

A transition to online teaching and learning of mathematics in Norwegian higher education institutions: the perspectives of lecturers and students

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Abstract

This paper reports a study of university lecturers' and students' experiences of teaching and learning mathematics following the abrupt requirement to switch to online teaching in 2020. A goal of the study is to share experiences that could be useful to improve the teaching and learning of mathematics in online settings. The qualitative research described is a phenomenological study and draws on interviews with ten mathematics lecturers and six undergraduate students who were enrolled in at least one university mathematical course during the lockdown in 2020. The interview data were analysed using a thematic approach. This paper reports findings regarding perceptions of lecturers and students about the challenges and benefits of online teaching and learning of mathematics, how the transition to online education has influenced assessment and sharing useful approaches for teaching and learning mathematics in online settings.

1. Introduction

In this paper, we report from a qualitative study of lecturers' and students' experiences following the sudden shift to online teaching and learning mathematics in Norway arising from the imposition in March 2020 of the COVID-19 lockdown. The main goal of the study was to learn from these experiences and disseminate examples of good practice and things to avoid. To achieve this goal, provisional findings were shared at an online seminar in June 2020. In this paper, we report findings from the interviews, which we believe to be of interest to a wider, international audience of mathematics lecturers and teachers.

On 12 March 2020, just 15 days after the COVID-19 virus appeared in Norway, the government ordered a complete and immediate ‘lockdown’ of all social gatherings, non-essential businesses and educational institutions. As in most countries, the national directive to ‘lockdown’ was made with very little warning that could enable special preparation. Schools and universities were expected to continue their work ‘online’. For the university employees, this meant working from ‘home office’. The institutions had just 3 days (which included a weekend) to make the transformation to providing education online, with the additional challenge that the preparation for the transformation also had to be done through digital communications. Fortunately for the Norwegian institutions and lecturers, the country is well connected through broadband and mobile communications and there existed a nationwide online network connecting educational institutions and services. In the years prior to the pandemic, Norwegian educational agencies had entered into agreements with providers of online video technologies and publishers. Thus, access to reasonably secure video networking, journals and books was already in place. Moreover, all universities had accumulated several years of experience streaming lectures (for synchronous viewing) and recording lectures (for asynchronous viewing).

Many of the challenges met by lecturers and students, possibly, could be predicted. Online learning in the context of flipped classrooms, MOOCs (Massive Open Online Course), and other distance education provision (e.g. the Open University in the UK, which has offered distance education for more than 50 years¹) have generated some relevant knowledge (e.g. Townsley, 2016; Trenholm *et al.*, 2019; Cevikbas & Kaiser, 2020). For instance, previous studies highlighted that designing an effective online environment for teaching and learning mathematics is a challenging task because of the sophistication of the new technologies and the nature of mathematics learning (e.g. Trenholm & Peschke, 2020). Additionally, the social nature of learning mathematics should not be overlooked when designing web-based courses (Stiles, 2000) and students should actively use online discussion forums (e.g. Engelbrecht *et al.*, 2020). In flipped classroom approaches, the video lectures that are prepared for students should not be long, it is recommended they have duration between 10 and 20 minutes (Cevikbas & Kaiser, 2020).

We approached the study by setting out from what might be predicted from published research findings and personal experience. The first step was to construct a protocol for semi-structured interviews that would encourage respondents, lecturers and students to expand on themes or introduce new themes in ways that we did not anticipate. Our concern was to avoid limiting our horizon to our existing conceptions; our hope then was to be able to make a fresh contribution to the body of evidence already available.

In this paper, we focus on the effectiveness of mathematics lecturers and students in the higher education sector in coping with the sudden transition to online teaching and learning during the first phase of responding to the lockdown, that is, the period mid-March to June 2020. The study focuses on lecturers’ and students’ experiences of the online approaches over a period of approximately 8 weeks prior to the end of semester assessment period; included in these 8 weeks are the 6 ‘working days’ holiday over Easter.² In the next section, we review the literature regarding online teaching and learning of mathematics in the context where the use of digital tools was planned in advance, not the pandemic

¹ However, it is recognised that the complexity of OU provision, the technical support available, production of bespoke resources and inclusion of physical meetings suggest the knowledge gained from the OU is not immediately relevant without considerable recontextualising.

² In Norway, higher education institutions take a mid-semester break from Monday before Easter up to and including the Monday immediately following Easter Day. In addition May 1, May 17 and May 21 were designated public holidays.

situation in which technology was integrated immediately with teaching and learning of mathematics without advance preparation.

2. Online teaching and learning of mathematics

Prior to the lockdown, the online medium had created new opportunities for teaching and learning, and teaching had changed to a mix of online and face-to-face teaching, and in some courses, into fully online teaching (Trenholm & Peschke, 2020). The high-speed internet and its accessibility have significantly influenced how two-way communications could happen among students and between instructors and students (Engelbrecht *et al.*, 2020). With the development of technologies in the past decade, what can be considered as a classroom/lecture has been extended, and it is now much harder to differentiate between inside and outside classroom and leisure and study time (Borba *et al.*, 2017). Teachers and lecturers are now facing a new generation of students who grew up in the digital era where computers, mobiles, the internet and online social media are integrated into their lives (Engelbrecht *et al.*, 2020).

In mathematics, online education was in its developmental phase with its own unique characteristics and differences from traditional teaching (Engelbrecht & Harding, 2005). Engelbrecht & Harding (2005) highlighted further: 'Although much has been done to develop a pedagogy for distance learning and also for computer-based learning, a pedagogy for driving online courses in mathematics is still only in its development phase' (p. 253). Nowadays, mathematics textbooks provide online resources (e.g. Thomas's calculus; Hass *et al.*, 2018) and learning management systems such as Canvas (<http://canvas.net/>) and Blackboard (<https://www.blackboard.com/>) are used to mediate online content delivery and assignment submission. Online mathematical learning resources challenged the traditional notion of mathematical knowledge 'flowing' from lecturer/teacher to students, and new approaches such as the flipped classroom have changed the role of lecturers and students (Borba *et al.*, 2017). The online mathematical learning resources need to be carefully designed and developed in order to foster meaningful understanding of mathematics. An important question that has been discussed between mathematics educators (e.g. Engelbrecht & Harding, 2005) and still relevant is 'does the *story* of math still come across when classes move online?' (Engelbrecht & Harding, 2005, p. 255). In this regard, Hopper (2001) questioned whether the online environment could provide a nurturing environment for students compared to face-to-face teaching and pointed out:

Even in a classroom wherein only the teacher speaks, there is a recognition of being physically present, of mutual awareness, and the student who merely listens attentively may in fact experience a highly intimate and satisfying learning and social transaction. (p. 41).

