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Reliable Underlay D2D Communications over Multiple Transmit Antenna Framework

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Abstract

Robust beamforming is an efficient technique to guarantee the desired receiver performance in the presence of erroneous channel state information (CSI). However, the application of robust beamforming in underlay device-to-device (D2D) communication still requires further investigation. In this paper, we investigate resource allocation problem for underlay D2D communications by considering multiple antennas at the base station (BS) and at the transmitters of D2D pairs. The proposed design problem aims at maximizing the aggregate rate of all D2D pairs and cellular users (CUs) in downlink spectrum. In addition, our objective is augmented to achieve a fair allocation of resources across the D2D pairs. Further, assuming elliptically bounded CSI errors, the formulation ensures maintaining signal to interference plus noise ratio (SINR) above a specified threshold. The derived optimization problem results in a mixed integer non-convex problem and requires exponential complexity to obtain the optimal solution. We perform a semi-definite relaxation (SDR) to handle the stochastic SINR constraints by using the S-Lemma, obtaining a number of linear matrix inequalities. The non-convexity is addressed by introducing slack variables and performing a quadratic transformation to obtain sub-optimal beamformers via alternating optimization. The solution for channel assignments to D2D pairs is obtained by convex relaxation of the integer constraints. Finally, we demonstrate the merit of the proposed approach by simulations in which we observe higher and more robust network throughput, as compared to previous state-of-the-art.

Index Terms

D2D communications, resource allocation, robust beamforming, semi-definite relaxation.

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I. INTRODUCTION

Robust transmit beamforming is recognized as a powerful technique to provide significant throughput gains in comparison to single antenna design [1]. However, most works in underlay D2D communications have considered single-antenna transmission, thus creating an opportunity for further investigation in a multi-antenna framework. The D2D communications in underlay configuration is a promising approach to improve efficiency in spectrum utilization by allowing simultaneous transmissions of existing cellular network and D2D pairs in the same spectrum [2], [3]. On the other hand, simultaneous transmissions in the same spectrum bands increase interference at the respective receivers which must be appropriately handled by devising judicious resource (power, channel) allocation algorithms. Introducing multiple antennas to transmit (beamforming) can further limit the interference and can act as an additional degree of freedom in devising resource allocation algorithms.

Resource allocation problems for underlay D2D communications have been extensively investigated under single antenna transmission in [2], [4], [5]. Considering simplicity in design,

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algorithms proposed in these works, restrict D2D pairs to access more than one channel. In contrast, improving throughput of D2D pairs by allowing access over multiple channels is studied in [6], [7]. It also important to note that these works assume perfect CSI in their problem formulation. Consideration of error in CSI while forming the resource allocation problem is considered in [8]–[10]. However, once again, these works limit investigation to single antenna transmission, leaving the scope for multi-antenna transmission which can be exploited to control the interference and improve the overall throughput of the network.

Considering transmission over multiple antennas, [6] presents a detailed analysis for joint beamforming in D2D underlay cellular networks. However, the analysis is restricted to a single D2D pair scenario under the additional assumption of perfect CSI. Scenarios with multiple D2D pairs are studied in [7], however, prefect CSI is also assumed to be available at the BS. Error in CSI due to quantization is considered in [11], where conventional maximum ratio transmission and interference cancellation techniques are exploited to compute the beamforming vectors. Design of robust beamformers for regular cellular communications has also been investigated in [12]. Under the assumption of Gaussian CSI errors, they propose several convex bounds to approximate the probabilistic rate outage constraints. In recent work, joint beamforming and power control strategies are studied in [13] under both perfect and erroneous CSI scenarios. In their formulation, the objective is to minimize the total transmit power of both BS and D2D pairs while ensuring quality of service (QoS) requirements. In conclusion, none of those previous works considers devising a robust beamforming design while performing resource allocation in underlay D2D communications, which is very relevant for maximizing aggregate network throughput.

In this work, we investigate the robust beamforming design problem in underlay D2D communications configuration under an erroneous (imperfect) CSI scenario. The main research contributions of this work are summarized as follows:

1) We formulate a robust beamforming design problem to maximize the aggregate rate of all D2D pairs and CUs while satisfying SINR to be above a specified threshold for both D2D and CUs. Under the assumption of CSI errors to be bounded within a specified ellipsoid, the proposed formulation maximizes the aggregate rate of the network in the worst case scenario of error in CSI. The objective of the design is also augmented to include the unfairness in channel assignment to D2D pairs. Further, our proposed formulation ensures higher throughput to D2D pairs by allowing simultaneous access of multiple channels to respective D2D pairs.
2) Our formulation leads to a mixed integer non-convex problem, for which we propose an algorithm to compute the power beamforming vectors and channel assignment to D2D pairs in a computationally efficient manner by exploiting SDR aided with a quadratic transformation. The power beamforming vectors and channel assignment are obtained by alternating optimization and convex relation of integer constraints, respectively.
3) In order to demonstrate the merits of our proposed formulation and the algorithm in reliably maximizing the aggregate rate of the underlay D2D communications network, we present Matlab based simulation results where we obtain a better performance than the-state-of-the-art alternatives.

