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ORIGINAL ARTICLE

Mathematics teaching development: learning from developmental research in Norway

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Abstract This paper is a product of mathematics teaching developmental research projects based on establishing communities of inquiry comprising schoolteachers working at all grades and university-based teacher educator/researchers (didacticians). The projects are established on the principle that teachers taking an inquiry stance in their practice can assert their agency to develop their teaching to enable improved learning experiences for their students. The educational context and the societal pressure to develop mathematics teaching in Norway where the projects were implemented are described. A case study of a group of upper secondary teachers who are working together with didacticians within the topic of algebra is briefly outlined. A descriptive analysis of the case calls into question the fundamental developmental principles upon which the projects are established. Teaching is observed to exist in alignment to regular practice through the interaction of strong constitutional, institutional, social and professional forces, which inquiry alone appears unable to realign. Teaching development occurs through a gradual extrapolation of practice as teachers implement approaches that they learn from the experience of others and imagine into their own practice.

Keywords Mathematics teaching development · Community of inquiry

1 Introduction

This paper reports a case study from mathematics teaching developmental research projects based on establishing communities of inquiry carried out in southern Norway. The case focuses on a group of upper secondary teachers and university didacticians (that is, teacher educators/researchers) engaged in discussions about teaching algebra in the teachers' classes. The case is constructed from data collected during the projects and was initially selected with the intention of pointing to evidence of the projects' positive impact on the teachers' practice. However, exploratory analysis of the data exposed evidence of the teachers' resistance to innovation. Since both positive and negative stories have the potential to expose valuable knowledge of teaching development activity, the case was re-analyzed to address the question: *What is interfering with creative innovation in teaching mathematics in a selected case of a group of upper secondary mathematics teachers?*

It is not being claimed that obstacles to teaching development have not been reported earlier. Schifter and Simon (1992), for example, observe:

> Attempting to modify one's classroom practice to be more consistent with insights derived from an inservice course can be a difficult and frustrating process. A great deal of learning in the context of the classroom is necessary. Each day students' unexpected thoughts, questions, and behaviors yield new information to be accommodated into practice. Such learning is frequently complicated by pressure to cover existing curriculum, lack of institutional support, resistance from students and parents, competing demands on teachers' time. Because of these obstacles, efforts to change instruction may be put off indefinitely or initial efforts that do not meet with instant success (probably the norm rather than the exception) may be abandoned. (p. 189)

Similar 'barriers' have been reported from the Cognitively Guided Instruction (CGI) research (Knapp &

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Peterson, 1995). More recent efforts with 'design research' have also experienced developmental obstacles as carefully developed and researched teaching designs are devolved to teachers beyond the developmental group, and in regular practice the design products are adapted and denatured to fit with the teacher's practice, rather than leading to an adaptation of teaching (Artigue, 2009).

Mathematics teaching developmental research in a community of inquiry frame is intended to address these problems. Regular teachers are placed in the designer and researcher role, through a process of collaborative and constructive inquiry into their own practice, and they are supported to meet the institutional and social constraints through engagement in communities. The significance of this current report is not in the teachers' reactions per se, it is rather that the teachers have these negative reactions in a developmental research context that was created, in part, to address these underlying issues.

The paper continues in Sect. 2 by outlining the Norwegian context within which mathematics teaching development projects have been established. The theoretical underpinning of these projects is outlined in Sect. 3, and the methodology and implementation is described in Sect. 4. The data generated and the analytic approach are explained in Sect. 5. The empirical substance of the paper is set out in Sect. 6 and discussed in Sect. 7.

2 National context

Mathematics teachers in Norway face pressures similar to those in other developed economies because of the underperformance of students in international studies (TIMSS, PISA, TIMSS Advanced, TEDS-M) (Grønmo & Onstad, 2009, 2012; Kjærnsli & Roe, 2010; Kjærnsli, et al., 2007). National tests of students' performance in mathematics (at grades 4, 7 and 10) and the Norwegian Mathematics Councils' test of undergraduates' mathematical skills (Nortvedt, 2012) also reinforce the perception that too many Norwegian students do not develop expected competencies in mathematics. Additionally, there is an unacceptably high dropout rate from post-compulsory school education (grades 11 to 13), part of the blame for which is attached to perceptions of the quality of teaching and learning mathematics in compulsory education (UD, 2006).

The school curriculum in Norway, as in many countries, has been marked by considerable instability over the last 50 years with four major curriculum changes (Breiteig & Goodchild, 2010). Underpinning each of these changes were fresh theoretical perspectives of learning, educational goals and values. Political and cultural factors have also contributed to the shifting challenges faced by teachers. Two fundamental principles of Norwegian society are incorporated within the curriculum: 'inclusion' and 'equal opportunity.' Inclusion is taken to mean that all activities are open to all, irrespective of gender, physical or mental impairment, ability, economic, social or ethnic background, etc. Discrimination is not permitted on any grounds, and within schools this means that students' attainment cannot be taken into account when forming classes. The result is 'inclusive education' that requires teachers at all levels to engage with classes that include the entire attainment range for an age group and students with mild to severe (social, emotional, cognitive and physical) learning needs. Teachers are expected to adapt teaching to the individual needs of students in their classes, while ensuring their lessons are inclusive and that all students have 'equal opportunities' to learn according to each student's individual development. Teachers need to accommodate the changing demands of a volatile educational context within their practice and meet major challenges in managing, motivating and supporting the learning of a very diverse student group (KD, 2006). This is the context of the mathematics teaching development described in this paper.

