Mathematical Tasks from the Teachers' Point of View

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A Multiple Case Study of Teachers' Goals in Norwegian Vocationally Oriented Classrooms

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I want to dedicate this PhD to Solrun Opheim. My mother, friend, and greatest supporter in the whole process with the PhD. She sadly passed away before it was submitted, but I know she cheers me on from above.

Summary

This study reports on the teachers' perspectives when it comes to mathematical tasks, and changes teachers make in their everyday classroom. Research within mathematics education is a complex field with many different factors, and there has been a huge effort by researchers to develop and improve teaching in mathematics. Still, it turns out that this is not easily transferred to the classrooms (Artigue, 2008; Breiteig & Goodchild, 2010). Past experiences also reveal that new curriculums are not implemented as intended (Breiteig & Goodchild, 2010). There is a perceived disconnection between practice and research which has vexed education for a very long time (Silver & Lunsford, 2017). To address these issues and understand the teachers' perspectives, two research questions were formulated:

1. What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom?
2. What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration?

The research is designed as a multiple case study (Stake, 2006), where the phenomenon to be studied is the teachers' descriptions of mathematical tasks they want to use in their classrooms, and the cases are four teachers in the context of their classes and the schools they work at. Each case consists of a task design process, which include designing tasks, refining them, implementing, and evaluating the tasks.

I have used techniques from grounded theory in the analysis process, and conducted open coding based on the ideas from Glaser and Strauss (1967). Through the inductive analysis process, I have identified three different dimensions of how the teachers describe mathematical tasks: *Outcome of tasks*, *Characteristics of tasks*, and *Students' reactions to tasks*. The data was further analyzed with respect to the change sequences for each design process conducted with the teachers, using the Interconnected Model of Professional Growth developed by Clarke and Hollingsworth (2002).

The answers to the two research questions are clearly intertwined, because the teachers' descriptions of mathematical tasks are linked to their rationales for initiating changes. According to the findings in this research project, teachers describe mathematical tasks mostly by the desired outcome of the tasks. These desired outcomes of tasks are related to their students and the need to resolve

three types of classroom issues: *work*, *motivation*, and *understanding*. However, there were also some aspects the teachers might struggle with that could hinder certain types of mathematical tasks. These were *didactics*, *communication*, and *mathematics*. The teachers describe mathematical tasks that might help them resolve and change issues in their classrooms. They want mathematical tasks that will help them get their students to work, to be more motivated or to gain a better understanding. These are teachers' rationales for initiating changes. However, some of the teachers are also making changes to improve one or more of the teacher aspects they might struggle with. This is evident when analyzing the change processes through the Interconnected Model of Clarke and Hollingsworth (2002). This research project has shown that the Interconnected Model of Clarke and Hollingsworth (2002) also can be useful for analyzing change processes from the teachers' perspective in the classroom when designing and implementing mathematical tasks. However, I argue that such an analysis requires an expansion of the Interconnected Model, to include the student domain.

Contents

1 Introduction

This dissertation focuses on the teachers' perspectives when it comes to mathematical tasks they use to offer students an opportunity to learn, and changes to those tasks teachers make for their everyday use in their classroom. Four teachers working in upper secondary school participated in the research project and the results are based on an analysis of a one-year collaboration between each of the teachers and one researcher. I will, in this introductory chapter, provide a rationale for a focus on the teachers' perspective (Section 1.1), present the research questions (Section 1.2), give a description of the research setting and the methods used (Section 1.3) and finally round off the chapter by presenting the structure of the dissertation (Section 1.4).

1.1 Understanding the Teacher and Teaching

When conducting research, implementation and use of the results are, of course, important to the researcher, but it is not always easy to accomplish this to the extent one might want. Research within mathematics education is a complex field with many different factors, and there has been a huge effort by researchers to develop and improve teaching in mathematics. Still, it turns out that this is not easily transferred to the classrooms (Artigue, 2008; Breiteig & Goodchild, 2010). Past experiences also reveal that new curriculums are not implemented as intended (Breiteig & Goodchild, 2010). There is a perceived disconnection between practice and research which has vexed education for a very long time (Silver & Lunsford, 2017).

This challenge of implementation of research in mathematics education is part of what sparked my interest when setting up my own research design. How could I conduct research which I felt both mattered and was implemented? What could be the reasons behind difficulties with implementation? One of my concerns was a lack of understanding of how teachers might impact implementation. Could it be that a greater knowledge and understanding of teachers might improve implementation of research in mathematics education? Even if both researchers and teachers have children's learning in focus, a teacher has a lot more practical obstacles to take into consideration, and this might not always be evident for the researcher (Ruthven & Goodchild, 2008). In addition, teachers experience the constraints of institutional expectations, but teachers are also continuously developing their knowledge on teaching by cases from their classrooms (Hundeland, 2011). Ruthven and Goodchild (2008) advocate researchers to acknowledge the craft knowledge of teachers, and this is something I want to contribute to with my research.

During the past decades, there has been an increasing interest in the research community to better understand teachers and teaching. Sfard (2005) reports on a decisive shift in research focus towards more articles on teachers and teacher practice around the millennium shift. Skott, Mosvold, and Sakonidis (2018) examine twenty years of research by the European Society for Research in Mathematics Education (ERME), and comment on how this shift of research focus towards teachers and teaching has gained even further momentum during the last years, not least in Europe. The first Congress of the European Society for Research in Mathematics Education (CERME) in 1998, had only one thematic working group on the topic: 'From a study of Teaching Practices to issues in Teacher Education'. This group has now been split into several others, and in CERME 9 and 10 there have been three different thematic working groups dedicated to teachers and teacher practice (Skott et al., 2018). Even though there has been a distinctive increase in research on teachers and teaching practice, there are many unresolved issues. The tension between research and practice has both been recognized in the mathematics education community, and it has given rise to much discussion. This persistent attention to the topic, even in the face of little evidence that the relationship has improved over time, suggests a deeply rooted, resilient belief and hope among scholars in mathematics education that it is both feasible and valuable to create a productive interface between research and practice in the field (Silver & Lunsford, 2017).

With this research project, my aim is that through a greater understanding of the teachers' perspective, there is also a greater chance of creating a productive interface between research and practice, which can help to reduce the research-practice gap. A combination of more insight into the teachers' perspective and what changes they are likely to make to mathematical tasks, will be an important contribution to the research field.

1.2 The Research Questions

With the aim of understanding the teachers' perspective, I have defined two research questions which guide the research presented in this dissertation.

1. What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom?

2. What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration?

I choose to focus on mathematical tasks in my collaboration with the teachers, since findings from the TIMSS advanced 2008 study (Mullis, Martin, Robitaille, & Foy, 2009), show that the most dominating activity in Norwegian classrooms is by far solving mathematical tasks. This is also a predominant classroom activity in other countries than Norway (Mullis, Martin, Robitaille, & Foy, 2009). In addition to explore how teachers describe mathematical tasks they want to use, I want to use the opportunity to also analyze the self-initiated changes teachers make when designing and implementing mathematical tasks, and the rationales they express for this.

1.3 The Research Setting, and Methods Adopted

Four teachers teaching mathematics in vocational classes at upper secondary school, volunteered to participate in this research project. They were offered help to design mathematical tasks they would want to use in their classrooms, and in return, I as a researcher, would in the process learn about what they looked for in tasks. My intentions were not to change the teachers, but to help the teachers make changes to their teaching which they might not otherwise have the time or resources to do.

The research is designed as a multiple case study (Stake, 2006), where the phenomenon to be studied is the teachers' descriptions of mathematical tasks they want to use in their classrooms, and the cases are four teachers in the context of their classes and the schools they work at. Each case consists of a task design process, which include designing tasks, refining them, implementing, and evaluating the tasks. By using such a design, I get access not only to which tasks the teachers claim they want to use, but also to their reflections when implementing and refining the tasks – thus connecting theory and practice.

All conversations with the teachers were recorded and analyzed. Since my aim was to describe the teachers' perspective of mathematical tasks they wanted to use in their classroom, I needed an inductive approach to analyze the data. I have therefore used techniques from grounded theory, and conducted open coding based on the ideas from Glaser and Strauss (1967). I elaborate on the details of this work in the methodology chapter. Through the inductive analysis process, I have identified three different dimensions of how the teachers describe mathematical tasks, and these are presented in Chapter 7.

Given a research design which is not aimed at changing the teachers or setting guidelines for how the teaching should be changed, this data material gives an opportunity to analyze the teacher-initiated change processes in the classroom. I have therefore analyzed the change sequences for each design process conducted with the teachers, using the Interconnected Model of Professional Growth developed by Clarke and Hollingsworth (2002).

1.4 The Structure of the Dissertation

Following this introductory chapter, I present the Norwegian school system as the context of the research study. This is followed by Chapter 3, where relevant theoretical perspectives are presented, and I position the research theoretically. I start this chapter by investigating teachers' impact on learning and what "good" teaching can be considered as, from the teachers' perspective. This is followed by a theoretical presentation of tasks and task design dilemmas, set in an international context. The theory section is rounded off by addressing teacher change and presenting a theoretical framework for investigating teacher change. Chapter 4 sets out the methodology of this study and presents the methods used, but also places the research within a research paradigm and argues for how this guides the choices made. The cases are presented in the same chapter, and trustworthiness and ethical considerations are discussed. The four cases are presented one by one in Chapter 5 and the tasks which were designed are presented in Chapter 6. Analysis with respect to research question 1 is described in Section 7.1, and analysis with respect to research question 2 is presented in Section 7.2. A discussion of both these analyses is then articulated in Chapter 8. Chapter 9 summarizes the findings and includes a discussion of strengths and limitations of the research. In addition, pedagogical implications and needs for further research are discussed.

2 The Norwegian School System

A case cannot be fully understood without context, and I will therefore elaborate on the context the teachers in this research project work within, which is the Norwegian school system. Getting teachers to collaborate closely with a researcher for a long period of time is not necessarily easy, given that time is a recurrent issue for many teachers and time pressure is a real day-to-day classroom experience with which teachers must live (Assude, 2005; Jordfald, Nyen, & Seip, 2009; Leong & Chick, 2011). However, the Norwegian school system might be one of the reasons such a research design is possible to conduct, and where teachers are willing to collaborate and spend time on such a project. I will in this chapter elaborate on the Norwegian school system to provide context for the research, but also on how the system might encourage teachers to invest their time in such a research project. At the end of this chapter, I will in addition discuss how some of the issues evident in the Norwegian school system also apply to teachers from other countries and therefore can be viewed as internationally relevant issues.

2.1 General Overview

In Norway there are 10 years of compulsory education. The children start school the year they turn six years old and have ten years of schooling before they leave at the end of lower secondary school. Mathematics is a compulsory subject throughout all these school years. After the children graduate lower secondary school, they can choose to continue their schooling at an upper secondary school. All people between the ages of 16 and 19 have a statutory right to upper secondary education and training, and today almost everyone continues into upper secondary schooling because it is becoming more and more difficult to get a job without it. In Figure 2.1, there is a sketch of the Norwegian school system and the possibilities the students have for continuing schooling after each completed level. The Norwegian terms of each level are written in the brackets, and the arrows represent possible movement between the various types of education. While most of the educational programs in Figure 2.1. are possible to attend as an adult, I have marked them with the most common age of those attending.

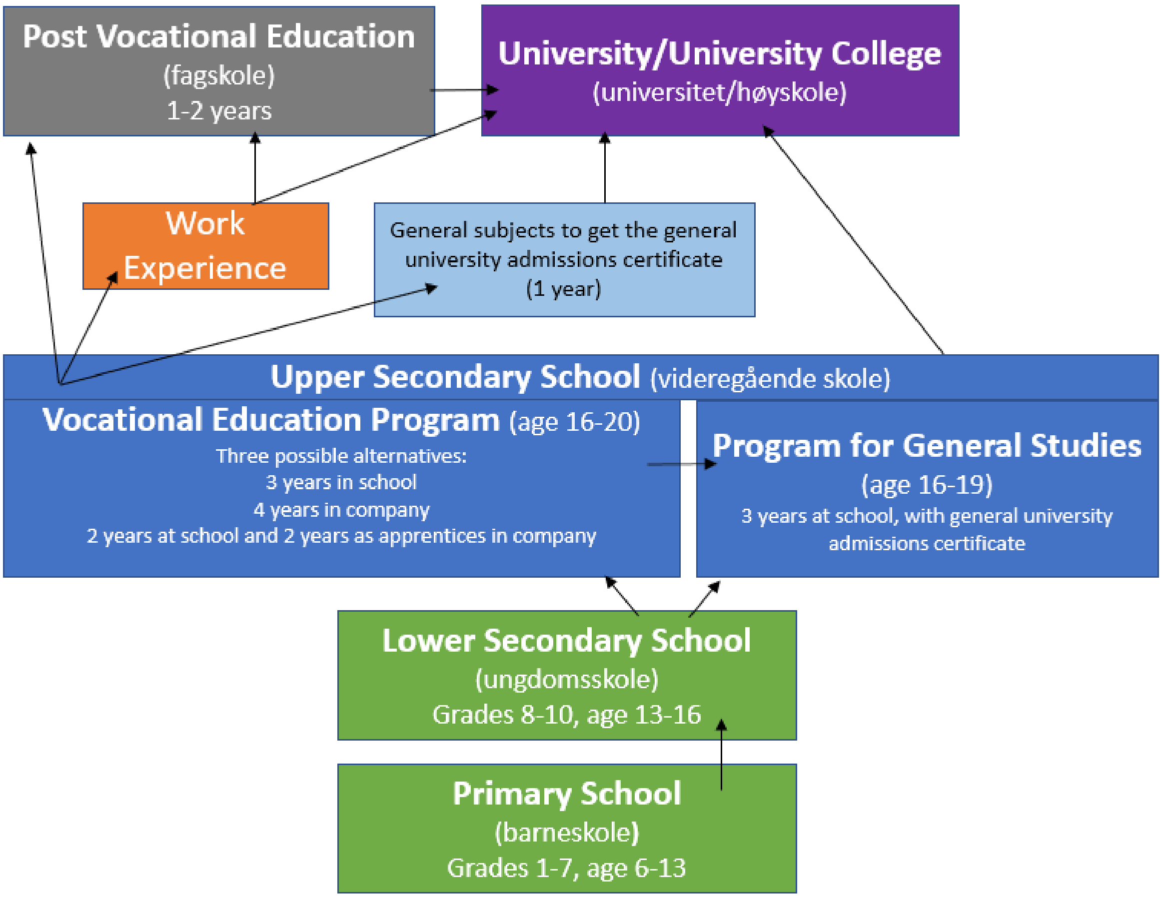

Figure 2.1: An overview of the Norwegian school system

The green boxes at the bottom of Figure 2.1 represent the compulsory years of school. Having completed the compulsory years, the students may choose to apply for upper secondary school, which is represented by the dark blue boxes. When applying for upper secondary school, the students can make a choice between general studies or vocational educational programs. By choosing general studies, the students will attain a general university admissions certificate. If they choose a vocational education, they will after two years of schooling and two years of apprenticeship achieve a trade certificate (they can also achieve this by four years in a company or three years in school). If these students change their minds and want to continue to higher education, they can take another year of general subjects to get a general university admission certificate (the light blue box in Figure 2.1). There is also a possibility to take further vocational education, either directly after upper secondary school or after some work experience. This is called post vocational education and is represented in the grey box in Figure 2.1.

During the two years of schooling in the vocational education and training, the students have both common core subjects and program subjects in addition to some hours set aside to project work. The distribution of these teaching hours per year, can be seen in Table 2.1. Mathematics is one of the common core subjects. During the first year of upper secondary school the students have general subjects and introductory courses to different crafts and trades within their program, before they choose specialization the second year.

Table 5.3.2: distribution of subjects in curricula at the various levels of upper secondary IVET. teaching hours per year

Subject	Upper secondary level 1	Upper secondary level 2	Apprenticeship
Common core subjects	336	252	according to collective agreements on working hours
Programme subjects	477	477	
In-depth study project	168	253	

Table 2.1: Distribution of subjects at the various levels of upper secondary vocational education and training (ReferNet Norway, 2010)

2.2 The Classes and Schools in this Project

Two of the teachers in this research project, Hanna and Sven, are working at an upper secondary vocational school, and their students are in their first year. That is the year they have mathematics as one of the compulsory common core subjects. The government has expressed that mathematics should be related to employment in these mathematics courses; however, there are challenges related to how to achieve this. The textbook has some vocationally related tasks, but it is limited. Also, the curriculum is general, and it is the same for all vocational programs, so the teachers would have to adapt and adjust the curriculum and make tasks which are relevant for the specific vocation their students are training for. However, most teachers who are teaching mathematics at vocational schools are mathematics educators and have limited knowledge about the vocation the students are being educated for. In addition, the mathematics teachers often teach many different vocational classes which can range from Building and Construction, Restaurant and Food Processing to Design and Craft, all within the same school year. This makes it challenging for the mathematics teachers to achieve enough knowledge about the students' vocational programs to make relevant vocational mathematical tasks. Another challenge is that the mathematics course is placed in the students' first year of vocational education, and most of the students do not have any previous knowledge of the vocation they are about to learn through schooling. Hence, it is difficult to create mathematics tasks relevant to a vocation the students do not know themselves yet and do not have many references to.

The other two teachers in the research project, Roger and Thomas, work in a different part of the vocational education sector. Roger teaches in the post vocational education, where the students have already achieved a trade certificate some years ago, but they are now back for further education. The classes Roger teaches in this research project take a preparatory mathematics course before they can start on an engineering degree at a university. Roger has therefore a group of students who are older than the typical upper secondary school student. Thomas is teaching the same course as Roger, but his group of students is a bit different since a trade certificate is not required to get admitted into the course. Most of his students are about twenty years old but did not take the mathematics and physics courses during secondary school needed for engineering programs at a university. The mathematics content in this preparatory course is not vocationally oriented towards engineering but is more a general course to give a mathematical foundation before university studies.

In addition to the differences between the schools as described above, there are some political principles which is a significant part of the context the teachers work in. I will therefore shortly describe some of the political ideals when it comes to education in Norway.

2.3 Comprehensive School and Political Ideals

Education for all and equality are important concepts in the Norwegian educational policy across political party lines, with a goal to reduce social inequality (Markussen, Frøseth, & Sandberg, 2011). The Norwegian Directorate for Education and Training has written this explanation for the term equity in education in Norway:

> Equity means to provide equal opportunities in education regardless of abilities and aptitudes, age, gender, skin colour, sexual orientation, social background, religious or ethnic background, place of residence, family education or family finances. Equity in Education must therefore be understood on the system level, using a national perspective based on overriding legislation, regulations and syllabuses, and on an individual level,

> adapting the education to individual abilities and aptitudes. To ensure Equity in Education for all, positive discrimination is required, not equal treatment. Equity in Education is a national goal and the overriding principle that applies to all areas of education (The Norwegian Directorate for Education and Training, 2008).

As a result of this focus on equality of opportunity in education, the Norwegian educational system has "the comprehensive school" as an important political ideal (Department of Education and Training, 2006-2007). There are very few special needs schools in Norway, and every student is entitled to be in a regular classroom. As a result, a typical class in Norway will include high and low achievers, children having different physical and psychological diagnoses, and special needs. The Education Act (1998) specifies: "Education shall be adapted to the abilities and aptitudes of the individual student, apprentice and training candidate" (§1-3). All students are entitled by law to experience education adapted according to their abilities, and this is the teacher's responsibility. The Norwegian Directorate for Education and Training has elaborated on what is meant by adapted education:

> - the school owner (the local or county authority), and the administration and staff at the educational institution must undertake to provide satisfactory and adequate teaching based on the individual's abilities and aptitudes. Adapted education involves choosing teaching material, methods and structures to ensure that each individual develops the basic skills and satisfies the competence objectives. This means that the teaching must be adapted on the individual and group levels. Adapted education does not mean that all teaching is individualized, but that all aspects of the learning environment take the variations among the students and apprentices into consideration (The Norwegian Directorate for Education and Training, 2008).

The political concept of the comprehensive school in Norway has influenced upper secondary education, and since 1994 every teenager has a statutory right of secondary schooling regardless of abilities and academic results. Consequently, almost every teenager now starts upper secondary school in Norway and many of the low achieving students apply for the vocational programs (Department of Education and Training, 2006-2007). The Education Act also applies to upper secondary school, and adapted teaching is a requirement for all the courses in the vocational programs. As a result, many teachers take it personally when their students fail subjects or quit secondary school. They have a goal to give adapted teaching to all their students, but the classes are diverse, and the pace of a course can be viewed quite different from student to student.

So, while Hanna and Sven work with a diverse group of students both when it comes to knowledge and motivation, Roger and Thomas are in a different position. Their classes are more homogeneous, and their students are motivated to pass the course so they can start their university studies. In addition, there are no requirements by law for adapted teaching in the classes of Roger and Thomas. Roger even expresses in our first conversation about tasks how it is a good thing if some of the students drop out in the beginning of the school year if this is because they are not motivated to work:

> My students are motivated for mathematics. This year, more students than what has been usual have dropped out of the course, but I'm happy with that. The students who thought this was going to be easy and were not interested in putting in the effort are gone, and now I'm left with a rather mature group. The students I have now work really hard (*first conversation about tasks, Roger*).

So, the four teachers in this research project teach in relatively different contexts. This diversity was not intentional in the research design, but a result of practical adjustments which is further explained in the methodology chapter. I will complete this section about the Norwegian educational system by referring to accountability and how teachers are accountable in the Norwegian school system.

2.4 Teacher Accountability

Norway is a country where accountability systems have never been approved for use in the education sector, even if there are some accountability devices in local quality-assurance systems (Christophersen, Elstad, & Turmo, 2010). Still, the Norwegian Prime Minister said in a speech in 2008 that "Teachers should have a clear responsibility for what students learn in school" (Christophersen et al., 2010, p. 2). It has not been explicitly stated clear how a Norwegian teacher can be made accountable for students' learning, and Christophersen et al. (2010) argue that this is not possible. The Prime Minister's speech might indicate a political shift when it comes to accountability in the future, but as of 2012/2013 when the data were collected, the focus in Norwegian schools was still on adapted teaching and not on the students' test results.

This is further reinforced by what the teachers in this research project focus upon in the conversations. None of the teachers express that they are worried about the exam, and Hanna hardly mentions the final exam at all. The person who mentions the exam most often is Roger, and then it concerns whether a task is relevant with respect to the exam or not. He might reject tasks based on them not being relevant enough for the exam. However, none of the teachers are talking about consequences of their students doing well or badly on the final exam, but they are often expressing worries about not all of the students understanding the mathematics. From this, I assume the teachers do not feel their work is solely being judged by exams or national tests, and that they might even feel more accountable towards their students than towards the government.

2.5 Summary

The Norwegian school system provides context for the teachers in this project, and this context varies. While Hanna and Sven teach heterogeneous mathematics classes for vocational students where the students differ in motivation and academic abilities, Roger and Thomas have more homogeneous mathematics classes with hard working students who aim at engineering studies at a university. While this group of students must succeed on their exam to get admitted into the University, Hanna's and Sven's students have various reasons to attend school, and some of them might just be there because they do not see other options. The comprehensive school as a political ideal lays the foundation for the teachers to focus on adapted teaching and how to motivate and include all students. It is the teacher's responsibility to teach according to the students' abilities. On the other hand, teachers are not measured on their students' exam results, and accountability systems have not been approved for use.

The Norwegian system provides an opportunity to study teachers coping with diverse mathematics classes and adapted teaching, without a sole focus on teaching for the exam. At the same time, the issues these teachers deal with are universal and thus relevant for the international community.

Having presented the context the teachers work in, I will in the next chapter present theoretical perspectives providing knowledge of what is already known through previous research, thus guiding the analysis of the data generated through the collaborations in this research project.

3 Theoretical Perspectives

I will in this chapter present relevant theoretical perspectives for investigating my two research questions, which are

1. What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom?
2. What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration?

Because ontological and epistemological assumptions will influence the collaboration with the teachers, I start this chapter by theoretically positioning myself and the research project in Section 3.1. This is followed by presenting theories on teachers in the classroom and their impact on learning in Section 3.2. The next section (3.3) presents research on 'good' teaching from the teachers' perspective. This gives me the opportunity to discuss and compare my findings with research that also focus on the teachers' perspective. In Section 3.4 I present theories on tasks, which gives me a theoretical frame to discuss the findings in research question 1. The last section is 3.5, which presents theories on teacher change and Sub-Section 3.5.2 presents the framework I use in the analysis.

3.1 Theoretical Positioning of this Research Project

This is a research project where the researcher has an important role in interpreting the teachers' requests for mathematical tasks, and design suggestions for tasks based on these requests. I will therefore present the underlying theoretical perspectives guiding this research project and the researcher, so the reader might herself determine the possible impact it might have on the results. The grand theoretical assumptions of the researcher and this research project builds upon a social-constructivist perspective. One of the leading contributors to constructivism is von Glasersfeld (1987, 1988), who is seen as representing radical constructivism. This theory is based on two tenets:

- Knowledge is not passively received but actively built up by the cognizing subject;
- The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality (von Glasersfeld, 1988, p. 162).

The first tenet rejects the notion of knowledge as being transferable and claims that knowledge is something each individual actively constructs. The second tenet displays an ontological view of knowledge as being subjective. It is the learner's interpretations of the world that is in focus, thus meaning that the learner has the principal role in the learning process.

This point of view has been criticized for "the cognizing subject emphasizes its individuality, its separateness, and its primarily cognitive representations of its experience" (Ernest, 2010, p. 41). In other words, the world exists only through personal interpretations, and how can we then explain interpersonal communication? This critique together with the entry of Vygotsky's work into the field in the mid-1980s, leads to what Lerman describes as "the social turn" in mathematics education (Lerman, 2000). There are some questions that are difficult to answer within a radical constructivist view, such as: "Why do school mathematics and the curricula repeatedly fail minorities and first peoples in numerous parts of the world?" (Sriraman & English, 2010, p. 26).

Constructivism viewed through its interaction with the theories of Vygotsky, leads to the notion of social constructivism. Bauersfeld (1995) is one of the main contributors to this direction. He claims that "the lonesome child does not develop" (Bauersfeld, 1995, p. 138). In this perspective, there is a shift of focus from the sole individual to the classroom as a cultural and a social environment that is important for learning. It is still based on the epistemological view that it is the individual that interprets and constructs knowledge based on experiences in the social context, so this theory of learning might be viewed as a combination of radical constructivism and a socio-cultural perspective (Goodchild, 2001).

3.2 The Teacher in the Classroom

A perspective of wanting more research on teachers and how they might influence students has been a national focus in Norway as well as an international focus. The Norwegian Ministry of Education announced in 2007 that they wanted research on the connection between tangible teacher competencies and students' learning, and this resulted in a systematic review report published in 2008 (Nordenbo, Larsen, Tiftikci, Wendt, & Østergaard, 2008). The Norwegian Government recognized how research studies point to the teacher being the single most influential factor on students' learning and wanted this connection to be further explored.

The report by Nordenbo et al. (2008) states that it is not possible to do a systematic synthesizing in the form of meta-analysis because there has not been conducted randomized, controlled experiments on the topic for the last ten years, so they have instead used a procedure which is described as narrative synthesis in systematic research reviews. One of the purposes of the report was stated as: "Which dimensions of the pedagogical staff's competencies in kindergarten and school can, through effect studies, be detected to contribute to the learning of children and young people?" (My translation, Nordenbo et al., 2008, p. 18) However, the authors of the report realized quite early that a purpose expressed this way, builds upon a theory of how teachers' competencies influence students' learning which can be illustrated as Figure 3.1 below:

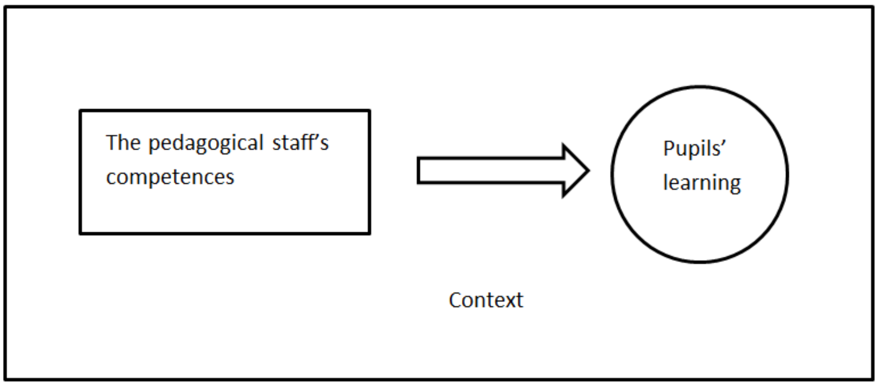

Figure 3.1: How teachers' competencies influence students' learning (Nordenbo et al., 2008, p. 46).

According to Nordenbo et al. (2008), the illustration in Figure 3.1 models how teachers' competencies influence students' learning in a contextual setting. However, even if this illustration takes context into account when examining the influence teachers' competencies have on students' learning, it does not capture the complexity of the pedagogical reality in a typical school class, which is something the authors of the report acknowledged rather early. They point to how all individuals in a class, both teachers and students, influence each other and how a classroom is built upon a complex system of social interactions and relationships, which the model above omits. Based on this, Nordenbo et al. (2008) chose to use a more complex model which they found in the work of Muijs and Reynolds (2002), when reviewing the research on teachers' influence on students learning. This model distinguishes between a teacher's personality, teacher beliefs, teacher behaviors, teacher subject and student achievement. In addition, the arrows go both back and forth, indicating a mutual influence and not just an effect in one direction (see Figure 3.2):

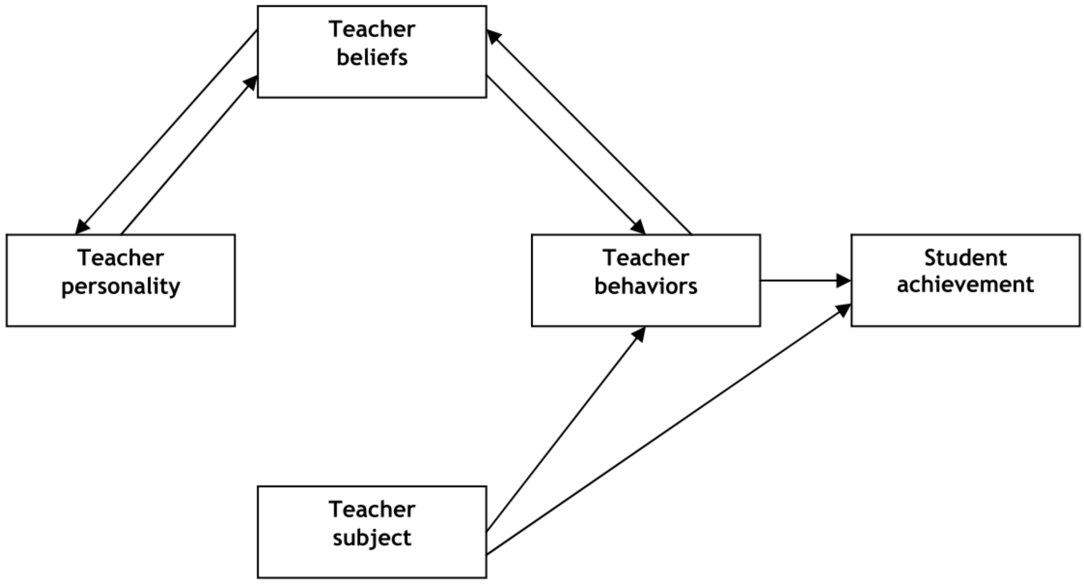

Figure 3.2: Complex model of how teachers' influence students' learning (Nordenbo et al., 2008, p. 47).

The model in Figure 3.2 can be viewed as a more detailed representation of the previous model presented by Nordenbo et al. (2008), however this model is still not taking into account social interactions and relationships in the classroom. Nevertheless, the model above is trying to capture how a teacher's competencies are complex, which might again influence student achievement.

Several researchers have addressed the complexity of teachers' competencies and how this might affect students' learning. Building on the work of Shulman (1987), who described how teachers need specific types of knowledge when teaching, Ball, Thames, and Phelps (2008) elaborated on what these types of teacher knowledge are when it comes to mathematics teaching. Although, the work of Ball et al. (2008) has been widely cited, Ball is now advocating a perspective where we need to go beyond examining teachers' knowledge and beliefs and instead look at what is actually going on in classrooms. During her plenary speech at the 13th International Congress on Mathematical Education in Hamburg 2016, her focus was that we now need to move on and more closely examine the work teachers do in the classroom and try to capture the complexity of this (Ball, 2017). The past focus on teachers' competencies and knowledge can impact what is going on in the classrooms; however, there is more to it.

The range of various learning theories indicates that we cannot agree upon how learning happens and exactly what it is. Giving solid evidence of learning is challenging and giving solid evidence of how a certain way of teaching will lead to learning is even more challenging. So, the question is to what degree a teacher can be held accountable for the students' learning or lack of learning. This is an issue which Christophersen et al. (2010) investigated. The aim of their study is expressed this way:

> The principal purpose of this study is to estimate the effect of the quality of the teaching (as assessed by the students) on the learning outcomes of the students (measured by their grades in science after term 1 in the first year of high school). We also estimate the effect of student motivation, engagement, and self-discipline on learning outcomes as well as their possible interrelationships with teacher quality (Christophersen et al., 2010, p. 414).

Christophersen et al. (2010) define quality teaching as the teacher's ability to enable the student to perform better than the student would have done without the teacher's influence. The main finding of this study is that the direct influence of teachers on the learning outcomes of 16-year-olds is limited (Christophersen et al., 2010). They make a point that given the age of the student, their knowledge is the product of 10,5 years of cumulative science teaching, so the high school teacher in their research has only contributed to roughly 5 % of this. However, this raises the question of how accountable a teacher can be hold for their students' learning, which is one of the points made by Christophersen et al. in their article. Despite of their results of no significant correlation between teaching and learning, they find that quality teaching can have substantial impact on student motivation, engagement, and self-discipline:

> If the statistical associations between adolescent students' perceptions of their classroom engagement, quality of teaching and responses to their teacher, and their own achievements in science represent causal relationships, our main finding is that holding the high school science teacher directly accountable for student learning outcomes is highly problematic. Still, the teacher's influence on student motivation, engagement, and self-discipline is substantial after only

teaching the students for half a school year. Once again, these aspects influence student learning outcomes to a large extent, and carry a rather optimistic message: the teacher can also influence the students' engagement in science significantly in a relatively short period of time (Christophersen et al., 2010, p. 422).

Even if this research is situated in the teaching of science, the results might also be applicable to the mathematics classroom since the focus is on how to estimate the effect of quality teaching on students learning. However, Christophersen et al. (2010) make a point that there are grounds to hypothesize that the more logical-sequential the structure of the subject is, the greater the influence of quality teaching might be. The reason they give for this, is that knowledge might be easier for the students to achieve on their own in subjects like history, while the students are more dependent on a teacher to explain difficult tasks and methods in mathematics. Having said all of this, it is also important to notice that the authors of this research article express that "their hypothesis is causal in nature and the results should be interpreted very cautiously. The cross-sectional nature of the data collection precluded causal analysis and inferences" (Christophersen et al., 2010, p. 423). So, even if they have not managed to prove a causal link between quality teaching and students' learning, it is not given that it does not exist.

All teachers in this research project work with students who have more than ten years of schooling in mathematics behind them. It is therefore not fair to hold them accountable for the total level of mathematical understanding their students have. However, it is important to notice the impact they can have on motivating their students, which can be an important factor in vocational secondary schools which struggle with high drop-out numbers and only 62 % completing within five years (Statistics Norway, 2019).

3.3 'Good' Teaching from the Teachers' Perspective

Just as the aim of this research project is to better understand the teachers' perspective on mathematical tasks, Brown and McIntyre (1993) aimed at understanding what good teaching is from the teachers' perspective. Brown and McIntyre (1993) point to research which suggests that what teachers do depends a lot on their thinking, which makes access to teachers' thinking an important aspect for researchers. However, this can be a challenging task. Ernest (1989) argues how a teacher's beliefs are not necessarily developed into fully articulated philosophies. In addition, Berliner (1988) has researched the differences between novice and expert teachers, and he found that expert teachers tend to focus on atypical situations. The normal is something they take for granted, so the teachers are more likely to report their thinking on atypical situations. These are issues which are important to consider when trying to capture and report the teachers' perspectives. As researchers, we are interested in not only atypical situations, but also the everyday thoughts and decisions teachers make. This means we need to pay attention not only to what teachers say, but also to what they are not talking about. While the experienced teachers might be more inclined to talk about special situations, what they are not focusing on in the collaboration, can give information on what they view as normal and taken for granted.

Brown and McIntyre (1993) argue how research which is grounded in the teachers' practice is needed:

> …, since it seems impossible to have direct access to teachers' thinking while teaching, it is important that theoretical accounts of teachers' classroom thinking should be grounded in teachers' own ways of making sense of the particular things they do and achieve in their teaching (Brown & McIntyre, 1993, p. 12).

This focus on the teachers' perspective is the starting point for Brown and McIntyre's research. Their aim was to identify the characteristics of the professional craft knowledge teachers use. Because of this focus on the teachers, Brown and McIntyre had to disregard theoretical models of what 'good teaching' is, because this would be a sort of judgment by researchers and not necessarily the same perspective as the teachers. Likewise, they would not look at the teaching as a sort of process-product research, where the product of the teaching is used as a measure of the quality of the teaching. Exam results are often used this way. All in all, it was the teachers' choices and valuing of the teaching which Brown and McIntyre were seeking (Brown & McIntyre, 1993). However, this meant they had to think differently than many other research projects when finding the expert teachers they wanted to work with. This issue was resolved by asking the students to identify teachers, who were in their opinion, conducting good teaching and to explain what it was in their teaching the students considered as good. This way, students' motivation and other aspects than just the exam results were considered when selecting the teachers.

Brown and McIntyre were not interested in the exceptional cases of teaching but wanted a deeper understanding of the expert teachers' day-to-day issues and what made these teachers good at handling such issues. Therefore, they used several criteria for analyzing and constructing their framework to avoid being misled by the out of routine events. These criteria were: 1. Any generalization must be directly supported by evidence. 2. Any generalization must relate to what is normal practice. 3.Where generalization goes beyond one teacher and one occasion, this must be supported by evidence from each teacher and each occasion. 4. The relationship between generalizable isolated elements must be supported by evidence. 5. The framework should not discount any part of the teachers' accounts. 6. Any theoretical account they provide of how teachers think, has to be recognized and accepted as a balanced account by the teachers themselves (Brown & McIntyre, 1993). The focus on the teachers' perspective is evident in all of Brown and McIntyre's criteria when analyzing and building their framework. On every level of the framework, the results must be empirically supported and relate to the normal practice of the teacher. In addition, their theoretical framework must in the end both be recognizable and accepted by the teachers and not just by the research community.

These are thoughts and criteria which are important in my research project as well, except for criterion number 2, because my aim is not to describe normal practice. The focus of my research is to determine what the teachers want to do differently and is thus deviating from their normal practice. This difference underpins how my research is complementary to Brown and McIntyre's research. Both research projects focus on the teachers' perspective, but they aim at identifying and describing normal, good teaching, while my aim is to identify and describe what the teachers are not content with and want to change.
Brown and McIntyre identified what they named Normal Desirable State of Student Activity (NDS) as the most obvious common feature of the different teachers' accounts. When the teachers were asked about their teaching, their response were almost always about what their students were doing. The teachers would evaluate the lessons with respect to whether the students were acting in the ways the teacher would see as routinely desirable. These NDSs would vary from teacher to teacher and could also change quite markedly from one stage of the lesson to another as the lesson progressed. However, NDS is the dominant

generalizable concept used by teachers in evaluating their own teaching (Brown & McIntyre, 1993).

Brown and McIntyre noticed through their research, that the teachers were describing some sort of outcomes instead of characteristics of what they were being asked about, which was good teaching. Brown and McIntyre described in detail numerous examples of what teachers saw as normal and desirable patterns of classroom activity, because they recognized how the maintenance of NDS was important to the teachers. This raised at the same time the question whether the teachers were concerned about the students' learning, since this was not expressed through NDS. Brown and McIntyre partly argued how this is due to the type of data they were collecting. The focus of their research was on how the teachers construed their classroom teaching, and thus the focus was on the activity. Brown and McIntyre assumed that in the teachers' prior planning, they would have focused more on what they wanted their students to learn. This is where my research project provides additional data to the work of Brown and McIntyre. Where Brown and McIntyre focused on the act of teaching, this research project focuses on mathematical task characteristics and the data material is collected from the design process, implementation, and evaluation. So, the data in this research project includes the phases of prior planning, where Brown and McIntyre assumed more focus from the teachers on what students should learn.

3.4 Tasks

Tasks are an intertwined part of teaching and the mathematics classroom. I am in the following relating tasks to the broad definition which can be found in Oxford dictionary: "A piece of work to be done or undertaken" ("Task", 1998). Hence a mathematical task will be a piece of work to be done that is related to mathematics. Even though one could assume many years of research in mathematics education already had given us the most ideal mathematics tasks, there has instead been a growth of research activity and publications on the topic. According to Watson and Ohtani (2015b), who are editors of the book Task Design in Mathematics Education an ICMI study 22, this includes the work of task designers, tasks and task adaptation in the classroom and comparisons of textbooks. Also, they point out how task design is a core issue in research about learning, and tasks have a major influence on assumed findings about student capability (Watson & Ohtani, 2015b).

In the Norwegian mathematics classroom, students' individual work on tasks has a predominant role, and the students spend a lot of time solving textbook tasks (Bergem, Kaarstein, & Nilsen, 2016; Nordahl, 2012). However, there is relatively little emphasis on introductions and summaries, and not much time is spent on cognitively challenging tasks and problem solving (Bergem et al., 2016). With more than 60 % of the Norwegian students' time spent individually working on tasks (Bergem et al., 2016), tasks are unquestionably an important aspect of mathematics teaching to investigate. This is further emphasized by how the four teachers in this research project responded when they were asked how they think students learn. All of them responded that the students only learn when they solve tasks, and that they need to have hands-on experience.

The study of mathematical tasks is a comprehensive area of research. I have chosen to organize this section using the focus areas presented in the book Task Design in Mathematics Education and ICMI study 22 (Watson & Ohtani, 2015a). This book is the outcome of a process aiming to produce an up-to-date summary of relevant research about task design (Watson & Ohtani, 2015b). From reviewing research, the international program committee, identified five themes and called for papers for the conference on Task Design in Mathematics Education which was held in Oxford in 2013. In the aftermath of the conference, these five themes were altered to more closely represent the scholarly work undertaken at the conference and subsequently. The five themes are: Frameworks and principles for task design, The relationship between task design, anticipated pedagogies, and student learning, Accounting for student perspectives in task design, Design issues related to text-based tasks, and Designing mathematics tasks: The role of tools. I have chosen to focus on the first two themes in this thesis. Although the last three themes are highly relevant for task design in general, they are investigating detailed aspects which are not relevant for my research.

3.4.1 Frameworks and Principles for Task Design

Design of mathematical tasks is a priority for many researchers, and there are several research designs with task design in focus, among others design-based research, developmental research, and didactical engineering. However, there are differences in the role theory has in various types of research. Kieran, Doorman, and Ohtani (2015) articulate this distinction as design as intention and design as implementation. Concerning design as intention, theory and design principles play an important role in designing tasks that are tested and further developed through implementation. The role of theoretical tools is thus important in the initial design and is emphasized. On the other hand, there are research projects where the design and implementation of tasks are used to further develop local instruction theories, thereby being referred to as design as implementation. While these research projects might have a theoretical starting point, which could qualify them as design as intention, the main aim is to develop theory through implementation. This distinction between types of research on mathematical tasks, resonates with how Prediger, Gravemeijer, and Confrey (2015) claim there are two arch types of design research: "one that primarily aims at direct practical use, and one that primarily aims at generating theory on teaching learning processes" (p. 880). The same authors express how theory in design research is used both prospectively and reflectively, that is to inform the design, but is also further developed in retrospective reflections (Prediger et al., 2015).

Since theory can have various purposes when used in research on task design, one helpful way of presenting theoretical perspectives is by the level of the theories used, more specifically distinguishing between grand theories, intermediate theories and domain specific/local instruction theories. Behind all mathematical task designs lies a theory about how students learn mathematics, whether it is explicitly or implicitly expressed. This could be cognitive theories, social-constructivism, socio-cultural learning theories and so on. All of them provide an overarching frame for how students learn. However, the challenge is that a theory about learning does not automatically transfer into a theory of instruction, or how to design a task. It is therefore often necessary to use intermediate theories which have a more specific focus, when designing mathematical tasks. Some examples of intermediate theories which are used for task design are Realistic Mathematics Education (Treffers, 1987), the Theory of Didactical Situations (Brousseau, 1997), The Anthropological Theory of Didactics (Chevallard, 1992), Cultural-Semiotics Theory (Radford, 2003), Commognitive Theory (Sfard, 2008) and many more. While all intermediate theories are developed within the tenets of a grand theory, the intermediate theories include explicit heuristics and design principles. Consequently, design-based research using a framework from an intermediate theory, is often categorized as design as intention.

The use of theories and frameworks in the research project presented herein, differs from most research where task design is an important element of the research design. The focus is neither on design as intention nor design as implementation; the design of tasks is merely used as a tool to gain access to the teachers' descriptions of tasks they want. While I, as a researcher design tasks for the teachers, I try to understand and interpret their wishes and withhold my own preferences. Still, the resulting tasks are likely to be designed in a mix of my own social-constructivist perspective and my interpretations of the teachers' wishes, combined with my knowledge of intermediate theories and their design principles.

3.4.2 The Relationship Between Task Design, Anticipated Pedagogies, and Student Learning

Teachers make task design choices based on their mathematical and pedagogical knowledge, but also in anticipation of how students will respond to tasks. Sullivan, Knott, and Yang (2015) point to researchers who have developed frameworks to investigate and understand these processes. Hill, Ball, and Schilling (2008) describe two types of knowledge relevant to converting tasks to use in the classroom: subject matter knowledge and pedagogical content knowledge. Subject matter knowledge includes common content knowledge, specialized content knowledge, and knowledge of the mathematical horizon. On the other hand, pedagogical content knowledge includes knowledge of content and teaching, knowledge of content and students, and knowledge of curriculum. The sum of these types of knowledge informs teachers' decisions in the classroom. A teacher might have strong knowledge of the mathematical content itself but have weak knowledge of how students learn the content, or vice versa (Hill et al., 2008). Gueudet, Pepin, and Trouche (2012) consider the complexity of implementing tasks and describe documentational genesis as the two-way process of which tasks are not only interpreted by the teacher, but also influence the decisions teachers make.

When designing tasks there are several pedagogical dilemmas which need to be considered. Barbosa and de Oliveira (2013) presented five dilemmas associated with mathematical task design in a group of researchers and teachers. These dilemmas are not only design considerations, but can be used as ways of evaluating adequacy of tasks (Barbosa & de Oliveira, 2013).

1. Context as a Dilemma

A task can on one hand be purely mathematical, and on the other hand be set in a realistic context. While a realistic context might foster student engagement, it can also detract from the potential of the task to achieve the intended learning (Sullivan et al., 2015). In the Norwegian curriculum for mathematics in the vocational education program at secondary school, there is an explicit emphasis on relating the mathematics to real life. One of the headings in the curriculum are: “Numbers and algebra in practice”, which among others include the competence goal: “interpret and use formulas that apply to day-to-day life and working life” (The Norwegian Directorate for Education and Training, 2006). However, it is relevant to note that the use of context may cause challenges as well. Some studies have shown that not all students perform better with contextualized mathematical problems, and this might be related to the socioeconomical status of the student (Sullivan et al., 2015). It is therefore not given that designing a context-specific mathematical task, will ensure its accessibility for all students. There is also a potential for the context to limit the potential for students to generalize solutions (Sullivan et al., 2015).

School mathematics has three standard aims, according to Ernest (2015): ‘functional numeracy’, ‘practical, work-related knowledge and skills’, and ‘advanced specialized knowledge’. Since the research presented in this report is conducted in vocational classes, I would expect the teachers to have more of a practical perspective on the nature of mathematics in the tasks they ask for. However, there is not always a clear line between mathematics addressing a practical perspective and specialized mathematical goals. That is, being good at problem solving, may be viewed as an important skill from both perspectives.

2. Language as a Dilemma

The language in a task serves at least two purposes. On one hand, mathematical precision is desirable; on the other hand, students need clarity to support learning (Sullivan et al., 2015). These are not necessarily contradictory, but both need to be considered when designing tasks. This is especially relevant when considering student groups including students who do not have Norwegian as their first language.

3. Structure as a Dilemma

Structure as dilemma refers to the degree of openness in tasks. This might refer to openness in various ways. It could be the task formulation which is open (described as open-start), it could be openness with respect to a variety of approaches (described as open-middle), and those that have a range of solutions (open-ended). While Barbosa and de Oliveira (2013) describe the dilemma as structure, Sullivan et al. (2015) argue that the dilemma can be considered as much a function of the task outcome as it is the structure.

> In this dilemma, the consideration is that specific questions can be posed which, on one hand, scaffold student engagement with a task in a more prescribed way and, on the other hand, allow students greater opportunity to make strategic decisions on pathways and destinations for themselves. (Sullivan et al., 2015, p. 93).

Tasks with a high degree of openness are often referred to as rich tasks, among other characteristics (Foster & Inglis, 2017). The Norwegian Directorate for Education and Training (2015) describe rich tasks as problem solving tasks offering opportunities to discuss solution strategies and mathematical concepts with peers. They have listed seven bullet points and claim that a rich task should:

- introduce important ideas or solution strategies
- be easy to understand and everyone should be able to get started and have possibilities to work with it (low threshold).
- be perceived as a challenge, require effort, and be allowed to take time to solve.
- be solved in several different ways, with different strategies and representations.
- be able to initiate an academic discussion that demonstrates different strategies, representations, and ideas.
- be able to function as a bridge builder between different academic areas.
- be able to lead students and teachers to formulate interesting new problems (What if…? Why is it so that…?)

(The Norwegian Directorate for Education and Training, 2015, p. 2. Translated by me)

They do not give any examples of rich tasks but claim that rich tasks are self-differentiating because of the low threshold and possibilities to expand the task (The Norwegian Directorate for Education and Training, 2015).

4. Distribution as a Dilemma

Distribution as a dilemma refers to what is expected to be taught in a task; what content should be selected and focused on (Barbosa & de Oliveira, 2013). This distribution is according to Barbosa and de Oliveira (2013) a function of the cognitive demand of tasks and can be related to the Mathematical Task Framework developed by Stein, Smith, Henningsen, and Silver (2000). The framework can be used to analyze the cognitive demand of mathematical tasks and whether the cognitive demand is being maintained through implementation in the classroom. The framework can be helpful to distinguish between tasks of lower cognitive demand and tasks that are more cognitively challenging. The former are tasks that require the student to recall what has been memorized, or to perform a procedure without any connections, while the latter has no clear solution path obvious for the student. Tasks characterized as problem solving in the literature are of high cognitive demand. They require the student to be creative in figuring out a way to solve the task. However, it is worth noting that a task cannot be analyzed with respect to cognitive demand without context. While multiplying 12 and 5 should be a routine task of low cognitive demand for a 12-year-old, the same task could be of high cognitive demand for a 6-year-old. Analyzing mathematics tasks in textbooks and the tasks students are working on in the classroom, reveals that many of the tasks are of lower cognitive demand (Brändström, 2005; Hiebert & Stigler, 2000; D. L. Jones & Tarr, 2007). An analysis of mathematics textbooks from lower secondary school in Norway, concludes that between 83 % and 94 % of the tasks are of lower cognitive demand (Johnsen & Storaas, 2015). So, if the teachers mostly rely on tasks from the textbooks, it seems that the students will mostly work on tasks of lower cognitive demand.

