# Performance Analysis of User-Centric SBS Deployment with Load Balancing in Heterogeneous Cellular Networks: A Thomas Cluster Process Approach

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**Abstract**

In conventional heterogeneous cellular networks (HCNets), the locations of user equipments (UEs) and base stations (BSs) are modeled randomly using two different homogeneous Poisson point processes (PPPs). However, this might not be a suitable assumption in case of UE distribution because UE density is not uniform everywhere in HCNets. Keeping in view the existence of nonuniform UEs, the small base stations (SBSs) are assumed to be deployed in the areas with high UE density, which results in correlation between UEs and BS locations. In this paper, we analyse the performance of HCNets with nonuniform UE deployment containing a union of clustered and uniform UE sets. The clustered UEs are considered to be modeled according to Thomas cluster process, and random UEs are assumed to be deployed via homogeneous PPP. The SBSs are considered to be deployed at the center of the UE clusters, which results in user-centric SBS deployment. We derive outage probability and rate coverage of the proposed model. Furthermore, to improve the network performance, the impact of association biasing is also assumed. Our results show that the user-centric SBS deployment outperforms the conventional HCNets model. Increase in association bias upto a certain value results in performance improvement of the proposed user-centric HCNet.

*Keywords:* Heterogeneous cellular networks, Stochastic geometry, Nonuniform user distribution, Thomas cluster process, Rate coverage, User-centric SBS deployment, Capacity driven small cells.

## 1. Introduction

### *1.1. Motivation and Related Work*

Leveraging small base stations (SBSs) deployment in the coverage area of macro base stations (MBSs), the network performance gain of heterogeneous cellular networks (HCNets) is improved by eliminating coverage holes. Stochastic geometry (SG) is an efficient tool to model and analyze HCNets accurately and tractably. According to a recent forecast [1], the data traffic volume in the future cellular networks is

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predicted to grow beyond the 1000× wireless capacity demand. One of the biggest challenges for the cellular operators is to fulfill the customers' demand while ensuring profit and energy efficiency [2, 3]. According to the previous practice, reduced cell size and densification of SBSs in the coverage umbrella of MBSs has resulted in 2700× gain [4]. HCNet is expected to be a part of future cellular networks to improve the coverage and throughput [5].

Closed-form expressions for downlink coverage probabilities are derived for single tier cellular networks in [6], and for multi tier cellular networks in [7, 8] by assuming the base stations (BSs) and user equipments (UEs) distributed via Poisson point processes (PPPs). The rate analysis of multi-tier HCNet is performed in [9], which is further extended in [10] by considering various special cases of interference and path loss to ensure tractability. The delay outage in HCnet with spatio-temperol traffic is studied in [11] where UEs are spatially modelled using PPP while the packet arrival at each UE is assumed to be Bernoulli random process. The performance of HCNets is explored for line-of-sight (LOS) and non line-of-sight (NLOS) transmission and Rayleigh-Rician fading environment to investigate the effects of blockage on the coverage probability [12, 13]. Following maximum power based UE association, most of the UEs connect with MBS due to its high transmit power. This results in imbalanced UE distribution in multi tier networks. Hence, cell range expansion (CRE) is used in HCNets to offload a fraction of MBS associated UEs to SBSs to improve system capacity through efficient utilization of SBSs' radio resources [14, 15]. In [16], the authors analyze CRE with resource partioninig and observe significant improvement in the coverage probability, rate coverage and energy efficiency. All these works model the locations of BSs and UEs of different tiers in HCNets according to independent homogeneous PPP [17]. However, according to 3GPP simulation model, the assumption of uniform UEs might not be accurate because a fraction of UEs are more closely packed at certain areas in the network, such as railway station, shopping malls and crowded buildings. Similarly, certain UEs in the network do exist at random locations, such as pedestrians on the street and highway UEs. Nonuniform UE distribution in single tier cellular network is investigated in [18], in which the UEs are deployed using the conditional thinning property of PPP. Based on the correlation coefficient, the coverage probability of correlated and uncorrelated UEs in the vicinity of SBSs is studied in [19]. Using system level simulations, it has been proved that high performance is achieved if the correlation between UEs and BS locations is considered [20]. The MBS distribution according to PPP is a quite reasonable assumption because it provides coverage over large area due to high transmission power of MBS. However, SBSs are needed to be deployed in the high density clustered UE areas or hotspots to eradicate outage spots in the network. Such deployment enables capacity-driven SBSs in HCNets, which results in the correlation between UEs and SBSs [21].

The authors in [7]-[17] consider that the locations of BSs and UEs are independent of each other, which ensures tractability by characterizing the distance distribution using homogeneous PPP. The tractable analysis becomes difficult in case of independent UE and BS locations. The dependent UE and BS analysis has been reported in [22], where the authors distribute UEs according to Type-1 process, wherein a single UE is placed at random location in each cell. The multi-integral expression of link distribution of Type-1 process is derived in [23]. The uplink analysis of Type-1 process using meta distribution is studied in [24] by assuming the interferer points as a non Poisson process. Using pair correlation function, the authors in [25] derive tight expression for meta distribution of downlink signal-to-interference-ratio (SIR) in a typical cell scenario. The SIR meta distribution in case of multi tier cellular networks is derived in [26]. All the aforementioned literature seeks to overcome the difficulty in the calculation of SIR meta distribution expressions, which becomes even worse in case of HCNets. In [27], the authors develope an uplink model for a single tier cellular network based on Johnson-Mehl cell with coupled UE and BS locations. The downlink $K$ tier HCNet model is studied in [28] for SIR meta distribution and CRE.

The user-centric SBS based HCNet model proposed by 3GPP is a dominant theme in the 5G architecture and beyond, to avoid coverage holes and increase network capacity [29]. Such capacity-driven SBS deployment (CDSD) in HCNets is underway, which is assumed in 3GPP simulation models to deploy SBSs in high density UE areas [30] to serve a cluster of UEs located in the closed proximity of SBSs. Using Poisson cluster process (PCP), the authors in [31] analyze the coverage probability of clustered UEs located around MBSs in a single tier network. The authors in [32] extend the analysis for comprehensive fading environment. To fill the gap between 3GPPP simulation model and SG based HCNet model, the authors in [33] consider correlation between UEs and BS locations. The performance of $K$ tier HCNets is analyzed based on coverage probability, where the locations of UEs are modeled according to PCP around PPP distributed SBSs, and MBSs are assumed to be deployed according to homogeneous PPP. The authors in [34] extend the work in [33] for different cases of UE and BS distributions. Both UEs and SBSs at the hotspots are distributed using PCP. An expression for downlink coverage probability is derived in terms of sum-product function under maximum SIR association. The impact of decoupled access based connectivity for the clustered UEs based HCNet is studied in [35]. The authors in [36] extend the work in [33] for coverage probability analysis in millimeter wave based HCNets. A detailed literature summary is given in Table 1.

### *1.2. Novelty and Contributions*

In this paper, we focus on the performance evaluation of user-centric SBS based HCNets while assuming mixed UE distribution, which contains clustered as well as random UEs. This is because the assumption of only clustered UEs might not be enough to capture the real scenario, as a portion of UEs in the network may be located at random positions outside the clusters. There are two special cases of PCP, i.e., Matern cluster process (MCP) and TCP. Both differ from each other in terms of UE distribution within a cluster. TCP distributes daughter points (i.e., UEs) around the parent point (i.e., BS) using Gaussian distribution, while in MCP daughter points are uniformly distributed within a circular disc around the parent points. In this paper, we use TCP because it captures the real UE cluster without limiting the cluster members to a circular disc and is more tractable compared with MCP. The performance of HCNets with mixed UEs is evaluated with an impact of load balancing using CRE, in terms of outage probability and rate coverage.

The main contributions of this paper are as follows.

1. We consider nonuniform UE distribution, which is more realistic as compared to the uniform UE distribution approach. For nonuniform UE modeling, we use TCP where UEs are distributed around SBSs. Using this scheme, the performance of the network is significantly improved as compared to conventional uniform UE distribution with no correlation between UEs and SBSs.
2. Besides nonuniform UE distribution, we also assume that a fraction of uniformly deployed UEs is randomly distributed throughout the network in less populated areas. Furthermore, we characterize the network performance parameters for the randomly deployed UEs using PPP. Similarly, we also derive network performance parameters for the clustered UEs separately. The network performance gain of mixed UE distribution, including a set of clustered as well as uniformly distributed UEs, is derived and compared with uniformly distributed UEs and clustered UEs separately.
3. We derive expressions for the outage probability and rate coverage while assuming both uniform and clustered UE distributions. We compare the outage probability and rate coverage of the proposed mixed UE model with uniform only and clustered UEs models.
4. The random UEs located outside a cluster might experience low coverage, therefore, we consider small cell association biasing in the proposed model. Results show that the overall network performance is improved with increasing SBS transmit power. However, beyond a certain biasing threshold, no further enhancement in the network performance is observed due to increase in the interference.

Table 1: Literature review

| **References** | **Network model** | **Uniform UEs** | **CDSD with PCP based UEs distribution** | **CDSD with mix UEs distribution** | **SIR Meta distribution** | **CRE** | **Rate analysis** |
|---|---|---|---|---|---|---|---|
| [6] | Downlink & single tier | Yes | No | No | No | No | No |
| [9]-[17] | Downlink & HCNet | Yes | No | No | No | No | No |
| [18] | Downlink & single tier | No | No | No | No | No | No |
| [22]-[27] | Downlink analysis | No | No | No | Yes | No | No |
| [24] | Uplink+downlink & single tier | No | No | No | Yes | No | No |
| [27] | Uplink & single tier | No | No | No | Yes | No | No |
| [28] | Downlink & HCNet | No | No | No | Yes | Yes | No |
| [31]-[32] | Downlink & single tier | No | Yes | No | No | No | No |
| [33]-[35] | Downlink & HCNet | No | Yes | No | No | No | No |
| [36] | Millimeter waves HCNet | No | Yes | No | No | No | No |
| This paper | Downlink & HCNet | Yes | Yes | Yes | No | Yes | Yes |

Table 2: Notation summary

| **Notation** | **Description** |
|---|---|
| $\Phi_i^{oa}, \Phi_i^{ca}$ | $i$th tier open access and closed access homogeneous PPPs $\forall i \in K$ |
| $\lambda_i^{oa}, \lambda_i^{ca}$ | $i$th tier open access and closed access BS density $\forall i \in K$ |
| $\lambda_u^{\text{ppp}}$ | Random UE density |
| $\sigma$ | Variance of UEs in the cluster |
| $\alpha$ | Path loss exponent |
| $K$ | Number of BS tiers |
| $\mathcal{N}_t$ | Thermal noise in dBm |
| $P_i$ | Transmit power of $i$th tier BS in dBm $\forall i \in K$ |
| $N$ | Number of UEs per cluster |
| $\zeta_i$ | SINR threshold of $i$th tier BS $\forall i \in K$ |
| $\psi_i$ | Rate threshold of $i$th tier BS $\forall i \in K$ |
| $\mathbb{P}(\mathbb{E}_{\Phi_i})$ | $i$th tier association probability $\forall i \in K$ |
| $\mathbb{O}_i^{\text{ppp}}, \mathbb{O}_i^{\text{tcp}}$ | $i$th tier outage probability of uniformly distributed and clustered UEs $\forall i \in K$ |
| $\mathbb{O}_t^{\text{ppp}}, \mathbb{O}_t^{\text{tcp}}$ | Total outage probability of uniformly distributed and clustered UEs |
| $\mathbb{R}_{\text{i}}^{\text{ppp}}, \mathbb{R}_i^{\text{tcp}}$ | $i$th tier rate coverage of uniformly distributed and clustered UEs $\forall i \in K$ |
| $\mathbb{R}_t^{\text{ppp}}, \mathbb{R}_t^{\text{tcp}}$ | Total rate coverage of uniformly distributed and clustered UEs |
| $\mathbb{O}_t^{\text{m}}, \mathbb{R}_t^{\text{m}}$ | Total outage and rate coverage of mixed (uniform plus clustered) UEs |
| $\mathbb{P}_{(\text{ppp})}$ | Probability that UE is randomly selected from uniformly distributed UEs |
| $\mathbb{P}_{(\text{tcp})}$ | Probability that UE is randomly selected from clustered UEs |
| $W$ | Available bandwidth |
| $B_1$ | MBS biasing power |
| $B_2$ | SBS biasing power in dB |

### 1.3. Paper Organization

The rest of the paper is organized as follows. The system model comprising of BS and UE distribution models is discussed in Section 2. Association probabilities, followed by outage probability and rate coverage analyses of uniform and nonuniform UE based HCNet models are presented in Section 3 and Section 4, respectively. Numerical results are presented in Section 5 and the paper is concluded in Section 6.

