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Teaching for mathematical literacy: School leaders' and teachers' rationales

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Abstract
This article reports a qualitative inquiry into school leaders' and teachers' rationales for teaching to develop students' *mathematical literacy*. The study is rooted in an exploration of the meanings that the school leaders and teachers hold about the term mathematical literacy. Six leaders and three grade 9 mathematics teachers from three schools were interviewed. Analysis framed within cultural-historical activity theory indicates that mathematical literacy is perceived as a desired outcome of schooling, and that teaching for mathematical literacy is connected to school leaders' and teachers' contradictory rationales for teaching. The rationales are connected to use value, meaning, teaching practice, teacher competences and knowledge, and universality.

Keywords: Cultural-historical activity theory, mathematical literacy, mathematics education, teaching

Introduction

The Programme of International Student Assessment (PISA) has provoked much interest and discussion about the notion *mathematical literacy*(ML)(Fried & Dreyfus, 2014). The Organisation for Economic Co-operation and Development (OECD)(2012, p. 25) defines ML as

> an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

In spite of the international rooting of the definition in the OECD-PISA study, ML has no universally accepted meaning and attempts have been made to clarify the expression (Fried & Dreyfus, 2014). In its broadest sense, ML concerns the ability to perform, understand, and use mathematics in daily life(Colwell & Enderson, 2016). It concerns the ability to use mathematical knowledge and competence to formulate and solve mathematical problems in a range of different situations. Sfard (2014) explains ML as the ability to participate in mathematical communication whenever this is necessary for our understanding and manipulation of the world. It is therefore important that students learn how to speak mathematically, when to speak mathematically, and what to speak mathematically about.

Mathematics education literature contains several notions related to ML. Some authors use concepts like ML, *numeracy*, and *quantitative literacy* synonymously, while others distinguish between them (Niss & Jablonka, 2014). Other related concepts are *critical mathematical numeracy*(e.g. Frankenstein, 2010), *mathemacy* (e.g. Skovsmose, 2011), *matheracy* (e.g. D'Ambrosio, 2007), and *statistical*

literacy(Watson, 2011). Still, the different interpretations of these concepts have in common that they stress awareness of the usefulness and ability to use mathematics in different areas (Niss & Jablonka, 2014).

Another problematic issue with the term ML is that although it is widely used internationally, it lacks non-English equivalents (Jablonka, 2015). This means that it is difficult to translate the concept into other languages and retain the meaning. In some languages, the word literacy has such a narrow meaning that it can be impossible to convey the broad meaning intended by PISA (Stacey & Turner, 2015). For example in Spanish, French, and the Scandinavian languages, literacy is linked to very basic reading and writing abilities. As a result, concepts like *mathematical competence* and *mathematical culture* are used instead to avoid the narrow connotations of the term literacy in educational debates (Stacey & Turner, 2015).

PISA's reports comparing students' performance have been influential in shaping educational policies in several OECD countries, and curriculum developers/reviewers have tried to reflect PISA competences in their national curricula. In the latest curricular reform in Norway (LK06), there is an explicit attempt to align with PISA frameworks by including basic skills in all subject syllabuses (Breakspear, 2012; Det kongelige utdannings- og forskningsdepartement, 2004). The basic skills correspond to the English notion *literacy* (Det kongelige utdannings- og forskningsdepartement, 2004). The five basic skills are *reading, writing, oral skills, digital skills*, and *numeracy*.

> Numeracy means applying mathematics in different situations. Being numerate means to be able to reason and use mathematical concepts, procedures, facts and tools to solve problems and to describe, explain and predict what will happen. It involves recognizing numeracy in different contexts, asking questions related to mathematics, choosing relevant methods to solve problems and interpreting validity and effect of the results. Furthermore, it involves being able to backtrack to make new choices. Numeracy includes communicating and arguing for choices by interpreting context and working on a problem until it is solved.
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> Numeracy is necessary to arrive at an informed opinion about civic and social issues. Furthermore, it is equally important for personal development and the ability to make appropriate decisions in work and everyday life. (Utdanningsdirektoratet, 2012, p. 14)

The lack of a universal understanding of ML and the range of similar notions affect teaching to promote ML. Colwell and Enderson (2016) studied pre-service teachers' perceptions of ML to inform program changes in teacher education in the USA. The pre-service teachers emphasised writing, communication, and application skills as important factors of ML. However, the pre-service teachers were uncertain of how to integrate such practices into their teaching. In the Norwegian context, an evaluation of LK06 shows that the basic skills have been understood in a more confined way than intended (NOU 2015:8, 2015), and Grønmo (2014) calls for discussion and measures on how to implement the basic skills in a satisfactory way. This is consistent with the findings in Haara, Bolstad, and Jenssen's (2017) review of empirical studies of ML. If students are to become problem solvers that use concepts, procedures, facts, and tools to reason, describe, explain, and predict in various contexts, teachers need to implement instructional techniques to promote this development. To support implementing such techniques, it is valuable to investigate and build on teachers' own understanding of practice in relation to the notion ML.

Teachers experience tension between wanting students to have time to "understand" the mathematics while making sure they "cover the syllabus" for the upcoming test. This tension between performance on tests and depth of understanding is one that reaches out beyond the classroom, implicating policy, texts, tests, and assessments (Williams, 2011). Hence, the immediate needs of performance conflicts

with future use demands. Studying how to teach for ML is therefore of interest both in a Norwegian and an international context.