5. Levels of Interactions as a Dilemma

By levels of interactions, Barbosa and de Oliveira (2013) refer to interactions between teacher and students. They argue that while a closed task is often viewed as something students should solve on their own, more open tasks require more involvement from the teacher, due to less scaffolding in the task itself. Sullivan et al. (2015) make an additional claim that “this can be interpreted to mean that the task does not exist by itself, but its implementation is influenced by the nature of the intended or anticipated interactions between the teacher and students when they are engaged with the task” (Sullivan et al., 2015, pp. 93-94).

In addition to the five design dilemmas presented, the teacher’s role in adapting a task developed by others or by taking part in the design process, will influence the design and implementation of tasks in the classrooms (Sullivan et al., 2015). There is substantial evidence that when teachers implement tasks, they might subvert the aims of the task’s designer, such as lowering the cognitive demand of the task, as before mentioned. Henningsen and Stein (1997) have shown how tasks categorized as high cognitive demand, can be reduced to routine tasks through implementation. For example when students get frustrated over challenging tasks, ask the teacher for help, and in the process of helping the students, the teacher reduces the cognitive demand of the task. Likewise, Franke et al. (2009) point out that teachers have difficulties following up on student ideas. Even when teachers are positive and want to ask students about their mathematical thinking and to understand their perspective, they struggle with the follow up.

However, it also seems that involving the teachers in considerations of design issues, can affect the potential of the task. Therefore, Sullivan et al. (2015) argue that “rather than fearing that teacher adaptations may limit the potential of the task, as is assumed by some designers, involving teachers as far as possible in the intentions of the designer can enhance the implementation of the task” (Sullivan et al., 2015, p. 103). Teachers will also take classroom culture into consideration when designing and implementing tasks in their classrooms. The prevailing classroom culture can have a significant impact on implementation of tasks, since student practices and expectations in the classroom depend on the establishment of social and sociomathematical norms (Sullivan et al., 2015). These norms take time both to develop and to change. In the research project reported in this thesis, the mathematics classes are newly put together and most of the students do not know each other or the teacher from before. Nevertheless, all students bring with them at least ten years of experience of being a student in a mathematics classroom and enter the new class with corresponding expectations.

Therefore, it is not sufficient to design ‘good’ tasks, we need to learn more about how we can help teachers implement tasks as intended. While tasks of low cognitive demand are easy to implement, the challenges of implementing rich tasks and tasks with high cognitive demand are much greater. Through the research project reported here, we gain insight into what teachers might look for in tasks they want to use. This knowledge will in turn provide understanding of how to implement more tasks of higher cognitive demand in the classroom. I assumed the teachers would ask for something different than just tasks of low cognitive demand. The textbooks already provide those type of tasks, thus there is no need for me to design more tasks of the same type.

3.5 Teachers' Change

In this research project, the aim was not to change the teachers, but to provide help designing tasks they were missing and wanting to use in their mathematics classroom, thereby learning more about the teachers' perspective. Now, even if these were the aims of the research, it would be naive to assume that a close collaboration over a school year would not entail some type of changes. Because I do not aim to change the teachers, the wanted changes they express implies a genuine wish for change.

In this section I will discuss various perspectives on teacher change and present some of the challenges and possibilities we know of. I have chosen to use the word 'change', meaning to become different. With respect to teachers' change, I would distinguish between a teacher changing and a teacher who is changing their practice. It is in my opinion not given that a teacher changing practice results in the teacher changing herself, or vice versa. For instance, I am not assuming that a teacher who starts using the software GeoGebra in her classroom simultaneously changes her belief about technology as a tool to learn mathematics. Change in practice might lead to a change in the teacher's beliefs, but not necessarily. Teacher change as a term is being used interchangeably about both teacher change and teacher changing their practice. However, the distinction between the two is important in this research project. The aim was not to change the teachers, but to help them design preferable mathematical tasks they would want to use in their classrooms. So, the assumption of my research design is that the teachers' practice is not completely aligned with their knowledge and beliefs about how they would want to teach mathematics. This is not to say that the teachers did not change at all throughout our collaboration, it was just not the initial aim of the research design. Therefore, I want to be explicit about the nuance between changing a teacher and changing the teacher's practice.

I have chosen to use the word ‘change’ as opposed to some scholars who prefer the terms teacher learning or professional growth (Clarke & Hollingsworth, 2002; Goldsmith, Doerr, & Lewis, 2014). I use the word change to be able to discuss observable or stated transformations in the teachers’ practice and/or knowledge, without interpreting beyond what is initially observable. In a review article on teacher change, Goldsmith et al. (2014) prefer to use the word teacher learning which is to be broadly considered as including “…changes in knowledge, beliefs, and/or practice (including both practice within the classroom and in related settings, such as planning or reflecting on practice outside of the classroom)” (Goldsmith et al., 2014, p. 6). Even though I agree with the importance of examining change in a broad aspect of a teacher’s every day, I want to avoid using the word learning. This is to avoid misunderstandings with respect to learning theories concerning learning as a term. I want to be able to describe change as it is expressed or acted out, before going into epistemological and ontological discussions beyond the change. I view change as a more neutral word, and this is therefore my choice of wording.

To summarize, in this research project I am investigating change as to become different, but without making claims about how long lasting these changes are. I am distinguishing between teachers’ changing practice and teachers changing themselves.

3.5.1 Teacher Change – What We Know

Understanding teacher change has been an aim of researchers in mathematics education for many years. There is a perceived research-practice gap (Silver & Lunsford, 2017) and efforts are made to understand why teachers are not making more research-based changes. Although I prefer to use the words teacher change, I need to relate my research to relevant literature, and then teacher learning is often used as a concept. In the review article on mathematics teachers’ change by Goldsmith et al. (2014), the authors reviewed 106 articles which were written between 1985 and 2008. Based on this, they suggest three main points which capture what we know about mathematics teachers’ learning (Goldsmith et al., 2014). The first of these points is that learning tends to occur incrementally and iteratively. Research supports that teacher learning is a complex process, and that “changes in teachers’ mathematical knowledge, beliefs, dispositions, and opportunities to learn from colleagues often occur in sequential increments, with small advances in any of them depending on advances in the others” (Goldsmith et al., 2014, p. 20). While this conclusion is not new, Goldsmith et al. (2014) want to emphasize this aspect since none of the intervention studies they reviewed prospectively laid out an iterative, multidomain theory of action for the intervention.

The second main point according to Goldsmith et al. (2014) of what we know about mathematics teachers' learning, is that "intervention impact varies across individuals and contexts" (Goldsmith et al., 2014, p. 20). While most approaches to professional learning are effective in some circumstances, they are also ineffective in others. Teachers respond in diverse ways to professional learning opportunities. However, several researchers have identified characteristics of professional learning which are associated with teachers' reports of learning, and how systemic factors might impact this (Goldsmith et al., 2014).

The third main point of what we know about mathematics teachers' learning, is that "existing research tends to focus on program effectiveness rather than on teachers' learning" (Goldsmith et al., 2014, p. 21). So, most intervention studies treat teachers' learning as an indicator of whether a professional development program has been effective, and not as the primary object of inquiry. Without a focus on the mechanisms and processes of teachers' learning, there is little knowledge on how teachers develop knowledge, beliefs, or instructional practices (Goldsmith et al., 2014).

Based on these results from the review on teachers' learning, Goldsmith et al. (2014) suggest the following implications for future research: "Develop standards for descriptions of professional development programs, develop shared conceptual frameworks, constructs and measures, and support varied types of studies" (Goldsmith et al., 2014, pp. 23-24). The last point about supporting varied types of studies, is a recognition of the complexity of the field and the need for a deeper understanding. For instance, they point out how a challenge of large-scale studies which mostly rely on self-report data of teachers making changes, is that the teachers' perceptions might not align with those of mathematics education researchers (Goldsmith et al., 2014). Building knowledge about teachers' learning thus requires a varied set of research approaches, and the research reported in this thesis contributes with a methodology giving insights into the teachers' self-initiated changes to mathematical tasks and classroom practice.

3.5.2 A Framework for Teacher Change

After examining many years of research on teacher change, and based on empirical data from three large professional development studies, Clarke and Hollingsworth (2002) have developed a model to understand teachers' change. Having noted the clear ineffectiveness of teacher change when viewed as something being done to teachers where the teacher's role is passive, they have shifted focus to change as a complex process which involves learning and they aim to model this in a useful and fruitful way (Clarke & Hollingsworth, 2002). According to Fullan (2001), many professional development programs have focused on changing teachers' beliefs and attitudes, with an expectation that this would lead to a change in classroom practice. On the other hand, some researchers such as Guskey (2002), argue that significant changes in teachers' beliefs and attitudes are only likely to take place after experiencing improved student learning outcomes. While these perspectives differ in the sequencing of change, the perspectives are both linear in form which means one change leads to another change in a specific order. Guskey's (2002) model of teacher change is represented in Figure 3.3, where he presents the process of change starting with some kind of professional development, leading to a change in teachers' classroom practices, making the teacher experience firsthand a change in student learning outcomes. This may in turn lead to a change in teachers' beliefs and attitudes.

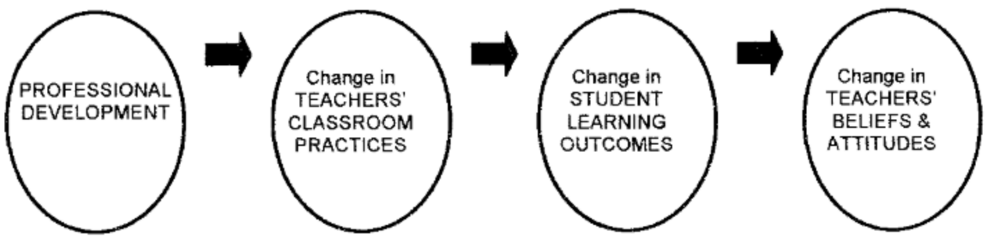

Figure 3.3: Guskey's model of teacher change (Guskey, 2002)

Clarke and Hollingsworth (2002) argue that a linear model of teacher change does not capture the complexity of these processes, and they have developed an alternative model named the Interconnected Model of Professional Growth (IMPG). This model (Figure 3.4) includes four distinct domains which encompasses the teacher's world, and Clarke and Hollingsworth argue how change in one domain might lead to change in any of the other domains (Clarke

& Hollingsworth, 2002). The four major domains are the Personal Domain (teacher knowledge, beliefs, and attitudes), the External Domain (sources of information, stimulus, or support), the Domain of Practice (professional experimentation) and the Domain of Consequence (salient outcomes).

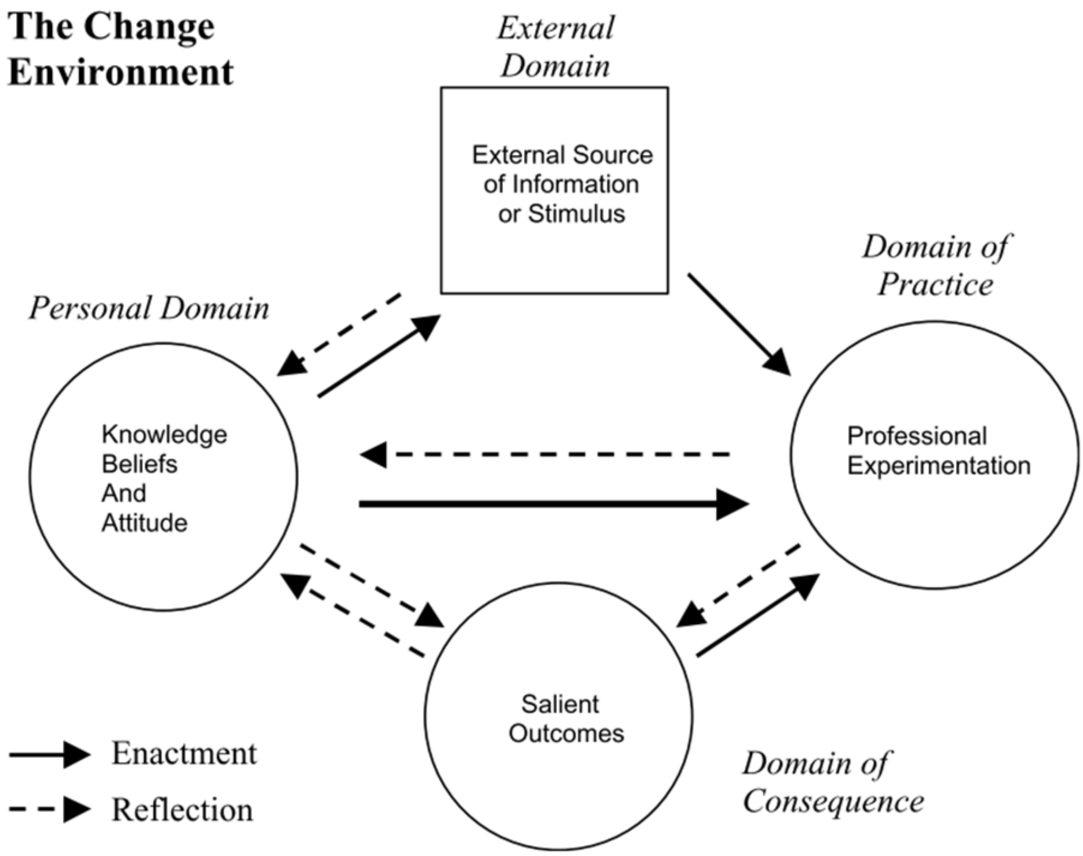

Figure 3.4: The change environment (Clarke & Hollingsworth, 2002)

There are two types of domains represented in the model. While three of the domains are part of the teacher's personal world of practice, the external domain is not included in the personal world and is therefore illustrated with a square in the model. The external domain represents all kind of external sources of information or stimulus, such as exams, curriculums, professional development programs, textbooks, and school culture. In this research project, the researcher and everything presented by her, is part of the external domain for the teachers when collaborating. However, the researcher is not the only external input. The teachers also relate to the mathematics textbook, they need to take the curriculum and the exam into consideration and many other aspects. Since the external domain can include various sorts of stimulus, it is not given that one is able to detect all of them and how they influence the teachers' world.

When it comes to the three domains which are part of the teacher's personal world of practice, one of them is named the personal domain. This domain includes various aspects of personal attributes of the teacher, such as knowledge, beliefs, and attitudes. Theoretical frameworks which focus on this domain, are for instance Shulman's (1987) conceptualization of pedagogical content knowledge and Ball, Thames and Phelp's (2008) mathematical knowledge for teaching. Clarke and Hollingsworth (2002) do not distinguish between the different aspects of the personal domain, and do not make any claims as to certain attributes being more important than others. That is, they just recognize that they are various aspects of the personal domain. As previously stated, I have in this research study not tried to influence or change the teachers' personal domains. However, the personal domain can still be influenced by the teachers' experiences from our collaboration process.

Another domain which is part of the teacher's personal world of practice in Clarke and Hollingsworth's (2002) model, is the domain of practice. This domain represents a teacher's professional experimentation. In the earliest versions of this model, Clarke and Peter (1993) describe the domain of practice as the enactment of teacher knowledge and beliefs, and where the classroom situation is perceived as problematic, it becomes classroom experimentation. They assert this experimentation is always present to some degree, and that teachers continuously work to improve their practice (Clarke & Peter, 1993). However, they also point out that "it may be that a teacher lacks either the expertise or the knowledge of possible alternatives required to engage in effective experimentation" (Clarke & Peter, 1993, p. 171). The domain of practice is where I have challenged the teachers through this research project, and my hope is to assist them in designing changes in classroom practice according to their wishes. By encouraging them to ask for help to design mathematical tasks they want to use in the classroom, they are making changes to their practice. So, this is the domain where the teachers are challenged in this project, however, they decide to what degree they want to make changes in practice. I am designing tasks for them, but it is not given that they will make any changes to their teaching and how they present the tasks. It is important to point out that Clarke and Hollingsworth are explicit about this domain not being limited to just classroom experimentation, but to all forms of professional experimentation (Clarke & Hollingsworth, 2002). I will therefore analyze both the implementation of tasks in the classroom and the design process of the tasks as professional experimentation.

The third domain Clarke and Hollingsworth (2002) refer to as part of the teacher's personal world of practice, is the domain of consequence. This domain includes what the teacher views as salient outcomes. Learning is an example of this, but the domain of consequences includes many more aspects which a teacher wants to achieve. It could be teacher control, student motivation, engaging classroom discussions and so on. According to Clarke and Hollingsworth (2002), the "significance of the designation 'salient outcomes' lies in the need to acknowledge that individuals (teachers) value and consequently attend to different things (they consider different things salient)" (p. 954). So, this is not an objective evaluation of important aspects of students learning mathematics, this is the teacher's personal perspective on which outcomes are salient to her.

While the four domains of this model are analogous to aspects of other models on teacher change, it differs in how it identifies multiple growth pathways between the domains. According to Clarke and Hollingsworth (2002), an important aspect of the Interconnected Model of Professional Growth, is the non-linear nature and how it recognizes the complexity of professional growth as an iterative and continuing process of learning. The model also includes arrows representing the mediating processes of reflection and enactment as the mechanisms by which change in one domain leads to change in another domain (Clarke & Hollingsworth, 2002). So, when change occurs in one domain, the teacher might reflect on this change, which might lead to change in another domain. However, changes in one domain might also lead to change in another domain through enactment. Clarke and Hollingsworth have chosen the word enactment to distinguish it from acting, because the teacher is acting on something she knows, believes, or has experienced. Both reflection and enactment are to be viewed as active processes by the teacher.

The four domains in the Interconnected Model are all encompassed in the change environment. Teachers are part of a school community, they have colleagues, various opportunities for professional development, and many aspects of their work life that influence changes they are making. This is what Clarke and Hollingsworth (2002) refer to as the change environment. The environment teachers work in can act to facilitate or constrain teacher growth.

Theoretically Clarke and Hollingsworth argue that the Interconnected Model can be interpreted as consistent with both a cognitive and a situative perspective on learning. Instead of adopting a specific perspective on learning, they claim that this model can represent cognitive learning theory if teacher growth is viewed as development of knowledge. On the other hand, the model can also represent a situative perspective if teacher growth is viewed as development of practice. Clarke and Hollingsworth ascribe the consistency of the model with both theoretical perspectives as illustrating the complementarity of the two perspectives, as much as the conformity of the model to a coherent theory of learning (Clarke & Hollingsworth, 2002).

The Interconnected Model of Professional Growth has been developed over time by Clarke and Hollingsworth (2002), first providing empirical grounds for the domains and mediating processes, but then looking more closely into the order in which change occurred. As a result, Clarke and Hollingsworth are proposing a distinction between change sequences and growth networks. They define a change sequence as "consisting of two or more domains together with the reflective or enactive links connecting these domains, where empirical data supports both the occurrence of change in one domain and their causal connection" (Clarke & Hollingsworth, 2002, p. 958). It is not given that change in one domain leads to change in another domain, so there must be a link in order to use the term change sequence. However, even if a change sequence is identified, this might be a single instance of experimentation which does not provide long lasting changes. Three examples of what change sequences might look like are given by Clarke and Hollingsworth (2002) in Figure 3.5.

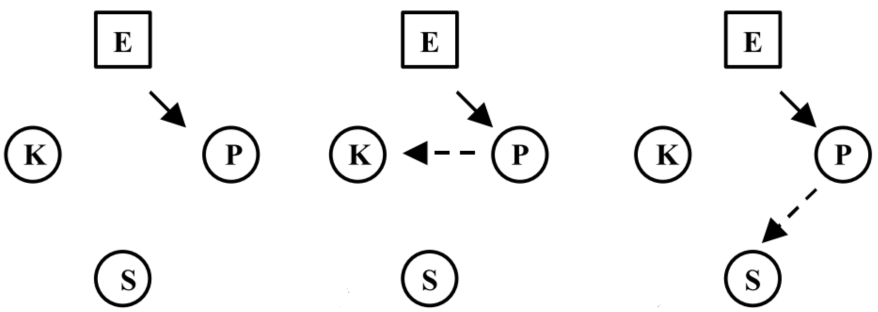

Figure 3.5: Three examples of change sequences (E = external domain; P = professional experimentation; S = salient outcomes; K = knowledge beliefs and attitudes (Clarke & Hollingsworth, 2002, p. 959)

These three diagrams indicate that a change in the external domain can lead to a change in the domain of practice through enactment. In the first diagram nothing more happens. In the second diagram the change in the domain of practice leads to a change in the personal domain through reflection. The last diagram shows how change in the domain of practice leads to a change in the domain of consequences through reflection. So, change sequences can be identified as change in one analytical domain leading to change in another domain through enactment or reflection, but there is no evidence that these changes continue beyond one or two changes.

Clarke and Hollingsworth want to identify more than just change sequences, and they use the term ‘growth’ for more lasting change. This underpins the notion of growth as being on-going changes. To identify growth, they require that the data must demonstrate the occurrence of change that is more than momentary and thus can be viewed as more lasting change. If the data can provide evidence of long-lasting change in a change sequence, this can be termed as a growth network. Three examples of what growth networks might look like are given by Clarke and Hollingsworth (2002) in Figure 3.6. They are all representing various elements of a teachers’ personal growth.

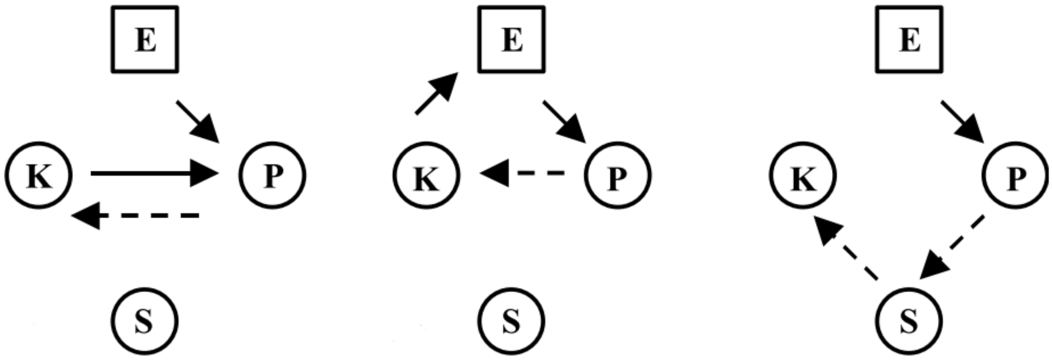

Figure 3.6: Three examples of growth networks (E = external domain; P = professional experimentation; S = salient outcomes; K = knowledge beliefs and attitudes (Clarke & Hollingsworth, 2002, p. 959)

The first diagram in Figure 3.6 is an example of a teacher who is doing ongoing refinement of practice. Changes are made in the domain of practice through enactment, and this again leads to change in the personal domain through reflection. This is an on-going process the teacher is working on (Clarke & Hollingsworth, 2002). The second diagram in Figure 3.6 illustrates how the same teacher continuously seeks new strategies. Changes in the personal domain leads to changes in the external domain through enactment, which again leads to changes in the domain of practice and changes in the personal domain through reflection, that is, a continuous process of developing new strategies (Clarke & Hollingsworth, 2002). The last diagram illustrates a long-term change to knowledge and beliefs of the same teacher. Experimentation in the domain of practice leads to changes in what the teacher views as salient outcomes, which in turn leads to changes in the teacher's knowledge, beliefs and attitudes through reflection. Again, this was an ongoing process (Clarke & Hollingsworth, 2002).

I have chosen to use the Interconnected Model of Professional Growth developed by Clarke and Hollingsworth (2002) to analyze the change processes throughout the collaboration between the teachers and me. This provides a framework to investigate and answer my second research question which is: What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration? The inter-connected model allows me to investigate not only the changes which are made, but also how these might be linked together. Thus, providing me with an understanding of which changes might foster or restrain other changes, which will further allow me to discuss the teachers' rationales for initiating changes during the collaboration.

3.6 Summary

This chapter has provided theoretical aspects on the teacher in the classroom and how she can impact students' learning. In addition, another research project focusing on the teachers' perspective has been presented, and how this was used to identify what the teachers considered as 'good' teaching. Next, theories on mathematical tasks were presented before I rounded off by presenting theoretical perspectives on teacher change and a theoretical framework for analyzing teacher change. I will in the next chapter present the methodology of this research, including the methods used and the theoretical underpinnings for these methods.

4 Methodology

The aim of this research is to understand the teachers' perspective, and two research questions guide the research presented in this dissertation.

1. What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom?
2. What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration?

This research is conducted within an interpretive research paradigm, based on a constructivist epistemology and a subtle realist ontology. The research strategies applied are both an abductive research strategy and a retroductive research strategy. I will in Sections 4.1 and 4.2 present and elaborate on the research paradigm and research strategies in this research project. This is followed by an overview of the generated data in Section 4.3, before an introduction of the four cases in Section 4.4. I present in Section 4.5 how techniques from grounded theory were used, before I reflect on trustworthiness of the research design in Section 4.6, and ethical considerations in Section 4.7

4.1 Interpretive Research Paradigm

All research is conducted within a research paradigm, whether the researchers are explicit on the matter or not. A research paradigm is here defined as the underlying theoretical and methodological perspectives through which the research is approached (Blaikie, 2007). A research paradigm overarches the aim of the research, formulation of research questions, selection of research strategies, and the kind of research outcomes which can be achieved based on ontological and epistemological assumptions. The research presented in this dissertation is conducted within an interpretive research paradigm. A fundamental tenet of the interpretive research paradigm is that there is a difference between the natural and social sciences (Blaikie, 2007). While a natural scientist can study nature from the "outside", this is not enough in the social sciences from an interpretive perspective. According to interpretivism, the study of social phenomena requires an understanding of the social world that people have constructed and which they reproduce through their continuing activities. However, people are constantly involved in interpreting and reinterpreting their world – social situations, other people's actions, their own actions, and natural and humanly created objects. They develop meanings for their activities together, and they have ideas about what is relevant for making sense of these activities. In short, social worlds are already interpreted before social scientists arrive (Blaikie, 2007, p. 124).

So, when studying the social world, we are studying an already interpreted reality and this 'reality' is what we need to make sense of. This view on reality calls for a further explanation on the ontological and epistemological underpinnings of such a research paradigm. I have positioned this research within a constructivist epistemology and a subtle realist ontology. On an imagined continuum, there are two extremes in social research when it comes to ontological positions. On the one hand there is the realist position, which assumes there exists a reality that can be studied independent of human activities. On the other hand, there is the idealist position which assumes that the external world is just appearances and has no independent existence apart from our thoughts. While an interpretive paradigm is closer to the idealist ontology, there are nuances also within an interpretivist paradigm. This research is based on the assumption that there exist some kind of independent, knowable phenomena, but this is not something we have direct access to. We must always rely on cultural assumptions to study them. This ontological perspective is referred to as subtle realism, and recognizes that all knowledge is a human construction, but also acknowledges that there exist independent and knowable phenomena (Blaikie, 2007).

A constructivist epistemology is often connected with an idealist ontological position but can also encompass subtle realism. Regardless of ontological nuances, research based on constructivism seeks to examine how social actors construct their knowledge and how they view the world. This will not be possible solely by outside observation but requires methods to gain further insights into the social actors thinking and reasoning.

To summarize, this research is set within an interpretive paradigm based on a constructivist epistemology and a subtle realist ontology. This sets the ground for the research strategies adopted in this study, which is elaborated in the next section.

4.2 Research Strategies

There are two research questions guiding this research, and they are investigated by adopting two different research strategies: an abductive and a retroductive research strategy, respectively. The first research question is: What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom? This research question does not just set out to investigate types of mathematical tasks but is specified to what teachers want to use in their classroom. Such a formulation resonates with the interpretive paradigm, emphasizing how various actors might not look for the same characteristics in mathematical tasks. Such a research question is also in line with using an abductive research strategy (Blaikie, 2007). According to Blaikie (2007), an abductive research strategy "…involves constructing theories that are derived from social actors' language, meanings and accounts in the context of everyday activities" (p. 89). For this research, the context of everyday activities is the mathematics classroom, and the social actors are the teachers. So, to answer the research question, I need to design a way to generate data which gives insight into not only what teachers might say they want, but to also understand their wishes concerning their classrooms and their everyday life. To accomplish this, I have used a combination of interviews and collaborative task design processes. By offering to design mathematical tasks the teachers want to use in their classroom, and be part of the implementation and evaluation process, I get access to the teachers' perspectives set in their everyday context.

One of the challenges when conducting abductive research, is that much of the activity in social life is routine, and thus conducted in a taken-for-granted, unreflective manner (Blaikie, 2007). This might also be a challenge in this research project, given how busy a teacher's workday is and many of the work tasks must be routinely done. To encourage the teachers to participate in this study in a reflective manner, the process started with an interview designed for the teachers to reflect on their practice and students' learning. Another aspect of the research design which calls for reflection, is that the teachers must explain to the researcher what type of mathematical tasks they want and what changes they want to make when they are refined. Because they are not designing the tasks themselves, they are forced to articulate what they want, but also what they do not want when presented for a task or a design idea.

With an abductive research strategy, the focus should be on the teachers' everyday concepts and meanings. Therefore, I cannot use already developed characteristics from the literature on mathematical tasks to analyze and categorize what the teachers are asking for. I need to examine what the teachers express in their own words. To accomplish this, I have used various techniques from grounded theory and a process of open coding (further elaborated in Section 4.5). Due to this process, I could present a descriptive answer to the first research question: What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom? These descriptive results are further analyzed across cases and discussed with respect to theory, to gain a deeper understanding of the teachers' descriptions of mathematical tasks.

Having conducted abductive research, it is not uncommon to use the results and look further into explanatory mechanisms using retroductive strategies (Blaikie, 2007), which is what I have chosen to do in this research project. Although the initial research focus was on the characteristics of mathematical tasks, this choice of method led to a collection of research data which can inform about teacher-initiated change processes. Given this possibility, the second research question was formulated to investigate the possible reasons behind the characteristics of tasks the teachers were describing: What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration? I am not only looking at the characteristics of the tasks themselves, but to the whole process of change initiated by the teacher. To accomplish this, a model for teacher change by Clarke and Hollingsworth (2002) was used to analyze the dynamics around the tasks the teachers were asking for. This resonates with a retroductive research strategy, aiming to discover underlying mechanisms to explain observed regularities (Blaikie, 2007).

4.3 An Overview of the Generated Data

A collaboration was initiated with four teachers working with students at upper secondary school in vocational mathematics classes. The four teachers were offered that I, as a researcher, would help designing any types of mathematics tasks they would like to use in their classrooms and which they did not already have. The benefit for the teachers would thus be tasks they would want to use, while the researcher would gain information about what the teachers were looking for. Note that consequently, it is not given that the mathematical tasks developed in this project are examples of best practice, but they are examples of my interpretation of what some teachers expressed a wish for.

The task design process went through several stages with each of the teachers, which can be summarized in the following points:

- Ask the teacher what kind of mathematical tasks she wants.
- Design a first draft of the task.
- Present and discuss the task with the teacher.
- Refine the task.
- Observe implementation of the task in the classroom.
- Evaluate the implementation together with the teacher.

Because of practical considerations, tasks we had developed through these stages, were seldom further developed through repeated implementation and redesign. However, I, as a researcher, learned from the process and the teachers' evaluations when designing the next task. This led to a development in our collaboration throughout the school year.

In addition to the task design process, I conducted a semi-structured interview with the teachers. Both interviews and the various stages of the design process were video- or sound-recorded, along with the observed implementations in the classroom. An overview of how many recorded hours and minutes there is of each type of data, is presented in Table 4.1

	Semi-structured interview	**Task design process**	**Recordings in the classroom**	**Recordings of discussions**	**Total recordings**
Roger	1 h and 32 min	2 · 0.5	0 min	4 h and 42 min	6 h and 14 min
Thomas	48 min	1.5	59 min	2 h and 03 min	3 h and 50 min
Hanna	44 min	4	6 h	8 h and 46 min	15 h and 30 min
Sven	25 min	2	1 h and 12 min	3 h and 25 min	5 h and 2 min

Table 4.1: Overview of the generated data

To the very left in Table 4.1, are the pseudonyms I have given the four teachers in this research project, and on the top of the table are descriptions of different types of data which have been collected. The first column gives the length of the semi-structured interview with each of the teachers. The interview with Sven is short compared to the others, since we already had a first talk about what kind of tasks he would like. During this talk, Sven answered and addressed many of the issues I asked about later in the interviews. The interview with Roger stands out as especially long compared to the others, which can be explained by more small talk and digressions. The second column in Table 4.1 indicates how many times I completed the several stages of the task design process together with the teachers. In the case of both Roger and Thomas there are half numbers, as we only completed the first stages of the task design process, and I do not know how the implementation went and there was no joint post evaluation. The third and fourth columns give information about how many hours and minutes that have been recorded of classroom's implementation and discussions with the teacher outside the classroom, respectively. The fifth column contains the total length of sound- and video-recordings of the collaboration with the given teacher. With this quick overview of what type of data that has been collected, I will in the next section elaborate on the theoretical reasons behind these choices and present the cases.

4.4 The Cases

As stated before, the aim of this research project is to gain an understanding of how teachers describe mathematical tasks they want to use in their classrooms. This is the phenomenon to be studied, or the quintain[1] (Stake, 2006). To study this quintain, I have investigated four cases. The cases consist of teachers working at upper secondary schools and teaching mathematics courses for students who are taking vocational education. This context was chosen because it is a part of the Norwegian school system with many challenges and high drop-out rates (Statistics Norway, 2019), and I therefore assumed there would be teachers wanting to make changes to their teaching. To get a broad understanding of the quintain, I wanted to also investigate cases which I assumed would have another perspective. I therefore contacted teachers who were looking to make changes to their teaching, but also teachers who were content with their teaching and was not looking to make significant changes. Contact with all four teachers were made through an acquaintance with a broad network who knew many teachers in the area.

[1] Stake argues that the word representing the collective target of the case studies needs to be generic, and is therefore using the word 'quintain' (Stake, 2006)

Even if the cases are chosen in combination to provide a greater understanding of the quintain, the cases are interesting and have value in themselves as well. Each teacher (case) has her own story, which should be understood in the context she works. I have used interviews and collaborative design processes to gain insight into the teachers' perspectives of tasks they want to use in their classrooms. However, the findings from investigating these cases cannot be understood without the context in which each of the teachers work. Each of the teachers are therefore shortly presented here, explaining the context in which they work. The teachers are numbered but have also been assigned pseudonyms. Following each presentation of a teacher, I have made a schematic overview of the type of data generated with respect to this teacher, and the duration of the recordings. I have also listed a column with a reference to which section in chapter six the tasks can be found. After the four cases have been presented in Sections 4.4.1 to 4.4.4, I elaborate on how the semi-structured interviews were conducted in Sub-Section 4.4.5.

4.4.1 Teacher 1, Roger

Roger is an experienced teacher who has taught mathematics and science for many years. He describes himself as a traditional teacher and was the first person I contacted. This year he only taught one class in mathematics. The class is a group of relatively mature students (about 30 years mean age), who have a vocational education but have decided to go back to school to qualify for an engineering degree course. The course is optional, but contains the mathematics required from secondary school to start on an engineering degree. The students seem motivated by how they all pay attention and take notes when the teacher is talking. It is the first time this teacher is teaching this specific course. When looking through the textbook, he does not immediately see tasks he wants different, because he does not see what kind of tasks would better prepare them for the exam. However, he still says he is open for suggestions. An overview of the data generated in the case of Roger are presented in Table 4.2.

Date	***General***	Section	Length
31.11.12	Semi structured interview (sound)		1:31:40
08.10.12	First conversation about tasks, part 1 (video)		8:58
08.10.12	First conversation about tasks, part 2 (video)		43:01

12.10.12	Handwritten field notes from class observation		
07.11.12	Mail discussion about teaching and learning mathematics in media		
29.01.13	Mail discussion about him as a teacher		
	Integration tasks	6.8	
09.01.13	Discussion of integration tasks (sound)		39:28
22.01.13	I present integral tasks (sound)		2:03:43
23.01.13	Integral tasks with my comments on how the teacher reacted		
	Logarithm tasks	6.6	
26.02.13	I present logarithm tasks (sound)		1:07:18
	Logarithm tasks		

Table 4.2: Data generated in the case of Roger

4.4.2 Teacher 2, Thomas

Thomas is a colleague of Roger, and they work at the same school. He teaches physics and mathematics, and this year he only has one class in mathematics. It is the same type of course as Roger, but the students are younger. They have just finished their vocational schooling and have little work experience before taking this optional course in mathematics. Their aim is to take the mathematics they need to start on an engineering degree. The teacher has taught this course many years and is familiar with the curriculum and the textbook. He is rather busy but has agreed to help me in my research. However, he stated rather early that he thought for instance five design processes would be too much, but three might be a possibility. When it comes to tasks he would like to change, he points out some tasks that are in a 'real' context, but he feels they are constructed and not good. He would like some tasks that are more 'down to earth'. An overview of the data generated in the case of Thomas is presented in Table 4.3.

Date	***General***	Section	Length
16.11.12	Semi-structured interview (sound)		47:49
08.11.12	Field notes of informal conversation		
18.01.13	Handwritten field notes from classroom observation		
	Logarithm tasks	6.6	

09.01.13	Discussion of logarithm tasks (sound)		44:30
05.02.13	I present logarithm tasks (sound)		43:58
08.03.13	Implementing logarithm tasks (video) Short evaluation of the tasks in the end		59:35
	Logarithm tasks		
	Trigonometry tasks	6.7	
10.04.13	Mail where Thomas describes what he wants in trigonometry tasks		
18.04.13	I present trigonometry tasks (sound)		37:01
	Trigonometry tasks		
24.04.13	Mail where Thomas postpones implementation due to the exams		

Table 4.3: Data generated in the case of Thomas

4.4.3 Teacher 3, Hanna

Hanna is a female teacher who works in another school than the first two. She teaches a compulsory course in mathematics for students who are becoming carpenters and the same course for students who are planning to be chefs. She especially wants help with the carpenters' class. Early in our conversations, she states that she is interested in this project because she wants help to change and to teach better. In the previous year she and some colleagues designed some tasks where they tried to make them more relevant to the students' vocations, but she did not feel they were successful in increasing the motivation of the students. She wants to change her teaching and her main aim is to increase the motivation of the students. Hanna looks for tasks that can function as an introduction to a new topic, so that the students can start to discover instead of her just telling them. The data generated in the case of Hanna is presented in Table 4.4.

Date	***General***	Section	Length
01.11.12	Semi-structured interview (sound)		43:57
08.10.12	First conversation about tasks		52:37
	A4-task	6.1	
15.11.12	Refining A4-task (sound)		1:16:17
20.12.12	Implementing A4-task part 1 (video)		38:38
20.12.12	Implementing A4-task part2 (video)		42:45

	Task A4-format		
20.12.12	Evaluating A4-task (sound)		1:04:56
	Rope and Area task		
27.11.12	I present rope and area task (sound)		1:09:08
	Rope task	6.4	
30.11.12	Implementing rope task part 1 (video)		48:37
30.11.12	Implementing rope task part 2 (video)		00:24
30.11.12	Implementing rope task part 3 (video)		32:35
30.11.12	Teacher comments implementing rope task (sound)		09:09
	Area task	6.2	
07.12.12	Implementing area task part 1 (video)		43:39
07.12.12	Implementing area task part 2 (video)		38:09
07.12.12	Teacher comments implementing area task part 1 (sound)		12:19
07.12.12	Teacher comments implementing area task part 2 (sound)		01:06
11.12.12	Evaluating rope and area task part 1		1:00:17
11.12.12	Evaluating rope and area task part 2		20:03
	Index task	6.5	
23.04.13	I present index task (sound)		1:06:10
	Index task		
22.05.13	Implementing index task part 1 (video)		36:35
22.05.13	Evaluating implementation of index task part 1 (sound)		25:48
24.05.13	Implementing index task part 2 (video)		40:01
24.05.13	Implementing index task part 3 (video)		37:57
24.05.13	Teacher comments implementing index task part 2 Sound)		12:30
24.05.13	Evaluating implementing index task part 2,3 (sound)		55:45

Table 4.4: Data generated in the case of Hanna.

4.4.4 Teacher 4, Sven

Sven is a male colleague of Hanna, and he is teaching a compulsory mathematics course for students who want to become hairdressers or something else within design and crafts. His motivation for participating in the research is to get tasks he is happier to use in the classroom. He does not like the textbook, but he feels his time is limited for making changes himself. He wants tasks that are more relevant for the students, but also tasks that provide a more conceptual understanding of the topics. Data generated in the case of Sven is presented in Table 4.5.

Date	***General***	Section	Length
27.11.12	Semi-structured interview (sound)		24:32
11.10.12	1st conversation about tasks (sound)		26:13
27.11.12	Talk about future collaboration (sound		08:45
16.11.12	Reflections around teacher and class dynamics		
	A4-task	6.1	
31.10.12	Discussion of proportion task (sound)		20:06
07.11.12	Refining A4-task part 1 (sound)		36:52
07.11.12	Refining A4-task part 2 (sound		13:31
	Task A4-format		
09.11.12	Reflections around designing the A4-task		
13.11.12	Evaluating A4-task (sound)		37:30
	Area task	6.2 and 6.3	
20.11.12	I present area task (sound)		45:07
	Web page with animated areas		
	Area task 1st version		
22.11.12	Implementing area task part 1 (video)		37:39
22.11.12	Implementing area task part 2 (video)		34:44
22.11.12	Reflections from visiting supervisor after implementation of area task		
27.11.12	Evaluating area task (sound)		17:25
	Area task revised		
07.12.12	Comments about revising the area task		

Table 4.5: Data generated in the case of Sven

4.4.5 The Semi-Structured Interviews

Given my research aim is to understand how teachers describe mathematical tasks they want to use in their classrooms, I wanted to acquire research knowledge on characteristics of tasks before I conducted semi-structured interviews. This is what Kvale and Brinkmann (2009, p. 106) refer to as subject matter knowledge. Therefore, I spent time reviewing the issues prior to designing the interview guide and conducting the interviews. To have subject matter knowledge is even more important when conducting semi-structured interviews, because one needs to be able to react and follow relevant comments and answers on the spot during the interview. Reviewing characteristics of mathematical tasks made me aware that it would not be possible to use concepts from the research literature, because the wording and formulations are so far from the focus and everyday language of the teachers. This meant that I needed to listen to the teachers and be aware of their wording, but at the same time keep the concepts from the research literature in the back of my mind.

The format of the semi-structured interview should be open enough to allow the teachers to speak rather freely and the researcher to follow up on answers without keeping strictly to the interview guide, but at the same time it should provide enough structure to be able to contrast the four teachers' answers and get feedback being relevant to my research questions (Bryman, 2008). I conducted a type of interview that Kvale and Brinkmann (2009) refer to as conceptual interview, i.e., an interview with the purpose of conceptual clarification. This is the type of interview which has been conducted in this research study, and the questions in the interview guide have all been designed to clarify the teachers' concepts of what 'good' teaching and learning is, and what kind of mathematical tasks can help fulfilling this. I will, in the following, set out the questions sequentially as they were presented to the teachers with comments on what they were designed to accomplish. It is worth noting that the initial research design had more focus on teachers' beliefs, which is recognizable in how the questions were formulated.

What is your background? (Education and experience)

This question is posed to learn about where the teacher is coming from. It might give me more insight into beliefs they have and choices they make in the classroom. What type of teaching they have been used to themselves in their own education and what type of knowledge and experience they have.

For how many years have you taught mathematics to vocational students?
This question is asked to ensure that I know the amount of experience the teachers have in vocational schools, since this might be different from other school experiences.

Which textbook do you use, and how satisfied are you with it?
This question is asked to get an understanding of how they are working today, but also a sense of how much they would like to change. Through this question I get to know to what degree they are using the textbook, and if they are happy with how they are using it today.

What does a 'typical' lesson in mathematics look like when you are responsible for it?
This question is asked because it is not given that what one would like to do in the classroom is what is enacted. With this question, I might learn more about a possible discrepancy between the teacher's beliefs about teaching and learning, and what happens in the classroom. This might also give me some insight into external constraints the teacher is experiencing.

How do you think students learn mathematics best?
This is a question related to the teacher's beliefs about teaching and learning. Hopefully, she does not feel obliged to express the exact same as what she does in the classroom. Hence this question following the question about what a typical lesson in mathematics looks like.

How important do you think the teaching is as opposed to the tasks in a mathematics lesson?
This question was included to give an impression of how linked the teachers felt that teaching and tasks were in the mathematics classroom. For instance, does the type of tasks they have access to constrain how they teach? Or do they feel the tasks are not so important, because of how they teach?

What are your strengths as a teacher?

This question is to get a feeling of what they think they succeed at. It gives me knowledge of whom they are as persons, but also on what they find important.

What are your challenges as a teacher?
This gives me some of the same information as the question above, but more detailed information about what they find difficult, but still important. This might be the point where they are the most willing to change.

Do you have an example of a task you like? Why?
This is to get some information which might help me in the design process, but also to get a hands-on example of a desired task.

Anything else you would like to add?
This is to give the teacher the opportunity to bring up issues or worries she might have come across during the interview (Kvale & Brinkmann, 2009).

4.5 Techniques from Grounded Theory

While grounded theory is a complete research methodology, I am not making any claims of adapting the whole methodology. However, my research focus and research questions made it apparent that I needed to use a grounded approach when analyzing my data, if I wanted to capture the teachers' perspectives. I had initially intended to analyze my data using well established categories from the research literature on mathematical tasks, however I quickly realized how this would be problematic. When talking to the teachers, their vocabulary is different than the theoretical terms, so if I would use previously theoretically developed categories it would imply analysis and interpretation even at the very beginning of coding my data material. I therefore decided to abandon the plan of using categories from the research literature on mathematical tasks, and instead use techniques inspired by grounded theory. I will in the following section explain how I use the work of Glaser and Strauss (1967), how I interpret the techniques and how I practically have employed them on my data. I have basically adapted many of the methods described in Corbin and Strauss' book: Basics of Qualitative Research (2008), but I have not adapted the whole methodology and I will explain how my approach might differ from theirs.

4.5.1 Designing the Study

When conducting grounded theory, there is a set of essential methods, with one being theoretical sampling (Teppo, 2015). In addition, Teppo (2015) argues that a crucial aspect of grounded theory research is the concurrent and continuous nature of data generation and analysis. Since the goal in grounded theory is to develop theory based on empirical data, analysis of data needs to guide further data collection. So, one starts by collecting data which one believes might be interesting and relevant for the research topic, and then analyzing this data should guide further data collection. This is what is referred to as theoretical sampling by Corbin and Strauss (2008). Based on the concepts developed from the analysis, one decides on what kind of data one would need to further develop and understand this concept. For instance, if I planned my research based on grounded theory, I might start collaborating with one teacher, design tasks and analyze our collaboration, before deciding on how to continue the data collection. My analysis of the first collaboration might have led me to a focus on vocationally oriented tasks, and I might have decided to collaborate with a teacher working in general studies to see if this was a unique characteristic from a vocational teacher's perspective, or if there might be more nuances.

Given that I had planned to initiate a collaboration with all four teachers at the same time, I cannot claim that my research design is grounded theory. Ideally, I should have used the first collaboration to determine what type of further data I would need and which teachers to collaborate with. Instead, I carefully selected different teachers with whom I had an extensive collaboration with over a school year, assuming it would provide me with both breadth and depth to my data without having to go back and collect more. However, I would say there was an alternative theoretical sampling in the design processes of tasks, where a task might be viewed as a sample. The new tasks were designed based on the outcomes of the previous tasks, and thus further developed. In addition, I have to a certain degree done theoretical sampling when analyzing my data, which I will describe in the next section.

4.5.2 Description of my Process of Analysis

The initial focus of my analytical process was to answer the first research question: What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom? I have already elaborated on what type of data I collected and the reasons to answer this research question. In addition, I have explained the need of an open coding process when examining these data. I will now explain how this was practically done, while the details of which type of categories that were developed and why, are explained in Chapter 7.

All conversations with the teachers were audio recorded and the implementations in the classrooms were video recorded. While video recordings would have given more information about gestures during the design process with the teachers, it would also take more time to set up and felt more intrusive than audio recordings. The semi-structured interviews were video recorded, but in the further collaboration I chose only to record audio. However, since there are many more participants during implementation in the classroom and much happening, I video recorded all implementations of the tasks. This provided a possibility to watch parts of a video of the implementation together with the teacher to discuss, although this was not done.

Both audio and video recordings were imported into NVivo[2] and data reduced concurrently with data generation. Data reduction was done by listening to the recordings while keeping my research focus in mind. I further split the recordings into parts, mostly between two and seven minutes long, depending on the content. For instance, one part could be the teacher talking about how a student asked him a question about a task, and then I would mark a new part if the teacher started talking more generally about students who struggle. So, I tried to break down my data into parts of a more manageable size and closely related in topic. In addition, I wrote down what was happening and what was being talked about in every partition. The data reductions are detailed facilitating retrieval of relevant data material at a later point, but they are also the first phase of analyzing, serving as a guide, focus and help when collaborating with the teachers. As I was the only researcher, collaborating with four teachers at the time, I realized it was not possible to transcribe all the recordings while I was generating data. The data reductions were less time demanding than a full transcript, but at the same time a way of organizing the data material.

At first, I intended to transcribe the data material later, and then analyze the transcript, but I started gradually to question this decision. There are many ways of transcribing, from discourse analysis where intonation is included (Linell, 1998), to transcripts including non-verbal communication like gestures (Radford, 2003). However, no matter how detailed the transcript is, something is

[2] NVivo is computer software designed for qualitative analysis.

lost when communication is written down in transcripts. Hence, I decided to analyze the original audio recordings directly, instead of using transcripts. I used the data reductions to navigate in the data material to find relevant parts for the analysis. All the data had been imported into NVivo, which allows a close connection between data reduction and the original audio recordings. I could click on a segment in the data reduction and immediately get to the corresponding audio recordings.

Doing data reduction helped me organize the data in such a way that I could easily retrieve important elements, but it also helped me to recall what happened in the discussions with the teachers, and thus guided me, to some extent, in the further collaboration. Great amounts of data were collected, including the tasks, oral and written communications with the teachers, videos of implementing the tasks and interviews with the teachers. It was not possible to analyze in detail more than 30 hours of video and audio recordings, thus some choices had to be made. I started a process of open coding combined with writing memos (Corbin & Strauss, 2008). The codes were made as close to the teachers' everyday language as possible, while I used the memos to write down thoughts on possible connections I might notice, or themes which I thought might emerge. I started the coding process with the interviews of the teachers, assuming I would get the richest data from the descriptions of mathematical tasks they would prefer to use in their classrooms, since they could express themselves in general during the interviews. Having analyzed these interviews, my memos guided me to which parts of the data to analyze next to obtain a greater understanding of the concepts. So even if my research was not designed strictly as a grounded theory methodology, I used the principles of theoretical sampling within the data I had collected. In addition, I had the possibility to contact the teachers I had collaborated with for further data or comments if needed.

When analyzing my data, I used several techniques from grounded theory. In addition to a certain level of theoretical sampling with my body of collected data, I did open coding, used the constant comparative method, axial coding and writing memos as part of the process of generating theory from my data. To help me in this process, I used NVivo software to organize my data and analysis. The qualitative software provided some affordances in the analytical process, especially when it came to the constant comparative method. Given my data reduction, which has previously been described, I could easily move back and forth in my data and get direct access to the original recordings from conversations with the teachers. For illustration, when I wrote a memo on how a teacher is commenting on the students' activity in the classroom and how happy he is with increased activity. Using a constant comparative method, I now wanted to go to the other parts of the data to see what this teacher had expressed about students' activity, and what other teachers might have said on the topic. With the help of the data reductions, I could easily find the relevant passages, which in turn were linked to the audio-recordings. So, the NVivo software program helped me organize my data in such a way that constant comparison became manageable across the whole data set.