Consistent with the above concerns, recently, Trenholm *et al.* (2019), after reviewing fully online undergraduate mathematics instruction based on large-scale research between 2000 and 2015, concluded that fully online mathematics teaching has not been successful compared to face-to-face teaching. Some aspects of face-to-face teaching are not available in online education; however, this should not prevent lecturers from thinking about creating opportunities for students to develop their relational understanding of mathematics. In an online setting, lecturers need to think carefully about how they can balance lecturer- and student-centred activities and plan how students could interact with content, lecturer and peers (Engelbrecht & Harding, 2005).

New technologies (e.g. tablet PCs) and online platforms (e.g. STACK) allow new and faster teaching and assessment methods; symbolic, graphical and interactive content can now be included in online mathematical courses (Galligan *et al.*, 2010; Voskoglou, 2019). Voskoglou (2019) highlighted that animated figures and mathematical representations could help students better understand mathematical concepts and solve mathematical problems. Karal *et al.* (2015) pointed out the importance of using digital

ink technology in online mathematics courses to present concepts, symbols and solution process steps to facilitate communication, interaction and students' participation in online mathematics courses. They further reported that lecturers' negative attitudes towards online mathematics courses have changed after experiencing using this technology, and they felt more comfortable in the online setting.

However, previous studies (e.g. Wallace, 2003; Trenholm & Peschke, 2020) reported that designing an efficient and effective online learning environment is challenging for mathematics lecturers, considering the nature of mathematics learning and the sophistication of the new technologies. Additionally, Trenholm & Peschke (2020) reported that fully online mathematical courses are not appealing to many undergraduate mathematics students.

Benefits and challenges of online versus face-to-face teaching have been discussed in the literature (e.g. Wallace, 2003; Engelbrecht & Harding, 2005; Basilaia *et al.*, 2020; Trenholm & Peschke, 2020). In terms of the benefits of online teaching, we can highlight that they are more accessible (from anywhere and anytime) than face-to-face teaching and that commuting to the office or lecture room is unnecessary (Jones, 2015). Challenges include the requirement of having certain equipment (e.g. computer, webcam, a stable internet connection and touchable screen or a tablet with a digital pen if it is required to share work with others) (Engelbrecht & Harding, 2005), good technological knowledge (Trenholm & Peschke, 2020) and lack of face-to-face contact (Jones, 2015). Stiles (2000) reported that not recognising the social nature of learning and replicating the traditional didactical approaches when using technology are among the main issues in designing web-based courses.

One of the tools used in online education to communicate is the opportunity to create discussion forums within online platforms. Student engagement in such discussion forums is very important as they can be exposed to different perspectives towards the mathematical concepts by their peers, which could help them develop a conceptual understanding of mathematics (Petty & Farinde, 2013). If students do not engage actively with their peers, they are more likely to achieve only a surface understanding of mathematics (Petty & Farinde, 2013). Finally, Lane & Ríordáin (2020) reported that some teachers believe online tutorials fit well with the direct instruction approach where the teacher provides clear explanations of the topic, clear steps to solve mathematical problems and examples that help them understand the involved theory. They further highlighted that these teachers believe many students prefer this method over inquiry-based approaches.

3. Technological pedagogical content knowledge (TPACK): a framework for integrating technology in teacher knowledge

TPACK (Mishra & Koehler, 2006; Koehler & Mishra, 2009) is promoted as a framework that focuses on the importance of technology in teacher knowledge while highlighting the complexity of teacher knowledge. It was developed over a 5-year design experiment that focused on how teachers and lecturers at schools and universities could develop their teaching with technology (Mishra & Koehler, 2006). The TPACK framework was designed in a context of development being gradual, evolutionary and on a voluntary basis, rather than the step-change enforced by the introduction of COVID-19 restriction measures. Additionally, it was developed in the context of using technology in an educational environment that is predominantly based on the physical presence of the learners rather than in an entirely distance learning setting (where technology is an essential starting point to access learning and teaching). TPACK has been frequently used in pre-university contexts and teacher education (e.g. Zambak & Tyminski, 2020). Furthermore, a systematic literature review (Mourlam, 2017) has shown that it has also been used in more than 40 studies in higher education. This framework has been used to

develop lecturers' TPACK through different means such as design-based activities (e.g. [Rienties et al., 2013](#)) and workshops ([Archambault et al., 2010](#)), and to characterise the nature of lecturers' TPACK (e.g. [Alzahrani, 2014](#)). Since the COVID-19 pandemic, the TPACK framework has been used in several studies to explore teachers' preparedness to integrate technology in teaching in schools (e.g. [Fuad et al., 2020](#)) and universities (e.g. [Li et al., 2021](#); [Scherer et al., 2021](#)); however, our literature search indicated that those studies were not related to teaching and learning of mathematics at higher education. Despite the usefulness of TPACK and drawing the attention of educators across different fields, it received a number of criticisms, for example, the practicality of the framework has been questioned and some scholars believed the definitions of the knowledge types are insufficient and could be more precise (See [Willermark, 2018](#)).

This framework claims that at the centre of effective teaching with technology are three core types of knowledge: content, pedagogy and technology ([Mishra & Koehler, 2008](#)). The TPACK approach does not consider these three knowledge types in isolation and highlights that the relationships among them are equally important ([Mishra & Koehler, 2008](#)). Additionally, these three types of knowledge might interact differently across different contexts, and therefore, different approaches might be taken when integrating technology into practice in various contexts ([Mishra & Koehler, 2008](#)). The TPACK framework acknowledges the importance of social and contextual factors and how they impact teachers' decisions on integrating technology in their teaching ([Koehler & Mishra, 2009](#)). Some institutions are very supportive of the inclusion of technology in teaching, and teachers are provided with the necessary means for integrating technology into their practice. In contrast, others limit the scope for introducing new and emergent technologies into teaching and learning. The extent to which the context encourages and facilitates the integration of technology into teaching influences how teachers adapt their practice to incorporate technology.

