The Performance of the Volatility-Targeting Strategy

The Impact of the Volatility Forecasting Methodology and Trading Frequency on the Performance of the Volatility-Targeting Strategy

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Abstract

Many researchers analyzed the performance of the volatility-targeting strategy (see, e.g., Moreira and Muir (2017) or Harvey, Hoyle, Korgaonkar, Rattray, Sargaison, and Hemert (2018)). It is a very tempting strategy as it reduces exposure to risk, especially during crises. Typically, it works pretty well on equities and gives higher Sharpe ratios and lower drawdowns. However, the latest studies (see, e.g., Zakamulin (2019), Liu, Tang, and Zhou (2019), Cederburg, O'Doherty, Wang, and Yan (2020), or Bongaerts, Kang, and van Dijk (2020)) showed some challenges in using the strategy by practitioners. This study aims to answer the questions: does the volatility-targeting strategy outperform the buy-and-hold strategy, what volatility forecasting method to use, how often to rebalance, and does it work for other asset classes? Based on the findings from the conducted study, the main conclusion is that the volatility-targeting strategy does not consistently beat the buy-and-hold strategy. The strategy works well for equities in volatile periods in history, but it may be challenging to implement with success ex-ante. The findings are mixed for bonds. However, the active strategy does not work for commodities and currencies.

Key words: volatility-targeting portfolios, risk managing

JEL classification: G11, G17

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1 Introduction

The tradeoff between risk and reward is one of the main exercises in finance. Some investors believe that it is rational to expect a higher reward for higher risk. It seems to be intuitive and logical. Merton (1980), among others, note that the market volatility should be positively related to the excess market return (the return on a stock portfolio minus the risk-free rate). On the contrary, French, Schwert, and Stambaugh (1987) study the Standard and Poor's (S&P) composite portfolio in the years 1928-1984 and report the regression result which demonstrates that reality may be the opposite - the relation between the market volatility and the excess market return is negative.

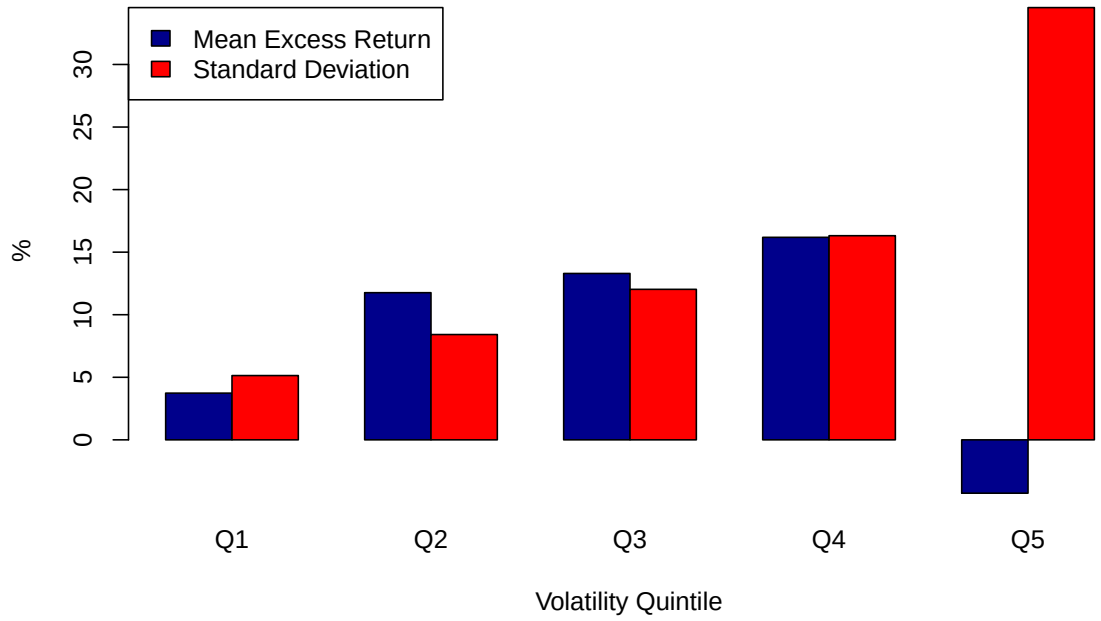
Figure 1 shows the annualized volatility and the annualized mean excess return when sorting on the previous month's volatility. The data are for all U.S. equities from 1926 to 2020. Volatility is divided into five states: very low (5%), low (8%), medium (12%), high (16%), and very high (35%). The mean return shows no clear pattern across different quintiles. The first quintile has a mean return of 4%, the second 12%, the third 13%, the fourth 16%, and the last one -4%. Those results illustrate that the volatility does not go in pair with returns for the equity market and when volatility is very high, the returns are negative in this dataset.

The findings are also confirmed in a study done by Hocquard, Ng, and Papageorgiou (2013), where the researchers go deeper into volatility regimes and asset returns. They divide volatility into a low, medium, and high state. In the low state with 6% volatility, the return is 19%, in the medium state with 12% volatility, the return is 7%, and in the high state with the volatility of 33%, the return is -38%. Harvey et al. (2018) and Moreira and Muir (2017) also note that higher volatility does not necessarily give a higher return in the stock market.[1]

Academics and practitioners try to find strategies that help to avoid this negative relationship between volatility and return, especially in the high state of volatility. Diversification is one of the attainable solutions. However, Hocquard et al. (2013) suggest that diversification does not work well during crises because all assets are correlated when there is a crisis. Moreover, it is especially during crises that many investors have challenges with large drawdowns. The profit acquired through many years of a bull market is gone or significantly declined after a (shorter) period of a bear market. Managing risk of momentum is the next potential remedy, which concentrates on eliminating the distress anomaly documented by Campbell, Hilscher, and Szilagyi (2008) (see, for example, Barroso and Santa-Clara (2015), Daniel and Moskowitz (2016), and Eisdorfer and Misirli (2020)). However, Dunn (2000), among others, believes that momentum is only a statistical illusion. In such circumstances, the volatility-targeting strategy is another possible solution to the problem with negative returns when volatility is very high. It is a way of

[1]The situation may be different for other asset classes, e.g., bonds.

Figure 1: Risk/Return Tradeoff for Equities



Quintile Analysis for Equities All U.S. (1926-2020). The figure shows the mean excess return and the volatility (both annualized) when sorting on the previous month's volatility.

risk management. In crisis time (when the volatility is usually high) is better to reduce exposure to risky assets or stay out of the market and avoid drawdowns. When one looks at the existing literature, the interest for volatility targeting among academics began around the dot-com bubble in the late 1990s, followed by the crash (see, e.g., Fleming, Kirby, and Ostdiek (2001)) and exploded after the global financial crisis in years 2007-2008 (see, e.g., Collie, Sylvanus, and Thomas (2011); Butler and Philbrick (2012), and Albeverio, Steblovskaya, and Wallbaum (2013)). The crises were more frequent - maybe a new normal.

The idea of a volatility-targeting strategy is to reduce the exposure to risky assets when the volatility is high. Furthermore, to increase the exposure to those assets when the volatility is low. The question scholars and practitioners ask is: does it work, and if yes, when and for what asset classes it works best. Hallerbach (2012) studies EURO STOXX 50 Index in the years 2003-2011. The researcher advocates that the better the volatility forecasts are, the higher the Sharpe ratio. His main conclusion is that volatility weighting over time increases the Sharpe ratios.

This master thesis examines if the volatility-targeting strategy is a good solution to the negative relationship between risk and return when volatility is high. The active strategy strongly

depends on accuracy in volatility forecasting, see, e.g., Hallerbach (2012). The first question is, then, what kind of volatility forecasting is best to use. We concentrate on two types of moving averages models - Simple Moving Average (SMA) and Exponentially Weighted Moving Average (EWMA) with different lengths of the estimation window. Furthermore, we try to find out if some of them work better than others. Second, the volatility forecast accuracy decreases with the horizon. So the more frequently one trade, the better. However, every transaction has its cost, and costs reduce the returns. We want to find out how often it is optimal to trade: daily, weekly, or monthly in order to achieve the optimal tradeoff between transaction costs and forecast accuracy, see, e.g., Zakamulin (2019). Third, we check if and when the volatility-targeting strategy gives better performance statistics (the Sharpe ratio) and lower risk than the buy-and-hold strategy. Fourth, Harvey et al. (2018), among others, report that the strategy works for equities but not for bonds, commodities, and currencies. We examine the same asset classes, yet the former study is extended by some other periods and assets to see if it is still valid.

When conducting an empirical study, there are analyzed time-series in four different asset classes: equities, bonds, commodities, and foreign exchange rates. The results from the conducted study show that, first, the volatility-targeting strategy does not consistently outperform the passive benchmark. It can depend on the asset class, region, method of volatility forecasting, frequency of trading, period, and target volatility. Second, the active strategy reduces risk, as the target volatility is constant and usually lower than the benchmark. Third, there is no clear pattern if some tested volatility forecasting models work better than others. However, the most frequent best results for gross Sharpe ratio (without transaction costs) gives the simples forecast - SMA-10, a simple moving average of ten days. The results are valid for U.S. equities and bonds analyzed in the study. As for trading frequency, when one uses SMA-10 with weekly trading for U.S. equities or SMA-10 with daily trading for U.S. bonds, one may achieve the best possible Sharp ratio among examined approaches. Fourth, like in all the previous studies, the volatility-targeting strategy does not work for commodities and currencies in this thesis. However, it may outperform the benchmark, especially during crisis times for some bonds and equities.

The rest of the thesis is organized as follows. Section 2 reviews the previous research on the analyzed strategy. Section 3 describes the datasets and methodology used in this empirical study. The empirical results are presented in Section 4. Section 5 summarise and concludes the study.

2 Literature Review

In this section, there are presented studies related to the volatility-targeting strategy in chronological order. As mentioned in the introductory section, academics took their interest in volatility timing around the dot-com bubble in the late 1990s and the later crash.

The first one is an article written by Fleming et al. (2001). They are concerned if those times standard volatility models have economic value. Their examination finds that conditional volatility timing strategies outperform the unconditionally efficient static portfolios with the same target expected return and volatility. Two years later, Fleming, Kirby, and Ostdiek (2003) go even deeper into the topic and advocate that going from daily to intraday returns makes the strategy even better.

Collie et al. (2011) find that a volatility-responsive asset allocation policy can lead to a more consistent outcome. And even a better risk-return tradeoff. They argue that if "market volatility is itself volatile," then the risk can be highly variable over time. So it makes sense to use a dynamic asset allocation policy and not a static one.

Hallerbach (2012) demonstrates that volatility weighting over time gives a higher Sharpe ratio, even if the mean return is constant. He claims that the better volatility smoothing is, the higher the risk-adjusted performance is. He also notes that not restricting leverage allowed in the strategy gives an even higher Sharpe ratio.

Hocquard et al. (2013), in their compelling study, suggest that diversification is not a suitable risk control mechanism during crises. It turns out that historical correlations between different asset classes break down during crises. Inspired by Taleb (2007), they advocate the importance of mitigating fat-tails risk (dramatic drawdowns) to achieve investors' long-term financial goals. It is not that the higher volatility, the higher return. As mentioned in the introductory section, they report that in the high volatility state, when the volatility is 33%, the return is -38%.[2] They focus on an approach that gives returns that have constant volatility.

After the dot-com bubble burst of 2000 and the subprime mortgage crisis of 2007-2008, many investors became more interested in protecting their portfolios from significant losses due to similar, coming market crashes. Albeverio et al. (2013) discuss that rising volatility can be a good indicator for a falling market. They analyze that the volatility-targeting portfolio may improve the risk-return relationship of an investment portfolio. Nevertheless, they also note some drawbacks of this strategy. Volatility-targeting works well in a falling market with high volatility or in a rising market with low volatility. However, when, e.g., the market is falling

[2]Figure 3 in their paper.

and volatility is low, there will be a high weight of equities, which are losing their value. They suggest that the volatility-targeting approach to asset allocation should be combined with other strategies.

Using Monte Carlo simulations Perchet, de Carvalho, Heckel, and Moulin (2015) identify two effects with the largest explanatory power of higher Sharpe ratio and reduced drawdowns: volatility clustering and fat tails in return distributions. They report that the volatility-targeting strategy works best for equities and high-yield corporate bonds because they have the strongest volatility clustering, fat tails, and negative relationship between returns and volatility.

The study of Moreira and Muir (2017) confirms earlier findings that taking less risk during high volatility periods increases portfolio performance. They explore the mean-variance trade-off of volatility-managed portfolios for the market factor, size, value, momentum, profitability, return on equity, investment, and betting-against-beta factors in equities and currencies carry trade. Their findings suggest that equity portfolios, by taking less risk during times with high volatility, increased Sharpe ratios and produced significant utility gains for mean-variance investors. They explain the increase in Sharpe ratios by changes in volatility that are not offset by proportional changes in expected returns. In other words, the Sharpe ratio shows a ratio between return (nominator) and risk (denominator). Volatility does not strongly forecast future returns, so if one lowers the risk exposure when volatility is high, the Sharpe ratio will increase.

Harvey et al. (2018) widen the body of knowledge by testing the volatility targeting across more than 60 assets in the U.S. market. Similar to previous research, their findings indicate that this strategy works best for risky assets. They document that the volatility-targeting strategy reduces tail risk across all analyzed asset classes. However, the Sharpe ratios improve only for equities and corporate credit.

Grobys and Äijö (2018) look into local risk factors in international equity markets. They extend the Moreira and Muir (2017) study by exploring the risk-return tradeoff in Europe, Asia, and Japan. They are motivated by Fama and French (2017), who find that global risk factors do not include regional risk factors.[3] Grobys and Äijö (2018) note that volatility managing strategy seems to work in Europe and Asia, but there is no evidence for the Japanese market.

Zakamulin (2019) observes that the portfolio manager needs to revise his or her positions as often as possible if he or she wants to benefit from risk control given by the volatility-targeting strategy entirely. He suggests that frequent trading can give substantial transaction costs. The volatility-weighting-over-time mechanism offers some control over portfolio turnover. His

[3]They claimed that average stock returns for North America, Europe, and Asia increase with the book-to-market ratio (B/M) and profitability and are negatively related to investment.

modification is achieved by adding two features: a tuning parameter and a no-transaction region around the targeted risk exposure. The researcher reports that the proposed strategy outperforms both the conventional volatility-targeting strategy and the buy-and-hold strategy.

The question that many investors ask is: does the strategy work in real life (that is, out-of-sample)? Recent studies suggest that the strategy does not work well in practice since it is challenging to implement. Liu et al. (2019) find that volatility timing strategies suffer from look-ahead bias. They correct the bias and notice significant drawdowns in almost all cases. They claim that the strategy exceeds the market only during financial crisis times. They conclude that investors could not easily beat the market by only timing the market. Cederburg et al. (2020) have a similar conclusion that the active volatility managed portfolios do not systematically outperform passive portfolios, and they are not implementable in real-time. The researchers criticize the paper of Moreira and Muir (2017) as it gives the impression that the volatility-managed equity strategies consistently outperform a buy-and-hold strategy.

Bongaerts et al. (2020) come to much the same conclusion as Liu et al. (2019) and Cederburg et al. (2020), that volatility-targeting strategy does not consistently outperform the passive strategy. Moreover, they state that publications of Moreira and Muir (2017) and Harvey et al. (2018) are subject to look-ahead bias as the scaling factor is constructed ex-post and with high portfolio turnover and leverage. Furthermore, they also propose a better solution - to adjust risk exposures only in the extremes during high and low volatility states; otherwise, the exposure is unscaled. They call it conditional volatility-targeting.

The subject of volatility-timing is widely discussed in academia. Partctionaries use to say that "timing is everything," and there were recently more financial crises that caused substantial losses. It is not surprising that the paper of Moreira and Muir (2017) about successful strategy received attention in the financial press and even impacted industry application.[4] One can almost only register positive opinions on managing volatility until 2018. First, the latest publications (e.g., Liu et al. (2019), Cederburg et al. (2020), and Bongaerts et al. (2020)) come with some criticism and some researches, e.g., Zakamulin (2019) and Bongaerts et al. (2020) even try to improve the conventional volatility-targeting strategies.

[4] *Representative examples of recent press coverage include "Re-assessing the classic risk-return tradeoff," The Financial Times, March 9, 2016 and "When markets get scary, panicking is smart," CNBC, March 23, 2016. For an example of volatility management in practice, BlackRock offers the following description of the investment strategy for its Managed Volatility V.I. Fund: "In periods of heightened volatility, the portfolio will de-risk into less volatile assets like fixed income and cash and re-risk when market turbulence subsides."* are all examples given by Cederburg et al. (2020).

3 Data and Methodology

3.1 Data

All the data used in this study come at a daily frequency, although monthly data are often available for a longer period. Nevertheless, we choose to use daily data because our pre-research findings show that daily data work much better than monthly data to obtain responsive volatility estimates. Earlier studies support this, e.g., Fleming et al. (2003) used intraday returns, which made the strategy work even better. Sample periods have different beginnings (January 1, between 1927 and 1992), but all end December 31, 2020. All stock returns are total returns that include dividends. Equities data are obtained from Kenneth French's online data library[5] and Yahoo Finance[6]. Bonds data come from Yahoo Finance. Federal Reserve Economic Data[7] is a source of commodities and foreign exchange rates data. All the necessary details are shown in Table 1.

The risk-free daily rate of return comes from the data library of Kenneth French. It is proxied by the Treasury bill rate. It is used for all U.S. data. For European and Asian stocks, it is used an adequate risk-free rate of return for those regions also from Kenneth French's data library. Using only one appropriate risk-free rate maintains consistency and makes it easier to compare empirical results across assets and asset classes.

3.2 Volatility Forecasting

There is a large strand of literature on volatility forecasting, e.g., Poon and Granger (2003), and Andersen, Bollerslev, Christoffersen, and Diebold (2005), among many others, review some methods. Most of the over a thousand published papers have two main conclusions in common. First, the higher the data frequency, the better. With daily data, one forecast better than with monthly frequency, see, e.g., Fleming et al. (2001). Second, the shorter the forecast horizon, the higher the accuracy of forecasting. When one has daily data and forecast volatility for one day, the forecast is better than, for example, using the same daily data and one forecast for one month, see, e.g., Zakamulin (2019).

Brailsford and Faff (1996) try to determine if relatively complex models are superior to simpler alternatives. They do not find convincing evidence. They conclude that "volatility forecasting is a notoriously difficult task." That is why we decided to concentrate on the simplest volatility forecasting models, which are time-series forecasting based on historical volatility. To

[5] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[6] https://finance.yahoo.com
[7] https://fred.stlouisfed.org

Table 1: Securities, Sample Periods, and Data Sources

Asset Class	Period	Source
Equities		
Equities (All U.S.)	1927-2020	K. French website
Equities (All U.S.)	1927-1959	K. French website
Equities (All U.S.)	1960-1990	K. French website
Equities (All U.S.)	1991-2020	K. French website
Value stocks (Portfolios Formed on Book-to-Market - Lo20)	1927-2020	K. French website
Growth stocks (Portfolios Formed on Book-to-Market - Hi20)	1927-2020	K. French website
Small stocks (Portfolios Formed on Size - Lo20)	1927-2020	K. French website
Large stocks (Portfolios Formed on Size - Hi20)	1927-2020	K. French website
European stocks (Fama/French European 3 Factors)	1991-2020	K. French website
Asian stocks (Fama/French Asia Pacific ex-Japan 3 Factors)	1991-2020	K. French website
North American stocks (Fama/French North American 3 Factors)	1991-2020	K. French website
The Walt Disney Company (DIS)	1963-2020	Yahoo Finance
The Boeing Company (BA)	1991-2020	Yahoo Finance
Intermediate-term bonds		
Vanguard GNMA Fund Investor Shares (VFIIX)	1981-2020	Yahoo Finance
Vanguard Intermediate-Term Treasury Fund Investor Shares (VFITX)	1992-2020	Yahoo Finance
Long-term bonds		
Vanguard Long-Term Investment-Grade Fund Investor Shares (VWESX)	1981-2020	Yahoo Finance
Vanguard Pennsylvania Long-Term Tax-Exempt Fund Investor Shares (VPAIX)	1987-2020	Yahoo Finance
Commodities		
Gold (GOLDPMGBD228NLBM)	1988-2020	Fred Database
Crude Oil (DCOILBRENTEU)	1988-2020	Fred Database
Foreign Exchange Rates		
U.S. / Euro Foreign Exchange Rate (DEXUSEU)	2000-2020	Fred Database
Norway / U.S. Foreign Exchange Rate (DEXNOUS)	1972-2020	Fred Database

The frequency of data used is daily. Some data were available in earlier periods. Data before the start of sample periods are used to initialize volatility measures.

be specific, it is Simple Moving Average (SMA) volatility, described in Section 3.2.1, and Exponentially Weighted Moving Average (EWMA volatility), described in Section 3.2.2. But of course, more complex models like ARCH class conditional volatility models, e.g., Autoregressive Conditional Heteroscedasticity (ARCH(q)) and Generalized Autoregressive Conditional Heteroskedasticity GARCH(p,q)[8] or stochastic volatility models are also possible to use. Even though SMA and EWMA are not the most sophisticated models, accuracy is high enough, and calculations are fast.

[8]In general, ARCH models are models built on the observation that high (low) volatility tends to be followed by high (low) volatility. This characteristic is known as volatility clustering or conditional heteroskedasticity. The main feature of ARCH models, proposed by Engle (1982), is that the variance of the error term in period t depends on the square of the error term in the period (t-1). In GARCH models, proposed by Bollerslev (1986), the error's variance is associated with both previous period errors and its own variance.

3.2.1 SMA Volatility

SMA means Simple Moving Average. This forecasting methodology relies on moving averages with fixed, equal weights $w = \frac{1}{N}$ of squared periodic returns. More details about SMA volatility are described in Longerstaey and Spencer (1996), Alexander (2008), and Zakamulin (2017).

The estimate of SMA volatility (volatility forecast) at time t+1 over N days is computed as:

$$\hat{\sigma}^2_{t+1} = \frac{1}{N} \sum_{k=0}^{N-1} r^2_{t-k}, \tag{1}$$

where r_{t-k} denotes the day $t-k$ return.

Note that

$$\hat{\sigma}^2_t - \sigma^2_{t-1} = \frac{r^2_t - r^2_{t-N}}{N}. \tag{2}$$

Therefore the recursive formula for SMA is

$$\hat{\sigma}^2_t = \sigma^2_{t-1} + \frac{r^2_t - r^2_{t-N}}{N}. \tag{3}$$

It is faster to use Equation 3 instead of using Equation 1.

We examine an SMA of 10, 20, 40, 60, and 90 days in this study. Those are the same number of days as used for EWMA days' half-life.[9]

3.2.2 EWMA Volatility

An Exponentially Weighted Moving Average (EWMA), similarly to SMA, is historical volatility. It is more advanced than simple volatility. As before, good sources to find out more about EWMA volatility are publications of Longerstaey and Spencer (1996), Alexander (2008), and Zakamulin (2017).

EWMA puts more weight on the more recent observations than simple volatility, which weights all past periods equally. In the case of extreme returns move in the past, they become less important in the average as the data window slides along. As a result, EWMA is smoother than SMA volatility, see Figure 3.

Assuming daily data, the estimate of EWMA volatility (volatility forecast) at time t+1 forecast is given by

[9] Harvey et al. (2018) also use 10, 20, 40, 60, and 90-day half-life.

$$\hat{\sigma}^2_{t+1} = \frac{\sum_{k=0}^{N-1} \lambda^k r^2_{t-k}}{\sum_{k=0}^{N-1} \lambda^k}, \tag{4}$$

where λ is constant, called the smoothing or the decay constant, r_{t-k} denotes the day $t-k$ return, and N is the number of days (size of the estimation window).

For simplicity of computations, $N \to \infty$ and, therefore, the volatility can be computed using the following equation:

$$\hat{\sigma}^2_{t+1} = (1-\lambda) \sum_{k=0}^{\infty} \lambda^k r^2_{t-k} \tag{5}$$

Equation 6 presents the recursive formula used to compute the EWMA volatility:

$$\hat{\sigma}^2_t = (1-\lambda) r^2_{t-1} + \lambda \sigma^2_{t-1}. \tag{6}$$

Instead of using Equation 4, it is faster to use Equation 6 in computations.

Longerstaey and Spencer (1996) applied this methodology to both daily and monthly returns. They found out that the smoothing constant for the daily data set is 0.94, and the smoothing constant for the monthly data set is 0.97. As there are used only data at daily frequency, we go a step further and use a half-life of EWMA volatility.

Half-Life of EWMA Volatility

Note the coefficients in front of returns: 1, λ, λ^2, ..., λ^k, and so on. The half-life is the time lag at which the exponential weights decay by one half, that is

$$\lambda^k = \frac{1}{2} \iff k = -\frac{ln2}{ln\lambda} \iff \lambda = (\frac{1}{2})^{\frac{1}{k}}$$

This study will use a half-life of 10, 20, 40, 60, and 90 days. Figure 3 shows a half-life of 10, 40, and 90 days with λ equal to 0.933 ($\lambda = (\frac{1}{2})^{\frac{1}{10}}$) for 10 days, 0.983 ($\lambda = (\frac{1}{2})^{\frac{1}{40}}$) for 40 days and 0.992 ($\lambda = (\frac{1}{2})^{\frac{1}{90}}$) for 90 days.

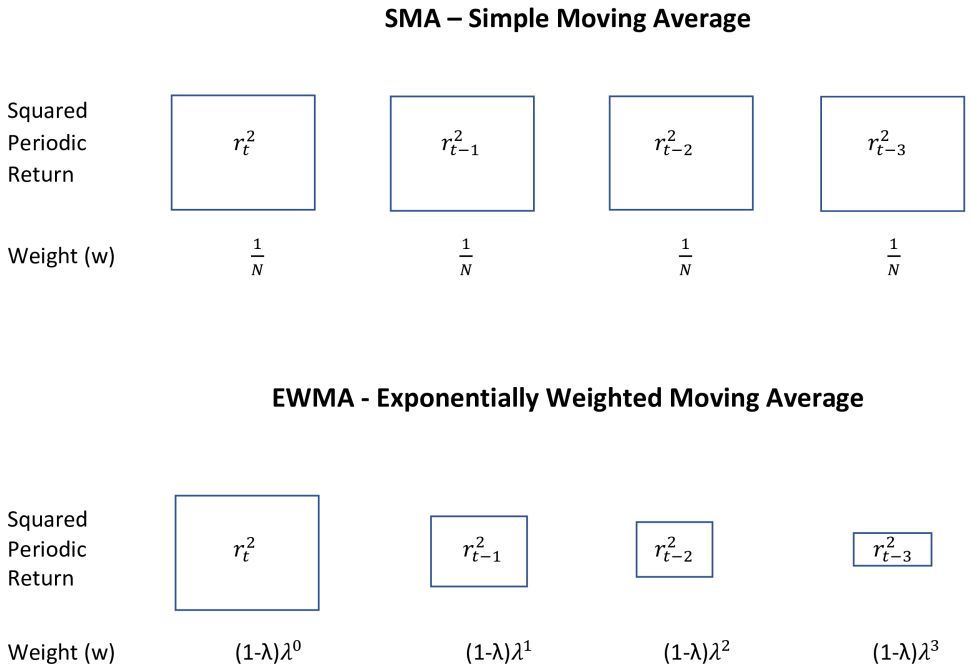
3.2.3 Summary SMA and EWMA Volatility

Both SMA and EWMA are weighted averages of squared returns. Figure 2 shows the main difference between the SMA and EWMA volatility - weights. SMA is an equally weighted moving average. It means that it weights every component of squared periodic return with the same weight $w = \frac{1}{N}$. EWMA has different weights for different squared periodic returns (exponentially weighted). The most recent return has weight 1, the next λ, then λ^2, and so on. The last component has the lowest weight and, in that way, matters least for the EWMA volatility

forecast. EWMA favors more recent returns and gives a quicker signal to recent changes.

