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Investigating the affordances of a flipped mathematics classroom from an activity theoretical perspective

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Flipped Classroom as a pedagogical framework has gained popularity at secondary and tertiary levels of mathematics education, but there is a lack of research based on a solid theoretical foundation. This article considers the flipped mathematics classroom from the perspective of affordances and cultural–historical activity theory. The empirical background is based on semi-structured interview data from eight first-year computer-engineering students following 1 year of flipped classroom teaching. The thematic analysis of the data indicates that the flipped format offers a range of affordances at various levels of the activity system. This article advances research on affordances for mathematical learning in a flipped classroom pedagogical frame, presenting operational affordances out-of-class, action affordances at the mathematical task level and finally activity affordances at the collective level.

1. Introduction—the flipped classroom approach

Recently, traditional lecture-based undergraduate teaching has been challenged by various learner-centred initiatives that focus on students' autonomy and community of learning through inquiry (Love *et al.*, 2014). These approaches emphasize students' participation in problem-solving and discussion-based learning activities, as opposed to teacher-centred transmission of skills (Hamdan *et al.*, 2013). As such, students' collaboration becomes an important arena for learning, where the teacher is more a facilitator and active listener to students' ideas than a lecturer (Stephan, 2014). These initiatives are partly driven by the recognition that such inquiry-oriented instruction often leads to improvement on many scales (Rasmussen and Wawro, 2017).

The pedagogical framework Flipped Classrooms (FC) is a type of teaching in-class that spends the majority of the time on students' own problematizing of the topic at hand. This is achieved by moving the lecturing part of the instruction out-of-class through the use of instructional videos. Although in-class activities may take many forms, the majority of implementations in mathematics utilize collaborative group work for various types of problem-solving (Lo *et al.*, 2017). As such, FC can be placed under the umbrella of student-centred frameworks, where the bulk of direct instruction is performed in the preparatory phase through videos. The idea is to be able to attend in-class sessions with a basic understanding of concepts that are explored at greater depth facilitated and supported by the instructor (Bergmann and Sams, 2012). Although there have been a considerable number of studies on FC recently, the field is still considered under-researched and under-theorized (Muir and Geiger, 2016). Thus, the aim of this study is to contribute to research on the characteristics of mathematics learning and teaching in a FC environment. The empirical background for the article is a class of first-year Norwegian students in computer engineering being subject to FC teaching in mathematics throughout a whole year of studies.

The article is structured as follows: Firstly, I introduce the theoretical framing of the research, where the concept of affordances and constraints is considered from the perspective of activity theory. Then, I consider the context of the study, and the method for analysing the data. The results of analysing the interview data are then presented. Finally, I discuss the results in light of the theoretical framework.

2. Theoretical background

This section will provide a brief overview of the concept of affordance and how it can be connected to Leontjev's activity theory. Furthermore, a literature review is performed, outlining major contribution to research on affordances in connection to the digital tools, mathematics education and flipped classroom. Finally, I introduce the research question for this article.

Gibson (1977) introduced the term 'affordance' to capture the relationship between organisms, in this case human beings, and the environment. He defined affordances as 'what it offers the animal, what it provides or furnishes, whether for good or ill' (ibid, p. 67). Through breaking the dichotomy of the subjective and objective characterizing the separation between the organism and the environment, he greatly contributed to the development of an ecological psychology emerging as an alternative to the dominant behaviourist thinking at the time (Bærentsen and Trettvik, 2002). According to Gibson, affordances exist independently of the observer, but they need to be perceived to be realized. Moreover, affordances refer to *action possibilities*, that is, what the observer can do with the object. These action possibilities relate to the capabilities of the actor. For example, a knee-height horizontal surface affords sitting for a human, but not for a most other animals which cannot sit. Norman (1988) developed the term further to include the notion of cultural conventions to make the term more nuanced. For example, if a human from pre-historic times encountered a bottle, she would probably not perceive the affordance of drinking from it, since she did not participate in a society where this behaviour was culturally embedded. Norman was also interested in the *perception* of the actor (or user), rather than the invariant properties of the object or tool. He argued that goals, culture and past experience greatly influence the perception of affordances. Analysing affordances also involves the identification of constraints (Norman, 1999). Constraints will restrict possible interactions with the environment. Similar to affordances, these can either be inherent properties or imposed deliberately if the object is designed to avoid certain undesired interactions by the user.