[Mishra & Koehler \(2006\)](#) have developed further [Shulman's \(1986\)](#) construct of pedagogical content knowledge (PCK). The TPACK framework comprises seven components ([Figure 1](#)). Three components are those introduced by [Shulman \(1986\)](#); these are content knowledge (CK), pedagogical knowledge (PK) and PCK. [Mishra and Koehler](#) then introduce four components to accommodate the introduction of technology; these are technological knowledge (TK), technological content knowledge (TCK), technological pedagogical knowledge (TPK) and TPACK. The detailed description of these seven components is available in [Koehler & Mishra \(2009\)](#) and [Mishra & Koehler \(2006\)](#).

In the following, we describe the TPACK component to some extent. [Koehler & Mishra \(2009\)](#) highlighted that the development of teachers' TPACK is critical for teaching effectively with technology. Subject matter experts, IT experts or educators with a low understanding of the subject matter typically do not hold this type of knowledge. The TPACK component goes beyond the three mentioned types of knowledge (i.e. content, technology and pedagogy) and comprises

... an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones. ([Mishra & Koehler, 2006](#), p. 1029).

Effective teachers use their TPACK anytime they teach and are well aware that no single technological solution works for every situation ([Koehler & Mishra, 2009](#)).

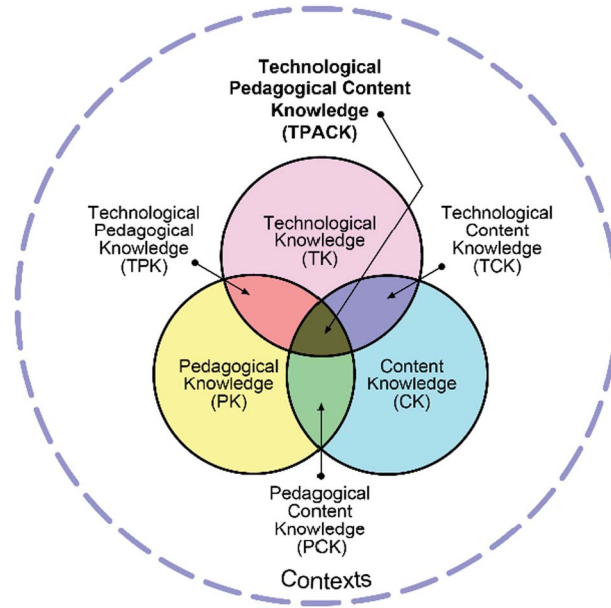


FIG. 1. *The TPACK framework*. Reproduced by permission of the publisher, © 2012 by tpack.org.

4. Research design and methodology

As part of our study of lecturers and students sudden transition to online teaching and learning of mathematics as a consequence of the COVID-19 pandemic, we recognised an opportunity to explore how the challenges perceived by students and lecturers are related to different aspect of mathematics lecturer's knowledge. Consequently, the research questions we address in this paper are as follows:

1. How did Norwegian university mathematics lecturers and their students experience the sudden transition to online teaching when universities were forced to lockdown due to a global health threat?
2. What might contribute to the development of mathematics lecturers' knowledge that could better equip them to teach mathematics in an online environment?

The qualitative study reported here took a phenomenological approach. In this approach, researchers describe 'the lived experiences of individuals about a phenomenon as described by participants' (Creswell, 2014, p.14). Online teaching is not new, and as stated above, there already exists a growing body of literature on this matter. However, the situation we were studying was characterised by the time-limited and restricted opportunity that lecturers had to prepare themselves for the change. Moreover, there was no consideration given to lecturers' or students' willingness to adopt digital approaches or adapt to the changes required. Furthermore, the transition was being made in a period of lockdown that prevented direct physical, social interaction.

We conducted 16 interviews with lecturers and students to inquire directly into their experiences. A strategic/convenience approach to sampling in choosing the lecturers and students to invite to the interviews was taken. We contacted lecturers employed in universities throughout Norway who were known to us through our existing network of mathematics lecturers and, through the lecturers, their students who were on courses they had taught online. Our network of lecturers is constructed through

several means, principally the Norwegian Mathematics Council (NMC) and MatRIC, a Norwegian Centre for Excellence in Education that focuses on mathematics teaching and learning in higher education. We wanted to be sure of the representation of a wide spectrum of opinion, and thus we approached the lecturers who we believed might adapt quickly and others reluctantly and perhaps with some resistance to the imposed transition. Further, we wanted to include lecturers on mathematics major courses as well as service courses. We are reasonably confident that we managed a broad representation, but we do not claim that any one informant was representative of a definable subset of mathematics lecturers in Norwegian higher education institutions. Therefore, care should be taken in interpreting the findings as these lecturers might have a keen interest in mathematics teaching and have developed their TPACK more than lecturers who have not engaged with, for example, MatRIC activities or NMC discussions. Another limitation of this study is that student learning assistants involved in mathematics teaching during the pandemic were not interviewed as part of this study. We acknowledge the importance of their perspectives and encourage further research on this matter.

4.1 Data analysis

An inductive thematic approach (see Braun & Clarke, 2006) was taken to the analysis of the interview data. In this approach, the identified themes are strongly connected to the generated data and the researchers do not try to fit the data to a preexisting coding frame (Braun & Clarke, 2006). This approach was taken as opposed to a deductive thematic approach because of the novelty of the COVID-19 pandemic situation. Therefore, during the data analysis process, we have not used different knowledge types of TPACK, and this framework was used later, when discussing the findings in the Discussion section. The main phases of thematic analysis were carried out in this study, including familiarising with the generated data; creating initial codes; searching, reviewing, defining and naming themes; and producing a report (Braun & Clarke, 2006).

One point to highlight is that before conducting the data analysis and when designing the study, we were interested in identifying what the lecturers and students have to say and share with us about the following issues:

- Challenges of online teaching (both synchronous and asynchronous approaches) and learning of mathematics;
- Benefits of online teaching and learning for students and lecturers (if any);
- Assessment of student learning during the lockdown period; and
- Useful approaches for teaching and learning of mathematics online (if the participants identified any).

