

Dynamic Asset Allocation Strategies Based on Volatility and Turbulence

Performance comparison 1973-2014

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This master's thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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Preface

This thesis is written as the final part of the master's degree in Economics and Business Administration, with specialization in financial economics, at the University of Agder. The workload in total corresponds to 30 credits, and thereby covers one whole semester. During the whole process, I've learned a whole lot about the art of scientific writing for finance, and also various features of the computational software R.

The first individual I want to express my sincere gratitude to, is my supervisor, professor Valeriy Zakamulin, for great guidance and constructive, as well as quick feedback throughout the course of this semester. Particularly I consider his remarks on the first draft as very valuable. Yet, any possible errors or inadequacies contained herein remain my own responsibility.

Due to the immense amount of time I've had to devote to this thesis, there have been periods where I've been awfully absent-minded and self-centred. At this point, I want to thank my boss, Kristin Eskeland, for her patience during these periods where I've only been able to contribute to a minor amount of work, in proportion to what I generally do, which indeed has caused some extra difficulties for her little store. I also owe gratefulness to my colleagues at the same place. In addition I want to place on record, my sense of appreciation towards my family and friends for their encouragement and support. In particular I want to thank my grandmother, Mari Stavenes, for providing me with peaceful accommodation and working environment in times when I needed to concentrate extra hard on completing the thesis.

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June, 2015

Abstract

The main purpose of this thesis is to evaluate the performance of the dynamic turbulence targeting asset allocation strategy, and then match it up against the volatility targeting strategy, which is the dynamic benchmark, plus the classic buy-and-hold equally-weighted strategy, which is the static benchmark portfolio. The analysis is conducted across four data sets, all of which are based on U.S. stock indices, using an out-of-sample period from 1973 to 2014, and also sub periods 1973 to 1990 and 1991 up to 2014. Performances are first of all determined and assessed by the use of the Sharpe and Sortino ratios. In addition, the Sharpe ratios of the three strategies are tested, in order to identify whether they truly are statistically significantly distinguishable from one another. The empirical findings of this examination suggest that the turbulence targeting portfolio outperforms the two other portfolios, as stated by the numerical values of the Sharpe and Sortino ratios. Though, we cannot conclude that the Sharpe ratio of the turbulence targeting portfolio is statistically significantly higher than those of the benchmark portfolios. We also document that turbulence and volatility both are negatively related to future excess returns.

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1 Introduction

1.1 Background and literature review

Financial investments are to a large degree focused on how you should balance your expected return with risk. The main goals for a rational investor are to maximize expected return, and at the same time keep the risk as low as possible. In order to do so, there are plenty of different strategies to choose from, of which most of them are categorized as diversification or hedging.

When regarding diversification, this is an expression that we see in a lot of different contexts, as well as in finance. For instance, in marketing, diversification would be to introduce a new product to a new market, while in corporate finance a well-diversified enterprise would be an enterprise that are represented in several different markets. Typically, enterprises which are heavily involved in shipping (which is a very volatile branch), would also have other, more steadily performing divisions that can offset possible losses from the shipping division during troughs in the shipping market. By using this strategy, the enterprise would still be able to gain substantially when the shipping industry is at a peak.

Much of the same reasoning can be used in financial investments, where we often talk about diversification across assets (i.e. asset allocation), which is how you proportion different asset classes (equity, fixed-income (bonds), cash) in your portfolio. Stocks are usually more volatile (risky) than bonds, so the question would typically be how you should allocate a portfolio so that you can exploit the largest upturns in the stock market and, at the same time, avoid the worst and most critical downturns.

For many years, the most used asset allocation strategies were the static ones, i.e. strategies with a chosen allocation that would stay the same, even during peaks and troughs. And in the period from around 1940 and almost to 1970, these strategies were in fact sufficient. Nevertheless, from 1970 and onwards things were about to change.

Leland (1984) mentions the following drawbacks with static strategies: First, the initial simulations, which make the basis of how a portfolio should be allocated, might be based on some particular estimated parametrical magnitudes. These simulations will probably include a confidence interval which will tell us that returns under a specific

level are completely unrealistic. However, if these parameters change later on, the simulations would no longer be valid. E.g. if the volatility one year is a lot higher than the volatility used in the simulations, then the confidence interval of this portfolio would consequently be much wider.

Second, these strategies do not offer any assurances of some kind. This has been proved during several recessions through history. Jacobsen (2009, p. 1) also came up with the following message after the financial crisis of 2007 and 2008: "strategic asset allocation is not static asset allocation". And the reason is simple: If markets don't behave statically, an asset allocation strategy cannot be static. Still, there might be static strategies that can offer almost risk-free returns, but in these cases you will have considerable opportunity costs related to the gains (from the more risky assets) that you've missed out on.

As early as in the 70s investors recognized the fact that static allocations didn't perform well, especially during peaks and troughs. In the oil crisis in 73-74 it was observed that even fairly conservative static allocation strategies failed to obtain positive returns. Perhaps this was a wake-up-call for the financial investors and academics, because new methods concerning asset allocation were discovered some years later; the *dynamic* asset allocation strategies.

These dynamic strategies have evolved quite a lot, particularly in the recent years, due to crises such as the dotcom-bubble crash and the global financial crisis. Also, because of these crises, the dynamic strategies have received even more attention during the last 6-7 years. Speaking of crises; Jacobsen (2009) specifically studied the concept of asset allocation during crises and stated that people should not become too smug with their asset allocation strategies. In this case, smugness can be related to the static asset allocation strategies where the investor has the same asset allocation over a longer period of time, regardless of how the market conditions are. Regarding dynamic strategies, Jacobsen declare the following: "*True* asset allocation is *dynamic*, it is prospective, and it manages risks that extend beyond simple style and size exposure. These are simple things an advisor can do to reduce risk and add value to client portfolios" (Jacobsen, 2009, p. 12).

Regarding the actual execution of such dynamic strategies, it is quite common that these portfolios are based on some particular market measure, and thereby reallocated according to the level of this measure. Volatility is a popular measure to use

in this respect, since it's expected to be negatively correlated with market returns. A dynamic strategy based on volatility is the originator to the so called *volatility targeting* (or sometimes volatility responsive) portfolio strategies, and at this point I will present some studies related to this type of dynamic strategy.

First, Collie, Sylvanus and Thomas (2011) studied a simplified volatility responsive strategy, that would invest; 30 per cent in stocks (and 70 per cent in bonds) if the volatility was high at the start of a given month; 50/50 if the volatility level was moderate; and finally, 70/30 for low-volatility periods. Studying a period from April 1979 to June 2011, they concluded that fixed weights (static) asset allocation strategies didn't provide a steady risk/return ratio over a given period of time. The fixed weights strategy had a clear tendency of having high volatility when the market volatility was high. When the market volatility was low, fixed weights strategy had slightly lower volatility than the dynamic one. Then they argued that volatility-sensitive investors instead should adopt the dynamic volatility responsive asset allocation strategy. Another interesting implication from their paper is that returns for one period are not a good predictor for returns in the next period, while this *is* the case for volatility. Since the volatility seems to be easier to predict, this makes it appealing for dynamic strategies. Though, what is not appealing for this purpose, is that they fail to find any obvious connection between quarterly returns and the quarterly volatility from the previous period.

Then, another study conducted by Albeverio, Steblovskaya and Wallbaum (2013) aimed to compare a volatility targeting strategy, which was based on a predetermined volatility target value, with a pure equity index such as the S&P 500. Investigating the monthly performances from 1963 to 2008, conclusions were that "the [volatility targeting] strategy works well in specific market environments such as a falling market accompanied by high volatility levels or a rising market accompanied by low volatility levels" (p. 10). However, as a drawback, they mention that when we, for instance, face a falling market with low volatility, the strategy will have a tendency of giving more weight to stocks, even though that most possibly will weaken the overall portfolio performance. They also argue that it would be advantageous to merge the strategy with other asset allocation strategies. And, a more general disadvantage of their approach is related to their use of monthly rebalancing, which makes the strategy more vulnerable to short-term market shocks. The latter problem can of course be the case for any

dynamic strategy which rebalances at a monthly interval.

In a newer study conducted by Zakamulin (2014) he uses a long-only variant of the volatility responsive strategy as a dynamic benchmark of which he compares it to another dynamic portfolio based on a target value of the *unexpected* volatility. Both these portfolios are also compared to a static benchmark portfolio, consisting of 50 per cent stocks and 50 per cent of a risk-free asset. As expected, the volatility-responsive strategy performs better than the static strategy, at least from 1991-2012 and 1970-2012. But from 1970 to 1990, the performances of the two are more ambiguous, with Sharpe ratios and alpha values which differ in their conclusions. In essence, the volatility responsive portfolio only outperforms the passive portfolio in the 2000s. It is also interesting to note that the unexpected-volatility-responsive strategy outperforms the volatility-responsive strategy as well as the static strategy in all periods considered, supported by dominating Sharpe ratios and alpha values.

Chew (2011) uses an annual target equity volatility of 16 per cent for his volatility targeting portfolio. This is then matched up against a static 80/20 portfolio and a dynamic constant proportion portfolio insurance (CPPI) strategy, over the decade of the 2000s. Not surprisingly, the target volatility asset allocation strategy tackled the volatile period during the global financial crisis in the most satisfying manner. What is also quite interesting in this paper, is that Chew emphasizes the importance of "taking accurate measure of investor's risk tolerance" (p. 12) when it comes to the assessment of the volatility targeting strategy, as compared to other strategies. This is a vital consideration which is easy to forget in between the range of conclusions which often are based on concrete, numerical results. In other words, one should not forget the human aspect among all the facts and figures.

It is also worth mentioning that Kirby and Ostdiek (2010) generalized the volatility targeting strategy to account for N different assets (not only stocks and fixed income), finding the weight of a given asset based on the variance level of that asset as opposed to the sum of the N different variances. They found that the volatility timing strategy they proposed outperformed the naïve diversification strategy ($1/N$) for several data sets, even when relatively high transaction costs were implemented to the model. Although not directly relevant for this thesis, this study does show that it is possible to apply a volatility targeting strategy for a higher number of assets ($N > 2$), with successful results.

The results of Kirby et al. were strengthened by the fact that both Giese (2012) and Ilmanen and Kizer (2012) arrived at similar conclusions. Based on all these empirical results, Hallenbach (2012) also chose to derive a mathematical proof to show that volatility targeting indeed improves the Sharpe ratio (and information ratio), using only a few assumptions. Although several studies have shown that this active strategy does not necessarily improve the Sharpe ratio (compared to some passive portfolio), Hallenbach's results do provide for some theoretical foundation in a field that mainly has been characterized by empiricism. Another of his conclusions, which is a more obvious one (at least from a practical point of view), is that the Sharpe ratio increases the better the volatility forecasts are.

However, Zakamulin (2015) derived a solution for an optimal dynamic diversification strategy, assuming that the mean returns and variance-covariance matrix (of returns) are completely predictable. His findings suggest that the most advantageous time diversification strategy is one that allocates assets so that the portfolio's exposure to risk follows the inverse of the portfolio's variance. This conclusion implies that the volatility-targeting approach is not optimal. Still, when he compares the optimal solution strategy with its volatility targeting counterpart, it is revealed that there are only marginal differences in the performances of the two, when considering a real market situation (note also that both strategies perform statistically significantly better than the static one).

Despite that several of these studies mention quite a lot of advantages with the volatility timing/targeting/responsive strategies, there are still some empirical conclusions which tell us that during some periods, and under some specific market conditions, these strategies do not perform as well as we might hope when we compare them to some passive (static) strategies. At this point it makes sense to introduce the concept of *turbulence* as a market measure. Kritzman and Li (2010) defined turbulence as "a condition in which asset prices, given their historical patterns of behaviour, behave in an uncharacteristic fashion, including extreme price moves, decoupling of correlated assets, and convergence of uncorrelated assets" (p. 30). In their study, they argued that the measure of turbulence captured higher-dimensional information, that returns to risk were lower during turbulent periods, and that turbulence is very persistent. They also introduced what they called "turbulence-resistant portfolios", that are dynamic portfolios conditioned on turbulence, and showed that such a portfolio performed better

than its unconditioned counterpart, both during the turbulent out-of-sample periods, and during other non-turbulent periods as well.

Regarding the performance of the above-mentioned naïve strategy (equally weighted portfolio); in a study conducted by DeMiguel, Garlappi and Uppal (2009), they were not able to find sufficient statistical evidence that the optimized portfolios achieved significantly higher Sharpe ratio than the equally weighted strategy. A paper by Plyakha, Uppal and Vilkov (2012) shows that the equally weighted portfolio outperforms value and price-weighted portfolios, and they also attempt to discover *why* this is the case. Kritzman, Page and Turkington (2010)¹, on the other hand, presented results which were in disfavour of the naïve strategy, where it was out conquered by both a minimum-variance portfolio and a mean-variance portfolio. Even though the studies related to the equally weighted portfolio approach differ slightly in their implications, there are still no doubt that this strategy is a good representative for passive portfolios, and hence, it's also satisfactory as a benchmark of which we can compare the dynamic portfolios with.

1.2 Research motivation and structure of this thesis

Inspired by Kritzman and Li's paper from 2010, the main motivation of this thesis is to inspect the out-of-sample performance of a turbulence targeting asset allocation strategy, and compare this to both a dynamic benchmark, the volatility targeting strategy, and indeed a passive benchmark, the naïve equally weighted portfolio. First of all, the portfolios will be evaluated according to the Sharpe and Sortino ratios. Then I will test if the Sharpe ratios of the dynamic portfolios are statistically significantly different from the one of the equally weighted portfolio. I will apply the same procedure to test if the Sharpe ratios of the turbulence and volatility targeting portfolios are significantly distinguishable as well. Regarding returns, I will calculate the monthly mean, and also the standard deviations, skewnesses and kurtoses, of which the latter three are terms that can, more or less, be related to the portfolios' downside and upside risk. The three portfolios will be tested across different data sets in order to gain robustness of the results, and also for three different periods: Total period 1973-2014 and sub periods 1973-1990 and 1991-2014. I will also execute a simple linear

¹ In order to avoid confusion on the two "Kritzman et al." papers from 2010: This is the last time I refer to the Kritzman, Page, and Turkington paper in this thesis, so from now on, every time I refer to "Kritzman et al. (2010)", I actually refer to the Kritzman and Li paper, which incidentally also was published in 2010.

regression in order to provide some evidence to whether the monthly volatility and/or turbulence is applicable as predictor for the excess return (which is the equally-weighted returns of the respective data sets minus the return rate of a risk-free asset) for the subsequent month. Finally, the absorption ratio (Kritzman, Li, Page and Rigobon, 2011) is estimated in order to detect periods of high systemic risk. Though, failing to find any significant relationship between the daily equally-weighted returns and the absorption ratio, the opportunity of using this measure as a target for a dynamic targeting portfolio is disregarded. At this point I will conduct a principal component analysis as well, in order to detect differences in the data sets' patterns, which first of all is to identify how the risk of the data set portfolios are spread across different components.

The execution of the turbulence and volatility targeting strategies will be based on a similar framework, so differences in their performances will first of all be related to the market measures themselves, and their interaction with the returns in question. Both strategies will allow for borrowing, short sales, buying stocks on margin, or whatever way an investor chooses in order to use leverage.

The thesis will be arranged in the subsequent manner: In Chapter 2, I will introduce the data sets I will make use of when comparing the performances of the static and the dynamic strategies, while in Chapter 3 I give an overview of the methods that are relevant. Further, in Chapter 4, I will use statistical software R to conduct some tests that will reveal how the different active asset allocation strategies performs in comparison to the static benchmark strategy. In this chapter I will also take a brief look at the data itself; some words on the estimation, and also the behaviour, of the market measures in question (i.e. volatility and turbulence). Estimation and interpretation of the absorption ratio, together with a principal component analysis, and indeed some predictive regressions, are also included in this chapter. Afterwards, in Chapter 5, I will discuss weaknesses of the strategies and compare my results to the most relevant previous studies. While Chapter 4 is heavily based on theory, Chapter 5 takes a slightly more practically-oriented approach. Finally, in the sixth and last chapter, I will sum up and make a conclusion.

2 Data

The data I will be using in this thesis is based on the daily returns of four different data sets from Kenneth French's data library². In addition, I will apply the monthly risk-free returns from the Fama/French 3 factors data set, obtained from the same data library. These risk-free returns are used to compute the excess returns, which again are used in the computation of the Sharpe and Sortino ratios.

All data sets initially range from sometime during 1926 and up to the middle of the 2010s, but I have chosen to limit the sample period to January 4, 1960 to December 31, 2014, which is quite in line with the time periods that has been studied in previous papers, and it is also a period which solely consists of weeks with 5 working days. It is advantageous to use daily observations so that we can estimate more precise figures for the volatility, although it does imply that we have to deal with a vast amount of observations.

Data set (daily interval except where noted)	<i>N</i>	Time period
10 Industry Portfolios	10	Jan 4, 1926-Dec 31, 2014
30 Industry Portfolios	30	Jan 4, 1926-Dec 31, 2014
25 Portfolios Formed on Size and Momentum	25	Jan 4, 1926-Dec 31, 2014
25 Portfolios Formed on Size and Book-to-Market	25	Jan 4, 1926-Dec 31, 2014
Fama/French 3 Factors (monthly interval)	-	July 1926-March 2015 ³

Table 2.1: Data sets employed in this study

Please note that the initial data sets have been edited so that they only account for the period 1960 to 2014, meaning that the data files in use in this thesis is not equivalent to the original ones we downloaded directly from French's web page in the first place.

The returns in the data sets are computed on the basis of the following formula

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}} + \frac{div_t}{p_{t-1}} \quad (2.1)$$

where r_t = return at time t
 p_t = price at time t
 div_t = dividends at time t

² See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

³ Only used for the purpose of the risk-free returns, hence the dimension is not relevant. This data set is later delimited to the same monthly interval as the estimates of the return, volatility and turbulence.