In the process of open coding, I tried my best to use in vivo codes, meaning to use the teachers' own wording in the code. My rationale for this was to limit my own perception on the codes and let the teachers' words be used instead. Even if being conscious of this, I still struggled doing so from time to time. An example would be how I used the code learning and tried to fit this in with the other codes through axial coding. I found this difficult and at some point, I realized that the teachers never used the word learning. They might use words like understanding, aha moments and so on, but they did not say learning. Having discovered this, I went back to my data even more cautious about letting the teachers' words be heard and not my own interpretations. This was an ongoing process for a long time, where I switched between open coding, constant comparison, writing memos and axial coding; working to make it all fit together and to represent the teachers in the project. The details of this process are elaborated on in Sub-Sections 7.1.1 and 7.1.2

4.5.3 Theoretical Sensitivity and Theoretical Integration

Grounded theory has often been misunderstood as researchers having to enter the field of study as tabula rasa, not knowing anything about the theories relevant to the field of study, however this is neither likely nor necessary (Teppo, 2015; Vollstedt, 2015). The idea is to not start your research with an extensive literature review, not only because it might affect and limit how you view your data, but also because a grounded approach might take you in a whole different direction where your initial literature review might not even be relevant anymore. This was, to some degree, what happened in my research, where I had assumed that teachers' beliefs would affect the outcome, and thus spent time reading up on theories about teachers' beliefs. However, the data did not support such a focus, and this was not as relevant to my research as I initially planned.

Even if a researcher conducting grounded theory does not start the research by doing an extensive literature review on the topic, the researcher still brings with her knowledge and experiences which might influence the research and analysis. This is part of the researcher's theoretical sensitivity (Vollstedt, 2015), and should not be ignored when reporting the research. Despite the goal of grounded theory to generate new theory from the data itself, a researcher's knowledge and background will to a certain degree influence how they assess the data and which things that might spark an interest to look closer at. In my own research, I have a background as a teacher, and this might lead me to notice elements in the data which might not be as noticeable if the researcher had a background as a mathematician instead. In the first books about grounded theory, theoretical issues like ontology and epistemology were not openly discussed or expressed (Glaser & Strauss, 1967; Strauss & Corbin, 1990). However, in the later editions and books published by the next generation of grounded theorists, there is a stronger focus on theoretical underpinnings, and these vary. Teppo (2015) describes how the interpretive frameworks include pragmatism, symbolic interactionism, constructivist grounded theory and situational analysis. It therefore seems reasonable to claim that there is not one specific interpretative theoretical framework that must underpin the analysis when using techniques from grounded theory. However, it does make evident the importance of the researcher stating her own ontological and epistemological point of view when conducting such an analysis.

The interpretive theoretical framework is part of the researcher's theoretical sensitivity when analyzing the data, but it could also influence the choices made at the point of theoretical integration. At the point where the researcher has constructed codes, categories and axial coding, the next natural step is to seek theoretical sampling in the research literature being relevant to the findings. What type of research literature the researcher turns to, can be influenced by the researchers ontological and epistemological beliefs. Therefore, I started the methodology chapter by accounting for the research paradigm underpinning the research reported in this dissertation.

4.6 Trustworthiness

This research was conducted within an interpretive paradigm, and this influence the perception of what can be agreed upon and known. As elaborated on in the introduction to the methodology chapter, this research is designed within a belief that all knowledge is human construction, which in turn bears consequences to what can be known. According to Blaikie (2007) based on constructionism: “The only criteria available are those that can be agreed upon, through negotiation and argument, by a community of scientists at a certain time, in a certain place, and under certain conditions” (p. 23). Albeit my perspective is not so strict, given my position as a subtle realist, I still emphasize communication of the research process to the reader. To make this process open and trustworthy, is part of the research rigor.

When conducting research that is dependent on both process and context, trackability becomes an important aspect of trustworthiness. Trackability refers to the research being reported so scrupulously and candidly that it can be retraced, or virtually replicated by other researchers (Gravemeijer & Cobb, 2006). This means reporting both on failures and successes on the procedures followed, and the reasons for choices being made. It is also important to be explicit about the criteria and type of evidence used when analyzing (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). Throughout the research I keep a research journal where I make notes of thoughts, ideas and so on to help me document the whole process together with audio or video recordings of conversations with the teachers and classroom implementation. In addition, I strive to report on the methods and techniques for analyzing as thoroughly as possible, so that the reader can make up her own mind if my conclusions might make sense.

Triangulation is another important aspect of achieving trustworthiness. Researchers deal with a lot of impressions, and they need some way of assuring their interpretation of the meaning. The process of gaining these assurances is called triangulation (Stake, 2006, p. 33). By using multiple sources for data collection, I can triangulate when analyzing the data. That is why I collect data from interviews, collaboration processes, and implementation in the classrooms.

Since I work so closely with the teachers, member checking is a natural means for me to use for assessing trustworthiness. By member checking, I refer to the teachers reading my interpretations and analyses of the situations they have been part of. It might be that we understand situations differently, but member checking provides an opportunity to discuss and agree upon the written exposition. All the teachers get the opportunity to read, comment, adjust or contradict what is written in this text.

When working on single cases that are related to both context and process, generalizability is a challenge. Therefore, theory is such an important part of the research design. By placing the design and analysis in a broad theoretical context, it is possible to generalize by showing how the study is a paradigmatic case of the phenomenon under investigation (Shavelson, Phillips, Towne, & Feuer, 2003).

4.6.1 Interrater Reliability

Another aspect of trustworthiness is whether another coder would agree on how the data were coded, in other words how good the interrater reliability is. The dimensions and categories which I have created from the open coding process previously described, are meant as a help to understand and answer my research question about what characterizes teachers' descriptions of mathematical tasks they want to use in their classroom. An overview of the dimensions and categories can be found in Sub-Section 7.1.2. The categories and dimensions are not meant as an interpretation of what the teachers express, but a way of organizing their response and utterances about mathematical tasks to make an analysis of what their focus is. All the data in this research project have been collected and analyzed by the author, however I wanted to test the categories I developed with help from a research colleague. My aim with this testing, was to see if another researcher would assign the categories to the same type of statements as I would, and even more importantly, if the other researcher felt the categories were sufficient to organize the descriptions of mathematical tasks the teachers expressed. The three dimensions can work as a framework for teachers' perspectives on mathematical tasks, however, I am not claiming that there might not be a need for other types of categories in addition to the ones I have presented. If one changes the group of students or the teachers, there might be a need for more categories than those I have presented, in the same way as some of my categories would be superfluous. An example of this could be the subcategory vocational, which is an important category for teachers teaching in vocational classrooms but might not be relevant for a teacher teaching a group of seven-year-olds. So, when I am doing a test on the interrater reliability on the categories I have created, I am not trying to make any claims of these categories being exhaustive. I am instead testing whether another researcher who analyzes some of my data material the same way as I did, will agree upon how to use the categories I have created.

The first phase of this process was to train the other researcher in my categories. To do this, I started by stating my research question and explaining how she should listen for statements made by the teachers concerning mathematical tasks. I explained how she would need to not only identify positive comments, but also negative or neutral comments. Even if my research question addresses what type of tasks the teachers want to use, identifying what they do not want is helpful in identifying the borderlines of what the teachers are looking for. Having explained the research question and what types of statements to look for in the data material, I followed this by elaborating on my own coding process and how this had resulted in the categories and dimensions I had created. After talking my colleague through this, I gave her the written explanation of the codes and categories, which is presented in Sub-Sections 7.1.1. and 7.1.2. Finally, I gave her a screen shot of the dimensions and categories from my work in NVivo, to provide her with an orderly overview of the categories and where they belong. When choosing sections of the data material for my colleague to code, I decided to choose one excerpt from each of the teachers, to obtain a certain width in type of data material. My own process of open coding was done several months earlier, so I could not remember details from this coding process. However, I used the previous coding as a guide to find parts of the data material for my colleague to code, looking for sections containing many codes ensuring she would listen to relevant data for coding. In total, I chose four segments of data for my colleague, ranging in time from four minutes and 20 seconds to seven minutes and 12 seconds. In total, my colleague had 24 minutes and 29 seconds of data to code, with parts from all the four teachers, and with different type of data sources like the semi-structured interview, discussions of what type of tasks the teachers want, and refining tasks. My colleague was asked to do the coding in the same way as I had, which meant listening to the recording of the conversations with the teachers and assigning coding to the sound, but technically marking it in the data reduction text, to refer to it more easily. All of this was done in NVivo, and I listened and re-coded the same parts to make it easier to compare and discuss.

After both me and my colleague had conducted the first coding, we realized there had been some misunderstandings when we compared codes. The screenshot made of my categories and dimensions in NVivo, contained the Norwegian naming I had used in the process of coding; however, I had made some clarifying changes to the wording when translating it to English. For instance, I had used the wording: wanted outcome of tasks in my analysis process. Realizing how the teachers were not only expressing what they wanted but could evaluate the outcome of tasks as both positive, negative, or neutral, I changed the wording to outcome of tasks in the text. However, my colleague, who coded using my categories, saw the wording wanted outcome of tasks, so she did not assign any codes in this dimension if the teachers did not talk about an outcome they wanted. As a result, our coding deviated on some of the categories used, but at the same time we discovered that we were not disagreeing on what to code and not even the coding when we discussed it. The only thing we needed to look more closely into, was the difference between the two dimensions: outcome of tasks and students' reactions to tasks. Why are they different and how to distinguish them? These discussions helped me to reformulate some of the descriptions of the categories and dimensions to clarify them.

Since I now had clarified the description of the categories, we decided to code another small data segment, just to assure we agreed when interpreting the data. I chose a section from a source which contained an evaluation of implementing a task, since this was a type of source we had not previously analyzed in this interrater reliability process. The section lasted for three minutes and 28 seconds, and we agreed on all the categories we assigned to the data. We even discussed a statement which we both had been uncertain about how to code but had ended making the same choice.

This process with my colleague helped me clarify some of the descriptions of the categories I had created, but also reassured me that another researcher can use these categories to analyze this type of data material.

4.7 Ethical Considerations

The University of Agder has made a commitment to notify and get approval for all our research containing sensitive data from the Data Protection Official for Research (NSD). This office has a mandate to ensure the personal protection of people involved in research (NSD, 2012), and are thus ensuring a certain level of ethical considerations from researchers. The ethical considerations that NSD demands to approve research are similar, but even more stringent than those stated by the British Educational Research Association (BERA, 2011).

Even if NSD approves my research and I follow their guidelines, there are many difficult ethical issues that might arise throughout my research. There is no way of safeguarding all ethical challenges, but I quote Pring (2004) on this matter: “Right action, in complex moral deliberations, stems from the right dispositions” (p. 150). Good planning and careful considerations of possible challenges might help to make the right choices when ethical problems arise.

Anonymity of the research participants is an ethical consideration that is often emphasized, and there are many guidelines about how this can be accomplished (BERA, 2011; NSD, 2012; Pring, 2004). Given the research design, I propose this is still a difficult issue even if I follow all the guidelines, because I work with a small number of teachers. Even if I use pseudonyms in my written work, colleagues and people in the area will be able to identify the teachers, since there are only four of them. The teachers will be a great resource, and it is important to me that I value their time and contribution and show them respect in my work also when publishing.

I know from my own time as a teacher, that we all have good and bad days. We also continue to develop throughout our working lives, and our opinions and thoughts might change. So, what do I do if I encounter a situation where I know reporting what happened might place the teacher in a bad light, but at the same time it is important for the trustworthiness of the research? This is a dilemma that is likely to happen and is therefore important to consider in advance.

Acknowledging the ethical dilemmas I might face throughout my research with respect to anonymity, I have used the opportunity to discuss and address these issues with colleagues at my university, but also with fellow researchers at summer schools and conferences. None of us have a clear answer on how to best address these issues, but we have tried to suggest several possible actions.

One of them is to work with more teachers than I actual report data from. This way, it might be more difficult to identify the actual teachers I work with. Even if this might be helpful in some research designs, I am not sure it will be very helpful in my design. Looking into teachers’ choices about teaching and learning is quite personal, and I believe this will make it relatively easy to recognize the teachers even if I only publish the work of three out of five. At the same time, my research design is demanding when it comes to data collection, so I believe too much time will be spent on work I will not publish, if I solve the dilemma this way.

Another solution we have discussed, is that I implement the designed tasks in the classroom myself and use the teacher as a critical friend during the process.

This way, my own mistakes in the classrooms will be reported instead of the teacher's. One challenge with this solution is that I might obtain very different data if I am doing the implementation myself instead of the teacher. Another possible solution is to divide the implementation of the tasks among researcher and teacher, making it more difficult to identify who is doing the implementation.

The last solution that has been discussed, and the one I chose to follow, is to keep the research design, but to inform the teachers about the challenges of anonymity and discuss continuously how to solve dilemmas as they come along. In addition, my focus in the analysis is on the conversations with the teachers, hence the implementation in the classroom serves only as a reference point. I will in the next section address the ethical dilemmas and the possible solutions with regards to ethical theory.

4.7.1 My Ethical Dilemmas in Light of Ethical Domains and Assessment

There are many ways of handling ethical dilemmas, and the choices one makes can be seen in light of different ethical domains for assessing. According to Pojman (1997) there are four domains of ethical assessment, and they are "types of action, consequences, character and motive" (p. 16). These four domains can be seen as two different types of ethics. Various types of actions and consequences are principle-based theories (normative ethics), while the last two are virtue-based theories (Beach, 1996). I will in the following discuss my ethical dilemmas with respect to these four domains.

One way of dealing with ethical dilemmas is to consider different types of actions, and a simple way of categorizing these is according to what is right and wrong. Theories that emphasize the nature of the act are called 'dentological', and these theories hold that there is something inherently good or bad in different acts (Pojman, 1997). One example of such a perspective is the Ten Commandments in the Bible. These commandments rule against actions which are viewed as bad and something one should never do, as well as exhort other actions as something one should do, like observe the sabbath and honor one's mother and father. There might still be an issue to determine what might be seen as right or wrong. Kant's solution to this can be summarized in the statement: "Act only on that maxim whereby you can at the same time will that it would become a universal law" (Pojman, 1997, p. 18). So, by Kant's perspective, one's actions should follow the guidelines of whether this action is something one also would want everyone else to do.

If I look at the ethical dilemmas in my research from this perspective, there are some decisions that are made only with the perspective of what is right or wrong. One example of this, is the research participants' right to withdraw at any given moment without having to give any explanation. Another example is the participants' voluntary and informed consent. I am not video filming any student or teacher without having a signature of consent. These examples are so strongly embedded in how I see 'good' research that I do not even consider them dilemmas. The only way I am considering these issues, is by bearing in mind that I should always treat my research participants fairly and with respect, so that they do not feel the need to withdraw from the research, which would obviously impact my work negatively. However, I have accepted that this might happen regardless of my actions, and thus ought to be considered.

Another domain of ethical assessment is to solve ethical dilemmas by considering different consequences. The most famous of these theories is Utilitarianism, fronting that decisions should be based on an analysis of what action will produce the greatest happiness for the greatest number (Pojman, 1997). An example of the difference in choice between a dentological and utilitarian perspective could be if a person is given the choice of stealing food from a rich person and giving it to many people who are starving to save their lives. From a dentological perspective, you might say that stealing is among the wrongs one should never do, but from a utilitarian perspective it would be worth stealing because of the gain of saving many other peoples' lives.

I feel that I balance these two perspectives in some of the ethical issues I face in my research. Anonymity of the research participants is a principle that should never be compromised unless the participants themselves waive the right. At the same time, I have described above that this is extremely difficult to fully accomplish in my research, because of the small number of teachers I work with. If I should decide on this based on a dentological perspective, the consequence would be that my research is not possible to conduct. So, by deciding to follow through with my research, I have brought the dilemma into the dimension of considering consequences. I would still claim that it is not a purely utilitarian approach, because I hold the possible negative impact on the teachers at a greater concern than my own research. Thus, I will claim that my decisions are based on considering consequences with a dentological perspective. After many thoughts and considerations, I decided to be open with the teachers about the challenge of anonymity and discuss this issue with them. I tried to help the situation by allowing the teachers to read the report before publishing, and suggest changes to the text, but this is also a matter of trust. All in all, the teachers are aware of the risk and problems with anonymity but are still willing to participate in the research. I believe this results from me setting up the research design to benefit the teachers, combined with a matter of trust, which brings me over to the next domain for ethical assessment.

Aristotle's ethics emphasize character or virtue, entailing that we can only ensure habitual right action when having good people (Pojman, 1997). This is an ethical domain that I find difficult to claim that I have used to solve an ethical dilemma, but I believe it is still in the back of my head when making decisions. I consider it important not only to be a researcher, but a researcher who can be proud and confident in all elements of my work. To me, this is related to Aristotle's ethical dimension of character or virtue, and I would say this is implicitly involved in all ethical decisions I make in my research. This is also a dimension which I believe is especially important when doing the type of research I am doing. The risk of lack of anonymity is something the research participants are willing to take, which I assume is due to the trust in my character or virtue. That is, they believe that I will treat them fairly and respectfully when dilemmas arise.

The last domain of ethical assessment that Pojman (1997) refers to is motive. I have already argued above that many of the ethical dimensions are intertwining each other, but motive is a domain which it is difficult to leave out of any ethical decision. This is also something that Pojman considers, and she claims that: "In a full moral description of any act, motive will be taken into consideration as a relevant factor" (Pojman, 1997, p. 19). I have stated my research motive clearly in a project description, and this has also been communicated to the teachers. One of my aims is to understand the teacher, but I am not trying to change her. This has turned out to be a challenge when one of the teachers has stated that she has a motive of getting help and to further develop herself as a teacher. So, our motives are to some point contradictory, and this is an ethical challenge. This is not a dilemma that is solved by one discussion and decision, but something I see as an ongoing process where we both must make some compromises. I am not trying to dictate or give her ready-made tasks and solutions, but at the same time I am open for discussions about teaching and learning. This way, I am trying to fulfill both our motives to some extent.

4.8 Summary

I have in this chapter presented the research paradigm and research strategies that have been used to answer the research questions. This was followed by an overview of the data collected and a presentation of the cases in the study. The analysis process further described, including how techniques from grounded theory has been used. The last parts of the methodology chapter dealt with aspects of trustworthiness and ethical considerations related to the research design.

I will in the next chapter present each teacher and describe the collaboration process on designing tasks. The cases are presented as a general presentation of the teacher, prior a description of each design process of tasks we went through together.

5 Presentation of the Cases

I will in this chapter present a summary of the collaboration with each of the four teachers who are participating in this research project. Roger is presented in Section 5.1, Thomas in Section 5.2, Hanna in Section 5.3, and Sven in Section 5.4.

Each of these sections are further divided into sub-sections where the teachers are first presented with a general overview of their background and their thoughts about teaching and learning of mathematics. This is followed by a presentation and description of our collaboration on designing the different tasks, before the collaboration in general is summarized. I have chosen to present all the tasks that were designed in collaboration with the teachers in Chapter 6. Several of the tasks have been used with more than one teacher, so to avoid repeating the tasks, they are gathered in a separate chapter, yet referred to when I present the cases.

5.1 Teacher 1: Roger

5.1.1 Background and Context

Roger is a teacher in his sixties and has extensive experience as a teacher in mathematics, chemistry, and physics. In addition to the theoretical subjects, he has also taught Electrical Engineering, Machinery Skills, and more practical courses. He has several university courses in both mathematics and science and a degree of cand. real. in chemistry. Cand. real. is an old Norwegian degree in mathematics or science of very high standard and quality which includes a dissertation as well as courses. A cand. real. degree is comparable to a master's degree today but was even more demanding. Most students used seven to eight years to meet the standards of a cand. real. degree. After Roger completed his education, he worked both as a research assistant and as a researcher in a private research department. However, a period in the industry with limited work, made him obtain further education in pedagogics and start working as a teacher at the vocational technical college where he still works today, and has been working for more than thirty years.

Roger refers to himself as a traditional teacher, and most of his lessons are based on expositions and examples presented from the front of the room. A typical lesson for him, would be to spend 45 to 90 minutes explaining and giving examples on how to work and solve mathematical tasks, followed by the students working on tasks. Roger is only teaching one class in mathematics this year. This class is a group of relatively mature students (about 30 years mean age), who have a trade certificate and have been working for a while but have now decided to go back to school to qualify for an engineering degree course. The mathematics course Roger is teaching is optional, but contains the mathematics required from secondary school to start on an engineering degree. It is the first time Roger is teaching this specific course.

I ask Roger if he is happy with the textbook they use, and he responds that it is ok. It is not perfect, but he has not seen any other books for this course to compare with. Roger says an important part of a textbook is to have many relevant tasks the students can work on, but he questions how good the textbook is to read to understand the mathematics. He assumes the students might find the textbook more useful as a sort of encyclopedia where they look up and check things, rather than to read it to develop an understanding of the topics.

When I ask Roger about his strengths as a teacher, his first comment is that he is almost always in a good mood and is enthusiastic for what he does. Enthusiasm is a word he mentions several times throughout the interview, and he emphasizes the importance of motivating the students. At the same time, he expresses how he is very happy with having a group of adult students, and how he does not really want to teach younger students if he can choose not to. He explains this as having to ‘babysit’ younger students more, and he does not enjoy that part of being a teacher. I ask him to elaborate on this difference in teaching adult students as opposed to 16-17-year-olds, and he says you must vary the teaching a lot more with the younger students and check their work more often, like having regular assignments they must submit. He prefers to teach more mature students where he does not have to worry about them putting in the required effort or not. He refers to the students he has now as a hardworking and dedicated group. Those students who thought this would be an easy course and were not willing to put in the work, have already dropped out at this point, which Roger considers as a good thing. When it comes to Roger’s challenges as a teacher, he admits writing like a ‘pig’ and sometimes being unstructured when writing on the blackboard. He also says he can relatively easily be led into digressions. However, he is not convinced that these challenges are something he wants to change, as he also sees a connection to the enthusiasm he has and finds important.

In Roger's opinion, the students learn only by solving tasks. Since he has previously talked about how he can give double lessons with lectures, I ask him to elaborate on this with respect to his belief that the students only learn mathematics when they solve tasks themselves. Roger responds by using a metaphor of fixing the engine of a car. The only way to learn to fix a car, is to do it yourself. However, it can be helpful to have someone point a finger and give you some ideas on where to start. That is what his lectures are – some help to get started and pointing out the essential elements before the students do the job themselves. It does not even have to be a teacher, it is possible to find this kind of help in a book or on the Internet as well, but students need someone or something to help them get started.

When I ask Roger if there are any tasks he would like to change or replace in the course he is teaching, he cannot immediately think of any. While looking through the textbook, the topics, and the tasks, he comments on both the tasks and what he thinks the students might struggle with. All the comments are related to types of understanding Roger thinks the students need to have on different topics. For instance, when it comes to logarithms, he refers to how the students need to realize there are more than one logarithm base, and how logarithms are the inverse of exponentials. However, even if Roger has a clear focus on students understanding the mathematics, he is also aware of the exam and what they would need to perform well. An example would be when he comments on tasks that are logarithmic equations and where the students are supposed to use the logarithmic rules to get an exact answer. Even if Roger comments on how it can be difficult to see the usefulness of these tasks and students are struggling with them, he also says this is all part of the game and he does not see how those tasks can be made 'fresher'.

Some of the tasks which Roger does not like are tasks set out to be realistic, but still are not. He gives an example from physics but cannot come up with an example from mathematics on the spot. In addition, he gives some examples of tasks which he finds badly formulated, and he does not like tasks providing several statements to evaluate the truthfulness of. There are also some tasks in the textbooks on logarithms which he does not like, because they might as well be solved on the calculator, but he just skips these tasks. So, even if there are some tasks he does not like, he is not sure about what to replace them with or what he would do differently. At the same time, he says there might be some tasks which would give the students better understanding of a topic, but he just does not see it.

Roger does not have any specific requests for new tasks he would prefer to use in the classroom, but at the same time he does not exclude that there might be tasks that provide better understanding. Based on this, we agree that I will present some examples of tasks on relevant topics, and Roger will have a look and decide if he wants to use them in the classroom.

5.1.2 Integral Tasks

Integrals was a topic where Roger thought there might be room for better tasks, even if he was not sure what they could be like. I decided to find a variety of different tasks on the topic of integrals, hoping to come up with something Roger might find interesting to use in the classroom. I presented ten different ideas for tasks on the topic, all of them described in Section 6.8 together with Roger's comments when I presented them. The tasks range from exploration in GeoGebra, multiple representations based on the principles of Swan (2008), exploratory task, compose and decompose (Bills, Bills, Mason, & Watson, 2004), a task which focuses on integrals as area (Orton, 1983), a practical task where using integrals to calculate the construction of a dam, and several tasks from the webpage of nrich.maths.org[3] where the students can explore different aspects of integrals.

Roger and I talked for almost two hours when I presented the different tasks on integrals, and he listened carefully to everything I said and asked follow-up questions like: What should be the learning outcome of this? He also reflected on how he plans to introduce and work on the integral topic. Still, in the end, he did not want to use any of the tasks. He did not always give a reason, but here are some of the reasons for not wanting the tasks. One of the tasks uses implication arrows which he has not taught his students. He did not see the task adding enough value to be willing to introduce implication arrows in his teaching. The task of composing and decomposing meant the students would have to work together in pairs. Roger's comment to this was that the task was fun enough, but he has never put people together in pairs to work, so they would be surprised by his change of pedagogics. They have never done this before, but he sees that the

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[3] I am providing the name of the task and a direct link as a reference when I present tasks from NRICH, however they are not in the reference list, since it would not provide any additional information.

idea could be good. Roger does not want to use GeoGebra in his teaching. He states that he will not use a computer to demonstrate stuff, even if he acknowledges that it could be somewhat useful. He is not used to GeoGebra and computers in teaching, and then when problems arise, he is unsure how to handle them. Roger still acknowledges how GeoGebra can be useful when it comes to visualizing certain things, and maybe especially the trigonometric functions. I make an offer to make a macro for him in GeoGebra, but he is still skeptical. He will have to use a projector and connect the computer to it, and this makes him hesitant. Roger questions if the extra time and burden will outweigh the gain.

There was a task which Roger liked, but it was not specific on integrals. The idea of the task is to set up two items on the floor a couple of meters apart, and name them *A* and *B*. The students are then asked to draw a diagram where the *x*-axis is the distance from *A*, and the *y*-axis is the distance from *B*. One person then walks straight lines between the two items, and the students draw the graph. This becomes quickly very difficult, given how one needs to relate to the two different variables at the same time. I presented this task, because drawing the integral function based on the function is not easy and my aim was to highlight, and thereby normalize, this difficulty. Roger responded positively to this task but commented how he did not see it relevant for integrals. However, he said he might want to use it at another time.

One of the tasks I presented, was an idea on how to use integrals to calculate how to construct a dam. This would be a type of realistic and relevant task, but Roger was still skeptical. Even though it is situated in a realistic context, Roger is not convinced this is how they work when constructing dams, and thus might be artificially realistic. He explains how his students have worked in vocational professions and in between them they have a lot of work experience in different professions. These students will react if something is presented as realistic when it is not how it is done in real life.

Roger did not comment on all the tasks or explain why he did not want to use them, but none of them sparked his interest enough to make him want to use the task on the topic of integrals. Still, he was open to look at more tasks on other topics, especially if his colleague Thomas (teacher 2) found some tasks he wanted to use. He knows Thomas has been involved in designing exam tasks on this course, hence he assumes that tasks Thomas would want to use in his mathematics class, would also be relevant for the exam.

5.1.3 Logarithm Tasks

I presented the logarithm tasks which were designed on Thomas' request (Section 6.6) for Roger. This time, Roger was more positive to the tasks than the last time I presented tasks. When I presented the pH-task, Roger's first comment was that this is a type of task that he likes, but not in this curriculum. He explains this by pH or logarithmic scales not being mentioned in the curriculum. The task which Roger especially liked, was the task addressing medicines and different calculations on half time. Still, he comments that he finds it difficult to use the task with his class now, because of limited time before the exam. He reflects on how this task might be suitable to use as a group task where the students can work on it without time restrictions. Even if Roger cannot say exactly when he will use the task, he seems positive to use it in the classroom.
One of the tasks I presented was from the webpage nrich.maths.org and is called Big, Bigger, Biggest (Retrieved from https://nrich.maths.org/386). The task asks students to compare three numbers and decide which one is the biggest, and which is the smallest of the following numbers:

$$2000^{2002} \qquad 2001^{2001} \qquad 2002^{2000}$$

This is a task where Roger's first comment was that the task is not suitable for vocational classes, but for people with special interest. He says the task is nice but too far from the real world for the vocational classes.

When it comes to the historical task with logarithms, Roger was not negative but says this type of task is easy for him to make himself, so he does not need the task I designed. He even comments how he has done this on the blackboard on some occasions. Roger continues to say that the same applies for the pH-task. That is, he could create such a task himself, and he would have changed subtask d) so that the volumes were the same, otherwise the students might think the pH would be an average instead of a logarithmic scale.

5.1.4 Summary

Even though I presented many tasks for Roger, and he even liked some of them, we never got to the point of implementing them in the classroom together. During one of our last talks, we were talking about clothing styles, and I commented friendly with a smile, how he seemed more willing to change his clothing style than his teaching style. Roger replied with yes, because I am rather confident that my teaching works for me, but I am more unsure about how I dress.

5.2 Teacher 2: Thomas

5.2.1 Background and Context

Thomas is a teacher in his late forties with a cand.scient. degree in physics. He did not plan to become a teacher when he first started his studies, but it grew on him, and he chose to take pedagogics after his degree to qualify for teaching. Thomas tells about how 40 % of those with a cand.scient. degree in physics became teachers when he studied, but today almost none of those with a master's in physics become teachers. They work in the industry instead, which Thomas views as problematic as he compares it to eating the seeds. He says we need people with a high degree in physics teaching in school, to recruit new people to the academic discipline. Thomas has been teaching for almost twenty years, teaching mostly physics, but also mathematics. It is his fifth year teaching the course he is teaching now. Thomas is teaching the same type of course as Roger, but the students are younger. They have just finished their vocational schooling and gotten their trade certificate. These students have little work experience before they take this optional course in mathematics. Their aim is to obtain the mathematics they need to start on an engineering degree at the university.

When asked about a typical lesson in his classroom, Thomas' first response is that it is not chaotic at least. He further describes his lessons in mathematics as very traditional where he introduces the topic of the day, gives a lecture on it, shows some examples which are then followed by the class working on related tasks. The students do not ask a lot of questions, which concerns Thomas since he finds it difficult to know if they understand the lecture or not. When the class is quiet, it is difficult to know if they follow the mathematics. Still, Thomas does not see any other options than to let them work on tasks and maybe issues then will surface. Thomas admits that he finds it difficult to discover when students do not understand the mathematics, but he is walking around in the classroom looking at the students work and try his best to help them understand.

Thomas views patience as his greatest strength as a teacher and says this is something he finds very important when it comes to teaching mathematics - to be patient with the student. Also, he is careful not to make students feel uncomfortable or ridiculed in the classroom, but to make them feel safe. Some years ago, he got feedback from some students that he was too strict and too sarcastic on some occasions. He took this feedback seriously, and he has worked on improving and changing how he acts and responds to the students. Communication is what Thomas finds most difficult in the classroom, to get the students to ask questions and communicate what they do not understand. Even this student group, who are planning to take an engineering degree, can ask why they need to learn the mathematics, which surprises him. At the same time, they do not always respond well if he gives them more practical tasks. Since he has a degree in physics, he has the knowledge to make the topics more realistic, but this often means more complex and difficult tasks and the students do not really like them. He has the impression that the students are happier getting tasks where they can use a method they already know. So even if this group of students are hardworking, he finds it difficult to motivate them.

He is not altogether happy with the textbook, and comments that teachers and textbooks never agree totally. To illustrate what he does not like, he mentions how the book explains the unit circle and trigonometry, but only parts of it, so it makes it difficult to fully understand the concept. In addition, he is not always happy with the sequencing of the book and uses vectors as an example. The book alternates between vectors in the plane and vectors in space. He would have preferred to complete vectors in the plane before moving on to vectors in space. When I ask him about tasks he does not like, he replies mathematical models which have nothing to do with the real world, as he sees it.

When I ask Thomas about how he thinks students learn best, his first response is how the students should be as prepared as possible before a lesson and work on the tasks that he gives them. He emphasizes the importance of working on tasks, that is what mathematics is all about. While continuing to reflect on the question, he also comments on how people might have different learning strategies, but his class is a rather homogenous group of students. He says these are his thoughts, but it is not easy to know what the right thing is when it comes to learning. Thomas also mentions how many try to visualize mathematics and try to see practical applications, and how this could be a motivator to learn more mathematics. In a more informal talk, Thomas has told about how he thinks it is easier to make mathematics alive and real at the lower levels, because the topics are easier to relate to the real world. At a higher level, the topics are more abstract.

5.2.2 Logarithm Tasks

They are almost done with logarithms as a topic, but the teacher comments on how he does not like tasks on logarithms like 'lynx population', because he does not find them realistic. He prefers logarithm tasks in the context of physics such as, the measurement of sound (decibel), tides and so on. The next topic with his class is vectors, but Thomas is happier with the tasks in the textbook on this topic. He suggests I can design some tasks on logarithms and trigonometric functions which they can use for the repetition period. Thomas's biggest issue with how the book presents logarithms, is the lack of motivation for why we need and use logarithms. It is presented just as playing with numbers. He would therefore like to use tasks which are more practical, where this is possible. In addition, he misses some historical perspective on logarithms, like how the tables were used to calculate.

When I present my suggestions for tasks to Thomas, I start by explaining that I have not completed them fully or translated them if the original was in English. The reason for not doing so, is to allow Thomas to provide input, and to save time if he does not like an idea. The first task I present is the one with the historical aspect on logarithms (Section 6.6). Thomas decides that instead of making the students calculate by using the table, he will allow them to use the calculators to find the logarithms. That way, the students can work on the idea, but avoid the difficulties by learning to use the table. The other tasks I present to Thomas are some tasks with a practical perspective, which are about acidity (pH value) and half-life for both radioactive substances and medications. In addition, I present a digital logarithmic scale where the aim is to hit the right spot, and a task about finding the biggest number. Thomas is positive to all the tasks and wants to use them. The tasks can be found in Section 6.6.

In the classroom, Thomas explains to the students how we have designed tasks for them, because we do not feel the tasks in the textbook are motivating enough. Mostly the tasks go straight for calculations, and do not expose the practical use you can have for logarithms. He emphasizes how in science; logarithms are one of the most important tools we have. In addition, the tasks are well within the curriculum.

It is relatively quiet in the classroom while the students work on the tasks, and it is difficult to hear any discussions on the solving process. After a while, Thomas goes through the task on radioactivity on the blackboard, asking them questions and trying to get some feedback and questions, but the student group does not respond much. Thomas also gives the students a starting point on the task about medication and half-life before he leaves them to work again. I ask Thomas how he likes the tasks, and if there is anything he would like to change, but he says he likes the tasks. He continues by saying that the way the implementation went, was no surprise to him. When a task gets practical and the students must figure out themselves how to set it up before calculating, they struggle. However, they also struggled with solving the equation after Thomas helped them and set it up on the blackboard. So, the tasks are challenging for the students, but Thomas is happy with the tasks.

5.2.3 Trigonometric Tasks

Thomas asked for trigonometric tasks that were relevant within physics/technology, so I presented many different tasks for him, all related to trigonometry (Section 6.7). Two of the tasks were macros in GeoGebra (Sub-Section 6.7.1). One with the aim of students exploring the trigonometric functions, and the other designed so the students should adjust a sine function to make it fit with tidal data. Thomas says this is something he can give his students, but he comments how the sine function is based on a different formula than the one in the textbook. The formula I presented for the sine function in the macro was $f(x) = A\sin(kx + c) + d$, but the textbook used the formula: $f(x) = A\sin(k(x - c)) + d$. The formula collection the students use, also has this other version of the formula, and this causes problems especially for the low achievers. Thomas prefers the version of the formula in the textbook, and I offer to change the macro in GeoGebra, so it uses the same type of formula.

In addition to the two tasks in GeoGebra, I presented tasks related to music, a pulsating star, and different weather phenomena. Thomas liked all of them and wanted to use them without making changes. He said some of them were similar to tasks in a task book they have, but he is still pleased with getting more tasks of this type. I added a task where the students were asked to reflect on solutions when adding a sine and a cosine function, and at first Thomas did not like this task. The task was formulated:

How many solutions has the equation $\sin x + \cos x = 2$ in the interval from 0 to 8π?

He commented how this is not part of the curriculum, but then he realized that the students should be able to reflect on it and conclude that there are no solutions. So, he wanted to use this task as well.

We completed these tasks so late that the normal teaching period was over, and the students were working towards their exams, and the first one coming up was one in physics. I was therefore never part of any implementations of these tasks, and we did not have the chance to evaluate them together.

5.2.4 Summary

Thomas mostly wanted tasks that were connected to realistic problems in physics and technology, but he was very clear on not wanting to use tasks which he thought were artificially real. Even if he requested tasks connected to realistic problems, he also problematized how students are not always happy with these tasks and find them more difficult. Still, it seemed like he wanted these types of tasks to motivate the students on the usefulness of mathematics. On the other hand, Thomas also seemed to like the more theoretical tasks I gave him, which forces students to reflect differently. He wanted to use all those tasks as well. Two of the tasks I presented for him were macros in GeoGebra, and he wanted to present them to his students even if I never saw him apply GeoGebra in the classroom. Thomas commented on how he has been using TI (Texas Instruments), but he can see how GeoGebra is more applicable, elegant and the curves look nicer. So, he is using an old technical tool which most other teachers have left behind, but he calls himself a conservative type and for his use it works well enough. At the same time, Thomas talks about how he feels computers and technology have become more and more important in mathematics.

5.3 Teacher 3: Hanna

5.3.1 Background and context

Hanna is in her forties and was suggested to me as a possible research informant by a colleague. She was presented as a proficient teacher who also wanted to make changes in her teaching. Hanna has a M.Sc. in biology and has in addition enough chemistry and mathematics to teach those subjects at upper secondary school (at least one year of university studies in a subject is required to qualify for teaching it at this level). She has just achieved her formal teaching competency in mathematics, even though she has been teaching it for several years.

Hanna has been working as a teacher for more than 15 years and started teaching science classes. However, it did not take long before she also taught mathematics, even if she did not have the official qualifications for it. She liked teaching mathematics, so she had for many years wanted to get the courses she needed to obtain the teaching qualification. She struggled to complete the courses in addition to her job, but her leader helped her, and she got admitted into a further education program and obtained her degree. Even if she now has the formal competency to teach mathematics, she has on several occasions expressed uncertainty of her own competency and does not really feel it is enough.

She works at a vocational secondary school and is teaching several different classes and subjects there. The data in this research project is collected from the first-year class ‘Building and Construction’, which she is teaching. The students following this program can continue in many different directions, and they may become craftsmen in as many as 18 different vocations. These include among others: road construction, bricklaying, carpentry, metal working and painting. During the first months of the school year all the different directions attend the same class, and they do not make any choices of specialization in one vocation until February. The first data collected from the collaboration with this teacher is therefore from a mixed class of all vocational directions. The data collected after February (The index task, Section 6.5), is from a class with students studying to be carpenters or painters. Becoming a carpenter is rather popular, so the general achievement level of the class is higher after the split in February.

Hanna describes her typical approach to mathematics classes as her explaining on the blackboard and then the students work on tasks, but this is a way of working that she wants to change. This type of teaching works better in the mathematics she teaches at the ‘supplementary program for general university and college admissions certification’, as these students are older and more disciplined. When I ask her why so much of her teaching is like this when she wants it to be different, she says it might be because she does not have a lot of education in mathematics, and feels she lacks knowledge and has a limited register. She also feels she lacks didactics in mathematics and has fewer tools than in science. She has a genuine wish to improve her mathematics teaching, but asks: ‘what do you do?’

Hanna expresses how it is important for her to be a friendly and patient teacher who wants the students to feel secure around her and to come to her if something is bothering them. That they feel it is ok to get help from her. Her aim is to be a friendly teacher and have a classroom where she can make jokes and have a good atmosphere with the students. If she has that, she also feels the lesson flows better, both with respect to explanations and lesson plans, than if she is a grumpy teacher. She does not worry about having classroom discussions and walks around the classroom trying to prompt discussions with the students as she goes along.

Hanna says she thinks students learn best by doing things themselves. She is adjusting and specifying this claim when it comes to quality versus quantity. It is not only about doing many tasks; it also depends on what type of tasks you are working on. Hanna has an opinion that if students can explore on their own and reach conclusions on their own, then the understanding will be retained better than if they just memorize without understanding why. She says that when it comes to just passing the exam for low achievers, one might think that it works with mechanical learning for a brief period, but the probability for them to forget this relatively quickly is high. However, if one manages to get them to do things on their own, to explore, then they will remember it longer because they understand. There has been a debate among the staff in the school concerning training procedures versus understanding, and the answer to this might differ across situations. If the aim is just to get someone to pass, procedures might be better, but then you more easily forget. In the end of our collaboration, Hanna expressed surprise over how most of the tasks we had designed were not open tasks, which she had assumed they would be if the students should explore on their own.

Hanna says she wants an introductory task; an activity where the students discover instead of her telling. She mentions several topics where this could be relevant such as area, Pythagoras and similarities. Hanna talks about area as an important concept for the carpenters, and it has a lot of practical elements which can be used with regards to their vocation as well. The teacher expresses that the task should be motivating, something that keeps the students going throughout the task. She also talks about a meaningful activity, so they understand a concept.

When I repeat the question of what kind of tasks she wants, she responds introductory tasks. She wants a way to introduce a topic without her being at the blackboard talking. The students should do an activity themselves, and when they are done with the activity, preferably many of them will understand the concept. The teacher talks about what she has tried before, like for instance tangrams which is a dissection puzzle where the students need to put together the pieces to form shapes, but she questions the transfer value. The students worked on the tangrams, but they are easily bored and restless. The teacher sees this as a challenge for herself as well, to not give the students too much help. Motivation is difficult - to get the students to want to explore. This will also differ between different classes.

5.3.2 The A4-task (Proportion)

Hanna said clearly that area is the most important topic to her; however, the first task is about similarities because this topic is relevant earlier in the school year. When we met, I had planned to discuss several ways of making tasks she might want to use, but I started off by showing her a task I had made together with her colleague (Section 6.1). She immediately liked it and wanted to use it. I tried to say we could make any changes she might want, but she is uncertain of what those changes might be and continued by saying this is one of her limitations - to see what might work or not. After some consideration, she chose to make a change to question six and make a more visual version of the task. The new formulation asks the students to place the different formats on top of each other's, so it is visually possible to see that they are similar and share the diagonal. Hanna wanted to use this task right away in a class she was to lecture in two hours, so we just sat down and completed the task together. I could not join her in the first class, because I did not have written permissions from those students to film, so I had to wait until she used it with the Building and Construction class.

When we evaluated the task together, Hanna's first concern was whether she said too much or too little. She wants the students to discover on their own, but she also wants to be sure they learn important concepts such as ratio, similarity and so on. She finds it difficult to balance how much she should talk in the introduction and for the summarizing. When it came to activity, she commented that there are some students that almost never do anything, and she hoped this task would get them to work more. It was still a struggle to get them to work, but they started on the task, which is an improvement. Hanna also had a long and nice talk with one of the students and tells that this has been difficult to achieve before. So, this task facilitated a conversation with this student and the student seemed engaged with the task. The teacher felt she had many good conversations throughout the lesson but was unsure about the summarizing. However, she comments that it might not be worth worrying about.

In the first class where she implemented the task, there was a girl who really 'blossomed' with the task and continued exploring on her own. She did not get the same extreme response by anyone in the class I observed. Hanna also felt she spent most of the time on the three first tasks and did not really get past them. We discussed if it might be helpful to summarize in between the tasks, but then they need to be physically separated on different sheets of paper. She wants to use the task again, and refer to it, but she is unsure if she understands the task well enough to get the students to work all the way through it.

5.3.3 The Area and Rope Task

The next time I met with Hanna to present ideas for tasks, I wanted to have many different options so I could get an impression of what she would prefer and go for. I had recently designed an area task together with her colleague (Section 6.2), but I deliberately waited to tell her about it because I did not want the 'easy' solution of just using a task she would not have to refine on her own.

One of the ideas I had for tasks were about the Pythagorean theorem and was related to practical issues for carpenters. She seemed somewhat positive, but then we just kept talking about other tasks and she never brought it up again. The next thing I presented was a rope task where one uses a rope which has the same length as one-self, to measure and create shapes with (Section 6.4). This was a task she immediately responded positively to and wanted to use. The task is not directly vocationally oriented, but it is exploratory. After we had talked a while around the details of this task, I also showed her the area task I had designed with her colleague (Section 6.2). As I suspected, she also wanted to use this task.

When it came to evaluating the rope task, Hanna was both happy and not happy with it. She thought some of the students worked well and used the opportunity to discover, while others were wasting time, doing things they were not supposed to. Hanna talked about the difficulty in getting the students to work and to be accurate, and said she is unsure if investigative tasks in this class will work. There are some students where she questions whether they have any curiosity on mathematical questions at all? But more students worked on this task than normally, so it is an improvement.

When I presented the area task, Hanna was enthusiastic and said that these are the kind of tasks she has been looking for. We did not make any changes from the way it was presented for the other teacher. Hanna was still happy with the area task when we were evaluating it and talked about some of the tasks being illustrative, making it easy to see that the area of the parallelogram equals the calculation of a rectangle. She commented that many students worked, including some of the students who are difficult to activate, hence this was the task she was most happy with so far.

One thing which surprised me, was when Hanna revealed how she had hoped none of the students would get around to task five, because she was not sure how to solve it herself. We had a closer look at the task together, and Hanna now expressed how the solution was actually very easy. She had just not had the time to look closer into the task before the lesson started but was still prepared to use it in the classroom.

5.3.4 The Index Tasks

When presenting my ideas for the last tasks for Hanna, I was really challenging her. She had expressed that she wanted something on the topic price index, because this is a concept which can be difficult to understand and which the students have not related well to. I had some ideas and presented the web pages of Statistics Norway, where one can find various prices and how they have developed over the years. I suggested using these pages and maybe relating it to building expenses, which is relevant for students who are becoming carpenters. At the same time, I said that this is something you would need to have ownership of yourself, and not just a task I can write down and hand it over to you. As a teacher, you will need to decide what kind of discussions you would like to initiate and how to use the web pages. Hanna accepted the challenge, and even if it took some time, she created this set of tasks all by herself and just asked me to check if it seemed ok (Section 6.5).

Hanna was not totally happy with the implementation, but she felt the class was engaged. She expressed that the first lesson was a bit chaotic, because even if she had written down point by point what the students should do, they did not read the information carefully. Therefore, she thinks they need even more structure. She regretted asking the students to find a house on the Internet, because too many of them spent too much time on this issue which was not important. Next time, she would just have given them a prospect of a house to start with.

However, even though it became a bit noisy, she felt that many of the students had some kind of understanding when she summarized on the blackboard.

5.3.5 Summary

Hanna is open to try new things and tasks in her classroom, and even hands out a task to the students which she has not had the time to solve herself. She explains how she is mostly teaching by giving lectures from the blackboard, but she wants to make changes. However, she is not sure what to do and expresses uncertainty when it comes to her own didactical knowledge in mathematics. Hanna asks for introductory tasks for mathematical topics and wants tasks that will get the students working. She believes the students will learn mathematics by doing it, and not just in a mechanical way but by making connections and developing a deeper understanding. Still yet, although Hanna was talking about area being an important concept for carpenters, she did not request changes to the area task to make it more vocationally oriented. This was the task she was happiest with, even if it was not vocationally oriented. So, it seems that she finds a task activating the students more important than a clear vocational connection.

5.4 Teacher 4: Sven

5.4.1 Background and context

Sven was asked to be a part of the research project on Hanna's request. She expressed a need for someone to collaborate with and discuss with both during the research project and afterwards, and Sven was a colleague she had been working with for some years. Sven was positive to be part of the research project, however he expressed this was something he did because he felt it would be beneficial for him as a teacher, and not because he had a heart of gold.

Sven is a man in his mid-thirties and highly educated with seven to eight years of higher education. He started taking university subjects in science and continued with an education which included both science and mathematics. In addition, he has a M.Sc. degree in mathematics education. Sven has worked for three and a half years at the secondary school where he is now employed, and has been teaching both science and mathematics mostly in vocational classes. Previously he has worked part time at another secondary school for adult students who are earning their certificate in general studies. In addition to his education and work background, Sven is part of the official group who makes the local exams for vocational students. This year, Sven is teaching mathematics in a vocational class of Design, Arts and Crafts, which mostly includes young girls wanting to become hairdressers. This is the class where I am following the implementations of the tasks we design. He also teaches a theoretical mathematics course for students attending General Studies at the same school, but I did not introduce myself to this class, due to positioning my research within a vocational context.

When talking, Sven expresses himself with certainty and gives an impression of knowing what he wants and does not want. He often refers to the official curriculum and how he interprets it and uses it in the classroom. None of the other teachers refer to the curriculum at the same detailed level. One of the reasons for Sven's extra emphasis on the curriculum, might be a result of his work in the group designing exams, since they must make sure the curriculum is well covered in the tasks. When we talk about the textbook they use in mathematics at his school, Sven is not happy with it, and explains this partly by how the book interprets the learning outcomes in the national curriculum differently to himself. While the textbook focuses on formulas and rules, Sven expresses how mathematics in these courses should be more related to practical and concrete cases. If the students have not learned how to solve equations by using algorithms during ten years of schooling, he does not see why he should be more successful this year. However, he says the students can figure out the mathematics when presented for a situation which might be modelled by an equation, and this is how he interprets the learning outcomes of equations for vocational students.

Sven expresses how some of his strengths as a teacher are that he is patient and accepts that not all students have to like mathematics. On the other hand, he finds the diversity in the vocational classes a challenge. The high achievers already know and understand the mathematics in this course, while the low achievers are struggling both with the mathematics and to engage in work.

When it comes to characteristics of tasks he wants to use in the classroom, he would like types of tasks where everyone can get started and where the task both challenges the high achievers, while the low achievers can attain some understanding while working on the tasks. He wants the students to understand and not just do the mathematics. As an example, he is frustrated about how many of the students memorize the area formulas instead of realizing that if you understand how to calculate the area of a triangle, you can calculate all other shapes too.

When talking about what type of tasks he wants to use in the classroom, he is not specific on the details, but describes them as a type of low threshold - high ceiling tasks where the students will gain mathematical understanding while working on them. On the other hand, when I ask Sven to give examples of tasks in the textbook he does not like, he gives some specific examples of characteristics in addition to more general descriptions. Overall, he is not happy with the textbook being rather mechanical in how it presents mathematics and tasks, and how the students are expected to learn a formula and then use it to solve tasks. In addition, he gives an example of a task he does not like for these reasons: It is a very long task, lots of text, not much air between the words and a long formula. Sven is, throughout our collaboration, specific and clear when it comes to what he does not believe in or does not think will work, and he has a focus on details as well as the bigger ideas.

Sven would like us to design tasks within proportions as a topic. He says this is a topic which you find across most of the curriculum topics, even if the students do not always see it that way. In addition to proportions, he mentions how geometry is a big topic in the curriculum, however they have already planned a collaboration with the vocational course making gingerbread houses. Still, he thinks calculating area might be a good topic, because he wants the students to understand how to do the calculations and not just memorize the formulas.

5.4.2 The A4-task (Proportion)

The first topic where Sven asked for tasks, was proportions because it can relate to so several parts of the curriculum. At this time, he wanted to link the tasks to similarities and scale. He needed the tasks already the following week, so we did not have a lot of time to design the tasks but decided to make the most of it within the time limit. One week later I presented the A4-task (Section 6.1) and explained my thoughts concerning how the students might explore on various levels. At this point, I had not formulated specific questions, just presented how different formats of papers are proportionally related and how it could be explored by calculating lengths and areas, and even prove the proportions between the sides using the Pythagorean theorem.

