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Grade 8 students appropriating Sankey diagrams: The first cycle in an educational design research

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Abstract

Many students do not experience usefulness in mathematics. To address this problem, we offered them a mathematical tool, Sankey diagrams, which is a flow chart appearing in news media to visualize social, industrial or environmental processes. We carried out an Educational Design Research (EDR) to develop and evaluate lesson materials about contextualized Sankey diagrams. We tested these materials with a class of grade 8 students and evaluated these on the feasibility of students' appropriation of the diagrams. In the lesson, we observed how students were able to read the Sankey diagrams, liked the societal processes visualized, yet did not fully grasp their mathematical properties. However, weeks later, the same students skillfully used Sankey diagrams in an unrelated project, which showed they needed time for their learning. Our contributions are that (1) grade 8 students can appropriate Sankey diagrams and use these in situations relevant to them, (2) design researchers should consider long-term learning effects beyond the experimental phase in EDR. We recommend educational designers to innovate curricula and introduce diagrams from news media to make students experience usefulness in school content.

Keywords: Appropriation, Educational Design Research, Sankey Diagram, Socio-Cultural Theory, Usefulness

In mathematics classrooms, students ask *"why do we need to learn this?"* (Hernandez-Martinez & Vos, 2018). With this question, they ask for the usefulness of the mathematical topics taught, for themselves, for others, for daily life or for work. Boaler (2000, p. 387) observed that mathematics teachers say, *"it will come in useful"*, offering a promise for the future without showing usefulness at that very moment. Moreover, some adults reinforce the idea of badness in mathematics by expressing *"it is a necessary evil to get through school"* (Antolin Drešar & Lipovec, 2017, p. 363) and that they only need elementary mathematics at their workplaces (Andreassen, et al., forthcoming). Niss (1994) observed that the uselessness of mathematics as experienced by students in classrooms paradoxically contradicts its importance as expressed by teachers, educational policy makers, and stakeholders in industries and sciences.

To improve students' perception of usefulness of mathematics, there have been calls to better connect school mathematics to the world outside school, in particular to workplaces (Kaiser, 2014). However, this connection is not without problems. Gainsburg (2008) observed that many mathematics

teachers lack knowledge of workplaces beyond school, but also have little experience in designing tasks that align with authentic workplaces and students' mathematical competencies and interests. When asked to design tasks related to local industries, Nilsen and Vegusdal (2017) observed teachers designing trivialized word problems, whereby the context was authentically situated in the enterprise, but the questions were inauthentic by asking for simple calculations. The tasks did neither assist students to better understand industrial processes, nor did they reflect the mathematical activities of workers (Vos, 2011, 2018).

Making students experience the usefulness of mathematics is not straightforward, since schools and teachers primarily prepare students for assessments, which often consist of short tasks about symbolic operations and algorithms in pure mathematics. By contrast, mathematics in industries, science and society (health, climate, etc.) is neither 'pure', nor readily understandable for outsiders, since it is increasingly more complex and 'hidden' in instruments and routines (Wake, 2014). Industries, science and society increasingly use large amounts of quantitative data, a process known as *datafication* (Mayer-Schönberger & Cukier, 2013), which are handled by mathematical formula and algorithms, a process known as *mathematization* (Gellert & Jablonka, 2007). Although often captured in tabulated forms, many data in industrial control rooms, in governmental or scientific reports, or in the media are visualized in diagrams showing trends, patterns, relations, correlations, and so forth (Kirk, 2019). These so-called *data visualizations* are graphical representations visualizing quantitative data or data sets. Data visualizations appear in different formats; the list of different formats increases continuously (Kirk, 2019; Tufte, 2001).

The growing numbers of different types of data visualizations in the public domain (newspapers, social media, etc.) has to do with (1) how they can convey information visually in more compact and structured ways than texts (Roth & McGinn, 1998), and (2) an increasingly more visual media landscape (Engebretsen & Kennedy, 2020). Nevertheless, the newer data visualizations are not (yet) included into mathematics curricula. In these curricula, on the one hand, diagrams are used to teach mathematical concepts, such as the use of histograms to teach the concepts of mean, quartile and standard deviation (Bakker, 2004). On the other hand, in statistics lessons, the diagrams that students are to learn are limited to only those that can be created with Excel, such as pie charts and bar charts (Prodromou & Dunne, 2017). In both cases, the focus is on prevailing mathematical and statistical content, and not on making use of new mathematical objects that are continuously developed outside schools. Therefore, we turned to the newer types of diagrams that presently emerge in the media; these have a clear mathematical nature, and simultaneously giving insights into complex social and industrial issues (Kirk, 2019). The combination of (1) being mathematical and (2) being useful for gaining insight into complex issues in real life could make these diagrams into enablers to convey the relevance of mathematics to students.

The present study aimed to find out whether some types of the newer, efficient, and colorful data visualizations could be learnt by students at grade 8 level. Since some of these data visualizations are displayed in mainstream media (Engebretsen & Kennedy, 2020), we anticipated that these could also be understandable to grade 8 students. Thus, we planned to introduce data visualizations of industrial and other complex societal processes to students and study their learning. From the range of formats for data visualizations, we selected a special type often used to visualize flows of goods, people or money, namely Sankey diagrams.

Sankey Diagrams

Sankey diagrams are flow charts, in which the width of flows is proportional to the quantity, whereas the length of a flow has no numerical meaning and can be drawn flexibly. Initially, Sankey diagrams were

created to visualize flows of people over time and space, like human migration across continents, and military movements in warfare (Tufte, 2001). Later, they were also used to visualize flows of goods, money, electrical energy (Tufte, 2001), or COVID-19 virus variants (Weise, 2020). Because Sankey diagrams enable the display of both relations between parts and quantities of these parts, they can be useful for visualizing complex processes in social, industrial or environmental contexts. As a result, they appear increasingly in news media, but also in research papers (Kirk, 2019). Below, we will present two examples.