Therefore, the outcomes of the thematic analysis on the interview data were reported in terms of these four main themes.

The Norwegian data protection agency (NSD³) has approved this research. NSD monitors the General Data Protection Regulations (GDPR) and ethical issues regarding the collection of personal data. Participation followed informed consent, and interviewees could not be identified from any element of the study (except, of course, in the interviews); records and memorandum from the interviews are anonymised, and only the researchers have accessed the generated data.

³ <https://www.nsd.no/en>

4.2 *The interviews*

We prepared schedules of interview questions, one each for the lecturers (e.g. *what are the greatest challenges that online teaching creates for you and for your students?*) and students (e.g. *how do you receive feedback from your lecturer in the current situation?*) (See appendix). These questions were open-ended and aimed at prompting respondents to describe their experiences of online mathematics teaching and learning. The questions were sent to respondents some days in advance of the interviews so that they could, and we hoped they would, prepare themselves in advance (this was also an element of ‘informed consent’, because informants could withdraw if they felt uncomfortable with any of the questions). About halfway through the lecturer interviews, we made a small modification to the schedule to pursue a theme that had emerged in the earlier interviews. Interviewees were guaranteed that the interview would not last longer than 30 minutes unless they wanted to say more.

The interviews were conducted by the first author, using Zoom. The second author sat in the background, microphone and video turned off, taking notes. At the beginning of the interview, respondents were asked if they would permit the interview to be audio recorded; they had already sent written consent that they agreed to this, but we wanted to be sure. All the interviews were terminated within 30 minutes as promised. Some interviews were followed by a brief discussion between the interviewee and both authors about issues raised in the interview. These follow-on discussions were also recorded. Notes from the interview were then sent to each respondent within 1 or 2 days for the respondent to check, change and add further detail if they wanted.

4.3 *Study participants*

The interviewees comprised 10 lecturers and 6 students who were drawn from 7 higher education institutions across Norway. The interviews were carried out during May 2020, at around the end of teaching in the spring semester. The lecturers had a range of experience teaching in higher education from approximately 1 to more than 30 years; male and female were represented in the ratio 7:3, respectively. Students also varied in age from early 20s to mid-30s and were studying on a variety of programmes including mathematics major, service mathematics for economics and engineers and teacher education.

5. Results

The analysis of the interview data was framed by the four issues set out above. These issues are related to the challenges and benefits of online education compared to traditional education and how assessment has changed due to this transition. They also consider practical approaches that can be used to improve the teaching and learning of mathematics in online settings.

5.1 *Challenges of online teaching and learning of mathematics*

Lecturers and students experienced several challenges and difficulties as a result of transitioning to online education. These challenges are outlined below.

5.1.1 *Challenges of learning mathematics during the lockdown period.* Both lecturers and students missed physical, social interaction and had concerns about lack of communication among students, lecturer and student learning assistants (SLAs), and between students themselves. The motivation for learning for a few students also decreased. For instance, a student described the situation as ‘quite terrible,

the motivation, the ability to communicate with other students, so it is really hard', and one lecturer highlighted: '[Greatest challenge] It is the distance, I cannot see them [students] . . . body language, and hope they look at you. It is easier when you are in the auditorium and take a test question . . .'. Another lecturer further pointed out the importance of receiving feedback from students for lecturers:

They [students] have all the feedback they need if they asked for it. But what is more important is what feedback I am receiving from them. That is more worrying for me because I am worried what they are doing, where they are, whether they are working. I cannot ask them; I cannot see them.

Lecturers additionally highlighted their concern about the lack of students' engagement with online teaching (online lectures, seminars and group sessions). Some students experienced difficulty in reaching out for help from their lecturer and SLAs; they were hesitant to make contact, and in many cases, the means for contacting (e.g. Facebook) would entail a loss of anonymity and invasion of privacy that present an obstacle. For instance, one lecturer highlighted:

I know that some students feel like it is hard to ask for help. It is hard to sit at home by yourself and just to reach out to me as the lecturer or to the student assistants. I think it is nerve-racking for a lot of students.

Students preferred to talk to their classmates (colleagues) about the (mathematical) difficulties they were experiencing; classmates were easily accessible. The students interviewed further elaborated that they missed the opportunities of working together with their colleagues. Students missed receiving real-time (synchronous) feedback through interaction with their colleagues compared to the past when they worked together at the university. One student highlighted:

There is more stress because you . . . do not have anybody else to lean on or to work together with. So, I think it is an important thing with this kind of study to work together with other people, you learn, you help each other on different issues.

Some also told that they had not received enough feedback on their learning from their lecturers and SLAs. Consequently, students reported that they had difficulties in monitoring their progress. For instance, one student pointed out:

I think some of the stress I have been experiencing is that I do not know how my learning is compared to the others. If I have understood enough or if I am way far beyond everyone else and also if I maybe have missed out on something that someone else has realised something crucial.

Students also reported that they had difficulties in adjusting to the new learning environment (i.e. studying at home), especially because of the distractions at home, some student respondents told how prior to the lockdown they had made an effort to keep home life and university work separate, so home represented a place to relax, not work. They further highlighted that taking more responsibility for their learning and establishing a new learning routine were challenging for them.

5.1.2 Challenges of online teaching. We provide the challenges of online teaching for asynchronous and synchronous approaches separately.

5.1.2.1 Asynchronous approach. The lecturers experienced several difficulties in terms of preparing the teaching materials. The lecturers who recorded their teaching reported that they missed student feedback and found it a bit strange to be talking without a physically present listener. For instance, a lecturer stated:

. . . when you are in a situation with students and talking and giving lectures in front of students there is a dynamic, right? You are communicating with people and of course this is completely cut out of the equation. I am just by myself talking to a machine, explaining to a machine how to create confidence intervals. I do not get any feedback, so I am losing this interaction with the students and I am getting used to it, but it feels artificial.

Additionally, some lecturers faced technical issues with IT devices, for example, if they wanted to write mathematical text to be recorded, without access to a writing tablet or touch screen computer. Editing recordings could also be time consuming; one lecturer told how he would start recording and continue until a mistake, at which point he would pause and start from just prior to the error and so on. When the ‘lecture’ was completed, he would then edit out the errors and join the required parts into a continuous sequence.