## 2. System Model

In this section, we present the proposed HCNet model including the distribution of different tier BSs and UEs throughout the network followed by channel modeling and path loss statistics. We also discuss various assumptions for the proposed system model.

### 2.1. BS Distribution Modeling

We consider a $K$ tier downlink HCNet in which the coverage of MBSs is overlaid by SBSs following user-centric deployment as shown in Fig. 1 and Fig. 2 . BSs of each tier are distributed according to homogeneous PPP $\Phi_i$ and BSs in different tiers differ from each other in terms of transmit power $P_i$ and deployment density $\lambda_i > 0$. All BSs in $i$th tier $\forall i \in K$ are assumed to have identical signal-to-interference plus noise ratio (SINR) thresholds and transmit power levels. Furthermore, the BSs in each tier are divided into sets of open access and closed access BSs represented by $\Phi_i^{oa}$ and $\Phi_i^{ca}$ $\forall i \in K$, respectively, such that $\Phi_i = \Phi_i^{oa} \cup \Phi_i^{ca}$. The densities of open access BSs and closed access BSs are given by $\lambda_i^{oa}$ and $\lambda_i^{ca}$, respectively, such that the total density of BSs in the $i$th tier $\lambda_i = \sum_{j\in\{\mathrm{oa,ca}\}} \lambda_i^j$, $\forall i \in K$. The SBSs are assumed to be deployed at center of high density UE clusters.

### 2.2. UE Distribution Modeling

Unlike uniformly distributed UEs studied in the literature with no correlation between UEs and BSs, this paper considers user-centric SBSs approach, in which UEs and SBSs are correlated because the SBSs are deployed in the area where the user density is high. The high density UEs are considered to be located in the form of clusters or hotspots, which results in nonuniform UE distribution. Furthermore, it is assumed that the SBSs are deployed at the center of these UE clusters. The clustered UEs follow PCP distribution represented by $\Phi_u^{\mathrm{tcp}}$, where SBSs are assumed to be the parent points located at the center of clusters. UEs are distributed around SBSs in the form of cluster as daughter points or cluster members. The cluster serving SBS[1] is assumed to be a member of open access SBSs.

$\Phi_u^{\mathrm{tcp}}$ follows TCP in which cluster member UEs are scattered around cluster serving SBS according to Gaussian distribution with variance $\sigma$. For simplicity, in this paper, we assume that variance $\sigma$ and number of UEs per cluster $N$ are fixed as shown in Fig. 2. The density of the clustered UEs throughout the network is $\lambda_u^{\mathrm{tcp}} = N\lambda_i^{oa}$. The correlation of SBSs and clustered UEs reflects the scenario of SBS deployment in the hotspots to provide maximum coverage. The probability density function (PDF) of TCP based scattered UEs around SBSs with random distance vector $Z$, $f_Z(z)$, is given by [37]

$$f_Z(z) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{z^2}{2\sigma^2}\right), \qquad z \in \mathbb{R}^2.$$

[1]The terms cluster serving SBS and cluster center SBS are used interchangebly throughout the paper.

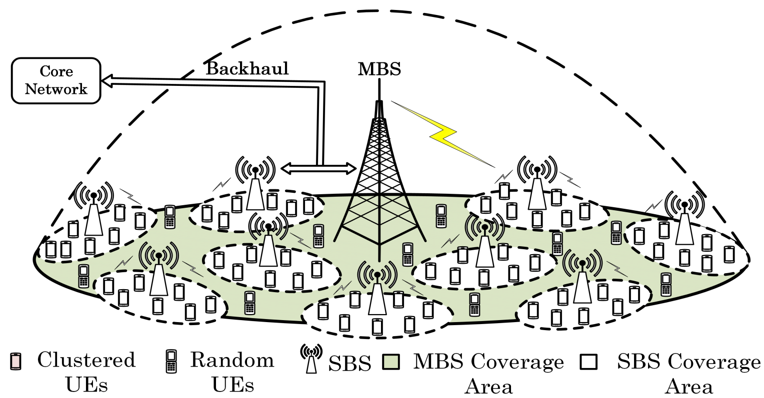


Figure 1: Two tier user-centric HCNet model: Users follow mixed distribution (uniformly deployed UEs along with clustered UEs) and SBSs are assumed to be deployed at hotspots.

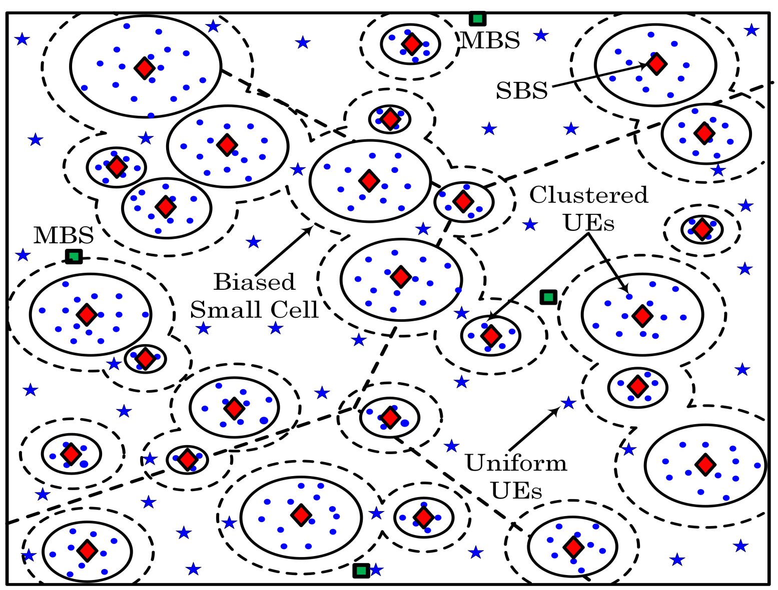


Figure 2: A two tier HCNet model: MBSs (represented by squares) and SBSs (represented by diamonds) are distributed according to two independent PPPs. Dash-dotted lines and circles represent coverage area of MBSs and SBSs, respectively. UEs follow mixed distribution (uniformly deployed UEs along with clustered UEs), whereas dots represent clustered UEs around SBSs and stars represent uniformly distributed UEs.

Besides clustered UEs located at hotspots, a fraction of uniform UEs, like pedestrian and highway users, also exist in the network. The analysis of such uniform UEs is already available in the literature [10] and can be best modeled according to homogeneous PPP $\Phi_u^{\text{ppp}}$ with density $\lambda_u^{\text{ppp}}$ . The overall mixed UE distribution is formed by the superposition of clustered and uniform UEs.

Without loss of generality, downlink analysis of clustered UEs is performed for a randomly selected UE, referred to as the typical UE, from the randomly selected cluster, known as representative cluster.

Let $X_0$ be the distance between the cluster serving SBS and the typical UE in the cluster, then the marginal PDF and complementary cumulative distribution function (CCDF) of $\Phi_u^{\text{tcp}}$ are derived by integrating the joint PDF of location $X_0$ and its polar coordinates over $2\pi$ [38], which are given, respectively, as

$$\text{PDF}: \quad f_{X_0}(x_0) = \frac{x_0}{\sigma^2}\exp\left(-\frac{x_0^2}{2\sigma^2}\right), \quad x_0 \geq 0, \tag{1a}$$

$$\text{CCDF}: \quad F_{X_0}(x_0) = \exp\left(-\frac{x_0^2}{2\sigma^2}\right), \quad x_0 \geq 0. \tag{1b}$$

For simplicity, we consider this single SBS as a subset of SBS tier to analyze the performance of SBS located at the center of cluster separately. The SBS tier is the union of representative cluster center SBS located at $x_0$ $\{\text{SBS}_0\}$ and a set of SBSs located at $x_i$ $\{\text{SBS}_i\}$ outside the representative cluster. Similarly, $\{\text{SBS}_0\}$ is treated as a separate tier, called 0th tier, consisting of a single SBS. The indices set of all tiers in the network can be written as $\mathcal{K} = 0 \cup K = \{0,1,2,..,K\}$. For stationary PPP, the origin can be shifted to the UE location without changing the statistics of locations via PPP according to Slivnyak's theorem [37]. The BSs in the $i$th tier are distributed via homogeneous PPP $\Phi_i$, $\forall i \in K$ and the nearest BS is located at distance $X_i$ from the typical UE. The PDF and CCDF of $X_i$ can, respectively, be written as

$$\text{PDF}: \quad f_{X_i}(x_i) = 2\pi\lambda_i x_i \exp\left(-\pi\lambda_i x_i^2\right), \quad x_i \geq 0, \tag{2a}$$

$$\text{CCDF}: \quad F_{X_i}(x_i) = \exp\left(-\pi\lambda_i x_i^2\right), \quad x_i \geq 0. \tag{2b}$$

### *2.3. Channel Modeling and Path Loss Statistics*

Without considering the shadowing effect, the received power by the typical UE located at a distance $x_i$ from $i$th tier BS is given as $P_x(x_i) = P_i B_i g_i x_i^{-\alpha}$, where $P_i$ is the transmit power of $i$th tier BS, $g_i$ denotes the Rayleigh fading gain, $B_i$ is the biased power factor, and $\alpha_i > 2$ represents the path loss exponent for $i$th tier BS $\forall \alpha_i = \alpha$. The averaged maximum biased received power (BRP) based association strategy is assumed, in which the typical UE associates with the nearest BS from $i$th tier open access BSs. The closed access BSs only contribute to interference in the network. The location of the nearest serving $i$th tier BS is given as $x_i^* = arg \max\limits_{x_i \in \Phi_i} P_i B_i g_i x_i^{-\alpha}$. The downlink SINR for the typical UE from the BS at $x_i$ is given as

$$\text{SINR}(x_i) = \frac{P_i B_i g_i ||x_i||^{-\alpha}}{\mathcal{N}_t + I_t}, \tag{3}$$

where $I_t$ denotes the total interference and is given as

$$I_t = \sum_{j \in \mathcal{K}} \sum_{x_j \in S} P_j B_j g_j x_j^{-\alpha}. \tag{4}$$

Here, $S = \{\Phi_j^{oa} \cup \Phi_j^{ca}\} \backslash x_j^*$, and $\mathcal{N}_t$ denotes system added thermal noise. The association event , $\mathbb{E}_{\Phi_i}^{\mathrm{m}}$, based on maximum BRP strategy for clustered and uniform UE sets in the network is given as [38]

$$\begin{aligned}\mathbb{E}_{\Phi_i}^{\mathrm{m}} &= arg \max_{i \in \mathcal{K}} P_i B_i x_i^{-\alpha} \\ &= \bigcap_{i \in \mathcal{K}} \left( X_j > \left( \frac{P_i}{P_j} \right)^{1/\alpha} X_i \right), \ \forall \mathrm{m} \in \{\mathrm{ppp}, \mathrm{tcp}\},\end{aligned} \tag{5}$$

where $j$ represents all other BSs (other than the associated BS) in the $i$th tier, from which the typical UE receives maximum power. Using (5), we analyze the association probability of the typical UE from the uniform as well as clustered UEs' set. It is worth noting that the cluster serving BS is the subset of SBS tier, therefore, $P_0 = P_i$.

In the following sections, we analyze the association probability, outage probability, and rate coverage of the uniform UEs followed by the clustered UEs of the proposed system model.

## 3. Analysis of Uniformly Distributed UEs

In this section, we consider that both UEs and SBSs are uniformly distributed according to two independent PPPs in the entire 2D plane. This section mainly focuses on the brief analysis of the uniform UE set in the proposed model presented in Section 2. The analysis follows the approach similar to the one in literature (see e.g., [6], [7], [10]). The performance of our proposed model is compared with these results while considering both uniform and clustered UE distributions to highlight the improvement in the network performance through user-centric SBS deployment.