Teaching for ML requires a notion of what it means to be mathematically literate. As noted above, literacy has no equivalent word in Norwegian, and the notion of literacy is not made explicit in the Norwegian curriculum. However, the development of students' ability to use mathematical procedures and tools to solve problems in different contexts is an explicit goal. Thus, it is relevant to study the influence of the OECD competence framework on Norwegian school education. In addition, knowledge about the Norwegian education system may contribute to insights in international educational contexts. According to Sfard (2014, p. 141) "the question of how to teach for ML must be theoretically and empirically studied. When we consider the urgency of the issue, we should make sure that such research is given high priority."

In the educational context of the school and classroom, school leaders and teachers are the decision-makers, planners, and organisers of activity. They focus their plans and actions towards a desired outcome. Hence, school leaders and teachers have rationales for teaching mathematics. To learn more about teaching for ML, this study seeks to investigate the rationales for teaching in the context of their understanding of ML.

School leaders' and mathematics teachers' understanding of ML involves some prior knowledge of the concept, different interpretations of the notion, and the aspects of mathematical knowledge and skills it encompasses. Their understanding includes their ideas about how ML is related to or manifested within the curriculum. It also includes their rationales and goals for teaching so that students develop ML. The study reported here focuses on how the concept ML is understood in Norwegian lower secondary schools, and the research question addressed in this article is:

> What are school leaders' and teachers' rationales for teaching with respect to their understanding of ML?

Theoretical Perspectives

In this article, students' ML development is conceived as a goal of a sub-set of teachers' actions. In this respect, "education" and schooling are taken as a culturally and historically situated activity. In other words, the study is framed within cultural-historical activity theory (CHAT).

In CHAT, *activity* is the unit of life that is mediated by mental reflection, which functions to orient the subject in the world of objects (Roth & Radford, 2011). Activities answer to a subject's specific need. This need stands behind the activity *object* or *motive*. Object/motive consist of material reality and its ideal reflection in consciousness and between current and future material/ideal states. In other words, the object/motive drives activity from the experienced "here and now" to a desired future state envisaged in the imagination. Hence, it consist of both the object-sensory practical activity and the ideal object reflected in consciousness during activity.

The main thing that distinguishes one activity from another is the difference in their objects/motives (Leont'ev, 1978). In the activity of education, students pursue different objects/motives. Students performing the same mathematical task, one with the ability to use mathematics in everyday life as motive, the other with the motive of achieving examination success in the subject, engage in different activities. The object/motive that drives learning activity is accessible to students only as an outcome of the activity. Students cannot know what they are supposed to learn before they have learnt it. They cannot recognise the objects/motives on their own, and the teachers cannot tell them. The object/motive emerges through the teacher's and the student's joint action. In learning activity, the

teachers have to take on the regulative function that in other productive human activities exist in the known object/motive (Roth & Radford, 2011). Teachers have to facilitate students' engagement in activities concerned with developing ML. The question is which activity the students engage in, and, therefore, which motives they take up and pursue.

*Actions*are initiated by the object/motive and translate activity into reality (Leont'ev, 1981). An action is directed toward a conscious *goal*. Several different goals and actions can relate to the same object/motive, but are not equal to it. ML contains goals connected to mastery of mathematical procedures, understanding of mathematical concepts, and the ability to use all of this in different contexts. Several actions can contribute to reaching these goals. For students in the process of learning, there is an absence of a concretisation of the object/motive. Learning goals are stated by the curriculum (and the teacher), and not by the students themselves. Therefore, their actions do not and cannot make sense. "Students may realize a task without taking up the object/motive, in which case they do not expand their action possibilities in the intended way, and do not learn what they are invited to learn" (Roth & Radford, 2011, p. 97). An example is students using rote-learned procedures to solve mathematical tasks without understanding the underlying processes and structures. Teachers have to launch an objectifying process where room is created for joint work for the object/motive to emerge for the student. The teacher's actions have to open up new possibilities for student action.

The intention behind ML is to enhance students' possibilities for action in their everyday life. It emphasises the use value of mathematics. Williams (2011) argues that within CHAT values must have a crucial role in shaping subjective needs, and that values are bound up in ideal outcomes and subjectively perceived needs mediated by cultural norms. If the object is a mathematical task, the goal of the acting subject may be to understand mathematics or to get the correct answer. In his study, the *use value* of mathematics was discussed in relation to mathematics in daily life (such as shopping) and connections between mathematics and vocational activity. Williams also found that students talk of a kind of currency of mathematics qualifications and grades. This currency is required for entry to universities and courses, and will eventually result in a respectable career. Williams sees this currency of mathematics as akin to *exchange value*. Exchange value is a quantitative relation between two values in use. "As use values, commodities are, above all, of different qualities, but as exchange values they are merely different quantities, and consequently do not contain an atom of use value" (p. 28). In relation to Ernest (2004), this can be denoted as utility, meaning narrowly conceived usefulness that can be demonstrated in the short term. He contrasts utility with relevance. Relevance is relative according to the person is using it. It is a relation between an activity or object, a subject, and a goal. Varying goals give rise to different conceptions of relevance in mathematics education (Ernest, 2004). Hence, teaching with respect to use value is complex.