Figure 1 was made by the first author using a free web-based drawing program (SankeyMatic ©, http://sankeymatic.com). It shows a Sankey diagram of the industrial processes regarding milk; it is based on authentic data from the Norwegian dairy enterprise Tine AS (2017). The diagram shows how milk comes from cows and goats, is transported to the dairy enterprise, where it is processed and ends in packed products of milk, yoghurt, butter or ice cream that can be sold to consumers. Sankey diagrams are, just like pie charts or histograms, members in the 'family' of *data visualizations*, also known as statistical graphics. Sankey diagrams can be used when all data are measured with the same unit (e.g., liters, persons, rupiah, Watt). Sankey diagrams can visualize a whole and its parts, with possibilities for displaying parts-of-parts, and for weaving different parts to make a new whole. Figure 1 is based on milk quantities, whereby the widths of the flows represent quantities in millions of liters.
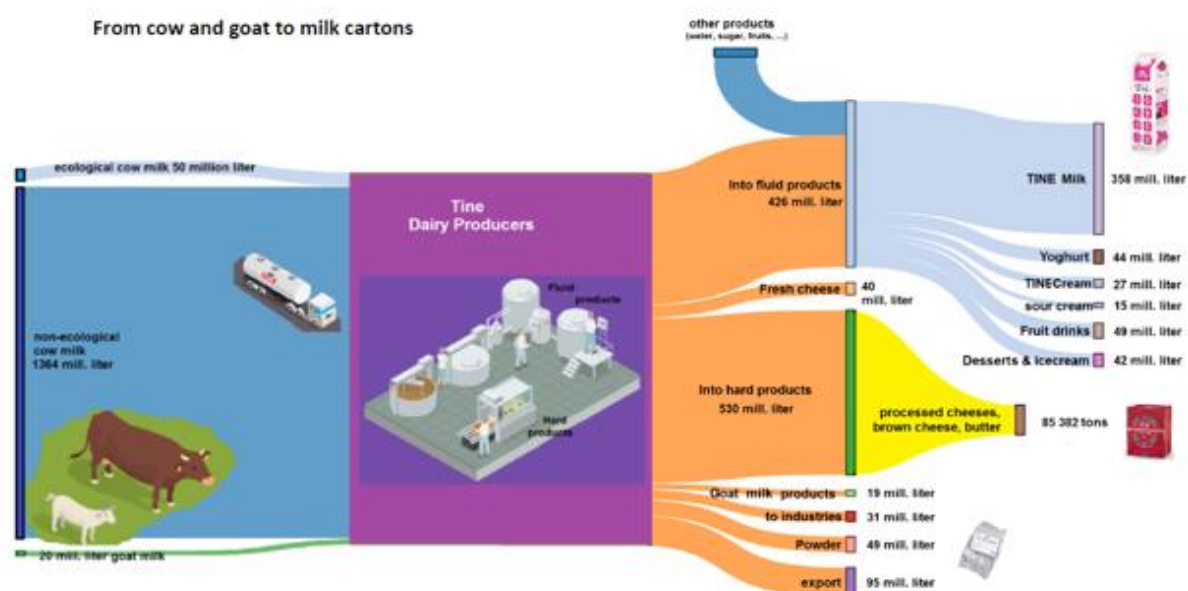

Figure 1. Sankey diagram of milk flow in a dairy enterprise

Figure 2 shows another Sankey diagram made by the first author. It visualizes a social process, namely Norway's population growth 1978-2018. The diagram is based on authentic data from Statistics Norway in 2018 (https://www.ssb.no/en). The width of the flows represents numbers of persons. The diagram covers four decades and visualizes the natural changes of a population in each decade: there are newcomers in the population (births, immigrants) and leavers (deaths, emigrants). When looking at the widths of flows, one can see that births and deaths remained relatively stable over the four decades. However, both emigration and immigration increased over the same period, whereby the latter was the primary reason for Norway's population growth.
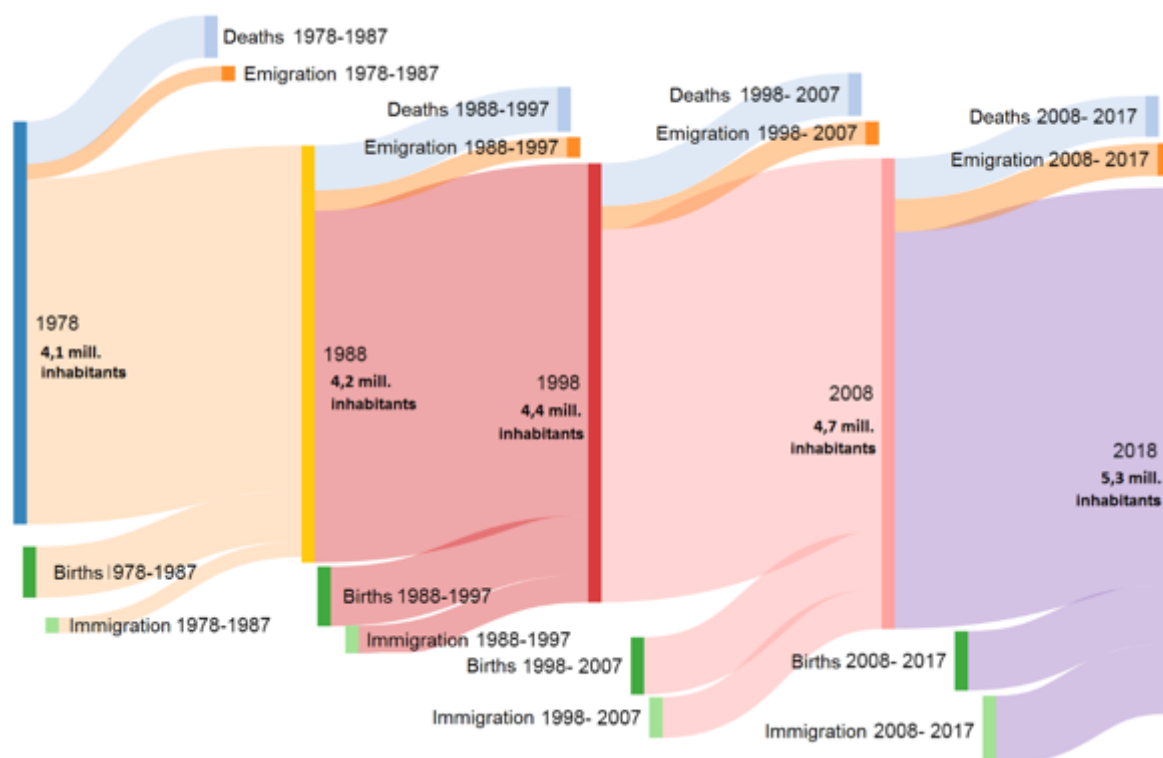
Figure 2. Sankey diagram visualizing Norway's population growth 1978-2018

Figures 1 and 2 illustrate that Sankey diagrams can visualize both industrial and societal processes. Both figures are based on authentic data and both display processes in the Norwegian environments of our students. We anticipated they would be familiar with these, yet not fully know them in more detail. More generally. we considered Sankey diagrams to be versatile, mathematical tools useful to visualize industrial and social processes. This property could assist students in seeing mathematics as a toolbox with useful tools that assist in better understanding the world around them. As far as we could trace, there is no published research on the learning of Sankey diagrams, neither by middle school students nor by older learners. However, their presence in newspapers and other media suggests that they should be quite accessible. Thus, we aimed to find out to what extent students could learn to read and handle this useful, mathematical tool.