Sven listened and looked at the tasks and then expressed his concerns. He thought the task was too difficult for the students and commented that classroom discussions are not possible in this class. Sven explained this by how the students do not want to engage in classroom discussions, and how he has not prioritized this in the brief time he has taught the class. In addition, he commented that many possibilities in a task are nice, but it is important to have a starting point that everyone can master. At this point in the collaboration, I realized how different it can be to design a mathematical task for another teacher than it would have been to design for my own teaching. Whereas I would have focused on classroom discussions and getting the students to talk, this is something Sven had chosen not to prioritize. He knew it would be beneficial, however since he had this class for about seven months only, and they struggled with mathematics in several ways, he had to make some choices and a focus on classroom discussions was not prioritized.

Even if Sven had some initial concerns about the A4-task, he expressed that it could be a good starting point. However, he wanted it to be clearly formulated with explicit goals, otherwise he said it can be hard to get the students to work. The students had not shown any willingness to explore, so they needed clear and specific questions, and clear instruction on what is expected from them. Otherwise, they will complain that it is too difficult and just give up - even the high achievers. As a result, we started the task by making a table the students were supposed to fill out, with specific directions. Further, a series of questions was designed for the students to realize the relationship between lengths and areas of the different paper formats (Section 6.1).

The plan was that I should observe the implementation of the task, but I was unfortunately not able to do this. However, Sven and I got the chance to talk and evaluate the implementation of the A4-task later that same day. He told how the students started working straight away and that the group of students were more active than usual. This was positive, and all the students could manage to do something. However, he would have changed the wording in task 2 from asking the students how much the area increases, to how many times larger the area is. This is because too many of the students misunderstood and wrote the difference instead of multiplying. Another change Sven would like to make to the task, is to start with the relationships between the sides instead of the areas, because this is closer to how the curriculum is interpreted, and thus what they might get on the exam.

Some of the students were using too many digits when calculating, which gave them problems when trying to generalize what was happening. Many of the students answered task 5 wrongly by suggesting four and then eight, when the teacher questioned their first answer. This is a task where they easily could have tested their answers by placing A4-papers on an A0-paper, and I asked Sven if any of them attempted to do so. Sven responded no and explained how it is very difficult to get the students to try things out. He has tried to show students how they can use sketching and other methods as an aid in solving mathematics, but he experienced that they seldom choose to do so, and even resisted this way of working. He thinks it might be related to this being an unfamiliar way of working for the students during their schooling in mathematics.

Sven did not get the time to summarize the lesson, which he sees as important when working on tasks like this, especially with respect to the curriculum and the exam. He says this is something he will follow up the next lesson, and his goal is that all the students should know how to use proportions to calculate unknown sides.

5.4.3 The Area Task

The next topic Sven wanted to focus on, was area and an understanding of how one can use triangles to calculate other areas. Even if Sven is rather specific on what he wants in the task, I present several ideas for him, so he has some choices. One idea is to use a rope to explore circumference and area (like the task I designed together with Hanna, Section 6.4). I also show him a webpage with different animations of how we can calculate different areas, which he likes. In addition to this, I show him the tasks on parallelogram, trapezium and the four identical triangles, which end up being part of the final task (Sections 6.2 and 6.3). However, Sven wants an even lower threshold as the starting point for the task and suggests a rectangle. One of his goals with this first task with a rectangle, is for all the students to be familiar with having to use different measurements for area than for length.

When working together with Sven, he has just as much ownership of the final task design as I have. He even comments at some point that he worries he is taking too much control on how to formulate and what questions and tasks to use. I tell him not to worry, because these tasks are his, and it is important that he is happy with them. So, even if I present the first ideas, Sven adds subtasks and formulations, and seems confident in what he thinks might work and what he wants. When it comes to the area task, he decides not to use GeoGebra to explore, but comes up with the idea of using two triangles to calculate a parallelogram as a subtask. In addition, he wants every task on a separate piece of paper. His reasons for this, are both so that he can pace how the students work to make it easier to summarize together with the whole class, but also so he can differentiate by giving high achievers different tasks in between. He is for instance using the task with identical triangles for this purpose. When it comes to the trapezium task, Sven wants us to write some more subquestions besides asking them to calculate the area in as many ways as possible. He thinks this wording might be too open and thus would leave the students not knowing what to do. So, he suggests that we also ask them to measure and find all the lengths they need to calculate the area.

I observed the implementation of this task in a vocational class, but he also used the same task in a general mathematics class at the same school. However, this class I did not observe because I did not have any confirmed consents from these students. He thought the task worked very well in the general mathematics class and well in the vocational class. Sven made some changes to the task, and one of them was to give the students millimeter (graph) paper when they were solving the first task with the rectangle. He also added several subquestions to task 2 with the parallelogram (Section 6.3). Previously when having this topic, he had presented the formulas prior giving the students tasks. Some students finished the tasks within five minutes, and then he spent the rest of the time on group explanations, explaining to those who did not understand. However, with this task, almost everybody worked, and he got the impression that some of the low achievers had some aha moments indicating that they gained new insights, and this is something they do not experience normally. At the same time, there was also a girl who asked what she was supposed to do even before she had had a proper look at the task. Sven describes this girl as a high achiever in mathematics, but not showing much interest in mathematics. She is normally looking for a way to finish as quickly as possible and becomes negative if the task is too open.

5.4.4 Summary

Sven liked both tasks we designed and plans to continue using them in his teaching. However, when asked to compare the two tasks, he preferred the area tasks. His reason was that the A4-task is a bit more difficult for them when it comes to generalizing their newfound understandings to textbook tasks. So, he saw the area task as having more impact when it came to the curriculum and exam. However, both tasks had a low entry point where everyone could get started, which is something he finds valuable. Since he only taught the vocational class until February, he did not feel the need for making more tasks after the area task. He already felt he had good tasks when it came to personal economy, which was the last topic before the exams.

5.5 Summary of the Collaboration with the Teachers

The four teachers are teaching very different classes, even if all classes are vocationally oriented. Roger and Thomas are teaching an optional mathematics course, but something the students need to complete if they want to study to become an engineer at a university. So, these groups of students are highly motivated, not only to pass this course, but also to attend further mathematics courses at the university. This contrasts the vocational classes Hanna and Sven are teaching. Their students are younger and some of them are probably not motivated neither for mathematics nor school in general. However, since it is difficult to get a job without education, most youngsters start at upper secondary school regardless of motivation. So, even though all the four classes are vocational of some kind, they are quite different when it comes to how motivated and committed the students are. This is also reflected in how diverse the classes are. Roger's and Thomas' classes are quite homogenous, while Hanna's and Sven's classes are rather diverse. Their classes include students who have done well in school, are hard workers and want to excel in their vocation, but also students who have been struggling in school, who did not understand mathematics at lower secondary school and are not interested in making any effort. These classes are a lot more diverse than Roger's and Thomas' classes. The difficulty level of the curriculum is also different. While Roger's and Thomas' classes are integrating, calculating with logarithms and trigonometric functions among other topics, Hanna's and Sven's classes are basically just getting a repetition of what they already should have learned in mathematics at lower secondary school.

All the four teachers explain how they mostly teach by lecturing at the front of the classroom, followed by giving the students tasks to work on. However, both Hanna and Sven express that they are not happy with this and would like to change their teaching approach. This is also evident in the mathematical tasks they request, for instance when Sven expresses how he would like a task where he could pull more back as a teacher and Hanna talks about wanting an introductory task instead of her presenting the new topic. Roger and Thomas do not express any wish to change from the teaching style of lectures given from the front of the room. However, all the teachers express that they think students learn when working on tasks and doing things themselves. This is something all the teachers view as important.

I have now given presentations of my collaboration with each of the four teachers, and I have also presented a summary of some of the similarities and differences in the context the teachers work. The next chapter is a presentation of the tasks I designed together with the teachers.

6 The Tasks

I will in this chapter present the tasks I designed together with the teachers, but also some of the ideas I presented that were never implemented. I present tasks used in the collaboration with Hanna and Sven in Sections 6.1-6.5 (these have been translated from Norwegian) and with Roger and Thomas in Sections 6.6-6.8 (these were mostly originally in English and were translated into Norwegian). Here is a list over the tasks presented in this chapter:

6.1 A4-task, used by Sven and Hanna.
6.2 Area task, first version, used by Sven and Hanna.
6.3 Area task, revised version by Sven.
6.4 Rope task, used by Hanna.
6.5 Indexes, designed by Hanna.
6.6 Logarithm tasks, used by Thomas.
6.7 Trigonometric functions, used by Thomas.
6.8 Ideas for integral tasks for Roger, not used.

6.1 A4-task, Used by Sven and Hanna

Task 1: Fill in the table below.

	Length of		Ratio between the sides	Area
Paper Format	**Long side**	**Short side**	**Long side divided by short side**	**Long side multiplied by short side**
A0				
A1				
A2				
A3				
A4				
A5				
A6				

Task 2: **a)** How much does the area increase from A4 to A3?
b) How much does the area increase from A1 to A0?
c) Can you make a general statement on how the area increases?

Task 3: What is the ratio between the sides of a sheet?

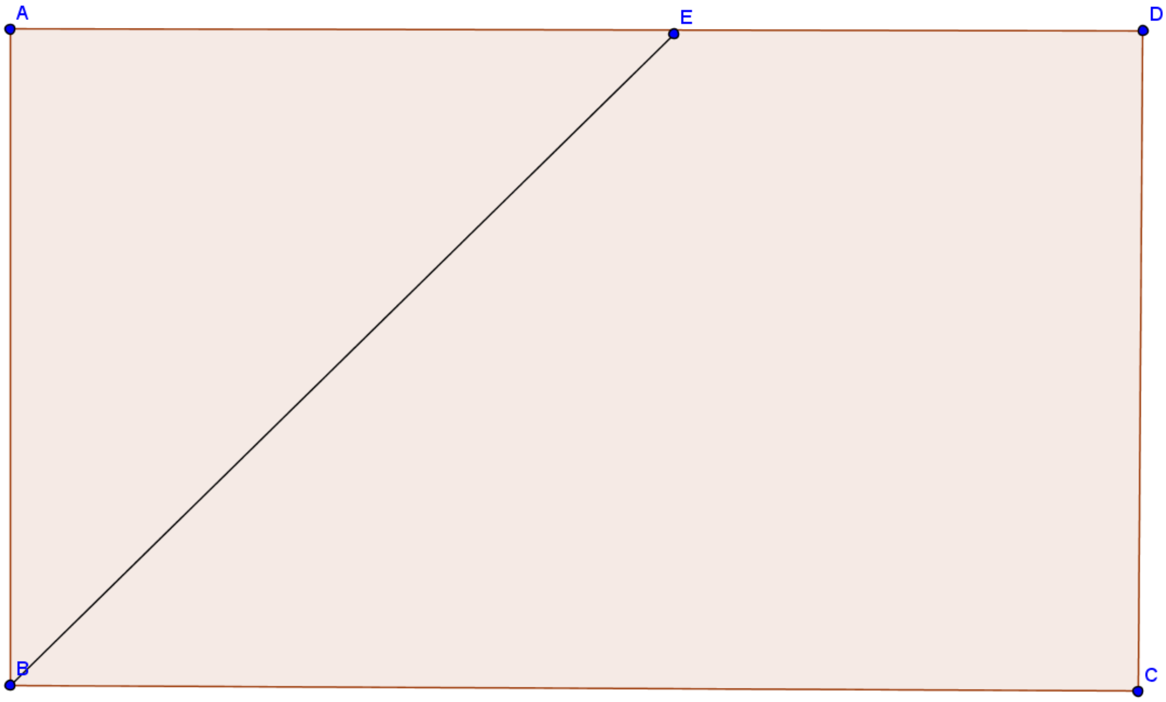
Task 4: **a)** What do you have to multiply the length of an A4 sheet by to get the length of an A2 sheet?

b) What do you have to multiply the length of an A6 sheet by to get the length of an A2 sheet?

Task 5: Both the length and the width of an A0 sheet are four times as long as the length and the width of an A4 sheet. How many A4 sheets do you need to make A0?

Task 6: Do the following folding, both with an A4 sheet and an A5 sheet. Fold the sheet so you get a square. That is, fold along the line BE as shown in the figure below.

Then fold so that corner C meets the point E.

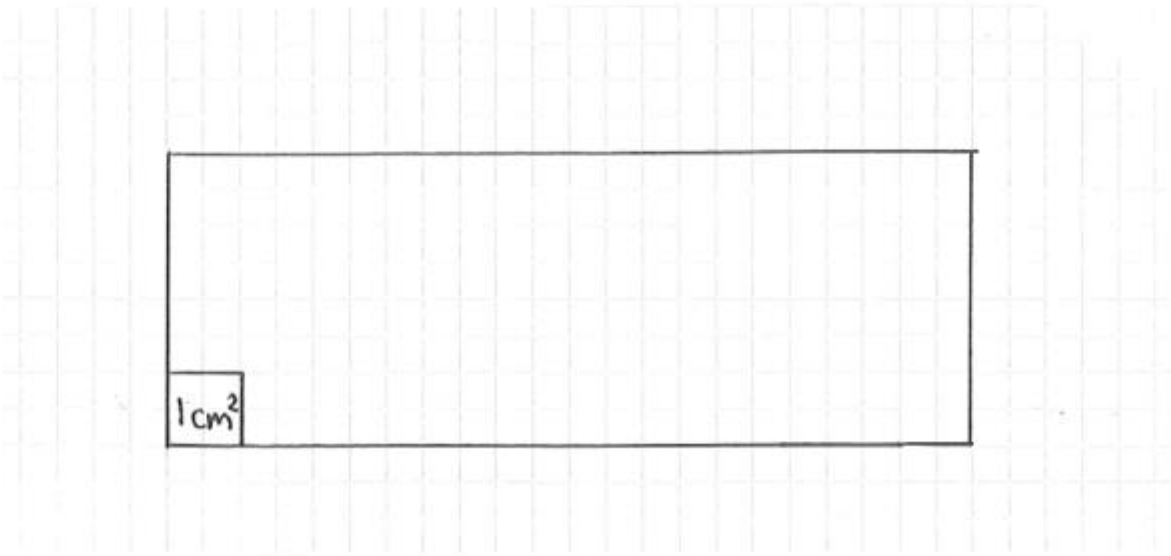


On the A4 sheet, the line BE will be 29.7 cm. Calculate how long the same line will be on the A5 sheet.

6.2 Area Task, First Version, Used by Sven and Hanna

Each of these tasks were presented on separate pieces of paper for the students, but for practical reasons I present them without the same spacing in this report.

Task 1: Rectangle



a) How many cm^2 fit into the figure?

b) What is the area of the rectangle?

c) What is the area in mm^2?

d) $1\ cm^2 =$ ___ mm^2

Task 2: Parallelogram

Draw a parallelogram and cut it out. Use a pair of scissors to make *one* cut so that you can assemble the two pieces into a rectangle.

Task 3: Triangles and parallelogram

Make two identical triangles and cut them out. Assemble so you get a parallelogram.

a) Calculate the area of one of the triangles.
b) What is the formula for the area of a triangle?

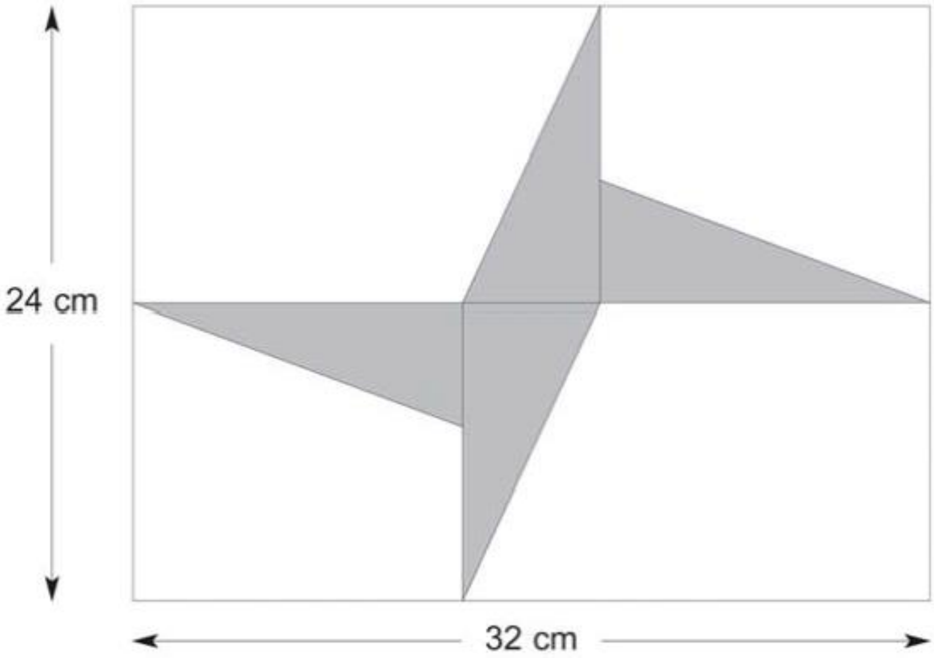
Task 4: Trapezium

Measure and find the lengths you need to calculate the area of the figure.

There are many ways of calculating the area. Find the area in as many ways as possible.

Task 5: Area

These are four identical triangles.



a) Calculate the area of one of them.

b) Calculate the area of the part of the figure that is not shaded.

6.3 Area Task Revised Version by Sven

Task 1: Rectangle

a) Draw a somewhat big rectangle on the millimeter paper that has been handed out. Draw along the thickest lines.

b) Use a ruler and be accurate when you draw.

c) Mark/draw a square one of the corners of the rectangle you have drawn. The area of the small square should be 1 cm^2.

d) How many of these «square centimeters» (1 cm^2) fit into the rectangle you have drawn?

e) What is the area of the rectangle?

f) What is the area in mm^2?

g) 1 cm^2 = ___ mm^2

Task 2: Parallelogram

a) Which measurements do you need to know to calculate the area of a rectangle?

b) What is the formula for the area of a rectangle?

c) Draw a parallelogram and cut it out. Use a pair of scissors to make *one* cut so that you can assemble the two pieces into a rectangle.

d) Use exercise c) to find the area of the parallelogram you drew.

e) In a parallelogram: which measurements do you need to know in order to calculate the area?

f) What is the formula for the area of a parallelogram?

Task 3: Triangles and Parallelogram

Make two identical triangles and cut them out. Assemble so you get a parallelogram.

a) Calculate the area of one of the triangles.

b) What is the formula for the area of a triangle?

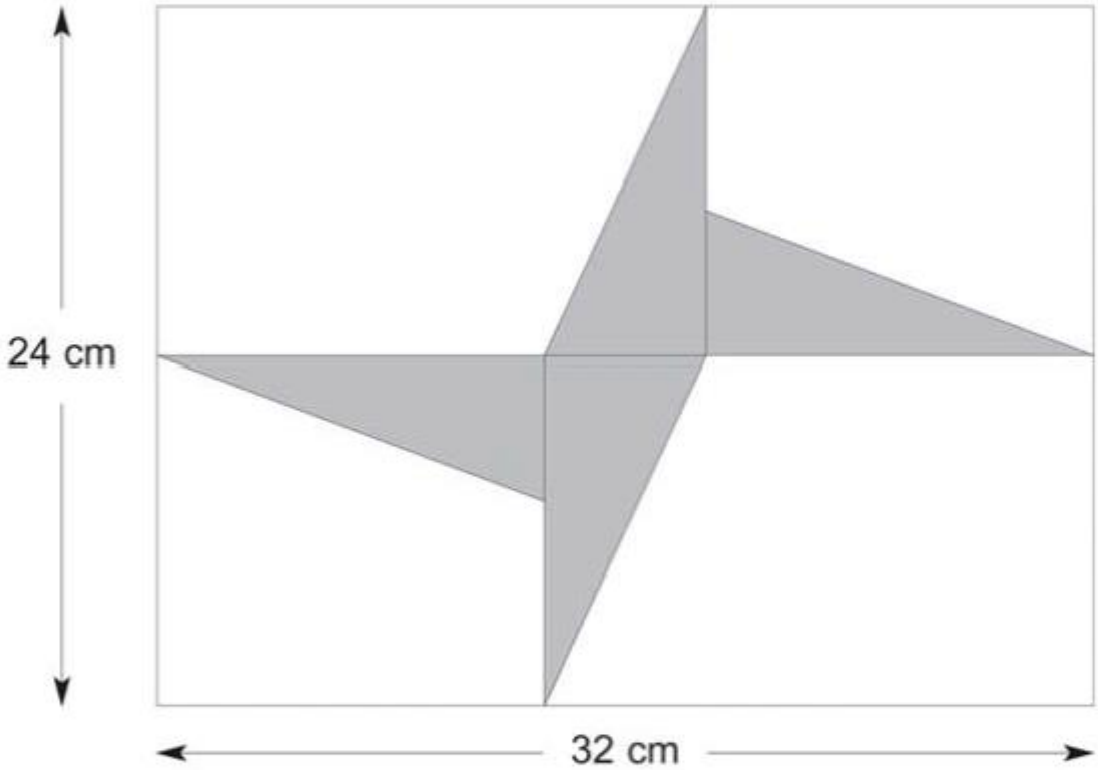
Task 4: Trapezium

Measure and find the lengths you need to calculate the area of the figure.

There are many ways of calculating the area. Find the area in as many ways as possible.

Task 5: Area

These are four identical triangles.



a) Calculate the area of one of them.

b) Calculate the area of the part of the figure that is not shaded.

6.4 Rope task

Task 1: Cut a piece of string so the length is the same length as your own height. How long is it in:

_____m? ______cm? _________mm?

Task 2: Use the string as the only aid to measure the length of___
[*Some students were measuring the length of the classroom, other students measured lengths in the hallway*].

Task 3:

a) Use the string to make a figure which has the area 1200 cm^2
b) Use the string to make a figure which has the area 100 cm^2
c) What is the minimum area you can make using your string?
d) What is the maximum area you can make using your string?

Task 4: Pair up with another student and make two similar figures out of your strings. Explain why they are similar.

Task 5: Use pieces of tape to divide your string into twelve parts that are the same size. Make a right-angled triangle, using the string where all sides must consist of «whole» parts.

6.5 Indexes

Task 1: Building a house

a) Enter the webpages of Statistics Norway, and find the **Construction cost index** for residential buildings http://ssb.no/priser-og-prisindekser/statistikker/bkibol
   (Do not close the tab when you have found it…)
b) Then enter the webpages of www.Finn.no [*biggest webpages for buying and selling in Norway – researchers comment*].
c) Go to new homes (http://www.finn.no/finn/realestate/newbuildings/browse1) and find a house that you would like to live in – in an area where you would like to live.
d) How much does this ready-to-move-in house cost today?
e) Go to the **Construction cost index** and use the calculator placed on the right side.
f) Use the calculator and explore:
   Approximately how much would the house you have chosen costed the year you were born?
g) How much more expensive (in Norwegian kroner and in percentage) is the house today?
h) Why do you think it is more expensive?
i) Explain the change as well as you can.
j) If you are to start your own company – what do you have to consider when you are calculating how much to charge your customers as the years are passing?
k) According to the Tenancy Act paragraph 4.2, one cannot increase the rent more than CPI.
   Find out what this means?

Task 2: Calculating with Indexes

Going through a calculating example on the blackboard.
The table below shows the price index for new detached houses in the period from 2003 to 2008

Year	2003	2004	2005	2006	2007	2008
Price index	120.1	124.3	134.6	140.3	152.6	170.1

A particular type of detached house cost 1 900 000 in 2003. What would an equivalent detached house cost in 2008?

The table below shows the consumer price index (CPI) from 1996 to 2008.

Year	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**
CPI	**95.3**	**97.8**	**100**	**102.3**	**105.5**	**108.7**	**110.1**	**112.8**	**113.3**	**115.1**	**117.7**	**118.6**	**123.1**

In the following tasks, you will need some of the consumer price indexes in the table.

3.1.8
Eivind bought new skis in 2003 for 1 490 kroner. How much does the same skis cost in 2008, if the price of the skies followed the consumer price index?

3.1.14
Miriam got 1000 kroner in pocket money in 2004. How much pocket money should she get the next year if her purchasing power should remain the same as in 2004?

6.6 Logarithm Tasks

6.6.1 Historical

Copyright picture: National Galleries of Scotland, Scottish National Portrait Gallery.

John Napier, baron of Murchiston (born 1550 – dead 4. April 1617) was a Scottish landlord and mathematician.

Napier is considered the inventor of logarithms. During his work of simplifying time-consuming calculations in navigation and trigonometry, he found that any number could be written as a power, and that for example multiplication can be transformed into a sum of the exponents of two powers with the same base. $4 \cdot 16 = 64$ can be written as $2^2 \cdot 2^4 = 2^6$, and the calculating will then be $2 + 4 = 6$ with the following calculation of $2^6 = 64$. Using this method, calculations at sea that would previously take an hour, would be reduced to minutes.

Napier died before the work was completed, and it was completed by Henry Briggs (1561-1630), Professor of geometry at Oxford. He further developed the concept and made tables with 10 as the base number. Therefore, logarithms with base 10, are today called Briggsian logarithms. In 1624 he published the book Arithmetica Logarithmica, which includes a table of the logarithms to numbers from 1 to 20 000.

Example of multiplication by the help of logarithms.

If you are to calculate $537.6 \cdot 2.642$ it can be transformed into logarithms, and then use the logarithms to calculate: $\log 537.6 + \log 2.642 = 2.7305 + 0.4219 = 3.1524$. If we take the antilogarithm to 3.1524, then we get 1420 which is equal to $537.6 \cdot 2.642$.

Example of finding roots by the help of logarithms.

The logarithmic tables can also be used to simplify the calculations of roots and exponents, like for instance the following expression:

$$X = \sqrt[3]{\frac{60.27 \cdot 70.34}{0.27}}$$

By the help of the calculation rules for Briggsian logarithms, this can be simplified to:

$\lg x = \frac{1}{3}(\lg 60.27 + \lg 70.34 - \lg 0.27) = \frac{1}{3}(1.78011191 + 1.847202364 - (-0.5686362358) = 1.39865017$

By taking the antilogarithm of 1.39865017, we further find the solution to the expression.

X = 25.04091361.

By the help of logarithmic tables, it was possible to simplify rather complicated calculations.

Use logarithms to solve the following tasks:

1. $\pi \cdot 236.7$
2. $\frac{20.36 \cdot 5.789}{1.309}$
3. $\sqrt[5]{70.36 \cdot 3.475}$
4. $(\frac{20.652}{3.456})^4$

6.6.2 Mixing pH.

(Retrieved from NRich: https://nrich.maths.org/6167)

The pH of a solution is defined using logarithms as

$$pH = -\log_{10}[H^+],$$

where [$\boldsymbol{H^+}$] is the concentration of $\boldsymbol{H^+}$ ions in mol/l of the solution.

Task 1:

a) Given that the pH of a beaker of pure water is 7, work out how many $\boldsymbol{H}^{+}$ ions there are in 1 litre of the water.

b) A strong acid has a pH of 2. If one litre of this acid is diluted with 1 litre of water, what is the pH of the resulting solution?

c) A strong acid has a pH of 1.3. If I have 100 ml of this acid, how much water needs to be added to create a solution of pH 2?

d) 400 ml of an acid of pH 3 is added to 300 ml of an acid of pH 4. What is the resulting pH?

6.6.3 Medicines and Half-life

(Retrieved from Nrich: https://nrich.maths.org/6457)

Drugs that are to be taken regularly by patients (such as anti-depressants) are often described as having a half-life: a time required for the body to clear half of the remaining levels of the initial dose of drug. For example, after one half-life, one half of the initial dose of drug remains in the body; after two half-lives, one quarter of the initial dose of drug remains in the body, and so on. As drugs are taken on a regular basis the levels in the body build up until steady minimum and maximum levels are reached.

The effective half-life of the drug Venlafaxine is about 12 hours. Suppose that a single dose of 100 mg of Venlafaxine is administered on Monday morning. On which morning will the level of the drug first have dropped below 10 mg?

Another tablet is given on Wednesday morning. What levels of the drug will be left in the body on Friday morning?

To be effective, drugs need time to reach steady minimum levels within the blood. If one of these tablets is given each morning, what will be the final steady minimum level?

If one of these tablets is given each morning and each evening, what will be the final steady minimum level?

Determining the correct dosages of drugs for individuals can be a difficult business, especially since it takes time for the drug levels in the body to reach stable levels. That is, changes in dose will only reach full effect several days later. In this second part, we look at the effects of Fluoxetine (otherwise known as Prozac) in the body.

Fluoxetine has a half-life of between 4 and 6 days, depending on the individual. What would be the stable, long term peak level of the drug for a patient taking a regular dose of 20 mg of fluoxetine per day?

To match this peak level, what equivalent weekly dose would need to be taken? In each case, what are the lowest and highest long-term levels of drug in the body? What issues might arise for the patient? Would missing a tablet cause problems?

6.6.4 Radioactivity

The half-life of radioactive cobalt is 5.27 years. Assume that after a nuclear accident, the level of cobalt radiation is 100 times as high as acceptable for humans to live there. How long does it take before the area is livable?

6.6.5 Big, Bigger, Biggest

(Retrieved from Nrich: https://nrich.maths.org/386)

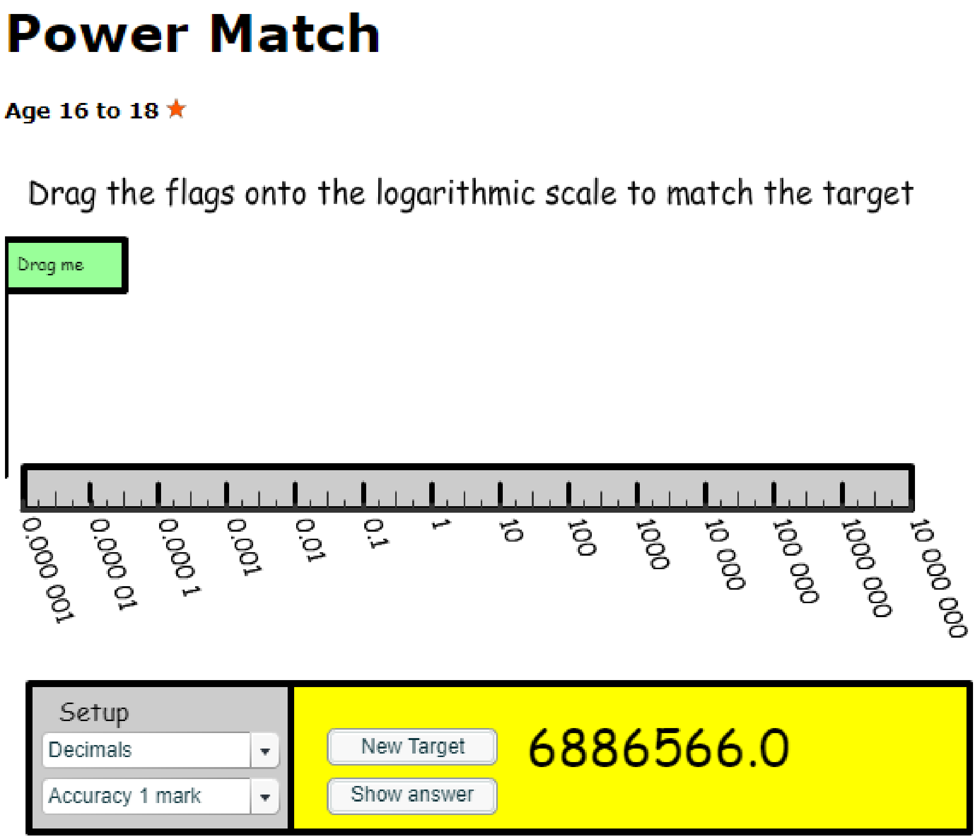
Which is the biggest and which is the smallest of these numbers?

$$2000^{2002} \qquad 2001^{2001} \qquad 2002^{2000}$$

How do they compare in magnitude?

6.6.6 Interactive Task

(Retrieved from Nrich: https://nrich.maths.org/6159)



6.7 Trigonometric Functions

1. The pulsating star Delta Cephei has a light intensity that swings between the extremes 4 ± 0.35 with a period of 5.4 days. Write the light intensity as a function of time using a sine function.

2. Create a function with the best fit to the graph of average temperatures in Kristiansand within a one-year period [*The students got links to webpages with weather forecasts*].

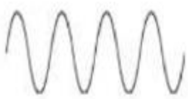
3. Use the information from this webpage to create a function that describes the number of sun hours in Kristiansand throughout a year: http://www.hvafor.no/oppslag/nar-er-soloppgang-og-solnedgang?location=Kristiansand&year=2013 [*The link is to a webpage*

including data on sunsets and sunrises throughout the year in Kristiansand].

4. How many solutions has the equation sin x + cos x = 2 in the interval from 0 to 8π?

1. Musical Tones

There is a scientific difference between noise and pure musical tones.

A random jumble of sound waves is heard as noise.	
Regular, evenly spaced sound waves are heard as tones.	
The closer together the waves are the higher the tone that is heard.	
The greater the amplitude the louder the tone.	

Trigonometric equations can be used to describe the initial behavior of the vibrations that give us specific tones, or notes.

a. Write a *sine* equation that models the initial behavior of the vibrations of the note G above middle C given that it has amplitude 0.015 and a frequency of 392 hertz.

b. Write a *sine* equation that models the initial behavior of the vibrations of the note D above middle C given that it has amplitude 0.25 and a frequency of 294 hertz.

c. Based on your equations, which note is higher? Which note is louder? How do you know?

d. Middle C has a frequency of 262 hertz. The C found one octave above middle C has a frequency of 254 hertz. The C found one octave below middle C has a frequency of 131 hertz.

i. Write a *sine* equation that models middle C if its amplitude is 0.4.

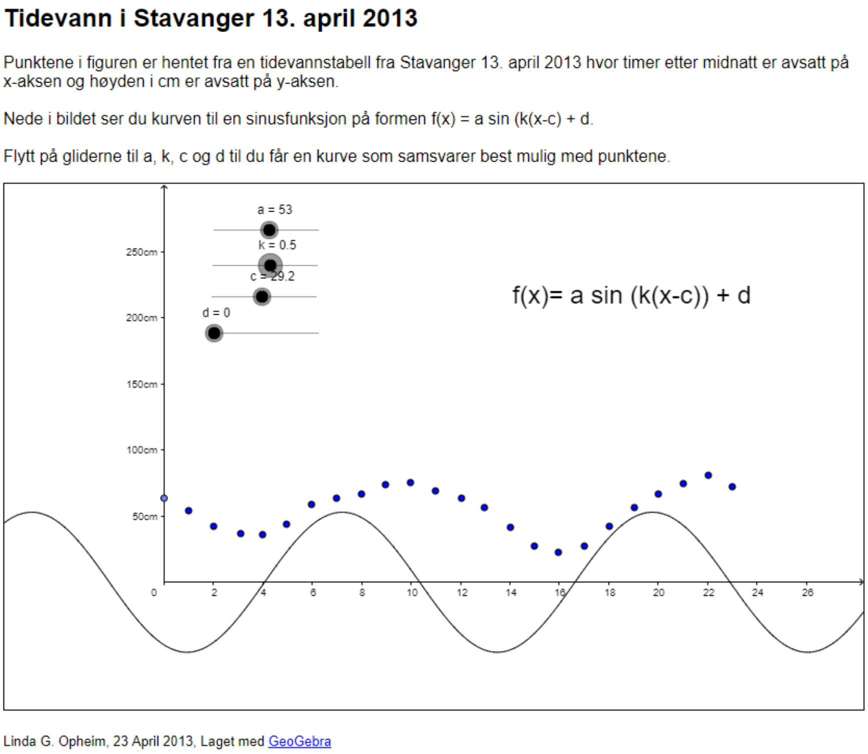
ii. Write a *sine* equation that models the C above middle C if its amplitude is one-half that of middle C.

iii. Write a *sine* equation that models the C below middle C if its amplitude is twice that of middle C.

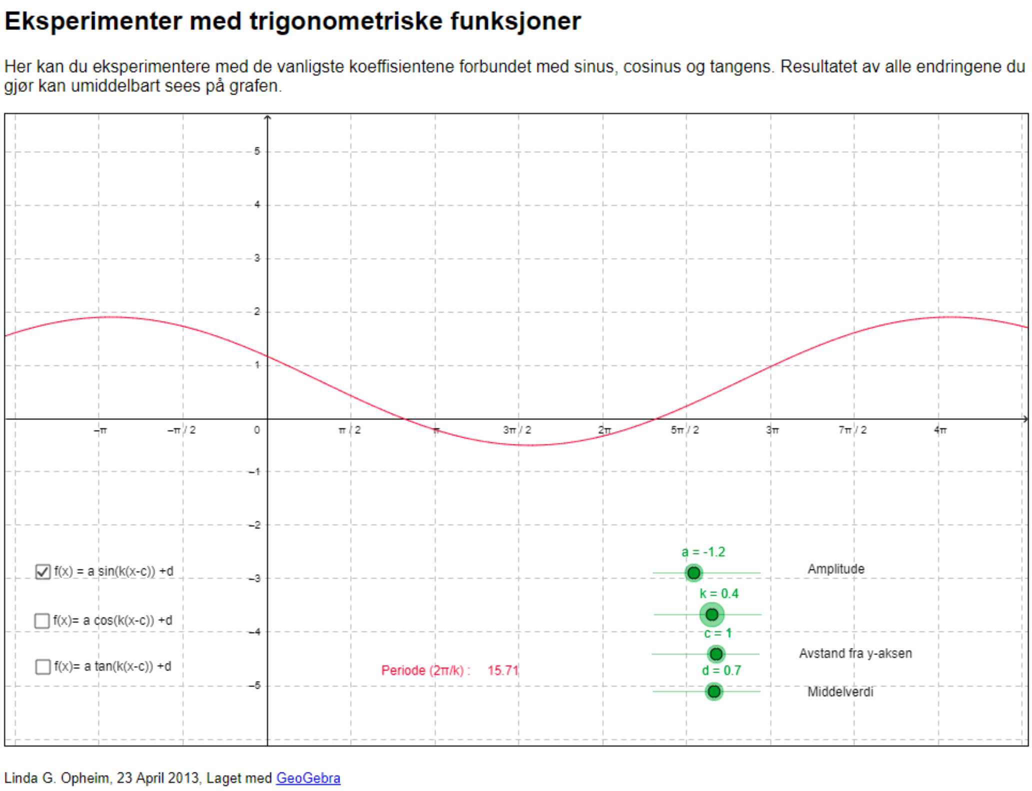
Retrieved from: https://www.georgiastandards.org/Georgia-Standards/Frameworks/Pre-Calculus-Unit-5.pdf

6.7.1 Screenshots of Two GeoGebra Resources Created by Me

Tidal water. The aim is to adjust the function to fit as good as possible to the tidal water data.



Experimenting with trigonometric functions. This is created so the students can experiment with the trigonometric functions.



6.8 Ideas for Integration Tasks for Roger

I tried to present a variety of tasks for teacher 1, but none of them resonated with him so that he would use them. Here is a list of tasks I suggested, and some of the teacher's comments.

1. Exploring in GeoGebra: I had made a macro where one has a function, the integral of the function and could explore the area. All of them with sliders. The idea was that either the students could explore, or he could use it for demonstration.
   Teacher's comment: He could see that GeoGebra might be beneficial to use on the trigonometric functions to show how the parameters influenced the graph, but he did not see the great advantage when it came to integrals. Also, he concluded that bringing a computer into the classroom would entail too much fuss, using the projector and so on - not worth it.

2. I presented a task which I designed based on the principles of multiple representations (Swan, 2008). I had graphs of $x, \frac{1}{x}, e^x, \cos x$ and their integrals and told him this could be expanded. My idea was to let the

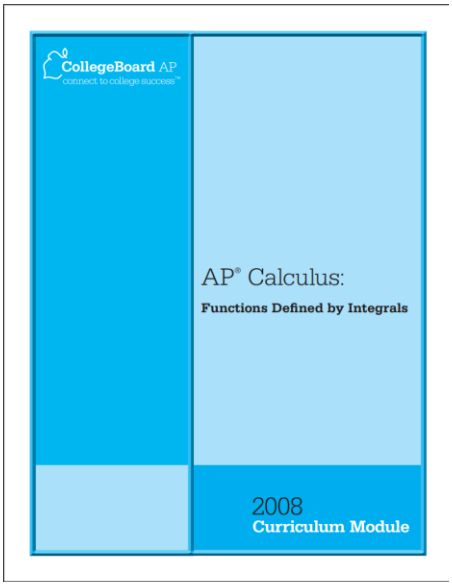
students combine these graphs, thereby ‘forcing’ them to consider how an area would be represented as a graph. I also presented many different ‘levels’ of difficulties, and they could be provided with the function expressions as well. It would also be possible to use different colors on the graphs in order to distinguish what was the $f(x)$ and what was $F(x)$.
Teacher’s comment: He liked the pair of $f(x) = x$ and $F(x) = \frac{1}{2}x^2$. The others he did not care much for. He asked if he could have this specific pair but didn’t make any promises whether he would use it or not.

3. The graph of an integral is not straight forward to draw from the graph of a derivative, and I assume many students become demotivated because they assume it should be easy, and yet they struggle. I therefore presented an idea for a task that makes these difficulties more obvious for everyone, even if the task does not have much to do with integrals. The idea is that I set up two items on the floor a bit apart, and name them *A* and *B*. I further ask the students to draw a diagram where the *x*-axis is the distance from *A*, and the *y*-axis is the distance from *B*. One person then walks straight lines between the two items, and the students draw the function of the graph. This quickly becomes difficult. The possibilities to expand this task are many. I have worked on it myself, and we spent about 1.5 hours as a group and still we had only just started.
   Teacher’s comment: This was a task the teacher liked, and he might want to use it. However, he did not find the task to fit with the topic he presented at the time.

4. I suggested an alternative way of practicing integration. That is, let the students work in pairs. Each student makes a function which he differentiates, and hands this over to his partner for integrating it again. You can then create a competition as well, urging the students to try to make a function which is so complicated that their partner gets stuck.
   Teacher’s comment: He never asks the students to work in pairs, and they would wonder about his change in pedagogical intentions if he suddenly made such a change.

5. The teacher starts to talk about how he does not like that the book introduces integration as the antiderivative and makes no mention of it being the area until 27 pages later. I therefore show him the tasks from Orton's (1983) article where the area is in focus.
   Teacher's comment: He does not respond much at all, and I do not pursue it.

6. I showed him a booklet with tasks on functions defined by integrals.

   (Pass, 2008) Retrieved from: http://apcentral.collegeboard.com/apc/public/repository/AP_CurricModCalculusFunctionsDefined.pdf
   Teacher's comment: Not much response and I do not pursue it.

7. I show the teacher this task:

Integral Arranging

Age 16 to 18 ★★

How might you sort these integrals into an order or different groups?

$$\int \frac{1}{1+x^2}\,\mathrm{dx} \qquad \int \frac{1}{1-\mathrm{x}^2}\,\mathrm{dx}$$

$$\int \frac{1}{(1+x)^2}\,dx \qquad \int \frac{1}{(1-x)^2}\,dx$$

$$\int \frac{1}{1+x}\,dx \qquad \int \frac{1}{1-x}\,dx$$

$$\int \frac{1}{\sqrt{1+x^2}}\,dx \qquad \int \frac{1}{\sqrt{1-x}}\,dx$$

$$\int \sqrt{1+x^2}\,dx \qquad \int \sqrt{1-x^2}\,dx$$

$$\int \sqrt{1+x}\,dx \qquad \int \sqrt{1-x}\,dx$$

Retrieved from: http://nrich.maths.org/6094 . The idea is to analyze various integrals.

Teacher's comment: Not much response.

8. This is another suggestion I showed him:

Mind Your Ps and Qs

Age 16 to 18 Short ★★

Here are 16 propositions involving a real number x:

$x\int_0^x y dy < 0$	$x > 1$	$0 < x < 1$	$x^2 + 4x + 4 = 0$
$x = 0$	$\cos(x/2) > \sin(x/2)$	$x > 2$	$x = 1$
$2\int_0^{x^2} y dy > x^2$	$x < 0$	$x^2 + x - 2 = 0$	$x = -2$
$x^3 > 1$	$\lvert x \rvert > 1$	$x > 4$	$\int_0^x \cos y dy = 0$

[Note: the trig functions are measured in radians]

By choosing p and q from this list, how many correct mathematical statements of the form $p \Rightarrow q$ or $p \Leftrightarrow q$ can you make?

It is possible to rearrange the statements into four statements $p \Rightarrow q$ and four statements $p \Leftrightarrow q$. Can you work out how to do this?

Retrieved from: http://nrich.maths.org/6382 .

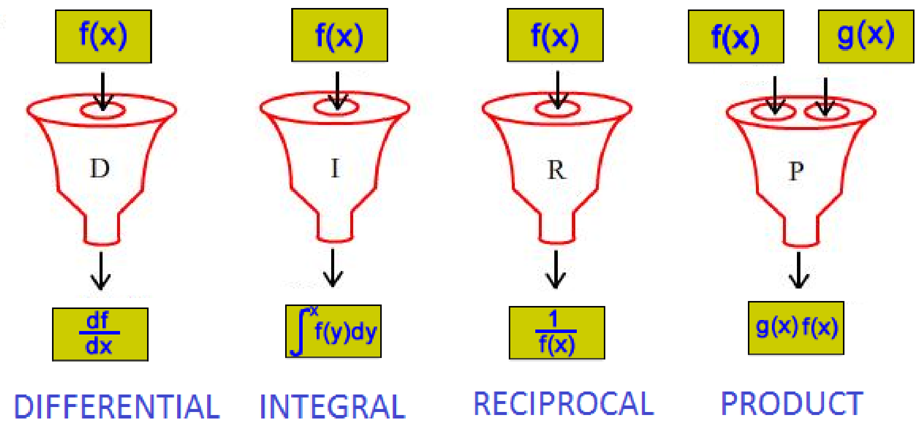
Teacher's comment: He has chosen not to focus much on implication arrows, so he feels the task will be wrong to use.

9. This is another task presented to Roger:

Calculus Countdown

Age 16 to 18 ★

In the game of *Calculus Countdown* you are given the following four machines into which you insert cards with functions written on them; the four machines chew up the input card(s) and spit out new cards with functions written on them. You can put any output cards back into the machines if you like. **The idea of the game is to hit certain target cards given a set of initial cards.**



Let's play a game. You are given the following initial seven cards (no constants of integration from the integral machine and no repeats of cards, other than the pair of e^xs)

x	x^2	4	$3x$	$\ln(x)$	e^x	e^x

Which of the following targets could you hit starting with these cards? You can use a fresh set of seven cards for each new target.

8	x^4	0.5	$x^6/36$	$-32/x^5$	$x(2-x)$

Can you make a smaller set of cards which could hit each of these targets?

Why not invent your own set of starting cards and targets?

Retrieved from: http://nrich.maths.org/6552 .

Teacher's comment: What is the learning outcome of this one? I answered that it is an alternative way of practicing integration, differentiation and so on. The teacher concluded that it is too difficult. There should be a starting point which is easy to see. I said that we can adjust the task so that he gets an easy starting point, but he was not interested.

10. I mentioned the possibility of a 'dam-task', but he was not eager on this. He worried the realistic setting would not be realistic enough, and that the students would react to this.

6.9 Summary

This chapter has provided an overview of the various tasks that were designed together with the teachers throughout the collaboration of this research project. Together with the presentation of the cases in Chapter 5, this gives an overall impression of the collaborative process of designing tasks to use in the teachers' classrooms. In Chapter 7, I will go more into details and analyze how the teachers are describing mathematical tasks and what type of changes the teachers are initiating.

7 Results

This chapter presents the analysis of the collaboration with the four teachers with respect to the research questions. The results are presented in two parts. Firstly, I present an analysis aimed to address the first research question: What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom? in Section 7.1. Secondly, I present an analysis to address the second research question: What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration? in Section 7.2. Direct quotes from the teachers are marked as citations, and the source is written in italic in parenthesis.

To answer the first research question, I have gone through a process of open coding of my collaboration with the four teachers, which is more thoroughly described in Sub-Section 7.1.1 below and Sub-Section 4.5.2 in the methodology chapter. Through this process I developed codes, categories, and dimensions to describe the mathematical tasks discussed with the teachers. The 305 open codes have been grouped into nine categories which belong to three dimensions. I use these categories to present what each of the four teachers are focusing on when it comes to tasks, both positively, negatively, and neutral.

In the second part of this results chapter (Section 7.2), I want to examine the process of change which is happening throughout my collaboration with the teachers. The aim of this research design was never to change the teachers, but to help the teachers make changes to their teaching which was in line with how they want to teach in their classrooms. Most of the research on teacher change stems from analyzing the effect of professional development programs, and how and to what degree the teachers make changes in their practice because of them (Clarke & Hollingsworth, 2002; Sztajn, Borko, & Smith, 2017). While this is important knowledge, we might understand even more of teacher change by investigating the change processes initiated by teachers themselves. Therefore, this research study provides valuable additional information on teacher change. I am using a well-known model for studying teacher change developed by Clarke and Hollingsworth (2002) and present my analysis in Section 7.2.

7.1 Teachers' Descriptions of Tasks

During the collaboration process with the teachers, we discussed mathematical tasks on many occasions. The teachers were interviewed and asked about what kind of tasks they liked and did not like, but they were also part of the design-, implementation- and evaluation process. While I presented ideas and first drafts of tasks based on the teachers' requests, the teachers refined them and implemented the tasks in the classroom. The implementation was further evaluated and new insights from this process were used in the next task design process. The following analysis is based on all these types of data collection to give a more holistic perspective than what would be provided by interviews only, to address the research question: What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom? Sub-Section 7.1.1 presents details on choices I had to make during the open coding process and how these have been resolved. It gives the details on how I went from the open codes to creating categories and dimensions. This is followed by Sub-section 7.1.2 where I present how codes were grouped into each of the categories and how these again were distributed into three distinct dimensions. Sub-Sections 7.1.3-7.1.8 present the analysis of the teachers' descriptions of mathematical tasks.

7.1.1 The Process of Open Coding and Creating Categories

All the recorded data have been coded using open coding, as explained in the methodology chapter. Trying to lose as little context as possible and to avoid too many interpretations at the initial stages, I coded in vivo, using the teachers own words and phrasings as much as possible. I only deviated from in vivo coding when the teachers' phrasings became too long, and I felt a need to summarize. An example of how I have summarized a coding instead of coding it purely in vivo, is how I coded the quotation: "Preferable where an understanding builds up while the student is working on the task, and that this understanding doesn't come from the teacher and down followed by practice". I chose the code 'understanding through working on the task' on this statement, trying to stay as close as possible to the wording of the teacher, but at the same time summarizing.

If the teachers are just repeating or rephrasing what they are looking for in mathematical tasks, I do not code it twice. An example of this is the quotation: "They need to understand that there is more than one logarithm, and that logarithms are the opposite of exponential, so they understand what it's about". Even though the teacher here mentions understanding twice, I only code it once as understanding, because I consider it to be a repetition, elaboration, and not a new comment. However, it is not always easy to know if something a teacher is saying is just a rephrasing, or if the teacher is intentionally repeating issues because this is something they want to emphasize. So, the exact frequency of codes could be questioned, depending on what the analyst interprets as being a repetition, a rephrasing, or a new statement. At the same time, I do not view the exact number of codes as being important in this analysis. I am not claiming to have arrived at saturation of the categories, because new categories might emerge if I was designing new tasks on new topics with different student groups and different teachers. Still, there is a lot of information and knowledge to gain from looking at the general tendencies in what type of categories the teachers' descriptions of tasks are within, and how this for some of the teachers, changes within the type of discussion we are doing. Assessing the distribution of codes and categories between the four teachers, is also helpful in determining differences and similarities between them.

7.1.2 The Codes, Categories and Dimensions

I will in the following elaborate on the choices I, as a researcher, made when creating codes, categories and dimensions and the reasoning for why these are representative for how the teachers described characteristics of mathematical tasks. To help the reader in following how the codes, categories and dimensions are linked together, I start by presenting a schematic overview in Table 7.1.