Each data set are constructed based on stocks from major stock exchange indices NASDAQ, NYSE and AMEX, which also indicates that the results of this thesis is dependent on US stocks only, and that these data sets are reliable and highly comparable to each other. I will now give a brief description of each data set to highlight the differences between them.

The data set 10 Industry Portfolios includes daily returns from ten different industries: durable (food, textiles etc.) and nondurable (cars, furniture etc.) goods, manufacturing (trucks, planes etc.), energy (oil, gas, coal), hi-tech (computers, software etc.), telecom (telephone and television distribution), shops (wholesale and retail), health (healthcare, medicine and drugs), utilities and *other* industries (mines, construction, hotels, finance, etc.). Although being a rather compact and sometimes deficient set, it is commonly used by scientists. For instance, DeMiguel et al. (2009) found that for this particular data set, it was not possible to find statistically significant differences between the Sharpe ratio of the $1/N$ strategy and the Sharpe ratio of all the other, more sophisticated strategies. Hence, this data set might "favour" the $1/N$ strategy in this study as well. In addition to DeMiguel et al. (2009), this data set is also applied by Kirby et al. (2012) and Kritzman et al. (2010), when taking into consideration the studies I mentioned in the literature review in Chapter 1.

The data set 30 Industry Portfolios⁴ does, not surprisingly, consist of daily returns from an entire 30 different industries, making it a lot more comprehensive than the data set described above. Most of the industries mentioned in parentheses in the previous paragraph are now represented by their own columns of returns, which should imply that this data set is highly representative for the market as a whole. For the total list of industries included, I refer to the industry definition list that is available at the website of Kenneth French (see footnote 1). Studies conducted by Zakamulin (2015) and Kritzman et al. (2010) utilized this data set.

The third data set of average value weighted daily returns, consists of 25 portfolios formed on size and momentum, where size indicates market capitalization, while momentum signifies that an asset's price is more likely to move in the same direction for a longer period, rather than to vary its directions. The data is constructed

⁴ The main reason why I use the 30 industries portfolio as opposed to the larger dimensional industries portfolios, such as the ones with 38, 48 or 49 industries, is simply owing to the fact that these data sets have a considerable amount of missing values for the time period I will be using, which make them more cumbersome to work with, and possibly less reliable as well.

on a 5 by 5 basis, meaning that the returns are first divided into five groups of firms sorted by the level of market capitalization, whereby each of these groups are individually sorted into five subgroups based on the degree of momentum in the returns from period $t-250$ (minus approximately one year from the present point in time) to $t-21$ (around one month backwards in time compared to the current time t), providing for 25 columns in total. Zakamulin (2015) makes use of this data set as well.

The final daily returns set consists of 25 portfolios shaped on size and the level of the book-to-market ratio. In the same way as the size and momentum based data set, this is also put up in a 5 by 5 manner, first sorted by size and then sorted from low (overvalued stocks) to high (undervalued stocks) book-to-market ratios. Kirby et al. (2012) used this data set in their study of volatility timing strategies and pointed out that "sorting firms on these criteria is known to produce a large cross-sectional dispersion in average returns" (p. 22), i.e. we should expect substantial variability in the returns among these portfolios. In addition, Zakamulin (2015) and DiMiguel et al. (2009) employed this data set for their simulations.

3 Methodology

In this chapter I will go through the methods I will be using in order to compute the results that are given in Chapter 4. Computation itself is done in R, and these programming procedures are given in Appendix C. Note that the possible effects of transaction costs and taxation are disregarded in this thesis.

3.1 Central moments

In this section⁵, I will introduce the four central moments, which are the mean, variance, skewness and kurtosis, respectively. The two first-mentioned are relevant for several aspects of this study, including turbulence and volatility estimation, together with the Sharpe and Sortino ratios. The two latter-mentioned will be used to examine the distributional properties of a given portfolio's monthly returns. These measures could also give indications on what type of risk the portfolios are exposed to; downside or upside risk. For that matter, I will also describe the Jarque-Bera test of normality.

⁵ Section 3.1 is mostly based on Tsay (2005) pages 8-10.

3.1.1 Mean and variance

Assume that we're dealing with a random sample X of T different observations, i.e. $X = (x_1, \dots, x_T)$. In order to make this more tangible, we could, for instance, say that X is a series of monthly returns. The first central moment, the realized mean of X , which we will denote as $\hat{\mu}_x$, is given by

$$\hat{\mu}_x = \frac{1}{T} \sum_{t=1}^T x_t, \quad (3.1)$$

while the second moment, the variance, $\hat{\sigma}_x^2$, is defined as

$$\hat{\sigma}_x^2 = \frac{1}{T-1} \sum_{t=1}^T (x_t - \hat{\mu}_x)^2. \quad (3.2)$$

However, for our purposes, the standard deviation is even more interesting than the variance. As we know from introductory statistics, the standard deviation is given by

$$\hat{\sigma}_x = \sqrt{\hat{\sigma}_x^2}. \quad (3.3)$$

The standard deviation is frequently used as a measure of financial risk, and since it is being used as the targeting measure for one of the dynamic strategies in this thesis, I will come back to this concept later on.

To be more specific, the realized mean (of returns) in this thesis is computed at a monthly interval. For comparative purposes, it would be convenient to express both the mean returns and the standard deviation of returns in annualized figures. Using the same notation as above, the annualized mean, represented as $\bar{\mu}_x$, is computed as follows:

$$\bar{\mu}_x = 12\hat{\mu}_x, \quad (3.4)$$

while the annualized standard deviation, $\bar{\sigma}_x$, is found in the subsequent manner:

$$\bar{\sigma}_x = \sqrt{12}\hat{\sigma}_x. \quad (3.5)$$

3.1.2 Skewness and kurtosis

Using the same notation as Tsay (2005), we can define the skewness (which is the third central moment) of a continuous random variable X as

$$S(X) = E \left[\frac{(X - \mu_x)^3}{\sigma_x^3} \right]. \quad (3.6)$$

For a normal distribution, the skewness would be equal to nought, since such a variable is expected to have a distribution that is completely symmetrical. As we can see, this formula will penalize abnormal observations. The skewness can take both negative and positive values, of which both cases would indicate that the distribution is "skewed", either to the left (negative values) or the right side (positive values). For practical purposes, a left-skewed distribution of returns would indicate that there is a greater risk of tremendously negative outcomes (i.e. greater downside risk), but also that the returns most frequently are slightly positive. The tendencies are the opposite for right-skewed distributions, implying less downside risk, a characteristic which would captivate risk-averse investors (Sortino and Satchell, 2001).

The kurtosis, on the other hand, gives us information on how the distribution is curved. It can either be; *leptokurtic*, indicating that it has fatter tails and a more sharp-pointed peak, or; *platykurtic*, which basically implies the opposite specifications. Using same notation as above, the kurtosis is defined as

$$K(X) = E \left[\frac{(X - \mu_x)^4}{\sigma_x^4} \right]. \quad (3.7)$$

A normal distribution does have a kurtosis of 3. When the kurtosis is larger than 3, we're dealing with a leptokurtic distribution. For instance, this means that most historical returns will be mutually grouped around the mean. Yet, the fatter tails suggest that extreme deviations is, to some extent, likely to happen as well. For platykurtic distributions, the kurtosis is smaller than 3. This entails that the historical returns are generally considered to be less risky, due to the fact that the distribution has fewer large deviations than in the case of leptokurtosis. Unexpected incidents, such as the financial crisis, is less likely to happen. Though, in this section it should also be mentioned that loads of empirical studies have shown that several categories of financial returns are leptokurtic (Righi and Ceretta, 2012).

Applied in practice, both kurtosis and skewness are estimated with foundation in the estimated values of the first two central moments, and they can also straightforwardly be extended to account for T observations. The skewness for a sample of T observations is

$$\hat{S}(x) = \frac{1}{(T-1)\hat{\sigma}_x^3} \sum_{t=1}^T (x_t - \hat{\mu}_x)^3, \quad (3.8)$$

while the kurtosis for T observations is

$$\hat{K}(x) = \frac{1}{(T-1)\hat{\sigma}_x^4} \sum_{t=1}^T (x_t - \hat{\mu}_x)^4. \quad (3.9)$$

Jarque-Bera normality test

By combining properties of the kurtosis and skewness measures, Jarque and Bera (1987) came up with a test statistic in order to test for normality of X . The test is pursued with normality as the null hypothesis; $H_0: S(x) = (K(x) - 3) = 0$; and with a test statistic that is computed as follows:

$$JB = \frac{(\hat{S}(x))^2}{6/T} + \frac{(\hat{K}(x) - 3)^2}{24/T} \quad (3.10)$$

As before, T equals the total number of observations, and the expressions in the denominator are the variances of $\hat{S}(x)$ and $\hat{K}(x)$ when normality is assumed. The JB test statistic is chi-squared distributed and H_0 is rejected if p_{JB} is smaller than the level of significance.

3.2 Capital allocation

Assume that we can choose between two assets; one risky and one risk-free⁶. Weight denoted by w , equals the proportion invested in the risky asset B , which has returns of r_B , while $1-w$ is the amount invested in the risk-free asset, with r_f as the rate of return. In other words, we're dealing with a *long only* portfolio where the sum of the weights is

⁶ It is in fact the case that the allocation methods applied in this thesis, both for the volatility targeting and turbulence targeting strategies, assumes only one risk-free and one risky asset. Hence, I will in Section 3.2 describe the very basics of this concept. Also, since the only strategy with multiple (at least more than 2) risky assets, is the simple $1/N$ portfolio, I will not consider the theories of multiple asset allocation.

equal to 1, meaning the investor only invests his own wealth in the assets (he doesn't need to raise a loan). The expected return of the portfolio p would be

$$\begin{aligned} E(r_p) &= w \cdot E(r_B) + (1-w) \cdot r_f \\ E(r_p) &= r_f + w(E(r_B) - r_f), \end{aligned} \tag{3.11}$$

and the standard deviation (risk) of portfolio p is

$$\sigma_p = w\sigma_B. \tag{3.12}$$

Rewrite equation (3.12) with respect to w , substitute this expression for w in equation (3.11) and you will end up with

$$E(r_p) = r_f + \left(\frac{E(r_B) - r_f}{\sigma_B} \right) \sigma_p, \tag{3.13}$$

which is the equation for the *capital allocation line*, that is a representation of the different ways an investor could allocate his wealth in the risky and risk-free asset. The expression in the parenthesis correspond to the *Sharpe ratio*, i.e. the price of risk, and the slope of the capital allocation line. All points on this line will have the same Sharpe ratio, indicating that the Sharpe ratio will stay at the same level, regardless of how the portfolio is allocated (as long as the risk/return point occurs somewhere along the capital allocation line). In essence, an investor who is assumed to seek the mean-variance optimum wants to maximise this measure when he chooses how he wants to allocate his portfolio. This follows from the *mean-variance criterion*⁷. Being an important measure for this thesis, I will go deeper into the subject of the Sharpe ratio in the subsequent section.

⁷ Portfolio A dominates portfolio B if; $E(r_A) \geq E(r_B)$ and; $\sigma_A \leq \sigma_B$
 Given that; $E(r_A) > E(r_B)$ or/and; $\sigma_A < \sigma_B$

3.3 Performance measures

In this section I will describe some measures that I will use to evaluate and compare the performances of the different portfolio strategies which I will elaborate more on in Section 3.4. First subsection describes the Sharpe ratio, while the second explains the Sortino ratio.

3.3.1 Sharpe ratio

The Sharpe ratio, also known as the reward-to-volatility ratio, was formulated by William Sharpe in 1966. It measures the size of excess return (reward given to the investor for tolerating risk) in proportion to the degree of risk (volatility). Excess return is computed by subtracting a risk-free rate of return from the return of the given portfolio. The most common measure of risk is the standard deviation of the portfolio. Thus, the estimated *monthly* Sharpe ratio of a portfolio p (denoted as $\hat{S}R_p$) can be computed in the following way:

$$\hat{S}R_p = \frac{\hat{\mu}_p - r_f}{\hat{\sigma}_p} \quad (3.14)$$

where $\hat{\mu}_p$ = realized mean return of portfolio p
 r_f = return of a risk-free asset
 $\hat{\sigma}_p$ = realized standard deviation of portfolio p

Now, recall from Section 3.2 how the expected return and standard deviation are defined for a portfolio p with one risky asset B and one asset which is risk-free. Then we substitute the realized equivalents of the expressions from equations (3.11) and (3.12) into equation (3.14) and get:

$$\begin{aligned} \hat{S}R_p &= \frac{r_f + w(\hat{\mu}_B - r_f) - r_f}{w\hat{\sigma}_B} \\ \hat{S}R_p &= \frac{w(\hat{\mu}_B - r_f)}{w\hat{\sigma}_B} \\ \hat{S}R_p &= \frac{\hat{\mu}_B - r_f}{\hat{\sigma}_B} \end{aligned} \quad (3.15)$$

This is a simple proof which shows that for such a portfolio, the allocation does not affect the level of the Sharpe ratio. The same accounts for the Sortino ratio, a risk-to-reward measure I will discuss further in Subsection 3.3.2.

Also, in order to once again obtain figures that are, to a larger extent, comparable to former studies in this field, the annualized Sharpe ratio would be more suitable. According to equations (3.4) and (3.5), the annualized Sharpe ratio of portfolio p , $\bar{S}R_p$, should be expressed as⁸:

$$\bar{S}R_p = \frac{(\hat{\mu}_p - r_f) \cdot 12}{\hat{\sigma}_p \cdot \sqrt{12}}$$

$$\bar{S}R_p = \hat{S}R_p \sqrt{12} \quad (3.16)$$

When comparing the performance of different portfolios, the best performing portfolio would be the one with the highest Sharpe ratio, i.e. the portfolio with the highest excess return for a certain level of risk. Although it has its faults (see below), the Sharpe ratio is an extremely popular measure, among practitioners as well as academics. As long as one is familiar with its weaknesses, it's easy to interpret, and also quite straightforward to estimate. Being one of the most common performance measures, used in numerous previous empirical studies and such, it is natural that it soldiers on, since new studies often will use exactly this measure in order to compare their results with the former outcomes in a convenient manner.

Hypothesis testing with the Sharpe ratio

Assume that we're dealing with two different portfolios, labelled '1' and '2'. It is *expected* that portfolio 1 and 2 would have different Sharpe ratios. However, one could actually enquire whether these measures are *significantly* different by using the test given by Jobson and Korkie (1981), which was corrected by Memmel (2003). The null and alternative hypotheses are as follows:

$$H_0 : \hat{S}R_1 - \hat{S}R_2 = 0 \quad \wedge \quad H_1 : \hat{S}R_1 - \hat{S}R_2 \neq 0,$$

⁸ Although this is a common way of annualizing the Sharpe ratio, one should be aware of some of the implications made in Lo (2002), where he points out that monthly Sharpe ratios can be converted into annual terms, but only in some particular situations.

while the test statistic is given by:

$$\hat{z} = \frac{\hat{S}R_1 - \hat{S}R_2}{\left(\frac{1}{T} \left[2(1 - \hat{\rho}^2) + \frac{1}{2} (\hat{S}R_1^2 + \hat{S}R_2^2 - 2\hat{S}R_1\hat{S}R_2\hat{\rho}^2) \right] \right)^{0.5}} \quad (3.17)$$

where \hat{z} = realized standard normal distributed test statistic
 T = number of observations
 $\hat{\rho}$ = realized correlation coefficient

Assuming normally distributed variables, the p-value is estimated on the basis of the z-value given in equation (3.17). If the p-value is lower than some critical value α , the null can be rejected at a α per cent significance level, indicating that the data generates sufficient evidence against the hypothesis that the Sharpe ratios of two different portfolios are similar. In this thesis it would be natural to compute this p-value to see whether the Sharpe ratio of the dynamic turbulence aiming portfolio is statistically significantly different from both the passive and dynamic benchmark portfolios, meaning that we have to conduct the test twice.

Disadvantages of the Sharpe ratio

It is often the case that the assumption of normality is violated when we're dealing with financial returns. Consequences of non-normality can be quite severe for the implications that can be drawn from the Sharpe ratio estimates. For instance, Gatfaoui (2010) concluded in his study from 2010, that non-normality in an asset's returns did provide for changes in the performance rankings given by the Sharpe ratio, as compared to an unbiased Sharpe ratio in a Gaussian setting.

Moreover, the Sharpe ratio fails to make a distinction between positive and negative variation, penalizing both directions equally. Although, here we should keep in mind that the Sharpe ratio initially was introduced as a reward-to-volatility ratio (not reward-to-risk), and one should therefore be aware of the fact that volatility is not *necessarily* the same as risk.

It is also a problem that for negative returns, the level of the Sharpe ratio makes no sense. The general advice for such cases, would be to just disregard the Sharpe ratio.

And finally, Schwager (1996) mentions that the Sharpe ratio does not

discriminate between irregular and successive losses. This is a direct consequence of the fact that it only uses the standard deviation of the period as a whole.

3.3.2 Sortino ratio

As implied above, the Sharpe ratio is often criticised for its use of the standard deviation. Problem is that the standard deviation punishes both downside risk (the amount an investor risks to lose) and upside return potential equivalently.

As a response to this, Brian Rom and Frank Alphonse Sortino introduced a "mean-lower partial moment ranking ratio"⁹ already in the beginning of the 1980s. Though, what later became known as the Sortino ratio, was not officially established until 1994, by Sortino and Price (1994). The Sortino ratio attempts to deal with this standard deviation-problem by introducing *lower partial moment* (LPM) as an alternative measure of risk. Then, by taking the square root of the LPM we get a measure of downside risk. The main difference from the normal standard deviation is that we now only focus on observed values below the mean. Based on that assumption, we can write the expression for the realized downside risk of portfolio p , \hat{DR}_p , as this:

$$\hat{DR}_p = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n \min(\hat{\mu}_p - r_f, 0)^2} \quad (3.18)$$

where n = number of observations below the mean

In practice, the downside risk is the risk that rational investors fear the most. Hence, we will now substitute the standard deviation of equation (3.14) with the expression for the downside risk from equation (3.18), to obtain the Sortino ratio:

$$S\hat{O}R_p = \frac{\hat{\mu}_p - r_f}{\hat{DR}_p} \quad (3.19)$$

This ratio can be annualized by using the same method as the one we used for the Sharpe ratio above:

$$S\bar{O}R_p = S\hat{O}R_p \sqrt{12} \quad (3.20)$$

⁹ Which in fact was the proposed name for the ratio by Sortino himself, "but Brian [Rom] didn't think that name would catch on" (Sortino, 2010, p. 23). Hence, Rom suggested to name it the Sortino ratio.