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II. System Model

The underlay D2D communications scenario under a multiple transmit antenna framework in downlink spectrum[1] is shown in Fig. 1. We assume that the BS have K_B transmit antennas to communicate with N_C single antenna CUs through N_C downlink channels. In order to avoid confusion in notation, CUs (equivalently, channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. The

[1]Without loss of generality, the same formulation and algorithm design developed here, can be also applied to the uplink spectrum.

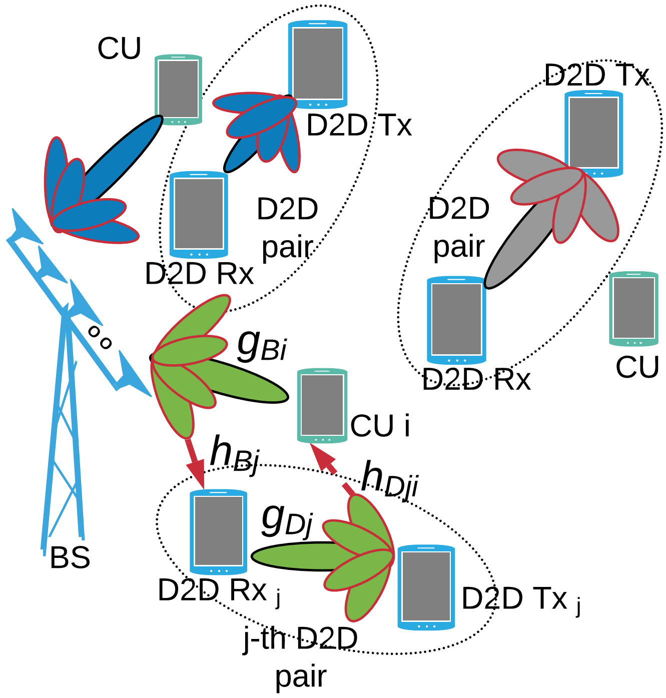

Fig. 1: Illustration of the overall system model.

D2D pairs wishing to communicate over the aforementioned N_C channels are indexed by $\mathcal{D}$ = $\{1, ..., N_D\}$. Similarly, we assume that the D2D transmitters have K_D transmit antennas to communicate with their respective single antenna D2D receivers[2].

The channel between the BS and the i-th cellular user (CU) is denoted by $\boldsymbol{g}_{B_i} \in \mathbb{C}^{K_B \times 1}$. Similarly, the channel between the j-th D2D pair is denoted by $\boldsymbol{g}_{D_j} \in \mathbb{C}^{K_D \times 1}$. The interference channel between the BS and the receiver of the j-th D2D is denoted by[3] $\boldsymbol{h}_{B_j} \in \mathbb{C}^{K_B \times 1}$. Similarly, the interference channel between the transmitter of the j-th D2D pair and the i-th CU is denoted by $\boldsymbol{h}_{D_{j,i}} \in \mathbb{C}^{K_D \times 1}$. Here, we assume that the CUs provide limited cooperation in estimating the gain of the interference channel (as expected in practice). Thus, if $\tilde{\boldsymbol{h}}_{D_{j,i}}$ denotes the estimate of the interference channel gain with error $\boldsymbol{e}_{ji}$, then the correct channel gain can be defined as $\boldsymbol{h}_{D_{j,i}} = \tilde{\boldsymbol{h}}_{D_{j,i}} + \boldsymbol{e}_{ji}$. This error vector is assumed to bounded within a specified ellipsoid, i.e., $\boldsymbol{e}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{e}_{ji} \leq 1$ where, $\boldsymbol{Q}_{ji} \in \mathbb{H}^{K_D}$, $\boldsymbol{Q}_{ji} \succeq \boldsymbol{0}$ specifies the size and shape of ellipsoid, and $\mathbb{H}^{K_D}$ is the space of $K_D \times K_D$ Hermitian matrices. The additive white noise power is denoted by N_0.