Outcome of tasks		210	Characteristics of tasks		49	Students' reactions to tasks		46
Activity		***38***	**Didactical characteristics**	**14**		**Students' reactions**	**46**	
Understanding		***38***	**Mathematical characteristics**	**2**				
Goal of the task		***6***	**Practical considerations**	**33**				
Diversity		***24***						
Relevance		***104***						
Exam/ curriculum	22							
Connecting mathematical topics	11							
Practical use	60							
Vocational	11							

Table 7.1: A presentation of the codes, categories, and dimensions. The total number of codes in each dimension are the numbers in the top row, additional numbers are the distribution among categories and subcategories.

The three dimensions are written on top of each column, and the numbers besides them refer to how many codes have been assigned to each dimension. Below the dimensions, in each column, are the categories belonging to the dimensions. The categories are written in bold text, and the numbers represent the number of codes belonging to each category. One of the categories, ‘Relevance’, has in addition been divided into four subcategories.

When describing and talking about mathematical tasks, the teachers are not just referring to characteristics of the tasks. In my data I have, through open coding, identified three different dimensions the teachers are referring to when describing tasks. These dimensions are *characteristics of tasks*, *outcome of tasks* and *students’ reactions to tasks*. Characteristics of tasks is a dimension that contains all the teachers’ comments on specific didactical or mathematical characteristics of the task. In addition, I have included practical considerations, entailing specific characteristics of the tasks the teachers describe, that are not explicitly justified didactically or by classroom management. This does not mean that such reasons for the practical considerations do not exist, the teachers have just not expressed them.

However, instead of describing characteristics of tasks, the teachers often describe tasks by wanted outcome, thus I have added outcome of tasks as a second dimension. The type of comments the teacher would make with respect to the outcome of tasks, could for instance be to get the students more active, to keep them working, to gain understanding, to have aha moments, to motivate the students and so on. The rationale for the third dimension, i.e., students’ reactions to tasks, is that the teachers often evaluate a mathematical task based on their expected or perceived reactions of their students. Though it is possible to argue that the students’ reactions to the tasks is the outcome of the task, I chose to distinguish these types of statements. The teachers’ statements that I have grouped into the dimension of students’ reactions to tasks, are context related to this specific group of students. It is not given that the teacher would make the same decisions and statements if the group of students were different. For instance, while classroom discussions are not viewed as possible with one of the classes in this research project, the teacher might use classroom discussions in a third-grade class he had followed since they started school. Also, when it comes to the statements belonging to outcome of tasks, these statements express what type of reactions the teacher wants from the students when working on these tasks, or how they will evaluate the impact of the tasks on the students later. This is different from making changes to or describing tasks based on the students' reaction to them. I have therefore ended up with the three dimensions characteristics of tasks, outcome of tasks and students' reactions to tasks.

These dimensions contain several categories and codes from my data material, and I will present the categories below, and describe how and why I have created them based on the coding process. In total, I assigned nearly 400 open codes to the data material, mostly in vivo as previously explained. However, during the analysis process the number of codes were reduced because I realized that the codes were not really describing mathematical tasks. For example, statements I coded with the word time, where the teachers described how lack of time might result in giving the students other types of tasks or teaching than what they would have preferred with more time on their hands. Although this is related to what types of tasks the students are working on, it does not describe what types of tasks the teachers would want to use, or what types of tasks they would not want to use, so these codes were not helpful in answering the research question. The comments where related to why they appreciated this collaboration, and not about rejecting tasks in the classrooms because they would be time demanding. I therefore removed these and some other irrelevant codes, which resulted in a total of 305 codes divided between the three dimensions.

The 305 codes included many sub-sets that shared similar meanings, such as the codes: active students, they get active right away, the students got quickly started, the students get started on working and so on. So, I grouped these types of codes, and ended up with the category activity. Notably, the word activity is the everyday meaning of the word, and not the theoretical construct. I chose to use the word activity, because this is the word closest to the wordings of the teachers[4].

Another word that the teachers used frequently during our conversations was the word 'understanding'. They could talk about how it was a goal that the students should understand through working on a task, but they would also talk about improved or prolonged understanding. In addition to explicitly using the word understanding, I have interpreted teacher's expressions like: students having to reflect, having an aha moment, tasks which can expose misconceptions etc. as being about understanding, thus belonging to the category *understanding*.

[4] In Norwegian, the teachers used the word 'aktivitet' and talked about how 'aktive' the students were.

The teachers are sometimes talking about different types of understanding, for instance memorizing for a test as opposed to prolonged understanding. However, I have chosen not to distinguish between types of understanding, because the important part for this research, is how a teacher relates the task to understanding and not how the teacher defines understanding. In addition, it is not always easy to know what type of understanding the teachers are talking about if they do not elaborate. It would therefore be difficult to make a distinction between types of understanding in most of the cases. There are also examples of codes that I have assigned to the category understanding, which are more indirect like the open code: *not a focus on formulas, but how squares can be divided into triangles to calculate the area*. I have interpreted this as being about understanding, the teacher has just specified what type of understanding she is looking for.

Some of the teachers refer to diversity in the class on several occasions, when they are talking about what types of mathematical tasks they want. Examples of such codes are *a starting point that everyone can master, at the same time challenge the high achievers*, *everyone can manage something* and so on. I have grouped these codes into a category which I have named *diversity*. However, as well as including codes that refer to the class as being diverse, I have also included codes that state the opposite. An example is the code *homogeneous student group*. I have chosen to include these codes as well because they relate to diversity, since they highlight challenges the teachers claim they are not having, which they attribute to a lack of diversity.

Another category I have created, is the category *students' reactions*. This category mainly contains codes of a limiting character when it comes to mathematical tasks. When I presented tasks for the teachers, they would sometimes say this type of task will not work because the students would react in a certain way. So, this is a category containing the teachers' perceptions or expectations of the students' reactions to tasks. Some examples of these types of codes are *the students want it the way they are used to, students quickly ask for help, the students give up easily and the students expect the teacher to give the answer*. These codes were created when I discussed tasks with the teachers before implementation, but I have also used this category on teachers' comments when we evaluated the implementation.

I have named a category *practical considerations*. This category contains codes related to practical adjustments the teachers want to make to the tasks. Some examples of these codes are *structure, squares in the rectangle*, *divide* *subtasks to different sheets of paper and clear formulations*. So, the common denominator for this category is details that the teachers want to add or change to tasks. I have chosen to name the category practical considerations, because the requests are specific and not directly related to the mathematics. This category could have been named didactical considerations because I assume most of these codes express considerations related to didactical or classroom management. However, the teachers are not necessarily expressing this, hence I have chosen to use the name practical considerations to avoid interpreting the teachers' expressions when naming the category.

Relevance is a category containing four subcategories. My reason for naming the category 'relevance', is because all statements I have coded and placed in this category refer to how the mathematical task is connected to more than just the mathematical topic the students are working on. The four subcategories are *connecting mathematical topics*, *vocational*, *practical use*, and *exam/curriculum*. Connecting mathematical topics is a category containing codes like tasks can be used to many different things and relate it to many different topics and tasks which can connect different topics. Herein, I have coded statements from the teachers where they describe wanting tasks that highlight how something is relevant for many mathematical topics. Proportions is a specific example of this, i.e., that Sven wanted the students to experience proportions as a recurring theme. *Vocational* is a category including all the codes that are related to referring to the mathematical tasks having a vocational aspect. All the teachers' students are in vocational classes, and I have in this category only included codes that contain the word vocational. While it is possible that a practical task also is vocationally related, it is not given that a task with a practical use is connected to their future vocation. I have therefore chosen to create a category named *practical use*. Some examples of codes from this category are *tasks which can connect the practical with the theoretical*, *practical perspective*, *realistic and relevant tasks*, *authentic* and *meaningful tasks*. These codes have been applied on statements where the teachers refer to tasks being connected to real and practical settings in some way, or that the teachers are criticizing the setting as being falsely realistic. The final subcategory of relevance is the category *exam/curriculum,* where I have only included codes that contain the words exam and/or curriculum, so I have done minimal interpretation on this code. However, these codes are related to teachers talking about how the exam or the curriculum is relevant with respect to the tasks we are discussing.

Sometimes the teachers are explicit about the mathematical characteristics of the tasks, and I have therefore created a category which is named *mathematical characteristics*. Examples of the few codes I have placed in this category are: *has a mathematical basic principle he wants to focus on* and *too much focus on implication arrows*. Despite few examples of statements I have coded and grouped into this category, I need it to distinguish these statements from more didactical comments. The mathematical characteristics contains statements from the teachers on specific mathematics they want to include, yet without expressing how the students should learn this. The category of *didactical characteristics* on the other hand, includes codes that I have assigned to statements the teachers have made about what type of mathematical task they want or do not want. Some examples of this type of codes are *open tasks*, *historical perspective*, *inquiry*, *illustrative* and *creative*. These codes are all coded in vivo, and the teachers have been using these words, which are also familiar terms in research on mathematical tasks.

When the teachers described mathematical tasks they preferred to use or did not like, they often describe the imagined outcome of the task and not the characteristics of the task itself. Activity, understanding, diversity and relevance are all categories within the dimension of outcome of tasks. Still, there were other statements related to the outcome of the task as well, that did not fit into any of the categories mentioned above. Thus, I chose to construct the category *goal of the task* for these codes. Examples of codes I have assigned to the category goal of the task, are *task as an introduction to the topic*, *tasks demanding an academic overview* and *limit the need of teacher help*. So, these codes have been assigned to the rest of the statements where the teacher sees an outcome of the task beyond just solving it and are using this to describe the task.

The open coding process shows why it was important not only to use task characteristics from theory when trying to capture the teachers' perspectives. That is, the teachers are just as likely to describe the tasks based on the outcome and what they want to achieve by using the task, or by how their students might react to the task. This makes it challenging when collaborating and designing tasks for the teachers, because we might not agree whether a task will achieve what they are asking for or not. At the same time, it yields a lot more possibilities for different suggestions and discussions, thus providing richer data to analyze.

There are some differences between the four teachers' focus, and I will in the following present a more detailed analysis of how each of them describes mathematical tasks they would want to use in their classrooms.

7.1.3 Presentation of the Analysis of the Teachers' Descriptions

This section presents an analysis of how each of the teachers described mathematical tasks. This is done with respect to all the three dimensions and the categories belonging therein. I present what the teachers focus on when describing tasks and how I have coded this, whether these codes are from several sources or just a few, and whether there might be a change in the type of descriptions the teachers are using throughout our collaboration. The codes, categories, and dimensions I have presented so far, do not say anything about whether the teachers talked about these characteristics in a positive, negative, or neutral way. I have therefore chosen to add an analysis which examines the grading of the categories. This is done by evaluating whether a teacher makes a positive comment (+) about a task, or if the teacher makes a negative (-) comment, rejecting or criticizing the task. This is represented in a table after each analysis of how the teachers describe tasks, and the details of the analysis follows the table. I have also added a neutral column (0) to the table since it is not always possible to determine whether a comment is purely positive or negative. Notably, this table should not be read as an overview over whether the teachers see the categories as being positive or negative in themselves. The table only represents whether a teacher uses the category as a positive argument for wanting or approving a task, or if the teacher rejects a task based on these categories. So, the table can be seen as an overview of the characteristics of the teachers' discourse on mathematical tasks.

For the reader, the first part where I analyze the teacher's descriptions of tasks gives the details on which codes, categories, and dimensions I could identify in the collaboration with each teacher. Categories and dimensions are written in italic to help the reader navigate. In the next part where I analyze the grading of these characteristics, there will be a repetition of the same characteristics, yet including more details on whether these were positive, negative, or neutral comments. While it might seem repetitive, dividing the analysis this way, it makes it easier for a reader to find relevant information without having to read all the details.

After presenting an analysis of the collaborations with each of the teachers, I will give a summary where I discuss similarities and differences across the four teachers.

7.1.4 Roger's Description of Tasks

Even though Roger and I never implemented any tasks, we had many discussions about mathematical tasks and what Roger preferred and what he did not like about various tasks. In most of our conversations, Roger described tasks in such a way that I would code the statements with *outcome of tasks*. 33 statements have codes belonging to the dimension of *outcome of tasks*, while five codes belong to the dimension of *characteristics of tasks* and only one statement has been coded with *students' reactions to tasks*. In Table 7.2 there is a schematic overview of the number of statements in each dimension and category from my collaboration with Roger.

Outcome of tasks	**33**	**Characteristics of tasks**	**5**	**Students' reactions to tasks**	**1**
Activity	***0***	**Didactical characteristics**	**1**	**Students' reactions**	**1**
Understanding	***8***	**Mathematical characteristics**	**2**		
Goal of the task	***1***	**Practical considerations**	**2**		
Diversity	***0***				
Relevance	***24***				
Exam/ curriculum	8				
Connecting mathematical topics	0				
Practical use	14				
Vocational	2				

Table 7.2: Overview of how Roger's statements have been coded and organized into the three dimensions which in turn contains categories and subcategories. The total number of codes in each dimension are the numbers in the top row, but the distribution among categories and subcategories are also shown.

I will in the following elaborate on what type of statements that were assigned to the various categories. Within the dimension *of outcome of tasks*, none of Roger's statements were coded with respect to *activity* or *diversity* as categories. However, I made eight codes that were grouped into the category of *understanding*. The statements from Roger that were coded with *understanding*, are about Roger wanting the students to understand a specific topic when working on tasks, understanding the tasks themselves, needing an understanding to solve a task, or being open to the question if there might be tasks that would help the students to improve their understanding of the topic. There is one statement in my collaboration with Roger that was coded as the category *goal of the task*, where he questions whether a task is exciting or not.

In the category *relevance* there were 24 statements coded from the conversations with Roger. Eight of these codes are from the subcategory *exam/curriculum*. Roger rejects some tasks because he does not feel they are relevant enough with respect to the exam or curriculum, and he also makes several comments about the students needing certain tasks to prepare for the exam. I get the impression that even though Roger emphasizes understanding, based on codes in the understanding category, he is also aware of the exam and how to help his students excel on it. I have not coded any statements from Roger with *connecting mathematical topics*. Even if he talks about how the students need to understand the relationship between differentiation and integration for instance, he does not specifically link these comments to tasks. There are only two coded statements which are placed in the subcategory *vocational*. In one of these statements, Roger says the students should understand the tasks and the topics due to their vocational knowledge. The other statement is Roger claiming a task not being suited for a vocational class, but more for those with special interests. When asked to elaborate, Roger adds how the task is not related to real life and how he thinks the students would be more interested in tasks like the one on medications (Sub-Section 6.6.3), because of the connection to real life. The last 14 codes were placed in the subcategory *practical use*. Roger came up with positive comments about some of the tasks because they were practical or realistically oriented. However, he does not like tasks that set out to be realistic but are not. Especially with respect to his student group, which he comments will see straight through those attempts if the tasks are not the way things are done in real life. I assume Roger is referring to the type of tasks that in the literature are referred to as "'dressing up' of purely mathematical problems in the words of an other discipline or of everyday life" (Blum & Niss, 1991, p. 40).

During our collaboration, Roger made several comments about tasks that I coded and grouped into the dimension of *characteristics of tasks*. In total, five statements have been coded within this dimension. Two of these codes, I have assigned to the subcategory *mathematical characteristics*. One of those are from when Roger talks about wanting tasks with a different focus on introducing integration as the opposite of differentiation. He wants to use the concept of areas earlier. The other code that belongs to the subcategory of *mathematical characteristics,* is assigned to a statement where Roger rejects a task because it contains some mathematics that he has not focused on. One statement from Roger was coded as being a *didactical characteristic,* and that is when he would change the numbers in a logarithm task, arguing it would be better pedagogically. This is a statement that could also be coded with *practical considerations,* but since Roger expresses clearly his didactical reasons for it, I have coded it as a *didactical characteristic*. The subtask Roger is talking about can be seen in Figure 7.1, and Roger says pedagogically he had preferred the volumes to be the same, because the students might then assume that the answer would be 3.5, which it is not since it is logarithmic.

d) 400ml of an acid of pH 3 is added to 300ml of an acid of pH 4. What is the resulting pH?

Figure 7.1: Subtask d) in the 'Mixing pH' in Logarithm Tasks (Sub-Section 6.6.2)

Roger made two comments that were coded and grouped into the category of *practical considerations*. Once he expressed how the formulation of a tasks is not good, and on another occasion, he talked about how tasks should be recognizable for the students.

There is only one code that was assigned to the dimension of *students' reactions*. Roger says that not everything is comprehended by the students when he explains, so maybe different tasks would give them a greater understanding. Roger refers to understanding here, but I have also coded this statement with *students' reactions*, because the students' reactions and lack of understanding is what motivates him for possibly new tasks.

When analyzing my collaboration with Roger, I have not focused on whether the codes are coming from many different sources or just a few. The reason for this, is that we never got to the point of implementation and evaluation, so all the sources are from the discussion phases of our collaboration. Thus, it is difficult to conclude about a development in how Roger expresses what he likes and does not like in mathematical tasks. Still, Roger is using a type of discourse about tasks which is similar to the other teachers in this research project. Instead of talking about specific characteristics of the tasks, he is more inclined to talk about tasks with respect to the wanted outcome of the task. Since Roger is mostly referring to tasks in both a positive and negative way with respect to the outcome of the task, it is difficult to know exactly what such a task would have to look like for him to want to use it. However, based on our conversations, I would say the tasks would need to improve the students' understanding on a topic, be relevant for the exam and curriculum, and fit into Roger's teaching style. This is a conclusion based on how Roger has rejected tasks that have included software or mathematical concepts he has not previously used in his classroom.

Roger's Grading of the Categories

Table 7.3 represents an overview of whether Roger commented of the categories in a positive, negative, or neutral way. The details of this analysis follow below the table.

Categories	**+**	**-**	**0**
Activity	0	0	0
Understanding	2	1	5
Goal of the task	0	1	0
Diversity	0	0	0
Exam/curriculum	0	7	1
Connecting mathematical topics	0	0	0
Practical use	4	8	2
Vocational	0	1	1
Didactical characteristics	1	0	0
Mathematical characteristics	1	1	0
Practical considerations	1	1	0
Students' reactions	0	1	1

Table 7.3: Overview of the grading of Roger's categories.

Understanding

Roger refers to understanding on eight separate occasions throughout our collaboration. I have chosen to mark one of the comments as negative. On this occasion, Roger appears to be thinking aloud about the balance between understanding and just asking the students to follow a formula when it comes to integration:

> So how much understanding should you focus on, as opposed to just saying use the formulas? I am not really happy about the way the book introduces the topic because of this, but I often change the sequencing if I feel like it. Maybe the people writing the textbook knows more pedagogics than me, but it's also about how you feel about it. The concept of integration is just something 'out in the sky' without linking it to areas under the graph (*presenting integral tasks*).

Although Roger starts by debating how much focus there should be on understanding versus using a formula, he follows this by criticizing the textbook for having too much focus on the formulas. In his opinion, the concept of integration does not make sense unless connecting it to areas under the graph. I have therefore marked this statement from Roger as negative, where he criticizes the tasks in the textbook for having too little focus on understanding the concept.

Twice, Roger talks positively about mathematical tasks where a certain level of understanding is necessary to solve the tasks. One of these is when I present the task where the students should draw the graph of someone walking between two points (Section 6.8). Roger responds positively to the challenges in the task and talks about how he likes to focus on logic in his teaching. He does not say he wants to use the task, but he does not explain why he rejects it either. Still, based on his reaction to the task, Roger is positive regarding a need for understanding when solving it. There is also another occasion where he evaluates a task positively with respect to understanding. He gives an example of a task he likes, even if his students complained that it was too difficult. Roger’s reasons for liking this task, was:

> … because they must use many different elements of knowledge and they have to figure out what to use when. But I consider this as an ‘A-task’. I like creative tasks where they have to look for something, but this is the last task on a test and only that (*semi-structured interview*).

This example is a task from physics, but Roger said he also likes these types of tasks in mathematics. He could just not come up with an example from mathematics on the spot. Roger emphasizes how he likes that the task demands creativity and connects different elements of the students' knowledge. He also makes it clear that he does not expect all the students to be able to solve such a task, and this is something for the top achievers in the class. The example task Roger gave, was from a test and some of his comments should be seen in light of this. However, Roger makes it clear that he values students who need to be creative and show understanding when solving tasks.

The last five statements from Roger that I have categorized into 'understanding' are statements that I cannot determine as either positive or negative comments with respect to a task. Roger talks about whether the students will understand what is asked of them in a task, or how they should understand specific topics in mathematics or having a basic understanding of how the mathematical topics are linked together. So, these five statements are connected more to the students' understanding in general than to an evaluation of tasks.

Goal of the Task

In our very first talk about tasks, Roger looks through the textbook they are using and comments on some of the tasks there. In the chapter concerning logarithms, he says: "It's a question whether these are exciting or not, or if the students understand logarithms after doing them" (*our first talk about tasks*). I have chosen to interpret this comment about one of the goals of the tasks being whether it is exciting, as a negative comment to the task. This is done by reading the statement into the further context of the situation in addition to the intonation of the comment. When I ask Roger a few minutes later if he wants to change some of the logarithm tasks, he does not want to make changes. Although he refers to some of the tasks as meaningless, like before mentioned, he says it is also part of the game to learn how to do this. All in all, I interpret that Roger views the tasks as not very exciting and meaningless, and this is something he sees as a negative characteristic of the tasks.

Exam/Curriculum

Roger brings up the exam as an issue on several occasions throughout our collaboration, and almost every time he is rejecting the use of a task. I include one quote from Roger that I have marked as neutral, because Roger was expressing uncertainty of what type of task he might want. "I am not really sure what kind of tasks I would want… but I have to keep the exam in mind" (*first* *conversation about tasks*). So, this is an example of Roger expressing uncertainty of tasks he could ask for, but he is making a clear statement that the exam is important when he considers tasks. Still, given how I am analyzing these comments as positive, negative, or neutral with respect to the teachers evaluating tasks, I have to mark this one as neutral.

The other seven comments made by Roger where he refers to either the exam or the curriculum, has been marked as negative. This is because he rejects using tasks by arguing they are not relevant enough for the exam curriculum. In our first conversation about tasks, he is uncertain of what tasks might work better than the once he has, but this comment is made referring to the exam. When I ask Roger directly if he would have made some changes if he did not have to worry about the exam, he responds: "If I could have made the exam myself, I would have made some changes. Focused more on some topics than others, but I can't because of the exam" (*first conversation about tasks*). Roger does not elaborate exactly on what changes he would have made, except a change of focus between topics. When I try to ask him for more details on changes he might have made, he just says it would be too hypothetical. So, already from our first conversation about mathematical tasks, Roger makes it clear that it is important for him to consider the exam when he is evaluating tasks. Three of the statements that were coded as exam/curriculum, are from this first conversation. In addition, Roger mentions the exams or curriculum also when I present integral and logarithm tasks to him. When I present the integral tasks, Roger is straight forward about the importance of the exam and the curriculum: "There is a defined curriculum and an exam, and then you have to work on tasks that are similar to this exam, even if they are not so fun" (*I present integral tasks*). Here, Roger is clear on the importance of preparing the students for the exam and giving them tasks that are similar to what they will get on an exam. Thus, he rejects both integral tasks and logarithm tasks I suggest. An example from the logarithm tasks is Roger's reaction when I present the pH-task. "These are absolutely tasks I would like, but not in this curriculum, because pH is not mentioned, nor logarithmic scales" (*I present logarithm tasks*). Here, Roger is clear and specific about how logarithmic scales are absent in the curriculum, and in addition that pH is not mentioned. I assume the last comment about pH is about the curriculum in chemistry or another science subject, since Roger has been teaching more than mathematics. So, this is an example of how a teacher might not only evaluate the relevance of a task in light of the curriculum in mathematics, but also the curriculum in other relevant subjects.

Practical Use

Roger refers to the practical perspective of a task or how realistic he finds the task, on several occasions. In total, there are 14 statements that were coded within this category. These statements about practical use were marked both as positive (4) and negative (8) in addition to two statements that were marked as neutral. Even if Roger sometimes uses a practical perspective as a reason for criticizing or rejecting a task, he might also be positive to a task due to the practical perspective. This is a coherent perspective from Roger, even if it is used both positively and negatively. While he can acknowledge qualities in a task because he finds it relevant and realistic, he can also criticize other tasks for being artificially real, in his opinion. When he uses the practical use as a positive characteristic of a task, I have marked the comment as positive. An example of such a statement is from our conversation on logarithm tasks (Section 6.6), where I had presented a medicine task, among others: “This task is kind of philosophical… society related, and at the same time they get to practice some standard mathematics. So that way, it is useful on some areas” (*I present logarithm tasks*). In the end, Roger did not use this task during our collaboration, but he expressed several quality characteristics of it. He liked the combination of the task being society related, philosophical, but at the same time challenging students to do some mathematics. Even though Roger did not use any of the tasks I presented throughout our collaboration, there were a total of four times when he expressed how a practical perspective could be positive, but he would also criticize tasks that were designed to be realistic without really being it, in his opinion. The following quote was chosen because Roger gives explicit examples of how it can be both a positive and negative characteristic.

> There was a task where they used functions of the third degree to model hotel guests, but that is just rubbish. It’s a so-called practical task… But with integrals, you can make tasks on water flooding in and out of bathtubs and so on. It becomes real (*discussion of tasks integration*).

The first part of this quote is an example of how Roger is not happy with tasks that he views as artificially realistic. However, the last part of the quote contains an example of what type of contexts Roger views as being truly realistic. So, this quote shows how ‘practical use’ as a category can be both positive and negative when evaluating mathematical tasks, but the teachers’ opinion about the topic is still consistent. However, there is one occasion where Roger expresses that he does not want a practical perspective on certain tasks within the topic functions. He is assessing some tasks in the textbook about functions, and he comments how he likes these tasks because the students need some kind of overview to solve them, like what is solvable or not. Now, even if this is a task that Roger likes, he does not want a practical perspective on these types of tasks: “I don’t like a practical perspective on these tasks, because it just confuses them” (*first conversation about tasks*). Roger does not further elaborate on why he thinks it would confuse the students, but this might be related to the same issues as Thomas has brought up on some occasions; that a practical perspective on mathematical tasks can make them more difficult and complex. So, it is not automatically given that tasks with a practical connection are more useful, it depends on the goal of the task. In this case, Roger likes the task because it challenges the students to use several types of knowledge and evaluations to solve the task. He might think that there is enough within this task, so that adding a practical element might take the focus away from the parts of the task he likes. In other words, making the task more difficult, and maybe even more challenging to understand the mathematics in it.

This is not the only time Roger mentions how a practical perspective can make the task more difficult. In our first conversation about tasks, Roger refers to tasks in the textbook regarding trigonometric functions and how some of them are related to real life, for instance temperature: “But some of these tasks are also very difficult and many of the students have struggled with them. The practical perspective makes them even more difficult” (*first conversation about tasks*). Here again, Roger brings up the issue of how tasks can become more difficult if you add a practical perspective to them. Both Roger and Thomas raise this as an issue, but this has not been a concern in the collaboration with Hanna and Sven. Especially Sven talked about how a practical perspective on the tasks might make the tasks easier to solve for the students. The teachers have different groups of students and a different curriculum, but how can it be that they have such diverging opinions on the difficulty level of practical oriented mathematical tasks? Is this a result of the teachers’ perceptions, or do the mathematical tasks become more difficult at higher level mathematics? Are there any differences between mathematical topics, or is it the same regardless of topic?

The rest of the statements from Roger that were marked as negative in the practical use category, are examples of Roger rejecting tasks because they are not realistic enough. At one point he elaborates more on one of the reasons it is important to him that the tasks are not artificially realistic: “These students have worked for some years, and they can often comment on what is actually being done in practice, and they are mostly right” (*I present integral tasks*). With this group of students, the stakes are even higher when it comes to realism in tasks, because they have practical experience and can more easily see-through attempts of making mathematical tasks real if this is not how it is actually being done in practice. This was Roger’s issue concerning the dam task with integrals. Although it would be possible to calculate how thick the dam would need to be using integrals, Roger doubts this is how dam constructors do it in practice, and this is something he would expect his students to know. So, Roger is careful using mathematical tasks that might only be theoretically realistic, that is being a ‘dressed up task’ (Blum & Niss, 1991). If the tasks are not practically realistic, he does not want to use them with this group of students.

Vocational

There are only two occasions where Roger talks about the vocational aspect of tasks, and one of these was marked as a neutral comment. Roger is neither positive nor negative to a specific task, but comments on how his students have had so much mechanics that vectors should be a topic they would understand quite well. While it is possible that Roger is referring to mechanics as a school subject, I have chosen to code this as vocational. This is because Roger is referring to the students’ knowledge within a physics/engineering topic as giving them an understanding of a mathematical topic. Since I know this student group has been working as skilled workers for some years, mechanics has most likely been part of their workday as well. However, Roger’s comment is different from what I would expect when it comes to mathematical tasks and vocational aspects. Instead of arguing for how the tasks should have a vocational aspect to motivate the students, Roger refers to how the students should have a good understanding of the mathematical topic, due to their vocational background.

At one point in our collaboration, Roger rejects a task, as he finds it not being suitable for this type of students. The task we discussed at the time was

Big, Bigger, Biggest which can be seen in Figure 7.2, which requires an understanding of logarithms to be able to solve.

Big, Bigger, Biggest

Stage: 5 ★

Which is the biggest and which the smallest of these numbers?

$$2000^{2002} \qquad 2001^{2001} \qquad 2002^{2000}$$

How do they compare in magnitude?

Figure 7.2: Big, Bigger, Biggest from the website nrich.maths.org

I presented this task for Roger, and our conversation continued like this:

Roger: This is a task that is not suitable for vocational classes. This is a task for people with special interest.

Researcher: So, the vocational students do not have special interest?

Roger: Not on this stuff. They could be fascinated by the medication task because it is more related to real life. This task is nice, but for people with special interest (*I present logarithm tasks*).

This is the only time during our collaboration that Roger expresses that his group of students would have different interests than other students in mathematics. Roger has previously not seemed worried about presenting theoretical mathematics to the vocational students, but then it has been part of the curriculum. This task differs from the others through being purely theoretical and at the same time different from the type of tasks they would get on an exam. So, I assume Roger thinks this a type of mathematical task that is too far from this group of students' interest area.

Didactical Characteristics

There is only one statement from my collaboration with Roger that was coded as a didactical characteristic, and I have chosen to mark this as positive when it comes to evaluating tasks. This is because Roger is suggesting an improvement to the task, by changing some numbers. So, his comment is a proposed improvement, which he therefore views as positive.

Mathematical Characteristics

Roger rejected a task I presented, since it included a mathematical concept he had not prioritized in his teaching. The task I presented was retrieved from the webpage nrich.no and is presented in Figure 7.3.

Mind Your Ps and Qs

Stage: 5 Short ★★

Here are 16 propositions involving a real number x:

$x\int_0^x y\,dy < 0$	$x > 1$	$0 < x < 1$	$x^2 + 4x + 4 = 0$
$x = 0$	$\cos(x/2) > \sin(x/2)$	$x > 2$	$x = 1$
$2\int_0^{x^2} y\,dy > x^2$	$x < 0$	$x^2 + x - 2 = 0$	$x = -2$
$x^3 > 1$	$\lvert x \rvert > 1$	$x > 4$	$\int_0^x \cos y\,dy = 0$

[Note: the trig functions are measured in radians]

By choosing p and q from this list, how many correct mathematical statements of the form $p \Rightarrow q$ or $p \Leftrightarrow q$ can you make?

It is possible to rearrange the statements into four statements $p \Rightarrow q$ and four statements $p \Leftrightarrow q$. Can you work out how to do this?

Figure 7.3: Mind your Ps and Qs from the website nrich.maths.org

The task is about evaluating functions and integrals, and to use implication arrows determining whether it is a one-way implication or whether it goes both ways. When showing the task to Roger, he responds:

> I haven't focused much on implication arrows… I don't feel this is something I should use now and make it an important topic. What is the point of this? There has to be some kind of learning effect if this is something I should move into (*I present integral tasks*).

From Roger's response, I deduce that he is not comfortable introducing implication arrows as a new mathematical notation without a sound reason. He reflects on what kind of learning effect there might be, but is not convinced that it is worth it. This can be related to the work of Brown and McIntyre (1993), where they emphasize how a change has to be viewed as clearly superior to the established practices, to justify the teacher to reject what she already knows and does (Brown & McIntyre, 1993). Roger is hesitant to introduce implication arrows, and questions if this task is superior to what he is already doing. Since he is not convinced the task will give a greater learning effect, he rejects it.

I have on one more occasion coded a statement by Roger as belonging to mathematical characteristic, and I have marked this as a positive comment. When we talk about integration, Roger explains how he would prefer the concept of area introduced in the beginning of an integral introduction. He finds it difficult to come up with a good introduction to integrals but thinks area should be part of it. Although Roger is not talking about a specific task, he is talking about how introductory tasks should include linking area under the graph to integration. Therefore, I interpret this as a positive comment on what integration tasks should include.

Practical considerations

During the semi-structured interview, Roger mentioned twice some practical considerations when it comes to mathematical tasks. One of these was marked as positive and the other as negative. The positive comment was when Roger talked about how tasks should be recognizable. I have marked this as positive, since this is a characteristic that Roger wants included in tasks. He does not elaborate, but I assume he refers to how tasks on the test and tasks in the classroom should not differ too much. In other words, the task in itself should not be a surprise to the student.

The negative comment Roger made about a practical consideration, was concerning how the tasks are formulated:

> There are some tasks where the formulation is not good (refers to physics), and this is something you can also find in mathematics. I can't from the top of my mind remember a task in mathematics that I don't really like… There are some tasks where the students should evaluate the correctness of some

> statements, and this is a kind of task I don't use on tests. That's not my type of tasks (*semi-structured interview*).

Roger teaches physics as well as mathematics, and his first examples of tasks are from physics. He uses some time before he recalls tasks he does not like in mathematics, but then he remembers some evaluation tasks. There are some statements listed, and the students should determine which statements that are true, and which are false. Roger does not like these types of tasks, because it becomes more semantics than mathematics, in his opinion, and he sometimes gets the feeling that the textbook authors are trying to trick the students.

Students' Reactions

Roger does not come across as a teacher who worries a lot about the students' reactions, and I have only categorized two of his statements as students' reactions to tasks. In our first conversation about tasks, Roger reflects on possible improvements. "There isn't everything that goes through to the students when I explain, so it could possibly be that some different tasks might increase their understanding which I don't see" (*first conversation about tasks*). Roger admits that there are occasions where he does not feel the students really understand, and that there might exist mathematical tasks which could be helpful. This is an example of how the students' reactions make Roger more open to implementing some new tasks, but it is not linked directly to any tasks. I have therefore chosen to mark this comment as neutral.

The other comment about students' reactions, was used by Roger to reject a task I presented about composing and decomposing integrals, working in pairs:

> The task is fun enough, but I have never put people together in pairs to work, so they will be surprised by my change of pedagogics. They have never done this before. But I see that the idea could be good (*I present Integral tasks*).

Here, Roger refers to how his students would be surprised by this change, and therefore the statement was coded as students' reactions to tasks. However, I am not convinced that Roger is too worried about the students' reactions concerning working in pairs. I assume this statement is more about what he is comfortable doing in the classroom. At the same time, this may express the importance of being perceived as a consistent teacher.

Summary

Collaborating with Roger provided valuable insight into the perspectives of a teacher who experiences success in his mathematics classroom. Because of this, he is more reluctant than his colleagues to try out various new tasks, but he is also the teacher with the most to "lose". If he does not see enough added value for his students' learning in a task as opposed to the risk of failure in implementation, he does not include it in his teaching just to be nice to the researcher. Roger's feedback on tasks made it even more evident how the other teacher asked for tasks that could help them improve their teaching, since Roger did not have such a need.

7.1.5 Thomas' Descriptions of Tasks

During the collaboration with Thomas, I only managed to be part of implementing and evaluating mathematical tasks once, but in addition we designed and refined tasks within trigonometry as well as logarithms. Throughout our collaboration, there were two statements which were coded as belonging to the dimension *students' reaction to tasks*, eight of Thomas' statements were coded and placed in the dimension of *characteristics of tasks,* while in the dimension *outcome of tasks,* there are 40 codes from the discussions with Thomas. Table 7.4 shows, a schematic overview of the number of statements in each dimension and category from the collaboration with Thomas.

Outcome of tasks		40	Characteristics of tasks	8	Students' reactions to tasks	2
Activity		*1*	Didactical characteristics	2	Students' reactions	2
Understanding		*7*	Mathematical characteristics	1		
Goal of the task		*0*	Practical considerations	5		
Diversity		*1*				
Relevance		*31*				
Exam/ curriculum	3					
Connecting mathematical topics	0					
Practical use	25					
Vocational	3					

Table7.4: Overview of how Thomas' statements have been coded and organized into the three dimensions which in turn contain categories and subcategories. The total number of codes in each dimension are the numbers in the top row, yet the distribution across categories and subcategories is also shown.

I will in the following elaborate on what type of statements that have been assigned to the various categories and subcategories. The 40 codes belonging to *outcome of tasks*, are found in four separate sources. However, most of them are from the semi-structured interview (16) and our discussion of what Thomas would like when it comes to logarithm tasks (15). When it comes to the categories within the dimension of *outcomes of tasks*, most of the codes belong to *relevance* as a category. There are a total of 31 statements which were coded within this category. Further, 25 of these codes belong to the category named *practical use*. Some of these codes were assigned to statements where Thomas criticizes tasks for appearing to be realistic, while they are artificially real in his opinion. At other times, he expresses how he misses a more practical perspective on tasks and the mathematical topics, and this is something he would have liked. Also, he discusses how tasks that are more practically or realistically oriented can be experienced as more difficult by the students. Many of the statements that were grouped into the category *practical use* from the discussions with Thomas, are somewhat similar. For instance, Thomas questions the textbook's presented motivation for logarithms, and misses a focus on why we need the Euler number. He wants a practical link to these mathematical topics, among others. Both examples are thematically similar when it comes to what Thomas wants from the task, which is a practical connection. The only difference is the mathematical topic. This is one of the reasons there are so many codes in the category of *practical use*, and it might even seem excessive since some of them are thematically alike. However, I have tried to stay as close as possible to the teacher's wordings while coding, so the thematic resemblance is something I have realized at a later point in the analytic process. At the same time, I think the repetition of wanting practical links to tasks on several types of mathematical topics, tells a story. Given how often Thomas mentions this and relates it to several topics, this is something that I would say he finds important.

When it comes to the other subcategories of *relevance*, three of Thomas' statements were coded as related to the subcategory *exam/curriculum*. One of these statements is Thomas' reaction to a task I presented for him, where he replies how this is not in the curriculum until he later realizes how the students should be able to reason their way to an answer. I am therefore not interpreting Thomas' response as him refusing a task because he does not find it relevant with respect to the curriculum, but as firstly rejecting it because he did not think the students would have the knowledge to solve it. The other two statements related to *exam/curriculum,* is one comment on how there is too much included in the curriculum and one comment on how he tries to give task examples on the blackboard that are relevant for the exam. I would say Thomas does not give an impression of being too concerned by the exam and curriculum when designing new tasks with me, but this might be related to him being part of making the exam for his students. He knows what type of tasks they make and can even make changes to the exam himself.

No statements were coded within the subcategory *connecting mathematical topics* in my conversations with Thomas, but three statements were coded as *vocational*. These statements are related to chemistry and physics, but he is not just talking about the theoretical field of the subjects. Thomas mentions how some of these students might become engineers in for instance chemistry and biochemistry.

Most of the codes from the dimension of *outcome of tasks* have been grouped into the category *relevance,* as I have explained. However, there are some codes belonging to other categories in this dimension. Only one statement has been coded as *activity,* and this is Thomas explaining how it is not a problem for him to get the group of students to work. Likewise, there is only one code belonging to the category of *diversity*, and this code was assigned to a statement made by Thomas on how his class is rather homogenous and diversity is not really an issue.

There are no statements that were coded within the more general category *goal of the task*. The last seven codes belonging to the dimension of *outcome of tasks*, have been grouped into the category *understanding*. These codes were assigned to comments Thomas made about how the tasks in the textbook are composed so that they do not follow the idea through to the end, and therefore makes it difficult for the students to fully grasp the concept. Another comment is about using tasks to help expose students' possible misunderstandings. Further, one of Thomas' statements is from when we design a logarithm task, and he suggests we let the students use the calculator so they can work with the idea, but do not have to struggle with all the technical difficulties.

In the dimension of *students' reaction to tasks*, there are only two statements from the conversations with Thomas that were coded as belonging here. At one point, Thomas explains how the students find realistic tasks more challenging, and how they prefer a task that is already set up so they know how to solve it. The other statements that I have coded as *students' reaction to tasks*, is from when Thomas is talking about how the students not really check if they have understood the tasks and solved them correctly. The focus is rather on getting the tasks done, than being sure they are correct. Based on these comments from Thomas, it seems like he has a perception of his students preferring routine tasks that are not especially cognitively demanding. This impression is reinforced by Thomas explaining how his students do not check if their thinking is correct when solving the tasks. It seems like his students are more concerned with getting many tasks done, rather than understanding what they are doing, that is, a focus on production rather than learning as described by Doyle (1983).

Throughout the collaboration with Thomas, there are eight statements that were assigned with codes belonging to the domain *characteristics of tasks*. One of these codes was assigned to the category *mathematical characteristics*. This is when Thomas talks about the possibility of making tasks using half-life as context but specifies how this can be done within both integration and differentiation as mathematical topics. There are two codes that were grouped in the category *didactical characteristics*. One of them is from Thomas wanting a historical introduction to a task, and the other is about Thomas aiming for tasks that are recognizable for the students. There were five statements from the discussions with Thomas that were coded and grouped into the category *practical considerations*. These codes were assigned to Thomas' comments about wanting to change the sequencing of subtasks in a task, not being happy about the students having too many different textbooks, which makes it chaotic, and the sequencing of topics in the textbook. In addition, he wants to make changes to how a formula is presented to the students.

Throughout the collaboration with Thomas, he has mostly focused on the practical or realistic aspect when describing tasks. However, even if this has been a focus, he has not solely talked about this as a positive aspect of tasks. He has also problematized how students find realistic tasks more difficult to solve. This can be related to Verschaffel's (1999) descriptions of issues students might struggle with when solving mathematical application problems. Nonetheless, he asks me to design tasks with a practical perspective in our collaboration. Thomas has expressed how the students can sometimes be hard to motivate for learning parts of the mathematics, and he has talked about how a practical perspective might be motivating.

It has sometimes been difficult to group the codes from my conversations with Thomas, for example when he criticizes the textbook for not highlighting why we need the Euler number. At first glance, I thought this was about wanting the students to have a better understanding of the Euler number, and thus I could have placed the code in the category *understanding*. However, Thomas does not mention understanding at this point, and later he talks about how a practical connection in the task might be viewed as motivating by the students. So again, this is an example of how important it is to not interpret too much too early when trying to capture the teachers' perspective. Thomas might, of course, also want the students to have a deeper understanding of the Euler number, but this is not the reason he his giving at this point.

Thomas' Grading of the Categories

Table 7.5 presents whether Thomas spoke of the categories in a positive, negative, or neutral way. The details of this analysis follow below the table.

Categories	+	-	0
Activity	0	0	1
Understanding	3	1	2
Goal of the task	0	0	0
Diversity	0	0	0
Exam/curriculum	1	1	1
Connecting mathematical topics	0	0	0
Practical use	8	14	3
Vocational	3	0	0
Didactical characteristics	1	0	1
Mathematical characteristics	0	1	0
Practical considerations	0	4	1
Students' reactions	0	1	1

Table 7.5: Overview of the grading of Thomas' categories.

Activity

Thomas does not mention activity much in our collaboration, and the one time he does, he expresses that it is not a problem. Since Thomas is not referring to any specific task, I have marked this comment as neutral. I asked Thomas if it was easy to get the students to work, and he responded: "Yes! It might sometimes be a little loud when they work and discuss, but then I just ask them to keep it a bit down. They are not chatting about everything else; they are working" (*semi-structured interview*). Here, Thomas is clear about how it is not a problem to get this group of students to work on tasks. This might also explain why this is the only statement I have coded with activity in our collaboration, because this is not an issue for him.

Understanding

Thomas talked positively about mathematical tasks with respect to understanding on three separate occasions. All these comments were related to how he finds it positive that the students need to reflect to be able to solve the task. One example is when we made a historical task about logarithms, and Thomas realized the logarithm tables were not so easy to use. We then made some changes so that they could solve tasks using the logarithms, but without having to use the tables.

“That way they can work on the ideas, but don't have to learn to use the table” (*I present logarithm tasks*). Here, Thomas expresses that he is pleased with how the task challenges the students to work on the ideas of calculating using logarithms, but without having to learn to use the logarithm tables. Another example of how he expresses understanding as a positive element of tasks, is his reaction when I presented the task about adding sine and cosine functions (Section 6.7). “No, this is not in the curriculum. But.. (thinks) … the students should be able to reflect on this and solve it… I can take this task as well” (*I present trigonometry tasks*). First, Thomas rejects the task because he assumes it requires knowledge the students do not have to solve it. However, he wants to use the task when he realizes it should be possible for them to reflect and find a solution.

Diversity

The only time Thomas refers to diversity in our conversations, he explains how it is not a problem in his class and that it is a homogenous group of students: “But this group of students is relatively homogeneous, they are focused on working and doing tasks, just like I learned mathematics at secondary school” (*semi-structured interview*). Here, Thomas elaborates on how this group of students are both homogenous and hard working. Consequently, he does not have to design tasks to motivate the students to work or to adapt tasks both to high achievers and low achievers. I have therefore marked diversity as a neutral comment with respect to mathematical tasks, as diversity is not an issue for Thomas in his classroom.

Exam/Curriculum

Thomas does not refer to the exam or curriculum on many occasions, but when he does, he makes it clear that both the exam and curriculum are important in his planning. I have marked one comment as grading tasks positively, one as negative and one as neutral. During our talks, he expresses disagreement with the curriculum, and how it sometimes includes elements that he views as unnecessary in an intensive course:

> This is an intensive course, and I wonder why they include so much, so many sections that are unnecessary. For example, integration methods, numerical integration which is a huge job to do by hand, and this is something computers do (*semi-structured interview*).

Thomas clearly does not agree with many of the elements included in the curriculum but mentioning them also means he pays attention to the demands of the curriculum. I have marked this comment as neutral when it comes to grading of tasks, because he is criticizing the curriculum and not any specific task. The comment, which was marked as negative, is from an occasion when Thomas was about to reject a task (Big, Bigger, Biggest, Sub-Section 6.6.5) claiming it required knowledge which was not included in the curriculum. This statement can of course be understood as Thomas not wanting tasks he thinks the students will not be able to solve, and not necessarily that he is negative to a task not being explicitly mentioned in the curriculum. However, to Thomas, these perspectives might be viewed as equivalent; the curriculum is a description of what the students should know at this point.

At one point, Thomas also made a comment about trying to give the students examples that are relevant for the exam. This is something I have graded as a positive comment about tasks, because he expresses a wish for using some tasks and examples that are related to the type of tasks they will get on the exam.

Practical use

During the collaboration with Thomas, practical use is the code that has been used most often by far. In total, I have assigned 50 separate codes to the discussions with Thomas, and half of them were marked as practical use. Eight of these were marked as positive comments, 14 as negative comments and three as neutral. However, Thomas' expressions about tasks with respect to the practical perspective are consistent, even if they are divided between positive and negative comments. The positive comments are statements where he expresses how he wants a practical perspective on tasks within a topic. For instance, when we talk about possible tasks in the logarithm chapter, Thomas explains:

> I want more practical tasks where this is possible. This is why I feel the logarithm chapter is worse than the trigonometric chapter. There is no reasoning as to why the students need logarithms (*discussion of tasks logarithms*).

Here, Thomas gives a general statement of wanting more practical tasks when it comes to logarithms, and he even explains why he finds this important. He wants the practical perspective so that the students feel a need for learning about logarithms. The other statements that were marked as positive when it comes to a practical perspective on tasks, concerns Thomas wanting either a specific practical connection to a topic, or he is praising a task due to its practical connection.

The statements that were coded as negative, are Thomas' comments on how tasks have either an unrealistic practical perspective or lack a practical perspective. Most of these comments are made with respect to specific tasks in the textbook, but Thomas also gave a general description of mathematical tasks he does not like when asked about it in the interview: "Yes, these mathematical models that has nothing to do with real life at all" (*semi-structured interview*). Here, Thomas is clear about not liking tasks that aim to relate to real life, but which he finds unrealistic. This is a standpoint that he consistently holds throughout our conversations, and comments on several tasks in the textbook that he does not view as realistic. The statements connected to a practical perspective that were marked as neutral, are statements concerning the practical perspective, but not entailing a positive or negative comment about tasks. For instance, Thomas refers to how many students find logarithms difficult: "Many students struggle with logarithms because they don't understand the point of it" (*discussion of tasks logarithms*). I have coded this statement 'practical' because Thomas expresses an issue when the students are working on tasks within logarithms as a topic. However, I have also marked this as neutral, because he is not praising or rejecting any specific tasks.

Even if Thomas' statements regarding the practical perspective on tasks are consistent, he has on two occasions made reflections on how he finds this a complicated issue:

> My students are going to be engineers, which is a practical profession. So, when I teach, I feel this topic [logarithms] becomes too theoretical. It is the easiest for me as a teacher, because the students are more likely to complain about a task being too difficult when it is more realistic (*I present logarithm tasks).*

Although Thomas wants tasks with a realistic, practical perspective, he is also aware that the students are not too happy with those type of tasks, as they often find them more difficult. Thomas told me how he, with a background in physics, could have presented many of the topics in a more realistic way for the students. However, he struggled with doing it in a way that would not make the students protest because the tasks became more demanding. These challenges recognized by Thomas can be related to those described by Verschaffel (1999).

Vocational

On three occasions when we discussed tasks, Thomas referred to the vocation the students would have in the future. All these comments were positive, and he expressed approval of tasks referring to how this is a relevant topic for some of the students' vocational goal. When I presented the task about medications and half-life, Thomas responded: "This task is somewhat like the other task about half-life, but this is the type of stuff we can use. Some of them might become engineers in chemistry or biochemistry" (*I present logarithm tasks*). Even if Thomas is not specifically mentioning the students' future vocation frequently, he is clear that tasks that are relevant for their future vocation is positive.

Didactical Characteristics

On two occasions, Thomas made comments that I have interpreted as didactical characteristics of the task. I have marked one of these comments as positive, because Thomas asked for a historical introduction to the tasks on logarithms. He does not explain why, but I interpret it as a didactical choice situating logarithms in a context. The other statement that I marked as a didactical characteristic, is when Thomas talks about how he tries to teach in alignment with the textbook:

> I emphasize that the teaching is related to the tasks, otherwise the students complain that we haven't seen this before. Because sometimes the book can be misleading. The examples don't fit the tasks. Sometimes the tasks are even related to topics that come later in the book (*semi-structured interview*).

In this statement, Thomas points to how tasks should not be a surprise for the students. Consequently, he tries to give examples and teach in a way that will prepare them for the tasks they are going to solve.

Mathematical Characteristics

There is only one time I have marked a statement as a mathematical characteristic during my collaboration with Thomas, and I have graded it as a negative comment about tasks. That is because Thomas questions why the textbook is not using integration and differentiation as mathematical topics to solve tasks on half-time.

Practical Considerations

Most of the five statements that were marked as practical considerations, are graded as negative. Thomas is on one occasion asking for the sequencing of subtasks within a task to be changed, and I marked that as a neutral practical consideration since he does not explain why. The other four statements are critical reflections of the textbook, but also concerning one of the tasks I presented for him. I made a resource in GeoGebra for Thomas regarding adjusting a sine function to measures of tide in Bergen, Thomas looks at it, and responds:

> It uses a different formula than the book. This is a problem, because the formula collection the students use, is also different from the one in the book. The low achievers struggle with this (*I present trigonometry tasks*).