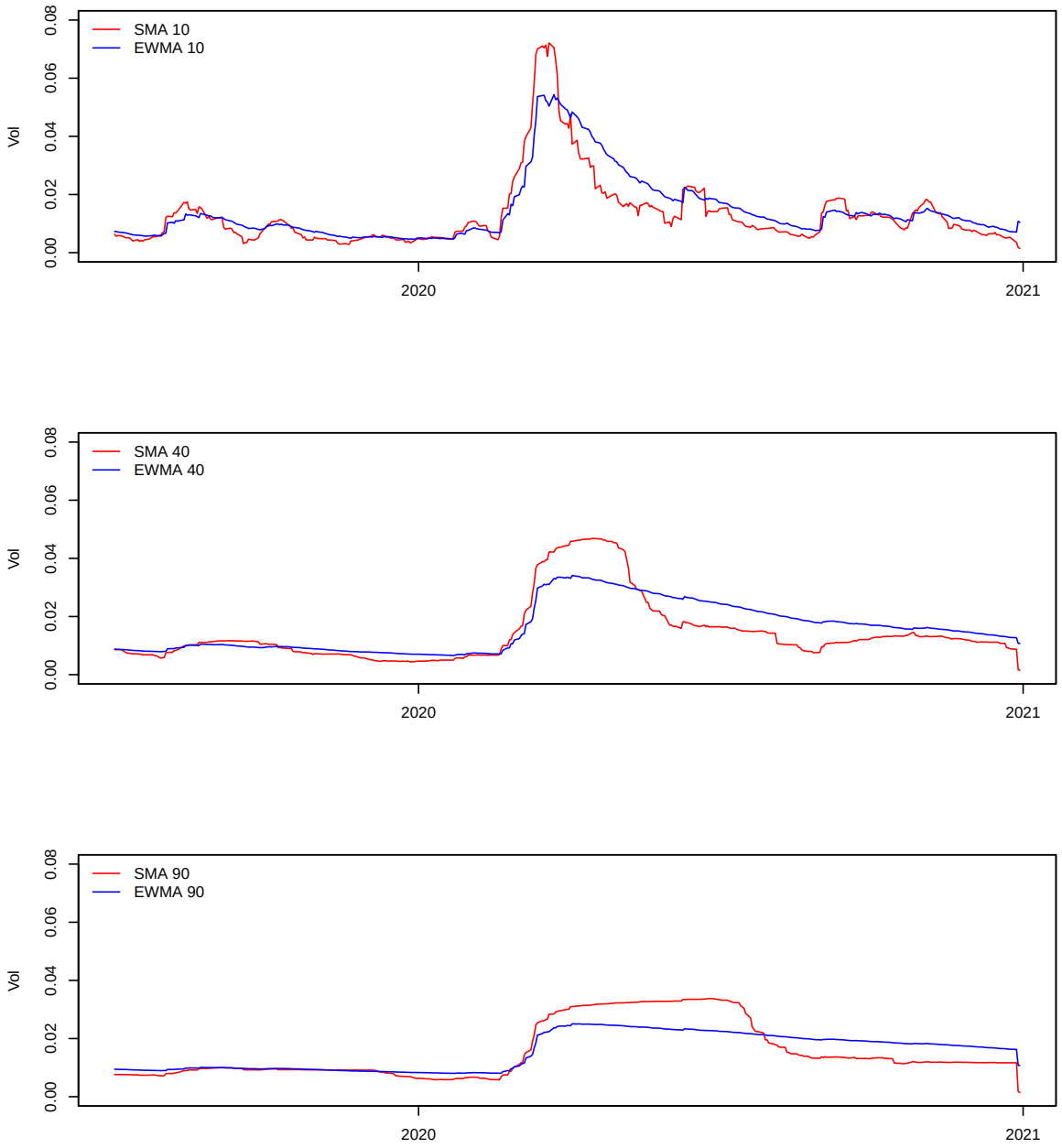
Figure 3 on page 18 visually presents the difference between the SMA volatility and the EWMA volatility at three different time intervals: 10, 40, and 90 days. EWMA smooths the volatility more than SMA. Moreover, as one goes from 10 to 90 days, the volatility is lower for both estimates. Figure 3 shows the period between July 1, 2019 - December 31, 2020. A time when volatility was high in some periods due to the covid-19 pandemic worldwide. According to Alexander (2008), SMA can have a "ghost" of some previous events (days with extremely high volatility). That "ghost" often has nothing to do with the actual market conditions. However, SMA weights are equal, no matter if the event was, e.g., three months ago, like for SMA-90. The situation is different for EWMA, that what happened three months ago is not as relevant as the latest moments in the 90-day volatility series.

Figure 2: SMA vs. EWMA



The first panel shows SMA volatility with fixed, equal weights $w = \frac{1}{N}$ of squared periodic return. The second panel presents EWMA volatility that puts more weight on the more recent observations. The newer the observation, the higher the weight.

Figure 3: Graphs of Different Volatility Forecasts



Dataset used: equities (All U.S.) between July 1, 2019 - December 31, 2020. The first panel presents SMA and EWMA volatilities at 10, second 40, and third 90 days.

3.3 Volatility-Targeting Strategy

The volatility-targeting strategy is an active strategy. According to Perchet et al. (2015), it is about rebalancing between a risky asset and a risk-free asset to target a constant level of risk over time. Zakamulin (2014) writes that the principle of the strategy is to reduce exposure to the risky asset when volatility is high and increase the exposure when volatility is low. Compared

to a buy-and-hold strategy, this strategy is known to improve the Sharpe ratio and reduce drawdowns. Among others, Collie et al. (2011), and Albeverio et al. (2013) describe the piratical implementation of this strategy.

Denoting by w_t the weight of a risky asset in the active portfolio, the return on the active strategy (passive portfolio, which is a benchmark for the strategy) and the risk-free asset is given by

$$r_{a(t)} = w_t r_{p(t)} + (1 - w_t) r_{f(t)}, \tag{7}$$

where $r_{p(t)}$ is the return on the buy-and-hold strategy with weight w_t, and $r_{f(t)}$ is the return on the risk-free asset with weight $(1 - w_t)$.

One can compute the volatility of $r_{a(t)}$ as follows:

$$\sigma(r_{a(t)}) \approx w_t \sigma(r_{p(t)}). \tag{8}$$

The total volatility is the sum of two components: volatility of the risky asset and the risk-free asset in the active portfolio. However, the last component - the volatility of $(1 - w_t) r_{f(t)}$ is very low and can be omitted.

Furthermore, there is an assumption that the volatility of the active strategy should be equal to the target volatility:

$$\sigma(r_{a(t)}) \approx \sigma^{target}, \tag{9}$$

then when Equations 8 and 9 are combined, one receives the following solution:

$$w_t = \frac{\sigma^{target}}{\hat{\sigma}_t}, \tag{10}$$

where $\hat{\sigma}_t$ is forecasted volatility using either SMA or EWMA. Both models are described earlier; see Sections 3.2.1 and 3.2.2.

As already mentioned above, the portfolio consists of a risky asset[10] and a risk-free asset. Since it is an active portfolio, the weight of the risky asset is changed at the end of the period. The risky asset weight for the next period is determined by

$$w_t = \min\left(\frac{\sigma^{target}}{\hat{\sigma}_t}, \quad w^{max} \right), \tag{11}$$

where σ^{target} is the target volatility, $\hat{\sigma}_t$ is the forecasted volatility for day t, and w^{max} is the

[10] In this study: equities, bonds, commodities, and foreign exchange rates.

maximum allowable exposure to risky assets. Here, we notice that if the volatility forecast for the next period turns out to be smaller than the target volatility, the weight of stocks will be greater than 100%. This means that the strategy requires borrowing. In this study, we deal with that by limit the weight of a risky asset by 150%.[11] The weight of a risk-free asset is computed as $1 - w_t$.

There are used both SMA and half-life of EWMA volatility for this strategy in the research. The target volatility in this study is 10%, and it is the same as in the paper of Harvey et al. (2018). However, bonds and currencies also have different target volatility, as volatility of examined bonds and currencies is lower or close to 10%.

3.4 Sharpe Ratio

The Sharpe ratio is a performance measure. It helps investors evaluate portfolio performance and better understand the risk-return tradeoff, one of the main exercises in finance. It was formulated by Sharpe (1966).

Mathematically Sharpe ratio is expressed as the average return earned in excess of the risk-free rate per unit of volatility (total risk):

$$SR = \frac{E[r_p - r_f]}{\sigma_p}, \tag{12}$$

where r_p is the return on a portfolio p, r_f is the return on a risk-free asset, $E[r_p - r_f]$ is the expected excess return, and σ_p is the standard deviation of portfolio p.

It is usual to compare the performance of two or more portfolios. A portfolio with the highest and statistically significant Sharpe ratio is the best performing portfolio. It is essential to test the Sharpe ratio. In order to find out if the measure is significantly different, one can use the test given by Jobson and Korkie (1981) with the Memmel (2003) correction. The null and alternative hypothesis are as follows:

$$H_0 : SR_a = SR_p \quad and \quad H_A : SR_a > SR_p,$$

where SR_a is the Sharpe Ratio of active strategy and SR_p is the Sharpe ratio of passive strategy (benchmark). This is a one sided-test.

[11] (Albeverio et al.,2013) use maximum weight of 200%. Hallerbach (2012) studied EURO STOXX 50 Index in the years 2003-2011. He suggests that restricting leverage allowed in the strategy gives a lower Sharpe ratio. So if there are no restrictions or allowed leverage is, e.g., 300%, the Sharpe ratios would be even higher in this master thesis.

The test statistic is:

$$z = \frac{SR_a - SR_p}{\sqrt{\frac{1}{T}[2(1-\rho) + \frac{1}{2}(SR_a^2 + SR_p^2 - 2SR_aSR_p\rho^2)]}}, \tag{13}$$

where ρ denotes the estimated correlation coefficient between returns over the sample size, T. z is asymptotically distributed as a standard normal. If the p-value is lower than the predeterminate significance level (usually $\alpha = 0.05$), the null hypothesis can be rejected. That indicates that the data generates sufficient evidence against the hypothesis that the Sharpe ratios of two different portfolios are similar.

3.5 Transaction Costs

Zakamulin (2019) suggest that transaction costs usually depend on who is trading. Large institutional investors have low transaction costs of about 0.1%, sometimes less. At the same time, small individual investors have high transaction costs of about 0.5%-1.5% plus some fixed fees. Harvey et al. (2018) use very low transaction cost estimates.[12] It is 1.0 bp (or 0.01%) for equities and 0.5 bp for bonds. We use higher transaction cost estimates, which are expressed as a fraction of notional value traded:

- Equities - 10.0 bps (or 0.1%)
- Bonds - 5.0 bps (or 0.05%)

We do that in order to see the impact of transaction costs better. Nevertheless, our solution is still a simplification and does not capture the complexity of the real world. For example, the underlying assumption is that the portfolio of all U.S. equities is the same in 1927 and 2020. There is no transaction cost assigned to the changes in the benchmark portfolio.

However, in Section 4 and Appendix A, performance tables present only descriptive (or performance) statistics gross of transaction costs. Still, results are pretty similar on a net basis, only a little bit worse.

Net Sharpe ratios for commodities and foreign exchange rates are not tested, as the results are very clear/poor already for gross Sharpe ratios and seldom statistically significant.

[12] Some other researchers use a conservative and complex approach like, e.g., Bongaerts et al. (2020) - 25 bps or Asness, Frazzini, Israel, and Moskowitz (2015) - 18 bps for trading equity factors. Another, e.g., Moreira and Muir (2017), use even three different scenarios with 1 bps, 10 bps, and 14 bps.

3.6 Frequency of Trading

The frequency of trading is relevant because of the tradeoff between transaction cost and forecast accuracy. Usually, the forecast accuracy decreases with the horizon, and the best forecast is for the shortest horizon, i.e., in this case, daily.[13] However, with daily trading, the amount of transaction cost is significant.

Many papers like, e.g., Moreira and Muir (2017) use SMA-22 days (or one month) volatility computed using daily data, but the trading is once a month. Harvey et al. (2018) simulate daily trading. So the question is: How often one has to trade? We try to answer this question by investigating trading executed each k-th day: daily (1 day), weekly (5 days), and monthly (21 days).

The average turnover per month is presented in the tables with performance statistics in Section 4 and Appendix A. Notice that an unscaled position in a particular asset will have zero turnovers.

3.7 Performance Statistics

In this study, there are used some of the typical descriptive statistics. Some of them are also performance statistics. Both names are used in this text: descriptive and performance statistic, but here they mean the same measures. A brief overview of used statistics is in Table 2. Moreover, some of them are explained in previous sections. In most cases, these statistics are calculated at a monthly frequency. Sharpe ratio gross and net help evaluate the volatility-targeting strategy's performance, while the volatility of volatility, maximum drawdown, and left tail are risk measures.

The Volatility of Volatility (Vol of Vol)

The volatility of volatility is the statistic used by Harvey et al. (2018) that directly shows the results of the volatility-targeting strategy when the risk exposure should be constant; in this analysis, 10% volatility for equities and commodities, 5% for currencies, and 2.5 - 4% for bonds.

Maximum Drawdown (MDD)

Maximum drawdown (MDD) is one of the most prevalent risk measures among investors,

[13] As already mentioned, Fleming et al. (2003) indicate that going from daily to intraday returns, i.e., volatility forecast, makes the volatility timing strategies even better.

probably because it is an indicator of downside risk.[14] It measures the largest single drop from peak to bottom in the value of a portfolio before a new peak is attained. The formula for the MDD is

$$MDD = \frac{P-L}{P},$$

where P denotes peak value before the largest drop and L is the lowest value before a new high is established.

Average Turnover (Turnover)

Average turnover is used to account for transaction costs. Here deducted from Equation 7

$$r_{a(t)} = w_t r_{p(t)} + (1-w_t) r_{f(t)} - |w_{t-1} - w_t| tc, \tag{14}$$

where $|w_{i-1} - w_i|$ is turnover and tc stands for transaction costs.

Turnover is the sum of all $|w_{i-1} - w_i|$ where w_i is the weight of risky assets in the portfolio. Average turnover per month is computed by taking the mean of turnover and multiple it by 21.

Left Tail (LT)

Standard deviation (variance) is the traditional measure of risk. It increases when either the left tail or right tail (or both) of the probability distribution increases. However, the risk comes from the left tail of the distribution where the negative returns (losses) are located. The right tail of the distribution, the return potential, has nothing to do with risk. The idea is that a proper risk measure must be related only to the downside risk, that is, to measure potential losses. Longerstaey and Spencer (1996) observe that market risk is recently almost synonymous with the term Value at Risk.

According to Longerstaey and Spencer (1996), VaR is the amount that a portfolio might lose, with a given probability $(1-\alpha)$, over a given period. α here is 0.05 and 0.01. The time horizon in this study is counted in years.[15] VaR is defined as a threshold value. The VaR of x at the confidence level $(1-\alpha), \alpha \in (0,1)$ is computed as

$$VaR_\alpha(x) = -F^{-1}(\alpha),$$

where F^{-1} is the inverse of the distribution function F.

[14] According to Magdon-Ismail and Atiya (2004), it shows how sustained one's losses can be.

[15] The exact period is given in Table 1, and headings of each performance table in Section 4 and Appendix A.

However, the value itself is not in focus in this study, as VaR says nothing about the magnitude of the potential loss. VaR is used to calculate the conditional VaR (CVaR)[16], called left tail (LT). Harvey et al. (2018) use the same name for CVaR. Alexander, Coleman, and Li (2006) support this and advocate that CVaR (left tail) compared to VaR is more attractive since it is a coherent risk measure.

As mentioned before, investors see risk as left tail risk - a downside of an investment. A substantial right tail is welcome as it gives higher returns, not losses. That is why we concentrate here on the left tail, as it represents risk. Nevertheless, notice that usually, when the left tail is smaller, the right tail is smaller also. The statistic is calculated by first finding Value at Risk at the 5th and 1st quantile and then taking the mean of all the returns below Value at Risk (VaR). It is formally given by

$$CVaR_{\alpha}(x) = -E[x|x \leq -VaR_{\alpha}(x)],$$

which means that *CVaR* gives the expected loss in the worst α cases.

Table 2: Description of Performance Statistics

Statistics	Abbrev	Description
Sharpe ratio	SR (Gross)	Details in section 3.4; p-value from the SR test is in brackets
Sharpe ratio Net	SR Net	SR net of transaction costs; transaction cost estimates are in section 3.5
Mean return	Mean	Mean return (annualized)
Volatility	Vol	Standard deviation computed by using monthly returns (annualized)
Volatility of volatility	Vol of Vol	Standard deviation of the volatility estimate above
Maximum Drawdown	MDD	The largest single drop from peak to bottom in the value of a portfolio
Average turnover	Turnover	Average turnover per month; details in section 3.6
Left tail	LT	Mean of returns below the 1st and 5th percentile

Performance Statistics used in tables in Section 4 and Appendix A.

[16]Other names of the same measure are the expected shortfall (ES), the mean shortfall, the average VaR (AVar), or the Expected Tail Loss.

4 Empirical Results

4.1 Equities

This part of the thesis presents some specifics about using the volatility-targeting strategy for equities. It starts with a deep analysis of equities (All U.S.) in 1927-2020 in Section 4.1.1. After that, the main focus will be on the highest/best Sharpe ratios and risk measures. We will try to find out which scaling and what frequency of trading gives the best performance. Value and growth stocks are described in Section 4.1.2, small and large stocks in Section 4.1.3, European, Asian, and North American stocks in Section 4.1.5, and some individual stocks in Section 4.1.4.

4.1.1 Equities (All U.S.)

Unscaled Equity Returns since 1926

Figure 4 plots daily and monthly U.S. equity returns in excess of the risk-free rate for the 1926-2020 period. Harvey et al. (2018) present similar graphs and suggest that it is clear that volatility tends to cluster. In the time during the Great Depression in the 1930s, the volatility of returns is consistently high. The same is true for other crises like the dot-com bubble in the late 1990s, the global financial crisis in 2007-2009, and the corona crisis in 2020. Black Monday (October 19, 1987) was the most damaging day in the analyzed period. It is also clearly visible. Engle (1982) is a pioneer in work on autoregressive conditional heteroscedasticity (ARCH) models, which show the persistence of volatility. His research underpins Harvey et al. (2018) observation about volatility clustering.

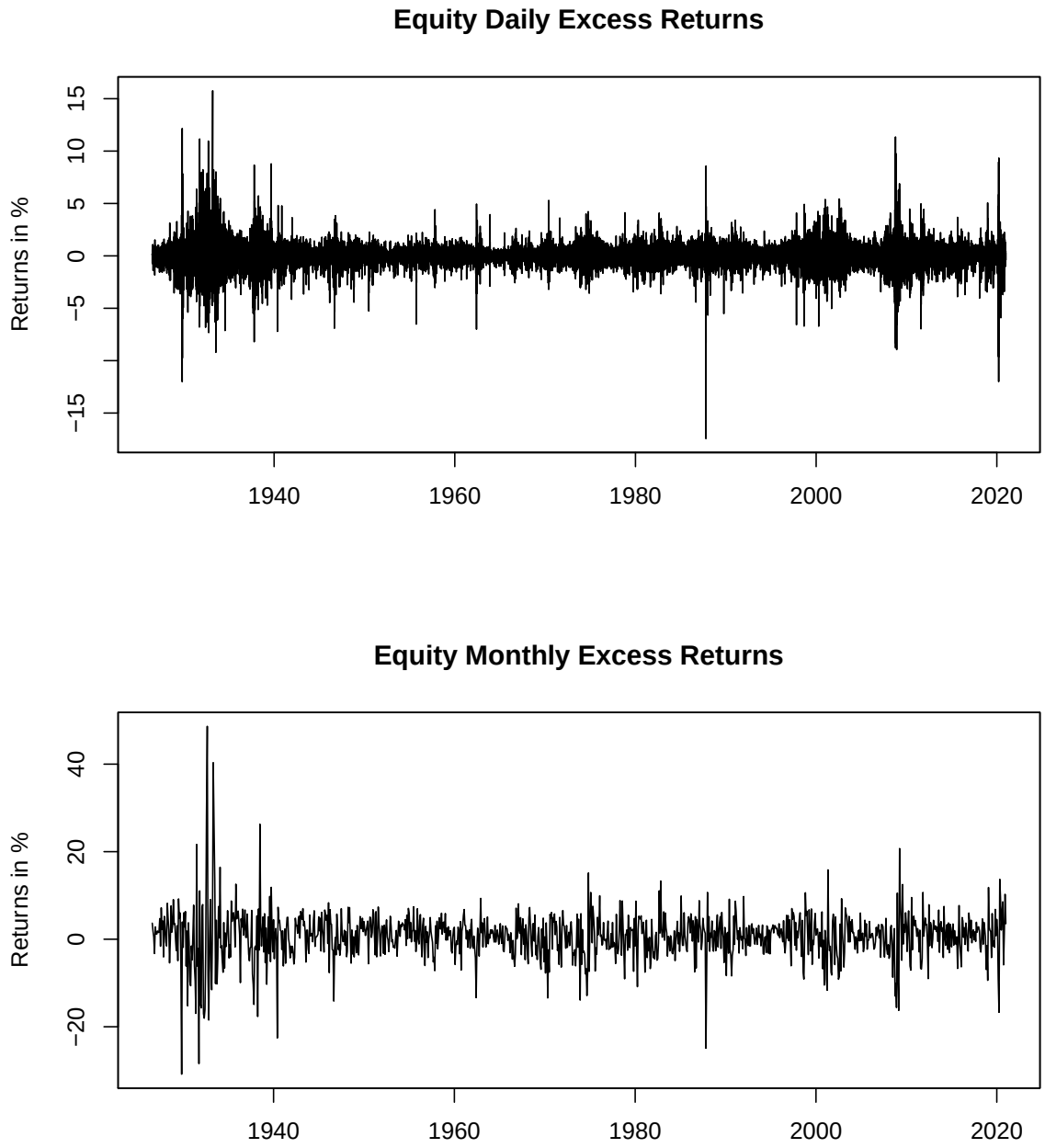
Performance of Volatility-Scaled Equity Returns

Table 3 presents performance measures and descriptive statistics for unscaled (top row) and volatility-scaled (other rows) Equities (All U.S.) (1927-2020). For scaling, there is used either SMA volatility or EWMA volatility. For details on volatility forecasting, see Section 3.2. One can observe that both gross and net Sharpe ratios of active strategy are always higher than the buy-and-hold strategy for the whole analyzed period. The risk is lower for the volatility-targeting strategy than for the benchmark. Both volatility of volatility, maximum drawdown, and left tail decreased in value after implementing active strategy.

Concrete improvements are analyzed here. The gross Sharpe ratio improves from 0.44 (unscaled) to between 0.50 and 0.62 (volatility scaled). Harvey et al. (2018), Moreira and Muir (2017), and Dopfel and Ramkumar (2013) also find that volatility targeting improves the Sharpe ratio for equities since 1927. The net Sharpe ratio[17] improves to between 0.46 and 0.56 (volatil-

[17] See details about used transaction cost in Section 3.5.

Figure 4: Equities All U.S. Returns (1926-2020)



The panels show daily and monthly U.S. equity returns in excess of the risk-free rate for the 1926-2020 period. No volatility scaling was applied.

ity scaled). Interestingly, the best and statistically significant gross Sharpe ratio is for SMA-10 volatility with weekly trading. However, for the net Sharpe ratio, it is EWMA-60 with weekly trading and EWMA-90 with daily and weekly trading. The realized volatility is slightly above the target of 10%. However, the volatility of volatility is substantially lower. It goes down from 9.25% (unscaled) to 2.90%-4.37% (scaled). The mean return of unscaled equities (11.39%) is

slightly higher than for scaled equities (9.31%-10.83%). Maximum drawdown is substantially lower for scaled equities (22.88%-46.59%) than unscaled (62.89%). The same is about the left tail. The performance measures show clear outperforming of volatility-targeting strategy to benchmark.

Although the robustness check gives not that clear evidence in all analyzed periods, the performance statistics for 1927-1959 show even better Sharpe ratios for the volatility-targeting strategy. This was the historical period with the highest volatility. At this time, there was a Great Depression. However, for the years 1960-1990, is not found higher and significant evidence for that. For years 1991-2020, Sharpe ratios are mostly higher but not significant for scaled equities. For detailed performance measures, see Appendix A, Tables 24, 25, and 26. Those findings are consistent with those of Liu et al. (2019). In their article, the authors pointed out that volatility-timing strategies outperform the benchmark only during the financial crisis period.[18]

Figure 5 shows a comparison of unscaled and volatility-scaled returns. The latter uses a volatility estimate based on an SMA of 10 days with weekly trading. In the top panel, there are plots of cumulative returns. One can observe that volatility-scaled investment generally outperformed unscaled investment. The impact of volatility scaling is illustrated in the bottom panel. The realized volatility of volatility-scaled returns is much more stable over time, and the volatility of volatility is 3.15%, while the volatility of volatility for unscaled investment is 9.25%.

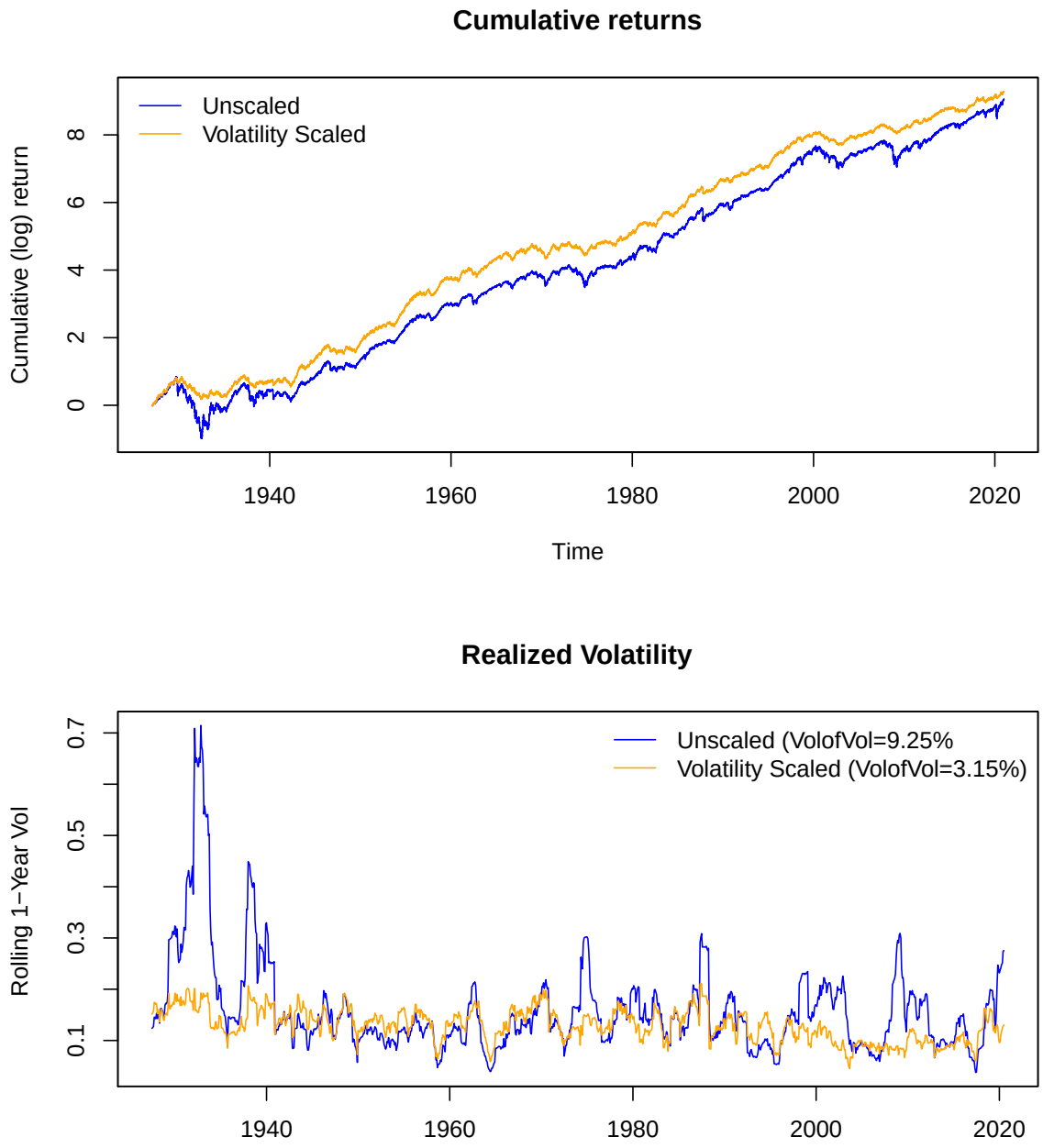
[18]The researchers break down their entire sample period into approximately 20-year subperiods. They find that the volatility-managed portfolio underperformed the market almost half of the time. However, during the recent financial crisis, the strategy outperformed the benchmark.

