Exploring Affordances of an Online Environment: A Case-Study of Electronics Engineering Undergraduate Students' Activity in Mathematics

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Abstract Online learning environments are being used for teaching and learning of mathematics at university level. Exploiting the potential of digital technology, these Internet-based environments administer computer-generated homework, assistance and feedback for students. This article presents a case-study of a small group of undergraduate engineering students' learning activity in mathematics in an online environment. The study focuses on students' interactions with the online environment to make sense of the affordances of this environment. Utilizing multiple sources of data aid in analyzing the intentional and the operational aspects of students' interactions with several resources in this environment. With regard to both of these aspects, the affordances are thus viewed as features of the environment which support students' engagement with the mathematical tasks. The analyses show that the students incorporated several online resources for solving the tasks posed in the automated system. Students met requirements of final answers in the automated system through varying sequences of mathematical operations for the posed tasks. The conditions of the automated system as well as the rules of the collective activity system played a role in students' interactions with the mathematical tasks.

Keywords Online learning activity · Calculus · Engineering students · Interaction with resources · Affordances

Introduction

In recent years, online education has become a common feature of university level courses (Rosa and Lerman 2011). While several Internet-based applications are being employed to facilitate the process of teaching and learning of mathematics, personalized learning environments (PLEs) mark the latest trend in e-learning (Borba et al. 2016; Gadanidis and Geiger 2010). The PLEs represent the automated online systems which not only deliver the instructional materials but also provide tailored assistance to students. So far, there is a dearth of research exploring the potential of such environments for students' learning of mathematics and students' interactions with these online environments (Borba et al. 2016; Webel, Krupa, and McManus 2017).

To address these gaps, this article seeks to characterize undergraduate engineering students' activity in an online learning environment (Engeström 1987; Leont'ev 1978), which involves Pearson's MyMathLab (MML) as a PLE and a collection of electronically accessible resources (e.g., tutorial videos, notes). MML is an automated system which serves as an online platform for homework and assessments for students and provides assistance and feedback through its built-in functions. The aim of this article is to illuminate the affordances of this environment for students' learning activity.

An online environment (or the PLE) has previously been defined as the collection of "tools, artifacts, processes, and physical connections that allow learners to control and manage their learning" (Borba et al. 2016, p. 602). Learning in such an environment involves "focusing on the appropriation of tools and resources by the learner" (Buchem, Attwell, and Torres 2011, p. 1). In this article, I will focus on students' interactions with the constituent resources of the environment during their online learning activity in mathematics.

Students' Activity in an Online Environment

This study adopts the theoretical perspectives of cultural-historical activity theory (CHAT) (Engeström 1987; Leont'ev 1981) which is rooted in the sociocultural theory of learning and development (Vygotsky 1978). The concept of activity was introduced by Leont'ev (1978) to represent the subject-object interaction mediated by tools. Mediation refers to the intermediate position of tools between the subject and the object of an activity. An *activity* is realized when a *subject*, an individual or a group, acts on an *object*, through *tools*, in order to transform it into an *outcome*. The *object*, material or ideal, is closely linked to the need behind the *activity* and differentiates one *activity* from another. Leont'ev devised a theoretical model explaining macrostructure of human activities (Fig. 1).

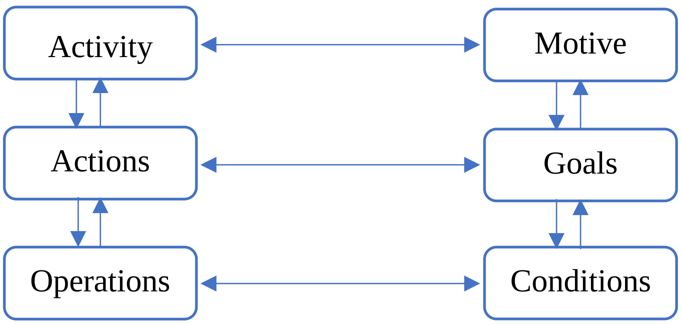


Fig. 1 Hierarchical levels of an activity (Leont'ev 1981)

In this model, Leont'ev (1981) discerned three hierarchical layers of human functioning at which an *activity* can be analyzed: the *activity* itself, the *actions*, and the *operations*. At the top level, the whole *activity* is viewed to be directed towards the *object*, which serves as the driving force or *motive* for the activity. It is through the lower levels that the *object* is transformed into the desired outcome. The middle level corresponds to goal-directed actions which realize the activity; the *goals* and *actions* represent the functions formerly merged in the motive. The bottom level concerns the *operations* "which depend directly on the conditions under which a specific goal is to be achieved" (Leont'ev 1974, p. 27). The nature of *operations* is also related to the conditions of the tools in use. Initially, the *subject* performs an *action* being conscious of the minute details concerning its execution. With enough practice, the *action* takes the form of a subconscious *operation*. The newly formed *operation* becomes part of another *action* which has a broader scope. If *conditions* concerning the execution of this *operation* change, it rises to the level of conscious *action* again. These changes are also resonated at the upper level of *activity* where the *object/motive* is reflected, questioned and transformed accordingly. The boundary between these levels of *activity* is dynamic – changing and developing all the time.

Kuutti (1996) notes that action-operation dynamics portray a basic feature of development in human functioning, and "to become more skilled in something operations must be developed so that one's scope of actions can become broader (p. 31)". Relevant to the tool-mediated learning actions in mathematics, Leont'ev (1974) specified, "when one uses a calculating device to solve a problem, the action is not interrupted by this extracerebral link; the action is realized through this link, as it is through its other links" (p. 27). Regarding operations, he wrote, "assume that a man was confronted with the goal of graphically representing some kind of dependences . . . . to do this, he must apply one method or another of constructing graphs – he must realize specific operation" (Leont'ev 1978, p. 66).

In this article, undergraduate students' (*subject*) activity in a Calculus (*object*) course mediated through several resources in an online environment is under consideration. The notion of resources corresponds to Wartofsky's primary artifacts, "those directly used in . . . production" (Wartofsky 1979, as cited in Engeström 2014, p. 49)", in accordance with Anastasakis, Robinson, and Lerman (2017). In the present case of students' activity, such production may be understood as to reaching the goals of the actions like solving the tasks. Leont'ev's model of activity (Fig. 1) is utilized in analyzing the structure of students' activity in relation to their interactions with resources. Leont'ev (1978) discussed that "a tool considered apart from a goal becomes the same kind of abstraction as an operation considered apart from the action that it realizes" (p. 65). In this view, I link students' use of resources with the action-goal layer (Fig. 1) i.e. the actions performed by using various resources and associated goals with incorporating those resources. The operation-condition layer is then analyzed to make sense of the nature of (mathematical) operations conditioned by those resources.

Leont'ev (1981) asserted that analysis of human action is not complete without considering it into the system of societal relations, and he described human activity to be "a system in the system of the social relations" (p. 47). On these lines, Engeström (1987) devised a unified model of collective activity system incorporating multiple mediations through tools and social relations in human activities (see Fig. 2). Engeström (2014) wrote, "the object-oriented and artifact-mediated collective activity system is the prime unit of analysis" (p. xvi). The model (Fig. 2) represents the "most simple unit that still preserves the essential unity and the integral quality behind the human activity" (Engeström 2014, p. 65).