The term ML concerns the use value of mathematics in everyday life and includes using mathematical *tools*. Tools are used for some purpose, in order to achieve something. They are embedded within a cultural-historical form of thinking (Roth & Radford, 2011). Tools can be external items(e.g. a calculator or an abacus), thinking tools (e.g. different forms of representations such as graphs and algebraic expressions), and communicative tools (e.g. language, text, and speech). They assists us to see something through something or someone else, in other words the tools *mediate*. Tools can also mediate mathematical understanding. Mathematical tools help us describe, explain, and predict phenomenon, and to understand the world. Mathematics, written language, speech, gestures, and every sign system are communicative systems developed for different purposes. They are tools to activities. To make clear the historical intelligence embedded in tools requires that other people who know this intelligence helps us acquire it (Radford, 2008).

Activity is social, and *communication* is an indissoluble part of the activity process. In ML the ability to formulate, interpret, reason, describe, and explain refer to different forms of communication. As

stated in the introduction, ML involves communicating mathematically in order to understand and manipulate the world. Communication is a system of goal-directed and motivated processes, which ensure the interpersonal components of activity. It is through communication that ideas are shared, strategies developed, and projects carried out (Mellin-Olsen, 1987).

Language and concepts are important for communication and learning. Language *mediates* significations, or meanings, which constitutes a practical consciousness for others and constitutes one of the main contents of collective consciousness. "As meaning exists in the form of language, language is shared socially as an objective reality. The meaning which language conveys, however, is interpreted subjectively by the individual (Mellin-Olsen, 1987, p. 44). Concepts are the result of the objectification of historically achieved significations. A word reflects the social, political, and theoretical position of the person uttering it. Knowing, as an outcome of learning, refers to the possibilities that become available to the participants for thinking, reflecting, arguing, and acting in a certain historically contingent cultural practice (Roth & Radford, 2011).

Radford (2008) defines learning as the social process of *objectification* of those external patterns of action fixed in the culture. Learning is not merely acquiring, possessing or mastering something, but seeking to find "something" in culture. It is a subjective awareness of cultural objects. Related to ML, it involves recognising the role that mathematics plays in the world. Objectification entails the process of *subjectification*—i.e., the becoming of the self. In this process, the learner objectifies cultural knowledge and finds himself objectified in a reflective move. This is the making of the subject, and it is the outcome of the act of learning. In knowing mathematics, the student enters into a historically mediated relationship with mathematics and other people. This historically mediated relationship not only makes mathematics noticeable to the student, but also the student to himself through the available forms of subjectivity and agency of the culture. Hence, it enables the student to make well-founded judgements and decisions in everyday life. Objectification and subjectification should be seen as two mutually constitutive processes leading to students' engagement with cultural forms of thinking and a sensibility to issues of interpersonal respect, plurality, and inclusiveness (Radford, 2008).

Hence, CHAT embodies both the individual and the society as a unity. The individual acts on society at the same time as s/he becomes socialised into it (Mellin-Olsen, 1987). ML concerns the individual's ability to act in and be a part of society by knowing mathematics.

Students learn about what is important knowledge, expectations, future prospects etc. for their local community and they develop rationales for learning. School may or may not be part of these rationales (Mellin-Olsen, 1981). The *S-rationale* is the rationale for school learning. "It is the rationale for learning evoked in the pupil by a synthesis of his self-concept, his cognition of school and schooling, and his concept of what is significant knowledge and a valuable future, as developed in his social setting" (Mellin-Olsen, 1981, p. 357). The conception of what is significant knowledge will differ between geographical regions and between social classes. Students can face contradictory rationales for learning according to their relation to different social groups. Additionally, there is a rationale for learning related to school as an instrument for a good future or qualifying the students so that they obtain a good price for their commodity of labour. Mellin-Olsen (1981) calls it the *I-rationale*. The I-rationale creates learning that shows no interests in the content itself, rather the purpose is to demonstrate knowledge to obtain good marks or a degree. This could mean rote learning of mathematics procedures and facts. The optimal situation is when the S- and the I-rationales coincide. This is when the curriculum that leads to good marks (the procedures and facts) is the same as that which the students experience as significant knowledge (knowledge useful in everyday life). However, the most common situation in the classroom is when the S- and the I-rationales overlap. The teachers' task is to make this overlap as large as possible.

The task of making the overlap is challenging due to students' differing S- and I-rationales. In addition, teachers' conflicts regarding teaching for understanding and teaching to cover the syllabus suggest that teachers also have rationales for teaching. This study seeks to investigate school leaders' and teachers' rationales for teaching for ML.

Research Design

According to Roth and Radford (2011), language is the vehicle of consciousness and words constitute aspects of consciousness. By conducting interviews with school leaders and mathematics teachers, I wanted to learn about their rationales for teaching, and their object/motive, actions, and goals in relation their understanding of ML.

However, words are addressed to an interlocutor, and will depend on the interlocutor's social role/status. Therefore, words are not solely a property of the person uttering them. In my research, the school leaders' and teachers' responses will be affected by my role as a researcher and teacher and the social relationship between them and me. Another researcher might get different answers or interpret the answers differently.

Subjects

This study aims to investigate school leaders' and teachers' rationales for teaching with respect to their understanding of ML.I conducted interviews with six school leaders (three male and three female) and three grade 9 mathematics teachers (one male and two female) in three schools in Western Norway. The schools' number of students on roll range from 220 to 370 and all 3 schools teach grades 1 through 10. The school leaders have previous experience as teachers. All the participants have more than ten years of experience from working in school.

