Using Primary and Secondary Market Movements to Construct an Optimal Time-Series Momentum Strategy

A Replication Study

NASIR MOHAMUD YUSUF AND HAVIN SAMIR

SUPERVISOR
Valeriy Ivanovich Zakamulin

University of Agder, 2021
School of Business and Law
Department of Economics and Finance

Abstract

Time series momentum (TSM) strategies is a topic that has been analyzed in numerous academic journals; often the results of the studies imply that TSM outperforms the benchmark (buy-and-hold strategy). Nevertheless, most of the research covers primary trends as proposed by the Dow Theory. We implement a new TSM strategy that in addition to the primary trends, also considers the secondary trends in the Dow Theory. This TSM strategy is then applied to various look-back periods(speed), including predetermined static speeds, and dynamic speeds. The latter in which we use back-testing to find optimal speeds for different market states (bull, bear, correction, and rebound), and implement the speeds in subsequent periods with forward-testing. The TSM strategies are applied on international market indices, and the Sharpe ratio for each strategy reveals that the dynamic speed strategies dominate in terms of performance.

Table of Contents

Introduction

In this thesis, we seek to replicate the results and findings reported in the paper "Momentum Turning Points" by the authors Garg, Goulding, Harvey, and Mazzoleni (2019). In the DOW Theory on stock price movement, the first of the basic six tenets outlines how the market has three movements: a main primary movement, a medium secondary one, and finally, a short minor movement. The primary movement lasts anywhere from a year to several years, the secondary a couple of days to a dozen months, and the short swing from a few hours to a month. Most of the research in trend-following strategies relies primarily on exploiting the primary movement as the foundation of its research. However, Garg et al. (2019) seek to use a combination of the primary and secondary market movements.

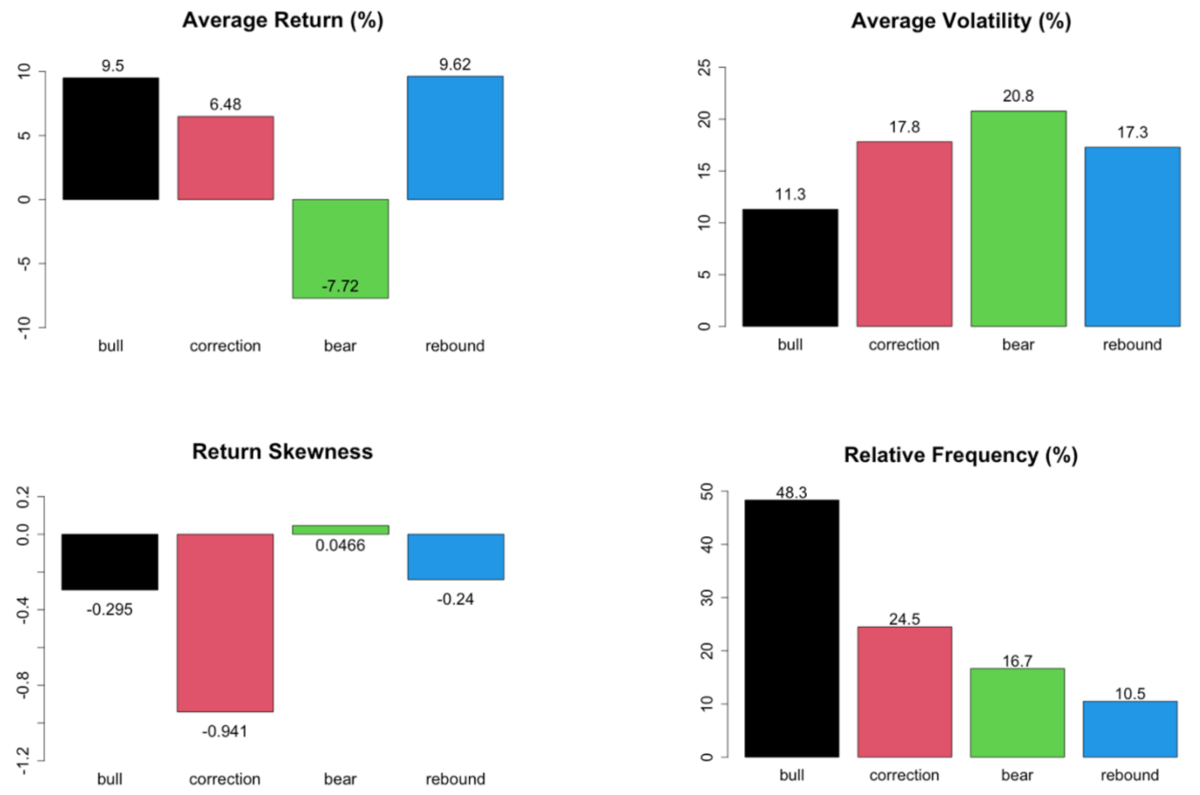
In the last couple of decades, there has been an increased interest in trend-following strategies both among professional investors as well as academics. Numerous papers have been published in academic journals and they often find that trend-following strategies simply outperform buy-and-hold strategies. In the literature, we are presented with strong proof of expected returns varying over time (Fama and French, 1988, and Cochrane, 2011). We are also provided the premise that trends that persist over time are supported by the research; asset returns measured over the recent past, usually a year or so, are positively correlated with the future returns of an asset (Jegadeesh and Titman, 1993; 2001, Asness, 1994, Conrad and Kaul, 1998, Lee and Swaminathan, 2000, and Gutierrez and Kelley, 2008). Moskowitz, Ooi, and Pederson (2012) show and provide evidence that TSM strategies can exploit these trends by having a lookback period of 12 months and finding persistence in returns that reverses over longer time horizons.

However, there has been a lack of research into what happens when a trend that has persisted over a period breaks, meaning what happens when the trend which the time-series strategy was relying on breaks. These so-called "momentum turning points" often occur before a change in a trend, often a reversal, in either an uptrend or downtrend situation or they can simply be noise, which in turn can lead to harmful bets which may be costly. Such bets can occur because we simply cannot observe the sign of the expected returns, and by extension, we cannot observe the persistence of the trend. Instead, time series momentum (TSM) strategies mainly rely on actual realized returns, which often reflect mixtures of both trend and noise. TSM strategies are based on the sign of the trailing return over a set lookback period; if the sign is positive, the TSM strategy would take a long position, whereas if the sign is negative, it would take a short position. The sign of the trailing return is referred to as the momentum signal.

In this thesis, we implement TSM strategies with various look-back periods (also referred to as *speed*) in which we examine a trend. The speed of the momentum signal tries to find a balance between reducing the noise and being able to react quickly if momentum changes. This tension manifests itself in various ways; either the momentum signal tries to reduce the influence of noise by having a long lookback period of a year but is then slow to react to new changes (i.e.: turning points) or it gets drowned out by the noise and reacts erroneously if it has a short lookback period of a month for example. Herein comes the main aspect of what this thesis is trying to solve; the trade-off between being able to react quickly to momentum breaks while avoiding being influenced by unnecessary noise. Respectively the two error types are failing to react to turning points, a Type II error, and reacting to noise when no turning point has occurred, a Type I error. Type II error usually occurs when a long lookback period is considered, and Type I error usually occurs when a short lookback period is considered. To differentiate between trends with long and short lookback periods we create two TSM strategies: SLOW and FAST, where the former has a 12-month lookback period and the latter 1 month.

In this thesis, the concord or discord between SLOW and FAST is used to differentiate trends and turning points. If a bet is placed when the two strategies disagree, that is when one strategy takes a long position while another takes a short position, this usually indicates a turning point. On the other hand, when both strategies take a short(long) position that is very likely indicating that the market is in a downtrend(uptrend).

Figure 1: U.S Stock Market States



Notes: This figure reports the 1) conditional average, 2) the conditional volatility, 3) the conditional skewness of monthly aggregate U.S stock market returns, considering the market state in the prior month, over the last 50-year evaluation period. In addition, the relative frequency of each market state is also reported. A month ending at date t is classified as Bull if both the trailing 12- month return, $r_{t-12,t}$, and the trailing 1-month return, $r_{t-1,t}$, are nonnegative. A month is classified as a Correction if $r_{t-12,t} \geq 0$ but $r_{t-1,t} < 0$; as Bear if $r_{t-12,t} < 0$ and $r_{t-1,t} < 0$; and as Rebound if $r_{t-12,t} < 0$ but $r_{t-12,t} \geq 0$. Market returns are U.S excess value-weighted factor returns (Mkt-RF) from the Kenneth French Data Library.

The thesis defines four different market states corresponding to the different combinations of signals. When both the SLOW and FAST signals agree on the direction of a trend, we refer to them as a “Bull” or “Bear” state, depending on whether the agreement between the signals is to take a long or short position. These labels refer to phases of uptrend and downtrend and are considered as the primary movements of trends as described by the DOW Theory. However, when the signals disagree, we call it a “Correction” state if SLOW momentum indicates a long position and a “Rebound” if it indicates a short position. Likewise, these loosely map to the possible occurrence of turning points from either an uptrend to a downtrend or vice-versa and are considered to be secondary movements.

In Figure 1, over a 50-year period of the US stock market, its conditional behavior of the average, volatility, and skewness of returns in months following each of the four market states, are summarized. Bull states, which are the most frequent market states, are followed by relatively high returns and low volatility, 9.5% and 11.3% respectively, as expected. Bear states, on the other hand, are followed by negative average returns and the highest volatility -7-71% and 20.8% respectively, of the four market states. Correction states are followed by gradually worsening average returns, increased volatility over time, 6.48% and 17.8% respectively, and they are mostly lead-ups to a Bear state. Rebounds in a similar fashion are followed by average returns, of 9.62%, and skewness of -0.24, similar to Bull phases but with the caveat of having increased volatility of 17.3%, and tends to lead to a Bull phase. Bull has a skewness of -0.295, making Bull and Rebound state slightly negatively skewed. Correction is moderately negatively skewed with skewness of -0.941. Bear is the only market state that has a slightly positive skew of 0.0466. Bull accounts for 48,3% of the frequency, while Bear accounts for 24.5%, and finally, Rebound and Correction in tandem account for over ⅓ of the remaining frequency. That means, in 35% of the cases in the examined 50-year period of the

U.S stock market, there were momentum turning points for 35% of it. It is of paramount importance to be able to tap into those periods and have an effective strategy in place.

Following the original paper, we have set out to replicate the same results and findings. We explore two main strategies: a *static* speed strategy and a *dynamic* speed strategy. With the former, we investigate static intermediate speeds with fixed proportion blends of FAST and SLOW strategies. We examine return and volatility risk characteristics. Then, we show drivers of market beta and alpha of the TSM strategies of various speeds. Finally, we examine the tail behavior of these various TSM strategies. As for the latter, we examine the merits of dynamic strategies whose speeds vary over months depending on observed market states. This is done as a way to solve the uncertainties associated with Correction and Rebound states. Thus, instead of using the same speed during one of these states, we find that a change of speed following each state proves to be more effective. We use an estimation window, where we examine the optimal speeds for these market states and implement them in a subsequent, evaluation window. Finally, we set out to test the external validity of these results by testing the strategies on the international market (outside of the U.S stock market).

The structure of this thesis is as follows. This section which we titled *Introduction* is where we present the topic and main concepts and aims of the thesis. We follow this with a *Literature Review* where we examine the current field of research on TSM strategies, and we try to relate that to our thesis. These two sections lay a foundation for the reader to understand the following sections. We present the data we have used in each table and where it is extracted from in section *Data*. In the section *Static and Dynamic Speed Strategies* we explore the two main strategies in this thesis. First, in each subsection, we explain the concept of TSM strategies in association with momentum signals, and then we explain the characterization of speed in the creation of SLOW and FAST TSM strategies. In the subsection *Static Speed Selection* we create various intermediate-speed strategies by combining SLOW and FAST, and in *Performance of Static Speed Strategies,* we analyze the performance of each strategy by comparing the Average Return, Volatility, Sharpe Ratio, Market Timing, Skewness, and Maximum Drawdown. In subsection *Return and Risk,* we look at the cycle-conditional average returns and the variance of returns for each intermediate-speed strategy. In subsection *Marker Timing* we provide a novel decomposition of alpha and beta, into a market timing and volatility timing component, and a static, market timing, and volatility timing component respectively. We follow this by examining the role of each component, in alpha and beta. In subsection *Tail Behaviour* we examine the Cycle-Conditional Market Return Distributions and delve into the topic of skewness. In subsection *Dynamic Speed Selection* we create an optimal dynamic-speed strategy, apply the strategy in windows of various lengths, and analyze the efficiency of strategy by looking at the ratio of the realized Sharpe ratio to the ex-post Sharpe ratio. In section *International Markets* we apply the two main strategies (static-speed and dynamic-speed strategies) on international market indices, and find that the results are valid in the international market as well. Finally, in the section Conclusion, we provide a conclusion where we sum up the thesis to reiterate the main results and findings.

Literature Review

TSM trading is a style of trading based on two different premises. The first being that expected returns vary across time, which is strongly supported by empirical evidence in the literature (Cochrane, 2011), and the second that there is a positive covariance in asset returns from a period to another (Moskowitz et al., 2012). TSM strategies make use of these facts and give investors profitable returns which are largely dependent on how well that strategy reacts to the market, that is whether it overestimates or underestimates in the most profitable fashion (Xue-Zhong, Li, and Wang, 2015). It has been tested across many different asset classes as well as across different asset groups (Moskowitz et al. 2012). Signs have some persistence over a period, which is referred to as a trend. We use an asset´s historical returns to predict its future performance and in this way construct a so-called TSM strategy (Moskowitz et al., 2012).

TSM strategies are not without their detractors, Huang, Li, Wang, & Zhou (2019) argue that the predictability of a 12-month TSM as captured by a forecasting regression does not appear to be statistically significant and on that note, the profitability of a diversified TSM portfolio with 1-year lookback is entirely due to its static tilt (i.e, its net long positions). We argue, however, that despite a net positive static tilt, TSM portfolios need not have positive betas to the underlying asset. Applying the novel decomposition of TSM beta and alpha, proposed by Garg et al. (2019) we can explain the role of innate volatility which may explain the disparity between its static tilt and beta. Also, as Huang et al. (2019) point out, it would be worthwhile to try and see different horizons than 12-months which this thesis does.