In this case, the risk-free rate is often referred to as the minimum acceptable return. Regarding the downside risk measure, this is also related to skewness, which we defined in Section 3.1. For instance, if the returns are characterized by positive skewness, most of the portfolio variability would consist of the risk of upward movements. Accordingly, the downside risk would be relatively low, providing for a better portfolio performance, according to the Sortino ratio. In such cases, the Sharpe ratio would underestimate the portfolio's performance. For negative skewness, on the other hand, the Sharpe ratio tends to overestimate the performance of the strategy.

Regarding the pros and cons of the Sortino ratio as compared to the Sharpe ratio, these are mostly related to the benefits and drawbacks of the downside deviation and standard deviation measures. Ridley (2004) mentions that the Sortino ratio is less efficient in cases where the fund (or asset) has extraordinarily low volatility levels, since this will imply that the downside risk measure is computed on the basis of very few observations. Also, someone might argue that upside deviations also is a sign of risk, and hence, they favour the use of the ordinary Sharpe ratio instead.

3.4 Asset allocation strategies

This section covers the asset allocation strategies that are relevant for this study. First, Subsection 3.4.1 describes the only static strategy, namely the equally-weighted portfolio. As is the case for all static strategies, it will follow the same allocation strategy during the whole sample period.

Then, Subsections 3.4.2 and 3.4.3 incorporate the concepts of the two dynamic strategies, that is the volatility and turbulence targeting portfolios. Dynamic asset allocation is an active strategy where the investor persistently reallocate his portfolio in accordance to some specific measure in the market, such as in this study; volatility or turbulence. While the first-mentioned is a pretty recognizable term in finance, the latter one is not. Hence, I will devote a larger amount of space to the definition of turbulence, than for the volatility. Also, for simplicity, I will use the equally-weighted portfolio's returns to represent the risky asset in the two dynamic strategies, since both the volatility targeting and the turbulence targeting approaches I use in this thesis are assumed to include one risky asset, and one risk-free.

3.4.1 Equally-weighted portfolio

As indicated earlier, this strategy allocates an equal fraction $1/N$ to each of the N obtainable risky assets. With w_i equal to $1/N$, the expected returns of the equally-weighted portfolio (EWP) are therefore given by

$$E[r_{EWP}] = \frac{1}{N} \sum_{i=1}^N E[r_i]. \quad (3.21)$$

Being equally weighted, the returns of the naïve strategy is in fact the equivalent to the mean return of N different assets. Another implication of this strategy is that it's only dedicated to risky assets, as opposed to the two dynamic strategies we will examine. According to its definition, there is no doubt that this is a very simple strategy, which also is one of its main benefits. It does not involve any estimations of some sort, implying that it's less costly to conduct, and risk of estimation errors is negligible. Also, many studies suggest that it works well as a benchmark strategy, of which we could compare other, probably recently developed strategies with.

3.4.2 Volatility targeting portfolio

The volatility-responsive strategy rebalances the portfolio in order to maintain the same volatility level as the initial portfolio. This protects the portfolio from getting exceptionally large volatility during recessions.

Estimating the volatility

As I briefly brought up in Chapter 2, I will use daily returns to calculate the monthly volatility. The realized daily volatility for period i can simply be calculated in the following manner:

$$\hat{\sigma}_i^d = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (r_t - \hat{\mu}_i)^2} \quad (3.22)$$

where $\hat{\sigma}_i^d$ = estimated daily volatility for period i
 r_t = returns for a given day
 $t = [1,2,3,4 \dots T]$ = number of days in a given period
 $\hat{\mu}_i \approx 0$ = daily realized mean return for period i

Due to the approximation of the daily mean return, we will often (in practice at least) use the following approach:

$$\hat{\sigma}_i^d = \sqrt{\frac{1}{T} \sum_{t=1}^T r_t^2} \quad (3.23)$$

Yet, computational software R will automatically adopt the approach from equation (3.22). To convert the daily volatility into monthly volatility, I just multiply the number I get from equation (3.22) with the square root value of the N days that there are in a month m

$$\hat{\sigma}_i^m = \hat{\sigma}_i^d \cdot \sqrt{N} \quad (3.24)$$

Predicting the volatility

In order to implement the volatility targeting strategy, one also has to predict the volatility for the next period. In this thesis I will make use of the naïve prediction method (also called the random walk), which simply says that the predicted volatility for the next period $i + 1$ is equal to the volatility in the current period i

$$\hat{\sigma}_{i+1} = \sigma_i \quad (3.25)$$

This is an approach which often is used as a benchmark in forecast comparison. The main advantage of this method, which is also the main reason why I use it as opposed to the more sophisticated methods, is that it's very cost-efficient in use. Also, implications of several former comparative studies, all of which aim to find the most accurate volatility forecasting model, are highly ambiguous. For instance, Poon and Granger (2003) wrote a rather comprehensive paper in 2003, where they summarized results from 93 different studies related to this given topic. First of all, they concluded that, yes, financial volatility *is* possible to predict, but to what extent is still not known for sure. Then, another implication of their study is that "historical volatility methods [which, among others, include the naïve prediction model] work equally well compared with more sophisticated ARCH [autoregressive conditional heteroscedasticity] and SV [stochastic volatility] models" (Poon et al., 2003, p. 507).

In this thesis, I will not examine the forecasting accuracy of the random walk model. Nevertheless it is reassuring to observe from former studies that this method,

despite its simplicity, does not necessarily perform worse than its more refined counterparts.

Implementing the strategy

If you have two assets, say stocks and a risk-free asset, one way to estimate the weight of stocks in the portfolio would be to divide the target volatility by the forecasted volatility:

$$w_{s,t} = \frac{\sigma^*}{\hat{\sigma}_t} \quad (3.26)$$

where $w_{s,t}$ = weight of stocks at time t

σ^* = target volatility

$\hat{\sigma}_t$ = forecasted volatility

T = number of time periods

Note first that the level of target volatility actually can be an arbitrary chosen value. In principle we can choose whatever value we want, it's only a yardstick (constant), and the value of this will not affect the performance measures of the portfolio.

Interpretation-wise; if the forecasted volatility for the next period turns out to be larger than the target volatility, the weight of stocks would be reduced, which seems to be a sensible response. However, by using this simple approach, we will almost immediately stumble upon an issue; that is, if the target volatility is larger than the forecasted volatility, the weight of stocks would be larger than one, indicating that we should implement a strategy that could require borrowing. Hence, this violates the borrowing restriction property. Still, this restriction is not totally rigid. We might very well relax the borrowing restriction property and use equation (3.26) to compute the weight of stocks for the volatility targeting strategy. Notice that when using this approach, we should be aware of the fact that the portfolio most probably will be substantially riskier than a portfolio which takes care of the borrowing restriction assumption. In other words, this strategy is not a decent alternative for the faint-hearted and risk-averse investors.

Giving a short interpretation of equation (3.26), we see that if the predicted volatility is lower than the target volatility, the strategy implies that the investor should borrow money in order to invest an even larger amount in stocks, due to the fact that stocks, at the moment, are considered to be relatively less risky, at least compared to the

target risk (volatility). If the forecasted volatility is approx equal to the target volatility, the investor should invest 100 per cent in stocks. And finally, if $\hat{\sigma}_t > \sigma^*$, the investor ought to invest a smaller amount in stocks, and invest the remaining percentage (up to 100 %) in the risk-free asset.

Scenarios	Stock allocation
$\hat{\sigma}_t \gg \sigma^*$	$w_{S,t} \rightarrow 0$
$\hat{\sigma}_t > \sigma^*$	$0 < w_{S,t} < 1$
$\hat{\sigma}_t = \sigma^*$	$w_{S,t} = 1$
$\hat{\sigma}_t < \sigma^*$	$1 < w_{S,t} < \infty$
$\hat{\sigma}_t \ll \sigma^*$	$w_{S,t} \rightarrow \infty$ ¹⁰

Table 3.1: Allocation properties of the volatility targeting portfolio

Finally, the return of this strategy follows from the simple methods we discussed in Section 3.2. Also, recall that the return of the risky asset is transferred directly from the return of the naïve asset diversification strategy. This means that the volatility targeting returns are given by:

$$E[r_{vol}] = w_{S,t} E[r_{EWP}] + (1 - w_{S,t}) r_f \quad (3.27)$$

Using this approach, we have to assume that the borrowing rate (when $w_{S,t} > 1$) is equivalent to the risk-free rate of return. Assume an investor who has \$10 000 of which he wants to invest in risky stocks and risk-free bonds. Then, for instance if $w_S = 1.4$, he would have to borrow $0.4 \cdot \$10\,000 = \$4\,000$ at an interest rate identical to the risk-free rate and invest the amount $1.4 \cdot \$10\,000 = \$14\,000$ in stocks.

3.4.3 Turbulence targeting portfolio

Chow, Jacquier, Lowrey, and Kritzman first introduced the concept of financial turbulence in 1999. Mathematically, the turbulence is based on a measure initiated by Mahalanobis as early as in 1936, which again was based on a paper written by the same Mahalanobis, from 1927. This measure is known as Mahalanobis' distance. The aim of Mahalanobis' distance is to detect how far a given observation is from a sample mean

¹⁰ If this property causes some problems, or if one simply wants to use a more cautious strategy, one could put a borrowing restriction, at some level, into equation (3.26). This alternative approach will impose a borrowing restriction at the level of $w_{S,t}^{\max}$, which in principle can be set to any value. For an illustration of this method, see for instance Albeverio et al. (2013) where they enforced a restriction such that maximum exposure to the risky asset would never exceed 200 per cent.

compared to the sample's variance. In financial terms, and for "turbulence purposes", this observation will typically be a return of some sort. In more intuitive terms, one can say that the turbulence measures the distance, which is normalized by the standard deviation, between a given level of return and the mean value, i.e. the center of the returns mass. For illustrational purposes, one can consider a simplified two-dimensional case where we assume that the returns of the two assets have no covariation. For this particular scenario, the turbulence at time t is computed as follows:

$$d_t = \left(\frac{r_{t,1} - \mu_1}{\sigma_1} \right)^2 + \left(\frac{r_{t,2} - \mu_2}{\sigma_2} \right)^2 \quad (3.28)$$

where d_t = financial turbulence at time t
 $r_{t,i}$ = asset return at time t for asset i , where $i = [1,2]$
 μ_i = mean return for asset i , where $i = [1,2]$
 σ_i = standard deviation of $r_{t,i}$

Now it's easy to see that if the observed returns of assets 1 and 2 deviates by a large margin from the sample's mean returns, a margin which also is a lot larger than the standard deviations, then we are dealing with a relatively high turbulence value. This also indicates that we can use this measure to detect outliers in a sample. Moreover, equation (3.28) states that the turbulence, by definition, will increase as the dimensionality increases, *ceteris paribus*.

Estimating the turbulence

In order to compute the turbulence for multivariate purposes, with n different assets that are correlated in some way, Chow et al. (1999) defined the financial turbulence at time t , denoted d_t , as:

$$d_t = (r_t - \mu)' \Sigma^{-1} (r_t - \mu) \quad (3.29)$$

where d_t = financial turbulence at time t (scalar)
 $r_t = (r_{1t}, r_{2t}, r_{3t}, \dots, r_{nt})$ = vector of n different historical asset returns
 $\mu = (\mu_1, \mu_2, \mu_3, \dots, \mu_n)$ = mean value of historical returns of each asset
 Σ^{-1} = inverse of the variance-covariance-matrix of historical returns

While r_t is already given by the data set, both the mean value and the variance-covariance matrix have to be estimated. When estimating these figures, I will apply a

rolling window approach to make sure that both the mean value and variance-covariance are updated according to the relevant point in time. Assuming a rolling window of T periods, the realized mean value of a given asset at time t is calculated in the subsequent manner:

$$\hat{\mu}_t = \frac{1}{T} \sum_{i=t-T}^{t-1} r_i \quad (3.30)$$

In rough terms, the variance-covariance matrix is a way of expressing the variance to multiple dimensions. For instance, when we are dealing with a historical returns matrix with n columns (i.e. n assets with their respective returns), the general n -dimensional variance-covariance matrix will look like this:

$$\Sigma = \begin{pmatrix} \text{Var}[r_1] & \text{Cov}[r_1, r_2] & \text{Cov}[r_1, r_3] & \cdots & \text{Cov}[r_1, r_n] \\ \text{Cov}[r_2, r_1] & \text{Var}[r_2] & \text{Cov}[r_2, r_3] & \cdots & \text{Cov}[r_2, r_n] \\ \text{Cov}[r_3, r_1] & \text{Cov}[r_3, r_2] & \text{Var}[r_3] & \cdots & \text{Cov}[r_3, r_n] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[r_n, r_1] & \text{Cov}[r_n, r_2] & \text{Cov}[r_n, r_3] & \cdots & \text{Var}[r_n] \end{pmatrix}$$

Using the same rolling window approach as I did for the mean, the variance-covariance matrix at time t is given by:

$$\hat{\Sigma}_t = \frac{1}{T-1} \sum_{i=t-T}^{t-1} (r_i - \hat{\mu}_t)' (r_i - \hat{\mu}_t) \quad (3.31)$$

A variance-covariance matrix is non-singular, i.e. it is a square matrix that is invertible. Now, assume a covariance matrix, Σ , of n dimensions. Then, inversion implies finding a square matrix, let's just call it Λ , so that the following property holds: $\Sigma\Lambda = \Lambda\Sigma = \mathbf{I}_n$. When applied in practice, the inverse matrix is often denoted by the symbol of the given matrix, raised to the power of minus one, as seen from equation (3.29). The inverse of the covariance matrix is sometimes called the *precision matrix*. While the ordinary covariance matrix measures how the variables are scattered (spread) around the mean, the precision matrix measures how tightly clustered the variables are around the mean.

With a rolling window ranging from $t - T$ to $t - 1$ (i.e. one lookback period), the turbulence measure will always be estimated using the latest figures of the means and the variance-covariance matrices.

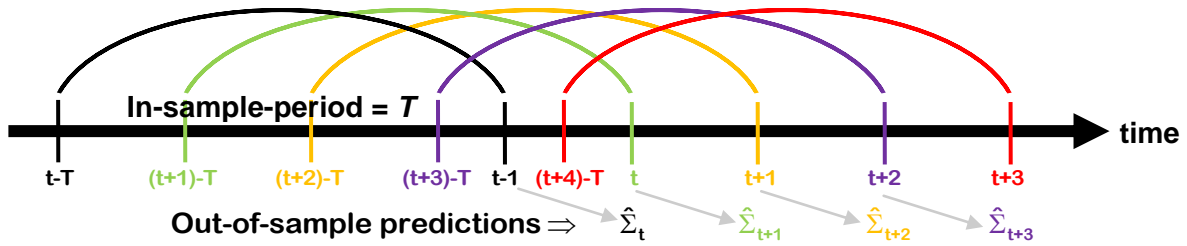


Figure 3.1: Out-of-sample prediction – variance-covariance matrix

The figure above illustrates an example on how the rolling window approach works. In the in-sample-periods, we estimate the predictions of the variance-covariance matrix for the period which is straight after the end of the in-sample-period. The colours in the figure show how this process goes on. For instance, the predicted covariance matrix for time $t+1$ is actually based on the covariance matrix which was estimated for the in-sample-period from time $(t+1)-T$ to t . This process can obviously be generalized further ahead in time than this example, which only goes to time $t+3$.

Now, back to the theories regarding turbulence. Based on the properties of the variance-covariance matrix, we can now say that turbulence relies heavily on both the individual variance of all variables, and also the covariance in between them, making it a more comprehensive measure than the standard deviation.

Studies have shown that relatively high historical values of this measure coincided with periods that are generally considered to be turbulent, i.e. periods characterized by irregular returns, abnormal asset correlations, illiquidity, risky asset devaluation and unusual high degree of risk aversion. Then, if we recall from above that turbulence (or in fact Mahalanobis' distance) could be a method to detect outliers in a sample, the results of these studies make sense, because such an outlier might very well be one of those "irregular returns" that we observe from time to another.

Finally, to sum up this brief review of the turbulence measure, I will emphasize two important empirical features of turbulence, both mentioned by Kritzman et al. (2010). First empirical feature says that "returns to risk are substantially lower during turbulent periods than during nonturbulent periods, irrespective of the source of turbulence" (Kritzman et al., 2010, p. 34). Also, figure 5 (p. 35) in this paper gives an excellent illustration of this statement. The second feature is that turbulence is extremely "sticky". It might turn up in an unpredicted manner, but when it does, it will typically carry on for a number of weeks.

Implementing the strategy

A turbulence responsive asset allocation strategy would typically be a portfolio that adjusts its weight in stocks according to the predicted level of turbulence. As for the volatility, we will also use the naïve prediction method here, which says that $\hat{d}_t = d_{t-1}$. The aim of the strategy would be to keep the portfolio at a predetermined target turbulence level. There may be several possible ways of achieving this, but the method I will be using is very similar to the method I used for the volatility targeting strategy:

$$w_{S,t} = \frac{d^*}{\hat{d}_t} \quad (3.32)$$

Due to its similarities with the volatility targeting strategy, the general behaviour of this strategy is quite similar as well: If the predicted turbulence turns out to be higher (lower) than the target turbulence, the fraction of stocks will be reduced (increased). How the strategies' behaviour will differ, will therefore depend on how their underlying market measures, i.e. the volatility and the turbulence, behave over time.