### *3.1. Association Probability and Statistical Distance of Serving BS*

Using the notion of association event in (5), the association probability is defined as the probability that a randomly selected UE from uniform UEs' set is associated with a BS belonging to $i$th tier for $\forall i \in \mathcal{K}$. Association probability of uniform UEs distributed via PPP, $\mathbb{P}(\mathbb{E}_{\Phi_i}^{\mathrm{ppp}})$, is given as

$$\mathbb{P}(\mathbb{E}_{\Phi_i}^{\mathrm{ppp}}) = 2\pi\lambda_i \int_0^\infty x_i \exp\left\{ -\pi \sum_{\substack{j \in K \\ j \neq i}} \lambda_i \left( \frac{B_j P_j}{B_i P_i} \right)^{2/\alpha_i} x_i^2 \right\} dx_i. \tag{6}$$

### *3.2. Outage Probability of Uniform UEs*

The outage probability can be defined as the probability that the received SINR of a typical UE is less than the predefined target SINR threshold of the $i$th tier BSs, given that the typical UE is served by the $i$th tier BS. Based on the law of total probability, the total outage probability of typical UE from all tiers of BSs in the network, $\mathbb{O}_t^{\mathrm{ppp}}$, is written as

$$\mathbb{O}_t^{\mathrm{ppp}} = \sum_{i=1}^{K} \mathbb{P}(\mathbb{E}_{\Phi_i}^{\mathrm{ppp}}) \mathbb{O}_i^{\mathrm{ppp}}. \tag{7}$$

The per tier outage probability of uniformly distributed UEs, $\mathbb{O}_i^{\mathrm{ppp}}$, in the proposed HCNet model with a set of open access and closed access BSs, is given by (8), where $\mathcal{T}_i$ is the target SINR threshold, $\mathcal{B}_i = \frac{B_j}{B_i}$, $\mathbb{Z}(\mathcal{T}_i, \alpha_i, \mathcal{B}_i) = \frac{2\mathcal{T}_i}{\alpha_i - 2}\, {}_2F_1\left[1, 1 - \frac{2}{\alpha_i}; 2 - \frac{2}{\alpha_i}; -\frac{\mathcal{T}_i}{\mathcal{B}_i}\right]$, and $\mathbb{Q}(\mathcal{T}_i, \alpha_i) = \mathcal{T}_i^{2/\alpha_i} \frac{2\pi \csc(\frac{2\pi}{\alpha_i})}{\alpha_i}$. Here, ${}_2F_1\,[.]$ is Hypergeometric function.

$$\mathbb{O}_i^{\text{ppp}} = 1 - \frac{2\pi\lambda_i}{\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{ppp}})} \int_0^\infty x_i \exp\Bigg\{ - \frac{\mathcal{T}_i \mathcal{N}_t x_i^{\alpha_i}}{P_i} - \pi \sum_{\substack{j\in K \\ j\neq i}} \left(\frac{B_j P_j}{B_i P_i}\right)^{2/\alpha_i} \Bigg( \lambda_j^{oa} \Big[ \mathcal{B}_i^{2/\alpha_i} + \mathbb{Z}\big(\mathcal{T}_i, \alpha_i, \mathcal{B}_i\big) \Big] + \lambda_j^{ca} \mathbb{Q}\big(\mathcal{T}_i, \alpha_i\big) \Bigg) x_i^2 \Bigg\} dx_i. \tag{8}$$

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$$\mathbb{R}_i^{\text{ppp}} = \sum_{n>0} \mathcal{P}_i(n) \frac{2\pi\lambda_i}{\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{ppp}})} \int_0^\infty x_i \ \exp\Bigg\{ - \frac{\big(2^{\frac{\psi_i \hat{L}_i}{B}} - 1\big)\mathcal{N}_t x_i^{\alpha_i}}{P_i} - \pi \sum_{\substack{j\in \mathcal{B} \\ j\neq i}} \left(\frac{B_j P_j}{B_i P_i}\right)^{2/\alpha_i} \Bigg( \lambda_j^{oa} \Big[ \mathcal{B}_j + \mathbb{Z}(\mathcal{R}_i, \alpha_i, \mathcal{B}_i) \Big] + \lambda_j^{ca} \mathbb{Q}\big(\zeta_i, \alpha_i\big) \Bigg) x_i^2 \Bigg\} d_{x_i}. \tag{11}$$

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*3.3. Rate Coverage Probability of Uniform UEs*

The rate coverage is defined as the probability that the random UE associated with $i$th tier BS achieves a rate greater than the predefined target rate threshold $\mathcal{R}_i$. Similar to (7), the total rate coverage of uniform UEs is given as

$$\mathbb{R}_t^{\text{ppp}} = \sum_{i=1}^{K} \mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{ppp}}) \mathbb{R}_i^{\text{ppp}}, \tag{9}$$

where rate achieved by the typical UE associated with $i$th tier BS is denoted by $\mathbb{R}_i^{\text{ppp}}$. According to the mean load approximation in [16], the average load of the $i$th tier BS, $\hat{L}_i^{\text{ppp}}$, is given as

$$\hat{L}_i^{\text{ppp}} = 1 + \frac{1.28\lambda_u^{\text{ppp}} \mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{ppp}})}{\lambda_i}. \tag{10}$$

Following the mean load approximation in (10), per tier rate coverage of a randomly selected UE given that the UE associates with $i$th tier BS is given in (11), where $\mathcal{P}_i(n) = \mathbb{P}(L_i^{\text{ppp}} = n)$ denotes probability mass function (PMF) of the cell load of the randomly selected $i$th tier BS.

In the next section, detailed analysis of clustered UEs based on user-centric SBS deployment is presented.

## 4. Analysis of Clustered UEs

In this section, the nonuniform UEs are assumed to be located in the form of clusters around SBSs in the HCNet model presented in Section 2. The SBSs are deployed following the user-centric deployment scheme at the center of cluster. The UEs are distributed via TCP at the hotspots following Gaussian distribution around SBSs with variance $\sigma_i$. Furthermore, it is assumed that $\sigma_i$ is same for all clusters in the network. The analysis is carried out for a typical UE located at randomly selected representative cluster. Moreover, this section focuses on derivation of per tier UE association, outage probability, and rate coverage for the clustered UEs in the network.

### 4.1. Association Probability of Clustered UEs

Based on the association event in (5), the conditional association probability of $i$th tier BS in case of clustered UEs is presented in Lemma 1.

**Lemma 1.** *The conditional association probability of a randomly selected UE from a representative cluster, with the ith tier open access BS, is given as*

$$\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{tcp}}) = \begin{cases} \frac{1}{\sigma_i^2}\int\limits_0^\infty x_0 \exp\Big\{-\pi\sum\limits_{\substack{j\in\mathcal{K}\\ j\neq i}} \lambda_i^{\text{oa}}\Big(\frac{B_jP_j}{B_0P_0}\Big)^{2/\alpha_i} x_0^2 - \frac{x_0^2}{2\sigma_i^2}\Big\} dz_0 & \text{if } i=0, \\ 2\pi\lambda_i^{\text{oa}}\int\limits_0^\infty x_i \exp\Big\{-\pi\sum\limits_{\substack{j\in\mathcal{K}\\ j\neq i}} \lambda_i^{oa}\Big(\frac{B_jP_j}{B_iP_i}\Big)^{2/\alpha_i} x_i^2 - \Big(\frac{B_jP_j}{B_iP_i}\Big)^{2/\alpha_i}\frac{x_i^2}{2\sigma_i^2} - \pi\lambda_i^{\text{oa}}x_i^2\Big\} dx_i & \\ & \text{if } i\in K, \end{cases} \tag{12}$$

*where $B_0 = B_i$ and $P_0 = P_i$ in case if $i = 0$ because the cluster serving SBS is a subset of SBS tier.*

PROOF OF LEMMA1. See Appendix A.

### 4.2. Statistical Serving Distances for Clustered UEs

In this subsection, we derive the PDF of serving distance from typical UE conditioned that the typical UE associates with $i$th tier BS. Let the serving distance, given that an association event has occurred, is denoted by the random variable $Y_i = X_i|\mathbb{E}_{\Phi_i}^{\text{tcp}}$. The PDF of $Y_i$ for clustered UEs in the vicinity of SBS is presented in Lemma 2.

**Lemma 2.** *The PDF of distance, $f_{Y_i}^{\text{tcp}}(y_i)$, of typical UE belonging to TCP distributed clustered UEs with ith tier BSs, given that association event $\mathbb{E}_{\Phi_i}^{\text{tcp}}$ has occurred, is expressed as*

$$f_{Y_i}^{\text{tcp}}(y_i) = \begin{cases} \frac{y_0}{\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{tcp}})\sigma_i^2} \exp\Big\{-\pi\sum_{\substack{j\in\mathcal{B}\\ j\neq i}} \lambda_i^{oa}\Big(\frac{B_jP_j}{B_0P_0}\Big)^{2/\alpha_i} y_0^2 - \frac{y_0^2}{2\sigma_i^2}\Big\}, & \text{if } i=0, \\ \frac{2\pi\lambda_i y_i}{\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{tcp}})\sigma_i^2} \exp\Big\{-\pi\sum_{\substack{j\in\mathcal{B}\\ j\neq i}} \lambda_i^{oa}\Big(\frac{B_jP_j}{B_iP_i}\Big)^{2/\alpha_i} y_i^2 - \pi\lambda_i y_i^2 - \Big(\frac{B_jP_j}{B_iP_i}\Big)^{2/\alpha_i}\frac{y_i^2}{2\sigma_i^2}\Big\}, & \text{if } i\in K. \end{cases} \tag{13}$$

PROOF OF LEMMA2. See Appendix B.

Using (12) and (13), derived in Lemma 1 and Lemma 2, respectively, the per tier outage probability of the clustered UEs is derived and presented in the next subsection.

### 4.3. Outage Probability of Clustered UEs

The outage probability of TCP distributed clustered UEs is defined as the probability that the randomly selected UE from representative cluster experiences SINR less than the target SINR threshold $\Omega$. The total outage probability of clustered UEs from $K$ number of BS tiers in the network, $\mathbb{O}_t^{\text{tcp}}$, is given as

$$\mathbb{O}_t^{\text{tcp}} = \mathbb{P}(\mathbb{E}_{\Phi_0}^{\text{tcp}})\mathbb{O}_0^{\text{tcp}} + \sum_{i\in K}\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{tcp}})\mathbb{O}_i^{\text{tcp}}, \tag{14}$$

where $\mathbb{O}_0^{\text{tcp}}$ denotes the outage probability of typical UE from the cluster serving SBS located at the center of cluster, and $\mathbb{O}_i^{\text{tcp}}$ is the outage probability of the typical UE from the $i$th tier BS excluding the cluster center SBS, i.e., $i \in K$ and $i \neq 0$. As mentioned earlier, the UE can only connect with the $i$th tier open access BSs, however, closed access BSs act as interferers. The total interference in the network is, therefore, the sum of interferences from open access and closed access BSs, i.e.,

$$I_t = \sum_{\substack{i \in \mathcal{K} \\ j \in \Phi_i}} \mathcal{I}_{(i,j)}^{oa} + \sum_{\substack{i \in \mathcal{K} \\ j \in \Phi_i}} \mathcal{I}_{(i,j)}^{ca}. \tag{15}$$

The per tier outage probability of the typical UE belonging to TCP distributed clustered UEs given that the UE is served by $i$th tier BS, $\mathbb{O}_i^{\text{tcp}}$, is given as

$$\begin{aligned} \mathbb{O}_i^{\text{tcp}} =& \mathbb{E}\big[\mathbb{P}\big\{\text{SINR}(x_i) < \Omega_i\big\}\big] \\ =& 1 - \int_0^\infty \mathbb{P}\big\{\text{SINR}(x_i) \geq \Omega_i\big\} f_{X_i}^{\text{tcp}}(z_i) dz_i. \end{aligned} \tag{16}$$

Using (16) along with per tier association probability and the PDF of serving distance from Lemma 1 and Lemma 2, receptively, per tier outage probability is derived in Theorem 1.

**Theorem 1.** *Per tier outage probability of a typical UE from clustered UE set in the proposed system model, given that the UE associates with ith tier open access BS, is given as (20). where* $\mathbb{Z}(\Omega_i, \alpha_i, \mathcal{B}_i) = \frac{2\Omega_i B_i^{2/\alpha_i - 1}}{\alpha_i - 2}$ ${}_2F_1\left[1, 1 - \frac{2}{\alpha_i}; 2 - \frac{2}{\alpha_i}; -\frac{\Omega_i}{\mathcal{B}_i}\right]$, *and* $\mathbb{Q}(\Omega_i, \alpha_i) = \Omega_i^{2/\alpha_i} \frac{2\csc(\frac{2\pi}{\alpha_i})}{\alpha_i}$. *Here,* ${}_2F_1\,[.]$ *is the Hypergeometric function and* $\mathcal{B}_i = \frac{B_j}{B_i}$. *The lower limit in the second integral is* $ll = \left(\frac{B_j P_j}{B_i P_i}\right)^{1/\alpha_i} y_j$.