In addition, a couple of researchers of momentum strategies Goyal and Jegadeesh (2017) argue that the net dollar exposure of a momentum strategy is a key determinant to its profitability. In other words, adding that same net dollar exposure to for example a cross-sectional strategy would give similar profits. However, this argument does not apply in the case of a single asset like ours in which cross-sectional (CS) strategies simply are either not defined or trivial. There is much similarity between this thesis's design and that of one found in CS literature. For example, market states are also employed by Cooper, Gutierrez, and Hameed (2004), Daniel, Jagannathan, and Kim (2012), and Daniel and Moskowitz (2016). However, there are differences, for example, Cooper et al. (2004) use a slow three-year trailing return to define their market states while we use SLOW and FAST trailing return signals with a trailing return of 12 and 1 month respectively.

Liu and Zhang (2008) have shown that in CS momentum, setting winners load temporarily on the growth rate of industrial production and that macroeconomic risk profits can explain more than half of CS momentum profits, as opposed to TSM strategies where the specific strategy employed determines the amount of profit gained as it can be adjusted constantly to react to current market states (Xue-Zhong et al., 2015).

In the literature, there is also a different type of investment strategy often mentioned alongside momentum trade strategies which is the moving average (MA) strategies. MA strategies tend to be a bit more complicated and involved than a momentum-following strategy. MA strategies are price-based strategies (aka. technical strategies) which similarly to trend following strategies seek to outperform the market (the simple buy-and-hold strategy, based on the efficient market hypothesis) on a risk-adjusted basis (Killagen, 2012). While there are several variations of such strategies, the most basic and widely used is the simple MA strategy wherein one buys security/asset once it starts to trade above the average closing prices from a specified lookback period of days and or months, and then sells the security once it falls below that same average.

MA and TSM strategies are of course closely related as the returns generated by either method are such that it frequently has a correlation that is an excess of 0.8 (Marshall, Nguyen, Visanalatochi, 2017). In the paper by Marshall et al. (2017), they found that there are some differences between the two methods. The relation between the TSM strategy and MA rules is that TSM rule entry and exit signals are generated when the MA changes direction. This means that TSM takes a longer time to react than MA rules, to give either buy or sell signals. This is obvious because a price change is more likely to result in price moving above (or below) the MA. A price change will cause MA to issue an entry or exit signal, rather than to change its direction completely (like it would for a TSM signal). Unlike CS momentum, which generates its buy and sell signals based on the return of a security relative to other securities in its sector, neither MA nor TSM strategies are susceptible to a crash risk since both exit long positions before sustained market downturns.

The literature has been pretty conclusive that both MA strategies and TSM strategies had significantly better risk-adjusted performance than buy-and-hold strategies. In the case of MA strategy, a study by Killgalen (2012) showed that across three different asset classes (equities, currencies, and commodities), MA strategy outperformed the buy-and-hold strategy consistently regardless of whether the study used 7, 9, 11- or 13-month variables to calculate the MA. For TSM strategies, a study by Antonacci (2013) showed the same results with 12-month absolute momentum improving return and lowering risk. Our thesis differs from both scenarios in that instead of just examining the primary movements of the market as outlined by the DOW Theory, we are looking at both the primary and secondary movement, so we have a lookback period of 12 months on one hand and a lookback period of 1 month on the other and we use the mix of these two to synthesize TSM strategies.

This thesis is also linked to the literature by Moreira and Muir (2017) regarding volatility-managed portfolios. The empirical regularity of negative correlation between stock market returns and volatility underscores the association between TSM and volatility-managed strategies. Thus, we will examine if market or volatility timing plays a larger role in driving positive returns. It's also important to note regarding market timing, that it is essential to be constantly receiving information on the state of the market to be able to react quickly enough when change occurs. Both Da et al. (2014) and Lim et al. (2016) observe that TSM returns are highest for stocks that have a continuous stream of information projected to the investor.

Finally, TSM literature has produced many recent studies with a focus on the application of TSM to aggregate factors. For example, a recent paper by Ehsani, Linnainmaa (2019) shows strong evidence of TSM across equity factors that appear to overtake and absorb the CS momentum risk factor, and following that it might be worth more since the TSM factor adds value to many different investment strategies (Gupta and Kelly, 2019).

Data

Given that we are replicating the results reported in Momentum Turning Points by Garg et al. (2019), we use the same data. The data used to compute and create Figure 1 and 2, and Table 1 through 7, is the excess equity market return in the US. This is secondary data which is excerpted from the Kenneth R. French Data Library. The data is composed of excess returns (rm – rf) from the dataset Fama/French 3 Factors. This includes all firms in NYSE, AMEX, and NASDAQ. Our unit of analysis will therefore be monthly excess equity market returns. We apply the strategies to an international market for external validity. Thus, for Table 7, we have used returns of country indices for various countries including Canada, Norway, Australia, France, Germany, Italy, Japan, and Spain. The index returns are from the Kenneth R. French Data Library, and the risk-free rate is from the central bank of the respective country. The data covers the period from February 1980 to December 2018. We have decided to use this period for the sake of consistency since the study we are replicating also used this period.

Static and Dynamic Speed Strategies

In this section, we introduce the Time Series Momentum (TSM) Strategy and explain the motivation for using this strategy. We explore primary and secondary trends and attribute them to various market states. With market states in mind, we create a TSM strategy that considers both the primary and secondary trends. We create TSM strategies that capture both trends by combining long and short lookback periods (speeds). From this, we first apply various predetermined static speeds and analyze the performance of each strategy. Specifically, we examine Return and Risk, Market Timing, and Tail Behavior. Then we apply dynamic speeds; we use back-testing to find the optimal speeds during each market state in an estimation window, and then apply these speeds, by forward-testing, in their respective market states in the subsequent period, in an evaluation window.

Time Series Momentum Strategy

The premise is that there seem to be trends in returns over time, meaning that it would be possible to predict an uptrend or downtrend based on historical returns. If there has been an uptrend(downtrend) the past year, it would be reasonable to assume that this uptrend(downtrend) will continue. The TSM strategy is based on this. We create a momentum signal,

$$\mathrm{MOM_N} = \sum_{i=1}^{N} r_i \, . \tag{1}$$

This shows the sum of returns for *N* months. If N = 12, this would imply that the lookback period of the momentum signal is 12 months. This means that when attempting to predict an uptrend or downtrend, we determine whether the sum of returns for the last 12 months is positive or negative. If the sum of the MOM_{12} strategy is positive(negative), this would mean that it's reasonable to assume that the returns would continue to stay positive(negative) in the subsequent period. In other words, we look for the momentum signal,

$$MOM_N : \begin{cases} > \ 0 \ \rightarrow Buy \\ \leq \ 0 \ \rightarrow Sell \end{cases} , \tag{2}$$

and accordingly determine to either buy if the sign is positive or to sell if the sign is negative. This means that there is always either an uptrend or downtrend, which we refer to as a Bull or Bear market, respectively. These market states are primary trends and are often seen when the market is in either market state for a longer period; from 12 months to several years.

Momentum Turning Points

The TSM strategy mainly considers primary trends and attempts to predict a Bull or Bear market. TSM strategies and other trend-following strategies in the field have mainly operated like so, with a focus on primary trends. Nevertheless, during a Bull or Bear market, there will be periods of times where returns "break" from the trend and turn negative or positive, respectively. These "breaks" or changes in trends are referred to as momentum turning points. Momentum turning points have two possible outcomes: (1) they appear to constitute a change in a trend or (2) they are simply noise. The former signifies that if in a Bull market, a momentum turning point will act as a signal and imply that the current trend is breaking. The latter signals noise and imply that the current trend will continue into the subsequent period. Lack of certainty as to which of the outcomes is the most likely one makes the TSM strategy likely to place bad bets and can be very costly. This is due to the TSM strategy (1) being prone to be influenced by noise and (2) not being able to react fast (if there is a change in a trend).

In trying to solve the problems associated with TSM strategy and momentum turning points, we first look at the first problem: *prone to be influenced by noise*. We implement a lookback period of 12 months. The advantage of a long lookback period is that the lookback period is long enough to not be influenced by noise. Meaning that if we look at the sum of returns over a year, it is likely that this trend will continue into the subsequent period. Therefore, if we detect a momentum turning point, we are somewhat certain that this turning point signals a change in a trend. In other words, a long lookback period will reduce any noise in returns. The downside, however, is that the strategy will be slow to react to changes in trends. This strategy will therefore hereby be referred to as SLOW.

The second problem we are dealing with, and an issue which SLOW was not able to account for, is: *not being able to react fast*. Accordingly, we shorten the lookback period to 1 month, as it will allow for faster detection of changes in returns. The downside is that since we are dealing with such a short lookback period a momentum turning point might as well be noise, which the TSM strategy is falsely reacting to. This means that a shorter lookback period reacts fast but is more influenced by noise. This strategy will therefore hereby be referred to as FAST.

When we are dealing with a FAST strategy, we are simultaneously dealing with secondary trends. Secondary trends are smaller and more short-term trends within the primary trends. That is, in a year of Bull/Bear market, we will observe periods, perhaps months, of returns changing signs. In a Bull market, when the price drops and the MOM sign is negative, the market is correcting itself, and these states are referred to as Correction. Whereas, when in a Bear market, prices go up and the MOM sign is positive, there is a Rebound. In other words, we are talking about short-term trends within a Bull or Bear market.

Characterizing speed

The framework we use for SLOW and FAST is:

$$w_{SLOW,t} := \begin{cases} +1 & if\ r_{t-12,t} \geq 0 \\ -1 & if\ r_{t-12,t} < 0 \end{cases}, \tag{3}$$

$$w_{FAST,t} := \begin{cases} +1 & if\ r_{t-1,t} \geq 0 \\ -1 & if\ r_{t-1,t} < 0 \end{cases}, \tag{4}$$

where trailing return,

$$r_{t-N,t} = \sum_{i=1}^{N} r_i \,. \tag{5}$$

Trailing return is, at time *t*, defined as the sum of returns for period *t* – *N*. For N = 12, this would give the trailing 12-month return, $r_{t-12,t} = \sum_{i=1}^{12} r_i$, and for N = 1, this would give the trailing 1-month return, $r_{t-1,t} = \sum_{i=1}^{1} r_i$. The purpose of the trailing return is to serve as a momentum signal as defined in (1). $w_{SLOW,t}$ is the weight of the SLOW strategy at date *t*. If the trailing 12-month return, $r_{t-12,\,t}$ is positive, then the strategy will take a long position (+1), whereas if it is negative, it would take a short position (-1). The same is true for the weight of the FAST strategy, $w_{FAST,t}$. It will take a long (+1) position if the trailing 1-month return is nonnegative and a short (-1) position if it is negative.

We specifically use a 12-month lookback period and not longer for SLOW because a 12-months horizon is the standard lookback period in the TSM literature and profitability associated with this lookback horizon appears to be statistically significant (Moskowitz et al, 2019). Additionally, the correlation between a 24-months and 12-months TSM portfolio is greater than 0.6 (Garg et al., 2019). The reason we a use 1-month horizon for FAST is first that since we are dealing with monthly data, this would be the shortest time horizon. Second, a 3-month TSM portfolio showed a greater correlation with a 12-month momentum than with a 1-month momentum (Garg et al., 2019).

In this thesis, we seek to implement a TSM strategy which not only recognizes the primary trends but also takes into account the secondary trends. Thus, we need a TSM strategy that is fast enough to react to a momentum turning point and simultaneously is not influenced by noise. If we only consider the SLOW or FAST strategy, we are prone to get the two following errors:

Type I error - reacting to noise when a turning point has not occurred (FAST)

Type II error - failing to react to a turning point when it occurs (SLOW)

To detect a turning point in trend from noise we will explore the four combinations of agreements and disagreements between the SLOW and FAST momentum strategies; Bull, Bear, Rebound, and Correction. The four combinations of SLOW and FAST and the market states are displayed in Table 1.

Table 1: Market States as Combinations of SLOW and FAST

SLOW / FAST	$if\ r_{t-12,t} > 0$ $w_{SLOW,t} = (+1)$	$if\ r_{t-12,t} < 0$ $w_{SLOW,t} = (-1)$
$if\ r_{t-1,t} > 0$ $w_{FAST,t} = (+1)$	**BULL**	**REBOUND**
$if\ r_{t-1,t} < 0$ $w_{FAST,t} = (-1)$	**CORRECTION**	**BEAR**

Notes: This table shows the four possible combinations of market states depending on the SLOW and FAST momentum strategies. SLOW and FAST are defined as follows. If the trailing 12-month return, $\boldsymbol{r_{t-12,t} > 0}$, is nonnegative, then the weight of the SLOW strategy, $w_{SLOW,t}$, takes a long (+1) position, and otherwise, it takes a short (-1) position. If the trailing 1-month return, $\boldsymbol{r_{t-1,t} > 0}$, is nonnegative, then the weight of the FAST strategy, $w_{FAST,t}$ takes a long position (+1), and otherwise, it takes a short (-1) position. The market states are defined as follows. Bull: $w_{SLOW,t} = w_{FAST,t} = +1$, Bear: $w_{SLOW,t} = w_{FAST,t} = -1$, Correction: $w_{SLOW,t} = +1\ and\ w_{FAST,t} = -1$, and Rebound: $w_{SLOW,t} = -1\ and\ w_{FAST,t} = +1$.