Returning to the ecological origins of Gibson, the intention of the term was to describe how affordances emerge in perception from the relationship between observer, object and environment. However, Gibson's view of affordances focuses on operational/functional aspects of the environment, without including the important impact of the socio-cultural context. As such, a new direction in the view of affordances has developed recently, one which attempts to merge cultural–historical activity theory with the notion of affordances. In Pedersen and Bang (2016) and Bærentsen and Trettvik (2002), affordances are considered from the perspective of Leontjev's activity theory, which is a three-layer hierarchical model consisting of operations, actions and activity. Leontjev (1977) attempted in this model to explain human behaviour in a wider perspective of collaborative activity. This model is depicted in Fig. 1, consisting of individuals' *actions* (at a conscious level) and *operations* (at a more subconscious

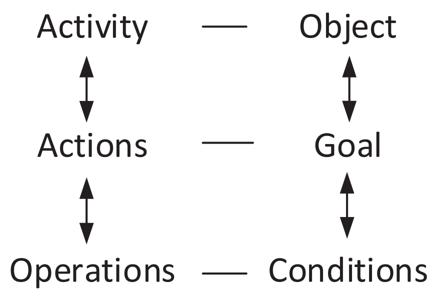

FIG. 1 Leontjev's activity model (Koschmann *et al.*, 1998).

level) organized in an *activity* at the social level. Furthermore, the Activity is motivated by an object of the community, while actions are individually goal driven. The operations that a certain action consists of are shaped by various conditions in the environment. Object, goal and conditions are interrelated equivalent to 'what I want' (goal) is related to 'what is needed' (object) and 'how to get it' (operations).

I have chosen to operationalize this model in terms of the affordances of a FC as follows:

- *Activity* considers affordances at the collective level, perceived by actors as the common object to learn mathematics. These emerge through norms and collaboration.
- This object is achieved through solving mathematical tasks at the *action* level. The goal for these actions is the joint understanding of how the tasks may adequately be solved, pointing towards affordances emerging at the mathematical task level.
- Actions consist of *operations* performed by the students at a functional level in interaction with technological artefacts. The operations are conditioned by the usability and availability of certain artefacts, highlighting affordances that emerge at the technological level.

I start my review of previous research by considering the design of digital tools since much of the research in this field seems to include cultural aspects of affordances, considered to be an important aspect for this article.

2.1. *Research on affordances in human–computer interaction*

Norman (1999) pioneered the focus on *usability* aspects of affordances in digital tools. His interest was in interface elements, which directly suggested suitable actions, informing design aspects of the digital tool. An area that has gained much from embracing the concept of affordances is the study of human–computer interaction (HCI) and digital tools (Bærentsen and Trettvik, 2002, Turner and Turner, 2002, Kirschner *et al.*, 2004, Hadjerrouit, 2017, Chiappini, 2012, Conole, 2013). Some of these studies consider socio-cultural aspects of affordances like Chiappini (2012) where 'cultural affordances' appears as a term to capture the cultural objectives underlying a digital learning tool for teaching and learning of algebra. Kirschner *et al*. (2004) define various types of affordances in connection with digital tools. *Technological affordances* relate to the usability of a tool and how it may induce and invite specific learning behaviours. Learners perceive *social affordances* when they through engaging with the tool experience encouragement to interact with peers. Somewhat similar, they define *educational affordances* as those characteristics of an artefact that invites particular kinds of learning, like collaborative learning. To which degree these types of affordances becomes salient is determined by factors like for example expectations, prior experiences and focus of attention, and as such they appear highly relative to how the learning activity is realized. Turner and Turner (2002) considered collaborative virtual environments, and how they could embed certain *cultural affordances*. They define cultural affordances as features in the artefact that either through its making or its use has been endowed with values from the culture. In many cases, these affordances can only be recognized by a member of the culture that created it. For example, most people in modern society cannot perceive the affordances of a slide rule to calculate logarithms. Similarly, providing a computer to a student from the 19th century would pose the same problem if she was requested to use it for logarithmic calculations.

FC do utilize digital tools to convey out-of-class content, and in this respect, HCIs are certainly an important part of the FC framework. As such, the HCI approach may inform many of the design aspects of out-of-class interfaces. Previous studies have pinpointed the need for students to be able to access the videos effortlessly for them to engage with this part of the FC learning cycle (Wong, 2016). Most universities and secondary schools employ various types of learning management systems (LMS) for the delivery of educational content. The choice of a LMS designed with usability for FC in mind can be essential for successful implementation of this framework. Such systems can provide ease of access on several platforms in addition to features like quizzes in between videos and the feedback to the teacher on problematic areas with the out-of-class learning.