Scenarios	Stock allocation
$\hat{d}_t \gg d^*$	$w_{S,t} \rightarrow 0$
$\hat{d}_t > d^*$	$0 < w_{S,t} < 1$
$\hat{d}_t = d^*$	$w_{S,t} = 1$
$\hat{d}_t < d^*$	$1 < w_{S,t} < \infty$
$\hat{d}_t \ll d^*$	$w_{S,t} \rightarrow \infty$

Table 3.2: Allocation properties of the turbulence targeting portfolio

The return of this strategy, denoted r_{tur} , is given in the same way as the volatility targeting strategy:

$$E[r_{tur}] = w_{S,t} E[r_{EWP}] + (1 - w_{S,t}) r_f \quad (3.33)$$

3.5 Simple linear regression model

One of the main reasons why we would want to implement dynamic asset allocation strategies that target turbulence and volatility, is due to the fact that we expect some kind of relationship between these market measures and the level of stock returns. Hence, in this section I will give a brief review of some basic concepts related to linear regression.

3.5.1 Estimating the model

For our purposes, the simple regression model with one explanatory variable is highly sufficient. At time t , the general version of this model is given by the following equation:

$$y_t = \alpha + \beta x_t + \varepsilon_t \quad (3.34)$$

where y is the stochastic dependent variable while x is the deterministic independent variable. ε is a random error term (residual), which indicate the difference between the value estimated by the model and the actual observed value.

There are some important assumptions related to this model. First, we have to assume linearity. Then it is assumed that the expected value of the parameter ε is zero for all t . Third assumption is that all observations should have equal variance, i.e. homoscedasticity. Fourth, all error terms are uncorrelated. Finally, the residuals should be normally distributed. Initially, I will not conduct any tests related to these assumptions, unless it should be the case that my simple regressions deliver some very unexpected or bizarre results.

The aim of this ordinary least squares (OLS) model is to minimize the sum of the squared residuals with respect to both $\hat{\beta}$ and $\hat{\alpha}$ in order to get the levels of β (slope) and α (intercept) so that the final estimated line is as close to the real data as possible. Without further derivation¹¹, the solution to these minimization problems are the following (each 'bar' denotes a mean value):

$$\hat{\beta} = \frac{\sum (x_t - \bar{x})(y_t - \bar{y})}{\sum (x_t - \bar{x})^2} = \frac{Cov[x_t, y_t]}{Var[x_t]} \quad (3.35)$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} \quad (3.36)$$

where the upper expression represents the estimate of the true (but unknown) parameter, while the lower one is the slope coefficient estimator. In this thesis, we will pay most attention to the former of these two.

¹¹ For the whole derivation of this minimization problem, see for instance Brooks (2008) p. 81.

3.5.2 Predictive regression

As mentioned in the introduction to this section, I will apply regression to detect possible relationships between (monthly) turbulence and return, and the same regarding volatility and return. To be more specific, such relationships can be related to whether current values of turbulence or volatility are capable of predicting future rate of returns. Accordingly, I will introduce two predictive variants of the simple regression model, both of which are more specifically related to our intentions than the one from equation (3.34):

$$r_t - r_{f,t} = \alpha + \beta\sigma_{t-1} + \varepsilon_t \quad (3.37)$$

$$r_t - r_{f,t} = \alpha + \beta d_{t-1} + \varepsilon_t \quad (3.38)$$

Equations (3.37) and (3.38) regress the monthly return of a risky asset, in excess of the risk-free return, on the lagged values of volatility and turbulence, respectively. This way we can find out whether there are any relationship between past values of these measures and the current excess return. Now, for the sake of illustration, consider only the relationship between the present excess return and the lagged turbulence. Assuming β is negative, then, the higher the turbulence is, the lower is the excess return, and vice versa if β is positive. If β equals zero, there are no linkages between the two measures. These properties can be used to project future excess returns based on the current value of the turbulence (obviously, the same applies for the volatility as well).

3.6 Data analysis

In this section I will review some concepts related to multidimensional data set analysis. Using several data sets, ranging from 10 to 30 different columns of returns, it would be interesting to examine these underlying data to see if there are any differences in between them that are worth mentioning. Also, the absorption ratio (below) should give an indication on the general fragility of the market throughout a given period, assuming that we have at least one data set which is quite representative for the total market.

3.6.1 Absorption ratio

The absorption ratio was introduced by Kritzman, Li, Page and Rigobon in 2011, in a paper that dealt with the implementation of principal component analysis (PCA) in

portfolio management. In this paper they define absorption ratio as an implied measure of *systemic risk*. To put it briefly, if the systemic risk in a financial system is high, then the risk of facing a total collapse of the entire financial system is highly enhanced.

Further, Kritzman et al. (p. 113, 2011) state the reason why the absorption ratio is a good technique to determine systemic risk: "The absorption ratio captures the extent to which markets are unified or tightly coupled. When markets are tightly coupled, they are more fragile in the sense that negative shocks propagate more quickly and broadly than when markets are loosely linked." To get a further understanding of how the absorption ratio can measure this, I will bring in the main ideas of a PCA.

3.6.2 Estimating the absorption ratio: Principal Component Analysis

A natural point of departure when we apply a PCA, is to define the variance-covariance matrix. Assume we have a data set of returns from n different assets. Thus, we have a n -dimensional covariance matrix with $n!/[n(n-1) \cdot 2]$ different covariance values (Smith, 2002), in the same way as the one we defined above:

$$\Sigma = \begin{pmatrix} \text{Var}[r_1] & \text{Cov}[r_1, r_2] & \text{Cov}[r_1, r_3] & \cdots & \text{Cov}[r_1, r_n] \\ \text{Cov}[r_2, r_1] & \text{Var}[r_2] & \text{Cov}[r_2, r_3] & \cdots & \text{Cov}[r_2, r_n] \\ \text{Cov}[r_3, r_1] & \text{Cov}[r_3, r_2] & \text{Var}[r_3] & \cdots & \text{Cov}[r_3, r_n] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[r_n, r_1] & \text{Cov}[r_n, r_2] & \text{Cov}[r_n, r_3] & \cdots & \text{Var}[r_n] \end{pmatrix}$$

First we should notice that this is a square $n \times n$ matrix, meaning that it's possible to calculate an assembly of n eigenvectors and n eigenvalues. An eigenvector \mathbf{v} of the covariance matrix Σ has the following property:

$$\Sigma \mathbf{v} = \lambda \mathbf{v} \tag{3.39}$$

In other words, if a square $n \times n$ matrix Σ is multiplied by a non-zero $n \times 1$ vector \mathbf{v} , and we thereby get a new vector which is a λ -multiple of the same vector \mathbf{v} , we can define \mathbf{v} as an eigenvector. The multiple is classified as an eigenvalue. Each eigenvector will have an associated eigenvalue. And if you multiply an eigenvector by the square root of its corresponding eigenvalue, you will obtain the *factor loadings*, which basically show the correlation of a given variable with an underlying eigenvector (or preferably, the correlation with a *principal component* eigenvector, which is defined further down in this text).

In more plain words, we can say that an eigenvector corresponds to a direction into a n -dimensional space. For instance, in a simple 2-dimensional (x, y) -plane, a $\begin{pmatrix} 7 \\ 9 \end{pmatrix}$ vector represents a line which starts out from the origin and takes a direction so that it intersects the point where x equals 7 and y equals 9. Before I introduce the intuition behind eigenvalues, it is worth noticing that all eigenvectors' lengths should be scaled so they have a length of one. This is a way of normalizing the eigenvectors. How this is done, is shown in Appendix B. Keep in mind that the software I'm using, R, does this operation automatically.

An eigenvalue, on the other hand, tells us how much of the total variance that is explained by a given eigenvector (direction). The eigenvector with the highest eigenvalue is known as the *principal component*, which also brings us to the main point of a principal component analysis, i.e. to find patterns in the data we have in hand. After we have found the eigenvalue of each eigenvector, we can order the eigenvectors from highest to lowest eigenvalue. Furthermore, the eigenvalues are a measure of significance, so we can now choose to leave out the eigenvectors with least significance. By doing this, we will also reduce the dimensions of the final data set. This is also the point where we return to the absorption ratio. According to Kritzman et al. (2011), the absorption ratio is the same as the variance explained by a restricted number of eigenvectors. Again, following the example of Kritzman et al. (2011), I will restrict the number of eigenvectors to one-fifth of the number of assets involved. Based on this, the equation for the absorption ratio, is given by

$$AR = \frac{\sum_{j=1}^k \lambda_j}{\sum_{j=1}^n \lambda_j} \in [0,1] \quad k = [0.2n] \quad (3.40)$$

where AR = absorption ratio
 n = number of assets or indices
 k = (restricted) number of eigenvectors used to compute the AR
 λ_j = eigenvalue of the j 'th eigenvector

If we for instance take the data set 10 industry portfolios (see Chapter 2) into consideration, an absorption ratio close to 1 would practically imply that only a couple of components are necessary to explain the total variance of the entire portfolio. This will also indicate that the different industries are closely coupled together and if one of

the industries experiences a massive drop in returns, or perhaps even a total collapse, the rest of the industries would most likely go down the drain as well. If the absorption ratio lies closer to 0.5, this total market collapse scenario is less likely to happen, and even if one industry should collapse, there's a smaller probability that the other industries will face the same problems, by means that the market in total stands a greater chance of getting recovered.

4 Results

4.1 Data analysis

4.1.1 Absorption ratio: Estimation and interpretation

The *daily* absorption ratio is first computed on the basis of a variance-covariance matrix, which is estimated over a rolling-window of 10 years, i.e. 2520 days. Then, the matrix's eigenvalues are computed, which is simply done by a built-in function in R called `eigen`. Finally, a *fixed* number of these eigenvalues are summed up and divided by the sum of the *total* amount of eigenvalues. For instance, for the 30 industry data set, each industry's returns represent one column in the data set matrix, meaning that the covariance matrix is of dimensions 30 by 30, and accordingly we have 30 eigenvectors with 30 corresponding eigenvalues. Following the same procedure as the one Kritzman et al. (2011) applied, this means I will be dividing the sum of the six largest eigenvalues (i.e. 20 per cent of 30) by the sum of all 30 eigenvalues. Note that both plots below are estimated on the basis of the 30 industry portfolios.

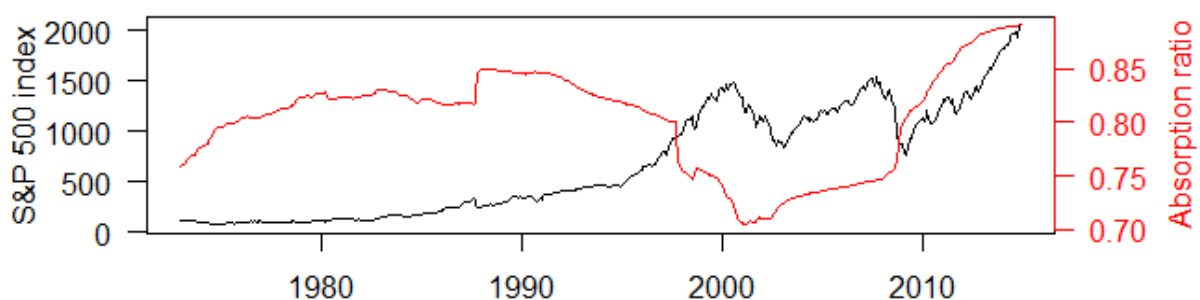


Figure 4.1: Monthly absorption ratio and S&P 500 index – 1973-2014

The main idea is that the absorption ratio has large values in periods with high systemic risk. At first glimpse, what we can see from Figure 4.1 that this measure at least has a slight tendency of shifting upwards in times when the estimated returns are in a trough.

The most apparent examples of this are the *Black Monday* in 1987, and when the financial crisis escalated around 2008. For both these cases, the absorption ratio exhibits an almost vertical upward shift.

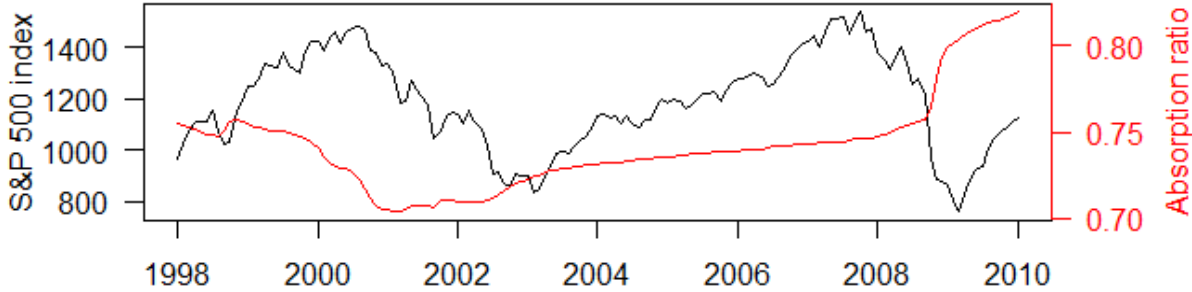


Figure 4.2: Monthly absorption ratio and S&P 500 index – 1998-Jan 2010

And especially when we take a more thorough look at the rather turbulent period from 1998 to the beginning of 2010 (which is the same period as the one Kritzman et al. (2011) examined), it is easy to deduce that the absorption ratio has a tendency of being relatively high in the periods when the stock index is fairly low, and vice versa. At most, the six principal components explain over 80 per cent of the total data variance, which is during the financial crisis.

For the exact period which Kritzman et al. (2011) inspect, the absorption ratio (or measures derived from it, like the standardized shift of the absorption ratio) might very well be a variable that is suitable as a targeting variable for some sort of dynamic portfolio (in the same way as the volatility and turbulence are applied as targeting measures in for two different dynamic portfolios in this thesis). Though, when we consider a longer time period, as for instance 1973-2014, we see from Figure 4.1 that the relationship between the stock index and the absorption ratio is more ambiguous. For curiosity I regressed daily equally-weighted returns (from the 30 industry set, which is believed to represent the total market rather well) on the daily absorption ratio (based on the same data set) and ended up with a model with literally no explanatory power and a highly insignificant regressor with a coefficient approximately equal to zero. Of course note that this doesn't have to be the case for other data sets and periods, after all Kritzman et al. (2011) did indeed find an inverse relationship between some US stock price index and the absorption ratio.

Still, even if we're not using the absorption ratio as a direct target for dynamic portfolios, it's an interesting measure when it comes to data analysis. From the figures in Table 4.1, we get an indication on how closely unified the different data sets are.

~ 1973-2014 ~	
Portfolio data set	Mean absorption ratio
10 industry	0.809
30 industry	0.800
25 formed on size and momentum	0.931
25 formed on size and boot-to-market	0.925

Table 4.1: Monthly mean absorption ratio for all data sets

Again, the sum of the 20 per cent of the largest eigenvalues are divided by the total variance. As expected, the two data sets which are partly sorted on size and partly on momentum and book-to-market, achieves the highest absorption ratios. As expected, because both data sets' returns are first sorted into five groups based on firm size. Thus, one of these groups contains five columns of returns from the biggest firms, and it is very likely to assume that the variance of the biggest firms' returns would explain a large proportion of the total variance. Hence, there would only be a few components necessary to explain as much as 93.1 and 92.5 per cent of the overall variance. In general one could state that these two data sets in fact are completely inappropriate when it comes to the estimation of the absorption ratio, mostly because they usually would imply extremely high levels of systemic risk. This is so due to how the data sets' returns are sorted initially, which make them incompatible as market portfolio representatives.

4.1.2 Principal component analysis

The PCA procedures one has to go through in order to estimate the absorption ratio does also have quite a few interesting features related to analysis of the underlying returns data. In this subsection, I will apply, and in fact comment briefly, on some of these statistical features. All *plots* in this subsection are based on the 10 industry portfolios daily returns, and they are included in order to illustrate and clarify the main points of this conceptual area.

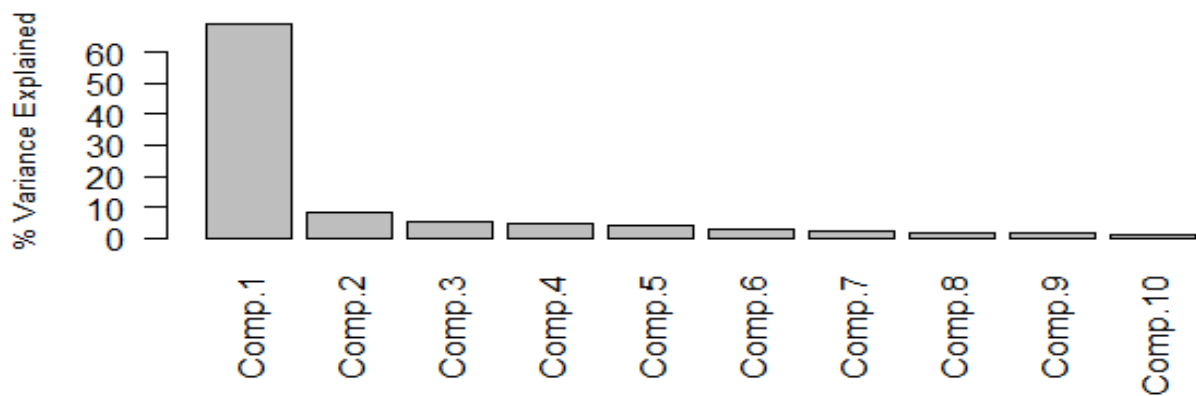


Figure 4.3: Percentage variance explained by each component – 1973-2014

Over the total period, the first principal component accounts for around 70 per cent of the total variance in this particular data set. In other words, it is believed that the 10 industries' returns, in general, are quite closely coupled together, at least when we look at the explained variance for the whole period 1973-2014. The second principal component does only explain around 8 per cent.

