Dynamics of undergraduate engineering students' learning activities in mathematics in an online and in a blended environment

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Preface

Prior to starting my PhD in mathematics education, I taught mathematics at school and undergraduate levels in Pakistan. During my teaching years in schools, which made four years in a row before I taught undergraduate mathematics for a year, I used digital tools in my classroom activities in different manners. Although I observed growing interest and involvement of students in these activities, I got the impression that I did not have enough knowledge and experience about the use of technology from a student learning perspective. During that time, I became curious to know more about if and how the technology could be used optimally in the teaching activities such that the students could get enhanced learning experiences in mathematics.

When University of Agder (UiA) announced a three-year PhD position in mathematics education where a possible focus could be investigating the impact of digital tools, I considered it as a great opportunity to explore more about students' learning in technology rich environments. The work with this thesis has provided me a wonderful opportunity to study several theoretical perspectives about human learning in general and those related to finding impact of technology-based tools. This has not only fed my curiosity, but I have found it a great experience for my own learning and in my development as a researcher. Several people have been very supportive in my PhD process to whom I am very grateful. First, I am extremely thankful to my principal supervisor, Professor Martin Carlsen, for his continuous support throughout my PhD in the form of encouragement, written comments on my drafts, and assistance where I needed guidance. Thank you for always being there whenever I needed support. I would like to extend my deepest gratitude to my co-supervisor, Professor Frode Rønning, for being a great mentor and providing me insights on some of the most challenging questions I faced during my PhD. I had so much to learn from you being an experienced university-level mathematics educator, from your assertive personality, and your disposition towards reason and precision. Thank you also for managing to travel to UiA for the supervision meetings and at other important events of my PhD.

I would like to extend sincere thanks to Professor Simon Goodchild and MatRIC for arranging seminars and workshops offering an arena for useful discussions with other PhD students and experienced researchers in mathematics education. These allowed me to further develop my research in significant ways.

I am also thankful to all the teachers who taught the courses I took as part of my PhD, including Said Hadjerrouit, Simon Goodchild, John Monaghan, and Pauline Vos. I am also thankful to all PhD students in mathematics education and colleagues at the department of mathematical sciences for a supportive and conducive research environment. Thanks to the participating students and the teacher for their cooperation and coordination in the data collection which was central to the execution of my research project.

I would like to thank my parents and siblings for their love, support, and encouragement at times when I needed it most. Special thanks to my sister, Ayesha, with whom I was able to discuss matters related to my research and PhD in general.

I moved to Norway for pursuing my PhD which has been both exciting and challenging. I am grateful to the Norwegian Flø family in Kristiansand, with whom I stayed during my PhD period, for being so compassionate that I did not feel lonely in the new country. I can not forget my friend, Neha Agnihotri, for being such a nice company and for her supportive attitude. There are many others who have been supportive in one or another way, thanks to all of you.

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Abstract

This thesis reports from a research study which investigates the role of technology rich environments in undergraduate engineering students' processes of learning mathematics. The research is founded within a naturalistic research paradigm (Lincoln, 2007) and adopts a case study design (Yin, 2014). The two cases under consideration comprise a small group of undergraduate electronics engineering students, selected as participants, from an online and a blended learning environment (Borba et al., 2016). The first case study of online environment incorporates an online system, MyMathLab, for homework and assessment, and tutorial videos for the lectures in a calculus course. The second case study of blended environment involves group work using paper and pencil and face-to-face lectures instead of tutorial videos in the subsequent course for the same class. The incorporation of digital systems in mathematics, particularly with regard to students' interactions, has not been researched enough (Borba et al., 2016). Qualitative approaches were adopted to collect and analyse the data, and empirical material was collected through multiple methods including student observations, interviews, field notes, and students' weekly reports.

The aim of the thesis is to analyse students' interactions with these environments and to get insights into the factors which play a role in students' engagement with mathematics. In doing so, the study explores dynamics that underlie students' learning activities including students' interactions with available resources, and macro and micro conditions of the learning environment. Cultural historical activity theory (CHAT) was used as an overarching theoretical framework for conceptualising students' learning activities and for analysing the data.

The thesis consists of four empirical studies which also portray the development of my work. The preliminary research report (Study 1) focuses on the manner in which the online environment afforded execution of mathematical competencies (Niss, 2003; Niss & Højgaard, 2011). The unit of analysis in this study was mediated action (Wertsch, 1998). Study 2 investigates the manner in which the students used several resources in their online work. The documentational approach to didactics (Gueudet & Pepin, 2016) was here used as the theoretical framework to analyse the nature of students' techniques (Artigue, 2002) associated with several resources.

Study 3 explores affordances (Bærentsen & Trettvik, 2002) of the online environment through the holistic perspective of CHAT. In particular, Engeström's (1987) extended triangular model of an activity system and Leont'ev's (1974) hierarchical models of activity have been combined to analyse the macro and micro aspects of students' activity in the online environment. The results illustrate that the macro conditions of the learning environment related to the organisation of the course play a part in how students interact with the mathematical tasks. Further, the results illustrate that the micro conditions of the online system led the students to focus only on getting the final answers of the mathematical tasks. The results show that the availability of the powerful computing tools also affects the manner in which students engage with mathematical tasks. The engagement depends upon the nature of tasks in relation to the functionalities of the tools. The study suggests the need to design such tasks which invite students to explore involved mathematical properties by using the dynamic properties of the digital tools instead of solely computing the final answers.

Study 4 focuses on students' reasoning processes with the use of both digital resources and paper and pencil. The data from both case studies, online and blended environment, were used in this study. By leaning on an existing research framework for mathematical reasoning (Lithner, 2008), the reasoning processes were modelled through a cultural historical perspective on learning (Engeström, 1987). The developed model of the reasoning processes is illustrated Figure 5 in this thesis. The model facilitated the analysis of the factors of the learning environment which played a part in students' reasoning.

Overall, the thesis contributes to research in mathematics education, and suggests various implications for the use of technology in mathematics instruction generally and particularly for undergraduate engineering mathematics. Implications are also suggested for further research concerning the use of technology-based tools in mathematics education.

Contents

1 Introduction

This doctoral thesis explores dynamics of undergraduate engineering students' activities while learning mathematics in technology enhanced environments. The thesis is based on four empirical studies which explore different facets of students' activities.

This introductory chapter provides the background, rationale, and overview of the thesis. Section 1.1 presents a brief overview of research on technology use in mathematics education followed by the significance of technology use in engineering mathematics in Section 1.2. Section 1.3 provides details of the learning environments which serve as the empirical basis of my research. After that, the aims of the research and research questions are presented in Section 1.4. Finally, Section 1.5 gives an overview of the thesis.

1.1 Technology in mathematics education

With the rapid growth in the field of technology, human activities are becoming increasingly digitalized. This transformation has impact on the education sector and also on mathematics education. A wide variety of technology-based tools are used in today's mathematics classrooms which may facilitate the processes of learning and teaching. In accordance with Hoyles and Noss (2009), the technology itself is unlikely to influence the mathematical development in any significant ways. It is how it is designed to support learning and how it is embedded in activities designed with some specific learning objectives that are crucial. Mathematical learning and cognition are linked with the physical and virtual tools through which mathematics is mediated.

The huge variety of digital tools available at present offer various functionalities which are relevant to teaching and learning of mathematics. For instance, Hoyles and Noss (2009) suggested the following four categories that distinguish between different ways in which digital tools have potential to affect mathematical cognition: i) dynamic and graphic tools; ii) tools that outsource processing power; iii) new representational infrastructure; and iv) tools focusing on connectivity and shared mathematics. This categorisation is still useful after a decade although one might find examples of tools that fall under multiple categories at the same time. The first category of dynamic tools offer dynamic, graphic, and interactive functionalities through which learners can explore mathematical objects and relationship among them. These functions provide students and teachers with opportunities to draw attention to those factors which remain unnoticed and to make explicit those which are often left unobserved (Noss & Hoyles, 1996). Examples of dynamic tools include GeoGebra, Cabri Geometry, and The Geometer's Sketchpad.

The key utility of the second category, tools that outsource processing power, is that the mathematical computation is taken over by technology. Computer algebra systems (CAS) such as Maple, Mathematica, Mathcad, Matlab, and Maxima represent examples of outsourcing tools. The third category of tools, such as programming languages, offer new representational infrastructure in relation to the paper and pencil media and thus have potential to affect students' meaning making. The last category of tools is based on connectivity and offer a platform for collaboration between participants of the classroom and offer opportunities for communicating mathematical ideas, results, and reflections, both synchronously and asynchronously. The past years have shown tremendous development in these internet-based tools including recent functions of sharing mathematical content and automation of the online activities through assistance and feedback. The online system, MyMathLab, used in this research can be taken as an advanced example of such tools (for more details of MML, see Section 2.1.1). In the research reported in this thesis, tools including dynamic software (GeoGebra, first category), a computer algebra system (Maxima, second category), and an online interactive system (MyMathLab, fourth category) come under consideration.

It is pointed out in several studies (e.g., Borba et al., 2016; Pepin, Choppin, Ruthven, & Sinclair, 2017) that the research in mathematics education is not keeping pace in exploring all aspects of implementation of different digital technologies. There are significant challenges linked to the use of technology in the learning and teaching of mathematics as well as to the related research. The most obvious difficulty is the rapid advance of the technology itself (Hoyles & Noss, 2009). The advanced digital tools afford new functionalities which requires adopting new research perspectives including theoretical and methodological stances within a short time span. For instance, Artigue (2002) points out challenges linked to implementation of computer algebra systems (CAS), among others, as to be handling aspects of tools including the complexities regarding the use of the tools themselves and the mathematical needs of their use, the problems arising with their connection to paper and pencil techniques, and their institutional management. Some of these aspects have also emerged in my research when students employed several tools in their learning activities in mathematics.

1.2 About the use of technology in engineering mathematics

Engineering mathematics concerns applying mathematics to complex real-world problems. It is considered as a branch of applied mathematics which combines the theoretical and practical aspects of mathematics relevant to engineering and industry problems. It may seem that the engineers are more interested in applying the mathematical identities than studying the mathematical basis of these identities and the relationship among them. At the outset, it may look as if the mathematics is like a toolbox for the engineers who apply it in the practical contexts. Kent and Noss (2000), however, argue that the metaphor of application is not straightforward. Mathematics is shaped by its applications and it takes on meaning which is derived from the setting in which it is used. Therefore, there are certain aspects which demand attention while looking at the application metaphor such as: What is it that is applied? To what exactly is it applied? Do the different people, such as mathematicians and engineers, consider the applications in a similar manner or not? Moreover, previous research studies (for example, Gynnild, Tyssedal, & Lorentzen, 2005; Hirst, Williamson, & Bishop, 2004) also illustrate that engineering students experience difficulties in transferring specific mathematical skills into different practical contexts.

In todays' technology enhanced professional environments, professional engineers report the use of technology-based tools in order to solve mathematical tasks at work (Van der Wal, Bakker, & Drijvers, 2017). During their education, engineering students also get experience with more and more sophisticated technological tools such as Mathcad, Mathematica, Matlab, and Maple, etc. Kent and Noss (2000) highlight that the design of the technological tools as well as the didactical activities shape engineering students' interactions with mathematics. These researchers argue that there is a need to consider the epistemology instead of restricting the issue to the use of technology in mathematics. The computational power underlying some of these tools let the students use mathematics in unprecedented manner through push button functions. In such cases, an appropriate question to ask is how the connections between mathematics and engineers can be made visible by using the computer software tools. The problem of technology use also comes down to what mathematics, and in what forms, should be visible for engineers since different technological tools offer different symbolic and graphic representations.

1.3 The online and blended learning environment

The interaction and feedback possibilities in emerging digital systems allow creation of online and blended learning environments for students (Borba et al., 2016). The implementation of such systems leads to qualitatively different patterns of interactions between teachers and students, and between students and mathematics (Pepin et al., 2017), than the usual classroom environment. There exists little to no research on students' interactions with mathematics in such environments (Borba et al., 2016).

In this thesis, a cultural practice of incorporation of a digital system, MyMathLab (for details, see Section 3.4). for undergraduate engineering mathematics is studied. During two consecutive semesters of 2017, the structure of mathematics courses for a class of undergraduate electronics engineering shifted from an online to a blended learning environment. This research study explores a small group of students' activities in both learning environments. The learning environment is theorised through a cultural historical perspective (Cole, 1996; Engeström, 2014). Specifically, it is characterised by the extended triangular model of an activity system (Engeström, 1987) (see Section 2.2.4, Section 2.2.5, and Study 3). This holistic view of the learning environment incorporates and takes the role of tools as well as the role of social relations into account for studying human learning and cognition.

1.4 Aims and research questions

The thesis aims to explore the impact of the learning environments, online and blended, for students' engagement with mathematics. Different aspects of students' activities are explored in order to make sense of the role of these environments. These research aims were formulated as the following research issues:

A. How do undergraduate engineering students engage with mathematics in an online and a blended learning environment?
B. How do factors from these learning environments contribute to students' learning activities in mathematics?

To comply with the empirical context, the aforementioned aims are operationalised into the following research questions (RQs), which have been addressed in the four resulting studies.

RQ1: What traces of mathematical competencies are observed in students' work when they practice mathematics digitally?

RQ2: How does this environment afford the execution of these mathematical competencies?

RQ3: How do engineering students incorporate resources during their work in an online learning environment?

RQ4: How do a small group of undergraduate engineering students interact with an online environment in their mathematical learning activity?

RQ5: In what manner does this environment afford students' engagement with mathematics?

RQ6: How do a small group of undergraduate engineering students accomplish mathematical reasoning processes in an online and a paper and pencil environment?

The 6 research questions are addressed in four empirical studies in the following manner: Study 1 addresses the RQ1 and RQ2, Study 2 addresses RQ3, Study 3 addresses RQ4 and RQ5, and Study 4 addresses RQ6.

1.5 Overview of the thesis

The thesis consists of six chapters: Introduction (Chapter 1), Theoretical foundations (Chapter 2), Methodological approaches (Chapter 3), Summary of research papers (Chapter 4), Conclusion and discussion (Chapter 5), and Implications of research (Chapter 6).

In the present chapter, I have provided the background and the aims of this research in relation to the empirical context. Chapter 2, elaborates the conceptual framework (Lester, 2005) which underlines the theoretical basis of this research study. For theorisation of the research problem(s), this chapter provides the details of the used theories as well as an argument for selecting the particular theories. The discussion of all the important terms and theoretical concepts relevant for the research study is included in this chapter, which consists of four sections. The first Section 2.1 begins by providing theoretical foundations of the online digital curriculum resources, leading to an account of the documentational approach in mathematics education and its use in the present research, and ends by offering a rationale for adopting the cultural historical activity theory (CHAT) (Engeström, 2014; Leont'ev, 1974). Next, in Section 2.2, CHAT is elaborated with a historical perspective by emphasizing the contributions by Leont'ev and Engeström as well as perspectives from researchers employing

CHAT in mathematics and science education (Jaworski & Potari, 2009; Roth, 2012, 2014) and human computer interaction (Nardi, 1996; Kaptelinin, 1996). While CHAT serves as an overarching theoretical framework, it allows for analysing the societal and tool dimensions of students' activities whereas there remains a room for interpretation of students' work in mathematics. I attend to the issue of characterising students' work with mathematics in Section 2.3. To achieve this purpose, I utilise the competence framework (Niss & Højgaard, 2011) (Study 1), action–operation dynamics in CHAT (Study 3), and then mathematical reasoning (Lithner, 2008) which is also a strand of the competence framework (Study 4). After that, I elaborate the theorisation of the mathematical reasoning processes through a cultural historical perspective (Leont'ev, 1974). The concept of affordance (Study 3) is explained in Section 2.4. In the end, I present the previous research literature relevant to the research foci in my thesis in Section 2.5.

Chapter 3 addresses the methodological approaches adopted in this research. The overall orientation of the research as being conducted within the naturalistic research paradigm is elaborated (Section 3.1) and the relationship between the theory(ies) and research is explicated (Section 3.2). The qualitative research strategy and the case study design are discussed in Section 3.3. The details about the two case studies regarding participants, the research method and the context are provided in Section 3.4. The strategies for analysing data in the four studies are outlined in Section 3.5. Section 3.6 gives the details of the mathematical topics involved in this research. The issues concerning quality of research and research ethics are addressed in Section 3.7 and Section 3.8.

In the subsequent Chapter 4, I present summaries of the four research studies. Chapter 5 presents the finding of the research (Section 5.1–5.5). I reflect on the effects of the choice of the theoretical framework as well as the methods with respect to the research findings (Section 5.6). After that, the limitations of the presented research are discussed in Section 5.7.

Chapter 6 presents implications of the research for further research (Section 6.1) and implications for instruction (Section 6.2).

2 Theoretical foundations

My research project focuses on undergraduate engineering students' learning of mathematics within activities in order to study the role of the conditions of the learning context. This chapter addresses the theoretical basis of my research. The chapter is divided into four sections. Section 2.1 addresses the general issues concerning theorisation of the inquiry. The section presents the theoretical foundations of personalised learning environments involved in this research and explains the rationale for first using the documentational approach to didactics and later adopting CHAT. Section 2.2 presents CHAT through a historical perspective and elaborates its key components and ideas pertinent to human learning. Section 2.3 describes the way in which students' learning in mathematics is characterised in present research. The concept of the affordances is elaborated in Section 2.4. The research literature is presented in Section 2.5.

2.1 Theorising the inquiry—learning mathematics while interacting with resources

In my thesis, I incorporate various ideas from relevant empirical research as well as from theories, which can be encapsulated as the conceptual framework of this research. In the following text in this section, I first elaborate the notion of conceptual framework and then provide further details and justifications regarding the need for the conceptual framework in the context of my research. Section 2.1.1 elaborates the idea of personalised learning environment, which makes an integral part of my research. Section 2.1.2 discusses the concepts from documentational approach to didactics in my research, and Section 2.1.3 argues for the shift to the cultural historical approach.

According to Lester (2005), a theoretical framework guides the research by its dependence on formal theory, i.e. the "theory which has been developed by using an established, coherent explanation of certain sorts of phenomena and relationships" (p. 458). However, a single theory may not cope with the complexity of the realistic research problems. Compliance with one theory may lead to preferring some specific aspects while exempting or ignoring other aspects that may also be relevant to the actual research problem. To address such concerns in mathematics education research, Lester suggests devising a conceptual framework, which may be based on "different theories, and various aspects of practitioner knowledge, depending upon what the researcher can argue will be relevant and important to address about a research problem" (p. 460).

The conceptual framework is "a skeletal structure of *justification,* rather than a skeletal structure of *explanation*" (p. 460). The justification refers to the rationale or "the argument that the concepts chosen for investigation, and any anticipated relationships among them, will be appropriate and useful given the research problem under consideration" (p. 460). The conceptual framework should also explain why a research question is being proposed in a particular manner and why certain factors (e.g. context, rules, resources) are more important than others while answering this question. The conceptual frameworks in mathematics education can also be exemplified as the models created by integrating ideas from different theories (see Lesh & Sriraman, 2005). The models serve as systems of thinking about problems of mathematics learning. Models are considered bigger than the individual theories in the sense that they aim at solving the problems which lie outside the realm of particular theories. They are also perceived smaller than the theories, as they are created for specific purposes in specific situations. The models are considered "purposeful, situated, easily modifiable, sharable, re-usable, multi-disciplinary, and multi-media chunks of knowledge" (Lesh & Sriraman, 2005, p. 502).

2.1.1 Digital curriculum resources and personalised learning environments

In the process of teaching and learning, students and teachers make use of the curriculum resources to interact with mathematics. These resources can be categorised into text resources (e.g., textbooks), material resources (e.g., calculators), and ICT-based resources (e.g., interactive textbooks) (Pepin & Gueudet, 2018). Pepin et al. (2017) put forth the notion of digital curriculum resources as "organised systems of digital resources in electronic formats that articulate a scope and sequence of curricular content" (p. 697). These resources facilitate in and focus on the process of teaching and learning by providing sequenced curricular content in electronic formats in a particular course of study (e.g., calculus) (Pepin et al., 2017). The attention to sequencing of content differentiates these resources from other types of digital resources (e.g., software). Ruthven (2018) argues that digital materials, due to advanced user interface and provisions of instantaneous feedback, lead to qualitatively different forms of interactions between the user and the medium than their non-digital counterparts.

Choppin and Borys (2017) conducted a survey on the existing trends on the design and dissemination of digital curriculum resources in mathematics education. The researchers identified different discourses around the design and use of curriculum materials emerging from the following four different perspectives: designer, policy, user, and private. The designer perspective conceptualises the design features based on research on learning and learning systems and places strong emphasis on the nature of tasks and collective interactions within in these materials. The policy perspective emphasizes the low cost of the materials and their ability to customize the content. The user perspective concerns the implementors of the digital media in the classrooms and emphasizes the pragmatic aspects of the digital materials to fit into existing practices as well as the abilities to reduce the management demands. The private perspective concerns the organisations which create the digital materials as products to appeal to the consumers. This perspective emphasizes the improvement of management practices about instruction and assessment. The digital curriculum materials emerging from the private sector are further divided into the following four categories: (1) comprehensive learning management systems with embedded digital content; (2) adaptive programs emphasising the mastery perspective for learners who progress through content via a series of assessments; (3) collection of lessons in form of presentations or videos linked to the practice problems and assessment problems; and (4) curated content embedded in an online assessment and curriculum resource site.

The online repositories with static content, known as learning management systems (LMS) or virtual learning environments (VLEs), mark the earlier forms of digital curriculum resources. Lately, adaptive programs emphasising the mastery perspective embed more sophisticated features have emerged. Such programs are also termed as personalised learning environments (PLEs) (Borba et al., 2016). Based on the concept of personalisation, these online systems incorporate dynamic features of digital technologies to provide learner-centered instruction. That is, these environments offer adapted feedback and tailored help according to the learners' needs. Buchem, Attwell, and Torres (2011) describe a personalised learning environment as a "concept related to the use of technology for learning focusing on the appropriation of tools and resources by the learner" (p. 1). My research involves the implementation of a personalised learning environment, namely Pearson's MyMathLab (see details in Section 3.4.2).

PLEs are widely employed in teaching and learning of mathematics at university level. These systems can assist lecturers in assigning and marking the homework and assessments, thus aiding in efficiently managing their time. Students can also get instant help and feedback during their homework which might not always be the case in the lecture halls with a large number of students. However, the nature of the feedback and the kind of assistance depend upon the design of a learning environment. Moreover, these programs are implemented in varying manners in mathematics courses. For instance, a trend involving employment of PLEs in college-level mathematics courses, is widespread in the USA (Twigg, 2011; Webel, Krupa, & McManus, 2017). The mathematics courses are administered in computer labs with computer-based learning environments (PLEs) as the main source of instruction. During these sessions, the instructors remain present during the course sessions in case if students need further assistance in their work. My research also involves the implementation of a PLE (MML) creating the online environment such that all the homework and tests in one calculus course were administered through this program.

2.1.2 The documentational approach in mathematics education for studying students' rationale of using several resources

The goal of my research project is to investigate engineering students' learning of mathematics in the online environment created through employing MyMathLab and other resources (see details in Section 3.4.1). In an initial inquiry (Study 2), I explore students' interaction with several resources and their rationales of using them while working with mathematics in the online environment. This study (Study 2) draws upon *the documentational approach to didactics* (Gueudet, Pepin, & Trouche, 2012; Gueudet & Trouche, 2009). Below, I provide a brief overview of the theoretical framework and I attend only to those ideas from the theory which I incorporated in my Study 2.

The documentational approach is grounded on Rabardel's work (2002) and extends the instrumental approach (Trouche, 2004) in mathematics education. The documentational approach conceptualises a teacher's work as an interplay between a teacher and the set of resources in his/her use (Monaghan, Trouche, & Borwein, 2016). Recently, students' use of resources has also been studied through this approach (e.g., Gueudet & Pepin, 2016, 2018). A resource is conceptualised as "both noun and verb, as both object and action that we draw on in our various practices" (Adler, 2000, p. 207). Therefore, this idea of a resource takes into account various means including material, human and cultural resources such as language, time, mathematics teachers, etc that intervene in students' or teachers' activity.

Furthermore, a resource is never considered in isolation but in its relationship to a wider set of resources (Gueudet & Trouche, 2009). When an individual uses a resource in a particular manner, the resource evolves into a *document*. This process is called documentational genesis. In my Study 2, I used the noun sense of the *resource* and distinguished between classical and digital resources in students' use. The subject of interest was to analyse the mediation of these resources in students' work in mathematics. Therefore, I utilised the concept of techniques (Artigue, 2002) which denotes the manner of solving the mathematical tasks. The instrumented techniques refer to the techniques involved in the use of tools. As a first step of the inquiry, my initial focus was the general manner of using several resources in undergraduate engineering students' mathematics work instead of focusing on specific resources and particular mathematical topics. Through students' rationales linked to several resources, the mediation through resources was analysed as to be both pragmatic and epistemic. Pragmatic mediation refers to the practical uses of the tools to reach the solutions and get through the processes in solving tasks. Epistemic mediation refers to the uses of resources in order to learn the mathematical concepts.

2.1.3 Rationale for adopting a cultural historical approach—shifting the focus to the learning environment

I figured out during the data collection that the context of my study was under change owing to the expected variation in the structure of the mathematics course. This change later led to the shift from the online environment to the blended environment (for details of these environments, see Section 3.4). The new setting related to the blended environment did not require using resources similar to the online environment. I anticipated that this variation would lead to different forms of students' activity, and therefore, I considered to keep track of the contextual conditions. I gave initial thoughts to using CHAT in order to study the effect of these changes in the learning environment on students' way of working in mathematics. Roth, Lee, and Hsu (2009) regard cultural historical activity theory as "a theory for praxis as much as it is a praxis of theory" (p. 154). That is, it enables to make sense of human practice in real settings. Along these lines, the aim of the further research was thus to systematically analyse the practices as well as the influence of the change in the setting on students' work in mathematics (Study 3 and Study 4).

Roth (2014) points out that CHAT requires a holistic approach, that is, it "integrates body and mind, on the one hand, and individual and collective, on the other hand" (p. 14). Moreover, CHAT acknowledges the cultural origins of human learning and cognition. The following comment by Roth (2012) clarifies the cultural historical stance on human learning and cognition: "[C]ultural-historical activity theory does not require us to make hypotheses about the contents of peoples' minds but asks us to study societal relations that are the origin of anything that might be attributable to the individuals and their minds (p. 102)".

These expositions suggested considering the contextual factors as these could trigger certain forms of actions on the part of the students. The results of Study 2 also pointed towards the role of contextual factors in the way students interacted with the resources. Therefore, I expanded the focus of my research from considering the students' use of resources to embracing the whole context in which these resources were embedded since I was to analyse students' learning. In other words, the unit of analysis was expanded from studying the interaction among student-tool-mathematics (Study 2) to the interaction among student-context-mathematics (Study 3). In this regard, the cultural historical approach could facilitate in making sense of the context while making an account of the resources as well (for details, see section 2.2.4). I figured out that the research questions could be well answered by using CHAT within the context of my research project. The attribution to social and cultural origins of learning and cognition in CHAT also resonated well with my own views. It can be noted that I also took the contextual aspects into account in the first two studies (Study 1 and Study 2) although in these studies I did not utilise the CHAT. The next section provides an exposition of the conceptualisation of context in CHAT and the role of the context in human learning.

Gueudet and Trouche (2012) suggested using the documentational approach to didactics along with CHAT in order to make sense of the social aspect in teachers' activity with the resources. It implied that the similar use could be possible for students' activity as in my research.

2.2 Cultural historical activity theory (CHAT)

CHAT traces back to dialectical materialism and the pioneering work of Vygotsky. Engeström (2001) notes that the development of CHAT can be organised into three generations. First generation CHAT was founded by Vygotsky (see e.g., 1978).

Further developments by Leont’ev (1974) and Engeström (1987) are known as second generation of CHAT. The third generation is based on subesquent developments by Engeström (2001). Roth (2014) suggests that CHAT can be organised into four generations. First generation was developed by Vygotsky and second generation is based on further developments by Leont’ev. The third generation consists of two strands: the Helsinki verson (Engeström, 2001), and the Berlin version by Holzkamp and colleagues. The fourth generation works by Roth and Radford (2011).

In this thesis, however, I follow Engeström’s categorization, and the second generation of CHAT guides present research. In what follows, a historical account of the development of the second generation is provided (Section 2.2.1–Section 2.2.4) Key ideas from the second generation of CHAT pertinent to human learning incorporated in this research are also elaborated (Section 2.2.5–Section 2.2.6).

2.2.1 Learning as mediated action—First-generation CHAT

Vygotsky’s (1978) sociocultural approach is considered as the first generation of CHAT. Learning is considered as an interplay between individuals and their culture. Knowledge does not lie in peoples’ minds rather it circulates between people when they communicate. It is created in social interactions when people convert their experience and reflections into language (Säljö, 1999). Learning is seen as mediated through tools and artefacts which are developed in the culture. The idea of mediation is usually illustrated by the mediational triangle (Figure 1) in which tools take the intermediate position between the subject and the object of learning. Learning refers to how individuals appropriate the tools for thinking and acting which exist in a culture or society (Wertsch, 1991). In the first-generation CHAT, the focus is on individuals performing actions in a sociocultural setting (Engeström, Miettinen, & Punamäki, 1999).

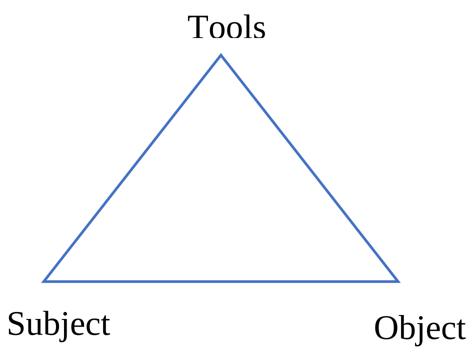

Figure 1: Triangular model illustrating the idea of mediation.

Kozulin (1998) notes that “Vygotsky … made a principal distinction between the ‘lower’, natural mental processes of perception, attention, memory, and will, and the ‘higher’ or cultural psychological functions that appear under the influence of symbolic tools” (p. 14). Furthermore, “Vygotsky’s research program included studies of the transition from the natural to the cultural psychological functions of memory, perception, tension, will, counting and speech. These studies were conducted in three directions: instrumental, developmental, and cultural historical” (p. 16). The cultural historical dimension is what features the second generation of CHAT.

2.2.2 Towards the concept of activity—Second-generation CHAT

As discussed above, the second-generation CHAT is based on the work of Leont'ev (1974) and Engeström (1987) and incorporates “societal, cultural, and historical dimensions into an explication of human mental functioning” (Roth & Lee, 2007, p. 189). Engeström (2014) discusses that the discourse in the first generation cultural-historical approach has largely been on development of higher psychological functions. The later generations of CHAT allow expanding the analysis to horizontal directions by considering “issues of subjectivity, experiencing, personal sense, emotion, embodiment, identity, and moral commitment” (Engeström, 2014, p. xv).

In line with first-generation, social interaction plays an important role in second-generation CHAT. Human psychological processes are regarded as originated in mutual interaction of individuals, as interpsychological processes which at a later stage happen to be carried out by individuals taking the form of intrapsychological processes. Social consciousness and language are considered as the origin of individual consciousness. Tools are considered of utmost importance in this regard. Tools are sociohistorically formed means and modes through which a person is connected to other people and assimilates the experiences of humanity.

The concept of activity, put forth by Leont’ev (1974), refers to the subject–object interaction mediated through tools as well as societal relations. The object is related to the need behind the activity. In my research, the activity is conceptualised as the students’ interaction with mathematics (object) in their mathematics course(s).

Leont’ev (1981a) argues that “human individual’s activity is a system in the system of the social relations (p. 47)”, and individual actions are “senseless and unjustified (p. 47)” without taking the collective activity into consideration. Leont’ev maintains that individual activity would not exist and would not have any structure if removed from the system of social relationships. However, it would be inadequate to consider that society is the external world to which humans must adapt to survive, “rather these social conditions carry the motives and goals of the activity, its means and modes. In a word, society produces the activity of the individual it forms.” (pp. 47-48). He, however, stresses that the activities do not embody the structure of a society and its culture. That is, there are complicated processes and transformations that play a part in the realisation of activities. Such transformations, he maintains, remain unrevealed in studying the individual mind in social world, and can be discovered by investigating the genesis of human activity and its inner structure.

Leont'ev (1981a) points out that the circle of internal mental processes in broken in external object-oriented activities through which humans encounter other humans as well as tools. In other words, thinking develops when individuals engage in practical and object-oriented activities. The following example by Leont’ev underlines the importance of incorporating the practical activity in psychological research.

> Let us examine a very simple process: the perception of an object’s elasticity. This is an external motor process by which the agent enters into practical contact with the external object. This process can be aimed at carrying out the noncognitive and practical task of transforming the object’s shape. Of course, the image that emerges here is mental, and therefore a proper object of psychology. However, in order to understand this image, I must study the external, practical process by which it is generated. Whether or not I want this and whether or not it corresponds to my theoretical views, I must include the agent' s external, practical action in my psychological research. (p. 52)

Summing up, the second-generation CHAT calls for shifting the focus of analysis from the individual tool-mediated action to the level of collective human activity. The following two subsections elaborate on the theoretical models of activities which serve as the tools for analysis in this research.

2.2.3 Structure of an activity—Leont’ev’s hierarchical model

Leont'ev (1974) presented a theoretical model which explicates the structure of human activities with regards to human functioning (Figure 2). According to this model, human functioning can be seen at three hierarchical layers (Figure 2). The top level concerns the whole activity which is driven by an object-related motive. In this sense, the object serves as the true motive of an activity, gives direction to an activity, and distinguishes one activity from another. In the present research, the motive as to leaning of mathematics links to the object ‘mathematics’ of students’ activities under consideration (Study 3).

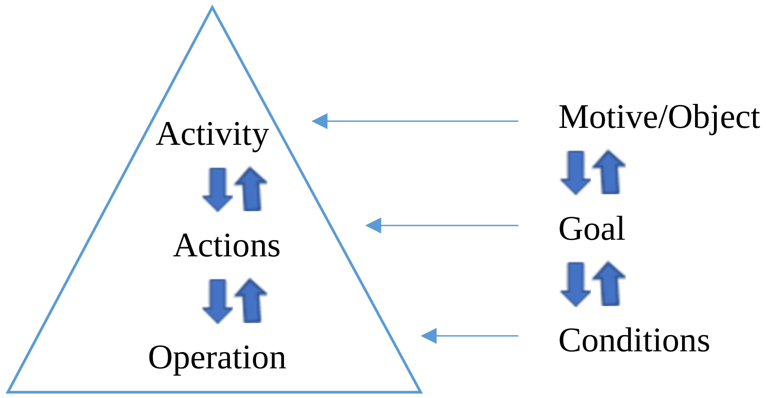

Figure 2: Hierarchical levels of an activity (Leont'ev, 1974).

Leont'ev (1981a) wrote, “the object of activity emerges in two ways: first and foremost, in its dependent existence as subordinating and transforming the subject’s activity, and secondly, as the mental image of the object, as the product of the subject’s detecting its properties” (p. 48). Thus, there cannot be an activity without a motive; the activity which is seemingly unmotivated has the motive concealed subjectively or objectively.

The middle level concerns the individual actions, which translate an activity into reality. The development of an activity into separate actions often results due to division of labour among the participants of an activity. The social relations “leads to isolation of the separate partial results, which are achieved by the separate participants in the collective labour activity, but do not in and of themselves satisfy their needs” (p. 60). The separate partial results are referred to as goals to which actions are directed.

The actions are also energized by the motive, and “isolation of goals and the formation of actions subordinated to them lead to a division of functions that were formerly interwoven in the motive” (Leont'ev, 1981a, p. 60). Selection and conscious perception of the goals are by no means automatic or instantaneous acts. Rather they are a long process of testing goals through actions and fleshing them out. He explained, an “important aspect of the process of goal formation is making the goal concrete or selecting the conditions of its attainment” (p. 62). The goal can exist in isolation from the situation in the consciousness of the subject whereas the action cannot.

In present research (Study 3), the examples of identified actions in students' learning activities include: reading through the textbook, searching for formulas, getting questions from the textbook, working on homework, and solving questions. The examples of identified goals are to solve the tasks in an assessment, to recall certain topics, and to get the general idea of the topic.

An action has operational aspects which are defined by the objective circumstances under which the action is carried out. The bottom level (Figure 2) deals with the operational aspects or operations, i.e. the methods by which goal-directed actions are carried out. The conditions of the environment and specifically the tools affect how the operations are executed. If the goal remains the same and the conditions concerning the attainment of actions changes, then the operational compositions also change. In Study 3, I also look at how the availability of tools in the online environment affects the execution of the operations in students' activity. If the students engage in the acts of solving an integration task in MyMathLab and Maxima, they need to perform some operations. The focus at this level is to identify what operations are formed in students' activity when they employ these tools, and in turn how these affect their engagement with mathematics.

2.2.4 The activity system—Engeström's extended triangular model

Engeström (1987) contributed to the further development of CHAT based on his interpretation of Leont'ev's (1974) work. He presented an extended triangular model of an activity system (Figure 3), which incorporates the tool mediation and the societal mediations in an activity, as proposed by Leont'ev. This model is obtained by adding societal dimensions in the triangular model of tool mediation (Figure 1). In this triangle, "the visible tip of the iceberg of collective activity" refers to the tool mediations whereas "the hidden bottom part" (Engeström, 1990, p. 172) refers to societal mediations through rules, division of labour, and the community. The upper sub-triangle (subject-tool-object) depicts the individual actions whereas the bottom sub-triangles represent the societal relationships influencing the individual actions.

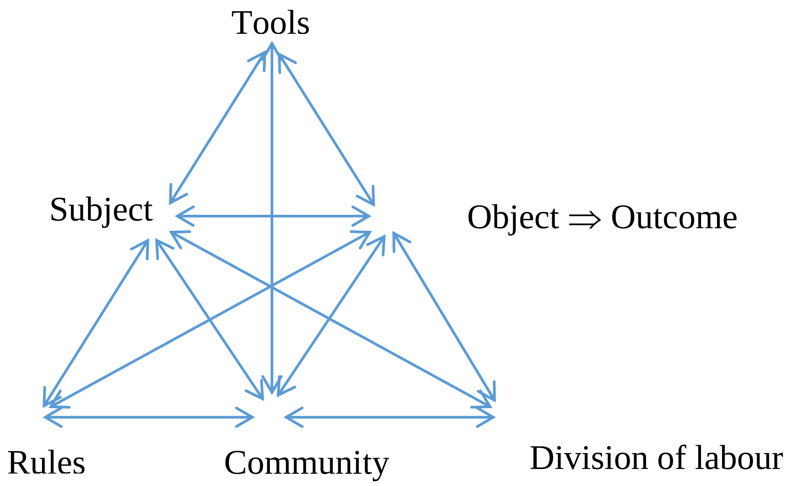

Figure 3: The human activity system adapted from Engeström (1990).