Theoretical Framework

Our study was about students' *learning*. We perceived this from a socio-cultural perspective (Vygotsky, 1978), which considers students within their social and cultural context, and their learning as *appropriation*. Appropriation is a socio-cultural process mediated by, among others, teachers and tools, such as worksheets, pens and calculators. *"It involves taking what someone else produces during joint activity for one's own use in a subsequent productive activity"* (Moschkovich, 2004, p. 51). Researchers can observe successful appropriation when students use the learnt content in new situations, as if it was their own (Vos & Roorda, 2018). In our study, we aimed for students to appropriate Sankey diagrams as tools in a written communication between a creator and readers. This meant that the appropriation process would start from students being readers of readymade diagrams, such as in Figures 1 and 2, before becoming creators of their own diagrams. This learning sequence, from reading to creating, builds

on Curcio (1987), Friel, Curcio and Bright (2001), and Hasan's (1996), who stated that in learning to understand written symbols, diagrams and texts, the reading and interpreting precedes the writing and creating.

For both readers and creators of Sankey diagrams, there are several features at play.

1. Sankey diagrams are part of a *mathematical culture*, which has certain conventions and definitions. This feature entails that both readers and creators must understand that in Sankey diagrams, the widths of flows matter and not their lengths; moreover, the widths must represent the quantities proportionally.

2. Sankey diagrams are *models of real-world phenomena*, such as an industrialized dairy production or a nation's population growth. In other words, the diagrams visualize and represent quantitative aspects in real-world phenomena. This real-world feature entails that the real-world data need to be clearly visible and should be displayed with explicit measurement units (liters, inhabitants). In other words, Sankey diagrams are not abstract, algebraic objects without units.

3. Sankey diagrams are part of a *written communication* between readers and creators, which means readers can expect that the visualization is truthful and accurate, whilst the creator should follow certain conventions, not only the mathematical conventions (see above), but also reading conventions, for example, regarding the readability of letters, or that flows should, in general, go from left-to-right, and that the 'looks' of the diagram are clear and attractive to readers. It is in this written communication that the readers can experience the usefulness, namely that the diagrams assist them to better understand the described phenomenon. It is also in the written communication that the creators can experience the usefulness, namely as tools for conveying information to others.

The research question guiding our study was: *to what extent is it feasible that grade 8 students can appropriate Sankey diagrams?* This question focused on feasibility, that is, whether a carefully designed introduction to Sankey diagrams could lead to students using them in new situations. We deferred the question whether students perceived the Sankey diagrams as useful until after feasibility was established.

METHODS

Our research question on the *feasibility* of study students' appropriation of Sankey diagrams begs for concrete evidence in practice. Thus, we needed a research approach to design lesson resources and empirically try these in classroom practice. This research approach is known as *Educational Design Research* (EDR), also known as design-based research or design and development methodology (Cobb et al., 2003; Design-Based Research Collective, 2003; Plomp & Nieveen, 2013; Van den Akker et al., 2006). This research approach combines theory and practice, that is, it is both scientific and productive for developing innovative, educational resources, such as worksheets or instructional videos. EDR involves a sequence of cycles, which start from a preliminary design of the resources, then the resources are experimentally tried in a school-based intervention and thereafter, retrospectively analyzed and formatively evaluated, after which the resources are re-designed, and a new cycle can start. EDR takes place in naturalistic settings, in which practitioners and researchers collaborate and aim for the co-construction of new knowledge.

EDR is applied in research on mathematics education to improve the teaching and learning of mathematics. For example, EDR is applied to design and evaluate new teaching approaches to curriculum content, for instance, to rather start the learning of certain mathematical concepts from informal contexts that students are familiar with (e.g., Muttaqin et al., 2017; Prahmana et al., 2012), to include digital tools (e.g., Bakker, 2004), to insert intermediate learning steps (e.g., Doorman & Gravemeijer, 2009), or to apply alternative instructional approaches such as group work and students' presentations to deepen the learning (e.g., Putri & Zulkardi, 2018). EDR can also be applied to develop and evaluate culturally relevant test items for standardized assessments like PISA (e.g., Nizar et al., 2018; Oktiningrum et al., 2016; Zulkardi & Kohar, 2018). In our case, we applied EDR to design and evaluate new, out-of-curriculum content to show them that mathematical tools can be useful and relevant to better understanding real-life phenomena.

Based on the theoretical framework described above, the first author made a *preliminary design* of a booklet of seven pages on Sankey diagrams. Its validity was determined as follows:

1. Content validity: the booklet was designed for a *hypothetical learning trajectory* (Simon, 2014), in which students were to start from *reading* Sankey diagrams before *creating* these (Curcio, 1987; Friel et al., 2001; Hasan, 1996). The booklet contained five tasks, each of which contained several questions. The first three tasks were based on ready-made Sankey diagrams (among which those in Figures 1 and 2). For these reading tasks, the design was based on a framework for students' reading of diagrams (Friel et al., 2001). It distinguishes three levels: (1) to read the data, that is, reading pointwise information from the diagram; (2) to read between the data, that is, connecting different data in the diagram with each other and/or connecting data in the diagram to their context; (3) to read beyond the data, that is, extrapolating, reflecting, and so forth. The fourth task was 'hybrid' offering students a ready-made Sankey diagram, based on which they were to create another one for a similar phenomenon. Here, students were taken 'by the hand' to create their first Sankey diagram. In the final fifth task, students were to create one for themselves for a phenomenon of their choice, which aimed at providing us with information regarding students' appropriation of Sankey diagrams.

2. Construct validity: the booklet was piloted with a 15-year-old friend, which helped to improve wordings and to add more tasks at the lower level of the framework by Friel et al. (2001).