PROOF OF THEOREM1. See Appendix C.

*4.3.1. Bounded Outage Probability of Clustered UEs*

The expression of outage probability with double integrals derived in (20) is not in closed form. The second integral is due to the consideration of interference from the cluster center SBS. Therefore, for further simplification, the upper and lower bounds on the outage probability are derived in this subsection. The bounds on the outage probability are derived either by ignoring or considering maximum interference from the cluster center SBS. The bounded outage probability can be written as

$$\mathbb{P}(\mathbb{E}_{\Phi_0}^{\text{tcp}})\mathbb{O}_0^{\text{tcp}} + \sum_{i \in K} \mathbb{O}_i^{\text{L}} \leq \mathbb{O}_t^{\text{tcp}} \leq \mathbb{P}(\mathbb{E}_{\Phi_0}^{\text{tcp}})\mathbb{O}_0^{\text{tcp}} + \sum_{i \in K} \mathbb{O}_i^{\text{U}}, \tag{17}$$

where $\mathbb{O}_i^{\text{L}}$ and $\mathbb{O}_i^{\text{U}}$ denote the lower and upper bounds on the per tier outage probability, respectively. $\mathbb{E}_{\Phi_0}^{\text{tcp}}$ and $\mathbb{E}_{\Phi_i}^{\text{tcp}}$ represent the association event with the 0th and the $i$th tier BS, respectively. By bounding the interference from cluster center SBS, bounds on outage probability can be derived as

$$\begin{aligned} \mathbb{O}_i^{\text{U}} =& 1 - \frac{2\pi\lambda_i}{1+\Omega_i} \int_0^\infty y_i \exp\bigg\{ - \frac{\Omega_i \mathcal{N}_i y_i^{\alpha_i}}{P_i} - \pi \sum_{\substack{j \in K \\ j \neq i}} \left(\frac{B_j P_j}{B_i P_i}\right)^{2/\alpha_i} \bigg( \lambda_j^{\text{oa}} \big[\mathcal{B}_i + \mathbb{Z}(\Omega_i, \alpha_i, \mathcal{B}_i)\big] + \\ & \lambda_j^{\text{ca}} \mathbb{Q}\big(\Omega_i, \alpha_i\big)\bigg) y_i^2 - \frac{x_0^2}{2\sigma_i^2} \bigg\} dy_i, \end{aligned} \tag{18}$$

$$\mathbb{O}_i^{\text{tcp}} = \begin{cases} 1 - \frac{1}{\mathbb{P}(\mathbb{E}_{\Phi_0}^{\text{tcp}})} \int\limits_0^\infty \frac{y_0}{\sigma_i^2} \exp\Bigg\{ - \frac{\Omega_i \mathcal{N}_t y_0^{\alpha_i}}{P_0} - \pi \sum\limits_{\substack{j\in K \\ j\neq i}} \left(\frac{B_j P_j}{B_i P_i}\right)^{2/\alpha_i} \Bigg( \lambda_j^{\text{oa}} \big[\mathcal{B}_i + \mathbb{Z}(\Omega_i, \alpha_i, \mathcal{B}_i)\big] + \\ \qquad \lambda_j^{\text{ca}} \mathbb{Q}(\Omega_i, \alpha_i)\Bigg) y_0^2 - \frac{y_0^2}{2\sigma_i^2} \Bigg\} dy_0, \text{if } i = 0, \\ 1 - \frac{2\pi\lambda_i}{\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{tcp}})} \int\limits_0^\infty \int\limits_{ll}^\infty y_i \exp\Bigg\{ - \frac{\Omega_i \mathcal{N}_i y_i^{\alpha_i}}{P_i} - \pi \sum\limits_{\substack{j\in K \\ j\neq i}} \left(\frac{B_j P_j}{B_i P_i}\right)^{2/\alpha_i} \Bigg( \lambda_j^{\text{oa}} \big[\mathcal{B}_i + \mathbb{Z}(\Omega_i, \alpha_i, \mathcal{B}_i)\big] + \\ \qquad \lambda_j^{\text{ca}} \mathbb{Q}\big(\Omega_i, \alpha_i\big)\Bigg) y_i^2 - \frac{x_0^2}{2\sigma_i^2} \Bigg\} - \frac{x_0}{\sigma_i^2 \left(1 + \Omega_i x_0^{-\alpha} \left(\frac{B_j P_j}{B_i P_i}\right) y_j^{\alpha}\right)} dx_0 dy_i, \text{if } i \in K. \end{cases} \tag{20}$$

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$$\mathbb{O}_i^{\text{L}} = 1 - 2\pi\lambda_i \int\limits_0^\infty y_i \exp\Bigg\{ - \frac{\Omega_i \mathcal{N}_i y_i^{\alpha_i}}{P_i} - \pi \sum_{\substack{j\in K \\ j\neq i}} \left(\frac{B_j P_j}{B_i P_i}\right)^{2/\alpha_i} \Bigg( \lambda_j^{\text{oa}} \big[\mathcal{B}_i + \mathbb{Z}(\Omega_i, \alpha_i, \mathcal{B}_i)\big] +$$

$$\lambda_j^{\text{ca}} \mathbb{Q}\big(\Omega_i, \alpha_i\big)\Bigg) y_i^2 - \frac{x_0^2}{2\sigma_i^2} \Bigg\} dy_i. \tag{19}$$

PROOF OF (18) AND (19). See Appendix D.

*4.4. Rate Coverage of Clustered UEs*

In this subsection, we derive the rate coverage probability of a typical UE belonging to cluster members in the proposed $K$ tier HCNet model. Using the approach similar to the one in (9), the total rate coverage, $\mathbb{R}_t$, from $\mathcal{K}$ number of tiers in the network, where $\mathcal{K} = 0 \cup K = \{0, 1, 2, .., K\}$, is given as

$$\mathbb{R}_t^{\text{tcp}} = \mathbb{P}(\mathbb{E}_{\Phi_0}^{\text{tcp}})\mathbb{R}_0^{\text{tcp}} + \sum_{i\in K} \mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{tcp}})\mathbb{R}_i^{\text{tcp}}. \tag{21}$$

Here $\mathbb{R}_0^{\text{tcp}}$ is the rate coverage when the typical UE is associated with cluster serving SBS located at the center of the representative cluster and $\mathbb{R}_i^{\text{tcp}}$ is the rate coverage when the UE is associated with $i$th tier BS. Assuming the fully loaded scenario, the average ergodic rate coverage of the TCP distributed clustered UEs is expressed as

$$\begin{aligned} \mathbb{R}_i^{\text{tcp}} &= \mathbb{P}\Bigg\{ W \log_2 \Bigg\{ 1 + \text{SINR}_i(x) \Bigg\} \geq \psi_i \Bigg\} \\ &= \mathbb{P}\Bigg\{ \text{SINR}_i(x) \geq 2^{\frac{\psi_i}{W}} - 1 \Bigg\}. \end{aligned} \tag{22}$$

Per tier average ergodic rate coverage of the typical UE belonging to clustered UEs' set is given in Theorem 2.

$$\mathbb{R}_i^{\text{tcp}} =$$

$$\begin{cases} \frac{1}{\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{tcp}})} \int_0^\infty \frac{y_0}{\sigma_i^2} \exp\Bigg\{ - \frac{\Psi_i(\psi_i, W)\mathcal{N}_t y_0^{\alpha_i}}{P_i} - \pi \sum_{\substack{j\in K\\ j\neq i}} \left(\frac{B_jP_j}{B_iP_i}\right)^{2/\alpha_i} \Bigg(\lambda_j^{\text{oa}}\Big[\mathcal{B}_i + \mathbb{Z}\big\{\Psi\big(\psi_i, W, \mathcal{B}_j\big), \\ \qquad \alpha_i, \mathcal{B}_i\big\}\Big] + \lambda_j^{\text{ca}}\mathbb{Q}\Big\{\Psi\big(\psi_i, W\big), \alpha_i\Big\}\Bigg) y_0^2 - \frac{y_0^2}{2\sigma_i^2}\Bigg\} d_{y_0}, \text{ if } i = 0, \\ \frac{2\pi\lambda_i}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})} \int_0^\infty \int_{ll}^\infty y_i \exp\Bigg\{ - \frac{\Psi_i(\psi_i, W)\mathcal{N}_t y_i^{\alpha_i}}{P_i} - \pi \sum_{\substack{j\in K\\ j\neq i}} \left(\frac{B_jP_j}{B_iP_i}\right)^{2/\alpha_i} \Bigg(\lambda_j^{oa}\Big[\mathcal{B}_i + \mathbb{Z}\Big\{\Psi\big(\psi_i, L_i^{\text{tcp}}\big), \\ \qquad \alpha_i, \mathcal{B}_i\Big\}\Big] + \lambda_j^{ca}\mathbb{Q}\Big\{\Psi\big(\psi_i, W\big), \alpha_i\Big\}\Bigg) y_i^2\Bigg\} \frac{x_0}{\sigma_i^2\left(1+\Omega_i x_0^{-\alpha}\left(\frac{B_jP_j}{B_iP_i}\right)y_j^{\alpha}\right)} dx_0 dy_i, \text{ if } i \in K. \end{cases} \tag{23}$$

---

**Theorem 2.** *Per tier rate coverage, $\mathbb{R}_i^{\text{tcp}}$, of a typical UE belonging to representative cluster members from $\Phi_u^{\text{tcp}}$, conditioned that the typical UE is served by ith tier BS is given as (23).*

In (23), $\mathbb{Z}\Big\{\Psi\big(\psi_i, W, L_i^{\text{tcp}}\big), \alpha_i, \mathcal{B}_i\Big\}$ and $\mathbb{Q}\Big\{\Psi\big(\psi_i, W\big), \alpha_i\Big\}$ are the same as already defined in Theorem 1, by substituting $\Omega_i$ with $\Psi(\psi_i, W) = 2^{\frac{\psi_i}{W}} - 1$.

PROOF OF THEOREM2. See Appendix E.

Utilizing outage probability and rate coverage derived in Theorem 1 and Theorem 2, respectively, the rate coverage of the proposed mixed UEs based HCNet is derived in Subsection 4.5.

### *4.5. Outage Probability and Rate Coverage of Mixed UEs in HCNet*

In this subsection, we derive the overall outage probability and rate coverage for the proposed system model (as explained in Section 2) containing uniform and clustered UEs. The UE distribution is formed by the superposition of PPP and TCP distributions in $\Phi_u^{\text{tcp}}$ and $\Phi_u^{\text{ppp}}$, respectively. The analysis is performed for a randomly selected user from any of the two UE sets. There is a probability that the typical UE selected for analysis belongs to any of the two UE sets. Based on the counting measure, the probability, $\mathbb{P}_{(\text{ppp})}$, that the typical UE is selected from a uniform UE set $\Phi_u^{\text{ppp}}$ is given as

$$\mathbb{P}_{(\text{ppp})} = \frac{\lambda_u^{\text{ppp}}}{\lambda_u^{\text{ppp}} + \sum_{i\in K} N_i \lambda_i^{\text{oa}}}. \tag{24}$$

Similarly, the probability, $\mathbb{P}_{(\text{tcp})}$, that the typical UE is selected from clustered UE set $\Phi_u^{\text{pcp}}$ is given as

$$\mathbb{P}_{(\text{tcp})} = \frac{N_i \lambda_i^{\text{oa}}}{\lambda_u^{\text{ppp}} + \sum_{i\in K} N_i \lambda_i^{\text{oa}}}. \tag{25}$$

The outage probability and rate coverage of mixed UEs are the combination of the outage probability and rate coverage of the clustered and unifrom UEs, conditioned that the randomly selected UE belongs to

Table 3: Simulation parameters

| Simulation parameters | Values |
|---|---|
| $\lambda_i^{oa}, \lambda_i^{ca}$ | 1,100 |
| $\lambda_u^{\mathrm{ppp}}$ | 100 |
| $\alpha_i$ | 3.5 |
| $\mathcal{K}$ | 2 |
| $\mathcal{N}_t$ | -174 dBm |
| $P_i$ | $\{53, 33\}$ dBm $\forall i \in K$ |
| $N$ | 10 |
| $\psi_i$ | 2 Mbps |
| $W$ | 10 MHz |
| $B_1$ | 0 dB |
| $B_2$ | [-10:5:30] dB |

any of the two UE sets. Hence, based on (7), (14), (24), and (25), the total outage probability, $\mathbb{O}_t^{\mathrm{m}}$, of mixed UEs in the proposed setup is written as

$$\mathbb{O}_t^{\mathrm{m}} = \mathbb{P}_{(\mathrm{ppp})}\mathbb{O}_{\mathrm{t}}^{\mathrm{ppp}} + \mathbb{P}_{(\mathrm{tcp})}\mathbb{O}_{\mathrm{t}}^{\mathrm{tcp}}. \tag{26}$$

Similarly, the rate coverage of UEs belonging to mixed UEs set, $\mathbb{R}_{\mathrm{t}}^{\mathrm{m}}$, is given as

$$\mathbb{R}_t^{\mathrm{m}} = \mathbb{P}_{(\mathrm{ppp})}\mathbb{R}_{\mathrm{t}}^{\mathrm{ppp}} + \mathbb{P}_{(\mathrm{tcp})}\mathbb{R}_{\mathrm{t}}^{\mathrm{tcp}}, \tag{27}$$

where $\mathbb{R}_t^{\mathrm{ppp}}$ and $\mathbb{R}_t^{\mathrm{tcp}}$ are given in (11) and (23), respectively.