We presume that when the market is in a Bull or Bear state, that the market is in an uptrend or downtrend respectively. In this case, both FAST and SLOW strategies agree; the trailing 12-month return, $\boldsymbol{r_{t-12,t}}$, and the trailing 1-month return, $\boldsymbol{r_{t-1,t}}$, are both nonnegative (negative), and thus both SLOW and FAST strategies will take a long position (+1) (short position (-1)). This suggests that that the market is in a Bull (Bear) state and that the trend will stay somewhat consistent. A Rebound state signifies a change in trend from negative to positive and a possible lead up to a Bull state. This is suggested by the weights of SLOW and FAST; the trailing 12-month return, $\boldsymbol{r_{t-12,t}}$, is negative, and the trailing 1-month return, $\boldsymbol{r_{t-1,t}}$, is nonnegative. This suggests that the primary movement, a downtrend, is subject to change, and this is indicated through the secondary movements. The opposite is true for a Correction state; the trailing 12-month return, $\boldsymbol{r_{t-12,t}}$, is nonnegative, and the trailing 1-month return, $\boldsymbol{r_{t-1,t}}$, is negative. This suggests that the primary movement is an uptrend and that the secondary trend indicates a change in this trend. This means that the uptrend can change and lead up to a Bear state. Rebound and Correction states are therefore useful when detecting noise from a trend.

This also suggests that in Rebound and Correction states, the market is either in a turning point and SLOW has failed to reflect this (Type II error), or the market is still in a trend phase and FAST has falsely reacted to noise (Type I error). A composition of SLOW and FAST strategies, to optimize reaction time and reduce noise is therefore of use.

Static Speed Selection

In trying to avoid the errors occurring, while also considering both the primary and secondary trends, we define a continuum of intermediate-speed strategies with signal speeds between SLOW and FAST,

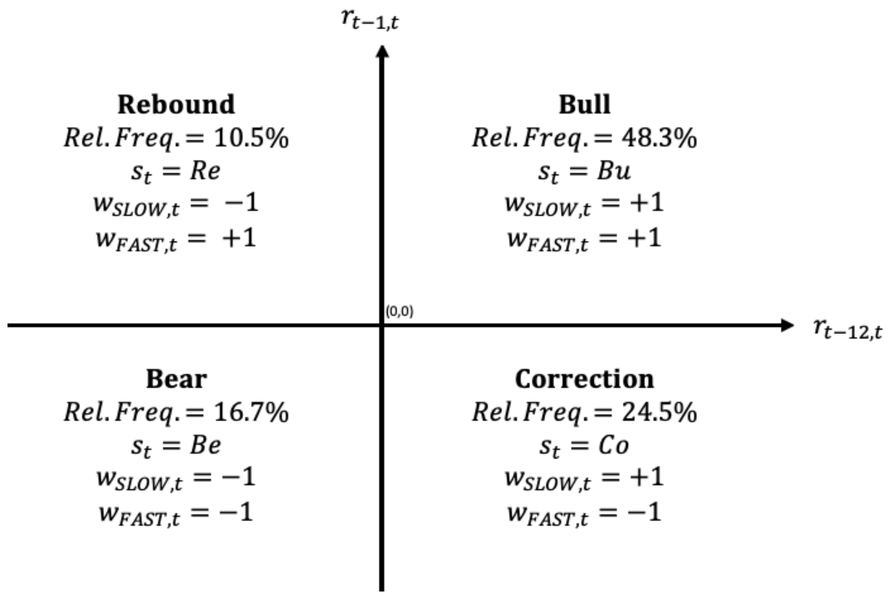
$$w_t(a) := (1-a)w_{SLOW,t} + a\, w_{FAST,t}, \tag{6}$$

$$r_{t+1}(a) := w_t(a)r_{t+1} = (1-a)r_{SLOW,t+1} + a\, r_{FAST,t+1}, \tag{7}$$

where w_t signifies the weights of the strategy at time t, while a is the speed parameter which is a scalar a = [0,1]. The speed represents the lookback period, with 0 being equivalent to a 1-month momentum (or FAST) and 1 being equivalent to 12-months momentum (or SLOW). Thus, we have a strategy whose weight w is determined by the speed a. At time t, for $a = 1$, we have $w_t(1) = w_{FAST,t}$, or in other words, the weight is composed solely of the FAST momentum. Whereas for $a = 0$, we have $w_t(0) = w_{SLOW,t}$, and the weight is composed solely of the SLOW momentum (the weights for SLOW and FAST, $w_{SLOW,t}$ and $w_{FAST,t}$, are defined in (3) and (4). By multiplying the weight strategy (5) with realized returns , r_{t+1}, at time $t+1$ we create a TSM portfolio strategy (6).

This strategy also allows for the creation of intermediate speed strategies. If a = ¼, and SLOW signals a long position (+1) and FAST signals a short position (-1) then we have: w(¼) = ¼ (+1) + ¾ (-1) = -½. Indicating a lower magnitude short position. If a = ½, and we have disagreement between SLOW and FAST, then we have: w(½) = ½ (+1) + ½ (-1) = 0. Indicating that the portfolio is out of the market completely. This contrasts with a TSM strategy that is solely based on trailing returns, for instance, a MOM_6 strategy that signals buy or sell based on the sum of the returns for the respective horizon. This strategy would not include the signal of longer or shorter horizons and potential disagreement between the two. This means that the strategy would not be able to scale down its position and would simply indicate a short or long position without being able to differentiate between turning points and noise (Garg et al., 2019).

Figure 2: the Stock Market States as a Function of Momentum



Notes: This figure shows the market states (Bull, Bear, Correction, and Rebound) as a function of momentum, as well as the relative frequency of each market state in the U.S stock market in a 50-year evaluation period from 1969-01 to 2018-12. For each respective market state, abbreviations are used: $s_t \in \{Bu, Be, Co, Re\}$. The x-axis represents the 12-month trailing return, $r_{t-12,t}$, at date *t*. The y-axis represents the 1-month trailing return, $r_{t-1,t}$, at date *t*. If both axes are positive, then both $w_{SLOW,t}$ and $w_{FAST,t}$ will take a long (+1) position, placing Bull as a market state in the upper-right quadrant of the figure. A month is classified as (1) Bear if both axes are negative, (2) Correction if $r_{t-12,t} > 0$ and $r_{t-1,t} < 0$, and (3) Rebound if $r_{t-12,t} < 0$ and $r_{t-1,t} > 0$. The data is extracted from the Kenneth R. French Library.

Furthermore, a blend of SLOW and FAST momentum will suggest the market state which it is currently inhabiting. As specified earlier the four possible market states are Bull, Bear, Correction and Rebound. In order to uphold the assumption of a sustained trend, we use

s_t to denote the market state at date *t*. That is, we label a month ending at date *t* as Bear if both SLOW and FAST trailing returns are negative (this is depicted in Figure 2, at the lower-left quadrant of the figure). Likewise, we explore the other combinations of agreement and disagreement between SLOW and FAST. These are depicted in Figure 2 in the following order (going clockwise): Rebound, Bull, and Correction. As can be seen in the figure the relative frequency of Bull is 48.3 %, indicating that Bull has been the most common market state (this reflects the risk premium allowed by the U.S Stock Market) during a 50-year period from 1969 to 2018. The bear market has not been very common, and this is reflected by its relative frequency of only 16.8 %. The remaining market states, Correction and Rebound, collectively account for 34.8 %. This means that once every third month we can expect SLOW and FAST momentums to disagree and therefore suggest two different positions. The relatively high frequency of these market states is mainly why we have a high focus on Correction and Rebound. Market states are also important to consider, due to the influence they appear to have on the Sharpe ratio and the skewness. This also opens up the possibility of actively adjusting the speed parameter ("speed timing") based on the market state. This is a challenge that has not been undertaken in the literature.

Performance of Static Speed Strategies

In this part, we start by analyzing the SLOW and FAST strategies, as well as other intermediate-speed strategies, and compare them to the buy-and-hold/market strategy. For each static-speed strategy, we will examine the *Return and Risk, Market Timing,* and *Tail Behavior*. These results will be displayed in Table 2. Next, we will decompose the *Average Returns* and *Variance of Returns,* for each static-speed-strategy, to *Unconditional* and *Cycle-Conditional* decomposition (market state).

Table 2: Performance Summary by Speed

	Market	$a = 0$ SLOW	$a = \frac{1}{4}$	$a = \frac{1}{2}$ MED	$a = \frac{3}{4}$	$a = 1$ FAST
Return and Risk						
Average (%)	5.91	6.46	6.17	5.88	5.59	5.30
Volatility (%)	15.64	15.62	12.72	11.60	12.74	15.66
Sharpe Ratio	0.38	0.41	0.49	0.51	0.44	0.34
Market Timing						
Average Position	1.00	0.46	0.39	0.32	0.25	0.18
Market Beta	1.00	0.15	0.05	-0.04	-0.13	-0.23
Alpha (%)	0.00	5.58	5.85	6.12	6.39	6.66
Alpha t-statistic		2.54	3.24	3.71	3.57	3.07
Tail Behavior						
Skewness	-0.54	-0.43	-0.13	0.02	0.03	0.15
Max. Drawdown (%)	54.36	43.43	37.97	34.43	34.07	44.53
Average/ |Max.DD|	0.11	0.15	0.16	0.17	0.16	0.12

Notes: This table reports the average return, volatility, Sharpe ratio, average position, market beta, alpha, alpha t-statistic, skewness, maximum drawdown, and the ratio of the average return to the absolute value of maximum drawdown for the benchmark (buy-and-hold/Market) strategy and TSM strategies of various speeds ($a \in [0,1]$). At time t, if trailing 12-month returns, $r_{t-12,t} > 0$, then the SLOW strategy takes a long (+1) position in the subsequent period, otherwise, it takes a short (-1) position. At time t, if trailing 1-month return, $r_{t-1,t} > 0$, then the FAST strategy takes a long (+1) position the subsequent period, otherwise, it takes a short (-1) position. The intermediate-speed TSM strategies are formed by combining SLOW and FAST strategies as follows: $r_{t+1}(a) := w_t(a)r_{t+1} = (1-a)r_{SLOW,t+1} + a\, r_{FAST,t+1}$, where r_{t+1} is the U.S market return (Mkt-Rf), extracted from the Kenneth R. French Data Library, and covers 1969-01 to 2018-12.

The "Market" represents a passive investment strategy, where an investor buys in January 1969 and holds, regardless of fluctuations in returns, until December 2018. This serves as a benchmark and will be referred to as the "Market" or the *buy-and-hold* strategy in this thesis. The TSM portfolio strategies of various static speeds and are defined in (6).

Return and Risk: In this subsection, we look at the average return, volatility, and Sharpe ratio of the various strategies reported in Table 2. The average return is defined as the annualized percentage of realized returns to the number of observations,

$$\mu_p = \frac{r_{p,t+1}}{N} * 12 * 100, \tag{8}$$

where $r_{p,t+1}$ is the returns of the respective portfolio at time $t + 1$, and *N* is the number of observations of returns in the respective strategy. The volatility is defined as the square root of the annualized variance. The variance,

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2 , \tag{9}$$

measures how returns are dispersed around the mean. *N* is the number of observations, x_i is the value of the i^{th} element (data point), and μ is the mean/average return. The annualized volatility in percentage is defined as

$$\sigma_p = \left(\sqrt{\sigma^2 * 12}\right) * 100. \tag{10}$$

σ_p is the volatility of the portfolio *p*. The volatility captures variations of returns over time and the standard deviation from the expectation. It is therefore an appropriate statistical measure to use when assessing risk in an investment strategy. High volatility represents high fluctuations in returns or remarkable changes in returns. Low volatility implies a rather stable movement in returns and is associated with low risk. There are, however, portfolios and stocks that are highly volatile but also have very high average returns. Therefore, to accurately analyze the performance of a strategy, it´s not sufficient to choose the strategy with the highest average return or the lowest volatility. For this reason, we use the ex-post Sharpe ratio. The Sharpe Ratio measures the historic average differential return per unit of historic variability of the differential return (Sharpe, 1994). This means that the average return is adjusted according to the volatility it has. We define the Sharpe ratio as

$$SR_p = \frac{\mu_p}{\sigma_p} , \tag{11}$$

where μ_p is the annualized average return, and σ_p is the annualized volatility of portfolio *p*. The Sharpe ratio is the average risk-adjusted return. A Sharpe ratio greater than 0 signifies that the risk-adjusted investment will likely give higher returns, than the risk-free rate. While a Sharpe ratio of less than 0 signifies that the risk-adjusted investment will likely give lower returns than the risk-free rate.

There are only two strategies that yield higher average returns compared to the benchmarks of 5.91%, and these are the SLOW strategy and the intermediate strategy (speed equal to 0.25), with average returns of 6,46% and 6,17% respectively. FAST shows the lowest average return with 5.30%. Therefore, in terms of average returns, SLOW is the best performing strategy, and FAST is the worst (performing even worse than the market). The average return of the other intermediate-speed strategies appears to be in a place in between the average return of SLOW and FAST.

All strategies, except for FAST (volatility of 15.66%), seem to have lower volatility than the market, although the volatility for SLOW is not vastly different from the market with a difference of 0.02% (15.62% and 15.64% respectively). The intermediate-speed MED strategy has the lowest volatility (11.60%), whereas the other two intermediate strategies with speeds 0.25 and 0.75 seem to have rather similar volatility of 12.72% and 12.74% respectively.

Solely based on the average returns we conclude that SLOW is the best performing strategy, FAST is the worst, and the intermediate-speed strategies perform more or less somewhere in between SLOW and FAST. We also conclude that SLOW and FAST have the highest risk and seeing from a risk-averse point of view, the intermediate-speed MED strategy (lowest volatility) is considered to be the best choice. In order to find the best performing strategy, we must not only consider which strategy has the highest average returns or the lowest volatility. Since volatility is associated with risk, a strategy that has high average returns, but also higher volatility, may generate a greater loss than a strategy with lower volatility and average returns. Therefore, it is necessary to adjust the performance of a portfolio for the excess risk that is taken by the investor. To look for the highest average return, while also taking into account the risk that follows, we use the Sharpe ratio as a performance measure. That is, we look at the excess return relative to its volatility. With this in mind, we can see that SLOW still performs better than the market with a Sharpe ratio of 0.41 compared to that of the market of 0.38. The lowest Sharpe ratio is found in FAST, performing worse than the market with a Sharpe ratio of 0.34; this may be due to Type I error occurring often. The intermediate-speed strategies are the highest performing (Sharpe ratio of 0.49 and 0.44 for intermediate strategies with speeds 0.25 and 0.75 respectively), with MED appearing to have the highest Sharpe ratio of 0.51.