3. Face validity: the booklet was reviewed by the collaborative teacher of the participating class (see below) and the second author, who is an experienced designer of curriculum materials (Frejd, 2017; Frejd & Muhrman, 2022).

With the improved material (Vos, 2019), we organized the design experiment in a grade 8 class to formatively evaluate the feasibility of students' appropriation of Sankey diagrams through these materials. As explained before, at this stage, we preferred to limit the data collection and focus only on the *feasibility* of students' appropriation. If this first cycle in the EDR was reasonably successful, we could thereafter organize new cycles of the EDR and carry out more in-depth and scaled-up studies on students' perceptions and meaning making during the appropriation process. For the first cycle, it sufficed to conduct a non-immersive data collection (without cameras or audio recordings), and limit the data collection to observations, fieldnotes, and students' written answers in the worksheets.

The *design experiment* took place at an urban school in Norway within a mathematics lesson of 90 minutes with a 10-minute break. There were 17 students at the 8th grade level, who worked in six groups of two or three students. The first author was the teacher; she shortly introduced herself and the

way or working, namely that students were to work on the tasks in groups, with each group only having one booklet to impose collaboration. She did not explain the Sankey diagrams and handed out the booklet. She and the collaborative teacher were available for questions and walked around to observe the progress of the students.

During the design experiment, the first author made observations and field notes. Afterwards, she wrote further reflections and collected the worksheets with students' written answers. For the *retrospective analysis*, we followed the sequence of tasks to reconstruct the feasibility of students reading and interpreting the given Sankey diagrams (Curcio, 1987; Friel et al., 2001; Hassan, 1996), and starting to create their own, which would yield evidence of the feasibility that they could appropriate Sankey diagrams, that is, make these into 'their own' for use in new situations (Moschkovich, 2004).

RESULTS AND DISCUSSION

The first task in the booklet dealt with Norway's population growth 1978-2018, which was visualized through Figure 2. For every decade, the diagram shows in-flows of births and immigrants, and out-flows of deaths and emigrants. There was one task associated to this diagram, consisting of a dozen questions. The first questions were about reading data in the diagram, then came more questions on reading between the data, until ultimately there were questions asking students to summarize in their own words the factor(s) that contributed to Norway's population growth over the four decades.

Figure 3. Tasks on \<country\>'s population dynamics (for the translation, see the text below)

Figure 3 shows the first four questions, with some typical student answers. Questions a) and b) asked: *a) how many citizens lived in Norway in 1978?* and *b) how many citizens lived in Norway in 1988?* These are typically 'read the data' questions, and all student groups found the correct answers of 4.1 million and 4.2 million. The ensuing question was *c) how much was the population growth between 1978 and 1988?* This question asked to 'read between the data', since the answer could not be read directly out of the diagram but needed combining two different figures. All student groups correctly deducted the answer, 4.2 million – 4.1 million = 0.1 million. Thereafter came a more complex 'read between the data' question, in which data were offered that were not shown in the diagram. It said:

d) Between 1978 and 1988 there were 0.2 million immigrants, 0.2 million emigrants and 0.4 million deaths. How many newborns were there between 1978 and 1988?

Four out of six student groups did well, understanding that the population growth between 1978 and 1988 of 0.1 million had to be caused by births, immigration on the one hand making for an increase, whilst on the other hand, emigration and deaths would cause a decrease. They reasoned as follows:

1. increase in the population comes from births and immigration, thus, increase = births + 0.2
2. decrease in the population comes from emigration and deaths, thus, decrease = 0.4 + 0.2
3. the population growth between 1978 and 1988 (= 0.1 million, see question c) comes from the difference between the increase and the decrease.
4. we can then calculate, 0.1 = increase – decrease   →   0.1 = (births + 0.2) – (0.4 + 0.2)
   →   0.1 = births – 0.4   →   births = 0.5 million

We observed that two groups were unable to find the above, or a similar, line of thought. They could neither connect the question to the diagram, nor distinguish between increasing and decreasing factors. They did as show in Figure 3, answering that there were 3.3 million births. When one group was asked to explain their thinking, they explained that they added all numbers in the column (0.2 million + 0.2 million + 0.4 million = 0.8 million) and then subtracted this sum from the first given number in the task (4.1 million), "because you then find the remaining fourth group", which to them looked reasonable as 3.3 million. When asked why they added the numbers in the column, they said that this is what they had learnt to do with numbers in columns. This justification showed how the students looked for superficial 'cues', which are apparent signs that point at what is expected of them (Boaler, 2000); in this case, students perceived the numbers in a column as a 'cue' to perform an addition. At this stage, they were still unaware of the width of flows; if there were 3.3 million newborns, the flow for births would have been much thicker than the flow for deaths. Additionally, at the start of this task, they probably were still unaware of the largest subgroup in a population, namely those who are neither emigrant, immigrant, newborn or dead, in other words, those who are 'static' over decades. This awareness developed soon after, when working further on subsequent questions in this task. More importantly, we observed that this initial lack of oversight and their error to question d) did not hinder them to answer subsequent questions correctly. It meant that stumbling over task d) did not block progress.

In the subsequent questions to the population growth in Figure 2, we noted that students started to get a better overview and became better in connecting the flows meaningfully to factors of population growth. This meant that with the first questions in this task, the diagram was still quite complex, and that students needed time to 'find their way' in the diagram. The dozen questions on this diagram enabled them to subsequently better 'see' the patterns in the diagram and distinguish details from the holistic process of population growth. When they had finished all questions to this diagram, the two groups that erred on question d) were asked to have a new look at this question. At that stage, all were able to reason correctly and showed understanding of the diagram. One group reasoned visually that the widths of the flow of newborns between 1978 and 1988 was approximately of the same magnitude as the width of the flow representing deaths in the same decade, so the answer should be approximately 0.4 million and not more than 3 million. The other group corrected their error by using the context of population growth, namely that between 1978 and 1988, the emigration and immigration cancelled each other out, so for the growth of 0.1 million, the births had to be that 0.1 more than the deaths (0.4 million deaths + 0.1 million growths = 0.5 million newborns).

The second task was about the Sankey diagram visualizing the flows of milk in the dairy enterprise (Figure 1). There were five questions, which started, again, from 'reading the data' and 'reading between the data'. We observed that students pointed at the different flows of what went in and what went out. There final two questions dealt with properties of width of the flows in centimeters. All student groups managed well, although all of them resisted to calculate the exact width; they rather estimated it roughly or measured it with a ruler.