According to Engeström (1990), the upper part of this model refers to individual tool-mediated actions which are "the visible tip of the iceberg of collective activity (p. 172)" whereas "the hidden bottom part (p. 172)" refers to societal mediations in the form of rules, division of labor, and community. The rules represent the explicit or implicit norms which needs to be followed during an activity and thus affect the realization of the activity. Division of labor specifies the way in which whole task of the activity is divided among the participants to reach the outcome. The community signifies the other human beings with which the subject has direct or indirect relations.

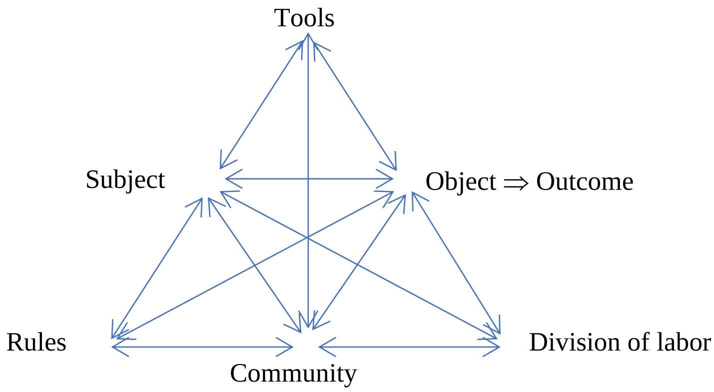


Fig. 2 The extended triangular model of human activity system (Engeström 1990)

According to Cole (1996), "in activity theory . . . contexts are activity systems" (p. 141). In this study, the online learning environment is characterized using Engeström's model (see Fig. 2). Engeström (2014) suggests analyzing the relationships between elements of the activity system by considering the systematic whole. In this regard, the model facilitates in analyzing the dynamics of students' activity with regards to features of the learning environment.

Engeström (2014) specified, "we may well speak of the activity of the individual, but never of individual activity" (p. 54). With reference to Roth (2012), the dialectical stance of CHAT "allows us to understand the person as a singularity and as collective phenomenon simultaneously without reducing it to one of its observable moments" (p. 97). In this sense, a student is considered as both an individual and a collective subject whose activity is regulated by features of the joint activity system.

An Activity-Theoretical Perspective on Affordances

The concept of *affordance* was introduced by Gibson (1977) to denote the action possibilities provided by the environment to an agent. The affordances are constituted in the meaningful relationship between the agent and the environment. According to Greeno (1994), the affordances are realized when attributes of the environment relate to the capabilities of the agent in such a way that an activity is supported. This view of affordances concerns the operational aspects of activity. Bærentsen and Trettvik (2002) argue for an activity-theoretical perspective for studying affordances of the environment. This perspective suggests considering the needs as well as the capabilities of the agent in relation to attributes of the environment. According to Bærentsen and Trettvik (2002), the affordances of computer software and programs should be studied in the processes of object-oriented activities of the intended users of such programs. Also, in addition to operational aspects, motivational and intentional aspects of users' activities should also be considered.

Studying affordances for students' mathematical activity in an online environment is essential to figure out the learning opportunities in such environments. Leont'ev (1981) argued that the external objective activity has particular implications for the inner psychological activity as, "mental reflection or consciousness is generated by the agent's objective activity" (p. 52). With regard to the role of the environment, Leont'ev (1981) stressed that "society produces the activity of the individual it forms", in the sense that, "social conditions carry the motives and goals of the activity, its means and modes" (p. 48). However, he emphasized that human activity is not the simple personification of the relations of society and its culture. There are complex transformations which need to be discovered through investigating the genesis of activities.

With these considerations, the research questions posed in this study are as follows.

RQ1: How do a small group of undergraduate students interact with an online environment during their learning activity in mathematics?

RQ2: In what manner does this environment afford students' learning activity in mathematics?

To answer RQ1, I first characterize the collective activity system in the present situation (Engeström 1987). Next, I investigate the structure of students' activity with regards to their interactions with this environment (Leont'ev 1981). In particular, I explore students' goals for which they use certain resources in their learning actions and analyze how this environment conditions the operational level of students' activity. Consequently, I discuss the answer to RQ2 i.e. the affordances of this environment in view of intentional and operational aspects of students' activity.

Previous Research Concerning Online Environments in University Mathematics

Several studies have sought to evaluate the impact of automated systems quantitatively by analyzing examination grades, cost effectiveness, and passing rates (e.g., Callahan 2016; Jonsdottir, Bjornsdottir, and Stefansson 2017; Kodippili and Senaratne 2008; Potocka 2010). Krupa, Webel, and McManus (2015) compared the impact of computer-based (CB) and face-to-face (F2F) instruction in an intermediate college algebra course. They used a quasi-experimental match design with the sample consisting of three levels of participants enrolled in the course. At the first level, they compared the exam results of two large groups ($N_{F2F} = 192, N_{CB} = 134$), and the second level included some other student-level predictors ($N_{F2F} = 73$, $N_{CB} = 50$). The third level concerned the quantitative analysis of students' solution strategies for some open response ($N_{F2F} = 38$, $N_{CB} = 24$). The results on the first two levels showed that students from the CB group performed better on the exam whereas they showed limited ability to interpret and relate algebraic equations to contextual situations. To follow up, Webel et al. (2017) investigated the implementation of a Math Emporium (ME), a model for teaching and learning of mathematics using computer-based programs, in an introductory college algebra course using mixed methods. They investigated: (1) whether the emporium is more helpful to a certain group of students; (2) the nature of mathematical learning in this setting; and (3) the students' perceptions about the emporium style courses. Webel et al. (2017) concluded that the emporium style served the students with higher mathematics achievement and those who less strongly believed that mathematics is about memorizing. Their findings suggested that the setting enabled students to focus on getting correct answers more than developing algebraic meanings. Regarding students' perceptions, they found that some students did not like the autonomy and flexibility offered by this setting. These findings led the researchers to question if examination grades and passing rates are the appropriate indicators of the impact of such settings. They recommended that future studies should focus on students' interactions and mathematical reasoning afforded by these environments.

With regards to students' activity in online environments for mathematics, Cazes, Gueudet, Hersant, and Vandebrouck (2006) focused on university students' strategies for different kinds of tasks posedin three Electronic-exercise bases (EEB) – similar to automated system. Through direct observation of individual students' work and electronically generated activity logs of their activity in these programs, they observed that students often developed unexpected strategies. The study took place during the experimental implementation of such environments and the conditions within each automated system affecting students' solution strategies were discussed.