3 Developmental research projects

This paper reports from projects that are framed to meet the challenges of the national context outlined above. The immediate goal is to support and enable mathematics teachers in their efforts to work on and develop their own practice. The strategy for achieving this goal is to develop communities of inquiry comprising didacticians and teachers, who will have a research role within their own school-based practice. The ultimate goal is for students to enjoy improved experiences and opportunities to learn mathematics. The projects set out from the premise that sustained and durable transformation of teaching can be achieved by enabling teachers to set their own developmental goals, make and implement their own plans and reflect on outcomes and feedback into their own practice. By contrast, it is asserted that an intervention that only offers teachers tips and recipes is of limited effectiveness and fails to prepare teachers to sustain their developmental activity and generate new ideas beyond the life of the intervention.

The research has focused on three broad developmental questions:

- How can didacticians learn (about) effective ways of working with teachers to enable teachers to conceptualize approaches to teaching that will result in students' learning mathematics with understanding and fluency?
- How can teachers learn (about) effective ways of working with students to enable students to learn mathematics with understanding and fluency?
- How can students learn mathematics? (Jaworski, 2006).

This paper focuses on the first of these.

Teaching is perceived as a cultural practice that takes place in communities (Gellert, 2008), rather than an individual and independent practice that takes place in isolation behind closed classroom doors. Recent initiatives from the Norwegian Department of Education (KD, 2012) also acknowledge the importance of community in school and in-service professional development. Making reference to a review by Stoll, et al. (2006), the Department acknowledges: "Schools that have a well-functioning professional community appear to be more able to create good learning opportunities for students than schools where teachers work more individually" (KD, 2012, p.9, author's translation). Further, it is recognized that changing practice entails the risk that novel approaches do not work as intended, and for a teacher a poor lesson can be a very public and embarrassing display of failure. The mutual collaboration of teachers, with didactician support, can reduce the sense and actuality of risk, inspire courage to be innovative and support teachers in implementation of, and reflection on, fresh approaches.

The projects seek to develop professional collaborative communities of teachers (with didacticians) and use 'inquiry' as a developmental tool and as a tool for studying the development of learning, teaching and the developmental process. The projects have been conceived within a framework of community of practice theory (CPT, Wenger, 1998) and with a local theory of inquiry as the agent of development. In the following, teaching is presented as the practice of a community of teachers and in Wenger's terms: *learning teaching* is a process of identity formation, through 'modes of belonging': *engagement*, *imagination* and *alignment* (Wenger, 1998, pp. 173 & 174).

3.1 Teaching from the perspective of community of practice theory

Teachers and teaching are a community of practice defined by their *mutual engagement* in the *joint enterprise* of formal education, and their *shared repertoire* (Wenger 1998, p. 73) of schooling (curriculum discourse and requirements, teaching approaches, resources, teaching schedules, experiences of working with learners and other teachers). Learning *teaching* is a transformation of identity and 'involves an interaction between experience and competence' (ibid., p. 214); it is a process of 'building an identity' (ibid., p. 145) of one who belongs to the community of teachers/teaching practice. Teachers *engage* through active involvement with other teachers in the processes of negotiating what it means to teach. Through the exercise of *imagination* they can envisage the world of other teachers' lives (practices and classrooms) and thus through *extrapolating* (ibid. p. 173) from their own practice they enter further into participation within the community of practice. Essentially, they *align* themselves with the goals, values, beliefs and activities of the practice; they 'do what it takes' (ibid., p. 179) to be a teacher.

Establishing mathematics teaching developmental research projects within a framework of CPT entailed an initial phase of community building. Teachers and didacticians came together to create a new community of practice in which the enterprise was teaching development. In other words, to engage in a recursive process in which the practice itself is continually under review and changing.

3.2 Inquiry as a developmental tool in a community of practice

The operative instrument for the recursive process of teaching development is 'inquiry'. Inquiry is taken as an active process that motivates a bringing to awareness (conscientization, Freire, 1972) of the contradictions and tensions of practice. Inquiry is also a tool that enables the person or community to work on resolving the contradictions and tensions that they experience. As the implementation of the projects is described below, more will be explained about the notions of inquiry as process and inquiry as tool. The introduction of inquiry into CPT, and thus the creation of a 'community of inquiry,' has significant theoretical consequences. Jaworski (2006) argues that through taking 'inquiry' as a stance or 'way of being', the mode of belonging *alignment* is transformed into *critical alignment*. Practice ceases to be something to which one aligns by subordinating one's agency to the goals of the practice unreflectively; the participant becomes an active agent, negotiating changes to the practice through critically reflective engagement.