Thereafter came a task on a Sankey diagram showing how students in grade 10 in Norway chose different directions for their further education, like for example, vocational streams (health care, agricultural or technical trades, arts & media) or theoretical studies preparing for a university education. The students found this really interesting, since they were to also chose one of these directions at the end of middle school. The students easily answered to the questions, and one group wrote down that the information could also have been given through a pie chart, which showed that they could 'read beyond the data'. These first three tasks and students' answers thereon showed the feasibility of students reading and interpreting Sankey diagrams that were given ready-made to them (Curcio, 1987; Friel et al., 2001; Hasan, 1996). So far, they were not yet asked to create their own.

Thereafter came a task about traditional and energy saving light bulbs, whereby a Sankey diagram was ready-made offered about a traditional light bulb of 40 Joule. The diagram visualized that out of the 40 Joule, only 2 Joule is needed for light energy whilst the remaining energy is wasted on heat. In this task, students were to draw a similar Sankey diagram for an energy saving light bulb, which used 20 Joule, of which 2 Joule was needed for making light. However, all students struggled with this task and asked us what 'light energy', 'heat energy', 'Joule' and 'J' was. They thoughts that 'heat energy' was *"the energy that a person gets when the sun shines on your skin"*; the students had not yet learnt the concept of 'energy' in physics. Consequently, most student groups struggled to understand the given diagram, and subsequently to structure the given data about the energy-saving bulb (20 J, of which 2 J for light and the remainder 18J is wasted on heat). This meant that their lack of understanding the energy situation in light bulbs hindered their reading and their creation. Only two groups were able to draw the required, second Sankey diagram. It was obvious that this task needed redesign or replacement by another task.

The final task asked students to "describe what a Sankey diagram is and what it can be used for", and to "create a Sankey diagram on something of your choice". The first question was left empty by many; the student group in Figure 4 answered *"how many in our class that go to sport or hobbies"*. So, they neither stated the property of the widths of flows, nor how Sankey diagrams can be used for many different phenomena. To the second question, this student group (see Figure 4) drew a sketchy diagram of leisure activities showing they were 17 students, of whom two played handballs, nine played football, two were riding, and four did "something else". The flows for the smaller groups were somewhat thinner, but the flow for the nine students in football was hardly wider than the flow representing the four students doing "something else". So, at best we can say that the students made a naive Sankey diagram that discerned a whole and its parts.

a) Beskriv hva et flytdiagram er og hva det kan brukes til.

Hvor mange i klassen som går på sport/hoobxer

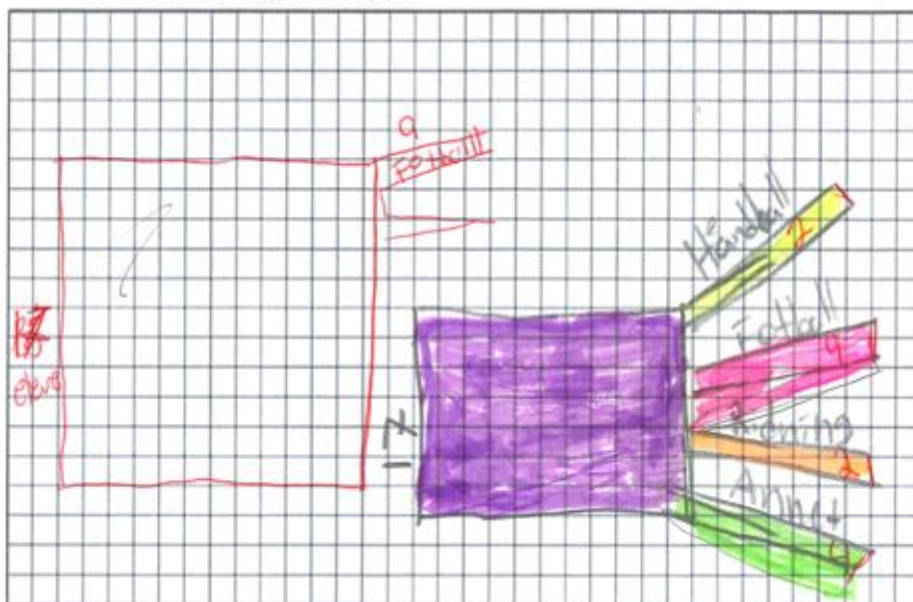b) Lag et flytdiagram til noe du velger.

Figure 4. Tasks on defining Sankey diagrams and creating one (for the translation, see the text below)

Other student groups made similar, naïve diagrams, for example of how they spent their pocket money (on sweets, entry tickets to an event, savings). All their diagrams showed a whole and its parts, which could also have been visualized through a pie chart. Their answers showed that they had appropriated that Sankey diagrams could display parts of a whole, but they could not well put properties of a Sankey diagram into wording. We also observed that all students had needed the full 90 minutes for the five tasks, and probably proper work on the final task required much more time. So, time pressure could have caused the sketchy, naive Sankey diagrams. Also, we realized that students did not create the diagrams because they felt a need to use them, but because the task in the worksheet asked them to do so. Thus, at this stage it remained unclear to us as researchers whether they could also use them in new situations without being prompted, that is: spontaneously. So, they could read and sketch a Sankey diagram, but the naïve products did not clearly indicate that the students really had made the Sankey diagrams into 'their own' (Moschkovich, 2004).

Our study could have stopped here, as often happens in EDR, when we, as researchers only observe the students during the lesson that we have designed without possibility to observe how students' thinking and learning further develops thereafter. However, in the present study, we coincidentally met the same students again two months later on a different occasion. The following account was neither expected, nor part of the formal data collection, but it supplements it in an insightful way. It is based on written notes from the first author, and an article published in a local newspaper (Nødland Skogedal, 2019).

In the two months after the 90-minute lesson described before, the same group of students engaged in an environmental project about improving the waste sorting by citizens. This project was

initiated by the local recycling enterprise, which burns all non-recyclable waste that is collected in the city. This enterprise had asked the grade 8 students at this particular school to find out how citizens could better sort their waste, so they would no longer burn recyclable materials like paper and plastics. After two months, the students were to present their findings to the leaders of the enterprise. A day before, the teacher of the class invited the first author to also attend students' presentation without revealing his reason for the invitation, but the first author happily obliged.