From a CHAT perspective, Rønning (2017) explored the influence of such an automated program (Maple T.A.) on undergraduate engineering students' engagement with mathematics. The data set in this study included six surveys of large cohorts ($n > 500$) followed by focus-group interviews between the years 2013 and 2016. Students' responses were used to analyze the factors pertinent to the collective activity system affecting their *actions* while participating in the activity. Rønning (2017) discussed that the system promoted quest of correct answers among students which hindered the deep learning of mathematics.

The brief literature review presented above indicates lack of research on students' interactions with the resources during their learning activity in online environment in mathematics. In particular, the analysis of students' activity in such settings taking into consideration the macro and micro-level factors (cf. Jaworski and Potari 2009) has, to the best of my knowledge, not been done so far. As an example, in case of a blended learning environment, a partially relevant study (Anastasakis et al. 2017) focused on students' interactions with several resources at the action-goal layer of their activity i.e. the type of resources used by undergraduate students and the relationship between students' goals and their choice of resources. Anastasakis et al. (2017) surveyed a cohort of 201 engineering undergraduate students followed by interviewing 6 students to get a deeper insight. From the survey responses, they found that students incorporated institutionally provided resources dominantly but also used some other resources such as online videos, WolframAlpha, and online encyclopedias. They concluded, from the analysis of interviews, that students' choice of resources was driven by exam-related goals. The operational details of students' activity were not addressed in this study.

Differentiating between different types of resources (e.g., social, material, digital), a strand of research (see Gueudet and Pepin 2016) focuses on students' use of resources in mathematics. From this strand, a relevant report in the context of university mathematics by Gueudet and Pepin (2018) investigated how university students interact with several resources in their general mathematical work. Through case-studies, Gueudet and Pepin (2018) observed discrepancies between students' actual use of several resources and the lecturers' expectations of students' use of those resources. With regard to evaluating impact of automated systems on students' learning, Gueudet (2006) suggested that the students' activity with such resources should be observed at two levels: the particular exercise level when students solve the task, and the global level i.e. patterns of work during a session.

This article adopts a holistic perspective on students' activity with special attention to students' interactions with the resources in an online environment. That is, the micro aspects concerning operational characteristics (Fig. 1) of students' activity and macro aspects of the collective activity system (Fig. 2) have been combined.

Methodology

Context

This study was carried out at a Norwegian university administering several engineering programs at undergraduate level. The students from an undergraduate electronics engineering program participated in this study. An online learning environment was created for the students in their calculus course. This course spans both semesters of the first year of the program. The study took place during the second semester.

In this course, instruction, homework and assessments were administered electronically. Lectures were provided to students in the form of tutorial videos. The tutorial videos were created by the lecturer and were recorded using a document camera capturing his writing-activity on paper accompanied by the explanation. Each tutorial video dealt with specific topics from the textbook and contained explanations of those topics. The written notes associated with these videos were also made available for students through the learning management system (LMS) used at the university. Face-to-face interactions with the lecturer were possible in case students required additional help, and they could contact the lecturer electronically or in person.

Homework and assessment were conducted through Pearson's MyMathLab (MML), based on the textbook *Mathematics for engineers* by Croft and Davison (2015). Each week's homework in MML was linked to specific sections in the textbook. MML aids the users in solving tasks through two embedded functions: 'help me solve this' and 'view an example'. The former option breaks down a similar task into several steps and prompts students to perform calculations in each step. The latter option illustrates a worked example. In addition, it provides feedback by indicating that the answer is correct or wrong. In case the answer is wrong, it offers hints about the solution procedure.

Three formative tests were administered through MML in this course. The course involved a group project in which students were required to make a question bank on the topic of integration and program those questions using Maxima – a computer algebra system (CAS). The final examination was also in a digital format allowing the use of resources. The final grade was calculated from a weighted average of tests, project work and the final examination.

Research Design and Methods

This research is founded within a naturalistic research paradigm (Guba and Lincoln 1982) in the sense that participants' everyday work in a natural setting is observed. Four students (pseudonyms: Per, Jan, Tor, and Ole) volunteered to participate in this research. Following a case-study research design

(Yin 2014), the case under consideration is the activity of the small group of participants in the online learning environment.

In order to understand an activity system, Engeström (1999) recommended that the researcher should look at the system from the above and at the same time through the eyes and interpretations of a subject, thereby complementing the system view and the subject's view. Nardi (1996) articulated general methodological implications deriving from the principles of activity theory for empirical research in the field of human-computer interaction. First, the frame of analysis should be long enough to understand the *subject's object*. This implication arises from the claim that the activities are long term formations and the objects are transformed into outcomes through a process of several phases. Second, the attention must be given to broad patterns or bigger picture of the activity instead of narrow episodic fragments. The small episodes may prove useful, but not in isolation from the overall situation. Third, various methods for collecting data should be used without unjustified reliance on any one form of the data. Fourth, the researcher should be committed to understand the object from the subjects' perspective.

The methods used for data collection in this study are in line with the considerations discussed above. Multiple methods including observations of students' group work, weekly journals, semi-structured interviews and field notes were used to collect the data. Weekly journals and interviews facilitated in gaining students' input regarding their interaction with the resources. Observational data provided micro details of students' activity concerning mathematical operations and corresponding conditions in this environment. The data were collected during the spring of 2017.

In weekly journals, students were asked to specify the resources they used and how they used each resource in their work. Only three of the participants (Per, Jan, and Tor) submitted the journal regularly. For observations, participants were requested to work together on campus for approximately an hour-long session in one week. During these sessions, they worked on their weekly assignments (homework, tests or the project) and communicated with each other in Norwegian. With the progression of the course, the participants' activity was becoming increasingly computer-based. I asked them to record their computer screen activity using Camstudio,[1] a freeware screen recorder. Semi-structured interviews were held to complement the data from journals and observations to gain further details about their usage of resources. The interviews were conducted in English. I kept field notes when I visited the students on campus.

Data Analysis

The field notes, semi-structured interviews, students' journals and my own observations aided in identifying elements of the collective activity system (Fig. 2) in the present setting (see Table 1). The rules, community and division of labor were mainly identified from my observations in the form of field notes and through interviews with students. The resources and the outcome were identified through students' journals.

For the analysis of the action-goal layer in Leont'ev's model (Fig. 1), weekly journals and interviews served as the main sources of data. The individual students' journals were analyzed to identify various

[1] http://camstudio.org/

manners in which each resource was used by the group of students collectively. In the first step, I extracted each students' descriptions linked to each resource from every journal and listed them across the resources in a single document. In the next step, I discerned students' goals and actions linked with each resource from those descriptions. Leont'ev (1974) defined an action to be "a process that is structured by a mental representation of the result to be achieved, i.e. a process structured by a conscious goal" (p. 23). In this sense, a statement such as "to try to understand how to calculate the length of a line" refers to the *goal* that the student wanted to achieve by incorporating a particular resource in her action. The statement such as "I got the questions from the book as well as some help with formulas" points to the actions mediated through the book. In some cases, I delineated the actions and the goals from single statements where applicable. Often, students also described some other aspects regarding their general manner of work organization such as their strategies, deviations in plans, and comments regarding the nature of resources. I extracted students' comments about the resources to see how they perceived each resource. The collective summary of the use of resources is presented in Table 2. The entries in Table 2 are not shared among the three participants.