The introduction of inquiry thus marks a paradigm shift. CPT is based on a perception of the participant being aligned to the practice goals. However, the introduction of 'inquiry' is based on the belief that members of the community have the agency to work on and develop their practice through individually chosen goal-directed actions. Thus, they 'liberate' themselves from the oppression of the contradictions and tensions of the practice and it can be argued that developmental research is consistent with a critical research paradigm (Goodchild, 2011). Further, the social-practice discourse has been transformed to embrace a new language of 'contradictions', 'tensions', 'agency', 'goals' and 'development'.

Aligning to the practice may simply entail accepting the contradictions and tensions, and adapting oneself as well as possible, especially in a very stable school or group of teachers that offers few resources that stimulate imagination. For example, a teacher who espouses personal beliefs about teaching for understanding succumbs to an overfull syllabus and high stakes examinations and adopts an approach based on example and routine exercises that favors quick gains that are just as quickly forgotten after the examination. The teacher has subordinated personal beliefs to the system and, possibly, student expectations. The foregoing example of contradictions arises in the relation between external demands and the inner beliefs of the teacher. However, contradictions can be experienced within the repertoire of the practice. For example, in a workshop group activity, upper secondary teachers discussed briefly a textbook chapter on 'experimental geometry' that was intended especially for the lower attaining grade 11 students. The teachers expressed their feeling that it is strange that the chapter, which is designed so that students will experience mathematics as 'fun' should be placed at the end of the book, when the students are about to finish their study of mathematics. The teachers also observed that examination questions based on this chapter are difficult to predict, with the implication that the 'fun' chapter is transformed into a source of difficulty and increased challenge.

Being critically aligned to the practice means that one does not have to be satisfied with the status quo (such as following the strict order of chapters in the textbook). Tensions and contradictions become the source of creative innovation. As Fullan writes: "tensions must be reconciled into powerful new forces for growth and development" (Fullan 1993, p. 4). The adoption of an inquiry stance offers participants increased awareness of the contradictions of practice through their critical alignment, and inquiry is the tool used to initiate and manage creative innovation.

4 Mathematics teaching developmental research

Developmental research entails a methodology based upon interacting cycles of research and development (Goodchild, 2008; Gravemeijer, 1994) as illustrated in Fig. 1. Global theories, such as CPT, are developed through the introduction of local theory, such as a theory of inquiry, thus contributing to theoretical development as briefly explained above. The research cycle informs a development cycle, as theory is transformed by thought experiment into implemented action plans. The development cycle from which this paper reports is based on principles from design research and didactical engineering, Japanese lesson study, learning study and more generally collaborative action research (Fernandez, 2005; Kelly et al. 2008; Marton, et al., 2004; Ruthven & Goodchild, 2008). Central to this is the implementation of an inquiry cycle. This begins with reflection on learning goals and an epistemological analysis of the mathematics to be learned, then collaborative planning for the classroom, implementation of the planned lesson with observation by colleagues (and/or didacticians), review and critical reflection, and reporting and feeding back into subsequent inquiry cycles. Teachers' participation in cycles of inquiry provides opportunities for learning and development of craft knowledge. Didacticians' participation and their own systematic inquiry into the developmental process contribute to the development cycle and informs the theoretical development of the research cycle.

This paper reports from mathematics teaching developmental research projects based at the University of Agder pursued over the past 8 years. The research has received financial support from the Research Council of Norway and the Competence Development Fund of Southern Norway. Over the years there has been some minor variations in focus (e.g., use of ICT) and participation (e.g., grade levels included). Core participation has included teachers from primary grade 1 through to upper secondary (grade 13). The implementation of the developmental research will be described in rather general terms and minor variations mostly reflect the breadth of participation and focus of each project; more detailed information can be found in Jaworski, et al. (2007).

The projects were set up to establish communities of inquiry, with collaboration and co-learning between teachers, and between teachers and didacticians. Mathematical tasks are suggested by didacticians as starting points for teachers' discussion and, with didactician support, to design classroom activity. A goal was to achieve a developmental approach to practice that will be sustainable beyond the life of the projects. Teachers thus engage in the design process rather than just receiving the outcome of the design process carried out elsewhere. The inquiry cycle outlined above begins in project workshops and continues through subsequent phases in the teachers' schools and classrooms. Review, critical reflection and reporting take place in a subsequent workshop. When the opportunity arises, didacticians visit schools to observe in classrooms as teachers implement lessons inspired and planned as a

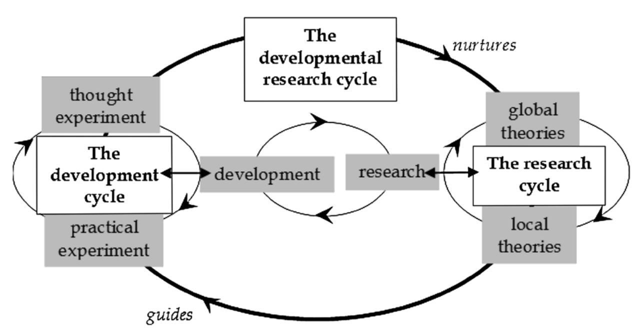


Fig. 1 The developmental research cycle (Goodchild, 2008, p. 208)

project activity. 'Opportunities' arise from teachers' specific invitations and didacticians' availability.