~ 1973-2014 ~				
Amount of eigenvectors	10 industry	30 industry	25 on size and momentum	25 on size and book-to-market
10 %	0.693	0.668	0.839	0.854
20 %	0.773	0.757	0.920	0.911
30 %	0.827	0.811	0.944	0.933
40 %	0.875	0.855	0.955	0.945
50 %	0.917	0.893	0.964	0.955
60 %	0.942	0.923	0.974	0.968
70 %	0.964	0.949	0.983	0.980
80 %	0.979	0.970	0.988	0.986
90 %	0.992	0.987	0.993	0.992

Table 4.2: Daily variance explained for all data sets

This table provides the variance explained for different amount of eigenvectors used. Recall that the eigenvectors are sorted according to their levels of significance (i.e. eigenvalue levels). The first row represents variance explained by the 10 % ($N/10$) of eigenvectors with the largest eigenvalues, the second applies the $N/5$ number of eigenvectors with the largest eigenvalues, and so on. By definition, the second row is equal to the absorption ratio, as we defined it in Section 3.6. Note that these are daily figures, hence there might be some variations compared to the figures in Table 4.1.

In particular, if we consider the two data sets to the right in this table, we would

see that around half of the components (eigenvectors), i.e. the ones with the lowest eigenvalues, only account for about 5 per cent of the total variance. At this point one can therefore choose to ignore these components in order to simplify and reduce the dimension of the data sets, without losing too much information.

At this point we can also compute to what degree the N different variables correlate with the N eigenvectors, i.e. the factor loadings. A graphical illustration can also be made for the 2D-case, which includes two principal axes that each represent the directions of the two principal components. A 3D plot is also feasible, but in general it does not add much extra information.

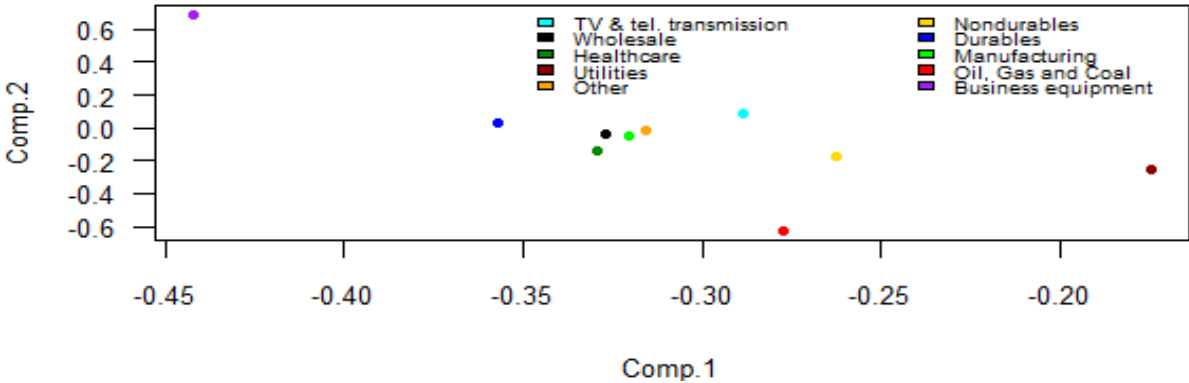


Figure 4.4: Factor loadings (two principal axes) – 1973-2014

Each point in Figure 4.4 represents a factor loading, which shows how a given industry is related to either the first principal component ("Comp.1"), or the second ("Comp.2").¹² This plot (and loadings in general) can give us additional illustration on how the different industries' data seem to be coupled together, and for this particular case we can also see quite fast how the data seems to create a horizontal line which goes in the same direction as the principal component (Comp.1) axis does. This proves why the explained variance of component 1 is as high as 70 %. In practice, such a component might be represented by some sort of market portfolio.

4.2 Estimation process and outcomes

In this section I will give a short review on the estimation procedures for the out-of-sample monthly returns, volatility and turbulence, respectively. Throughout this section, I will centre my attention towards the period which covers the years from 1973 to 2014 (if else, this will be remarked explicitly). The average number of days in a year is, as near

¹² For a plot with three principal axes, I refer to Figure A-1.

as makes no difference, 252 days, while the corresponding number for each month is 21. This indicates I will not distinguish between leap years and ordinary years, and neither between the different number of days in the calendar months. The total period (including in-sample period) does *not* include any years where six workdays in a week were the applicable standard. Hence, there will not be any obvious estimation errors that are related to dislocation of time.

I will also add some supplementary plots here and there in this section in order to give an intuitive illustration of some of the concepts we go through. And finally, for further details on the computation itself, I refer to Appendix C.

4.2.1 Return

The returns from the data sets mentioned in Chapter 2, are initially on a daily basis and divided into N different columns of returns, either depending on size, momentum, type of industry or a combination of those criteria. In order to make these returns appropriate for the methods I'll be using for the volatility and turbulence targeting strategies (see Subsections 3.4.1 and 3.4.2), I have to reduce their number of dimensions, from N to one. The single column that remains will represent the risky asset for these two dynamic strategies. The dimensions are reduced simply by taking the mean daily return of the N different columns. As we recall from Subsection 3.4.1, this will also correspond to the returns of the naïve diversification strategy. In the end, each monthly return are estimated from the cumulative daily returns, on a rolling interval of 21 days.

4.2.2 Volatility and turbulence

First, the *daily* volatility is computed by taking the standard deviation of the equal-weighted returns, from each and every data set individually. Then, based on equation (3.24), this daily standard deviation is multiplied by the square-root of the average number of days in a month (21), to convert it into monthly figures.

The turbulence estimation is based on a rolling window of 120 months (i.e. 2520 days). First, I calculate the necessary figures, which is; the mean daily return of the N different columns of returns in the given data set; *and* the inverse of the variance-covariance matrix. Second, I subtract the mean return from the daily returns on a monthly interval. Large deviations here will provide for high figures of turbulence. Third, I use these figures to compute the turbulence in accordance with equation (3.29).

I've also added a plot which aims to compare the turbulence and volatility levels. These figures are based on the 30 industries data set, which, for purely illustrational purposes, works as a representative for all four data sets.

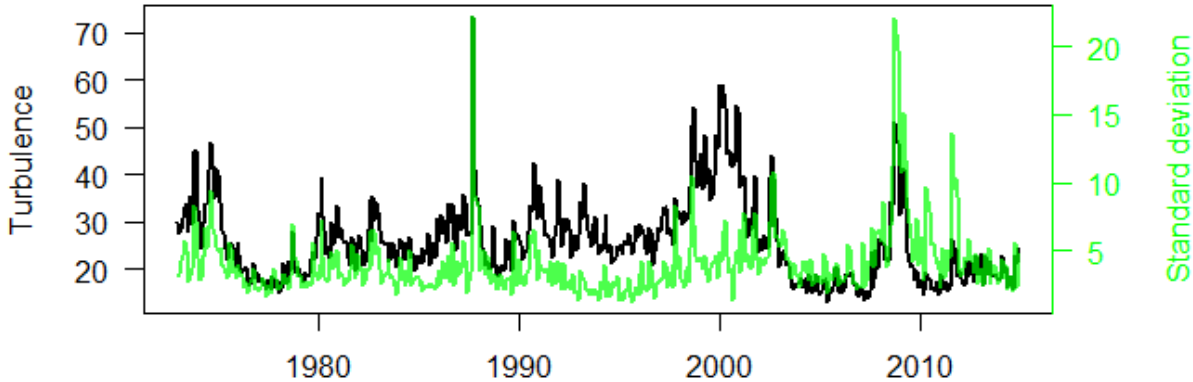


Figure 4.5: Monthly volatility and turbulence – 1973-2014

Comparing the turbulence and volatility levels, it is apparent that the turbulence measure shares quite a lot of similarities with the volatility. The most striking common feature is the spike they both exhibit in the end of the 1980s, which was due to the *Black Monday*. This refers to a day in October 1987 which is, percentage-wise, considered to be the worst stock market downturn in history. Hence, both volatility and turbulence ascended to sky-high levels in this particular month.

Also, in more recent years, we've had times which are regarded to be both turbulent and volatile, such as the recession following the IT bubble crash in the early 2000s, and the subprime mortgage crisis (or simply the financial crisis) from 2007 to 2009. Take notice that the turbulence exhibits a relatively higher peak during the dot-com bubble burst than during the financial crisis (although it *is* high in both periods, it has to be said), while the opposite is the case for the volatility. So, there are in fact some differences in the plots here and there, and, only by the look of it, it seems like the turbulence is somewhat more persistent than the volatility.

Finally, Table 4.3 shows how these market measures varies across the different data sets. While the volatility exhibits relatively identical behaviour, the turbulence appears to increase, when the dimension of the data set increases, which is in line with what was clearly observed in the simplified composition used in equation (3.28). So, in

principle, for data sets with very high dimensions, the turbulence measure would exhibit quite large values.¹³

~ 1973-2014 ~			
Panel A: 10 industry portfolios			
	Return (%)	Standard dev. (%)	Turbulence
Mean values	1.01	3.98	14.92
Minimum values	-27.46	1.34	6.29
Maximum values	13.71	22.32	48.92
Panel B: 30 industry portfolios			
Mean values	1.05	4.08	26.00
Minimum values	-31.95	1.29	13.20
Maximum values	16.80	22.19	73.34
Panel C: 25 portfolios formed on size and momentum			
Mean values	1.15	4.03	22.86
Minimum values	-30.51	1.19	12.65
Maximum values	20.65	22.45	62.63
Panel D: 25 portfolios formed on size and book-to-market			
Mean values	1.12	3.85	22.95
Minimum values	-30.61	1.12	12.28
Maximum values	16.60	20.69	65.09

Table 4.3: Monthly return, standard deviation and turbulence – main features

4.3 Volatility and turbulence as predictors for excess returns

In this section I will investigate the actual relationship between monthly excess returns, and monthly volatility and turbulence, where the focus primarily lies on whether lagged values of these two market measures are able to predict the returns, at least to some extent. The returns will be based on the equally-weighted returns from the four data sets described in Chapter 2. To determine this possible relationship, the excess returns are regressed on volatility and turbulence, respectively, using the simple linear regression model. Here it is important to note that both returns and volatility values are in percentage terms (i.e. multiplied by 100). Again, to gain the strength of this enquiry, I will consider the total period, plus the same sub periods as in the former section. The results are presented in Table 4.4 below, where the coefficient of the explanatory variables (volatility and turbulence) are included, together with the p -values of these in

¹³ If this feature should cause some trouble, one could alternatively opt for the method applied by Kinlaw and Turkington (2014), where they divide the turbulence measure, as of equation (3.29), by the number of assets (N).

order to establish if they are statistically significantly different from zero, at a significance level of 5 per cent. Finally, the table contains the R-squared measure (R^2), which tells us to what extent the given model explains the total variation in the regressand.

	Volatility:			Turbulence:		
	$r_t - r_{f,t} = \alpha + \beta\sigma_{t-1} + \varepsilon_t$			$r_t - r_{f,t} = \alpha + \beta d_{t-1} + \varepsilon_t$		
Panel A: 10 industry portfolios						
	$\widehat{\beta}$ -coeff.	p-value	R^2	$\widehat{\beta}$ -coeff.	p-value	R^2
1973-2014	-0.626	0.000	0.096	-0.152	0.000	0.034
1973-1990	-0.878	0.000	0.108	-0.221	0.002	0.046
1991-2014	-0.548	0.000	0.104	-0.121	0.004	0.029
Panel B: 30 industry portfolios						
1973-2014	-0.642	0.000	0.093	-0.083	0.002	0.019
1973-1990	-1.086	0.000	0.138	-0.140	0.010	0.030
1991-2014	-0.509	0.000	0.085	-0.062	0.035	0.015
Panel C: 25 portfolios formed on size and momentum						
1973-2014	-0.515	0.000	0.063	-0.127	0.000	0.027
1973-1990	-1.338	0.000	0.164	-0.251	0.000	0.066
1991-2014	-0.402	0.000	0.053	-0.083	0.039	0.015
Panel D: 25 portfolios formed on size and book-to-market						
1973-2014	-0.679	0.000	0.102	-0.174	0.000	0.049
1973-1990	-1.384	0.000	0.177	-0.292	0.000	0.077
1991-2014	-0.572	0.000	0.102	-0.139	0.000	0.043

Table 4.4: Excess returns regressed on volatility and turbulence

As we see from our results, all beta coefficients are significant at a 5 per cent level. The p-values are in general very small, indicating that the lagged turbulence and volatility regressors at least have some importance in the prediction of the subsequent excess return. Also, all coefficients exhibit negative signs, which mean that volatility and returns, and indeed turbulence and returns, have negative relationships. Regarding volatility and excess returns, this negative relationship is as one would anticipate, when we base our expectations on the conclusions of the majority of previous studies. Same accounts for the turbulence, although one should be aware of the fact that there are a lot less previous research in this field. This negative relationship with returns is after all one of the main empirical features of turbulence, as stated by Kritzman et al. (2010). With respect to the R-squared figures, we see that the volatility apparently is better fitted to explain the variability of future excess returns than the turbulence. It is also

worth pointing out that, in general, both volatility and turbulence predictors achieve the highest R-squared during the sub period 1973-1990, a period where stocks mostly performed worse compared to the other periods examined.

In conclusion, we can say that there are a negative relationship between the variables we have looked at, which insinuate that monthly volatility and turbulence both can be used to predict future returns. This also supports the argument that these variables are useful as targets for dynamic asset allocation strategies. Still, it is important to note that these simple predictive models we have been estimating are not particularly solid, and especially the models which use turbulence as predictor exhibits very low coefficients of determination.

4.4 Performance evaluation

In this section I will evaluate the performances of the different portfolio strategies. First of all, this is done by assessing the annualized Sharpe and Sortino ratios. The p-value of the Jobson-Korkie-Memmel-test is computed as well, to see whether the Sharpe ratios of the two dynamic strategies, independently, are statistically distinguishable from the static benchmark portfolio. The same test will be applied to see if the same accounts for the Sharpe ratio of the turbulence strategy when compared to the dynamic benchmark strategy (the volatility targeting approach). For both cases, I will use a significance level of 5 per cent.

I will also evaluate statistical concepts related to how the returns are distributed, such as the skewness and the kurtosis. At this point I've conducted the Jarque-Bera normality test too, again using a 5 per cent level of significance, to see if there actually are statistical support that the returns are normally distributed.

Note that the monthly target (benchmark) turbulence is set to 25, while the corresponding number of the volatility is 5 per cent. These are both regarded as relatively high target values, and would most likely be inappropriate for the risk-averse investors, since the active portfolios quite often will suggest exposures to the risky asset at way over 100 per cent. But, even though these benchmark values do have an effect on the return and volatility levels, we should bear in mind that they will not have any influence on the risk-adjusted performance of these active strategies.

The out-of-sample simulations are carried out for the period from January 1973 up to December 2014. In order to provide more robustness to my results, I will also

compute the same measures as the ones I mentioned at the beginning of this section for two sub-periods. These sub-periods are 1973-1990 and 1991-2014, where both are characterized by having longer bull periods (80s and 90s), and indeed longer bear periods (70s and 2000s) as well.

Before I present the tables which summarize the empirical results, I will go through the system of notations. These symbols and abbreviations will be in force throughout all the tables in this section.

Abbreviation	Complete expression
Pas	Passive portfolio. Also expressed as the static, the equally-weighted, the naïvely diversified, or the buy-and-hold portfolio.
Vol	Active (or dynamic) volatility targeting portfolio.
Tur	Active (or dynamic) turbulence targeting portfolio.
JB-test	Jarque-Bera-test.
JK-test	Jobson-Korkie-test (note: correction by Memmel (2003) is included).

Table 4.5: Abbreviations and notations

~ 1973-2014 ~

Panel A: 10 industry portfolios			
	Pas	Vol	Tur
Mean returns (%)	12.18	13.88	18.74
Standard dev. (%)	16.03	21.69	28.20
Sharpe ratio	0.45	0.41	0.49
<i>p-values JK-test, Pas:</i>	-	0.53	0.48
<i>p-values JK-test, Vol:</i>	0.53	-	0.16
Sortino ratio	0.62	0.58	0.69
Skewness	-1.19	-0.78	-1.03
Kurtosis	5.11	2.59	4.38
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel B: 30 industry portfolios			
	Pas	Vol	Tur
Mean returns (%)	12.64	13.78	13.08
Standard dev. (%)	17.91	23.93	17.73
Sharpe ratio	0.43	0.37	0.46
<i>p-values JK-test, Pas:</i>	-	0.38	0.54
<i>p-values JK-test, Vol:</i>	0.38	-	0.15
Sortino ratio	0.59	0.51	0.64
Skewness	-1.21	-0.78	-1.17
Kurtosis	5.71	2.81	5.36
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel C: 25 portfolios formed on size and momentum			
	Pas	Vol	Tur
Mean returns (%)	13.77	15.20	14.72
Standard dev. (%)	19.49	28.94	21.06
Sharpe ratio	0.45	0.35	0.46
<i>p-values JK-test, Pas:</i>	-	0.19	0.80
<i>p-values JK-test, Vol:</i>	0.19	-	0.07
Sortino ratio	0.64	0.49	0.65
Skewness	-0.92	-0.86	-0.97
Kurtosis	4.24	3.94	3.73
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel D: 25 portfolios formed on size and book-to-market			
	Pas	Vol	Tur
Mean returns (%)	13.46	16.09	14.72
Standard dev. (%)	18.32	28.38	19.73
Sharpe ratio	0.46	0.39	0.49
<i>p-values JK-test, Pas:</i>	-	0.31	0.44
<i>p-values JK-test, Vol:</i>	0.31	-	0.08
Sortino ratio	0.64	0.54	0.70
Skewness	-1.16	-0.87	-0.97
Kurtosis	4.71	3.88	3.58
<i>p-value JB-test</i>	0.00	0.00	0.00

Table 4.6: Performance measures and statistical figures – 1973-2014

First of all, note that the mean returns and standard deviations are in fact just straws in the wind; they may give a slight indication on the risk-to-return proportion, but still, in general, I will not give emphasis to any interpretation of these. The reason for this is that they both are highly dependent on the level of stock exposure, which again is extremely dependent on the volatility and turbulence target levels, i.e. σ^* and d^* . Recall that these target values are only arbitrary chosen values, and that the weight in stocks does not affect the most important measures of the ones we consider, namely the risk-adjusted performance measures.