5.1.2.2 Synchronous approach. The lecturers who delivered the lectures online also reported that it was difficult for them to receive feedback from students as many students turned their camera off and only communicated with the lecturers through a ‘chat’ function. If the streamed lectures were to be recorded, because of the EU GDPR universities required that students’ cameras and microphones were switched off. If students had revealed themselves by voice or image during a lecture, this would need to be edited out before the recording could be posted online for asynchronous viewing. One lecturer commented on the ‘intensity’ of streamed (synchronous) lectures and the additional concentration required by students. In this case, the lecturer explained that she would not lecture for 45 minutes without a break or change to some other type of activity, such as giving students a task to work on by themselves for a few minutes.

5.2 *Benefits of online teaching and learning of mathematics*

The participants mentioned several benefits for online education. These benefits can be categorised as *benefits of online teaching for lecturers* and *benefits of online learning for students*.

5.2.1 Benefits of online teaching for lecturers. In relation to benefits for lecturers, those who recorded their lectures mentioned that these recorded lectures could be used later for teaching purposes. Additionally, in online education, lecturers do not need to be physically present on campus, and this would make it easier for them to travel, (e.g. participation in seminars and conferences) without changing their lecturing schedules as they can deliver their lectures digitally. Those who used recorded streamed lectures from a previous year perceived that they now have better control over their time as they do not need to deliver the teaching again and can spend that time on other tasks. One lecturer also highlighted that he now has more freedom to use approaches he believes helpful for his students.

5.2.2 Benefits of online learning for students. In terms of benefits for students, lecturers mentioned that online courses could be available to everyone, and students can watch the recorded lectures many times and at their own pace. Additionally, some lecturers perceived the fact that students must take more responsibility for their learning is a benefit of online education, other lecturers did not refer to this as either a benefit or otherwise. However, whereas students clearly recognised their increased level of responsibility for their own learning, few described it as beneficial. Additionally, one lecturer mentioned that in the current situation, students have more awareness of the learning materials uploaded to the Learning Management System (LMS) for the course compared to pre-lockdown.

The interviewed students also highlighted some benefits of online education, while some mentioned that online education has no benefits for learning mathematics. Students highlighted that they could watch the recorded lectures many times and at their own pace. Their use of time is more flexible than when they must attend the university, and they could work at their own pace. For instance, one student highlighted:

When it is at home, I can do it in my own pace. I can hear the PowerPoints over and over. I just kind of get it so much more, it makes more sense, and it feels like much easier, so the lectures have been like ten times more learning after the lockdown . . .

Some students felt they were more productive and had more time to focus on learning because of the lockdown and no social gatherings. It should be pointed out that this is not a feature of online learning but a result of lockdown measures. It might be reasonable to assume that with the easing of lockdown this advantage will disappear although some online learning will remain. Furthermore, a few students mentioned they felt less stressed compared to the past and one student highlighted that it is easier to ask for help online because they could access the mathematics support centre immediately rather than going from home to university to seek advice.

5.3 *Assessment during the lockdown period*

The summative assessment (final examination) changed from supervised examination to home examination across Norwegian universities because of the COVID-19 outbreak. During the interviews, we realised that lecturers shared concerns about the possibility of students' communication during the examination. However, one believed that in the long term, such communications might even be beneficial for student learning. The lecturers were divided in their decision to take one of two different approaches in designing questions for final examinations.

A group of lecturers mentioned that they would not change the nature of assessment questions for two main reasons: The teaching has not changed significantly, and many students prepare for examinations using the published past papers. Therefore, it is unfair to make significant changes to the examinations and create additional stress for students. The other group of lecturers decided to make some changes to the examination questions to reduce the possibility of cheating and because students would have access to resources such as textbook, lecture notes, and the internet.

Their adjustments entailed including more open-ended and analytical questions in contrast to computational and closed questions. For instance, a lecturer highlighted:

The written examination will be the usual six hours, done at home, with some additional time for uploading answers. There has been some attempt to make the questions a bit different because students will have access to books, papers, documents, and online resources. So the attempt is to change how questions are asked and what students are required to do. Fewer questions that require just calculation, more requiring explanation . . . More emphasis on explanations and arguments.

In our interviews with students, we noticed that students had expected some changes in the examination questions because of the possibility of student cheating, and that creates extra stress for a number of them. However, some students reported that they did not experience many changes in the examination questions compared to the past papers they had used in their preparation.

Some other approaches were also chosen by the lecturers to reduce the possibility of cheating. The second predominant one after changing the nature of the assessment question was to increase the time pressure by adding more questions in the examination compared to the previous years. This potentially eliminates classmates' readiness to help others because they would lose valuable time. During the interviews with students, we realised that some students felt the time pressure as an additional stress factor. For example, one student pointed out: 'In another subject, some students had a 24-hour exam which was very stressful and seems unhealthy for students mentally'. Furthermore, one student highlighted that he could not finish all of the examination questions: 'It was quite a long exam, I did not have time to answer all the questions'.

The following two approaches to reduce the possibility or impact of cheating were mentioned only by single lecturers. One said that he produced different versions of each question, and students were randomly assigned one version of each question. Another lecturer explained that he changed the weighting of summative assessment (from 70% to 30%) and course projects (from 30% to 70%) in

students' final grades. One of the interviewed students reacted negatively to this change and argued that if students knew that course projects would have a higher weight in the final grades, they would have invested more effort into them.

There were also some changes in the grading system of university mathematical courses in Norway. Some lecturers decided to change from letter grade (A to F) to pass or not pass. The other group decided to keep the letter grading system to keep students motivated to work hard and study more to improve their final grade. From the students' points of view, one student mentioned that: 'If students are actively cheating, the letter grade is not a good idea and it is a good decision to change to pass/not pass'. One disadvantage of changing from the literal A–F grades to simple pass/not pass was felt by students who had registered to retake the examination with the hope of improving their grade.