The model also portrays the complex mediational structure of human activities. The sub-triangles point to multiple mediations in an activity system. For example, rules mediate between subject and community, and the community mediates between subject and the object, and so on. In this model, the *rules* represent norms, conventions, or social traditions that are established by the community to govern its members (Engeström, 1998). The *Rules* regulate an activity by setting standards for human actions. Researchers in mathematics education have distinguished between implicit and explicit rules (Núñez, 2009). Explicit rules refer to the conditions set by authorities. Examples of explicit rules can be assessments, format of examination, including groupwork in the university students' activity. Implicit rules refer to normative understandings regarding acceptable and unacceptable argumentation in a mathematics classroom, for example, social and sociomathematical norms (Yackel & Cobb, 1996). The *community* signifies other members of the society which have direct or indirect interaction with the subjects. The examples of community for students' activities can be their class fellows, teacher, group members, and other students at the university. *Division of labour* specifies the way in which participants in an activity divide the task to reach the object of the activity.

According to Engeström (2014), this model is the simplest unit that carries "the essential unity and the integral quality behind the human activity" (p. 65). The nodes of the triangular model represent the minimum constituent elements to be taken into consideration while analysing a concrete human activity system. Transforming this model to concrete activities, inner relationships between the constituent element of the activity system as well as the historical change in an activity system can be analysed.

Engeström (1987) proposed looking at such relationships by considering the systematic whole rather than just separate connections. Leont'ev (1981a) also emphasized that the analysis of an activity should not aim for separating living activity but revealing the inner relations which characterise it. Multi-voicedness is an essential feature of activities in that it contains the variety of viewpoints of the participants of an activity. Furthermore, layers of historically accumulated artefacts, rules and patterns of division of labour are also significant parts of an activity system.

Engeström (2014) writes, “we may well speak of the activity of the individual, but never of individual activity; only actions are individual” (p. 54). At a later point, he cites Leont’ev, “when we are dealing with joint activity, we can with full justification speak of a *collective subject* or of a total subject of this activity, whose interrelation with the ‘individual’ subjects can only be comprehended through a psychological analysis of the structure of the joint activity” (p. 57). The demarcation between the individual and the collective subject relates to the level at which the activity system is analysed. That is, the subject is considered as individual at the level of actions whereas it is considered as collective at the level of collective activity. This point becomes clearer in Section 2.2.6.

According to Cole (1996), the context of learning and teaching can be determined by the activity system while using CHAT. In my research, I use the term *learning environment* which is characterised through the activity system (Figure 3). In Study 3, the elements of the learning environment are described through the extended triangular model of the activity system. That is, the rules, division of labour, and community are described at the level of the mathematics course. It allowed in analysing the macro structure of students’ collective activity in the mathematics course. The micro analysis is done through using Leont’ev’s hierarchical levels of the activity (see for example, Jaworski & Potari, 2009). Combining the macro and micro view facilitated in zooming in and out in the activity. In turn, it made it possible to trace the effects of the collective conditions on students’ micro level interactions with the resources and hence with mathematics.

2.2.5 The mediational means—Tools, resources, or instruments

My study involves a variety of mediating means which students interacted with in their learning activity in mathematics. Therefore, I attend to the concept of mediating means in light of the existing literature in mathematics education and in CHAT. In this thesis, I use alternating terms such as tools, resources, and instruments. According to Monaghan et al. (2016), an artefact is a material object which is made by humans for specific purposes. When the artefacts have some users and the purpose of the use linked with them, they are considered as tools (Trouche, 2004). Tools and artefacts can be considered as two faces of the same coin. That is, when an artefact is used in a goal-directed action, it becomes a tool (Cole, 1996). Also, “artefacts are often, but not always, the physical objects” (Drijvers et al., 2009, p. 108). In this sense, an artefact can be a paper, compass, or even an algorithm. The instrument, according to Trouche (2004), are those artefacts which the subjects have integrated in their activities. This suggests that tools and instruments are used interchangeably in the literature with similar meanings. The meaning of the term ‘resource’ is already discussed in Section 2.1.2.

As seen earlier, the tool mediations occupy a vital position in the studies of human activities (Figure 3). In categorising the mediational means, CHAT emphasizes the manner in which the mediational means objectify human needs in addition to their properties itself. In first generation CHAT, Vygotsky (1978) distinguishes between two types of tools: material/physical, and psychological. Material or physical tools refer to the artefacts which people use to accomplish tasks at hand, e.g., a ruler for drawing lines or geometrical shapes, a pencil for writing. The psychological tools are artificial formations directed towards the mastery of mental processes. Examples of these can be: “language, various systems for counting, mnemonic techniques, algebraic symbol systems, works of art, writing, schemes, diagrams, maps, and mechanical drawings, all sorts of conventional signs, etc.” (Vygotsky, 1981, p. 137). The distinction between the two kinds of tools lies in the fact that the former are “directed toward producing one or another set of changes in the object itself” whereas the latter “directs the mind and behaviour” (p. 138).

In the case of computers and information and communication technologies, Säljö (1999) argues that they represent an example of an interrelationship between physical and psychological tools, on the one hand. A computer can be perceived as a physical tool when using the push button functions and, at the same time, as a psychological tool when using several incorporated features such as figures, simulations, computations, etc. On the other hand, the emerging software with features of feedback and response can be perceived halfway between a human and a tool. The feedback has potential to invoke reflections on the part of the learner, which may be qualitatively different in each software. This argument applies to the digital system under consideration in this thesis (see Study 3) which is a virtual learning environment.

In second generation CHAT (Engeström, 2014), the notion of tools is viewed at three levels which corresponds to Wartofksy's primary, secondary and tertiary artefacts. Primary artefacts are those which are used in production. Cole (1996) argues that the idea of a primary artefact is closely linked to the concept of artefact as a matter transformed by prior human activity. The examples of primary artefacts given by Wartofsky as "axes, clubs, needles, bowls" (Wartofsky, 1979, p. 201) point to their use in material production, whereas in the production of social life, primary artefacts are thought of as "words, writing instruments, telecommunication network" (p. 121). Secondary artefacts are used in preserving and transmitting modes of action or praxis through which the production in carried out. That is, secondary artefacts are "reflexive embodiments of forms of actions …. and therefore understood as images of such forms of actions—or, if you like, pictures or models of them" (Wartofsky, 1979, p. 201). The mode of these representations may be gestural, oral, or visual. However, these representations are not the mental entities as residing in the mind, rather they are externally embodied representations.

Engeström (2014) deems secondary artefacts as parallel to Vygotsky's psychological tools as "the essence of psychological tools is that they are originally instruments for cooperative, communicative, and self-conscious shaping and controlling of the procedures of using and making technical tools" (p. 49). Examples of secondary artefacts, according to Cole, include "recipes, traditional beliefs, norms, constitutions, and the like" (Cole, 1996, p. 121).

In a nutshell, CHAT suggests viewing tools and resources with respect to their incorporation at the functioning levels in the activity. A resource may therefore be considered as a primary artefact or a part of a secondary artefact according to how it objectifies the human needs. In Study 3 in which I study students' incorporation of several resources in the activity of learning mathematics, each resource objectifies different purposes at different moments in the students' activities. For instance, when a website is used in searching for relevant information for solving a task, it is involved in the mode of carrying out an action, and thus the website is considered as a secondary artefact. Similarly, a resource such as GeoGebra facilitates in executing mathematical operations. It can thus be regarded as a primary artefact as well as partially a secondary artefact as it is involved in the models for carrying out actions. However, I confined the analysis to the incorporation of primary artefacts (Wartofsky, 1979). The notion of tools in relation to the activity under consideration is further treated in the upcoming Section 2.2.6.

Kaptelinin (1996) discusses that the notion of tool mediation becomes problematic in the case of virtual environments as the border between the tool and reality is merged. In Study 3, for the operationalisation of a personalised learning environment (MML), while the system is considered as a resource for production in the students' activity, other conditions of the environment are incorporated in the analysis of the activity system as seen as contributing to the rules and division of labour. In Study 4, I also consider the incorporation of digital tools in secondary artefacts (Wartofsky, 1979), i.e. as modes of actions.

2.2.6 Learning while participating in object-oriented activities

Engeström (2014) discusses that the object-oriented activities serve as methodology for learning activities. In an educational institute such as a school, reading, writing, communicating with language, and mathematics constitute examples of object-oriented activities in which learning is manifested. My study involves mathematics as the object of students' learning activity in a university setting. In this regard, Roth (2014) writes, "activity theory explains learning as a by-product in the production of grades. It does not account for mathematical activity as if it could occur outside and independent of the schooling context (p. 13)".

These claims point to the significance of considering the activity system while analysing students' learning. Now, the question arises about what is meant by learning in CHAT. To answer this question, I turn to Engeström (2014) who, drawing upon Bateson's (1972) theory of learning, argues that an activity involves three types of human learning. Each type corresponds to a layer in Leont'ev's (1974) model of human functioning (see Figure 2) and to three different levels of human subjects—nonconscious, individual and collective. The corresponding elements of an activity system at each of the three levels are illustrated in Table 1.

The bottom layer of operations relates to Learning I (Bateson, 1972). Learning I is equivalent to the formation of nonconscious operations "in the course of simple adaptation to existing external conditions" (Engeström, 2014, p. 115). The object presents itself as mere resistance. The instrument at this level refer to Wartofsky's primary artefacts, i.e. those used in the production. The subject uses the instrument upon the object making repetitive corrections. The object and the instrument are not consciously separated by the subject. The instrument and the object both are

Table 1: The proposed hierarchical structure of activity (Engeström, 2014, p. 122)

Subject	Instruments	Object	Community	Rules	Division of labour
Collective subject	Methodology, Ideology	We in the world	Societal network of activities	Societal (state, law, religion)	Societal division of labour
Individual subject	Models	Problem task	Collective organisation	Organisational rules	Organisational division of labour
Nonconscious	Tools	Resistance	Immediate primary group	Interpersonal rules	Interpersonal division of labour

considered as given. The possible examples involving Learning I can be thought of as when a student applies an algorithm or performs simple mathematical operations such as an addition or subtraction algorithm with respect to three-digit numbers. Provided that the student is familiar with the algorithm and the subtraction concept, he or she applies this algorithm (tool) on the given numbers (object) to obtain the outcome. In this process, the student performs nonconscious operations involving simple calculations such as $5-4$ or $3-2$, while new operations are formed simultaneously. The examples in terms of use of physical tools can be thought of as using a pencil to write an intended expression. In this process, the subject deals with simple conditions such as the thickness of the paper and sharpness of the pencil to reach to the object of having written the expression. The nonconscious operations are formed which deal with being able to use paper and pencil for writing. Another example can be thought of when using the calculator for performing mathematical operations such as addition, multiplication, and so on.

The formation and execution of the action–goal layer in Leont'ev's scheme (see Figure 2) involves Learning II which corresponds to the individual subject (see Table 1). In Learning II, the object is perceived as a problem demanding specific efforts from the subject, who is no more a nonconscious agent but an individual under conscious self-assessment. The instruments at this level are deemed as Wartofsky's (1979) secondary artifacts, which "refer to the preservation and transmission of the acquired skills or modes of action or praxis by which this production is carried out. Secondary artefacts are therefore representations of such modes of action" (Wartofsky, 1979, p. 202). The instrument is found by the subject. The instrument is regarded as a model which is a tacit representation of the way of accomplishing the tasks produced on the basis of Learning I.

Furthermore, the automatic operations are also formed at the action–goal layer, i.e. from the top down. These operations, in contrast to nonconscious operations, are in principle capable of becoming subjected to conscious elaboration in case the normal conditions of performance are altered. It means that Learning I and II are intertwined and cannot be separated.

Depending on the method of finding the instruments, Learning II can be regarded as a) reproductive, or b) productive. In the reproductive type, Learning IIa, the instrument is found through trial and error and with blind search among previously known means. In the productive type, Learning IIb, the instrument is invented through experimentation. In this sense, the former leads to empirical generalisations and the latter may lead to the theoretical generalisations. From Learning I to Learning II, from specific to the general, represents a developmental step in an activity. For example, if the students solve a mathematical task by using the previously known methods or through trial and error methods, they engage in the reproductive learning (IIa). On the other hand, if the students invent the methods themselves by careful experimentation which is also mathematically grounded, they engage in productive learning (IIb).

In Learning III, the collective subject becomes conscious and gains an imaginative and thus potentially also a practical mastery of whole systems of activity in terms of the past, present, and the future (Engeström, 2014). Learning III results due to the double bind situations. At this level, the problem or the task itself must be created. The subject is immersed in the object system and the quality of the subject changes radically. Learning III may now be characterized as the construction and application of world outlooks, ideologies, however, it is not confined to imaginary production.

From the discussion so far and Table 1, it can be deduced that the Learning III concerns the network of activities whereas Learning I and Learning II concern a single activity system. According to Engeström (2014), Learning I and Learning II represent what is commonly understood as learning while Learning III represents development (Vygotsky, 1978) which involves both Learning I and Learning II.

In my thesis, I confine my attention within the separate activity systems which concern an online and a blended learning environment (for details, see in Section 3.4). That is, the object of interest is the dynamics within activity systems instead of the interaction between the activity systems. In the language of CHAT, it is to say that the interest is to capture learning (Learning I and Learning II) which is woven into action–operation dynamics as evident through the foregoing discussion. Therefore, the focus remains on the bottom two layers (see Table 1) while analysing students' learning activity in mathematics (Study 3 and Study 4). In particular, I analyse action and operation dynamics associated with the use of available resources which correspond to individual and nonconscious subjects (Table 1). Through this analysis, I make sense of the role of the factors such as tools and social actors in students' learning of mathematics.

2.3 Characterising students' work in mathematics

From the discussion so far, it can be noted that CHAT provides the methodology of learning irrespective of the object of learning. Mathematics being the object of an activity has its own inner structure and characteristics. In the field of mathematics education, learning of mathematics is characterised in several ways. Among others, the competence framework (Niss & Højgaard, 2011) puts forth eight significant areas encapsulating the learning of mathematics (Figure 4). In what follows, I briefly describe the constituents of the framework and its use in my thesis. In Section 2.3.1, I give an account for the competence framework of Niss and Højgaard (2011). In Section 2.3.2, I discuss the notions of mathematical reasoning and argumentation. In Section 2.3.3, I argue a link between mathematical reasoning and the learning environments that I study.

2.3.1 Competence framework

The notion of mathematical competence is considered as "a well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge" (Niss & Højgaard, 2011, p. 49). The eight sub-competencies, as depicted in Figure 4, comprise two groups. The first group (to the left) concerns asking and answering questions, in, with, about mathematics, whereas the second group (to the right) concerns dealing with mathematical language and tools.

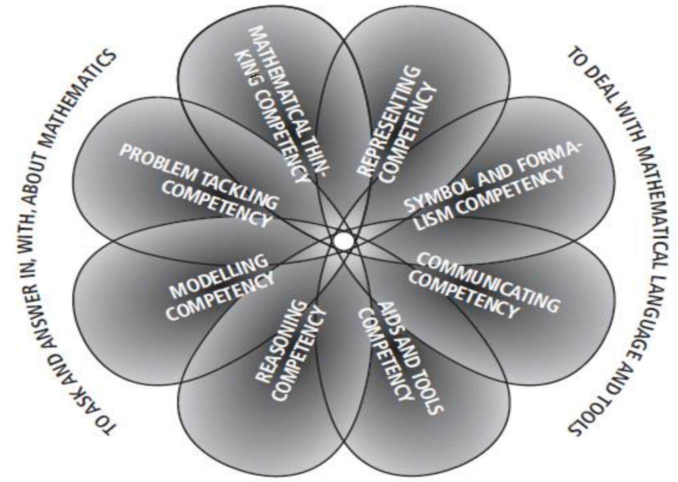

Figure 4: A visual representation of the competence framework (Niss & Højgaard, 2011, p. 51).

The first group of asking and answering questions comprise the sub-competencies of mathematical thinking, problem tackling, mathematical modelling, and mathematical reasoning. The mathematical thinking competency deals with familiarity with the type of questions that characterise mathematics, the ability to pose effective questions, and the awareness about the type and form of the expected answers. The problem tackling competency comprises with the ability to identify, formulate, and specify the scope of different mathematical problems. The modelling competency deals with performing active modelling in the given contexts, analysing the properties of existing models, assessing their range of application, and identifying the conditions of their validity. Mathematical reasoning competency deals with devising formal and informal arguments in support of claims about the mathematical tasks and following and assessing mathematical reasoning put forward by others both in written and oral forms.

The second group of dealing with mathematical language and tools comprises the competencies of representing, symbol and formalism, communicating, and using aids and tools. The competency of representing deals with the ability to interpret and utilise different forms of representations of mathematical objects and phenomena including symbolic, algebraic, visual, and verbal depictions. The competency of symbol and formalism deals with the ability to interpret symbol and formal mathematical language, including symbolic statements, expressions, and formulas, and to translate back and forth between mathematical symbolic language and natural language. The competency of communicating deals with expressing oneself in various ways including oral, written, and visual, and with different level of theoretical or technical precision about mathematical matters. The competency of tools and aids deals with having awareness about relevant tools used in mathematics and using those tools reflectively while being aware of their possibilities and limitations in certain situations.

Niss and Højgaard (2011) point out that the competency description of mathematics can be used to describe the subject in two different ways. It can be applied normatively, i.e., for determining the main instruments in mathematics curricula. Such normative use of the competence framework can be seen in the mathematics curricula for engineering education (see Alpers et al., 2013). The mathematics curricula for engineering mathematics (Alpers et al., 2013) suggest integrating technology in mathematics courses for engineers in order to enhance students' mathematical competencies. The other use of competencies is suggested as descriptive, i.e. to describe the ongoing teaching and learning of mathematics.

The description of individual competencies by Niss and Højgaard (2011) includes the word 'ability', which hints that the competencies are properties of the individuals. In this regard, Niss and Højgaard state that the use of the word 'ability' is merely an alternative way to denote 'being able to' due to linguistic substantive and that the term is "by no means a psychological term aimed at referring to a person's mental personality" (p. 50). In light of these suggestions, I used the competence framework as an analytical tool to look at how students engaged with mathematics in the online environment (for details about the online environment, see section 3.4) in Study 1. The focus was on how the aspects of the environment *enabled* the students to execute different competencies.

2.3.2 Mathematical reasoning and argumentation

In Study 4, I narrow down the focus to mathematical reasoning as the characterisation of students' engagement with mathematics, which is a strand of the competence framework (see Figure 4). The following text sheds light on the perspectives about mathematical reasoning taken in my research.

In the competence framework, mathematical reasoning is regarded as "a chain of arguments …. in writing or orally, in support of a claim" (Niss & Højgaard, 2011, p. 60). Lithner (2000) conceptualises mathematical reasoning as "the line of thought, the way of thinking, adopted to produce assertions and reach conclusions" (p. 166). The

argumentation, thus, is considered as “the part of the reasoning that aims at convincing oneself, or someone else, that the reasoning is appropriate” (p. 166).
Lithner (2008) devised a research framework to characterise university students’ mathematical reasoning. In his framework, solving a task is seen as comprising four steps:

i) identifying a problematic situation,
ii) choosing (recalling, guessing, discovering, etc.) a strategy (local approach or the general procedures),
iii) implementing the strategy, and
iv) reaching to a conclusion.

The reasoning is involved in the second and the third step of the above task solving sequence. The second step may be supported by predictive argumentation, i.e. why the selected strategy will solve the task. The third step includes verificative argumentation, i.e. why the selected strategy did solve the task.

Lithner’s framework takes into account any type of reasoning as long as it is sensible to the reasoner. That is, it does not confine reasoning to the sense of a strict proof. On the basis of the mathematical nature of the assertions and claims, the reasoning is characterised as either imitative or creative. The reasoning based on surface properties of involved mathematical components is classified as imitative reasoning. In the imitative type of reasoning, the epistemic value, the degree of trust (likely, true, absurd, unreal, or obvious) the reasoner has in a claim, lies in the authority of the sources of the imitated information. The imitative reasoning is further classified into memorized reasoning, algorithmic reasoning, familiar algorithmic reasoning, delimiting algorithmic reasoning, and guided (text or teacher) algorithmic reasoning.

In contrast, a sequence of reasoning is considered as creative mathematically founded reasoning (CMR) if it fulfils the following criteria: creativity, plausibility, and anchoring of arguments in intrinsic properties of the mathematical components involved. Plausibility refers to distinguishing a more reasonable guess from a less reasonable guess, in predictive and verificative argumentation, based on deductive logic (in a less strict sense than proof). The epistemic value of CMR lies in the plausibility and the logic of the reasoning itself (Lithner, 2008).

2.3.3 Processes of mathematical reasoning with respect to the role of the learning environment

In Lithner's framework (2008), reasoning is manifested in the steps of selecting and implementing strategies while solving a task. It is believed that the environment in which students practice mathematical tasks plays a role on the formation of reasoning. However, the manner in which the learning environment affects the reasoning has not been investigated. In my Study 4, I seek to investigate the role of the learning environment in the formation of students' mathematical reasoning. To achieve this purpose, the reasoning has to be considered as a process instead of a product in such a way that the students are engaged with the reasoning in their learning activities. To trace the roots of the reasoning process in the environment, I make use of the principles of CHAT that facilitate in analysing reasoning with regards to the conditions of the environment (discussed in detail in section 2.2.6). In the following text, I provide details about how reasoning is conceptualised as a process (depicted in Figure 5) in the light of CHAT.

In my research, the process of reasoning is seen as comprising the main steps of selecting and implementing strategies, described by Lithner (2008) as the significant steps in the sequence of task solving activity. These steps are crucial as they involve the decisions pertinent to solving a task. A mathematical task can be seen as a *problem* for a student in the perspective of CHAT, which corresponds to the level of action–goal of the students' activity. To solve this problem (task), it is required that the student develop a model of the necessary step of the process (see Section 2.2.6). The *model* is similar to what is called a strategy in Lithner's framework. In turn, solving a task comprises, in CHAT terms, a combination of action(s) and operations.

Concerning the reasoning process, the step of *selecting a strategy* is parallel to *selecting a model* for carrying out the action(s) involved in solving the task (more details in section 2.2.6), which is entailed in the action–goal layer of the activity. The analysis of this layer enables to make sense of the basis of the selection of the models, i.e. if those are created or selected from the previously known ones. In turn, it facilitates in making sense of the factors of the environment that contribute to the selection of models, if any.

The step of *implementing the strategy* involves executing mathematical operations. This step is thus considered parallel to the operation–condition layer (Leont'ev, 1974), which relates to the use of tools. This step enables to analyse the role of available tools in the execution of the operations while implementing the strategy.

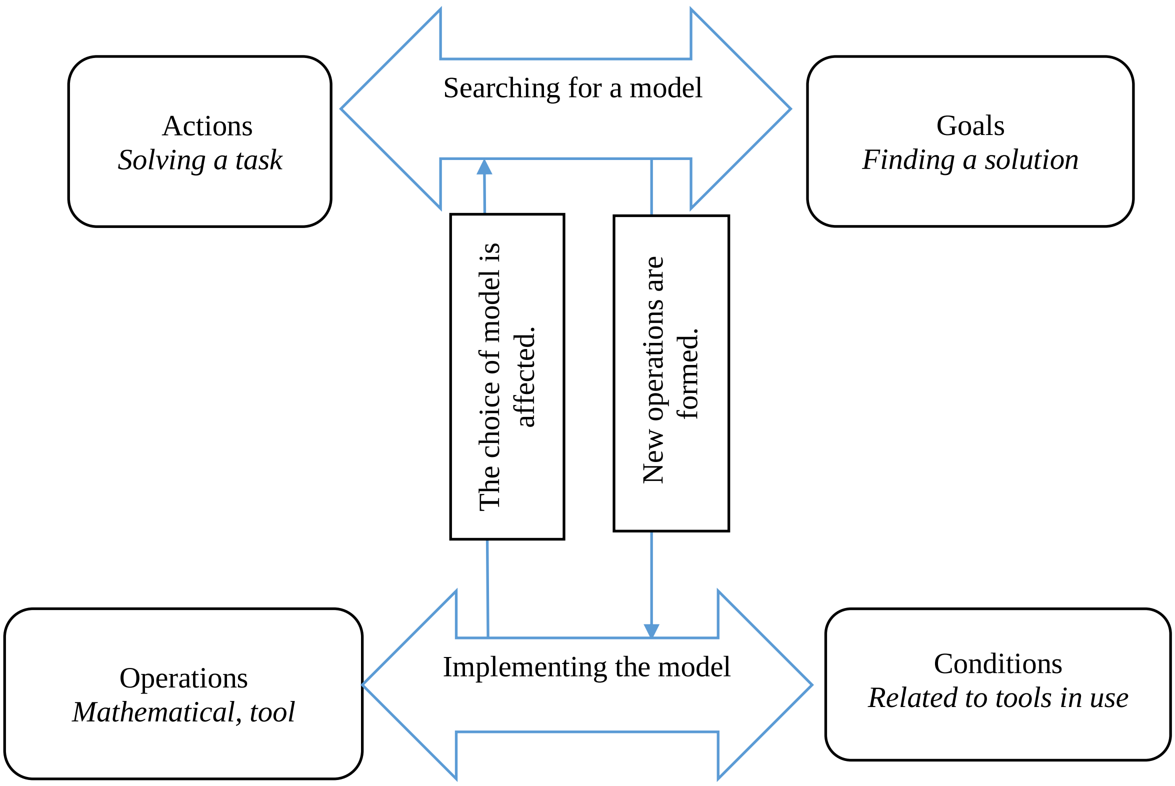

Figure 5: A model for reasoning processes in mathematics.

The principle of transformation between actions and operations in CHAT (see section 2.2.3) implies that the steps of selecting and implementing strategies are connected. That is, the initially selected models, upon enough execution and over time, result in the formation of consciously controlled operations. Conversely, based on implementation of the models at the operation–condition layer, the forthcoming selection of models is affected. The visual representation of the process of reasoning is shown in Figure 5.

As shown in the foregoing text, the model builds upon and extends an existing research framework concerning mathematical reasoning (Lithner, 2008). The model adds another dimension of the effect of the learning environment into the analysis of reasoning, thereby considering reasoning as a process. The above model serves as a tool to analyse the formation of reasoning processes and allows looking at the role of the learning environment (tasks, tools, etc.) in this regard.

2.4 The concept of affordance

This section explains the concept of affordance which concerns Study 3 in this thesis. The concept of *affordance* was first introduced by Gibson (1977) to denote the relationship between an organism and its environment. The affordances are characterized as the action possibilities provided by the environment.

Greeno (1994) argues that an affordance has to be a property of an object that interacts with a property of an agent in such a way that an activity is supported. In other words, those attributes of the environment that contribute to the kind of interaction that takes place between the agent and the environment. A system that provides an affordance through certain attributes does not imply that the activity will occur, although it contributes to the possibility of the activity. Other aspects of the agent's activity, including motivation and perception, are contributing factors in realisation of affordances. Motivation relates to what the person is doing at a general level and to the needs of the individual. For instance, if a person is engaged in the activity of going to attend a class, then the action of entering into the classroom is a significant part of that activity. It will make the person attentive to the aspects of the environment such as a door for passing from one side of the partition to another. The affordance of passing is only realized when the person's width corresponds to the width of the door.

In the case of university students' mathematical activity, lectures, for instance, might provide affordances such as: attending the lectures, interact with the lecturer and classmates on a regular basis, questioning about the mathematics, or exploration of mathematical concepts. There are many factors involved for these affordances to be realized. On the one hand, if such opportunities exist in the lectures depending upon how the lectures are being organised. On the other hand, the realisation of affordances depends if students' needs correspond to the content and organisation of the lectures.

The issue of perception relates to how the agent picks up information about affordances of the environment. Greeno (1994) argues that the affordance of passing through a door is perceived by picking up visual information that specifies the physical width of the door. However, the affordance of a mailbox for posting letters is not directly perceived by the physical attributes of the mailbox. For the classification of this function of the mailbox, the required information has to be in the form of symbols or words which would mediate the perception of the physical box as a mailbox.

Similar to the above considerations, Bærentsen and Trettvik (2002) advocate an activity-theoretical perspective of affordances that considers the motivational level as well as the operational level of an agent's activity in the study of affordances. The consideration of functional facets such as passing through the doorway or grasping an object are called operational aspects of affordances. These aspects only relate to the operational level of the activity and thus to the physical attributes of the environment. The motivational aspects of affordances refer to the agents' motives for using the artefacts in their actions. Different motives of different users may lead to uses other than the intended uses of an artefact. For instance, a hammer has at least an intended use while many other uses are possible depending upon the user's activity. Motivational aspects are thus important to investigate all the uses of an artefact in an activity. The motivational aspects relate to the goal-directed actions in an agent' activity. In the case of computer programs as involved in this study, Bærentsen and Trettvik argue: "It is often assumed that the motivational or goal-directed structure of the users' activity is identical to the designer's or it is assumed that it is given by the design of the operational structure. But as clearly documented by Suchman (1987) this is not necessarily so, and unless we understand what motivates people to use something, we cannot even begin to understand why they succeed or fail to realize it" (Bærentsen & Trettvik, 2002, p. 59).

It follows that the intended affordances as well as the actual affordance of a computer software in use are important to consider in order to make sense of the effectiveness of a software. In line with these suggestions, I analyse the affordances of an online system (Study 3) considering the intentional and operational aspects of the students' activity. I relate the motivational aspects with the action–goal layer of the students' activity whereas the operational aspects are studied through the analysis of the operation– condition layer of the students' activity.

2.5 Research related to the use of technology-based tools in students' learning of mathematics

In this section, I present a short research literature review by dividing it around two themes: student's use of digital and classical resources and its link with certain aspects of students' learning activities (2.5.1), and the relationship between mathematical reasoning and the digital tools and other aspects such as tasks (2.5.2). The studies given an account of below, are considered highly relevant for my research, as these studies theoretical stances, research foci, and partly types of students, coincide with mine.

2.5.1 Students' use of classical and digital resources

Several studies have investigated students' use of multiple resources, such as institutionally provided classical and digital resources, and other available resources in mathematics. For instance, Gueudet and Pepin (2018) explored, through a case study design, how university students interacted with several provided resources in their mathematics work. The study focused on the link between the rules set by the institution and the teachers about the students' use of resources and the students' actual use of those resources. Adopting, among others, the Documentational Approach to Didactics, similar to the theoretical approach I adopted in Study 2, Gueudet and Pepin observed discrepancies between students' actual use of provided resources, such as textbook, lectures, own notes, and MyMathLab, and lecturers' expectations with respect to their use. For example, the lecturers expected the students to focus on the content of the lectures particularly the proofs whereas the students searched for solved examples. Similarly, the students did not make much use of the digital resources offered by the institution, but they used other Internet-based resources to search for the methods to work on mathematical tasks. Gueudet and Pepin suggested the need for the teachers to be more explicit about their expectations of the use of provided resources as well as Internet resources and to support an appropriate use by the students.

Anastasakis, Robinson, and Lerman (2017) investigated the links between undergraduate engineering students' goals and their choice of resources. The data were collected through a survey from 201 students followed by in-depth interviews of six students. The study utilised Leont'ev's (1974) model of the structure of an activity and mainly focused on the action–goal layer of the participants' activities, similar to my use of this theoretical approach in Study 3. The analysis of the survey responses showed that while students used external resources to some degree including online videos, WolframAlpha (an online computing engine), and online encyclopaedias, they dominantly incorporated the institutionally provided resources such as textbooks, the university's virtual learning system, the mathematics support centres, etc. The interviews further illustrated that the students' choices of resources were driven by exam-related goals.

Concerning digital resources, Jaworski, Robinson, Matthews, and Croft (2012) focused on the incorporation of the dynamic software called GeoGebra in the mathematics teaching and learning of undergraduate engineering students. The study addressed the issues between teachers' intentions behind the incorporation of GeoGebra and students' responses, engagement, and performance regarding this approach. The elements of the collective activity systems (Engeström, 2014) for both teachers and students were analysed. The action and operation layers of students' activities of learning mathematics, using Leont'ev's (1974) hierarchical model of an activity, were also identified. Thus, Jaworski et al. (2012) adopted the similar theoretical approach as I did in Study 3. The results showed that there were differences in teachers' and students' ways of perceiving the value and the quality of understanding, and thus of perceiving the purpose of, the incorporation of GeoGebra. The researchers concluded that the teachers needed to make the expectations and purposes regarding the use of GeoGebra clear to the students. Another study, by Rønning (2017), explored the effects of an automated program called Maple T.A. on undergraduate engineering students' ways of working in mathematics. Maple T.A. presents a set of problems or tasks to students and can evaluate the solutions using the incorporated functionality of Maple CAS. This computer program was used to administer mathematics tests to the engineering students. The data in this study were collected through six surveys from large cohorts of students (N > 500) followed by focused group interviews. The survey responses were analysed against the backdrop of the collective activity system (Engeström, 2014) in which the students participated, and the factors affecting students' engagement of mathematics were identified. Regarding the effect of Maple T.A., Rønning concluded that the system promoted the quest for final solutions among the students, which in turn affected their deep learning of mathematics.

The impact of the online and automated systems for homeworks in students' learning of mathematics has been explored in various manners in previous research. For instance, a strand of research employed quantitative approaches for evaluating the effectiveness of online systems. (e.g., Callahan, 2016; Jonsdottir, Bjornsdottir, & Stefansson, 2017; Kodippili & Senaratne, 2008; Potocka, 2010). These studies determined the effectiveness of these systems by adopting criteria such as grades, cost effectiveness, and passing rates. In a comparative study, Krupa, Webel, and McManus (2015) analysed the impact of computer-based (CB) and face-to-face (F2F) instruction in a college algebra course. They used a quasi-experimental match design and compared the student related predictors at three levels. The first level concerned the comparison of the exam results of two large groups ($N_{F2F} = 192$, $N_{CB} =134$), and the second level included some other student-level predictors ($N_{F2F} = 73$, $N_{CB} = 50$). The third level concerned quantitative analysis of students' solution strategies for some open response tasks ($N_{F2F} = 38$, $N_{CB} = 24$). The results showed that the CB group performed better on the exam than the F2F group. The analysis of students' responses to the open-ended tasks, however, indicated that the students from CB group experienced problems in interpreting and relating algebraic symbols to contextual situations effectively in comparison to their F2F peers. The students in CB group were better able to select appropriate symbolic procedures, whereas students in F2F group adopted the guess and check methods and were less likely to get the correct answer. Moreover, the students in CB groups were less persistent in attempts to solve contextual problems in comparison to their peers in F2F group. Overall, the study raised concerns about the students' use of algebraic symbols in new situations in both settings.

Webel et al. (2017) performed a study that investigated the implementation of a Math Emporium (ME), an instructional model for teaching and learning of mathematics using computer-based programs. ME was used with students in an introductory college algebra course and the study took a mixed methods approach to data collection and data analysis. Webel et al. focused on the following three themes: (1) whether the emporium is more helpful to a certain group of students; (2) the nature of mathematical learning in this setting; and (3) the students′ perceptions about the emporium style courses. Webel and colleagues concluded that the emporium style served the students with higher mathematics achievement and those who less strongly believed that mathematics is about memorising. The findings of their study also suggested that the setting enabled students to focus on getting correct answers more than developing algebraic meanings. Regarding students′ perceptions, Webel et al. found that some students did not like the autonomy and flexibility offered by this setting. These findings led the researchers to question if examination grades and passing rates are the appropriate indicators of the impact of such settings. The researchers recommended that future studies should focus on students′ interactions and mathematical reasoning afforded by these environments.

The interactional stance for exploring the impact of some online and automated systems has also been adopted in some past studies (e.g., Cazes & Vandebrouck, 2013; Gueudet, 2006). For instance, Cazes, Gueudet, Hersant, and Vandebrouck (2006) investigated university students′ strategies while working on different tasks posed in three electronic-exercise bases (EEB), that are programs similar to automated systems such as MML. The study took place during an experimental implementation of EEB environments. The data were collected through direct observations of individual students' work on the assigned tasks and through electronically generated logs of students' activities. The results of this study illustrated that the students' actual strategies may differ from the expected strategies in EEBs. Cazes et al. (2006) recommend these environments for direct application task, those in which the strategies are given, as there were found no mismatches between students' activities and the effective ones in such task situations. In the case of advanced tasks, the tasks in which the strategies were not explicitly provided through the help functions, and students often developed unexpected strategies. This is due to the reason that programs EEBs could not evaluate students' strategies as the programs only evaluate the final responses.

2.5.2 Mathematical reasoning and the use of digital tools

The relationship between the availability of resources, including classical resources (e.g. textbooks) and digital resources (e.g. GeoGebra), and students' mathematical reasoning has also been the object of study in previous research. For instance, Granberg and Olsson (2015) explored the effect of using GeoGebra on upper secondary school students' collaboration and creative reasoning. The study involved 36 students and was conducted during a 45-minute long session outside the classroom. The participating students worked in pairs and solved a task on linear functions using GeoGebra. The findings illustrated that GeoGebra facilitated the students in visualising, testing, and monitoring while sharing their ideas, strategies, and the state of the problem with each other. GeoGebra served as an interactive partner for students to visualise their strategies. Moreover, since GeoGebra did not offer feedback in the form of final solutions, the students had to interpret and evaluate the feedback through verificative arguments (cf. Lithner, 2008) as to why an idea did or did not work. The students' evaluation thus served as the basis for their creative reasoning. Zembat (2008) compared pre-service teachers' mathematical reasoning in a paper and pencil environment with their mathematical reasoning in a technology supported learning environment. The technological environment comprised of The Geometer's Sketchpad, spreadsheet, and T1 83 calculator. In this study, the analysis focused on the following three types of reasoning: analytical reasoning relating to the use of mathematical formulas, creative reasoning referring to invention of methods for thinking about problems, and practical reasoning referring to the practical applications of the involved concepts. Zembat found that the participants' focus was confined to analytical reasoning in the paper and pencil environment. However, the participants' focus was shifted from analytical reasoning to practical reasoning in the technology supported learning environment. This shift was possible due to the facility of the tool functions such as dragging, graphing, etc in The Geometer's Sketchpad. Consequently, the pre-service teachers' misconceptions about the concepts of function, slope and derivative were revealed in the paper and pencil environment while these were partly overcome in the technological environment. This was because the availability of the tools in the technologically enhanced environment allowed the teachers to reason in alternate ways about the mathematical object in contrast to the paper and pencil environment.