The numerical results and discussion of the proposed model are presented in Section 5.

## 5. Numerical Results and Discussion

In this section, the performance of user-centric SBS based HCNet and the effect of biasing power are presented. The proposed model is validated by comparing the analytical results with Monte Carlo simulations given in Fig. 3. The simulations are performed for a two-tier HCNet by averaging 10000 iteration for each target SINR value. The considered simulation area is $\pi(1000)^2$. The densities of open access MBS and SBSs ($\lambda_m^{\mathrm{oa}}, \lambda_s^{\mathrm{oa}}$) are set as 1 and 100 per simulation area, respectively. The density of closed access BSs ($\lambda_m^{\mathrm{ca}}, \lambda_s^{\mathrm{ca}}$) is assumed to be equal to open access BSs in each tier. For the ease of access, other simulation parameter values are listed in Table 3, which are borrowed from the state-of-the art [10, 33, 34].

### 5.1. *Effect of Biasing on Association Probability of Clustered UEs*

Per tier association probability is compared with cluster size $\sigma$ for different values of SBS biasing power $B_2$ in Fig. 4. For $B_1 = B_2 = 0$ dB (i.e., the unbiased case), the association probability of UEs with cluster serving SBSs decreases with increase in the cluster size. However, the association probability of UEs with MBSs and SBSs located outside representative cluster increases for large cluster. This is due to the fact that in large clusters, UEs are located farther from cluster serving SBS. Hence, UEs receive more power from MBS and SBSs located outside the representative cluster. Due to low received power from cluster center SBS, fewer UEs get associated with the cluster center SBS as the cluster size increases. For a larger cluster size, the UEs receive more power form SBSs located outside the cluster and, hence, the association

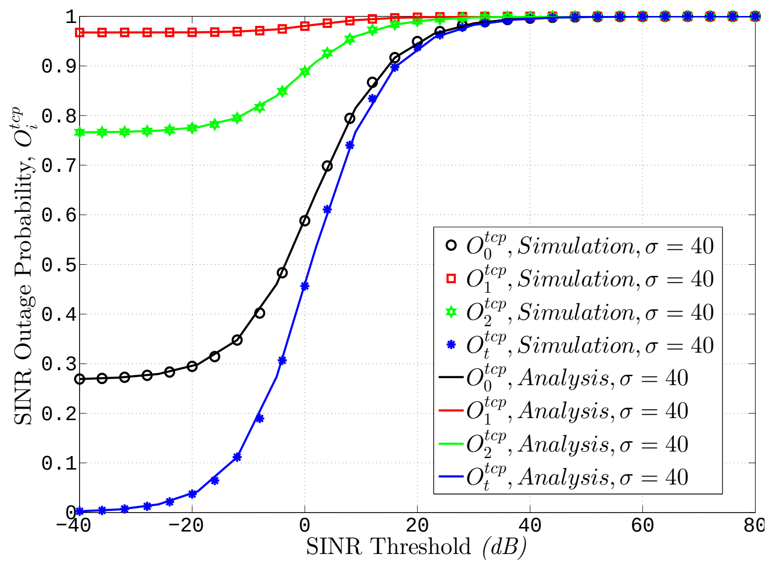


Figure 3: Per tier outage probability vs SINR threshold of clustered UE set.

with SBSs located outside the cluster increases with increase in the cluster size. The effect of $B_2$ can be interpreted for two cases.

**1: Small clusters** ($\sigma \leq 25$)**:** For smaller cluster size, the impact of SBS biasing is negligible on per tier association probability because if $\sigma$ is lower, UEs are closely located with the respective cluster serving SBS.

**2: Large clusters** ($\sigma > 25$)**:** As the cluster size increases, UEs are located farther from the respective cluster serving SBS, hence, higher transmit power is needed to offload UEs to SBSs. It can be seen in Fig. 4 that changing $B_2$ from 5 dB to 30 dB results in higher UE association with cluster serving SBS and the SBSs located outside representative cluster, while association with MBS decreases with increase in $B_2$.

### *5.2. Effect of Cluster Size and Biasing on Outage Probability*

For 0 dB SINR threshold, per tier outage probability vs SINR threshold of cluster and mixed UE based HCNet model with $\sigma = 40$ and $N_i = 10$ is given in Fig. 5. In case of clustered as well as mixed UEs, the outage probabilities of MBS and SBSs located outside the representative cluster are higher as compared to the cluster serving SBS. The outage probability increase for higher target SINR threshold. The total outage probability for mixed UE model is higher as compared to clustered UEs model.

The total outage probability versus SINR threshold plots of clustered UEs and mixed UEs for different cluster sizes are shown in Fig. 6. For comparison, we also plot outage probability of a conventional HCNet model with uniformly distributed and uncorrelated UE and BS locations. In Fig. 6 the outage probability increases with increase in cluster size for both models and converges to conventional PPP distributed UEs and BSs based HCNet model. This is due to the fact that, for a higher cluster size, i.e., $\sigma > 100$, the cluster behavior of UE locations diminishes and UEs become almost uniformly distributed throughout the network. The total outage probability of mixed distributed UEs is higher for a smaller cluster size than clustered UEs. For a higher cluster size, a small difference between the outage probability of clustered UEs and mixed UE model is observed. In Fig. 7, the per tier outage probability vs $\sigma$ of both models with fixed SINR threshold of

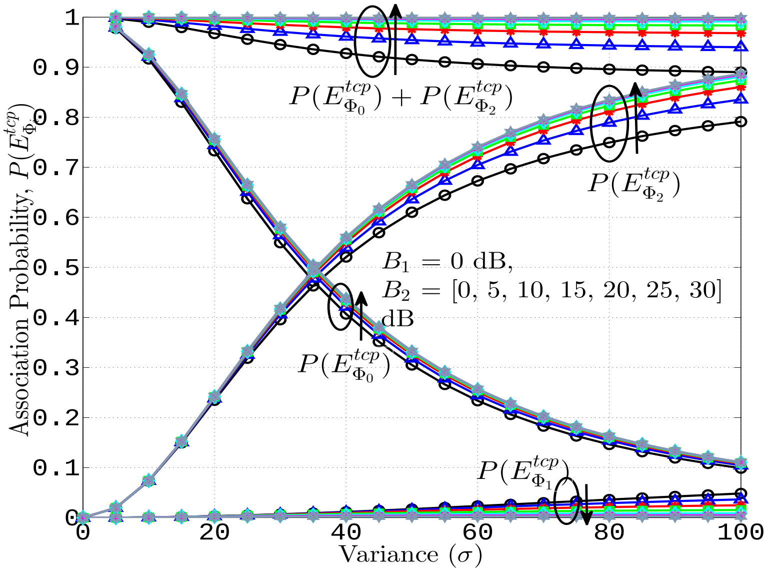


Figure 4: Association probability versus cluster size for different small cell biasing power for TCP distributed UEs.

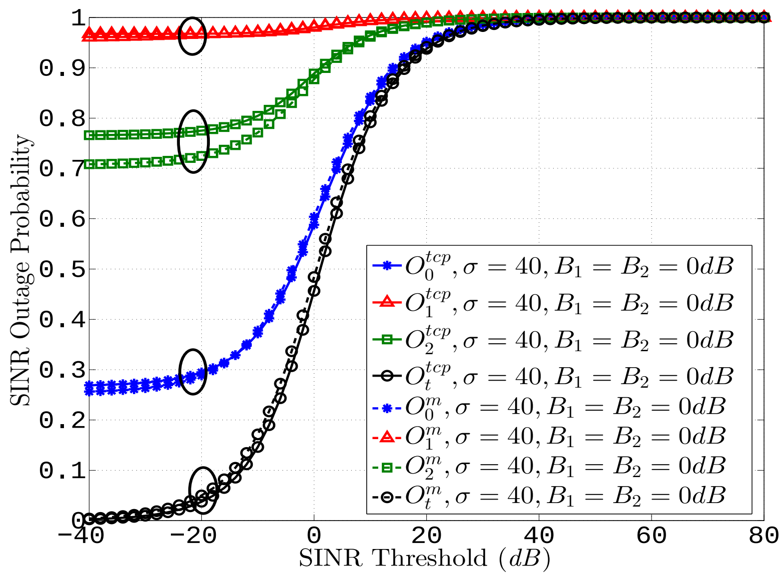


Figure 5: Per tier outage probability versus SINR threshold for TCP and mixed distributed UEs with $N_i = 10$.

0 dB is compared. The outage probability in both the cases is less than that for conventional PPP distributed UE model. For higher $\sigma > 120$, the outage probability in both the UE models becomes approximately equal to the conventional PPP based UEs model. The outage probability in case of mixed UE based model is higher than the clustered UE case because in the latter case all the UEs are located close to the cluster center SBS and, hence, receive SINR greater than the threshold. For the mixed UE case, the effect of uniformly distributed UEs located outside the cluster are also taken into account.

The impact of association biasing on the per tier outage probability of clustered UEs and mixed UEs based HCNet is given in Fig. 8 and Fig. 9, respectively. In both the models, the outage probability decreases with increase in small cell biasing $B_2$, i.e., 10 dB. $B_2$ beyond 10 dB has a negligible effect on outage probability for cluster serving SBS. The total outage probability of clustered UEs decreases with increase in $B_2$. In case of clustered UEs based model, the biasing does not affect the performance beyond a certain biasing value because the closer UEs in the network get enough coverage, and any further increase in biasing results in increased interference. In case of mixed UEs, as shown in Fig. 9, the total outage probability decreases with increase in $B_2$ and no significant decrease in outage occurs with $B_2$ beyond 10 dB because of sufficient SINR reception at UE. The random UEs outside cluster center SBS also get enough coverage with increased biasing values, therefore, the overall outage decreases with increase in biasing.

### *5.3. Effect of UE Variance and Association Biasing on Rate Coverage Probability*

The rate coverages of clustered UEs and mixed UEs (TCP along with PPP) for different value of $\sigma$ are shown in Fig. 10 and Fig. 11, respectively, by setting 2 Mbps rate threshold. Fig. 10 shows that the rate coverage of clustered UEs decreases with increase in cluster size and small difference is observed for higher cluster size and higher rate threshold. Similarly, in Fig. 11, the rate coverage of mixed UEs, also decreases with increase in cluster size. The rate coverage in case of mixed UEs is lower than clustered UEs for smaller clusters and becomes equal for higher cluster size. This is because the distribution of UEs in both mixed and clustered case converges to PPP and becomes approximately uniform. In case of a smaller cluster size, the load on the SBS is lower compared with the larger cluster because fewer UEs associate with the cluster center SBS, hence, the UEs experience higher ergodic rate. The rate in case of mixed UE based HCNet model is lower compared with the clustered UEs based HCNet because a portion of uniformly distributed UEs also connects with SBSs, which results in increased cell load.

The rate coverage for different values of $B_2$ for clustered and mixed UE model is plotted in Fig. 12 and Fig. 13, respectively. The rate coverage increases with increase in $B_2$ keeping $B_1 = 0$ dB. Increasing $B_2$ beyond 10 dB has no impact on rate coverage in case of clustered UEs. Similarly, from Fig. 13 it can be observed that the increase in $B_2$ results in improved rate coverage even for $B_2 = 20$ dB. The significant improvement in case of mixed UEs is observed because the rate coverage of random UEs is also taken into account, which is located randomly outside clusters.