In terms of formative and diagnostic assessments, some mathematical courses had assignments and projects. Some of these were delivered before and some after the lockdown. Online assessment platforms (e.g. STACK) were also used in some courses. In those courses that online assessment platforms were not used, students highlighted that they did not receive enough feedback on their assignments. They reported that the majority of the feedback they received was only whether the assignment is approved or not approved. They further elaborated this was the case before and after the lockdown. Students also explained that they knew they could ask for more detailed feedback, and it was up to them to take the initiative to ask.

5.4 *Useful approaches for teaching and learning of mathematics online*

In the following, we report the useful approaches that can be used to enhance students' experiences of learning mathematics in an online setting at tertiary level. These are based on the analysis of the interview data, especially the challenges reported by students and lecturers, and also what lecturers and students believe would be helpful for students.

In terms of teaching, it seems students would benefit from more opportunities for working together in an online environment. This could be facilitated by having more online group sessions as part of the teaching offered to students (e.g. using breakout rooms in Zoom). Another approach to facilitate communication between students is to design compulsory group assignments and projects. If students have those tasks, they might be more motivated to reach out to other students to study and work together. Using digital ink technologies was also found useful for communicating mathematics in synchronous teaching. For instance, one lecturer was pleased with using an iPad for teaching mathematics:

... surprised by how well it [online teaching] works especially with this iPad I have ... I can mirror the screen ... I can write here real time and if they [students] have a question I can just show them ...

Concerning providing feedback to students, we could make opportunities for students to ask questions anonymously (e.g. Piazza), so students' reluctance to ask questions decreases, and they can ask questions without being afraid of being judged by their lecturers, SLAs and other students. However, we suggest that the responses provided by the lecturers and SLAs be identifiable. Lecturers mentioned that students ask questions using a wide range of platforms (e.g. email and LMS). It would also be beneficial that lecturers and SLAs re-post all of the questions they receive in a single platform, so, other students can access those questions and answers- other students might have the same questions. Minimising the response time from the lecturers and SLAs would also be beneficial for students. Online assessment platforms (e.g. STACK) can also be used to give students instant feedback on their learning. This also saves some time for SLAs and lecturers if a bank of relevant questions has been developed previously for the course or has been adapted from the existing questions on the platform. Providing comprehensive feedback on students' assignments is also very important. As stated, students complained about not

receiving enough feedback from lecturers and SLAs on their assignments. SLAs might benefit from receiving some training in this regard as they typically mark assignments in many institutions.

Several students suffered from a lack of motivation to study and missed physical social interaction with their colleagues, lecturers, and SLAs. Therefore, motivating students to keep studying is very important, and as two lecturers highlighted in the interviews, sometimes lecturers need to ‘be a social counsellor!’:

... I think my job as a lecturer is not just to relay information from brain to brain, but also to be a social counsellor and to encourage them [the students] ... I consider that part of my job description. And that has been less ... The typical student, I think needs a little bit of encouragement, needs a little bit of face to face to know that the teacher cares ...

In this regard, one lecturer who was using recorded (asynchronous) lectures mentioned that he held ‘social meetings’ on Zoom, once a week with students to talk about how to study and how to get through difficulties, and to motivate them to keep working and making progress. He reported that those social meetings were better attended compared to online lectures:

... I get no contact from the students. So ... we had at least once a week a social meeting, just an open meeting, all the students could make contact with me and my assistant. We sat there for two hours talking to them [students] about how to study in this situation ... to comfort them, to say that we understood that you are stressed, maybe we can fix this in the fall ... I think it is very important to have such open social meetings.

6. Discussion

This study reports the experiences of university lecturers and students of teaching and learning of mathematics following the sudden requirement to change to online education in 2020. Regarding the first research question, the benefits and challenges described in previous studies regarding online education were also identified in our study. Similar to the previous studies (e.g. Wallace, 2003; Engelbrecht & Harding, 2005; Petty & Farinde, 2013; Basilaia *et al.*, 2020), we found that in online education, many lecturers and students missed face to face contact and had difficulty with engaging with technology for teaching and learning of mathematics.

The useful approaches that have been identified in this study for teaching and learning mathematics in online settings are in line with the existing literature in (mathematics) education. First of all, in line with the literature (e.g. Galligan *et al.*, 2010; Karal *et al.*, 2015), new technologies, including tablets and digital ink technologies, were found useful by lecturers for communicating mathematics and increasing students’ participation in online settings. As Karal *et al.* (2015) highlighted, ‘digital pen technology plays a positive role in the enhancement of interaction between the elements of an online learning environment by providing real-time feedback to students and permitting to digitise mathematical concepts’ (p. 319). Secondly, in terms of *mathematics lecturers as social counsellors* and their roles in paying attention to social and affective aspects of learning mathematics, in the recent studies that have been published in mathematics education since the COVID-19 pandemic, the importance of caring teaching has been pointed out. Even new terms such as panic-gogy (Panic+ pedagogy) have emerged, which includes but is not limited to ‘understanding students’ practical resources and problems, including availability of devices and the internet, family responsibilities, students sent home who need to find a new place to live, and financial constraints’ (Engelbrecht *et al.*, 2020, p. 836). Thirdly, the findings indicate that computer-aided assessment systems such as STACK were found helpful to provide more comprehensive feedback on student mathematical learning, as highlighted in the literature (e.g. Sangwin & Grove, 2006). Fourthly, regarding the suggestion of including compulsory group assignments, care should be taken when using such an initiative as there is some evidence that a number of students do not collaborate

enough with their group members towards completing these assignments (e.g. Putra *et al.*, 2020). Fifthly, using breakout rooms has also been recently highlighted (Lishchynska *et al.*, 2021) as one of the ways to facilitate peer-to-peer interactions because of several reasons, such as less anxiety for speaking and more opportunities for focused discussion compared to whole-class discussion; however, Lishchynska *et al.* (2021) reported that this approach might not be found helpful by all university students and concluded: ‘breakout rooms can be effective for sharing work, discussions and receiving feedback (from peers or the lecturer) but seem to work better for structured tasks where work can be shared effectively’ (p. 431). Similar to our findings, providing opportunities for students to ask questions anonymously to address some students’ shyness were perceived as useful by students in Lishchynska *et al.* (2021) in Ireland. In the UK, similarly, Gilbert *et al.* (2021) reported that it seems students with anxiety or negative attitudes towards mathematics were using the online support more than the physical support centre in the past. They highlighted these students ‘found comfort in hiding behind cameras that were switched off, communicating instead through the text chat’ (p. 313). In the following, we reflect on the theoretical framework, TPACK, and discuss the findings in light of the seven components of this framework in response to the second research question.