What we first observe is that the turbulence targeting strategy consistently performs better than both the active and passive benchmark portfolios, across all four data sets, and according to both the Sharpe ratio and the Sortino ratio. Still, in all four data sets, there are insufficient statistical evidence that the Sharpe ratio of the turbulence strategy in fact is significantly distinguishable from the ones of the naïve diversification and volatility-targeting strategies. There are however a couple of incidents where the Sharpe ratios of the turbulence and volatility targeting strategies are significantly different at a 10 per cent level. This brings us to the second observation, which is the fact that the volatility targeting strategy achieves the lowest performance measures for all data sets. Although former studies have shown that the volatility targeting, and also volatility *responsive* (which is a long-only approach) strategies first of all have their strength in the decade of the 2000s, these results are rather unexpected. However, the differences in Sharpe ratios between this strategy and the naïve diversification one are not statistically significant, either at a 5 or 10 per cent level.

Regarding skewness, all strategies are consistently displaying negative values, and as a result of this, they are skewed to the left (i.e. the left tail of the distribution is longer). This indicates that downside risk is, to some extent, higher than the upside risk. Hence, we also observe that the *relative* difference between the Sharpe and Sortino ratios for the equally-weighted strategy, is slightly less the more negative the skewness is. As was commented on in Section 3.3, this means that the Sharpe ratio would have a propensity to overrate the performance of the negatively skewed portfolio. It is also worth mentioning that the volatility targeting strategy consistently exhibits the lowest skewness. Though, here it is important to point out that lowest skewness does not necessarily imply that this strategy has the lowest downside risk. Similar skewnesses of two strategies do not imply that their downside risk levels are equivalent as well, and

vice versa (Sortino et al., 2001).

Moving on to the topic of kurtosis and we see that the strategies are not even close to fulfil the assumption of normal distribution, that is, having an *excess kurtosis*¹⁴ equal to zero. Instead, these figures varies between values of 2.81 and up to 5.71. Accordingly, with positive excess kurtoses, the distributions of each portfolio's returns are leptokurtic. Hence, they are expected to have fatter tails than a normal distribution has, providing for a greater probability that they contain extreme values, which in this case primarily are extremely negative values.

As was intimated above, none of the distributions are believed to be normally distributed. This is also supported by the Jarque-Bera test where we can reject the null hypothesis of normality at very low significance levels. When it becomes so palpable that we are dealing with non-normal distributions, we should take the Sharpe ratio with a pinch of salt, and probably emphasize the Sortino ratio to a greater degree.

¹⁴ By default, R uses excess kurtosis, which is the kurtosis minus 3, and I will therefore use this measure throughout this section. For the data to be normally distributed, the excess kurtosis has to be equal to nought.

~ 1973-1990 ~

Panel A: 10 industry portfolios			
	Pas	Vol	Tur
Mean returns (%)	12.06	12.70	15.02
Standard dev. (%)	17.25	25.10	29.19
Sharpe ratio	0.25	0.20	0.25
<i>p-values JK-test, Pas:</i>	-	0.44	1.00
<i>p-values JK-test, Vol:</i>	0.44	-	0.20
Sortino ratio	0.34	0.27	0.35
Skewness	-1.06	-0.49	-0.58
Kurtosis	7.62	4.49	5.18
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel B: 30 industry portfolios			
	Pas	Vol	Tur
Mean returns (%)	12.68	13.27	12.50
Standard dev. (%)	19.35	27.89	18.73
Sharpe ratio	0.25	0.20	0.25
<i>p-values JK-test, Pas:</i>	-	0.42	0.98
<i>p-values JK-test, Vol:</i>	0.42	-	0.25
Sortino ratio	0.34	0.27	0.35
Skewness	-1.11	-0.52	-0.84
Kurtosis	8.21	4.61	7.05
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel C: 25 portfolios formed on size and momentum			
	Pas	Vol	Tur
Mean returns (%)	12.82	14.23	13.61
Standard dev. (%)	19.98	33.94	22.56
Sharpe ratio	0.25	0.19	0.26
<i>p-values JK-test, Pas:</i>	-	0.35	0.89
<i>p-values JK-test, Vol:</i>	0.35	-	0.16
Sortino ratio	0.34	0.26	0.36
Skewness	-1.01	-0.68	-0.82
Kurtosis	6.96	5.34	6.23
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel D: 25 portfolios formed on size and book-to-market			
	Pas	Vol	Tur
Mean returns (%)	13.40	15.97	14.23
Standard dev. (%)	19.31	33.43	21.47
Sharpe ratio	0.29	0.24	0.30
<i>p-values JK-test, Pas:</i>	-	0.50	0.81
<i>p-values JK-test, Vol:</i>	0.50	-	0.22
Sortino ratio	0.39	0.34	0.42
Skewness	-1.12	-0.66	-0.83
Kurtosis	7.59	5.34	6.08
<i>p-value JB-test</i>	0.00	0.00	0.00

Table 4.7: Performance measures and statistical figures – 1973-1990

~ 1991-2014 ~

Panel A: 10 industry portfolios			
	Pas	Vol	Tur
Mean returns (%)	12.26	14.77	21.53
Standard dev. (%)	15.08	18.78	27.46
Sharpe ratio	0.62	0.64	0.68
<i>p-values JK-test, Pas:</i>	-	0.92	0.52
<i>p-values JK-test, Vol:</i>	0.92	-	0.64
Sortino ratio	0.88	0.90	0.97
Skewness	-1.32	-1.18	-1.42
Kurtosis	8.33	6.66	9.48
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel B: 30 industry portfolios			
	Pas	Vol	Tur
Mean returns (%)	12.60	14.17	13.51
Standard dev. (%)	16.79	20.51	16.97
Sharpe ratio	0.58	0.55	0.63
<i>p-values JK-test, Pas:</i>	-	0.80	0.54
<i>p-values JK-test, Vol:</i>	0.80	-	0.48
Sortino ratio	0.81	0.76	0.88
Skewness	-1.30	-1.18	-1.48
Kurtosis	8.89	7.01	9.58
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel C: 25 portfolios formed on size and momentum			
	Pas	Vol	Tur
Mean returns (%)	14.48	15.92	15.56
Standard dev. (%)	19.15	24.60	19.90
Sharpe ratio	0.61	0.53	0.64
<i>p-values JK-test, Pas:</i>	-	0.54	0.68
<i>p-values JK-test, Vol:</i>	0.54	-	0.32
Sortino ratio	0.88	0.74	0.92
Skewness	-0.83	-1.08	-1.12
Kurtosis	7.40	8.74	7.05
<i>p-value JB-test</i>	0.00	0.00	0.00
Panel D: 25 portfolios formed on size and book-to-market			
	Pas	Vol	Tur
Mean returns (%)	13.51	16.18	15.09
Standard dev. (%)	17.58	23.97	18.35
Sharpe ratio	0.61	0.56	0.67
<i>p-values JK-test, Pas:</i>	-	0.67	0.36
<i>p-values JK-test, Vol:</i>	0.67	-	0.29
Sortino ratio	0.85	0.77	0.96
Skewness	-1.19	-1.19	-1.11
Kurtosis	7.64	8.53	6.83
<i>p-value JB-test</i>	0.00	0.00	0.00

Table 4.8: Performance measures and statistical figures – 1991-2014

On the whole, the results from the sub-periods 1973-1990 and 1991-2014 seem to support the results of the total period. That is, the turbulence targeting portfolio consistently accomplishes the highest level of the performance measures. Although there are a couple of incidents where the Sharpe ratio is similar to that of the passive counterpart. When we rely on the Sortino ratio, it apparently performs best in all periods and for all data sets. But again, we cannot make any hasty conclusions solely based on these numbers, since none of the p-values obtained from the Jobson-Korkie-Memmel-test seem to give statistical support that the Sharpe ratios in fact are significantly different from one another.

We should also bring up that the volatility targeting approach is, at least to a larger degree, compatible with the two other strategies when we consider the latest of the sub-periods. This is mostly due to its strong performance in the 2000s, and is also in line with the implications of some of the studies I mentioned in Chapter 1; the volatility targeting strategy performs best in the 2000s, otherwise its performance compared to some passive strategies, are rather ambiguous. For instance, a quick re-simulation based on the 30 industry set, shows that volatility targeting achieves a Sortino ratio of 0.141 in the volatile period from 2001 to 2009, while the corresponding number of the static strategy is only 0.098 (turbulence targeting still best at 0.178).

5 Discussion

The plain figures and its implications were mentioned and discussed in Chapter 4, so in this chapter I will go through some possible weaknesses of the methods I have applied in this thesis, and also try to establish *why* the volatility targeting strategy performs somewhat worse than one would expect, and *why* the turbulence targeting strategy again and again performs better than the volatility targeting approach. Some of these discussions will also take a more practical point-of-view, in order to make the overall reflections more nuanced. I will also compare my results with a couple of previous studies.¹⁵

Even though one of the main objectives of an active portfolio strategy should be to safeguard the investor against extreme losses, we still see a tendency that one-time

¹⁵ Bear in mind that there are only a few former studies which apply a nearly equivalent variant of the volatility targeting portfolio as the one I use. Hence, there are quite a lot of the previous studies mentioned in Chapter 1 that are not directly comparable to this study. Regarding turbulence targeting, there is only one former study that is relevant, at least to some degree, namely Kritzman et al. (2010).

freak events which occur very unexpectedly, cause a lot of trouble for the strategies I have assessed in this thesis. The most simple explanation to this, is that since we use the naïve prediction method, we will not be able to capture the *current* extreme levels of turbulence and volatility. We will have to make do with the turbulence and volatility levels from the *previous* period, which will not be as extreme. Then the inevitable happens; the strategy will suggest to place a relatively large amount of the investor's disposable wealth in stocks. The investor might even be suggested to buy stocks on margin, take up a loan or short sell stocks in order to invest over 100 per cent in stocks. Then, when the market falls, the investor would have to deal with a loss larger than the size of the actual decline in stock value. To illustrate, consider Black Monday in October 1987, together with the month before and the month after this incident. This example is based on the 30 industry returns data set. The volatility and turbulence target values obviously have a huge power of influence on the stock exposures of both strategies, but we will consider both risk (standard deviations) and returns in this modest example.

~ 1987 ~			
	September	October	November
Returns (%)	-1.95	-31.95	7.87
Standard deviation (%)	4.64	22.19	9.08
Turbulence	27.43	73.34	41.17
Volatility targeting:			
Stock exposure	1.48	1.08	0.23
Returns (%)	-3.09	-34.46	2.05
Standard dev. (%)	6.85	23.90	2.05
Turbulence targeting:			
Stock exposure	0.90	0.91	0.34
Returns (%)	-1.71	-29.07	2.91
Standard dev. (%)	4.17	20.22	3.10

Table 5.1: Reallocation during Black Monday

As we see from the figures we have gathered, both strategies struggle during such tremendous events as this one, as was also suggested above. And what makes the dynamic strategies perform even worse, is that in the following month, the returns are clearly positive. Due to the fact that we use the random walk model to forecast, the stock allocations for November are based on the turbulence and volatility levels from October, which were dreadfully high. Then recall that the target values we are using are 5 per cent for the volatility and a turbulence of 25. Accordingly, exposure to stocks turned out to be very low for both strategies in November, causing them to lose out on the market upturn. Again it is important to note that the randomly chosen target values obviously

have a huge power of influence on the strategies' stock exposures (and thereby its levels of returns and standard deviations), but when we consider the Sortino ratio (which is not affected by the stock allocation levels) we see that it is negative, by a long way, for both strategies.

At this point, the most feasible "solution" would be to blame the prediction method for the portfolio misallocations. But it's not that simple; even though the prediction method which is used in this thesis, is a very straightforward and "naïve" approach, studies have shown that this method in fact performs quite well when it comes to volatility forecasting (see for instance "lecture notes 14", Zakamulin, 2014). Also, it is by no means necessarily the case that the more advanced prediction methods would produce better forecasts, especially when it comes to one-time events, such as the Black Monday, which are almost impossible to predict nevertheless. There are also acknowledged theories, such as the efficient market and random walk hypotheses, which take this concept even further, saying that stock values follow a completely random path and hence, they're impossible to predict.

Bring to mind that the volatility targeting portfolio is a strategy which allows for borrowing, without any restrictions at all. Hence, it becomes fairly risky, and at some points, extremely risky. Maximum exposure to stocks when we consider the whole out-of-sample period, is around 400 per cent for all data sets, which is immense. For instance, the maximum stock weight for the 30 industry set is 388 per cent, and this is even in a month with negative stock returns. Even if we were to change the volatility target value, it will be the case that the strategy's *largest* stock exposure occurs in a period with *negative* returns. Obviously, if this strategy exhibits several cases such as this one, it's easy to understand why it performs worse than the two others. Though, for practical purposes, we have to keep in mind that if an investor wants to avoid such high exposures to the risky asset, we could impose a borrowing restriction at some level. The volatility target value can also be adjusted so that it suits a given investor's particular demands. Although its drawbacks, the different types of volatility targeting strategies have in fact received quite a lot of commendation by researchers, in the last 5-10 years.

For instance, Zakamulin (2014) tested a long-only volatility responsive strategy (which is believed to perform roughly in the same way as the approach we use) for approximately the same periods as has been applied in this thesis, also using one risk-free and one risky asset (based on two major stock indices), and showed that this

dynamic portfolio outperformed the static 50/50 portfolio from 1970 to 2012, mainly due to its performance in the 2000s. Receiving Sharpe ratios of 0.43 and 0.46, using stock indices S&P 500 and DJIA, respectively, this is slightly higher than the mean Sharpe ratio from the four data sets I examined (0.38). Though, its Sharpe ratio never turned out to be statistically significantly distinguishable from its passive counterpart. Hence, there are reasons to believe that these results seem to be quite in line with my own.

It's definitely not a straightforward task to find out why this strategy underperforms during the periods and data sets we have examined, and why the turbulence strategy generally achieves better results. But in general, volatility targeting are not well suited for periods characterized by low volatilities and low returns. As a practical illustration, we could take a closer look at such a period which occurred in the beginning of 1993.

	~ 1993 ~			
	January	February	March	April
Returns (%)	-0.02	-0.30	3.17	-1.76
Standard dev. (%)	1.31	3.59	2.91	2.83
Turbulence	27.02	34.08	30.81	38.00
Volatility targeting:				
Stock exposure	2.46	3.81	1.39	1.72
Returns (%)	-0.38	-1.75	4.32	-3.21
Standard dev. (%)	3.23	13.67	4.05	4.87
Turbulence targeting:				
Stock exposure	1.04	0.93	0.73	0.81
Returns (%)	-0.03	-0.26	2.39	-1.38
Standard dev. (%)	1.36	3.32	2.13	2.29

Table 5.2: Reallocation during period with low volatility and low returns

Again, these figures are based on the 30 industry portfolios, using target values of 0.05 for volatility and 25 for turbulence. Note that the tendency is very similar in the other data sets as well. This shows how the volatility targeting strategy assigns high weights in stocks, even though the stock returns mostly are slightly negative. This happens simply due to the fact that $\hat{\sigma}_t \ll \sigma^*$. And yet again this strategy takes a position in stocks of nearly 400 per cent in a period with negative returns. From a practitioner's viewpoint, such misjudgements can be quite fatal. The turbulence strategy takes a more moderate exposure in stocks and thereby reduces the loss. Comparing the risk and return of both strategies, there's no doubt that the volatility targeting strategy performs a lot worse. Not implying that this minor sub period is representative for the total period

performance, but it can possibly give a signal on why the turbulence targeting strategy consistently outperforms its volatility-aiming counterpart (in a couple of cases, its Sharpe ratio is even statistically significantly higher, at a 10 per cent level). What might provide us with additional information on this matter, are the following plots (based on the 30 industry set) which illustrate how the volatility targeting (black curve) and turbulence targeting (grey curve) strategies reallocate every month, for the same two sub periods as we applied in Chapter 4. Target values are still 25 for the turbulence and 5 per cent for the volatility. Again, changing these would obviously change the stock allocation levels, but the general tendency of ups and downs in the stock exposure, would still be the same.

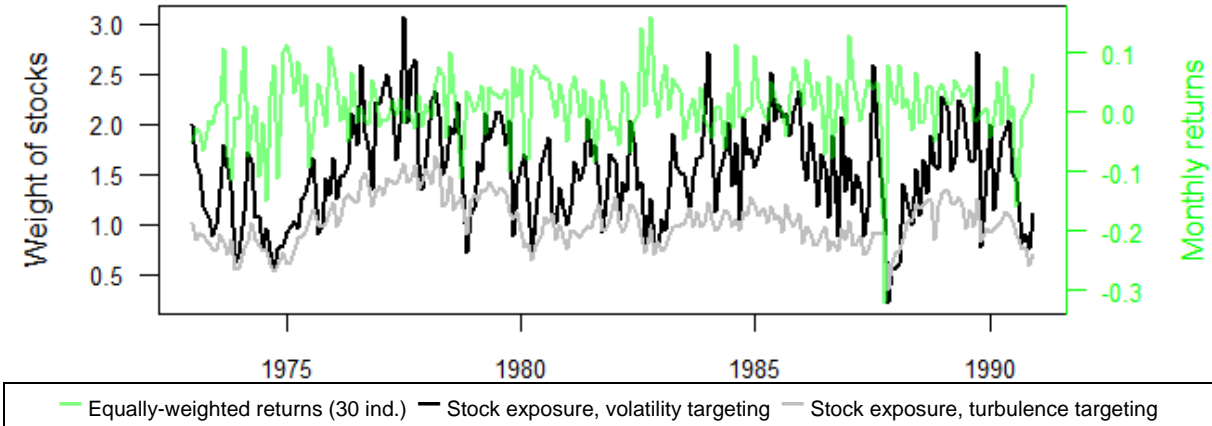


Figure 5.1: Stock exposure of dynamic strategies – 1973-1990

It really shows that the levels of stock exposure for volatility targeting strategy fluctuate a whole lot more than what is the case for its turbulence aiming counterpart. This is so first of all due to the underlying behaviour of the respective market measure which these dynamic portfolios reallocate according to. The turbulence targeting strategy does indeed follow a very steady path, but it still manages to reallocate to very low levels when needed.