Regarding the operation-condition layer in Leont'ev's model (Fig. 1), the operational details are considered as "not often consciously reflected by the subject" (Engeström 2014, p. 54). Nardi (1996) discussed that some minute details about the operations can be retrieved through careful questioning during interviews. In this study, students' responses in the journals and interviews did not account for the operational details. For such details, video-recorded observations of the group work were utilized.

During the group work sessions, students worked independently for significant amount of time interacting with their computer screens. The discussions were initiated when they faced some problem, for instance, when the feedback from the program was difficult to comprehend. For the analysis, I first searched for the episodes with relatively active communication among the group members. Five out of seven group work sessions were translated into English by a native speaker of Norwegian. Further, I selected one episode for the purpose of illustration from the twelfth week when the activity system had developed enough. The episode is selected as it involves: the use of various resources in participants' work, and varying conditions in the sequence of tasks thus ensuring variation and richness in mathematical contents. I utilized the screen recording as well for the analysis of this episode.

Results and Discussion

The following sections present the answer to RQ1. The answer to RQ2 is presented in the last section.

Characteristics of the Collective Activity System

The analytical account of the characteristic elements of the students' collective activity system (see Fig. 2) in the present setting is given in Table 1. The collective activity system is conceptualized at the level of mathematics course. Therefore, the object of the activity is considered to be including topics covered in the course (see Table 1). In addition to the provided resources, the three students reported using a variety of other resources during their learning activity (see Table 1). Division of labor in this case made students in charge of their own learning process. Students had more choices to make in terms of selecting resources, suitable time, and place to work. The lecturer's duties in the course were mainly performed electronically. The explicit rules at the level of activity, mainly the test-deadlines, aided in maintaining students' pace with the course. The test scores were also included in the aggregation of the final grade; therefore, students were motivated to complete their homework in order to take tests before the deadlines. The implicit rules correspond to the specifications in MyMathLab, i.e. the manner in which it conditioned the micro interactions at the level of tasks. For instance, the number of attempts allowed, the form in which it required solution of tasks, the nature of feedback, and the syntax in which it accepted the answers.

Table 1 Elements of the collective activity system

Elements of activity	Analytical description
Subject	A group of electronics engineering students
Tools	Tutorial videos, Textbook, MyMathLab features, lecturer's notes, Maxima, own notes, MatRIC TV[a], YouTube, GeoGebra, STACK environment[b], WolframAlpha[c], Mathway[d] other calculators, and Internet (Google search)
Object	Calculus (differentiation, applications of differentiation, integration, applications of integration, and sequences and series)
Outcome	Learning Calculus, passing the exam, getting good grades
Rules	Work on homework, test deadlines, final digital examinations, specifications in MML
Division of labor	Students' work according to the rules of the course taking the responsibility for own learning. Lecturer organizes the online course making use of MML program by integrating it with the tutorial videos. MML features aid in distribution and collection of homework and providing instant help and feedback to students; other resources (Maxima, Internet, calculators) affect the manner in which students engage with mathematical tasks.
Community	Other engineering students, lecturer

[a] https://www.matric.no/tv; An online repository of short mathematical videos for first-year undergraduate students in Norway aimed to support their transition from upper secondary school to university; [b] A computer aided assessment platform which they were required to use in their project; [c] https://www.wolframalpha.com; [d] https://www.mathway.com

Students' Interaction with the Environment – Actions, Goals and Resources

The collective summary of three participants' weekly journals illustrating the action-goal layer in participants' activity (Leont'ev 1981) is presented in Table 2.

Regarding the provided resources, Per and Jan reported textbook use repeatedly in their actions as a means to get questions (during their project), to find mathematical formulas related to the tasks, and to acquire help on specific topics. Tor, however, did not report using the textbook in the journals, he rather reported using the lecturer's notes. The only form of lecturing in this course was through the videos, and the goals associated with the use of this resource were linked with learning of certain mathematical topics. For instance, Jan used the videos with the goals: "to try to understand how to calculate…", and "to understand the calculation behind the math". I noticed a gradual decrease in the use of videos through the students' weekly journals, and I therefore held a semi-structured interview to know more about this trend. I asked the participants regarding their manner of working on the homework tasks to which Per responded first, followed by Tor and Jan.

Per: These topics I think are quite hard to learn all by yourself. When I get a new topic, I first try to solve it myself, if I can't do that I try to look at the examples in MML… and if I don't completely understand the examples I take a look at Olav's (lecturer) video…mainly the examples' videos because then I get to see the practical kind of way to do…to solve questions.

Tor's response was somewhat similar as follows:

Int: Did you use any video while working on last week's homework?

Tor: No, I think MML seemed sufficient so far.

Int: Ok. So which resource did you use for getting introduction to the new topic?

Tor: I tried first MML but it went fine so I just carried on. …I check the notes and watch the videos if I get stuck.

While Jan responded as follows.

Jan: I did not watch that many videos. I mostly use MML and just see the examples …and if I can't get it from there then I go to…to the book because it is faster… and eventually go to the videos if I do not get constructive help from there.

These excerpts from the interviews indicate participants' preference for MML features. As Per mentioned, "When I get a new topic … I try to look at the examples in MML". Tor stated, "MML seemed sufficient so far" and "I tried first MML but it went fine" while Jan mentioned "I mostly use MML". Tor wrote in a journal, "it's a more powerful tool and it's easier to attain help and information online". This preference for MML may be attributed to the immediate help available in the program for the tasks at hand whereas in the textbook and in the videos, students were required to search for the relevant information themselves.

Wertsch (1998) argues that the analysis of the goals of mediated action depends on the circumference of the context under consideration. In the case of multiplying two numbers, he explicated, the goal will be " 'to get the right answer within the confines of a particular way of setting up the problem' (i.e., using Arabic numerals, using the syntax of multiplication outlined, not using a calculator, and so forth)" (p. 33). Moreover, "the goal of obtaining the right answer needs to be coordinated with other aspects of the sociocultural setting as well" (Wertsch 1998, p. 34).