6.1 Content knowledge

In our study, students did not comment on the CK of mathematics lecturers. This could be due to the fact that the majority of mathematics lecturers in Norway have a PhD in mathematics or mathematics education and therefore are well prepared with the necessary mathematical knowledge (CK) for teaching the courses. Furthermore, we noticed that the lecturers’ CK informs them that mathematics is not a ‘spectator sport’, mathematics entails cognitive activity, and mathematical communication requires action. We did notice, by its absence, that students did not remark about their SLAs’ inadequate CK. In earlier work (in progress) we have noted that students can sometimes be critical of their SLAs’ CK. However, during the lockdown, it seems the lecturers had been able to organise their SLAs in ways that did not make lack of CK evident, because the lecturer was always available (and unseen) when SLAs were active to respond to SLAs’ questions or take over with a student that was posing a question the SLA could not handle.

6.2 Pedagogical knowledge

Students raised issues about mathematics lecturers’ PK. In detail, students highlighted that insufficient feedback on their learning was provided, especially on their assignments. Students further elaborated that this was not an issue that only appeared after transitioning to online education; it has existed before.

6.3 Pedagogical content knowledge

Our interview data reveal that the lecturers had PCK that tells them that if students are to learn mathematics, they need to be active, and many lecturers will incorporate approaches into their practice to motivate students’ active learning, for example, by facilitating and promoting reflection and metacognitive activity. In this context, for many students, mathematics needs to be developed synchronously with the student engaged; mathematics writing, by the lecturer, or by the student, or SLA needs to happen in real-time (synchronously) with the oral or textual explanation.

6.4 *Technological knowledge*

TK is beyond the traditional definition of computer literacy and is about developing understanding of ways in which information technology could facilitate or hinder achieving a goal (Koehler & Mishra, 2009). For many lecturers, TK has expanded during the lockdown. Many more are familiar with online video lectures (synchronous) or recording lectures for asynchronous viewing. For some, it has been a steep learning curve, for others a natural development. Benson & Ward (2013) highlighted that developing TK along with PK is necessary for building TPACK.

6.5 *Technological pedagogical knowledge*

Some experienced lecturers mentioned that their use of technology in teaching was limited before the COVID-19 pandemic. They further elaborated that it takes time for them to learn how to work with online platforms for teaching, such as using breakout rooms in Zoom. Additionally, some lecturers were not familiar with online platforms that can be used for communications between students, lecturers and SLAs (e.g. Piazza), another indication of lack of TPK. However, there was also evidence of TPK to keep recordings relatively short or in synchronous presentation to break up a more extended session by incorporating different types of activities for students.

6.6 *Technological content knowledge*

In the context of mathematics, TCK is about how technology and mathematics are reciprocally associated and influence and constrain each other (Koehler & Mishra, 2009). No specific issue was identified from the study finding regarding the mathematics lecturers' TCK. This could be because mathematics lecturers are typically well equipped with the knowledge of how to use mathematical software (e.g. MATLAB and Maple) to explore, manipulate and represent mathematical objects during their education towards becoming mathematicians and also during their academic careers.

6.7 *Technological pedagogical content knowledge*

In relation to TPACK, as one lecturer put it, the mathematics does not change; it is only a matter of thinking how to present the mathematics using the new medium:

Every new way of teaching makes you think differently . . . write books you have to do things differently, make videos . . . teach 500 students you have to do it differently. And now I have to do things differently. Like I say, the mathematics does not change . . .

Lecturers used a number of approaches to use the technology to enable them to present the mathematical content formally: using lectures recorded from a previous year, recording their lectures using PowerPoint and Beamer (a LaTeX document class for creating presentation slides), using a writing tablet (iPad) and pen, rigging up a webcam to emulate a document camera.

Furthermore, a theme stands out particularly, and that is in the online situation, many students and lecturers prefer the synchronous sharing of mathematical writing by students and lecturer. Regarding the issue of developing mathematical knowledge in a technological context, the great challenge many students and lecturers face is the sharing of mathematics 'in progress' synchronously. Many students and lecturers want to be able to write mathematics, and they want to see the mathematics being written as it also has a synchronous spoken commentary, and they want to be able to question as the mathematics is being written. Moreover, they want to be able to write on the same, shared surface. Lecturers and

students have come close to this ideal, by using a writing tablet, using a webcam, showing their text to the computer camera or taking a photograph of their text with a smartphone. However, none of these solutions enables the synchronous sharing of writing on the same surface. It seems a possible solution does exist, for example, this has recently been added within an upgrade of Zoom, but it was not available to any of the respondents in our study. Nevertheless, any solution is likely to entail all students and lecturer having access to a writing table or computer with a touch-sensitive screen.

7. Conclusion

In this paper, we shared how Norwegian university mathematics lecturers and their students experienced the sudden transition to online teaching when universities were forced to lockdown due to a global health threat. We have reported students and lecturers' perceived challenges and what they perceived beneficial in such setting. Furthermore, when we embarked on this study, we hoped that it would expose some innovative practices that effectively address challenges faced by lecturers and students when mathematics is taught online. This hope has only been fulfilled in part. It does appear that there is a considerable range and availability of online-digital solutions to many of the issues lecturers and their students have met. However, the strongest message that comes through the interviews, with both students and their lecturers, is that the real-time interaction possible through physical meetings has been the greatest loss. Lecturers want face-to-face contact with their students; students want to be able to work together in groups where they share the same space. We have also discussed the findings in term of the seven components of TPACK framework in response to the second research question and found that the challenges reported were more related to PK, TPK and TPACK of mathematics lecturers. We would like to conclude this paper by highlighting that the development of TPACK in all its complexity could be facilitated by well-functioning learning communities in which lecturers can share their experiences, challenges and innovations in the social space created by coffee and lunch breaks. The lockdown has raised awareness of technological solutions to teaching and learning; perhaps, it also has helped us to value more highly those occasions in which we can physically meet and enjoy relaxed, informal professional conversations.