Table 3: Performance Statistics for Equities - All U.S. (1927-2020)

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.44		11.39	18.58	9.25	62.89	0	-11.47	-6.54
Scaled with SMA-10 Volatility (10-day window)									
Daily	**0.59** (0.01)	0.47 (0.36)	10.35	13.21	2.90	22.88	115.28	-6.48	-4.21
Weekly	**0.62** (0.00)	**0.55** (0.04)	10.83	13.53	3.15	27.94	66.86	-6.76	-4.30
Monthly	0.48 (0.24)	0.46 (0.39)	9.86	14.52	4.37	46.59	27.00	-8.32	-4.73
Scaled with SMA-20 Volatility (20-day window)									
Daily	**0.57** (0.01)	0.50 (0.16)	9.93	12.95	2.91	24.86	60.76	-6.41	-4.10
Weekly	**0.57** (0.01)	0.53 (0.06)	10.11	13.13	3.13	27.37	37.03	-6.69	-4.18
Monthly	**0.54** (0.04)	0.52 (0.09)	9.95	13.43	3.47	31.85	22.16	-7.31	-4.38
Scaled with SMA-40 Volatility (40-day window)									
Daily	**0.56** (0.01)	0.53 (0.06)	9.74	12.68	3.06	27.44	31.60	-6.43	-4.04
Weekly	**0.56** (0.01)	**0.54** (0.03)	9.85	12.79	3.18	29.24	20.16	-6.65	-4.10
Monthly	0.52 (0.07)	0.51 (0.11)	9.64	13.19	3.66	33.56	13.29	-7.25	-4.31
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.56** (0.01)	**0.54** (0.03)	9.73	12.63	3.16	30.09	21.55	-6.45	-4.03
Weekly	**0.56** (0.01)	**0.55** (0.02)	9.82	12.72	3.29	32.09	14.21	-6.66	-4.08
Monthly	0.51 (0.10)	0.50 (0.14)	9.48	13.10	3.78	35.53	9.98	-7.30	-4.28
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.55** (0.02)	**0.53** (0.04)	9.57	12.54	3.25	32.55	14.72	-6.59	-4.05
Weekly	**0.55** (0.02)	**0.54** (0.03)	9.63	12.63	3.40	34.19	10.04	-6.79	-4.09
Monthly	0.50 (0.14)	0.49 (0.18)	9.31	12.95	3.80	36.13	7.41	-7.42	-4.28
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	**0.57** (0.01)	0.50 (0.14)	9.82	12.73	3.01	26.38	58.22	-6.35	-4.01
Weekly	**0.56** (0.01)	0.53 (0.06)	9.88	12.88	3.23	31.96	31.40	-6.64	-4.09
Monthly	0.52 (0.07)	0.51 (0.12)	9.63	13.17	3.54	32.89	15.98	-7.20	-4.27
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**0.57** (0.01)	**0.53** (0.05)	9.67	12.45	3.09	28.96	30.36	-6.29	-3.93
Weekly	**0.56** (0.01)	**0.54** (0.03)	9.71	12.54	3.22	31.86	17.66	-6.52	-4.00
Monthly	0.52 (0.07)	0.51 (0.10)	9.46	12.82	3.54	32.89	10.43	-7.06	-4.17
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	**0.57** (0.01)	**0.55** (0.02)	9.59	12.18	3.20	28.53	15.56	-6.30	-3.89
Weekly	**0.56** (0.01)	**0.55** (0.01)	9.61	12.23	3.26	29.86	9.70	-6.48	-3.93
Monthly	**0.53** (0.04)	0.52 (0.06)	9.39	12.45	3.52	31.56	6.33	-6.92	-4.07
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	**0.57** (0.01)	**0.56** (0.01)	9.54	12.02	3.29	29.07	10.37	-6.32	-3.87
Weekly	**0.56** (0.01)	**0.56** (0.01)	9.56	12.06	3.33	30.08	6.79	-6.47	-3.90
Monthly	**0.53** (0.03)	**0.53** (0.04)	9.37	12.23	3.54	31.16	4.60	-6.83	-4.02
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	**0.57** (0.00)	**0.56** (0.01)	9.47	11.87	3.43	29.52	6.94	-6.35	-3.86
Weekly	**0.56** (0.01)	**0.56** (0.01)	9.47	11.91	3.46	30.25	4.82	-6.47	-3.89
Monthly	**0.54** (0.02)	**0.53** (0.02)	9.32	12.04	3.62	30.91	3.34	-6.77	-3.98

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR. SR highlighted in red points to the lowest SR.

Figure 5: Cumulative Returns and Realized Volatility for Equities (All U.S.) (1927-2020)



The top panel shows the growth of wealth from the volatility-targeting portfolio (scaled) versus the benchmark (unscaled). The bottom panel shows the unscaled and scaled volatility (SMA-10) with weekly trading. Both over the period 1927-2020.

4.1.2 Value and Growth Stocks

In this section, value and growth stocks are in focus. As before for all U.S. equities, the volatility-targeting strategy outperforms the passive strategy. All Sharpe ratios are always higher for scaled portfolios than for unscaled. Furthermore, SMA-10 with weekly trading gives outstanding results.

Here are described specific improvements. The value stocks give the best and statistically significant gross Sharpe ratio (0.59) for SMA-10 and weekly trading. The Sharpe ratio of unscaled investment is 0.42. There is a few best and statistically significant net Sharpe Ratio with a transaction cost of 10.0 bps. They are 0.53 and are for SMA-10, SMA-60, EWMA-60, and EWMA-90 with a weekly frequency. For the last two also with daily frequency. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 4 presents the details. The volatility of volatility is substantially lower. It goes down from 8.51% (unscaled) to 2.78%-3.58% (scaled). The mean return of unscaled equities (11.25%) is higher than for scaled equities (8.97%-10.56%). Maximum drawdown is also lower for scaled equities (25.20%-37.55%) than unscaled (56.39%). The same is about the left tail risk. It goes down from -11.55% to -7.07% - -6.06% for 1% and from -6.76% to -4.56% - -3.79% for 5%. Like for the market portfolio, the performance measures show clear outperforming of volatility-targeting strategy to benchmark. Risk measures indicate a lower risk for scaled equities.

The growth stocks give the best and statistically significant gross Sharpe ratio (0.65) for SMA-10 and weekly trading. The net Sharpe ratio is best and statistically significant for SMA-40 with weekly trading (0.62). The Sharpe ratio of unscaled investment is 0.48. Other risk measures indicate lower risk when using active strategy. Table 5 presents the details. The volatility of volatility is substantially lower. It goes down from 17.49% (unscaled) to 3.06%-4.79% (scaled). The mean return of unscaled equities (15.81%) is higher than for scaled equities (10.09%-11.56%). Maximum drawdown is also lower for scaled equities (24.28%-56.48%) than unscaled (115.32%). The same is about the left tail risk. It goes down from -16.04% to -8.35% - -6.09% for 1% and from -8.86% to -4.76% - -3.79% for 5%. The performance measures show clear outperforming of volatility-targeting strategy to benchmark for the market portfolio and value stocks. Also, the risk is lower.

The growth stocks have higher volatility than the value stocks. One can observe that the volatility-targeting strategy reduces more risk for growth stocks. The maximum drawdowns are notably more reduced. The same applies to risk in the left tail and volatility of volatility. However, the empirical results are pretty similar to all U.S. equities analyzed in Section 4.1.1.

Table 4: Performance Statistics for Equities - Value Stocks (1927-2020)

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.42		11.25	18.88	8.51	56.39	0	-11.55	-6.76
Scaled with SMA-10 Volatility (10-day window)									
Daily	**0.55** (0.01)	0.43 (0.46)	9.88	13.02	2.78	25.20	114.11	-6.35	-4.16
Weekly	**0.59** (0.00)	**0.53** (0.03)	10.56	13.29	2.98	27.14	65.42	-6.52	-4.22
Monthly	0.50 (0.08)	0.47 (0.18)	9.79	13.92	3.58	33.90	26.00	-7.67	-4.56
Scaled with SMA-20 Volatility (20-day window)									
Daily	**0.54** (0.02)	0.47 (0.17)	9.56	12.74	2.78	27.29	59.20	-6.22	-4.02
Weekly	**0.55** (0.01)	**0.52** (0.04)	9.85	12.88	2.97	28.38	35.63	-6.41	-4.09
Monthly	**0.51** (0.04)	0.49 (0.08)	9.61	13.14	3.22	31.69	20.75	-7.05	-4.29
Scaled with SMA-40 Volatility (40-day window)									
Daily	**0.53** (0.01)	0.50 (0.06)	9.41	12.48	2.98	29.56	29.98	-6.24	-3.96
Weekly	**0.54** (0.01)	**0.52** (0.02)	9.57	12.56	3.07	30.52	18.96	-6.40	-4.01
Monthly	**0.51** (0.04)	0.50 (0.07)	9.41	12.90	3.41	34.37	12.37	-6.95	-4.20
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.54** (0.01)	**0.52** (0.03)	9.44	12.42	3.06	31.92	20.33	-6.24	-3.95
Weekly	**0.54** (0.01)	**0.53** (0.01)	9.54	12.48	3.15	33.15	13.20	-6.41	-3.99
Monthly	0.50 (0.06)	0.49 (0.09)	9.22	12.76	3.49	37.55	9.23	-6.98	-4.17
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.53** (0.01)	**0.51** (0.03)	9.27	12.27	3.11	34.60	13.78	-6.36	-3.96
Weekly	**0.53** (0.01)	**0.52** (0.02)	9.34	12.33	3.20	35.35	9.28	-6.52	-4.00
Monthly	0.48 (0.10)	0.47 (0.12)	9.02	12.56	3.45	36.19	6.76	-7.07	-4.16
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	**0.55** (0.01)	0.49 (0.11)	9.55	12.50	2.91	28.70	55.68	-6.10	-3.91
Weekly	**0.55** (0.01)	**0.52** (0.04)	9.64	12.62	3.12	33.40	29.72	-6.37	-3.99
Monthly	**0.51** (0.04)	0.50 (0.07)	9.40	12.84	3.28	32.43	14.70	-6.89	-4.16
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**0.54** (0.01)	**0.51** (0.04)	9.36	12.21	2.98	31.22	28.68	-6.06	-3.85
Weekly	**0.54** (0.01)	**0.52** (0.02)	9.40	12.28	3.10	33.69	16.46	-6.27	-3.91
Monthly	**0.50** (0.04)	0.49 (0.07)	9.17	12.50	3.30	33.58	9.53	-6.76	-4.07
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	**0.54** (0.01)	**0.52** (0.01)	9.25	11.94	3.07	31.61	14.48	-6.08	-3.81
Weekly	**0.54** (0.01)	**0.53** (0.01)	9.28	11.98	3.11	32.14	8.94	-6.24	-3.85
Monthly	**0.51** (0.03)	**0.50** (0.04)	9.06	12.15	3.30	31.88	5.74	-6.66	-3.98
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	**0.54** (0.00)	**0.53** (0.01)	9.19	11.78	3.14	30.38	9.67	-6.11	-3.79
Weekly	**0.54** (0.00)	**0.53** (0.01)	9.21	11.80	3.16	30.39	6.30	-6.24	-3.83
Monthly	**0.51** (0.02)	**0.51** (0.03)	9.03	11.95	3.33	31.07	4.20	-6.58	-3.93
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	**0.54** (0.00)	**0.53** (0.01)	9.11	11.62	3.25	29.53	6.49	-6.14	-3.79
Weekly	**0.53** (0.00)	**0.53** (0.01)	9.12	11.64	3.26	30.05	4.52	-6.25	-3.81
Monthly	**0.51** (0.02)	**0.51** (0.02)	8.97	11.76	3.40	30.80	3.05	-6.53	-3.90

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR. SR highlighted in red points to the lowest SR.

Table 5: Performance Statistics for Equities - Growth Stocks (1927-2020)

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.48		15.81	28.06	17.49	115.32	0	-16.04	-8.86
Scaled with SMA-10 Volatility (10-day window)									
Daily	**0.60** (0.03)	0.48 (0.47)	10.79	13.98	3.06	24.28	112.51	-6.65	-4.26
Weekly	**0.65** (0.00)	0.58 (0.06)	11.56	14.45	3.34	30.09	65.14	-6.89	-4.34
Monthly	0.55 (0.15)	0.52 (0.26)	10.86	15.54	4.79	56.48	25.67	-8.35	-4.76
Scaled with SMA-20 Volatility (20-day window)									
Daily	**0.62** (0.01)	0.56 (0.10)	10.72	13.56	3.11	27.67	56.14	-6.36	-4.08
Weekly	**0.64** (0.01)	**0.60** (0.03)	11.05	13.81	3.35	32.73	33.90	-6.58	-4.14
Monthly	**0.62** (0.01)	**0.60** (0.03)	11.00	14.14	3.86	37.91	19.27	-7.09	-4.31
Scaled with SMA-40 Volatility (40-day window)									
Daily	**0.61** (0.01)	**0.58** (0.05)	10.42	13.25	3.21	28.01	27.93	-6.20	-3.98
Weekly	**0.63** (0.01)	**0.61** (0.01)	10.71	13.40	3.36	30.85	17.86	-6.33	-4.02
Monthly	**0.60** (0.02)	**0.59** (0.03)	10.67	13.92	4.04	38.27	11.46	-6.94	-4.20
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.62** (0.01)	**0.60** (0.02)	10.48	13.20	3.34	30.36	18.80	-6.24	-3.96
Weekly	**0.63** (0.01)	**0.62** (0.01)	10.67	13.31	3.49	33.27	12.38	-6.39	-4.00
Monthly	**0.59** (0.03)	**0.58** (0.04)	10.45	13.74	4.07	39.73	8.23	-6.95	-4.15
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.59** (0.03)	**0.58** (0.04)	10.13	13.08	3.46	32.59	12.56	-6.39	-3.98
Weekly	**0.60** (0.02)	**0.59** (0.03)	10.29	13.19	3.61	35.40	8.68	-6.52	-4.01
Monthly	0.57 (0.06)	0.57 (0.07)	10.16	13.53	4.10	39.77	6.10	-7.00	-4.14
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	**0.62** (0.01)	0.57 (0.08)	10.52	13.26	3.12	28.15	51.80	-6.18	-3.95
Weekly	**0.63** (0.01)	**0.60** (0.03)	10.65	13.46	3.27	31.29	27.68	-6.40	-4.02
Monthly	**0.60** (0.02)	**0.59** (0.03)	10.65	13.84	3.93	39.09	13.38	-6.91	-4.18
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**0.62** (0.01)	**0.59** (0.03)	10.33	12.96	3.21	29.94	26.39	-6.10	-3.88
Weekly	**0.62** (0.01)	**0.61** (0.02)	10.44	13.10	3.35	32.38	15.14	-6.29	-3.93
Monthly	**0.60** (0.02)	**0.59** (0.03)	10.40	13.46	3.91	38.04	8.62	-6.75	-4.06
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	**0.62** (0.01)	**0.61** (0.02)	10.21	12.72	3.40	30.82	13.10	-6.09	-3.83
Weekly	**0.62** (0.01)	**0.61** (0.01)	10.30	12.82	3.52	32.54	8.16	-6.25	-3.86
Monthly	**0.61** (0.01)	**0.60** (0.02)	10.28	13.10	3.93	35.97	5.13	-6.59	-3.96
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	**0.62** (0.01)	**0.61** (0.01)	10.15	12.58	3.55	30.70	8.67	-6.11	-3.81
Weekly	**0.62** (0.01)	**0.61** (0.01)	10.23	12.68	3.66	32.07	5.75	-6.24	-3.84
Monthly	**0.61** (0.01)	**0.60** (0.01)	10.22	12.90	3.98	34.84	3.72	-6.51	-3.91
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	**0.62** (0.01)	**0.61** (0.01)	10.09	12.47	3.73	30.64	5.74	-6.14	-3.79
Weekly	**0.62** (0.01)	**0.61** (0.01)	10.15	12.55	3.83	31.85	4.16	-6.24	-3.81
Monthly	**0.61** (0.01)	**0.61** (0.01)	10.16	12.73	4.07	34.72	2.71	-6.44	-3.87

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR. SR highlighted in red points to the lowest SR.

4.1.3 Small and Large Stocks

As in previous sections, we examine if the active strategy outperforms the passive strategy, and if yes, what volatility-scaling and trading frequency gives the best results. In general, one can observe the same pattern of Sharpe ratios as for equities analyzed before. However, the net Sharpe ratio for SMA-10 with daily trading is lower for scaled small stocks (0.42, not statistically significant) than for unscaled ones (0.48).

Here follows the description of some concrete improvements. The small stocks give the best and statistically significant gross Sharpe ratio (0.61) for EWMA-60 and EWMA-90, both with weekly trading. The Sharpe ratio of unscaled investment is 0.48, precisely like growth stocks (see table 5). EWMA-60 with weekly trading and EWMA-90 (with all trading regimes) also give the best and statistically significant net Sharpe ratio of 0.60. SMA-10 with weekly trading is no longer the best option to use for small stocks. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 6 presents the details. The volatility of volatility is substantially lower. It goes down from 18.33% (unscaled) to 4.22%-6.18% (scaled). The mean return of unscaled equities (15.52%) is higher than for scaled equities (10.27%-11.15%). Maximum drawdown is also lower for scaled equities (30.29%-66.74%) than unscaled (127.46%). The same is about the left tail risk. It goes down from -13.94% to -8.97% - -6.77% for 1% and from -7.82% to -4.05% - -3.79% for 5%. Most of the performance measures show outperforming of volatility-targeting strategy to benchmark for most of the times.

The large stocks give the best and statistically significant gross Sharpe ratio (0.61) for SMA-10 and weekly trading. This is consistent with all U.S. equities, growth, and value stocks. The net Sharpe ratio is best and statistically significant for EWMA-60 with daily and weekly trading and EWMA-90 with daily trading (0.55). The Sharpe ratio of unscaled investment is 0.43. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 7 presents the details. The volatility of volatility is substantially lower. It goes down from 8.86% (unscaled) to 2.84%-4.03% (scaled). The mean return of unscaled equities (10.98%) is slightly higher than for scaled equities (9.10%-10.45%). Maximum drawdown is also lower for scaled equities (22.62%-43.36%) than unscaled (60.44%). The same is about the left tail risk. It goes down from -11.38% to -7.13% - -6.13% for 1% and from -6.52% to -4.61% - -3.78% for 5%. As for most of the analyzed equities so far, the performance measures show clear outperforming of volatility-targeting strategy to benchmark.

Small stocks show some similarities with growth stocks and large stocks with value stocks. It is not surprising because growth stocks are usually small stocks and value stocks are usually large stocks.

Table 6: Performance Statistics for Equities - Small Stocks (1927-2020)

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.48		15.52	30.87	18.33	127.46	0	-13.94	-7.82
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.53 (0.20)	0.42 (0.19)	10.35	17.40	4.22	30.29	102.48	-6.95	-4.39
Weekly	0.54 (0.14)	0.48 (0.50)	10.93	18.54	5.19	53.27	64.10	-7.61	-4.62
Monthly	0.50 (0.34)	0.48 (0.50)	11.04	19.77	6.11	53.49	26.65	-8.97	-5.07
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.57 (0.07)	0.51 (0.29)	10.58	17.18	4.32	34.84	52.30	-6.77	-4.26
Weekly	0.55 (0.11)	0.51 (0.27)	10.75	17.98	5.27	53.65	35.20	-7.34	-4.42
Monthly	0.56 (0.09)	0.53 (0.17)	11.15	18.74	5.87	54.53	21.98	-8.09	-4.66
Scaled with SMA-40 Volatility (40-day window)									
Daily	**0.58** (0.04)	0.55 (0.11)	10.66	17.18	4.63	38.85	27.35	-6.86	-4.22
Weekly	**0.57** (0.05)	0.55 (0.10)	10.82	17.69	5.21	52.05	19.45	-7.25	-4.34
Monthly	0.54 (0.14)	0.53 (0.20)	10.86	18.47	6.00	58.99	13.74	-8.07	-4.58
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.60** (0.02)	**0.58** (0.04)	10.84	17.19	4.86	44.79	18.65	-6.97	-4.21
Weekly	**0.59** (0.03)	**0.57** (0.05)	10.96	17.66	5.50	59.31	13.82	-7.35	-4.31
Monthly	0.55 (0.10)	0.54 (0.14)	10.89	18.30	6.14	64.40	10.03	-8.06	-4.51
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.58** (0.03)	0.56 (0.06)	10.62	17.12	5.00	48.82	12.80	-7.16	-4.23
Weekly	**0.57** (0.05)	0.56 (0.08)	10.74	17.56	5.64	62.12	9.93	-7.51	-4.31
Monthly	0.54 (0.14)	0.53 (0.17)	10.69	18.12	6.18	66.74	7.62	-8.15	-4.49
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.55 (0.11)	0.50 (0.37)	10.27	17.06	4.49	40.36	48.40	-6.79	-4.18
Weekly	0.56 (0.07)	0.53 (0.17)	10.65	17.50	4.93	49.28	29.13	-7.21	-4.30
Monthly	0.55 (0.12)	0.53 (0.19)	10.78	18.19	5.72	56.16	15.75	-7.92	-4.52
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**0.57** (0.05)	0.54 (0.12)	10.42	16.85	4.69	45.81	25.71	-6.81	-4.11
Weekly	**0.58** (0.03)	0.56 (0.07)	10.70	17.19	5.10	54.44	16.75	-7.18	-4.20
Monthly	0.56 (0.07)	0.55 (0.10)	10.78	17.82	5.80	61.05	10.43	-7.80	-4.39
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	**0.59** (0.02)	**0.58** (0.03)	10.57	16.65	4.98	49.28	13.71	-6.87	-4.07
Weekly	**0.60** (0.01)	**0.59** (0.02)	10.77	16.93	5.35	56.97	9.60	-7.15	-4.13
Monthly	**0.58** (0.03)	**0.57** (0.04)	10.82	17.40	5.88	61.63	6.53	-7.62	-4.27
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	**0.60** (0.01)	**0.59** (0.02)	10.60	16.52	5.18	49.86	9.39	-6.91	-4.06
Weekly	**0.61** (0.01)	**0.60** (0.01)	10.78	16.75	5.49	56.72	6.94	-7.13	-4.11
Monthly	**0.59** (0.01)	**0.59** (0.02)	10.83	17.12	5.92	59.87	4.91	-7.50	-4.21
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	**0.60** (0.01)	**0.60** (0.01)	10.58	16.38	5.41	51.03	6.50	-6.96	-4.05
Weekly	**0.61** (0.01)	**0.60** (0.01)	10.75	16.59	5.67	56.94	5.12	-7.13	-4.09
Monthly	**0.60** (0.01)	**0.60** (0.01)	10.82	16.88	6.02	59.23	3.69	-7.41	-4.17

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR. SR highlighted in red points to the lowest SR.

Table 7: Performance Statistics for Equities - Large Stocks (1927-2020)

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.43		10.98	17.82	8.86	60.44	0	-11.38	-6.52
Scaled with SMA-10 Volatility (10-day window)									
Daily	**0.57** (0.01)	0.45 (0.34)	10.09	12.64	2.84	22.62	113.81	-6.32	-4.12
Weekly	**0.61** (0.00)	**0.54** (0.02)	10.65	12.88	3.00	25.82	65.69	-6.58	-4.19
Monthly	0.48 (0.20)	0.45 (0.34)	9.59	13.75	4.03	43.36	26.49	-8.09	-4.61
Scaled with SMA-20 Volatility (20-day window)									
Daily	**0.56** (0.01)	0.49 (0.13)	9.70	12.37	2.82	25.28	60.09	-6.26	-4.01
Weekly	**0.57** (0.01)	**0.53** (0.04)	9.91	12.48	2.98	27.72	36.19	-6.51	-4.08
Monthly	**0.53** (0.03)	0.51 (0.07)	9.71	12.75	3.26	31.44	21.31	-7.13	-4.28
Scaled with SMA-40 Volatility (40-day window)									
Daily	**0.55** (0.01)	**0.52** (0.05)	9.55	12.08	2.95	28.29	31.19	-6.27	-3.95
Weekly	**0.56** (0.01)	**0.54** (0.02)	9.68	12.16	3.02	30.22	19.70	-6.48	-4.01
Monthly	**0.51** (0.05)	0.50 (0.08)	9.44	12.52	3.44	33.83	12.87	-7.07	-4.21
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.56** (0.01)	**0.53** (0.02)	9.56	12.04	3.05	31.04	21.26	-6.30	-3.94
Weekly	**0.56** (0.01)	**0.54** (0.01)	9.65	12.09	3.14	33.16	13.92	-6.50	-3.99
Monthly	0.50 (0.07)	0.49 (0.10)	9.30	12.44	3.57	36.63	9.73	-7.12	-4.18
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.54** (0.01)	**0.53** (0.02)	9.41	11.95	3.13	33.27	14.50	-6.42	-3.96
Weekly	**0.54** (0.01)	**0.53** (0.02)	9.48	12.00	3.22	35.00	9.78	-6.61	-4.00
Monthly	0.49 (0.10)	0.48 (0.13)	9.14	12.30	3.58	35.34	7.18	-7.22	-4.18
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	**0.57** (0.01)	0.50 (0.09)	9.69	12.13	2.91	27.21	57.90	-6.19	-3.91
Weekly	**0.56** (0.01)	**0.53** (0.03)	9.74	12.26	3.10	32.49	30.87	-6.46	-3.99
Monthly	**0.51** (0.05)	0.50 (0.09)	9.43	12.51	3.32	31.69	15.52	-7.02	-4.17
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**0.56** (0.01)	**0.53** (0.03)	9.52	11.86	2.97	29.86	30.13	-6.13	-3.85
Weekly	**0.56** (0.01)	**0.54** (0.02)	9.54	11.94	3.09	32.95	17.31	-6.36	-3.91
Monthly	**0.51** (0.05)	0.50 (0.07)	9.25	12.17	3.33	32.16	10.13	-6.88	-4.08
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	**0.56** (0.00)	**0.54** (0.01)	9.42	11.58	3.04	29.30	15.39	-6.14	-3.81
Weekly	**0.56** (0.00)	**0.54** (0.01)	9.43	11.63	3.08	30.08	9.51	-6.31	-3.85
Monthly	**0.52** (0.03)	**0.51** (0.04)	9.18	11.82	3.30	29.26	6.17	-6.75	-3.99
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	**0.56** (0.00)	**0.55** (0.01)	9.36	11.42	3.10	27.53	10.33	-6.16	-3.79
Weekly	**0.55** (0.00)	**0.55** (0.01)	9.36	11.46	3.12	27.73	6.71	-6.30	-3.83
Monthly	**0.52** (0.02)	**0.52** (0.03)	9.16	11.61	3.31	27.29	4.49	-6.67	-3.94
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	**0.55** (0.00)	**0.55** (0.00)	9.27	11.27	3.21	25.82	6.87	-6.19	-3.78
Weekly	**0.55** (0.00)	**0.54** (0.01)	9.26	11.30	3.22	26.36	4.75	-6.31	-3.81
Monthly	**0.52** (0.02)	**0.52** (0.02)	9.10	11.43	3.38	27.09	3.27	-6.61	-3.91

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR. SR highlighted in red points to the lowest SR.

4.1.4 Individual Stocks

So far, the volatility-targeting strategy mostly works for portfolios of U.S. stocks. We wanted to check also some individual stocks and randomly chose the Walt Disney Company in the period 1963-2020 and the Boeing Company in the period 1991-2020. The Walt Disney Company has no statistically significant Sharpe ratios. The Boeing Company has some statistically significant gross Sharpe ratios. The highest one is for SMA-10 with daily trading (0.57), while the unscaled Sharpe ratio is 0.38. SMA-10 is similar to the previous observation of the different U.S. indexes in 1927-2020, but it was best to trade weekly for the different portfolios.

Here comes a deeper analysis of descriptive statistics and risk measures. SMA-20 with weekly and monthly trading and EWMA-10 with weekly trading give the best, but not statically significant, gross (0.54) and net (0.52) Sharpe ratios for The Walt Disney Company. For the net Sharpe ratio, also EWMA-20 with weekly trading gives 0.52. This is higher than unscaled investment, which gives a Sharpe ratio of 0.47. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 8 shows the details. The volatility of volatility is substantially lower. It goes down from 10.57% (unscaled) to 2.38%-3.46% (scaled). The mean return of unscaled equities (19.08%) is much higher than for scaled equities (9.48%-11.18%). Maximum drawdown is lower for scaled equities (18.99%-35.03%) than unscaled (68.89%). The same is about the left tail risk. It goes down from -18.16% to -7.34% - -5.52% for 1% and from -10.85% to -4.39% - -3.43% for 5%.

As mentioned before, the SMA-10 with daily trading gives the best and statistically significant gross Sharpe ratio of 0.57 for The Boeing Company. The best, but not statistically significant, net Sharpe ratio (0.51) is for SMA-20 with daily and weekly trading. At the same time, the unscaled investment in The Boeing Company gives a Sharpe ratio of 0.38. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 9 presents the details. The volatility of volatility is substantially lower. It goes down from 12.07% (unscaled) to 2.26%-4.05% (scaled). The mean return of unscaled equities (14.70%) is higher than for scaled equities (7.24%-9.49%). Maximum drawdown is substantially lower for scaled equities (23.84%-36.63%) than unscaled (82.67%). The same is about the left tail risk. It goes down from -20.52% to -7.23% - -5.82% for 1% and from -11.77% to -4.37% - -3.58% for 5%.

It is, of course, challenging and even impossible to generalize with only two stocks. This is just a sample to see how the volatility-targeting strategy works for random individual stocks. However, one can observe that the Sharpe ratios are always higher when using the volatility-targeting strategy than just the buy-and-hold strategy. Also, risk measures are consistent with previous observations for the U.S. market. Especially maximum drawdowns are dramatically reduced and with even lower turnover than for portfolios of stocks.

