandgap of $E_g = 1.17\ eV$ for a Sun concentration factor of $X = 1000$. Due to having taken fewer approximations in Eqs. (10) and (30), the energy loss due to unabsorbed photons and the power out efficiency obtained in this work are slightly higher than the ones achieved in Ref. [2]. The sum of the two occurring thermalizations gives a fraction of the incident solar radiation that agrees with the results obtained in Ref. [2]. This is also the case with the new expression

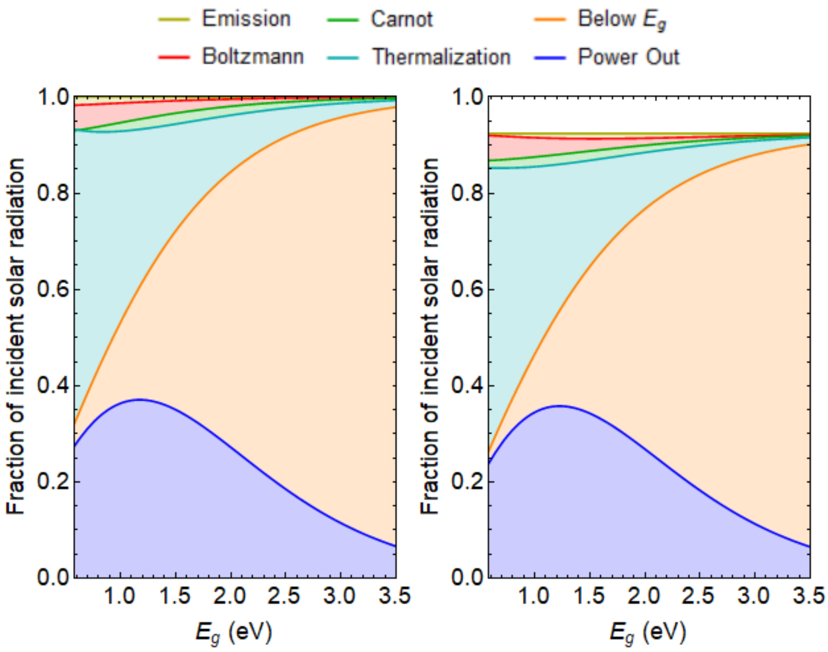

Figure 3: Comparison between this work (left) and Ref. [2] (right) of the intrinsic losses and power out efficiency as a function of E_g at Sun concentration factor of $X = 1000$.

Table 1: Fraction of solar radiation for all losses and power out efficiency at $E_g = 1.17\ eV$ and X = 1000.

X = 1000 Mechanism	Fraction of solar radiation This Work	Fraction of solar radiation Ref. [2]
Power out	0.371	0.357
Below E_g	0.235	0.180
Thermalization	0.327	0.322
Carnot	0.021	0.021
Boltzmann	0.035	0.035
Emission	0.011	0.009
Total	1.000	0.924

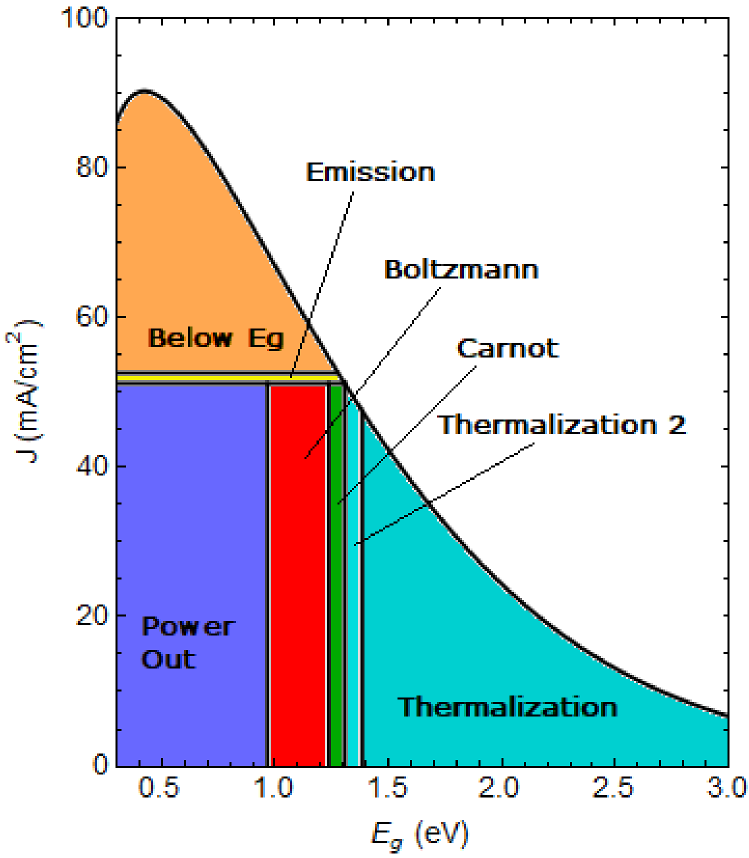

Figure 4: Intrinsic losses for a solar cell with optimal bandgap of $E_g = 1.31\ eV$ and a Sun concentration factor of $X = 1$. This type of plot was inspired by Ref. [2]

that combines emission, Carnot and Boltzmann losses, L_{CBE}. Overall, the sum of the output power and all the losses gives significant improvement with respect to the previous model, adding to 100% of the incident solar radiation.

All the expressions for the intrinsic losses presented in this work as well as the output power are also presented in Fig. 4, where a plot of the optimal current in Eq. (9), J_{opt}, as a function of the bandgap energy is showed. The intrinsic losses and output power are evaluated at a Sun concentration factor of $X = 1$, so that the Boltzmann loss is easier to notice.

6 Conclusions

With the purpose of improving the expressions for the intrinsic losses of single-junction solar cells given in Ref. [2], we have presented new analytical expressions. Our approach has its starting point in Ref. [3] where Lambert's W function was used in order to find analytical expressions for both the optimal voltage and current. We have made use of Lambert's W to find expressions for some of the fundamental energy losses. In the thermalization loss, the fermionic nature of electrons and holes was also accounted. A second thermalization, occurring in the process of carrier extraction, has been discussed. An expression that combines emission, Carnot and Boltzmann losses has been presented. Each contribution to the combined loss has also been identified. As shown in Tab. 1, our results show a significant improvement with respect to the previous model, adding up to 100% of the incident solar radiation.

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