2.2. Research on affordances in mathematics education

The concept of affordances has been utilized to inform research in mathematics education in a range of different fields, not merely from the digital tool aspect. Watson (2004) investigated how the classroom can be associated with Gibson's ecology metaphor. The classroom community can be seen as a system that offers certain kinds of interaction possibilities, including certain norms regulating the behaviour of the interaction. As such, Watson's interest lies in unveiling how the whole classroom contributes to learning, as opposed to individuals' actions. She continues to focus on affordances and constraints of mathematical tasks and finds support in the notion of learning as improved participation in interactive systems through becoming better attuned to constraints and affordances of activities (Greeno, 1998). This work on tasks is followed up in Watson (2007) where she develops an analytical instrument which identifies mathematical affordances in public tasks, questions and prompts of the mathematical classroom. Gresalfi *et al*. (2012) also considered the ecology of the classroom, in the study of how the teacher can have significant impact on the way students engage with tasks. In their analysis, they considered the dynamic relation between the environment (the tasks, teachers and peers) and the student as crucial for the learning outcome. Boaler (1999) considered various classroom incidents and showed how affordances and constraints of formalized mathematics classrooms, to which students become attuned, developing students' identities peculiar to certain communities of practice. The use of affordance theory shows that students' development and use of the knowledge attained in the classroom had little relation to real-life situations later in their careers. Randahl (2012) studied affordances and constraints of the mathematical textbook for first-year engineering students from epistemological, cognitive and didactical aspects of learning. She found that the formal/deductive presentation of mathematics in calculus textbooks somewhat constrained students' ability to engage with the topics.

2.3. Research on affordances in FC

Muir and Geiger (2016) investigated the effect of the FC approach of teaching mathematics to a 10th-grade class in Tasmania. They performed surveys and interviews to investigate the nature of both students and teachers perceived benefits with the FC approach and found informants to be generally positive and engaged. This study seems to be of particular interest for considering aspects of engagement and motivation among students towards FC in secondary mathematics education, but no consideration of cultural aspects appears in their analysis. Bormann (2014) did a literature review of the affordances of FC viewed from the perspective of student engagement and achievements, in addition to contrasting FC affordances with traditional teaching at the graduate, undergraduate and secondary levels. He found that FC teaching provided opportunities for differentiated learning, engaging in-class activities, greater quality of learning and meta-perspectives of how they learned in addition to being better prepared for a post-study collaborative job environment. However, even if both these studies claim to be using the notion of affordances to theorize their studies, there seem to be no consideration of what the construct offers in terms of explanatory power towards the flipped mathematics classroom. Thus, there is a lack of research operationalizing affordances of the flipped mathematical classroom. In particular, there are no studies considering this from an activity theoretical perspective and the factors that may enable or hinder learning at a collective and individual level.

As such, the main purpose of the study is to uncover which affordances emerge in the activity system of FC consisting of students in their pursuit to learn mathematics. The research question can thus be framed as follows:

Which perceived affordances and constraints for mathematical learning emerge in the activity system of the flipped classroom at the university level?

These affordances and constraints are sought through the analysis of students' interviews. Furthermore, attention is given to how their perceived affordances align with opportunities for learning originally intended through the design of the FC, in addition to results from previous research.

3. Methodology

This section presents the context of the study, before providing insight towards the research design, including implementation details of the flipped mathematics classroom. Furthermore, I present the design of the interview guide and how the interviews were conducted, before finally discussing the method utilized in analysing the data.

3.1. Research design

Empirical data for this study was collected from interviewing eight students in a class of 15 subject to the flipped classroom model of teaching throughout their first year of mathematics courses. The students in this 2017/2018 cohort were following a bachelor degree in computer engineering at a Norwegian university. The first course (Mathemathics-1) in the autumn term was calculus-based, while the second in the spring term (Mathematics-2) drew on various topics like series, Laplace transform, Fourier series and Linear Algebra. Both courses were 10 European Credits (ECTS) and were obligatory for the bachelor degree. Before I initiated the FC teaching, I made an effort to explain how the FC framework was implemented in their teaching and what expectations I had towards preparation and in-class group work.

Preceding each in-class session, a corresponding out-of-class session was presented to the students in Campus Inkrement, which fulfils the role of the LMS in this case. The web-based tool is designed based on FC principles, allowing the teacher to highlight video watching statistics for the individual student. From a student perspective, Campus Inkrement provides the opportunity to give feedback on how well the student understood the current topic on a scale 1–5. In addition, self-perceived effort can be reported on a similar scale. The student also has the opportunity to ask for further guidance from the teacher on specific topics.

The videos presented the mathematics in a chalk-and-talk fashion (Artemeva and Fox, 2011), where I utilized screen-capturing software to record hand-writing and voice on a virtual blackboard. The lectures were 5–15 min in length, and each out-of-class session consisted of 3–5 of these. The videos considered mostly mathematical results and how to utilize these for mathematical problem solving through calculated examples. This choice was intended to make the video homework manageable in the limited out-of-class time.

When students came to class, the session was usually organized by an introductory talk by me followed by students' group work on a set of tasks. The talk at the beginning of the lesson considered the main topics from the videos. This provided an opportunity for students' initial participation towards the topic, since I usually would make this talk highly interactive, asking questions on the main ideas from the videos. The tasks students engaged with were either from a modelling perspective, designed with a realistic mathematics education[1] perspective in mind or a collection of tasks from the textbook. The textbook tasks for each session were organized by increased difficulty, starting in a more procedural fashion and ending with conceptually oriented tasks. Students spent the session working in groups, predominantly assembled by themselves, while I walked around and assisted. The work with the tasks would occasionally be interrupted by a whole-class walk-through if multiple groups struggled on the same task. A summing up at the end of the session would sometimes be done if I considered it necessary.