Thomas criticizes the formula which is used in the task and would like it to be the same as the students meet in the textbook. The other comments being about practical considerations, are directed towards the textbook. Thomas is for instance not happy with the sequencing of some of the topics in the book, and he comments about the sequencing of subtasks within a task.

Students' Reactions

Thomas is negative regarding teaching his students everything about how they used logarithmic tables before, since it is complicated. He does not want to burden his students with all of it. I have therefore marked this comment as a negative reaction to a task. The other time Thomas mentions his students' reactions, it is a reflection on how his students often calculate without reflecting on their answers. This is a comment that I have marked as neutral since Thomas neither rejects nor praises any tasks due to the students' reactions. He rather comments about how many of the students often work on mathematical tasks.

Summary

Collaborating with Thomas provided valuable insight into how a teacher with hard working students still looked for ways to motivate them in mathematics. He is a teacher with lots of subject knowledge in mathematics and physics but has less didactical knowledge and experiences. Still, he does his best to articulate and ask for mathematics tasks that might help his students see the importance of mathematics and also to motivate them to explore more on their own.

7.1.6 Hanna's Descriptions of Tasks

Throughout the conversations with Hanna, there is a clear majority of statements being categorized in the dimension *outcome of tasks*. In total, 83 codes were created within this dimension. In comparison, 23 codes were created within the dimension *characteristics of tasks* and 25 codes were assigned to the dimension *students' reactions to tasks*. So, mostly Hanna does not describe specific characteristics of mathematical tasks but is rather describing them with respect to what type of outcome she wants or what kind of outcome she experienced that they provided. In Table 7.6 there is a schematic overview of the number of statements in each dimension and category from the collaboration with Hanna.

Outcome of tasks		**83**	**Characteristics of tasks**	**23**	**Students' reactions to tasks**	**25**
Activity		***28***	**Didactical characteristics**	**10**	**Students' reactions**	**25**
Understanding		***12***	**Mathematical characteristics**	**0**		
Goal of the task		***4***	**Practical considerations**	**13**		
Diversity		***8***				
Relevance		***31***				
Exam/ curriculum	4					
Connecting mathematical topics	6					
Practical use	15					
Vocational	6					

Table 7.6: Overview of how Hanna's statements have been coded and organized into the three dimensions, which in turn contain categories and subcategories. The total number of codes in each dimension are the numbers in the top row, yet the distribution among categories and subcategories are also shown.

In the following, I will elaborate on what type of statements that have been assigned to the various categories and subcategories. When looking more closely at what type of outcomes Hanna talked about, there are 28 codes that are related to the category *activity*, and these 28 codes are distributed across ten different sources of data. In other words, Hanna has mentioned activity in many of our conversations. So, designing tasks that make the students active and working, is something Hanna emphasizes throughout our collaboration. Another category which is frequently represented in the discussions with Hanna, is *relevance*, which contains a total of 31 codes. However, not all subcategories of relevance are mentioned equally often. There are only four times the codes are related to the *exam or curriculum*, and the category *vocational* was just represented with six codes in the data material. However, 15 codes were assigned to the category *practical use*. Thus, while Hanna refers to tasks being connected to practical aspects or being realistic, she does not necessarily focus specifically on the tasks being vocationally related. She does not seem too worried about the exam they ultimately need to pass, either. When it comes to the last subcategory *relevance,* which is *connecting mathematical topics*, Hanna's statements were coded within this subcategory six times.

Diversity is a category that only has eight codes assigned in the data material. Half of these codes are from the same source, being the conversation with Hanna where we evaluated the task about proportions. So, Hanna has not specifically asked for tasks to deal with diversity issues to any extent but has evaluated a task with respect to how it worked for both high and low achievers. However, it does not seem that diversity is among the issues which concerns Hanna the most, based on how few times she talks about it during our collaboration.

Understanding is a category that was assigned 12 codes, yet these 12 codes are spread across six various sources and understanding is mentioned by Hanna both when we discuss what tasks to design, and when evaluating the implementation of the tasks. In the dimension *outcome of tasks*, there is one last category that contain outcomes that do not fit into the other subcategories. This category was named the *goal of the task*, including four codes from the collaboration with Hanna. The codes are related to Hanna's wish for introductory tasks for some topics. While she sometimes elaborates on what she wants to achieve with an introductory task, she does not always specify beyond naming it an introductory task.

When looking more closely at the 23 codes in the category of *characteristics of tasks*, none of Hanna's statements are purely related to *mathematical characteristics*. However, there are ten codes assigned to the category *didactical characteristics,* and 13 codes that were categorized as *practical considerations*. All the codes in the category *practical considerations* were assigned to conversations with Hanna where we were evaluating or refining tasks. The only exception is one comment from the interview where Hanna commented on a task in the textbook that she thought was well structured. However, based on the analysis of the conversations with Hanna, it seems like statements that can be coded in the category *practical considerations* are more likely to emerge during an evaluation of tasks rather than during the design process. When it comes to the ten codes in the category *didactical characteristics,* Hanna uses a variety of words. She talks about open tasks, inquiry, exploring, visualizing, creative and investigating. However, six codes, i.e., more than half of these codes, are also from conversations where we are evaluating tasks. So, Hanna is in general more specific on characteristics of tasks when she evaluates them, than when she describes what she is looking for. Maybe this is a result of her own stated lack of didactical confidence in mathematics. That is, she is reluctant to ask for specific characteristics due to uncertainty, yet she has more confidence in commenting them in retrospect. However, it might also be that Hanna is not giving specific characteristics of the tasks because she is either assuming it to be implicit when she explains the outcome she wants, or that she is deliberately not giving specific characteristics because she wants to be open to all possibilities that might help her achieve what she aims for.

There were 25 codes assigned to the dimension *students' reactions to tasks*. These codes come from six separate sources, however most of them were assigned to statements Hanna made when we evaluated the implementation of tasks. 21 of the codes are from these phases of our collaboration, and they were assigned to statements Hanna made about the difficulties she faces when implementing tasks that we have designed. Ten of the codes are from the evaluation of the rope and area task, but there were also nine codes assigned to the evaluation of the index tasks. These were the implementations where Hanna was not happy with how everything went, and the codes belonging to the dimension *students' reactions to tasks,* reflect this. When it came to Hanna's comments when we evaluated the implementation of the rope and area task, most of the comments belonging to students' reactions were related to the rope task. She did not experience the students as accurate when they worked, and many of them struggled on task 3 where they should use a string to make a figure with a given area (Section 6.4). Some of the students completed the task but did not check or make sure their answer was correct, which it was not. She did not feel that the students engaged in the task and she even questions whether they have any mathematical curiosity at all. To summarize, Hanna experienced many of the students' reactions to the rope tasks as negative, and she did not feel competent to help them past their struggles. She was happier with the implementation of the area task, but questions if the students have enough ownership in the process of the area task. She questions if more 'heavy' investigational tasks would be possible in this class, given how difficult it is to get them to do something as concrete as the area task. From the evaluation of implementing the index task, the codes that were assigned to statements belonging to the category *students' reactions* are mostly related to how the students are not doing what they are supposed to. Hanna refers to the students not really reading the information they are given, but just skimming through to get started. In addition, she says the students are not very good at finding information on their own, and often they just do other things.

Among the codes belonging to the category *students' reactions*, there are some that were assigned to Hanna talking about how one of the students tried to take control over classroom discussions. In addition, she explains how a mathematical table can be frustrating for some of the students with a more practical approach, because they like to take short cuts and figure things out on their own. Another code from the category *students' reactions*, is when Hanna talks about how the students did not experiment when using the index calculator, so it did not work. The last code when we were evaluating the index task, was assigned to when Hanna summarized the lesson by saying that the students were too noisy, but also blaming herself for not managing the class better. There were also two codes that were assigned to the category *students' reactions* from when Hanna and I evaluated the A4-task. One of the codes was assigned to a statement Hanna made about a positive reaction from a student on the A4-task. Hanna told about how this student blossomed while working on the A4-task, working independently, and seeing the relationships. This student is a low achiever and part of a group with special needs in mathematics, so Hanna was positively surprised by how this student reacted to the task. The other code I assigned to the category *students' reactions*, was a more general comment from Hanna on how students expect credit for just writing something on a test, regardless of whether there is some value in the writings or not.

The four last codes in the category *students' reactions* were from an evaluation of implementing mathematical tasks, but here Hanna refers to something they have tried before. That is, designing tasks that were more vocationally relevant, but where the students did not respond well to the tasks. Both statements from the semi-structured interview are related to this, as well as the statement coded when we had our first talk about what type of tasks she would like. There is only one code in the dimension *students' reactions to tasks* that was assigned to a statement prior to a task evaluation. This statement was when Hanna commented how we should not give the students too much information in an image, because then they might not be interested in doing the task.

To summarize, it seems that Hanna is making comments that are coded to be within the domain of *students' reactions to tasks*, more or less solely when evaluating tasks, and when the implementation has not succeeded, in her opinion. She does not make statements that I have coded and categorized in this domain that are used to reject certain tasks or subtasks.

Hanna's Grading of the Categories

Table 7.7 presents an overview of whether Hanna spoke of the categories in a positive, negative, or neutral way. The details of this analysis follow below the table.

Categories	+	-	0
Activity	20	8	0
Understanding	10	2	0
Goal of the task	4	0	0
Diversity	3	2	3
Exam/curriculum	1	3	0
Connecting mathematical topics	6	0	0
Practical use	11	3	1
Vocational	2	0	4
Didactical characteristics	9	1	0
Mathematical characteristics	0	0	0
Practical considerations	2	2	6
Students' reactions	1	23	1

Table 7.7: Overview of the grading of Hanna's categories.

Activity

Hanna mentions activity frequently throughout our collaboration, and I have marked most of these statements as positive with respect to tasks. Hanna asks for tasks that will make the students active and this is one of her main characteristics of a mathematical task she wants in her classroom. She makes this clear on many occasions and here is an example from when we evaluated the rope and area task:

> The goal with the task, in addition to a different introduction of the topic, is also to engage the low achievers to make them more active. I got more students active, but the ones who are the most resistant don't do much no matter what. But this was maybe a bit more successful on the area task (*evaluating rope and area task*).

Hanna expresses active students as a goal with the task, as well as evaluating the success of a task by how active her students were. The excerpt from our conversations above, is a typical example of Hanna's positive comments about mathematical tasks with respect to activity. I have also marked eight of Hanna's comments as negative, and these are from when Hanna criticizes the implementation of a task because the students are not as active as she would have hoped, or it could be that she is worried about getting the students active on a certain type of task. For instance, throughout our collaboration, it surfaced that Hanna initially had thought the tasks should be open with many solutions, but this is not the case with many of the tasks we have designed together. Hanna is reflecting upon this in the excerpt below:

> But in this task, they must do something, and it is an activity, which is what I have asked for. I feel this area task works as an introduction task for area. It is questionable if you use 'heavier' investigational tasks, if it is going to work in this class when I see how much I struggle just to get them to do something as concrete as this (*evaluating rope and area task*).

Here, Hanna is reflecting on how the students are active and are working, even if it is not an open task. She also questions whether more open, investigational tasks are even possible with this class because she finds it challenging to get them working.

Understanding

Hanna expresses how she wants tasks where the students develop an understanding through working on them. Ten of the comments about understanding were graded as positive, because this is a characteristic Hanna wants in a task and something she evaluates tasks by. The two comments that were graded as negative when it comes to understanding as a characteristic of tasks, are because Hanna criticizes some tasks that do not require understanding, in her opinion. An excerpt from a conversation with Hanna that shows both an example of her being negative and an example of how she wants a focus on understanding, comes from our first conversation about tasks: "But the tasks in the book are dry, mechanical and procedural learning. Here is the formula and then practice it. Maybe one should get some more understanding into the tasks?" (*first conversation about tasks*). Although Hanna does not explain what she means by getting some more understanding into the tasks, I assume it is as opposed to her description of the textbook tasks which she did not like, i.e., tasks that only demand mechanical and procedural thinking.

Goal of the Task

In the first conversation with Hanna about what kind of tasks she wants, she explained how she wanted some type of introductory tasks. Three of the statements which were coded with goal of the task have this focus, and all of them are graded as positive since this is something Hanna wants.

> Introductory tasks. A way to introduce the topic without me being at the blackboard talking. They should do an activity themselves. And when they are done with the activity, many of them understand the concept (*first conversation about tasks*).

Hanna explains how she wants tasks to introduce a new mathematical topic instead of her talking and explaining at the blackboard. She also expresses how she wants the students to gain understanding through working on these tasks. On one occasion, Hanna talked about wanting tasks that motivate the students to explore, and this is the fourth statement I coded as a positive goal of the task.

Diversity

Hanna only once talks about the group as diverse, and thus needing tasks that will work for a diverse group. However, she talks about both low achieving students and high achievers, meaning several types of students, as opposed to the class as a diverse group. She addresses the diversity indirectly by focusing on various achievement levels of students. I have marked three of Hanna's statements as positive when it comes to diversity, because Hanna gives examples of positive effects for specific groups of students or wants to use a task with specific groups of students. When we refined the task about proportions, Hanna commented:

> I would like to try this one. I have one of those basic skills groups in the program for Restaurant and Food Processing. But I don't think they will get as far as to the square [task 7], but this they should be able to do [the measuring in the beginning] (*refining A4-task*).

Here, Hanna talks about wanting to use this task with a group of students with special needs, in addition to her regular groups of students, hence confirming this is a task that can be used in diverse groups. The statements from Hanna that I have marked as negative with respect to diversity, are when she comments on specific groups of students where things are not always working well. For instance, Hanna was not pleased with how some of the low achievers worked on the A4-task:

> Many of them got lost in some ridiculously long sequences of decimals, here. That they don't have the ability to understand that it's not so interesting, and these are very low achieving students. I saw this on one of the students on the basic skills group. He is very low achieving, and he used a looong time to write down all the digits from the calculator. He doesn't have the ability to see what is relevant here. What it's important to work on. So that's part of the problem for many of the low achieving students (*evaluating A4-task*).

Although Hanna expresses a general challenge for low achieving students in mathematics, I have marked it as a negative comment about the task because it surfaced for some of the students when working on this task. I have also marked some of the statements about diversity as neutral. These statements are more about how to adjust the presentation of the tasks to meet the diversity of the group: "I'm uncertain what to do with the students who have finished a task. Should they get the next one or should I summarize in between every task?" (*implementing area task*). When implementing the area task, Hanna realized that the students were working at different paces and was not sure how to deal with it. These are the type of statements that I have marked as neutral concerning diversity because she is not making any positive or negative judgement of the task itself.

Exam/Curriculum

Hanna does not talk frequently about the exam or curriculum but mentions it on some occasions. Most of these statements were graded as neutral comments because she just explains how things are, or what they are going to do. For instance, when we evaluated the A4-task, Hanna rounded up by saying: "And now they have to do some calculating on similarities, and how to use it on tasks they get on tests and the exam" (*evaluating A4-task*). Here, Hanna tells me how the students also need to work on similarities in contexts that are relevant for tests and exams. She was happy with the tasks we designed and implemented, but also wants the students to work on tasks like those they will get on the exam.

Three of Hanna's statements that were coded with exam/curriculum, are of this type. The one I graded as positive, is when Hanna comments that by doing this task, we have also fulfilled one of the requirements in the curriculum. "We have now used a consumer calculator, which the curriculum requires" (*evaluating index task*). I have interpreted this as a positive comment about the task because we are fulfilling parts of the curriculum requirements.

Connecting Mathematical Topics

Hanna says she wants tasks that can connect different mathematical topics, and I have graded all her statements regarding this as positive. The reason for this, is that she either says she wants this in the tasks we design, or because she evaluates tasks positively if they accomplish to connect mathematical topics. Hanna tells me about how she and some colleagues have talked about this:

> We have been discussing if we could make some exploratory tasks related to indexes, ratio, scale, proportions and so on in the beginning of the school year. These are topics that come back throughout the whole curriculum, and if they have a good teaching sequence in the beginning, they can relate to it throughout the school year. So far, we have just conducted brainstorming around the topic, so nothing is ready, but we want the students to experience that all these topics are basically the same (*I present index task*).

Both Hanna and her colleagues want to use tasks that can help the students to see and connect mathematical topics; to realize they are basically the same.

Practical Use

Hanna mostly talks about tasks which are connected to practical use or can be perceived as meaningful by the students, in a positive way. This is a characteristic that Hanna looks for in mathematical tasks. All the 11 statements that were graded as positive when it comes to practical use, are about Hanna asking for this to be included in a task or she speaks positively about tasks having such a perspective. For instance, Hanna tells me about a similarity task she has used and why she likes it:

> I like this task because the students will react if a person is 36 cm high, for instance. They will not necessarily react if the length of a side in a triangle is a

> little off, so many of them add instead of multiplying (*evaluating rope and area task*).

Here, Hanna gives an example of a task where the practical context will, in her opinion, help the students to evaluate their answer. So, she not only asks for a practical perspective in the tasks, but she also gives examples of how they can be helpful. In addition, she mentions meaningful tasks on several occasions, linking this to a practical perspective.

Three of the coded statements were graded as negative with respect to practical use. Two of these statements are about tasks from the textbook that Hanna claims do not make sense to the students, while the last statement is about how a practical context also can be challenging. “I have previously gotten the class to make personal budgets and to calculate expenses and incomes. The problem is that many of the students don't believe they have expenses” (*I present index task*). This is an example where a practical perspective can be challenging because the students do not recognize this as their reality. If they live at home and their parents are paying, they do not think of these as expenses they have in their personal experience.

The last statement that was coded with practical use was graded as neutral, because Hanna just tells me about how students sometimes ask her why they need to learn this. I am interpreting this as the students requesting a practical necessity for the mathematics, but Hanna is not evaluating tasks by this comment, hence I marked it as neutral.

Vocational

I have chosen to mark only two of the four statements as positive with respect to the vocational aspect. This is because there is a requirement of having a vocational perspective in these mathematics classes, so just referring to a vocational aspect in a task, means I will mark the statement as neutral. However, Hanna is on two occasions elaborating a bit more, and gives the impression that she views linking vocation to mathematical tasks as positive:

> I have been inspired by investigational tasks through the courses I have taken at UiA, and in the workshops [research community lead by UiA] where we have talked about both linking it to vocation and inspire the low achievers. I want to accomplish something like this (*evaluating rope and area task*).

Here, Hanna describes how the University has inspired her to want investigational tasks that are also linked to the students' vocation. She is explicit about how a vocational aspect is a positive element in mathematical tasks for these students.

Didactical Characteristics

Throughout our collaboration, Hanna mentions several didactical characteristics of tasks. She talks about investigative, open, creative, explorative, and varied tasks and teaching. I have marked almost all her statements about didactical characteristics as positive because she talks about wanting tasks with these types of characteristics. The only statement that I marked negative, was when she told me about tasks she had made for a low achieving group of students on Pythagoras which she was happy with, but said in the end: "This approach is maybe not creative enough and I am not sure if the students have enough ownership in the process" (*evaluating rope and area task*). Here, Hanna reflects on whether her experienced success with these tasks is only related to the students being able to follow instructions, or if they also gain an understanding of the concept. She uses the word creative, which I have coded as a didactical characteristic. I cannot be sure how she defines creative, but from the context it seems that she is evaluating the students' possible ownership in the task.

Practical Considerations

Most of the statements that were coded as practical considerations, are graded as neutral. This is because Hanna only describes practical adjustments she wants to make to tasks but is not elaborating on why or if she thinks this improves the task. For instance, when we were in the process of refining the A4-task, we discussed the possibility of giving task seven orally, and Hanna says she could also draw it on the blackboard and explain what the students should do. This is a type of statement that I have marked as neutral since Hanna is not explicit on whether it is an improvement of the task or not.

The two statements that were graded as positive, are because Hanna comments on some practical elements of a task in a positive way. For instance, she says she likes that there was one task on every sheet of paper on the area task. This was a change to the task originally initiated by her colleague Sven, but Hanna likes it. The two statements that were graded as negative, are from when

Hanna talked about giving too much freedom to the students in the index task: “Just giving them the option to choose a house, makes it difficult. I think they need even more structure. It takes too much time looking around for houses” (*evaluating index task*). Hanna expresses frustration because the students spent, in her opinion, too much time on finding a listed house instead of working on the mathematics. When I ask her if she would rather just give them the details about a house next time she uses this task, she says yes. Also, the other comment, which was graded as negative, was from the same evaluation of the index task, concerning not formulating the question specifically enough. Hanna thought the questions should have been more concrete.

Students’ Reaction

Almost all the statements that were coded as students’ reactions, were also graded as negative. This is because most of the statements are about Hanna evaluating how tasks went where the students did not react as she had anticipated or hoped. An example of this is from when we evaluated the implementation of the index task:

> Even if the task is formulated with bullet points and is concrete, they don't follow it. Most of them probably don't read it carefully. They just look down and find a couple of links [to the Internet]. So, I miss the structure that I hoped for (*evaluating index task*).

Here, Hanna describes how the students are not reading the information as carefully as she wished for, which in turn creates a problem. So, many of the statements about students’ reactions being marked as negative, does not necessarily mean that the students are negative to the task, rather on many occasions the students are not reacting or behaving the way the teacher expected. However, there are also a couple of examples where the students expressed that they did not like a task.

I have graded one statement as neutral, and that is when Hanna explains that if you ask students what type of teaching they learn the most from, you often get blackboard lectures as an answer. Hanna is not attributing any positive or negative value to this, but just explains the type of response students are likely to give. The one statement about students’ reactions that I have marked as positive, is from a situation where a student positively surprised Hanna:

> One of the girls in the basic skills group really blossomed with this task [A4-task]. She did everything on her own and saw the relationships and so on. I didn't get this 'extreme' response in the big class, but some of the high achievers worked well and got furthest and could express some thoughts around it (*evaluating A4-task*).

Hanna used the A4-task in the small basic skills group first and experienced how one of the girls really blossomed, took responsibility, and worked on this task. So, this is an example of a very positive student reaction in Hanna's opinion

Summary

The collaboration with Hanna provided valuable insight into what a teacher who really wants to further develop herself as a teacher, might ask for in tasks. Hanna wanted to learn, and she wanted to improve her teaching in mathematics. This made her open to try out various tasks, although not all of them were successfully implemented in her opinion.

7.1.7 Sven's Descriptions of Tasks

Sven mostly describes mathematical tasks by their outcome, just like the three other teachers in this research project. In total, I have coded conversations with Sven with codes belonging to the dimension *outcome of tasks* 54 times. In contrast, I have coded the same data with 18 codes belonging to the dimension *students' reactions to tasks* and 13 codes belonging to the dimension *characteristics of tasks*. Table 7.8 presents a schematic overview of the number of statements in each dimension and category from the collaboration with Sven.

Outcome of tasks	54	Characteristics of tasks	13	Students' reactions to tasks	18
Activity	*9*	Didactical characteristics	1	Students' reactions	18
Understanding	*11*	Mathematical characteristics	2		
Goal of the task	*1*	Practical considerations	10		
Diversity	*15*				
Relevance	*18*				
Exam/ curriculum	7				
Connecting mathematical topics	5				
Practical use	6				
Vocational	0				

Table 7.8: Overview of how Sven's statements have been coded and organized into the three dimensions, which in turn contain categories and subcategories. The total number of codes in each dimension are the numbers in the top row, yet the distribution among categories and subcategories are also shown.

In the following I will elaborate on how I have grouped the codes assigned to statements from the collaboration with Sven into categories and subcategories. When looking more closely at the categories belonging to the dimension *outcome of tasks*, *activity* contains nine codes. However, most of these codes were assigned to one source, which is the evaluation of the A4-task. This source was coded six times with *activity*, while the three last codes were divided between the interview, discussion of area task and evaluation of area task. *Understanding* is another outcome that was frequently mentioned throughout the conversations with Sven. In total, *understanding* is a category containing eleven codes from four sources of the data material. The codes are from all the types of data sources: the semi-structured interview, discussion of tasks, and evaluation of the implementation of tasks.

Relevance is a category that contains a total of 18 codes. Five of these codes belong to the category *connecting mathematical topics*. However, even if there are only five of these codes, they are from various sources, being discussion of tasks, refining tasks, and evaluation of tasks. So, even if *connecting mathematical topics* have not been mentioned so frequently, it has been brought up on various occasions. Seven of the codes from the discussions with Sven belong to the category *exam/curriculum*, and these come from three separate sources. These codes were assigned to statements where Sven talks about textbook authors' interpretations of the curriculum, as opposed to his own interpretation and the interpretations of the group he is part of, who design the vocational exams. When it comes to *practical use*, I have coded six statements in the data material within this category. However, none of the statements during my collaboration with Sven contains codes belonging to the category *vocational*.

When it comes to diversity in the classroom, and to what degree the mathematical tasks meet these issues, I have assigned a total of 15 codes belonging to the subcategory *diversity*, being derived from six different sources. So, diversity is an issue that Sven refers to frequently. The last category belonging to the dimension *outcome of tasks*, is the category the *goal of the task*, where I have grouped codes that do not fit into the other categories. Sven has made one statement which was coded and placed here, and that is about him wanting a task where he can be more in the background as a teacher while the students are working on the task.

In the dimension of characteristics of tasks, Sven has made two comments that were coded and grouped in the category *mathematical characteristics,* and both are related to how the area of a triangle stays the same if the height and baseline are the same. One of the comments was made when we discussed and designed the area task, and the other was made when evaluating the same task. When it comes to the category *didactical characteristics*, there is only one code assigned, being the statement where Sven talks about the textbook not giving examples of entities that were not proportional, and thus not exposing the students to this. There are ten codes that were placed in the category of *practical considerations*, because Sven has not explicitly stated a didactical reason for his choices. Seven of these codes are from the process when we designed or refined tasks and three of them are from when we evaluated the area task. Still, just looking at the source of where the statements are from, can give a false impression of the statements being evaluating comments, while the statements were made with respect to further development of the task. Sven had used the same task with another class and had made some further improvements on the task which he told me about when we evaluated the implementation of the area task. So, all the codes belonging to the category *practical considerations*, are from the designing stages of the tasks.

There is a total of 18 statements from my collaboration with Sven that were coded within the dimension *students' reactions to tasks*. These codes are distributed among four sources, but most of them (14) belong to when Sven and I were refining the first task we designed, and from evaluating it. I interpret this decline in the frequency of these specific codes in the collaboration with Sven as a result of me learning more about what Sven saw as limitations and possibilities in his student group. The statements coded within *students' reactions*, with respect to our work on the first task, were related to how classroom discussions were not possible, how the task was too difficult, and similar codes expressing limitations of the task. So, when I presented the next ideas for tasks on area, I tried to consider what Sven had previously stated would not work well with his group of students, and it seemed like I managed to design tasks that Sven found better suited for his classroom and students.

When it comes to the collaboration with Sven, his statements are consistently coded across the various stages of our collaboration. Most of the categories can be found in all the types of sources I have, whether it is the discussion or refining of tasks, or the evaluation of tasks. However, there is one category where the codes did not emerge until we started refining the tasks we planned to use, and that was the category *students' reactions*. I do not think this is because Sven's opinions changed, but the issues did not become relevant to discuss before we had the first drafts of the tasks in front of us. In general, Sven gives an impression of being clear on what he wants and being consistent in wanting this.

Sven's Grading of the Categories

Table 7.9 presents whether Sven spoke of the categories in a positive, negative, or neutral way. The details of this analysis follow below the table.

Categories	+	-	0
Activity	9	0	0
Understanding	7	2	2
Goal of the task	1	0	0
Diversity	12	1	2
Exam/curriculum	6	1	0
Connecting mathematical topics	5	0	0
Practical use	6	0	0
Vocational	0	0	0
Didactical characteristics	0	1	0
Mathematical characteristics	0	1	1
Practical considerations	10	0	0
Students' reactions	0	12	6

Table 7.9: Overview of the grading of Sven's categories.

Activity

I have graded all the comments Sven made about activity as positive characteristics of the tasks. Mostly, these comments are when Sven positively evaluates mathematical tasks by how the students are more active and self-going when working on the task. An example of this is when Sven sums up the benefits of the A4-task: "To summarize: the benefits are they get quickly started, everyone can manage something, and they get active straight away" (*evaluating A4-task*). Here, Sven is explicitly describing tasks where students get active straight away as positive. The only comment Sven makes about activity, with respect to tasks, which is slightly different, is when he suggests another wording in the area task to help the students get started:

> How about: Which lengths do I need to know in order to calculate the area? I worry that if you only ask them to calculate the area in as many ways as possible, the students won't know how to do it. It becomes too open, so they need something to get them started (*discussion of area task*).

I have marked this comment as positive with respect to activity because Sven suggests a change in the wording that he anticipates will make it easier for the students to get started with the task.

Understanding

When Sven talks in general about what he wants in a mathematical task, understanding is a central concept. He expresses how he wants tasks where the students can work, and thereby gain an understanding, and he evaluates tasks we have implemented with respect to whether the students seem to understand the mathematics or not. I have therefore marked seven of the comments about understanding as positive. An example of how Sven talks about understanding in our collaboration is: “I experienced that some of the low achievers seemed to get some aha-moments, and this is not something I experience normally” (*evaluating area task*).

The two comments that were marked as negative with respect to understanding and mathematical tasks, are when Sven criticizes some textbook tasks for a lack of focus on understanding. He talks about some tasks where the students can just solve them mechanically, but he is also criticizing how the textbook presents some concepts like percentage factor. He says: “The high achievers manage to use it, but don't understand it, and the low achievers don't understand anything and can't use it either” (*semi-structured interview*). So, Sven does not like tasks which encourages students to solve them by using a formula for growth factor because the students do not understand it. I have therefore marked this comment as negative.

There are also two comments which were marked as neutral. One of them is because Sven talks about how some students realized a connection when working on a task, but he does not evaluate this explicitly as something positive or negative about the task. The other comment is from the evaluation of the A4-task, where I asked Sven if he thinks the students got an understanding of how to use proportions to calculate unknown sides through working on the task, and Sven responds: “It seemed like it, given what they managed to do. If they got better and more lasting understanding of the concept is something that time will show” (*evaluation of A4-task*). Since Sven says he cannot know at the given time if the understanding the students got from working on this task is better or more lasting, I have marked the comment as neutral.

Goal of the Task

During the first conversation about tasks with Sven, he describes wanting tasks that make it possible for him to pull back a as a teacher: “I would like tasks where I can pull back a bit as a teacher, and the low achievers can have a starting point while the high achievers can at the same time get a challenge” (*first conversation about tasks*). Since this is a characteristic of a task Sven wants, I have marked this comment as positive.

Diversity

Diversity is a topic that Sven brings up on many occasions during our collaboration. Grading the comments about diversity as either positive, negative, or neutral gives only a partial picture. That is, sometimes Sven speaks positively about a task having a starting point that everyone can master: “It's nice to have many possibilities, but it's important to have a starting point that everyone can master” (*refining A4-task*). However, on other occasions Sven is positive because a task has elements which will challenge the high achievers, but at the same time will be too difficult for some of the students. When I asked if one of the solutions we had presented for the A4-task would be too difficult for the students, Sven replied: “For some of them, but at the same time I need something for those who finish quickly. This is normal” (*refining A4-task*). At the same time, Sven is explicit about how the area task worked especially well for some of the low achievers: “Low achievers that are still willing, gain from this type of tasks more than others” (*evaluating area task*). So, while Sven expressed several times that tasks that would get all the students started and at the same time challenge the high achievers was positive, he would also consider it as positive if either group could benefit. It did not always have to be a task for all students. Altogether, I have marked 12 comments as positive with respect to diversity.

There are two comments that were marked as neutral, and these are because Sven describes the diversity in the class but is not linking it directly to a specific mathematical task. For instance, this is how Sven describes some of his challenges with this group of students: “The high achieving pupils know the book from before, and they get bored. The low achievers are struggling with the basic” (*first conversation about tasks*). Since Sven talks in general and is not relating this comment about diversity in the class to any mathematical task, I have marked it as neutral. There are two general comments like this that were marked as neutral.

There is one occasion where I have marked a comment as negative, with respect to diversity. This is when Sven talks about the tasks in the textbook and explains why he is not happy with it: "On some topics I feel it lacks challenges. It's an ok book for low achievers, but it doesn't have much for the high achievers" (*semi-structured interview*). I have marked this comment as negative, since Sven expresses he is not happy with all the tasks in the textbook because they, in his opinion, lack challenges for the high achievers.

Exam/Curriculum

There are seven comments that were coded as related to the exam or curriculum during the collaboration with Sven, and all but one of them are marked as positive. This is because Sven either evaluates tasks as good because they are relevant for the exam/curriculum, or because Sven wants to adjust tasks for the same reasons. An example of a comment that I marked as positive is when Sven talks about the A4-task:

> What I want everyone to manage, is to find the ratio and use it to find unknown sides. That's what the curriculum demands. If they manage this with complicated numbers as in this task, I think it is very good (*refining A4-task*).

Sven compares the expected learning outcome of the task to what the curriculum demands, and he is happy with the task's relevance with respect to the curriculum.

There is one comment that I have marked as negative, and this comes from when we evaluated the A4-task and I asked Sven if there was anything he would have liked to change. Sven responds: "I wouldn't have started with the area, but with the relationship of the sides. It's because it's closer to how the curriculum is interpreted and tasks they might get on the exam" (*evaluation of A4-task*). Since Sven says there is something he would like to be different in the task due to the curriculum, I have marked this as a negative comment about the task.

Connecting Mathematical Topics

In total five statements from the collaboration with Sven are coded as talking about connecting mathematical topics, and all of them are graded as being positive. This is because Sven talks about wanting tasks that can help the students connect what they learn to many different mathematical topics. Sven expresses it like this:

> I'd like them to see connections. First, they have proportionality, and then whoops! They are going to calculate indexes. But that is proportionality. Fractions and… it's possible to make so many connections. And not the least similarities and scale, scaling (*first conversation about tasks*).

Sven talks about how the book presents various topics without highlighting connections, but he wants his students to experience that proportionality is a concept being relevant within most of the topics they work on, and he mentions several of them. All the five statements that have been coded as positive with respect to connecting mathematical topics, entail Sven talking about wanting the students to see connections across the mathematical topics. Another example is when he asks for tasks that can help students realize how they can develop formulas themselves for calculating area of various figures, using already known facts.

Practical Use

There are six comments that were coded as practical use, and they are all graded as positive. This is because Sven expresses that a task with a practical perspective is better for his students. For instance, Sven talks about how many of his students have failed so many times already with the type of tasks in the mathematics book:

> So, I try to make my own tasks which appeal to the practical side of them. Many times, they can do it if they can relate to it. I try to make tasks that connect the practical with the theoretical, but I have yet to crack that code (*first conversation about tasks*).

Sven is clear about thinking some of his students will benefit from a practical perspective on the mathematical tasks, but he can also find it challenging to design tasks that appeal to their practical side. However, since all his comments about a practical perspective in mathematical tasks are positive, these statements have all been graded as positive.

Didactical Characteristics

Only once during the collaboration with Sven he made a statement that was coded as a didactical characteristic, and I have graded this as a negative comment. This is because he is criticizing some tasks in the textbook in the chapter on proportionality: “There are NO tasks in the book where the entities are not proportional. So, when one has these kinds of tasks, they are always proportional” (*semi-structured interview*). Sven is here criticizing a didactical choice the textbook has made, entailing not to include tasks where the entities are not proportional.

Mathematical Characteristics

I have coded two statements with respect to mathematical characteristics in the collaboration with Sven, and one of these is marked as neutral and the other as a negative comment about tasks. The statement which has been coded as neutral, is when Sven talks about needing to use GeoGebra as a tool to show students that the area is always the same in a triangle if the height and baseline are the same. He is here talking about how to present a mathematical characteristic in a task but is not being explicitly negative or positive.

However, the other statement that has been graded as negative, is because Sven criticizes that a mathematical characteristic in a task we design is not easily generalized to also account for triangles: “When they have realized in the A4-format series that there is a constant ratio, it is not so easy to generalize this as also being true for similar triangles” (*evaluating area task*). Since Sven has previously talked about generalizing across topics as something he wants his students to do, I have graded this statement as a negative evaluation of this aspect of the task.

Practical Considerations

All ten statements that were coded as practical considerations during the collaboration with Sven, have been graded as positive characteristics of mathematical tasks. This is because they are all statements where Sven wants to make practical adjustments to the tasks we are working on. For instance, Sven was clear about how the structure was important when we designed tasks:

> It must be very structured. They are not taking any initiative whatsoever to explore on their own. They have not been raised to do this throughout their

> schooling, and I have not taken that fight either. So, they need very clear goals to work towards (*refining A4-task*).

Sven made several comments like the one above, about how we needed to be clear, structured, and not ambiguous when formulating the tasks to the students. They needed to know what they were supposed to do. Another example of a practical consideration Sven wanted, was related to how he wanted to structure a lesson and to summarize in between subtasks: “With respect to summarizing, it’s a good idea to work only one task at the time. So, I would like the tasks on different sheets of paper” (*refining area task*). Dividing the task into separate sheets of paper so that Sven could organize when students worked on which task, was one of many practical considerations he asked for in the tasks we designed together.

Students’ Reactions

Throughout the collaboration with Sven, there are a total of 18 statements that were coded as students’ reactions. Of these, 12 are graded as negative comments about tasks and six are viewed as neutral. The ones that are graded as negative, are because Sven either rejects parts of a task or wants to change it, due to how he expects the students will react. For instance, he mentions several times that there are aspects he thinks will be too difficult for the students. An example is: “The students will react to the difficulty of the numbers” (*refining A4-task*). So, the statements that have been graded as negative, are because Sven wants to make changes to tasks because he assumes they will be too difficult for the students. There are also six statements that have been graded as neutral, and this is because Sven’s comments about the students’ reactions are not directly related to a specific task. For instance, Sven explains some of the difficulties his students struggle with when we are evaluating the A4-task:

> They also have many negative experiences with mathematics throughout their schooling, so they give up quickly when things get difficult. They just give up and say: ‘I don’t understand anything’. This goes for even the high achievers. If they don’t manage straight away, they ask the teacher (*evaluating A4-task*).

This quotation is made during the evaluation of the A4-task, but Sven’s comments about the students reacting negatively to challenges are not directly related to the task we designed. It is more of a general statement, and I have therefore chosen to grade it as a neutral comment. The other statements that have been graded as neutral are of the same kind. That is, Sven explains challenges in how the students react but does not link them to specific tasks.

Summary

The collaboration with Sven provided valuable insight into what an experienced teacher with clear didactical goals would ask for in tasks. Sven was confident in what he wanted to achieve in the classroom but restricted by time and a lack of what he considered good mathematical tasks. He was therefore clear and consistent in communicating what he wanted throughout the collaboration.

7.1.8 Cross case analysis of the Teachers' Descriptions of Tasks

I start this section by reminding the reader of the results of the analysis presented previously in this chapter, which is summarized in Table 7.10.

Outcome of tasks		**210**	**Characteristics of tasks**	**49**	**Students' reactions to tasks**	**46**
Activity		***38***	**Didactical characteristics**	**14**	**Students' reactions**	**46**
Understanding		***38***	**Mathematical characteristics**	**2**		
Goal of the task		***6***	**Practical considerations**	**33**		
Diversity		***24***				
Relevance		***104***				
Exam/ curriculum	22					
Connecting mathematical topics	11					
Practical use	60					
Vocational	11					

Table 7.10: A presentation of the codes, categories, and dimensions. The total number of codes in each dimension are the numbers in the top row, yet the distribution among categories and subcategories is also shown.

The teachers are fairly consistent when it comes to their focus and what type of categories their statements about mathematical tasks fit into. By that, I mean that three of the teachers use the same type of expressions no matter what part of the collaboration process we are at. However, Hanna is an exception here. From my collaboration with Hanna, almost all the statements belonging to the dimension characteristics of tasks, are from when we refine or evaluate tasks (Sub-Section 7.1.6). When she explains tasks she wants, she is almost solely describing them by the outcome of the tasks she wants. Hanna and Thomas are the only teachers who have expressed uncertainty when it comes to didactics in mathematics (Sub-Sections 5.3.1 and 5.4.1). While Hanna is explicit on not feeling she has enough didactical education and knowledge in mathematics, Thomas expresses some difficulties in discovering misconceptions among the students. In my collaboration with Hanna, it became evident how the codes belonging to the dimension characteristics of tasks are in the refining or evaluation phases (Sub-Section 7.1.6), but in my collaboration with Thomas there are not many codes from this dimension (Sub-Section 7.1.5). It might be that Thomas does not see as many different possibilities when it comes to mathematical tasks, given how he responded to the question in the interview about how he thought the students learn best. His answer was mostly related to working habits, more than to diversity in tasks, and he referred to his own schooling and what he was used to when learning mathematics in school, on a couple of occasions (Sub-Section 5.3.1). If one does not know there are other types or possibilities of tasks, it is not easy to ask for either. On the other hand, Hanna has been part of some collaboration projects in mathematics education with researchers from a university, hence I assume she knows about many types of tasks and ways of teaching. So, maybe Hanna avoided using specific characteristics because she was uncertain what to ask for, and instead specified the outcomes she wanted from the tasks. This way, she gave me more of the responsibility to design a task with 'good' characteristics. In the end of our collaboration, she expressed surprise over how most of the tasks we had designed were not open tasks, which she had assumed they would be if the students should explore on their own (Sub-Section 5.4.1). So, Hanna had made some assumptions which she did not express in the early phases of our collaboration, because she took them for granted.

While Hanna and Thomas have said there are some didactical issues they struggle with, both Roger and Sven seem more confident on these issues. Both are consistent when describing tasks throughout our collaboration. When it comes to Roger, he is a teacher who is happy with the way things are now. His students are doing well on the exam, and he gets positive feedback from the student group on his teaching. It might seem like Roger has found what works in his teaching with this student group and is therefore confident in types of tasks and what types of characteristics these tasks need to have. Sven, on the other hand, is not happy with how his class is doing and expresses there are many ways to improve (Sub-Section 5.5.1). However, it also seems like he has a clear opinion of what would be helpful; he is just limited by time and other constraints. Thus, Sven is consistent when it comes to what he asks for, because he has a clear opinion on what will work and what will not work.

If we look past how the teachers have various levels of confidence in what they do and would want to change, there are also some differences in what type of outcomes they focus on when it comes to mathematical tasks. Even though Roger and Thomas have a different type of student group than Hanna and Sven, all teachers mostly describe mathematical tasks by the outcome of the tasks. Thomas wants tasks being practical or realistic and expresses how he hopes this could be motivating for the students (Sub-Section 7.1.5). Hanna often refers to getting the students active and keep them working on tasks but does not express many concerns about diversity (Sub-Section 7.1.6). At the same time, diversity is an issue Sven regularly introduces as an outcome he wants from tasks (Sub-Section 7.1.7). Roger does not have any specific new outcomes he wants from the tasks but is open if new tasks would give his students a better understanding of the topic in some way (Sub-Section 7.1.4). This diversity in what type of outcomes the teachers are looking for in tasks, can be explained by what they are struggling with and wanting to improve in their teaching. So, the tasks are just one part of their world of teaching, and when describing what types of tasks they would prefer to use in their classrooms, they describe tasks that would help them overcome some of their teaching challenges.

When it comes to the frequency of codes belonging to the dimension of students' reactions to tasks, there is a marked difference between Roger and Thomas on one side, and Hanna and Sven on the other hand (Tables 7.2, 7.4, 7.6 and 7.8). Roger and Thomas make very few comments that are coded within this dimension, and the few codes assigned there, are mostly expressing how the students' reactions do not worry them. However, both Hanna and Sven make many statements that I have coded and categorized in the dimension of students' reactions to tasks. I have categorized 18 out of 85 codes from my data material with Sven as belonging to the dimension students' reactions to tasks, and 25 out of 131 codes from my data material with Hanna. This means that approximately 20 % of the codes from these two teachers are within this dimension, while the other two teachers hardly mention it at all. However, there is also a difference between Hanna and Sven when it comes to when they are making such statements. While 24 of the 25 coded statements from Hanna are from evaluating tasks (Sub-Section 7.1.6), 14 of the 18 coded statements from Sven are from when we are refining tasks prior to implementation (Sub-Section 7.1.7). This difference might be a result of experience and didactical skills in the mathematics classroom. While Sven wants to make changes before the tasks are implemented, based on his knowledge and perception of how his class will react, Hanna does not do the same. This might be because Hanna is not able to predict how her students will react in the same way as Sven does, but it might also be a result of Hanna being willing to take more chances in trying out new things. Even if she has made many statements when evaluating the tasks, that have been assigned to the domain of students' reactions to tasks, she also blames herself for not managing some of these reactions better. So, it seems that Hanna is to some degree working on developing her own skills as a teacher, and therefore she might be willing to try out a greater variety of tasks in the classroom. However, there is a question about how many times she might be willing to try out certain types of tasks if she experiences that the class does not want to explore or engage in the task. She has already made a comment on how she does not think 'heavy' investigational tasks are possible in this class (Sub-Section 5.4.3). So, it could be that Hanna's responses might become more like Sven's responses in some years, having tried out several types of tasks and concluded what will not work and what might work with this type of class. Nonetheless, maybe the difference in Hanna's and Sven's response when it comes to students' reactions to tasks are just a matter of where they are in their own professional development as a mathematics teacher.

The research question which has guided this analysis, is: What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom? I have described how the teachers mostly characterize tasks by the outcome of the task. There are three main issues from the mathematics classroom these teachers consider when they ask for tasks. All issues are related to the students, and these are:

- Work
- Motivation
- Understanding

While Thomas and Roger do not worry about their students working on the mathematics (Sub-Sections 5.1.1 and 5.3.1), it is a concern for Hanna and Sven (Sub-Sections 5.4.1 and 5.5.1). However, Sven seems confident he can get the students to work, he just struggles to get some of them started. Hanna worries about some students not being willing to work at all. Thus, both Hanna and Sven ask for tasks that will help to get the students to start working.

Motivation and work are often related terms, yet I have chosen to distinguish between them because the teachers do so. While it might be difficult to find examples of motivated students who do not work hard, the other way around is possible. Thomas says his students are hardworking, but he is still looking for tasks which might motivate them (Sub-Section 5.3.1).

The last point is understanding, meaning the teachers' own conception of what type of understanding is important for their students. Understanding might therefore differ from teacher to teacher. All teachers in this research project talk about their students' understanding (see analysis in Sub-Sections 7.1.4, 7.1.5, 7.1.6 and 7.1.7). Sven wants tasks that help the students develop an understanding on the topic while working on them, and Hanna talks negatively about tasks that only encourage a mechanical understanding. Thomas sometimes worries whether the students understand or not when he explains the mathematics, and Roger says he is open to tasks that might provide his students with a better understanding.

These three bullet points summarize the issues the teachers consider when they ask for mathematical tasks that can fulfill a need for them. Roger is happy with all three aspects, and therefore has less need for new tasks. His students work hard, he talks about them as motivated, and they understand the mathematics. Understanding is the only aspect where Roger says there might be tasks that could improve his students' understanding, but he never finds tasks where he concludes with this. There are some tasks that he is positive to and considers using, but that is more about him liking the task, rather than the task meeting a need (Section 5.1).

Thomas' group of students are hardworking, so he does not worry about this aspect. However, both motivation and understanding are aspects that he seems to find possible to improve. He is explicit about wanting tasks with a practical perspective because he thinks it might motivate the students. In addition, he seems worried that the students sometimes work on tasks without always understanding the mathematics (Sub-Section 7.1.5).

Both Hanna and Sven express concerns about all three aspects. The slightly different focus concerning what they ask for, might be related to which of the aspects they view as most critical, and what they feel a need to address first.

While the three bullet points I have presented sum up the issues the teachers consider when they ask for tasks, there are some teacher aspects that can limit the possibilities in mathematical tasks. I am describing these aspects due to reasons the teachers gave for rejecting tasks or issues they worry about throughout our conversations. These are:

- Didactics
- Communication
- Mathematics

Each of these aspects can limit what type of mathematical tasks the teachers want to use. The teachers who describe themselves as needing to improve their didactical skills are open to a wide range of tasks, because they look for opportunities to improve their didactical choices in the classroom. On the other hand, the teachers with more didactical skills are confident in what characteristics they look for in a task and are thus less open to other options. Hanna and Thomas are the two teachers in this project who have expressed most uncertainty about didactical choices and they are also the two teachers who accepted and used more or less all the suggested tasks I presented for them. Hanna talks about this already during the semi-structured interview and says she feels she lacks didactics in mathematics and is uncertain about how to improve her mathematics teaching (Sub-Section 5.3.1). Thomas is not using didactics as a specific word but talks about how he finds it difficult to discover when the students do not understand the mathematics and to motivate his students (Sub-Section 5.2.1).

Communication is another aspect influencing characteristics of tasks the teachers want to use in their classroom. To lead classroom discussions in these classes is a challenge for Sven and Thomas, and they are asking for rather detailed written tasks. For instance, Thomas would like a historical task about logarithms (Sub-Section 5.3.2). I designed such a task, which Thomas uses, and I present the same task for Roger at a later point. Roger comments that he does not need such a task, because he can easily make it himself (Sub-Section 7.1.4). I interpret this as Roger not seeing a point in writing down such a task when he can talk about it freely with his students. Sven also wants to add text to some of the tasks I designed (Sub-Section 7.1.7). While he summarizes tasks with the students, he wants probing questions for them to reflect on the mathematics when they work on solving the task. He does not want tasks that require classroom discussions, so there needs to be a certain degree of text in the tasks.

I have included mathematics as a third aspect, although the teachers in this project do not talk much about it, because they all have a high degree of mathematical skills. However, we know from the literature that a lack of mathematical knowledge might cause teachers to reject certain tasks (Hill, Schilling, & Ball, 2004). I have therefore included it as an aspect that needs to be considered.

These three aspects of teacher skills can limit what type of tasks teachers are willing to use in their classrooms, however they are not always evident when the teachers describe what they look for in tasks. The aspects mostly surface when I present suggestions to tasks, or in the evaluation phases of the collaboration. They are often formulated as concerns about how the students might react to the tasks.

All three aspects that I have described, can be linked to previous theory about teachers' knowledge and competence for teaching, like the work of Hill et al. (2008) and instructional dialogues as described by Leinhardt and Steele (2005). Nevertheless, there is a difference, which is why I have chosen to use slightly different wording when describing them. This concerns the teachers' subjective confidence and skills in these areas, which can be different from how an outsider might evaluate their competence and skills. However, the teachers' perception of these aspects might lead them to accept or reject mathematical tasks. I will further discuss these findings in Sub-Section 8.1.4.

To summarize, the teachers are considering three aspects of their students and classrooms when they ask for mathematical tasks: work, motivation and understanding. They want tasks that can help them to fulfill these aspects. However, they might reject tasks based on three various aspects: didactics, communication, and mathematics. It is worth noting that these aspects are not necessarily visible unless the teachers are presented with tasks they are asked to use. The teachers are also, to some degree, using didactical and mathematical characteristics of tasks when they ask for tasks, but that is not their primary way of expressing themselves.

Having analyzed how the teachers describe mathematical tasks they prefer to use in their classrooms, I would also like to present an analysis of the teachers' willingness to change. Since my analysis of the teachers' characteristics of tasks shows how the tasks are, for them, only one part of their teaching that is not possible to separate from the rest, I want to use the same data to analyze how this can be related to the teachers' willingness to change. This analysis is presented in Section 7.2.

7.2 The Teachers' Change Sequences

My second research question is: What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration? In order to answer this research question, I will in this section analyze my collaboration with the teachers using the Interconnected Model of Professional Growth (IMPG), developed by Clarke and Hollingsworth (2002) and previously presented in Section 3.5 This model of understanding teacher change is based on three large empirical studies investigating the impact of professional development programs on teachers and their teaching. Clarke and Hollingsworth (2002) claim that one of the strengths of their model is that it can be used to understand the smaller changes teachers experience in their everyday classroom, because the model is not dependent on the external domain. My research provides empirical data of a collaboration between researcher and teacher where the researcher does not have an agenda for change, but where it is the teacher who initiates changes in their practice and can thus add value to the IMPG.