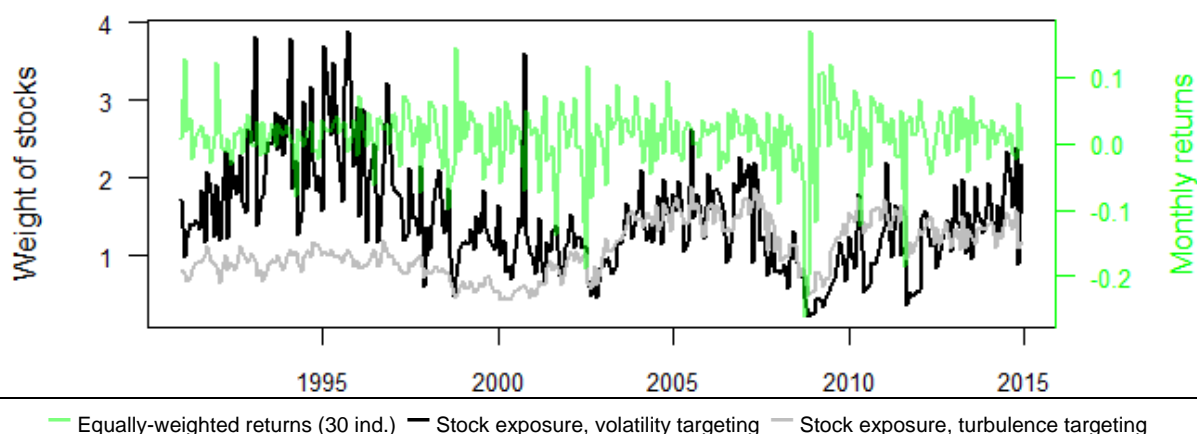


Figure 5.2: Stock exposure of dynamic strategies – 1991-2014

The same tendency applies for sub period 1991-2014. Again, notice that these plots only provide us with slight indications on how these two dynamic strategies allocate from month to month. In order to provide more firmly evidence of this observation, one could measure the stock exposure variability simply by taking the standard deviation of the stock exposure levels from 1973 to 2014. This is done for all data sets, and for a wide range of different volatility target (σ^*) and turbulence target (d^*) values.

~ 1973-2014 ~				
Panel A: Volatility targeting – stock exposure variability				
Target values	10 industry	30 industry	25 on size and momentum	25 on size and book-to-market
$\sigma^* = 0.01$	0.11	0.12	0.15	0.15
$\sigma^* = 0.03$	0.34	0.37	0.44	0.45
$\sigma^* = 0.05$	0.57	0.61	0.73	0.75
$\sigma^* = 0.10$	1.14	1.22	1.47	1.49
$\sigma^* = 0.20$	2.28	2.44	2.93	2.99
Panel B: Turbulence targeting – stock exposure variability				
$d^* = 5$	0.13	0.06	0.06	0.06
$d^* = 15$	0.38	0.18	0.18	0.17
$d^* = 25$	0.64	0.31	0.31	0.28
$d^* = 50$	1.27	0.61	0.61	0.56
$d^* = 70$	1.78	0.85	0.86	0.78

Table 5.3: Stock exposure variability for dynamic strategies

There's no doubt that the volatility targeting portfolio generally varies a lot more than its turbulence aiming counterpart. And especially when using the naïve prediction method, it is reasonable to assume that a strategy which exhibit large variations in stock exposure, is more likely to exhibit large losses as well. Due to its quite poor performances in general, we have reasons to believe that the volatility targeting strategy

does not manage to capture the upturns as well as it should either.

So, in general, the turbulence strategy never varies as much (as the volatility targeting) when it comes to reallocation, with the lowest (highest) weight in stocks always higher (lower) than the corresponding number of its dynamic equivalent. This is so due to the general behaviour of the turbulence as an underlying market measure. Obviously, this can affect the portfolio's results in a negative way if it "refuses" to go lower than a given percentage in stocks, no matter what the circumstances are. Also, regarding the estimation of the turbulence, there could probably be some fluctuations on the basis of what method one uses to estimate the forecasted variance-covariance matrix. Yet, at this point I could mention that Engel and Gizon (1999) assessed different time-series models that could be used to predict the variance-covariance matrix and concluded that the simpler models achieve equally good results as the more sophisticated models. Although I cannot generalize these results (due to the fact that there are a lot of similar studies, and some may come up with other conclusions), they can give a some indication. What is also sure, is that the turbulence value will also exhibit some variations depending on the length of the lookback period. For that matter, I did a few re-estimations of the turbulence based on different lengths of the in-sample period, and the differences in the realized figures were minor. Hence, I will not discuss this topic further. Regarding previous research on the topic of turbulence targeting portfolios, there are very few to choose from, and none of them are directly comparable to the approach applied in this thesis. Yet, Kritzman et al. (2010) built a turbulence-resistant portfolio and found out that it achieved higher return-to-risk than the other strategies he considered. Quite consistently, the same is the case for the turbulence targeting strategy in this paper as well (although we did not manage to find statistical evidence that it was performing significantly better than the volatility targeting and naïve diversification portfolios).

6 Summary and conclusion

Let's initiate this chapter with a summary of the main points of this thesis. First, we did a rather general analysis on the systemic risk levels from 1973 to 2014, which was mainly based on the absorption ratio, as defined by Kritzman et al. (2011). Assuming that the 30 industry portfolios are the most representative data set of the four we have (in order to represent the total market), we did find that upward shifts of this measure undeniably

coincided with incidents where the market returns dropped massively. Though, considering the whole period for this particular data set, we failed to find any direct relationship between daily returns and absorption ratio. Hence, we did not use this measure as a targeting measure for a dynamic portfolio, unlike what was the case for the volatility and turbulence. Finally, a short data analysis also revealed that the two data sets where the returns were (partly) sorted by firm size should not be used in order to estimate the absorption ratio, since they would result in artificially high figures (which is not a surprise, due to the nature of the data sets versus how the absorption ratio is defined).

Then, we briefly described the estimation procedures related to the computation of monthly returns, volatility and turbulence. We also compared the turbulence measure with the volatility (though in a rather superficial manner), and find some tendencies which suggested that the turbulence might be somewhat more persistent than the volatility. In this section we also estimated and compared the monthly return, volatility and turbulence across the different data sets and find that the volatility and return levels were quite consistent, while the turbulence level increased as the dimensionality of the data set increased.

In the following section we carried out two different predictive regressions for the excess returns; one using the lagged value of volatility as predictor, and one by applying the lagged turbulence as a predictive variable. This was conducted on all four data sets, for periods 1973-2014, 1973-1990 and 1991-2014, and the results were highly consistent; all regressors were significant at a 5 per cent level. Even though the R-squared was quite low for some of the regressions, we did find a clear negative relationship between turbulence and volatility values from the previous period, and returns from the current period. Hence, we found some evidence on the relationship between these variables which would make the turbulence and volatility measures even more appealing as targeting figures for dynamic portfolio purposes.

This brings us to what is considered to be the main motivation of this thesis; the assessment of the dynamic volatility and turbulence targeting strategies, and the static $1/N$ strategy. Mean returns, standard deviations, Sharpe ratios, Jobson-Korkie-Memmel p-values, Sortino ratios, kurtoses and skewnesses were computed for all three strategies, for the same three periods as mentioned in the previous paragraph, and for the same four data sets. If we were to assess the portfolios' performances purely based

on the level of the Sharpe and Sortino ratios, the turbulence targeting strategy seems to perform slightly better than the two other portfolios.. The results are indeed consistent, since the turbulence targeting portfolio achieves the highest levels of the performance measures when we take the whole period 1973-2014 into consideration, as well as when we examine the two different sub periods. Yet, we cannot draw any sharp conclusions based on this alone, especially since the Jobson-Korkie-Memmel test in fact propose that the Sharpe ratio of the turbulence targeting portfolio never is significantly different from the Sharpe ratio of the passive $1/N$ portfolio strategy. In Chapter 5 we also made an effort in order to deduce why the volatility targeting portfolio clearly underperforms according to the results in Section 4.4. Applying a more practically-oriented discussion, we found that the volatility targeting strategy exhibited much larger variability in stock exposure over time than the turbulence strategy, and insinuated that this, accompanied with the naïve prediction method, might be the reason why this strategy achieves rather unsatisfactory results in this particular research setting.

As a final remark, it should be noted that the findings in this thesis does not confirm anything else than the fact that especially the concept of turbulence targeting strategies has to be examined further, and more thoroughly, in order to gain robustness of the results. When we inspect this field, there are indeed several factors one can consider in addition to the ones already examined in this thesis, all of which can have significant impacts on the final results; one could study other data sets, additional time periods, vary the length of the in-sample period, one could also apply other forecasting methods, and one could test for forecasting errors, just to mention a few ideas for further investigation.

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Appendices

A Supplementary data

A-1 Omitted figures

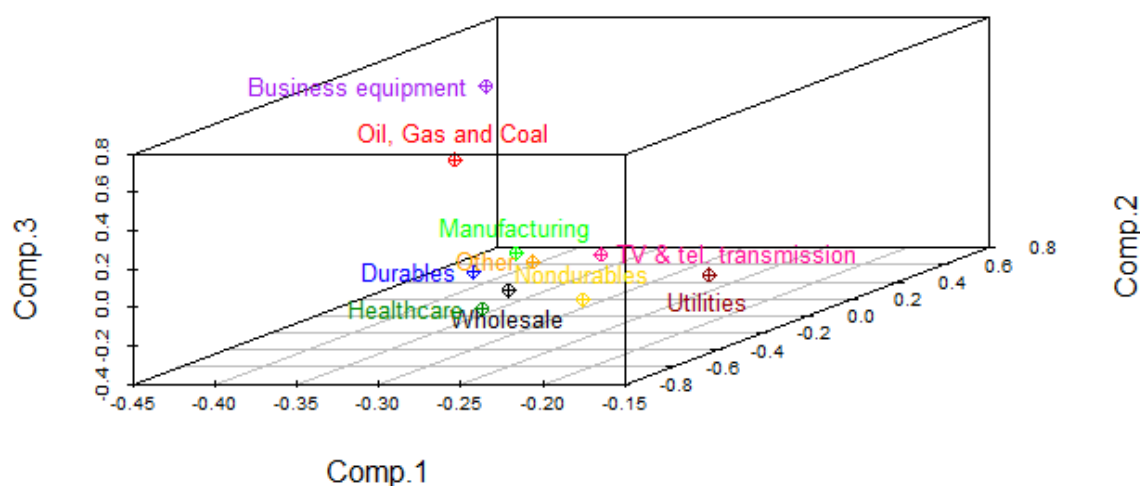


Figure A-1: Factor loadings (three principal axes) – 1973-2014

A-2 Omitted tables

~ 1973-2014 ~		
Panel A: 10 industry portfolios		
	Vol	Tur
Mean stock allocation	1.51	1.89
Min. stock allocation	0.22	0.51
Max. stock allocation	3.74	3.97
Panel B: 30 industry portfolios		
Mean stock allocation	1.50	1.05
Min. stock allocation	0.23	0.34
Max. stock allocation	3.88	1.89
Panel C: 25 portfolios formed on size and momentum		
Mean stock allocation	1.62	1.18
Min. stock allocation	0.22	0.40
Max. stock allocation	4.20	1.98
Panel D: 25 portfolios formed on size and book-to-market		
Mean stock allocation	1.67	1.16
Min. stock allocation	0.24	0.38
Max. stock allocation	4.45	2.04

Table A-1: Dynamic asset allocation features¹⁶

~ 1973-2014 ~			
Panel A: 10 industry portfolios			
	Pas	Vol	Tur
Monthly min. return (%)	-27.46	-29.72	-45.65
Monthly max. return (%)	13.71	19.32	22.53
Panel B: 30 industry portfolios			
Monthly min. return (%)	-31.95	-34.46	-29.07
Monthly max. return (%)	16.80	22.83	15.29
Panel C: 25 portfolios formed on size and momentum			
Monthly min. return (%)	-30.51	-46.49	-33.51
Monthly max. return (%)	20.65	33.08	17.45
Panel D: 25 portfolios formed on size and book-to-market			
Monthly min. return (%)	-30.61	-45.84	-31.13
Monthly max. return (%)	16.60	30.23	16.43

Table A-2: Monthly minimum and maximum returns of each strategy¹⁶

B Mathematical appendix

B-1 Finding eigenvectors and eigenvalues¹⁷

Consider the same case as in equation (3.39), but this time the square matrix is denoted by A :

$$Av = \lambda v \tag{B.1}$$

On the basis of the properties of the identity matrix, denoted I , we can rewrite (B.1) in the subsequent manner:

$$\begin{aligned} \lambda Iv &= Av \\ \lambda Iv - Av &= 0 \\ (\lambda I - A)v &= 0 \end{aligned} \tag{B.2}$$

Assuming that $\lambda I - A$ is a singular matrix, v is the eigenvector of A . Then, by solving the equation $|\lambda I - A| = 0$, we can find the eigenvalues as well.

¹⁶ Target values $\sigma^* = 0.05$ and $d^* = 25$.

¹⁷ Section C-1 is based on Musau's lecture notes from the course SE-409: Quantitative Methods in Economics and Finance (2013).

B-2 Normalizing an eigenvector

Based on Jeffrey (2001), normalization of an eigenvector is done in the following way; first, consider a 2-dimensional example where the vector $k = (x, y)$ equals a point K in the (x, y) -plane. Then, the length of vector k , denoted as $\|k\|$, is computed by applying Pythagoras' theorem (which also can be generalized to higher dimensions):

$$\|k\| = \sqrt{x^2 + y^2} \quad (\text{B.3})$$

Finally, we divide the original vector by its length so that we obtain a normalized vector with a length of 1:

$$\frac{\begin{pmatrix} x \\ y \end{pmatrix}}{\sqrt{x^2 + y^2}} = \begin{pmatrix} \frac{x}{\sqrt{x^2 + y^2}} \\ \frac{y}{\sqrt{x^2 + y^2}} \end{pmatrix} \quad (\text{B.4})$$

Or, given in simpler terms:

$$\frac{k}{\|k\|} = \hat{k} \quad (\text{B.5})$$

where \hat{k} denotes the normalized vector.

C R-programs

C-1 Estimating monthly turbulence, standard deviation and returns

```
1 # Clear environment
2 rm(list=ls(all=TRUE))
3 # Loading essential packages
4 library("quadprog")18
5
6 # Read data set consisting of daily returns
7 Ret <- read.table("10ind_1960-2014.txt"19, header=TRUE)
8 Ret <- Ret[,2:ncol(Ret)]/100
9
10 # Defining number of years and days for the total period in question
11 NYears <- 55 # Jan 2, 1960 up to Dec 31, 2014
12 NDays <- dim(Ret)[1]
13 # Computing the average number of days in a month for the period 1960-
    2014
14 nDays.Month <- round(NDays/NYears/12)
15 nMonths <- NYears*12
16 nDays.Year <- nDays.Month*12
17 # Define lookback period
18 nYears <- 10 # number of years
19 nMonths.lb <- nYears*12 # number of months
20 lb.period <- nYears*nDays.Year # number of days
21
22 # Compute number of months in the OOS period
23 nDays.OOS <- NDays-lb.period
24 nMonths.OOS <- round(nDays.OOS/nDays.Month)
25
26 # Define number of rows, columns and observations
27 nRow <- nrow(Ret)
28 nCol <- ncol(Ret)
29 n <- nRow-lb.period
30
31 # Reserve space for monthly data
32 ret <- rep(0,nMonths.OOS) # returns
33 std <- rep(0,nMonths.OOS) # volatility
34 tur <- rep(0,nMonths.OOS) # turbulence
35
36 for(i in 1:nMonths.OOS) {
37   # Define the start and end points of the periods
38   start.lb <- (i-1)*nDays.Month + 1 # start of lookback period
39   end.lb <- start.lb + nMonths.lb*nDays.Month - 1 # end of lookback
    period + 1 month
```

¹⁸ Ref. Turlach and Weingessel (2013).