The three schools cooperate with the author's university teacher education programme. They were therefore recruited as an outcome of acquaintance. I first contacted the school leaders and they recruited the teachers. The criteria for selection of teachers were that they were teaching grade 9 (students aged 14-15 years) mathematics the current school year, and that they agreed to participate. All participants received an information letter explaining my interest in studying their understanding of concepts in policy documents regarding mathematics teaching and learning. Because of the lack of a Norwegian equivalent and a universal understanding of ML, I did not include this notion in the letter. In addition to the information letter and e-mail correspondence, I also attended meetings with them to ensure informed consent.

Mathematics teachers plan and conduct teaching in order to enable students to obtain the goals stated in the syllabus. In Norway, mathematics is a discrete subject within the curriculum. The syllabus contains sets of competence goals, which the students are to obtain. The goals are connected to the different mathematical topics. Trough grades 1 to 10, students are expected to study mathematics on average about 11 hours each week (given that one school year consists of 38 weeks). As noted above, numeracy is expected to permeate the whole curriculum. Teaching is likely influenced by how the teacher interprets, understands, and conceptualises the ideas and concepts in the syllabus and the textbook. In Norway, the school leaders have pedagogical, administrative, and staff responsibilities at the school. School leaders are responsible for students' learning environments and outcomes, and expected to make professional decisions rooted in subject knowledge. They are also responsible for school development. In this way, the school leaders have to prioritise the issues worked with and the extent to which they are emphasised. It is expected that the school leaders' priorities influence the teaching.

Design and Procedures

The school leaders have the pedagogical responsibility at the schools, but the teachers conduct the teaching. I was therefore interested to explore both groups' understanding of ML. I conducted individual semi-structured interviews with all nine participants. I developed an interview guide with questions and topics I wanted them to consider/reflect upon but without a predetermined sequence. I used the same interview guide for all interviews to get perspectives from the different groups' standpoint. I asked four questions about ML. First, I asked whether they had heard about ML. I wanted them to give their own explanation of the concept. Second, I presented the OECD definition of ML and asked them to comment on it. Third, I asked whether this definition corresponds with the Norwegian curriculum. Fourth, I asked how they would conduct teaching in order to develop students' ML. Each interview lasted for about one hour. The interviews were video recorded and transcribed.

Process of Analysis

This study was exploratory and data was analysed qualitatively using an inductive approach. I analysed the data using the computer-assisted qualitative data analysis software NVivo. First, the transcribed data were categorised according to the questions I asked in the interviews. In this way, I had some general structuring principles, and I focused the analysis on the answers to the questions concerning ML.

Second, I engaged in multiple close readings and interpretations of the data. I tried to get an overall understanding of the data and to identify text segments. Text segments are statements connected to a specific topic or issue. They contain school leaders' and teachers' meanings and concernsrelated to the four questions about ML. Sometimes whole sections of transcripts elaborated on the same issue, and sometimes only short sentences. In this way, the text segments differ in length.

Third, I grouped together text segments containing similar topics or issues to make broader categories. The broader categories were developed with respect to key themes in the text segments. The categories are*use value, meaning, teaching practice, teacher competences and knowledge*, and *universality*. The categories are closely connected, and not mutually exclusive. Table 1 shows category descriptions and the categories' relation to the theory outlined in section 2.

Table 1. Category descriptions and their relations to theory

Category	Description	Theory
Use value	Examples of how and in what areas mathematics is used. Examples of teaching activities that show mathematics in context. Related to the justification and relevance of mathematics as a school subject, and highlighting contextualisation of mathematics.	Subjective needs and goals for teaching and learning mathematics. Connected to I-rationales and the exchange value of mathematics, or S-rationales and use value of mathematics.
Meaning	Concerns meaning making and the ability to understand and communicate. Students should develop this ability. It relates to use value in the sense that ability to communicate has use value, but in a more abstract way than i.e. to use mathematics in an occupation.	Communication and language as mediator of cultural-historical aspects of mathematics. Mathematical language can be perceived as both I- and S-knowledge.
Teaching practice	Concerns what teachers do, or should (not) do, generally, in teaching mathematics. It includes	Actions related to goals and rationales.

	class organisation, teaching materials and factors influencing practice (like curriculum, textbooks, etc.), and the way teaching is conducted. It relates to use value and meaning, but is more general.	
Teacher competences and knowledge	Concerns knowledge and skills needed by the teacher. It involves subject knowledge, professional knowledge, and personal qualities. It relates to teaching practice, but focuses on the teacher rather than teaching.	Goals and rationales for teaching.
Universality	Everything is linked together. Teaching, learning, mathematics, and the world are all part of a whole.	Seeing mathematics as part of the cultural-historical activity. Both I- and S-knowledge are valued.

After developing the categories, I coded the transcriptions again using these categories. To test for reliability, I provided a colleague with a sample of the transcriptions and my preliminary category descriptions. We discussed the data sample and the category descriptions to get a mutual understanding of them. She then engaged in the data sample with respect to the coding and categories. We then compared and discussed our coding, and agreed on some category revisions. Finally, we both coded the data using the revised categories. The inter-rater reliability test in NVivo showed a 92, 2% agreement (Kappa value 0, 47).

Categories of Rationales for ML

In this section, I will describe the results of analysis as five categories based on the responses given to questions about the OECD ML definition, teaching for ML, and correspondence between ML and the curriculum. I present the results with respect to the two groups as a whole. However, not every individual commented on all the specific issues.