Other aspects of the learning environment, such as properties of the tasks in textbooks (Lithner, 2003) and in assessments (Boesen, Lithner, & Palm, 2010), have also been linked with the nature of students' reasoning. Norqvist (2018) studied the role of different kinds of task which is partly involved in my Study 4. Norqvist compared the effect of three kinds of tasks involving creative reasoning, algorithmic reasoning, and explained reasoning. The study included administering pre- and post-tests for three group of students, each of whom worked on creative, algorithmic, and explained algorithmic reasoning tasks. The statistical analysis of the three groups' performance on the pre- and post-tests illustrated that the students who practiced creative mathematical reasoning tasks outperformed those who practiced explained reasoning and algorithmic reasoning tasks. Also, the added explanation did not contribute to increase the learning efficiency of algorithmic reasoning tasks. The use of programming and its link with the reasoning is not studied, according to my knowledge.

3 Methodological approaches

This chapter deals with the methodological approaches concerning my research project. The chapter opens with a presentation of the research paradigm in Section 3.1. Next, Section 3.2 elaborates on the relationships between theory and methodology, followed by an elaboration of the adopted research strategy and research design contained in Section 3.3. Section 3.4 further elaborates on the context and the research methods adopted to collect the data for research. This section also sheds light on my role as a researcher during the process of data collection. Subsequently, Section 3.5 briefly addresses the strategies concerning the analysis of data within the arising studies. Section 3.6 is devoted to the mathematical context of this research. Section 3.7 discusses quality criteria of the research with respect to the conducted studies, followed by ethical considerations in Section 3.8.

3.1 Research paradigm

A research paradigm is thought of as "a network of coherent ideas about the nature of the world and of the functions of the researchers which, adhered to by a group of researchers, conditions the patterns of their thinking and underpins their research actions" (Bassey, 1999, p. 42). In other words, research paradigms represent various established models in a research community within which researchers align themselves, according to their beliefs, to make sense of the phenomena to be investigated. A paradigm is a set of assumptions related to ontology, epistemology and the methodology of the research (Lincoln & Guba, 2000). The ontology is concerned with the postures on reality, the epistemology with the ways of coming to know the reality, and the methodology with the means of knowing the reality (Lincoln, 2007).

The research reported in this thesis is situated within a naturalistic research paradigm (Lincoln, 2007), also referred to as an interpretative, constructive or constructionist paradigm (Bryman, 2015). This position regards the difference in the subjects of social sciences and the natural sciences in the sense that human beings—unlike the objects of the natural sciences—have meanings associated with the social reality which serve as the basis of their actions. The research questions as well as the theoretical stance in my study allow me to take a constructionist stance because the elements of the students' activity system in mathematics are hard to quantify objectively and some sort of subjectiveness has to be involved.

With regard to the ontological assumptions in the naturalistic paradigm, reality is seen as extending beyond the measurable variables to also include the constructions of the perceptions and actions of the human beings. The social constructions hold an equal ontological status as physical realities. “Constructions are the mental and sensemaking processes and products which humans engage as they make sense of, and organize, the physical realities, sensory data, situations, contexts, experiences, attitudes, values, beliefs, expectations, and the like which swirl around them” (Lincoln, 2007, Metaphyics and the Paradigm section, para. 1). My research also considers the reality as to be comprising social constructions in addition to physical quantities. For instance, while studying the students’ use of resources (Study 2, Study 3) or the reasoning processes (Study 4), I consider actions, operations, contextual conditions, etc. as comprising the reality.

The epistemological issues determine “the ways of coming to know the reality” (Bryman, 2015, p. 24). In a naturalistic paradigm, an interpretivist stance is required to reach to research findings. Bryman (2015) notes that in interpretative research, three levels of interpretations are involved. At the first two levels, the researcher provides the interpretation of others’ interpretations. That is, the constructivist researcher grasps meanings and interprets those from the subjects’ point of view. At the third level, “the researcher’s interpretations have to be further interpreted in terms of the concepts, beliefs, and literature of a discipline” (Bryman, 2015, p. 28). In this sense, the constructionist account of a social phenomenon is also dependent on a researcher’s way of perceiving the reality. In my work, in study 3 and study 4, the third level is achieved when I interpret participants’ interpretations according to the theoretical framework of CHAT.

The methodology or the means of coming to know the reality in my research relates to the research questions, theoretical guidelines, and the context of research. The forthcoming sections address these relationships in detail.

3.2 Relationship between theory and methodology

As discussed in the previous chapter, the conceptual framework of this research incorporates multiple theories (see Chapter 2, Section 2.1). The use of each theory can be regarded as what Simon (2009) calls lenses: “When one looks at a situation through a particular theoretical lens, some phenomena are prominent while others are not (e.g., cultural practices from a sociocultural perspective, prior knowledge from a cognitive perspective)" (Simon, 2009, p. 482). With respect to my research, the theoretical lens of documentational approach in didactics aided in characterising students' use of resources (Study 2). In Study 1, the first generation of CHAT was adopted to analyse execution of mathematical competencies when students incorporated resources in their learning actions. In Study 3 and Study 4, I employ the theoretical lens of second generation CHAT (Engeström, 1987; Leont'ev, 1974) to characterise the inner dynamics of the students' activity system in varying manners. In particular, Study 3 focuses on the role of resources at the action and operation layers, whereas Study 4 looks at the processes of reasoning in relation to the conditions of the environment.

Overall, second generation CHAT acts as the overarching theoretical framework to characterise the practice under consideration. CHAT guides on certain aspects of the research methodology. First, in this regard, is the recommendation that taking an activity system as a unit of analysis should complement the system view and the subject's view. To achieve this purpose, the researcher looks at the system from above and at the same time through the eyes and interpretations of a subject, the member of the local activity, to understand the activity system (Engeström, 1999). In Study 3, while looking at the students' use of resources, I incorporate students' input regarding the usage of resources while keeping in view the whole activity system (cf. Figure 3).

Second, CHAT proposes studying the dynamic motion in activities across time and generations. Human activities are considered as evolving over long periods of time. They are not short-lived events or actions which have a clear-cut temporal beginning and an end. Studying an activity system historically requires appropriate periodization, which means that the stream of historical events must be divided into larger patterns which have meaningful characteristics of their own (Engeström & Miettinen, 1999). Shchedrovitskii (as cited in Engeström, Miettinen, & Punamäki,1999), a Soviet activity theorist concerned with the development of collective activity systems, pointed out that "it is quite natural to endeavour to represent reproduction as *cycles* resulting in the formation of a new social structure on the basis of some preceding one" (p. 33). These cyclic time structures called *expansive cycles* need not to be repetitive but can lead to the emergence of new structures.

The new activity system does not emerge out of the blue, but it requires reflective analysis of the existing culturally advanced models and tools that offer ways out of the internal contradictions. According to Engeström (2001), expansive transformation takes place when "the object and the motive of the activity are reconceptualised to embrace radically wider horizon of possibilities than in previous mode of the activity" (p. 137). Engeström (1999) recommends that studying *expansive cycles* is the best methodology for understanding transformations going on in the human activity systems. He wrote:

> I want to suggest that such a methodology is best developed when researchers enter actual activity systems undergoing such transformations. I am not suggesting a return to naive forms of action research, idealizing so-called spontaneous ideas and efforts coming from practitioners. To the contrary, the type of methodology I have in mind requires that general ideas of activity theory be put to the acid test of practical validity and relevance in interventions that aim at the construction of new models of activity jointly with the local participants. (p. 35)

The empirical basis of this thesis involves changes in an activity system which may account for an expansive cycle as discussed above (see more details in Section 3.4). Furthermore, I draw upon the following four methodological implications, suggested by Nardi (1996), which she derives from CHAT for conducting research in human–computer interaction. Firstly, the research frame of analysis should be long enough to understand the user's object. This implication stems from the claim that the activities are longer term formations and the objects are transformed into outcomes through a process of several phases rather than single steps. This suggestion led me to consider the data set spanned over long time when analysing certain aspects of the activity system, for instance, student's use of resources (Study 3). Secondly, the attention must be given to broad patterns or the bigger picture of activity rather than narrow episodic fragments. The small episodes may prove useful, but not in isolation from the overall situation. This second suggestion led me to choose the events for analysis in relation to the whole activity system (Study 3). Thirdly, the data collection techniques should be varied including video observations, interviews, historical materials without undue reliance on any one of the data sets. The data is collected using multiple methods (cf. Section 3.4). The fourth and the last consideration underscores that a researcher should be committed to understand the object from the user's point of view. Nardi (1996) writes:

> It is severely limiting to ignore motive and consciousness in human activity and constricting to confine analyses to observable moment by moment interaction. Aiming for a broader, deeper account of what people are up to as activity unfolds over time and reaching for a way to incorporate subjective accounts of why people do what they do and how prior knowledge shapes the experience of a given situation is the more satisfying path in the long run. (p. 94)

This last suggestion implied taking students' views into consideration while analysing their use of several resources in their actions (see Study 3).

3.3 Research strategy and research design

A research strategy describes the general orientation to the research. The nature of the research questions addressed in this thesis requires adopting a qualitative research strategy (cf. Bryman, 2015). The research design serves as a "blueprint" for any research which serves not only as a work plan but also safeguards against the situation in which evidence does not address the initial research questions (Yin, 2014). According to Yin, the three factors which are important to consider while selecting a research design are: (a) the type of research questions; (b) the extent to which a researcher has control over actual behaviour events; and (c) the degree of focus on contemporary or historical events (p. 9). Moreover, a case study research design is preferred when "a 'how' or 'why' question is being asked about a contemporary set of events, and over which the researcher has little or no control" (p. 14). This is because the 'how' and/or 'why' questions require more explanations and need exploring the operational links traced over time instead of measuring just frequencies. Such questions can be answered using a case study, an experiment, or a history (Yin, 2014). In case study research, the researcher is interested in exploring the unique features of the case whereas in other designs, the interest is to generate statements applicable regardless of time and space (Bryman, 2015). The case study design facilitates me to address the research questions formulated for my thesis (see Section 1.4). It enables me to make sense of affordances of an online environment through studying students' interactions with the resources (Study 3) and to analyse the factors in the environment which contribute to students' reasoning processes (Study 4).

According to Yin (2014), multiple-case studies with more than one carefully chosen case generate more compelling results than single-case studies. The selection of the cases should be done using replication designs instead of sampling design. That is, the cases should be selected based on some theoretical interests rather than on the surface properties of the cases such as the number of participants or the context. Each case is carefully selected so that it "either predicts similar results (a literal replication) or predicts contrasting results but for anticipatable reason (a theoretical replication)" (p. 57). A further suggestion concerns the number of cases deemed necessary in a multiple case study design, which depends on the number of replications considered necessary to acquire the desired level of certainty about the findings of the research.

A two-case study research design has been adopted in this thesis. This is also in line with methodological implications of CHAT. Two phases of an activity system or the expansive cycles (see details in Section 3.4) serve as the cases under consideration. The point of distinction between the two cases is the varied structure of the mathematics course during two consecutive semesters. Each case answers different research questions. The first three studies (Study 1, Study 2, and Study 3) arise from the first case study, while the fourth (Study 4) concerns analyses of both case studies.

The sampling in the case studies reported in this thesis is done at two levels: the context, and the participants (Yin, 2014). The first level of sampling is considered as purposive in the sense that the context is selected that is relevant for answering the research questions. The next level of sampling of participants is considered as random. The students who were willing to take part in the study were involved as participants. The drawback of such sampling at the participant level is that it becomes difficult to generalise the results to a broader population. The sampling serves the purposes of my research which is to document the processes of students' learning and significant factors which play a role in these processes. Generalisation in a quantitative sense is thus neither possible nor of particular interest in my research.

3.4 Context

The techniques for collecting data are considered as research methods. The examples of research methods can be questionnaires, structured observations, participant observations, etc. (Bryman, 2015). As stated earlier, the research reported in this thesis is based upon two case studies. In each case study, I used a different scheme of data collection methods which related to the addressed research questions as well as to the context of the research. In this section, I present the details about the context and the methods of data collection in each of the case studies, the online environment (3.4.1), specifications of the online, interactive educational system used (3.4.2), an account of the tutorial videos available for the students (3.4.3), and an account of the blended environment in the second case study (3.4.4) In the end, I also reflect on my role being the researcher during the process of conducting the research (3.4.5).

3.4.1 Case study 1—The online environment

The first round of data collection took place in Spring 2017. All the students from the class of electronics engineering were informed about the project. These students were in their second semester of an undergraduate engineering programme. Four students, Per, Jan, Tor, and Ole volunteered to participate in the study (See Appendices 1 for Letter of information and Letter of consent).

The mathematics course was organised by offering lectures in the form of tutorial videos. The homework and assessments were administered through MyMathLab (details in section 3.4.2). The final examination was digital, and students had access to the provided resources (textbook, calculator) as well as to the resources on Internet. The course spanned over the first two semesters (Autumn 2016, Spring 2017) in the first year of the undergraduate electronics engineering program. In the second semester (Spring 2017), the topics included differentiation, applications of differentiation, integration, applications of integration, and sequences and series.

The participants were requested to work together on campus. For the group work sessions, the group study rooms on campus were used. I used to book the group study room and inform the students through email. Those sessions were video recorded. As the course progressed, students' activity was becoming increasingly computer-based. I then asked the students to record their screen activity using the freeware, Camstudio (see Appendix 4 for the Camstudio settings).

Weekly journals were included to explore students' use of tutorial videos and I asked them to specify the manner in which they used videos or other resources in their work. After about two weeks, I provided them a table format for the weekly journals (attached as Appendix 3). Three of the students submitted their weekly journals. The journals were sent to me through email. In total, I received 34 journals (Per: 9, Jan: 12, Tor: 9). Semi-structured interviews were also conducted occasionally in order to complement the data in journals. The interviews were also focused on students' use of resources and the general manner of their work organisation (see Appendix 7 for the interview guide). In the group work sessions, students communicated with each other in Norwegian. The weekly journals and the interviews were conducted in English.

3.4.2 Specifications of MyMathLab

MyMathLab (MML) is an online interactive and educational system designed by Pearson Education to accompany its published mathematics textbooks. The system serves as a personalised learning environment (Borba et al., 2016) for mathematics. In both phases of the data collection, the homework and assessments were administered through MML.

In MML, the set of tasks in each homework is linked with corresponding sections in the textbook (Croft & Davison, 2015). The *Question Help* feature offers five options through a drop-down menu: help me solve this, view an example, textbook, calculator, and print. Through *help me solve this,* the learner receives the step by step assistance to solve the task at hand. The learner needs to perform the involved operations in order to proceed through the task. The *view an example* feature offers a solved example with slight changes in involved numbers or function in the task. The features, *textbook, calculator* and *print*, direct the users to these resources. Upon solving the task, a feature *check answer* specifies whether the answer is correct. If the answer is not correct, the feedback is provided by specifying the strategy to solve the task. If the answer is correct, the learner is proceeded to the next task.

3.4.3 Tutorial videos

The tutorial videos provided in this course replaced the lectures. The videos were created by the lecturer using a document camera. Each video was linked to a section in the textbook (see Figure 6). The videos were of varied length depending upon the topics the videos dealt with. Moreover, mathematical concepts were explained, and examples were provided in the videos.

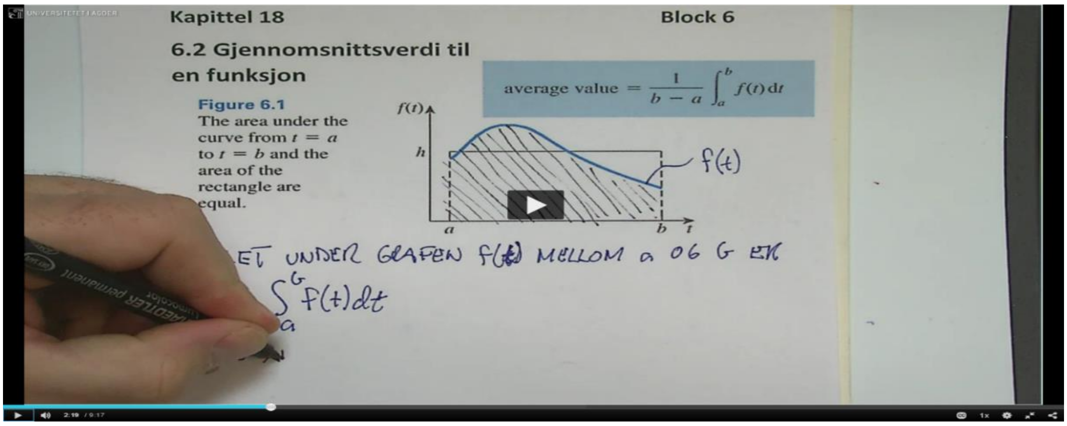

Figure 6: A screenshot from a tutorial video provided in the course.

3.4.4 Case study 2—The blended environment

The second phase of the data collection was carried out in Autumn 2017, in the same class. The three participants from the previous phase, Per, Jan and Tor, continued their participation in my research. Only one participant, Dag, was new in this phase of the data collection. The participants were now in their third semester of their studies.

The topics included in this course were: differential equations, functions of several variables and partial differentiation, Laplace transformation, and introduction to Fourier series and Fourier transformation. The organisation of this course differed from the previous course in certain aspects. This course included the weekly face-face lectures instead of video lectures. The group work was included occasionally during the lectures. The lecturer assigned the mathematical tasks and encouraged the students to work collaboratively on the tasks using paper and pencil. Moreover, the final examination was changed to written format in this semester. The homework and the tests were administered through MML as in the previous course.

For collection of data, I video recorded the lectures, as well as the group works, assigned corresponding with the lectures. During the group work sessions, two LiveScribe[1] Smartpens were provided to the participants. The LiveScribe Smartpens capture the writing activity along with the sound and record them synchronously in the form of pencasts. When students used the Smartpens during their group work, it facilitated in tracing their writing on the paper together with their utterances. Participants also worked in pairs, each having one pen, leading to more interactions among the group members. Semi-structured interviews were also conducted during this semester. The interviews were focused on the organisation of the course in both of the semesters, the differences and similarities in the use of resources in students' work. An overview of the data collected can be seen in Appendix 2.

Apart from these two case studies, I also collected data from another group of students from the electronics engineering students during Spring 2018. Those data have not been analysed so far due to the limitations of time as well as of personal resources for doing the translations to the English language.

[1] https://us.livescribe.com/

3.4.5 My role as a researcher

When I began collecting the data, I had completed six months of the PhD program. At that time, I had very limited knowledge of the Norwegian language. During the video observations, when students communicated with each other in Norwegian, I could observe to some extent what the students were doing but I was unable to grasp all of the exchanges among them. Therefore, I was mostly an observer, and I did not intervene in their mathematics work. However, I used to have short discussions at the end or the beginning of the sessions depending upon if I had to convey some further plans or wanted to ask what they were doing in those sessions. I believe that my presence along with the video camera might have had some effect on their ways of working and on the group dynamics. For instance, I observed in the initial sessions of group observation that the students were apparently conscious of being recorded with the video camera. Such an effect due to the presence of the researcher is known as the Hawthorne effect in the literature (cf. Sowder, 1998).

The lecturer enrolled me in MyMathLab as a student, so I had access to the weekly assignments in MML, important dates and tests in the progression of the course. Nevertheless, it was through students' journals and the screen recordings that I was able to make sense of their task solving processes. Towards the end of the first phase of data collection, I had a meeting with the lecturer of the course and informed him about the general observations I had at that time. The lecturer, probably based on my observations and own reflections, revised the structure of the course which concerns the second case study. I was informed about the changes in the upcoming course structure during the first phase of data collection.

3.5 Structuring and analysing the data

In the four studies arising from this research, the methods of analysing differed according to the foci of the addressed research questions. The data sources were selected and utilised accordingly. For instance, Study 3 concerns a holistic view of students' activity afforded in the online environment, and all the sources of data from the first case study were utilised for the purpose of analysis in this study. Below, I provide details of how each source of data was analysed and in each study.

3.5.1 Students' weekly journals and semi-structured interviews

The students' journals were analysed in Study 2 and Study 3. Study 2 focuses on students' use of resources and the manner in which these resources mediated between the mathematics and the students in their mathematics work. The main sources of data utilised were the weekly journals and one semi-structured interview. The students' responses regarding their use of several resources in their work were analysed, to differentiate between the epistemic and the pragmatic mediation by way of the resources used. For instance, when a student wrote that he used the tutorial videos to understand a concept. This was identified as an epistemic mediation. The uses such as the "got the questions", "double-checked the answers" were regarded as pragmatic mediation. From one semi-structured interview, I compared the students' use of MyMathLab and tutorial videos. From the second phase, the interviews have not been included in the analysis so far.

In Study 3, using the CHAT framework, the journals were analysed to make sense of the uses of resources of the action–goal layer of the activity. Through students' responses, I identified students' actions as something which was done by students. The indicators such as "I solved…" and "I got information…" were used to identify students' actions related to each of the resources. The goals were identified from those responses where they expressed what they wanted to achieve by using each resource. The responses such as "Use(d) the textbook to find the formulas for the different expansion series" indicated about the action as well as the goal of the student. Appendix 3 provides the document containing analysis of students' journals. The reason why the journals were used are theoretical. As indicated elsewhere, the actions are considered as in the conscious awareness of subjects of activity in contrast to the operations (Leont'ev, 1981b).

3.5.2 Video-recorded observations

The video data from the first case study were transcribed in English by a native speaker of Norwegian (see Appendix 6, for example). The screen recordings were attached to the respective video recordings. From the second phase, three group works were transcribed and translated into English (see Appendix 6). Electronic copies of the task sheets and the solution sheets were created. The mathematics tasks which the students solved in the three group work sessions can be seen in Appendix 5. Also, Livescribe PDFs and pencasts of students' written work were collected.

Study 1 focuses on the execution of mathematical competencies in the online environment. The primary data set used in this study was the video recordings of student group work from the first case study. From those sessions, the instances where the students used various digital resources were identified and analysed.

Study 4 focuses on students' reasoning processes (see section 2.3.2) in an online environment and a paper and pencil environment. The video recorded observations from both cases were utilised in this study. The video recorded group works from both case studies served as the primary source of data in this study. I search for instances of mathematical reasoning in students' work by going through transcriptions of video recordings, screen recordings, and students' written work (for details, see the summary of Study 4). According to Figure 5, the steps of selecting and implementing models were analysed. In this regard, the instances of students'work on complete tasks were searched from both environments. After that, the focus was placed on the differences between the two steps among the two settings.

3.6 The mathematical context

As mentioned above, the data collected in both case studies cover a wide range of mathematical topics. However, the instances for analysis were selected according to the foci of the research questions associated with each of the four studies. These instances (Study 1, Study 3, and Study 4) encompass students' work on techniques of integration, applications of integration, and stationary points of the functions of two variables. The subsections below treat these topics separately.

3.6.1 Integration by parts

Integrating by parts refers to a technique for finding a primitive of a given function. The following formula, based on the product rule for differentiation, is applied to carry out integration by parts,

$$\int u\left(\frac{dv}{dx}\right)dx = uv - \int v\left(\frac{du}{dx}\right)dx. \qquad (1)$$

In this formula, the integrand is seen as the product of the terms u and $\frac{dv}{dx}$. The term u must be differentiable in order to find $\frac{du}{dx}$. The term $\frac{dv}{dx}$ must be integrable to be able to find v. The intention is to replace the given integral on the left-hand side of (1) with the one on the right-hand side, selecting u and $\frac{dv}{dx}$ so that the integral on the right hand side becomes simpler than the initial integral. As an example, consider $\int xe^x dx$. Here, both the terms x and e^x are differentiable and integrable functions. If we let $u = e^x$, and $\frac{dv}{dx} = x$. Then, $\frac{du}{dx} = e^x$, $v = \frac{x^2}{2}$. Using (1), we get

$$\int xe^x dx = e^x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot e^x dx$$
$$= \frac{x^2 e^x}{2} - \frac{1}{2} \int x^2 e^x dx.$$

The integral on the right-hand side is more complicated than the one on the left-hand side, so this was not a wise choice. Now, let $u = x$, and $\frac{dv}{dx} = e^x$. Then, $\frac{du}{dx} = 1$, $v = e^x$. Using (1), we get

$$\int xe^x dx = xe^x - \int e^x \cdot 1 dx$$
$$= xe^x - e^x.$$

3.6.2 The integral as the limit of a sum

This treatment defines the integral as the limit of a sum. This method relates to the concept of Riemann sum introduced by German mathematician, Bernhard Riemann (Bressoud, 2019). The idea behind this treatment is illustrated below.

Consider the graph of a positive function $y(x)$ as shown in Figure 7 (a) below. An approximation to the area under the curve between $x = a$ and $x = b$ can be found by first dividing the area into strips of equal width δx and then approximating these strips by rectangles. The area of one such rectangular strip, with its left endpoint at $x = x_k$ will then be y$(x_k)(\ \delta x)$ (Figure 7(b)).

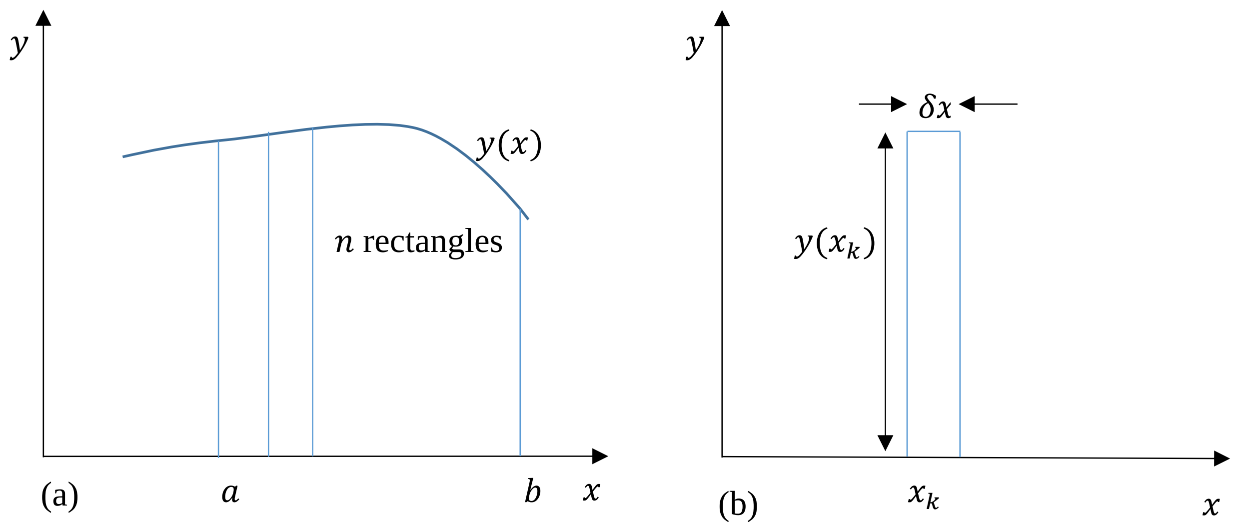

Figure 7: (a) Approximation of area by rectangles, (b) dimensions of one rectangle (Croft & Davison, 2015, p. 861).

Adding the areas of all the rectangular strips formed between $x_1 = a$ and $x_n = b - \delta x$, the sum $\sum_{k=1}^{n} y(x_k)(\delta x)$ provides an estimate for the area under the curve. This sum is a special case of Riemann sum. In this sum, when the width of the rectangles gets smaller, i.e. δx approaches zero and the number of rectangles n tends to infinity, the limiting value of the sum provides the exact area under the curve. Thus, the limiting value, $\lim_{n\to\infty} \sum_{k=1}^{n} y(x_k)(\,\delta x)$, becomes equal to the definite integral of the function $y(x)$ from a to b. A function is said to be integrable if the limit, $\lim_{n\to\infty} \sum_{k=1}^{n} y(x_k)(\,\delta x)$ exists. And,

$$\lim_{n\to\infty} \sum_{k=1}^{n} y(x_k)(\,\delta x) = \int_a^b y(x)dx$$

Thus, a definite integral can be expressed as a limit of a sum using the above treatment. Study 3 involves students' work on a task requiring the limit of sums method to solve a definite integral.

3.6.3 Using integrals to compute volumes: The Disk Method and the Shell Method

The idea of seeing the definite integral as the limit of a sum lies behind the application of integrals to compute areas, volumes, and other physical quantities. Below, the two methods for finding volumes, commonly termed as the Disc Method and the Shell Method, are treated in detail. These methods are relevant to the students' work in the first case study (see Study 3).

In these methods, the solid whose volume is to be found is generated by revolving an area bounded by the curve between two points about a given axis. This solid is, therefore, termed as a solid of revolution. Consider the area bounded by the curve $y = f(x) > 0$, $x = c \geq 0$, $x = d > c$, and the x-axis (see Figure 8(a)) and let this area be rotated about the x-axis. The volume of the resulting solid can be seen as a sum of circular disks of equal thickness δx and radius y (see Figure 8(a)).

The volume of one such disc formed at $x = x_k$ becomes $\pi y_k^2 \delta x$. Adding the volumes of these vertical discs provides an estimation of the total volume of the solid. If the number of the circular disks $n \to \infty$, that is the thickness of one such disk $\delta x \to 0$, the limit of the sum provides the exact volume of the solid.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \pi y_k^2 \delta x$$

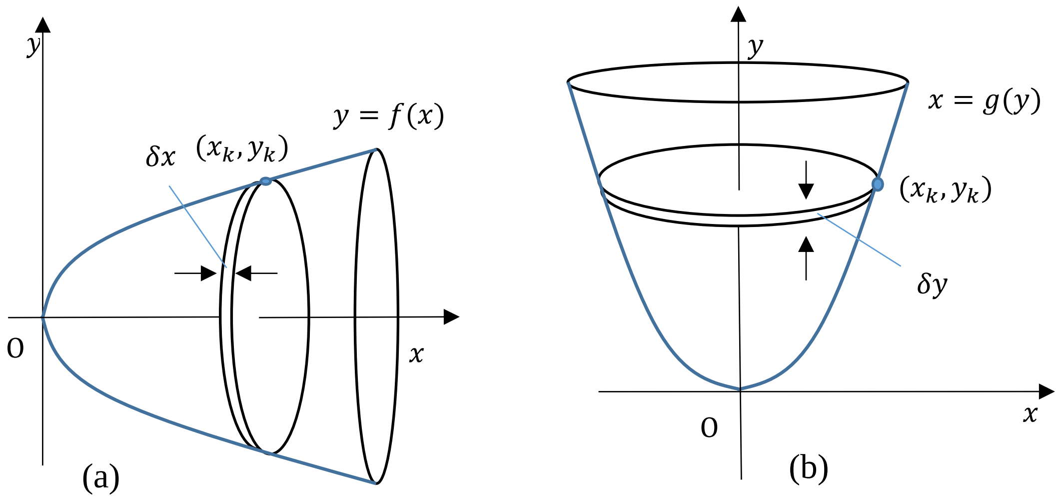

Figure 8: An illustration of the Disk Method when the solid is formed by revolution around (a) x -axis, (b) y-axis (adapted from Croft & Davison, 2015, p. 871).

Using the idea that the limit of a sum is equal to an integral, this summation equals the integral $\int_c^d \pi y^2 dx = \int_c^d \pi f(x)^2 dx$. The volume, therefore, can be found by evaluating this integral. This method is termed the Disc Method on account of the discs yielding the whole solid.

Consider, the curve of the equation of the form $x = g(y) > 0$ and the area bounded by $x = g(y) > 0$, $y = c \geq 0$, $y = d > c$, and the y-axis (see Figure 8(b)) and let this area be rotated about the y-axis. The volume of the resulting solid can now be seen as a sum of circular disks of equal thickness δy and radius x (see Figure 8(b)). The formed disks will be horizontal, and therefore the integration will be carried out along the y-axis. The role of x and y will be interchanged in the integration. The integral formula will thus become $\int_c^d \pi x^2 dy = \int_c^d \pi g(y)^2 dy$.

The Shell Method is applicable when the generated solid of revolution (see Figure 9) can be seen as a sum of cylindrical shells. Suppose that the same area i.e., the area between the curve $y = f(x) > 0$ and the x-axis, say from $x = c \geq 0$ and $x = d > c$, is rotated around the y-axis (see Figure 9), the generated solid in this case (shown with dotted lines in Figure 9) can be seen as composed of cylindrical shells of thickness δx.

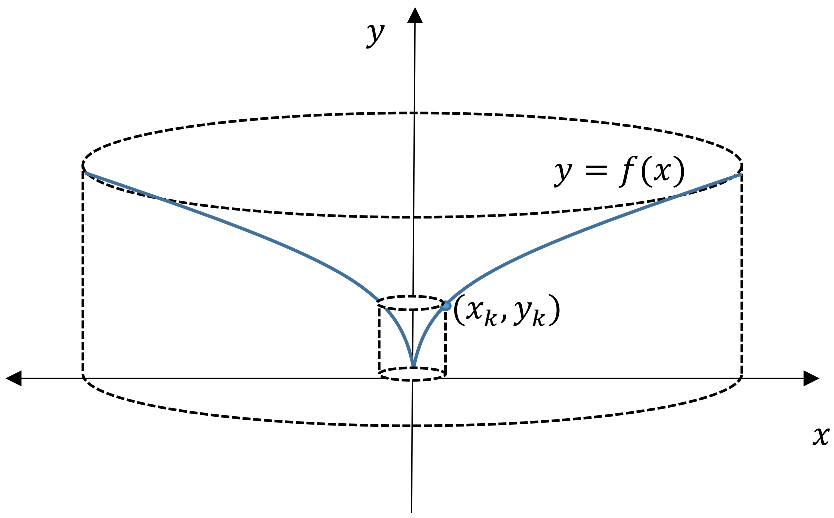

Figure 9: An illustration of the Shell Method of revolution.

The volume of one such shell formed at $x = x_k$ having radius x_k and width δx will be $2\pi x_k \cdot f(x_k) \cdot \delta x$. The sum of these volumes when $\delta x \to 0$, then becomes

$$\lim_{n\to\infty} \sum_{k=1}^{n} 2\pi x_k f(x_k)\delta x,$$

which gives the total volume of the solid, as the integral $\int_c^d 2\pi x f(x)dx =$ $2\pi \int_c^d x f(x)dx$.

Consider the case when an area is bounded by a curve with equation of the form $x = g(y) > 0$ and the y- axis, say from $y = c \geq 0$ and $y = d > c,$ is rotated around the x-axis . In this case, the cylindrical shells will have radius $y = y_k$ and height $x_k (=$ $g(y_k))$. The volume of one such shell with thickness δy will be $2\pi y_k \cdot g(y_k) \cdot \delta y$. Following the same steps as above, the integral will take the form $2\pi \int_c^d y f(y)dy$.

From the above illustrations of the Disk Method and the Shell Method, it can be noted that the integration is carried out along the axis of revolution in the Disk Method. In contrast, in the Shell Method, the integration is carried out perpendicular to the axis of revolution.

3.6.4 Stationary points of a function of two variables

In the second case study, one task involves investigating the nature of stationary points of a function of two variables (see Appendix 5). At stationary points, the graph of a function is neither increasing nor decreasing. That is, the gradient of the function at these points is the zero vector. The location of these points for a given function is found by equating where the first-order partial derivatives of the function are zero. That is

$$\frac{\partial f}{\partial x} = 0, \qquad \frac{\partial f}{\partial y} = 0$$

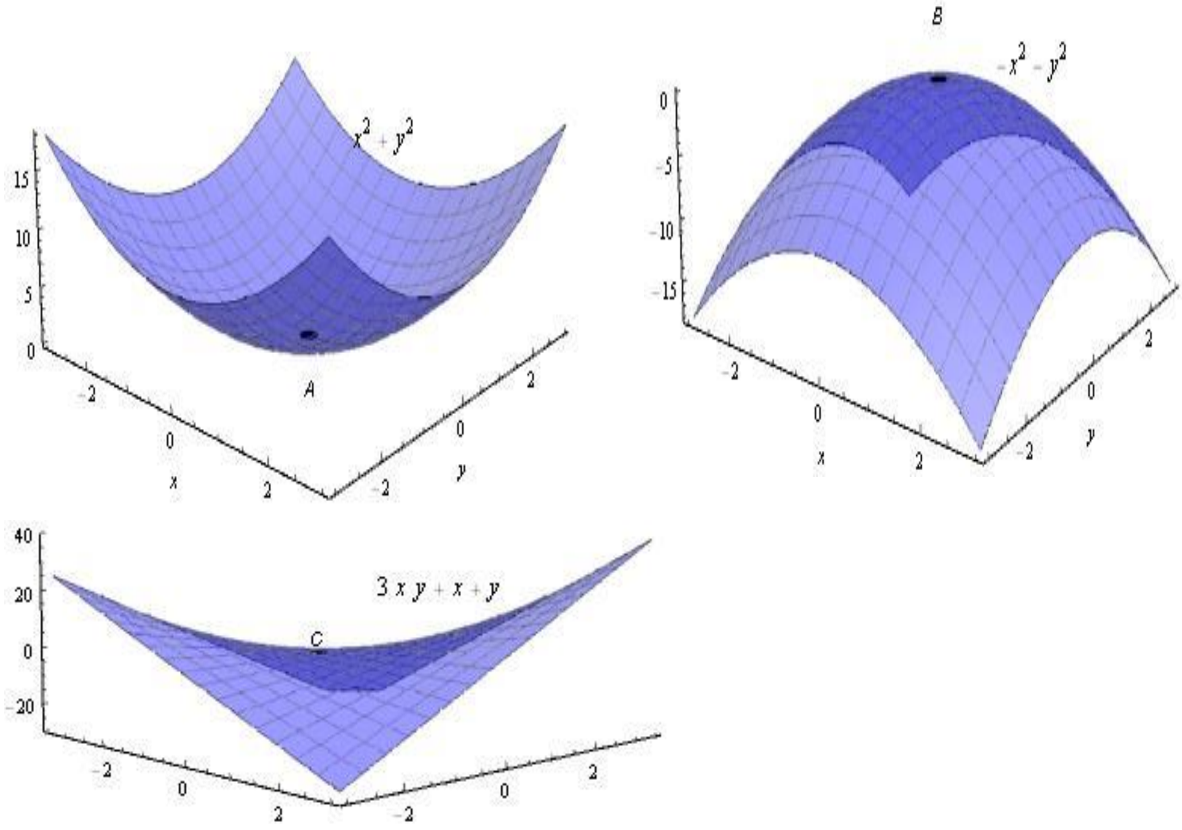

Figure 10: A is a minimum point of $f(x, y) = x^2 + y^2$; B is a maximum point of $f(x, y) = -x^2 - y^2$; and C is a saddle point of $f(x, y) = 3xy + x + y$.

Moreover, the graph of a function at stationary points either has the highest value, lowest value, or saddle-like behavior. Accordingly, the points are termed as a local maximum point, a local minimum point, or a saddle point. For illustration purposes, the maximum point, the minimum point, and the saddle point for three different functions are shown in Figure 10.

After finding stationary points using the first-order partial derivatives, the nature of the stationary points can be investigated through the following test (Croft & Davison, 2015).