The 2017/2018 class was the last cohort of in all three consecutive cohorts subject for research on the flipped classroom pedagogy. As I had the dual role of both being an insider (teacher) and outsider (researcher), I consider myself being a participant observer throughout the whole study (Bryman, 2008). As such, this study is placed in the naturalistic or ethnographical research paradigm (Moschkovich and Brenner, 2000). Research on and experience from the previous two cohorts informed the design of the interview guide tailored for this study.

3.2. *The interviews*

An interview guide was constructed based on uncovering students' impressions of affordances and constraints of the FC implementation they had been subject to. As indicated by the research question above, I attempted to shed light on intentionally designed affordances in addition to previously known affordances of the flipped format (Muir and Geiger, 2016, Bormann, 2014). Important examples of such affordances related to previous research is *teacher access*, *interactivity*, *preparedness*, *personalization* and *engagement*. Questions covered aspects about the pedagogical structure, participation during in-class and out-of-class activities, about the role of the teacher, the classroom session, about the technological implementation and issues about the mathematical learning activities. The guide put emphasis on querying about the presentation of mathematical content through the out-of-class videos and the students' collaboration on tasks in-class. Eight participants were picked for the interview, amounting to about half of the students. Since the interviews were performed in the end of the spring term, I had a good

[1] Realistic Mathematics Education is an instruction theory in mathematics, where 'realistic' situations are given a prominent position in the learning process. These situations serve as a source for initiating the development of mathematical concepts, tools and procedures. From this initial situational understanding, students gradually develop more formal an less context specific mathematization VAN DEN HEUVEL-PANHUIZEN, M. & DRIJVERS, P. 2014. Realistic Mathematics Education. *In:* LERMAN, S. (ed.) *Encyclopedia of Mathematics Education.* Dordrecht: Springer Netherlands, ibid.

personal knowledge of the various types of personality of the students, making it easier for me to choose informants that would fit on a range of these dimensions:

- Ability to make critical remarks.
- Attendance in-class (coming to sessions) and out-of-class (having prepared watching videos).
- Engagement during in-class sessions.
- Performance in mathematics. The performance measure considered was mainly based on the final grade from the previous mathematics course (Mathemathics-1) the students had attended.

To avoid the influence of my role as a teacher in the interview situation, I asked an independent researcher to conduct these, not affiliated with final assessment of the students. Furthermore, the students interviewed were informed that recordings would not be subject for investigation before the final exam in Mathematics-2. This strategy is similar to the one followed by Strayer (2007) and Tawfik and Lilly (2015) in their qualitative studies on Flipped Classroom on statistics courses in tertiary education.

3.3. Method and data analysis

The eight audio-recorded interviews were transcribed verbatim and were subjected to thematic analysis (Braun and Clarke, 2006). This method is utilized for identifying, analysing and reporting patterns or themes within the data. Thematic analysis allowed me to approach the data with an activity theoretical presumption, tuning in on properties of the data that informed me on these perspectives. As a result, the purely inductive paradigm of grounded theory was rejected (Corbin and Strauss, 2008). Accepting a theoretical framing of the data allowed me, as a researcher, to attain an active role in identifying patterns aligned with activity theoretical principles. Although the interviews were coded individually, the themes were sought on a global basis, combining results from the whole corpus of data.

Thematic analysis includes the phases of researcher familiarization, generating codes, searching for themes, reviewing themes, defining and naming themes and writing a report (Braun and Clarke, 2006). Familiarization was achieved by writing a summary of important points noted when listening to the audio files. Before initiating the coding process, I conducted a literature review of previous research on flipped classroom (see previous section), basing initial codes on already known affordances. In all, 27 such codes were proposed as a starting point for the analysis, where six of these were constraints. Throughout the analysis, I kept an open mind towards emergent codes and I identified 25 more codes in the process of coding the eight interviews. Coding was performed by marking related passages in the transcriptions. Some of the initial codes from the literature review were not detected in the data and discarded in the subsequent analysis. Others were found to illustrate the same phenomenon and concatenated/renamed into new codes during the analysis.

An example of how the analysis was performed is highlighted in the following quote:

Pete: ‘Many enjoys the quick walkthrough of the main points in the videos during the beginning of the lesson. Then we also get the chance to clear up things that might have been unclear. Then we do the tasks. He often highlights an example as well, and that is very good.’