Table 8: Performance Statistics for Equities - The Walt Disney Company (DIS) (1963-2020)

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.47		19.08	30.89	10.57	68.89	0	-18.16	-10.85
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.51 (0.29)	0.43 (0.26)	10.69	12.75	2.74	26.30	79.61	-6.56	-4.08
Weekly	0.51 (0.28)	0.46 (0.46)	10.92	13.09	3.28	28.43	47.42	-6.86	-4.21
Monthly	0.52 (0.22)	0.50 (0.31)	11.18	13.51	3.46	35.03	17.72	-7.34	-4.39
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.52 (0.20)	0.48 (0.42)	10.35	11.62	2.39	22.42	35.48	-5.95	-3.77
Weekly	0.54 (0.11)	0.52 (0.21)	10.66	11.70	2.50	23.44	22.40	-6.13	-3.83
Monthly	0.54 (0.12)	0.52 (0.17)	10.76	12.09	2.85	26.41	12.16	-6.55	-3.96
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.52 (0.15)	0.51 (0.24)	10.19	11.27	2.45	23.63	16.52	-5.75	-3.60
Weekly	0.53 (0.14)	0.51 (0.19)	10.25	11.29	2.53	24.22	11.07	-5.85	-3.63
Monthly	0.50 (0.29)	0.49 (0.34)	10.07	11.68	2.94	29.39	6.72	-6.22	-3.75
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.51 (0.21)	0.50 (0.29)	9.97	11.18	2.61	25.59	10.84	-5.73	-3.57
Weekly	0.52 (0.16)	0.51 (0.21)	10.11	11.20	2.66	26.10	7.85	-5.82	-3.59
Monthly	0.49 (0.31)	0.49 (0.35)	9.92	11.33	2.80	25.74	4.83	-6.16	-3.68
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.49 (0.30)	0.49 (0.36)	9.71	10.84	2.44	22.35	7.03	-5.69	-3.54
Weekly	0.50 (0.24)	0.50 (0.29)	9.82	10.82	2.43	22.19	5.65	-5.75	-3.55
Monthly	0.50 (0.27)	0.49 (0.30)	9.86	10.99	2.65	25.13	3.60	-5.99	-3.62
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.53 (0.12)	0.50 (0.30)	10.30	11.29	2.40	22.95	30.72	-5.71	-3.61
Weekly	0.54 (0.11)	0.52 (0.19)	10.38	11.30	2.41	23.37	17.36	-5.78	-3.64
Monthly	0.51 (0.25)	0.50 (0.31)	10.19	11.74	2.86	26.93	8.44	-6.31	-3.80
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.53 (0.13)	0.51 (0.22)	10.05	10.92	2.38	21.82	15.18	-5.57	-3.51
Weekly	0.53 (0.13)	0.52 (0.18)	10.08	10.90	2.37	21.27	9.50	-5.65	-3.53
Monthly	0.50 (0.27)	0.49 (0.31)	9.90	11.20	2.68	24.15	5.28	-6.03	-3.64
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.51 (0.17)	0.50 (0.22)	9.80	10.58	2.39	20.81	7.50	-5.52	-3.45
Weekly	0.51 (0.17)	0.51 (0.21)	9.81	10.57	2.38	19.50	5.48	-5.59	-3.47
Monthly	0.50 (0.27)	0.49 (0.30)	9.72	10.75	2.57	22.28	3.23	-5.80	-3.53
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.50 (0.22)	0.50 (0.26)	9.64	10.41	2.44	20.33	4.96	-5.52	-3.43
Weekly	0.50 (0.22)	0.50 (0.26)	9.65	10.41	2.45	19.27	4.22	-5.58	-3.45
Monthly	0.49 (0.29)	0.49 (0.31)	9.60	10.54	2.60	21.21	2.47	-5.69	-3.49
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.49 (0.29)	0.49 (0.32)	9.49	10.26	2.55	19.56	3.28	-5.54	-3.43
Weekly	0.49 (0.29)	0.49 (0.33)	9.49	10.27	2.56	18.99	3.45	-5.58	-3.44
Monthly	0.49 (0.33)	0.49 (0.35)	9.48	10.37	2.67	20.19	1.99	-5.63	-3.47

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 9: Performance Statistics for Equities - The Boeing Company (BA) (1991-2020)

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.38		14.70	30.98	12.07	82.67	0	-20.52	-11.77
Scaled with SMA-10 Volatility (10-day window)									
Daily	**0.57** (0.04)	0.49 (0.14)	9.49	12.45	2.55	23.84	80.33	-6.69	-4.20
Weekly	0.54 (0.07)	0.49 (0.14)	9.24	12.82	2.88	25.53	47.17	-7.23	-4.37
Monthly	0.46 (0.22)	0.44 (0.27)	8.15	12.67	4.05	36.63	16.53	-7.23	-4.36
Scaled with SMA-20 Volatility (20-day window)									
Daily	**0.55** (0.04)	0.51 (0.09)	8.61	11.23	2.26	24.51	35.45	-6.09	-3.82
Weekly	0.53 (0.06)	0.51 (0.09)	8.49	11.38	2.46	25.86	22.42	-6.30	-3.92
Monthly	0.45 (0.22)	0.44 (0.26)	7.63	11.37	2.69	28.63	11.46	-6.68	-4.04
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.51 (0.06)	0.50 (0.09)	8.01	10.74	2.32	25.88	16.47	-5.94	-3.71
Weekly	0.51 (0.07)	0.50 (0.09)	8.01	10.90	2.49	27.36	11.19	-6.14	-3.77
Monthly	0.49 (0.10)	0.48 (0.12)	7.83	10.95	2.66	29.47	6.58	-6.35	-3.84
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.52** (0.05)	0.50 (0.07)	7.97	10.66	2.34	27.05	10.88	-5.96	-3.67
Weekly	0.51 (0.07)	0.50 (0.08)	7.90	10.73	2.45	27.92	7.88	-6.10	-3.72
Monthly	0.50 (0.07)	0.50 (0.08)	7.91	10.82	2.68	31.12	4.79	-6.30	-3.77
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.51** (0.05)	0.50 (0.07)	7.85	10.60	2.57	29.12	7.13	-6.03	-3.67
Weekly	**0.51** (0.05)	0.50 (0.06)	7.91	10.63	2.62	29.31	5.67	-6.18	-3.70
Monthly	0.50 (0.06)	0.49 (0.07)	7.82	10.77	2.92	32.91	3.55	-6.36	-3.75
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.52 (0.06)	0.49 (0.12)	8.13	10.89	2.38	25.27	30.63	-5.79	-3.71
Weekly	0.49 (0.12)	0.47 (0.17)	7.78	11.06	2.56	27.52	17.32	-6.15	-3.80
Monthly	0.46 (0.16)	0.46 (0.19)	7.61	11.09	2.76	29.59	8.00	-6.38	-3.87
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**0.52** (0.05)	0.50 (0.08)	7.90	10.52	2.36	26.64	15.10	-5.75	-3.60
Weekly	0.49 (0.09)	0.48 (0.12)	7.68	10.69	2.56	28.84	9.36	-6.01	-3.67
Monthly	0.48 (0.11)	0.47 (0.12)	7.62	10.74	2.70	30.94	4.91	-6.19	-3.73
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.51 (0.06)	0.50 (0.07)	7.68	10.29	2.41	28.04	7.48	-5.82	-3.58
Weekly	0.49 (0.08)	0.48 (0.09)	7.57	10.40	2.54	29.54	5.34	-6.01	-3.62
Monthly	0.48 (0.09)	0.48 (0.10)	7.50	10.49	2.72	31.80	2.98	-6.14	-3.66
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.49 (0.06)	0.49 (0.07)	7.54	10.19	2.47	28.31	4.97	-5.89	-3.58
Weekly	0.49 (0.07)	0.48 (0.08)	7.49	10.24	2.53	29.12	4.10	-6.03	-3.61
Monthly	0.47 (0.10)	0.47 (0.10)	7.39	10.37	2.75	31.54	2.30	-6.13	-3.64
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.48 (0.08)	0.48 (0.08)	7.37	10.11	2.59	28.21	3.31	-5.99	-3.59
Weekly	0.48 (0.08)	0.47 (0.09)	7.37	10.10	2.57	28.22	3.36	-6.08	-3.61
Monthly	0.46 (0.11)	0.46 (0.12)	7.24	10.28	2.83	30.87	1.85	-6.14	-3.64

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR; in yellow, the highest, but not statistically significant SR. SR highlighted in red points to the lowest SR.

4.1.5 European, Asian, and North American Stocks

The sample period for European, Asian, and North American stocks is much shorter than for the U.S. market, and it is in the years 1991-2020. In the analyzed period and the regional markets, there is no statistically significant Sharpe ratio.[19] This means that one can not say that the active strategy is better than the passive strategy.

Here comes a deeper analysis of descriptive statistics and risk measures. As mentioned before, the European stocks do not give statistically significant Sharpe ratios. The Sharpe ratio of unscaled investment is 0.37. The highest gross Sharpe ratio is 0.46, and it is for SMA-40 with daily trading. The best net Shape ratio is 0.43, and it is for SMA-40 and SMA-90 with daily trading. Most of the risk measures indicate lower risk when using the volatility-targeting strategy. Table 10 presents the details. The volatility of volatility goes down from 6.41% (unscaled) to 1.84%-4.16% (scaled). The mean return of unscaled equities (9.12%) is higher than for scaled equities (6.24%-7.49%). Maximum drawdown is not always lower for scaled equities (16.45%-40.83%) because the unscaled maximum drawdown is 35.18%. The left tail risk goes down from -11.55% to -7.66% - -5.80% for 1% and from -6.96% to -4.58% - -3.79% for 5%.

Also, Asian and Pacific ex-Japan stocks, in this period, give not statistically significant Sharpe ratios. The highest gross Sharpe ratio (0.55) is with daily trading for EWMA-10 and EWMA-20. The highest net Sharpe ratio (0.54) is for SMA-90 with weekly trading. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 11 shows the details. The volatility of volatility is substantially lower. It goes down from 7.95% (unscaled) to 2.45%-3.90% (scaled). The mean return of unscaled equities (11.42%) is higher than for scaled equities (7.98%-9.43%). Maximum drawdown is lower for scaled equities (22.91%-37.02%) than unscaled (46.47%). The same is about the left tail risk. It goes down from -11.12% to -8.00% - -6.41% for 1% and from -6.50% to -4.44% - -3.91% for 5%.

For North American Stocks is EWMA-40 with daily trading best for gross Sharpe ratio (0.63) and net Sharpe ratio (0.61), while Sharpe ratio of unscaled investment is 0.52. Like before for analysis in this period, the Sharpe ratio is not statistically significant. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 12 presents the details. The volatility of volatility goes down from 5.88% (unscaled) to 2.12%-4.40% (scaled). The mean return of unscaled equities (11.57%) is higher than for scaled equities (8.33%-9.35%). Maximum drawdown is not always lower for scaled equities (17.55%-40.36%) than for unscaled ones (28.91%). The left tail risk goes down from -11.78% to -9.18% - -6.28% for 1% and from -6.81% to -4.98% - -3.89% for 5%.

[19]This may be due to having a short historical period. Generally, the smaller the number of observations, the lower the test power.

Grobys and Äijö (2018) find that volatility-managing factors added value in Europe and Asia. In Europe, the increase in Sharpe ratios is the largest. Their sample period was from July 1990 until May 2017. This study is longer than theirs, and it confirms their findings to some extend. The North American stocks can have an even higher increase in Sharpe ratio for some of the volatility forecasting. However, there are also worse Sharpe ratios, especially if one takes the SMA-10 volatility and monthly trading. For all analyzed datasets, it produces a lower Sharpe ratio. The worse scaled Sharpe ratio for European stocks is 0.31, while the unscaled Sharpe ratio is 0.37. For Asian and Pacific ex-Japan stocks, the worse scaled Sharpe ratio is 0.40, while the unscaled Sharpe ratio is 0.48. Also, for North American stocks, the worse scaled Sharpe ratio is 0.42, while the unscaled Sharpe ratio is 0.52. Last but not least, there are no statistically significant Sharpe ratios for the analyzed period.

4.1.6 Summary for Equities

The summary of the findings of the Sharpe ratio is in Table 13. We find that SMA-10 and weekly trading are most frequent for gross SR for the analyzed U.S. equities. However, EWMA-60 and EWMA-90 are the most frequent/best forecasting methods for the net Sharpe ratio. Weekly trading is most frequent as for the gross Sharpe ratio, but right after is daily trading. Furthermore, the regional markets have not one best volatility forecasting method, as each of them is different with their dynamics. However, they have in common that the best results were achieved with daily trading. Monthly trading, also with SMA-10, EWMA-60, EWMA-90, and for all the regions, can give the worst Sharpe ratios and risk measurements.

Table 10: Performance Statistics for Equities - European Stocks (1991-2020)

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.37		9.12	17.03	6.41	35.18	0	-11.55	-6.96
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.39 (0.42)	0.26 (0.14)	7.05	11.93	1.97	17.07	117.47	-6.29	-4.19
Weekly	0.37 (0.49)	0.31 (0.27)	6.88	12.12	2.14	20.64	66.29	-6.48	-4.26
Monthly	0.31 (0.26)	0.29 (0.21)	6.50	13.26	4.45	40.83	25.89	-7.66	-4.58
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.43 (0.23)	0.37 (0.50)	7.34	11.24	1.84	16.45	57.94	-5.90	-3.96
Weekly	0.41 (0.30)	0.36 (0.48)	7.20	11.39	1.96	17.75	34.31	-6.06	-4.04
Monthly	0.34 (0.38)	0.34 (0.38)	6.59	12.01	3.00	27.83	19.98	-7.08	-4.36
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.46 (0.14)	0.43 (0.24)	7.49	10.93	2.03	17.34	28.19	-5.80	-3.89
Weekly	0.44 (0.19)	0.42 (0.25)	7.36	11.07	2.19	18.05	17.54	-6.00	-3.97
Monthly	0.35 (0.43)	0.34 (0.37)	6.43	11.95	3.65	33.37	11.25	-7.09	-4.26
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.44 (0.18)	0.42 (0.26)	7.27	10.83	2.07	17.08	19.05	-5.91	-3.90
Weekly	0.43 (0.22)	0.42 (0.25)	7.22	10.95	2.23	18.36	12.40	-6.12	-3.97
Monthly	0.36 (0.45)	0.36 (0.47)	6.70	11.85	3.65	33.24	8.48	-7.10	-4.22
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.44 (0.16)	0.43 (0.21)	7.29	10.73	2.21	18.79	12.91	-6.08	-3.93
Weekly	0.43 (0.22)	0.42 (0.23)	7.17	10.87	2.42	20.20	8.73	-6.30	-3.99
Monthly	0.35 (0.40)	0.34 (0.37)	6.42	11.89	4.16	36.38	6.01	-7.27	-4.23
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.44 (0.21)	0.38 (0.46)	7.26	10.95	1.84	16.64	53.49	-5.78	-3.85
Weekly	0.41 (0.32)	0.39 (0.38)	7.01	11.11	2.01	17.37	27.78	-6.01	-3.96
Monthly	0.36 (0.46)	0.34 (0.38)	6.29	12.04	3.58	32.53	13.81	-7.09	-4.25
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.45 (0.16)	0.42 (0.26)	7.27	10.59	1.90	16.57	26.71	-5.70	-3.79
Weekly	0.43 (0.23)	0.42 (0.25)	7.10	10.73	2.10	17.08	15.06	-5.93	-3.87
Monthly	0.34 (0.37)	0.36 (0.46)	6.41	11.63	3.55	32.20	8.76	-6.91	-4.13
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.44 (0.18)	0.42 (0.24)	7.06	10.28	2.04	17.00	13.39	-5.78	-3.79
Weekly	0.42 (0.24)	0.41 (0.26)	6.93	10.41	2.24	18.99	8.08	-5.99	-3.85
Monthly	0.37 (0.50)	0.36 (0.47)	6.37	11.12	3.40	30.63	5.09	-6.74	-4.05
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.42 (0.23)	0.41 (0.28)	6.85	10.12	2.16	18.24	8.87	-5.85	-3.79
Weekly	0.41 (0.29)	0.40 (0.32)	6.75	10.24	2.33	20.08	5.62	-6.02	-3.84
Monthly	0.36 (0.48)	0.36 (0.46)	6.30	10.80	3.25	29.28	3.66	-6.61	-4.00
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.40 (0.29)	0.40 (0.33)	6.66	9.99	2.31	19.41	5.88	-5.93	-3.81
Weekly	0.39 (0.35)	0.39 (0.38)	6.58	10.08	2.45	20.96	3.97	-6.06	-3.84
Monthly	0.36 (0.46)	0.36 (0.44)	6.24	10.50	3.13	27.86	2.63	-6.52	-3.97

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 11: Performance Statistics for Equities - Asian and Pacific ex-Japan Stocks (1991-2020)

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.48		11.42	20.25	7.95	46.47	0	-11.12	-6.50
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.53 (0.28)	0.41 (0.23)	9.14	14.53	2.63	22.91	118.08	-6.52	-4.20
Weekly	0.52 (0.35)	0.45 (0.36)	9.43	14.79	2.83	25.15	67.99	-6.92	-4.35
Monthly	0.40 (0.20)	0.38 (0.13)	8.34	15.81	3.90	34.26	26.92	-8.00	-4.64
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.54 (0.26)	0.47 (0.46)	8.97	14.15	2.56	25.82	61.65	-6.50	-4.08
Weekly	0.54 (0.26)	0.49 (0.44)	9.06	14.31	2.80	27.96	36.90	-6.91	-4.20
Monthly	0.46 (0.38)	0.43 (0.28)	8.39	14.71	3.18	32.42	21.79	-7.57	-4.44
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.53 (0.26)	0.50 (0.41)	8.74	13.70	2.60	25.42	31.33	-6.57	-4.07
Weekly	0.54 (0.25)	0.51 (0.34)	8.84	13.80	2.78	26.19	19.86	-6.80	-4.16
Monthly	0.48 (0.49)	0.47 (0.43)	8.38	14.13	3.23	34.20	13.36	-7.34	-4.31
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.52 (0.32)	0.49 (0.43)	8.51	13.55	2.66	25.74	21.20	-6.68	-4.08
Weekly	0.53 (0.27)	0.51 (0.36)	8.70	13.65	2.79	27.24	14.32	-6.89	-4.15
Monthly	0.50 (0.39)	0.48 (0.48)	8.58	14.09	3.44	36.39	9.97	-7.42	-4.28
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.54 (0.20)	0.53 (0.26)	8.76	13.47	2.92	27.97	13.92	-6.73	-4.07
Weekly	0.55 (0.18)	0.54 (0.22)	8.82	13.56	3.05	29.10	9.67	-6.89	-4.12
Monthly	0.51 (0.33)	0.50 (0.39)	8.63	13.89	3.62	37.02	6.60	-7.37	-4.23
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.55 (0.20)	0.49 (0.46)	8.91	13.65	2.45	24.61	57.32	-6.43	-3.99
Weekly	0.54 (0.23)	0.50 (0.40)	8.89	13.73	2.60	24.46	30.70	-6.64	-4.07
Monthly	0.47 (0.43)	0.45 (0.38)	8.24	14.29	3.27	33.94	15.43	-7.39	-4.31
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.55 (0.20)	0.51 (0.34)	8.69	13.25	2.55	25.48	29.17	-6.41	-3.94
Weekly	0.54 (0.22)	0.52 (0.32)	8.70	13.33	2.67	25.47	16.93	-6.59	-4.01
Monthly	0.49 (0.44)	0.48 (0.50)	8.33	13.79	3.30	34.62	9.84	-7.26	-4.20
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.53 (0.23)	0.52 (0.31)	8.43	12.90	2.79	26.79	14.63	-6.50	-3.92
Weekly	0.53 (0.25)	0.51 (0.32)	8.42	12.98	2.90	26.77	9.00	-6.65	-3.97
Monthly	0.50 (0.41)	0.49 (0.45)	8.25	13.36	3.44	34.31	5.83	-7.19	-4.11
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.52 (0.29)	0.51 (0.35)	8.21	12.75	3.02	27.80	9.70	-6.60	-3.92
Weekly	0.51 (0.31)	0.50 (0.36)	8.21	12.83	3.13	27.99	6.26	-6.73	-3.96
Monthly	0.49 (0.44)	0.49 (0.46)	8.12	13.14	3.58	34.19	4.19	-7.16	-4.07
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.50 (0.37)	0.49 (0.42)	8.00	12.64	3.32	29.60	6.41	-6.70	-3.91
Weekly	0.50 (0.39)	0.49 (0.42)	8.01	12.71	3.42	29.75	4.43	-6.81	-3.95
Monthly	0.48 (0.48)	0.48 (0.50)	7.98	12.97	3.78	34.27	3.03	-7.15	-4.03

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 12: Performance Statistics for Equities - North American Stocks (1991-2020)

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.52		11.57	15.05	5.88	28.91	0	-11.78	-6.81
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.60 (0.23)	0.48 (0.34)	9.35	10.85	2.12	17.55	113.90	-6.47	-4.25
Weekly	0.58 (0.28)	0.49 (0.38)	9.27	11.14	2.48	22.63	65.10	-6.83	-4.36
Monthly	0.42 (0.16)	0.39 (0.11)	8.59	12.51	4.40	40.36	28.15	-9.18	-4.98
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.60 (0.24)	0.53 (0.48)	9.10	10.51	2.13	18.43	61.33	-6.45	-4.15
Weekly	0.60 (0.21)	0.52 (0.47)	9.27	10.51	2.22	19.84	37.11	-6.63	-4.21
Monthly	0.53 (0.48)	0.51 (0.44)	8.88	11.24	2.97	27.17	22.92	-8.02	-4.61
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.62 (0.14)	0.59 (0.25)	9.27	10.14	2.14	18.49	31.42	-6.50	-4.07
Weekly	0.61 (0.17)	0.58 (0.28)	9.21	10.17	2.21	18.66	19.49	-6.70	-4.14
Monthly	0.52 (0.49)	0.51 (0.43)	8.57	10.85	3.23	28.06	13.38	-8.07	-4.50
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.61 (0.17)	0.58 (0.25)	9.07	10.03	2.28	18.52	21.20	-6.51	-4.06
Weekly	0.60 (0.19)	0.57 (0.29)	9.08	10.09	2.35	18.75	13.81	-6.70	-4.11
Monthly	0.53 (0.47)	0.52 (0.49)	8.65	10.90	3.43	30.52	9.69	-8.06	-4.43
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.60 (0.19)	0.58 (0.25)	8.97	9.97	2.32	19.68	14.59	-6.69	-4.09
Weekly	0.58 (0.23)	0.56 (0.32)	8.90	10.04	2.43	19.62	10.12	-6.94	-4.16
Monthly	0.48 (0.33)	0.49 (0.35)	8.33	10.94	3.77	32.62	7.07	-8.32	-4.48
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.61 (0.20)	0.54 (0.43)	9.03	10.22	2.16	19.26	58.54	-6.34	-4.02
Weekly	0.60 (0.22)	0.56 (0.37)	9.05	10.26	2.29	18.52	30.61	-6.57	-4.10
Monthly	0.51 (0.45)	0.49 (0.34)	8.55	10.93	3.16	27.64	15.98	-7.95	-4.48
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.62 (0.14)	0.59 (0.25)	9.05	9.87	2.15	18.29	29.86	-6.28	-3.94
Weekly	0.61 (0.17)	0.58 (0.27)	9.01	9.92	2.25	18.08	16.91	-6.48	-4.00
Monthly	0.52 (0.47)	0.51 (0.44)	8.48	10.59	3.14	28.11	10.04	-7.78	-4.34
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.63 (0.11)	0.61 (0.16)	9.02	9.59	2.21	19.04	15.22	-6.30	-3.90
Weekly	0.61 (0.15)	0.59 (0.21)	8.95	9.65	2.30	19.08	9.21	-6.49	-3.95
Monthly	0.54 (0.44)	0.53 (0.44)	8.51	10.22	3.03	27.00	6.04	-7.52	-4.21
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.62 (0.11)	0.61 (0.14)	8.95	9.47	2.30	19.16	10.25	-6.33	-3.89
Weekly	0.61 (0.14)	0.59 (0.20)	8.87	9.53	2.38	19.18	6.48	-6.49	-3.93
Monthly	0.55 (0.35)	0.55 (0.38)	8.53	9.98	2.94	25.19	4.40	-7.30	-4.14
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.61 (0.12)	0.60 (0.14)	8.83	9.37	2.45	18.95	6.76	-6.37	-3.89
Weekly	0.60 (0.15)	0.58 (0.20)	8.75	9.42	2.50	18.96	4.55	-6.51	-3.93
Monthly	0.56 (0.31)	0.56 (0.33)	8.52	9.76	2.92	23.13	3.16	-7.10	-4.08

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 13: Securities, Sample Periods, and the Highest Sharpe Ratios for Equities

Asset Class	Period	SR unscaled	Best Gross SR	Scaling	Daily	Weekly	Monthly	Best Net SR	Scaling	Daily	Weekly	Monthly
Equities (All U.S.)	1927 -2020	0.44	**0.62** (0.00)	SMA-10		x		**0.56** (0.01)	EWMA-60		x	
								0.56 (0.01)	EWMA-90	x	x	
Value Stocks	1927 -2020	0.42	**0.59** (0.00)	SMA-10		x		**0.53** (0.03)	SMA-10		x	
								0.53 (0.01)	SMA-60		x	
								0.53 (0.01)	EWMA-60	x	x	
								0.53 (0.01)	EWMA-90	x	x	
Growth Stocks	1927 - 2020	0.48	**0.65** (0.00)	SMA-10		x		**0.62** (0.01)	SMA-60		x	
Small Stocks	1927 - 2020	0.48	**0.61** (0.01)	EWMA-60		x		**0.60** (0.01)	EWMA-60		x	
			0.61 (0.01)	EWMA-90		x		**0.60** (0.01)	EWMA-90	x	x	x
Large Stocks	1927 - 2020	0.43	**0.61** (0.00)	SMA-10		x		**0.55** (0.01)	EWMA-60	x	x	
								0.55 (0.00)	EWMA-90	x		
DIS	1963 - 2020	0.47	0.54 (0.11)	SMA-20		x	x	0.52 (0.21)	SMA-20		x	x
			0.54 (0.11)	EWMA-10		x		0.52 (0.19)	EWMA-10		x	
								0.52 (0.18)	EWMA-20		x	
BA	1991 - 2020	0.38	**0.57** (0.04)	SMA-10	x			0.51 (0.09)	SMA-20	x	x	
European Stocks	1991 - 2020	0.37	0.46 (0.14)	SMA-40	x			0.43 (0.24)	SMA-40	x		
								0.43 (0.21)	SMA-90	x		
Asian Stocks	1991 - 2020	0.48	0.55 (0.20)	EWMA-10	x			0.54 (0.22)	SMA-90		x	
			0.55 (0.20)	EWMA-20	x							
Nor.Am Stocks	1991 - 2020	0.52	0.63 (0.11)	EWMA-40	x			0.61 (0.16)	EWMA-40	x		

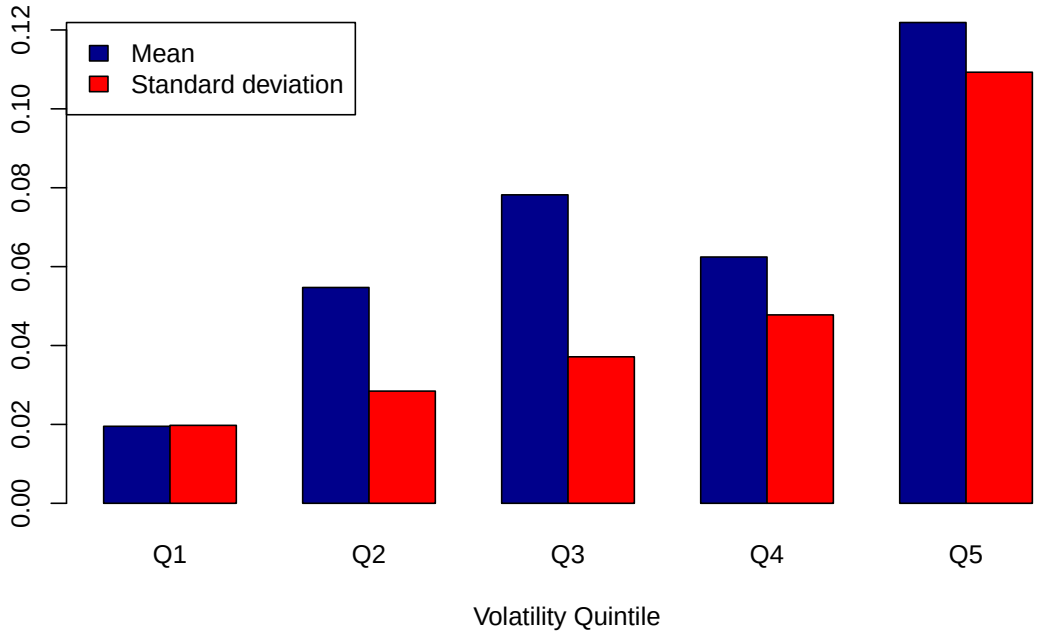
Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. Above the double line are stocks and portfolios from the U.S. market. Under the double line are indices from different regional markets.

4.2 Other Assets

4.2.1 Bonds

Figure 6 illustrates the annualized volatility and the annualized mean return when sorting on the previous month's volatility. It is a similar procedure as for Figure 1 on page 8. The mean bond returns seem to grow through different quintiles, except for Q4. The returns are highest in the high-volatility quintile. That is why it is not apparent that volatility scaling will affect the Sharpe ratio, according to Harvey et al. (2018).

Figure 6: Risk/Return Tradeoff for Bonds



Quintile Analysis for VFIIX (1981-2020). The figure shows the mean excess return and the volatility (both annualized) when sorting on the previous month's volatility.