An important initial phase in the projects has been community building. A requirement for participation is that from each school there should be at least three teachers and the support of the school principal. The goal is to ensure in each school a minimum critical mass to establish a school project community. Community building across the project schools and university is one of the functions of workshops, ranging from two to five each semester, in which teachers (about 30) and didacticians (about 10) come together. The workshops include small group collaboration in mathematical activity and planning mathematics tasks and lessons for the classroom. Also, there are plenary sessions for teachers to present project-related activity from their own classes, and didacticians' presentation of fundamental ideas (such as inquiry), research activity and mathematics.

It is intended that teachers and didacticians are equal partners in the process, but with different functions and responsibilities. Teachers are primarily responsible for their classes and didacticians take responsibility for planning the workshops. As indicated above, both have a research role and the aim is that teachers and didacticians collaborate in a co-learning partnership (Wagner 1997) in which developments of scholarly knowledge and craft knowledge are sought and equally esteemed. An important objective in the projects is that teachers will accept responsibility for their own knowledge creation and learning. To this end the projects seek to challenge tacit assumptions that didacticians know more or know better (they know 'different') and to reinforce teachers' agency and authority in their own practice. As will be evident from the following, this approach sometimes resulted in contradictions between teachers' and didacticians' expectations and intentions.

5 The generation of naturalistic data

Most events in the projects that include teachers, such as workshops, school meetings and the implementation of specially planned lessons, have a developmental purpose. A small number of events, such as group interviews in which didacticians ask teachers to report on their experiences in the projects, are set up with a research purpose. However, group interviews also serve the developmental aims as they create opportunities for teachers to reflect on their practice, and didacticians to review projects' plans and program. Very few events are staged only for the generation of data. Audio and/or video recordings are made of all events that take place in the projects; it is from these recordings and the documents used in each event that the data are generated. The data are thus qualitative and naturally occurring. Credibility and authenticity of the data are embedded in the events that were planned and implemented within the development cycle rather than the research cycle.

A vast amount of material, recordings and printed or written products has been generated and stored digitally; this provides the source of data for case studies. A focus is chosen for case study, this might be a particular event within the projects, such as a special lesson, or as described in the next section a (mathematical) theme that was deliberately pursued. Material related to the event or theme is identified and this becomes the initial set of data for analysis. As the analysis progresses, references to other events such as documents, etc. may be made and these are included in the dataset for the case. The goal is to expose how the teachers relate to project events and what meaning they bring to and make out of the activities in which they are invited to engage in project workshops. Evidence of the development and conditions of effective collaboration between didacticians is sought. By comparing events and teachers' responses to tasks and activities, explanations for differences in effectiveness may be conjectured. These conjectures may then be tested in further developmental activity, or against other episodes that can be explored in the data corpus. The intention is to contribute to an empirical knowledge base for the practice of mathematics teaching development, rather than the development of global theories of learning and development or the local theories of inquiry.

Analysis is approached within a symbolic interactionist frame and progresses through phases of exploration and inspection. An 'abductive' approach to exploration is adopted in which the intention is to gain knowledge and understanding of developmental processes by analyzing the data through a synthesis of open coding and established theory (Strauss and Corbin, 1998). Selection of material is followed by data reduction, which is a process of annotated indexing of the material undertaken to obtain an overview from a first comprehensive sweep of the data selected. Exploration is a protracted and recursive process in which the data are intensively and repeatedly studied in an attempt to establish the sense and meanings of the participants in the events. Inspection is then pursued by reviewing the data and interpretation through the lens of the theoretical framework—in the present case, CPT and particularly the notions of alignment and critical alignment. The qualitative approach is supported by continual memo writing. Reporting takes the form a narrative account based on evidence within the memos and focused on the analytical findings.

6 An example of the development of a theme (algebra) over a sequence of workshops

In this section, discussions between upper secondary teachers that occurred over a sequence of workshops focusing on algebra are reported. As explained at the outset, the 'case' used in this paper was selected while seeking examples of (effective) developmental collaboration. However, the initial exploratory stages of analysis revealed the episode to be disappointing because it appeared to contradict claims for effectiveness. Reflection on the episode led to the realization that it held useful lessons about engagement of didacticians with teachers in developmental research. Thus, the episodes were reconsidered to address the question: *What is interfering with creative innovation in teaching mathematics in a selected case of a group of upper secondary mathematics teachers?*

The episodes taken from the workshops do not, by themselves, expose an exciting story of positive engagement and lively development of teaching. They might even be interpreted as failure to meet the project goals. However, such an interpretation would be unjustified because the experience leads to learning, to greater awareness and to the realistic potential to change practice—by both didacticians and teachers. Developmental research is not only about doing things better, but it is also concerned with inquiry into the developmental process and identifying the affordances and constraints of practices that contribute to or impede the development.