This excerpt was given the code *reinforcing*. The student talks about how a particular topic is encountered multiple times and in various ways, reinforcing understanding. The code was initially considered part of a theme named ‘Facilitation of learning’, related to how the students considered the FC format to support their own learning processes. In all, nine such initial themes were found, ranging from themes like ‘Structure of FC teaching’ and ‘Social affordances’ at the collective level to ‘Engagement/motivation’ and ‘Technological affordances’ on a more individual level. The themes were initially unrelated to activity theory. In reviewing these themes, I sought an adaption to activity theory.

This caused many of the initial themes to break up, while others were kept as categories, being part of the new themes. Referring to the example above, the theme ‘Facilitation of learning’ was abandoned due to analysing the data again through an activity theoretical lens. The code *reinforcing* was then considered to fit into a category ‘Structural affordances’ which were further related to the theme *affordances for collective learning in-class*.

4. Results

Applying a socio-cultural approach to affordances of learning mathematics in a FC setting implies analysing the activity of humans within this framework. Building on the previous discussion in the section Theoretical Background, I draw on the hierarchical approach of Leontjev (1977) in the identification of the various types of affordances through the thematic analysis described above:

- Technical and functional affordances out-of-class
- Affordances of the mathematical tasks
- Affordances for collective learning in-class

The results given below are based on the report produced as the end product of the thematic analysis. Please note the following. (i) I use the term ‘affordance’ but this can be read as ‘perceived affordance’. (ii) Extracts from student interviews are translated from Norwegian and selected to be representative of a group of students (i.e. ‘outliers’ are not used). (iii) Names used in quotes are pseudonyms for the purpose of anonymization of the informants.

4.1. *Technological and functional affordances out-of-class*

The theme *technological and functional affordances out-of-class* relates to how the video as a medium and the design of the Campus Inkrement provides students with certain abilities to operate out-of-class learning. Through this instrumental aspect of students’ engagement with the out-of-class learning, two different types of affordances/constraints categories emerged from the data in this theme:

• **Technological affordances** are related to the video playback and how that eased the operation of the presented material. Related codes were *control*, *self-pacing*, *repetition* and *feedback*. Below are two examples from the interviews that were coded as *self-pacing*, one in all four codes in this category

Phillip (upon the affordance of turning up/down the speed on the videos): ‘Yes, very often I added 25% to the speed. I feel that it is easier to learn when you are able to “get it” at higher speeds. And if you don’t get it, you may pause, then you may turn down the speed, it’s really nice that YouTube has that function’. (The LMS utilized YouTube integration).

Pete: ‘ . . . you may look at the videos as many times as you want, and control the speed and take notes etc. You obtain a more customized learning experience that way’.

The last statement was also coded *repetition*; an affordance several students mentioned as beneficial for gaining personalized control of the presentation of the videos. Other opportunities at the technological level mentioned by the students were pausing the video for a break, watching them on their mobile phone and the ability for accessing them independent of time and place.

• **Functional affordances and constraints** concerns how the videos and Campus Inkrement as instruments mediate mathematical learning. Codes associated with this category were the following: *Dynamics of videos*, *preparedness for exam*, *visualization*, *videos preferred over book*, *minimum video learning*, *examples better in videos* and *interruption affect.* A code representing constraints were *confusing errors*. Several students reported that they found the videos easier to understand than reading the text-book, illustrated by this statement

Alex: 'It helps to see the calculation by pen, not just a picture where you get the explanation'.

Others claimed the English language being an obstacle to understand the mathematics in the textbook, making it necessary to 'read the whole chapter again to understand the thing'. This, it is claimed, made it more efficient to prepare watching a video on the topic than reading the textbook. Other students reported that videos were found useful as a medium for repetition purposes before final examination. Another functional affordance not intentionally thought of was related to students' hesitance to interrupt a lecture situation to repeat certain difficult passages. This discomfort was eased when the lecture was replaced by videos, as illustrated by this statement.

Matt: 'It's not possible in a lecture hall to interrupt and ask questions, and that is very important for me. You will be sitting there half an hour then and try to understand how that little bit worked out, and then you miss the other three or four steps'.

However, students also told us that the quality of the videos could pose a problem. Usually the videos were produced by myself 2 days before the in-class session, making it difficult to conduct a thorough quality assurance. Thus, there were occasions of inconsistencies in the calculations that were performed in the videos, a phenomenon several students reported as problematic for the learning outcome as illustrated by the following quote:

Pete: 'Sometimes the videos have errors in the calculations or it misses a character here and there'.

In a live lecture situation, if such errors during calculations would occur, the confusion would usually be remedied by interrupting the lecturer to ask questions about the calculations on the whiteboard.