We represent the assignment of channels to D2D pairs by the indicators $\{\beta_{i,j}\}_{i \in \mathcal{C}, j \in \mathcal{D}} \in \{0, 1\}$, where $\beta_{i,j} = 1$ when the i-th channel is assigned to the j-th D2D pair and $\beta_{i,j} = 0$ otherwise. In order to provide higher throughput to D2D pairs, we allow simultaneous access of multiple channels to a D2D pair, however, to restrict the interference among D2D pairs, access of more than one D2D pair is not allowed over a particular channel, i.e., $\sum_{j=1}^{N_D} \beta_{i,j} \leq 1, \forall i$.

[2]In general, a BS/D2D transmitter with multiple antennas can simultaneously communicate to multiple CUs/D2D receivers on a single channel; however, for simplicity in our analysis, we assume one CUs/D2D pair on every channel. With minor modification, the analysis can be extended to the multi-user case.

[3]In principle, $\boldsymbol{g}_{D_j}$ and $\boldsymbol{h}_{B_j}$ should also depend on the i-th channel, however, this subscript is dropped as the proposed scheme carries over immediately to accommodate such dependence.

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Finally, we denote the beamforming power vector of the BS to communicate with the i-th CU by $\boldsymbol{p}_{B_i} \in \mathbb{C}^{K_D \times 1}$ and for the j-th D2D pair on the i-th channel by $\boldsymbol{p}_{D_{j,i}} \in \mathbb{C}^{K_B \times 1}$. The respective transmit powers are constrained as $||\boldsymbol{p}_{B_i}||_2^2 \leq p_{B,\max}$ and $||\boldsymbol{p}_{D_{j,i}}||_2^2 \leq p_{D,\max}$. To ensure successful communication, the SINR is desired to be greater than $\eta_{D,\min}$ for D2D pairs and $\eta_{C,\min}$ for CUs.

III. Problem Formulation

In order to take into account the error in the estimate of the interference channels from D2D pairs to CUs, i.e., $\tilde{\boldsymbol{h}}_{D_{ji}}$, we formulate the beamforming design problem for the worst case error in $\tilde{\boldsymbol{h}}_{D_{ji}}$. Let $\Gamma(z) := \text{BW} \times \log_2(1+z)$ denote the rate obtained over channel bandwidth BW for the given SINR z. The total rate that can be achieved over every i-th channel is defined by $R_i := (1 - \sum_{j\in\mathcal{D}} \beta_{i,j})R_{C_{i,0}} + \sum_{j\in\mathcal{D}} \beta_{i,j}[R_{D_{j,i}} + R_{C_{i,j}}]$, where:

- $R_{C_{i,0}} := \Gamma(p_{B_{\max}}||\boldsymbol{g}_{B_i}||_2^2/N_0)$, rate of the i-th CU without assignment of D2D pairs, i.e., $\beta_{ij} = 0\ \forall j$.
- $R_{D_{j,i}} := \Gamma(|\boldsymbol{p}_{D_{ji}}^H \boldsymbol{g}_{D_j}|^2\ /(N_0 + |\boldsymbol{p}_{B_i}^H \boldsymbol{h}_{B_j}|^2))$, rate of j-th D2D pair when assigned with i-th CU, i.e., $\beta_{ij} = 1$.
- $R_{C_{i,j}} := \Gamma(|\boldsymbol{p}_{B_i}^H \boldsymbol{g}_{B_i}|^2/(N_0 + |\boldsymbol{p}_{D_{ji}}^H \boldsymbol{h}_{D_{j,i}}|^2))$, rate achieved by i-th CU when assigned with j-th D2D pair, i.e., $\beta_{ij} = 1$.

Finally, the aggregate network rate is defined as $R(\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D) := \sum_{i\in\mathcal{C}} R_i$, where, $\mathbb{B} := \{\beta_{i,j}\},\ \mathbb{P}_B := \{\boldsymbol{p}_{B_i}\},\ \mathbb{P}_D := \{\boldsymbol{p}_{D_{j,i}}\}\ \forall i \in \{1, \cdots N_C\}$ and $j \in \{1, \cdots N_D\}$.