6.1 Group discussions in workshops

At the end of the first year of the project, teachers asked for some changes in the project's implementation. In particular, they wanted workshops to be closely related to the school curriculum, rather than the more general focus on mathematical thinking and learning through inquiry that had been taken. Further, they wanted group activity within workshops to be based on teachers working at the same or adjacent grades, with more focus on preparing for classroom activity. It appears that they were keen to see the project better aligned to their regular practice. The events described below occurred in the second year of the project. The account starts with discussions between teachers from upper secondary schools in the context of two consecutive workshops that focused on teaching and learning algebra.

In advance of the 'algebra' workshops, didacticians sent out some tasks which were intended to stimulate discussion and initiate group work, both mathematical and preparing classroom activity, in the workshop. The tasks were chosen so that they could be attempted by the whole project community, i.e., including teachers of grade 1 to grade 13 students, and to demonstrate a strand of algebraic development through the school curriculum. They were not intended to be taken as suggestions for activities in the teachers' classes.

The workshop was planned with two small groups for the upper secondary teachers, but they decided to combine into one larger group including six teachers ranging from about 12 to 40 years' teaching experience at the upper secondary level, and four didacticians. Upper secondary teachers are generally well qualified, to master's level, in one of two subjects that they are expected to teach. The teachers had done as requested regarding the tasks sent in advance, but they had decided they would rather use the group time to discuss issues that were pertinent to their practice. They argued that bringing teachers together from different schools opened an opportunity to discuss common problems and how different teachers tackled these, in particular students' common, recurrent and persistent errors in algebra.[1] At strategic points in the following, extracts from memos produced during the analysis are included to demonstrate the process of building the narrative and make a clearer connection between the data and the interpretation.

Olav[2]: But I just thought about <u>typical problems, or errors</u>, that I have observed lately in tests and such. For example, the expression two x divided by x, they [students] have, two x over x, so they say we have two x on top, we take away one x, and so we have x left (*Olav writes*: $\frac{2x}{x} \to \frac{xx}{x} \to x$). Or if there is x plus four over x, here it is OK to simplify, we [students] cancel that [x] and that [x], (*Olav writes*: $\frac{4+x}{x} \to 4$) isn't it? … (*continues with another example*) … It is these things [errors] that <u>repeat</u>. And they occur in first class [grade 11], and again in the second class [grade 12], and they <u>persist</u>, we still see them in the third class [grade 13]. <u>No matter what we do, it seems that we cannot get rid of them</u>.

Memo: Through exploration, key themes emerge in Olav's statement (these have been underlined)—student errors, repeat, persist, resist efforts to get rid of them. These themes, inspected through the lens of the theoretical framework contribute to an evidential basis for interpretation. Olav experiences tensions and contradiction in his practice, between meeting the curriculum goals and his perceived ineffectiveness, or between his teaching performance and the students' accomplishment.

Osvald: It was a thought, as we were all together, that then, possibly some good ideas could arise. I have good experience with [using] THIS (*emphasised*). So another could say oh no, I

[1] Such errors in algebra are well documented in the literature, as discussed for example by Hewitt (2012).

[2] In the quotations pseudonyms are used for teachers and didacticians. Analysis is carried out in the original language, illustrative quotes are translated by the author. Text in square brackets has been added to improve sense, italic script is used as explanatory text.

have good experience with THIS. And I believe we feel that we fight [struggle with these errors] and many of us will come a little bit further. It is the same things that we have problems with every year, we have not been good enough.

Memo: Themes—opportunity to share, benefit from others' success, recurrent experience, struggle and fight, not been good enough (inadequacy). Inspected through the lens of the theoretical framework—there is evidence of mutual engagement in a common practice, the teachers understand each other and share similar experiences, they recognise the possibility of learning from each other because they can imagine the other teachers' classes and challenges to be like their own and will be able to extrapolate from others' descriptions of their practice. Once again there is evidence of an underlying contradiction between competence and accomplishment.
The teachers were seeking ways to achieve students' fluency and reliability in performing basic algebraic manipulation (Hewitt, 1996).

Didacticians (Leo and Ida) expressed their opinion that underlying the errors that had been described is students' development of procedural rather than conceptual understanding of the subject. The teachers agreed with this, but they argued that the problem originated earlier in students' learning, and at upper secondary level the only realistic approach is through repetition and practice. The teachers use this argument to counter didacticians' suggestion that they should attend to the development of students' conceptual understanding.

Leo: But don't some of the errors shown here occur because they have learned mechanical procedures without seeing what lies behind the letters? Also, they do not have the foundations, but they only have the surface, and they use rules ….

Olav: But it is always easy to push the responsibility downwards, isn't it. Universities and such complain about upper secondary school, and we [upper secondary school teachers] complain about lower secondary school, and lower secondary school [teachers] surely complain about primary school. But it is clear that conceptual understanding and such should come in much further down than with us. But, Stefan's point is that they [students] come to us and they cannot do it. And we expect in part that they should be able to do it, and so we go quite quickly through things. And so they lack conceptual understanding completely, it does not lie at the foundation.