Market Timing: Under market timing, we look at the average position, the market beta, and the alpha. The average position is defined as

$$average\ position = mean\left(\frac{r_{t+1}(a)}{r_{t+1}}\right) - 1. \tag{12}$$

$r_{t+1}(a)$ represents the realized returns from the TSM strategies (each with its respective speed *a)* at time $t+1$, while r_{t+1} represents the realized returns from the buy-and-hold strategy at time $t+1$. The average position is therefore defined as the mean of the ratio of the realized returns from the TSM portfolio strategies to the realized returns in the market, subtracted by one. An average position that is greater than 0 signifies the average percentage that the TSM strategy outperforms the buy-and-hold strategy. An average position of less than 0, signifies that the strategy performs worse than the buy-and-hold strategy. As can be observed in the results there is a positive static tilt in average position in all TSM strategies (this comes as no surprise given the high frequency of Bull states in the market). The average position for the TSM strategies decreases from SLOW to FAST, from 46% to 18% respectively.

Another method of measuring performance while adjusting for risk is using Jensen´s Alpha. To understand Jensen´s Alpha, we need to understand the Capital Asset Pricing Model (CAPM). CAPM is an extension of Harry Markowitz work (Markowitz, 2008), developed independently by William F. Sharpe (1996), John Lintner (1965), Jan Mossin (1966), and Jack Treynor (1961, 1962). The CAPM explains the equilibrium relationship between a single risky asset (or a portfolio) and the market return. The CAPM is defined as

$$E[r_i] = r_f + \beta_i \underbrace{\left(E[r_M] - r_f\right)}_{\substack{Excess\ Market \\ Return}}, \tag{13}$$

where r_f and $E[r_M]$, represent the risk-free rate and the expected return on the market portfolio respectively. $E[r_i]$ is the expected return of the risky asset (or portfolio) i. $(E[r_M] - r_f)$ is the excess market return. β_i represents the volatility associated with the risky asset (or portfolio) *i* as compared to the market return, and is defined as

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2}\ , \tag{14}$$

where $Cov(r_i, r_M)$ is the covariance between the return on the risky asset (or portfolio) *i* and the return on the market portfolio. In other words, β_i explains how the changes in the risky assets return relate to that of the market. σ_M^2 is the variance of returns of the market portfolio, that is how far the returns deviate from the average return. If β_i is equal to 1, it means that the changes in returns in the risky asset *i* strongly correlate with the changes in the market, and the risky asset has therefore a *systematic* risk. If β_i is greater (less) than 1, it means that the risky asset, in theory, is more (less) volatile than the market return.

Jensen (1967, 1969) proposed to add the y-intercept to the CAPM,

$$\underbrace{r_i - r_f}_{R_i} = \alpha_i - \beta_i \underbrace{\left(r_m - r_f\right)}_{R_M} + \epsilon_i, \tag{15}$$

$$R_i = \alpha_i - \beta_i R_M + \epsilon_i\ , \tag{16}$$

where R_i and R_M represent the excess returns over the risk-free rate for the risky asset *i* and the market portfolio respectively. The ϵ_i is the random error component of the risky asset *i* and is firm-specific with a mean of 0. The β_i is defined as

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma_M^2}, \tag{17}$$

where $Cov(R_i, R_M)$ is the covariance between the excess return of the risky asset and the excess return of the portfolio. α_i represents the y-intercept. A rearrangement of (11), while also considering the expected values of the returns, we arrive at

$$\alpha_i = E[r_i] - [r_f + \beta_i(E[r_{M]} - r_f) + E[\epsilon_i]], \tag{18}$$

since the expected value of the error term, $E[\epsilon_i]$, is 0, we get

$$\alpha_i = E[r_i] - \underbrace{[r_f + \beta_i(E[r_{M]} - r_f)]}_{CAPM}. \tag{19}$$

The second component in (19) is equal to the CAPM defined in (13), and the explanations for the symbols are therefore described in the respective section. α_i represents the abnormal or extra excess of the return of risky asset *i* predicted by the CAPM. Thus, an α_i greater than 0 (a positive alpha), signifies that the portfolio is earning excess returns and has "beat the market". Therefore, the higher the alpha is for a portfolio, the better.

We use the Ordinary Least-Squares (OLS) regression to calculate the values of alpha and beta. The modern OLS regression was developed by Ronald Aylmer Fisher (1922), combining the regression theory of Karl Pearson (1920) and George Udny Yule (1897) and the least-squares theory of Carl Friedrich Gauss and Adrien-Marie Legendre (Stigler, 1981). The OLS regression is a statistical technique that is used to model and analyze a linear relationship between one or more independent variables and a dependent variable. The method minimizes the sum of squares in the errors (difference between the predicted and the observed values). The OLS regression is defined as,

$$Y = \alpha + \beta X + \epsilon, \tag{20}$$

where α is the intercept. X is the independent variable and β is the coefficient of X (or the slope of the linear regression). Y is the dependent variable, while ϵ is an error term that captures the underlying relationship between Y and X. In this thesis, the dependent variable, Y, is the excess return of a TSM portfolio, while the independent variable, X, represents the excess return of the market portfolio as formulated in (16). The OLS regression is formed after the Best Linear Unbiased Estimator (BLUE) which is based on the Gauss-Markov theorem (Hill, Griffith, & Lim, 2016, p.62-63). The theorem states that the OLS regression is BLUE, this means that the linear regression produces unbiased estimates that have the smallest variance possible. Without delving too much into the details of this, the fact that the OLS is BLUE means that the coefficients calculated by the regression have the smallest variance possible and are therefore reliable (given that the classic assumptions of the OLS regression are fulfilled (Hill et al., 2016, p. 47)).

The alpha and beta inform us on how well our TSM strategy is doing compared to a normal buy-and-hold strategy, and how much risk is involved in doing so. As displayed in Table 2, the betas for the TSM strategies range from positive to negative from SLOW to FAST, from 0.15 to -0.23. TSM strategies are therefore in theory less volatile than the overall market, where intermediate-speed MED to FAST strategies are even negatively correlated with the market considering their volatility. The alpha for TSM strategies is all positive and increases from SLOW to FAST, from 5.58 % to 6.66 % respectively. This means that the FAST strategy has the highest alpha and performs 6.66% better than the market. Concerning statistical significance, we also look at the t-statistics for the alphas. Statistical significance in the results is necessary to make sure that the profitability attributed to the TSM strategies is not solely due to randomness and luck. As displayed in Table 2, the alpha t-statistic for all TSM strategies is above 1.96, rendering the results statistically significant at the 1 % level. The highest alpha t-statistic is found in the MED strategy with a t-statistic of 3.71, rendering it statistically significant at the 0.05 % level. A decomposition of alpha and beta will be laid out and examined more closely in section *Market Timing*, to explain why the profitability of the TSM strategies can be attributed to the predictability characteristic associated with TSM strategies.

Tail Behavior: Tail behavior is an important element to scrutinize; it can tell us whether there is a probability of a fat tail or not. It is ideal that a strategy provides high/positive returns for each unit of volatility, and simultaneously has a low maximum drawdown. We expect that the TSM strategies, as compared to the benchmark, will be composed of more desirable traits, such as positive skew and low maximum drawdown. A positive skew will signify a longer and fatter tail at the right side, suggesting a higher probability of the occurrence of positive returns. Higher returns for each unit of volatility will also generate a higher Sharpe ratio.

Skewness is the third moment of a statistical distribution (where the first and second moment is the average mean and the variance, respectively). Skewness is a measure of asymmetry in a probability distribution of returns. It measures how the distribution of a given dataset deviates from the normal probability distribution. A normal distribution appears as a bell curve, with a mean of 0, indicating that most of the data will occur near the mean, rather than far from the mean. It also implies that the probability of the occurrence of extreme values in either end (positive or negative) is the same, making the probability distribution symmetrical. Skewness is theoretically defined as

$$\mathbb{S}[X] = E\left[\frac{(X-\mu_x)^3}{\sigma_x^3}\right], \tag{21}$$

where X is a random variable, μ_x is the mean and σ_x^3 is the standard deviation of the random variable *X*. E is the expectation operator. Skewness can be positive, negative, or normally distributed. If skewness is between -0.5 and 0.5, it is regarded as approximately symmetrical. A skewness between -1 and -0.5 or 0.5 and 1, is considered moderately skewed. While a skewness less than -1 and greater than 1, signifies a highly skewed distribution. A positive skewness implies that the tail on the right side is longer or fatter. This means that although there is a higher frequency of negative (low) values occurring, the few occurrences of the positive values are very extreme. Positive skewness is usually preferred by investors, as there is a probability that the extreme values (or profits) that they might gain, most likely will make up for all the small losses that have occurred. A negative skewness implies that the tail on the left side is longer or fatter, and this suggests that there are frequent occurrences of positive (high) values, and few, but more extreme, occurrences of negative values.

Skewness is either positive or less negative for all TSM strategies compared to the skewness that is found in the Market portfolio. The Market portfolio has a moderately negative skewness of -0.54. SLOW has a skewness of -0.43, making it the TSM strategy with the highest negative skew, followed by the intermediate-speed strategy with a speed equal to 0.25 (a skewness of -0.13). The other TSM strategies, MED to FAST, are positively skewed. Although for intermediate-speed strategies with speeds of 0.5 and 0.75, skewness is only slightly positively skewed with skewness of 0.02 and 0.03 respectively, while skewness is at 0.15 for FAST. The skewness suggests that all TSM strategies are approximately normally distributed.

The maximum drawdown (MDD) is defined as

$$MDD = \max\left\{\frac{1 - R_j}{\max\{\prod_i^N R_j\}}\right\} * 100\,, \tag{22}$$

where R is

$$R_j = \prod_{i=1}^{N}(1 + r_j)\,. \tag{23}$$

R is the cumulative product of the returns, r, of portfolio *j*. N is the number of observations (returns) in the portfolio. The intuition behind (22) is that we look for the largest movement from a high value to a low value. This can more easily be formulated as

$$MDD = \frac{Trough\,Value - Peak\,Value}{Peak\,Value}\,, \tag{24}$$

where we look at the percentage decrease in a movement. The MDD, therefore, implies that we are looking at the maximum percentage decrease in returns.

The maximum drawdown for all strategies is lower than that of the benchmark. The lowest maximum drawdown is found in the intermediate strategies varying from -34.07 to -

37.96. This also implicates, as shown in the final row in Table 2, that the highest average returns per unit of maximum drawdown (defined as an average return over the absolute value of the maximum drawdown) are higher in intermediate strategies. The maximum drawdown for SLOW and FAST are rather similar; -43.43 % and -44.53 % respectively.

An overview of Table 2 shows that the intermediate-speed strategies have high returns, low volatility, and high Sharpe ratios. They also appear to have high and statistically significant alphas, in addition to relatively low systematic risk. Also, maximum drawdown is low in intermediate-speed strategies compared to SLOW, FAST, and the market. Garg et al. (2019) show that there is a higher correlation in all aspects of subsequent returns (returns, volatility, lower and upper tail) in the intermediate-speed strategies compared to SLOW and FAST. Specifically, (1) they find that there are weak positive correlations in returns and negative correlations in volatility for all TSM strategies. This means that there is a tendency to take positive positions when returns are positive and that there is a stronger tendency to take negative positions when volatility is high across all TSM strategies. Furthermore, (2) TSM strategies tend to be positive predictors of lower tail returns and negative predictors of upper tail returns. This means that there is a tendency to take negative/weaker positions when the subsequent returns are in the lowest 10 %. The opposite is true for the upper tail returns; there is a tendency to take negative/weaker positions even when the subsequent returns are in the highest 10 %, which implies missing out on potential profits. Lastly, (3) they show that there is a high correlation between TSM strategies and subsequent returns in both the lower and upper tails. This correlation is highest in the intermediate-speed strategies and especially in MED. This is the main reason why MED has such a high Sharpe ratio.

Return and Risk

In this section, we decompose the Average Returns and Variance of Returns of each TSM strategy into their conditional contributions succeeding each market state. The cycle-conditional returns are given by

$$r_{t+1}(a) = r_{t+1}1(a)_{\{Bu\}} + r_{t+1}1(a)_{\{Be\}} + r_{t+1}1(a)_{\{Co\}} + r_{t+1}1(a)_{\{Re\}}, \qquad (25)$$

with zero-one indicators for each market state at date $t+1$. The $r_{t+1}(a)$ signifies the cycle-conditional return for TSM strategy with speed a. {Bu, Be, Co, Re} signifies the market states Bull, Bear, Correction, and Rebound, respectively. The cycle-conditional returns for Bull will therefore be,

$$r_{t+1}(a) = r_{t+1}1(a)_{\{Bu\}} + r_{t+1}0(a)_{\{Be\}} + r_{t+1}0(a)_{\{Co\}} + r_{t+1}0(a)_{\{Re\}}$$

$$r_{t+1} = r_{t+1}1_{\{Bu\}}, \qquad (26)$$

The conditional contribution to each TSM strategy with speed a will, therefore, after a Bull state, be the sample estimate $E[r_{t+1}(a)1_{Bu}]$. Where E is the expectation operator.

The cycle-conditional variance of returns is defined as

$$Cycle - Conditional\ Variance = \sigma^2{}_{t+1}(a) * \underbrace{\frac{n_{\{s(t)\}}}{N}}_{\substack{relative \\ frequency}} * 12 * 100, \qquad (27)$$

where $\sigma^2{}_{t+1}(a)$ signifies the variance (see (9)) of TSM strategy of speed a, at time $t+1$. $n_{\{s(t)\}}$ is the number of observations of returns in market state $s(t) \in \{Bu, Be, Co, Re\}$. N is the total number of observations of returns in the respective strategy. Thus, variance is adjusted according to the relative frequency of each state, and multiplied with 12 and 100, to get the annualized percentage of the variance.