In this study, students' goals linked to the use of WolframAlpha, Mathway, and Maxima point to features of the collective activity system (see Table 2). The online resources WolframAlpha and

Table 2 Incorporation of resources in participants mathematical activity - Summary of students' journals

Resources	Goals for using each resource	Performed actions	Students' comments about resources
Textbook	To find formulas for specific topics, to understand a topic	Read through the book, found formulas to work on homework, got questions from book (during project)	
Maxima	To avoid calculating everything by hand, to solve problems in an easy way, to make the work easier in the long run	Programmed tasks in Maxima for the project, used while doing homework, solved tasks using Maxima	Programming in Maxima is hard but when it is done, all the problems are easy to solve
MatRIC videos	To recall certain topics	Skimmed through the video at an amplified speed	
MyMathLab	To learn how to solve problems, to get inspiration for making questions in the project, to get an overview before taking test	Worked on homework, learnt specific topic, solved some questions with higher difficulty	Powerful tool, easier to get help and information online
Lecturer's notes	To get the general idea of the topic		Tailored for the tasks at hand, the most relevant piece of information
WolframAlpha	To solve problems by using shortcuts	Used as a shortcut to get answers, compared answers got from Maxima, got help with solving difficult tasks	Easier to use than Maxima, faster than using calculator, useful when the answer is in the form of expression instead of numbers
YouTube videos	To recall a certain topic	Watched Maxima tutorials	
Mathway and other online calculators	To solve tasks in assessment	Solved questions	Severely increase the probability to get the correct answer, and therefore the overall score.
STACK	To make questions in STACK	Made some questions in STACK	
Internet	To learn Maxima, to search for how to solve the problems		
Tutorial videos	To learn rules and methods, understanding a specific topic, to recall previously done content	Watched to get information to complete homework	Easy to understand through videos

Mathway aid in the task solving processes. Tor reported using WolframAlpha and Mathway for solving the tasks in homework and tests. WolframAlpha was incorporated by Per and Jan to double check the answers, to solve the tasks by short-cut methods, and to get help with the difficult questions. Regarding Maxima, students learnt programming in Maxima as a part of the course, which they later used in their task solving activity in MML. Per and Jan started to make programs for each task in the homework with the goal to liberate themselves from calculations. Per inscribed in a weekly journal, “(I) used Maxima to make a program to solve the problems in an easy way. This is hard to make, but when it is done, all the problems are easy to solve”. Tor wrote, “if I could make a template for each question, then I would have severer [*sic*] advantage on the upcoming exam”.

Students’ use of these computing tools can be ascribed to the rules of the activity system. Within the confines of this setting, students had to learn mathematics with regards to the implicit conditions in MML. At the same time, they also had to take part in the digital examination, which was the explicit rule of their activity system. From students’ reports, it appears that the use of these resources let the students meet implicit as well as explicit rules of the activity system. Students’ motive in the activity is thus taken as to learn mathematics and to perform well on the tests and in the final examination.

The nature of Mathematical Operations in Students’ Online Learning Activity

This section focuses on incorporation of several resources (Maxima, GeoGebra, Internet and MML help) in mathematical operations in students’ activity (Leont'ev 1981). Below, I analyze a part of a group work session in which the participants began working on their weekly homework dealing with applications of integration. I divide the analysis with respect to the three kinds of tasks involved in the homework. While narrating the group work, I follow Per’s screen recording since he led the activity in the sense that he was ahead of the other participants.

Engaging with the Integral as Limit of a Sum. The first task required using the limit of sums for calculating the area under a curve (see Fig. 3). This task involves identifying the area under $y = x + 1$ between $x = 0$ and $x = 9$, dividing it into rectangles of equal width, and summing the areas of these rectangles. Applying the limit to the number of rectangles in the summation gives the definite integral $\int_0^9 (x+1)dx$. This value then represents the area under the curve. In MML, the worked example for this task suggested the sequence of involved mathematical operations.

Find the area under $y = x + 1$ from $x = 0$ to $x = 9$ using the limit of a sum.

Fig. 3 The first task

In this task, Per began by performing an operation in Maxima as observed through his screen recording (see Fig. 4). He entered the obtained number into MML which affirmed him that his answer was correct.

(%i1)	integrate (x+1, x, 0, 9);
(%o1)	$\frac{99}{2}$

Fig. 4 Per’s solution strategy using Maxima

Jan, who was working with his paper notebook while getting questions from the MML opened on his computer screen, posed a question regarding the first task to which Per responded as follows.

02 Per: […] You must take the integral from 0 to 9. Or from 0… From the smallest value to the largest value.

03 Jan: Yeah. You are to *split it up* [emphasis added].

04 Per: I don't think so.

The discussion stopped at this point and Jan continued working in his notebook. It appears that the two participants were performing different operations. Per's operation in Maxima let him find the required area by calculating the involved integral whereas the task required using method of the limit of sums. The automated system (MML), being the main source of help and assistance in this case, provided Per feedback that his answer was correct. Jan seemed to be following the steps suggested in MML (also in the textbook) as he pointed towards dividing the area into rectangles (03). As Per had reached the immediate goal of getting the final answer, he did not agree with Jan (03). From (04), it seems that Per was unaware that he missed the mathematical operations in this task.

The next three tasks in MML also concerned using the limit of sum method for calculating area under different curves. Per solved these tasks using the same command in Maxima.

Engaging with the Disk Method. The next task in MML dealt with the application of integration for finding the volume of a solid formed by revolving a given area around an axis (Fig. 5). This task involves identifying the area to be revolved bounded by $y = x^2$, $x = 1$ and $x = 7$, and then dividing it into the strips of infinitesimal width, say, dx. These strips, upon revolving around the x-axis, take the form of cylindrical disks of radius y and height dx. The volume of one such disk becomes $\pi y^2 dx = \pi x^4 dx$. The limit of the sums of these volumes becomes the integral $\int_1^7 \pi\, x^4\, dx$, which gives the volume of the whole solid.

Find the volume of the solid formed when the area under $y = x^2$ between $x = 1$ and $x = 7$ is rotated about the x-axis.

Fig. 5 The disk method task

```
(%i9)     integrate (%pi*(x^2)^2, x, 1, 7);
(%o9)     (16806π/5)
```

Fig. 6 Per's solution in Maxima

Upon getting this task in MML, Per's first action was reading in the book for a while where the disk method for finding the volume of revolution was given. Next, he calculated the involved integral by performing an operation in Maxima (Fig. 6) which resulted in the correct answer.

The next task was similar and Per obtained the solution by performing similar operation in Maxima for computing the integral. However, the subsequent task was phrased slightly differently (see Fig. 7).

Find the volume of the solid of revolution formed by rotating about the x-axis the region bounded by the curves $f(x) = 3x^2$, $y = 0$, $x = 1$, and $x = 4$.

Fig. 7 Another disk method task

This task asked for "bounded region" instead of "area under the curve". Therefore, it included four bounds on the area to be revolved instead of three in the previous tasks (see Fig. 5). In this sense, the conditions for reaching to the solution of this task were apparently different from the earlier tasks. In Per's actions, he adjusted his Maxima command which he used in the previous task (see Fig. 6) by halving the integrand (see Fig. 8). This action did not yield in the correct answer, and MML provided him the feedback (see Fig. 9).

```
(%i13)    integrate (%pi/2*(3*x^2)^2, x, 1, 4);
(%o13)    (9207π/10)
```

Fig. 8 Per's command in Maxima

Remember that, if f(x) is nonnegative and R is the region between f(x) and the x-axis from x=a to x=b, the volume of the solid formed by rotating R about the x-axis is given by $\int_a^b \pi\,[f(x)]^2\,dx$. Make sure that you are correctly setting up and evaluating the integral. Check your work carefully. Please try again.