Memo: Themes—students progress through grades and do not acquire competencies, responsibilities lie at other grades, need for remedial action. Inspected through the lens of the theoretical framework—there is evidence of imagining teachers at other grade levels with similar complaints and other teachers facing challenges like their own extrapolating from their own practice. Basic contradiction between the expected and necessary competencies of students and what they bring from previous grades.
They were agreed in their belief that there is only one solution: practice and memorizing. Students could be helped by using notation, or given strategies to test manipulation, but nothing could replace 'practice'.

Stefan: Algebra is one of the first places in mathematics that one has to drill, they must drill. Drill is a dangerous word in a workshop about inquiry, but here it is at home, that's a matter of fact.

Oswald: I agree with Stefan, you must drill it.

Memo: Stefan introduces an instructional activity 'drill' that he believes will be understood by the others, and contradictory to the project's basic principles. Inspected through the lens of the theoretical framework—Stefan is aligned to his teaching practice and critically aligned to the project. He further recognises, through an exercise of imagination that he and the other teachers are engaged in a common enterprise, with mutual experiences.
Despite the fact that he recognizes a contradiction between the project workshop context and his opinion, Stefan asserts that it is necessary to 'drill' algebraic manipulation, and he gains support for this from Oswald. Later in the conversation it becomes clear that by 'drill' Stefan means practice exercises in which algebraic manipulation is embedded in different contexts.
Ida asked how the teachers respond to students when these types of errors are observed, do they ask students to explain their working, or just point to the error and show the correction? They explained that it depends on the circumstances, whether they are working with individual students or the whole class. However, Stefan tells that it is his experience that when students are shown the correction, they respond "of course!" The implication he makes is that the issue at stake is not about the students'

understanding, but rather about memory; thus, perhaps, supporting his assertion that 'drill' is necessary. Another possibility, not considered in the discussion, is that students have not developed an internal monitor or the personal disposition to try to make sense of their mathematical activity and habits of reflecting and checking on their work.

The teachers wanted to learn from experiences of other teachers, and suggestions from didacticians were dismissed quickly. A fundamental contradiction in the teachers' argument might be expressed in the following manner: these student errors are persistent and resilient, no matter what the teachers do the errors recur, and the only approach is practice and memorizing. Even though this approach is perceived as ineffective, novel solutions or arguments presented by didacticians are summarily dismissed. The suggestions of didacticians are dismissed as unrealistic because of the time available, and curriculum and examination demands at the upper secondary level (stated elsewhere in the data). By the end of the group session, the teachers decided to report their discussion within plenary session by outlining the student errors they experienced and requesting teachers of other grades to come forward with suggestions for alternative approaches.

Didacticians were challenged by the perceived contradiction in the teachers' discussion and teachers' apparent rejection of the rationale for adopting approaches that would foster conceptual understanding. Didacticians wanted to present some new ideas for initiating class activity that would meet both the teachers' requirements and foster students' learning. Thus in advance of the second algebra workshop, a fresh set of tasks was sent out to schools for consideration in school teams prior to group discussion in the workshop; these are presented in Fig. 2. It must be emphasized that the tasks were intended to meet several goals. They had to be accessible for teachers working at a wide spread of grades (grade 1 through to 13) and give some common ground for discussion in the workshop plenary sessions. The tasks were intended primarily to 'trigger' discussion and developmental activity rather than to be assimilated into regular classroom practice.

e upper secondary teachers were not enthusiastic toward these suggestions. The first task was considered inappropriate for students at their level, it was better for lower secondary school when algebra was being introduced, and there was no discussion about extending the task to make it more challenging. The second task was based upon one of the errors discussed in the previous workshop. This task might be used to support students' development of an internal monitor to make mathematical processes more meaningful. However, this task came in for the heaviest criticism because it was felt that it would establish bad habits.

Stefan: I will advise most strongly against using that with (my) students.
Olav: You'll get them into bad habits.
Stefan: Exactly, … I would never use that in a lesson, not in that form anyway.

The third task was based on 'mixed numbers' (whole + fraction). This was dismissed because mixed numbers are excluded from upper secondary students' mathematics, and students are instructed not to express numbers that comprise whole and fraction parts as a 'mixed number'. The rationale given for this was that students easily confuse the mixed number with algebraic notation in which multiplication is an implied operation between the two parts, rather than addition.

Only the fourth task was considered suitable. This too related to the discussion in the previous workshop in which one of the teachers explained that she would approach algebra through geometry because it made the symbols more meaningful. Moreover, the original task had been discussed at length by the upper secondary teachers in a previous workshop, which focused on geometry (an account of this can be found in Berg, Fuglestad, Goodchild & Sriraman, 2012). The original task was presented without any 'help line' being shown; the task included a hint 'to draw a help line', that is an additional line that exposed the angle properties of parallels, transversals, and rectilinear figures more explicitly.