4.2. Affordances of the mathematical tasks

This theme considers affordances and constraints related to the individual sphere, concerning students'/teachers' practice-oriented actions at the mathematical task level. The affordances at this level are associated with individuals' meaning-making (providing opportunities for mathematizing) and engagement (being stimulated to do so by various means). As such, affordances associated with mathematical tasks consider how FC provides opportunities for spending class-time on tasks related to real-world applications, inquiry and modelling as opposed to lecturing only. Such affordances were associated with codes like *real-world applications*, *conceptual learning, variation*, *modelling skills*, *scaffolding*, *customization of tasks*, *teacher access*, *correction by teacher*, *teachers' active role* and the constraint code *inconsistency tensions*. A statement exemplifying the first of these codes:

Alex: 'The optimization tasks were a bit more about how these things work out in the real life. When you can tie it to the reality, it's not just number and magic'.

An important meaning-making issue for engineering students is to contextualize the mathematics, as illustrated by this quote.

As a teacher, I usually made an effort during in-class sessions to support students in their problem solving, not giving complete solutions when asked, but rather providing directions. Pete reflects on this:

Pete: 'He does not solve them for us, but he gives us a way of solving them'.

However, scaffolding sometimes feel 'awkward' for students if pursued too far:

Phillip: 'Well, he is active and walks around to the groups and considers the work that is being done. If you are preoccupied with the task and have an understanding of it, it's very nice, but if you are a bit slow, then it may become a bit awkward'.

Another important aspect of the tasks was the facilitation of progress. Usually, the set of tasks had a progression from a more procedural level at the beginning, usually directly related to the examples in the videos, towards a more conceptual level at the end of the collection. This feature seemed to have been recognized by some of the students:

Pete: ‘I felt that the tasks were well fitted to the videos and the progress through the curricula. The tasks had an increasing level of difficulty as you worked through them during the lesson. If we were just given random tasks from the book, then I think many would have dropped off. Therefore he has found tasks adopted to the level of difficulty we needed and to the curricula’.

As is mentioned by Strayer (2012), students may experience various forms of stress and confusion when being subjected to this new pedagogical framework, effectively hindering students in meaning-making and engagement with the topic. In some instances, students report tensions between out-of-class presentation and facilitated tasks in-class:

Matt: ‘There was one example where we were supposed to integrate over a period (referring to the calculation of Fourier coefficients). In one of the examples he used half of the period (meaning the examples in the video), while in an example on the blackboard he used a whole period. We used half of that lesson to make sense of this, and didn’t even manage to conclude anything. That lesson just confused everyone’.

Leonie (on conceptual tasks): ‘It depends how well you understood the topic. I remember that I had some problems with my understanding before I came to class, and then it was a bit over my head’.

Such inconsistencies and lack of understanding of the presentation in the videos seem to have led to unproductive and confusing sessions, constraining learning in a FC.

4.3. *Affordances for collective learning in-class*

This theme encapsulate affordances for learning that emerge in a collective setting, either caused by the facilitated group work or through the combined effect of videos and in-class activities. These appears as two categories (1) social affordances related to the various collaborative learning efforts of the students and (2) opportunities for learning enabled by the ‘new’ *norms* associated with FC as compared to more traditional lecture-based teaching.

• **Social affordances** concerns students’ comments on the importance of contributing verbally, how listening to multiple angles towards a solution process supports understanding, how they get a sense of contribution when helping their peers and collaborative learning in general. Codes associated with this category were *sense of contribution*, *collaborative learning*, *multiple angles*, *accomplishment of task*, *sense of achievement*, *verbal participation*, *critical skills*, *confidence* and *socially obliged*. A quote from Bill illustrates the code *sense of contribution*:

Bill: ‘I dared to ask questions, since I was always prepared for the lessons. This also gave me the opportunity to make myself useful in many situations. When some of my peers were unprepared I was able to assist them a bit’.

Other students expressed thoughts on the role of explaining to other members in the group, highlighting the idea of clarifying the topic for others as a way to make things more explicit for oneself. A third factor was considered by Emma, and coded *multiple angles:*

Emily: ‘It’s OK to collaborate in the sense that the person that understands the topic can explain it to the others in another way. If you can be explained it three different ways, it’s probable that one of them works for you’.

Students can be put in a challenging position during group work if they choose not to prepare properly before the in-class session, since they will not be able to contribute efficiently. Another equally important factor is that students feel obliged to work with tasks in a FC setting, as Pete expresses:

Pete: ‘If I was going to sit at home with the tasks, it wouldn’t have been much accomplished. But if you meet at the sessions, you will also feel a bit socially obliged to work with the tasks’.

This statement also expresses the importance of being positioned for efficient collaborative work with tasks, as this is imposed as a structural activity in-class, activity students otherwise may not prioritize. Thus, in addition to be given the code *socially obliged*, this statement qualified for the code *accomplishment of tasks.*

• **Structural affordances** capture the opportunities for learning that is enabled through the established norms of FC teaching, that is, affordances emerging at a systemic level in the activity system of FC. This is akin to the term Pedersen and Bang (2016) coins as *the affording of societal standards.* The most prominent of these norms is the rule that students should prepare for in-class activity watching the out-of-class videos provided. I also strongly advised the students to take notes from the videos. Associated codes were *reinforcing*, *preparing for work life*, *worked harder*, *facilitation*, *preparedness*, *community* and *note-taking*.