Fig. 9 Feedback from MML regarding disk method task

After looking at the feedback for a while, Per plotted the curve in GeoGebra, and then removed the $1/2$ in his Maxima command. Per reflected on these actions later, which can be seen in the excerpt below.

16 Per: This exercise here (showing his laptop screen). You are to integrate that formula and find the volume.

17 Ole: Mm

18 Per: And then $y = 0$, then I thought, rather than rotating it the whole way, you know, should just rotate it down till $y = 0$ because that is here. (Illustrating the revolution while making a gesture through his hands)

19 Ole: Mm.

20 Per: But that wasn't it, it was just as we do. Like on the previous task.

Here, Per's action of intercalating the $1/2$ factor in the integrand in his Maxima command were based on his misinterpretation of the conditions of this task. Instead of considering $y = 0$ as a bound on the region to be revolved as specified in the task, Per considered it as a bound on revolution. He thought that the area had to be revolved in such a way that it did not need to go below the x-axis (18). Assuming that the revolution stops halfway, and then the generated volume will also be halved, he multiplied the integrand by $1/2$ (see Fig. 8) which did not result in the correct answer. He then excluded the $1/2$ factor and obtained the correct solution. The Maxima command now had become similar to the one he used in the previous task (see Fig. 6).

Although Per seemed aware of the revolution involved in these tasks, he could not realize the implications of the slightly different formulation of both tasks. As the same operation let him reach the solution in both tasks, he reached to the faulty conclusion that they needed to do the same (process) as they did in the previous task (20).

Engaging with the Shell Method. The next task concerned finding the volume of a solid using the shell method of revolution (see Fig. 10). This task requires the identification of the region to be revolved and dividing it into rectangles of infinitesimal width dy, as in the disk method. The rectangles should then be revolved around the x-axis in such a way that the solid formed is a cylindrical shell (instead of a disk) of radius y, height x, and thickness dy. The volume of one such shell is $2\pi yxdy = 2\pi y(16y - y^2)dy$. The limit of sums of these volumes becomes the integral $2\pi \int_0^{16} y(16y - y^2)\, dy$, which gives the volume of the whole solid.

Use the shell method to find the volume generated by revolving the region bounded by $x = 16y - y^2$ and $x = 0$ about the x-axis.

Fig. 10 The shell method task

In this task, Per adjusted the Maxima command from the disk method task (see Fig. 8) by replacing the integral to $16x - x^2$ (see Fig. 11). The integral formula and the limits of integration remained the same. He then entered the obtained answer into the MML window which responded that the answer was not correct and provided him the feedback shown in Fig. 12.

```
(%i15)      integrate (%pi*(16*x-x^2)^2, x, 1, 4);
(%o15)      (17703π/5)
```

Fig. 11 Per's command in Maxima

Volume, V, as determined by the shell method with rotation about a line parallel to the x-axis is: $V = \int_a^b 2\pi \begin{pmatrix} \text{shell} \\ \text{radius} \end{pmatrix} \begin{pmatrix} \text{shell} \\ \text{height} \end{pmatrix} dy$. The limits of integration are y-values at which $x = 0$ and $x = 16y - y^2$ intersect.

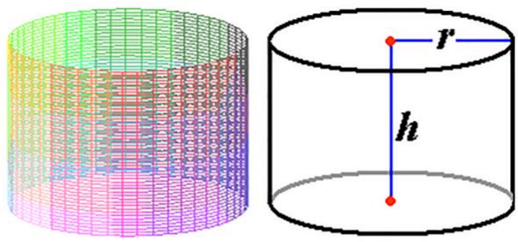
Fig. 12 Feedback in MML regarding the shell method task

Looking at the feedback for a while, Per opened GeoGebra and plotted the curve. He then searched on Google and found a Web page containing description concerning the shell method (see Fig. 13).

www2.bc.cc.ca.us/resperic/Math6A/Lectures/ch6/2/shell.htm

6.2 *Volumes of Revolution* (Part II) - Shell Method

In the disk method, we sum up the volumes of an infinite number of infinitesimally thin circular disks to find the total volume of a solid. The solid has been decomposed into stacked circular disks, and by integrating the disk volumes we obtain the total volume. In the washer method we decompose the solid into concentric "shells", each one a hollow tube infinitesimally thin. Here's one such shell, actually the outer tubular part of a right-circular cylinder:



The volume of a right circular cylindrical shell with radius *r*, height *h*, and infinitesimal thickness *dx*, is given by:

$$V_{\text{shell}} = 2\pi rh\,dx.$$

If one slits the cylinder down a side and unrolls it into a rectangle, the height of the rectangle is the height of the cylinder, h, and the length of the rectangle is the circumference of a circular end of the cylinder, $2\pi r$. So the area of the rectangle (and the surface of the cylinder) is $2\pi rh$. Multiply this by a (slight) thickness dx to get the volume.

Fig. 13 Web Page explaining the shell method

Meanwhile another participant, Ole, asked him about this task.

52 Ole: You didn't just put it into the calculator? (Referring to Maxima)

53 Per: No, it's something else, but it says nothing about it there.

54 Ole: Can't grasp why it is like that…

55 Per: Yes, $2\pi r$ times h. …That's not quite the same. (Reading from the Web page shown in Fig. 13)

In the next moment, he navigated back to the GeoGebra window and checked for the points of intersection of the curve with the y-axis (Fig. 14).