6.2 Perceptions of contradictions and tensions

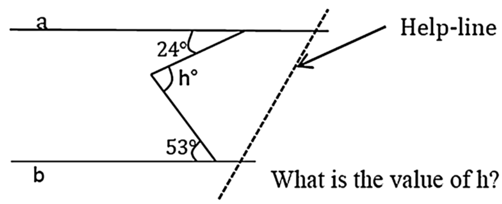
The discussion between the upper secondary teachers does not lead to a conclusion that could be described as 'positive' or 'encouraging'. However, their deliberation had an impact upon workshop activity and discussions between the group of teachers of grades 5–7 in which the outcome was both positive and encouraging.[3]

The teachers' reaction draws attention to the tension between the perceived agendas of teachers and didacticians. Elsewhere in the data generated from the projects, teachers repeatedly draw attention to the constraints within which they work—school time and preparation time, curriculum, textbook, examinations, students' expectations, and so on. Alignment to regular practice is not a simple matter of choice, but rather a position of equilibrium that is

[3] This is reported by Jørgensen, Steinsland and Solheim (2007). Unfortunately space prevents presentation of this contrasting case here.

Fig. 2 Tasks sent out to schools in advance of the second algebra workshop

1. I will make a frame around a rectangular picture. If the material for the frame is 1 cm broad, what is the length of frame material I need?
2. Is the following sometimes, always or never true? $\frac{x+4}{x} = 4$ Explain why.
3. $$2 + 2 = 2 \cdot 2; \quad 3 + 1\,{}^{1}/_{2} = 3 \cdot 1\,{}^{1}/_{2}; \quad 4 + 1\,{}^{1}/_{3} = 4 \cdot 1\,{}^{1}/_{3} \cdots \ \textit{true or false}?$$
   Can you continue this: 5 +, 6 +, 7 +; … 10 +; 59 +; -7 +; ${}^{2}/_{3}$ +
4.



A conversation in the staffroom: “A pupil came to me today and claimed that he had solved the ‘M’ problem with the help-line shown in this figure. Is it possible?

maintained by many intersecting constitutional, institutional, social, and professional forces. Teachers within the context of these constraining forces evaluate suggestions made by didacticians. Moreover, there could be the perception that didacticians work with theory and ideas rather than practice. The episode draws attention to the tension; it does not constitute new knowledge. Didacticians learn and develop new knowledge as they inquire into fresh approaches to resolve these tensions.

To be clear, the tension is not about the learning goals. There is no disagreement about the need for students to acquire procedural knowledge and develop fluency, accuracy, and reliability when working algebraically. Also, teachers and didacticians agree about the importance of facilitating the development of students’ conceptual knowledge. The issue at stake appears to be uncritical alignment with regular teaching practice that applies techniques used for refining physical skills. Adopting an inquiry stance means to challenge regular practice when it is ineffective, to reflect on the reasons why an approach might not achieve the intended outcome and to propose alternative approaches—this is critical alignment.

Training routine skills through practice exercises may be effective, but if the fluency and reliability of the skill depend on regular practice, then the exercises must be repeated as a matter of routine—whether it is a sporting activity or playing a musical instrument, or algebraic manipulation. However, such regular routine practice takes time, determination, motivation, and dedication. Moreover, mathematical procedures are not the same as ball control, physical fitness, or dexterity. Teachers find themselves in a double bind. The desired outcomes are students’ fluency and reliability in algebraic manipulation and meeting all the curriculum goals within the time available. Time spent practicing algebraic manipulation comes at a cost, less time to spend addressing other important areas of the curriculum.

The episodes reported above also expose a tension between two practices, the project practice and regular teaching practice. This is, perhaps, inevitable because alignment to the project entailed critical alignment to regular practice, and the forces holding teachers in alignment to their regular practice held them in critical alignment to the project. The project also challenged teachers in several ways because it did not fit their expectations; they had not experienced professional development in which teachers of all grades from 1 to 13 had been brought together. They were surprised with the principle of co-learning and the division of labor, as expressed by one primary grade teacher:

> Agnes: … in the beginning I struggled, had a bit of a problem with this because then I thought very much about you [didacticians] should come and tell us how we should run mathematics teaching. That was how I thought, you are the great teachers. … But now I see that my view has gradually changed because I see that you are the participants in this as much as we are. (Focus Group, Stjernen Primary School)

Three years after the episodes reported above, one of the teachers, Paul, who contributed to the group discussion in the algebra workshop, reflected on what he had learned from the projects. He has learned within his own practice and about the challenge of sharing lesson ideas with colleagues. Paul admits that at the outset he had a notion of inquiry, or investigative approaches, and used these as techniques within lessons, and as ways of introducing new concepts. However, he claims that his involvement in the projects has led him to appreciate that ‘inquiry’ is not merely a method or technique to be applied in discrete circumstances or short episodes, it is rather an attitude that influences and informs practice more generally.

> Paul: [Inquiry is] … a way to conduct oneself in a style of teaching, as a teacher. … I was very focused on inquiry that it was, in a way, equivalent to an activity within a lesson, or that it was a way to introduce new content. But I have had my eyes opened, … inquiry is also a way to conduct oneself, both students and oneself in the approach to teaching … Everything comes in the category inquiry. (Focus Group, Slottet Upper Secondary School)

Paul has also learned about the challenge of working with colleagues who may be skeptical about inquiry approaches. He recognizes the problems involved in sharing lesson ideas between colleagues and the need to have realistic expectations about what is possible. The projects set out to develop communities of inquiry within schools through which teaching development would be sustained beyond the life of the projects. Paul is an experienced teacher, with a master's degree in mathematics education. He works with older and more experienced mathematics teachers and with younger novices entering the profession. The insights that he has gained through the projects reveal his potential to make a sustained impact on teaching development within his school community over the many years of professional practice that lie before him.