The analyzed bonds in the sample periods have relatively low volatility. That is why we use about half of the bond's volatility as target volatility. The volatility targets in this section are between 2.5% and 4%. This is lower than the target volatility of 10% used for some other asset classes.

Intermediate-Term Bonds

In this section, Vanguard GNMA Fund Investor Shares (VFIIX) in the years 1981-2020 and Vanguard Intermediate-Term Treasury Fund Investor Shares (VFITX) in the period 1992-2020

are in focus. The applied volatility target is 2.5% for both funds.

Volatility scaling for VFIIX gives a better and statistically significant gross Sharpe ratio of 0.84, while unscaled bonds give a Sharpe ratio of 0.55. This is achieved with SMA-10 scaling with daily trading. After adding transaction cost, the Sharpe ratio goes down to 0.67, which is still higher than unscaled but not statistically significant. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 14 shows the details. The volatility of volatility goes down from 2.59% (unscaled) to 0.62%-1.06% (scaled). The mean return of unscaled bonds (6.96%) is slightly higher than for scaled bonds (5.33%-6.51%). Maximum drawdown is lower for scaled bonds (5.03%-6.47%) than unscaled (15.66%). The same is about the left tail risk. It goes down from -3.36% to -1.63% - -1.39% for 1% and from -1.75% to -1.03% - -0.81% for 5%.

In the case of VFITX, there is a statistically significant Sharpe ratio of 0.16 for gross Sharpe ratio (SMA-20 with monthly trading) and 0.18 for net Sharpe ratio (SMA-90 with daily and weekly trading and EWMA-20 with monthly trading). Both are lower than the Sharpe ratio of unscaled bonds, which is 0.33. There exist one higher gross Sharpe ratio of 0.37 for SMA-10 with daily trading. Other risk measures indicate only slightly lower risk when using the volatility-targeting strategy. Table 15 presents the details. The volatility of volatility goes down from 2.05% (unscaled) to 0.50%-0.67% (scaled). The mean return of unscaled bonds (2.77%) is slightly higher than for scaled bonds (1.50%-2.39%). Maximum drawdown is lower for scaled bonds (5.35%-7.99%) than unscaled (12.43%). The same is about the left tail risk. It goes down from -2.90% to -1.97% - -1.47% for 1% and from -1.72% to -1.1% - -0.87% for 5%.

Although the applied volatility target is the same for both funds (2.5%). The performance measures are very different. The VFIIX has similar volatility but a higher mean return than VFITX. The Sharpe ratio shows the ratio between excess return (numerator) and risk (denominator). It is understandable why there are different Sharpe ratios for two funds with the same risk but different returns. Nevertheless, risk measures are pretty similar for the analyzed intermediate-term bonds.

Figure 7 shows a comparison of unscaled and volatility-scaled returns. The latter uses a volatility estimate based on the SMA of 20 days with monthly trading. In the top panel, there are plots of cumulative returns. One can observe that volatility-scaled investment generally not outperformed the unscaled investment. The impact of volatility scaling is illustrated in the bottom panel. The realized volatility of volatility-scaled returns is much more stable over time, and the volatility of volatility is 0.65%, while the volatility of volatility for unscaled investment is 2.05%.

Table 14: Performance Statistics for Bonds - Vanguard GNMA Fund Investor Shares (VFIIX), (1981-2020) with the Target Volatility of 2.5%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.55		6.96	4.88	2.59	15.66	0	-3.36	-1.75
Scaled with SMA-10 Volatility (10-day window)									
Daily	**0.84** (0.00)	0.61 (0.30)	6.51	2.89	0.63	5.03	122.28	-1.54	-0.94
Weekly	**0.82** (0.00)	0.67 (0.10)	6.51	2.89	0.57	5.10	78.47	-1.56	-0.96
Monthly	0.53 (0.42)	0.49 (0.24)	5.59	3.03	1.06	8.80	22.24	-1.82	-1.03
Scaled with SMA-20 Volatility (20-day window)									
Daily	**0.76** (0.01)	0.64 (0.18)	6.05	2.74	0.65	5.30	57.02	-1.44	-0.87
Weekly	0.66 (0.09)	0.59 (0.30)	5.79	2.72	0.62	5.45	31.62	-1.46	-0.89
Monthly	0.55 (0.48)	0.51 (0.32)	5.47	2.74	0.76	6.22	16.62	-1.63	-0.92
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.68 (0.09)	0.62 (0.25)	5.73	2.64	0.66	5.59	29.20	-1.43	-0.86
Weekly	0.62 (0.20)	0.59 (0.35)	5.59	2.66	0.69	5.72	17.13	-1.46	-0.87
Monthly	0.55 (0.48)	0.53 (0.39)	5.41	2.66	0.74	6.11	9.73	-1.60	-0.90
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.63 (0.20)	0.59 (0.36)	5.57	2.60	0.68	5.55	19.45	-1.45	-0.86
Weekly	0.61 (0.25)	0.58 (0.36)	5.54	2.62	0.69	5.59	12.49	-1.47	-0.87
Monthly	0.56 (0.48)	0.54 (0.44)	5.40	2.61	0.74	5.77	7.27	-1.55	-0.89
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.56 (0.45)	0.53 (0.41)	5.38	2.55	0.69	5.49	12.92	-1.46	-0.86
Weekly	0.58 (0.39)	0.56 (0.49)	5.44	2.57	0.71	5.50	9.03	-1.48	-0.87
Monthly	0.53 (0.41)	0.52 (0.35)	5.35	2.58	0.73	5.75	5.61	-1.57	-0.90
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.70 (0.06)	0.58 (0.38)	5.81	2.63	0.63	5.21	54.97	-1.40	-0.85
Weekly	**0.70** (0.05)	0.63 (0.19)	5.82	2.63	0.62	5.28	31.79	-1.43	-0.86
Monthly	0.54 (0.45)	0.52 (0.33)	5.40	2.67	0.77	6.47	12.14	-1.59	-0.90
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.63 (0.20)	0.57 (0.44)	5.55	2.55	0.64	5.37	27.68	-1.40	-0.84
Weekly	0.63 (0.17)	0.59 (0.31)	5.58	2.56	0.65	5.48	16.79	-1.42	-0.84
Monthly	0.55 (0.46)	0.53 (0.38)	5.36	2.58	0.72	5.90	7.96	-1.54	-0.88
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.59 (0.32)	0.56 (0.46)	5.41	2.47	0.65	5.49	13.80	-1.40	-0.83
Weekly	0.60 (0.29)	0.58 (0.38)	5.44	2.48	0.67	5.56	8.73	-1.42	-0.83
Monthly	0.55 (0.48)	0.54 (0.43)	5.33	2.48	0.70	5.80	4.80	-1.50	-0.85
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.59 (0.35)	0.56 (0.45)	5.37	2.41	0.65	5.50	9.03	-1.40	-0.82
Weekly	0.59 (0.32)	0.58 (0.38)	5.40	2.42	0.66	5.55	5.86	-1.41	-0.82
Monthly	0.56 (0.48)	0.55 (0.49)	5.33	2.42	0.68	5.73	3.46	-1.48	-0.84
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.59 (0.34)	0.57 (0.40)	5.35	2.35	0.63	5.45	5.89	-1.39	-0.81
Weekly	0.59 (0.31)	0.58 (0.35)	5.38	2.36	0.64	5.49	3.92	-1.40	-0.82
Monthly	0.57 (0.41)	0.57 (0.44)	5.34	2.35	0.65	5.64	2.46	-1.45	-0.83

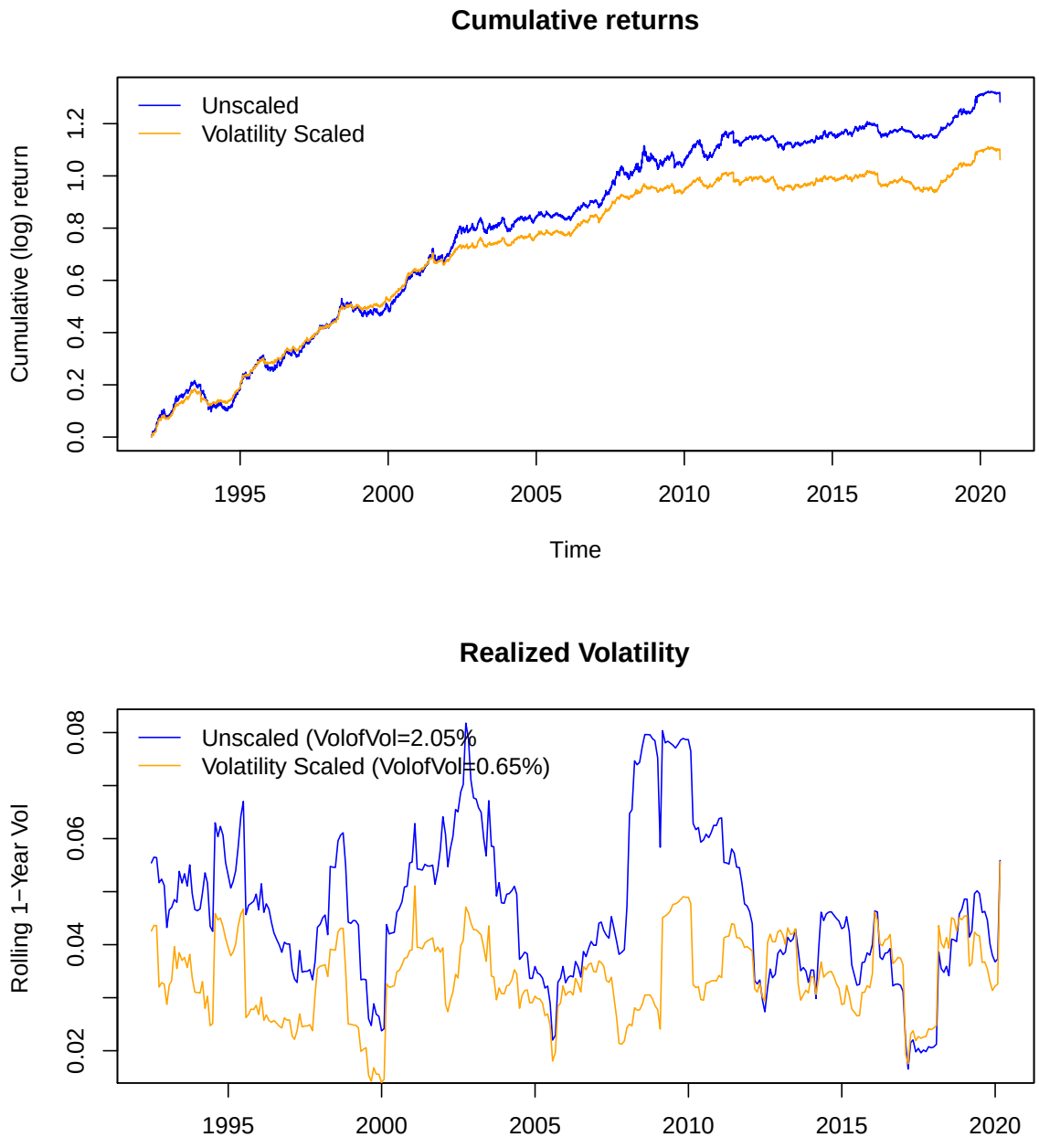
A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR; in yellow, the highest, but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 15: Performance Statistics for Bonds - Vanguard Intermediate-Term Treasury Fund Investor Shares (VFITX), (1992-2020) with the Target Volatility of 2.5%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.33		2.77	4.89	2.05	12.43	0	-2.90	-1.72
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.37 (0.33)	0.17 (0.08)	2.39	3.23	0.67	7.95	108.72	-1.83	-1.03
Weekly	0.30 (0.43)	0.19 (0.11)	2.17	3.16	0.63	7.99	60.60	-1.86	-1.04
Monthly	**0.10** (0.02)	**0.06** (0.01)	1.50	3.01	0.66	7.43	22.38	-1.97	-1.10
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.20 (0.12)	**0.10** (0.02)	1.81	3.04	0.61	7.73	51.24	-1.74	-0.99
Weekly	0.16 (0.06)	**0.10** (0.02)	1.69	3.05	0.64	7.84	29.53	-1.78	-1.00
Monthly	**0.16** (0.05)	**0.13** (0.02)	1.69	3.00	0.65	7.42	17.45	-1.88	-1.05
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.20 (0.08)	**0.15** (0.02)	1.77	2.89	0.57	6.82	24.75	-1.66	-0.96
Weekly	0.19 (0.06)	**0.16** (0.03)	1.74	2.88	0.59	7.00	14.93	-1.67	-0.96
Monthly	0.21 (0.10)	0.19 (0.07)	1.80	2.79	0.50	6.31	9.09	-1.73	-0.98
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.23 (0.13)	0.19 (0.06)	1.83	2.83	0.51	6.96	16.65	-1.65	-0.94
Weekly	0.21 (0.10)	0.19 (0.07)	1.80	2.84	0.54	7.24	10.30	-1.65	-0.94
Monthly	0.21 (0.10)	0.20 (0.08)	1.80	2.81	0.52	6.71	6.37	-1.71	-0.96
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.21 (0.09)	**0.18** (0.05)	1.78	2.85	0.56	7.32	11.46	-1.65	-0.93
Weekly	0.20 (0.08)	**0.18** (0.05)	1.75	2.86	0.58	7.48	7.52	-1.66	-0.94
Monthly	0.20 (0.08)	0.19 (0.06)	1.76	2.82	0.56	6.91	4.73	-1.69	-0.95
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.20 (0.11)	**0.11** (0.02)	1.79	2.95	0.54	7.67	46.44	-1.69	-0.95
Weekly	0.19 (0.08)	**0.14** (0.03)	1.75	2.94	0.57	7.51	24.84	-1.69	-0.96
Monthly	0.18 (0.06)	**0.16** (0.03)	1.73	2.81	0.51	6.49	11.59	-1.71	-0.97
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.20 (0.09)	**0.15** (0.03)	1.77	2.83	0.51	7.14	23.02	-1.61	-0.92
Weekly	0.20 (0.08)	**0.17** (0.04)	1.76	2.84	0.54	7.24	13.02	-1.62	-0.93
Monthly	0.20 (0.07)	**0.18** (0.05)	1.74	2.76	0.50	6.40	6.88	-1.66	-0.94
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.23 (0.11)	0.20 (0.06)	1.81	2.71	0.52	6.36	11.22	-1.55	-0.90
Weekly	0.23 (0.11)	0.22 (0.08)	1.82	2.72	0.55	6.54	6.62	-1.56	-0.90
Monthly	0.23 (0.11)	0.22 (0.08)	1.81	2.67	0.53	5.96	3.91	-1.59	-0.91
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.26 (0.19)	0.24 (0.13)	1.88	2.64	0.55	5.84	7.39	-1.51	-0.88
Weekly	0.26 (0.19)	0.25 (0.16)	1.89	2.65	0.58	6.03	4.48	-1.51	-0.88
Monthly	0.26 (0.18)	0.25 (0.16)	1.88	2.61	0.57	5.56	2.80	-1.54	-0.89
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.29 (0.32)	0.28 (0.26)	1.95	2.59	0.61	5.55	4.86	-1.47	-0.87
Weekly	0.30 (0.33)	0.29 (0.29)	1.96	2.60	0.63	5.73	3.04	-1.47	-0.87
Monthly	0.29 (0.32)	0.29 (0.29)	1.95	2.58	0.63	5.35	1.98	-1.50	-0.88

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR; in yellow, the highest, but not statistically significant SR. SR highlighted in red points to the lowest SR.

Figure 7: Cumulative Returns and Realized Volatility for Vanguard Intermediate-Term Treasury Fund Investor Shares (1992-2020)



The first panel shows the growth of wealth from the volatility-targeting portfolio (scaled) versus the benchmark (unscaled). The bottom panel shows the unscaled and scaled volatility (SMA-20) with monthly trading. Both over the period 1992-2020.

Long-Term Bonds

In this part, Vanguard Long-Term Investment-Grade Fund Investor Shares (VWESX) in years 1981-2020 and Vanguard Pennsylvania Long-Term Tax-Exempt Fund Investor Shares (VPAIX) in the period 1987-2020 are in focus. The applied volatility target is 4% for the former fund and 3% for the latter fund. As for intermediate-term bonds, the performance statistics are very different. VWESX has no statistically significant Sharpe ratios, while VPAIX has many. Besides, the highest significant gross Sharpe ratio for VPAIX (1.18) is higher than for unscaled bonds (0.69) and even for equities. The maximum Share ratio for unscaled stocks was 0.48 and for scaled ones 0.65. Moreover, there is another similarity to U.S. stocks. The highest Sharpe ratio for VPAIX (1.18) is also achieved with SMA-10, but this time with daily trading and not weekly like for equities. There is a general pattern for bonds that even net Sharpe ratio, with a transaction cost of 5.0 bps, is higher with daily trading.

Here comes a deeper analysis of descriptive statistics and risk measures. VWESX has an unscaled Sharpe ratio of 0.54 with SMA-10 scaling and daily trading, the best gross Sharpe ratio is 0.66, but it is not statistically significant at 0.05 level. The net Sharpe ratio is 0.58 with SMA-20 and daily trading. It is better than the unscaled Sharpe ratio, but again not statistically significant. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 16 shows the details. The volatility of volatility goes down from 2.99% (unscaled) to 1.19%-1.79% (scaled). The mean return of unscaled bonds (8.57%) is higher than scaled bonds (6.05%-7.05%). Maximum drawdown is lower for scaled bonds (7.71%-14.40%) than unscaled (20.20%). The same is about the left tail risk. It goes down from -4.82% to -2.73% - -2.06% for 1% and from -3.07% to -1.66% - -1.36% for 5%.

For VPAIX, the unscaled Sharpe ratio is 0.69. The statistically significant gross Sharpe ratio is 1.18 with SMA-10 and daily trading, much better than the unscaled Sharpe ratio. Also, the net Sharpe ratio of 1.05 with SMA-20 and daily trading is better than the unscaled Sharpe ratio. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 16 presents the details. The volatility of volatility goes down from 4.96% (unscaled) to 1.02%-1.48% (scaled). The mean return of unscaled bonds (8.57%) is higher than scaled bonds (4.46%-6.35%). Maximum drawdown is lower for scaled bonds (7.46%-12.23%) than unscaled (12.96%). The same is about the left tail risk. It goes down from -2.54% to -2.31% - -1.74% for 1% and from -1.32% to -1.20% - -1.07% for 5%.

Figure 8 shows a comparison of unscaled and volatility-scaled returns. The latter uses a volatility estimate based on the SMA of 10 days with daily trading. In the top panel, there are plots of cumulative returns. One can observe that volatility-scaled investment generally outperformed unscaled investment. The impact of volatility scaling is illustrated in the bottom

panel. The realized volatility of volatility-scaled returns is much more stable over time, and the volatility of volatility is 1.09%, while the volatility of volatility for unscaled investment is 2.35%.

Table 16: Performance Statistics for Bonds - Vanguard Long-Term Investment-Grade Fund Investor Shares (VWESX), (1981-2020) with the Target Volatility of 4%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.54		8.57	8.39	2.99	20.20	0	-4.82	-3.07
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.66 (0.07)	0.54 (0.48)	7.05	4.99	1.25	8.81	101.26	-2.36	-1.55
Weekly	0.64 (0.09)	0.56 (0.38)	6.93	5.09	1.33	10.12	58.34	-2.38	-1.56
Monthly	0.51 (0.35)	0.49 (0.22)	6.40	5.36	1.79	14.40	20.06	-2.73	-1.66
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.64 (0.06)	0.58 (0.29)	6.68	4.70	1.20	7.91	44.88	-2.12	-1.43
Weekly	0.59 (0.23)	0.55 (0.43)	6.48	4.78	1.29	8.94	25.85	-2.19	-1.45
Monthly	0.54 (0.45)	0.52 (0.34)	6.28	4.84	1.48	11.92	13.25	-2.38	-1.50
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.59 (0.19)	0.56 (0.37)	6.41	4.56	1.23	8.46	22.13	-2.12	-1.40
Weekly	0.57 (0.34)	0.55 (0.46)	6.30	4.60	1.26	9.09	12.85	-2.15	-1.41
Monthly	0.53 (0.44)	0.52 (0.37)	6.19	4.64	1.38	11.59	7.21	-2.31	-1.44
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.58 (0.27)	0.56 (0.40)	6.31	4.48	1.22	8.31	14.50	-2.12	-1.39
Weekly	0.56 (0.35)	0.55 (0.44)	6.26	4.53	1.24	8.81	8.76	-2.16	-1.40
Monthly	0.53 (0.43)	0.53 (0.37)	6.16	4.57	1.35	10.83	5.03	-2.25	-1.43
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.55 (0.43)	0.54 (0.47)	6.20	4.45	1.24	8.96	9.65	-2.13	-1.39
Weekly	0.55 (0.48)	0.54 (0.45)	6.18	4.48	1.26	9.20	6.16	-2.16	-1.40
Monthly	0.52 (0.37)	0.52 (0.33)	6.14	4.57	1.36	10.50	3.86	-2.29	-1.44
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.61 (0.12)	0.55 (0.44)	6.50	4.57	1.21	7.71	42.87	-2.07	-1.40
Weekly	0.59 (0.22)	0.56 (0.41)	6.41	4.62	1.23	8.02	22.70	-2.12	-1.42
Monthly	0.52 (0.34)	0.51 (0.27)	6.16	4.76	1.48	11.89	9.01	-2.35	-1.47
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.58 (0.25)	0.55 (0.45)	6.30	4.45	1.19	8.21	21.20	-2.06	-1.37
Weekly	0.57 (0.33)	0.55 (0.45)	6.26	4.48	1.20	8.53	11.71	-2.10	-1.38
Monthly	0.52 (0.37)	0.52 (0.31)	6.12	4.58	1.36	11.23	5.63	-2.27	-1.42
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.56 (0.39)	0.54 (0.49)	6.17	4.37	1.19	8.77	10.50	-2.08	-1.36
Weekly	0.55 (0.44)	0.54 (0.49)	6.15	4.38	1.20	8.98	5.99	-2.11	-1.37
Monthly	0.52 (0.35)	0.52 (0.32)	6.08	4.46	1.31	10.68	3.31	-2.22	-1.40
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.55 (0.47)	0.54 (0.44)	6.12	4.33	1.20	9.02	6.97	-2.10	-1.36
Weekly	0.54 (0.49)	0.54 (0.44)	6.11	4.34	1.21	9.18	4.08	-2.12	-1.37
Monthly	0.52 (0.33)	0.52 (0.31)	6.06	4.41	1.31	10.50	2.37	-2.21	-1.40
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.54 (0.47)	0.53 (0.41)	6.09	4.29	1.22	9.18	4.63	-2.11	-1.37
Weekly	0.54 (0.45)	0.53 (0.42)	6.08	4.30	1.23	9.30	2.81	-2.13	-1.37
Monthly	0.52 (0.33)	0.52 (0.31)	6.05	4.36	1.31	10.39	1.69	-2.20	-1.39

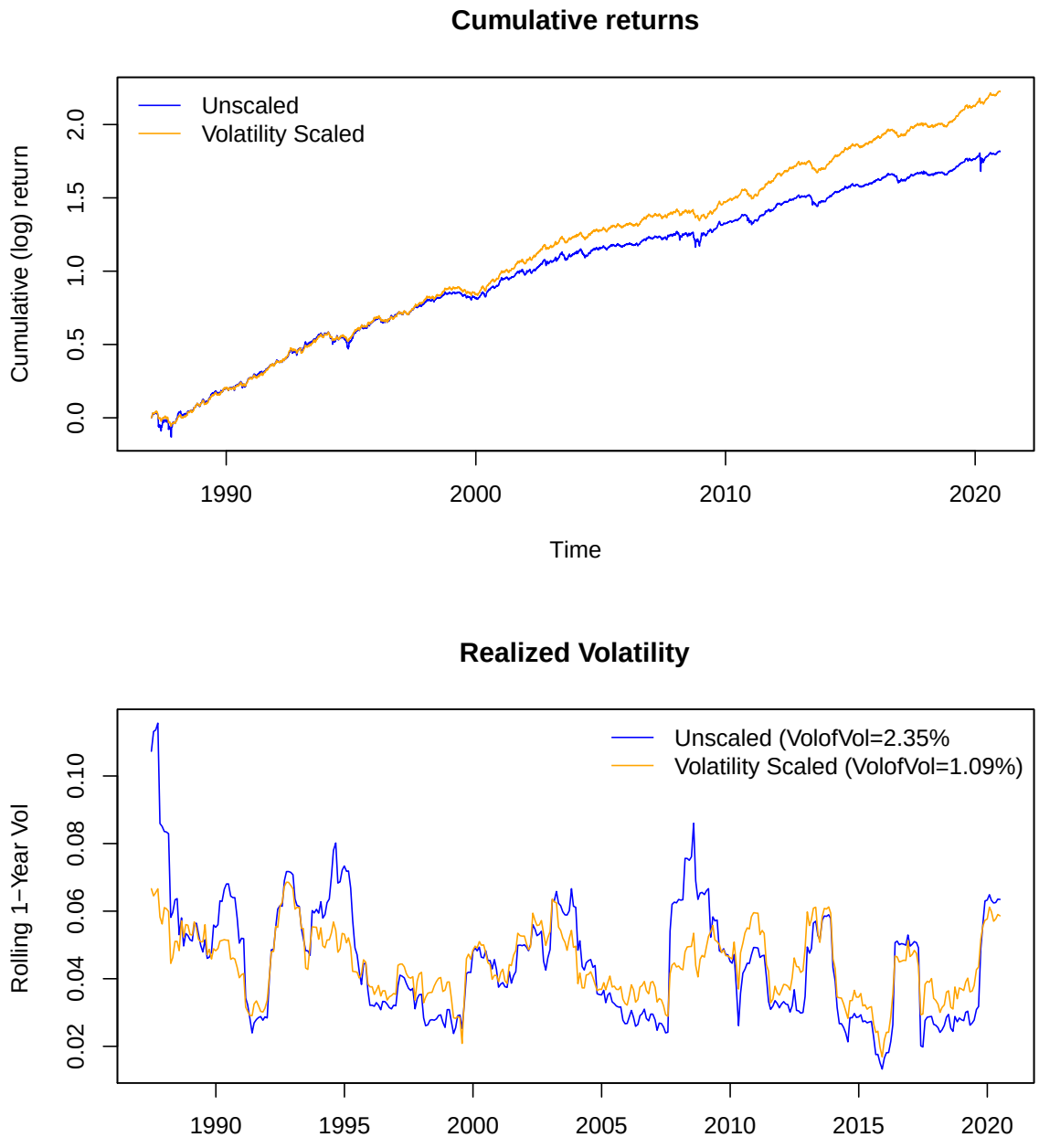
A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 17: Performance Statistics for Bonds - Vanguard Pennsylvania Long-Term Tax-Exempt Fund Investor Shares (VPAIX), (1987-2020) with the Target Volatility of 3%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.69		4.96	5.12	2.35	12.96	0	-2.54	-1.32
Scaled with SMA-10 Volatility (10-day window)									
Daily	**1.18** (0.00)	**0.95** (0.00)	6.35	4.70	1.09	7.46	124.92	-1.82	-1.12
Weekly	**1.12** (0.00)	**0.98** (0.00)	6.32	4.87	1.19	9.12	83.72	-2.09	-1.18
Monthly	**0.93** (0.00)	**0.89** (0.01)	5.56	4.81	1.48	12.23	25.98	-2.15	-1.18
Scaled with SMA-20 Volatility (20-day window)									
Daily	**1.17** (0.00)	**1.05** (0.00)	6.15	4.57	1.02	7.48	65.16	-1.74	-1.07
Weekly	**1.01** (0.00)	**0.94** (0.00)	5.73	4.73	1.16	9.11	38.50	-1.99	-1.13
Monthly	**0.91** (0.01)	**0.87** (0.02)	5.53	4.83	1.32	10.66	22.62	-2.16	-1.20
Scaled with SMA-40 Volatility (40-day window)									
Daily	**1.05** (0.00)	**0.98** (0.00)	5.69	4.50	1.10	8.86	36.08	-1.84	-1.09
Weekly	**0.92** (0.00)	**0.88** (0.01)	5.39	4.61	1.18	9.62	22.57	-2.04	-1.13
Monthly	**0.83** (0.05)	0.80 (0.09)	5.18	4.68	1.30	10.87	14.10	-2.19	-1.19
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.94** (0.00)	**0.89** (0.01)	5.33	4.46	1.13	9.54	25.74	-1.93	-1.11
Weekly	**0.84** (0.03)	0.81 (0.07)	5.12	4.56	1.20	10.16	16.59	-2.11	-1.15
Monthly	0.74 (0.28)	0.72 (0.37)	4.86	4.61	1.31	11.14	10.72	-2.25	-1.20
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.78 (0.13)	0.74 (0.25)	4.82	4.44	1.19	10.07	17.88	-2.00	-1.13
Weekly	0.73 (0.29)	0.71 (0.41)	4.76	4.52	1.25	10.55	12.83	-2.14	-1.16
Monthly	0.71 (0.41)	0.69 (0.49)	4.72	4.54	1.35	11.22	8.05	-2.22	-1.18
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	**1.06** (0.00)	**0.93** (0.00)	5.75	4.50	1.02	7.84	70.20	-1.82	-1.08
Weekly	**0.97** (0.00)	**0.90** (0.01)	5.58	4.62	1.12	9.50	40.91	-2.00	-1.13
Monthly	**0.85** (0.02)	**0.82** (0.05)	5.23	4.60	1.24	10.75	17.42	-2.16	-1.17
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**0.92** (0.00)	**0.85** (0.03)	5.26	4.43	1.08	8.66	37.69	-1.90	-1.09
Weekly	**0.86** (0.02)	0.82 (0.06)	5.15	4.51	1.15	9.81	23.10	-2.03	-1.12
Monthly	0.77 (0.13)	0.75 (0.20)	4.95	4.52	1.24	10.66	11.66	-2.21	-1.17
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.78 (0.13)	0.74 (0.27)	4.79	4.37	1.17	9.32	20.29	-2.00	-1.11
Weekly	0.75 (0.24)	0.72 (0.35)	4.74	4.42	1.22	10.00	12.82	-2.09	-1.13
Monthly	0.69 (0.48)	0.68 (0.39)	4.67	4.43	1.28	10.39	7.37	-2.26	-1.18
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.72 (0.36)	0.69 (0.49)	4.59	4.35	1.25	9.58	14.13	-2.05	-1.11
Weekly	0.69 (0.49)	0.68 (0.41)	4.57	4.38	1.28	10.08	9.13	-2.12	-1.13
Monthly	0.65 (0.23)	0.64 (0.18)	4.55	4.39	1.34	10.32	5.59	-2.30	-1.18
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.67 (0.39)	0.66 (0.28)	4.46	4.33	1.32	9.87	9.44	-2.08	-1.12
Weekly	0.66 (0.31)	0.65 (0.25)	4.46	4.35	1.35	10.25	6.25	-2.14	-1.13
Monthly	0.63 (0.11)	0.62 (0.08)	4.47	4.36	1.40	10.40	4.01	-2.31	-1.18

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR. SR highlighted in red points to the lowest SR.