In the environmental project, the students had re-worded the problem of the recycling enterprise into a simpler question, namely: *"How much waste in the black bins (for non-recyclable waste) at our school is recyclable, and therefore should be in other bins?"* If recyclable waste, such as paper and plastics, was thrown into the black bins, it would be burned rather than recycled. To study their question, the students worked in groups and overturned the black bins from the school yard and made one big pile of waste. They sorted this big pile meticulously into smaller piles of recyclable materials, making separate piles for 'organics & food', 'paper & cartons', 'metal & glass', 'plastics', and a final pile for 'unrecyclable waste'. They borrowed scales from the physics department and weighed the separated waste piles in grams. They repeated the exercise on consecutive days, and also at their homes. The students gathered all data and tabulated their results. In their presentation to the leaders from the recycling enterprise, they narrated that they had worked a lot in out-of-school hours. Also, they explained, that their teacher had been present, but only for support, and that all decisions had been theirs. The students also explained that they wanted their presentation *"must be professional, because it is for adults"*, and that they had learned about Sankey diagrams and wanted to use these. They proudly informed the audience that they had found a web-based drawing program, SankeyMatic © (http://sankeymatic.com), to make smooth looking Sankey diagrams.

With confidence and by taking turns, the students reported of their collaborative research, how they had overturned the black waste bins on different days and at their homes and analyzed the content for recyclable materials. For the results of each analyzed day, they had created a table and a Sankey diagram. One of the students' diagrams is in Figure 5.
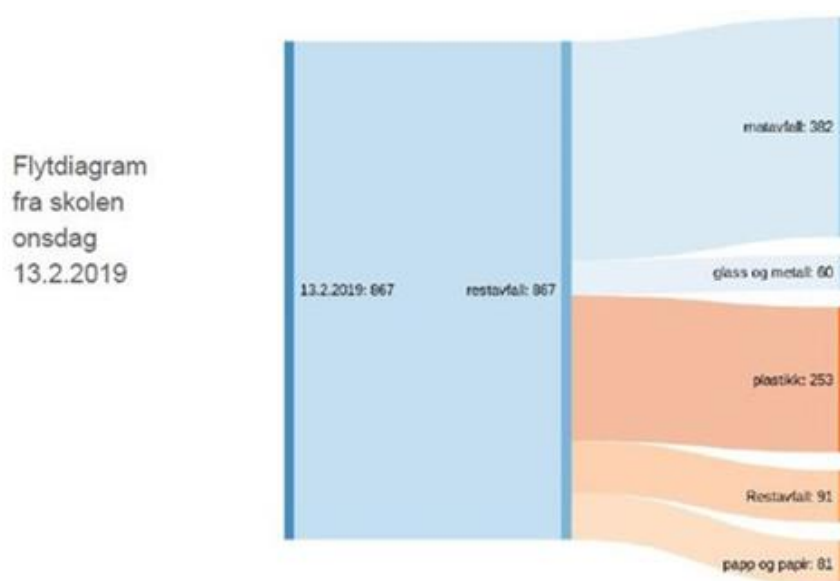

Figure 5. Students' Sankey diagram visualizing the recyclable waste (for the translation, see the text on the next page)

Figure 5 shows that the bins analyzed on that day consisted of 867-gram unsorted waste ("restavfall", on the left). They had sorted it and it turned out to be 382-gram food waste ("matavfall", right top), 60-gram glass and metal ("glass of metall", right, second from the top), 253 gram plastics ("plastikk", right middle), 81 gram paper and carton ("papp og kartong", right, bottom), and only 91 gram unrecyclable waste ("Restavfall", right, second from the bottom). Averaging over the days that they took samples, they found that the sorting was much better at their homes. More importantly, on average 90% of the waste in the school bins was recyclable and should have been put into recycling bins and not in the black bins for unsorted waste. However, these bins for separating waste were not present at their school yard. According to the students, a week before their presentation to the leaders of the recycling enterprise, they had selected a delegation of four students from their midst, which went to the school headmaster to plead for recycling bins in the school yard. With a smile, they explained that in their presentation to the headmaster, they had used the Sankey diagrams, because *"these would be good for convincing"*. As a result, they were promised new waste bins to improve the waste recycling at their school.

The presentation to the leaders of the recycling company did not show whether all students had appropriated the Sankey diagrams, and it did not clearly show the process that had led them to use the diagrams. However, it showed great progress between the naïve Sankey diagrams they made at the end of the 90-minute lesson and the diagrams displaying the sorting of the black bins into recyclable waste. Students' confidence in presenting the Sankey diagrams, their pride in how they had purposefully decided on using these (and not some other means to convince their audience), and their delight in having found a digital tool to create them, demonstrated that they felt a close connection to the Sankey diagrams – they had made them into 'their own' (Moschkovich, 2004). Being able to use the diagrams in a new situation testified of their appropriation of Sankey diagrams.

This study had the aim to offer middle school students mathematical tools that can be useful to them, in particular, that can assist them in better understanding real-life phenomena from their environment. We chose the mathematical tool Sankey diagram, because it is a tool that shows up in news media and is a versatile and powerful mathematical tool to visualize social, industrial, or environmental processes. To our knowledge, Sankey diagrams are not yet included in mathematics curricula, and we couldn't trace any research on teaching experiments with them. Therefore, we started a study guided by the research question: *to what extent is it feasible that grade 8 students can appropriate Sankey diagrams?*

To answer this question, we carried out an EDR (Cobb et al., 2003; Design-Based Research Collective, 2003; Plomp & Nieveen, 2013; Van den Akker et al., 2006), which consists of a cyclic process, in which innovative, educational resources are preliminarily designed, tested, retrospectively analysed, formatively evaluated, and thereafter improved for use in a subsequent cycle of the EDR. The cyclic interventions take place in naturalistic settings, that is in our case, in schools, and lead to both scientific and productive results. We contend that the 7-page worksheet (Vos, 2019) introducing Sankey diagrams to grade 8 students is innovative, because (1) our study is among the first to introduce Sankey diagrams to grade 8 students, (2) the diagrams are part of the newer, powerful mathematical tools used in news media to visualize complex phenomena (Engebretsen & Kennedy, 2020; Kirk, 2019), and (3) the worksheet adheres to the call to innovate mathematics education to better show the usefulness of mathematics in society to students (Kaiser, 2014; Niss, 1994). Based on our empirical results, (1) we found that certain questions in a worksheet may be answered incorrectly, but that this does not necessarily hinder students' progress and later insights, and (2) we gained the methodological insight that formative evaluations within EDR could look beyond the mere lesson in which the lesson material is

tried, since the learning may continue thereafter. Our results are practical yielding a worksheet, of which we observed that the students were interested in them, and of which the task on the light bulbs needs redesign. Also, the worksheet is adaptable to environments familiar to other target students; for instance, an educational designer can replace Norway's population growth 1978-2018 by Indonesia's population growth 1985-2025, or the industrial dairy process by that of a different, local enterprise.