The upper and lower bounds on outage probability versus $\sigma$ for mixed UE based HCNet are plotted in Fig. 14. It can be observed from Fig. 14 that the exact outage probability derived is closer to the lower bound of the outage probability. The bounds exist in case if the interference from the cluster center SBS is ignored or SBS is placed at the boundary of the cluster. The upper and lower bounds are tighter for a smaller cluster size compared with larger clusters. This is because for smaller clusters, the probability that UEs connect to cluster center SBSs is higher and, hence, the interference from the cluster center SBS diminishes. For a lager cluster size, the probability that UEs connect to the cluster center SBS decreases and, hence, the effect of interference from this SBS is taken into account, which results in loose upper bound on the outage probability.

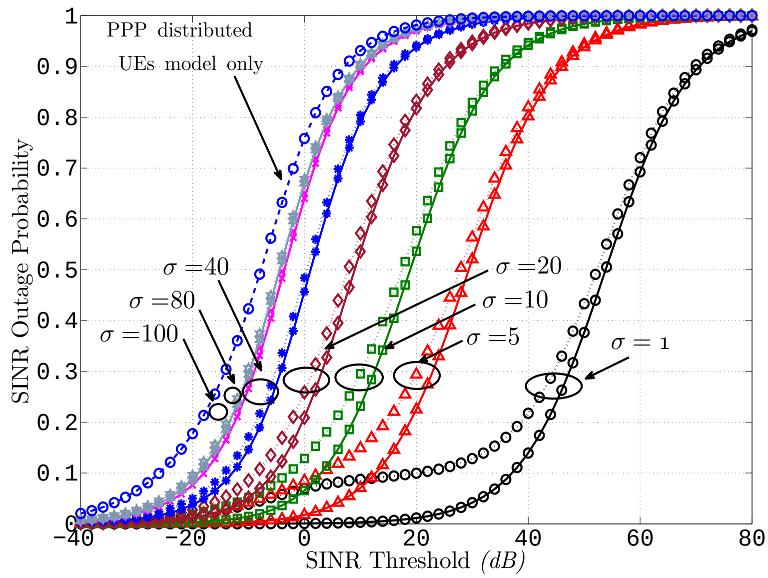


Figure 6: Outage probability versus SINR threshold for TCP distributed UEs and mixed UEs HCNets model.

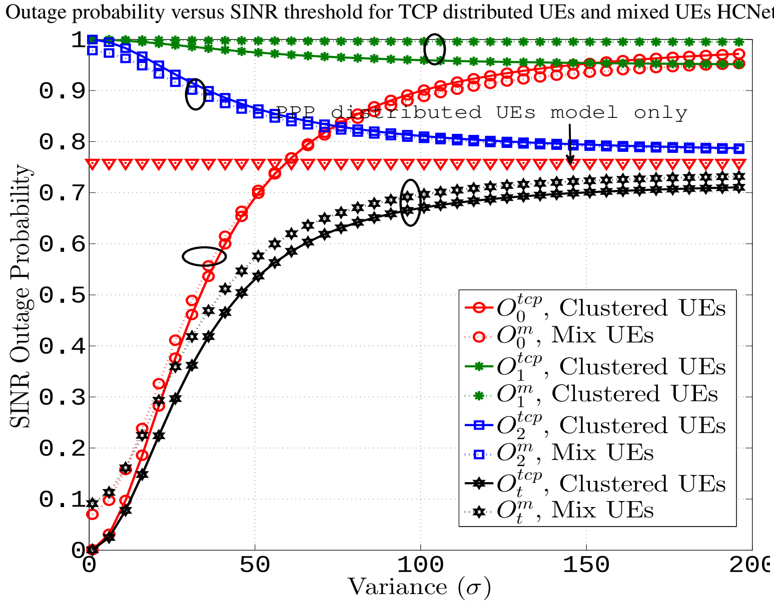


Figure 7: Per tier outage probability versus cluster size for different cluster size values in TCP distributed UEs.

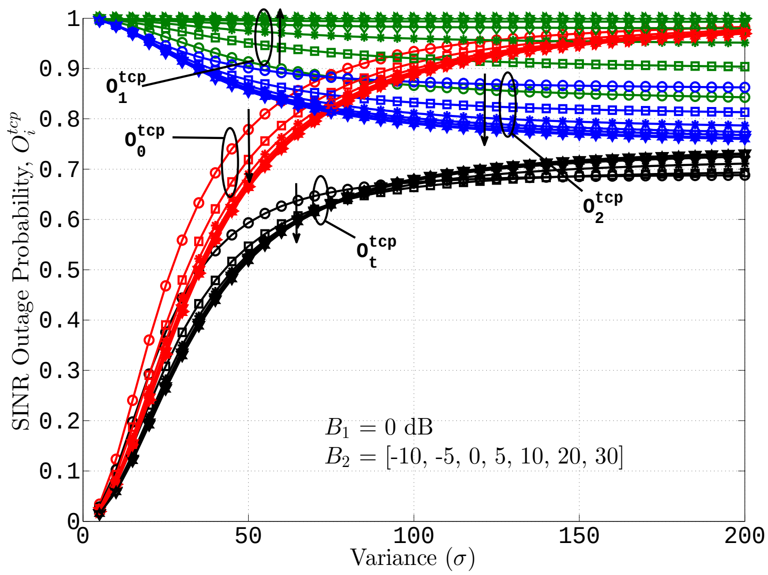


Figure 8: Per tier outage probability versus cluster size for different value of biasing power in TCP distributed UEs.

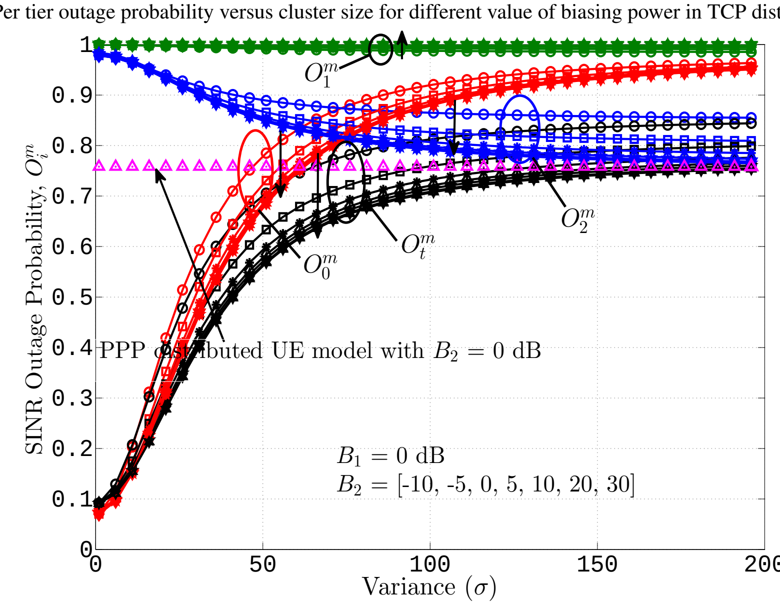


Figure 9: Per tier outage probability versus cluster size for different value of biasing power in mixed (TCP plus PPP) distributed UEs.

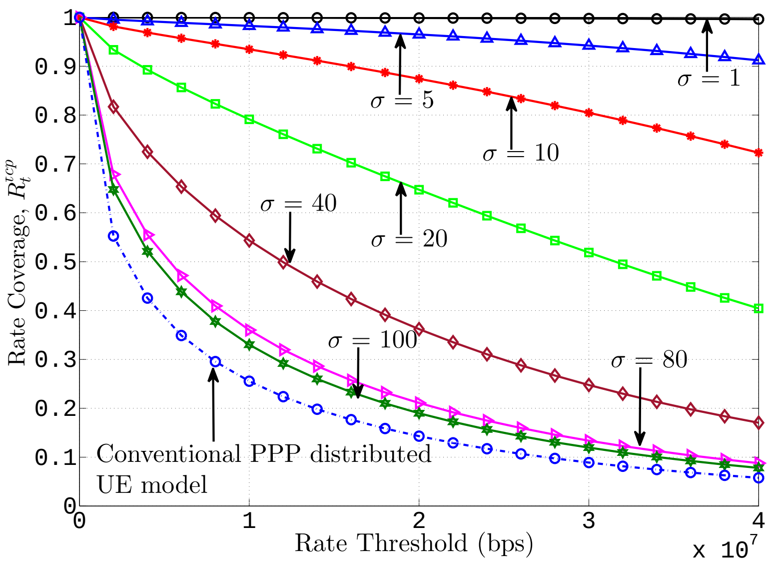


Figure 10: Rate coverage versus SINR threshold for various cluster size $\sigma$ without biasing of clustered UEs based HCNet.

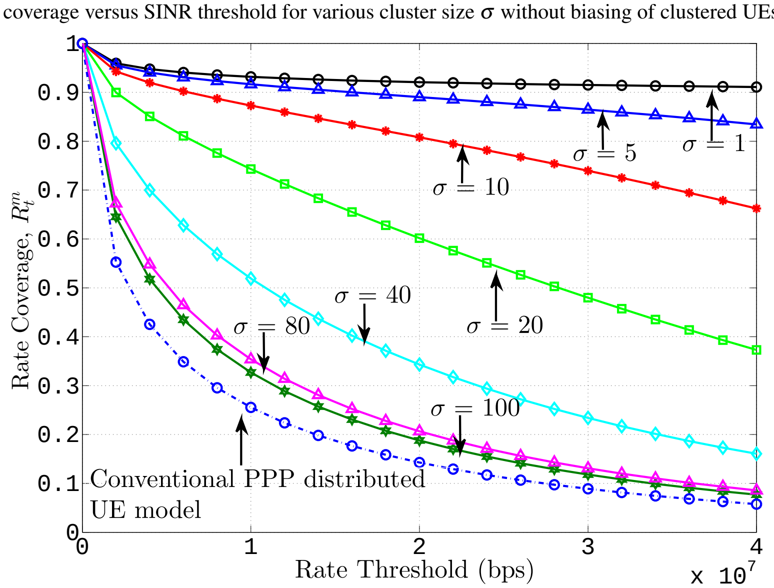


Figure 11: Rate coverage versus SINR threshold for various cluster size $\sigma$ without biasing in mixed UEs based HCNet.

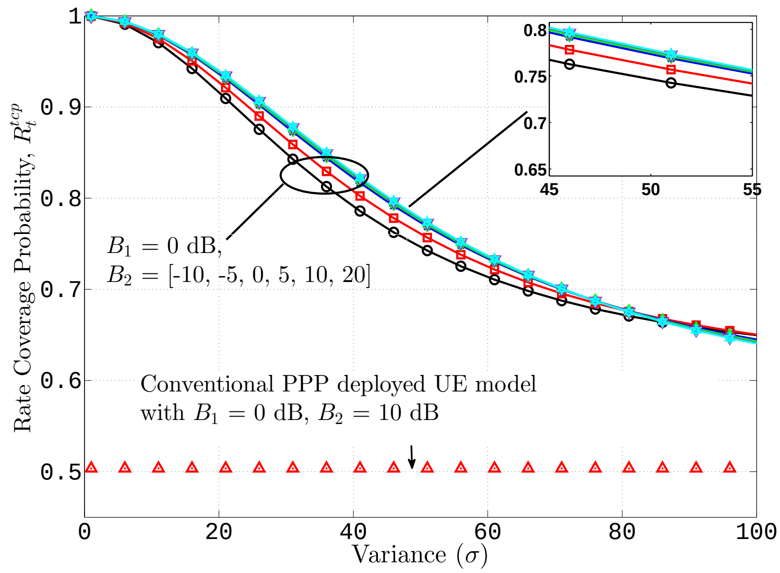


Figure 12: Rate coverage versus SINR threshold for various cluster size $\sigma$ with biasing in clustered UEs based HCNet.

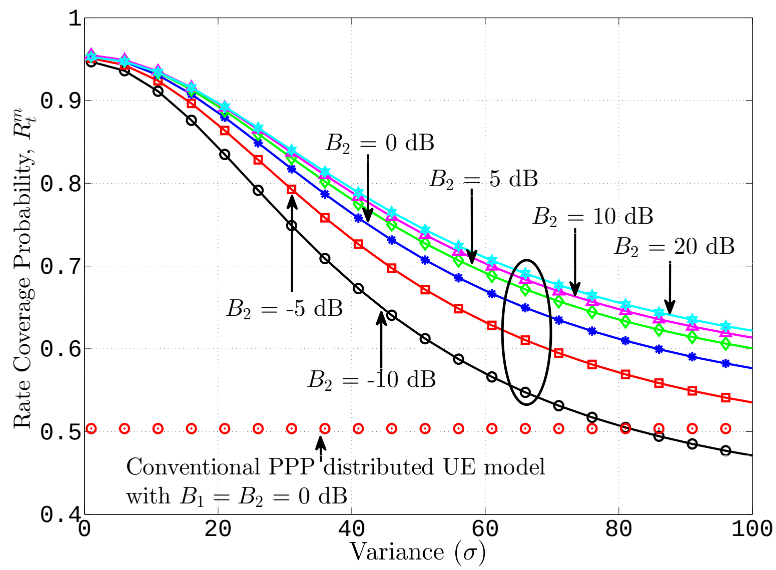


Figure 13: Rate coverage versus SINR threshold for various cluster size $\sigma$ with biasing in mixed UEs based HCNet.