For a stationary point (x, y), if

$$\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 < 0, \text{ } (x, y) \text{ is a saddle point}$$

$$\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0 \text{ and } \frac{\partial^2 f}{\partial x^2} > 0, \text{ } (x, y) \text{ is a minimum point}$$

$$\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0 \text{ and } \frac{\partial^2 f}{\partial x^2} < 0, \text{ } (x, y) \text{ is a maximum point}$$

$$\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = 0, \text{ no conclusion can be drawn}$$

3.7 Quality criteria in my research

According to Bryman (2015), three main criteria for evaluating the quality of quantitative, social research are reliability, replicability, and internal and external validity. Reliability addresses the question of whether the measures of the concepts under investigation are consistent. Replicability deals with the replication of the research, i.e. the research results must be replicable to address the concerns such as when the generated results do not match with the existing evidence. In order for a study to be replicable, the operationalisation of the research must be clear enough or consistent. That is, the study must be reliable in the selection of the measures to study. Thus, the criterion of replicability relates to that of reliability. Validity is concerned with the integrity of the conclusions that are generated from a piece of research.

In the case of qualitative research, the parallel of these three criteria are suggested as credibility, transferability, dependability and confirmability, which come under the umbrella term of trustworthiness (cf. Bryman, 2015). The criterion of credibility parallels the internal validity, which deals with the question whether the conclusions of the research about a causal relationship between two variables holds true. Transferability parallels the external validity, which concerns the issue if the findings apply to other contexts. Dependability parallels reliability and deals with the questions whether the findings can be applied at other times. Confirmability concerns objectivity of the research and relates to the question whether the researcher has allowed the personal values and beliefs intrude to a high degree.

In case study research design, as in my thesis, Yin (2014) calls for the following four quality tests: construct validity, internal validity, external validity, and reliability. These quality criteria may be achieved by adopting some tactics at different stages of the case studies. For instance, construct validity calls for developing the operational set of measures of the concepts for collecting the data rather than the subjective judgements; it parallels objectivity in Bryman's (2015) terms. It is achievable by using multiple sources of data which establish a chain of evidence which makes case study findings more convincing and accurate. If the triangulation of data sources is aimed to develop a convergent evidence, it helps to increase the construct validity. In the first case study, I used multiple methods of data collection, referred to as triangulation of methods (Yin, 2014), in order to develop converging lines of inquiry. These multiple data collection methods generated different data sources which aided in establishing the chain of evidence about the layers of the activity while analysing the dynamics of the activity system. For instance, in Study 3, the video recordings aided in looking at the operation–condition and the operational aspects of students' activity, and the students' journals were used to analyse the action–goal layer and the intentional aspects of the students' activity. The use of different sources of data was driven by the theoretical guidelines of activity theory to study the constructs of actions and operations (see also Section 2.2.3). The findings from both sources of the data converged in the sense that the stated actions and goals in the journals were traceable to their corresponding operations in the video data.

Internal validity is relevant when causal relationships need to be established, mainly in explanatory studies, by considering all the hidden variables thus avoiding the spurious relationships. Internal validity also improves if the construct validity of the research is clear and well established. I believe that this is clear in Study 3, which illustrates links between the levels (activity, action, and operation) of students' online learning activity.

Concerning external validity, Yin (2014) states that the purpose of case studies is not to generalize to a wider public as in the survey analysis. Rather, case studies are meant to produce rich descriptions of the cases being studied. The goal is to expand or generalize theories and not to extrapolate probabilities. Yin calls this analytic generaliation rather than statistical generalization. In my research, the aim is for analytic generalisation on account of the consideration of a specific learning environment and hence certain tools and conditions.

As mentioned, reliability concerns whether the operations of the study are repeatable. In case study research, achieving reliability is only possible when all the procedures are operationalised to the most possible level so that it is likely for someone else to be able to understand and repeat the study. The replication does not mean to replicate the results of one case study by doing another case study but doing the same case over again. I argue that I have made the operationalisations of the case studies understandable for replication, i.e. in terms of application of activity theory as discussed before.

3.8 Ethical considerations for my study

My PhD research involved voluntary participation from undergraduate electronics engineering students. Initially, the project was aimed at investigating the role of the tutorial videos in engineering students' learning of mathematics. The project was registered with NSD—Norwegian Centre for Research Data (http://www.nsd.uib.no/) and fulfilled their requirements. I informed all the students in the class of electronics engineering students about this project. I asked the students if they wanted to participate in my project. Only four students volunteered to participate in the project. These students signed the consent form (see Appendix 1). Also, they were informed that they could withdraw from the project at any later point (stated in the information letter).

In the first meeting with the four participants, I informed them more about the plans of my data collection and asked the students regarding their general manner of working in the course. My impression after the initial discussion was that the students were not using the tutorial videos in any rigorous manner. Rather they mentioned using MyMathLab and other resources (see Study 2, Study 3). Therefore, I decided to focus on other resources as well, in addition to the tutorial videos, in my project. In this meeting, the students also told me that there were no regular lectures and they preferred to work from home. Therefore, the weekly journals were added at this point to get glimpses into the use of resources in students' work in a systematic manner, which was not initially planned for.

Bryman (2015), with reference to Diener and Crandall, suggests to pay attention to the following four ethical concerns in a research project: (a) harm to the participants; (b) lack of informed consent; (c) invasion of privacy; and (d) involvement of deception. Considering my own research, I believe that the possible harm in my study could be the extra time they had to spend on writing the weekly journals in addition to their normal course work. My presence in the group room and my observations may also have caused disturbances in the students' mathematical work that may have not been there without my presence. The use of a video camera may likewise have caused stress amongst the students. However, my experience from being present in the group room with the camera tells me that this ethical principle was not violated. With respect to informed consent, I gave the students an information letter (Appendix 1) and I informed them orally as well. Regarding the invasion of privacy, no data were collected involving sensitive information. The names of the participants are anonymised, and other details are treated confidentially. For the last point, I informed students that the aim of the current project was to explore the role of the resources in their learning of mathematics. I argue that studies 1–4, as well as this extended abstract, reveal that this ethical principle is not violated in my research. I have indeed explored the role of resources and students’ use of these in their mathematics work.

4 Summary of research papers

In this chapter, I present summaries of the four studies which constitute the empirical research basis of this thesis. The studies are presented in the chronological order in which they were written. The sequence of the papers also portrays the path which I have taken during my PhD and the gradual development of myself as a researcher. Study 1 (Section 4.1) is a preliminary report based on the initial analysis from the first case study and focuses on the execution of mathematical competencies in the online environment. Study 2 (Section 4.2) focuses on students' use of resources in the online environment and adopts the theoretical framework of documentational approach to didactics. In Study 3 (Section 4.3), I focus on affordances of the online environment adopting a holistic view upon the activity system within which the students participate. The last summary, Study 4 (Section 4.4), focuses on the students' reasoning processes in the online environment and the paper and pencil environment and utilises the empirical material from both case studies.

4.1 Study 1: Mathematical competencies and e-learning: a case study of engineering students' use of digital resources[2]

This preliminary research report analyses the manner in which the online learning environment under consideration supports execution of mathematical competencies (cf. RQ1 and RQ2 in Section 1.4). The framework of mathematical competencies (Niss, 2003; Niss & Højgaard, 2011), as explicated in Section 2.3, was used to guide the analysis of mathematical aspects of students' work while working in a technological environment. The framework divides the mathematical competencies into two groups: the ability to ask and answer questions in and with mathematics, and the ability to deal with mathematical language and tools.

In this study, I only utilised video-recorded group works and looked for the short instances which provided evidence for the execution of mathematical competencies with the use of digital tools in students' work. The unit of analysis was taken as mediated action (Wertsch, 1991). The analyses of the selected instances show that

[2] This study was submitted and published as a preliminary research report in the proceedings of the Research in Undergraduate Mathematics Education (RUME) conference held in 2018.

multiple competencies were in action at the same time. However, in this initial analysis I kept my focus on distinguishing between the two sets of competencies rather than disentangling all the 8 competencies in Niss and Højgaard’s (2011) framework.

The analysis illustrated that the students in my study utilised multiple tools such as WolframAlpha, GeoGebra, and Maxima while solving the tasks in the online environment. The students used these tools to handle mathematical symbolism and formalism as well as for mathematical reasoning, and problem tackling. For example, when Maxima provided a solution in a form which the students could not comprehend, they used WolframAlpha to confirm if their solution obtained in Maxima was correct. WolframAlpha supported students’ reasoning about finding and validating the final answers. That is, the competency of using tools facilitated in executing these competencies from the first group. However, the analyses show that the students used these tools by relating to surface mathematical properties involved in the tasks. For example, the students used tools such as Maxima and WolframAlpha for deciding about the final solution, rather than exploring the mathematical aspects in depth. Based on rather short analyses, I conjecture that there were more traces of the second group of competencies in the students’ work and the environment afforded more of the use of different tools for solving the assigned tasks. Moreover, the use of resources had twofold effects for execution of mathematical competencies of mathematical reasoning, which related to the nature of the resources in the students’ use. That is, when the resources facilitated in calculating and providing answers, the possibilities of exploration of mathematical properties were limited. On the other hand, when the resources were used to comprehend the tasks at hand, there was more potential for engaging with the intrinsic mathematical properties.

The paper concluded with this reflecting question: How to devise a better systematic scheme for analysing mathematical competencies in this environment? My next goal was to come up with a narrowed down theme for analysing students’ engagement with mathematics in the online environment. This goal led me to conduct the research resulting in Study 2 below.

4.2 Study 2: Engineering students' engagement with resources in an online learning environment[3]

This study focuses on the undergraduate engineering students' manner of using resources in their online learning activity (cf. RQ3 in Section 1.4). The theoretical lens of documental approach to didactics (Gueudet & Trouche, 2009) has been used in this study. The data set used comprises students' journals, semi structured interview(s), and screen recordings of their online work in mathematics from the first case study.

Through these data, I analysed general features of students' techniques while solving the tasks in the online environment. A technique is regarded as the manner of solving a task (Artigue, 2002). Techniques are characterised as having either pragmatic or epistemic value based on the nature of the tool mediation as well as the engagement with the involved mathematics. A technique with a pragmatic value refers to the use of tools in reaching to end results or solution of the tasks. A technique with an epistemic value refers to making sense of the involved concepts while solving the tasks. In my study, I identified the pragmatic and the epistemic values of students' techniques through students' weekly journals, although the attention was not confined to particular mathematical tasks.

As evident from the previous section, the students in my study employed several other resources to solve the tasks, including online tools such as WolframAlpha, online calculators, YouTube videos, webpages, as well as programming in Maxima in their techniques, in addition to the provided resources including the textbook, tutorial videos and accompanying notes. The provided resources such as tutorial videos, textbook, and MyMathLab suggested the paper and pencil-based techniques for solving the given tasks in these resources. The analysis shows that the students opted more for the instrumented techniques with the progression of the mathematics course.

The students used the resources in various manners which linked to the potential and function of each of the resources. The students linked the use of WolframAlpha

[3] This study was published in the proceedings of the International Network for Didactic Research in University Mathematics (INDRUM) conference held in 2018.

and other computing tools with the pragmatic purposes. For instance, the students used WolframAlpha to doublecheck the answers obtained from Maxima, as a shortcut to solve the tasks and to get help with the difficult questions. The online calculators were used to solve the tasks in homeworks as well as tests which are considered as a pragmatic purpose. During a project in the mathematics course, students were required to program some tasks in Maxima. Later, they used Maxima to solve the assigned tasks in homeworks and tests. While students apparently associated the pragmatic purposes of obtaining the solutions with the use of Maxima, they expressed that it was hard for them to develop the Maxima code for the first time. Therefore, I concluded that the use of Maxima was not merely pragmatic as the students needed to comprehend the mathematical tasks as well as the programming language for making the programs. The students in my study preferred using MyMathLab over tutorial videos. They expressed that MyMathLab was the source of quick and most relevant help for them. This can be explained by the fact that the features in MyMathLab offered immediate assistance on the tasks at hand whereas the videos required more time in order to search for the required information. Also, they associated the use of tutorial videos with the epistemic purposes as they used the videos to make sense of the mathematical concepts.

The findings of this study suggest that the students' unexpected use of some resources (online calculators) and the techniques was linked to the conditions of MyMathLab and the general features of the setting, particularly the combination of the online homework and assessment.

4.3 Study 3: Exploring affordances of an online environment: a case-study of electronics engineering undergraduate students' activity in mathematics[4]

This study seeks to explore affordances (Bærentsen & Trettvik, 2002) of the online learning environment for undergraduate engineering students' activity in mathematics (cf. RQ4 and RQ5 in Section 1.4). This study adopts a holistic view on the online environment which involves administration of homework and assessments through MyMathLab, lectures through tutorial videos, and the final examination in an

[4] This study was published in International Journal for Research in Undergraduate Mathematics Education (IJRUME) in 2020 (vol: 6(1), pp. 42-64).

electronic format. Moreover, this study complements the findings from the previous studies (Study 1, Study 2) and analyses students' use of resources in greater depth.

In this study, students' choices of different resources and in turn their engagement with mathematical tasks, have been studied by considering both the macro features and the micro features of the learning environment. The macro view of the learning environment is taken as the level of the mathematics course. Engeström's (2014) extended triangular model of an activity system was used to conceptualise the learning environment and the triangular model was also used as the unit of analysis. The extended triangular model takes into account the societal mediations and tool mediations in human activities. The societal mediations, rules, community, and division of labour, were considered at the level of the mathematics course in this study. In this manner, it allowed analysing the macro features of the learning environment and their relationships with students' engagement with mathematics. The micro features in this study concern the conditions of the MyMathLab system as well as of other tools in students' use which affected the students' engagement with the mathematical tasks.

Leont'ev's (1974) theoretical model of activity structure facilitated in analysing the micro features of the students' interactions with several tools. According to this model (shown in Figure 2 in Section 2.2), an activity can be analysed at three levels: activity–motive, actions–goals, and operations–conditions. In my study, the two bottom layers were analysed using different sources of data which aided in making sense of both intentional and operational aspects of students' use of resources. I used students' journals for the analysis of their actions and goals, linked with the use of several reported resources.

As seen in the previous two studies (Study 1, Study 2), the students incorporated several tools and resources to solve the assigned tasks. The analysis of students' journals showed students' actions as well as goals linked to the use of each resource. The analysis led to the identification of the association of students' goals and actions with the elements of the collective activity system. For instance, the students' goals linked to Maxima were to make templates, to make the work easier for them, as well as to prepare themselves in accordance with the final digital examination. These goals pointed towards the explicit rule concerning the final examination in the digital format. This rule led the students to use this resource.

The intentional aspects of the students' selection of some resources pointed towards the features of the collective activity system, i.e. the conditions of the learning environment at the level of the mathematics course. The students used some of the resources such as Maxima in order to prepare themselves according to the conditions of the final examination as well as conditions of the homeworks and the tests. The operational aspects at the operation-condition level of the students' activity were analysed through students' recorded group work sessions. Particularly, the micro conditions of the MyMathLab system and other resources in the students' engagement with the mathematical operations in the assigned tasks have been analysed. The micro analysis shows that the conditions of MyMathLab allowed students to proceed through the tasks by providing the final solutions of the tasks regardless of the process of reaching to those solutions. In some instances, where students solved the mathematical tasks by using different resources, they did not engage with the mathematical operations according to the requirements of tasks. For instance, they computed an integral using Maxima without engaging with the required process of integration. The nature of the tasks in MyMathLab was also a contributing factor to the manner of the students' engagement with the mathematical operations. That is, with respect to the procedural tasks, which could be solved in single steps using online calculators, the involved mathematics was black-boxed (Anderson, 1999) for students as there remained no need for them to engage with mathematical operations. However, the students engaged relatively more with the mathematical operations in the tasks which demanded explorations instead of applying given procedures. In such tasks, the students utilised the potential of several tools for exploring relationships, visualising the mathematical objects, etc. instead of computing final solutions.

4.4 Study 4: Undergraduate engineering students' mathematical reasoning processes in an online and a paper and pencil environment [5]

This study seeks to characterise undergraduate engineering students' reasoning processes in an online environment and a paper and pencil environment (cf. RQ6 in Section 1.4). The aim of the study is to analyse the role of these environments in

[5] This is a manuscript under preparation and the manuscript is attached in the appendices in the form in which it currently is, at the time of delivery.

students’ mathematical reasoning (Lithner, 2003). In order to do so, the study leans on the existing research on mathematical reasoning (Lithner, 2008), and adds another dimension of the environment in the study of mathematical reasoning. This additional dimension of the environment was added using the action–operation dynamics from CHAT. The process of reasoning is seen as comprising the steps of selecting and implementing strategies in solving a task (Lithner, 2003). Through a cultural-historical perspective, solving a task is seen as a combination of actions and operations which entail the selection and implementation of strategies (see Figure 5, Section 2.3.3). The selection of strategy is considered as selecting a model that refers to the method for carrying out actions of solving the tasks. The implementation of strategy entails execution of mathematical operations involved in a task which relate to the conditions of the tools in use.

In CHAT, an action is considered as conscious whereas an operation is nonconscious or consciously controlled (Leont'ev, 1981b). Against this background, a mathematical operation may initially be an action which becomes a consciously controlled operation later. The nonconscious operations are also relevant for the execution of consciously controlled mathematical operations. In my study, the actions and operations are identified from the data in the form of students’ utterances, writings, button clicks within the software, shifting between the computer windows, etc. The distinction between the operations and actions has been made through the identification of students’ focus on some parts of the tasks. That is, those parts which came under discussion were considered as actions and others which were carried out without paying much attention were considered as operations.

The data utilised in this study were the video-recorded observations of students’ group work from both case studies. The analysis illustrates similarities as well as differences in the students’ processes of reasoning in the two environments, which relate to the nature of the tasks and the tools in students’ use. Regarding the nature of the tasks, the analysis shows that when the models were given in the form of mathematical formulas, the students engaged in implementing given formulas and hence in formation and execution of the involved operations.

However, if the models were not given, the students searched for them using the potential of the resources at hand and by engaging with the mathematical properties of the task. This example concerns the Shell Method task: Use shell method to find the volume generated by revolving the region bounded by $x = 16y - y^2$ and $x = 0$ about the x-axis.

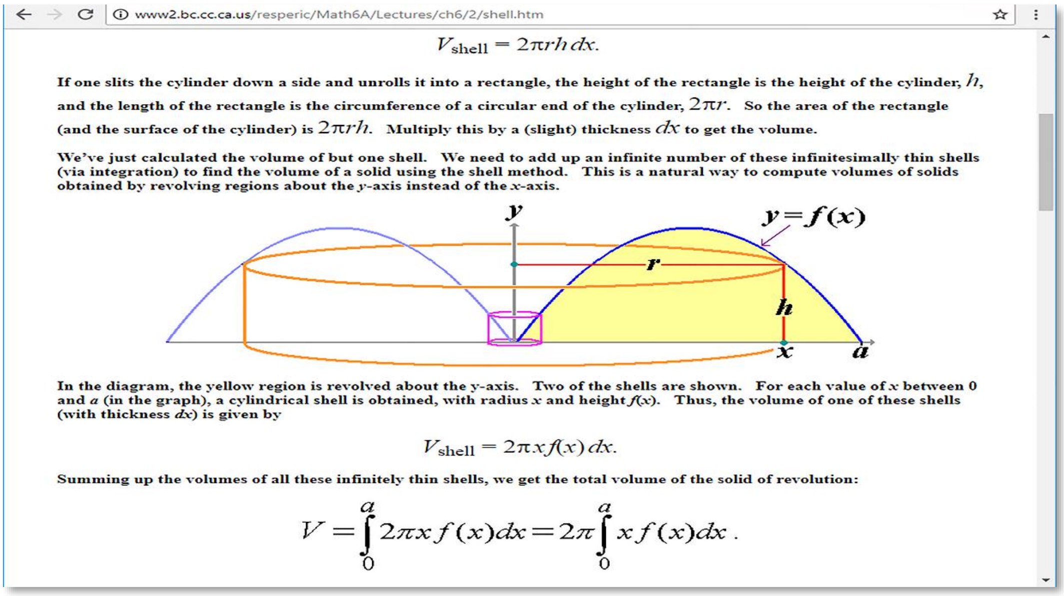

Figure 11: Per's search on Internet in the Shell Method task

When model was not available in the Shell Method task,Per first applied a model previously used in the Disk Method task without considering the involved mathematical aspects in the Shell Method.

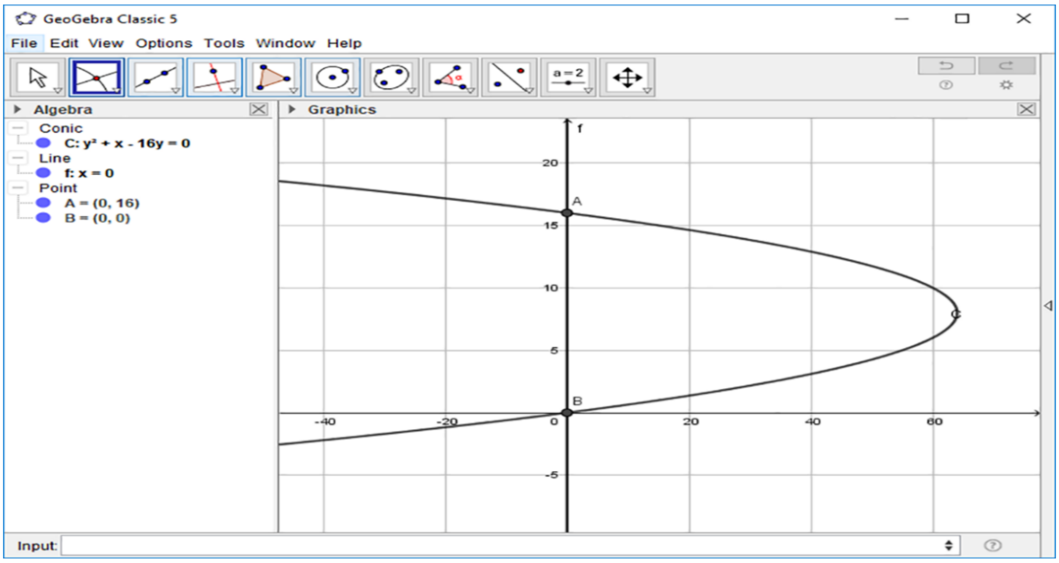

Figure 12. Per's use of GeoGebra in the Shell Method task

When this strategy failed, Per started to experiment by considering intrinsic properties to reach a solution of the task. In other words, the student started by a trial and error approach, and when this strategy did not work, he later started utilising the potential of the resources at hand, i.e. GeoGebra, Internet, and Maxima, to gradually make a new model and to reach to the solution of the task (see Figure 11 and Figure 12).

The analysis in this study illustrates that when the students solved tasks using the programming in Maxima, their actions were focused on finding commands to compute the integral. For instance, in the case of an integration task, the model they used comprised the conversion of an integral formula into programming in Maxima. The operations were executed in Maxima. The students obtained the correct solution, but they experienced trouble in validating the solution. This was due to the fact that the solution was in a different format due to appearance of additional terms in Maxima.

On the other hand, the students managed a similar situation successfully in the paper and pencil environment. Upon encountering a mistake in the final solutions, they checked back the performed operations and sorted out the mistake in the process. This contrast was explained by the fact that the mathematical operations were visible in the paper and pencil environment, which provided the students a basis to reason about the validity of the solutions. That is, the operations were visible and available for students to engage with contrary to the case of Maxima.

5 Conclusions and discussion

In this chapter, I discuss the results of my research with respect to the research literature presented and the theories adopted, and I present conclusions of my research. The chapter begins by revisiting the research questions in Section 5.1. In Sections 5.2–5.5, I summarise and discuss the findings with regards to the addressed research questions. This is done through a synthesis of concepts selected for analysis and the results presented in the four arising studies. In Sections 5.6, I reflect on the adopted theoretical and methodological approaches and their links with the research results. The chapter is ended with Section 5.7, which gives an account of the limitations of my research.

5.1 Revisiting the research questions

This thesis aims to provide deeper insights into the factors of the learning environment which play a role in undergraduate engineering students' engagement with mathematics. Two case studies have been performed which take into consideration an online learning environment and a blended learning environment. The case studies generate theoretical generalisations (Yin, 2014) which is in line with the goals of this research project. The research questions addressed in this thesis are given below.

RQ1: What traces of mathematical competencies are observed in students' work when they practice mathematics digitally?

RQ2: How does this (online) environment afford the execution of these mathematical competencies?

RQ3: How do engineering students incorporate resources during their work in an online learning environment?

RQ4: How do a small group of undergraduate engineering students interact with an online environment in their mathematical learning activity?

RQ5: In what manner does this environment afford students' engagement with mathematics?

RQ6: How do a small group of undergraduate engineering students accomplish mathematical reasoning processes in an online and a paper and pencil environment?

In the following sections, I outline the research outcomes with respect to the foci of each of the research questions presented above.

5.2 Mathematical competencies in the online learning environment—RQ1 and RQ2

To make sense of the way the online learning environment afforded students' engagement with mathematics, I utilised the lens of a competence framework (Niss & Højgaard, 2011). This competence framework divides the notion of mathematical competence into two subgroups: (1) asking and answering question in mathematics, and (2) using mathematical language and tools. In Section 2.3.1, I elaborated these subgroups and illustrated how this competence framework facilitated in characterising the studied students' mathematics work.

The online learning environment under consideration afforded students' use of many resources in their mathematics work, including WolframAlpha, GeoGebra, online calculators, Webpages, etc. (see further details in Section 5.3). I focused on the instances in which the students used several resources in solving the mathematical tasks. The analytical findings (see Study 1) illustrate that the students handled mathematical symbolism and related issues of mathematical language by simultaneously employing multiple tools. For instance, when WolframAlpha and Maxima were used together to solve one mathematical task and to validate the obtained solution of the task. In order to make sense of the output generated by Maxima, the students made use of WolframAlpha to check if the obtained solution was correct. I argued that the competency of tool-use was in action in these instances and the students used the functionalities of the tools to reason and argue about mathematical tasks.

Zembat's (2008) conclusion that a technological environment supported students' shifting from analytical reasoning to practical reasoning due to available functionalities of the tools is partially supported in my research. Although the functionalities of the tools were used by the students in my study to reason in practical ways, the objects of reasoning were often the outputs generated or the syntax related issues in the tools-in-use. Therefore, I regarded this approach as engagement with superficial features of the mathematical tasks at hand, as the students concentrated more on the use of tools and resources to tackle arising issues than to reason about the underlying mathematical properties.

Overall, the competencies of asking and answering questions were dominated by the tool using competency in the online environment (cf. Niss & Højgaard, 2011). This online environment facilitated the unfolding and revealing of the competencies of using mathematical language and tools to a greater extent than the competencies of asking and answering questions in mathematics. The results thus point toward the risk that using tools with computational power might hinder students' engagement with mathematical thinking, reasoning, and problem tackling. An implication which arises from this result is that the online environment should be designed in such a way that the students get to explore the intrinsic mathematical properties. That is, the students in the online environment should engage with the tasks involving competencies of asking and answering questions. The relationship between tools and tasks are of crucial importance to consider in this regard. By doing so, the recommendations by standards of the curriculum document for engineering mathematics regarding the use of technology can also be addressed in an appropriate manner (Alpers, 2011).

5.3 Engineering students' incorporation of resources in the online learning environment—RQ3

The third research question addresses the manner in which undergraduate engineering students incorporated resources in their mathematics work. The students' use of the resources will be discussed with respect to the distinction between pragmatic and epistemic purposes (Artigue, 2002), linked with the resources in students' techniques. In the online learning environment (for details, see Section 3.4.1), MyMathLab (MML) served as the central resource as it offered the tasks, assistance in solving the tasks, and feedback on the solutions of those tasks. In addition, the textbook, the tutorial videos, and the notes accompanying the videos were provided by the lecturer. The analytical findings (cf. Study 2) indicate that while students employed the provided resources in their general techniques (Artigue, 2002), they also employed other resources, including GeoGebra, WolframAlpha, Webpages, Maxima, YouTube videos, and online calculators. The analyses show different purposes linked with the students' use of these resources according to their needs and the functions of each resource. For instance, they used online calculators and WolframAlpha in order to produce solutions of the tasks at hand or for related purposes such as double checking the obtained solutions. Therefore, I described the students' use of Maxima, online calculators, and WolframAlpha as having pragmatic purposes (cf. Artigue, 2002). On the other hand, the use of the textbook was associated with both pragmatic and epistemic purposes in the parlance of Artigue, as they used the textbook to learn about mathematical concepts as well as to get the tasks and to check the solutions of those tasks. Moreover, through the analysis it also became evident that the students' use of the tutorial videos fulfilled epistemic purposes as they watched the videos to learn about the mathematical concepts. However, they preferred MML over watching the videos. That was due to the fact that MML provided immediate help and feedback related to the tasks at hand, whereas tutorial videos required using more time in order to fetch the required information. The results pinpointed that the students' use of the resources was related to their needs as well as to the functions of each resource. The resources, such as tutorial videos, linked with epistemic purposes were not extensively used compared with the ones linked with pragmatic purposes. My results thus coincide with the results provided in the study of Gueudet and Pepin (2016). The results point toward the needs to clarify the purpose of the use of each of the resources by the teachers, as suggested by Gueudet and Pepin (2018).

5.4 Engineering students' learning activity in an online environment—RQ4 and RQ5

To make sense of the factors of the online learning environment that played a role in students' engagement with mathematics, I adopted a holistic perspective of students' activity (Engeström, 2014; Leont'ev, 1974), i.e. Cultural Historical Activity Theory (CHAT). This perspective allowed combining the macro view and micro view of students' activity (Jaworski & Potari, 2009) in the online learning environment and facilitated in tracing the relationships between the system level factors and students' interactions with the mathematical tasks.

The analyses (see Study 3) illustrate that the macro conditions pertaining to the collective activity system (Engeström, 2014) played an important role in students' choice of resources and tools. In particular, the collective rule of the final digital examination led the students to opt for and use several resources, such as Maxima and the online calculators. For example, the analyses in Study 3 show that the students used Maxima with the goals to solve the tasks at hand and to make templates for the future use during the examination. The goal of examination was linked with the students' use of resources, a result confirming a finding in an earlier study by (Anastasakis et al., 2017). With the progression of the mathematics course, the use of Maxima and other online tools became dominant in students' work with the tasks.

The availability of powerful online tools allowed many action possibilities for students, from searching for required information and exploring mathematical properties to finding solutions of tasks. The resources such as GeoGebra aided in exploring the properties of the mathematical objects, whereas the online calculators provided final answers to the tasks. The Google search engine was used to search for various sorts of information, such as mathematical formulas and syntax in Maxima. In turn, the choices of these resources affected their micro level interactions with the mathematical tasks in MML.

Moreover, the micro conditions of MML along with the demands of the tasks also played a role in how students employed the resources in solving the tasks. In particular, the MML system allowed the students to proceed through the tasks by entering final solutions of individual tasks. Despite the fact that the system offered assistance in the process of solving tasks in the form of solution steps, the only requirement for the students was to provide the final answer of tasks. The analytical findings show that the students focused more on finding the solutions than to engage with the process. This concern of administering an online program is documented in earlier studies (cf. Rønning, 2017; Webel et al., 2017). The analysis indicates that students in my study obtained the final answers by varying techniques, such as using programming in Maxima, through the use of online calculators, or by following the solution steps given in MML. It showed that these students managed the search of symbolic procedures, also found by Krupa et al. (2015). However, most of the times, their activity deviated from the required activity. In line with the results of Cazes and Vandebrouck (2013) who observed that the students' activity was sometimes similar but often varied from the expected activity in the automated programs namely Electronic Exercise Bases (EEBs). However, the variation in Cazes and Vandebrouck's study did not involve the use of resources in students' activity, rather it referred to the deviation in the sequence of operations.

In some of the tasks where programming was used, the students had to convert mathematical formulas into Maxima codes which required tackling the syntax in Maxima. For example, in the task concerning the evaluation of an integral using the limit of sum method (see Section 3.6.2), students converted the integral formula into a Maxima code and obtained the final solution. As a result, the object of this task was shifted from the process of integration to the computation of the final answer. This shift was due to the fact that the students could not engage with the required process of integration using the limit of sum method and the involved operations. Similarly, with respect to the Disk Method (see Section 3.6.3), where the mathematical formula was stated together with the task, a conversion of the mathematical formula into Maxima code was required. Students' focus shifted from intrinsic mathematical properties to surface properties of these tasks. In contrast, the tasks that required use of the Shell Method, where the mathematical formula which concerned application of integrals such as in the Shell Method was not provided, the students had to first comprehend the mathematical task in order to write the command in Maxima. They needed to extract the relevant information from the task to use in the Maxima code in order to obtain the solution of the task. This allowed them to use the tools while engaging with the mathematical properties of the task. That is, they searched on the Internet for the correct mathematical formula by taking into account the demands of the task. Similarly, they used GeoGebra to find the points of intersection of the given curve with a coordinate axis, which later served as the limits of the integration.

It also emerged that the tasks in the MML program were mostly in the form of direct application tasks, with the terminology coined by Cazes and Vandebrouck (2013). Cazes and Vandebrouck found the automated programs more useful for direct application tasks, i.e. those in which the strategies were explicitly mentioned. The programs in their study administered several types of tasks including the direct application and more demanding tasks. They observed gaps between students' activity and expected activity in those tasks which were not direct application tasks, in those tasks where the information was not explicit, or when the tasks were difficult to comprehend in the programs. On the contrary, the tasks involved in MML, analysed in my research (Study 3), were mostly direct application tasks and there were often deviations found in students' activity from the expected activity. The gaps documented in Cazes and Vandebrouck's study were associated with deviation from the correct strategy in terms of incorrect sequence of operations, obstacles, etc., whereas the deviation observed in my study was the inappropriate use of the resources leading to skipping the mathematical operations. However, the difference between my findings and those of Cazes and Vandebrouck can be explained by two reasons. First, the tasks administered in EEBs were mostly of a technical nature. These tasks required actions such as dragging points on a graph, and therefore, the use of calculators and programming was omitted automatically. Moreover, in their study, the students worked in presence of teachers, which might have had an impact in keeping students to use other tools in solving the tasks.

The findings of my research suggest administering the tasks which are not direct application tasks in order to avoid the use of calculators and computing tools to reach the solutions. In this way, the students could engage with the objects of the tasks. The example of the Shell Method illustrates that the student used the tools with relatively more rigour. Moreover, the findings also point towards the need for the lecturers to give clear instructions regarding the task solving process in the online programs. The deviation in students' activity may also be minimised if the teachers remain present and monitor students' progress.

5.5 Processes of mathematical reasoning in an online environment and a paper and pencil environment—RQ6

As stated earlier, this thesis aimed to make sense of the undergraduate engineering students' engagement with mathematics in two different environments and to make sense of the contributing factors from both environments. The last research question (RQ6) utilises the construct of mathematical reasoning to characterise students' engagement with mathematics, and it is addressed in Study 4. The thesis thus offers empirical and theoretical insights into mathematical reasoning. Theoretically, leaning on the existing research on mathematical reasoning (Lithner, 2008), the processes of mathematical reasoning are documented in the light of the CHAT perspective (Leont'ev, 1974). This perspective facilitates in considering the mathematical reasoning as a process and accounts for the relevant conditions of the learning environment (tools, task, etc.) in the process. In Section 2.3.2, I discussed that mathematical reasoning processes comprise the steps of selecting a strategy and implementing the strategy while solving a task, in accordance with Lithner (2008). These steps are entailed in the action and operation layers of students' learning activities (Leont'ev, 1974), which are linked to students' goals and conditions of the environment. That is, the selection of a strategy takes place when students search for a method to carry out certain goal-directed actions for solving a task. The implementation of the strategy takes place through the execution of operations, which relate to conditions of the tools in use. A visual representation of this process is illustrated in Figure 5.

Lithner's (2008) reasoning framework, which serves as a basis of the development of reasoning processes in this thesis, differentiates between imitative and creative forms of student reasoning. The imitative reasoning involves applying previously known methods or the ones obtained from other resources, such as the textbook. In contrast, creative mathematical reasoning refers to the formation of new ways to approach the tasks (Lithner, 2000). Creative mathematical reasoning is the desired form of reasoning, and it is linked to improved mathematics learning of students (see, for example, Jonsson, Norqvist, Liljekvist, & Lithner, 2014). In this regard, a CHAT perspective on learning offers implications for the imitative and creative reasoning processes. It considers trial and error and experimentation approaches as determining factors for the quality of learning. Based on these considerations, I argued that experimentation is a necessary condition for the formation of new methods in creative mathematical reasoning processes. The experimentation approach deals with finding of new methods that work successfully while taking into account the mathematical aspects involved in the tasks. Careful experimentation is linked to creative reasoning when the intrinsic mathematical properties are considered to find new strategies. The trial and error approach addresses the implementation of previously known methods in new task situations without taking into account the details of the tasks. Therefore, I argue that the trial and error approach is linked to imitative reasoning as the underlying mathematical properties are not considered in this approach.

The empirical findings show similarities as well as differences between the processes of students' reasoning in the two learning environments. The selection of strategy was made in similar ways in both environments in all tasks except in the Shell Method task. The strategies were searchable in the given resources in almost all of the tasks except the Shell Method task. The implementation of strategies in both environments varied in both environments. In the online environment, the use of Maxima, as well as paper and pencil, was found.

In the online learning environment, where students used Maxima, the selection of strategy involved finding the mathematical formula to be used in Maxima. The implementation of the strategy involved converting the mathematical formula into programming language to make a program and executing the program. The extent to which the students attended to each step was found to be related to the demands of mathematical tasks. That is, in the case of tasks where the strategy was the direct conversion of a mathematical formula into a programming code, the students did not have to extract the information from the task. As an example, in evaluating the definite integral $\int_{-1}^{1} e^{-j\omega t}\, dt$, the selection of strategy involved the conversion of this integral formula into the following Maxima code: "j: sqrt (−1); A: e^(−j*w*t); integrate (A,

$t, -1, 1)$". The students searched for the Maxima syntax on the Internet to write this code. The implementation of this strategy was done in one click by running the command. On the other hand, in the Shell Method task, when the integral formula was not explicitly given in the task, the students had to search for the strategy. The strategy was the conversion of an integral formula into Maxima code. In order to find the integral formula, they needed to comprehend the relationship between the integration and the volume of the solid. In this task, the students extracted the information from the task and employed several resources to select and implement the strategy. The strategy was again implemented by running the devised command in Maxima.

Regarding the dynamic tools, a rather brief occurrence of the use of GeoGebra is found in my study. The student performed an operation crucial in the task solving. That is, GeoGebra facilitated in students' actions instead of taking over the computation or providing solutions, as Granberg and Olsson (2015) have also reported.

An important difference that became apparent in the two learning environments is the validation of the obtained solutions of the tasks. The students in my study experienced problems in reasoning about validity of the obtained answers when the operations were conducted in Maxima. In the paper and pencil environment, the students reasoned about the validity of the solutions by relating back to the performed mathematical operations.