Table 3: Market-Cycle Decomposition of Returns by Speed

Panel A: Average Returns						
		$a = 0$	$a = \frac{1}{4}$	$a = \frac{1}{2}$	$a = \frac{3}{4}$	$a = 1$
Average (%)	Market	SLOW		MED		FAST
Unconditional	5.91	6.46	6.17	5.88	5.59	5.30
Cycle-Conditional Decomposition						
Bull	4.59	4.59	4.59	4.59	4.59	4.59
Correction	1.59	1.59	0.79	0.00	-0.79	-1.59
Bear	-1.29	1.29	1.29	1.29	1.29	1.29
Rebound	1.01	-1.01	-0.51	0.00	0.51	1.01
Bull + Bear	3.31	5.88	5.88	5.88	5.88	5.88
Correction + Rebound	2.60	5.58	0.29	0.00	-0.29	-5.58
Panel B: Variance of Returns						
Variance (%)						
Unconditional	2.45	2.44	1.62	1.35	1.62	2.45
Cycle-Conditional Decomposition						
Bull	0.64	0.64	0.64	0.64	0.64	0.64
Correction	0.78	0.78	0.20	0.00	0.20	0.78
Bear	0.72	0.72	0.72	0.72	0.72	0.72
Rebound	0.32	0.32	0.08	0.00	0.08	0.32
Bull + Bear	1.36	1.36	1.36	1.36	1.36	1.36
Correction + Rebound	1.10	1.10	0.27	0.00	0.27	1.10

Notes: This table reports the unconditional, and cycle-conditional decomposition of average returns in *Panel A* and variance of return in *Panel B*, of the Market (buy-and-hold strategy) and TSM strategies of various speeds. The cycle-conditional decomposition reports the contribution following each market state for each strategy. The sum of these equal to their corresponding unconditional values in the first row of each panel. The SLOW strategy takes a long (+1) position if its 12-month trailing returns are nonnegative, and otherwise a short (-1) position. The FAST strategy takes a long (+1) position if its 1-month trailing returns are nonnegative, and otherwise a short (-1) position. The intermediate-speed TSM strategies are formed by combining SLOW and FAST strategies $r_{t+1}(a) := w_t(a) r_{t+1} = (1-a) r_{SLOW,t+1} + a\, r_{FAST,t+1}$. The market states are defined as follows. Bull: $w_{SLOW,t} = w_{FAST,t} = +1$, Bear: $w_{SLOW,t} = w_{FAST,t} = -1$, Correction: $w_{SLOW,t} = +1 \text{ and } w_{FAST,t} = -1$, and Rebound: $w_{SLOW,t} = -1 \text{ and } w_{FAST,t} = +1$. The market return is the U.S excess value-weighted factor return (Mkt-Rf), extracted from the Kenneth R. French Data Library, and covers 1969-01 to 2018-12.

Panel A in Table 3 covers the Market-Cycle Decomposition of Average Returns. The first row reiterates the average unconditional returns for each TSM strategy for reference. The first row also equals the sum of the four first rows under Cycle-Conditional Decomposition and the sum of the last two rows. The former shows each market state and their contributions, while the latter shows the sum of contributions from Bull and Bear, and Correction and Rebound collectively.

We quickly notice that the two market states seem to contribute the same regardless of the TSM strategy; after Bull and Bear market states, the conditional contribution across all TSM strategies are 4.59 and 1.29 respectively. The reason for this is that when in a Bull state, all TSM strategies take a long position meaning that all strategies have positive returns. In a Bear state, all TSM strategies take a short position, resulting in a positive return (which is the reason the sign is negative in the market and positive in TSM strategies).

As for SLOW and FAST strategies during Correction and Rebound states, there is some regularity; during a Correction state, the market is correcting itself in a state where SLOW is taking a long position while FAST is going short. This is represented in the contributions for SLOW and FAST of 1.59 and -1.59. Since FAST is taking a short position, it will miss out on positive returns which is the reason we see a negative sign. This coherence is also to be found after a Rebound state; a state where SLOW takes a short position and FAST a long position. Here the contributions for SLOW and FAST are -1.01 and 1.01 respectively, and here SLOW is missing out on returns since it is taking a short position. This consistency can also be found in the intermediate-speed strategies with speeds *a* equal to ¼ and ¾; after a Correction state the contributions are 0.79 and -0.79 respectively, and after a Rebound state the contributions are -0.51 and 0.51 respectively. After Correction and Rebound, the contribution, as with regards to the MED strategy, is 0.00. This is because the strategy exits the market and takes no position.

The net contributions after Bull and Bear states are 5.88 for all TSM strategies as compared to the Market with 3.31. The contributions after Correction and Rebound, however, are higher for the Market, a contribution of 2.60, than for the TSM strategies. Contributions after the aforementioned states for SLOW and FAST are 0.58 and -0.58 respectively, and for the other intermediate strategies with speed, *a* equal to ⅓ and ¾ are 0.29 and -0.29 respectively.

Panel B in Table 3 covers the *Variance of Returns*. This first row displays the variance of returns, which is the squares of the volatility reported in Table 2. The variance of returns in the market is 2.44, equal to the SLOW strategy and just below the FAST strategy with a variance of return of 2.45. The MED strategy has a variance of 1.35, which also happens to be the lowest. The other intermediate-speed strategies have a variance of 1.62. This means that there is generally speaking less risk associated with the intermediate-speed strategies.

Considering the cycle-conditional variance we are looking at the sample estimate of $Cov[r_{t+1}(a)1_{\{Bu\}}, r_{t+1}(a)]$. When examining the cycle-conditional contributions we can see the same regularity as in the results in Panel A. The variance of returns after a Bull and Bear state is the same across TSM strategies, with 0.64 and 0.72 respectively. After Correction states contributions on SLOW and FAST are 0.78 for both, and after Rebound states, contributions are 0.32 for both. As for the MED strategy, since it is out of the market, there is no risk attributed to the strategy, and the variance of returns is therefore 0.00. This explains why there is generally lower risk attributed to the intermediate-speed strategies. As for the remaining intermediate-speed strategies, after Correction states, contributions are 0.20 for both strategies, and after Rebound states, they are 0.08 for both.

The net contributions after Bull and Bear states are 1.36 across all TSM strategies, which is similar to the variance of returns in the market. After Correction and Rebound states, the contributions on SLOW and FAST are similar to that of the Market at 1.10. The other intermediate-speed strategies, excluding MED, have contributions equal to 0.27.

The disagreements between SLOW and FAST are the drivers of the variations in average returns and variances across the TSM strategies. This is shown by Garg et al. (2019), where the effect of such disagreements in determining Sharpe ratios, is manifested in a *disagreement multiplier D(a)*. This multiplier represents the ratio of the volatility of the market return to the volatility of the TSM strategy. The volatility of the TSM strategy more specifically is the volatility contributions from Bull and Bear, and Correction and Rebound states. There are two findings with this multiplier: (1) the risk-adjusted performance of the intermediate-speed strategies is greater than the average risk-adjusted performance of SLOW and FAST, and (2) intermediate-speed strategies tend to reduce volatility which is associated with the Correction and Rebound states.

Market Timing

The alpha is an indicator of how much a given strategy performs better than the benchmark, while the beta indicates a strategy´s volatility. In other words, the alpha represents the "excess return" and is a means to measure the performance of a portfolio as compared to the market return. The idea behind this is that the market is efficient and that any returns earned systematically that are above the market return as a whole are abnormal or in excess. The beta measures a portfolio's volatility compared to the systematic risk in the market. If a portfolio has a beta of 1, this means that said portfolio is strongly correlated with the market and incurs systematic risk. A beta of below 1 means that the portfolio, in theory, is less volatile than the market, and finally, a negative beta implies that the portfolio's volatility is negatively correlated with the market's systematic risk.

The determinants of market alpha and beta in momentum strategies are important to understand. In this section, we will examine what drives market alpha and beta. As shown by Figure 2, Bull states are the most frequent market state with a 46 % frequency, so almost half of the months in the period we examined. This indicates simply that trend-following in the U.S market has a positive static tilt, where the average position of the momentum strategy ranges from 46% to 18%. However, there is an indication that despite the positive static tilts, beta tends to be low in magnitude and range from 0.15 to -0.23, including negative point estimates for the intermediate-speed strategies.

This evidence can be understood through a widely used decomposition where we disentangle static and dynamic bets in expected returns in the following way:

$$r_{t+1}(a) = w_t(a) r_{t+1} = \underbrace{(w_t(a) - \mathrm{E}[w_t(\mathrm{a})])}_{dynamic} r_{t+1} + \underbrace{\mathrm{E}[w_t(a)]}_{static} r_{t+1}, \quad (28)$$

$r_{t+1}(a)$ is the realized returns at time $t+1$ for TSM strategy with speed a. $w_t(a)$ is the strategy weight at time t for TSM strategy with speed a. E is the expectation operator.

Here the first equality matches (7) and we have:

$$E[r_{t+1}(a)] = \underbrace{Cov[w_t(a), r_{t+1}]}_{market\ timing} + \underbrace{\mathrm{E}[w_t(a)]}_{\substack{static \\ dollar \\ exposure}} E[r_{t+1}], \quad (29)$$

due to $(w_t(a) - E[w_t(a)])$ being a mean of 0. The covariance above represents the share of the expected returns generated by dynamic bets that are reflected by the strategy weight, $w_t(a)$, while the second component which is the average strategy weight, attempts to summarize the static dollar exposure of the strategy overall.

This relates directly to what Huang et al. (2019) argue, which is that predictability, as obtained by regression, of 12-month TSM does not statistically outperform a non-predictive strategy based on historical data, even when using Sharpe Ratio as a performance measure. Furthermore, they argue that a TSM strategy may still be profitable and that this would however only be due to a positive static tilt, which is demonstrated with a higher mean. Therefore, they believe that a 12-month horizon TSM does not offer much alpha.

There is however an alternative view on the properties of trend portfolios. It's important before discussing this alternative view, to first bear in mind that it's unwise to dismiss trend signals solely based on forecasting regressions of excess returns. This is because static allocations do in fact constitute a large overall share of expected returns, with the market timing adding marginal significant returns in comparison. However, that is only in cases where market timing doesn't add negative betas with respect to the underlying market, which offset the beta of the static allocation. What this implies is a meaningful alpha as shown earlier in Table 2.

To understand the latter more clearly, we explain this evidence through further disentangling the static and dynamic components in the market covariance, beta, and alpha. The contemporaneous covariance between the speed strategy returns and the buy-and-hold market strategy can be decomposed in the following manner:

$$\mathbf{Cov}[r_{t+1}(a), r_{t+1}] = \mathrm{E}[w_t(a)]\mathrm{Var}[r_{t+1}] + \mathrm{Cov}[w_t(a), r_{t+1}]\mathrm{E}[r_{t+1}] + \mathrm{Cov}[w_t(a), (r_{t+1} - \mathrm{E}], \quad (30)$$

While the market alpha and beta can be respectively decomposed as follows:

$$\mathbf{Beta}[r_{t+1}(a)] = \underbrace{\mathrm{E}[w_t(a)]}_{\substack{static\\ component}} + \underbrace{\frac{\mathrm{Cov}[w_t(a), r_{t+1}]}{\mathrm{Var}[r_{t+1}]}\mathrm{E}[r_{t+1}]}_{\substack{market\ timing\\ component}} + \underbrace{\frac{\mathrm{Cov}[w_t(a), (r_{t+1} - \mathrm{E}[r_{t+1}])^2]}{\mathrm{Var}[r_{t+1}]}}_{\substack{volatility\ timing\\ component}}, \quad (31)$$

$$\mathbf{Alpha}[r_{t+1}(a)] = \underbrace{\mathrm{Cov}[w_t(a), r_{t+1}]\left(1 - \frac{(\mathrm{E}[r_{t+1}])^2}{\mathrm{Var}[r_{t+1}]}\right)}_{\substack{market\ timing\\ component}} - \underbrace{\frac{\mathrm{Cov}[w_t(a), \mathrm{E}[r_{t+1}])^2]}{\mathrm{Var}[r_{t+1}]}\mathrm{E}[r_{t+1}]}_{\substack{volatility\ timing\\ component}}, \quad (32)$$

The alpha (32) is decomposed into the sum of a market timing and a volatility timing component, and the beta (31) into the sum of a static, a market timing, and a volatility component. We can observe that in (31) in which we decompose the market beta of any of our momentum strategies, a static and a market timing component arise similar to the decomposition of the expected return in (29), wherein an additional component of volatility arises. The market timing component is the covariance between the strategy weights and returns, meaning the share of the expected return generated by dynamic bets of the signal. However, the volatility timing component is what reflects the predictability of strategy weights for subsequent return volatility. This means that if the momentum weights significantly covary with the subsequent return variance then it stands to reason that the beta of the momentum portfolio is not that well approximated by the beta of the average momentum position. This indicates that even though the market timing component could be small compared to a large positive static component, the volatility timing component can be relatively large, but of the opposite sign and just enough to offset the static component of the market beta. We can see this is the case in Table 3. Table 4 lays out the decomposition of the alpha and beta into the static (for beta), marketing timing, and volatility timing components. The first part of the table simply reiterates the results from Table 2.

Table 4: Beta and Alpha Decompositions by Speed

Market Beta and Alpha	$a = 0$ SLOW	$a = \frac{1}{4}$	$a = \frac{1}{2}$ MED	$a = \frac{3}{4}$	$a = 1$ FAST
Beta	0.15	0.05	-0.04	-0.13	-0.23
Alpha	5.58	5.85	6.12	.6.39	6.66
Alpha t-statistic	2.54	3.24	3.71	3.57	3.07
Beta Components					
Static	0.457	0.387	0.317	0.247	0.177
Market Timing	0.008	0.008	0.008	0.008	0.009
Volatility Timing	-0.315	-0.340	-0.365	-0.389	-0.414
Alpha Components (%)					
Market Timing	3.72	3.85	3.97	4.09	4.22
Volatility Timing	1.86	2.01	2.15	2.30	2.45

Notes: This table reports the sample market beta, alpha, and alpha t-statistic of monthly returns of momentum strategies of various speeds repeated from Table 1. This table additionally reports the (additive) decomposition of beta into static, market-timing, and volatility-timing components according to estimates of the terms in (11) and the (additive) decomposition of the alpha into market timing and volatility timing according to estimates of the terms in (12). The slow strategy weight applied to the market return in month $t + 1$, $w_{SLOW,t}$, equals (+1) if the trailing 12-month return is nonnegative and is otherwise (-1). The fast strategy weight, $w_{FAST,t}$, equals (+1) if the trailing 1-month return is nonnegative and is otherwise (-1). Intermediate-speed strategy weight, $w_t(a)$ are formed by mixing the slow and fast strategies with mixing parameter a: $w_t(a) = (1 - a)\ w_{SLOW,t}, +w_{FAST,t}$, for $a \in [0.1]$. strategy returns are formed as $r_{t+1}(a) = w_t(a)r_{t+1}$, where r_{t+1} is U.S excess value-weighted market factor return (Mkt-RF) from the Kenneth French Data Library. The evaluation period is from 1969-01 to 2019-12.