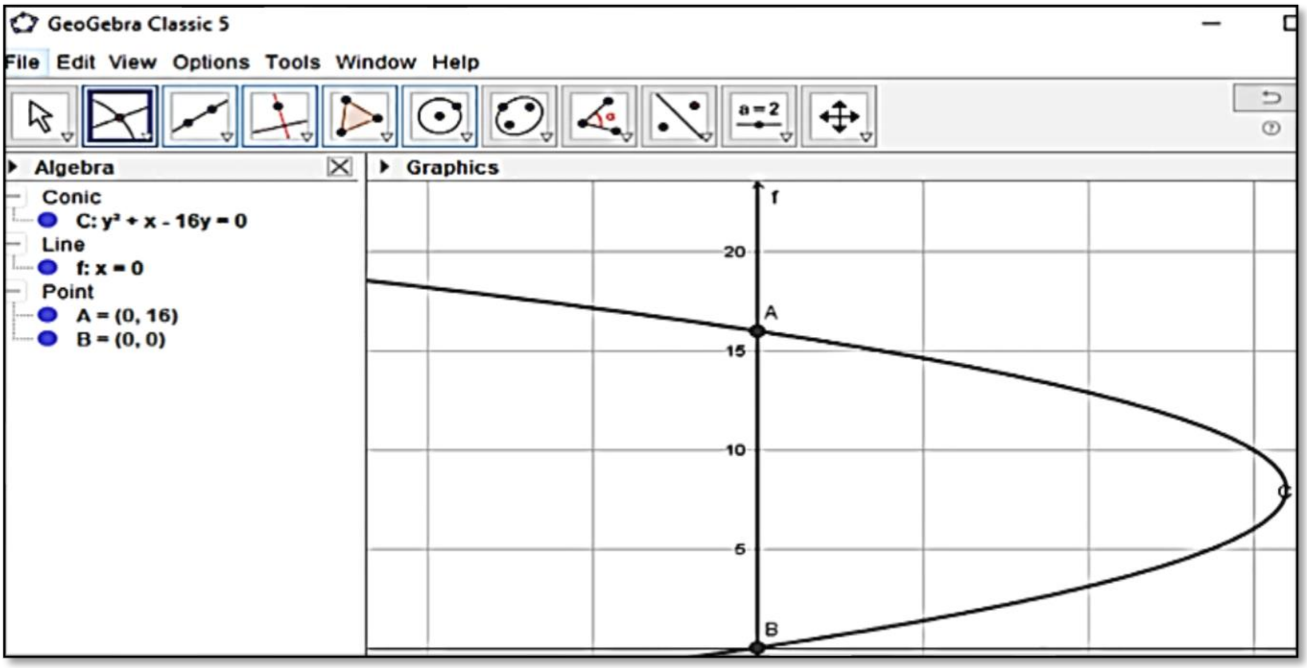


Fig. 14 Per's activity in GeoGebra

In his Maxima command (see Fig. 11), Per then changed the limits from 0 to 16 and inserted the term $2x$ in the formula which did not result in correct answer. Per navigated back to the Web page and scrolled down a bit (see Fig. 15).

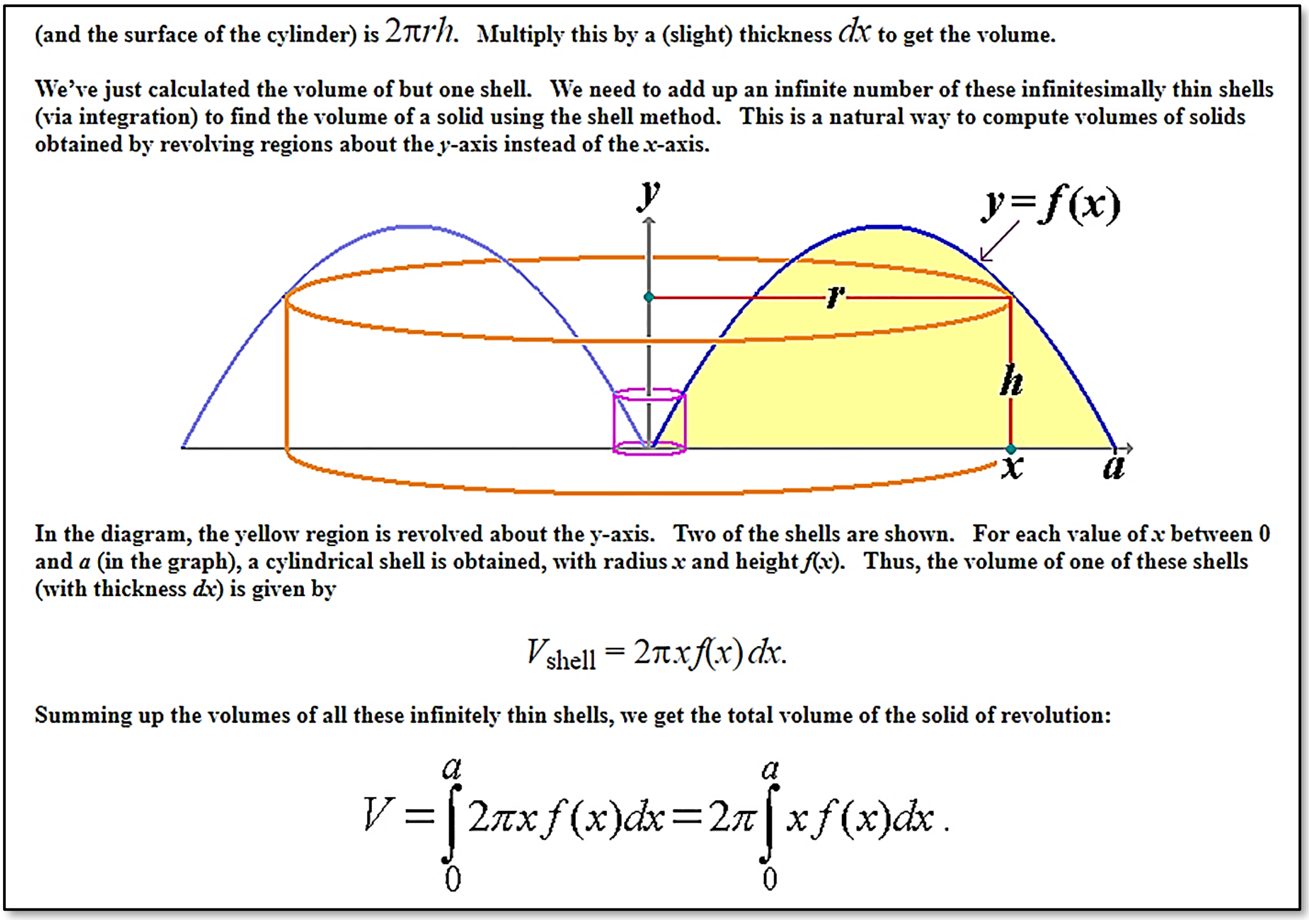
Jan posed a question at this moment.

66 Jan: Did you figure out that shell method?

67 Per: I'm reading on it, but I think I got it. The formula for it is $2\pi x$ times the function.

68 Jan: $2\pi x$ times the function. (Repeats the formula)

69 Per: But it is also like this. 2π …times r times h. Got to read a bit on it.


(and the surface of the cylinder) is $2\pi rh$. Multiply this by a (slight) thickness dx to get the volume.

We've just calculated the volume of but one shell. We need to add up an infinite number of these infinitesimally thin shells (via integration) to find the volume of a solid using the shell method. This is a natural way to compute volumes of solids obtained by revolving regions about the *y*-axis instead of the *x*-axis.

In the diagram, the yellow region is revolved about the y-axis. Two of the shells are shown. For each value of *x* between 0 and *a* (in the graph), a cylindrical shell is obtained, with radius *x* and height *f*(*x*). Thus, the volume of one of these shells (with thickness *dx*) is given by

$$V_{\text{shell}} = 2\pi x f(x)\, dx.$$

Summing up the volumes of all these infinitely thin shells, we get the total volume of the solid of revolution:

$$V = \int_0^a 2\pi x f(x) dx = 2\pi \int_0^a x f(x) dx .$$

Fig. 15 Scrolling down the Web page

Next, looking at the Web page, he removed the square in his Maxima command (see Fig. 11) and eventually got the correct answer. In the next moment, Ole again inquired about this question.