Figure 8: Cumulative Returns and Realized Volatility for Vanguard Pennsylvania Long-Term Tax-Exempt Fund Investor Shares (1987-2020)



The first panel shows the growth of wealth from the volatility-targeting portfolio (scaled) versus the benchmark (unscaled). The bottom panel shows the unscaled and scaled volatility (SMA-10) with daily trading. Both over the period 1987-2020.

Summary Bonds

In Table 18, there is a summary of the findings of the Sharpe ratio. Scaling with SMA-10 and SMA-20 gives the highest Sharpe ratio. Findings are very mixed as the periods of analysis also are. Sometimes the Sharpe ratio is significantly higher (like for VFIIX and VPAIX), and sometimes it is significantly lower (like for VFITX). Sometimes the results are not statistically significant at all. In the case of bonds in this sample with those volatility targets (between 2.5% and 4%), it looks like the higher frequency of trading (daily) is better than lower (monthly).

Harvey et al. (2018) report that unscaled U.S. bonds give a higher Sharpe ratio than scaled. With the target volatility of 10% for analyzed bonds, there is a slight difference between unscaled and scaled bonds. Sometimes active strategy has a significantly higher Sharpe ratio (VFIIX) and sometimes lower (VPAIX). In general, there is a small difference between gross and net SR as the trading costs are 5.0 bps. In the case of bonds with a target volatility of 10%, it looks like the lower frequency of trading is good enough. For gross and net Sharpe ratio, the most frequent trading with the best Sharpe ratio is monthly. For details, see Appendix A, Tables 27, 28, 29, 30, and 31.

Table 18: Securities, Sample Periods, and the Highest Sharpe Ratios for Bonds, Target Volatility: 2.5% - 4%

Bond	Period	SR unscaled	Best Gross SR	Scaling	Daily	Weekly	Monthly	Best Net SR	Scaling	Daily	Weekly	Monthly
VFIIX (2.5%)	1981 -2020	0.55	**0.84** (0.00)	SMA-10	x			0.67 (0.10)	SMA-10		x	
VFITX (2.5%)	1992 -2020	0.33	**0.16** (0.05)	SMA-20			x	**0.18** (0.05)	SMA-90	x	x	
			0.37 (0.33)	SMA-10	x			**0.18** (0.05)	EWMA-20			x
VWESX (4%)	1981 -2020	0.54	0.66 (0.07)	SMA-10	x			0.58 (0.29)	SMA-20	x		
VPAIX (3%)	1987 -2020	0.69	**1.18** (0.00)	SMA-10	x			**1.05** (0.00)	SMA-20	x		

Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. The applied volatility target is in the bracket under the bond ticker.

4.2.2 Commodities

In this section, there are analyzed two commodities with very different volatilities. In order to make the comparison meaningful, both are analyzed in the years 1988-2020. The first one is gold (GOLDPMGBD228NLBM). Gold has volatility of 15.41%, and volatility of volatility is 6.26% in this period. The second one is crude oil (DCOILBRENTEU). Crude oil has volatility of 39.12% and volatility of volatility 15.44%. Crude oil has higher volatility than gold.

Unscaled gold has a Sharpe ratio of 0.15, while the maximum scaled Sharpe ratio is 0.14, however not significant. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 19 shows the details. The volatility of volatility goes down from 6.26% (unscaled) to 2.50%-3.39% (scaled). The mean return of unscaled gold (5.05%) is higher than for scaled gold (3.30%-4.15%). Maximum drawdown is lower for scaled gold (20.94%-30.80%) than unscaled (38.51%). The same is about the left tail risk. It goes down from -9.71% to -6.80% - -5.53% for 1% and from -5.85% to -4.18% - -3.55% for 5%.

Unscaled crude oil has a Sharpe ratio of 0.22, while the maximum scaled Sharpe ratio is 0.23, however not significant. The highest statistically significant Sharpe ratio is 0.09. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 20 presents the details. The volatility of volatility is much lower. It goes down from 15.44% (unscaled) to 2.08%-3.76% (scaled). The mean return of unscaled crude oil (10.66%) is higher than for scaled oil (3.43%-5.55%). Maximum drawdown is lower for scaled oil (17.66%-31.81%) than unscaled (83.02%). The same is about the left tail risk. It goes down from -25.12% to -7.05% - -5.58% for 1% and from -13.90% to -4.43% - -3.67% for 5%.

Figure 9 shows a comparison of unscaled and volatility-scaled returns for crude oil. The latter of which uses a volatility estimate based on EWMA of 90-day half-life with monthly trading. In the top panel, there are plots of cumulative returns. In some periods, the volatility-scaled investment outperformed the unscaled investment. However, it seems like the buy-and-hold strategy gives a better cumulative return than the active strategy in the last years. The impact of volatility scaling is illustrated in the bottom panel. The realized volatility of volatility-scaled returns is much more stable over time, and the volatility of volatility is 3.76%, while the volatility of volatility for unscaled investment is 15.44%.

Table 19: Performance Statistics for Commodities - Gold (GOLDAMGBD228NLBM) (1988-2020)

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	P-value						1%	5%
Unscaled	0.15		5.05	15.41	6.26	38.51	0	-9.71	-5.85
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.12	0.35	4.19	12.05	2.50	20.94	116.05	-6.21	-3.96
Weekly	0.04	0.08	3.30	12.22	2.71	24.04	66.49	-6.58	-4.11
Monthly	0.11	0.32	4.15	12.34	3.39	30.80	24.81	-6.80	-4.18
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.11	0.32	4.07	11.55	2.72	23.35	57.57	-5.90	-3.76
Weekly	0.08	0.17	3.68	11.75	2.92	28.00	34.89	-6.21	-3.89
Monthly	0.11	0.29	4.05	12.03	3.34	29.86	20.64	-6.64	-4.07
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.14	0.42	4.26	11.19	2.73	24.80	29.46	-5.68	-3.65
Weekly	0.11	0.27	3.97	11.32	2.97	27.63	19.09	-5.93	-3.75
Monthly	0.12	0.35	4.15	11.37	3.21	27.49	12.77	-6.10	-3.87
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.13	0.41	4.21	10.99	2.86	26.15	19.83	-5.67	-3.65
Weekly	0.10	0.24	3.88	11.12	3.08	28.87	13.07	-5.90	-3.72
Monthly	0.10	0.24	3.90	11.18	3.28	27.56	9.49	-6.11	-3.86
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.09	0.20	3.77	10.72	3.00	25.86	12.85	-5.65	-3.64
Weekly	0.08	0.14	3.61	10.79	3.11	26.90	8.94	-5.78	-3.69
Monthly	0.09	0.18	3.74	10.84	3.25	26.35	6.43	-5.89	-3.78
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.11	0.29	3.97	11.06	2.43	22.47	54.36	-5.68	-3.66
Weekly	0.08	0.18	3.72	11.26	2.66	24.91	30.06	-5.95	-3.75
Monthly	0.11	0.30	4.02	11.40	3.11	27.84	14.67	-6.11	-3.86
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.10	0.25	3.87	10.75	2.62	24.23	27.50	-5.53	-3.58
Weekly	0.09	0.19	3.74	10.89	2.84	26.44	16.44	-5.71	-3.64
Monthly	0.10	0.24	3.87	11.01	3.12	26.54	9.70	-5.90	-3.74
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.10	0.22	3.81	10.54	2.90	25.09	14.11	-5.53	-3.55
Weekly	0.09	0.19	3.75	10.64	3.08	26.66	9.06	-5.65	-3.59
Monthly	0.10	0.22	3.85	10.74	3.27	25.66	6.12	-5.79	-3.68
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.10	0.20	3.79	10.41	3.04	25.07	9.48	-5.57	-3.55
Weekly	0.09	0.18	3.75	10.49	3.17	26.29	6.40	-5.65	-3.58
Monthly	0.10	0.20	3.81	10.57	3.33	25.54	4.53	-5.78	-3.66
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.09	0.16	3.73	10.26	3.14	25.19	6.41	-5.62	-3.56
Weekly	0.09	0.15	3.70	10.32	3.23	26.34	4.57	-5.69	-3.58
Monthly	0.09	0.16	3.72	10.38	3.35	25.69	3.37	-5.81	-3.65

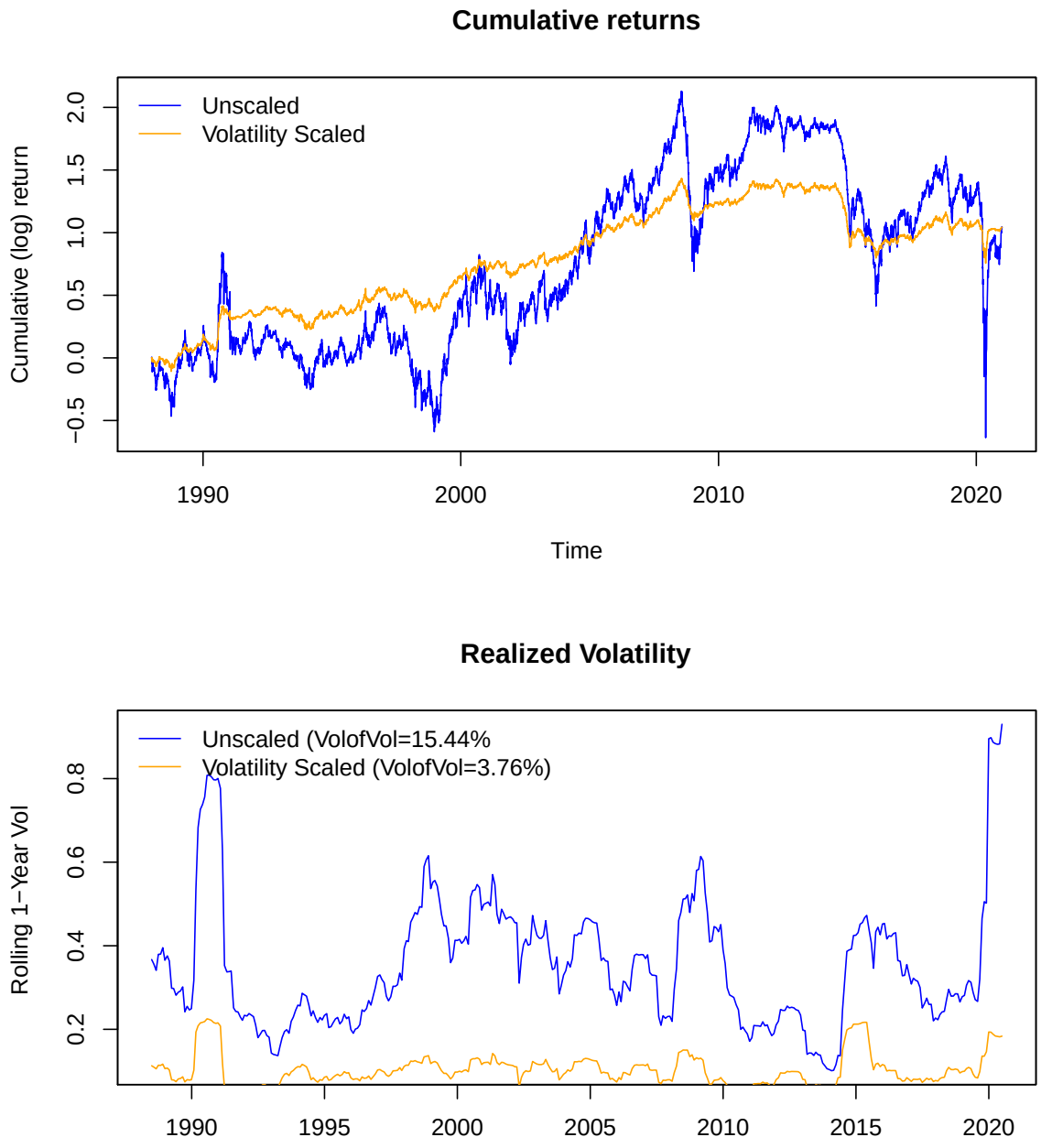
A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 20: Performance statistics for commodities - Crude Oil (DCOILBRENTEU), (1988-2020)

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	P-value						1%	5%
Unscaled	0.22		10.66	39.12	15.44	83.02	0	-25.12	-13.90
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.23	0.44	5.55	12.67	2.34	17.66	64.21	-6.19	-4.13
Weekly	0.22	0.49	5.45	12.87	2.46	20.04	35.90	-6.60	-4.26
Monthly	0.09	0.10	3.99	12.91	2.95	22.68	12.28	-7.05	-4.43
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.16	0.29	4.63	11.67	2.08	17.71	28.23	-5.70	-3.84
Weekly	0.12	0.17	4.22	11.68	2.16	19.04	16.25	-5.91	-3.91
Monthly	**0.06**	0.05	3.57	11.95	2.55	26.45	9.58	-6.45	-4.12
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.09	0.09	3.83	11.23	2.12	20.58	13.67	-5.75	-3.78
Weekly	0.08	0.07	3.73	11.27	2.19	21.61	8.93	-5.91	-3.84
Monthly	**0.04**	0.03	3.34	11.54	2.58	28.50	5.46	-6.26	-3.97
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.06**	0.03	3.48	11.12	2.16	22.09	9.04	-5.77	-3.78
Weekly	**0.06**	0.03	3.44	11.15	2.24	22.79	6.29	-5.88	-3.80
Monthly	**0.05**	0.02	3.44	11.45	2.77	29.60	3.87	-6.17	-3.88
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.05**	0.02	3.43	11.13	2.47	23.82	6.02	-5.96	-3.79
Weekly	**0.05**	0.02	3.40	11.18	2.56	24.49	4.86	-6.09	-3.82
Monthly	**0.06**	0.02	3.51	11.47	3.10	31.00	3.19	-6.34	-3.90
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.12	0.16	4.16	11.37	2.10	20.70	25.56	-5.58	-3.74
Weekly	0.12	0.16	4.15	11.35	2.14	20.55	13.32	-5.72	-3.78
Monthly	**0.07**	0.05	3.57	11.54	2.49	25.35	6.28	-6.15	-3.93
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.09	0.07	3.75	11.01	2.20	21.63	12.62	-5.59	-3.67
Weekly	0.08	0.06	3.71	10.99	2.22	22.07	7.34	-5.70	-3.71
Monthly	**0.06**	0.03	3.53	11.25	2.68	27.79	3.87	-6.01	-3.81
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	**0.06**	0.03	3.49	10.79	2.55	24.05	6.25	-5.69	-3.67
Weekly	**0.06**	0.03	3.43	10.80	2.58	24.41	4.46	-5.80	-3.69
Monthly	**0.07**	0.03	3.57	11.10	3.09	30.30	2.47	-5.98	-3.76
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	**0.06**	0.02	3.43	10.73	2.87	25.50	4.14	-5.81	-3.68
Weekly	**0.05**	0.02	3.37	10.75	2.91	25.78	3.68	-5.90	-3.70
Monthly	**0.08**	0.02	3.65	11.07	3.43	31.44	2.06	-6.01	-3.75
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	**0.06**	0.01	3.44	10.71	3.23	26.47	2.74	-5.95	-3.69
Weekly	**0.05**	0.01	3.37	10.73	3.26	26.65	3.26	-6.03	-3.71
Monthly	**0.09**	0.02	3.78	11.07	3.76	31.81	1.83	-6.09	-3.75

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR; in yellow, the highest, but not statistically significant SR. SR highlighted in red points to the lowest SR.

Figure 9: Cumulative Returns and Realized Volatility for Crude Oil (1988-2020)



The first panel shows the growth of wealth from the volatility-targeting portfolio (scaled) versus the benchmark (unscaled). The bottom panel shows the unscaled and scaled volatility (EWMA-90) with monthly trading. Both over the period 1988-2020.

Summary Commodities

Table 21 contains a summary of the findings of the Sharpe ratio. There is only a gross Sharpe ratio, as scaled commodities usually do not outperform unscaled commodities. Scaling with EWMA-90 and monthly trading gives statistically significant SR for Crude Oil (0.09), while the unscaled Sharpe ratio is 0.22. For gold, there is no better and statistically significant gross Sharpe ratio in the analyzed period. Harvey et al. (2018) report that unscaled commodities did not give much better performance statistics than scaled. This is consistent with the tiny sample in this study.

Table 21: Securities, Sample Periods, and the Highest Sharpe Ratios for Commodities and Foreign Exchange Rates

Asset	Period	SR unscaled	Best Gross SR	Scaling	Daily	Weekly	Monthly
Commodities							
Gold (GOLDPMGBD228NLBM) (10%)	1988 -2020	0.15	0.14 (0.42)	SMA-40	x		
Crude Oil (DCOILBRENTEU) (10%)	1988 -2020	0.22	0.23 (0.44)	SMA-10	x		
			0.09 (0.02)	EWMA-90			x
Foreign Exchange Rates							
U.S. / Euro Foreign Exchange Rate (DEXUSEU) (5%)	2000 -2020	-0.05	0.01 (0.24)	SMA-10	x		
Norway / U.S. Foreign Exchange Rate (DEXNOUS) (5%)	1972 -2020	-0.31	-0.37 (0.10)	EWMA-90	x	x	x
			-0.37 (0.17)	SMA-10		x	
			-0.39 (0.05)	EWMA-60	x		

Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. The applied volatility target is in the bracket under asset's name/ticker.

4.2.3 Foreign Exchange Rate

Two currencies are analyzed in this section. The first one is U.S. / Euro Foreign Exchange Rate (DEXUSEU) in the period 2000-2020, and the second one is Norway / U.S. Foreign Exchange Rate (DEXNOUS) in the period 1972-2020. The applied volatility target is 5%. Although the sample period is different, both currencies show similarities in unscaled returns and risk.

The highest Sharpe ratio for U.S. / Euro Foreign Exchange Rate is 0.01, however not statistically significant. The unscaled Sharpe ratio is -0.05. There is no statistically significant Sharpe ratio for this currency in those years. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 22 exhibits the details. The volatility of volatility goes down from 3.79% (unscaled) to 0.99%-1.25% (scaled). The mean return of unscaled currency (1.14%) is sometimes lower than for scaled currency (1.00% - 1.53%). Maximum drawdown is lower for scaled currency (8.50%-10.41%) than unscaled (18.27%). The same is about the left tail risk. It goes down from -4.76% to -2.83% - -2.42% for 1% and from -3.43% to -1.98% - -1.75% for 5%.

However, the highest Sharpe ratio for Norway / U.S. Foreign Exchange Rate is - 0.37, yet not statistically significant. The unscaled Sharpe ratio is -0.31. The statistically significant Sharpe ratio for this currency is -0.39 for EWMA-60 with daily frequency. Other risk measures indicate lower risk when using the volatility-targeting strategy. Table 23 presents the details. The volatility of volatility goes down from 3.07% (unscaled) to 1.34%-1.96% (scaled). The mean return of unscaled currency (1.10%) is always lower than for scaled currency (1.81% - 2.50%). Maximum drawdown is lower for scaled currency (12.69%-16.84%) than unscaled (22.04%). The same is about the left tail risk. It goes down from -5.79% to -3.72% - -2.74% for 1% and from -3.77% to -2.24% - -1.75% for 5%.

Figure 10 shows a comparison of unscaled and volatility-scaled returns for Norway / U.S. Foreign Exchange Rate. The latter of which uses a volatility estimate based on EWMA of 60-day half-life with daily trading. In the top panel, there are plots of cumulative returns. The volatility-scaled investment gives higher cumulative returns most of the time, as the mean return is always higher for volatility-scaled currency. The impact of volatility scaling is illustrated in the bottom panel. The realized volatility of volatility-scaled returns is slightly more stable over time, and the volatility of volatility is 2.56%, while the volatility of volatility for unscaled investment is 3.07%.

Summary Foreign Exchange Rate

Table 21 contains a summary of the findings of the Sharpe ratio for currencies with the

Table 22: Performance Statistics for Exchange - U.S. / Euro Foreign Exchange Rate (DEXUSEU), (2000-2020) with the Target Volatility of 5%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	P-value						1%	5%
Unscaled	-0.05		1.14	9.93	3.79	18.27	0	-4.76	-3.43
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.01	0.24	1.64	5.92	1.10	9.19	100.12	-2.80	-1.98
Weekly	-0.02	0.37	1.46	5.91	1.15	9.78	50.60	-2.79	-1.96
Monthly	-0.00	0.31	1.53	5.70	1.27	10.41	17.20	-2.83	-1.95
Scaled with SMA-20 Volatility (20-day window)									
Daily	-0.01	0.31	1.52	5.61	1.10	9.19	46.50	-2.56	-1.83
Weekly	-0.04	0.45	1.37	5.54	1.08	8.98	24.11	-2.55	-1.83
Monthly	-0.04	0.46	1.36	5.53	1.14	10.22	13.12	-2.67	-1.86
Scaled with SMA-40 Volatility (40-day window)									
Daily	-0.04	0.44	1.38	5.37	1.01	9.04	23.20	-2.47	-1.78
Weekly	-0.05	0.48	1.30	5.36	1.02	9.10	12.62	-2.48	-1.79
Monthly	-0.06	0.45	1.27	5.31	1.05	9.28	7.64	-2.55	-1.82
Scaled with SMA-60 Volatility (60-day window)									
Daily	-0.06	0.43	1.25	5.28	0.99	8.64	15.32	-2.49	-1.78
Weekly	-0.08	0.32	1.16	5.26	1.00	8.53	8.62	-2.50	-1.79
Monthly	-0.08	0.32	1.14	5.25	1.02	8.71	5.34	-2.57	-1.81
Scaled with SMA-90 Volatility (90-day window)									
Daily	-0.10	0.21	1.05	5.25	1.04	8.86	10.44	-2.51	-1.80
Weekly	-0.11	0.17	1.00	5.24	1.05	8.90	6.15	-2.52	-1.80
Monthly	-0.10	0.20	1.02	5.21	1.08	8.87	4.06	-2.57	-1.82
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	-0.04	0.48	1.35	5.45	1.01	8.67	45.30	-2.49	-1.79
Weekly	-0.05	0.50	1.32	5.43	1.02	8.97	20.80	-2.48	-1.79
Monthly	-0.03	0.42	1.39	5.34	1.09	9.77	9.48	-2.57	-1.81
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	-0.07	0.40	1.23	5.28	1.01	8.51	22.45	-2.43	-1.76
Weekly	-0.07	0.35	1.19	5.27	1.01	8.64	11.00	-2.44	-1.76
Monthly	-0.06	0.41	1.23	5.21	1.05	8.96	6.07	-2.51	-1.79
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	-0.08	0.29	1.15	5.18	1.06	8.50	11.09	-2.41	-1.75
Weekly	-0.09	0.25	1.11	5.17	1.07	8.55	5.83	-2.44	-1.76
Monthly	-0.08	0.29	1.14	5.14	1.10	8.81	3.64	-2.50	-1.78
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	-0.08	0.26	1.14	5.13	1.13	8.78	7.33	-2.42	-1.75
Weekly	-0.09	0.23	1.11	5.13	1.13	8.81	4.04	-2.45	-1.76
Monthly	-0.08	0.27	1.14	5.11	1.17	9.05	2.63	-2.50	-1.78
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	-0.08	0.24	1.14	5.09	1.23	8.99	4.82	-2.44	-1.75
Weekly	-0.09	0.22	1.12	5.09	1.23	9.02	2.83	-2.47	-1.76
Monthly	-0.09	0.22	1.11	5.09	1.25	9.05	2.23	-2.49	-1.77

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

target volatility of 5%. There is only a gross Sharpe ratio, as analyzed currencies show only negative unscaled SR. Scaling with EWMA-60 and daily trading gives a statistically significant Sharpe ratio for Norway / U.S. Foreign Exchange Rate (DEXNOUS) of -0.39, while the unscaled Sharpe ratio is -0.31. For U.S. / Euro Foreign Exchange Rate (DEXUSEU), there is