¹⁹ Has to be changed according to what data set we want to examine.


```

40 start.month <- end.lb + 1 # start of current month
41 end.month <- start.month + nDays.Month -1 # end of current month
42
43 # The following figures are needed to compute the turbulence
44 r.lb <- Ret[start.lb:end.lb,] # all columns
45 covmat <- cov(r.lb) # sample variance-covariance matrix
46 covmat.inv <- solve(covmat) # inverse of the covariance matrix
47 er <- colMeans(r.lb) # sample mean
48
49 # Select the returns for a given month
50 r.month <- as.matrix(Ret[start.month:end.month,])
51 rp.daily <- rep(0,nDays.Month)
52 for (j in 1:nDays.Month) { #
53 # compute the monthly turbulence
54 r <- r.month[j,] - er #
55 d <- r %%% covmat.inv %%% r
56 tur[i] <- tur[i] + d
57 # compute daily returns as a mean
58 rp.daily[j] <- mean(r.month[j,])
59 }
60 tur[i] <- sqrt(tur[i]) # monthly turbulence
61
62 # Compute the standard deviation in per cent
63 std[i] <- sd(rp.daily)*sqrt(nDays.Month)*100
64
65 # Compute monthly returns
66 r <- cumprod(1+rp.daily)
67 ret[i] <- r[nDays.Month] - 1
68 }
69
70 index <- cumprod(c(1, 1+ret))
71 ind.ts <- ts(index, start=c(1970,2), frequency=12)
72 ind.ts <- window(ind.ts, start=c(1973,1), end=c(2014,12))
73 year.start <- 1973
74 year.end <- 2015
75 plot(log(ind.ts), plot.type = "single", xlab=NULL, ylab=NULL,
76 col="green", lty=1, lwd=2, axes=FALSE) # plot monthly log-returns
77
78 # Construct time series objects
79 ret.ts <- ts(ret, start=c(1970,2), frequency=12)
80 Ret.ts <- window(ret.ts, start=c(1973,1), end=c(2014,12))
81 std.ts <- ts(std, start=c(1970,2), frequency=12)
82 Std.ts <- window(std.ts, start=c(1973,1), end=c(2014,12))
83 tur.ts <- ts(tur, start=c(1970,2), frequency=12)
84 Tur.ts <- window(tur.ts, start=c(1973,1), end=c(2014,12))
85

```

```

86 # Construct plot which compares volatility and turbulence level
87 par(mar = c(5, 4, 4, 4) + 0.3) # add extra space for volatility axis
  text
88 plot(Tur.ts, type="l",xlab=NULL,ylab="Turbulence",lwd=2,las=1) # plot
  turbulence
89 par(new = TRUE)
90 Col <- rgb(red=0,green=1,blue=0,alpha=0.7)
91 plot(Std.ts, type = "l", col=Col,axes = FALSE, bty = "n",
92      xlab = NULL, ylab = NULL, lwd=2) # plot standard deviation
93 axis(side=4, col.axis="green",at =
  pretty(range(std.ts)),col="green",las=1)
94 mtext("Standard deviation", side=4, line=3, col="green")
95
96 # Finding monthly data of a "risk-free" asset
97 data.ff3 <- read.table("ff3.txt", header=TRUE)
98 r.tbill <- ts((data.ff3[,5]), start=c(1926,7),frequency=12)
99 r.tbill <- window(r.tbill, start=c(1970,2),end=c(2014,12))
100
101 # Creating new data file
102 newdata <- data.frame(RET=c(ret.ts*100), TURB=c(tur.ts),
103                      STD=c(std.ts), RF=c(r.tbill))
104 write.table(newdata, file = "tur,ret,std,rf-10ind.csv"20,
105            sep = ",", col.names = colnames(newdata))

```

C-2 Estimating monthly absorption ratio, and PCA

Some parts of this program are based on blog posts from *Systematic Edge Blog* (2013) and *Systematic Investor Blog* (2012), while the rest is written from scratch based on relevant theories for absorption ratio/PCA.

```

1 rm(list=ls(all=TRUE))
2 library("quadprog")
3 library("scatterplot3d")21
4 library("TTR")22
5
6 # Read data set consisting of daily returns
7 Ret <- read.table("30ind_1960-2014.txt", header=TRUE)
8 Ret <- Ret[,2:ncol(Ret)]/100
9
10 # Defining number of years and days for the whole period in question
11 NYears <- 55 # Jan 2, 1960 up to Dec 31, 2014
12 NDays <- dim(Ret)[1]
13 # Computing the average number of days in a month for the period 1960-
  2014

```

²⁰ It is convenient to change the name of this file according to what data set we have used.

²¹ Ligges and Mächler (2003).

²² Ulrich (2013).

```

14 nDays.Month <- round(NDays/NYears/12)
15 nMonths <- NYears*12
16 nDays.Year <- nDays.Month*12
17 # Define lookback period
18 nYears <- 10 # number of years
19 nMonths.lb <- nYears*12 # number of months
20 lb.period <- nYears*nDays.Year # number of days
21
22 # Compute number of months in the OOS period
23 nDays.OOS <- NDays-lb.period
24 nMonths.OOS <- round(nDays.OOS/nDays.Month)
25
26 # Define number of rows, columns and observations
27 nRow <- nrow(Ret)
28 nCol <- ncol(Ret)
29 n <- nRow-lb.period
30
31 abs <- rep(0,n)
32 for(i in 1:n) {
33   start <- i
34   end <- i+lb.period-1
35   Return <- Ret[start:end,]
36   cov <- cov(Return)
37   eigenval <- eigen(cov)$values
38   sumeigenval <- sum(eigenval)
39   sumeigen20 <- sum(eigenval[1:(round(nCol/5))])# fixed number of
   eigenvectors
40   abs[i] <- sumeigen20/sumeigenval # absorption ratio with nCol/5
   eigenvector
41 }
42
43 abs.m <- rep(0,nMonths.OOS) # reserve space for monthly abs.ratio
44 for(i in 1:nMonths.OOS) {
45   start <- (i-1)*nDays.Month + 1
46   end <- i*nDays.Month
47   abs.d <- abs[start:end] # daily abs.ratio during a month
48   abs.m[i] <- (mean(abs.d)) # monthly abs.ratio
49 }
50 # Create ts object and plot
51 Start <- 1973
52 End <- 2014
53 abs.ts <- ts(abs.m, start=c(1970,2), end=c(2014,12), frequency=12)
54 Abs.ts <- window(abs.ts, start=c(Start,1), end=c(End,12))
55 plot(Abs.ts, ylab="Absorption ratio", col="red",
56       plot.type="single", xlab=NULL, las=1, ylim=c(range(Abs.ts)))
57
58 # Comparing level of absorption ratio with price level of S&P 500

```

```

59 Data <- read.csv("sp500.csv", header=TRUE)23
60 Data <- ts(Data[,2], start=c(1871,1), frequency = 12)
61 sp500 <- window(Data, start=c(Start, 1), end=c(End,12))
62 par(mar = c(5, 4, 4, 4) + 0.3) # add extra space for volatility axis
  text
63 plot(sp500, type="l", xlab=NULL, ylab="S&P 500 index", las=1) # plot
  returns
64 par(new = TRUE)
65 plot(Abs.ts, type = "l", col="red",
66       axes = FALSE, bty = "n",xlab = NULL, ylab=" ") # plot AR
67 axis(side=4, col.axis="red",at = pretty(range(abs.m)),col="red",
68       las=1) # add volatility axis
69 mtext("Absorption ratio", side=4, line=3, col="red")
70
71 # Regress daily returns on daily abs.ratio to find possible relations
72 ret <- rowMeans(Ret)
73 ret <- ret[(lb.period+1):nRow]
74 reg.abs <- summary(lm(ret ~ abs))
75 L1.abs <- lag(abs, k=1)
76 reg.L1abs <- summary(lm(ret ~ L1.abs))
77
78 # Short principal component analysis
79 ret.ts <- ts(Ret, start=c(1970,1), end=c(2014,252), frequency=252)
80 pc<-princomp(ret.ts) # define principal components
81 pc.var.expl. = pc$sdev^2 / sum(pc$sdev^2) # var. explained by each
  component
82 barplot(100*pc.var.expl., las=2, xlab='', ylab='% Variance Explained',
83         cex.lab=0.8); par(cex.axis=0.8)
84 nCol <- ncol(Ret)
85 # Explained variance for x/N number of components
86 round(rbind(sum(pc$sdev[1:round(nCol*0.1)]^2) / sum(pc$sdev^2),
87 sum(pc$sdev[1:round(nCol*0.2)]^2) / sum(pc$sdev^2),
88 sum(pc$sdev[1:round(nCol*0.3)]^2) / sum(pc$sdev^2),
89 sum(pc$sdev[1:round(nCol*0.4)]^2) / sum(pc$sdev^2),
90 sum(pc$sdev[1:round(nCol*0.5)]^2) / sum(pc$sdev^2),
91 sum(pc$sdev[1:round(nCol*0.6)]^2) / sum(pc$sdev^2),
92 sum(pc$sdev[1:round(nCol*0.7)]^2) / sum(pc$sdev^2),
93 sum(pc$sdev[1:round(nCol*0.8)]^2) / sum(pc$sdev^2),
94 sum(pc$sdev[1:round(nCol*0.9)]^2) / sum(pc$sdev^2)),digits=3)
95
96 loadings <- pc$loadings[]
97 x<-loadings[,1]
98 y<-loadings[,2]
99 colours<- c("gold","blue","green","red","purple","deeppink","black",
100            "green4","red4","orange")

```

²³ Data file from Robert Shiller's online data library: <http://www.econ.yale.edu/~shiller/data.htm>

```

101 name <- c("Nondurables","Durables","Manufacturing",
102           "Oil, Gas and Coal","Business equipment",
103           "TV & tel. transmission","Wholesale",
104           "Healthcare","Utilities","Other")24
105 plot(x, y, type='p',pch=20,col=colours,xlab='Comp.1',ylab='Comp.2',
106       cex.axis=0.8, cex.lab=0.8, las=1) # 2-d plot
107 legend('topright',cex=.6,legend=name[1:5],col=colours[1:5],fill=colours[1:5],bty='n')
108 legend('top',cex=.6,legend=name[6:10],col=colours[6:10],fill=colours[6:10],bty='n')
109 z<-loadings[,3]
110 s3d <- scatterplot3d(x, y, z, xlab='Comp.1', ylab='Comp.2',
111                    zlab='Comp.3',color=colours, pch = 10)
112 s3d.coords <- s3d$xyz.convert(x, y, z)
113 text(s3d.coords$x, s3d.coords$y, labels=name, col=colours, cex=.8,
114      pos=c(3,2,3,3,2,4,1,2,1,2)) # 3-d plot

```

C-3 Sharpe ratio, Sortino ratio and Jobson & Korkie test statistic

This entire program is made by Zakamulin (2014), and was obtained through participation in the course Computational Finance.

```

1 SR <- function(er) {
2   # computes the Sharpe ratio
3   return(mean(er)/sd(er))
4 }
5
6 SharpeTest <- function(ex1, ex2) {
7   # test for equality of two Sharpe ratios
8   # ex1 - excess returns to the first portfolio
9   # ex2 - excess returns to the second portfolio
10  # returns the p-value of the test
11  # the null hypothesis is rejected when p-value is small
12  if (length(ex1) != length(ex2))
13    stop("Different lengths of two returns!")
14  SR1 <- mean(ex1)/sd(ex1)
15  SR2 <- mean(ex2)/sd(ex2)
16  ro <- cor(ex1,ex2)
17  n <- length(ex1)
18  z <- (SR2-SR1)/sqrt( (2*(1-ro)+0.5*(SR1^2+SR2^2-2*SR1*SR2*ro^2))/n )
19  pval <- 2*pnorm(-abs(z))
20  return(pval)
21 }
22

```

²⁴ These names are only appropriate for the 10 industry data set, and used only for the sake of illustration. When analysing other data sets, these should be ignored. Same accounts for the legends in lines 107-108 and colours in lines 99-100.

```

23 SoR <- function(er) {
24 # computes the Sortino ratio
25 lpm <- pmax(-er,0)
26 lpm <- sqrt(mean(lpm^2)) # this is the lower standard deviation
27 return(mean(er)/lpm)
28 }

```

C-4 Portfolio performances

```

1 # Clear environment
2 rm(list=ls(all=TRUE))
3
4 # Loading essential packages and functions
5 source("performance.r")25
6 library("fBasics")26
7
8 # Read data set
9 data <- read.csv("tur,ret,std,rf-10ind.csv"27, header=TRUE)
10
11 # Define time period
12 Start <- 1973
13 End <- 2014
14
15 data <- ts(data, start=c(1970,2), frequency = 12)
16 data <- window(data, start=c(Start-1, 1), end=c(End,12))
17
18 start <- 13
19
20 # Assign names to each column
21 tur <- as.numeric(data[, "TURB"]) # turbulence
22 ret <- (as.numeric(data[, "RET"])/100 # stock returns
23 std <- (as.numeric(data[, "STD"])/100 # standard deviations
24 rf <- (as.numeric(data[, "RF"])/100 # risk-free returns
25
26 # Total number of observations
27 N <- length(tur)
28
29 # Compute returns of the different dynamic portfolio strategies
30 n <- N-start+1 # number of monthly portfolio returns
31
32 # Reserve space for...
33 r.vol <- rep(0,n) # volatility targeting portfolio returns
34 ws.vol <- rep(0,n) # weight of stocks in the vol.targeting portfolio
35 r.tur <- rep(0,n) # turbulence targeting portfolio returns

```

²⁵ Which is equivalent to the R-program in section B-3.

²⁶ Wuertz, Setz and Chalabi (2014).

²⁷ Same data file as the one that was made at the end of the program in Section B-1. Note that the name of this file has to be in accordance with the initial name that was given at lines 104-105 in Section B-1.

```

36 ws.tur <- rep(0,n) # weight of stocks in the tur.targeting portfolio
37 r.stc <- rep(0,n) # returns of the static, naïve portfolio
38 ws.stc <- rep(0,n) # weight of stocks in the static portfolio
39
40 std.target <- 0.05 # arbitrary (does not affect the Sharpe and Sortinos)
41 tur.target <- 25 # arbitrary (does not affect the Sharpe and Sortinos)
42
43 for (i in 1:n) {
44   month.end <- start + i - 2 # Index of the previous month
45   # Volatility targeting:
46   # Use volatility from the previous month as forecast for the present
   month
47   ws.vol[i] <- std.target/std[month.end]
48   r.vol[i] <- ws.vol[i]*ret[month.end+1] + (1-ws.vol[i])*rf[month.end+1]
49
50   # Turbulence targeting:
51   # Use turbulence from the previous month as forecast for the present
   month
52   ws.tur[i] <- tur.target/tur[month.end]
53   r.tur[i] <- ws.tur[i]*ret[month.end+1] + (1-ws.tur[i])*rf[month.end+1]
54
55   # Naïve strategy:
56   ws.stc[i] <- 1
57   r.stc[i] <- ws.stc[i]*ret[month.end+1] + (1-ws.stc[i])*rf[month.end+1]
58 }
59
60 # Define risk-free returns for the given sub period
61 rf <- rf[start:N]
62
63 # Compute the performance measures
64 SR.vol <- SR(r.vol-rf) # Sharpe ratio
65 SoR.vol <- SoR(r.vol-rf) # Sortino ratio
66
67 SR.tur <- SR(r.tur-rf)
68 SoR.tur <- SoR(r.tur-rf)
69
70 SR.stc <- SR(r.stc-rf)
71 SoR.stc <- SoR (r.stc-rf)
72
73 # Estimate p-value of Jobson&Korkie+Mommel test
74 pval.vol <- SharpeTest(r.vol-rf, r.stc-rf) # vol.targ. vs static
75 pval.tur <- SharpeTest(r.tur-rf, r.stc-rf) # turb.targ. vs static
76 pval.tur2 <- SharpeTest(r.tur-rf, r.vol-rf) # turb.targ vs vol.targ
77
78 # plot the weight of stocks of active portfolios and the returns level
79 ret.ts <- ts(r.stc, start=c(Start,1), frequency=12)
80 ws.ts <- ts(cbind(ws.vol,ws.tur), start=c(Start,1), frequency=12)

```

```

81 par(mar = c(5, 4, 4, 4) + 0.3) # add extra space for returns axis text
82 plot(ws.ts, type="l", xlab=NULL, ylab="Weight of stocks", lwd=2,
83     las=1,plot.type="single", col=c("black","grey")) # plot weights
84 par(new = TRUE)
85 # use a slightly transparent colour on the returns curve to avoid too
    much overlap
86 col1 <- rgb(red=0,green=1,blue=0,alpha=0.5)
87 plot(ret.ts, type = "l", col=col1,axes = FALSE, bty = "n",
88     xlab = NULL, ylab = NULL, lwd=2) # plot returns
89 axis(side=4, col.axis="green",at = pretty(range(r.stc)),col="green",
90     las=1) # add returns axis
91 mtext("Monthly returns", side=4, line=3, col="green")
92
93 # Gathering key numbers related to the performances of the strategies
94 round((colMeans(cbind(r.stc,r.vol,r.tur))*100)*12,digits=2)
95 round((colStdevs(cbind(r.stc,r.vol,r.tur))*100)*12^0.5,digits=2)
96
97 round(cbind(SR.stc,SR.vol,SR.tur)*12^0.5,digits=2)
98 round(cbind(pval.vol,pval.tur,pval.tur2),digits=2)
99 round(cbind(SoR.stc,SoR.vol,SoR.tur)*12^0.5,digits=2)
100
101 round(colSkewness(cbind(r.stc,r.vol,r.tur)),digits=2)
102 round(colKurtosis(cbind(r.stc,r.vol,r.tur),method="moment"),digits=2)
103 jarqueberaTest(r.stc);jarqueberaTest(r.vol);jarqueberaTest(r.tur)
104
105 round((colMins(cbind(r.stc,r.vol,r.tur))*100),digits=2)
106 round((colMaxs(cbind(r.stc,r.vol,r.tur))*100),digits=2)
107
108 # Gathering key numbers related to the (re)allocations of the
    strategies
109 round(colMeans(cbind(ws.vol,ws.tur)),digits=2)
110 round(colMins(cbind(ws.vol,ws.tur)),digits=2)
111 round(colMaxs(cbind(ws.vol,ws.tur)),digits=2)
112 round(colStdevs(cbind(ws.vol,ws.tur)),digits=2)
113
114 # Predictive regressions
115 Ret <- ret[start:(n+12)]*100
116 Std <- std[start:(n+12)]*100
117 Rf <- rf*100
118 L1.std <- lag(Std, k=1)
119 reg.vol <- summary(lm((Ret-Rf) ~ L1.std))
120 Tur <- tur[start:(n+12)]
121 L1.tur <- lag(Tur, k=1)
122 reg.tur <- summary(lm((Ret-Rf) ~ L1.tur))
123
124 # Mean, max, min. for return, volatility and turbulence
125 Data <- cbind(Ret,Std,Tur)

```



```
126 round(colMeans(Data),digits=2)
127 round(colMins(Data),digits=2)
128 round(colMaxs(Data),digits=2)
129
130 # The following simulations are used to study reallocation properties
131 s <- 177 # start-month
132 k <- s+2 # end-month
133 R.vol <- r.vol*100; R.tur <- r.tur*100
134 misc <-
135 round(rbind(Ret,Std,Tur,ws.vol,R.vol,ws.tur,R.tur)[,s:k],digits=2)
136 round(rowMeans(misc),digits=2)
137 round(rowStdevs(misc),digits=2)
```