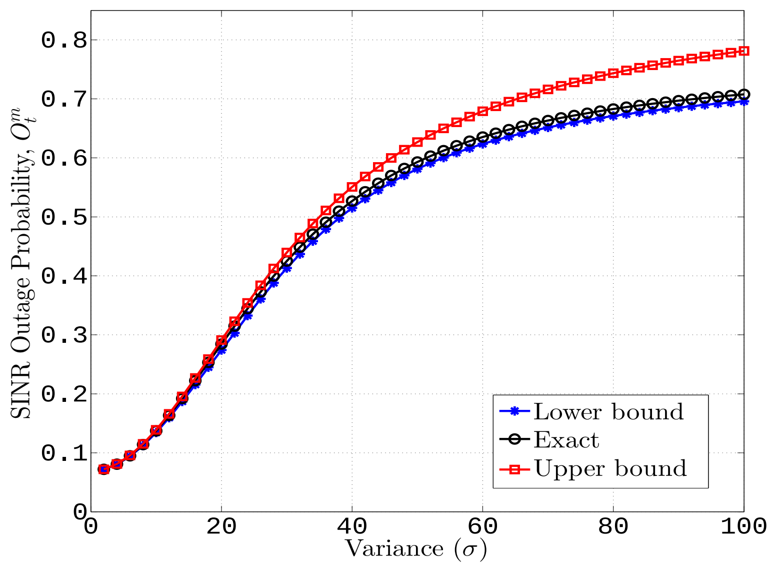


Figure 14: Bounded outage probability versus variance for mixed UE based HCNet.

### *5.4. Critical Discussion*

The performance analysis of the user-centric SBS based HCNet with mixed UEs, presented in the previous subsections, captures the realistic picture of UEs' existence in the network. Due to the presence of crowded UEs at different locations in the network, the user-centric SBS deployment improves the performance of HCNets significantly. The association bias or CRE of SBSs further boosts the performance of HCNets. In case of mixed UE based HCNets, the effect of association bias is higher compared to the clustered UEs. This is because in mixed UE based user-centric HCNet, the performance of UEs located outside the cluster is also taken into account. The cluster size has shown significant impact on the performance of HCNets. Due to the small transmission distance and higher association of UEs with cluster center SBSs, the outage probability is lower if the SBSs are deployed for smaller clusters compared to the larger clusters. For larger clusters, the performance of user-centric SBS based HCNets becomes nearly equal to the conventional uniform UE and SBS deployment in HCNets. The performance improvement is due to the captured correlation between UEs and BSs, which is not assumed in the case of conventional HCNets.

In the case of mixed UE based HCNet model, the overall UEs in the network is the superposition of clustered and uniform UE sets. A typical UE is randomly selected from two sets using selection probability. The uniform UE set is analyzed based on the conventional HCNet approach to ensure tractability.

The user centric SBS deployment has improved network performance compared with the conventional HCNet. The numerical results provide insights into the appropriate deployment of SBSs in HCNets. Keeping in view the energy consumption and performance of HCNets, the SBSs need to be deployed in crowded areas to provide better coverage to UEs in the network along with minimal contribution to overall network energy consumption.

The performance of user-centric SBS deployment for mixed and clustered UEs is evaluated based on two basic assumptions, which need to be addressed in the future extension of this work. The first assumption is that the cluster size of all the clusters in the network is assumed to be the same with fixed number of UEs per cluster. The second is that the SBSs are assumed to be deployed at the center of a cluster. For operators, the deployment of SBS at cluster center may not be practical in every case. The deployment of SBS at random distance away from the cluster center needs to be analyzed.

## 6. Conclusion

In this paper, we have analyzed the user-centric SBS deployment with nonuniform distribution of UEs including clustered and uniformly deployed UEs in two tier HCNets, which reflects a more realistic UE distribution scenario. The SBSs are placed according to user-centric deployment throughout the network. We have evaluated the outage probability and rate coverage of the proposed HCNet model. Our results show that deploying the SBSs at UE hotspots significantly improves the network performance compared with the conventional PPP based UE model due to correlation between UEs and SBSs. We also observed the effect of SBS association biasing on the outage probability and rate coverage. The SBS association biasing contributes to improved network performance upto a certain extent and any further increase in the biasing power does not affect the network performance. At the end, it is concluded that the user-centric SBS deployment significantly improves network performance with SBS association biasing.

## Appendices

### Appendix A. Proof of Lemma 1

Association probability of the clustered UEs can be written as

$$\begin{aligned}
\mathbb{P}(\mathbb{E}_{\Phi_i}^{\text{tcp}}) &= \mathbb{E}_{X_i}\Bigg\{\bigcap_{i\in\mathcal{K}} 1\Bigg(X_i > \left(\frac{B_jP_j}{B_iP_i}\right)^{1/\alpha_j} X_j\Bigg)\Bigg\} \\
&= \mathbb{E}_{X_i}\Bigg\{\prod_{\substack{j\in\mathcal{K}\\ j\neq i}} \mathbb{P}\Bigg(X_i > \left(\frac{B_jP_j}{B_iP_i}\right)^{1/\alpha_j} X_j\Bigg)\Bigg\} \\
&= \int_0^\infty \prod_{\substack{j\in\mathcal{K}\\ j\neq i}} F_{X_j}\Bigg(\left(\frac{B_jP_j}{B_iP_i}\right)^{1/\alpha_j} X_j\Bigg) f_{X_i}(z_i)dx_i.
\end{aligned}$$

Similarly, we can write the association probability of the typical UE with its cluster center SBS and with other BSs in the $i$th tier by setting $i = 0$ and $i \in K$, respectively, as given by (A.1) and (A.2), respectively, below.

*For* $0$*th tier, i.e.,* $(i = 0)$ :

$$\mathbb{P}(\mathbb{E}_{\Phi_0}^{\text{tcp}}) = \int_0^\infty \prod_{\substack{j\in K\\ j\neq i}} F_{X_0}\Bigg(\left(\frac{B_jP_j}{B_0P_0}\right)^{1/\alpha_j} X_0\Bigg) f_{X_0}(x_0)dx_0. \tag{A.1}$$

*For ith tier where $i \in \mathcal{K}$ and $i \neq 0$:*

$$\mathbb{P}\Big(\mathbb{E}^{\text{tcp}}_{\Phi_i}) = \mathbb{E}_{X_i}\Big\{\prod_{\substack{j\in\mathcal{K}\\ j\neq i}} \mathbb{P}(X_i > \Big(\frac{B_jP_j}{B_iP_i}\Big)^{1/\alpha_i} X_i\Big)\Big\}$$

$$= \mathbb{E}_{X_i}\Big\{\mathbb{P}\Big(X_i > \Big(\frac{B_jP_j}{B\ \ _0P_0}\Big)^{1/\alpha_i} X_j\Big) \prod_{\substack{j\in\mathcal{K}\\ j\neq i}} \mathbb{P}\Big(X_j > \Big(\frac{B_jP_j}{B_iP_i}\Big)^{1/\alpha_i} X_j\Big)\Big\}$$

$$= \int_0^\infty F_{X_0}\Big(\Big(\frac{B_jP_j}{B_0P_0}\Big)^{1/\alpha_i} X_i\Big) \prod_{\substack{j\in\mathcal{K}\\ j\neq i}} F_{X_i}\Big(\Big(\frac{B_jP_j}{B_iP_i}\Big)^{1/\alpha_i} X_i\Big) f_{X_i}(x_i)dx_i. \tag{A.2}$$

Substituting the CCDF and PDF using (1b), (2b), (1a), and (2a) into (A.1) and (A.2) completes the proof of Lemma 1).

## Appendix B. Proof of Lemma 2

The PDF of serving distances from $i$th tier BS is given by

$$f^{\text{tcp}}_{X_i}(x_i) = \frac{d}{dx}\Big\{\mathbb{P}\Big\{X_i > x|\mathbb{E}^{\text{tcp}}_{\Phi_i}\Big\}\Big\} = \frac{d}{dx}\Bigg\{\frac{\mathbb{P}\Big(X_i > \Big(\frac{B_jP_j}{B_iP_i}\Big)^{1/\alpha_j} X_j\Big)}{\mathbb{P}\big(\mathbb{E}^{\text{tcp}}_{\Phi_i}\big)}\Bigg\} \tag{B.1}$$

$$= \frac{1}{\mathbb{P}\big(\mathbb{E}^{\text{tcp}}_{\Phi_i}\big)} \frac{d}{dx}\Big\{\int_x^\infty \prod_{\substack{j\in\mathcal{K}\\ j\neq i}} F_{X_i}\Big(\Big(\frac{B_jP_j}{B_iP_i}\Big)^{1/\alpha_i} x_j\Big) f_{X_i}(x_i)dx_i\Big\}$$

$$= \frac{1}{\mathbb{P}\big(\mathbb{E}^{\text{tcp}}_{\Phi_i}\big)} \prod_{\substack{j\in\mathcal{K}\\ j\neq i}} F_{X_i}\Big(\Big(\frac{B_jP_j}{B_iP_i}\Big)^{1/\alpha_i} x_i\Big) f_{X_i}(x_i). \tag{B.2}$$

Similarly, the PDF of serving distances for $i = 0$ and $i \in K$ is given by (B.3) and (B.4), respectively.
*For 0th tier i.e., $(i = 0)$ :*

$$f_{X_0^{\text{tcp}}}(x_0) = \frac{1}{\mathbb{P}\big(\mathbb{E}^{\text{tcp}}_{\Phi_0}\big)} \prod_{\substack{j\in\mathcal{K}\\ j\neq i}} F_{X_o}\Big(\Big(\frac{B_jP_j}{B_0P_0}\Big)^{1/\alpha_i} x_0\Big) f_{X_0}(x_0). \tag{B.3}$$

*For ith tier where $i \in \mathcal{K}$ and $i \neq 0$:*

$$f^{\text{tcp}}_{X_i}(x_i) = \frac{1}{\mathbb{P}\big(\mathbb{E}^{\text{tcp}}_{\Phi_0}\big)} F_{X_0}\Big(\Big(\frac{B_jP_j}{B_0P_0}\Big)^{1/\alpha_i} x_0\Big) \prod_{\substack{j\in K\\ j\neq i}} F_{X_i}\Big(\Big(\frac{B_jP_j}{B_iP_i}\Big)^{1/\alpha_i} x_i\Big) f_{X_i}(x_i). \tag{B.4}$$

Substituting (1b), (2b), (1a), and (2a) into (B.3) and (B.4) completes the proof of Lemma 2.