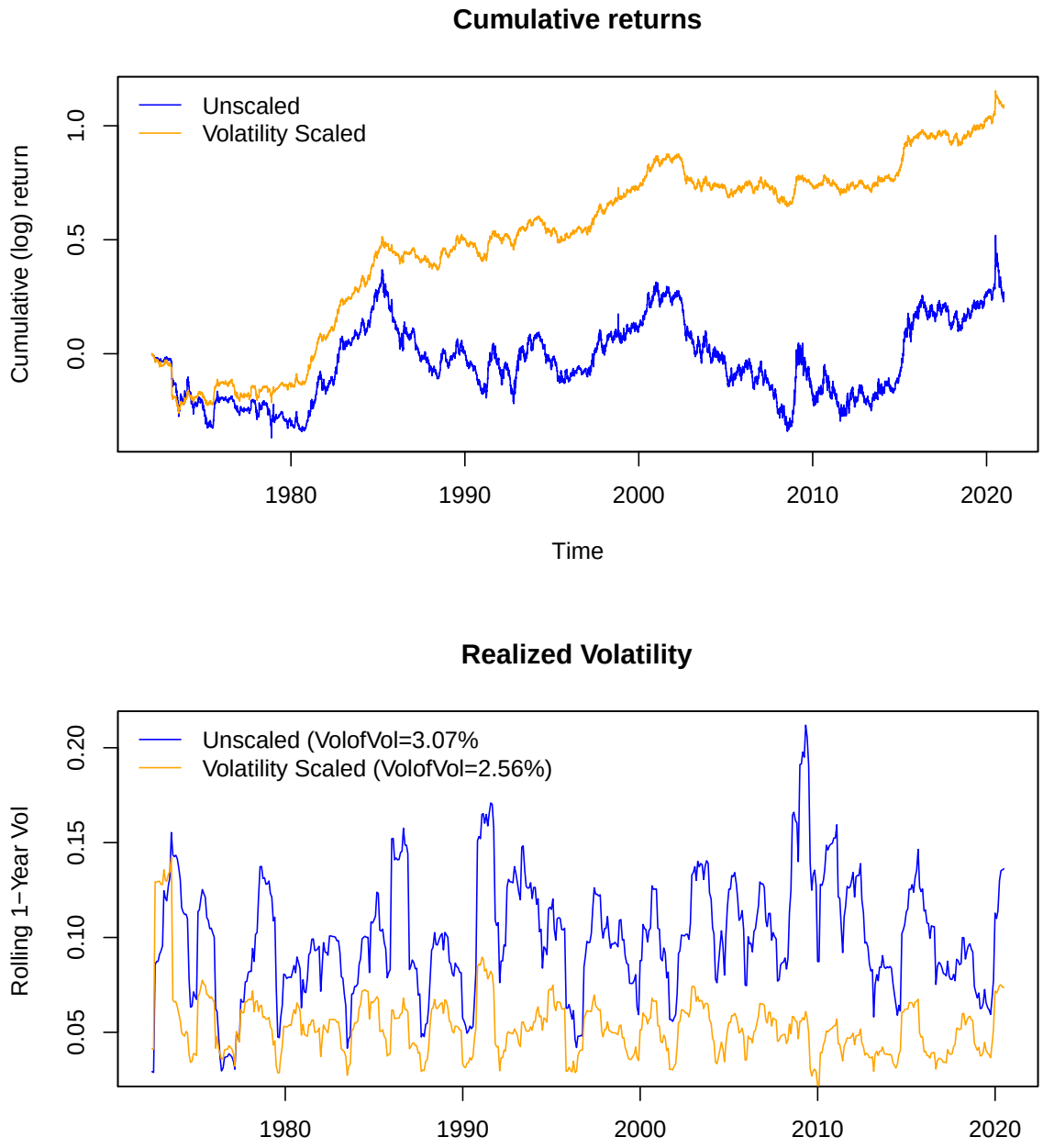
Table 23: Performance Statistics for Exchange - Norway / U.S. Foreign Exchange Rate (DEXNOUS), (1972-2020) with the Target Volatility of 5%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	P-value						1%	5%
Unscaled	-0.31		1.10	10.54	3.07	22.04	0	-5.79	-3.77
Scaled with SMA-10 Volatility (10-day window)									
Daily	-0.40	0.09	2.06	6.60	1.57	14.22	98.06	-3.03	-2.02
Weekly	-0.37	0.17	2.20	6.69	1.73	13.54	58.14	-3.12	-2.04
Monthly	-0.40	0.09	1.87	6.81	1.96	16.34	23.78	-3.72	-2.24
Scaled with SMA-20 Volatility (20-day window)									
Daily	-0.41	0.06	2.14	6.31	1.53	13.04	48.82	-2.89	-1.91
Weekly	**-0.42**	0.04	2.03	6.30	1.62	13.87	30.61	-3.07	-1.95
Monthly	**-0.43**	0.03	1.85	6.38	1.78	16.84	18.79	-3.54	-2.10
Scaled with SMA-40 Volatility (40-day window)									
Daily	**-0.44**	0.01	2.05	5.92	1.34	12.75	23.21	-2.82	-1.84
Weekly	**-0.44**	0.02	2.03	6.02	1.57	13.69	15.40	-2.96	-1.87
Monthly	**-0.46**	0.01	1.81	6.12	1.67	13.89	10.46	-3.33	-1.99
Scaled with SMA-60 Volatility (60-day window)									
Daily	**-0.44**	0.01	2.06	5.85	1.41	12.67	15.74	-2.85	-1.83
Weekly	**-0.44**	0.01	2.04	5.95	1.64	14.49	10.81	-2.97	-1.86
Monthly	**-0.43**	0.02	2.00	6.06	1.71	14.41	7.57	-3.26	-1.95
Scaled with SMA-90 Volatility (90-day window)									
Daily	**-0.40**	0.04	2.26	5.84	1.47	12.22	10.81	-2.83	-1.82
Weekly	**-0.40**	0.04	2.21	5.92	1.65	14.46	7.70	-2.93	-1.84
Monthly	**-0.40**	0.05	2.15	5.98	1.72	14.66	5.70	-3.17	-1.92
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	**-0.44**	0.01	2.04	6.02	1.44	12.69	42.95	-2.84	-1.84
Weekly	**-0.42**	0.02	2.12	5.97	1.42	13.17	23.97	-2.88	-1.85
Monthly	**-0.43**	0.02	1.99	6.06	1.58	14.02	12.75	-3.19	-1.95
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**-0.43**	0.01	2.14	5.85	1.52	13.90	21.76	-2.79	-1.79
Weekly	**-0.42**	0.02	2.17	5.83	1.51	13.56	13.09	-2.83	-1.80
Monthly	**-0.42**	0.03	2.10	5.90	1.61	13.99	7.92	-3.11	-1.89
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	**-0.41**	0.03	2.29	5.74	1.64	14.89	10.93	-2.77	-1.77
Weekly	**-0.40**	0.04	2.30	5.73	1.63	14.55	7.12	-2.80	-1.78
Monthly	**-0.40**	0.05	2.25	5.79	1.70	14.74	4.82	-3.04	-1.85
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	**-0.39**	0.05	2.39	5.68	1.69	14.91	7.37	-2.75	-1.76
Weekly	-0.38	0.06	2.40	5.67	1.67	14.55	5.06	-2.78	-1.77
Monthly	-0.38	0.07	2.35	5.72	1.73	14.69	3.57	-2.99	-1.83
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	-0.37	0.08	2.49	5.59	1.69	14.08	4.98	-2.74	-1.75
Weekly	-0.37	0.10	2.50	5.58	1.67	13.79	3.62	-2.76	-1.76
Monthly	-0.37	0.10	2.44	5.63	1.73	13.94	2.64	-2.94	-1.81

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR; in yellow, the highest, but not statistically significant SR. SR highlighted in red points to the lowest SR.

a better but not statistically significant gross Sharpe ratio in the analyzed period. In the same currencies, but with the target volatility of 10%, the Sharpe ratios are very similar to those with the target volatility of 5%. However, risk measurements show no big difference between scaled

Figure 10: Cumulative Returns and Realized Volatility for NOK/USD Exchange Rate (1972-2020)



The first panel shows the growth of wealth from the volatility-targeting portfolio (scaled) versus the benchmark (unscaled). The bottom panel shows the unscaled and scaled volatility (EWMA-60) with daily trading. Both over the period 1972-2020.

and unscaled currencies. For details, see Tables 32, and 33 in Appendix A.

Like with commodities, Harvey et al. (2018) report that unscaled foreign exchange rates give similar performance statistics to scaled ones. The study of Moreira and Muir (2017, page 1631) also finds that the volatility-timing strategy does not work for the currency carry trade.

5 Summary

First, the empirical results show that the volatility-targeting strategy does not consistently outperform the passive benchmark. It can depend on the asset class, region, method of volatility forecasting, frequency of trading, period, and target volatility. As Liu et al. (2019) suggest, the active strategy only outperforms the passive strategy during the financial crisis. Furthermore, if one uses Kenneth French datasets and start the analysis from the beginning of the series, around 1926, and to the current years, it will usually indicate that the volatility-targeting strategy works well for equities, see, e.g., Moreira and Muir (2017), Harvey et al. (2018) and this study (Sections 4.1.1, 4.1.2 or 4.1.3). However, the robustness check can lead to opposite results - the Sharpe ratio is worse for the active strategy, see, e.g., Liu et al. (2019), and robustness check for years 1960-1990 for U.S. market in this study (Table 25 in Appendix A).

Second, the volatility-targeting strategy reduces risk, as the target volatility is constant and usually lower than the benchmark. Harvey et al. (2018) conclude the same. For all the analyzed data in this study, the left tail has a lower risk. The volatility of volatility is also permanently reduced if the target volatility is lower than the buy-and-hold strategy. Maximum drawdowns are usually also reduced, however not always. The European and North American stocks can have larger drawdowns when scaled than unscaled; see Section 4.1.5.

Third, we wanted to examine if some of the volatility forecasting models work better than others. There is no clear pattern here. However, the most frequent best results for gross Sharpe ratio gives SMA-10 for U.S. equities and bonds analyzed in the study. Nevertheless, also the worst results can come from SMA-10. The difference makes the frequency of trading. When one uses SMA-10 with weekly trading for U.S. equities or SMA-10 with daily trading for U.S. bonds, one may achieve the best possible Sharp ratio among analyzed strategies. However, SMA-10 with monthly trading also gives the worst gross Sharpe ratios in this study. After considering transaction costs, the situation is even worse, as SMA-10 with daily trading can also give the worst net Sharpe ratio for equities and bonds in this research.

Last but not least, the volatility-targeting strategy does not work for commodities and currencies at all. Other researchers report similar results, e.g., Moreira and Muir (2017) and Harvey et al. (2018). However, it may outperform the benchmark, especially during crisis times for some equities and bonds. Bongaerts et al. (2020) propose how to correct the volatility-targeting strategy for better performance. They suggest that they can eliminate some downsides of the conventional volatility-targeting strategy, like high turnover and leverage. They believe that one should adjust risk exposures only in the extremes during high and low volatility states; otherwise, the exposure is unscaled. Their strategy is called conditional volatility-targeting. Zakamulin (2019) also comes with the promising idea of volatility weighting over time in the

presence of transaction costs. The volatility-weighting-over-time mechanism offers some control over portfolio turnover. His modification is achieved by adding two features: a tuning parameter and a no-transaction region around the targeted risk exposure.

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Appendices

A Appendix

Table 24: Performance Statistics for Equities - All U.S. (1927-1959), robustness check

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.47		11.92	23.38	13.19	62.89	0	-12.89	-7.50
Scaled with SMA-10 Volatility (10-day window)									
Daily	**0.79** (0.00)	**0.67** (0.03)	11.78	14.82	2.46	22.88	116.39	-6.91	-4.47
Weekly	**0.84** (0.00)	**0.77** (0.00)	12.71	15.07	2.69	26.10	70.32	-7.19	-4.55
Monthly	**0.69** (0.02)	**0.66** (0.03)	11.11	15.62	3.39	29.81	27.92	-8.42	-4.90
Scaled with SMA-20 Volatility (20-day window)									
Daily	**0.74** (0.00)	**0.68** (0.02)	10.85	14.36	2.46	24.86	61.47	-6.90	-4.36
Weekly	**0.74** (0.00)	**0.70** (0.01)	11.01	14.57	2.67	26.76	38.46	-7.23	-4.46
Monthly	**0.71** (0.01)	**0.69** (0.01)	10.92	14.84	3.08	30.35	23.34	-7.71	-4.61
Scaled with SMA-40 Volatility (40-day window)									
Daily	**0.76** (0.00)	**0.73** (0.00)	10.86	14.08	2.72	27.44	32.36	-6.92	-4.27
Weekly	**0.78** (0.00)	**0.75** (0.00)	11.18	14.20	2.88	29.24	21.19	-7.16	-4.33
Monthly	**0.75 (0.00)**	**0.73** (0.00)	11.11	14.42	3.20	32.58	13.72	-7.59	-4.49
Scaled with SMA-60 Volatility (60-day window)									
Daily	**0.79** (0.00)	**0.77** (0.00)	11.15	13.98	2.81	30.09	22.25	-6.96	-4.24
Weekly	**0.79** (0.00)	**0.77** (0.00)	11.25	14.06	2.98	32.09	14.90	-7.20	-4.30
Monthly	**0.74** (0.00)	**0.73** (0.00)	10.94	14.32	3.31	35.53	10.55	-7.59	-4.47
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.79** (0.00)	**0.77** (0.00)	10.88	13.90	2.98	32.61	15.16	-7.08	-4.24
Weekly	**0.79** (0.00)	**0.78** (0.00)	11.23	13.99	3.13	34.19	10.24	-7.27	-4.27
Monthly	**0.75** (0.00)	**0.74** (0.00)	10.89	14.18	3.25	34.71	7.60	-7.63	-4.42
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	**0.77** (0.00)	**0.71** (0.01)	10.97	14.12	2.55	26.38	58.60	-6.86	-4.24
Weekly	**0.77** (0.00)	**0.73** (0.00)	11.15	14.41	2.97	31.96	33.26	-7.29	-4.36
Monthly	**0.72** (0.00)	**0.70** (0.01)	10.66	14.31	2.96	30.28	17.20	-7.53	-4.45
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**0.78** (0.00)	**0.75** (0.00)	10.84	13.77	2.71	28.96	30.97	-6.83	-4.14
Weekly	**0.78** (0.00)	**0.76** (0.00)	11.01	13.91	2.94	31.86	18.88	-7.13	-4.22
Monthly	**0.74** (0.00)	**0.73** (0.00)	10.64	13.89	2.95	31.14	11.06	-7.33	-4.32
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	**0.80** (0.00)	**0.78** (0.00)	10.85	13.36	2.81	28.53	15.56	-6.79	-4.04
Weekly	**0.81** (0.00)	**0.79** (0.00)	11.00	13.39	2.88	29.26	10.09	-6.97	-4.08
Monthly	**0.77** (0.00)	**0.77** (0.00)	10.74	13.38	2.88	28.46	6.44	-7.15	-4.17
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	**0.81** (0.00)	**0.80** (0.00)	10.79	13.06	2.84	26.86	10.29	-6.77	-3.99
Weekly	**0.81** (0.00)	**0.80** (0.00)	10.91	13.07	2.87	27.04	7.01	-6.90	-4.02
Monthly	**0.79** (0.00)	**0.78** (0.00)	10.71	13.06	2.89	26.39	4.63	-7.09	-4.10
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	**0.80** (0.00)	**0.80** (0.00)	10.63	12.77	2.94	25.14	6.87	-6.77	-3.95
Weekly	**0.81** (0.00)	**0.80** (0.00)	10.73	12.77	2.95	25.14	4.95	-6.86	-3.97
Monthly	**0.79** (0.00)	**0.78** (0.00)	10.59	12.77	2.98	24.69	3.34	-7.06	-4.04

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR. SR highlighted in red points to the lowest SR.

Table 25: Performance Statistics for Equities All U.S. (1960-1990), robustness check

Trading	Sharpe Ratio Gross	Sharpe Ratio Net	Mean	Vol	Vol of Vol	MDD	Turnover	Left tail 1%	Left tail 5%
Unscaled	0.30		10.51	15.70	5.23	39.32	0	-7.70	-4.67
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.31 (0.45)	0.18 (0.07)	9.93	13.70	2.48	20.11	116.86	-5.82	-3.88
Weekly	0.32 (0.40)	0.25 (0.26)	10.14	14.09	2.76	21.69	65.73	-6.06	-4.00
Monthly	0.22 (0.14)	0.19 (0.07)	9.25	15.12	3.87	32.84	26.65	-6.73	-4.23
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.31 (0.44)	0.24 (0.23)	9.87	13.69	2.50	22.11	60.25	-5.62	-3.77
Weekly	0.31 (0.42)	0.27 (0.37)	9.97	13.74	2.64	22.53	35.88	-5.82	-3.84
Monthly	0.26 (0.26)	0.23 (0.16)	9.49	14.44	3.11	30.47	21.72	-6.35	-4.05
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.24 (0.20)	0.20 (0.09)	9.10	13.52	2.59	23.79	31.86	-5.67	-3.76
Weekly	0.25 (0.21)	0.22 (0.13)	9.17	13.53	2.66	23.63	19.98	-5.84	-3.84
Monthly	0.22 (0.12)	0.21 (0.08)	9.08	14.12	3.26	30.82	13.59	-6.33	-4.00
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.23 (0.14)	0.20 (0.07)	8.96	13.55	2.76	25.70	21.85	-5.72	-3.78
Weekly	0.24 (0.17)	0.22 (0.11)	9.09	13.53	2.78	25.35	14.24	-5.92	-3.83
Monthly	0.21 (0.07)	**0.20** (0.05)	8.90	14.04	3.47	34.17	10.07	-6.40	-3.99
Scaled with SMA-90 Volatility (90-day window)									
Daily	**0.20** (0.05)	**0.18** (0.03)	8.63	13.46	2.91	27.35	15.08	-5.89	-3.81
Weekly	0.21 (0.07)	**0.20** (0.05)	8.78	13.43	2.88	26.85	10.37	-6.09	-3.87
Monthly	**0.18** (0.02)	**0.17** (0.02)	8.57	13.89	3.53	34.20	7.33	-6.45	-3.99
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.27 (0.34)	0.20 (0.09)	9.40	13.52	2.60	24.03	59.18	-5.61	-3.74
Weekly	0.30 (0.48)	0.26 (0.29)	9.71	13.47	2.61	23.08	31.08	-5.72	-3.77
Monthly	0.26 (0.26)	0.24 (0.18)	9.39	13.97	3.15	29.96	16.17	-6.21	-3.94
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.24 (0.19)	0.20 (0.08)	9.05	13.33	2.71	25.52	30.83	-5.60	-3.71
Weekly	0.26 (0.28)	0.24 (0.18)	9.28	13.25	2.66	24.15	17.62	-5.72	-3.74
Monthly	0.23 (0.13)	0.22 (0.09)	9.04	13.71	3.20	30.71	10.96	-6.18	-3.90
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.21 (0.09)	**0.20** (0.05)	8.77	13.17	2.94	26.85	16.11	-5.67	-3.70
Weekly	0.23 (0.14)	0.22 (0.10)	8.96	13.08	2.86	25.11	9.91	-5.80	-3.73
Monthly	0.21 (0.06)	**0.20** (0.05)	8.78	13.45	3.36	30.79	6.88	-6.20	-3.85
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.21 (0.06)	**0.20** (0.04)	8.70	13.08	3.20	27.21	10.84	-5.73	-3.70
Weekly	0.23 (0.10)	0.22 (0.07)	8.87	12.99	3.11	25.40	7.02	-5.85	-3.72
Monthly	**0.21** (0.05)	**0.20** (0.04)	8.74	13.30	3.54	30.31	5.08	-6.17	-3.82
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	**0.21** (0.05)	**0.20** (0.03)	8.69	13.00	3.49	27.69	7.30	-5.82	-3.70
Weekly	0.22 (0.07)	0.22 (0.06)	8.82	12.92	3.40	25.94	5.00	-5.91	-3.72
Monthly	**0.21** (0.04)	**0.20** (0.03)	8.72	13.17	3.75	30.05	3.72	-6.18	-3.80

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR; in yellow, the highest, but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 26: Performance Statistics for Equities - All U.S. (1991-2020), robustness check

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.53		11.73	15.07	5.79	28.36	0	-11.90	-6.96
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.61 (0.23)	0.49 (0.35)	9.26	10.57	2.14	17.38	112.41	-6.42	-4.21
Weekly	0.61 (0.25)	0.54 (0.49)	9.27	10.70	2.32	20.31	63.61	-6.89	-4.29
Monthly	0.64 (0.16)	0.61 (0.23)	10.03	10.76	2.67	21.72	27.61	-7.41	-4.56
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.61 (0.23)	0.54 (0.47)	9.04	10.26	2.14	18.38	60.69	-6.41	-4.11
Weekly	0.62 (0.19)	0.58 (0.32)	9.28	10.26	2.20	18.46	36.92	-6.68	-4.19
Monthly	0.61 (0.21)	0.59 (0.29)	9.50	10.44	2.58	21.99	22.59	-7.36	-4.44
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.64 (0.13)	0.61 (0.23)	9.21	9.84	2.12	17.99	30.70	-6.45	-4.03
Weekly	0.65 (0.12)	0.63 (0.16)	9.37	9.82	2.17	18.45	18.90	-6.60	-4.08
Monthly	0.60 (0.22)	0.59 (0.27)	9.17	9.97	2.47	20.71	13.38	-7.25	-4.30
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.62 (0.16)	0.60 (0.23)	9.01	9.75	2.25	18.44	20.71	-6.44	-4.01
Weekly	0.63 (0.14)	0.62 (0.18)	9.15	9.72	2.26	19.17	13.40	-6.55	-4.05
Monthly	0.60 (0.21)	0.59 (0.25)	9.09	9.86	2.46	20.50	9.55	-7.08	-4.24
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.61 (0.18)	0.60 (0.24)	8.89	9.67	2.27	19.22	14.22	-6.62	-4.06
Weekly	0.62 (0.17)	0.61 (0.20)	9.00	9.64	2.30	19.65	9.72	-6.78	-4.09
Monthly	0.59 (0.23)	0.59 (0.26)	8.95	9.62	2.34	19.78	6.74	-7.23	-4.25
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.63 (0.17)	0.56 (0.39)	9.03	9.93	2.16	18.89	56.74	-6.30	-3.98
Weekly	0.63 (0.17)	0.60 (0.27)	9.14	9.83	2.07	17.95	30.06	-6.49	-4.05
Monthly	0.62 (0.18)	0.60 (0.23)	9.34	10.08	2.51	21.04	16.16	-6.99	-4.25
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.64 (0.13)	0.61 (0.22)	9.01	9.57	2.13	18.08	28.82	-6.22	-3.90
Weekly	0.64 (0.13)	0.62 (0.18)	9.07	9.50	2.07	18.48	16.38	-6.37	-3.95
Monthly	0.62 (0.16)	0.61 (0.20)	9.11	9.64	2.30	19.25	10.02	-6.81	-4.11
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.64 (0.11)	0.62 (0.15)	8.95	9.29	2.16	18.80	14.63	-6.23	-3.86
Weekly	0.64 (0.11)	0.63 (0.14)	8.96	9.27	2.16	19.17	8.89	-6.35	-3.90
Monthly	0.62 (0.15)	0.61 (0.17)	8.94	9.33	2.23	18.72	5.83	-6.74	-4.04
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.64 (0.10)	0.63 (0.13)	8.88	9.16	2.24	18.90	9.68	-6.25	-3.85
Weekly	0.63 (0.12)	0.62 (0.13)	8.86	9.17	2.26	19.13	6.16	-6.36	-3.88
Monthly	0.61 (0.16)	0.61 (0.17)	8.82	9.22	2.30	18.82	4.19	-6.70	-4.01
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.63 (0.10)	0.62 (0.12)	8.79	9.07	2.37	18.83	6.41	-6.29	-3.86
Weekly	0.62 (0.12)	0.62 (0.14)	8.75	9.10	2.41	18.95	4.35	-6.40	-3.88
Monthly	0.60 (0.17)	0.60 (0.18)	8.70	9.14	2.44	18.75	3.01	-6.68	-3.99

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 27: Securities, Sample Periods, and the Highest Sharpe Ratios for Bonds, Target Volatility: 10%

Bond	Period	SR unscaled	Best Gross SR	Scaling	Daily	Weekly	Monthly	Best Net SR	Scaling	Daily	Weekly	Monthly
VFIIX	1981 -2020	0.55	**0.63** (0.04)	EWMA-90		x	x	**0.63** (0.04)	EWMA-90		x	x
VFITX	1992 -2020	0.33	0.35 (0.17)	SMA-90		x		0.34 (0.21)	SMA-40			x
								0.34 (0.18)	SMA-60	x	x	x
								0.34 (0.20)	SMA-90	x	x	x
								0.34 (0.24)	EWMA-40			x
VWESX	1981 -2020	0.54	0.62 (0.06)	SMA-10	x			0.57 (0.29)	SMA-10	x		
								0.57 (0.30)	SMA-20	x		
VPAIX	1987 -2020	0.69	0.74 (0.16)	SMA-10			x	0.73 (0.18)	SMA-10			x
			0.68 (0.00)	EWMA-90			x	**0.68** (0.00)	EWMA-90			x

Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level.

Table 28: Performance Statistics for Bonds - Vanguard GNMA Fund Investor Shares (VFIIX) (1981-2020) with the Target Volatility of 10%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.55		6.96	4.88	2.59	15.66	0	-3.36	-1.75
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.54 (0.44)	0.53 (0.34)	7.61	6.28	3.02	19.33	17.38	-3.94	-2.26
Weekly	0.56 (0.42)	0.55 (0.49)	7.97	6.00	2.53	12.73	11.38	-3.90	-2.26
Monthly	0.58 (0.27)	0.57 (0.31)	8.21	6.06	2.62	14.78	5.52	-4.02	-2.28
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.55 (0.48)	0.54 (0.43)	7.67	6.24	2.95	18.77	8.48	-3.91	-2.26
Weekly	0.58 (0.26)	0.57 (0.31)	8.17	6.02	2.55	13.61	6.75	-3.89	-2.26
Monthly	0.58 (0.21)	0.58 (0.24)	8.17	6.03	2.56	15.19	4.56	-3.96	-2.28
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.61 (0.18)	0.60 (0.19)	7.91	6.03	2.61	13.96	4.30	-3.79	-2.24
Weekly	0.62 (0.06)	0.61 (0.08)	8.12	5.98	2.51	14.34	4.19	-3.80	-2.24
Monthly	0.61 (0.09)	0.61 (0.10)	8.08	6.01	2.55	15.55	3.11	-3.85	-2.26
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.59 (0.25)	0.59 (0.27)	7.82	6.01	2.58	14.21	2.99	-3.84	-2.25
Weekly	0.60 (0.14)	0.59 (0.16)	8.00	5.99	2.52	14.38	3.31	-3.85	-2.26
Monthly	0.61 (0.08)	0.61 (0.09)	8.08	6.01	2.54	15.11	2.46	-3.86	-2.26
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.59 (0.21)	0.59 (0.22)	7.89	6.01	2.57	14.99	2.21	-3.87	-2.26
Weekly	0.61 (0.09)	0.61 (0.10)	8.10	5.99	2.52	15.20	2.72	-3.88	-2.25
Monthly	0.61 (0.09)	0.60 (0.10)	8.09	5.98	2.51	15.65	1.86	-3.88	-2.26
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.57 (0.38)	0.56 (0.43)	7.72	6.12	2.77	16.45	8.80	-3.83	-2.24
Weekly	0.59 (0.15)	0.59 (0.19)	8.05	5.96	2.49	13.38	6.17	-3.81	-2.23
Monthly	0.60 (0.12)	0.60 (0.13)	8.07	5.95	2.48	14.77	3.71	-3.85	-2.25
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.59 (0.29)	0.58 (0.31)	7.78	6.00	2.59	14.43	4.58	-3.81	-2.23
Weekly	0.60 (0.11)	0.60 (0.12)	8.05	5.91	2.44	13.89	3.92	-3.80	-2.23
Monthly	0.60 (0.11)	0.60 (0.12)	8.03	5.91	2.44	15.01	2.60	-3.82	-2.24
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.60 (0.18)	0.60 (0.19)	7.87	5.89	2.44	14.70	2.48	-3.78	-2.22
Weekly	0.61 (0.07)	0.61 (0.08)	8.08	5.85	2.39	14.76	2.72	-3.78	-2.22
Monthly	0.62 (0.07)	0.61 (0.07)	8.07	5.85	2.39	15.45	1.87	-3.79	-2.22
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.61 (0.13)	0.61 (0.13)	7.93	5.84	2.40	15.04	1.79	-3.76	-2.20
Weekly	**0.62** (0.05)	0.62 (0.06)	8.09	5.82	2.39	15.09	2.29	-3.76	-2.20
Monthly	**0.62** (0.05)	0.62 (0.06)	8.08	5.82	2.39	15.66	1.54	-3.77	-2.21
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.63 (0.08)	0.62 (0.08)	7.99	5.78	2.34	14.53	1.21	-3.74	-2.19
Weekly	**0.63** (0.04)	**0.63** (0.04)	8.12	5.79	2.35	14.57	1.88	-3.74	-2.19
Monthly	**0.63** (0.04)	**0.63** (0.04)	8.11	5.77	2.34	14.92	1.26	-3.74	-2.19

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR. SR highlighted in red points to the lowest SR.