As discussed above, while using Maxima in the direct application tasks, the task for the students was to write the code and execute it. In such tasks, the students needed to deal more with the syntax in Maxima than with the involved mathematics as Maxima took over the computation. Therefore, they could not interpret the final solution, as discussed earlier. In the task where the solution method was not provided, the task for the student was more than converting a given mathematical expression into Maxima code. That is, they first had to search for the integral formula and comprehend it. This task demanded experimentation and therefore invited the students to creative reasoning to a certain degree. Although the issue of interpretation in these tasks did not arise in students' activity.

Based on these, I argue that the validation of solutions in the paper and pencil environment was achieved because the focus was on executing the operations. It made the operations visible, contrary to the case of using Maxima. Moreover, the students also were not familiar with the syntax in Maxima and therefore encountered issues in comprehending the outputs in Maxima. This also added to the difficulty in making sense of the solution. Here the argument by Kent and Noss (2000) is applicable, which calls for paying attention to epistemology instead of mere use of the tools so that the intended mathematical objectives do not become invisible or black-boxed (Anderson, 1999).

Previous research points out that the tasks play a role in students' reasoning as to be imitative or creative (Boesen et al., 2010; Lithner, 2003, 2008, 2017). For instance, the algorithmic tasks with the given strategies are not found fruitful for students' creative reasoning (cf. Norqvist, 2018). The results in my study show that the students started careful experimentation in their reasoning processes when the trial and error approach did not work. In the example where the Shell Method was to be used to solve the task, and the strategy was not searchable in the given resources, the students began solving the task with a trial and error use of a previously known formula, which did not result in a correct solution. Consequently, the students started experimenting, and they searched for the correct model by utilising the potentialities of the resources at hand while engaging with the mathematical properties of the task. That is, the students found the description of the Shell Method, the correct integral formula, and the limits of integration by using a step by step approach. I argue that the condition which prompted the experimentation and eventually, the formation of a new method for the students was the appearance of a different nature of the task in the task sequence.

5.6 Reflections on the use of theoretical perspectives and their link with the research findings

In this section, I reflect on possible consequences of the different theoretical frameworks adopted and the research methods used with respect to the findings of my research. I adopted CHAT as an overarching framework which led me to come up to conclusions of my research. CHAT, as Roth et al. (2009) call it, is not only a praxis of the theory but also a theory of praxis. It offered me an analytical mechanism to follow, by way of the extended mediated triangle (cf. Engeström, 2014) and the three-layer model (cf. Leont'ev, 1974) of structure of an activity. The combination of these two complementary models allowed me to focus at different levels of the activities from time to time and from study to study in my research.

CHAT also pointed towards the necessary dimensions to take into consideration for conducting research about engineering students' learning activities. With reference to Nardi (1996), the theory makes it possible to adopt multiple methods in order to conduct research about a particular phenomenon. On these lines, the theory offered me the methodology of my research. Adopting CHAT has led me to take a holistic approach (cf. Jaworski & Potari, 2009) where I counted on multiple sources of the data which added to the epistemological foundations of my research. For instance, if I had not included both sources of data including students' reports and the video recorded observations in the analysis, I could not have come up to draw the present conclusions about relationships among several aspects of the students' activity. The point I want to make is that the individual sources of data provided partial view of the students' activity. For instance, using one source of data, such as video-recorded observations, made me able to describe the students' work with the tools and tasks but without tracing the role of the factors from the collective activity system point of view. Similarly, when I analysed students' reports along with the interviews in Study 2, the results provided insights into aspects of the use of resources based on students' inputs. Although the results were partly similar to those in Study 3, they lacked the operational aspects of students' work in mathematics. In Study 2, the Documentational approach to Didactics allowed me to characterise students' use of resources in an effective manner. Due to the variations in the empirical context, I had to shift my focus in further studies on the wider perspective of students' activity.

Moreover, CHAT offers a general perspective on learning with regards to the interplay between the subject and the social aspects involved in the activities. The theory itself does not offer a characterisation of the learning of mathematics, in particular. Similar is the case with the analysis of mediated action which concerns the Study 1, that it does not specify the characteristics of 'mathematical' actions. Therefore, in my case, it was required to use middle range theories for the characterisation of learning of mathematics. For instance, in Study 1, I made use of the competence framework to characterise the ongoing activity. The use of the framework, although not in a quite extensive analysis, allowed to make sense of students' ongoing practices. This led me further narrow down the analysis in Study 4. In Study 4, I have used the reasoning framework (Lithner, 2003) which is established and recognised by mathematics education researchers. Using CHAT in parallel with the reasoning framework allowed me to analyse the genesis of students' reasoning in addition to the characterisation of the reasoning process. For the reasoning processes (Study 4), the purpose of my research was not only to characterise students' reasoning but to identify the contributing factors of the two learning environments. To achieve this purpose, an analytical tool was needed which could scrutinise the formation of students' reasoning in view of the conditions of the environments. That is, the one which viewed reasoning as a process instead of a product. The need for such an analytical tool corresponded with the empirical basis of this thesis as the students in my study were engaged in learning the concepts instead of being assessed on already covered topics. I created a systematic model (see Figure 5) by the synthesis of a research framework of reasoning (Lithner, 2008) and the principles of CHAT (cf. Study 4). This model complements the existing research (Granberg & Olsson, 2015; Kieran & Saldanha, 2005) in the sense that it adds an environmental dimension to the analysis of the mathematical reasoning process in a systematic manner. This model does not solely confine reasoning as the property of the learning student, but also to the conditions of the particular learning environment. The contributing factors from the environment become apparent through the analysis of students' realised activity.

5.7 Limitations of the research

The issue of limitations of research is important to attend to in order to evaluate the quality of the conducted research. The research reported in this thesis arises from in-depth analyses of two case studies involving specific contextual conditions. Both cases involve a small number of participants in the selected contexts. Therefore, the findings drawn from this research have clear contextual influence such as with respect to the automatic program used, the format of examination, the structure of the course, etc. The participant sampling was random whereas the context of research was purposefully selected in the case studies. Therefore, the results cannot be quantitatively generalised, for instance, to all engineering students' activity. Other participants could have organised their activity in a different manner, leading to different observations concerning their use of resources or the processes of reasoning. However, the conditions of the environment, which became evident, can be generalised in the sense that these point to the significant aspects of engineering students' activities. Due to such issues, the case studies are often criticised for the lack of generalisability of the results produced. In response to this criticism, Flyvbjerg (2006) points out that case studies provide a generalisation through the "force of example". This argument applies to my research, which offers an example of engineering students' activities and the conditions which became evident through the research. The generalisation can be considered as what Bassey (1999) calls “fuzzy generalisation” in the sense that the results arising from this case might occur in other situations as well.

Regarding the research methods for analysing students’ reasoning in my thesis, my choices have some limitations. The use of video-recorded observations in parallel with student written reports and semi-structured interviews revealed extensive insights into the engineering students’ use of resources and their difficulties in solving the provided mathematical tasks. However, stimulated recall interviews could have been conducted in order to investigate further the students’ perspectives on the actions and the operations they performed during their task solving. In this manner, the analysis could be further enriched, owing to additional information from the students. In the analysis that I performed for this thesis, certain aspects where the empirical material collected was not enough to conclude about students’ choices. On some occasions, I had to leave instances without interpretation. For instance, when the students claimed about an output in Maxima which was not apparent, this could have been taken up in a stimulated recall interview.

Another limitation of my research is related to my limited acquaintance with the students’ native language. Although I conducted the semi-structured interviews in English, I believe the interviews would have been more beneficial and purposeful if those had been conducted in the students’ native language. Moreover, I had to involve a native transcriber of the student dialogues, as these took place in Norwegian. Thus, I both had to trust that the transcriber transcribed the oral talk appropriately and to trust that the translation into English was appropriate. Some aspects of this limitation were addressed and met when certain student expressions and utterances were to be analysed in the studies. These analyses were discussed with my supervisors, both native Norwegian speakers and mathematics education researchers familiar with transcription issues as well as translation issues from Norwegian to English.

Another limitation with research involving humans as participants is what is called the Hawthorne effect (Cook, 1962). This relates to the influence of the researcher (and the video camera) on the manner of participants’ research responses, and it is considered as an unavoidable effect. However, I tried not to intervene in students’ work and let them work independently. I observed during the video recordings that the students engaged in their usual talks, which indicates that they seemed not to be inappropriately affected by my presence.

6 Implications

In this chapter I propose some implications of my research for further research in Section 6.1 and implications for the practice involving technology integration in the teaching and learning of (engineering) mathematics in Section 6.2.

6.1 Implications for further research

The four studies of this thesis attend to different objects of study which may all be further researched in the future with different perspectives.
Regarding engineering students' use of different resources in online learning environments, the data in this study comprised three students' reports about their use of several resources over the period of one semester. The study may be replicated by involving more participants, over longer periods of time, to analyse their use of resources to gain more insights about students' manner of working in todays' technology rich environments. Results of such studies might complement the findings of my research in this respect.

An object of study in this thesis is to analyse the affordances of the online learning environments regarding the engineering students' mathematical activity. Further research may be conducted by adopting the theory and methodology of CHAT involving different learning environments including the online systems and settings. Depending upon the researcher's interest and the object of study, the focus of research can also be narrowed down to micro interactions with mathematics in such environments.

Another important contribution of this thesis is the attribution of the learning environment in students' reasoning processes. This finding provides several prospects for further research. A prototype model of the reasoning processes (Figure 5) is presented in this thesis; a model that can be tested in other environments by including other participants. Using this model as an analytical tool, the role of different factors can be analysed by keeping some factors fixed and varying others. For instance, by keeping the tasks fixed and changing the dynamic tools, it may allow making sense of the effect of the tools in students' reasoning processes.

6.2 Implications for instruction

The findings of this thesis suggest the need to ensure students' engagement with the mathematical tasks for students to be able to practice different competencies. In this regard, the attention must be paid to (at least) two aspects: overall setting and the relationship between mathematical tasks and tools-in-use. The design of the collective setting is crucial for providing conditions for students to work according to objectives of the mathematics course. Particularly, the rules of the setting play an important role in how students' activity is realised. The format of the examinations and relevant conditions affects the manner in which students engaged with mathematical tasks.

Regarding the second aspect, students' interactions with the required mathematical objectives can be achieved if the potential of the available tools conforms with the task requirements. This research shows that the object of the task changes with the available tools in the environment. For instance, if the goal is to engage students in the process of integration, then using programming for computing integration will not automatically allow them to engage with the mathematical subtleties of the integration process and make sense of integration. The teachers need to be aware of all the possible ways in which the students may engage with the tasks and the aspects of mathematics that they will eventually be working with while using different technology-based tools (cf. Kent & Noss, 2000). In other words, the issue of black boxing needs to be considered while administering tasks in online environments. The tasks also need to be rich in the sense that they are well-grounded mathematically, and they can demand more than the application of known and devised algorithms.

Furthermore, when the students are invited to provide the final answers of the tasks, it causes them to shift their focus away from the process. In order to engage students in creative mathematical reasoning (cf. Lithner, 2008), the tasks are to be administered in such a way that the students are invited to find solution strategies themselves. That is, if solution methods or algorithms are provided to the students, they plausibly engage with the application of those methods. While earning algorithms is also considered significant in teaching and learning mathematics to some extent, the learning should not be reduced to only this manner of working with mathematics. On the other hand, if the students are invited to elaborate their methods of working on the tasks, it allows them to experiment with the tools at hand and gives them opportunities to engage with creative ways of working on the tasks. In this manner, the students may utilise the potential of the tools for engaging with the intrinsic mathematical properties of tasks. For instance, a dynamic geometry software such as GeoGebra, may be used to explore properties of the mathematical objects by visualising those properties. In a nutshell, for designing an online or a technological environment, it is essential to carry out an a priori analysis of the tasks in relation to the potential of the tools available, in order to ensure students' engagement with the mathematics incorporated in the tasks.

Regarding the use of programming mathematics, the research results imply the need to make students attain mastery of the language and syntax in the programs. In this way, the shift from the paper and pencil to using programming can be smooth in the sense that the students might also interpret obtained inputs. The research findings also illustrate the need for the lecturers to emphasise the involved mathematics more than the use of technological tools. The goal of technology use should be to enhance the learning environment and inquire about the mathematical properties more deeply. The teachers ought to emphasise the mathematical actions that can be performed through various tools. Moreover, collaboration and communication among the students about the involved mathematics need to be emphasised.

7 References

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*(3), 205–224.

Alpers, B. (2011). Studies on the mathematical expertise of mechanical engineers. *Journal of Mathematical Modelling and Application, 1*(3), 2–17.

Alpers, B. A., Demlova, M., Fant, C.-H., Gustafsson, T., Lawson, D., Mustoe, L., . . . Velichova, D. (2013). *A framework for mathematics curricula in engineering education: A report of the mathematics working group.* Brussels, Belgium: SEFI.

Anastasakis, M., Robinson, C. L., & Lerman, S. (2017). Links between students' goals and their choice of educational resources in undergraduate mathematics†. *Teaching Mathematics and its Applications, 36*(2), 67–80.

Anderson, J. (1999). Being mathematically educated in the 21st century: What should it mean? In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the mathematics curriculum* (pp. 8–21). London, UK: The Falmer Press.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical learning, 7*(3), 245–274.

Bassey, M. (1999). *Case study research in educational settings*. Buckingham, UK: Open University Press.

Bateson, G. (1972). *Steps to an ecology of mind*. New York, NY: Ballantine Books.

Boesen, J., Lithner, J., & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics, 75*, 89–105.

Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2016). Blended learning, e-learning and mobile learning in mathematics education. *ZDM Mathematics Education, 48*, 589–610.

Bressoud, D. M. (2019). *Calculus reordered: A history of the big ideas*. Princeton, NJ: Princeton University Press.

Bryman, A. (2015). *Social research methods*. Oxford, UK: Oxford University Press.

Buchem, I., Attwell, G., & Torres, R. (2011). *Understanding personal learning environments: Literature review and synthesis through the activity theory lens*. Paper presented at the PLE Conference 2011, Southampton, UK.

Bærentsen, K. B., & Trettvik, J. (2002). An activity theory approach to affordance. In S. Bødker, O. Bertelsen, & K. Kuutti (Eds.), *Proceedings of the second Nordic conference on Human–Computer Interaction (Aarhus, Denmark - October 19–23, 2002)* (pp. 51–60). New York, NY: ACM.

Callahan, J. T. (2016). Assessing online homework in first-semester calculus. *PRIMUS, 26*(6), 545–556.

Cazes, C., Gueudet, G., Hersant, M., & Vandebrouck, F. (2006). Using e-exercise bases in mathematics: Case studies at university. *International Journal of Computers for Mathematical learning, 11*(3), 327–350.

Cazes, C., & Vandebrouck, F. (2013). Student activities with e-exercise bases. In F. Vandebrouck (Ed.), *Mathematics Classrooms: Students' Activities and Teachers' Practices* (pp. 167–184). Rotterdam, The Netherlands: Sense Publishers.

Choppin, J., & Borys, Z. (2017). Trends in the design, development, and use of digital curriculum materials. *ZDM Mathematics Education, 49*, 663–674.

Cole, M. (1996). *Cultural Psychology: A once and future discipline*. Cambridge, MA: Harvard University Press.

Cook, D. L. (1962). The Hawthorne effect in educational research. *The Phi Delta Kappan, 44*(3), 116–122.

Croft, A., & Davison, R. (2015). *Mathematics for engineers* (4th ed.). Harlow, UK: Pearson Education Limited.

Drijvers, P., Kieran, C., Mariotti, M.-A., Ainley, J., Andresen, M., Chan, Y. C., . . . Leung, A. (2009). Integrating technology into mathematics education: Theoretical perspectives. In C. Hoyles & J.-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 89–132). New York, NY: Springer.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki: Orienta-Konsultit.

Engeström, Y. (1990). When is a tool? Multiple meanings of artifacts in activity. In Y. Engeström (Ed.), *Learning, working and imagining: Twelve studies in activity theory* (pp. 171–195). Helsinki: Orienta-Konsultit.

Engeström, Y. (1998). Reorganizing the motivational sphere of classroom culture: An activity-theoretical analysis of planning in a teacher team. *The culture of the mathematics classroom*, 76–103.

Engeström, Y. (1999). Activity theory and individual and social transformation. In Y. Engeström, R. Miettinen, & R.-L. Punamäki (Eds.), *Perspectives on activity theory* (pp. 19–38). Cambridge, UK: Cambridge University Press.

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of education and work, 14*(1), 133–156.

Engeström, Y. (2014). *Learning by expanding: An activity theoretical approach to developmental research* (2nd ed.). New York, NY: Cambridge University Press.

Engeström, Y., & Miettinen, R. (1999). Introduction. In Y. Engeström, R. Miettinen, & R.-L. Punamäki (Eds.), *Perspectives on activity theory* (pp. 1–16). Cambridge: Cambridge University Press.

Engeström, Y., Miettinen, R., & Punamäki, R.-L. (Eds.). (1999). *Perspectives on activity theory*. Cambridge, UK: Cambridge University Press.

Flyvbjerg, B. (2006). Five misunderstandings about case-study research. *Qualitative inquiry, 12*(2), 219–245.

Gibson, J. J. (1977). The theory of affordances. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting and knowing* (pp. 67–82). Hillsdale, NJ: Erlbaum.

Granberg, C., & Olsson, J. (2015). ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics software. *The Journal of Mathematical Behavior, 37*, 48–62.

Greeno, J. G. (1994). Gibson's affordances. *Psychological Review, 101*, 336–342.

Gueudet, G. (2006). Learning mathematics in class with online resources. In C. Hoyles, J.-b. Lagrange, L.H. Son, & N. Sinclair (Eds.), *Proceedings of the seventeenth study conference of the International Commission on Mathematical Instruction* (pp. 205–212). Hanoi: Hanoi Institute of Technology and Didirem Université Paris 7.

Gueudet, G., & Pepin, B. (2016). Students' work in mathematics and resources mediation at university. In E. Nardi, C. Winsløw, & T. Hausberger (Eds.), *Proceedings of the first conference of International Network for Didactic Research in University Mathematics (INDRUM)* (pp. 444–453). Montpellier, France: University of Montpellier and INDRUM.

Gueudet, G., & Pepin, B. (2018). Didactic contract at the beginning of university: A focus on resources and their use. *International Journal of Research in Undergraduate Mathematics Education, 4*(1), 56–73.

Gueudet, G., Pepin, B., & Trouche, L. (2012). *From text to 'lived' resources: Mathematics curriculum material and teacher development*. New York, NY: Springer.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*, 199–218.

Gueudet, G., & Trouche, L. (2012). Teachers' work with resources: Documentational geneses and professional geneses. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 23–41). Dordrecht, The Netherlands: Springer Netherlands.

Gynnild, V., Tyssedal, J., & Lorentzen, L. (2005). Approaches to study and the quality of learning: Some empirical evidence from engineering education. *International Journal of Science and Mathematics Education, 3*(4), 587–607.

Hirst, C., Williamson, S., & Bishop, P. (2004). A holistic approach to mathematics support for engineering. In C. Baillie & I. Moore (Eds.), *Effective learning and teaching in engineering* (pp. 100–121). Abingdon, UK: Routledge.

Hoyles, C., & Noss, R. (2009). The technological mediation of mathematics and its learning. *Human development, 52*(2), 129–147.

Jaworski, B., & Potari, D. (2009). Bridging the macro-and micro-divide: Using an activity theory model to capture sociocultural complexity in mathematics teaching and its development. *Educational Studies in Mathematics, 72*, 219–236.

Jaworski, B., Robinson, C., Matthews, J., & Croft, A. C. (2012). An activity theory analysis of teaching goals versus student epistemological positions. *International Journal for Technology in Mathematics Education, 19*(4), 147–152.

Jonsdottir, A. H., Bjornsdottir, A., & Stefansson, G. (2017). Difference in learning among students doing pen-and-paper homework compared to web-based homework in an introductory statistics course. *Journal of Statistics Education, 25*(1), 12–20.
Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. *The Journal of Mathematical Behavior, 36*, 20–32.
Kaptelinin, V. (1996). Computer-mediated activity: Functional organs in social and developmental contexts. In B. A. Nardi (Ed.), *Context and conciousness: Activity theory and human–computer interaction* (pp. 45–68). Cambridge, MA: The MIT Press.
Kent, P., & Noss, R. (2000). The visibility of models: using technology as a bridge between mathematics and engineering. *International Journal of Mathematical Education in Science and Technology, 31*(1), 61–69.
Kieran, C., & Saldanha, L. (2005). Computer algebra systems (CAS) as a tool for coaxing the emergence of reasoning about equivalence of algebraic expressions. In H. Chick, J. Vincent, & L. Melbourne (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (pp. 193–200). Melbourne, Australia: PME.
Kodippili, A., & Senaratne, D. (2008). Is computer-generated interactive mathematics homework more effective than traditional instructor-graded homework? *British Journal of Educational Technology, 39*(5), 928–932.
Kozulin, A. (1998). *Psychological tools: A sociocultural approach to education*. Cambridge, MA: Harvard University Press.
Krupa, E. E., Webel, C., & McManus, J. (2015). Undergraduate students' knowledge of algebra: Evaluating the impact of computer-based and traditional learning environments. *PRIMUS, 25*(1), 13–30.
Leont'ev, A. N. (1974). The problem of activity in psychology. *Soviet Psychology, 13*(2), 4–33.
Leont'ev, A. N. (1981a). The problem of activity in psychology. In J. V. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp. 37–71). Armonk, NY: M. E. Sharpe.
Leont'ev, A. N. (1981b). *Problems of the development of the mind*. Moscow, Russia: Progress.
Lesh, R., & Sriraman, B. (2005). Mathematics education as a design science. *ZDM The International Journal on Mathematics Education, 37*, 490–505.
Lester, F. K. (2005). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. *ZDM Mathematics Education, 37*(6), 457-467.
Lincoln, Y. S. (2007). Naturalistic inquiry. In G. Ritzer (Ed.), *The Blackwell Encyclopedia of Sociology*. doi:10.1002/9781405165518.wbeosn006
Lincoln, Y. S., & Guba, E. G. (2000). Paradigmatic controversies, contradictions, and emerging confluences. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of*

qualitative research (2nd ed., pp. 163–188). Thousand Oaks, CA: Sage Publications.
Lithner, J. (2000). Mathematical reasoning in task solving. *Educational Studies in Mathematics, 41*, 165–190.
Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics, 52*, 29–55.
Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*, 255–276.
Lithner, J. (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *ZDM Mathematics Education*, *49*, 937–949.
Monaghan, J., Trouche, L., & Borwein, J. M. (2016). *Tools and Mathematics: Instruments for learning*. New York, NY: Springer
Nardi, B. A. (1996). Studying context: A comparison of activity theory, situated action models and distributed cognition. In B. A. Nardi (Ed.), *Context and consciousness: Activity theory and human–computer interaction*. Cambridge, MA: MIT Press.
Niss, M. (2003). *Mathematical competencies and the learning of mathematics: The Danish KOM project.* Paper presented at the 3rd Mediterranean Conference on Mathematical Education, Greece.
Niss, M. A., & Højgaard, T. (Eds.). (2011). *Competencies and Mathematical Learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark* (Vol. 485). Roskilde, Denmark: Roskilde Universitet.
Norqvist, M. (2018). The effect of explanations on mathematical reasoning tasks. *International Journal of Mathematical Education in Science and Technology, 49*(1), 15–30. doi:10.1080/0020739X.2017.1340679
Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning cultures and computers*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
Núñez, I. (2009). Activity theory and the utilisation of the activity system according to the mathematics educational community [Special issue]. *Educate*, *7*(3), 7–20. Retrieved from http://www.educatejournal.org/index.php/educate/article/view/217
Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: Foundations for change. *ZDM Mathematics Education*, *49*, 645–661.
Pepin, B., & Gueudet, G. (2018). Curriculum resources and textbooks in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Cham, Switzerland: Springer.
Potocka, K. (2010). An entirely-online developmental mathematics course: Creation and outcomes. *PRIMUS*, *20*(6), 498–516.
Rabardel, P. (2002). People and Technology: A cognitive approach to contemporary instruments (translation of Les Hommes et les Technologies). Retrieved from https://halshs.archives-ouvertes.fr/file/index/docid/1020705/filename/people_and_technology.pdf

Roth, W.-M., & Radford, L. (2011). *A cultural-historical perspective on mathematics teaching and learning* (Vol. 2). Rotterdam: Sense Publishers.
Roth, W. (2012). Cultural-historical activity theory: Vygotsky's forgotten and suppressed legacy and its implication for mathematics education. *Mathematics Education Research Journal, 24*(1), 87–104.
Roth, W. (2014). Activity theory in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 11–15). Dordrecht, The Netherlands: Springer.
Roth, W. M., & Lee, Y. J. (2007). Vygotsky's neglected legacy: Cultural-historical activity theory. *Review of Educational Research, 77*(2), 186–232.
Roth, W. M., Lee, Y. J., & Hsu, P. L. (2009). A tool for changing the world: possibilities of cultural-historical activity theory to reinvigorate science education. *Studies in Science Education, 45*(2), 131–167.
Ruthven, K. (2018). Instructional activity and student interaction with digital resources. In L. Fan, L. Trouche, C. Qi, S. Rezat, & J. Visnovska (Eds.), *Research on mathematics textbooks and teachers' resources: Advances and issues* (pp. 261–275). Cham, Switzerland: Springer.
Rønning, F. (2017). Influence of computer-aided assessment on ways of working with mathematics. *Teaching Mathematics and Its Applications: International Journal of the IMA, 36*(2), 94–107.
Simon, M. A. (2009). Amidst multiple theories of learning in mathematics education. *Journal for Research in Mathematics Education, 40*, 477–490.
Sowder, J. T. (1998). Ethics in mathematics education research. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity. An ICMI study* (pp. 427–442). Dordrecht, The Netherlands: Kluwer.
Suchman, L. A. (1987). *Plans and situated actions: The problem of human-machine communication*. Cambridge: Cambridge University Press.
Säljö, R. (1999). Learning as the use of tools: A sociocultural perspective on the human–technology link. In K. Littleton & P. Light (Eds.), *Learning with computers: Analysing productive intervention* (pp. 144–161). London, UK: Routledge.
Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical learning, 9*(3), 281–307.
Twigg, C. A. (2011). The Math Emporium: A silver bullet for higher education. *Change: The magazine of higher learning, 43*(3), 25–34.
Van der Wal, N. J., Bakker, A., & Drijvers, P. (2017). Which techno-mathematical literacies are essential for future engineers? *International Journal of Science and Mathematics Education, 15*(1), 87–104.
Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Vygotsky, L. S. (1981). The instrumental method in psychology. In J. V. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp. 135–143). Armonk, NY: M. E. Sharpe, Inc.

Wartofsky, M. (1979). *Models: Representation and the scientific understanding*. Dordrecht, The Netherlands: Reidel.

Webel, C., Krupa, E. E., & McManus, J. (2017). The Math Emporium: Effective for whom, and for what? *International Journal of Research in Undergraduate Mathematics Education, 3*(2), 355–380.

Wertsch, J. V. (1991). *Voices of the mind: A sociocultural approach to mediated action*. Cambridge, MA: Harvard University Press.

Wertsch, J. V. (1998). *Mind as action*. New York, NY: Oxford University Press.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*, 458–477.

Yin, R. K. (2014). *Case study research: Design and methods* (5th ed.). Thousand Oaks, CA: Sage Publications.

Zembat, I. O. (2008). Pre-service teachers' use of different types of mathematical reasoning in paper-and-pencil versus technology-supported environments. *International Journal of Mathematical Education in Science and Technology, 39*(2), 143–160.

8 Appendices

8.1 Appendix 1: The letter of consent

Request for participation in the research project

"*University Students' Mathematics Learning Using Digital Tools: The Impact of Tutorial Videos*"

Background and purpose

The aim of my project is to find the impact of tutorial videos when used as the main source for teaching mathematics at undergraduate level. The idea is to explore such situations with respect to students' experiences with videos which are particularly designed for teaching purposes. It is of main interest in my study to see how such digital videos contribute to students' mathematical learning. The interactions between students, videos and mathematical concepts will be studied.

It is a PhD project which will be conducted at University of Agder within the scope of MatRIC, the Centre for Research, Innovation and Coordination of Mathematics Teaching. MatRIC is hosted by the University of Agder and it is the centre of excellence for mathematics education in Norway.

The requirement of the project is to observe those university students who are practicing this innovative method for learning mathematics through tutorial videos, and for this reason Electronics Engineering undergraduate students have been particularly requested to take part in this project.

What does participation in the project imply?

Participation in this project requires to be willing to take part in questionnaire survey, observations and interviews. The questions in survey and interviews will be concerned with the participants' experiences with videos in terms of mathematical learning. In the same manner, the group observations will also be carried out to explore the role of videos in students' mathematical sense-making. The data for observation will be collected via video recordings.

What will happen to the information about you?

All personal data will be treated confidentially. The data will be stored at University of Agder's system which will be password protected to ensure the confidentiality. Access to the collected data will be limited to me and my supervisors: Professor Frode Rønning and Associate Professor Martin Carlsen. If any further viewers will be

required to have access to this data at any point in future, participants will be asked before doing so.
The project was started in August, 2016 and it is scheduled for completion by August, 2019. However, I will start observations and other data collection in January, 2017. The collected video recordings will be stored till 2025 for further analysis and research. After 2025, all data collected will be anonymized.

Voluntary participation

It is voluntary to participate in the project, and you can at any time choose to withdraw your consent without stating any reason. If you decide to withdraw, all your personal data will be made anonymous.
If you would like to participate or if you have any questions concerning the project, please contact me or my supervisors.

Shaista Kanwal	Department of Mathematical Sciences,
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	University of Agder
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Frode Rønning	NTNU, Department of Mathematical Sciences.
Supervisor	Alfred Getz vei 1, Sentralbygg II, 7491 Trondheim
	Tlf: 73550256
	Email: Frode.Ronning@math.ntnu.no

The study has been notified to the Data Protection Official for Research, NSD - Norwegian Centre for Research Data.

Consent for participation in the study

I …………………. have received information about the project and,

a) I agree to participate in the questionnaire survey. Yes/No

b) I agree to participate in the interview and observation. Yes/No

----

(Signed by participant, date)

8.2 Appendix 2: Overview of collected data

Spring 2017

Semeste	Lectures	Group work observations	Screen recordings	Interviews	Weekly journals
Spring, 2017 Participants: 4	------	- 7 weeks - Approx. 50 min. each - working on weekly assignments	From week 9 onwards	- group discussions - almost during every observation	- Over the semester
Autumn , 2017 Participants: 4	- 5 lectures - Approx. 1 hr 30 minutes each	- 4 sessions of group work on tasks assigned by the teacher	- Over the semester while doing online homework remotely	One semi structured interview: Approx. 15 min with each participant	-----

Details of the data collected in Autumn 2017

Week	Lecture/topic		Groupwork	Interview
35	10:15-	Ch 12: Matrices	2 tasks within the lecture: 10 and 15 min	----
36	10:15-	Ch 13: Applications of matrices Block 1,2,3	1 task: 5 minutes	

37	10:15-	Ch13: Applications of matrices. Block 4,5	---------------	
38	10:15-	Ch 22: Laplace transformations Block 1 &2	---------------	
39	11	Ch 20: Differential equations	3 tasks about Matrices	
42		----------------	--------------	15 min each
44	Thursday	Revision of Kap. 20 Block 1-4	Tuesday 10.15-12:00	
			7-8 students in total Ch 20 Differential equations	
45		Ch. 20 block 5, 6	Ch 20 Differential equations Block 5 & 6	
46	Thursday 12:15-	Revision of Ch 21 & 24	-7 students Ch 21 & 24: Function of several variables and Fourier series	

8.3 Appendix 3: Student's weekly journals

Journal for learning mathematics, week___. Topic: Name:

Days	What have I done today?	Which of these resources have I used?			How did the use of each resource help in my work?
		Resources	Yes /No	Details and time spent	
Monday		Tutorial videos			
		My MathLab			
		Textbook			
		Any other			
Tuesday		Tutorial videos			
		My MathLab			
		Textbook			
		Any other			
Wednesday		Tutorial videos			
		My MathLab			
		Textbook			
		Any other			
Thursday		Tutorial videos			
		My MathLab			
		Textbook			
		Any other			
Friday		Tutorial videos			
		My MathLab			
		Textbook			
		Any other			

Analysis of journal data

Colour guide: The descriptions by Jan: Black ; Tor: Red; Per: Green

Aims/expectations/goals: Yellow

Actions/Operations: blue

Views about a resource: Grey

Overview of usage of resources

Resource	Using resources	General comments
Tutorial videos	- They could learn me the necessary rules and methods to solve the questions I got. I only watched those videos I thought could learn me a more difficult theme. by watching these two videos I got enough information to complete almost all questions for the work at week 3. Still there was some questions with higher difficulty at the end which I used MyLabs for.	In the videos he explains how to proceed in solving problems, with some videos containing multiple examples
	-I needed very much help with block 7, and some help understanding at block 8. -Watched the video to try understand how to calculate the length of a line, but will have to watch it some more before I understand it. - To understand the calculation behind the math - The hardest part with this week's exercises was that the problems tend to become really long and it was quite easy to make sloppy mistakes. I felt that the videos helped me the most this week, it was a bit hard to follow along with the examples in the textbook. It's a long time since I last did any integration problems, so I watched	From block 8 I learned that most answers come from rules, and is very hard to calculate by hand. I will probably need to watch it several times if we get more tasks at this theme. From block 7 I still did not understand how to calculate after watching the video's. I will have to study this part much more in order to solve it. - I easily understand it when someone explains me the way of solving a problem.

	lecturer's "Kapittel 17 1.2 Integrasjon" just to refresh the basic. -I needed a reminder of how to integrate by substitution.	
MatRIC videos	-I watched the videos at 1.25% speed because I didn't heave the need to learn it from scratch, I simply needed a reminder of how derivation worked. -thinking I should rejuvenate my feeble understanding of integration.	I don't watch the videos because I want to, but because I need to.
Own notes	-As help for the test, I used my previous notes from earlier homework, which is many similar tasks with different numbers. - In the start, it was hard to remember how to solve the problems, since it was 2 weeks since I last solved any derivation problems. But after looking through **my notes** and solving a couple of problems, I remembered most the problems.	
Textbook	-I (only) used the book to find rules on different integrations. Because it helped me with basic integration rules. - In order to understand more from each block I read through the little information I had from the book. - I got the questions from the book as well as some help with formulas. -Got many formulas to work with and help to use MAXIMA GUI. -Formulas for the homework this week - Used the textbook to find the formulas for the different expansion series.	

	Use the textbook to find the formulas for the different expansion series.	
Mathway, WolframAlpha	-It helped me with some more difficult integrations, where I couldn't see the solution. -After manually doing the problems for a while, I got the gist of it, and preceded by taking "shortcuts", using the calculators **WolframAlpha** to simply solve the problems for me. -Used Wolfram to compare answers we got in Maxima. -Used it only to double check my answers for the Stack project. - I mostly used Wolfram Alpha and/**or** **GeoGebra** to solve the problems. I also did some by hand to check the answer I got in WA or **GeoGebra**. The first time I did the test I got 80% right. And the second time I did it I got 93%. - I used it for the problems I didn't understand how to solve. Some problems I can't get right even if I get help from My Mathlab. Then I use wolfram alpha to compare my answer to what WA says the answer is. Then I can look for what I did wrong and how to get it right next time. I also used it for solving definite integrals because it takes so long time by hand and it is easy to do stupid mistakes.	-(Also,) great when I need an expression and not numbers for my answer. Very fast at definite integration. - I learned that wolfram is significantly easier to use than maxima. - Because it goes faster than punching in everything in on the calculator.
MyMathLab	- I learned the basics of integration. - Used it for inspiration to what tasks I could use for the project, when it came to different ways to integrate.	-I made a mental note concerning MyLabs: If only more mathematic subject were done this way. In my opinion it would be so much easier to learn and understand. I would argue

	- Worked on homework with the group - Worked on the homework for week 13, and made some progress. Further on I think I will need to read more in the book and see more videos. -Worked on my homework for ca. 2h and used the helping option for some questions - Got the gist of trapezium rule - I did a quick overview of the test. Which chapters I needed to recap and what tasks I would need to repeat before attempting the partial-test. - I opened homework and the partial test in different browsers to swap between for comparing questions. By comparing the questions in the test with the ones from homework, I can easily access help and/or examples to similar questions, and by compare results and to doublecheck what kind of answer they are looking for - if the given answer is similar or very different from the ones from homework I could quickly know if I'm very wrong or on the right track. Using this method gives me a great advantage both in efficiency and time used. - I used my mathlab quite active to help learn how to solve the problems. It took some time to get some problems right but I eventually understood how to solve them, I learned how to integrate by parts (I used it actively for this topic) and got a lot of help to understand how to solve the other topics for this week.	that the diversity and "fluidity" of challenges MyLabs might throw at you is superior to the standard textbook recipe. Where there is a set of tasks which increases in difficulty but with rigid variables, preventing the possibility to redo a certain problem without already knowing the answer and thereby weakening your understanding altogether. - Tried to solve some of the problems using maxima, but I considered learning mathlab instead. Given it's a more powerful tool and it's easier to attain help and information online. - Not much to report. Doing the homework without much trouble - Going for a new strategy: going slow and thorough, taking notes and making solutions as I go. The downside is that it takes forever. After an hour, I completed maybe 2-3 tasks. This was a bad idea, since I lack fortitude.