Use Value

The responses concerned with use value focused on utility. Both school leaders and teachers referred to students' request for justification of mathematical topics. The students want to know why they have to learn mathematics and how it will (or can be) useful to them, hence they ask for the use value of mathematics. The school leaders and teachers argued that teaching for ML should focus on how to use mathematics in societal, occupational and personal life. That is, try to relate to students' S-rationales. However, it is difficult to know what will or will not come of use in the students' future life. The school leaders and teachers thushighlighted the way of thinking about a problem and the ability to use mathematics as a tool as a goal for teaching.

Students have different S-rationales for learning, and this makes teaching challenging. The school leaders pointed to challenges with finding suitable contexts for teaching related to the use value of mathematics. "You can learn the area of… a building, and calculate it, but still the student doesn't see it, how it is in reality, and do not understand the concepts. (…) Because when you, as an adult, have made a judgement on… Right, you have an image of a concept in your mind, based on several experiences, right? Perhaps you have measured the size of a floor, and you know how many square meters you have to paint, and then, right… You have seen it practically, but an eight year old has not" (School leader A2).

The textbook is a much-used teaching tool. According to the school leaders, teaching should involve practical tasks, and not rely too heavily on solving routine tasks in the textbook. Teaching should aim to get the students to experience the use value of mathematics themselves: "To let them experience things, I think. We telling them does not always do the trick, but to let them experience that this was important. (…) For them to see that 'Wow! You know, now it was really helpful that I knew this'" (School leader B2).

The teachers were concerned about developing students' ability to see mathematics in everyday life: "I wish I was able to do what that sentence says; to assist individuals to recognise the role that mathematics plays in the world" (Teacher A). They gave examples from lessons where they had tried to do this. Regarding mathematics in everyday life, the school leaderscommented that the curriculum competence goals do not focus too much on the use value of mathematics. "I think you need to think very creatively about the competency goals before you're able to, to assist individuals to recognise the role that mathematics plays in the world" (School leader B2).

Meaning

The school leaders commented that mathematics can be seen as a language. They connected ML to the ability to use mathematical language and understand mathematical concepts. They gave several reasons for this, for instance that mathematics gives you a foundation in communication with others. "It's important for everyone to understand what we talk about, when we speak the language" (School leader A1).In this sense, mathematical language can be seen as S-knowledge and important in order to function socially. An important part of learning to speak the language is to learn the concepts. According to the school leaders, we often think that students understand concepts to a greater extent than they actually do. Hence, language is socially shared but meaning is interpreted by the individual.

As stated earlier, communication is important for developing and sharing ideas. The school leaders and teachers highlighted language as important in the learning process. They said that it is important to talk about mathematics. Students should put into words what they know and discuss and justify their methods and interpretations.

The school leaders connected ML to reading as a basic skill. They highlighted that reasoning, reflection and interpretation is important. "The capacity to formulate, interpret, for example, in various contexts. Reasoning. It was these things we just worked with… in our development project on reading in all subjects. (…) As I told you before, we've had a great focus on reflecting on a text. What lies between the lines? What…? You know? What's the message, really, instead of just finding the answer directly in the text, because that's often what happens in school, right? (…) There's too little focus on reasoning and reflecting and such" (School leader A2). Reasoning and reflecting concern the significations and meanings that language mediates.

Teaching Practice

The premises for teaching lies in the curriculum. The curriculum contains the object/motive for education, and different goals to satisfy the object/motive. The school leaders pointed to different aspects of the curriculum that affect teaching practice and the actions carried out to reach the goals. The Norwegian curriculum is structured in competence goals for each school subject. The textbooks often refer to these goals. The school leaders pointed to this as one reason for the heavy reliance on textbooks. "I don't think that we managed to crack the code with the curriculum. If we had cracked the code with the curriculum, I think maybe we had managed to put the textbook aside. But we were so focused on the learning goals, and the textbook contained learning goals, so it was an easy way out for us" (School leader B1). The textbook as a teaching tool's close connection to the teaching goals may have a limiting effect on the teachers' teaching actions.

Both school leaders and teachers commented that heavy reliance on textbooks does not support the development of ML. "You know, theory got too much attention in a way, and we put aside a lot, or some, of the practical. (…) Now it's the desk, and to sit and work with the textbook. I can see that the textbook steers too much, and we have to dare to put it away if we're to achieve that students get different skills, understanding, and competence" (School leader B1).According to both groups, interdisciplinary work and connecting different subjects is a way to develop ML. The teachers gave examples from their own teaching practice where they tried to connect mathematics with other school subjects. "When I teach arts and crafts I draw on mathematical knowledge, like with measuring, and to be accurate and... It's going to be a certain length in centimetres and we have to measure… If we're to find the middle… All the time" (Teacher C).

The school leaders connected the curriculum to ML through the basic skills framework. They also connected the basic skills to interdisciplinary work. The challenge is to implement it in the teaching practice in a natural way. "They have tried, with the curriculum, to include every subject in every subject. Or the basic subjects [Norwegian, English, and mathematics]. You know, to… That you should have basic skills in all areas, right. Reading, numeracy, in all subjects. However, I'm not sure that they have quite succeeded. You know… that it has worked. Because it's… I think that it's sometimes a bit artificial. Like, 'Oh, by the way, we have to remember to put in something about numeracy in this project about reading', right? Because we have to make sure that that goal is also reached. Instead of doing it in a more natural way" (School leader B2).