Firstly, the static component merely represents the average market position of the strategies. It is, nevertheless, obvious that the static component is significant, as it makes up almost as much of the beta as the volatility timing component (with the opposite sign) does.

Since there are no extreme values to be found in the decompositions, the betas for the various strategies are rather low in magnitude with the highest beta being 0.15 and the lowest -0.23. It is also noteworthy that as speed increases, the static component makes up a smaller portion of the beta and the volatility component, naturally, makes up a bigger portion. The market timing component in the beta is rather small for all strategies. This is not true for the alpha, where the market timing component makes up approximately ⅔, and the volatility component makes up the remaining. For an increase in speed, the latter component tends to increase as well. This conforms to the well-known empirical regularity that predicts a negative correlation between volatility and returns.

Furthermore, in the case of the alpha, it's broken down into its two additive components; the market timing component and the volatility timing component. This decomposition fully indicates that the two components can be the main drivers of the alpha of the momentum strategies.

To conclude we have demonstrated in our analysis, a decomposition of the beta and alpha into market and volatility timing components that gives us a different interpretation of the profitability than Huang et al. (2019). We can see that the volatility timing component reflects the predictability of strategy weights for the subsequent return volatility and appears to be a large part of what makes the alpha and beta. For beta, this means that the strategy is not well estimated solely by the average position. The decomposition showed that about 1/3 of the alpha is composed of the volatility component. These results appear to be statistically significant and suggest that profitability, unlike what Huang et al. (2019) concluded, can be attributed to a predictability characteristic with TSM strategy. The predictability characteristic of the volatility component also renders the argument of dollar exposure being a key determinant to profitability imprecise.

Relation of TS and volatility-managed portfolios

To start with the relation of TSM portfolios and volatility-managed (VOM) portfolios we need to look at the relevance of volatility timing for TSM portfolios. The contemporaneous correlation between stock market returns and their monthly volatility has been about -0.28 for the evaluated 50-year period. This empirical regularity indicates that the return of the market has some predictive ability of subsequent volatility in the coming months and is the first potential overlap between TSM strategies and volatility-managed portfolios.

A study by Moreira and Muir (2017) showed that managing the leverage of a strategy based on its trailing volatility could increase its Sharpe ratio and deliver alpha with respect to the underlying strategy. Their two main findings when it came to the U.S stock market were that trailing volatility tended to be uncorrelated to subsequent returns and that volatility was persistent at short horizons. Given these findings, investors should increase exposure to the stock market following low volatility states and decrease it following high volatility states. This advice is not unlike the findings of most TSM strategies where the prescriptive advice is going in during Bull states and out during Bear states. However, that's where the similarities end; TSM strategies combine both market and volatility timing components, with the former contributing to most of the alphas. While volatility-managed portfolios rely entirely on the volatility timing component, where for example Garg et al. (2019) find that the significant alpha of the VOM resulted in just about 2 % annualized in the evaluated 50-year period.

Tail Behavior

In order to analyze the distribution of returns, we will look at the *Cycle-Conditional Market Return Distribution*; the distribution of returns immediately following each market state. This will give us some explanation of the pattern that we found in the skewness for the various TSM strategies.

Looking at Table 5 we can see various percentiles of monthly return in months following each of the four market states. We can observe that Corrections introduce extreme outcomes and volatility even though most of its outcomes are positive; the median return is 1.07%. Yet, even though that's the case, extreme outcomes tend to be more extreme on the downside than the upside, so the losses are much bigger than the gains. The FAST strategy is the one that tends to flip Correction losses into gains by going short after Corrections due to being able to react quickly, which is probably what explains the slightly positive point estimate for skewness that is found in Table 2. FAST has full exposure to volatility from both Correction and Rebound states, in which the spread of returns is much larger on both the positive and the negative sides as opposed to Bull states. What´s interesting to note, however, is that the intermediate-speed strategies reduce their exposure to both volatility and extreme events associated with those two states. Particularly, we can see that MED strategy avoids this exposure altogether and has zero position in the months that follow Correction and Rebound states. This explains the 0 in skewness for this strategy in Table 2.

Table 5: Cycle-Conditional Market Return Distributions

Return Percentiles (%)	Bull	Correction	Bear	Rebound
MIN	-9.55	-23.24	-17.23	-10.35
P01	-7.88	-14.62	-12.79	-10.16
P05	-4.64	-7.14	-10.10	-8.41
P10	-3.37	-5.65	-8.05	-5.51
P25	-1.51	-2.07	-4.83	-2.43
P50	1.05	1.07	-0.89	1.15
P75	3.07	3.82	3.98	4.59
P90	4.68	5.84	6.82	7.24
P95	6.13	7.15	7.99	7.98
P99	7.21	11.79	13.68	10.61
MAX	9.59	12.47	16.10	11.30

Notes: Table 5 reports the various percentiles of monthly market returns in the months following each of the four different market states s_t, which are defined in terms of slow and fast momentum strategy positions in the following manner. The slow strategy weight applied to the market return in month $t+1$, $w_{SLOW,t}$, equals (+1) if the trailing 12-month return is nonnegative and is otherwise (-1). The fast strategy weight, $w_{FAST,t}$, equals (+1) if the trailing 1-month return is nonnegative and is otherwise (-1). Bull: $w_{SLOW,t} = w_{FAST,t} = +1$, Bear: $w_{SLOW,t} = w_{FAST,t} = -1$, Correction: $w_{SLOW,t} = +1 \; and \; w_{FAST,t} = -1$, and Rebound: $w_{SLOW,t} = -1 \; and \; w_{FAST,t} = +1$. MIN and MAX are the lowest and the highest observed monthly returns respectively. PXX is the XX-th percentile. For example, P95 is the 95th percentile of monthly returns. The market return is the U.S. excess value-weighted factor return (Mkt-RF) from the Kenneth French Data Library. The evaluation period is 1969-01 to 2018-12.

As is reported in Table 5, Corrections have the most extreme outcomes and volatility despite most outcomes being positive, the MIN (lowest observed return) here is -23.24%. The lowest observed returns are -9.55%, -17.23%, and -10.35% respectively for Bull, Bear, and Rebound. On the other hand, the MAX (highest observed return) obtained return is in a Bear state with a return of 16.10%. The highest observed returns are 9.59%, 12.47%, and 11.30%, for Bull, Correction, and Rebound respectively. All four market states are positive at the 50th percentile except for the Bear state.

The downside risk exposure after each market state might explain the pattern observed of maximum drawdowns across the different strategies as reported in Table 2. Also, the magnitudes of lower percentile (i.e., negative) returns are higher in most cases than magnitudes of symmetrically higher percentile (positive) returns, but because all TSM strategies go short after Bear states this downside becomes an upside. This is in line with what we find in Table 2, which is that all maximum drawdowns for the speeds are lower than the buy-and-hold market strategy. Intermediate-speed strategies are even more extreme as they further reduce downside exposure by scaling down after the Corrections and Rebounds mentioned earlier. This leads to intermediate-speed strategies having lower maximum drawdown and higher average returns per unit of absolute maximum drawdown as reported in Table 2.

Skewness: This section formalizes the relationship between the skewness of the MED strategy relative to the skewness of the SLOW and the FAST strategies in terms of MED's Sharpe ratio and the *disagreement multiplier*, *D(a)*. The connection between the Sharpe ratio and the skewness of a random variable is illustrated and shown in Lemma 3, which is applied to get the skewness decomposition.

Lemma 3. $for\ any\ Y\ such\ that\ its\ first\ three\ moments\ are\ defined\ and$ SD[Y]> 0, $then$

Skew[Y] = $\frac{E[Y^3]}{(SD[Y])^3} - \text{Sharpe}[Y](3 + \text{Sharpe}[Y])^2), where$ Sharpe[Y]= $\frac{E[Y]}{SD[Y]}$

The skewness decomposition is as follows. The skewness of $r_{t+1}(a)$ can be expressed in terms of the skewness of $r_{SLOW,t+1}$ and $r_{FAST,t+1}$, respectively, and a disagreement multiplier. An exact expression as well as an approximation for all $a \in [0, 1]$ based on $(E[r_{t+1}(a)]) \approx 0$ for a ∈ [0, 1]. In the special case of a = 1/2, we have

$$\text{Skew}[r_{t+1}(\frac{1}{2})] \approx \frac{1}{2}\Big(\text{Skew}[r_{SLOW,t+1}] + \text{Skew}[r_{FAST,t+1}])(D(\frac{1}{2}\Big))^3$$
$$+\ 3\text{Sharpe}[r_{t+1}(\frac{1}{2})]((D\left(\frac{1}{2}\right))^2\left[1 + \Big(\frac{\text{Sharpe}[r_{FAST,t+1}] - \text{Sharpe}[r_{SLOW,t+1}]}{2})^2\right] - 1\Big), \quad (33)$$

Wherein $D(\frac{1}{2})$ is defined for a = 1/2. This seems to show that the skewness of MED is usually scaled up relative to the average skewness of the SLOW and FAST strategies. As shown in Table 2, SLOW has a negative skewness of -0.43, while FAST has a positive skewness of 0.15. The average skewness is negative at -0.14. the disagreement multiplier $D(\frac{1}{2})$ = 1.34 amplifies the first term in (13) by a factor of ($D(\frac{1}{2})$)^2= 2.42, drawing its contribution to −0.34. The second term in (33) shifts this value to the right by just 0.36, yielding the small positive skewness of 0.02 for MED as shown in Table 2

Finally, a corollary to this result is, if both SLOW and FAST have nonnegative skewness and Sharpe ratios when applied in some market, then the skewness of MED strategy is going to be positive and higher than the maximum skewness of both SLOW and FAST.

Dynamic Speed Selection

Looking at the variations in the conditional returns we note that following Correction and Rebound, the returns percentiles are generally rather extreme, especially on the downside. We also saw from Table 2 that the intermediate-speed strategies reduce the downside risk exposure by scaling down following Correction and Rebound states. The effect of this is observed through the lower maximum drawdowns. This raises the question of whether, rather than always using the same speed, a strategy that changes speed depending on the market state that follows, could have improved performance. Instead of a strategy whose weights are predetermined, we would use a *dynamic* speed strategy; a strategy whose weights would be individually determined depending on the state it follows. In this section, we define the dynamic speed strategy, find the optimal speeds following each market state and then apply this on windows of various lengths. Finally, we analyze the performance of this strategy and test for efficiency.

The major difference between a dynamic-speed strategy and a static-speed strategy is that while the latter has a static speed parameter a, the speed a varies in the dynamic speed strategy according to the four different market states. So, the speed at date t is a function of the observable market state $s(t)$. The four market states in questions are Bull, Bear, Correction, and Rebound, and are abbreviated as $\{Bu, Co, Be, Re\}$ respectively. Thus, $s_{(t)} = Re$ means that we are in a Rebound state at time *t*. The $a_{s(t)=}\, a_{Re}$ is the parameter that will decide the blending between SLOW and FAST strategy weights in the following months. If the state remains the same at $t+1$, then we apply the same speed a_{Re}. If it shifts to Bull at $t+2$, then we apply a_{Bu} for the subsequent month and so on. The matching dynamic strategy returns for this is:

$$r_{t+1}(a_{s(t)}) = w_t(a_{s(t)})r_{t+1} = [(1 - a_{s(t)})w_{SLOW,t} + a_{s(t)}w_{FAST,t}]r_{t+1}, \qquad (34)$$

It’s important to note here that since $w_{SLOW,t} = w_{FAST,t}$ with magnitude 1 following Bull or Bear states, the dynamic weight in (34) is invariant to the values of a_{Bu} and a_{Be}. This means that $w_t(a)_{Bu} = 1$ after Bull for all a_{Bu} regardless and the same applies for Bear states, where $w_t(a_{Bu})$ = -1 after Bear for all a_{Be}. This means that the dynamic weight is only sensitive to the values of a_{Co} and a_{Re} following Correction and Rebound states. We will now establish the values of these state-conditional speed parameters that will maximize the steady-state Sharpe ratio of the dynamic strategy.

For an optimal dynamic-speed we need to choose a state conditional speed which will be applied following every occurrence of a state $s(t)$ to achieve the highest steady-state Sharpe ratio possible:

$$\max_{a_{s(t):s(t)\in\{Co,Re\}}} Sharpe\left[r_{t+1}\left(a_{s(t)}\right)\right]. \tag{35}$$

If $E[r_{t+1}|\, Bu]\, P[Bu] > \; E[r_{t+1}|\, Be]\, P[Be]$, then

$$a_{Co} = \frac{1}{2}(1 - \frac{\mathbf{E}[r_{t+1}^2]_{Be}^{Bu}\mathbf{P}[_{Be}^{Bu}]}{\mathbf{E}[r_{t+1}]Bu]\mathbf{P}[Bu] - \mathbf{E}[r_{t+1}]Be]\boldsymbol{P}[Be]}\frac{\mathbf{E}[r_{t+1}]Co]}{\mathbf{E}[r_{t+1}^2]Co]}, \tag{36}$$

$$a_{Re} = \frac{1}{2}(1 - \frac{\mathbf{E}[r_{t+1}^2]_{Be}^{Bu}\mathbf{P}[_{Be}^{Bu}]}{\mathbf{E}[r_{t+1}]Bu]\mathbf{P}[Bu] - \mathbf{E}[r_{t+1}]Be]\boldsymbol{P}[Be]}\frac{\mathbf{E}[r_{t+1}]Re]}{\mathbf{E}[r_{t+1}^2]Re]}, \tag{37}$$

is the unique state conditional pair that helps maximize (35). The above helps us specify the dynamic speed selections that maximize the steady-state Sharpe ratio in terms of state-conditional first and second population moments of market returns. Population values for the first and second moments in (36) and (37) are not observable. Therefore, we use historical estimates of these moments to approximate their values. “DYN” is used to denote the investable strategy that uses state-dependent speeds based on estimated versions of (36) and (37) using only data prior to strategy implementation, this means we have no look-ahead bias.