> Paul: One must try in a way to play one's cards right, because when folk … have the enthusiasm to try and make something [a novel lesson idea], one must make sure that one gets some time to work with it and do things carefully and not give too big expectations, especially right from the start. Because often I have experienced that when folk try some lesson idea, which could be something which others have created … it does not always go well. Because when one gets a lesson idea others have made for example, it can be that one presents it wrong or that one does it in a way that was not intended. And so one throws it (away) and says that it has not worked at all. It is, because doing things in a different way requires that one, not just the lesson idea is different, but one as teacher also stands in a different way. So there are many things to learn, and [one must not] expect too much too quickly. (Focus Group, Slottet Upper Secondary School)

Paul reveals sensitivity and empathy toward his colleagues as they accommodate new approaches into their practice. These are the same characteristics that have underpinned the didacticians' engagement with teachers in the projects.

7 Discussion and conclusion

I have chosen to focus on the discussions between teachers working at upper secondary level. These teachers were always more ready than teachers at other levels to express their criticism about proposed project activities. However, their commitment was sustained throughout the successive projects. It is possible that their critical alignment to the projects provided the most opportunities for learning, certainly didacticians' learning and, as indicated by Paul above, their own.

In section three, it was explained that the projects were established within communities of practice theory (CPT), and the introduction of inquiry signaled a paradigm shift. Inquiry, it was argued, should transform alignment to practice into critical alignment in which teachers realize their agency to address the tensions and contradictions that emerge in their practice.

The episodes above led to conjecture that the experience of other practitioners is of crucial importance when teachers evaluate alternative approaches for teaching mathematics. The key role of experience may be explained within communities of practice theory in terms of 'imagination' (Wenger, 1998). Teachers engage with the experience of other teachers by extrapolating from their own classroom experience, which offers a means of adopting new approaches into their own practice. Alternatively, experiential knowledge may be explained from methodological principles; teachers' reports emerge from the experience of others with whom they can identify themselves and whose reports from practice can be accepted as 'authentic' and 'trustworthy'. It appears that from the perspective of the practitioner the craft knowledge gained through practice is of greater 'weight' than scholarly knowledge, no matter how firmly rooted in empirical evidence and established theory.

The negative reaction of the upper secondary teachers is disappointing, but it is also extremely valuable because it illuminates important issues within developmental research. Through their rejection of suggested tasks, it is possible to understand the importance of presenting activities based on experience to which the teachers can relate. It is also possible to discern an unwillingness to engage with tasks that teachers perceive as belonging to grades below those at which they teach, even if engagement is about adapting the tasks to make them more challenging and appropriate for their level. It is necessary to find more evidence to support, or contradict, these initial conjectures.

The case draws attention to forces that combine to keep teachers aligned to their regular practice and the operative model of development is alignment to practice and extrapolation through the application of imagination, rather than critical alignment and innovation in practice. The teachers experience tensions and contradictions in their practice, and they are aware of the recurrent errors of their students and the constraints of their practice that prevent addressing the problems effectively. However, the teachers presented in this paper were not prepared to consider suggestions put forward by the didacticians. There may be many reasons for their reluctance, but the outcome is that creative innovation that results in goal-directed development of practice does not occur.

It is not being suggested here that establishing communities of inquiry is an ineffective approach to mathematics teaching development. Similar case studies of upper secondary teachers engaging in inquiry cycles within the projects, with positive outcomes, have been reported (Goodchild et al. 2013). There have been many such cycles within the life of the projects. However, it appears that the design cycle in the case reported here might have been better motivated if the presentation of initial ideas had been demonstrably rooted in practice as well as theory. The projects have been concerned in engaging teachers in a sustainable developmental process and there are indications this may have been achieved. The reflections of Paul, above, reveal that he has developed professional competencies and understanding that will enable him to lead sustained teaching development within his own school setting. There are also teachers' accounts of their engagement in inquiry activity reported in Jaworski et al. (2007); unfortunately for the international readership, these important accounts are in Norwegian.

The case exposes tensions between the implementation of the projects as conceived and motivated by the didacticians and received and pursued by the teachers. These tensions also have the potential to motivate creative innovation in terms of mathematics teaching developmental activity. Teachers will engage in inquiry cycle activity in their practice when the proposed activity can be imagined into their classroom as they extrapolate from their regular practice. It may be unreasonable, as a general rule, to expect teachers to develop their practice in a principled fashion by creating new lesson 'designs'. Perhaps, expecting teachers to engage in the design process from first principles is too ambitious. More attention needs to be given to initiating development from shared experience that has been gained within the same constitutional, institutional, temporal, and social constraints that teachers recognize in their own practice. Nevertheless, it is possible to establish teachers' communities of inquiry and these do have the potential for sustained teaching development beyond the lives of relatively short-term developmental projects.

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