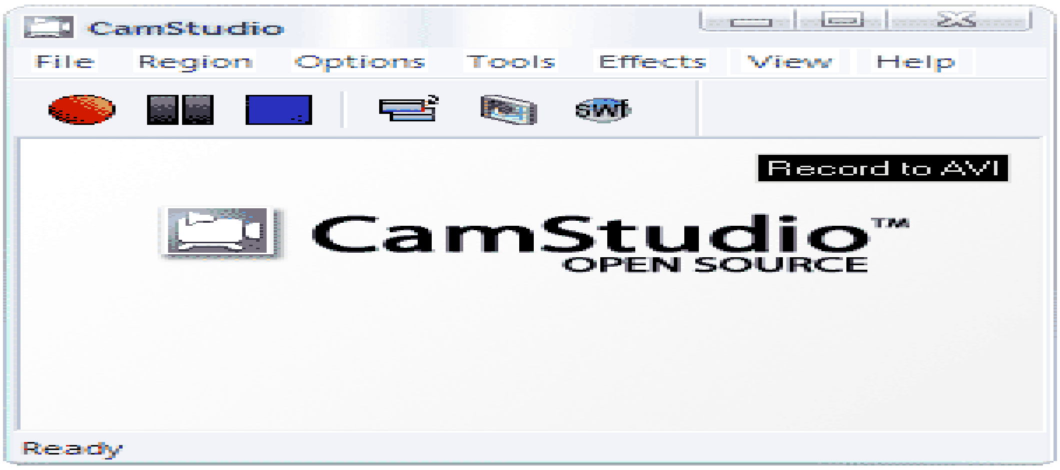
	- I need to use My MathLab for doing the homework	
Internet	-I made the first question in our stack project -I made the question 2 and 4 in our stack project. I also edited a bit on Tor's question 3, and added one more feedback. - Maxima is new to us so we used the internet to search for how to solve the problems and learn the maxima programming language.	
Maxima	-Did one of the tasks in our maxima project about trapezoid rule. -Solved some problems using maxima, but will need more learning before I can solve these problems with it. -With formulas from the textbook I solved almost every questions. -Made a formula to calculate the problems with ''binominal formula'', so I don't have to calculate everything by hand. -I somewhat understood the use for maxima. If used correctly it could save me a lot of trouble down the line -Tried to solve some of the problems using maxima, but I considered learning mathlab instead. Given it's a more powerful tool and it's easier to attain help and information online. -I also used Maxima to solve a handful of other tasks (in partial test) which were too complex for a simple browser calculator. -Tried to solve a few tasks in Maxima, but I reckoned that it would be swifter to do them the easier ones by hand.	-It would be easier in the long run. If I could make a template for each question, then I would have a severer advantage on the upcoming exam. I would only have to plot in the variables to get the correct answer on each question.

	-We are working on a project where we need to solve problems in maxima, so obviously we needed to use maxima to do this. -Tried to use maxima while doing the homework - I used Maxima to do the homework, and also to finish the Maxima project. - Tried to use maxima while doing the homework - I tried to learn some more Maxima while doing the homework. (This took some time) - Used maxima to make a program to solve the problems in an easy way. This is hard to make, but when it is done, all the problems are easy to solve.	
Lecturer's notes	-After stumbling on a few tasks, not being able to comprehend some of the tasks. I skimmed through Lecturer's notes -I read lecturer's notes on parametric derivation and skimmed trough a video with Per on the same topic. - Skimmed through and stole a few examples he had previously used. I learnt the formulas for integration or technique of partial integration -I skimmed through Lecturer's notes only to get a general idea of the subject (3 min)	-because those notes are "tailored" for the tasks at hand. - I also believe that I get a deeper understanding through continuously trying to understand and solve a problem, repeating it until completion. - Because it would be the most relevant piece of information it this subject.

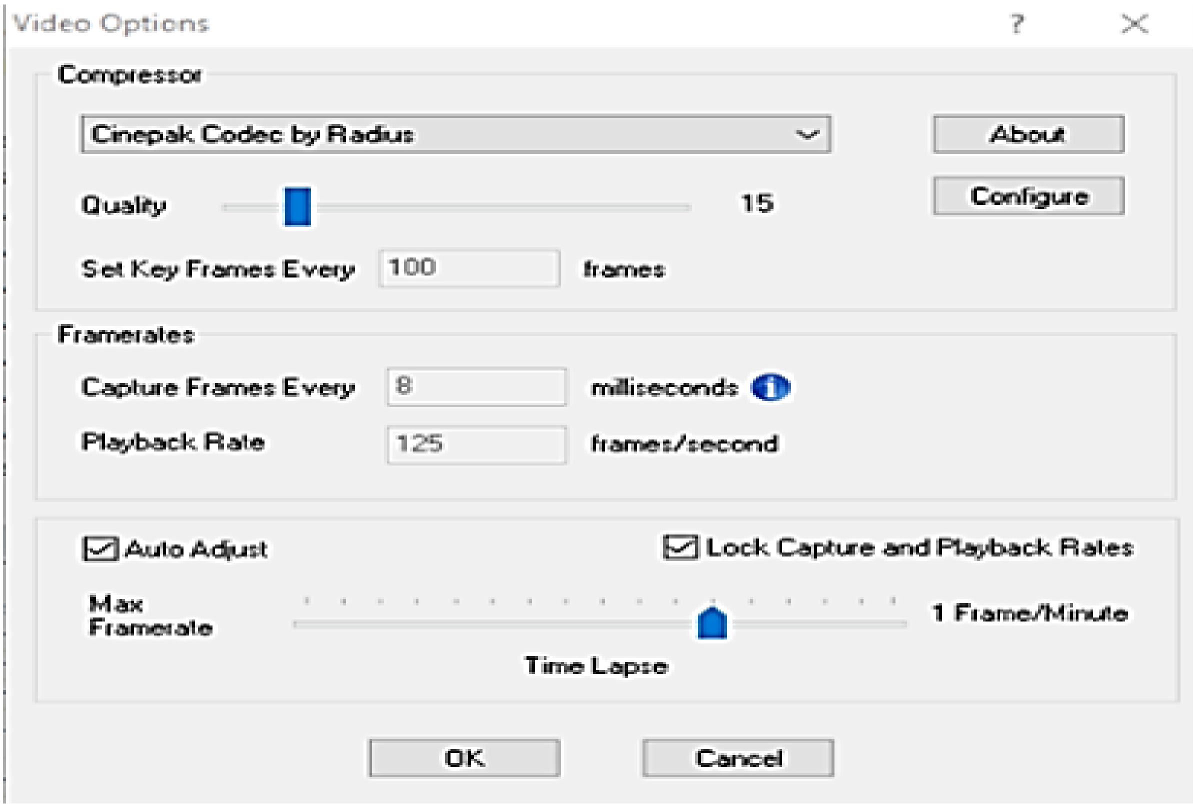
Youtube	-I have skimmed through a few videos on basic Maxima tutorials on Youtube, -I learned from the video how to partial integrate and to just refresh long forgotten memories	(Learned?)Not much, It would seem it is more proficient to simply “play around” on your own in maxima than to watch youtube-videos.
Mathway	-I used a website called mathway to solve almost every task in the partial-test. I did not learn anything doing this, but it severely increases the probability to get the correct answer, and therefore the overall score…. I googled trapezium- and simpson’s rule calculators instead of solving them by hand.	
Online calculators	-I found online calculators for Maclaurin and Taylor series which made some of the tougher questions significantly easier. I did try to solve them by hand first, only to get the feel and gist of it.	
Stacks	-We had a lecture in Stacks and afterwards we tried to make some easy questions using Stacks	

8.4 Appendix 4: Settings used for Camstudio

Interface



The settings used:



8.5 Appendix 5: Mathematical tasks used in the group work sessions, Autumn 2017

Week 39

Task 1a): Given matrices, A and B. Find product A and B and write up the transpose A^{T}.

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 5 & 4 & 2 \end{bmatrix}. \qquad B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \\ 2 & 1 \end{bmatrix}.$$

Task 1b): Here you find a little challenge.

(Remember: Area of a triangle made up of the vectors $\vec{a}$ and $\vec{b}$ is

$$A = \frac{1}{2}|\det(\vec{a}, \vec{b})|$$

You need to find the area of the triangle with the vertices at the points $c = (-1, 2)$, $d = (4, 8)$, and $e = (2, -3)$. Hint: Find two vectors and set them in the matrix form.

Task 2: Given the following matrix: $A = \begin{bmatrix} 0 & 3 & -6 & -14 \\ 3 & -7 & 8 & -5 \\ 3 & -9 & 12 & 9 \end{bmatrix}$. Find all the solutions of the system of equations $Ax = 0$. Also write the solution in vector form.

Task 3: Given following matrix. $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. Find eigen values and associated eigen vectors of the matrix.

Week 45

Task 1:

a) Write down the homogeneous form of the equation $\frac{d^2y}{dx^2} + 15\frac{dy}{dx} - 6y = 2sinx$

b) Which of the following are the constant-coefficient equations? Which are homogeneous?

i) $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 12y = e^{-13x}$

ii) $x\frac{d^2y}{dx^2} + 13y = 0$

iii) $\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 9x = 0$

iv) $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 10y = 0$

Task 2:

a) Obtain the general solution, that is the complementary function, of the homogeneous equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$

b) Obtain the general solution of the homogeneous equation $\frac{d^2x}{dt^2} + 11\frac{dx}{dt} + 30x = 0$

Task 3:

a) Find a particular integral for the equation $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 10x = 12e^{4t}$

b) Find the general solution of $\frac{d^2x}{dt^2} + 13\frac{dx}{dt} + 40x = 5t$

Week 46

Task 1:

a) Find all the second-order partial derivatives of $z = 3e^{2y}cos6x$.

b) Find all stationary points and determine the type of points for the function

$$f(x, y) = \frac{1}{x} + \frac{1}{y} - \frac{7}{xy}$$

8.6 Appendix 6: Examples of transcriptions

Phase 1

Transcription week 9, Participants: Per, Jan and Tor.

Length: 1:02:29 (screencast). 23:39+23:38+7:10 (room view).

Times below are screencast time. Screencast is of Per's screen. Screencast recording starts 35 seconds later than room recording.

Brief summary: Students are working on project in which they are supposed to program the tasks in Maxima.

What is said?		Comments on what is being done?
1	Per: I doubt that you have done much since last time.	Discussion regarding the programs, Maxima and STACK related to the project.
2	Tor: I actually tried visiting those… Matrix things.	
3	Per: Matrix?	
4	Tor: The thing that codes math.	*TRANSLATION NOTE: To be Vant to something is to be used to it. The letter u gives it the opposite meaning. 113tar113ring to something as uvant means that it is different from what the person speaking is used to, and will affect them in some way and to some extent. For instance, one may feel weird about what one is not used to, or as here, worry that one is less proficient with what one is used to.
5	Per: The stack thing?	
6	Tor: No, it's not stack yet.	
7	Per: Isn't it stack when you want to code math?	
8	Tor: No, the first code language, isn't it…	
9	Per: It is Maxima that…	
10	Tor (conclusively): Maxima!	
11	Per: Well, do you have a masters now.	
12	Tor: What?	
13	Per: Do you have a masters?	
14	Tor: No, it is complicated. I think it's so fucking *uvant**.	
15	Per: Yeah	
16	Tor: It's a stupid program, compared to Wolfram.	*TRANSLATION NOTE: In Norwegian there is one word for the singular version of the word you, and one for the plural. I will normally translate
17	Per: Yeah, Wolfram would have been nice… If we had it.	
18	Tor: Yeah [short laugh]	

19 Per: Shall we just finish that… ‘*Gruppe Primero*’ [reading heading of the document on screen]? Did you edit that? 20 Tor: We are group one, you know. 21 Per: Oh, right. Have you-all* sent a mail to that guy… 22 Tor: What? Oh, to Amar? Yes. 23 Per: Yeah, with group? 24 Tor: Yeah, I sent a mail right away. 25 Per: Okay. Have you [inaudible]? 26 Tor: That one we shall have, that one not. 27 Per: Yeah. 28 Tor: We’ll not have that one. 29 Per: A lot of those thing pops up where you need to press [inaudible] 30 Tor: Yeah, I pressed because I thought it was [inaudible]. 31 Per: Yeah, it’s often like that. 32 Tor: Okay.	the plural version of the word to you, but when it would be hard to tell from context that the person uses the plural version, I will instead translate to you-all.
33 Per (while scrolling through the document): So, we have done exercise 3… 34 Tor: How about those? [inaudible] 35 Per: You must have those. You are to delete that thing… Delete what comes up there 36 Tor: Alright. [Per is logging onto fronter] 37 Per: Microsoft [inaudible] have the kind of thing… A thing you have to have, you know… To get the program to work. 38 Tor: Yeah… That’s what’s so awful if you suddenly deleted something like that. Then…	Talking about the task 3 and then again about Maxima.

39 Per: Yeah. 40 Tor: [inaudible] 41 Jan: Do you have… You know the thing if you go to Maxima, then you can download… ? 42 Tor: Command line? I think that was a bit stro… A bit better for solving… 43 Per: Is it better? 44 Tor: Yeah… [inaudible] said that he had used it for something. [pause] But… [inaudible] I didn't get any more… 45 Per: Yeah… Yeah, I saw that last time, but…	
46 Tor: Who of you is it that have solved the… Is it you who have solved exercise 3 twice? 47 Per: Have we solved exercise 3 twice? 48 Tor: Yes. 49 Per: Let's see… [Checks document, timestamp **2:15**] Yes. Cause we have two different answers. 50 Jan: But that is correct. 51 Per: Is that correct? 52 Jan: At least according to the book I think it is correct. 53 Per: Okay, [scrolls up a bit] but do you see the difference there? 54 Jan: No, man, that's what I don't. There's no difference. 55 Per: There I have written that… Right, I forgot the multiplication sign… up there. Just wrote it wrong.	Discussion regarding exercise 3 (they solved it in Maxima and in Wolfram and get different answers)

Phase 2: an example

Transcriptions

Video: Week 45 – MVI 0080, MVI 0081 and MVI 0082

Persons: O: Lecturer, D: Dag, J: Jan, S: Shaista,

Transcriptions Keys:

…	Unfinished sentence
() or (uhørbart)	Unheard
(uhørbart ord)	One unheard word
(?)	Unsure of what the word is (said/translated)
words in red	Unsure of translation

Task 1 (see appendix 5)

c) Write down the homogeneous form of the equation $\frac{d^2y}{dx^2} + 15\frac{dy}{dx} - 6y = 2\sin x$

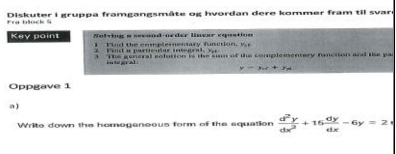
d) Which of the following are the constant-coefficient equations? Which are homogeneous?

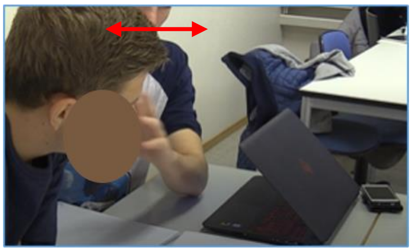
v) $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 12y = e^{-13x}$

vi) $x\frac{d^2y}{dx^2} + 13y = 0$

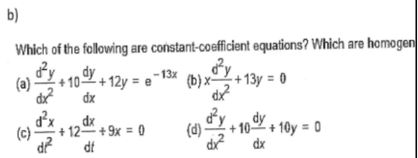
vii) $\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 9x = 0$

viii) $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 10y = 0$

Turn /Time	Person	What is said (Norsk)	What is said (English)	What is done.
0:20 Task 1	1 D	Ja, skal vi bare skrive…	Yes, shall we just write…	
	2 J	Ja, prøve på det.	Yes, lets try that.	
Task 1	3 D	Skal vi se. Den her… første oppgaven er å bare skrive den homogene formelen av den da.	Lets see. This one here… the first task is about just writing the homogeneous formula of that one then.	Points on task sheet  Diskuter i gruppa framgangsmåte og hvordan dere kommer fram til svar Fra block 5 Key point Solving a second-order linear equation 1 Find the complementary function, y_{cf}. 2 Find a particular integral, y_{pi}. 3 The general solution is the sum of the complementary function and the pa integral: $y = y_{cf} + y_{pi}$ Oppgave 1 a) Write down the homogeneous form of the equation $\frac{d^2y}{dx^2} + 15\frac{dy}{dx} - 6y = 2$

	4	J	Mhm.	Mhm.	
	5	D	Og det er jo... Da skal det være... bare y. Og så den deriverte av y.	And that is.... It is supposed to be... just y. and then the derivative of y.	
	6	J	Ja.	Yes.	
	7	D	Og så koeffisientene foran. Og så alt annet skal liksom være...	And the coefficients in front. And everything else shall be...	Gesture 
	8	J	Ja. Så det blir y i andre blir det ikke det, pluss...	Yes. So it will be y squared doesn't it, plus...	Points on task sheet
	9	D	Ja, det blir den der er lik 0 bare.	Yes, it will be this one equals 0.	Circles around a part on task sheet (right hand side of the equation
	10	J	Ja. Jeg skjønte det.	Yes. I understood it.	
	11	D	Ja, for etter at du har den kan du skrive den karakteristiske linja vet du.	Yes, because after you have this one, you can write the characteristic line you know.	
	12	J	Ja.	Yes.	
	13	D	Skal jeg bare skrive da?	Shall I just write then?	
	14	J	Ja, bare...	Yes, just...	
	15	D	Du kan skrive du og.	You can also write.	Starts writing
	16	J	Ja (uhørbart)	Yes ()	
	17	D	Jeg kan teste da. Du kan få lov til å prøve etterpå.	I can test then. You can try afterwards.	

	18 J	Ja, det er ikke så veldig farlig men…	Yes, it is not important, but…	
	19 D	Nei, men jeg har lyst til å se det…	No, but I want to see the…	
	20 J	(noe latter) (uhørt)	(some laughter) ()	
	21 D	Se… Skal jeg bare skrive svaret med en gang da?	See… Shall I just write the answer straight away then?	
	22 J	Eh….du må vel skrive opp… bare skrive det. Ja.	Eh… you need to write… just write that. Yes.	
01:53	23 D	Eh… Føler et press å skrive med denne pennen her.	Eh… feeling a pressure by writing with these pens here.	Keeps writing
	24 J	Mhm. (uhørbart) For å si det sånn.	Mhm (). To put it like that.	
02:11	25 D	Sånn.	Like that.	
	26 J	Ja. Vil det si at alle disse her kommer igjen? Og den er… koeffisient?	Yes. Does that mean that all these here come again? And this one is… coefficient?	Points at task sheet under the grey area
	27 D	Ehhhh… ja? Faktisk. Eller… jo, den…	Ehhh…yes? In fact. Or… yes, it…	Points at task sheet 
	28 J	(uhørbart) Jeg vet ikke hva som er kravet.	() I don't know what is the requirement.	
	29 D	Det burde gå fint det.	It should be fine.	
	30 J	Ja, det burde jo det. Jeg vet ikke om du kan skrive x foran, men det…	Yes, it should. I don't know if you can write x in front, but it…	

	31 D	Men går det an å… Ja, nei det…altså, det burde gå fint. Det 119tar bare 0…	But you can do… Yes, no it… well, it should be fine. It's only 0…	
	32 J	Mhm.	Mhm.	
	33 D	…foran den…det mellomste leddet der.	… in front of it… the middle term there.	
	34 J	Så står det jo en x og foran der da, men… Jeg vet ikke om det gjør noe.	Then it stands a x also in front of that, but…. I don't know if it matters.	Points at task sheet
	35 D	Ja, det er sant. Nei, jeg tror ikke den… noen av delene faktisk.	Yes, that is true. No, I don't it… some parts actually.	
	36 J	Nei (noe latter).	No (some laughter).	
	37 D	Hvis du deler med x på… her sånn, så får du… 13 eh… x foran den. Det er noe annet (uhørbart ord).	If you divide by x on… here, you will get… 13 eh… x in front of it. It is something else ().	Points at task sheet  b) Which of the following are constant-coefficient equations? Which are homogen (a) $\frac{d^2y}{dx^2}+10\frac{dy}{dx}+12y=e^{-13x}$ (b) $x\frac{d^2y}{dx^2}+13y=0$ (c) $\frac{d^2x}{dt^2}+12\frac{dx}{dt}+9x=0$ (d) $\frac{d^2y}{dx^2}+10\frac{dy}{dx}+10y=0$

8.7 Appendix 7: Interview guide

Autumn 2017

The following themes have been addressed in the interview with each student. Each theme was extended based on students' responses.

1. Which resources you use mostly in this mathematics course?
2. Leading to more questions about the nature of help from the resources as well as views on each resource. Do you use paper and pencil or digital tools for solving the tasks in MML?
3. Do you see any difference in ways of working on mathematical tasks in this course than in previous semester?
4. How do you see the inclusion of lectures in this course?

Study 1–4

Mathematical Competencies and E-Learning: A Case Study of Engineering Students' Use of Digital Resources

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This paper explores how an e-learning environment affords the execution of mathematical competencies in an undergraduate engineering context. Considering the students' mathematical practice as action mediated by the digital resources in a sociocultural sense, we employ the competence framework by (Niss & Højgaard, 2011) to make sense of students' learning. Case-study research design has been implemented to thoroughly observe the mathematical practices of a small group of participants. Observing students' group work and following their mathematical discussions elucidated the way this environment afforded the execution of competencies. Closer analysis revealed that the availability of online tools in this environment has the twofold effects on mathematical thinking, mathematical reasoning and problem-tackling competencies.

Keywords: Calculus, Engineering mathematics, E-learning, Mathematical competencies.

Introduction

The use of digital resources in mathematics education has started since the development of such tools and is still being researched to study its impact on mathematical learning. Increased dependence on digital tools for practicing mathematics is transforming the mathematics education, and to learn mathematics is not the same as it was before the introduction of digital technology. The use of digital resources is of particular relevance in engineering mathematics in the sense that modern-day engineers during their professional activities rely on technology for mathematical tasks (van der Wal, Bakker, & Drijvers, 2017). The framework for mathematics curricula in engineering (Alpers et al., 2013) also recommends how technology should contribute towards fostering the engineering students' mathematical competencies (Alpers et al., 2013). The notion of mathematical competence from the Danish KOM project (Niss, 2003; Niss & Højgaard, 2011) has been adopted to make sense of the engineering students' mathematical learning.

Previous research studies have also employed this competence framework, either to make sense of students' learning in mathematics or to analyse how these competencies are developed in particular situations or through certain activities. For instance, Jaworski (2012) used Niss's idea of mathematical competencies to design and analyse the tasks and to recognise the engineering students' mathematical learning. Jaworski pointed out that a potential use of the competence framework may be to create opportunities for students to achieve certain competencies (Jaworski, 2013). Furthermore, Albano and Pierri (2014) used a role play activity and identified the first-year engineering students' mathematical competencies through the questions students asked. Albano and Pierri concluded that students seemed to possess all the competencies by Niss (2003) which were evident through the words they used in their questions. García, García, Del Rey, Rodríguez, and De La Villa (2014) presented a model for the integrated use of CAS which they implemented and analysed in engineering classrooms. They suggested that the use of CAS in all learning and assessment activities has the potential to positively influence the development of mathematical competencies. Recently, Queiruga-Dios et al. (2016) analysed the development of mathematical competencies among industrial engineering students through their teamwork which included the use of CAS for solving mathematical problems as an integral part. While their main aim was to integrate these mathematical competencies with the required engineering competencies in Spain, they claimed that the students acquired all the mathematical competencies during this task.

Our study focuses particularly on nature of mathematical competence afforded by an e-learning environment. Realising the contemporary and the future state of mathematics education, we attempt to add to the research literature within the context of engineering mathematics education. In this paper, we analyse engineering students' engagement within a calculus course to report on how their mathematical competencies are supported within an e-learning situation. We attempt to answer the following research questions: What traces of mathematical competencies are observed in students' work when they practice mathematics digitally? How does this environment afford the execution of these mathematical competencies?

Theoretical perspective

We consider students' mathematical practice in the present situation as mediated action in sociocultural terms (Vygotsky, 1978). The provided resources which support the learning of mathematics serve as mediating artefacts between students and the mathematical concepts. The mediating artefacts used in the present situation are MyMathLab, tutorial videos, textbook, Maxima for programming, and other internet-based resources. The students' homework and eventually the students' assessments are done digitally. There were no regular face-to face lectures thus the situation is considered as e-learning in which students remotely work with the resources. A brief introduction of these resources follows.

MyMathLab is an online interactive learning environment for practicing mathematics digitally. While the main aim of this resource is to provide a platform for digital homework and assessments, it also facilitates in solving the tasks by providing illustrated worked examples and personalised feedback. The tutorial videos replace traditional university lectures and are linked topic-wise with the textbook sections. The videos are recorded by the mathematics teacher using a document camera, and they consist introduction to each mathematical topic along with worked examples. The tutorial videos and the homework in MyMathLab were clearly linked with the chapters in the textbook.

We employ the competence framework by Niss and Højgaard (2011) to make sense of engineering students' mathematical learning (Jaworski, 2012, 2013). The framework is complemented by sociocultural notion of resource mediation. The Danish KOM project (Niss & Højgaard, 2011) enlisted eight mathematical competencies, divided into two groups as follows (Figure 1):

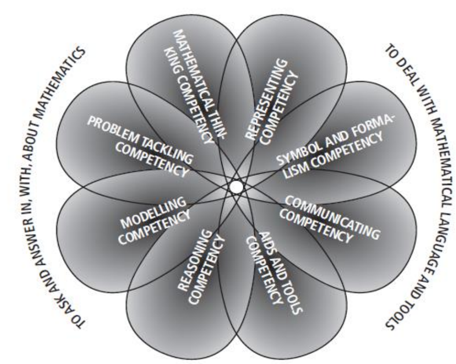

Figure 1: A visual representation of eight mathematical competencies (Niss & Højgaard, 2011, p. 51).

The Ability to Ask and Answer Questions in and with Mathematics

The first group comprises the competencies of mathematical thinking, mathematical reasoning, problem tackling, and mathematical modelling. Mathematical thinking competency involves “awareness of the types of questions which characterise mathematics” (Niss & Højgaard, 2011, p. 52) and “being able to recognise, understand and deal with scope of given mathematical concepts” (Niss & Højgaard, 2011, p. 53). Mathematical reasoning includes following and assessing chains of arguments, comprehending a mathematical proof, and devising formal and informal mathematical arguments (Niss, 2003). In the present study, the proofs were not a part of the mathematics curriculum. Thus, the reasoning competency is only observed within the context of problem solving. Mathematical modelling is neither a part of the curriculum in the present situation.

The Ability to Deal with Mathematical Language and Tools

The second group includes the competencies of representing mathematical entities, handling mathematical symbols and formalism, communicating in, with and about mathematics, and making use of aids and tools.

Research Design and Methods

This study is carried out following a case study design (Yin, 2013) and the data has been collected in a Norwegian public university. A small group of three male students, enrolled in the first year of an electronics engineering program, has been observed over the whole semester. The methods used to generate data include group observations, group interviews, individual weekly journals and field notes by the researcher.

For the participant observations, video recordings of their group work, and screen recordings to follow the activity on computer screens have been collected. Additionally, participants provided screen recordings of their individual work, and weekly journals containing self-reports about the use of resources for practicing mathematics. In this paper, we analyse three episodes of the students’ group work in order to look for how these competencies are supported in an e-learning environment.

Analysis

The two sets of competencies are not mutually disjoint, in general, and are intertwined which is evident from the so-called competency flower. Although each competency has a well-defined identity in theory, execution of each competency in practical will draw on some other competencies. This makes it empirically challenging to disentangle one competency from the others (Niss, Bruder, Planas, Turner, & Villa-Ochoa, 2016). We adhere to these considerations and our purpose here is to rather we look for possibilities in which e-learning influences each sets of competencies.

In the quest for finding correct answers to the given tasks in present situation, participants needed to go through certain procedures where they could demonstrate these competencies. Geogebra (https://www.geogebra.org/) and WolframAlpha (https://www.wolframalpha.com/) were main tools used by the students to make sense of various mathematical functions, checking for the functions’ behaviour and to look at the solutions of the tasks. Textbook served as a main written help material in terms of consulting for mathematical formulas, explanations or illustrations, and for checking whether their solutions were correct by comparing these with the answers to tasks provided in the end of textbook. At several occasions, the textbook served as an aid to get acquainted with the mathematical topics, as the students read the textbook to understand the mathematics. The introduction of Maxima was done in a project in this course, and the purpose was to make engineering students capable of using this programming language to solve mathematical problems thus it also served as a resource.

The exposure to Google and different online calculators, in this case, for finding solutions of the given tasks, has shared the role for computing and calculating the solutions. We noticed that in participants' arguments, the element of tool dependence was evident.

In this regard, WolframAlpha and GeoGebra have a central role, since it in the present situation supported students in making sense of the functions, expressions and mathematical concepts in different ways. For example, when the students were not able to solve an integral $\int_0^1 \frac{\sin(x)}{x} dx$ by programming with Maxima, they started wondering whether it was solvable at all, and they used WolframAlpha to make sense of the scope of the task or to know the answer:

Per: (…) Maybe it… (we) can't solve it? Have you tried Wolfram? [Per is addressing Jan and visits WolframAlpha website himself. Per has looked up $\int_0^1 \frac{\sin(x)}{x} dx$ on WolframAlpha (Figure 2)]

Per: No, you're supposed to get an answer.

In this example, when asked by Jan, Per was trying to handle the scope of this integral. He used WolframAlpha to see what this integral is all about, and based on the output, he decided that it could be solved. This example illuminates how the mathematical thinking and problem-tackling competencies are being executed along with the obviously observed aids and tools competency.

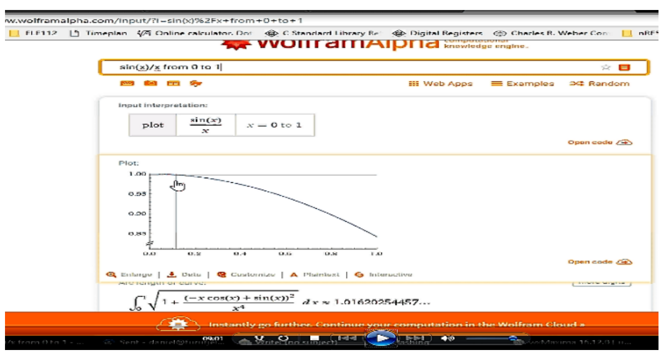

Figure 2: Screenshot of a participant's work on WolframAlpha.

The online tools mediated in the students' abilities to think and reason mathematically either by providing the complete calculations or the opportunities to explore the tasks at hand. By using paper and pencil techniques, both of these functions require a different kind of knowledge and skills as it says in the competence framework.

The following excerpt indicates how this environment is supporting the competencies of dealing with mathematical language and tools. While trying to solve a definite integral $\int_{-1}^{1} e^{-\sqrt{-1}wt}\, dt$ using Maxima, they got apparently a different outcome than what it said in the book.

Per: It is the same? It is the same thing, just written in a different way.
Jan: Yeah
Per: Simplify [Per tries to use the "simplify" command on the expression in Maxima]
Jan: Yeah, it just looks that much nicer when you do it in…
Per: In Wolfram.
Jan: Yeah. Yeah, or at that. Did you get…You got the same in Wolfram?

Per: Nnn… I haven't checked it. I assume I get what it says in the book.

[Per looks up $\int_{-1}^{1} e^{-\sqrt{-1}wt}\,dt$ in WolframAlpha.]

Per: Then I get sine w to…2 sine w divided by w, and that's exactly the same as it says in the book.

Jan: There, it… If you go back. Wolfram has moved -1 outside.

[Jan is trying to make Per aware how WolframAlpha has changed the representation.]

Per: Where?

Jan: Put the square root outside the parentheses.

Per: Yeah, but that's just if… I don't think it matters if...

[Meanwhile Per writes the original expression slightly differently in Maxima and gets the same output]

Per: It is exactly the same. I think it is correct.

Here, Per and Jan were trying to make sense of the different representations of the expression when both resources offered the result in a slightly different manner. The second set of competencies concerning representing mathematical entities, handling mathematical symbols and formalism, communicating in, with and about mathematics, and making use of aids and tools are *in action*.

An interplay of different resources had also been helping to approach a given task from different perspectives and to gain more information about the task in hand. Also, the use of Maxima apparently seemed as a short cut for getting ready-made answers. However, it has been observed that it required some effort from the students to decode the mathematical language into programming language.

Discussion

We intended to look for the execution of mathematical competencies in an e-learning environment in our case, and the findings of this study differ from the previous findings by (García et al., 2014). We found that while this environment supports some competencies, it does not ensure enhancing all of these in all learning environments. The way in which this online learning environment provides possibilities for practicing mathematics makes it different from the traditional way of doing mathematics in a paper and pencil environment.

For instance, from the first set, when the competencies of thinking and reasoning mathematically have to be executed in an online environment. We conjecture that the effects are twofold. On one hand, the resources are facilitating in computing, calculating and providing answers requiring less effort from the students thus limiting the possibilities for exploration. However, on the other hand, when used for comprehension of the tasks at hand they have potential to enhance the possibilities of exploration. We further observed that e-learning is certainly not on the same lines as it means to think and reason mathematically in a traditional way. In a traditional paper and pencil environment, students use their own knowledge and skills for performing the tasks at the hand.

The second set of competencies has more scope in the present context owing to the use of different tools and aids for practicing mathematics. When students used different tools for practicing mathematics, and each one of those tools uses different symbolism which provides some opportunities for the students to experience and handle varied mathematical formalism in a way.

Question for discussion: How to devise a better systematic scheme for analysing mathematical competencies in this environment?

References

Albano, G., & Pierri, A. (2014). Mathematical competencies in a role-play activity. In C. Nicol, P. Liljedahl, S. Oesterle, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 2, pp. 17-24). Vancouver: PME.

Alpers, B. A., Demlova, M., Fant, C.-H., Gustafsson, T., Lawson, D., Mustoe, L., . . . Velichova, D. (2013). *A framework for mathematics curricula in engineering education*. Brussels: SEFI.

García, A., García, F., Del Rey, Á. M., Rodríguez, G., & De La Villa, A. (2014). Changing assessment methods: New rules, new roles. *Journal of Symbolic Computation, 61*, 70-84.

Jaworski, B. (2012). Mathematical competence framework: An aid to identifying understanding? In C. Smith (Ed.), *Proceedings of the British Society for Research into Learning Mathematics* (Vol. 32(3), pp. 103-109). London: British Society for Research into Learning Mathematics (BSRLM).

Jaworski, B. (2013). Developing teaching of mathematics to engineering students: Teacher research, student epistemology and mathematical competence. In S. Coppe & M. Haspekian (Eds.), *Actes du Séminaire National de Didactique 2012* (pp. 5-14). Paris: Université Paris Diderot.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis & S. Papastravidis (Eds.), *3rd Mediterranean Conference on Mathematics Education, Athens* (pp. 115-124). Greece: Hellenic Mathematical Society and Cyprus Mathematical Society.

Niss, M., Bruder, R., Planas, N., Turner, R., & Villa-Ochoa, J. A. (2016). Survey team on: conceptualisation of the role of competencies, knowing and knowledge in mathematics education research. *ZDM, 48*(5), 611-632.

Niss, M., & Højgaard, T (Eds.). (2011). *Competencies and mathematical learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark.* Roskilde: Roskilde University.

Queiruga-Dios, A., Sánchez, G. R., Bullón-Pérez, J., Encinas, A. H., del Rey, A. M., & Martín-Vaquero, J. (2016). Case study: Acquisition of mathematical industrial engineering competences during the first year. In B. Alpers, U. Dinger, T. Gustafsson, & D. Velichová (Eds.), *Proceedings of the 18th SEFI Mathematics Working Group Seminar on Mathematics in Engineering Education* (pp. 138-142). Brussels: European Society for Engineering Education (SEFI).

van der Wal, N. J., Bakker, A., & Drijvers, P. (2017). Which techno-mathematical literacies are essential for future engineers? *International Journal of Science and Mathematics Education, 15*(1), 87-104.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Yin, R. K. (2013). *Case study research: Design and methods* (5th ed.). Los Angeles: Sage Publications.

Engineering students' engagement with resources in an online learning environment

Shaista Kanwal

Engineering students' engagement with resources in an online learning environment

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In this paper, we investigate how undergraduate engineering students interact with an online learning environment provided to them in a Calculus course. The constituent resources of this environment include tutorial videos, textbook and MyMathLab – an online interactive system for mathematics. A qualitative case study involving a small group of students has been conducted. We investigated which resources these students used and the manner in which they incorporated these resources in their online mathematical work.

Keywords: Students' interactions with resources, the role of digital and other resources in university mathematics education, mathematics for engineers.

INTRODUCTION

In recent years, digital resources are increasingly used for teaching and learning of mathematics (Borba et al., 2016; Pepin, Choppin, Ruthven, & Sinclair, 2017). The presence of wide range of digital resources in terms of their functionalities allows various possibilities of creating digital environments for students to learn mathematics. Each digital environment might afford unique interactive and learning opportunities; therefore, empirical research closely looking at students' engagement and the opportunities for their learning in such environments is well needed. This study deals with one digital learning environment provided to undergraduate engineering students for practicing mathematics. The aim is to explore students' interactions with the constituent resources of this environment to elucidate the learning opportunities in this environment.

Adler (2000) introduced the term *resource* to embrace several agents such as physical, human and cultural tools and aids intervening in a teacher's activity. In this paper, however, we distinguish between digital and classical resources and focus on students' work with resources. The use of digital resources is relevant in the context of engineering mathematics in the sense that engineers during their professional activities rely on technology for solving mathematical tasks (van der Wal, Bakker, & Drijvers, 2017). The framework for mathematics curricula in engineering education (Alpers et al., 2013) recommends the use of technology aimed at fostering engineering students' mathematical competencies. In the next section, we present the theoretical framework, and the subsequent section contains introduction to the constituent resources of the online learning environment.

THEORETICAL PERSPECTIVE

In order to study students' interactions with the resources, we employ the *documentational approach to didactics* (Gueudet, Pepin, & Trouche, 2012; Gueudet & Trouche, 2009) which is grounded on Rabardel's work (Rabardel, 2002) and enlarges the instrumental approach (Trouche, 2004) in mathematics education. One important distinction between the two approaches lies in the extension of the concept of *artefact*, in the former approach, to *resource* which allows considering wider set of materials intervening in the teachers' and students' activities. A resource can be conceptualised "as both noun and verb, as both object and action that we draw on in our various practices (Adler, 2000, p. 207)". Thus, the approach has the potential to take in consideration material, human and cultural resources such as language, time, mathematics teachers, etc. Moreover, a resource is never isolated but belongs to the wider set of resources (Gueudet & Trouche, 2009).

While one focus of this approach is on the teacher's work with the resources, the study of students' use of resources can provide the overview of their actual use (Gueudet & Pepin, 2016). Also, this approach has the potential to provide rich analyses if used to evaluate students' work in terms of interactions with different resource systems (Trouche & Pepin, 2014) or with a particular resource (Aldon, 2010). We will employ this approach to analyse how students interact with available resources.

In particular, we analyse students' techniques when working digitally in mathematics (Artigue, 2002). A technique is perceived as "a manner of solving a task (Artigue, 2002, p. 248)". While students work on mathematical tasks in a digital environment, they might adopt paper and pencil based techniques or instrumented techniques. The obvious and easily observable objective of each technique is to reach the goal of the activity i.e. to produce the results whereas the contribution of a technique to the learning of involved mathematical concepts might not be easily recognisable. The former corresponds to pragmatic value while the latter corresponds to epistemological value liked to each technique.

We seek to explore the kind of techniques implemented by the students in the digital environment to make sense of how students interact with this environment while working on mathematical tasks. Furthermore, realisation of the values attached to the students' instrumented techniques will also help to understand the role of digital resources in their learning (Guin, Ruthven, & Trouche, 2005). There are several resources involved in present situation, therefore, we confine to the general features of corresponding techniques in the present paper. By this, we mean to consider students' general organisation of digital work with several resources related to all contents in a Calculus course. We ask the following question: How do engineering students incorporate resources during their work in an online learning environment?