Our research built on calls for better connecting school mathematics to students' local environment and to make them experience usefulness in mathematics as a tool to better understand their environment. However, we did not yet ask students whether they experienced this usefulness; we first focused on the feasibility of them learning about Sankey diagrams and finding weaknesses in the worksheet. With an improved version of the worksheet, we could yield better results regarding the feasibility and then also better study students' experiences of the usefulness of mathematics. This cyclic process of designing and re-designing is an asset in EDR and the first cycle in our research is promising. We found in the final presentation of the students, that at least some extend, and at least for some of the students, their appropriation of Sankey diagrams was quite successful, and that the students made use of this usefulness by applying them as carriers of their message to the headmaster.

Our answer to the research question is based on different observations. The first observations pertained to the 90-minute lesson, which showed that the diagrams were truly new to the students, so the appropriation process really started from zero. We observed that at the start, the students found the diagrams complex and they needed time to get both a holistic overview and insights into the details. However, the worksheet also brought a new way of working, whereby students worked together on one booklet, and also, they could not look for superficial cues (numbers in a column should be added, see Figure 3) and make meaningless calculations as they were used in the mathematics classes. Instead, they had to make calculations that were to be meaningfully connected to the diagrams (e.g. immigrations and new-borns mean an increase in the population). However, when progressing in the booklet, all students showed appreciation of the different social and industrial processes that the diagrams displayed, and some student groups were able to draw a somewhat sketchy and naïve Sankey diagram about a situation of their choice. Thus, the lesson based on the designed worksheet yielded a commencing appropriation. A second observation, two months later, showed that students were able to create their own Sankey diagrams to visualize the enormous amount of wrongly binned waste in their school yard, and additionally, to use the diagrams for their communication with both the headmaster (to make their point that more recycling bins were necessary), and to the recycling enterprise on how the sorting could be improved. This was evidence of a successful appropriation, although we do not know to what extent this occurred with all students, or only with a few. This will be object of study in forthcoming cycles in the EDR.

Researchers aiming at innovating mathematics education are studying ways to connect school mathematics to the world outside school, such as Muttaqin et al. (2017) and Prahmana at al. (2012). Our study is in line with this research but adds to it. The significance of our contribution is that we offered students a new mathematical tool that can be used to holistically visualize complex processes. We distinguished this tool as an artefact within a mathematical culture that emphasizes its mathematical properties (e.g., widths of flows being proportional), as models to visualize and represent real-world phenomena, and as a tool for communication between a creator and readers. The latter is the reason for its practicality, which should assist students in experienced how mathematics can be relevant and useful (Hernandez-Martinez & Vos, 2018), not only for analyzing phenomena for oneself to understand, but also to visualize the phenomena in an attractive way for others to understand. The grade 8 students in our

study could appropriate the tool, and this enabled them to achieve ownership on an environmental problem and actively participate in issues from a world outside school.

CONCLUSION

The present study also has methodological significance. It shows that classroom-based research, whereby a researcher enters a mathematics classroom for a session, has limitations in capturing long-term learning processes. Appropriation is a process that can take a long time; a researcher and the video camera may no longer be present when the long-term learning results can be captured. In a study on learning of the derivative, Vos and Roorda (2018) showed that appropriation can take more than a year, so longitudinal research approaches can yield findings that otherwise remain obscured. In the present study, it was a coincidence that the appropriation of Sankey diagrams was captured two months later due to a close connection between the school and the first author. We recommend methodological contemplation on the tension between, on the one hand, the study of long-term learning processes in mathematics and, on the other hand, the fact that much research is to be completed within a short time span. The financial incentives for short term data collections bias short term learning results. The present study shows that long-term learning processes may be captured when researchers frequently visit schools and contribute to the teaching of mathematics there. However, it then remains a challenge to keep up methodological standards.

Finally, we recommend more research into the teaching and learning of data visualizations, in particular the use of mathematical tools for communicating, and also research on those diagrams that cannot easily be created with standard spreadsheet tools (Prodromou & Dunne, 2017). When such diagrams become part of the mathematics curriculum, they can assist students in gaining more holistic overviews of complex processes, which is a competency needed in a society that faces increasing datafication (Mayer-Schönberger & Cukier, 2013).

REFERENCES

Andreassen, I.S., Nilsen, H. K., & Vos, P. (forthcoming). *Characterizing the relevance of mathematics as perceived by grade 8 students in a workplace related project*.

Antolin Drešar, D., & Lipovec, A. (2017). Mathematical experiences and parental involvement of parents who are and who are not mathematicians. *Irish Educational Studies, 36*(3), 357–374.

Bakker, A. (2004). *Design research in statistics education: On symbolizing and computer tools* (Doctoral dissertation). University of Utrecht, The Netherlands.

Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *Journal of Mathematical Behavior, 18*(4), 379–397. https://doi.org/10.1016/S0732-3123(00)00026-2

Cobb, P., Confrey, J., diSessa, A., Lehrer, R. & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13. https://doi.org/10.3102/0013189X032001009

Curcio, F. R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education, 18*(5), 382–393.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5–8. https://doi.org/10.3102/0013189X032001005

Doorman, L. M., & Gravemeijer, K. P. E. (2009). Emergent modeling: Discrete graphs to support the understanding of change and velocity. *ZDM Mathematics Education, 41*(1), 199–211. https://doi.org/10.1007/s11858-008-0130-z

Engebretsen, M., & Kennedy, H. (Eds.) (2020). *Data visualization in society*. Amsterdam University Press.