The teachers have different views on whether the curriculum supports teaching for ML. On one hand, the teachers connected ML to the basic skills, and in this way also found support for interdisciplinary work. "We work with numeracy in all subjects, like we work with reading and writing in all subjects. And we try to be more conscious that, well, in social subjects for example, with tables, that's mathematics. We work with it in Norwegian, with study techniques, different ways to work with a text. And a table is a text too" (Teacher A). On the other hand, the teachers conceived the curriculum as focused on subject knowledge, specifying what students should learn in the different subjects and not on connections between subjects.

Teacher Competences and Knowledge

Both school leaders and teachers expressed concern about the increasing subject knowledge requirements for teachers. However, they had differing concerns. School leaders were worried that increasing requirements might be at the expense of good student-teacher relations. "What I fear a bit in the future, if it turns out as suggested, is that we'll get, like in the old days, more specialised subject teachers. I'm not sure that will benefit the student, because for me it's important to attend to the whole student, to see the whole picture, and to see the responsibility the school has for the student as a whole, and not just the one subject. I can go in and teach mathematics, and out again, barely knowing who the student is. Right? And just go on to the next class" (School leader C2). In addition, the school leaders were worried that school will miss out on good teachers because admission to teacher education requires higher mathematics grades than Norwegian.

The teachers were concerned that as teacher education becomes more specialised it will be challenging for the teacher to work interdisciplinary. The inclusion of mathematical topics in other school subjects can be challenging if mathematics is not part of the teacher's curriculum. "If we're going to accomplish that (ML)… Well, then we need a minimum in our education where we have the opportunity to work interdisciplinary. (…) We get more and more specialisation within subjects. And that can be a good thing. But you don't get that interdisciplinary… if you just specialise within a subject. So I'm thinking that there has to be a connection between what we're supposed to do and the

education. (…) It's difficult to implement if you don't have the competence to work interdisciplinary" (Teacher C).

According to the school leaders, teachers can have different intentions with their teaching and one needs to be conscious about what one wants to accomplish. "First and foremost there needs to be an awakening with the mathematics teachers, right? What do I want with this subject, and should I focus on this (points to the ML definition)? Should I… So… It depends on the person. Some teachers teach… as long as they get through the syllabus. Then they're in the clear, they have done it, and the students' results are insignificant. We have this kind of teachers. And it's difficult to shift the focus and to give them another approach, right? What do I really want to accomplish, is it that you've done your job or that the students understand?" (School leader A2).

Teachers' competences and knowledge also concerned textbook use. The school leaders commented that a reason for using the textbook might be low confidence with one's own competences. "We are strange, us teachers. We don't trust that we know our subject. We think that the textbook decides what the students should know" (School leader B1).

Universality

Universality consist of text segments suggesting that everything is connected. Teaching, learning, mathematics, and the world are all part of a whole. The school leaders commented that the curriculum is wide and the ML definition is wide. It has to be this way, because they are supposed to cover all students. The school leaders see ML as comprehensive, something that concerns all subjects, not just mathematics.

The teachers commented that mathematics is everywhere all the time. It is important in several contexts.Hence, mathematics is part of a cultural-historical activity.

Both the school leaders and the teachers expressed that ML is a desired outcome of mathematics education and something that they work with. "It is what teaching, or a subject is all about. That's what I think" (School leader B2). "Isn't this in fact what we're doing?" (Teacher B). This suggests that ML is a goal for their teaching actions.

Discussion

In the previous section, I described five categories of school leaders' and teachers' rationales for ML. The remaining part of this article will focus on interpreting the rationales in the categories in light of the previous outlined theoretical perspectives.

From the perspective of this article, the object/motive is mathematics education and a goal and desired outcome of teachers' actions is students' ML. By objectifying cultural knowledge embedded in mathematical concepts, procedures, facts and tools, the students find themselves objectified. They find agency and individual capacity to make well-founded judgements, and they recognise the role mathematics plays in the world. In this way, ML contributes to subjectification. Learning is both a process of knowing and becoming. The teaching process consists in offering the students rich activities where they encounter cultural objects supported by meaning in tools and social interaction.

Teachers plan different learning actions related to learning goals in order to satisfy the object/motive. Their planning relates to the goals in the curriculum, but also to their own goals for teaching. In the teachers' competences and knowledge category, the school leaders expressed that teachers need to be conscious about their goals for teaching. The teachers' S- and I-rationales influence teaching.

I see ML as the optimal situation where the S- and the I-rationales coincide. ML contains aspects of mathematics theory (concepts, procedures, tools), and the ability to use the theory in various contexts. Local, national, and international tests and examinations assess students' level of ML.ML mastery will lead to good marks. This connects ML to the I-rationale in the sense that knowledge can be perceived as instrumental. At the same time, ML is supposed to be valuable and useful for the future and life outside of the classroom. Knowledge should meet personal and societal needs. Hence, ML is also connected to the S-rationale. Therefore, teaching for ML means that the mathematical instruments taught in school are the ones perceived as useful in social life.

The school leaders and teachers highlighted that students often question the use value of mathematical topics and tasks. This may indicate students' search for an S-rationale. The challenge for the teacher is to relate the topic to the student's conception of use value. Hence, the use value category is related to the S-rationale for teaching. The school leaders and teachers focused on the utility aspect, although relevance and appreciation of mathematical ideas might also answer to students' S-rationales. The school leaders and teachers expressed that students must recognise that mathematics is important for their total life situation, not just life in school. The meaning category is closely connected to use value and the S-rationale. To learn to understand mathematical language and to interpret and reflect upon mathematical results is also important in society outside of school. As stated earlier, communication is important for sharing ideas, developing strategies, and carrying out projects, and hence is an important part of activity.