## Appendix C. Proof of Theorem 1

The outage probability of clustered UEs can be written as

$$\mathbb{O}_i^{\text{tcp}} = 1 - \int_0^{\infty} \Big\{ P\Big( \text{SINR}\big(x_i\big) > \Omega_i \Big) \Big\} f_{X_i}^{\text{tcp}}(z_i) dz_i, \tag{C.1}$$

where

$$\begin{aligned} \mathbb{P}\big(\text{SINR}(x_i) > \Omega_i\big) &= \mathbb{P}\left( \frac{P_i g_i x_i^{-\alpha_i}}{\mathcal{N}_t + I_t} > \Omega_i \right) \\ &= \mathbb{P}\left( g_i > \frac{\Omega_i \mathcal{N}_t x_i^{\alpha_i}}{P_i} \left( \sum_{\substack{i \in \mathcal{K} \\ j \in \Phi_i}} \mathcal{I}_{(i,j)}^{\text{oa}} + \sum_{\substack{i \in \mathcal{K} \\ j \in \Phi_i}} \mathcal{I}_{(i,j)}^{\text{ca}} \right) \right) \\ &= \exp\left( - \frac{\Omega_i \mathcal{N}_t x_i^{\alpha_i}}{P_i} \right) \prod_{\substack{j \in \mathcal{K} \\ j \in \Phi_i}} \mathcal{L}_{\mathcal{I}_{(i,j)}^{\text{oa}}} \left( \frac{\Omega_i x_i^{\alpha_i}}{P_i} \right) \prod_{\substack{j \in \mathcal{K} \\ j \in \Phi_i}} \mathcal{L}_{\mathcal{I}_{(i,j)}^{\text{ca}}} \left( \frac{\Omega_i x_i^{\alpha_i}}{P_i} \right), \end{aligned} \tag{C.2}$$

where $\mathcal{L}_{\mathcal{I}_{(i,j)}^{\text{oa}}}\big(.\big)$ and $\mathcal{L}_{\mathcal{I}_{(i,j)}^{\text{ca}}}\big(.\big)$ represent the Laplace transforms of interference from open access and closed access BSs, respectively. The Laplace transforms of interference from open access BSs can be written as

$$\begin{aligned} \mathcal{L}_{\mathcal{I}_{(i,j)}^{\text{oa}}} \left( \frac{\Omega_i x_i^{\alpha_i}}{P_i} \right) &= \mathbb{E}_{\mathcal{I}_{(i,j)}^{\text{oa}}} \left[ \exp\left\{ - \frac{\Omega_i x_i^{\alpha_i} \mathcal{I}_{(i,j)}^{oa}}{P_i} \right\} \right] = \mathbb{E}_{\Phi_i} \left[ \exp\left\{ - \frac{\Omega_i x_i^{\alpha_i}}{P_i} \sum_{i \in \mathcal{K}} P_j g_i ||x_i||^{-\alpha_i} \right\} \right] \\ &= \mathbb{E}_{\Phi_i} \left[ \prod_{x_i \in \Phi_i} \mathbb{E}_{g_i} \left\{ \exp\left( - \frac{\Omega_i x_i^{\alpha_i}}{P_i} P_j g_j ||x_j||^{-\alpha_i} \right) \right\} \right], \\ &\overset{(a)}{=} \mathbb{E}_{\Phi_i} \left[ \prod_{x_i \in \Phi_i} \frac{1}{1 + \frac{\Omega_i x_i^{\alpha_i}}{P_i} P_i ||x_i||^{-\alpha_i}} \right] \\ &\overset{(b)}{=} \exp\left\{ -2\pi\lambda_i \int_{(\frac{P_j}{P_i})^{1/\alpha_i} x_i^{-\alpha_i}}^{\infty} \frac{x_i}{1 + \frac{\Omega_i^{-1} x_i^{-\alpha_i}}{P_j} P_i ||x_i||^{-\alpha_i}} d_{x_i} \right\} \\ &\overset{(c)}{=} \exp\left\{ -\pi\lambda_i^{oa} \left( \frac{P_j}{P_i} \right)^{2/\alpha_i} \mathbb{Z}(\Omega_i, \alpha_i, \mathcal{B}_i) x_i^2 \right\}, \end{aligned} \tag{C.3}$$

where Step (a) follows Rayleigh fading assumption of channel gain and independence of PPP. Step (b) is obtained by using probability generating functional of PPP. Step (c) is obtained by using the same procedure of employing change in variable and integrating over the limits, as followed in the proof of Theorem 1 in [10].

As the Laplace transform of interference from closed access BSs is independent of $x_i$, it can simply be obtained by making the lower limit of the integral equal to zero in the case of open access BSs, and is given as

$$\mathcal{L}_{\mathcal{I}_{(i,j)}^{\text{ca}}} \left( \frac{\Omega_i x_i^{\alpha_i}}{P_i} \right) = \exp\left\{ -\pi\lambda_i^{\text{ca}} \mathbb{Q}(\Omega_i, \alpha_i) \left( \frac{P_j}{P_i} \right)^{2/\alpha_i} x_i^2 \right\}. \tag{C.4}$$

Using (C.1) and (C.2), the outage probability of the clustered UEs, given that the UEs associate with the BS belonging to $i$th tier, can be written as (C.5) and (C.6), respectively.
*For* 0*th tier i.e.,* $(i=0)$ :

$$\mathbb{O}_0^{\text{tcp}} = 1 - \int_0^{\infty} \exp\left(\frac{\Omega_i \mathcal{N}_t x_0^{\alpha_i}}{P_0}\right) \prod_{j\in\mathcal{K}} \mathcal{L}_{\mathcal{I}^{oa}_{(i,j)}}\left(\frac{\Omega_i x_0^{\alpha_i}}{P_0}\right) \prod_{j\in\mathcal{B}} \mathcal{L}_{\mathcal{I}^{ca}_{(i,j)}}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_0}\right) f_{X_0}^{\text{tcp}}(z_0) dz_0. \tag{C.5}$$

*For ith tier where* $i\in\mathcal{K}$ *and* $i\neq 0$*:*

Similarly, per tier outage probability, when the UE is connected to BS other than the SBS located at the center of representative cluster, can be written as

$$\mathbb{O}_i^{\text{tcp}} = 1 - \int_0^{\infty} \exp\left(\frac{\Omega_i \mathcal{N}_t x_i^{\alpha_i}}{P_i}\right) \mathcal{I}^{oa}_{(i,0)}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right) \prod_{j\in\mathcal{K}} \mathcal{L}_{\mathcal{I}^{\text{oa}}_{(i,j)}}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right) \prod_{j\in\mathcal{K}} \mathcal{L}_{\mathcal{I}^{ca}_{(i,j)}}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right) f_{X_i}(x_i) dx_i, \tag{C.6}$$

where $\mathcal{I}^{oa}_{(i,0)}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right)$ is the interference contributed by the BS located at the center of the representative cluster.

If the serving BS of the UE is other than the cluster center SBS and is located at distance $X_i$, then the cluster center BS also contributes to the total interference and, hence, the conditional PDF of the distance from the cluster center SBS can be written as

$$f_{X_0}\left(X_0|x_0 > \left(\frac{B_i P_i}{B_j P_j}\right)^{1/\alpha_i} x_i\right) = \frac{f_{X_0}(x_0)}{F_{X_0}\left(\left(\frac{B_i P_i}{B_j P_j}\right)^{1/\alpha_i} x_i\right)}. \tag{C.7}$$

The Laplace transform of interference from the cluster center SBS can be written as

$$\begin{aligned}
\mathcal{L}_{\mathcal{I}^{\text{oa}}_{(i,0)}}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right) &= \mathbb{E}_{X_0}\left[\mathbb{E}_{g_i}\left\{\exp\left(-\frac{\Omega_i x_0^{\alpha_i}}{P_i} P_j h_i ||x_0||^{-\alpha_i}\right)\right\}|X_0 > \left(\frac{P_i}{P_j}\right)^{1/\alpha_i} x_i\right] \\
&= \mathbb{E}_{X_0}\left[\frac{1}{1+\frac{\Omega_i x_i^{\alpha_i}}{P_i} P_i ||x_0||^{-\alpha_i}} |X_0 > \left(\frac{P_i}{P_j}\right)^{1/\alpha_i} x_i\right] \\
&= \int_l^{\infty} \frac{1}{1+\frac{\Omega_i x_i^{\alpha_i}}{P_i} P_i ||x_0||^{-\alpha_i}} f_{X_0}\left(x_0|X_0 > \left(\frac{B_i P_i}{B_j P_j}\right)^{1/\alpha_i} x_i\right) d_{x_0},
\end{aligned} \tag{C.8}$$

where $l = \left(\frac{B_j P_j}{B_i P_i}\right)^{1/\alpha_i} x_i^{-\alpha_i}$.

Substituting (C.3), (C.4) and (C.8) into (C.5) and (C.6) completes the proof of Theorem 1.

## Appendix D. Proof of (18) and (19)

The outage probability bounds can be derived based on the bounded interference from cluster center SBS. The Laplace transform of interference from cluster center SBS is given as

$$\mathcal{L}_{\mathcal{I}^{\text{oa}}_{(i,0)}}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right) = \mathbb{E}_{X_0}\left[\frac{1}{1+\frac{\Omega_i x_i^{\alpha_i}}{P_i}P_i||x_0||^{-\alpha_i}}|X_0 > \left(\frac{P_i}{P_j}\right)^{1/\alpha_i} x_i\right].$$

The typical UE experiences maximum interference from the cluster center SBS if the latter is located at the same distance as the serving BS from the typical UE, i.e., $x_0 = \left(\frac{P_i}{P_j}\right)^{1/\alpha_i} x_i$. Similarly, if the cluster center SBS is located farther away from the typical UE, i.e., $x_0 = \infty$, then no interference is incorporated by cluster center SBS. The interference from cluster center SBS can be upper bounded as

$$\mathcal{L}^{\text{U}}_{\mathcal{I}^{\text{oa}}_{(i,0)}}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right) \leq \left.\frac{1}{1+\frac{\Omega_i x_i^{\alpha_i}}{P_i}P_i||x_0||^{-\alpha_i}}\right|_{x_0=\left(\frac{P_i}{P_j}\right)^{1/\alpha_i} x_i} = \frac{1}{1+\Omega_i}. \tag{D.1}$$

The interference from cluster center SBS can be lower bounded as

$$\mathcal{L}^{\text{L}}_{\mathcal{I}^{\text{oa}}_{(i,0)}}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right) \geq \left.\frac{1}{1+\frac{\Omega_i x_i^{\alpha_i}}{P_i}P_i||x_0||^{-\alpha_i}}\right|_{x_0=\infty} = 1. \tag{D.2}$$

Substituting (D.1) and (D.2) in place of $\mathcal{L}_{\mathcal{I}^{\text{oa}}_{(i,0)}}\left(\frac{\Omega_i x_i^{\alpha_i}}{P_i}\right)$ in (C.6) completes the proof of (18) and (19), respectively.

## Appendix E. Proof of Theorem 2

The per tier rate coverage, defined in Section 4, for clustered UEs can be expressed as

$$\mathbb{R}_i^{\text{PCP}} = \mathbb{P}\left\{\text{SINR}(x_i) > 2^{\psi_i/B} - 1\right\}.$$

The rate coverage can be derived by replacing $\Omega_i = \Psi_i(\psi_i, W) = 2^{\psi_i/B} - 1$ and following the similar steps as followed in the proof Theorem 1 . Hence, we need to consider 0th tier as a separate tier consisting of single cluster center SBS.

*For* 0*th tier i.e.,* $(i = 0)$ :

When the typical UE connects with cluster center SBS, rate coverage of the typical UE can be written as

$$\mathbb{R}_0^{\text{tcp}} = \frac{1}{\mathbb{P}(\mathbb{E}_0^{\text{tcp}})}\int_{x_i>0} x_i \exp\Bigg\{-\frac{\Psi(\psi_i, W)x_i^{\alpha_i}}{P_0} - \pi\sum_{j\in\mathcal{K}}\left(\frac{B_j P_j}{B_0 P_0}\right)^{2/\alpha_i}\Bigg(\lambda_j^{\text{oa}}\Bigg[1+$$
$$\mathbb{Z}\Big(\Psi(\psi_i, W, \mathcal{B}_j), \alpha_i\Big)\Bigg] + \lambda_j^{\text{ca}}\mathbb{Q}\Big(\Psi(\psi_i, W), \alpha_i\Big)\Bigg)x_i^2\Bigg\}dx_i. \tag{E.1}$$

*For ith tier where $i \in \mathcal{K}$ and $i \neq 0$:*
Similar to Theorem 1, the rate coverage of $i$th tier BS, other than the cluster center SBS, is given by

$$\mathbb{R}_i^{\text{tcp}} = \frac{2\pi\lambda_i}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})} \int_{z_i>0} x_i \exp\Bigg\{ -\frac{\Psi(\psi_i, W)x_i^{\alpha_i}}{P_i} - \pi \sum_{j\in\mathcal{B}} \left(\frac{B_j P_j}{B_i P_i}\right)^{2/\alpha_i} \Bigg(\lambda_j^{oa}\Bigg[1+ \mathbb{Z}\Bigg(\Psi\big(\psi_i, W, \mathcal{B}_j\big), \alpha_i\Bigg)\Bigg] + \lambda_j^{ca}\mathbb{Q}\Bigg(\Psi\big(\psi_i, W\big), \alpha_i\Bigg)\Bigg) x_i^2\Bigg\} \mathcal{L}_{\mathcal{I}_{oa(i,0)}}\left(\frac{\Psi(\psi_i, W)x_i^{\alpha}}{P_i}\right) dx_i. \tag{E.2}$$

Following the same procedure as used in Theorem 1, and after simplification, we obtain (23).

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