The TPACK framework has drawn attention to the need to develop TK that meets the pedagogical needs which arise in the online mathematics teaching context. In response to the first research question, the crucial negative experiences of students and lecturers arose mainly from pedagogical issues, such as motivation, active engagement in discussion and dialogue and group interaction. As such this did not challenge lecturers' PK, but in many cases their TK and consequently their TPK did not support the pedagogy they knew to be important. This finding leads immediately on to address the second research question. Some lecturers are aware of emergent technologies and how these might be employed to address, especially the affective and social learning support structures students need in online situations. Without structured and targeted intervention knowledge spread through the higher education mathematics teaching community is likely to be slow. Perhaps the greatest contribution to the development of lecturers' TPACK will be through the sustained actions of the Norwegian Mathematics Council and MatRIC that will enable the ongoing community-based sharing of practices that emerge alongside the technological developments.

We would like to end the paper by highlighting that the findings reported here is based on the interviews conducted after the first wave of lockdown and the COVID-19 pandemic in Norway. If the interviews were conducted at a later stage, the findings could be different. For instance, Gilbert *et al.* (2021) reported that a semester after the provision of online mathematics and statistics support (MSS) in early 2021 (i.e. January and February 2021), the practitioners were more open to online teaching and learning of mathematics and statistics, and all participated practitioners mentioned that they would like to continue

offering some sort of online support to students. Gilbert *et al.* (2021) highlighted that ‘practitioners believe that MSS in the new normal will be a hybrid of face-to-face and online support but predominantly face to face’ (p. 312–313).

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Appendix

Lecturers' interview questions.

Thank you for agreeing to help us in this inquiry into online mathematics teaching and learning.

1. Please can you tell us, how many years' experience of teaching mathematics at higher education do you have?
2. Please will you describe your online mathematics teaching that you have been doing recently.
 - i. What course(s)? *How many students do you have in your courses?*⁴ *Do you have students assistant to help you with the courses?*
 - ii. What level?
 - iii. Types of provision—live lectures (Zoom, streamed, etc.) Seminar or group work?
 - iv. How do these work in practice?
 - v. What resources do you use to support teaching, and encourage students to use in their learning?
 - vi. What challenges have you experienced from using technology during your online teaching?
 - vii. *Would you encourage communication between students?*
 - viii. *In regular teaching, when physically present with the students, the teacher is receiving a range of feedback from the students that indicates their engagement—facial expressions, body language, etc. What do you do to get similar feedback when working online?*
3. Try to think of the 'typical' student that you teach, how has your approach to communicating mathematics to that typical student been affected by the change from being physically present to being online?
4. If you had experience of online teaching prior to this current crisis situation, please will you tell about this.

⁴ The italic parts in both lecturers and students' interviews were added to pursue a theme that had emerged in the earlier interviews.

5. How do you provide feedback to students in the current situation?
6. Have you introduced anything to your online teaching that has been innovative within your own practice, or in a wider sense of mathematics teaching generally?
7. What additional support or resources would be helpful to you or your students?
8. What are the greatest challenges that online teaching creates for you and for your students?
9. Does online teaching bring any benefits? If yes, what are the benefits?
10. What have you done to adjust the assessment methods to take account of the new teaching and examining arrangements?
 - i. Why?
 - ii. Did you consider doing anything different?
11. Overall, if it is necessary to continue with online teaching for a prolonged period, what do you think will be the long-term consequences for university mathematics education? *For teaching, learning, and assessment*

Thank you for your help in responding to these questions.

Students' interview questions.

Thank you for agreeing to help us in this inquiry into online mathematics teaching and learning.

1. Please will you describe your online mathematics learning that you have been doing recently.
 - i. What course(s)?
 - ii. What level?
 - iii. Types of provision—live lectures (Zoom, streamed, etc.) Seminar or group work?
 - iv. How do these support your learning?
 - v. What resources has your lecturer suggested you use to support your learning?
 - vi. What additional resources have you introduced for yourself?
 - vii. What do you do to support your own learning apart from what the lecturer tells you to do?
 - viii. What challenges have you experienced from using technology during the online teaching?
 - ix. *Communicating with other students to learn the topics?*
 - x. *What do you do when engaging with the lecturing material (whether online or offline)? Do you take notes or just listen, etc.?*
2. If you had experience of online learning prior to this current crisis situation, please will you tell about this?
3. In regular teaching, when physically present with the lecturer, there is a sense that the lecturer able to receive a range of feedback from the students that indicates their engagement—facial expressions, body language, restlessness, questions and answers hands raised, etc. Does your lecturer do anything to get similar feedback when working online?
4. How do you receive feedback from your lecturer in the current situation?
5. What are the greatest challenges that online learning creates for you?
6. Does online learning bring any benefits?
7. Overall, if it is necessary to continue with online teaching for a prolonged period, what do you think will be the long-term consequences for your learning of mathematics?
8. Are you aware of any changes to the assessment/examination requirement as a result of doing everything online?
 - i. How do you feel about this?
 - ii. *Have you changed the way you prepare for exam?*

9. Can you describe any additional stress you have experienced as a result of online teaching?

Thank you for your help in responding to these questions.

The following additional questions are only asked from a student who had special learning needs:

- How have you experienced the additional strain and stress in your studies and examination preparation?—if you could be specific like lack of concentration, perhaps disturbed sleep patterns causing increased tiredness, impact on engaging deeply with logical arguments and understanding, impact on memory and so on.
- The university has been made aware of your special and specific learning needs, so what has the university done to ensure that you study on a ‘level playing field’ with other students (in other words to mitigate the impact of the additional problems you face)?
- Have you felt that the support you have received has been appropriate, sufficient and effective?
- Have you done anything yourself to mitigate the impact? I cannot image what this might be, but maybe taking more exercise and trying to keep to a careful well-structured routine (I mention these things because it is what I am doing to reduce the increased stress that I have experienced—back in March I literally believed I had received a death sentence . . . to the point I even wrote a new ‘last will and testament’—it seems a bit extreme now, but it did have a significant calming effect at the time).
- What more could be done in the present situation, and if we have to continue with online provision during the autumn, to provide you, and others with learning difficulties, with the additional support you require?