THE SETTING

This study took place in a Norwegian public university during the spring of 2017. Undergraduate students enrolled in electronics engineering program participated in this study. In their Calculus course, students were offered an online learning environment such that they could work remotely by interacting with the provided resources. These resources were made available to them electronically to work and proceed through the course. There were no mandatory lectures, and they could access the lecturer in the case they needed additional support. The final examination was also in digital format where the students were allowed the access to tools and aids.

The resource system comprised MyMathLab environment, tutorial videos coupled with the notes, and the textbook. The students' homework and the formative assessments were administered online through MyMathLab system. MyMathLab is an interactive learning system for practicing mathematics online (figure 1). While this system provides an online platform for homework and assessments, it also facilitates students in solving the tasks by providing help and feedback. Students can seek help through utilising "help me solve this" or "view an example" functions in the system. The former lets the student solve a similar task by guiding on each step whereas the latter shows a similar worked-example. The interactive nature of MyMathLab system allows considering it as a resource which can potentially influence students' activity in this course.

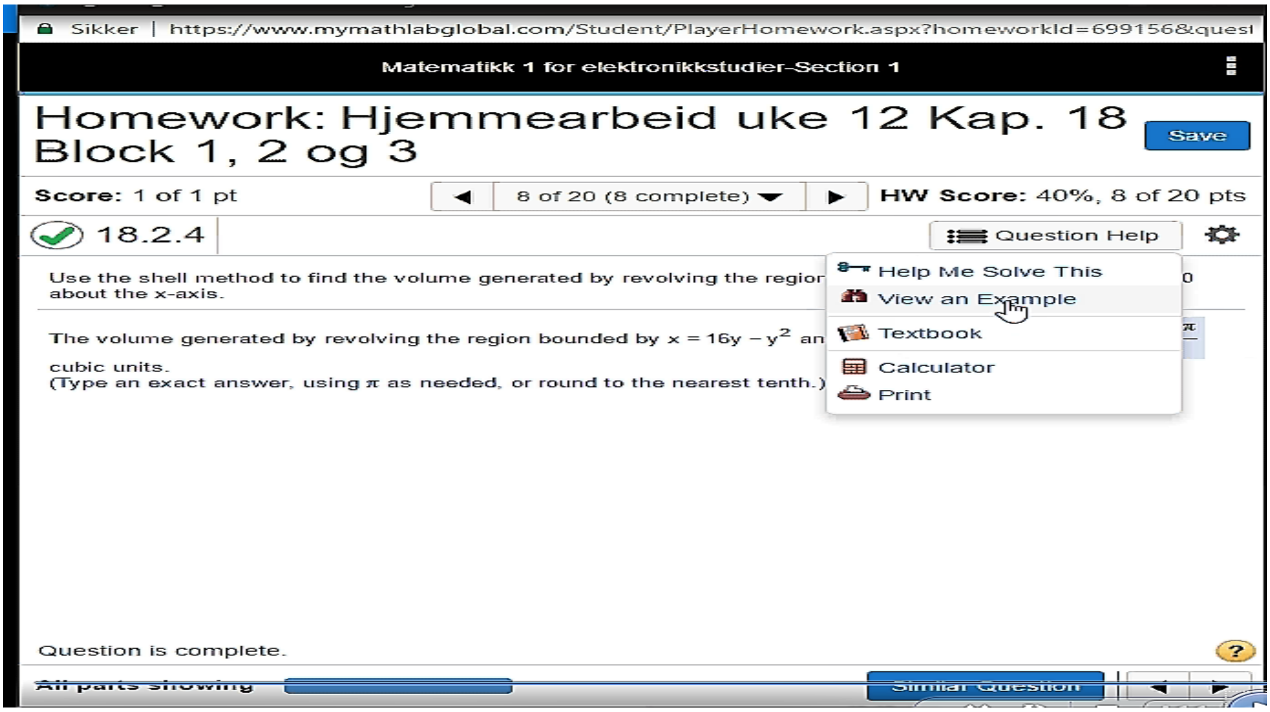

Figure 1. Interface of MyMathLab environment.

The tutorial videos are created by the lecturer, and recorded by using a document camera. Each video deals with a specific section in the book and is named accordingly. In these videos, the lecturer explained the topics in the book and worked through the relevant examples occasionally. The notes pertaining to the video tutorials were available online. The length of these videos varies depending on nature of the concerned topics. The tutorial videos replaced lectures and it was expected that students would watch the videos to learn mathematical topics. The textbook served as the central resource in the sense that MyMathLab and tutorial videos were based on contents in the book.

In this course, a compulsory task was the group project in which students were required to prepare a question bank related to integration. That question bank was needed to be programmed in the STACK environment, a computer aided assessment platform. Maxima is the programming language used in the STACK, thus they were required to learn Maxima to complete the project. The intention was to make students familiar with programming language and its use in mathematics.

RESEARCH DESIGN AND METHODS

The case study research design (Yin, 2013) has been followed in this study. A group of three students has been observed over the semester. The methods used to generate data include group observations, semi-structured interviews, individual weekly journals and field notes. Using multiple methods for data collection contributed to triangulation of data.

In order to be able to observe participants' activity, we requested them to work at campus each week for which they agreed. During these sessions, they worked on their routine work including homework and assessments. Video recordings of their group work accompanied with the screen recordings to follow the activity on their computer screens have been collected. Screen recordings of their individual work external to these group sessions have also been collected. Furthermore, weekly journals containing self-reports about their use of resources were included to get the detailed overview. The journal was provided to participants in tabular format which they filled and submitted electronically each week. In the journal, they were asked to specify the resources they used and state how the use of a particular resource helped them in their work each week. The semi-structured interviews were held occasionally to understand the emerging patterns in their use of resources. During the group work sessions, participants communicated in their native language whereas the interviews were held in English. Both the group sessions and the interviews were transcribed.

We analyse participants' weekly journals, a semi-structured interview in the middle of the semester, screen recordings, and the field notes for reporting on students' use of resources in their work. This interview is being counted on because the participants were inquired about the general manner in which they used the resources. The observations, screen recordings and the field notes are being counted on while identifying participants' techniques during their work.

ANALYSIS

Participants' weekly self-reports about use of resources

Table 1 presents the overview of participants' use of several resources as they reported in their journals. The manner in which they used them in their work and their evaluations of resources have been extracted from their journal inscriptions.

Resource used	How they incorporated resources in their work	Comments about resources (if any)
Tutorial videos	Watched to get information to complete homework	Easy to understand through videos
MatRIC videos	Skimmed through the video at amplified speed	
Own note	Used the already solved similar problems in the notes, to recall the problems (methods for solution)	
Textbook	Read through the book, found formulas to work on homework, got questions from book (during project)	
Maxima	Programmed tasks in Maxima for the project, used while doing homework, solved tasks using Maxima	Programming in Maxima is hard but when it is done, all the problems are easy to solve
WolframAlpha	Used as a shortcut to get answers, compared answers obtained from Maxima, got help with solving difficult tasks	Easier to use than Maxima, Faster than using calculator, useful when the answer is in the form of expression instead of numbers
MyMathLab	Worked on homework, learnt specific topic, solved some questions with higher difficulty	Powerful tool, easier to get help and information online
Internet		
Lecturer's notes		Tailored" for the tasks at hand, the most relevant piece of information
Youtube vidoes	Watched Maxima tutorials	
Mathway and other online calulators	Solved questions	Severely increase the probability to get the correct answer, and therefore the overall score.
STACKS	Made some questions in STACKS	

Table 1: Overview of participants' use of resources.

The three participants, Tor, Per and Jan, used MyMathLab almost every week because homework and assessments were required to be done in this system. As regards the textbook, Tor did not report the textbook in the journals rather he used the lecturer's notes. While in Per and Jan's weekly reports, they pointed out few ways in which they used the textbook on different occasions. The textbook served as a source of getting questions, checking answers to those questions, getting help with formulas, and going through examples in the book. During their project work, they consulted the book to take questions and subsequently checked the answers for those questions.

The tutorial videos were reported to be used by Jan and Per during their work. Jan occasionally watched the videos and when specifying about the kind of help, he used the word *understand* linked with this resource such as "to try to understand how to calculate…" and "to understand the calculation behind the math". Per has also mentioned the use of videos and commented, "I easily understand it when someone explains me the way of solving a problem". Tor did not mention any tutorial video provided by the lecturer, however he watched few videos on other platforms, MatRIC TV (an online resource containing videos aiming to support students in their transition from high school to university) and YouTube, once for getting introduction to *partial integration* and at another occasion to learn Maxima – the programming language.

It can be seen that participants used some other resources in their work such as online calculators, WolframAlpha, Maxima and internet (cf. Table 1). Tor named several online calculators including Mathway (https://www.mathway.com) and WolframAlpha (https://www.wolframalpha.com) to solve the tasks and to compare the answers they got in Maxima while working on the project. He mentioned that he used online calculators for saving time, however, he wrote, "I did not learn anything doing this, but it severely increases the probability to get the correct answer, and therefore the overall score". Wolfram Alpha has also been used by Per and Jan in order to verify whether the answers they got were correct. While working on the project, they picked some questions from the book and programmed in Maxima. To check the answers to those questions, they used WolframAlpha.

After completing the group project that involved learning Maxima, this programming language became an important resource for them to solve tasks in homework and assessments. Both Per and Jan began making programs for solving each task to liberate themselves from calculations. Per inscribed in a weekly journal, "(I) used Maxima to make a program to solve the problems in an easy way. This is hard to make, but when it is done, all the problems are easy to solve". Tor did not seem to use Maxima a lot, he spent some time on learning how to use Maxima for solving tasks in one week, and then spending some more time in the next week, he rather chose to focus on MyMathLab. He inscribed that, "it's (MyMathLab) a more powerful tool and it's easier to attain help and information online".

In response to a question about using videos in a semi-structured interview, Per explained his way of working on homework using the provided resources.

Per: These topics I think are quite hard to learn all by yourself. When I get a new topic, I first try to solve it myself, if I can't do that I try to look at the examples in MyMathLab… and if I don't completely understand the examples I take a look at Olav's (lecturer) video…mainly the examples' videos because then I get to see the practical kind of way to do..to solve questions.

Int: How would you rank the provided resources? Which one do you first consult with?

Per: First, I will try to do it myself because then I think I… remember and learn it the best because then I have to think and ….and if I can't do it that way…then I will try to look at example just to get a few hints. If that does not work then I watch the videos because I can't look at the notes (provided by the lecturer)…I have to get explanation of what he is doing step by step.

Tor's response was somewhat similar as he replied:

Int: Did you use any video while working on last week's homework?

Tor: No, I think MyMathLab seemed sufficient so far.

Int: Ok. So which resource did you use for getting introduction to the new topic?

Tor: I tried first MyMathLab but it went fine so I just carried on. …I check the notes and watch the videos if I get stuck..

Int: So, you turn to the videos when you get stuck.

Tor: When it is a new topic, then I just skim through his notes, but since we have integration from a couple of weeks now, I am pretty confident and go straight with it.

While Jan responded to the same question as follows.

Jan: I did not watch that many videos. I mostly use MyMathLab and just see the examples…and if I can't get it from there then I go to…to the book because it is faster… and eventually go to the videos if I do not get constructive help from there.

The participants preferred MyMathLab during their work for being the source of quick and most relevant help in comparison to the other available resources. This approach of working on the tasks saved them time and effort to search for the required piece of information from other resources such as the videos and the textbook. However, the use of MyMathLab can be considered more pragmatic as both Per and Jan mentioned that the kind of help they get from MyMathLab is in the form of examples which contributes more towards producing the results.

Another approach was to watch the videos when the help from MyMathLab was not *sufficient* as evident through participants' responses in the interview. The use of videos has not been preferred much but participants reported that they consulted the videos when they needed to *understand* something. As discussed earlier, the help and feedback in MyMathLab concern the task only as it offers the formula and solution-steps for the task. They might have needed to consult the videos to learn the concepts involved in those tasks in case when just knowing the solution steps in a question did not work. In the journal data, Jan and Per wrote that they used the videos to *understand* thus it indicates the epistemic value linked to usage of videos.

Observing participants' activity helped in finding that the use of different resources affected their manner of working on tasks i.e. techniques. We seek to categorise the participants' techniques pertaining to different resources they used, and by considering their motives behind use of each resource helped in recognising the pragmatic and epistemic value of their techniques. It is found that they increasingly used the digital tools to solve the tasks in MyMathLab environment with the progression in the course. This led to the use of more instrumented techniques instead of paper and pencil techniques promoted in the lecturer's videos and through MyMathLab. For instance, Tor mentioned in his weekly journals and it is observed in the screen recordings of his individual work that he used several calculators to work on homework as well as assessments. The participants themselves perceived this technique of using online calculators to solve the task as pragmatic.

Two of the participants used Maxima in their work as evident from journals and could be seen through the screen recordings of their work. They wanted to be pragmatic in order to make their future work easier. Making programs for each task for the first time can not be considered as merely pragmatic as Per mentioned that he found it hard. The difficulty in making programs may be linked to their knowledge of programming in order to code mathematical tasks. However, the extent to which it contributes epistemically in learning mathematics is not covered in present paper.

DISCUSSION AND CONCLUSION

In this study, we observed how a small group of three students interacted with the resources when provided with an online learning environment in their Calculus course. The environment allowed self-regulated learning and students could work remotely on their homework and assessments. To make sense of the opportunities for students' learning with resources in this environment, we explored their manner of incorporating the resources in general organisation of their digital work. Furthermore, we discussed the epistemic and pragmatic potential of participants' techniques.

In terms of resource usage and the corresponding techniques, participants opted for the resources and the techniques which were pragmatic in terms of producing results for the assigned tasks. Pragmatic techniques involved the use of online calculators, using help in the MyMathLab to produce the results for tasks. Watching videos for learning mathematical concepts seemed to be time consuming and hence not preferred much. Participants appropriated the programming language to work on the tasks with the motive to be more pragmatic and produce results easily in their work. An important factor which is likely to cause the preference for more pragmatic instrumented techniques was the online final examination where they could use the resources. As for students, it is quite important to prepare according to the examination to be able to score better.

This case study provides an example of a self-regulated learning environment created for students to work independently. Our findings suggest some general prospects which are worth paying attention when assigning online homework to students. Combination of an online homework with online examination is likely to cause students to use unexpected use of resources and techniques, for instance, online calculators and solution tools in the present case. This observation also relates to the nature of tasks posed in an online homework environment. Variety in the nature of tasks, such as open-ended tasks, may lead students to interact with resources epistemically.

REFERENCES

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*(3), 205-224.

Aldon, G. (2010). Handheld calculators between instrument and document. *ZDM, 42*(7), 733-745.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for mathematical learning, 7*(3), 245-274.

Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2016). Blended learning, e-learning and mobile learning in mathematics education. *ZDM, 48*(5), 589-610.

Gueudet, G., & Pepin, B. (2016). Students' work in mathematics and resources mediation at university. In E. Nardi, C. Winsløw, & T. Hausberger (Eds.), *Proceedings of the First Conference of the International Network for Didactic Research in University Mathematics (INDRUM 2016, 31 March-2 April 2016)* (pp. 444-453). Montpellier, France: University of Montpellier and INDRUM.

Gueudet, G., Pepin, B., & Trouche, L. (Eds.). (2012). *From text to 'lived' resources: Mathematics curriculum material and teacher development*. New York: Springer

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199-218.

Guin, D., Ruthven, K., & Trouche, L. (Eds.). (2005). *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument*. New York: Springer

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: foundations for change. *ZDM*, *49*(5), 645-661.

Rabardel, P. (2002). People and Technology: A cognitive approach to contemporary instruments (translation of Les Hommes et les Technologies). Retrieved from https://halshs.archives-ouvertes.fr/file/index/docid/1020705/filename/people_and_technology.pdf

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for mathematical learning*, *9*(3), 281-307.

Trouche, L., & Pepin, B. (2014). From instrumental to documentational approach: towards a holistic perspective of teachers' resource systems in higher education. *Research in Mathematics Education*, *16*(2), 156-160.

Exploring Affordances of an Online Environment: A Case-Study of Electronics Engineering Undergraduate Students' Activity in Mathematics[1]

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Abstract Online learning environments are being used for teaching and learning of mathematics at university level. Exploiting the potential of digital technology, these Internet-based environments administer computer-generated homework, assistance and feedback for students. This article presents a case-study of a small group of undergraduate engineering students' learning activity in mathematics in an online environment. The study focuses on students' interactions with the online environment to make sense of the affordances of this environment. Utilizing multiple sources of data aid in analyzing the intentional and the operational aspects of students' interactions with several resources in this environment. With regard to both of these aspects, the affordances are thus viewed as features of the environment which support students' engagement with the mathematical tasks. The analyses show that the students incorporated several online resources for solving the tasks posed in the automated system. Students met requirements of final answers in the automated system through varying sequences of mathematical operations for the posed tasks. The conditions of the automated system as well as the rules of the collective activity system played a role in students' interactions with the mathematical tasks.

Keywords Online learning activity · Calculus · Engineering students · Interaction with resources · Affordances

Introduction

In recent years, online education has become a common feature of university level courses (Rosa and Lerman 2011). While several Internet-based applications are being employed to facilitate the process of teaching and learning of mathematics, personalized learning environments (PLEs) mark the latest trend in e-learning (Borba et al. 2016; Gadanidis and Geiger 2010). The PLEs represent the automated online systems which not only deliver the instructional materials but also provide tailored assistance to students. So far, there is a dearth of research exploring the potential of such environments for students' learning of mathematics and students' interactions with these online environments (Borba et al. 2016; Webel, Krupa, and McManus 2017).

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To address these gaps, this article seeks to characterize undergraduate engineering students' activity in an online learning environment (Engeström 1987; Leont'ev 1978), which involves Pearson's MyMathLab (MML) as a PLE and a collection of electronically accessible resources (e.g., tutorial videos, notes). MML is an automated system which serves as an online platform for homework and assessments for students and provides assistance and feedback through its built-in functions. The aim of this article is to illuminate the affordances of this environment for students' learning activity.

An online environment (or the PLE) has previously been defined as the collection of "tools, artifacts, processes, and physical connections that allow learners to control and manage their learning" (Borba et al. 2016, p. 602). Learning in such an environment involves "focusing on the appropriation of tools and resources by the learner" (Buchem, Attwell, and Torres 2011, p. 1). In this article, I will focus on students' interactions with the constituent resources of the environment during their online learning activity in mathematics.

Students' Activity in an Online Environment

This study adopts the theoretical perspectives of cultural-historical activity theory (CHAT) (Engeström 1987; Leont'ev 1981) which is rooted in the sociocultural theory of learning and development (Vygotsky 1978). The concept of activity was introduced by Leont'ev (1978) to represent the subject-object interaction mediated by tools. Mediation refers to the intermediate position of tools between the subject and the object of an activity. An *activity* is realized when a *subject*, an individual or a group, acts on an *object*, through *tools*, in order to transform it into an *outcome*. The *object*, material or ideal, is closely linked to the need behind the *activity* and differentiates one *activity* from another. Leont'ev devised a theoretical model explaining macrostructure of human activities (Fig. 1).

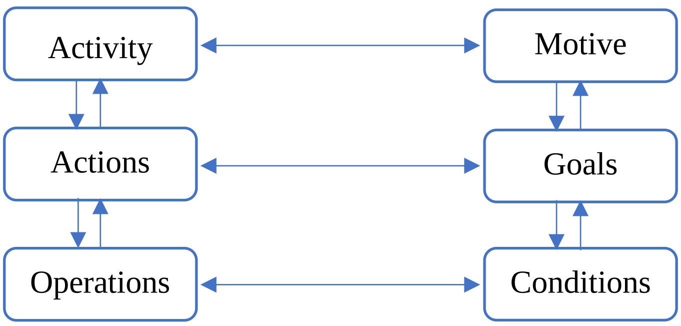


Fig. 1 Hierarchical levels of an activity (Leont'ev 1981)

In this model, Leont'ev (1981) discerned three hierarchical layers of human functioning at which an *activity* can be analyzed: the *activity* itself, the *actions*, and the *operations*. At the top level, the whole *activity* is viewed to be directed towards the *object*, which serves as the driving force or *motive* for the activity. It is through the lower levels that the *object* is transformed into the desired outcome. The middle level corresponds to goal-directed actions which realize the activity; the *goals* and *actions* represent the functions formerly merged in the motive. The bottom level concerns the *operations* “which depend directly on the conditions under which a specific goal is to be achieved” (Leont'ev 1974, p. 27). The nature of *operations* is also related to the conditions of the tools in use. Initially, the *subject* performs an *action* being conscious of the minute details concerning its execution. With enough practice, the *action* takes the form of a subconscious *operation*. The newly formed *operation* becomes part of another *action* which has a broader scope. If *conditions* concerning the execution of this *operation* change, it rises to the level of conscious *action* again. These changes are also resonated at the upper level of *activity* where the *object/motive* is reflected, questioned and transformed accordingly. The boundary between these levels of *activity* is dynamic – changing and developing all the time.

Kuutti (1996) notes that action-operation dynamics portray a basic feature of development in human functioning, and “to become more skilled in something operations must be developed so that one’s scope of actions can become broader (p. 31)”. Relevant to the tool-mediated learning actions in mathematics, Leont'ev (1974) specified, “when one uses a calculating device to solve a problem, the action is not interrupted by this extracerebral link; the action is realized through this link, as it is through its other links” (p. 27). Regarding operations, he wrote, “assume that a man was confronted with the goal of graphically representing some kind of dependences . . . . to do this, he must apply one method or another of constructing graphs – he must realize specific operation” (Leont'ev 1978, p. 66).

In this article, undergraduate students’ (*subject*) activity in a Calculus (*object*) course mediated through several resources in an online environment is under consideration. The notion of resources corresponds to Wartofsky’s primary artifacts, “those directly used in . . . production” (Wartofsky 1979, as cited in Engeström 2014, p. 49)”, in accordance with Anastasakis, Robinson, and Lerman (2017). In the present case of students’ activity, such production may be understood as to reaching the goals of the actions like solving the tasks. Leont’ev’s model of activity (Fig. 1) is utilized in analyzing the structure of students’ activity in relation to their interactions with resources. Leont'ev (1978) discussed that “a tool considered apart from a goal becomes the same kind of abstraction as an operation considered apart from the action that it realizes” (p. 65). In this view, I link students’ use of resources with the action-goal layer (Fig. 1) i.e. the actions performed by using various resources and associated goals with incorporating those resources. The operation-condition layer is then analyzed to make sense of the nature of (mathematical) operations conditioned by those resources.

Leont'ev (1981) asserted that analysis of human action is not complete without considering it into the system of societal relations, and he described human activity to be “a system in the system of the social relations” (p. 47). On these lines, Engeström (1987) devised a unified model of collective activity system incorporating multiple mediations through tools and social relations in human activities (see Fig. 2). Engeström (2014) wrote, “the object-oriented and artifact-mediated collective activity system is the prime unit of analysis” (p. xvi). The model (Fig. 2) represents the “most simple unit that still preserves the essential unity and the integral quality behind the human activity” (Engeström 2014, p. 65).

According to Engeström (1990), the upper part of this model refers to individual tool-mediated actions which are “the visible tip of the iceberg of collective activity (p. 172)” whereas “the hidden bottom part (p. 172)” refers to societal mediations in the form of rules, division of labor, and community. The rules represent the explicit or implicit norms which needs to be followed during an activity and thus affect the realization of the activity. Division of labor specifies the way in which whole task of the activity is divided among the participants to reach the outcome. The community signifies the other human beings with which the subject has direct or indirect relations.

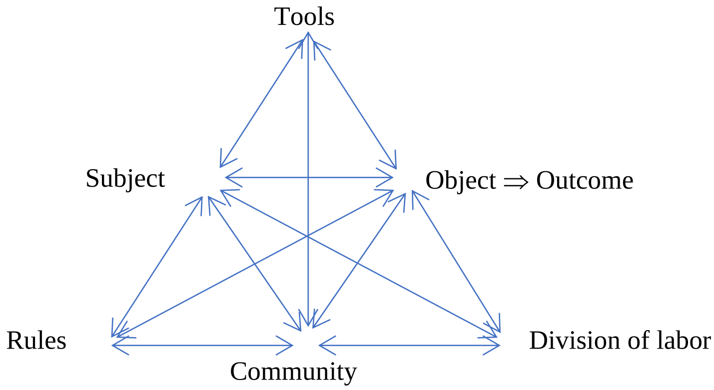


Fig. 2 The extended triangular model of human activity system (Engeström 1990)

According to Cole (1996), “in activity theory . . . contexts are activity systems” (p. 141). In this study, the online learning environment is characterized using Engeström’s model (see Fig. 2). Engeström (2014) suggests analyzing the relationships between elements of the activity system by considering the systematic whole. In this regard, the model facilitates in analyzing the dynamics of students’ activity with regards to features of the learning environment.

Engeström (2014) specified, “we may well speak of the activity of the individual, but never of individual activity” (p. 54). With reference to Roth (2012), the dialectical stance of CHAT “allows us to understand the person as a singularity and as collective phenomenon simultaneously without reducing it to one of its observable moments” (p. 97). In this sense, a student is considered as both an individual and a collective subject whose activity is regulated by features of the joint activity system.

An Activity-Theoretical Perspective on Affordances

The concept of *affordance* was introduced by Gibson (1977) to denote the action possibilities provided by the environment to an agent. The affordances are constituted in the meaningful relationship between the agent and the environment. According to Greeno (1994), the affordances are realized when attributes of the environment relate to the capabilities of the agent in such a way that an activity is supported. This view of affordances concerns the operational aspects of activity. Bærentsen and Trettvik (2002) argue for an activity-theoretical perspective for studying affordances of the environment. This perspective suggests considering the needs as well as the capabilities of the agent in relation to attributes of the environment. According to Bærentsen and Trettvik (2002), the affordances of computer software and programs should be studied in the processes of object-oriented activities of the intended users of such programs. Also, in addition to operational aspects, motivational and intentional aspects of users' activities should also be considered.

Studying affordances for students' mathematical activity in an online environment is essential to figure out the learning opportunities in such environments. Leont'ev (1981) argued that the external objective activity has particular implications for the inner psychological activity as, "mental reflection or consciousness is generated by the agent's objective activity" (p. 52). With regard to the role of the environment, Leont'ev (1981) stressed that "society produces the activity of the individual it forms", in the sense that, "social conditions carry the motives and goals of the activity, its means and modes" (p. 48). However, he emphasized that human activity is not the simple personification of the relations of society and its culture. There are complex transformations which need to be discovered through investigating the genesis of activities.

With these considerations, the research questions posed in this study are as follows.

RQ1: How do a small group of undergraduate students interact with an online environment during their learning activity in mathematics?

RQ2: In what manner does this environment afford students' learning activity in mathematics?

To answer RQ1, I first characterize the collective activity system in the present situation (Engeström 1987). Next, I investigate the structure of students' activity with regards to their interactions with this environment (Leont'ev 1981). In particular, I explore students' goals for which they use certain resources in their learning actions and analyze how this environment conditions the operational level of students' activity. Consequently, I discuss the answer to RQ2 i.e. the affordances of this environment in view of intentional and operational aspects of students' activity.

Previous Research Concerning Online Environments in University Mathematics

Several studies have sought to evaluate the impact of automated systems quantitatively by analyzing examination grades, cost effectiveness, and passing rates (e.g., Callahan 2016; Jonsdottir, Bjornsdottir, and Stefansson 2017; Kodippili and Senaratne 2008; Potocka 2010). Krupa, Webel, and McManus (2015) compared the impact of computer-based (CB) and face-to-face (F2F) instruction in an intermediate college algebra course. They used a quasi-experimental match design with the sample consisting of three levels of participants enrolled in the course. At the first level, they compared the exam results of two large groups ($N_{F2F} = 192, N_{CB} = 134$), and the second level included some other student-level predictors ($N_{F2F} = 73,\ N_{CB} = 50$). The third level concerned the quantitative analysis of students' solution strategies for some open response ($N_{F2F} = 38,\ N_{CB} = 24$). The results on the first two levels showed that students from the CB group performed better on the exam whereas they showed limited ability to interpret and relate algebraic equations to contextual situations. To follow up, Webel et al. (2017) investigated the implementation of a Math Emporium (ME), a model for teaching and learning of mathematics using computer-based programs, in an introductory college algebra course using mixed methods. They investigated: (1) whether the emporium is more helpful to a certain group of students; (2) the nature of mathematical learning in this setting; and (3) the students' perceptions about the emporium style courses. Webel et al. (2017) concluded that the emporium style served the students with higher mathematics achievement and those who less strongly believed that mathematics is about memorizing. Their findings suggested that the setting enabled students to focus on getting correct answers more than developing algebraic meanings. Regarding students' perceptions, they found that some students did not like the autonomy and flexibility offered by this setting. These findings led the researchers to question if examination grades and passing rates are the appropriate indicators of the impact of such settings. They recommended that future studies should focus on students' interactions and mathematical reasoning afforded by these environments.

With regards to students' activity in online environments for mathematics, Cazes, Gueudet, Hersant, and Vandebrouck (2006) focused on university students' strategies for different kinds of tasks posedin three Electronic-exercise bases (EEB) – similar to automated system. Through direct observation of individual students' work and electronically generated activity logs of their activity in these programs, they observed that students often developed unexpected strategies. The study took place during the experimental implementation of such environments and the conditions within each automated system affecting students' solution strategies were discussed.

From a CHAT perspective, Rønning (2017) explored the influence of such an automated program (Maple T.A.) on undergraduate engineering students' engagement with mathematics. The data set in this study included six surveys of large cohorts ($n > 500$) followed by focus-group interviews between the years 2013 and 2016. Students' responses were used to analyze the factors pertinent to the collective activity system affecting their *actions* while participating in the activity. Rønning (2017) discussed that the system promoted quest of correct answers among students which hindered the deep learning of mathematics.

The brief literature review presented above indicates lack of research on students' interactions with the resources during their learning activity in online environment in mathematics. In particular, the analysis of students' activity in such settings taking into consideration the macro and micro-level factors (cf. Jaworski and Potari 2009) has, to the best of my knowledge, not been done so far. As an example, in case of a blended learning environment, a partially relevant study (Anastasakis et al. 2017) focused on students' interactions with several resources at the action-goal layer of their activity i.e. the type of resources used by undergraduate students and the relationship between students' goals and their choice of resources. Anastasakis et al. (2017) surveyed a cohort of 201 engineering undergraduate students followed by interviewing 6 students to get a deeper insight. From the survey responses, they found that students incorporated institutionally provided resources dominantly but also used some other resources such as online videos, WolframAlpha, and online encyclopedias. They concluded, from the analysis of interviews, that students' choice of resources was driven by exam-related goals. The operational details of students' activity were not addressed in this study.

Differentiating between different types of resources (e.g., social, material, digital), a strand of research (see Gueudet and Pepin 2016) focuses on students' use of resources in mathematics. From this strand, a relevant report in the context of university mathematics by Gueudet and Pepin (2018) investigated how university students interact with several resources in their general mathematical work. Through case-studies, Gueudet and Pepin (2018) observed discrepancies between students' actual use of several resources and the lecturers' expectations of students' use of those resources. With regard to evaluating impact of automated systems on students' learning, Gueudet (2006) suggested that the students' activity with such resources should be observed at two levels: the particular exercise level when students solve the task, and the global level i.e. patterns of work during a session.

This article adopts a holistic perspective on students' activity with special attention to students' interactions with the resources in an online environment. That is, the micro aspects concerning operational characteristics (Fig. 1) of students' activity and macro aspects of the collective activity system (Fig. 2) have been combined.

Methodology

Context

This study was carried out at a Norwegian university administering several engineering programs at undergraduate level. The students from an undergraduate electronics engineering program participated in this study. An online learning environment was created for the students in their calculus course. This course spans both semesters of the first year of the program. The study took place during the second semester.

In this course, instruction, homework and assessments were administered electronically. Lectures were provided to students in the form of tutorial videos. The tutorial videos were created by the lecturer and were recorded using a document camera capturing his writing-activity on paper accompanied by the explanation. Each tutorial video dealt with specific topics from the textbook and contained explanations of those topics. The written notes associated with these videos were also made available for students through the learning management system (LMS) used at the university. Face-to-face interactions with the lecturer were possible in case students required additional help, and they could contact the lecturer electronically or in person.

Homework and assessment were conducted through Pearson's MyMathLab (MML), based on the textbook *Mathematics for engineers* by Croft and Davison (2015). Each week's homework in MML was linked to specific sections in the textbook. MML aids the users in solving tasks through two embedded functions: 'help me solve this' and 'view an example'. The former option breaks down a similar task into several steps and prompts students to perform calculations in each step. The latter option illustrates a worked example. In addition, it provides feedback by indicating that the answer is correct or wrong. In case the answer is wrong, it offers hints about the solution procedure.

Three formative tests were administered through MML in this course. The course involved a group project in which students were required to make a question bank on the topic of integration and program those questions using Maxima – a computer algebra system (CAS). The final examination was also in a digital format allowing the use of resources. The final grade was calculated from a weighted average of tests, project work and the final examination.

Research Design and Methods

This research is founded within a naturalistic research paradigm (Guba and Lincoln 1982) in the sense that participants' everyday work in a natural setting is observed. Four students (pseudonyms: Per, Jan, Tor, and Ole) volunteered to participate in this research. Following a case-study research design (Yin 2014), the case under consideration is the activity of the small group of participants in the online learning environment.

In order to understand an activity system, Engeström (1999) recommended that the researcher should look at the system from the above and at the same time through the eyes and interpretations of a subject, thereby complementing the system view and the subject's view. Nardi (1996) articulated general methodological implications deriving from the principles of activity theory for empirical research in the field of human-computer interaction. First, the frame of analysis should be long enough to understand the *subject's object*. This implication arises from the claim that the activities are long term formations and the objects are transformed into outcomes through a process of several phases. Second, the attention must be given to broad patterns or bigger picture of the activity instead of narrow episodic fragments. The small episodes may prove useful, but not in isolation from the overall situation. Third, various methods for collecting data should be used without unjustified reliance on any one form of the data. Fourth, the researcher should be committed to understand the object from the subjects' perspective.

The methods used for data collection in this study are in line with the considerations discussed above. Multiple methods including observations of students' group work, weekly journals, semi-structured interviews and field notes were used to collect the data. Weekly journals and interviews facilitated in gaining students' input regarding their interaction with the resources. Observational data provided micro details of students' activity concerning mathematical operations and corresponding conditions in this environment. The data were collected during the spring of 2017.

In weekly journals, students were asked to specify the resources they used and how they used each resource in their work. Only three of the participants (Per, Jan, and Tor) submitted the journal regularly. For observations, participants were requested to work together on campus for approximately an hour-long session in one week. During these sessions, they worked on their weekly assignments (homework, tests or the project) and communicated with each other in Norwegian. With the progression of the course, the participants' activity was becoming increasingly computer-based. I asked them to record their computer screen activity using Camstudio,[2] a freeware screen recorder. Semi-structured interviews were held to complement the data from journals and observations to gain further details about their usage of resources. The interviews were conducted in English. I kept field notes when I visited the students on campus.

Data Analysis

The field notes, semi-structured interviews, students' journals and my own observations aided in identifying elements of the collective activity system (Fig. 2) in the present setting (see Table 1). The rules, community and division of labor were mainly identified from my observations in the form of

[2] http://camstudio.org/

field notes and through interviews with students. The resources and the outcome were identified through students' journals.

For the analysis of the action-goal layer in Leont'ev's model (Fig. 1), weekly journals and interviews served as the main sources of data. The individual students' journals were analyzed to identify various manners in which each resource was used by the group of students collectively. In the first step, I extracted each students' descriptions linked to each resource from every journal and listed them across the resources in a single document. In the next step, I discerned students' goals and actions linked with each resource from those descriptions. Leont'ev (1974) defined an action to be "a process that is structured by a mental representation of the result to be achieved, i.e. a process structured by a conscious goal" (p. 23). In this sense, a statement such as "to try to understand how to calculate the length of a line" refers to the *goal* that the student wanted to achieve by incorporating a particular resource in her action. The statement such as "I got the questions from the book as well as some help with formulas" points to the actions mediated through the book. In some cases, I delineated the actions and the goals from single statements where applicable. Often, students also described some other aspects regarding their general manner of work organization such as their strategies, deviations in plans, and comments regarding the nature of resources. I extracted students' comments about the resources to see how they perceived each resource. The collective summary of the use of resources is presented in Table 2. The entries in Table 2 are not shared among the three participants.

Regarding the operation-condition layer in Leont'ev's model (Fig. 1), the operational details are considered as "not often consciously reflected by the subject" (Engeström 2014, p. 54). Nardi (1996) discussed that some minute details about the operations can be retrieved through careful questioning during interviews. In this study, students' responses in the journals and interviews did not account for the operational details. For such details, video-recorded observations of the group work were utilized.

During the group work sessions, students worked independently for significant amount of time interacting with their computer screens. The discussions were initiated when they faced some problem, for instance, when the feedback from the program was difficult to comprehend. For the analysis, I first searched for the episodes with relatively active communication among the group members. Five out of seven group work sessions were translated into English by a native speaker of Norwegian. Further, I selected one episode for the purpose of illustration from the twelfth week when the activity system had developed enough. The episode is selected as it involves: the use of various resources in participants' work, and varying conditions in the sequence of tasks thus ensuring variation and richness in mathematical contents. I utilized the screen recording as well for the analysis of this episode.

Results and Discussion

The following sections present the answer to RQ1. The answer to RQ2 is presented in the last section.

Characteristics of the Collective Activity System

The analytical account of the characteristic elements of the students' collective activity system (see Fig. 2) in the present setting is given in Table 1. The collective activity system is conceptualized at the level of mathematics course. Therefore, the object of the activity is considered to be including topics covered in the course (see Table 1). In addition to the provided resources, the three students reported using a variety of other resources during their learning activity (see Table 1). Division of labor in this case made students in charge of their own learning process. Students had more choices to make in terms of selecting resources, suitable time, and place to work. The lecturer's duties in the course were mainly performed electronically. The explicit rules at the level of activity, mainly the test-deadlines, aided in maintaining students' pace with the course. The test scores were also included in the aggregation of the final grade; therefore, students were motivated to complete their homework in order to take tests before the deadlines. The implicit rules correspond to the specifications in MyMathLab, i.e. the manner in which it conditioned the micro interactions at the level of tasks. For instance, the number of attempts allowed, the form in which it required solution of tasks, the nature of feedback, and the syntax in which it accepted the answers.