In the following quote, the student claims that the multiple points of contact with the mathematical topic in FC teaching provided the opportunity for 'better learning':

Matt (being questioned on preparing with the videos): 'The fact that you are introduced to the topic in advance means that it matures in your sub-consciousness for a while. Then you get the possibility to think it over, and forget it. And when you forget it, you have the possibility to refresh it again when you come to school, and then maybe learn it better'.

This statement was coded *reinforcing*. Several also reported note-taking as an important instrument to mediate out-of-class video content towards in-class activity. Pete explains:

Pete: 'Yes, I took notes. Wrote down what was done in the videos and did the tasks so that I remembered it better. In the lessons I used the notes to cross-check them when he talked about the topic. I also used them when I solved tasks, so that I could look back at examples in the videos. Then I remembered how to do it'.

There is of course nothing new in the fact that students' own notes from lectures can become vital in their work with collaborative task solving. However, since FC relies on students' operating on an individual level while out-of-class, as compared to the collective environment in-class, it seems reasonable that they need such instruments for connecting out-of-class and in-class learning activity.

5. Discussion

The results highlight affordances for mathematical learning in a FC at the three levels of Leonjev's activity model. This section will elaborate on the results of the thematic analysis, emphasizing the perceived affordances from students' own reported opportunities for learning. The analysis is drawn based on previous research on FC. Specifically, I attempt to highlight affordances emerging as new findings as compared to previous research studies on FC, focusing on how the activity theoretical approach might bring new perspectives on the affordances of FC in mathematics teaching at the university level.

The theme *technical and functional affordances out-of-class* depicts affordances at the operational level. Technological affordances like controlling the video speed and the ability to pause and go back to repeat unclear parts were considered valuable features for individualized presentation of the mathematics in the videos. This finding is in line with Hadjerrouit (2017), who emphasized that technological affordances are a crucial pre-requisite for any work with mathematics education using digital tools. More surprisingly was the fact that many students stopped utilizing the textbook after being introduced to the videos as a mediating artefact (Vygotsky, 1978). This seems to have been caused by the videos' ability to show the dynamics of the calculations including voice-over by the teacher. Previous studies show that engineering students often struggle to relate to the mathematical textbook as a source for mathematical learning (Randahl, 2012). Students may also feel uncomfortable asking questions in a lecture situation, disrupting the flow of the presentation. Not being able to ask questions to the lecturer is usually considered a drawback in FC, but several informants found it convenient to be able to avoid the embarrassment of interrupting the lecturer by utilizing pause and rewind on the videos. This was not an intended affordance, but it has been reported by students in previous research of FC (Love *et al.*, 2014). However, there is a risk that inconsistencies in the calculated examples in the videos posed a tension in students' sense-making during out-of-class activity. This finding is consistent with previous studies of FC involving another cohort of students at this campus (Fredriksen and Hadjerrouit, 2020). One could consider this a constraining factor in the out-of-class teaching, due to the inability of the teacher to correct such mistakes as one would in a live lecture situation if prompted by the audience.

At the action level, the theme *affordances of the mathematical tasks* considers opportunities FC may offer the students to achieve their learning goals through mathematization and motivation for working with tasks. Actions are considered to be the most conscious level in Leontjev's model of activity (Pedersen and Bang, 2016). As such, in a learning perspective, this can be viewed as the level of highest engagement and participation. The affordance of the flipped format to provide more space for motivating students through real-life modelling tasks was a feature designed for and enacted in several occasions throughout the FC teaching. The results showed that this was indeed an important motivational factor. Moreover, the availability of teacher support during problem solving was also appreciated by the participants, in addition to the possibility to ask questions about the videos in-class. These affordances are some of the key advantages of FC teaching (Hamdan *et al.*, 2013). The teachers' efforts directed towards the customization of tasks seem to influence on students' sense of progress in the problem solving. However, students seem to be very sensitive to inconsistencies at this level as well. If students meet a different world of symbols and nomenclature out-of-class as compared to in-class, confusing sessions may result as illustrated by the quote from Markus (see the subsection 'Affordances of the mathematical tasks'). This is consistent with findings in previous studies of FC, highlighting the importance of students' participation in-class forming a consistent extension with out-of-class video content (Fredriksen and Hadjerrouit, 2020, Fredriksen, 2020).