In Table 6, in order to test the performance of the dynamic strategies, we consider different windows with various lengths. The optimal speeds a_{co} and a_{Re} are computed based on returns from an estimation window. These speeds are then applied to the evaluation window. DYN shows the Sharpe ratios achieved with this strategy in the evaluation window. The “Oracle” OPT is the Sharpe ratio that would have been achieved ex-post, or in other words, the maximum Sharpe ratio that could have been achieved in the evaluation window. The last column shows the efficiency of the dynamic strategy by dividing the Sharpe ratio attained by the dynamic strategy, by the maximum achievable Sharpe ratio for the respective window (DYN/OPT ratio). Efficiency is above 90% for all window frames, with the highest rate being 98.8%.

Table 6: DYN Strategy Performance Over the Last 50 Years

DYN Strategy Estimation Window			**Evaluation**						
			Evaluation Window			Sharpe Ratio			
From (yr-mo)	To (yr-mo)	Length (yrs)	From (yr-mo)	To (yr-mo)	Length (yrs)	DYN	$(\hat{a}_{Co}\hat{a}_{Re})$	"Oracle" OPT	Efficiency DYN/OPT
1926-07	1968-12	42.5	1969-01	2018-12	50.0	0.52	(0.00, 0.58)	0.57	0.92
1926-07	1973-12	47.5	1974-01	2018-12	45.0	0.55	(0.07, 0.59)	0.57	0.96
1926-07	1978-12	52.5	1979-01	2018-12	40.0	0.61	(0.08, 0.65)	0.63	0.98
1926-07	1983-12	57.5	1984-01	2018-12	35.0	0.61	(0.22, 0.66)	0.62	0.99
1926-07	1988-12	62.5	1989-01	2018-12	30.0	0.69	(0.26, 0.69)	0.72	0.95
1926-07	1993-12	67.5	1994-01	2018-12	25.0	0.68	(0.11, 0.71)	0.68	0.99
1926-07	1998-12	72.5	1999-01	2018-12	20.0	0.56	(0.17, 0.69)	0.58	0.97
1926-07	2003-12	77.5	2004-01	2018-12	15.0	0.61	(0.16, 0.69)	0.62	0.98

Notes: This table reports the DYN momentum strategies Sharpe ratio and its efficiency when looking at different evaluation periods within the last 50 years. DYN is the dynamic (state-dependent) speed strategy. Based on the points in the estimation window the SLOW strategy weight applied to the market return in month $t + 1$, $w_{SLOW,t}$, equals (+1) if the trailing 12-month return is nonnegative and is otherwise (-1). The fast strategy weight, $w_{FAST,t}$, equals (+1) if the trailing 1-month return is nonnegative and is otherwise (-1). Bull: $w_{SLOW,t} = w_{FAST,t} = +1$, Bear: $w_{SLOW,t} = w_{FAST,t} = -1$, Correction: $w_{SLOW,t} = +1$ and $w_{FAST,t} = -1$, and Rebound: $w_{SLOW,t} = -1$ and $w_{FAST,t} = +1$. If either of the estimates $\hat{a}_{Co}$ or $\hat{a}_{Re}$ from equation (36) or (37) fall outside the unit interval [0, 1], then we set the value to the nearest endpoint, be it be 0 or 1. Strategy returns $r_{t+1}(a) = w_t(a)r_{t+1}$, where r_{t+1} is the U.S excess value-weighted market factor return (Mkt-RF) from the Kenneth French Data Library. OPT is the dynamic speed strategy that would have achieved the maximum Sharpe ratio, ex-post, state-dependent speeds of both strategies are fixed over the evaluation window. Efficiency DYN/OPT is the ratio of the Sharpe ratio achieved by the dynamic speed strategy to the maximum achievable Sharpe ratio, ex-post. That is, DYN/OPT examines how well the DYN strategy has done compared to the OPT strategy as a percentage of return acquired.

In summary, the premise of the dynamic strategy is that the optimal speed which we consider when trading, changes depending on which state (Bull, Bear, Rebound, and Correction) the market currently is in. As was explained earlier, the optimal speed when in either a Bull or Bear state always turns out to be 0.5, meaning a 6-month lookback period is the optimal window frame. As for the other states of Correction and Rebound, the optimal speeds are computed through back-testing. The optimal speeds are then applied to the strategy based on the state in which the market is. Although the dynamic strategy outperformed the static strategies, there were still certain static strategies that were able to outperform the benchmark. Additionally, t-statistics for the alpha of various strategies render the returns of the strategies statistically significant.

International Markets

Regarding the findings and results of this thesis, it is important to determine whether these results only apply to the U.S market or whether we can find the same results in other markets as well. In other words, we need to explore whether our results are valid or not. Since quantitative research typically has an inbuilt measure of standard error and the like which is widely acknowledged we need an additional method to verify the findings of this study.

Validity can often be divided into external and internal validity. Internal validity seeks to show that the explanation of the particular set of data we are dealing with can be sustained by the data and deals with accuracy (Cohen et al.,2007). External validity on the other hand deals with how well the results in a study can be generalized to a wider population of cases or situations and not just exclusively what is being researched at that very moment (Cohen et al., 2007). We attempt to test the external validity of the findings of this paper for the U.S stock market, by examining the empirical performance of TSM strategies of various static and dynamic speeds in different international equity markets. These markets are from Australia, France, Germany, Italy, Spain, Japan, Norway, and Canada. The results are reported in Table 7.

Table 7 shows the Sharpe ratios for the various static-speed strategies as well as for the dynamic strategy, applied to various international markets evaluated over a 15-year window frame. The last column uses the median of the speed-pair for the various international markets that are included. The speed-pair equals 0.90 and 0.30 for a_{Co} and a_{Re} respectively. As we can observe from the table, the Sharpe ratio for MED is higher than the average Sharpe ratios of

SLOW and FAST. In addition to that, in most countries when looking at the various static-speed strategies, intermediate ones exhibit the largest Sharpe ratio point estimates and beat both FAST and SLOW. This conveys to us that the conclusion we drew earlier of the intermediate-speed TSM strategies dominating in terms of performance, largely carries across international equity markets.

Table 7: Sharpe Ratios of Momentum Strategies in International Markets

	Sharpe Ratio							
	$a=0$	$a=\frac{1}{4}$	$a=\frac{1}{2}$	$a=\frac{3}{4}$	$a=1$			DYN Common
Country	SLOW		MED		FAST	DYN	$(\hat{a}_{Co}, \hat{a}_{Re})$	(0.90, 0.30)
CA	0.403	0.573	0.716	0.735	0.664	0.765	(1.00, 0.59)	0.778
NO	0.166	0.335	0.514	0.609	0.613	0.688	(1.00, 0.47)	0.631
AU	0.349	0.353	0.307	0.209	0.113	0.451	(1.00, 0.00)	0.394
FR	0.396	0.465	0.465	0.386	0.283	0.536	(1.00, 0.34)	0.550
DE	0.436	0.428	0.372	0.272	0.168	0.455	(0.44, 0.26)	0.467
IT	0.372	0.322	0.214	0.074	-0.041	0.372	(0.00, 0.00)	0.273
JP	0.539	0.465	0.330	0.162	0.011	0.539	(0.00, 0.00)	0.295
ES	0.176	0.229	0.268	0.264	0.233	0.328	(0.81, 0.37)	0.338

Notes: This table reports the Sharpe ratios for various strategies applied to different country equity markets evaluated over the 15-year period from 2004-01 to 2018-12. Static-speed strategy weights are made according to $w_t(a) = (1-a)w_{SLOW,T} + w_{FAST,t}$ for speed parameter $a \in [0.1]$, where, $w_{SLOW,t}$, equals (+1) if the trailing 12-month return is nonnegative and is otherwise (-1). The fast strategy weight, $w_{FAST,t}$, equals (+1) if the trailing 1-month return is nonnegative and is otherwise (-1). DYN strategy weights take the form $(1-a)w_{SLOW,T} + w_{FAST,t})$ where speed $a_{s(t)}$ on the four observable market states which are defined as follows. Bull: $w_{SLOW,t} = w_{FAST,t} = +1$, Bear: $w_{SLOW,t} = w_{FAST,t} = -1$, Correction: $w_{SLOW,t} = +1 \; and \; w_{FAST,t} = -1$, and Rebound: $w_{SLOW,t} = -1 \; and \; w_{FAST,t} = +1$. DYN speeds are based on point estimates of optimal state- depend speeds from equation (16) and (17), and the date is based on before the evaluation window beginning. If either of the estimates$\hat{a}_{Co}$ or $\hat{a}_{Re}$ fall outside the unit interval [0, 1], then we set the value to the nearest endpoint, be it be 0 or 1. The DYN common uses the median of DYN country speed-pair estimates, which are $\hat{a}_{Co} = 0.00$, and $\hat{a}_{Re} = 0.81$. Strategy returns $r_{t+1}(a) = w_t(a)r_{t+1}$, where r_{t+1} is the U.S excess value-weighted market factor return (Mkt-RF) from the Kenneth French Data Library. The highlighted points are the highest performing strategies for each country (the highlighted values under DYN common perform better than static-speed strategies).

When considering the static-speed strategies, it is evident that there is no single strategy that performs best for all markets. For Canada, for instance, the best performing static-speed strategy is a speed of 0.75. Whereas for Germany and Japan, the best performing strategies would be the SLOW strategy, and for France, the MED strategy. Nevertheless, the dynamic strategy dominates for all countries, except for Italy and Japan, where the Sharpe ratio is equal to that of the best-performing static-speed strategy which happens to be the SLOW strategy for both.

Finally, the speed-pairs for the dynamic common strategy also produce high Sharpe ratios, surpassing all static-speed strategies, and in some cases also the dynamic strategies, for all countries except for Italy and Japan. Even here it performed much better than the SLOW or FAST static-speed strategies. This largely holds for our findings of the optimal strategy being one that goes SLOW after Corrections and FAST after Rebound, which minimizes the weaknesses of both and maximizes their respective strengths

Conclusion

In this paper, we replicated the results and findings of the paper "Momentum Turning Points" by Garg et al. (2019). This thesis explored momentum turning points so the first thing we set out to do was to find a way to identify them. We defined SLOW and FAST strategies which respectively have lookback periods of 12 months and 1 month, these either indicate going long or short 1 unit. The agreement or the disagreement between these two signals gave rise to four different market states; Bull, Correction, Bear, and Rebound. These four states help view the challenges that momentum turning points pose for TSM strategies. We found that intermediate-speed strategies formed by blending SLOW and FAST strategies performed better as measured by the Sharpe ratio than the SLOW and FAST strategies. This is because intermediate strategies scale down their positions after Corrections and Rebounds. Thus, they reduce exposure to volatility without surrendering average returns compared to the FAST and SLOW strategies. Overall, they have higher Sharpe ratios, less severe drawdowns, higher significant alphas, and more positive skewness, making them the way to go for static-speed TSM strategies.

We further investigated the drivers of market beta and alpha of TSM strategies at various speeds. Here, we found that market timing and volatility timing played an important role in TSM. We found empirically that market timing, which reflects the covariance between strategy weight and subsequent market returns, accounts for about two-thirds of TSM´s alpha, with the remaining one-third being attributed to volatility timing, which reflects the covariance in strategy weights and subsequent market return volatility. We also showed that TSM strategies of all speeds have positive average exposures, meaning they go long more often than not (because Bull markets account for 48% of the frequency of the U.S market in the 50-year period). Surprisingly, their market betas are much lower than expected and predicted by these exposures. Beta estimates are near 0 for SLOW to intermediate-speed strategies and negative for FAST. With the decomposition of the beta, we have exposed that this disparity arises from the ability of TSM portfolios to time volatility.

Lastly, we tested a dynamic TSM strategy that changed speeds based on the market state to maximize its Sharpe ratio as opposed to a static strategy that holds the speed regardless of the market state. We showed that a dynamic strategy with a SLOW strategy following Correction states and a FAST strategy following Rebound states, improved not only the Sharpe ratio but also average returns per unit of drawdown risk. This is because following Corrections, it´s likely that Type I errors (false alarms) might dominate while following rebounds Type II errors (missed detection) might be more prevalent, this minimizes both errors. We tested the strategy on evaluation windows of various lengths and found consistent improvements compared to the static-speed strategies. Finally, we tested these findings across different international equity markets and showed that the same results hold.

References

Antonacci, G. (2013). Absolute Momentum: A Simple Rule-Based Strategy and Universal Trend-Following Overlay, https://dx.doi.org/10.2139/ssrn.2244633

Blitz, D., & van Vliet, P. (u.d.). Global Tactical cross Asset Allocation: Applying Value and Momentum Across Asset Classes. Journal of Portfolio Management, 35 (1), pp. 23-38, DOI: https://doi.org/10.3905/JPM.2008.35.1.23

Cohen, M. L., & Morrison, K. (2007). Research Methods in Education (sixth edition). Oxfordshire: Abingdon.