The students have S-rationales for learning. Therefore, they seek justification and use value of mathematics. However, it is difficult for the teacher to identify students' activities outside the classroom. Teachers have to observe the students' actions to learn something about their activity. The teachers can provide students with situations intended to initiate constructive activities, but the individual decides whether they will engage in them. By focusing on the use value of mathematics and real world contexts, the school leaders and teachers try to create educational situations that relate to the students activity. They try to help students endow conceptual objects of mathematics and culture with meaning. The school leaders and teachers commented that mathematics is a tool for solving problems and for our understanding of the world. By focusing on use value, teachers' try to help students acquire the cultural-historical intelligence embedded in mathematics and relate it to students' S-rationales.

The I- and S- rationales are connected to the object/motive of the activity. If ML is a desired outcome and goal for teaching, the teacher has to help students discover the object/motive of their actions. Interdisciplinary work and real life contexts are regarded as approaches to teaching students the use value of mathematics. They provide a means of materialising the object/motive for the students. The use of specific situations and contexts are designed to help students understand the use of mathematics in general. They are particular instances of the general objects/motives. However, the design and selection of teaching materials and tasks may involve consideration of their attractiveness to the student. Attractiveness and attention may lead to the false assumption that the presence of the curriculum materials in students' consciousness will lead to the intended learning. In fact, the elaborations that such materials include may actually detract learners from engaging in the real activity, that is, in discovering the real object of their activity. The inner actions that are to be structured by the students require the abstraction from the materially objective content of the presentations, and this abstraction is more difficult the richer the content is(Roth & Radford, 2011). For example, students may perceive practical tasks intended to highlight specific mathematical content as a fun break from the regular teaching activities, without reflections concerning the mathematics involved. Hence, the students realise the task without taking up the object/motive. In that case, they do not expand their action possibilities in the intended way and do not learn what was intended. That is, the teachers have an S-rationale for teaching, and they try to relate it to the

students' S-rationales, but if the students do not recognise the social significance of the mathematical content (related to their S-rationales), the task may be pursued with an I-rationale or be reduced to a welcome break. This is an important issue when it comes to teaching practice, and teachers need to consider this when they plan their teaching.

The teachers do not agree on whether the curriculum supports teaching for ML. This may relate to their rationales for teaching. On the one hand, the teachers said that the curriculum does not support ML and that the curriculum focuses on the individual subjects. I suspect that the rooting of this statement comes from the competence goals, which are presented subject by subject. In this sense, the curriculum represents the I-rationale for teaching. The curriculum goals state the knowledge students should attain. Teachers base the students' grades and examination results on their level of goal attainment. Therefore, the teachers have to make sure that they work on all the goals. If the teachers do not teach according to the specific topics in the curriculum, the students will miss out on opportunities for education and employment later in life. Hence, the I-rationale also relates to the exchange value of mathematics in that the student has to learn because it will pay out in terms of grades and exams.

On the other hand, the teachers said that the curriculum supports ML through the basic skills framework. The basic skills are fundamental to learning in school, work and social life. The basic skills represents the S-rationale where teaching means teaching something more than just school knowledge. Teaching should prepare students for life in the real world, to help them reason, interpret, reflect, and solve problems. Hence, this relates to use value and meaning.

The school leaders' statement about not managing to crack the code with the curriculum may relate to teacher rationales and their perception of the curriculum rationales. The competence goals represent I-rationales for teaching and the basic skills represent the S-rationales. The challenge is to get the two to coincide, or at least to make the overlap as great as possible.

The school leaders pointed to challenges in planning actions and goals that satisfy the object/motive of mathematics education. The curriculum goals guide the teaching and therefore the textbook becomes an important mediating tool in mathematics teaching. The textbooks can make teachers' lives easier when it comes to what to teach and what tasks to use. However, there was agreement among the participants that extensive textbook use does not satisfy the object/motive of mathematics teaching. To reach the goals involved in mathematics education requires more than solving routine textbook tasks. A heavy reliance on textbooks does not seem to relate to students activities and does not meet their S-rationale for learning. Textbook use was related to teachers' competences and knowledge. The school leaders suggested that one reason for textbook use might be that teachers do not trust their own professional competences. This supports the claim about challenges regarding teachers' S- and I-rationales and their challenges to make them overlap. If the teachers perceive the curriculum as focusing on I-knowledge, it will require a great deal of effort from the teachers to be able to present it for the students as S-knowledge. If this is the case, they may struggle to plan actions that answer to the object/motive of mathematics education. This could suggest that they do not have agency, and have not subjectified the teacher role.

Another suggested reason for textbook use was that teachers might have different perceptions about the object/motive of mathematics education. This relates to the school leaders' comment regarding curriculum interpretation and the heavy focus on the competence goals. The teachers do not agree on whether ML is a goal for education supported by the curriculum. The curriculum has to be interpreted, and the teacher has to recognise this as an object/motive when working with the curriculum. S/he will also be affected by other objects/motives in their hierarchy of objects/motives, and the related actions and goals.

The teachers must be conscious of their own objects/motives and how they relate to teaching actions and goals.