Table 29: Performance Statistics for Bonds - Vanguard Intermediate-Term Treasury Fund Investor Shares (VFITX) (1992-2020) with the Target Volatility of 10%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.33		2.77	4.89	2.05	12.43	0	-2.90	-1.72
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.32 (0.44)	0.31 (0.25)	3.42	6.98	2.58	15.92	17.37	-4.05	-2.42
Weekly	0.33 (0.45)	0.32 (0.40)	3.48	7.01	2.62	15.94	12.57	-4.08	-2.43
Monthly	0.32 (0.45)	0.32 (0.37)	3.42	6.87	2.57	16.39	7.37	-4.14	-2.46
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.32 (0.47)	0.32 (0.35)	3.44	6.90	2.48	15.14	9.73	-4.10	-2.44
Weekly	0.34 (0.35)	0.33 (0.45)	3.52	6.94	2.50	15.62	7.73	-4.09	-2.44
Monthly	0.33 (0.43)	0.33 (0.50)	3.49	6.91	2.56	16.30	5.73	-4.16	-2.47
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.33 (0.42)	0.33 (0.50)	3.50	6.95	2.53	15.82	5.14	-4.18	-2.47
Weekly	0.34 (0.27)	0.33 (0.34)	3.56	6.97	2.54	15.86	4.77	-4.18	-2.47
Monthly	0.35 (0.18)	0.34 (0.22)	3.60	6.90	2.49	15.55	3.33	-4.16	-2.46
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.34 (0.24)	0.34 (0.29)	3.57	6.94	2.52	15.61	3.63	-4.17	-2.46
Weekly	0.35 (0.17)	0.34 (0.21)	3.60	6.95	2.53	15.80	3.53	-4.16	-2.46
Monthly	0.34 (0.17)	0.34 (0.19)	3.60	6.96	2.55	15.96	2.32	-4.18	-2.46
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.34 (0.18)	0.34 (0.21)	3.60	7.03	2.64	16.61	2.28	-4.17	-2.47
Weekly	0.34 (0.17)	0.34 (0.20)	3.61	7.04	2.67	16.79	2.56	-4.17	-2.47
Monthly	0.34 (0.19)	0.34 (0.22)	3.59	7.06	2.70	16.91	1.72	-4.20	-2.48
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.32 (0.41)	0.31 (0.27)	3.42	6.90	2.49	15.04	10.06	-4.14	-2.46
Weekly	0.33 (0.37)	0.33 (0.48)	3.52	6.97	2.54	15.73	7.54	-4.12	-2.45
Monthly	0.32 (0.49)	0.32 (0.42)	3.45	6.89	2.52	15.72	4.71	-4.18	-2.47
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.33 (0.48)	0.32 (0.42)	3.48	6.96	2.56	15.57	5.49	-4.17	-2.47
Weekly	0.34 (0.29)	0.33 (0.37)	3.55	7.00	2.59	16.04	4.91	-4.15	-2.47
Monthly	0.33 (0.34)	0.33 (0.39)	3.53	6.95	2.57	15.94	3.14	-4.19	-2.48
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.33 (0.31)	0.33 (0.37)	3.54	7.04	2.66	16.34	2.81	-4.19	-2.48
Weekly	0.34 (0.24)	0.33 (0.29)	3.57	7.06	2.67	16.65	3.06	-4.18	-2.48
Monthly	0.34 (0.22)	0.34 (0.25)	3.58	7.04	2.67	16.59	2.01	-4.20	-2.49
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.33 (0.35)	0.33 (0.38)	3.54	7.10	2.74	16.87	1.82	-4.22	-2.50
Weekly	0.33 (0.28)	0.33 (0.33)	3.56	7.10	2.75	17.10	2.39	-4.22	-2.50
Monthly	0.33 (0.26)	0.33 (0.29)	3.56	7.10	2.75	17.06	1.58	-4.23	-2.50
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.33 (0.36)	0.33 (0.38)	3.54	7.17	2.84	17.51	1.05	-4.25	-2.52
Weekly	0.33 (0.29)	0.33 (0.34)	3.56	7.17	2.85	17.66	1.95	-4.25	-2.52
Monthly	0.33 (0.28)	0.33 (0.31)	3.56	7.17	2.85	17.65	1.23	-4.26	-2.52

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 30: Performance Statistics for Bonds - Vanguard Long-Term Investment-Grade Fund Investor Shares (VWESX) (1981-2020) with the Target Volatility of 10%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.54		8.57	8.39	2.99	20.20	0	-4.82	-3.07
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.62 (0.06)	0.57 (0.30)	10.08	9.90	2.09	16.42	89.31	-4.80	-3.31
Weekly	0.60 (0.15)	0.56 (0.33)	9.80	9.92	2.11	16.94	51.79	-4.85	-3.34
Monthly	0.55 (0.46)	0.54 (0.44)	9.52	10.63	3.30	34.59	19.29	-5.56	-3.50
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.60 (0.13)	0.57 (0.31)	9.78	9.92	2.07	16.43	50.03	-4.83	-3.31
Weekly	0.58 (0.21)	0.56 (0.33)	9.64	9.98	2.19	17.08	29.21	-4.94	-3.34
Monthly	0.56 (0.35)	0.55 (0.44)	9.53	10.25	2.75	26.77	16.13	-5.36	-3.43
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.58 (0.23)	0.56 (0.35)	9.55	9.89	2.24	17.94	27.07	-4.91	-3.31
Weekly	0.57 (0.30)	0.56 (0.38)	9.47	9.96	2.31	18.59	16.31	-5.00	-3.33
Monthly	0.54 (0.47)	0.54 (0.47)	9.36	10.19	2.81	27.78	9.94	-5.37	-3.40
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.57 (0.24)	0.56 (0.33)	9.52	9.88	2.32	19.28	18.77	-4.96	-3.32
Weekly	0.56 (0.35)	0.55 (0.41)	9.40	9.96	2.40	19.89	11.59	-5.04	-3.34
Monthly	0.54 (0.48)	0.54 (0.44)	9.29	10.13	2.76	25.95	7.26	-5.30	-3.40
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.55 (0.40)	0.55 (0.47)	9.33	9.87	2.42	19.93	13.04	-5.04	-3.33
Weekly	0.54 (0.48)	0.54 (0.47)	9.26	9.93	2.48	20.31	8.63	-5.12	-3.35
Monthly	0.53 (0.41)	0.53 (0.37)	9.22	10.10	2.76	25.20	5.66	-5.38	-3.42
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.59 (0.15)	0.56 (0.37)	9.69	9.86	2.13	16.76	53.49	-4.80	-3.29
Weekly	0.58 (0.23)	0.56 (0.35)	9.58	9.92	2.19	17.15	28.06	-4.91	-3.32
Monthly	0.54 (0.48)	0.54 (0.45)	9.37	10.26	2.86	28.41	12.17	-5.36	-3.41
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.57 (0.26)	0.55 (0.39)	9.47	9.81	2.22	18.12	28.77	-4.85	-3.28
Weekly	0.56 (0.33)	0.55 (0.41)	9.40	9.86	2.30	18.50	15.89	-4.95	-3.31
Monthly	0.54 (0.46)	0.53 (0.41)	9.25	10.13	2.78	26.85	8.15	-5.31	-3.38
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.55 (0.39)	0.55 (0.47)	9.29	9.81	2.39	19.55	14.99	-4.97	-3.30
Weekly	0.55 (0.44)	0.54 (0.50)	9.25	9.85	2.45	19.86	8.85	-5.05	-3.32
Monthly	0.53 (0.40)	0.53 (0.37)	9.17	10.04	2.78	25.59	5.13	-5.29	-3.37
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.55 (0.46)	0.54 (0.48)	9.23	9.83	2.51	20.26	10.56	-5.04	-3.32
Weekly	0.54 (0.50)	0.54 (0.46)	9.20	9.86	2.56	20.51	6.50	-5.10	-3.34
Monthly	0.53 (0.38)	0.53 (0.36)	9.14	10.02	2.82	25.20	3.91	-5.30	-3.38
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.54 (0.49)	0.54 (0.44)	9.19	9.84	2.63	20.83	7.29	-5.11	-3.34
Weekly	0.54 (0.44)	0.53 (0.42)	9.16	9.87	2.68	21.14	4.75	-5.16	-3.35
Monthly	0.53 (0.34)	0.52 (0.32)	9.12	10.00	2.89	25.02	2.93	-5.32	-3.39

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 31: Performance Statistics for Bonds - Vanguard Pennsylvania Long-Term Tax-Exempt Fund Investor Shares (VPAIX), (1987-2020) with the Target Volatility of 10%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	Net						1%	5%
Unscaled	0.69		4.96	5.12	2.35	12.96	0	-2.54	-1.32
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.71 (0.39)	0.70 (0.44)	6.04	7.39	3.07	17.43	6.85	-3.36	-1.87
Weekly	0.69 (0.48)	0.69 (0.46)	6.03	7.45	3.18	18.55	6.87	-3.47	-1.89
Monthly	0.74 (0.16)	0.73 (0.18)	6.28	7.56	3.39	19.28	3.35	-3.47	-1.89
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.71 (0.33)	0.71 (0.36)	6.11	7.40	3.08	17.22	3.53	-3.44	-1.89
Weekly	0.71 (0.37)	0.70 (0.41)	6.11	7.42	3.14	17.86	3.97	-3.49	-1.90
Monthly	0.72 (0.18)	0.72 (0.21)	6.25	7.47	3.23	18.35	3.02	-3.55	-1.91
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.69 (0.47)	0.69 (0.50)	6.08	7.47	3.22	18.90	1.82	-3.57	-1.91
Weekly	0.68 (0.35)	0.68 (0.31)	6.03	7.49	3.24	19.25	2.81	-3.63	-1.93
Monthly	0.68 (0.30)	0.68 (0.27)	6.12	7.43	3.18	19.28	2.08	-3.69	-1.95
Scaled with SMA-60 Volatility (60-day window)									
Daily	0.67 (0.26)	0.67 (0.25)	6.01	7.39	3.10	19.18	1.34	-3.65	-1.94
Weekly	0.66 (0.10)	0.65 (0.08)	5.95	7.40	3.11	19.25	2.28	-3.71	-1.95
Monthly	**0.66** (0.01)	**0.66** (0.01)	6.09	7.40	3.16	19.28	1.75	-3.79	-1.97
Scaled with SMA-90 Volatility (90-day window)									
Daily	0.66 (0.09)	0.66 (0.08)	6.01	7.48	3.25	19.21	1.00	-3.73	-1.96
Weekly	**0.65** (0.02)	**0.65** (0.02)	5.99	7.49	3.27	19.25	2.00	-3.77	-1.97
Monthly	**0.66** (0.00)	**0.66** (0.00)	6.12	7.51	3.32	19.28	1.47	-3.84	-1.99
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.69 (0.48)	0.69 (0.48)	6.03	7.46	3.16	18.05	3.90	-3.48	-1.90
Weekly	0.69 (0.49)	0.69 (0.47)	6.05	7.48	3.23	19.11	3.97	-3.52	-1.91
Monthly	0.68 (0.24)	0.68 (0.18)	6.22	7.54	3.36	19.28	2.63	-3.78	-1.97
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	0.68 (0.42)	0.68 (0.39)	6.04	7.47	3.21	18.85	2.09	-3.57	-1.92
Weekly	0.68 (0.39)	0.68 (0.35)	6.05	7.46	3.22	19.25	2.66	-3.60	-1.93
Monthly	**0.67** (0.03)	**0.67** (0.02)	6.18	7.50	3.30	19.28	1.91	-3.82	-1.98
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	0.67 (0.22)	0.67 (0.21)	6.07	7.57	3.36	19.21	1.18	-3.70	-1.96
Weekly	0.67 (0.17)	0.67 (0.15)	6.08	7.55	3.35	19.25	2.04	-3.72	-1.96
Monthly	**0.67** (0.01)	**0.67** (0.00)	6.19	7.56	3.39	19.28	1.46	-3.83	-1.99
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	0.67 (0.17)	0.67 (0.16)	6.13	7.63	3.47	19.21	0.73	-3.76	-1.98
Weekly	0.67 (0.11)	0.67 (0.09)	6.14	7.62	3.47	19.25	1.77	-3.77	-1.98
Monthly	**0.67** (0.01)	**0.67** (0.00)	6.23	7.63	3.49	19.28	1.24	-3.84	-2.00
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	0.68 (0.26)	0.68 (0.25)	6.22	7.65	3.52	19.21	0.35	-3.78	-1.98
Weekly	0.68 (0.17)	0.68 (0.14)	6.24	7.66	3.52	19.25	1.52	-3.79	-1.99
Monthly	**0.68** (0.00)	**0.68** (0.00)	6.25	7.67	3.55	19.28	1.07	-3.84	-2.00

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR; in yellow, the highest, but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 32: Performance Statistics for Exchange - U.S. / Euro Foreign Exchange Rate (DEXUSEU) 2000-2020 with the Target Voltility of 10%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	P-value						1%	5%
Unscaled	-0.05		1.14	9.93	3.79	18.27	0	-4.76	-3.43
Scaled with SMA-10 Volatility (10-day window)									
Daily	0.04	0.10	1.97	10.76	2.59	16.71	106.93	-4.99	-3.60
Weekly	-0.01	0.31	1.46	10.77	2.67	17.39	57.29	-5.07	-3.61
Monthly	0.00	0.24	1.60	10.65	2.91	20.08	22.36	-5.17	-3.64
Scaled with SMA-20 Volatility (20-day window)									
Daily	0.03	0.11	1.92	10.57	2.50	17.24	56.41	-4.83	-3.48
Weekly	0.01	0.20	1.65	10.47	2.46	17.55	29.99	-4.79	-3.48
Monthly	0.01	0.20	1.66	10.48	2.58	19.51	18.03	-5.02	-3.55
Scaled with SMA-40 Volatility (40-day window)									
Daily	0.01	0.19	1.64	10.25	2.35	16.75	30.03	-4.78	-3.44
Weekly	-0.01	0.26	1.51	10.24	2.36	17.02	16.30	-4.76	-3.45
Monthly	-0.02	0.32	1.39	10.23	2.40	17.20	10.76	-4.88	-3.51
Scaled with SMA-60 Volatility (60-day window)									
Daily	-0.02	0.32	1.39	10.12	2.30	15.80	20.46	-4.81	-3.45
Weekly	-0.03	0.39	1.28	10.10	2.29	16.06	11.60	-4.81	-3.46
Monthly	-0.04	0.45	1.19	10.11	2.35	16.37	7.38	-4.88	-3.50
Scaled with SMA-90 Volatility (90-day window)									
Daily	-0.06	0.44	1.04	10.10	2.35	16.83	14.48	-4.82	-3.49
Weekly	-0.06	0.38	0.95	10.10	2.37	17.16	8.65	-4.82	-3.49
Monthly	-0.07	0.36	0.91	10.08	2.42	17.77	5.73	-4.89	-3.52
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	0.00	0.21	1.62	10.37	2.36	16.57	59.38	-4.72	-3.44
Weekly	-0.02	0.32	1.42	10.31	2.38	17.23	28.58	-4.73	-3.45
Monthly	0.00	0.20	1.62	10.25	2.48	18.76	13.64	-4.84	-3.48
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	-0.02	0.35	1.35	10.17	2.30	16.09	31.36	-4.70	-3.42
Weekly	-0.04	0.44	1.22	10.13	2.31	16.30	16.00	-4.72	-3.43
Monthly	-0.03	0.38	1.29	10.08	2.35	16.84	8.94	-4.80	-3.47
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	-0.05	0.44	1.05	10.05	2.35	16.62	16.42	-4.69	-3.44
Weekly	-0.06	0.40	0.99	10.04	2.35	16.60	8.90	-4.72	-3.44
Monthly	-0.06	0.43	1.03	10.03	2.41	17.23	5.48	-4.80	-3.47
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	-0.06	0.36	0.95	10.03	2.43	17.00	11.21	-4.73	-3.45
Weekly	-0.07	0.34	0.92	10.02	2.44	17.07	6.29	-4.74	-3.45
Monthly	-0.07	0.35	0.93	10.03	2.50	17.71	4.01	-4.82	-3.48
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	-0.07	0.30	0.88	10.02	2.57	17.42	7.61	-4.78	-3.47
Weekly	-0.07	0.29	0.86	10.01	2.57	17.49	4.49	-4.80	-3.48
Monthly	-0.07	0.29	0.86	10.03	2.65	18.05	2.97	-4.87	-3.51

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in yellow shows the highest but not statistically significant SR. SR highlighted in red points to the lowest SR.

Table 33: Performance Statistics for Exchange - Norway / U.S. Foreign Exchange Rate (DEXNOUS), (1972-2020) with the Target Volatility 10%

Trading	Sharpe Ratio		Mean	Vol	Vol of Vol	MDD	Turnover	Left tail	
	Gross	P-value						1%	5%
Unscaled	-0.31		1.10	10.54	3.07	22.04	0	-5.79	-3.77
Scaled with SMA-10 Volatility (10-day window)									
Daily	-0.37	0.13	0.68	11.55	2.61	23.30	105.11	-5.23	-3.64
Weekly	-0.35	0.22	0.80	11.47	2.60	23.93	63.59	-5.33	-3.67
Monthly	-0.38	0.06	0.20	11.60	2.91	22.43	25.91	-6.05	-3.90
Scaled with SMA-20 Volatility (20-day window)									
Daily	-0.36	0.17	0.83	11.33	2.57	23.69	57.73	-5.20	-3.56
Weekly	-0.37	0.10	0.60	11.18	2.54	23.68	35.82	-5.37	-3.61
Monthly	**-0.39**	0.05	0.25	11.20	2.63	22.69	21.84	-5.87	-3.77
Scaled with SMA-40 Volatility (40-day window)									
Daily	**-0.40**	0.03	0.47	10.90	2.42	22.98	30.73	-5.21	-3.51
Weekly	**-0.39**	0.04	0.45	10.88	2.47	23.74	19.91	-5.34	-3.54
Monthly	**-0.40**	0.03	0.26	10.96	2.52	24.03	13.35	-5.67	-3.66
Scaled with SMA-60 Volatility (60-day window)									
Daily	**-0.39**	0.04	0.53	10.85	2.49	23.83	21.73	-5.27	-3.50
Weekly	-0.38	0.06	0.58	10.84	2.54	24.41	14.56	-5.33	-3.53
Monthly	-0.37	0.10	0.61	10.95	2.58	23.63	10.11	-5.64	-3.63
Scaled with SMA-90 Volatility (90-day window)									
Daily	-0.35	0.14	0.86	10.93	2.63	23.02	15.05	-5.26	-3.50
Weekly	-0.35	0.16	0.85	10.92	2.66	23.35	10.35	-5.31	-3.52
Monthly	-0.35	0.16	0.76	11.02	2.74	23.02	7.69	-5.61	-3.63
Scaled with EWMA-10 Volatility (10-day half-life)									
Daily	**-0.40**	0.04	0.48	11.04	2.44	23.25	58.57	-5.21	-3.51
Weekly	**-0.39**	0.04	0.47	10.94	2.43	23.71	31.85	-5.26	-3.52
Monthly	**-0.40**	0.03	0.30	10.97	2.54	22.63	16.12	-5.62	-3.65
Scaled with EWMA-20 Volatility (20-day half-life)									
Daily	**-0.39**	0.04	0.58	10.83	2.46	23.17	31.93	-5.18	-3.46
Weekly	**-0.38**	0.05	0.59	10.79	2.48	23.59	18.78	-5.23	-3.48
Monthly	**-0.38**	0.05	0.48	10.86	2.56	22.94	10.86	-5.56	-3.59
Scaled with EWMA-40 Volatility (40-day half-life)									
Daily	-0.36	0.10	0.82	10.72	2.56	22.36	17.28	-5.18	-3.45
Weekly	-0.36	0.12	0.84	10.72	2.59	22.50	10.83	-5.23	-3.47
Monthly	-0.35	0.13	0.78	10.79	2.66	22.07	7.05	-5.52	-3.56
Scaled with EWMA-60 Volatility (60-day half-life)									
Daily	-0.35	0.16	0.97	10.67	2.64	22.57	12.18	-5.21	-3.45
Weekly	-0.34	0.19	0.99	10.67	2.66	22.50	7.86	-5.25	-3.47
Monthly	-0.34	0.20	0.93	10.73	2.73	22.27	5.38	-5.51	-3.56
Scaled with EWMA-90 Volatility (90-day half-life)									
Daily	-0.34	0.21	1.05	10.59	2.73	22.66	8.34	-5.26	-3.47
Weekly	-0.34	0.23	1.06	10.59	2.74	22.61	5.58	-5.29	-3.48
Monthly	-0.33	0.25	1.00	10.64	2.79	22.47	3.97	-5.52	-3.55

A detailed description of statistics used is in section 3.7 and table 2 on page 24. Bold text indicates values of the Sharpe ratios (SR) that are statistically significant at the 5% level. SR highlighted in green shows the highest significant SR; in yellow, the highest, but not statistically significant SR. SR highlighted in red points to the lowest SR.

B Discussion Paper

One of the key concepts in the UiA School of Business and Law's mission statement and strategy is responsible. In this discussion paper, I present how I applied the concept to my master thesis. "Responsible" is a comprehensive concept. Here I concentrate on three possible meanings of the word. The first of the meanings is writing responsibly. The second is about being responsible for conducting reliable research. Which tells that one has to be aware of different methodologies and statistical tools. Being responsible can mean knowing the strengths and weaknesses of the used toolbox. The third is more connected to the field of the topic of this master thesis - finance. What does it mean to invest responsibly? All the three plausible definitions are connected with ethical challenges, as being responsible means thinking of the consequences of our actions to others and the environment. The rest of the document is organized as follows. First, I briefly present the topic of my master thesis. Second I describe some of the relevant theories. Then I discuss the concept "responsible" and conclude this discussion paper.

Many researchers analyzed the performance of the volatility-targeting strategy (see, e.g., Moreira and Muir (2017) or Harvey, Hoyle, Korgaonkar, Rattray, Sargaison, and Hemert (2018)). It is a very tempting strategy as it reduces exposure to risk, especially during crises. Typically, it works pretty well on equities and gives higher Sharpe ratios and lower drawdowns. However, the latest studies (see, e.g., Zakamulin (2019), Liu, Tang, and Zhou (2019), Cederburg, O'Doherty, Wang, and Yan (2020), or Bongaerts, Kang, and van Dijk (2020)) showed some challenges in using the strategy by practitioners. The idea of a volatility-targeting strategy is to reduce the exposure to risky assets when the volatility is high. Furthermore, to increase the exposure to those assets when the volatility is low. The question academics and practitioners ask is: does it work, and if yes, when and for what asset classes it works best. This study aims to answer the questions: does the volatility-targeting strategy outperform the buy-and-hold strategy, what volatility forecasting method to use, how often to rebalance, and does it work for other asset classes?

The tradeoff between risk and reward is one of the main exercises in finance. Some investors believe that it is rational to expect a higher reward for higher risk. It seems to be intuitive and logical. Merton (1980), among others, note that the market volatility should be positively related to the excess market return (the return on a stock portfolio minus the risk-free rate). On the contrary, French, Schwert, and Stambaugh (1987) study the Standard and Poor's (S&P) composite portfolio in the years 1928-1984 and report the regression result which demonstrates that reality may be the opposite - the relation between the market volatility and the excess market return is negative. The conclusion for investors is that stocks with low volatility earn higher risk-adjusted returns than those with high volatility, and one usually overpays for risky stocks.

In my master thesis, I examine if the volatility-targeting strategy is a good solution to the negative relationship between risk and return when volatility is high. The active strategy strongly depends on accuracy in volatility forecasting. The first question is, then, what kind of volatility forecasting is it best to use. I concentrate on two types of moving averages models - Simple Moving Average (SMA) and Exponentially Weighted Moving Average (EWMA) with different lengths of the estimation window. Furthermore, I try to find out if some of them work better than others. Second, the volatility forecast accuracy decreases with the horizon. So the more frequently one trade, the better. However, every transaction has its cost, and costs reduce the returns. I want to find out how often it is optimal to trade: daily, weekly, or monthly, in order to achieve the optimal tradeoff between transaction costs and forecast accuracy, see, e.g., Zakamulin (2019). Third, I check if and when the volatility-targeting strategy gives better performance statistics (the Sharpe ratio) and lower risk than the buy-and-hold strategy. Fourth, Harvey et al. (2018), among others, report that the strategy works for equities but not for bonds, commodities, and exchange rates. I examine the same asset classes, but the former study is extended by some other periods and assets to see if it is still valid.

Based on the findings from the conducted study, the main conclusion is that the volatility-targeting strategy does not consistently beat the buy-and-hold strategy. The strategy works well for equities in volatile periods in history, but it may be challenging to implement with success ex-ante. The findings are mixed for bonds. However, the active strategy does not work for commodities and currencies.

As mentioned in the introduction, ”responsible” is a comprehensive concept. First, it can be related to **writing responsibly**, which in the academic world is associated with the concept of ”standing on the shoulders of giants.” The concept is regarding knowledge about the ideas different people came with and published their thoughts or somehow got the credit. Massaro, Dumay, and Guthrie (2016) emphasize the importance of a structured literature review (SLR). They write that SLR is crucial to be aware of the current academic conversation and thought. One can then develop insights, critical reflections, future research paths, and research questions. Every (new) scholar should know that research is like joining a conversation that’s already been going on for years. Citation makes it possible to show respect to the ”giants” before us and not taking credit for their ideas.

Second, the concept of ”responsible” may also be connected to **methodology and statistics**. Are economics and finance a science or an art? Taleb (2005) is a former ”mathematical trader,” and he studies the problems of luck, uncertainty, probability, and knowledge. Taleb (2007) stated that (finance)people could not predict. The existing statistical toolbox is not good enough. Furthermore, my study uses use-inspired research. Stokes (2011) described

use-inspired research as a mix of basic (pure)[1] and applied research.[2] Those kinds of research can be tempting to show the results more favorable than reality is. The problem is well known as p-hacking. Of course, every scientist wants to use Popper (1963) falsification. However, the academics experience press to publish, which can make the p-hacking even more tempting. A scholar can show consistent (not confusing) and ”revolutionizing” research and be recognized. It is a part of being human. Furthermore, this part is recently also noticed in economics and finance. Behavioral economists claim that people are biased - have some systematic fails, see, e.g., Kahneman (2011) or Thaler and Sunstein (2009). Those biases can, among many other things, influence moral judgments. Shiller (2003) claim that behavioral finance, which includes perspectives from psychology and sociology, became the most vital research program from the 1990s. Shiller said to his students that it is also essential to do something ”good” out of ethical reasons and not only for profit.

That leads to the third possible meaning of ”responsible” - **responsible investment** in finance. Kuhn et al. (1970) wrote about ”paradigm shifts”. He meant that science can not be objective as there is always some context, and every researcher has his or her subjective worldview. It seems like ”shareholderism” is losing its dominating position as the world is challenged by global climate changes, growing inequality, and many others. Tietenberg and Lewis (2018) presented some possible solutions to those challenges, e.g., taxes. The shift is nowadays towards ”green” and responsible investments. ShareAction[3] defines responsible investment as ”an investment strategy which integrates environmental, social, and governance (ESG) factors into investment analysis and decisions.” Sullivan and Mackenzie (2017), among others, suggest that social, environmental, and economic goals harmonize with each other. Kramer and Porter (2011) stated that sustainable development (or creating shared value) is the path that businesses should follow. Many believe that by investing, people can shape the world also for good - the concept of ”money talks” receives a new, more positive meaning. By buying shares, investors ”vote”/choose the kind of companies they want to see in the future.

First, ethical challenges may be related to writing responsibly. Massaro et al. (2016) wrote that SLR is essential in scientific disciplines dominated by quantitative approaches. Moreover, no research is an island. My thesis builds on the work done by former researchers. When one looks at the existing literature, the interest for volatility targeting among academics began around the dot-com bubble crash in the late 1990s (see, e.g., Fleming, Kirby, and Ostdiek (2001)) and exploded after the global financial crisis in years 2007-2008 (see, e.g., Collie, Sylvanus, and Thomas (2011); Butler and Philbrick (2012), and Albeverio, Steblovskaya, and Wallbaum (2013)). I tried to follow the academic discussion with the outstanding help of my

[1] Basic (pure) research is made mainly to extend the body of knowledge.
[2] Applied research is made to be useful for practitioners.
[3] ShareAction is the UK registered charity. https://shareaction.org/

supervisor.

Second, my master thesis is built on mathematical calculations and statistical models. To find out if the active strategy outperforms the passive strategy, I use the Sharpe ratio. It is a performance measure, which helps investors evaluate portfolio performance and better understand the risk-return tradeoff, one of the main exercises in finance. It was formulated by Sharpe (1966). Falsification is essential, and the hypothesis is tested. If the p-value is lower than the predeterminate significance level (usually $\alpha = 0.05$), the null hypothesis can be rejected. That indicates that the data generates sufficient evidence against the hypothesis that the Sharpe ratios of two different portfolios are similar. Sometimes I received a statistically significant Sharpe ratio, but very often not. I also received the statically significant Sharpe ratio where it was not expected. The first reaction is always to double-check and quality checks the process. If there is no mistake, I see it, as Taleb's (2007) "black swan" and not trying p-hacking by changing the test or shortening or extending the sample.[4] The practice is also called for data-snooping, see, e.g., Sullivan, Timmermann, and White (1999), who tried to find a remedy for this practice. It means that one can fit the trading rule on the data. Taleb (2005) meant that the more one tries, the higher likelihood of finding a rule that worked on past data.

In the thesis, I try to predict volatility (risk). Brailsford and Faff (1996) tried to determine if relatively complex volatility forecasting models are superior to simpler alternatives. They have not found convincing evidence. They concluded that "volatility forecasting is a notoriously difficult task." That is why I decided to concentrate on the simplest volatility forecasting models, which are time-series forecasting based on historical volatility. Specifically, it is Simple Moving Average (SMA) volatility and Exponentially Weighted Moving Average (EWMA volatility). I do not hide that SMA and EWMA are not the most sophisticated models. However, accuracy is high enough, and calculations are fast. Moreover, in the end, the world is very complex, and it is impossible to predict exact future volatility. Economists are good at ex-post but not at ex-ante.

Third, responsible investment in finance. The famous investor Warren Buffet in the Berkshire Hathaway annual report for 2002, wrote that "derivatives are financial weapons of mass destruction." In my master thesis, I do not deal with derivatives (however, derivatives often follow my units of analysis: market indexes, stocks, bonds, commodities, and currencies). Nevertheless, in the volatility-targeting strategy, I allow only 150% leverage, making the strategy less "successful." Hallerbach (2012) studied EURO STOXX 50 Index in the years 2003-2011. The researcher indicated that the better the volatility forecasts are, the higher the Sharpe ratio. He also pointed out that restricting leverage allowed in the strategy gave a lower Sharpe ratio. So if there were no restrictions or allowed leverage was 300%, the Sharpe ratios would

[4]Sharpe ratio can be manipulated by, for example, lengthening the measurement interval or choosing a period for the analysis with the best potential Sharpe ratio.

be even higher in my master thesis. However, according to Principles for Responsible Investment (PRI),[5] leverage can also be dangerous: ”In times of market stress or illiquidity, excessive leverage across a large number of investment managers and financial institutions could harm the economy as a whole.” Risk management is crucial to avoid future bubbles and financial crises.

In this discussion paper, I presented how one can use the concept ”responsible” in practice when writing a master thesis. First, I explained the concept behind ”standing on the shoulders of giants.” Furthermore, I showed why it is not good to ”reinvent the wheel,” but to join the current discussion and try to contribute with some original and new ideas. Second, one has to be aware of the (statistical) tools one uses. The thesis is as reliable as the statistical tools used allow for. Third, I took a border look at the topic of economics and finance especially. Studying risk management is a critical topic. It can stabilize financial markets and let investors avoid large drawdowns, and destabilize the global financial markets as a whole and connected system. Healthy financial markets, which push the investments in the right direction, are crucial for the global economy, and they have waist consequences for the planet and all its residents.

[5]The PRI describes itself as: ”is the world’s leading proponent of responsible investment.” Retrieved from: https://www.unpri.org/download?ac=4155

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