Frejd, P. (2017). Mathematical modelling as a professional activity: Lessons for the classroom. In: Stillman, G., Blum, W., Kaiser, G. (Eds.), *Mathematical Modelling and Applications. International Perspectives on the Teaching and Learning of Mathematical Modelling* (pp. 371–388). Springer. https://doi.org/10.1007/978-3-319-62968-1_31

Frejd, P., & Muhrman, K. (2022). Is the mathematics classroom a suitable learning space for making workplace mathematics visible? – An analysis of a subject integrated team-teaching approach applied in different learning spaces. *Journal of Vocational Education & Training, 74*(2), 333–351. https://doi.org/10.1080/13636820.2020.1760337

Friel, S. N., Curcio, F. R., & Bright, G. W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education*, *32*(2), 124–158. https://doi.org/10.2307/749671

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education*, *11*(3), 199–219. https://doi.org/10.1007/s10857-007-9070-8

Gellert, U., & Jablonka, E. (2007). *Mathematization and demathematization: social, philosophical and educational ramifications.* Sense Publishers.

Hasan, R. (1996). Literacy, everyday talk and society. In R. Hasan & G. Williams (Eds.), *Literacy in society* (pp. 377–424). Longman.

Hernandez-Martinez, P. & Vos, P. (2018). "Why do I have to learn this?" - A study from mathematical modelling education about the relevance of mathematics. *ZDM Mathematics Education, 50*(1–2), 245–257. https://doi.org/10.1007/s11858-017-0904-2

Kaiser, G. (2014). Mathematical modelling and applications in education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 396–404). Springer. https://doi.org/10.1007/978-3-030-15789-0

Kirk, A. (2019). *Data visualization: A handbook for data driven design* (2nd ed.). Sage.

Mayer-Schönberger, V., & Cukier, K. (2013). *Big data: A revolution that will transform how we live, work, and think.* Houghton Mifflin Harcourt.

Moschkovich, J. N. (2004). Appropriating mathematical practices: Learning to use functions through interaction with a tutor. *Educational Studies in Mathematics, 55,* 49–80. https://doi.org/10.1023/B:EDUC.0000017691.13428.b9

Muttaqin, H., Putri, R. I. I., & Somakim. (2017). Design research on ratio and proportion learning by using ratio table and graph with oku timur context at the 7th grade. *Journal on Mathematics Education, 8(*2), 211–222. https://doi.org/10.22342/jme.8.2.3969.211-222

Nilsen, H. K., & Vegusdal, A. (2017). Mathematics at the enterprise: Industry, university and school working together to facilitate learning. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the 10th Congress of the European Society for Research in Mathematics Education* (pp. 1537–1544). Dublin, Ireland: European Society for Research in Mathematics Education. https://hal.archives-ouvertes.fr/hal-01937338

Niss, M. (1994). Mathematics in society. In R. Biehler, R. Scholz, R. Straesser, & B. Winkelmann (Eds.), *The didactics of mathematics as a scientific discipline* (pp. 367–378). Kluwer Academic Publishers.

Nizar, H., Putri, R. I. I., & Zulkardi (2018). Developing PISA-like mathematics problem using the 2018 Asian games football and table tennis context. *Journal on Mathematics Education, 9*(2), 183–194. https://doi.org/10.22342/jme.9.2.5246.183-194.

Nødland Skogedal, M. (2019). Åttendeklassinger har forsket på søppelsortering [Eighth grade students doing research on waste recycling]. *Agder-Næring, 2019*(1). https://issuu.com/gunnarskarpodde/docs/agdern_ring_01_2019_-_nett

Oktiningrum, W., Zulkardi, & Hartono, Y. (2016). Developing PISA-like mathematics task with Indonesia natural and cultural heritage as context to assess students' mathematical literacy. *Journal on Mathematics Education, 7*(1), 1–8. https://doi.org/10.22342/jme.7.1.2812.1-8

Plomp, T. & Nieveen, N. (Eds.) (2013). *Educational Design Research: An Introduction*. SLO.

Prahmana, R. C. I., Zulkardi, & Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education, 3*(2), 115–132. https://doi.org/10.22342/jme.3.2.1931.115-132

Prodromou, T., & Dunne, T. (2017). Data visualisation and statistics education in the future. In *Data visualization and statistical literacy for open and big data* (pp. 1–28). IGI Global.

Putri, R. I. I., & Zulkardi. (2018). Higher-order thinking skill problem on data representation in primary school: A case study. *Journal of Physics: Conference Series, 948*(1), 012056. https://doi.org/10.1088/1742-6596/948/1/012056

Roth, W.-M., & McGinn, M. K. (1998). Inscriptions: A social practice approach to representations. *Review of Educational Research, 68*, 35–59. https://doi.org/10.3102/00346543068001035

Simon, M. (2014). Hypothetical learning trajectories in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 354–356). Springer. https://doi.org/10.1007/978-3-030-15789-0

Tine AS. (2017). *Annual Report 2017*. Tine AS.

Tufte, E. R. (2001). *The visual display of quantitative information* (2nd ed.). Graphics Press.

Van den Akker, J., Gravemeijer, K., McKenney, S., Nieveen, N. (2006). *Education Design Research.* Routledge.

Vos, P. (2011). What is 'authentic' in the teaching and learning of mathematical modelling? In G. Kaiser et al. (Eds.), Trends in teaching and learning of mathematical modelling (pp 713–722). Springer. https://doi.org/10.1007/978-94-007-0910-2_68

Vos, P. (2018). "How real people really need mathematics in the real world" ─ Authenticity in mathematics education. *Education Sciences, 8*(4), 195. https://doi.org/10.3390/educsci8040195

Vos, P. (2019). *Flytdiagrammer*. University of Agder. https://www.academia.edu/41095841/Ark_Flyt_diagrammer

Vos, P., & Roorda, G. (2018). Students' readiness to appropriate the derivative - meta-knowledge as support for the ZPD. In Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education, Vol. 4 (pp. 387–394). PME.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. MA: Harvard University Press.

Wake, G. (2014). Making sense of and with mathematics: The interface between academic mathematics and mathematics in practice. *Educational Studies in Mathematics, 86*(2), 271–290. https://doi.org/10.1007/s10649-014-9540-8

Weise, E. (2020, March 27). 8 strains of the coronavirus are circling the globe. Here's what clues they're giving scientists. *USA Today*. https://www.usatoday.com/story/news/nation/2020/03/27/scientists-track-coronavirusstrains-mutation/5080571002/

Zulkardi, & Kohar, A.W. (2018). Designing PISA-like mathematics tasks in Indonesia: Experiences and challenges. *Journal of Physics: Conference Series, 947*(1), 012015. https://doi.org/10.1088/1742-6596/947/1/012015