> Activities represent how a particular individual decides to act in her world, according to the make-up of this world. Individuals do not always agree on which Activities are the important ones to carry through, or how to carry out any particular Activity for which the goal is agreed" (Mellin-Olsen, 1987, p. 37).

Even if the teachers and school leaders agree on ML as an object/motive, and on the curriculum goals, they may not agree on how to act in order to pursue these. One teacher's I-rationale can be another's S-rationale. Teachers with different rationales for teaching will plan their teaching actions differently. Another difficulty for the teacher is that it will vary within the same class what will pass as I-knowledge and what will pass as S-knowledge. This means that the teachers sometimes have to choose which group of students to favour.

To plan for the actions to reach the goals that satisfy the object/motive, teachers need relevant education. The teacher requirements must work together with the object/motive of mathematics education. The teachers' concern about more subject specialisation can also be connected to their rationales. The subject requirements for teachers do not correspond with the teachers' rationales for teaching. The increasing focus on subject knowledge may lead to less competence in interdisciplinary work. Teachers want knowledge about learning, pedagogy, students, about all aspects of teaching and learning that are important for their school community. Thus, they have an S-rationale for teaching. They perceive requirements as an I-rationale, something they have to do to qualify for teaching. The school leaders and teachers worry that in the next step this may lead to a focus on teaching I-knowledge rather than S-knowledge.

Usually the I- and S-rationales work together (Mellin-Olsen, 1987). The rationales for teaching is a combination of the two. The mathematics content is important both in terms of examinations and in itself. Teachers have to cope with curricula designed to gather all the students under one umbrella of knowledge and to provide a coherent education. "This is the major contradiction of the comprehensive school and the most severe problem the didactician faces when he attempts to design a curriculum which applies to all pupils" (Mellin-Olsen, 1981, p. 357).Teachers' and students' sometimes differing rationales make teaching challenging. "There is no lack of exercises in which the pupils experience what the numbers and their relations stand for. But it is often a coincidence whether or not the use of mathematics proves to be of any significance to the pupil" (p. 361). This depends on whether the student takes up the object/motive of teaching, and if the teacher's rationales matches the students'.

The school leaders and teachers want to teach mathematics that has use value for the students. They want the students to learn and experience mathematical meaning and the universality of mathematics. However, it can be challenging to teach the use value, meaning, and universality of mathematics. The mathematics subject content is stated in curriculum goals, which school leaders and teachers do not always feel match the mathematics they want to teach. This influences their teaching practice. The close connection between the curriculum goals and the textbook also affect teaching practice. They feel they need to finish the textbook to make sure they cover the syllabus. Teaching practice also connects to school leaders' and teachers' competences and knowledge. They experience increasing subject requirements, but these do not always correspond with the school leaders' and teachers' own rationales and perceptions about competences and knowledge important for teaching. Hence, school leaders and teachers experience contradictory rationales for teaching with respect to

their understanding of ML. Their contradictory rationales arise according to their relation to the curriculum, the students, the textbook, and the policy makers.

Concluding Remarks

In this article, I reported school leaders' and teacher rationales for teaching with respect to their understanding of ML. Although ML does not have an equivalent notion in Norwegian language, the school leaders and teachers seem to recognise the ideas connected to ML. They see ML as a desirable outcome, and an object/motive, of schooling. However, the specific content, actions and goals of the teaching and learning activity cause some challenges. Hence, the school leaders' and teachers' experience contradictory rationales for teaching for ML when it comes to use value, meaning, teaching practice, teacher competences and knowledge, and universality.

School leaders and teachers connect ML to their S-rationales for teaching mathematics. The S-rationales concern the use value of mathematics. School leaders and teachers want the students to recognise the role mathematics plays in the world, and to use mathematics as a tool for solving problems. Meaning is also part of their S-rationales for teaching mathematics. They see mathematics language, reflection, and conceptual knowledge as important to understand the world.

I-rationales are also connected to teaching for ML. These are connected to teaching practice. Curriculum goals do not always support school leaders' and teachers' goals to teach use value and meaning. Additionally, the school leaders and teachers experience increasing requirements for subject specialisation. These are connected to teacher competences and knowledge. They perceive these requirements as I-rationales because they are not directed to helping them become better teachers and improve their teaching pedagogy, but rather as qualification.

The school leaders' and teachers' rationales are concerned with all areas of teaching, not just subject issues. There seems to be challenges related to the overlap between school leaders' and teachers' S- and I-rationales for teaching for ML. This may suggest that ML is difficult to both understand and teach in a way that is consistent with curriculum goals, policy expectations, their own convictions, and students' requests.

This study focuses on rationales at the school leader and teacher level based on interviews. It isa multiple case study based on a convenience sample and the results are not generalizable. However, this study of teachers' and school leaders' rationales may contribute to knowledge about the complexity of teaching and learning mathematics in general, and teaching for ML in particular.

To teach for ML, school leaders and teachers need to be conscious about their rationales for teaching and to see the totality of mathematics education in addition to the particular lesson. There is a need for further research on this issue.

Further research should also focus on teachers' rationales related to their teaching practice. This may provide fruitful insight on how to teach for ML, and possible challenges. It will also be of interest to study students' rationales for learning with respect to ML. Student learning is what teaching is all about. By gaining more insight in teachers' rationales related to their teaching practice and students' rationales for learning mathematics, we might get closer to answering the question of how to teach for ML.

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