7.2.1 The Analyzing Process

I use the IMPG model when analyzing my collaboration with the teachers to identify change sequences. A change sequence is understood as the sequencing of changes between domains, and how these influence changes in other domains through reflection or enactment (see Section 3.5 for further elaboration).

I chose to use the task design process as a unit of analysis. So, for each teacher, I have analyzed the changes and identified change sequences from what the teachers ask for in a task, the refinement of the task, and implementation and evaluation of the task. Each design process has been analyzed and is represented with its own change sequences. In the article of Clarke and Hollingsworth (2002) they distinguish between a change sequence and a longer lasting professional growth. I have chosen to call it change sequences in my analysis, because I find it hard with my empirical data to support evidence for long lasting professional growth within the timespan of my research.

When analyzing, I have used the codes I developed through open coding, describing the characteristics of tasks the teachers want to use in their classrooms. Hence, these codes represent comments teachers have made with respect to mathematical tasks, which are what we have developed and thus made changes to during this collaboration. All codes that were marked either as positive or negative were written down, while I ignored the neutral ones. The reason why I use the positive or negative statements about tasks, is because the statements can be indicators of change. A positive comment means the teacher likes this characteristic of a task, while a negative statement might indicate rejection or wanting it different. The neutral comments do not provide information about change, as they are only identifying a characteristic of a task, not making it clear if this characteristic is positive or negative. In addition to using these codes, I went through all the conversations with the teachers, identifying phrasings that might indicate change. This could be phrasings like: Before I used to, but now… or I want to…. or I am not happy with… All these phrases were written down in addition to the codes already mentioned. For each design process, I further placed the statements in each of the four domains developed by Clarke and Hollingsworth, prior assessing how they were connected and influenced each other. For instance, when we evaluated the implementation of the area task, Hanna said the students were active, and that is a good thing. This is a statement that I placed in the domain of consequences. Given the setting, this was something Hanna said when evaluating the area task. Therefore, this comment about how it is positive that the students are active, is a reflection upon the task implemented in the domain of practice.

Based on the sequencing of when the teacher mentioned the various items, I made arrows to show when change occurred in the domains, through reflection or enactment in another domain. Since the domain of practice can be about both when we design the task and when it is implemented, I have used red numbers to indicate the arrows where changes occur during implementation or evaluation, otherwise the numbers are black.

In contrast to Clarke and Hollingsworth (2002), who adopt a rather large unit of analysis that includes the impact of a professional development program on a teacher, my unit of analysis is at a micro level analyzing the change sequence in a design process for a specific mathematical task. This led to the need of adding a fifth dimension to Clarke and Hollingsworth's (2002) IMPG. I have added the 'student domain', to capture more of the complexity when implementing tasks. While the domain of consequences makes sense to use when referring to the whole class, the teachers are on many occasions referring to only some of the students. So, while for instance good classroom discussions might be a salient outcome according to the teacher, a few students might counteract this. Consequently, the teacher might make some changes with respect to these few students, while continuing as planned with the rest of the group. For illustration, when I analyze the rope task with Hanna, she complains about how some students are not doing what they are supposed to, but at the same time she says the class in general is more active than usual. To highlight these differences, I added the student domain to the model. This domain is used when the teachers are referring to only parts of the class, or if the teacher refers to something students have said or asked for. For instance, if the students are asking for more textbook tasks.

I have collaborated with four teachers throughout this research project, but I have only analyzed the collaboration with three of them when it comes to change sequences. This is because the collaboration with Roger never reached any implementation of tasks or even refinement of them. I will in the following present the change sequences of each design process, having added arrows to describe the process. The arrows and the sequencing are then elaborated below each diagram followed by a summary. Sub-Sections 7.2.2-7.2.5 are presentations of the change sequences identified in the collaboration with Hanna. In Sub-Sections 7.2.6-7.2.7 the change sequences from the collaboration with Sven can be found, and Sub-Sections 7.2.8-7.2.9 contain the change sequences from the collaboration with Thomas.

7.2.2 Hanna: A4-Task

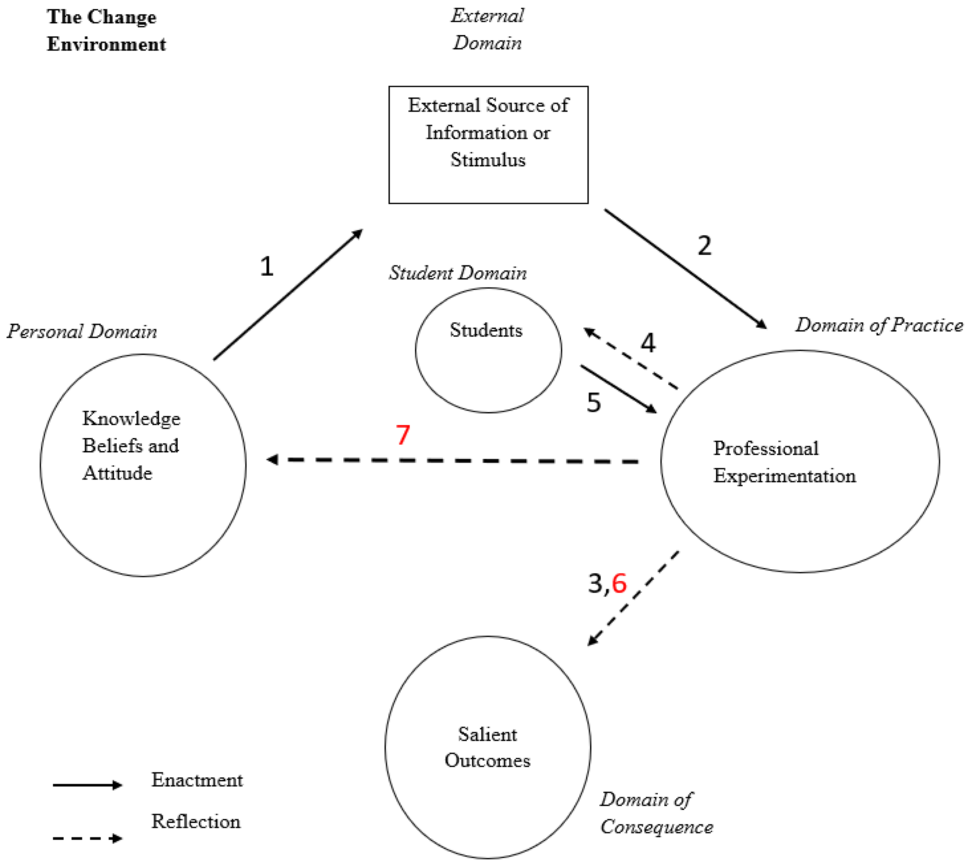


Figure 7.4: Hanna's change sequences on the A4-task.

Hanna’s change sequence on the A4-task is illustrated in Figure 7.4 above. All change sequences in this project starts with a change in the external domain when the researcher makes contact and challenges the teachers to describe mathematical tasks they want to use. Since this sequence applies for all cases, I have not included it in the diagram, I rather start the sequence by the teachers’ requests for tasks. The first step is when Hanna explains to me, as the researcher, what kind of tasks she wants (arrow 1). She wants introductory tasks where the students discover, tasks with practical views that might be relevant for their vocation, motivating tasks that keep them going, tasks that foster understanding, the students do an activity and then they understand the concept through this work, and a way to introduce the topic without her being at the blackboard. Hanna’s requests are many and general, entailing they are not just for the first task, but are relevant for the next tasks as well. So, Hanna enacts on her personal knowledge, attitudes, and beliefs and describes tasks to the researcher who is part of the external domain. This leads to a change in the external domain, where the researcher designs and presents the A4-task for Hanna. Having examined the task, Hanna decides she wants to use this task in her mathematics classroom. This is an enactment based on the change in the external domain, which leads to a change in the domain of practice (arrow 2). When studying the task, Hanna comments that this task can work as a bridge builder for proportions and similarities, and that she can refer to this task later. So, reflecting on the task, which is a change in the domain of practice, Hanna realizes how it can be used to achieve an understanding of the concept proportions and similarities, which is to her a salient outcome in the domain of consequences. So, the change in the domain of practice led to a change in the domain of consequences, based on Hanna's reflections (arrow 3).

Another reflection Hanna makes when further examining the task, is that she worries that subtask 6 (Section 6.1) might be difficult to motivate some of the students to get through. She anticipates that some of her students might stop working on this subtask, thus leading to a change in the student domain (arrow 4). Because of the anticipated change in the student domain, Hanna instigates a change in the task to make it more visual/practical. This is an enactment on the anticipated change in the student domain that leads to a change in the domain of practice (arrow 5).

The A4-task is now implemented in the classroom, and Hanna reflects on this afterwards. She talks about how this task gave her opportunities to talk with different students on how they were thinking and working. So, making a change in the practice domain and implementing the A4-task, led to a change in the domain of consequences which Hanna appreciates through reflection (arrow 6). At the same time, Hanna worries if she said too much when summarizing, and when introducing the task. She expresses how saying less is more in line with the students discovering on their own but is uncertain about the balance. Implementing the A4-task and thus making a change in the domain of practice, leads Hanna to reflect and make a change in the personal domain (arrow 7). She seems to adjust her thinking on how much she should summarize and talk to the students when they are working on tasks. This is a work in progress for Hanna, and further experience might lead to further changes in her personal domain.

The A4-task provides an example of change sequences that mostly goes on around the domain of practice, the domain of consequences and the student domain. Adjustments are made to the task with respect to diversity in the class, and some reflections are made with respect to salient outcomes. However, ultimately, Hanna also makes some reflections back to the personal domain where she might have started a line of thought that could continue beyond this task design. She reflects upon the balance between students discovering and her telling. This is not resolved in our discussions but may indicate that Hanna is in a continuing process of reflecting and improving herself as a teacher and how she teaches.

7.2.3 Hanna: Rope Task

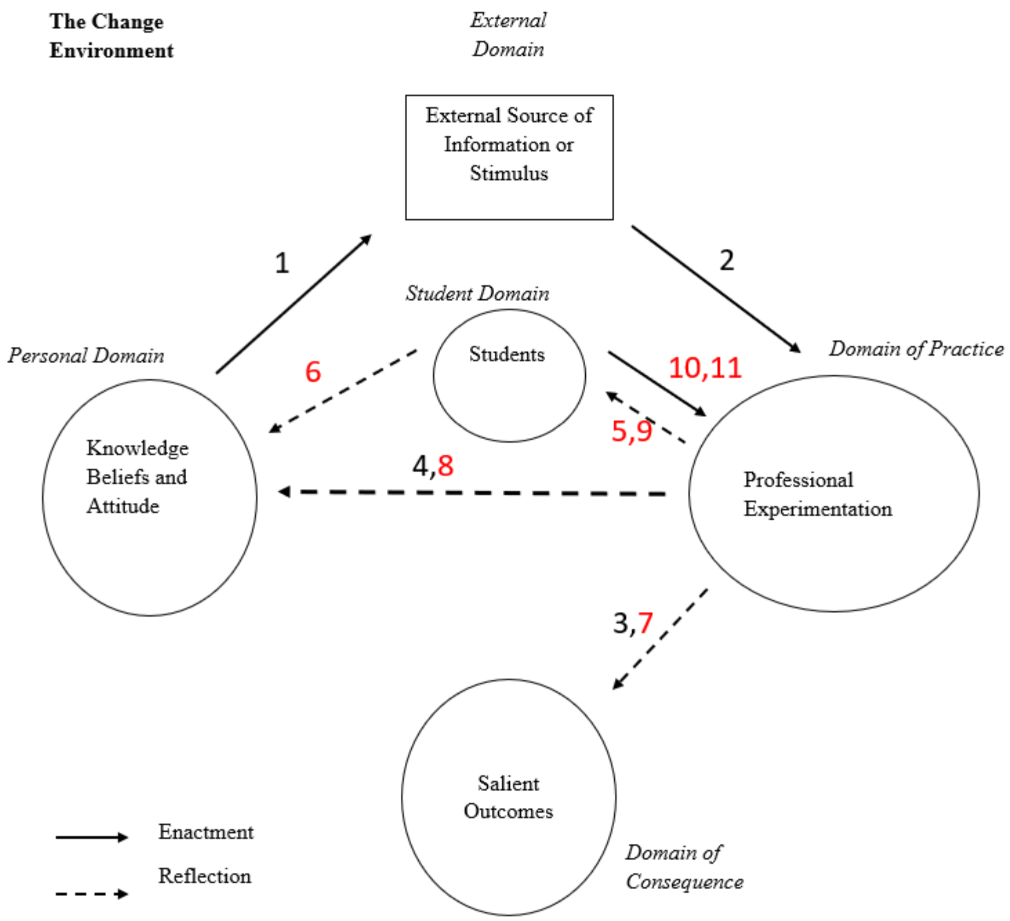


Figure 7.5: Hanna's change sequences on the rope task.

Hanna has made the same type of requests for the type of tasks she wants, as presented on the A4-task. This enactment from Hanna leads to a change in the external domain, and the researcher therefore presents the rope task for her (arrow 1). Hanna likes the task and decides she wants to use it, thereby making a change in the domain of practice through enactment upon the change in the external domain and the task that was presented for her (arrow 2). Hanna says she wants this task to get some activity in the classroom. So, she reflects upon the change in the domain of practice and expects a change in the domain of consequences that she wants (arrow 3). In addition, she reflects on how this task resonates with her personal domain when she says: “These are the kind of tasks I have been looking for” (arrow 4).

The rope task is further implemented, and Hanna reflects on the result of the implementation. The implementation of the rope task represents a change in the domain of practice, and Hanna reflects upon how this change made it challenging to get some of the students to do something, representing a change in the student domain (arrow 5). Further, Hanna wonders if she is good enough to get the students to do what they are supposed to. So, Hanna reflects whether this negative change in the student domain might be because she does not follow up these students sufficiently, and thus might need to make changes in the personal domain (arrow 6). At the same time, she says the class has been working more during this lesson than usual. So, Hanna reflects upon how the change in the domain of practice has led to a change in the domain of consequence that she likes (arrow 7). However, she wonders if investigative tasks in this class will work. She has thought of investigative tasks as an ideal type of mathematical task, but now it seems that she reconsiders this based on the implementation of this task. So, the change in the domain of practice leads to a change in the personal domain, through Hanna’s reflections (arrow 8). Some of the students struggled with task three, and some students did the task but did not check that their solution would work. Reflecting on how some of the students struggle with the task and make short cuts, Hanna realizes there is a change in the student domain that she is not content with (arrow 9). She decides to change task three from asking the students to use the rope to make an optional figure with a given area, to make a rectangle and calculate the area of it. The change in the student domain has therefore led Hanna to make a change in the domain of practice through enactment (arrow 10). In addition, she decides to move task four prior to task three, because many of the students managed task four, while they struggled with task three (arrow 11).

There are many arrows in these change sequences, and the first set of black arrows indicate that Hanna is happy with the task and reflects on why she likes the task and what she can achieve by implementing the task. However, there are many things going on when the task is implemented. While Hanna says the students were more active than usual, many things are happening that she is not so pleased with. The arrows and the sequencing of them provide insight into how Hanna is not blaming the problems on one specific thing but is reflecting on what changes can be done and what changes are necessary, in her view. It seems like Hanna takes a holistic perspective to what could be changed when things are working out as planned in the classroom.

7.2.4 Hanna: Area Task

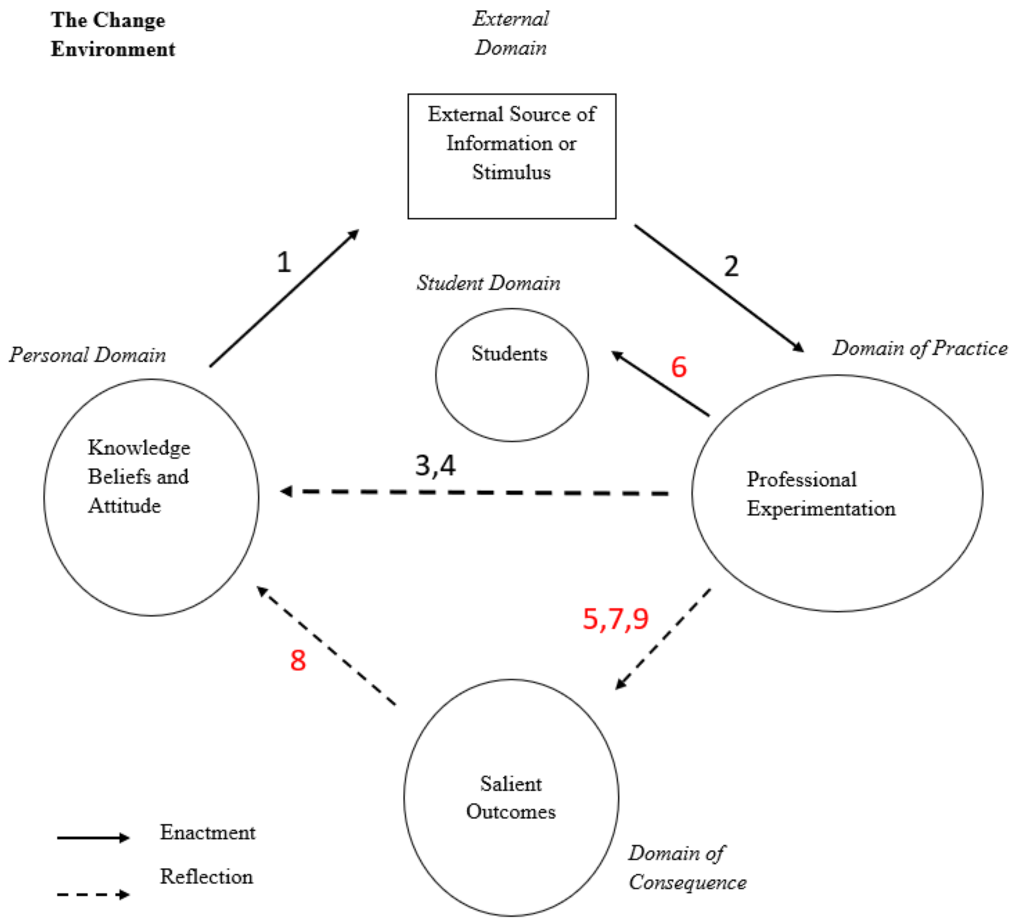


Figure 7.6: Hanna's change sequences on the area task.

Hanna tells me what type of tasks she wants, and this enactment from Hanna leads to a change in the external domain and the researcher therefore presents the area task for her (arrow 1). Hanna decides she wants to use this task, which is an enactment that leads to change in the domain of practice (arrow 2). The area task was presented for Hanna on the same day as the rope task, and she commented on both tasks that they were the kind of tasks she has been looking for. So, Hanna reflects on how this task resonates with her personal domain (arrow 3). When we look at the task, Hanna reflects on whether she should summarize between tasks or not but does not conclude during our talk. It seems like the task instigates reflections by Hanna on how she best should lead this lesson, thus bringing about a change in the personal domain (arrow 4).

After implementing the area task, Hanna says the students are active, and that is a good thing. The change in the domain of practice has thus led to a change in the domain of consequence which Hanna approves (arrow 5). In addition, Hanna says the students claimed to understand, so there is a change in the student domain as a result of the implementation of the task (arrow 6). Initially, Hanna wanted more investigative and open tasks, which she does not find the area task to be. This reflection upon the implementation of the area task, leads Hanna to make a change in what she would expect in the domain of consequences (arrow 7). Still, she likes the area task. So, by reflecting upon the change in the domain of consequences, Hanna adjusts her beliefs about characteristics of good mathematical tasks (arrow 8). She summarizes the implementation of the task and reflects upon the consequences of the change in the domain of practice upon the domain of consequences. Hanna concludes that the students must do something, it is an activity, and it works as an introductory task, which is what she asked for (arrow 9).

This is the task that Hanna liked the most, and this can be seen in the change sequences by how the arrows are only going in one direction. The changes that are made, are viewed as positive changes, and thus there is no need for making additional changes in any domains based on unsuccessful changes in other domains. So, when something is perceived as working, there is less need of making changes.

7.2.5 Hanna: Index Task

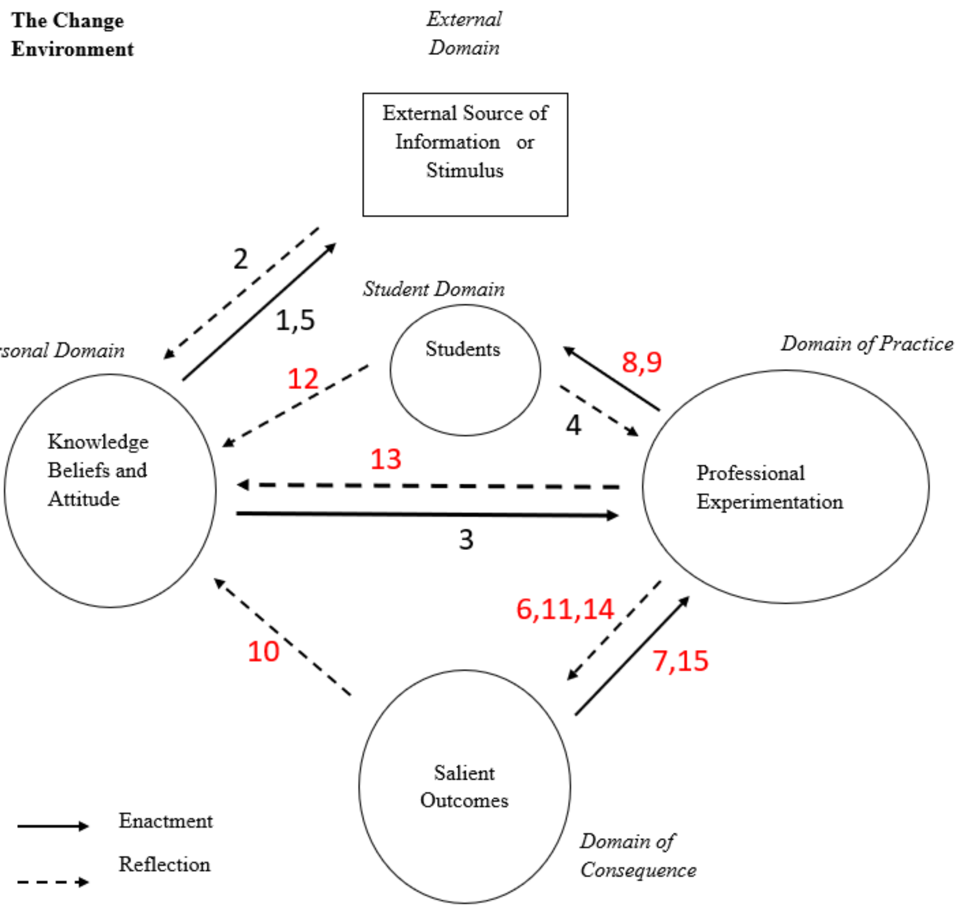


Figure 7.7: Hanna's change sequences on the index task.

Hanna asks for tasks she would like to use in the classroom and mentions index as a specific topic she wants help with. This enactment from Hanna leads to a change in the external domain, however it is not a readymade task (arrow 1). This time, the researcher does not design tasks, but instead I present ideas and resources for Hanna on the relevant topic. By reflecting on the ideas presented, Hanna says she thinks this is a brilliant starting point for a carpenter class. So, Hanna adds these ideas of tasks to her personal domain and combines it with her knowledge of the carpenter class (arrow 2). Based on these ideas and resources, Hanna designs the index task herself, which is an enactment that leads to change in the domain of practice (arrow 3). In this process, she worries that she might get some tough, difficult questions from some students. This is a reflection on how reactions in the student domain might lead to a change in the practice domain that she worries about (arrow 4). Because Hanna is uncertain if she has the knowledge to handle this situation, she asks the researcher for input and help with reflecting questions. So, based on Hanna's personal domain, she prompts a change in the external domain through enactment (arrow 5).

The index task is implemented over two lessons, and Hanna is happier with the second lesson than the first one. According to Hanna, the first lesson was chaotic, and the students did not read what they were supposed to. So, the implementation of the task, and thus the change in the domain of practice, lead to change in the domain of consequences that Hanna did not like (arrow 6). To fix this issue, Hanna says the task needs more structure, and it would be preferable to provide the students with a house (Section 6.5) rather than letting them search for one themselves on the Internet. These are changes Hanna wants to make to the task, being in the domain of practice (arrow 7). When implementing the task, the students reacted in various ways, and there were some interactions with the students that Hanna talks about afterwards. One challenge was a high achieving student who tried to control the class discussions and likes to argue (arrow 8), but she also talks about how this topic is interesting for the students (arrow 9). This time, Hanna was not happy with the conversations around the task. So, she reflects upon the conversations, which are part of the domain of consequence, and concludes that these are not in line with what she views as good mathematical conversations in the personal domain (arrow 10). Hanna thinks the idea for the index task was good, but the implementation was not. So, Hanna is not happy with the changes implementing the task brought to the domain of consequences (arrow 11). When Hanna reflects upon changes in the student domain, she does not believe that all the students understood the mathematics (arrow 12). Another reflection based on the implementation of the task and thus the change in the domain of practice, is that Hanna concludes she has not planned the lesson well enough (arrow 13). However, Hanna is more positive when finishing the second lesson on the index task. She felt many gained some kind of understanding when she summarized, hence this led to a change in the domain of consequences that she liked (arrow 14). By reflecting on the changes in the domain of consequences, she concludes that she would have made changes to the first lesson on indexes, but not the rest (arrow 15).

The change sequences for the index task have several similarities to the change sequences for the rope tasks. That is, there are many arrows going back and forth from the different domains. Just like the rope task, Hanna was not happy with everything when she implemented the index task. There were many positive things going on, but at the same time there were many things Hanna would have liked to change. Again, the change sequences shows that when in such a position, Hanna views the situation holistically, looking at several domains for what could be improved.

7.2.6 Sven A4 Task

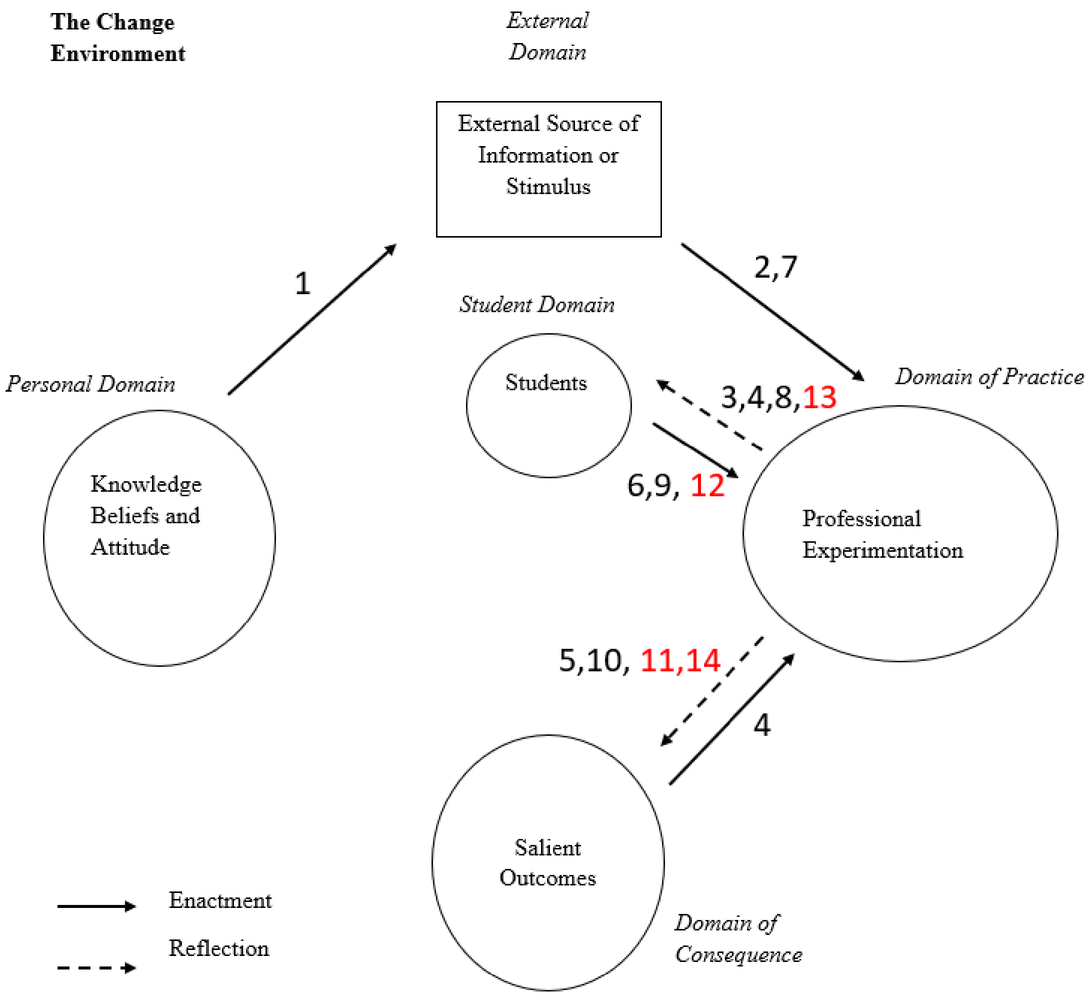


Figure 7.8: Sven's change sequences on the A4 task.

Sven wants tasks where he can pull back as a teacher and the students are more self-going, meaning tasks providing low achievers with a starting point and high achievers with a challenge. Preferably, an understanding builds up while the student works on the task. Proportions is a suitable topic for this aim and can be linked across topics. These explanations from Sven work as enactments on the external domain, leading the researcher to design tasks based on these wishes (arrow 1). I present the A4 task for Sven and he decides to use it, but firstly he has some elements he would like to adjust (arrow 2). He worries that the task is too difficult and therefore will have a negative impact on the students in the student domain (arrow 3). In addition, he says class discussions are not possible in this class, so this part of the change in the domain of practice will not go well in the student domain, according to his reflections (arrow 4). However, Sven says it is a good starting point for measuring and calculating the sides and areas, so this is a change in the domain of consequences that he likes (arrow 5). He says the task must be very structured, including specific wording and goals, as the students do not take initiative to explore on their own. So, these anticipated reactions in the student domain, lead to Sven wanting changes in the domain of practice (arrow 6). In addition, he suggests adding some subtasks being similar to the ones they might get on an exam. So, because of Sven's knowledge of the exam, which is part of the external domain, he makes changes to the domain of practice (arrow 7). Another change that Sven expects in the student domain, is that the students will react to the difficulty of the numbers (arrow 8), but we agree that he can use this as an opportunity to discuss measuring differences, which in turn leads to a change in the domain of practice (arrow 9). After we are done refining the task, Sven says that if everyone manages to understand proportions and use them to find unknown sides, then he is happy. So, this is, in his opinion, a salient outcome in the domain of consequences (arrow 10).

After implementing the A4 task, Sven says it worked well to get the students to start working, which is a change in the domain of consequences that Sven likes (arrow 11). However, he wants to change the wording from 'increase' to 'times', in task 2, because several students misunderstood this wording. Thus, by reflecting on how changes in the domain of practice lead to change in the student domain (arrow 12), Sven enacted on this and wanted to make changes to the task, which is the domain of practice (arrow 13). Sven says that it seemed like the students gained the knowledge he finds important. Also, the students engaged with the task quickly, everyone could manage something, and everyone became active without delay. So, by reflecting on the implementation of the task, Sven lists up changes in the domain of consequences that he likes (arrow 14).

The change sequences representing the A4-task designed and implemented with Sven, are different from Hanna's change sequences. Although there are many arrows, they are all more or less allocated between the domain of practice, the domain of consequence and the student domain. In addition, most of the arrows are from the design process and not from the implementation. This shows that Sven is confident in what he wants and has a clear opinion on why and what it might achieve. I could not detect any reflections on changes in the personal domain.

7.2.7 Sven: Area Task

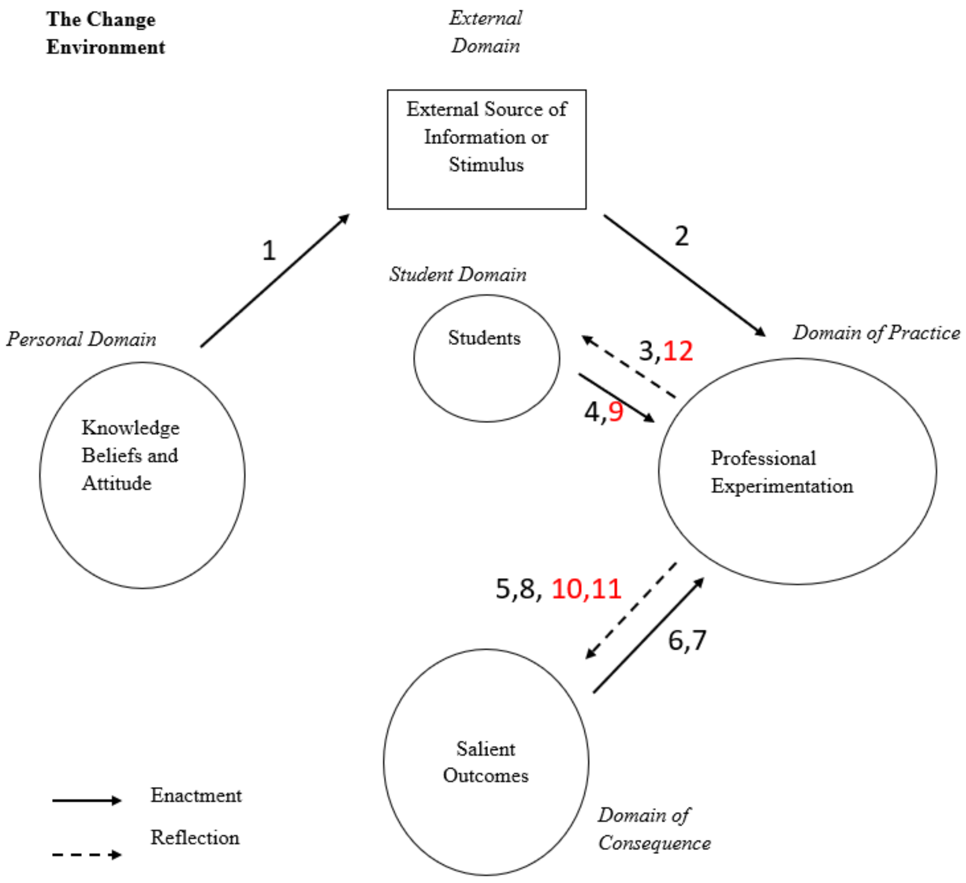


Figure 7.9: Sven's change sequences on the area task.

Sven wants a task in geometry on how to calculate area without providing formulas. As a result of his enactment on the external domain, the researcher presents the first draft of the area task (arrow 1). Sven wants to use the task in his classroom, being a change in the domain of practice (arrow 2), but he wants to do some adjustments first. According to Sven, many of the students do not know what area is, so by reflecting on the task, Sven knows that some of the students might struggle from the start. This is a change in the student domain that Sven wants to prevent (arrow 3). As a result, Sven wants to start with a rectangle and get the basics of the area concept from the very beginning of the task. Hence, this perceived challenge in the student domain leads to a change to the task in the domain of practice (arrow 4). One of the suggested subtasks in the area task, is to make the students cut a parallelogram and use it to find the area (Section 6.2), but Sven says he has tried this before and it did not work. So, by reflecting on this subtask, which is a change in the domain of practice, Sven concludes that it will lead to a change in the domain of consequences that he does not want. That is, he does not think it will work (arrow 5). Still, he says we can try by changing how the subtask is presented. We therefore put some effort into formulating wordings he is satisfied with. So, to avoid the change in the domain of consequences that worried Sven, we make changes to the task in the domain of practice (arrow 6). In addition, Sven wants to change the wording of 'calculating the area in as many ways as possible', to 'which lengths do I need to know to calculate the area' (arrow 7). Otherwise, the task becomes too open, they need something to get them started (arrow 8). Since Sven finds the task too open, which might lead to the students not getting to work, representing a change in the domain of consequences, he makes changes to the task in the domain of practice. It would therefore be reasonable to change the sequencing of arrow 7 and 8. However, Sven first presents the changes he wants to make to the task, and then explains why. I have decided to number the arrows in that order, because that is the chronological order of how he presented the arguments to me.

After implementing the area task, Sven made several small changes to the task. He added more questions to the parallelogram task and drew the rectangle in task 1 on graph paper, so the students can see what a mm^2 is. All these changes that Sven did to the task in the domain of practice, were due to the changes that he observed in the student domain (arrow 9). Sven says this task works better than when he does it the 'quicker' way, meaning to give the students the formula and use it to solve tasks. So, by reflecting on the change in the domain of practice, which was a result of implementing the area task, Sven expresses he likes the change it led to in the domain of consequences (arrow 10). He gave several reasons for why this task worked well: Everyone could get started on something and the students were a lot more self-directed than usual. In addition, he says this task is easier to generalize for the students, and therefore has more impact, which is another change in the domain of consequences that Sven approves (arrow 11). While it seemed like some students had aha moments, one high achiever who likes to finish quickly, reacted negatively (arrow 12).

Once again, there are many arrows in the change sequence above, but they are mostly located between three domains: the domain of practice, the domain of consequence and the student domain. In addition, many of the arrows are from the planning and designing phase. The two change sequences representing my collaboration with Sven are similar to each other, and at the same time different from those representing the collaboration with Hanna, where a lot of reflections happened after the implementation and all domains were represented.

7.2.8 Thomas: Logarithm Tasks

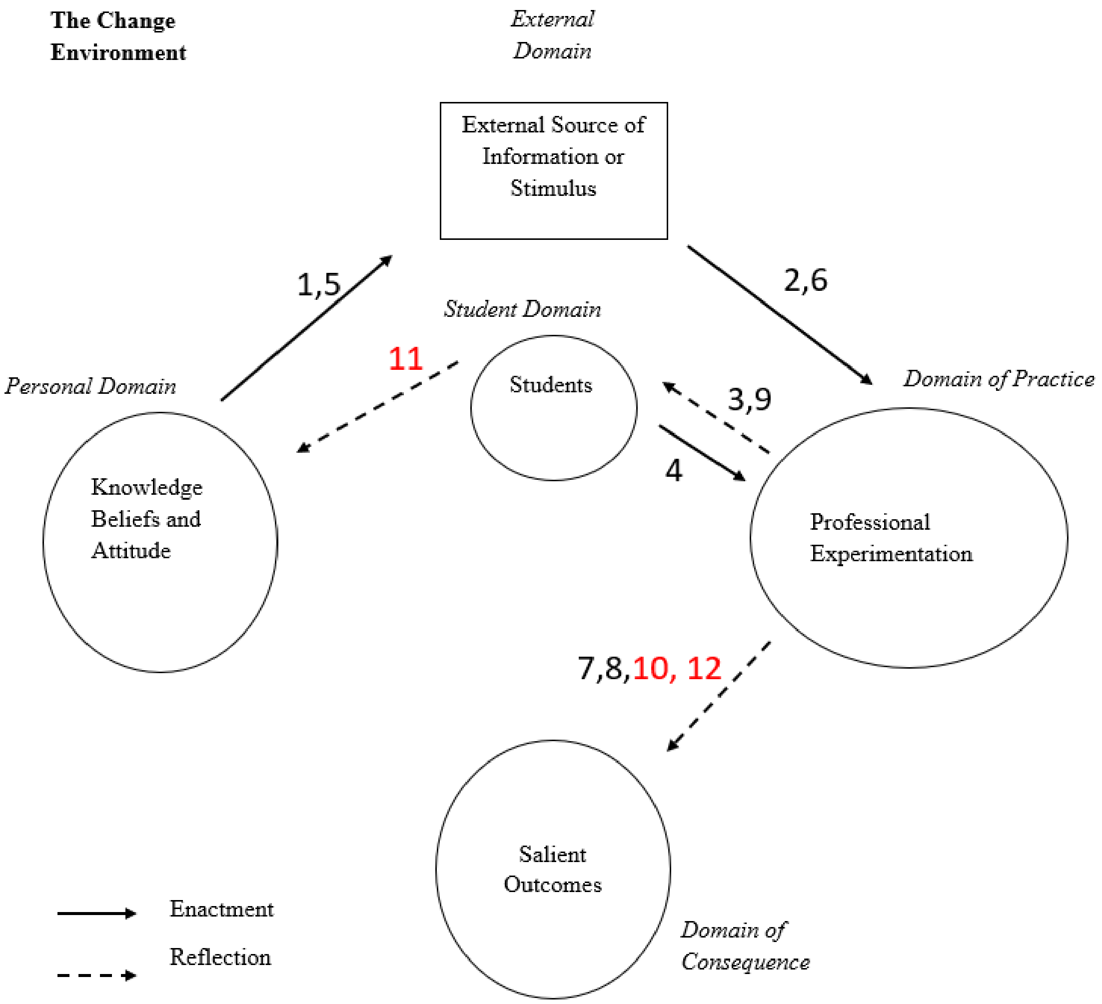


Figure 7.10: Thomas' change sequences on the logarithm task.

Thomas does not like the textbook tasks on logarithms, as he finds them semi-real, for instance a task about a lynx population. He would prefer tasks with physics, like tide level, measurement of sound (decibels) and so on. He says he wants more practical tasks where this is possible. In addition, he would have liked some historical perspectives on how logarithms were used before we had calculators. All these wishes are Thomas' enactment on the external domain, that lead to a change when the researcher designs and presents several logarithm tasks (arrow 1). Thomas decides he wants to use the tasks with his students (arrow 2). However, he realizes that using the logarithm table is a bit complicated, and he is skeptical to burden the students with all of it. So, Thomas reflects upon whether this would be too much for some of the students (arrow 3). Consequently, we decide to make a change to the task in the domain of practice, so that the students can use their calculators to find the logarithm instead of the tables (arrow 4). That way they can work on the idea, but do not have to learn to use the table. Thomas would have liked a short historical introduction to the logarithmic table, representing a request to the researcher in the external domain (arrow 5). Based on this request, I design and mail it to him, resulting in a change in the domain of practice (arrow 6). Thomas is positive to all the tasks on logarithms I present, and comments that he likes them, and that they might be vocationally relevant for some of the students who might become engineers in chemistry or biochemistry. So, these are changes in the domain of consequences that Thomas reflects upon as a result of implementing this task in the domain of practice (arrow 7). He also reflects on how the students are becoming engineers which is a practical occupation, yet he feels that when he teaches logarithms, it becomes too theoretical, which is an outcome in the domain of consequences that he otherwise would have liked (arrow 8). However, it is easier to teach that way because the students are more likely to complain that it is too difficult if it is more realistic. So, when he makes changes in the domain of practice to tasks that are more realistic, students in the student domain are more likely to complain (arrow 9).

After the tasks were implemented, Thomas said it is a good thing they got these tasks on logarithms, because the students needed the repetition. So, Thomas liked how the change in the domain of practice led to this outcome in the domain of consequences (arrow 10). Still, he also got the reaction he anticipated, that when a task is practical, and the students must set it up themselves, they struggle. This was a change in the student domain that was in line with what Thomas thought would happen (arrow 11). However, he does not want to change the tasks, he says they are nice. He therefore let the students struggle somewhat in the student domain (arrow 12).

The image of Thomas' change sequences on the logarithm tasks looks more like Hanna's change sequences than Sven's. However, there are some differences to what Hanna experienced. Although some of the students struggle when working on the tasks, this is in line with what Thomas expected when we designed the tasks. So, while he reflects on how the implementation went, he concludes that he likes the consequences and wants to keep the tasks as they are.

7.2.9 Thomas: Trigonometry Tasks

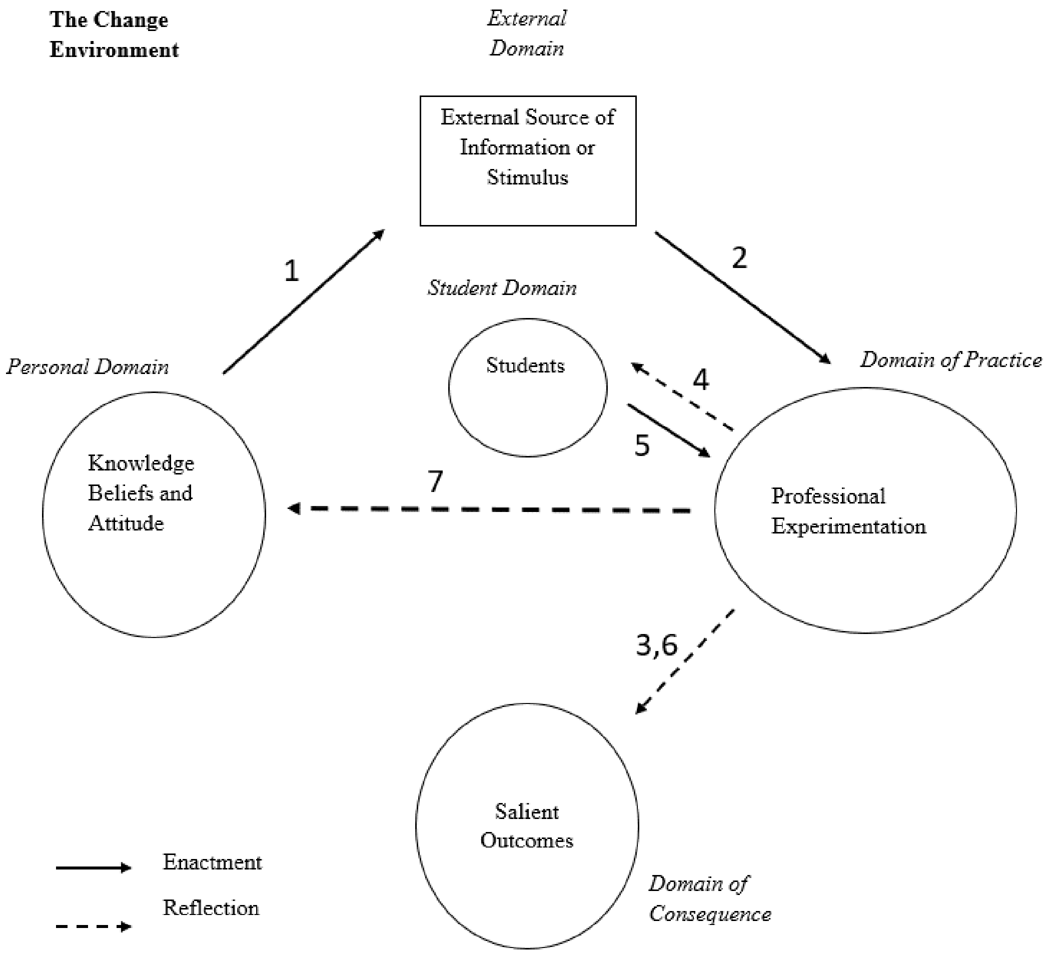


Figure 7.11: Thomas' change sequences on the trigonometry tasks.

Thomas asks for trigonometry tasks that are relevant within physics or technology. This enactment from Thomas on the external domain, leads to the researcher designing several tasks trying to fulfill these wishes, representing a change in the external domain (arrow 1). Thomas wants to use all these tasks with his students, representing a change in the domain of practice (arrow 2). Two of these tasks employ macros the researcher made in GeoGebra, that he is positive to and says it is something he can give to the students and not just use for demonstration. I interpret this as Thomas reflecting upon an outcome implementing this task will lead to, being a change in the domain of consequences he likes (arrow 3). On one of the macros in GeoGebra, Thomas comments that it uses a different formula for the sine function than the textbook. This is something that confuses the low achievers (arrow 4), and he would like it to be similar to the one in the textbook, which he prefers (arrow 5). Thomas comments that he likes that the students must model the functions in some of the tasks, instead of just calculating them, so this is a change in the domain of consequences that he likes (arrow 6). Thomas continues to reflect upon the use of ICT and how it seems to become more and more important in mathematics. He has used TI-calculator (Texas Instruments), and though he sees that GeoGebra is more applicable, elegant and the curves look nicer, TI works well enough, according to him (arrow 7). We did not reach the stage of implementing these tasks, so there is no evaluation process.

This change sequence does not contain many arrows, and just as for Hanna, this indicates Thomas being happy with the task. Except for a minor change in how to write a formula in order not to confuse the low achievers, Thomas wants the task as it is. Notably, as I was not part of the implementation, all the possible red arrows are missing from these change sequences.

7.2.10 Summary of the Teachers' Change Sequences

Clarke and Hollingsworth (2002) emphasize how their model has the potential to capture the complexity of change sequences, because the model is not linear and thus "recognizes the complexity of professional growth through the identification of multiple growth pathways" (Clarke & Hollingsworth, 2002, p. 950). Using the Interconnected Model to analyze the change sequences in the collaboration with the teachers in this project, makes the complexity behind the changes visible.

The arrows and changes in the analysis in this chapter are not always chronologically sequenced, thus making it difficult to see long change sequences. This is because the analysis has been done according to how and when the teachers talk about changes, and they are not necessarily presenting them sequentially and linking them together logically. The teachers might comment on how the implementation led to salient outcomes, then remember some difficulties with some of the students, which in turn leads to reflections upon possible reasons and solutions, and so on. It is not a structured, logical account of what changes led to what changes, rather it reflects the teachers' thoughts in the moment and what this meant to them. From the work of Brown and McIntyre (1993), we know that although teachers can give coherent reasons for their actions, it is not always easy for them to recall the sequencing of decision making they do in the classroom. It is therefore not surprising that the teachers' narratives about the change processes in this collaboration does not follow a chronological pattern.

The change sequences presented in this analysis have been made according to the teachers' expressions and reasonings. They have not been made as a result of the researcher's summary of what I thought as important and could recognize as changes. Thus, the analysis includes many arrows and change sequences for each design process, but this does not mean they are all linked together into one larger process of change. It is rather a picture of the diversity of things that teachers consider when they initiate changes or evaluate them, and how these are linked together. Hence, this is not growth networks as described by Clarke and Hollingsworth (2002), as there is no evidence of long-lasting changes. However, these analyses of the teachers' change sequences provide insight into which aspects teachers tend to or worry about when they make changes in the classroom, and it can give insight into what they might want to change.

Sven seems confident and clear about what he wants to change, and this is evident in the analysis of the change sequences in our collaboration. The arrows indicating change are linked between the domain of practice, the domain of consequences and the student domain. This is where Sven's focus is when we make changes to the mathematics classroom. He constantly works on finding ways to achieve salient outcomes where the students are willing to work and do not complain or protest too much. He knows what his salient outcomes are, but struggles to achieve all of them in the classroom. Design and implementation are therefore constantly evaluated against which of his salient outcomes that are achieved, and at the same time does not make the students react too strongly.

While the student groups of Hanna and Thomas are different, there are many similarities in the analysis of the change sequences in the collaboration with these two teachers. Thomas teaches a homogeneous group of hardworking students who aim to be engineers, while Hanna's students are diverse and many of them with little to no motivation to learn mathematics. Still, both Hanna and Thomas are open to new didactical ways of teaching that might improve their students' learning. This can be seen in the analysis of the change sequences, where the arrows are not only located around the domain of practice, domain of consequences and student domain, but also include the personal domain. Hanna and Sven evaluate their own actions and whether they could make changes to how they act and teach to further improve the mathematics lessons, in addition to make changes to the other analytical domains. So, these diagrams of the change sequences of the collaborations with the teachers, can give insight into which aspects of the teachers' personal world they are looking to change.