70	Ole:	Did you get exercise 8 right, with the shell method?
71	Per:	Yeah. I did it just now.
72	[]	
73	Ole:	Is it to find the bounds or something?
75	Per:	Yeah, you find that out by… where y and $x = 0$ intersect.
76	Ole:	So, you need to have $x = 0$ and $y = 0$?
77	Per:	It's like this, kind of. When it intersects there. (Points at his GeoGebra window exactly where the curve intersects y-axis (see Fig. 14))

In this task, Per began by trying the similar operation used in the disk method task which did not result in the correct answer. The feedback from the program offered the integral formula (see Fig. 12) which required using the *shell height* and *shell radius* for calculating its volume. For applying this formula, one needs to know *where* and *how* the shell is formed, which was not discussed in the feedback.

It may be due to this lack of clarity in the feedback which conditioned to Per's action of searching the Internet. He opened a Web page which contained details regarding: the mathematical formula for calculating the volume of a shell $V_{shell} = 2\pi rhdx$, discussion of how the shell is formed by revolving an area around the y-axis, and the derivation of the integral formula, $2\pi \int_0^a xf(x)dx$ (see Fig. 13 and Fig. 15). Per initially tried to comprehend which one of these two formulae was relevant to the task or why these two were different (55 and 69). Per used the correct integral formula in his Maxima command and obtained the volume of revolution. The axis of revolution in the Web page illustration was the *y*-axis whereas the task in MML concerned revolution around the *x*-axis. Per managed this by using dummy variables in his Maxima command. For the limits of integration, Per employed GeoGebra to find the points of intersection of the given curve with the *y*-axis.

Summarizing the activity. The three kinds of tasks analyzed above can be thought to embody development in terms of the involved mathematical operations. The first task, for instance, introduces that an integral is equal to the limit of sums. The next two tasks involved this idea as an operation while widening the scope of its application to the case of finding volumes. The shell method task involves progression in the involved revolution in the disk method task.

The above analysis show that the students performed different sequences of mathematical operations in these tasks. In the first task, for instance, Jan seemed to be performing the sequence of operations suggested in MML. Per, however, skipped required mathematical operations and obtained the solution by employing Maxima. The Maxima command was concerned with computing the value of the involved integral. In the case of disk method tasks, Per again calculated the involved integrals through Maxima. In the shell method task, the integral to be calculated was not given explicitly. In this task, Per found the integral formula by searching on the Internet and calculated the integral in Maxima. The limits of integration were found by using GeoGebra.

By employing Maxima, Per met the requirement of final solutions to proceed through the tasks in MML. The mathematical operations were not necessarily in accordance with the requirements of the tasks. The conditions in MML were not concerned whether the students realized the involved mathematical operations to reach to the solutions of the tasks.

Affordances of the Online Environment

In this study, I set out to investigate students' interactions with an online environment during their learning activity in mathematics to make sense of affordances of this environment. The online environment under consideration involves implementation of an automated system (MML) with specific contextual aspects i.e. rules, division of labor, and community (see Table 1). The automated system (MML) offered the tasks, worked-examples with the sequence of mathematical operations, and instant feedback for regulating the students' online learning activity. The implementation of

MML together with the contextual aspects (rules, division of labor, community) of this setting afforded self-regulated learning for students.

Concerning intentional aspects of students' interactions, students reported in their journals the use of some other resources in addition to the provided resources (see Table 2). The finding regarding undergraduate students' use of explanatory YouTube videos, Web pages and WolframAlpha is consistent with an earlier study (Anastasakis et al. 2017). In this study, the students also reported the use of Maxima, GeoGebra and online calculators in their activity. The students incorporated these resources in their learning actions with the goals to get immediate assistance and to prepare according to the final digital examination. The role of examination in shaping the students' use of resources is also reported in other studies (Anastasakis et al. 2017; Gueudet and Pepin 2018).

Regarding the operational aspects, the incorporation of several resources afforded various actions and operations (Leont'ev 1974) conditioned by the nature of each resource. In case of Web pages or videos, for instance, the afforded actions were making sense of the involved mathematical concepts. The use of calculators was linked to short-cut methods for solving the tasks posed in the program. The closer analysis of students' activity showed that the individual students worked on the same tasks by performing different mathematical operations. The automated system offered the relevant sequence of mathematical operations for the posed tasks while the students did not necessarily follow those steps. This result is also supported in the study by Cazes et al. (2006) that students' activity deviated from the desired mathematical activity. In the present study, the use of powerful computing tools affording the solution of tasks in single steps also led to diverting students' attention from the required mathematical operations in those tasks.

With respect to the conditions within automated system, the observed deviation in students' realized activity can be attributed to two specifications in MML. Firstly, the acceptance of the final answers in MML without accounting for the process of getting those solutions led students to focus more on getting the correct answers, which is also reported in the study by Rønning (2017). The program allowed students to proceed even when the mathematical operations were not in accordance with the demands of the tasks. Secondly, the mathematical tasks posed in the program could be solved using online calculators. This led to realization of students' actions and goals linked to solving the tasks while the operations were performed by the powerful computing tools. In this regard, Borba (2007) asserted that the nature of available media conditions the mathematical tasks. To explicate, Borba (2007) argued that a task such as "draw the graph of a function" represents an obstacle for students in a paper-and-pencil environment because students need to find the coordinates to plot the curve. The same task does not represent an obstacle in a technologically rich environment. Therefore, it needs to be shifted to an open-ended task such as "why does the graph of a function behave in a particular way?" in order to realize a meaningful obstacle. Thus, employing powerful computing tools for solving the procedural tasks may lead the mathematics to be black-boxed (Anderson 1999).

The participants in this study were undergraduate engineering students. It is generally recommended to integrate technology in mathematics courses (Alpers et al. 2013) to prepare future engineers according to the professional needs of today's technologically rich work environments. According to previous research, professional engineers emphasize the significance of mathematics for analytical and logical thinking although they reported using technology for mathematical tasks at work (Van der Wal, Bakker, and Drijvers 2017). In this view, the emphasis on the processes of solving the tasks instead of using the powerful computing tools needs to be ensured in order for involved mathematics not to be black-boxed for students.

From the features of the collective activity system, the rule concerning digital examination together with the conditions of the automated system led to students' choice and use of resources such as Maxima and calculators. That is, the students used these resources to meet the requirements of the MML and to prepare for the final digital examination. In turn, it affected students' engagement with the mathematical tasks.

The automated systems serve as the platform for managing (delivering, assigning, and evaluating) the homework and tests electronically. The findings of this study suggest that the implementation of this environment does not ensure that the students engage with the mathematical tasks in the expected manner. In addition to the conditions within the automated system, contextual aspects pertinent to students' activity, examination in the digital format as found in this case, also play an important role in students' interactions with this environment.

This study investigates implementation of an automated system for undergraduate mathematics in a specific manner: the digital final examination, and the division of labor managed through the resources in an online environment. Also, the findings are based on the analysis of the small number of participants' activity. Other students' activity in similar contexts may not unfold in the particular manner as observed in this study. However, the present study contributes to make explicit the role of factors at the wider level of the activity system in students' interactions with the automated system. The theoretical stance of CHAT (Engeström 2014; Leont'ev 1974) capturing the collective activity system in addition to the micro details of interactions offers a systematic way to analyze affordances of such systems for students' learning activity.

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