Cooper, M., & Gutierrez, R. (2004). Market States and Momentum. Journal of Finance, 59(3), pp.1345-1365, DOI: https://doi.org/10.1111/j.1540-6261.2004.00665.x

Da, Z., Gurun, U. G., & Warachka, M. (2014). Frog in the Pan: Continuous information and momentum. The review of financial studies, 27(7), pp. 2171-2218, DOI: https://doi.org/10.1093/rfs/hhu003

Daniel, K. D., Ravi, J., & Soohun, K. (2012). Tail Risk in Momentum Strategy Returns. National Bureau of Economic Research, http://www.nber.org/papers/w18169.pdf

Daniel, K., & Moskowitz, T. J. (2016). Momentum crashes. Journal of Financial Economics, 122(2), pp. 221-247, DOI: https://doi.org/10.1016/j.jfineco.2015.12.002

Ehsani, S., & Linnainmaa, J. T. (2020). Factor Momentum and the Momentum Factor. Journal of Finance, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3014521

Fisher, R. A. (1922). The Goodness of Fit of Regression Formulae, and the Distribution of Regression Coefficients. *Journal of the Royal Statistical Society*, 85(4), pp.597-612, DOI: https://doi.org/10.1111/j.2397-2335.1922.tb00832.x

Garg, A., Goulding, C. L., Harvey, C. R., & Mazzoleni, M. (2019). Momentum turning points. Available at SSRN 3489539.

Goyal, A., & Jegadeesh, N. (2018). Cross-Sectional and Time-Series Tests of Return Predictability: What is the Difference? The Review of Financial Studies, 31(5), pp. 1784-1824, DOI: https://doi.org/10.1016/j.pacfin.2020.101458

Gupta, T., & Kelly, B. (2019). Factor momentum everywhere. The Journal of Portfolio Management, 45(3), pp. 13-36, DOI: https://doi.org/10.3905/jpm.2019.45.3.013

He, X. Z., & Li, K. (2015). Profitability of time series momentum. *Journal of Banking & Finance*, *53*, pp.140-157, DOI: https://doi.org/10.1016/j.jbankfin.2014.12.017

Huang, D., Li, J., Wang, L., & Zhou, G. (2019). Time Series Momentum: Is it there? Journal of Financial Economics, 135(3), pp. 774-794 doi:https://doi.org/10.1016/j.jfineco.2019.08.004

Jegadeesh, N., & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. The Journal of Finance, 48(1), 65-91, https://doi.org/10.2307/2328882

Jegadeesh, N., & Titman, S. (2001). Profitability of Momentum Strategies: An Evaluation of Alternative Explanations. The Journal of Finance, 56(2), pp. 699-720, https://www.jstor.org/stable/222579

Killgallen, T. (2012). Testing the Simple Moving Average across Commodities, Global Stock Indices, and Currencies. The Journal of Wealth Management, 15 (1), pp. 82-100, doi: https://doi.org/10.3905/jwm.2012.15.1.082

Litner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. The Review of Economics and Statistics, 47(1), pp. 13-37, doi: https://doi.org/10.2307/1924119

Lim, B., Zohren, S., & Roberts, S. (2019). Enhancing time-series momentum strategies using deep neural networks. The Journal of Financial Data Science, 1(4), pp. 19-38, doi: https://doi.org/10.3905/jfds.2019.1.015

Liu, L. X., & Zhang, L. (2008). Momentum Profits, Factor Pricing, and Macroeconomic Risk. The Review of Financial Studies, 21(6), pp. 2417-2448, doi: https://doi.org/10.1093/rfs/hhn090

Marshall, B., Huy, N. N., & Visaltanachoti, N. (2016). Time Series Momentum and moving average trading rules. Quantitative Finance, 17(3), pp. 1-17, doi: https://doi.org/10.1080/14697688.2016.1205209

Moreira, A., & Muir, T. (2017). Volatility-Managed Portfolios. The Journal of Finance: The Journal of The American Finance Association, 72(4), pp. 1611-1644, doi: https://dx.doi.org/10.2139/ssrn.3385377

Moskowitz, T. j., Hua, Y., & Pedersen, L. H. (2012). Time series momentum. Journal of Financial Economics, 104(2), pp. 228-250, doi: https://dx.doi.org/10.2139/ssrn.2089463

Mossin, J. (1966). Equilibrium in a Capital Asset Market. The Review of Economics and Statistics, 34(4), pp. 768-783, doi: https://doi.org/10.2307/1910098

Pearson, K. (1920). Notes on the History of Correlation. *Biometrika*, 13(1) pp. 25-45, doi: https://doi.org/10.2307/2331722

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Con-ditions of Risk. *Journal of Finance*, 19(3) pp. 425-442, doi: https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

Sharpe, W. F. (1994). The Sharpe Ratio. *The Journal of Portfolio Management*, 21(1), pp. 49-58, doi: https://doi.org/10.3905/jpm.1994.409501

Stigler, S. M. (1981). Gauss and the Invention of Least Squares. *The Annals of Statistics*, 9(3) pp. 465-474, doi: https://doi.org/10.1214/aos/1176345451

Treynor, J. L. (1961). Market Value, Time, and Risk, The Review of Economics and Statistics, 52(1), pp. 87-99, doi: https://doi.org/10.2307/1927602.

Treynor, J. L. (1962). ack Treynor's 'Toward a Theory of Market Value of Risky Assets', doi: https://dx.doi.org/10.2139/ssrn.628187

Yule, G. U. (1897). On the significance of Bravais' formula for regression. Fellows of the Royal Society, 60, pp. 308-323, http://www.jstor.org/stable/115878

HAVIN SAMIR

In this discussion paper I write about my Master´s thesis in conjunction with my Master´s Programme in Business Administration. I delve into the topics of innovation, sustainability, and international, and how my master thesis upholds these themes. I also write briefly about how the use of R programming in my thesis, has served as a valuable skill for me. Finally, I end the discussion paper with a short summary of the discussion paper, as well as about my general experience of the Master Programme.

Master´s Thesis

The theme of our Master Thesis is based around trend-following strategies. The thesis seeks to replicate the results reported in Momentum Turning Point by the authors Garg, Goulding, Harvey, and Mazzoleni (2019). Time-series momentum (TSM) strategies is a trend-following strategy built upon the premise that there are trends in returns over time, and with statistical analysis it is possible to predict the movement of the trend. As the name suggests, time-series data is used when analyzing TSM strategies. This is in contrast to cross-sectional momentum strategies, that use cross-sectional data by looking at several assets and predicting which will be perform best and worse. TSM strategies use an assets historical returns, to predict an up- or downtrend. In order to evaluate a momentum strategy, the performance is typically compared to a benchmark. The benchmark in TSM strategies is usually a portfolio which is based on the buy-and-hold strategy.

TSM strategies is a topic that has been analyzed in numerous academic journals; often the results of the studies imply that TSM outperforms the benchmark. Nevertheless, most of the research covers primary trends as proposed by the DOW Theory. In the DOW Theory on stock price movement, the first of the basic six tenets outlines how the market has three movements: a main primary movement, a medium secondary one, and finally a short minor movement. The primary movement lasts anywhere from a year to several years, the secondary a couple of days to a dozen months, and the short swing from a few hours to a month. Most of the research in trend following strategies relies primarily on exploiting the primary movement as the foundation of its research.

We implement a new TSM strategy that in addition to the primary trends, also considers the secondary trends in the DOW Theory. During a trend (up- or downtrend) there will be "breaks" or changes in the trend, and these are referred to as momentum turning points. Momentum turning points have two possible outcomes: (1) they appear to constitute a change in a trend or (2) they are simply noise. Lack of certainty to which of the outcomes will occur, we create two strategies: SLOW with a lookback period of 12 months, and FAST with a lookback period of 1 month. The advantage of SLOW is that the lookback period is long enough to not be influenced by noise, but the downside is that the strategy will be slow to react to changes in trends. The FAST strategy with its shorter lookback period will react fast but will also be more influenced by noise. By combining these two strategies, we are better able to detect noise from a turning point.

We attribute primary and secondary trends to various market states by looking at the four combinations of agreement and disagreements between SLOW and FAST. If the trailing return for both strategies are positive (negative), both will take a long (short) position, indicating a Bull (Bear) market. If SLOW takes a long position and FAST a short position, then we are in a Correction state (suggests that the primary movement is an uptrend and that the secondary trend indicates a change in this trend). If SLOW takes a short position and FAST a long position, then we are in a Rebound state (suggests that the primary movement, a downtrend, is subject to change, and this is indicated through the secondary movements). This TSM strategy is then applied to various look-back periods(speed), including predetermined static speeds, and dynamic speeds. The latter in which we use back-testing to find optimal speeds for the different market states and implement the speeds in subsequent periods with forward-testing. The TSM strategies are applied on international market indices, and the Sharpe ratio for each strategy reveals that the dynamic speed strategies dominate in terms of performance.

Innovation and Sustainability

There has been a lot of research done on cross-sectional strategies. Cross-sectional strategies are based on analysis of various assets or portfolios. Lookback periods of 3-12 months are typically considered, and based on the returns over the chosen period, the highest performing assets or portfolios are bought (a long position is taken), and the lowest performing are sold (a short position is taken). Time-series momentum strategies uses an asset´s historical returns. With a determined lookback period (usually 12 months, but can extend up to three years), the strategy takes a long position (buys the asset) if the trailing return of that period is positive and takes a short position (sells the asset) if the trailing return of that period is negative. Thus, time-series momentum strategies only consider the primary movements of the price of an asset, as described in the DOW Theory (explained under the section *Master Thesis)*.

Our thesis, a replication of Momentum Turning Point by Garg et al. (2019), sought to combine the primary and secondary movements in order to create a new time-series momentum strategy. As the primary movement of asset price covers a long period of time (from 12 months to several years), a strategy that uses a long lookback period will be great at detecting long-term trends. The strategy will, however, miss out on short-term gains and suffer the losses. It will also be slower to react to a change in a trend, and thus incur even more losses up till the point where the downtrend has persisted long enough for the strategy to notice a downtrend. This can lead investors to place bad bets and can be very costly. This strategy is therefore not very *sustainable*, first as it´s not very effective (slow to react), and consequently can be very costly for investors, and second, since it´s slow to react, the average return may decrease in the time it takes for the strategy to react. Lastly, there is high volatility associated with the strategies, and are therefore not very sustainable in a long-time horizon perspective.

Our thesis suggested looking at a strategy with a short lookback period as well, and in our case, we chose a 1-month lookback period. This is because mainly due to our unit of analysis being monthly returns, and 1 month is the shortest lookback period. The reason we look at this is to avoid the problems associated with a long lookback period. A strategy with a short lookback period will be fast to react, that is, if there are changes in the price movement in a month (secondary movements), the strategy will be able to detect it right away. This eliminates the problem of slow reaction time associated with the former strategy. There is, however, a downside to this strategy as well; this increases the strategy to be influenced by noise (movements that misrepresent the underlying trend). Thus, we combine these two strategies, with long and short lookback periods, and create various strategies with different lookback periods in order to find the optimal strategies. That is, we take into account both primary and secondary movements in asset price as described in the DOW Theory, in order to create new time-series momentum strategies. This is something that has not been explored in the literature before and can therefore be considered *innovative*.

Thus, by combing primary and secondary movements, we created time-series momentum strategies with various lookback period. The method was twofold, we first (1) explored static lookback periods; we created five time-series momentum strategies, each with a different lookback period, and second (2) we created a strategy with a dynamic lookback period that is dependent on the market state. The latter strategy was created because by analyzing the frequency of the market states, we found that Correction and Rebound states account for one-third of the total time. Instead of continuing with the same lookback period after one of these market states, perhaps a change would be more effective.

Thus, we created a dynamic strategy whose lookback period varies based on which market state it is in. Utilizing Sharpe ratio as a performance measure, we were able to achieve more than 90 % efficiency with this strategy. This strategy turned out to be much more *sustainable* than what has been explored in the literature. First, the strategies have the right combination of long and short lookback periods, making it more effective by being able to detect noise from a change in trend. Second, there were high returns and low volatility associated with the strategies, making them more sustainable.

International

The topic of momentum strategies is an *international* topic. The topic has been researched because there is a drive/curiosity in academics and in the financial world in whether or not there is a method of "beating the market". Since momentum strategies are based on historical data analysis, the main tool in the research is statistics, and mathematics. It is the language of science, and anyone who understands mathematics will be able understand the momentum strategies created. The strategies which we created in our thesis, were also applied in international markets in order to test for external validity. The dynamic lookback period strategy dominated in terms of performance in most international markets and is therefore applicable in other countries as well. The findings might also be of interest among academics, as well in the finance world, since the strategies explored in our thesis have not been explored before and are in that sense innovative.

During my master's programme, I have had many interesting courses. A course which I found particularly interesting is *Computational Finance and Portfolio Management* by Professor Valeriy Zakamulin. In this course I was introduced to R programming, something I have found to be very useful. I have used R programming as the main tool for the empirical part. I used it to create functions and analyse time-series data. Implementing programming in the Business Administration programme is something I find particularly valuable for me as a student. Economics, finance and technology are more intertwined now than ever before. Technology is being used to further improve decision making in Economics, for instance in Business Intelligence, Big Data, CRM, and ERP-systems. I learned about these fields of technology in economics in an IT-course (IS-406-1 Enterprise Systems) and found it very valuable for my field of study. It is for that reason that I hope that programming and coding, as well as courses with a focus on the technology progress in business, continue to be included in the Business Administrations Programme. I have therefore also learned a valuable skill that I think will come in handy when I start working.

Summary

The aim of our thesis was to replicate the results and findings reported in *Momentum Turning Point* by Garg et al. (2019). We implemented a new strategy that takes into account the primary and secondary movements as described in the DOW Theory. This contrasts with what has been normally done in momentum strategies, where they mainly consider the primary movements of trend. The strategy is there innovative in some sense. The findings of our thesis suggest that the dynamic lookback period strategy returns a Sharpe ratio of more than 90% efficiency. Thus, there is high average returns and low volatility associated with the new strategy, which suggests that it is more sustainable as well. The strategies were also applied on the international market for external validity and proved to be effective also in the international market. Lastly, I feel that the courses I´ve had during my Master´s Programme, have given me a solid foundation of knowledge in economics and finance.

References

Garg, A., Goulding, C. L., Harvey, C. R., & Mazzoleni, M. (2019). Momentum turning points. Available at SSRN 3489539.