If everything goes well during the implementation of a mathematical task, there are few arrows to indicate changes. An example of this is the change sequence from the logarithm tasks with Thomas (Sub-Section 7.2.8). There are several arrows indicating changes in the design and planning phase, but almost no arrows after the implementation. This is because it went as Thomas had expected, more or less. So, he comments on which salient outcomes the implementation of the tasks led to, but there is no need for further refinements or changes. However, there are also examples of tasks that were more problematic to implement, e.g., the change sequence from the index task with Hanna (Sub-Section 7.2.5). This is by far the change sequence with the most arrows indicating changes, and also the task with the least successful implementation, according to Hanna. So, the more problems an implementation faces, according to the teacher, the more arrows there will be in the change environment where they are evaluating and considering improvements.

7.3 Summary of the Results

I have in this chapter presented an analysis of the collaboration with the teachers to investigate the two research questions guiding this research. In Section 7.1, I investigated "What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom?" According to my findings, the teachers mostly describe mathematical tasks by the wanted outcomes of the task. The outcomes the teachers focused on, could be summarized into three issues that were all related to their students: hard work, motivation, and understanding. These were the three issues the teachers wanted mathematical tasks to help them to resolve. In addition, my findings provide evidence of teacher aspects that can hinder implementation of some types of mathematical tasks. These aspects are didactical skills, communicative skills, and mathematical skills. Notably, these aspects mostly did not surface during the interviews with the teachers, but when the teachers were presented for suggested mathematical tasks.

In Section 7.2, I investigated "What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration?" The findings show that the changes initiated by the teachers are related to the issues they want to resolve (Sub-Section 7.1.8), but also to the teacher aspects which might hinder implementation (Sub-Section 7.1.8). However, using the Interconnected Model provides insight concerning whether the teachers aim to change these aspects or not. This is evident in whether the arrows of change include the personal domain, or only the student domain, the domain of practice, and the domain of salient outcomes. The analysis in Section 7.2 also provides insight into the complexity of how teachers make and evaluate changes, and that this is not a clear chronologically sequenced pattern of decision making.

The results of the analysis in Chapter 7 will in the next chapter be discussed in light of previous theories and findings.

8 Discussion

The aim of this research project was to learn more about the teachers' perspective with respect to mathematical tasks they want to use in their classroom. In addition to exploring how teachers characterize mathematical tasks they want to use, I wanted to utilize the opportunity to analyze the changes teachers make when designing and implementing mathematical tasks as well. Based on this, I formulated the following research questions to guide the research.

1. What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom?
2. What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration?

In Section 8.1, I will discuss results from the analysis with respect to the first research question. The focus in this section is thus on how teachers express what kind of tasks they prefer to use in their classrooms and dilemmas they might face. In Section 8.2, the discussion is focused on the change processes identified through this collaboration.

8.1 Characteristics of Mathematical Tasks

I will in the following discuss my results with respect to the theory on mathematical tasks presented in Section 3.4, to provide theoretical insights into my findings related to research question 1. As accounted for in Section 3.4, the 22nd ICME (International Commission on Mathematical Instruction) study had task design in mathematics education as the area of interest and identified five themes to produce an up-to-date summary of research on the area (Watson & Ohtani, 2015a). The five themes are: Frameworks and principles for task design, The relationship between task design, anticipated pedagogies, and student learning, Accounting for student perspectives in task design, Design issues related to text-based tasks, and Designing mathematics tasks: The role of tools. I have chosen to focus on the first two themes in this thesis. Although the last three themes are highly relevant for task design in general, the data generated through this research does not provide enough details within these themes. While the teachers in this research project often refer to their students when we design tasks, the student's themselves have not been interviewed and therefore do not have a voice of their own in this data material. Thus, the results of this research project do not provide data to discuss the student perspective in task design. Likewise, there was little discussion of elements concerning text-based tasks and the role of tools during the collaboration with the teachers.

I structure the discussion the same way as I structured the presentation of theory. That is, first I discuss my results with respect to frameworks and principles for task design, prior elaborating on design as desired outcome in this research project. Further, I discuss the results with respect to various design elements of tasks, which was presented in Sub-Section 3.4.2. Lastly, I discuss salient outcomes of tasks and teacher constraints.

8.1.1 Frameworks and Principles for Task Design

From a researcher's perspective on task design, mathematical tasks are designed based on theoretical frameworks and principles, yet the role of theory differs across various types of research. Kieran et al. (2015) explain how there is a distinction between design as intention and design as implementation, which was elaborated in Sub-Section 3.4.1. While it is not expected that the teachers articulate design theories or learning theories when we design tasks in this research project, they bring with them assumptions on how students learn into our collaboration. When I talked to the teachers about how they think students best learn mathematics, they all responded that they learn from working on tasks. None of them claimed the lecturing part was important for the students' learning, and they all emphasized the students as active learners. When I challenged Roger about this, since he can spend hours lecturing, he explained that the lectures are to be viewed only as guidance and help. The real learning happens when students work on tasks (Sub-Section 5.1.1). Hanna and Sven could also talk about how the students might benefit from a teacher helping them to summarize, but in general all four teachers were clear on how the students learned mathematics through working on tasks. So, they all express a perspective on learning as the learner having to play an active part in constructing their knowledge.

While other researchers go further in identifying teachers' underlying learning theories by observing their teaching or asking them to fill out carefully constructed surveys, I have chosen not to follow up on this. For instance, while it is possible to argue that Sven does not value learning as socially constructed, because he did not want to use classroom discussions, he explains otherwise when we talk. He struggled to get classroom discussion going in this class, and he knew it would have taken him time and effort to get there. With little time and many issues to deal with, he had chosen not to focus on this. At the same time, the students were seated so that they could talk and discuss among themselves when solving tasks. So, through this research project, I will argue that many of the choices the teachers made, that maybe could have been attributed to learning theories, were more about their personality and what they might find challenging or comfortable to do in the classroom.

When it comes to task design and frameworks and principles behind it, Kieran et al. (2015) argue that there is a difference in the role of theory in various types of research, and they distinguish between design as intention and design as implementation, which I have previously elaborated in Sub-Section 3.4.1. The difference is that in design as intention, theory and design principles play an important role when designing tasks, while in design as implementation, the focus is to further develop local instruction theories based on the implementation. While the teachers in this project do not have the theoretical prerequisites to use theory as a researcher when designing tasks, they still have a third focus that I will describe as design as desired outcome. I will elaborate on this third focus in the next section.

8.1.2 Design as Desired Outcome.

Through my analysis of the collaboration with the teachers presented in Section 7.1, I organized how the teachers talked about mathematical tasks into three dimensions with corresponding categories. I remind the reader of these results by presenting them again in Table 8.1 below.

Outcome of tasks	210	Characteristics of tasks	49	Students' reactions to tasks	46
Activity	***38***	**Didactical characteristics**	**14**	**Students' reactions**	**46**
Understanding	***38***	**Mathematical characteristics**	**2**		
Goal of the task	***6***	**Practical considerations**	**33**		
Diversity	***24***				
Relevance	***104***				
Exam/ curriculum	22				
Connecting mathematical topics	11				
Practical use	60				
Vocational	11				

Table 8.1: Schematic overview of the dimensions, categories and sub-categories presented in Section 7.1.

The details of the categories are described in Section 7.1, and I will now apply a more holistic view on the results. When looking beyond the specifics of what the teachers are asking for, there is a system in how the teachers express their wishes. About 70 % of the statements do not concern specific characteristics of mathematical tasks, but rather the outcome of the task. So, when the teachers ask for tasks, they mostly talk about what they want to achieve in the classroom through using a task, rather than specifying characteristics of the task itself.

As presented in Section 3.3, Brown and McIntyre (1993) conducted research where the aim was to understand the teachers' perspective on teaching. Brown and McIntyre identified through their research how teachers were more likely to talk about the outcome of their teaching, rather than describing characteristics of teaching. They also noticed how the teachers did not talk much about the students' learning but instead described normal desirable states of students' activity (NDS), like for instance the students as active and working. Brown and McIntyre assumed that these findings might be different if they had collected data differently. For instance, they assumed the teachers would talk more about the students' learning if they had been part of the planning process for the lessons.

The research project presented here, collected different types of data than Brown and McIntyre (1993). While they used teacher-interviews combined with classroom observations and stimulated recalls, I engaged in the task design process together with the teachers. This way, I would get access to test the teachers' ideas in the classroom and their evaluation of what worked and did not work. My data are therefore complementary to Brown and McIntyre's data. In Brown and McIntyre's work, the teachers give their accounts of good teaching and what they do to be successful. My data, on the other hand, are stories from the teachers' perspective on what they would like to change in their teaching, and I follow this teacher-initiated change process. So, while the focus is still the teachers' perspective, I get insight into the planning process of the teachers through our joint task design activity. However, the type of accounts the teachers give regarding characteristics of tasks they want to use in their classrooms, are similar to what Brown and McIntyre found through their work. That is, the teachers I collaborate with mostly describe the tasks through the outcome they want to achieve from the task.

Both the research of Brown and McIntyre (1993) and this research project have identified how teachers are inclined to talk about the outcome of teaching or tasks, rather than characterizing them. When researchers address teachers, it can be helpful to use a discourse that resonates with how the teachers think and work. In other words, expressing what tasks can accomplish before following up with characteristics might help to get teachers' attention.

8.1.3 Design Elements of Tasks

As presented in Sub-Section 3.4.2, a designer must consider several pedagogical dilemmas when designing mathematical tasks. One of the recurring issues that were debated at the ICMI Study Conference on task design in mathematics education, was that:

> while the mathematics exemplified by the task is central, there are many other important considerations in designing tasks, especially when designers wish to anticipate and encourage particular pedagogical choices (Sullivan et al., 2015, p. 91).

The teachers in this research project have mostly asked for tasks with respect to the desired outcome of the task. As described in Sub-Section 3.4.2, pedagogical choices in the design process of tasks must take several dilemmas into account. Although the teachers in this project are not necessarily explicit about considering dilemmas in our conversations, the theory presented in 3.4.2 gives a range of design considerations that must be addressed whether it is intentional or not. I have therefore chosen to discuss the findings in this research project with respect to the five dilemmas presented by Barbosa and de Oliveira (2013) (Sub-Section 3.4.2). These five dilemmas are: context as a dilemma, language as a dilemma, structure as a dilemma, distribution as a dilemma, and levels of interactions as a dilemma.

Context as a Dilemma

Because the teachers in this research project were working with student groups taking some kind of vocational education, I had expected a focus on connecting the mathematical tasks to their vocation. Although this was mentioned by all the teachers at some point, it was less emphasized than anticipated. The reason for this, can be related to how use of context is not unambiguous. While a realistic focus might foster engagement, there are also studies showing that not all students perform better with contextualized mathematical problems (Sullivan et al., 2015). The teachers in this research project expressed several concerns and issues when it came to realistic contexts. Thomas talked about how a realistic context often made the tasks more difficult for the students, and that they did not like this (Sub-Section 5.2.1). These difficulties can be related to Verschaffel's (1999) descriptions of issues students might struggle with when solving mathematical application problems. While Thomas, with background from physics, had the knowledge to make more realistic tasks, he hesitated to do so, because he believed that the students preferred the familiar tasks that resemble an example task. At the same time, Thomas asked for realistic tasks within several topics (Sub-Section 5.2.1) and assumed some students might find them motivating. So, for Thomas, context was a dilemma. He wanted more realistic tasks but was also aware that some of the students would find them more difficult, and thus could react negatively.

Both Roger and Thomas were concerned with what they called artificially real tasks (Sub-Sections 5.1.1 and 5.2.2). That is, tasks that the designer tries to make within a realistic context, but it does not resemble what is actually being done in real life. This type of tasks is in the literature referred to as "'dressing up' of purely mathematical problems in the words of an other discipline or of everyday life" (Blum & Niss, 1991). This was something Roger and Thomas found problematic, especially when working with students who might have craft knowledge on the topic.

Neither Hanna nor Sven had much focus on tasks designed in a realistic context. The reader can see this by studying Tables 7.6 and 7.8, where only some of the statements have been coded as *practical use* or *vocational*. However, this lack of focus on realistic context could be a result of them having tried out a project where they designed several vocationally oriented tasks, but still had issues. The students did not accept the realistic context and treated them instead like any other mathematical task they met in the classroom. This experience might have had an impact on Hanna and Sven, making them realize a realistic context is not necessarily enough. At the same time, they spoke positively of using realistic context and that it might help students understand and solve tasks. For instance, Sven talked about how solving equations by using algorithms could be difficult for the students, but they would be able to solve the same task if it was presented realistically (Sub-Section 5.4.1).

Language as a Dilemma

While there were a few students who used Norwegian as an additional language in these classes, the teacher never talked about extra challenges with respect to students who do not have Norwegian as their first language. There was neither any emphasis on using more everyday language as opposed to a scientifically correct and specific language. The only dilemmas that I could detect with respect to language, was about giving clear and explicit instructions regarding what the students should do. However, I attribute this more to classroom management than to language as a dilemma.

Structure as a Dilemma

According to Barbosa and de Oliveira (2013), structure as a dilemma refers to the degree of openness in tasks. As described in Sub-Section 3.4.2, this can be related to tasks having open-start, open-middle, or being open-ended. The design dilemma is whether to use specific questions to scaffold student engagement in more prescribed ways or allowing students to make strategic choices on their own.

Neither Roger nor Thomas made any requests for open tasks or rejected tasks because they were too open. Although they did not specifically ask for open tasks, they reacted positively to some of the tasks that were open. An example from the collaboration with Roger is the task I presented where the students were to investigate various graphs (Section 6.8). The idea of the task is to set up two items on the floor a couple of meters apart, and name them *A* and *B*. The students are then asked to draw a diagram where the *x*-axis is the distance from *A*, and the *y*-axis is the distance from *B*. One person then walks straight lines between the two items, and the students draw the graph. This is a task that to a certain degree has open-start, since the graphs are a result of how the person walks. At the same time, there are a range of solutions depending on which aspects of the graphs the students want to investigate and categorize in the end. Roger rejected most of the ideas I presented for tasks (Sub-Section 5.1.4), but it does not seem to be because of the degree of openness in the task itself, since he liked a task like the one described above.

Likewise, Thomas was positive to tasks I presented that were open. Not all the tasks during our collaboration were open, but some of the tasks about logarithms and trigonometric functions (Sections 6.6 and 6.7) were open-middled, since no solution strategy was presented, and the tasks could be approached in various ways. Thomas liked the tasks, but he did comment that when a task gets practical and the students must figure out themselves how to set it up before calculating, they struggle (Sub-Section 5.2.1). So, Thomas seems to be conscious of structure as a dilemma and how a higher degree of openness in tasks can be motivating because of student autonomy, but also makes the tasks more difficult because they must figure out which strategies to use.

Both Hanna and Sven were skeptical if the tasks became too open. Sven wanted to change some of the tasks before implementation and give the students more direct instructions on what to do. This happened already with the first task we designed, which was the A4-task (Section 6.1). Sven was clear and said that even if many possibilities in a task is nice, it is important to have a starting point that everyone can master (Sub-Section 5.4.2). He wanted the task to be clearly formulated with explicit goals, since it otherwise could be hard to get the students to work. Even though I tried to take this into account when I designed the next task for Sven, he made some adjustments on the Area task (Sub-Section 5.4.3) as well. For instance, the students were asked to calculate the area in as many ways as possible on the trapezium subtask, but Sven wanted more subquestions to this task. His reasoning for this, was that the wording might be too open and thus would leave the students not knowing what to do (Sub-Section 5.4.3). Even though Sven reduced some of the openness in the tasks, and especially with respect to those being open-started, both the A4-task and the Area task are mostly open-middled. That is, the students must use their own strategies to find the solutions.

Hanna did not reject any tasks due to openness before implementation, but she commented on several occasions that it might be beneficial to make the tasks less open in a revised version. The Rope task had some subtasks that were open-started, where the students should use their string to measure various lengths of their own choice (Section 6.4). While Hanna thought some of the students worked well on this task, others waisted, in her opinion, time doing things they were not supposed to. She said she is unsure whether investigative tasks will work in this class (Sub-Section 5.3.3). So, Hanna reflects upon whether this group of students need more direct instructions on what to do, instead of investigating on their own. Likewise, Hanna wanted to remove some of the openness in the Index task (Section 6.5) when we evaluated it. She had given the students the opportunity to find a house they liked on the Internet, but said that next time, she would just give them a prospect of a house to start with (Sub-Section 5.3.4). Hanna said she had written down point by point what the students should do on the Index task, but it was still a bit chaotic during the lesson, since the students did not read the information carefully. Therefore, Hanna thinks the class needs even more structure (Sub-Section 5.3.4).

The difficulties the teachers in this research project mention with respect to open tasks, are consistent with the findings of Klein and Leikin (2020), who argue how these difficulties are linked to conceptions related to teaching:

> It seems that opening tasks requires flexibility at multiple stages: flexibility is required of the person who poses the tasks, of the person who solves them, and when implementing the tasks in the classroom. We can guess that this is the reason why teachers tend not to use OTs (open mathematical tasks) in teaching if they are not instructed to (Klein & Leikin, 2020, p. 362).

While all teachers in this research project used and liked mathematical tasks with some degree of openness, as I have shown above, there were also some issues that surfaced. Thomas' students reacted negatively when they had to find their own strategies. Sven claimed it could be hard to get some of the students to work, and Hanna wanted to revise and add more structure to the task to get the students working on what they were supposed to. When flexibility is required of the person who poses the tasks, of the person who solves then, and when implementing the tasks in the classroom, this makes it a complex process as pointed out by Klein and Leikin (2020). However, the teachers in this project took part in designing tasks that might help both themselves and their students to become better at solving tasks that are more open. While almost none of the tasks where open-started and only some of them where open-ended, many of the tasks where open-middled, as described above. This allows both the students and the teacher to work on tasks and develop some of the flexibility needed as described by Klein and Leikin (2020). Therefore, while the degree of openness in tasks always will be part of structure as a dilemma when designing tasks, it is worth noting that opening parts of mathematical tasks, might help both students and teachers to further develop their flexibility needed to work on fully open tasks.

Tasks with a high degree of openness are often referred to as rich tasks, among other characteristics (Foster & Inglis, 2017). As presented in the theory Sub-Section 3.4.2, The Norwegian Directorate for Education and Training describe rich tasks as problem solving tasks offering opportunities to discuss solution strategies and mathematical concepts with peers. They list rich tasks as an example of mathematical tasks under the heading: "Be conscious in choosing tasks" (The Norwegian Directorate for Education and Training, 2015, my translation), which is one of several principles the Directorate presents as guiding 'good' mathematics teaching. Rich tasks are described using the following seven bullet points, and the Directorate claim a rich task should:

- introduce important ideas or solution strategies.
- be easy to understand and everyone should be able to get started and have possibilities to work with it (low threshold).
- be perceived as a challenge, require effort, and be allowed to take time to solve.
- be solved in several different ways, with different strategies and representations.
- be able to initiate an academic discussion that demonstrates different strategies, representations, and ideas.
- be able to function as a bridge builder between different academic areas.
- be able to lead students and teachers to formulate interesting new problems (What if…? Why is it so that…?) (The Norwegian Directorate for Education and Training, 2015, p. 2. Translated by me)

While none of the teachers in this research project used the words rich tasks or listed all the bullet points, there are many similarities between what the teachers described and some of the bullet points. I will in the following present examples that illustrate how the teachers ask for elements in tasks that can be linked to some of the bullet points above. For instance, behind the category diversity there were several formulations almost identical to the second bullet point – which represents a task with low threshold and high ceiling. A concrete example of this, is when Sven summarizes the benefits of the A4-task (Sub-Section 7.1.7): “To summarize: the benefits are they get quickly started, everyone can manage something, and they get active straight away” (evaluating A4-task). Although he does not mention the word understanding, he talks about the importance of all students getting started and being able to manage something.

Also, Hanna’s wish for introductory tasks (Sub-Section 5.3.1) can be viewed as a request for a task that accomplishes the first bullet point above – introduce important ideas or solution strategies. She explains that she wants an introductory task where the students discover instead of her telling. When it comes to bullet point six, several of the teachers in this research study asked for tasks that could connect different mathematical topics. Therefore, 11 codes were assigned to the sub-category connecting mathematical tasks, as shown in Table 8.1. In addition, when the teachers were asking for tasks that would get the students active, this could be comparable to bullet points five or seven, as these bullet points require students to actively take part in solving and further developing tasks.

As shown above, the teachers in this research project request many characteristics in mathematical tasks that are comparable to most of the bullet points from the Norwegian Directorate for Education and Training describing what a rich task should achieve. On the other hand, most of the tasks we designed during this research study cannot be defined as rich tasks since they do not fulfill all bullet points. As previously presented in Sub-Section 7.1.8, there are some teacher aspects that can limit possibilities in tasks, thus making it difficult for a teacher to implement tasks fulfilling all the listed bullet points as presented by the Norwegian Directorate of Education and Training.

While using mathematical tasks that fulfill all the bullet points describing rich tasks, as presented by the Norwegian Directorate of Education and Training, can help foster mathematical learning, it does not help if teachers reject tasks because they do not master some of the bullet points. Just like openness in tasks adds to the complexity of teaching as described by Klein and Leikin (2020), using rich tasks fulfilling all characteristics, adds to the complexity of teaching as well. The teachers in this project articulate how they want tasks that fulfill several of the bullet points, yet they do struggle with some of the bullet points. Consequently, they might reject some mathematical tasks. This will be further discussed in the next Sub-Section (8.1.4).

Just like there exists tasks with a varying degree of openness (open-started, open-middled, open-ended and combinations of these), I would argue it is important to have tasks that can help teachers work on developing some of the bullet points from the characteristics of rich tasks, without having to work on all of them at once. While rich tasks might be an ideal to work for, the teachers also need tasks that can help them develop these skills together with their students. If the teachers are presented with mathematical tasks that focus on only some of the outcomes associated with rich tasks, it can allow teachers to improve their teaching step by step. Therefore, when rich tasks are promoted, teachers can also find alternative tasks that only fulfill some of the outcomes of rich tasks. This way, a learning trajectory, aiming to use more rich tasks in the classrooms, would be accessible for mathematics teachers.

Distribution as Dilemma

Distribution as a dilemma refers to what is expected to be taught in a task; what content should be selected and focused on (Barbosa & de Oliveira, 2013). This distribution is according to Barbosa and de Oliveira (2013), a function of the cognitive demand of tasks and can be related to the Mathematical Task Framework developed by Stein et al. (2000). The framework describes mathematical tasks as a hierarchy of tasks that develop from *memorization* to *procedures without connections* to *procedures with connections* to *doing mathematics*. While I have not conducted a complete analysis of the tasks in this project with respect to their cognitive demand, it is possible to do some general reflections on how the teachers considered this dilemma.

None of the teachers in this research project asked for tasks that can be categorized purely within the lower cognitive demands, that is *memorization* or *procedures without connections*. This might be because the textbook already provides many tasks of this kind, but the teachers were explicit about wanting something more. Hanna talked about the dilemma, which has been debated at her school. That is, if you want a low achiever to just pass the exam, it might help with training procedures for a period, but then the student is more likely to forget in the long run (Sub-Section 5.4.1). Opposingly, Hanna wants tasks where the students can develop an understanding that is long lasting (Sub-Section 5.4.1). This request for tasks coincides with higher cognitive demands in the Mathematical Task Framework because it means the students will need to make some type of connections when solving tasks.

Sven describes tasks he wants as a type of low threshold – high ceiling, where everyone can get started on something, but at the same time the high achievers can be challenged (Sub-Section 5.4.1). The ‘low threshold’ means that not all parts of the tasks are cognitively challenging since everyone in the classroom should be able to start working on it. At the same time, Sven shares his frustration over how many of the students memorize the area formulas instead of realizing that if you understand how to calculate the area of a triangle, you can calculate all other types too (Sub-Section 5.4.1). Hence, Sven requests tasks where all students can develop their mathematical understanding and not just memorize procedures. This means he wants tasks of higher cognitive demand.

Thomas asked for tasks that were connected to realistic problems in physics and technology (Sub-Section 5.2.4). At the same time, he commented how his students seem happier getting tasks where they can use a method they already know (Sub-Section 5.2.1). Distribution as a dilemma therefore seems to be an issue Thomas is concerned about, and while he would like to give his students tasks of a higher cognitive demand, the students often react negatively to it. This was also something Thomas commented after the logarithm tasks were implemented. He said the tasks were challenging for the students, but he liked them (Sub-Section 5.2.2).

All three teachers who requested tasks, asked for tasks of higher cognitive demand with respect to the Mathematical Task Framework. This might indicate that many teachers want to challenge their students through tasks but might find it difficult to follow through on it. Thomas was explicit about this dilemma, and this is what he wanted help with, even if he has a degree in physics himself and the knowledge to connect many topics to practical issues.

Levels of Interactions as a Dilemma

Levels of interactions concerns, according to Barbosa and de Oliveira (2013), interactions between teacher and students. On one hand, a closed task is often viewed as something a student should solve on her own, while an open task requires more help and involvement from the teacher. In general, levels of interactions “can be interpreted to mean that the task does not exist by itself, but its implementation is influenced by the nature of the intended or anticipated interactions between the teacher and the students when they are engaged with the task” (Sullivan et al., 2015, pp. 93-94).

Sven is the teacher who is most explicit about what he wants in the tasks with respect to levels of interactions. He says he wants tasks where he can ‘pull back’ more as a teacher, and the students start to work without needing help (Sub-Section 7.1.7). However, I attribute this to him wanting the students to be more independent and learn to work on their own, and not to avoid interacting with them. He expresses his concern of students being too passive when we refine the A4-task:

> It must be very structured. They are not taking any initiative whatsoever to explore on their own. They have not been raised to do this throughout their schooling, and I have not taken that fight either. So, they need very clear goals to work towards (*refining A4-task*).

I will therefore argue that Sven’s main goal when it comes to interactions, is to get the students more actively involved in the mathematics lessons, thus wanting to pull more back as a teacher at this point, not always being the driving force in the classroom himself.

Hanna on the other hand, wants to interact with her students through discussions and wants a friendly and secure atmosphere. She walks around the classroom trying to prompt discussions with the students as she goes along (Sub-Section 5.3.1). However, she sometimes struggles on how to structure the levels of interactions. When it came to the index task, she said the students were engaged, but the first lesson was a bit chaotic, according to Hanna (Sub-Section 5.3.4). I will therefore say that Hanna wants tasks providing levels of interactions where both the teacher and the students are active, but she is still working on improving the quality of these interactions.

Thomas utters concerns about little response from his students when he lectures, and he wants the students to interact more (Sub-Section 5.2.1). However, he is unsure on how to achieve this. When the students worked on the logarithm tasks, he went through the task on radioactivity on the blackboard, asking them questions and trying to get some feedback and questions, but the student group did not respond much (Sub-Section 5.2.2). In hindsight, I could have designed tasks for Thomas focusing more on making the students verbally engaged, but I did not think of it at the time. Also, because he did not explicitly ask for that when he requested tasks.

Roger did not ask for specific tasks, so I cannot say much about his perspective on this dilemma when it comes to task design. However, he did talk about how he could easily be led into digressions when he was teaching (Section 5.1). I interpret this as there are interactions between him and the students, and not just him explaining and the students working on tasks.

In general, all the teachers in this project want to interact with their students, but they might be at various points concerning the feasibility of such interactions. While Sven wants to pull more back as a teacher to get the students engaged, Hanna works on how to structure the interaction to be more productive.

8.1.4 Salient Outcomes of Tasks and Teacher Constraints

Already in 1986, Guskey pointed out that: "For the vast majority of teachers, becoming a better teacher means enhancing the learning outcomes of their students" (Guskey, 1986, p. 6). According to Harootunian and Yargar (1981), teachers determine their success in the classroom not only by improved student achievement, but by the maintenance of student involvement. In this sub-section I will show how these perspectives resonate with the findings of this research project. The teachers ask for mathematical tasks based on what they perceive as salient outcomes, and these are related to their students' involvement and understanding. As presented in Sub-Section 7.1.8, the main issues the teachers want to resolve through mathematical tasks are related to their students, and these are:

- Work
- Motivation
- Understanding

The first two of these issues can be related to aspects of student involvement, as described by Harootunian and Yargar (1981), while the last is an example of improved learning outcome, which Guskey (1986) pointed out as the goal of most teachers. Also, Clarke and Hollingsworth (2002) emphasize what teachers consider as salient outcomes but argue that we need to acknowledge that teachers value and attend to different things. These finding have thus been consistent for at least forty years, and all four of the teachers in this research project refer to these aspects at some point, as described in Sub-Section 7.1.8. However, they are not necessarily considering all three of them when they describe characteristics of mathematical tasks they want to use in their classroom. They prioritize the issue(s) they perceive themselves to struggle the most with in the classroom. During the semi-structured interview (Sub-Section 5.2.1) Thomas talked about finding it difficult to motivate the students, and asked for logarithm tasks being practical, so the students can experience why we need logarithms and trigonometric functions (Sub-Section 5.2.2). Hanna and Sven referred to all three issues in our conversations, but the starting point was to get the students to work. Sven wanted tasks where the students could get started on their own and he could pull more back as a teacher (Sub-Section 7.1.7), while Hanna talked about introductory tasks and tasks that would get the students active (Sub-Section 7.1.6). Roger was content with all three issues in his classroom, and therefore did not ask for specific characteristics in mathematical tasks. So, it is not just the group of students that determines what characteristics in mathematical tasks the teachers ask for, but just as much which of these three issues the teachers struggle the most to overcome themselves. That is, the teachers will ask for mathematical tasks that can help them solve didactical issues in their mathematics classroom.

In addition to the three issues the teachers consider when they ask for tasks, there are some teacher aspects that seem to limit the possibilities in mathematical tasks that were also presented in Sub-Section 7.1.8. These are:

- Didactics
- Communication
- Mathematics

The research literature has pointed out how a lack of pedagogical and mathematical knowledge might limit teachers (Ball et al., 2008). However, there have also been researchers using Bandura’s theories on self-efficacy, which can shortly be defined as: “an individual's judgments of his or her capabilities to perform given actions” (Schunk, 1991, p. 207). These researchers examine how a teacher’s perceived confidence as a mathematics teacher might influence their teaching, also linking this to mathematical competence (Xenofontos & Andrews, 2020). I have chosen to avoid the words knowledge and self-efficacy when I describe the teacher aspects that can limit the type of tasks we implemented during the collaboration in this research project. While it is possible to argue for these theoretical constructs in my data, I would say my data provides evidence that there is more to these aspects than what can be explained by knowledge or self-efficacy. Hanna describes uncertainty about her didactical skills in mathematics (Sub-Section 5.3.1). However, she has just completed further education that qualifies her as a secondary mathematics teacher, and the courses she has taken includes both mathematics and didactics. She has also been part of inquiry-based collaborative research projects with the local University (Sub-Section 5.3.1). From an outsider perspective, she should have more than enough mathematical and didactical knowledge to be confident and do a good teaching job in the classroom. However, maybe the courses she has taken and the research projects she has been part of, have made her aware of the complexity of inquiry-based teaching, and she needs time to develop skills as a mathematics teacher. Knowing mathematics and didactics is not the same as acting this out in the classroom with students. I have therefore chosen to use the term didactics on this aspect, because it concerns not just what you know and what you believe you can do, but also developing didactical skills that can only be done in the mathematics classroom over time.

Thomas and Sven are both, to some degree, struggling with classroom discussions and the communication with their students. Sven said classroom discussions were not possible (Sub-Section 5.4.2) and Thomas talked about how it can be difficult to know if the students understand when he lectures (Sub-Section 5.2.1). However, both are, according to my impression, clear and eloquent when communicating with me during our collaboration. So, I would say they both have knowledge and confidence in talking and communicating but struggle to communicate with some of their students. It is not given that a middle-aged man with a master’s degree in mathematics education instantly can communicate well with 16-year-old girls wanting to become hairdressers. These communicative skills might take a long time to develop, and it is not given that this is something all teachers will master fully with all type of students. Sven is aware of this issue and explains that with the limited time he has with this student group, and the students’ lack of knowledge within key topics, he has not prioritized developing classroom discussions further (Sub-Section 5.4.2).

The third teacher aspect that can limit implementation of tasks is mathematics. All four teachers in this research project had many years of education within mathematics, so this was not a predominant aspect herein. However, there was one incident with Hanna when we evaluated the area task (Section 6.2). She told me she was a bit nervous for when the students would get to task five, because she had not had the time to solve it herself and was uncertain how to approach this task (Sub-Section 5.3.3). Still, she chose to include the subtask when she gave it to her students, which I argue is an example of mathematical skills in the classroom. Although Hanna has not prepared a solution of the task before the lesson starts, she is confident enough in her mathematical skills to assume that she will find a way of solving it together with her students. Even if she expresses nervousness about her students working on the task, her choice of using it says a lot about her mathematical skills with this group of students.

While we have known for a long time that teachers are looking for mathematical tasks that can improve student involvement and understanding (Guskey, 1986; Harootunian & Yargar, 1981), this research provides a more detailed perspective on how different teachers will focus on various issues in the classroom, further building on the work of Clarke and Hollingsworth (2002), who argue teachers' value and attend to different things. The teachers' focus in this research project seems to depend on what didactical issues they find the most essential to address. In addition, the teachers' choice of mathematical tasks might be limited due to skills that they have not fully developed together with this group of students. It might be beneficial to design mathematical tasks taking these aspects into account, and thereby use tasks as one of several means to foster teacher learning and development together with their students.

I have in Section 8.1 discussed the findings in this research project with respect to the first research question: What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom? I will in Section 8.2 discuss the findings in this research project with respect to the second research question: What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration?

8.2 Change

I will in the following sections discuss the findings in this research project in light of theory on teacher change. This is to provide theoretical insights into my second research question: What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration? I will in Sub-Section 8.2.1 discuss what types of changes the teachers are initiating and their rationales behind it. Further, I discuss how I interpret the domain of consequences in the Interconnected Model of Clarke and Hollingsworth (2002) with respect to other models of teacher change. I use this discussion to argue why I need to add the student domain to the model in Sub-Section 8.2.3.

8.2.1 Various Types of Changes Initiated by the Teachers

When discussing teacher change, there is a difference between changing the teacher and changing the teaching. Some researchers argue that changing teachers' beliefs will change their teaching, while other researchers assume it is the other way around. If one changes classroom practice, it can lead to a change in teachers' beliefs and attitudes. According to Clarke and Hollingsworth (2002), change is a complex process that needs to be analyzed through an interconnected model. Having used Clarke and Hollingsworth's model to analyze the change processes initiated by the teachers in this research project, the difference between changing the teacher and changing the teaching became evident. While Sven is making many changes, they are all located in either the domain of practice, the domain of consequence or the student domain (Sub-Section 7.1.7). He never shares reflections on his personal opinions or whether he should make any changes in what he does. Sven comes across as a confident teacher who has a clear view on what he thinks works in the classroom and what his own strengths and limitations are. However, he has not managed to get enough of the type of tasks he wants in order to teach the way he wants and is therefore seeking to make changes in practice.

While Sven wants to make changes to his teaching, Hanna wants to change both her teaching and herself as a teacher (Sub-Section 4.7.1). Early in the collaboration, she explicitly said that she wanted to develop as a teacher through participating in this project. This is also evident when examining the analyzed change sequences from our collaboration, which I present later in this chapter. The arrows go between all the domains, and when she is not happy with what is going on, she also reflects on her own impact on the situation and what she might be able to change.

Thomas is not as frustrated as Hanna when something goes wrong and the students complain, but he wants to improve his teaching (Sub-Section 5.2.1).

However, unlike Sven, Thomas also reflects on how what happens in the classroom aligns with his own thoughts, and how he might change his behavior to improve the teaching. He told me about a concrete situation a long time ago when he changed his behavior in the classroom, based on feedback from students (Sub-Section 5.2.1). While Sven comes across as confident in who he is as a teacher and with a plan to change the teaching, Thomas comes across as more inquiry-oriented on both levels. While having confidence in himself as a teacher and his experience, Thomas is still open to make changes both to his teaching and to himself as a teacher. However, he is not familiar with various options to facilitate changes.

To summarize, using the Interconnected Model of Professional Growth (IMPG) developed by Clarke and Hollingsworth (2002) to analyze the teachers' own processes of change, gives a lens to examine what kind of changes the teachers are making and their rationales behind the changes. There is a difference in making changes to their teaching practice versus trying to develop themselves as teachers. With the help of the IMPG, these nuances become evident when examining the change sequences.

8.2.2 The Domain of Consequences

According to Clarke and Hollingsworth (2002) the four domains in their model are "analogous (but not identical) to the four domains identified by Guskey (1986)" (p. 950). However, they do not further elaborate on the differences, but they later claim that "The Interconnected Model incorporates all previous linear models" (Clarke & Hollingsworth, 2002, p. 959). I would still say there is a qualitative difference from some of the previous models on teacher change, like the one developed by Guskey (1986). While Guskey (1986) in his model (Figure 3.3) refers to change in the learning outcomes of students and how improved learning among the students are important for the teacher, Clarke and Hollingsworth (2002) refer to change in what the teacher views as salient outcomes. While improved learning among students can be viewed as a result of changed practice, I argue that a change in the teacher's view of salient outcomes is about changing the teacher.

This difference I have described above, illustrates how there is not always a clear line between a teacher making changes to practice and a teacher changing beliefs and knowledge. When Clarke and Peter (1993) first presented the four analytical domains that would later be fully developed to the Interconnected

Model, they used the name “domain of inference” (p. 170). Already, they were explicit about how this domain concerned the teacher’s evaluation of practice:

> Those professional outcomes to which the teacher attaches value constitute the mediating domain by which classroom experimentation is translated into changed teacher knowledge and beliefs. These valued outcomes may include student learning, teacher satisfaction, teacher planning effectiveness and efficiency, reduced teacher classroom stress, and increased student and teacher classroom enjoyment (Clarke & Peter, 1993, p. 170).

This seems to be a broadening of professional outcomes compared to previous models on teacher change, because Clarke and Peter are not only including student learning, but also aspects directly related to the teacher, like teacher satisfaction. However, Guskey (1986) also included more than just improved students’ learning. He elaborated that students’ learning not only included students’ improved scores:

> But they can also include students’ attendance, their involvement in class sessions, their motivation for learning, and their attitudes toward school, the class, and themselves. In other words, learning outcomes include whatever evidence a teacher uses to judge the effectiveness of his or her teaching (Guskey, 1986, p. 7).

So, both models include several aspects of consequences due to professional experimentation, but there is a difference in what the teacher focuses on. In Guskey’s (1986) model, it is about what evidence the teacher uses to judge changes in classroom practice, while Clarke and Peter (1993) point out that this is about which professional outcomes the teacher attaches value to. This is a perspective that has followed the analytical domain through further refinement of the model and is also found in Clarke and Hollingsworth’s (2002) interconnected model. They are explicit about why this perspective is important: “The significance of the designation ‘Salient Outcomes’ lies in the need to acknowledge that individuals (teachers) value and consequently attend to different things (they consider different things salient)” (Clarke & Hollingsworth, 2002, p. 954).

Clarke and Hollingsworth (2002) are explicit about the empirical foundations for their model and illustrate each component by empirical data. This might also explain their extension of the domain of consequence as more than improved student learning. There were aspects from their empirical data that did not fit into the previous models on teacher change. This is one of the challenges when developing models of change processes in the classroom. On one hand, the point of a theoretical model is to disregard data that is not important for the perspective being studied, thereby illuminating key aspects. On the other hand, simplifying too much might result in losing some of the complexity of what happens in the classroom. As a result, collecting various types of empirical data might raise the need for adjusting some models due to the aspects being studied. This might also account for the differences in the models of Guskey (1986) and Clarke and Hollingsworth (2002), and further my need for making additions to Clarke and Hollingsworth's model. Guskey (1986) is clear about the limitations of the model he presents:

> Note that this model is not necessarily novel and does not explain or account for all of the variables that might be associated with the teacher change process. Its simplicity is not meant to impugn the complexity of the issues involved or the inherent interrelationships among components. Rather, the model is offered primarily as an ordered framework by which to better understand trends that appear to typify the dynamics of the teacher change process (Guskey, 1986, p. 7).

Guskey's point with his model for teacher change is primarily to emphasize the sequencing of teacher change. That is, a teacher is most likely to change her beliefs after she sees evidence of improved student learning, and the consequences this might have for professional development programs.

Clarke and Hollingsworth (2002) want to emphasize the multidimensional aspects of teacher change in addition to study what type of changes that might lead to long lasting professional growth. Their model is set in the teachers' world, and all analytical domains are viewed from the teachers' perspective. Therefore, their model also needs to consider that what a teacher views as salient outcomes might change over time. So, the main purpose of their model is to analyze, predict or interrogate the teachers' change processes over time, and what might lead to long lasting changes.

In Guskey's (1986) model, the domain of consequences is viewed more as an objective result of changes in classroom practice. Students' improved scores on tests or increased participation in classroom activities are possible to measure and compare. However, the domain of consequences in Clarke and Hollingsworth's (2002) model is a subjective perspective from the teacher concerning what she views as salient outcomes.

Like Clarke and Hollingsworth (2002) used their empirical data to adjust the Interconnected Model to take into account what the teacher views as salient outcomes, I have used the empirical data from this research project to adjust the model to include the student domain. I will elaborate on this in the next section.

8.2.3 Adding the Student Domain to the IMPG

As previously mentioned, I have used the Interconnected model of professional growth differently than Clarke and Hollingsworth (2002) when analyzing the data in this research project. I have analyzed the changes initiated by teachers in their everyday classroom on a micro level, while Clarke and Hollingsworth have used it to examine the impact of professional development programs. This change in analytical level brought about a different need in my analysis. The aim of this research project was to capture the teachers' perspectives, and to analyze their change process. Central in the teachers' vocabulary are the students, and I would lose some aspects if I interpreted the teachers' talk about their students to either belonging to the domain of consequence or the domain of practice. Especially since students might react positively or negatively to changes happening in the domain of practice, and the teachers might adjust in the domain of practice as a result.

Especially when it came to the change sequences for teachers seeking to change their teaching, like Sven, the change sequence would lose most of the dynamics if the student domain was not there. With the student domain as a part of the model, it becomes visible how Sven makes changes based on some specific students and their reactions, while he also makes changes based on salient outcomes for the class as a whole (Figures 7.5 and 7.6).

The teachers' rationales for making changes are related to their students. I showed in the analysis in Section 7.1 that the teachers mostly refer to the outcome of tasks when they describe tasks they want to use in their mathematics classrooms. I further argue in 7.1.8 that what the teachers ask for in tasks can be summarized into three issues that are all related to their students, i.e., work, motivation, and understanding. In the same section I point out three aspects of teacher skills that can limit which tasks the teachers will use, being didactics, communication, and mathematics. Some of these aspects are expressed through referring to the students, like when Sven talks about classroom discussions not being possible with this group of students. If the aim is to capture the teachers' rationales for making changes, I therefore argue that the student domain needs to be part of the change model. When placing the student domain, I chose to place it in the middle of the model, as this is where the student is situated from the teachers' perspective. That is, the student is always in the center.

Goldsmith et al. (2014) concluded in their review on teachers' learning that there was a need for varied types of research studies, because of the complexity of the field and the need for a deeper understanding. They explained this by how the teachers' perceptions might not align with those of mathematics education researchers. The research study presented in this dissertation provides an insight into what teachers view as important, and how their focus might differ from mathematics education researchers', even though we have the same goal, being students learning mathematics.

I have in Chapter 8 discussed my findings with respect to other theoretical perspectives and I will summarize and conclude these findings in the next chapter.

9 Conclusion and Implications

The aim of this research project was to focus on the teachers' perspectives when it comes to mathematical tasks, and changes teachers make in their everyday classroom. To address these issues, two research questions were formulated:

1. What characterizes teachers' descriptions of mathematical tasks they want to use in their classroom?
2. What rationales do teachers express when they initiate changes to mathematical tasks during the collaboration?

The answers to these two research questions are clearly intertwined, because the teachers' descriptions of mathematical tasks are linked to their rationales for initiating changes. According to the findings in this research project, teachers describe mathematical tasks they want to use in their classrooms differently than researchers and didacticians do. Instead of using task characteristics like open tasks, rich tasks and so on, the teachers mostly describe tasks by the desired outcome of the tasks. About 70 % of the teachers' statements are not about specific characteristics of mathematical tasks but rather address the outcome of the task. So, when the teachers ask for tasks, they mostly talk about what they want to achieve in the classroom through using a task, rather than specifying characteristics of the task itself.

Looking further into what type of outcomes the teachers describe they want to achieve from the tasks, I found that they were all related to their students, and could be summarized into three issues:

- Work
- Motivation
- Understanding

The teachers would ask for tasks that would help them accomplish didactical issues they were struggling with in their classrooms (Sub-Section 7.1.8). For some of them, the first point was to get their students to work in the mathematics classroom (Hanna and Sven, 7.1.8), others would focus on motivating their students (Thomas, 7.1.8), and they would also talk about tasks that could help their students develop an understanding of the topic (Hanna and Sven, 7.1.8).

Some of the teachers in this project wanted tasks that could help them with all of these issues (Hanna and Sven, 7.1.8), others were only worried about some of them (Thomas, 7.1.8), while one teacher was content with all issues and did not ask for specific changes (Roger 7.1.8).

When designing and implementing the tasks together with the teachers, other aspects surfaced that could hinder certain types of mathematical tasks. I have summarized this into three teacher aspects:

- Didactics
- Communication
- Mathematics

These three aspects of teacher skills could restrict what type of tasks we might design and use in the classroom. For instance, the teachers who were not as comfortable with communicating with this specific student group, asked for more text in the tasks. They would also avoid tasks that required class discussions. I refer to these aspects skills because it is not just about a lack of knowledge or self-efficacy, rather it is about developing certain skills in various contexts that can differ across teachers and classrooms.

The three student issues the teachers want to resolve in combination with the three teacher aspects that might cause the teachers to reject certain tasks, are also the rationales behind the teacher-initiated change processes. The teachers describe mathematical tasks that might help them resolve and change issues in their classrooms. They want mathematical tasks that will help them get their students to work, to be more motivated or to gain a better understanding. These are teachers' rationales for initiating changes. However, some of the teachers are also making changes to improve one or more of the teacher aspects they might struggle with. This is evident when analyzing the change processes through the Interconnected Model of Clarke and Hollingsworth (2002), like I have done in Section 7.2. The arrows indicating change sequences go between all five domains, including the personal domain, when the teachers work to improve themselves as teachers and not just their teaching. Likewise, when a teacher aims to improve her teaching, but is not explicitly working on personal development, the arrows indicating changes go between the domain of practice, the domain of consequences and the student domain. There are seldom arrows including the personal domain.

Through this research project, I have shown that the Interconnected Model of Clarke and Hollingsworth (2002) also can be useful for analyzing change processes from the teachers' perspective in the classroom when designing and implementing mathematical tasks. However, such an analysis requires an expansion of the Interconnected Model, to include the student domain. This is because the students are an important reference point for the teachers in all changes they make in the classroom. The teachers' focus on the students in this research project, might be especially strong due to the Norwegian cultural context the teachers are working within. As described in Section 2.3, the national political-cultural emphasis in Norway is on education for all and equality of opportunity in education. It is therefore expected of the teachers to focus on the students and each individual's learning.

9.1 Strengths and Limitations

As previously described in Section 4.1, this research has been conducted within an interpretive research paradigm and the results must be critically evaluated from this perspective. I have in Section 4.6 elaborated on various measures undertaken throughout the research process to ensure trustworthiness to the results and conclusions. Within the interpretive research paradigm, there are also epistemological and ontological assumptions as I have explained in Section 4.1. I have positioned this research within a constructivist epistemology and a subtle realist ontology, meaning I recognize that all knowledge is a human construction, but also acknowledges that there exist independent and knowable phenomena (Blaikie, 2007).

The data generated through this research come from collaborations with four teachers, and as described in the conclusion, there are differences in what the teachers ask for in mathematical tasks and the rationales behind the changes they initiate. The results must be viewed in the context the teachers work in, and with respect to who the teachers are. Conducting similar research with mathematics teachers working in primary school, or even at other vocational schools, might result in another distribution of the codes and could yield new codes. However, despite the variation in focus on the type of characteristics of tasks the teachers request, there is a clear inclination to describe tasks by the desired outcome. This is a result that is consistent among all teachers in the research project, and it resonates with findings from previous research on teachers' perspectives (Brown & McIntyre, 1993). As a result, this is a finding that I claim is generalizable in how teachers describe characteristics of mathematical tasks.

Lincoln and Guba (1986) argue how inquiry is value-bound, entailing that in research projects as the one reported in this dissertation, there is value-pluralism. Both the researchers' values and the teachers' values will influence the research and the results. It is therefore the researcher's responsibility to present all participants' values as truthfully and conscientious as possible. Since the researcher's values will influence the results, I have reported on the theoretical positioning of this research project in Section 3.1, and the methodological positioning in Section 4.1. This gives the reader a possibility to view the analysis, discussion, and findings in light of the values of the researcher conducting the research.

However, it is not enough to position the researcher's values, but also to present the participants' values. Lincoln and Guba (1986) use the concept fairness and define it as: "a balanced view that presents all constructions and the values that undergird them" (p. 79). The research reported herein, is the result of a research project designed to fulfill several of the criteria of fairness. For instance, to balance the power between researcher and participants, the researcher did not go into this collaboration as an expert on mathematical tasks but asked the teachers what kind of tasks they wanted. The collaboration with the teachers yielded many hours of data, but the conversations were coded in vivo to keep the teachers' wordings for as long as possible. The process of grouping the codes into categories and dimensions has been described, and the interpretations are explained, this is to represent the teachers' values as fairly as possible. When analyzing the change processes and the teachers' rationales for making the changes, I added a fifth domain to the Interconnected Model developed by Clarke and Hollingsworth (2002). This was a result of fairness to the teachers' values and perspectives because the students were always in the center of their decisions, and I therefore needed to add a student domain to the model.

9.2 Implications

This research project has identified how teachers are inclined to talk about the outcome of tasks, rather than characterizing the tasks, and this can be valuable for those that focus on teachers' professional development and the research community to account for. When researchers address teachers, it might be helpful to use a discourse that resonates with how the teachers think and work. That is, by expressing what tasks can accomplish before following up with characteristics might help getting teachers attention.

Another aspect that researchers need to consider when designing tasks for teachers, is how the teachers focus on their students and how a task might help them resolve didactical issues in the classroom. While learning and understanding are important to the teachers, so is motivating the students and making them work hard. Preferably, the teachers look for tasks that can help them fulfill all these aspects.

Task designers also need to consider if there are teacher aspects that might hinder implementation. These aspects may not only be related to knowledge and self-efficacy, but to where the teachers are when it comes to developing skills together with a certain group of students. While these aspects might cause teachers to reject certain tasks, it is also possible to view this as a window of opportunity to design tasks that might help teachers further develop these aspects.

The findings in this research project provides additional knowledge regarding teacher change. Through using the Interconnected Model developed by Clarke and Hollingsworth (2002), I have provided evidence that the model is useful also to analyze the process of design, implementation, and evaluation of mathematical tasks. However, this type of analysis required a fifth domain in the model, which I have named the student domain.

9.3 Further Research

As described in the introduction, there has been a shift in research over the last couple of decades, implying that more researchers focus on the mathematics teachers in the classrooms. Based on a review of the literature on tasks, de Araujo and Singletary (2011) concluded that teachers' perspective on tasks were lacking. Since then, several researchers have worked with mathematics teachers as partners in tasks design, as reported in a review article by Jones and Pepin (2016). However, de Araujo and Singletary (2011) also argued that "carefully listening to teachers describe their practice is an excellent place to start" (p. 1214). This research project contributes to this field, but there is still a need for additional knowledge concerning the teachers' perspective. Knowing how teachers and researchers use various discourses when talking about tasks, this should be further investigated and explored to see if a change in discourse by researchers might impact implementation in the classrooms. The findings in this research project also provides increased understanding regarding the teachers' rationales for initiating changes, but there is still a need for additional research addressing this perspective. Knowing more about teacher-initiated changes is likely to be valuable, and accounting for the teachers' perspectives and rationales when designing tasks or professional development programs, may result in more successful tasks and programs.

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