Table 1 Elements of the collective activity system

Elements of activity	Analytical description
Subject	A group of electronics engineering students
Tools	Tutorial videos, Textbook, MyMathLab features, lecturer's notes, Maxima, own notes, MatRIC TV[a], YouTube, GeoGebra, STACK environment[b], WolframAlpha[c], Mathway[d] other calculators, and Internet (Google search)
Object	Calculus (differentiation, applications of differentiation, integration, applications of integration, and sequences and series)
Outcome	Learning Calculus, passing the exam, getting good grades
Rules	Work on homework, test deadlines, final digital examinations, specifications in MML
Division of labor	Students' work according to the rules of the course taking the responsibility for own learning. Lecturer organizes the online course making use of MML program by integrating it with the tutorial videos. MML features aid in distribution and collection of homework and providing instant help and feedback to students; other resources (Maxima, Internet, calculators) affect the manner in which students engage with mathematical tasks.
Community	Other engineering students, lecturer

[a] https://www.matric.no/tv; An online repository of short mathematical videos for first-year undergraduate students in Norway aimed to support their transition from upper secondary school to university; [b] A computer aided assessment platform which they were required to use in their project; [c] https://www.wolframalpha.com; [d] https://www.mathway.com

Students' Interaction with the Environment – Actions, Goals and Resources

The collective summary of three participants' weekly journals illustrating the action-goal layer in participants' activity (Leont'ev 1981) is presented in Table 2.

Regarding the provided resources, Per and Jan reported textbook use repeatedly in their actions as a means to get questions (during their project), to find mathematical formulas related to the tasks, and to acquire help on specific topics. Tor, however, did not report using the textbook in the journals, he rather reported using the lecturer's notes. The only form of lecturing in this course was through the videos, and the goals associated with the use of this resource were linked with learning of certain mathematical topics. For instance, Jan used the videos with the goals: “to try to understand how to calculate…”, and “to understand the calculation behind the math”. I noticed a gradual decrease in the use of videos through the students’ weekly journals, and I therefore held a semi-structured interview to know more about this trend. I asked the participants regarding their manner of working on the homework tasks to which Per responded first, followed by Tor and Jan.

> Per: These topics I think are quite hard to learn all by yourself. When I get a new topic, I first try to solve it myself, if I can’t do that I try to look at the examples in MML… and if I don’t completely understand the examples I take a look at Olav’s (lecturer) video…mainly the examples’ videos because then I get to see the practical kind of way to do…to solve questions.

Tor’s response was somewhat similar as follows:

> Int: Did you use any video while working on last week’s homework?
>
> Tor: No, I think MML seemed sufficient so far.
>
> Int: Ok. So which resource did you use for getting introduction to the new topic?
>
> Tor: I tried first MML but it went fine so I just carried on. …I check the notes and watch the videos if I get stuck.

While Jan responded as follows.

> Jan: I did not watch that many videos. I mostly use MML and just see the examples …and if I can’t get it from there then I go to…to the book because it is faster… and eventually go to the videos if I do not get constructive help from there.

These excerpts from the interviews indicate participants’ preference for MML features. As Per mentioned, “When I get a new topic … I try to look at the examples in MML”. Tor stated, “MML seemed sufficient so far” and “I tried first MML but it went fine” while Jan mentioned “I mostly use MML”. Tor wrote in a journal, “it’s a more powerful tool and it’s easier to attain help and information online”. This preference for MML may be attributed to the immediate help available in the program for the tasks at hand whereas in the textbook and in the videos, students were required to search for the relevant information themselves.

Wertsch (1998) argues that the analysis of the goals of mediated action depends on the circumference of the context under consideration. In the case of multiplying two numbers, he explicated, the goal will be “ ‘to get the right answer within the confines of a particular way of setting up the problem’ (i.e., using Arabic numerals, using the syntax of multiplication outlined, not using a calculator, and so forth)” (p. 33). Moreover, “the goal of obtaining the right answer needs to be coordinated with other aspects of the sociocultural setting as well” (Wertsch 1998, p. 34).

In this study, students’ goals linked to the use of WolframAlpha, Mathway, and Maxima point to features of the collective activity system (see Table 2). The online resources WolframAlpha and

Table 2 Incorporation of resources in participants mathematical activity - Summary of students’ journals

Resources	**Goals for using each resource**	**Performed actions**	**Students’ comments about resources**
Textbook	To find formulas for specific topics, to understand a topic	Read through the book, found formulas to work on homework, got questions from book (during project)	
Maxima	To avoid calculating everything by hand, to solve problems in an easy way, to make the work easier in the long run	Programmed tasks in Maxima for the project, used while doing homework, solved tasks using Maxima	Programming in Maxima is hard but when it is done, all the problems are easy to solve
MatRIC videos	To recall certain topics	Skimmed through the video at an amplified speed	
MyMathLab	To learn how to solve problems, to get inspiration for making questions in the project, to get an overview before taking test	Worked on homework, learnt specific topic, solved some questions with higher difficulty	Powerful tool, easier to get help and information online
Lecturer’s notes	To get the general idea of the topic		Tailored for the tasks at hand, the most relevant piece of information
WolframAlpha	To solve problems by using shortcuts	Used as a shortcut to get answers, compared answers got from Maxima, got help with solving difficult tasks	Easier to use than Maxima, faster than using calculator, useful when the answer is in the form of expression instead of numbers
YouTube videos	To recall a certain topic	Watched Maxima tutorials	
Mathway and other online calculators	To solve tasks in assessment	Solved questions	Severely increase the probability to get the correct answer, and therefore the overall score.
STACK	To make questions in STACK	Made some questions in STACK	
Internet	To learn Maxima, to search for how to solve the problems		
Tutorial videos	To learn rules and methods, understanding a specific topic, to recall previously done content	Watched to get information to complete homework	Easy to understand through videos

Mathway aid in the task solving processes. Tor reported using WolframAlpha and Mathway for solving the tasks in homework and tests. WolframAlpha was incorporated by Per and Jan to double check the answers, to solve the tasks by short-cut methods, and to get help with the difficult questions. Regarding Maxima, students learnt programming in Maxima as a part of the course, which they later used in their task solving activity in MML. Per and Jan started to make programs for each task in the homework with the goal to liberate themselves from calculations. Per inscribed in a weekly journal, "(I) used Maxima to make a program to solve the problems in an easy way. This is hard to make, but when it is done, all the problems are easy to solve". Tor wrote, "if I could make a template for each question, then I would have severer [*sic*] advantage on the upcoming exam".

Students' use of these computing tools can be ascribed to the rules of the activity system. Within the confines of this setting, students had to learn mathematics with regards to the implicit conditions in MML. At the same time, they also had to take part in the digital examination, which was the explicit rule of their activity system. From students' reports, it appears that the use of these resources let the students meet implicit as well as explicit rules of the activity system. Students' motive in the activity is thus taken as to learn mathematics and to perform well on the tests and in the final examination.

The nature of Mathematical Operations in Students' Online Learning Activity

This section focuses on incorporation of several resources (Maxima, GeoGebra, Internet and MML help) in mathematical operations in students' activity (Leont'ev 1981). Below, I analyze a part of a group work session in which the participants began working on their weekly homework dealing with applications of integration. I divide the analysis with respect to the three kinds of tasks involved in the homework. While narrating the group work, I follow Per's screen recording since he led the activity in the sense that he was ahead of the other participants.

Engaging with the Integral as Limit of a Sum. The first task required using the limit of sums for calculating the area under a curve (see Fig. 3). This task involves identifying the area under $y = x + 1$ between $x = 0$ and $x = 9$, dividing it into rectangles of equal width, and summing the areas of these rectangles. Applying the limit to the number of rectangles in the summation gives the definite integral $\int_0^9 (x+1)dx$. This value then represents the area under the curve. In MML, the worked example for this task suggested the sequence of involved mathematical operations.

Find the area under $y = x + 1$ from $x = 0$ to $x = 9$ using the limit of a sum.

Fig. 3 The first task

In this task, Per began by performing an operation in Maxima as observed through his screen recording (see Fig. 4). He entered the obtained number into MML which affirmed him that his answer was correct.

```
(%i1)    integrate (x+1, x, 0, 9);
(%o1)    99/2
```

Fig. 4 Per's solution strategy using Maxima

Jan, who was working with his paper notebook while getting questions from the MML opened on his computer screen, posed a question regarding the first task to which Per responded as follows.

02 Per: […] You must take the integral from 0 to 9. Or from 0… From the smallest value to the largest value.

03 Jan: Yeah. You are to *split it up* [emphasis added].

04 Per: I don't think so.

The discussion stopped at this point and Jan continued working in his notebook. It appears that the two participants were performing different operations. Per's operation in Maxima let him find the required area by calculating the involved integral whereas the task required using method of the limit of sums. The automated system (MML), being the main source of help and assistance in this case, provided Per feedback that his answer was correct. Jan seemed to be following the steps suggested in MML (also in the textbook) as he pointed towards dividing the area into rectangles (03). As Per had reached the immediate goal of getting the final answer, he did not agree with Jan (03). From (04), it seems that Per was unaware that he missed the mathematical operations in this task.

The next three tasks in MML also concerned using the limit of sum method for calculating area under different curves. Per solved these tasks using the same command in Maxima.

Engaging with the Disk Method. The next task in MML dealt with the application of integration for finding the volume of a solid formed by revolving a given area around an axis (Fig. 5). This task involves identifying the area to be revolved bounded by $y = x^2$, $x = 1$ and $x = 7$, and then dividing it into the strips of infinitesimal width, say, dx. These strips, upon revolving around the x-axis, take the form of cylindrical disks of radius y and height dx. The volume of one such disk becomes $\pi y^2 dx = \pi x^4 dx$. The limit of the sums of these volumes becomes the integral $\int_1^7 \pi\, x^4\, dx$, which gives the volume of the whole solid.

Find the volume of the solid formed when the area under $y = x^2$ between $x = 1$ and $x = 7$ is rotated about the x-axis.

Fig. 5 The disk method task

```
(%i9)    integrate (%pi*(x^2)^2, x, 1, 7);
(%o9)    (16806π/5)
```

Fig. 6 Per's solution in Maxima

Upon getting this task in MML, Per's first action was reading in the book for a while where the disk method for finding the volume of revolution was given. Next, he calculated the involved integral by performing an operation in Maxima (Fig. 6) which resulted in the correct answer.

The next task was similar and Per obtained the solution by performing similar operation in Maxima for computing the integral. However, the subsequent task was phrased slightly differently (see Fig. 7).

Find the volume of the solid of revolution formed by rotating about the x-axis the region bounded by the curves $f(x) = 3x^2$, $y = 0$, $x = 1$, and $x = 4$.

Fig. 7 Another disk method task

This task asked for "bounded region" instead of "area under the curve". Therefore, it included four bounds on the area to be revolved instead of three in the previous tasks (see Fig. 5). In this sense, the conditions for reaching to the solution of this task were apparently different from the earlier tasks. In Per's actions, he adjusted his Maxima command which he used in the previous task (see Fig. 6) by halving the integrand (see Fig. 8). This action did not yield in the correct answer, and MML provided him the feedback (see Fig. 9).

```
(%i13)    integrate (%pi/2*(3*x^2)^2, x, 1, 4);
(%o13)    (9207π/10)
```

Fig. 8 Per's command in Maxima

Remember that, if f(x) is nonnegative and R is the region between f(x) and the x-axis from x=a to x=b, the volume of the solid formed by rotating R about the x-axis is given by $\int_a^b \pi\, [f(x)]^2\, dx$. Make sure that you are correctly setting up and evaluating the integral. Check your work carefully. Please try again.

Fig. 9 Feedback from MML regarding disk method task

After looking at the feedback for a while, Per plotted the curve in GeoGebra, and then removed the $1/2$ in his Maxima command. Per reflected on these actions later, which can be seen in the excerpt below.

16	Per:	This exercise here (showing his laptop screen). You are to integrate that formula and find the volume.
17	Ole:	Mm
18	Per:	And then $y = 0$, then I thought, rather than rotating it the whole way, you know, should just rotate it down till $y = 0$ because that is here. (Illustrating the revolution while making a gesture through his hands)
19	Ole:	Mm.
20	Per:	But that wasn't it, it was just as we do. Like on the previous task.

Here, Per's action of intercalating the $1/2$ factor in the integrand in his Maxima command were based on his misinterpretation of the conditions of this task. Instead of considering $y = 0$ as a bound on the region to be revolved as specified in the task, Per considered it as a bound on revolution. He thought that the area had to be revolved in such a way that it did not need to go below the x-axis (18). Assuming that the revolution stops halfway, and then the generated volume will also be halved, he multiplied the integrand by $1/2$ (see Fig. 8) which did not result in the correct answer. He then excluded the $1/2$ factor and obtained the correct solution. The Maxima command now had become similar to the one he used in the previous task (see Fig. 6).

Although Per seemed aware of the revolution involved in these tasks, he could not realize the implications of the slightly different formulation of both tasks. As the same operation let him reach the solution in both tasks, he reached to the faulty conclusion that they needed to do the same (process) as they did in the previous task (20).

Engaging with the Shell Method. The next task concerned finding the volume of a solid using the shell method of revolution (see Fig. 10). This task requires the identification of the region to be revolved and dividing it into rectangles of infinitesimal width dy, as in the disk method. The rectangles should then be revolved around the x-axis in such a way that the solid formed is a cylindrical shell (instead of a disk) of radius y, height x, and thickness dy. The volume of one such shell is $2\pi yxdy = 2\pi y(16y - y^2)dy$. The limit of sums of these volumes becomes the integral $2\pi \int_0^{16} y(16y - y^2)\, dy$, which gives the volume of the whole solid.

Use the shell method to find the volume generated by revolving the region bounded by $x = 16y - y^2$ and $x = 0$ about the x-axis.

Fig. 10 The shell method task

In this task, Per adjusted the Maxima command from the disk method task (see Fig. 8) by replacing the integral to $16x - x^2$ (see Fig. 11). The integral formula and the limits of integration remained the same. He then entered the obtained answer into the MML window which responded that the answer was not correct and provided him the feedback shown in Fig. 12.

```
(%i15)        integrate (%pi*(16*x-x^2)^2, x, 1, 4);
(%o15)        (17703π/5)
```

Fig. 11 Per’s command in Maxima

Volume, V, as determined by the shell method with rotation about a line parallel to the x-axis is: V = $\int_a^b 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix} \begin{pmatrix}\text{shell}\\ \text{height}\end{pmatrix} dy$. The limits of integration are y-values at which $x = 0$ and $x = 16y - y^2$ intersect.

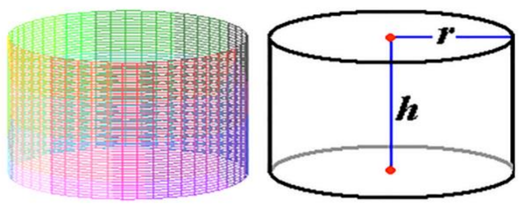
Fig. 12 Feedback in MML regarding the shell method task

Looking at the feedback for a while, Per opened GeoGebra and plotted the curve. He then searched on Google and found a Web page containing description concerning the shell method (see Fig. 13).

www2.bc.cc.ca.us/resperic/Math6A/Lectures/ch6/2/shell.htm

6.2 *Volumes of Revolution* (Part II) - Shell Method

In the disk method, we sum up the volumes of an infinite number of infinitesimally thin circular disks to find the total volume of a solid. The solid has been decomposed into stacked circular disks, and by integrating the disk volumes we obtain the total volume. In the washer method we decompose the solid into concentric "shells", each one a hollow tube infinitesimally thin. Here's one such shell, actually the outer tubular part of a right-circular cylinder:



The volume of a right circular cylindrical shell with radius *r*, height *h*, and infinitesimal thickness *dx*, is given by:

$$V_{\text{shell}} = 2\pi r h\, dx.$$

If one slits the cylinder down a side and unrolls it into a rectangle, the height of the rectangle is the height of the cylinder, h, and the length of the rectangle is the circumference of a circular end of the cylinder, $2\pi r$. So the area of the rectangle (and the surface of the cylinder) is $2\pi r h$. Multiply this by a (slight) thickness dx to get the volume.

Fig. 13 Web Page explaining the shell method

Meanwhile another participant, Ole, asked him about this task.

52 Ole: You didn't just put it into the calculator? (Referring to Maxima)

53 Per: No, it's something else, but it says nothing about it there.

54 Ole: Can't grasp why it is like that…

55 Per: Yes, $2\pi r$ times h. …That's not quite the same. (Reading from the Web page shown in Fig. 13)

In the next moment, he navigated back to the GeoGebra window and checked for the points of intersection of the curve with the y-axis (Fig. 14).

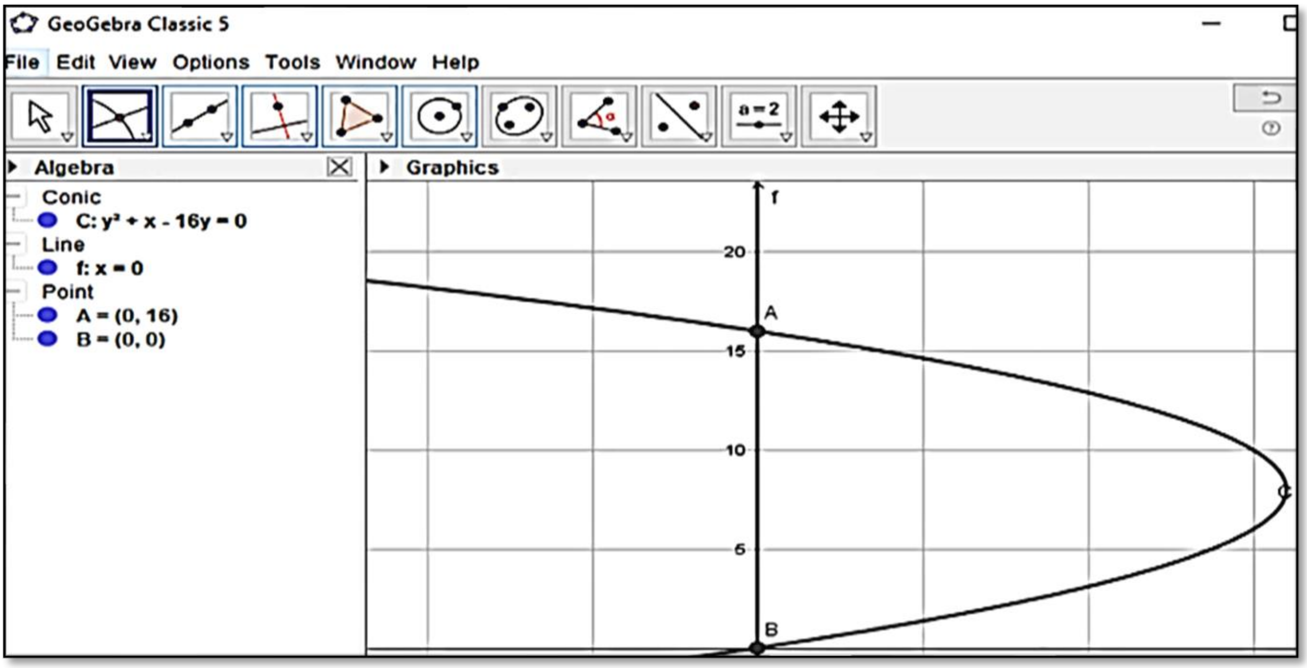


Fig. 14 Per's activity in GeoGebra

In his Maxima command (see Fig. 11), Per then changed the limits from 0 to 16 and inserted the term $2x$ in the formula which did not result in correct answer. Per navigated back to the Web page and scrolled down a bit (see Fig. 15).

Jan posed a question at this moment.

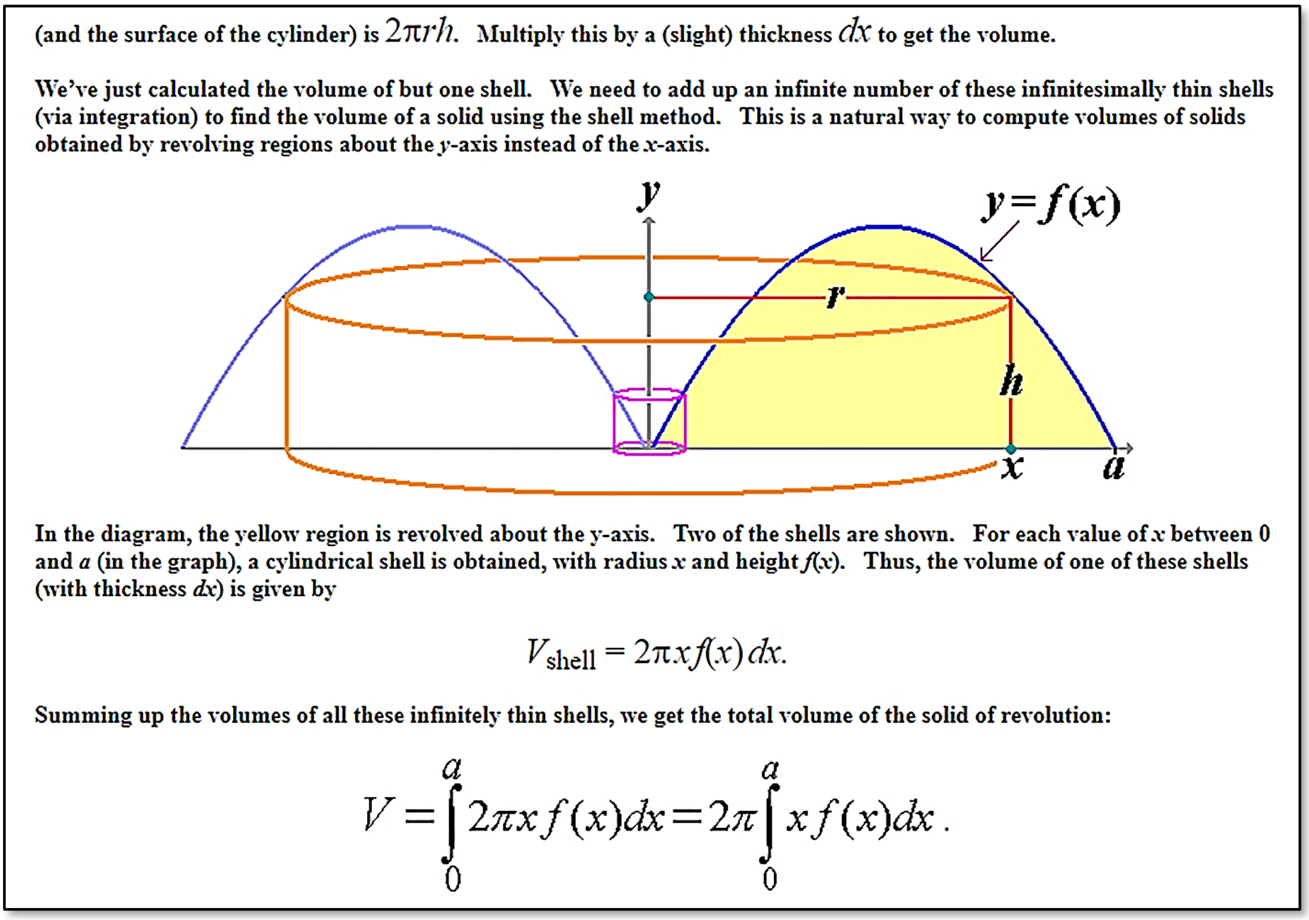
66 Jan: Did you figure out that shell method?

67 Per: I'm reading on it, but I think I got it. The formula for it is $2\pi x$ times the

function.

68 Jan: $2\pi x$ times the function. (Repeats the formula)

69 Per: But it is also like this. 2π …times r times h. Got to read a bit on it.



(and the surface of the cylinder) is $2\pi rh$. Multiply this by a (slight) thickness dx to get the volume.

We've just calculated the volume of but one shell. We need to add up an infinite number of these infinitesimally thin shells (via integration) to find the volume of a solid using the shell method. This is a natural way to compute volumes of solids obtained by revolving regions about the y-axis instead of the x-axis.

In the diagram, the yellow region is revolved about the y-axis. Two of the shells are shown. For each value of x between 0 and a (in the graph), a cylindrical shell is obtained, with radius x and height $f(x)$. Thus, the volume of one of these shells (with thickness dx) is given by

$$V_{\text{shell}} = 2\pi x f(x)\,dx.$$

Summing up the volumes of all these infinitely thin shells, we get the total volume of the solid of revolution:

$$V = \int_0^a 2\pi x f(x)dx = 2\pi \int_0^a x f(x)dx .$$

Fig. 15 Scrolling down the Web page

Next, looking at the Web page, he removed the square in his Maxima command (see Fig. 11) and eventually got the correct answer. In the next moment, Ole again inquired about this question.

70 Ole: Did you get exercise 8 right, with the shell method?

71 Per: Yeah. I did it just now.

72 []

73 Ole: Is it to find the bounds or something?

75 Per: Yeah, you find that out by… where y and $x = 0$ intersect.

76 Ole: So, you need to have $x = 0$ and $y = 0$?

77 Per: It's like this, kind of. When it intersects there. (Points at his GeoGebra window exactly where the curve intersects y-axis (see Fig. 14))

In this task, Per began by trying the similar operation used in the disk method task which did not result in the correct answer. The feedback from the program offered the integral formula (see Fig. 12) which required using the *shell height* and *shell radius* for calculating its volume. For applying this formula, one needs to know *where* and *how* the shell is formed, which was not discussed in the feedback.

It may be due to this lack of clarity in the feedback which conditioned to Per's action of searching the Internet. He opened a Web page which contained details regarding: the mathematical formula for calculating the volume of a shell $V_{shell} = 2\pi rhdx$, discussion of how the shell is formed by revolving an area around the y-axis, and the derivation of the integral formula, $2\pi \int_0^a xf(x)dx$ (see Fig. 13 and Fig. 15). Per initially tried to comprehend which one of these two formulae was relevant to the task or why these two were different (55 and 69). Per used the correct integral formula in his Maxima command and obtained the volume of revolution. The axis of revolution in the Web page illustration was the *y*-axis whereas the task in MML concerned revolution around the *x*-axis. Per managed this by using dummy variables in his Maxima command. For the limits of integration, Per employed GeoGebra to find the points of intersection of the given curve with the *y*-axis.

Summarizing the activity. The three kinds of tasks analyzed above can be thought to embody development in terms of the involved mathematical operations. The first task, for instance, introduces that an integral is equal to the limit of sums. The next two tasks involved this idea as an operation while widening the scope of its application to the case of finding volumes. The shell method task involves progression in the involved revolution in the disk method task.

The above analysis show that the students performed different sequences of mathematical operations in these tasks. In the first task, for instance, Jan seemed to be performing the sequence of operations suggested in MML. Per, however, skipped required mathematical operations and obtained the solution by employing Maxima. The Maxima command was concerned with computing the value of the involved integral. In the case of disk method tasks, Per again calculated the involved integrals through Maxima. In the shell method task, the integral to be calculated was not given explicitly. In this task, Per found the integral formula by searching on the Internet and calculated the integral in Maxima. The limits of integration were found by using GeoGebra.

By employing Maxima, Per met the requirement of final solutions to proceed through the tasks in MML. The mathematical operations were not necessarily in accordance with the requirements of the tasks. The conditions in MML were not concerned whether the students realized the involved mathematical operations to reach to the solutions of the tasks.

Affordances of the Online Environment

In this study, I set out to investigate students' interactions with an online environment during their learning activity in mathematics to make sense of affordances of this environment. The online environment under consideration involves implementation of an automated system (MML) with specific contextual aspects i.e. rules, division of labor, and community (see Table 1). The automated system (MML) offered the tasks, worked-examples with the sequence of mathematical operations, and instant feedback for regulating the students' online learning activity. The implementation of MML together with the contextual aspects (rules, division of labor, community) of this setting afforded self-regulated learning for students.

Concerning intentional aspects of students' interactions, students reported in their journals the use of some other resources in addition to the provided resources (see Table 2). The finding regarding undergraduate students' use of explanatory YouTube videos, Web pages and WolframAlpha is consistent with an earlier study (Anastasakis et al. 2017). In this study, the students also reported the use of Maxima, GeoGebra and online calculators in their activity. The students incorporated these resources in their learning actions with the goals to get immediate assistance and to prepare according to the final digital examination. The role of examination in shaping the students' use of resources is also reported in other studies (Anastasakis et al. 2017; Gueudet and Pepin 2018).

Regarding the operational aspects, the incorporation of several resources afforded various actions and operations (Leont'ev 1974) conditioned by the nature of each resource. In case of Web pages or videos, for instance, the afforded actions were making sense of the involved mathematical concepts. The use of calculators was linked to short-cut methods for solving the tasks posed in the program. The closer analysis of students' activity showed that the individual students worked on the same tasks by performing different mathematical operations. The automated system offered the relevant sequence of mathematical operations for the posed tasks while the students did not necessarily follow those steps. This result is also supported in the study by Cazes et al. (2006) that students' activity deviated from the desired mathematical activity. In the present study, the use of powerful computing tools affording the solution of tasks in single steps also led to diverting students' attention from the required mathematical operations in those tasks.

With respect to the conditions within automated system, the observed deviation in students' realized activity can be attributed to two specifications in MML. Firstly, the acceptance of the final answers in MML without accounting for the process of getting those solutions led students to focus more on getting the correct answers, which is also reported in the study by Rønning (2017). The program allowed students to proceed even when the mathematical operations were not in accordance with the demands of the tasks. Secondly, the mathematical tasks posed in the program could be solved using online calculators. This led to realization of students' actions and goals linked to solving the tasks while the operations were performed by the powerful computing tools. In this regard, Borba (2007) asserted that the nature of available media conditions the mathematical tasks. To explicate, Borba (2007) argued that a task such as "draw the graph of a function" represents an obstacle for students in a paper-and-pencil environment because students need to find the coordinates to plot the curve. The same task does not represent an obstacle in a technologically rich environment. Therefore, it needs to be shifted to an open-ended task such as "why does the graph of a function behave in a particular way?" in order to realize a meaningful obstacle. Thus, employing powerful computing tools for solving the procedural tasks may lead the mathematics to be black-boxed (Anderson 1999).

The participants in this study were undergraduate engineering students. It is generally recommended to integrate technology in mathematics courses (Alpers et al. 2013) to prepare future engineers according to the professional needs of today's technologically rich work environments. According to previous research, professional engineers emphasize the significance of mathematics for analytical and logical thinking although they reported using technology for mathematical tasks at work (Van der Wal, Bakker, and Drijvers 2017). In this view, the emphasis on the processes of solving the tasks instead of using the powerful computing tools needs to be ensured in order for involved mathematics not to be black-boxed for students.

From the features of the collective activity system, the rule concerning digital examination together with the conditions of the automated system led to students' choice and use of resources such as Maxima and calculators. That is, the students used these resources to meet the requirements of the MML and to prepare for the final digital examination. In turn, it affected students' engagement with the mathematical tasks.

The automated systems serve as the platform for managing (delivering, assigning, and evaluating) the homework and tests electronically. The findings of this study suggest that the implementation of this environment does not ensure that the students engage with the mathematical tasks in the expected manner. In addition to the conditions within the automated system, contextual aspects pertinent to students' activity, examination in the digital format as found in this case, also play an important role in students' interactions with this environment.

This study investigates implementation of an automated system for undergraduate mathematics in a specific manner: the digital final examination, and the division of labor managed through the resources in an online environment. Also, the findings are based on the analysis of the small number of participants' activity. Other students' activity in similar contexts may not unfold in the particular manner as observed in this study. However, the present study contributes to make explicit the role of factors at the wider level of the activity system in students' interactions with the automated system. The theoretical stance of CHAT (Engeström 2014; Leont'ev 1974) capturing the collective activity system in addition to the micro details of interactions offers a systematic way to analyze affordances of such systems for students' learning activity.

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References

Alpers, B. A., Demlova, M., Fant, C.-H., Gustafsson, T., Lawson, D., Mustoe, L., et al. (2013). *A framework for mathematics curricula in engineering education: A report of the mathematics working group.* Brussels, Belgium: SEFI.

Anastasakis, M., Robinson, C. L., & Lerman, S. (2017). Links between students' goals and their choice of educational resources in undergraduate mathematics†. *Teaching Mathematics and its Applications, 36*(2), 67-80.

Anderson, J. (1999). Being mathematically educated in the 21st century: What should it mean? In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the mathematics curriculum* (pp. 8-21). London: The Falmer Press.

Bærentsen, K. B., & Trettvik, J. (2002). An activity theory approach to affordance. In S. Bødker, O. Bertelsen, & K. Kuutti (Eds.), *Proceedings of the second Nordic conference on Human-Computer Interaction (Aarhus, Denmark - October 19 - 23, 2002)* (pp. 51-60). New York: ACM.

Borba, M. C. (2007). Humans-with-media: A performance collective in the classroom. In G. Gadanidis, & C. Hoogland (Eds.), *Proceedings of the Fields Institute Digital Mathematical Performance Symposium* (pp. 15-21). Toronto: Fields Institute.

Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2016). Blended learning, e-learning and mobile learning in mathematics education. *ZDM Mathematics Education, 48*(5), 589-610.

Buchem, I., Attwell, G., & Torres, R. (2011). *Understanding personal learning environments: Literature review and synthesis through the activity theory lens.* Paper presented at the PLE Conference 2011, Southampton, UK.

Callahan, J. T. (2016). Assessing online homework in first-semester calculus. *PRIMUS, 26*(6), 545-556.

Cazes, C., Gueudet, G., Hersant, M., & Vandebrouck, F. (2006). Using e-exercise bases in mathematics: Case studies at university. *International Journal of Computers for Mathematical Learning, 11*(3), 327-350.

Cole, M. (1996). *Cultural Psychology: A once and future discipline*. Cambridge, MA: Harvard University Press.

Croft, A., & Davison, R. (2015). *Mathematics for engineers* (4th ed.). Harlow, England: Pearson Education Limited.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki: Orienta-Konsultit.

Engeström, Y. (1990). When is a tool? Multiple meanings of artifacts in activity. In Y. Engeström (Ed.), *Learning, working and imagining: Twelve studies in activity theory* (pp. 171-195). Helsinki: Orienta-Konsultit.

Engeström, Y. (1999). Activity theory and individual and social transformation. In Y. Engeström, R. Miettinen, & R.-L. Punamäki (Eds.), *Perspectives on activity theory* (pp. 19-38). Cambridge, England: Cambridge University Press.

Engeström, Y. (2014). *Learning by expanding: An activity theoretical approach to developmental research* (2nd ed.). New York: Cambridge University Press.

Gadanidis, G., & Geiger, V. (2010). A social perspective on technology-enhanced mathematical learning: From collaboration to performance. *ZDM Mathematics Education, 42*(1), 91-104.

Gibson, J. J. (1977). The theory of affordances. In R. Shaw, & J. Bransford (Eds.), *Perceiving, acting and knowing* (pp. 67-82). Hillsdale, NJ: Erlbaum.

Greeno, J. G. (1994). Gibson's affordances. *Psychological Review, 101*, 336-342.

Guba, E. G., & Lincoln, Y. S. (1982). Epistemological and methodological bases of naturalistic inquiry. *Educational Communication and Technology Journal, 30*(4), 233-252.

Gueudet, G. (2006). Learning mathematics in class with online resources. In C. Hoyles, J.-b. Lagrange, L.H. Son, & N. Sinclair (Eds.), *Proceedings of the seventeenth study conference of the International Commission on Mathematical Instruction* (pp. 205-212). Hanoi: Hanoi Institute of Technology and Didirem Université Paris 7.

Gueudet, G., & Pepin, B. (2016). Students' work in mathematics and resources mediation at university. In E. Nardi, C. Winsløw, & T. Hausberger (Eds.), *Proceedings of the first conference of International Network for Didactic Research in University Mathematics (INDRUM)* (pp. 444-453). Montpellier, France: University of Montpellier and INDRUM.

Gueudet, G., & Pepin, B. (2018). Didactic contract at the beginning of university: A focus on resources and their use. *International Journal of Research in Undergraduate Mathematics Education, 4*(1), 56-73.
Jaworski, B., & Potari, D. (2009). Bridging the macro-and micro-divide: Using an activity theory model to capture sociocultural complexity in mathematics teaching and its development. *Educational Studies in Mathematics, 72(2),* 219-236.
Jonsdottir, A. H., Bjornsdottir, A., & Stefansson, G. (2017). Difference in learning among students doing pen-and-paper homework compared to web-based homework in an introductory statistics course. *Journal of Statistics Education, 25*(1), 12-20.
Kodippili, A., & Senaratne, D. (2008). Is computer-generated interactive mathematics homework more effective than traditional instructor-graded homework? *British Journal of Educational Technology, 39*(5), 928-932.
Krupa, E. E., Webel, C., & McManus, J. (2015). Undergraduate students' knowledge of algebra: Evaluating the impact of computer-based and traditional learning environments. *PRIMUS, 25*(1), 13-30.
Kuutti, K. (1996). Activity theory as a potential framework for human-computer interaction research. In B. A. Nardi (Ed.), *Context and consciousness: Activity theory and human-computer interaction* (pp. 17-44). Cambridge, MA: MIT Press.
Leont'ev, A. N. (1974). The problem of activity in psychology. *Soviet Psychology, 13*(2), 4-33.
Leont'ev, A. N. (1978). The problem of activity and psychology. In *Activity, consciousness, and personality* (pp. 45-74). Englewood Cliffs, NJ: Prentice-Hall.
Leont'ev, A. N. (1981). The problem of activity in psychology. In J. V. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp. 37-71). Armonk, NY: M. E. Sharpe.
Nardi, B. A. (1996). Studying context: A comparison of activity theory, situated action models and distributed cognition. In B. A. Nardi (Ed.), *Context and consciousness: Activity theory and human-computer interaction*. Cambridge, MA: MIT Press.
Potocka, K. (2010). An entirely-online developmental mathematics course: Creation and outcomes. *PRIMUS, 20*(6), 498-516.
Rønning, F. (2017). Influence of computer-aided assessment on ways of working with mathematics. *Teaching Mathematics and its Applications, 36*(2), 94-107.
Rosa, M., & Lerman, S. (2011). Researching online mathematics education: Opening a space for virtual learner identities. *Educational Studies in Mathematics, 78*(1), 69-90.
Roth, W. (2012). Cultural-historical activity theory: Vygotsky's forgotten and suppressed legacy and its implication for mathematics education. *Mathematics Education Research Journal, 24*(1), 87-104.
Van der Wal, N. J., Bakker, A., & Drijvers, P. (2017). Which techno-mathematical literacies are essential for future engineers? *International Journal of Science and Mathematics Education, 15*(1), 87-104.
Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
Webel, C., Krupa, E. E., & McManus, J. (2017). The Math Emporium: Effective for whom, and for what? *International Journal of Research in Undergraduate Mathematics Education, 3*(2), 355-380.
Wertsch, J. V. (1998). *Mind as action*. New York: Oxford University Press.
Yin, R. K. (2014). *Case study research: Design and methods* (5th ed.). Thousand Oaks, CA: Sage Publications.