The theme *affordances at the collective level* captures how FC norms connects out-of-class and in-class activities together and, in doing so, creates space for collaborative learning (Watson, 2007). This theme is associated with the *activity* level in Leontjev's model of activity, where students collaborate towards the common goal of learning mathematics. A fundamental principle of FC teaching is to require students to prepare for the lesson through the out-of-class video-preparation. This is a vital prerequisite for the ability to shift from teacher-centred lecturing to student-centred problem-solving (Hamdan *et al.*, 2013). The results show that FC in-class activity affords a range of collaborative learning opportunities based on this transition, consistent with the findings in Gresalfi *et al.* (2012). Furthermore, note-taking seemed to be important for mediating procedural knowledge between in-class and out-of-class spheres. A key finding relates to how the implementation of FC structure provided students an opportunity to interact with the mathematical topics on multiple occasions and in various ways during a FC session, initially through the more procedural presentation in the videos, then through repeating main points at the beginning of the in-class session, and finally through collaborative working with the tasks at a more conceptual level. This was considered advantageous for reinforcing the mathematical concepts, a finding also reported in Love *et al.* (2014).

5.1. Limitations

In this study, I had the dual role of being both a researcher and the teacher. As such, the interpretation of the results may have been coloured by the direct participation of the researcher in the study. Another limitation is the coding process, which has only involved myself. However, a team of researchers have supported me in the process of considering appropriate themes, and the further alignment of results with theory. Furthermore, one may argue that this study is a single qualitative case study of a small group of students in Norway, with results not necessarily applicable to other settings. However, there exist a range of consistent findings in previous research indicating external validity of the results, see for instance Giannakos *et al*. (2018), Lo *et al*. (2017), Franqueira and Tunnicliffe (2015).

These results are based on a cohort of students of no more than 15 students. In tertiary educational settings, cohorts may consist of several hundred students. Adopting FC to these contexts will of course be a different story, where one probably will need to break the large number of students into smaller sub-cohorts with the additional help of teacher assistants. However, if the course structure is based on out-of-class content delivered prior to in-class collaboration on tasks, the affordances reported here should apply.

6. Conclusion

It is well-known through previous research that transition from upper secondary to university mathematics can pose severe problems for many students (Hourigan and O'Donoghue, 2007). As such, including videos as an additional supporting medium in the preparation for classes that focuses on deeper conceptualization of the mathematics can remedy some of these obstacles. Collective learning in-class also seems to provide means for students to get the assistance through questioning peers and the instructor, forming a second important facilitating aspect of learning in a FC. However, concepts may only be explored at deeper levels if necessary procedural knowledge has been established. The videos could form an important role in preparing students at this stage. Also, note-taking can form an important mediating instrument between the two learning arenas of FC. However, constraints for conceptual learning may emerge if activities in-class appear disconnected from out-of-class preparation.

From the outset, FC designs may appear confusing with the need to combine the joint effect of two learning arenas, the out-of-class and in-class worlds. However, considering affordances of a mathematical FC through an activity theoretical perspective provides the opportunity to observe aspects of learning from a comprehensive viewpoint. As such, separating the affordances and constraints into the various levels of the activity can bring along a deeper understanding of the mechanisms involved in the learning through the abstraction involved. These levels form a *conceptual hierarchy* (Niss, 2007), organizing the affordances of the FC into a meaningful and coherent structure adopted to the human activity of mathematical learning. Thus, one may observe a "red thread" in the activity: (1) out-of-class videos and quizzes have the role of *conditioning* procedural learning at an *operative level*, (2) mathematical problem solving in-class forms the *goal-driven actions* towards procedural and conceptual learning at an individual level and (3) the norms and rules of the FC *activity level* connect the out-of-class preparative stage with in-class collaboration towards the common *objective* of learning mathematics. This hierarchy paves the way for understanding the role of the affordances emerging at the various levels, and how they need to connect to form a coordinated platform for mathematical learning. As an illustration, the concept of *social and structural affordances* aided the understanding of how the norms of in-class collaborative problem solving could form a social obligation towards accomplishing tasks, leading to students' enhanced mathematical activity. Such reflections on the qualitative data emerge based on applying the theoretical lens of activity theory and may pave the way for further development of future FC designs.

Recognizing the wide variety of possible implementations of FC at the university level, there may be equally many possibilities for failing. I do not claim that all affordances at the three activity levels reported in this paper apply to every possible realization of FC. My prediction would be that affordances at the Activity level in the model will be the most generalizable, with decreasing relevance as one goes downwards in the hierarchy of the model towards the Actions an Operations levels. The reason for this is the tight connection at these levels towards implementation details such as task design and layout of the presentation of videos and quizzes. For example: Many implement a requirement to pass a set of out-of-class quizzes to ensure the preparedness of students, which might lead to other affordances not seen in this study.

Summarizing, the results from this study shows that care should be taken towards consistency and layout of in-class sessions, focusing on tasks in-class that extend out-of-class content. An initial repetition phase of main points from the videos seems to have an important effect for connecting out-of-class and in-class activity. Another important instrument for retention was students' note-taking during video-watching, which could be utilized to ease the problem-solving process. Lastly, facilitating students' collaborative participation in groups and whole-class discussions seem to be an important structural component for meaningful activity in-class.

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