In order to have fairness in channel assignment, we introduce a secondary objective that penalizes greedy channel assignments to the D2D pairs. We also define the unfairness measure $\delta(\mathbb{B}) = 1/(N_D c^2) \sum_{j=1}^{N_D} (x_j - c)^2$ along similar lines to [14], [15], where $x_j := \sum_{i=1}^{N_C} \beta_{ij}$ is the number of channels assigned to the j-th D2D pair; and where $c := N_C/N_D$ is the fairest assignment. Summing up, the overall problem considering the worst case error in estimation of interference channel, can be formulated as:

$$\underset{\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D}{\text{maximize}}\ R(\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D) - \gamma\delta(\mathbb{B}) \tag{1a}$$

$$\text{subject to } \beta_{i,j} \in \{0, 1\},\ \sum_{j=1}^{N_D} \beta_{i,j} \leq 1\, \forall i; \tag{1b}$$

$$||\boldsymbol{p}_{B_i}||_2^2 \leq p_{B,\max}\, \forall i,\ ||\boldsymbol{p}_{D_{j,i}}||_2^2 \leq p_{D,\max}\, \forall j, i; \tag{1c}$$

$$\frac{|\boldsymbol{p}_{B_i}^H \boldsymbol{g}_{B_i}|^2}{N_0 + |\boldsymbol{p}_{D_{ji}}^H \boldsymbol{h}_{D_{j,i}}|^2} \geq \eta_{C,\min} \quad \text{if } \beta_{ij} = 1,\ \forall i, j \tag{1d}$$

$$\frac{|\boldsymbol{p}_{D_{ji}}^H \boldsymbol{g}_{D_j}|^2}{N_0 + |\boldsymbol{p}_{B_i}^H \boldsymbol{h}_{B_j}|^2} \geq \eta_{D,\min} \quad \text{if } \beta_{ij} = 1,\ \forall i, j \tag{1e}$$

$$\boldsymbol{h}_{D_{j,i}} = \tilde{\boldsymbol{h}}_{D_{j,i}} + \boldsymbol{e}_{ji}, \quad \boldsymbol{e}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{e}_{ji} \leq 1, \quad \forall i, j \tag{1f}$$

The regularization parameter $\gamma > 0$ is selected to balance the trade-off between aggregate rate and fairness in channel assignment. Problem (1) is a non-convex mixed-integer program, which involves exponential complexity. In next section, we discuss the proposed strategy by exploiting semi-definite relaxation and quadratic transformation.

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IV. Proposed Optimization Algorithm

The complexity to obtain the solution of (1) can be reduced by decomposing the problem into multiple sub-problems of lower complexity. We first re-express the aggregate throughput as:

$$R(\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D) = \sum_{i\in\mathcal{C}} \left[\sum_{j\in\mathcal{D}} \beta_{i,j} v_{i,j}(\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}) + R_{C_{i,0}} \right] \tag{2}$$

where $v_{i,j}(P_{B_i}, P_{D_{ji}}) := R_{C_{i,j}} + R_{D_{j,i}} - R_{C_{i,0}}$ represents the rate increment due to the assignment of channel i to the D2D pair j relative to the case where the channel i is only used by the CU. Next, notice that the objective of (1) with the substitution of (2) can be equivalently expressed by replicating $\{\boldsymbol{p}_{B_i}\}$ with multiple auxiliary variables $\{\boldsymbol{p}_{B_{ij}}\}$ and removing the constant terms from the objective function. The resulting problem can be stated as:

$$\begin{aligned} &\underset{\mathbb{B},\mathbb{P}_B,\mathbb{P}_D}{\text{maximize}} \quad \sum_{i\in\mathcal{C}} \sum_{j\in\mathcal{D}} \left[\beta_{i,j} v_{i,j}(\boldsymbol{p}_{B_{ij}}, \boldsymbol{p}_{D_{ji}})\right] - \gamma\delta(\mathbb{B}) \\ &\text{subject to} \quad (1b), (1c), (1d), (1e) \text{ and } (1f) \end{aligned} \tag{3}$$

To recover the optimal $\{\boldsymbol{p}^*_{B_i}\}$ of (1) from the optimal $\{\boldsymbol{p}^*_{B_{i,j}}\}$ of (3), one only needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $\boldsymbol{p}^*_{B_i} = \boldsymbol{p}^*_{B_{i,j}}$. If no such a j exists, i.e. $\beta_{i,j} = 0\ \forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power $\boldsymbol{p}^*_{B_i} = p_{B,\max}$.

In addition, we can also notice that (3) decouples across i and j into $N_C \times N_D$ power allocation sub-problems and a final channel assignment problem. Then, for each i, j, the power allocation sub-problem can be stated as:

$$\begin{aligned} &\underset{\boldsymbol{p}_{B_{ij}},\boldsymbol{p}_{D_{ji}}}{\text{maximize}} \quad R_{C_{i,j}} + R_{D_{j,i}} \\ &\text{subject to} \quad (1c), (1d), (1e) \text{ and } (1f) \end{aligned} \tag{4}$$

which should be solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$. The subsequent channel assignment problem is discussed in section IV-C. We can notice that problem (4) is still a non-convex stochastic problem. In the next section, we perform SDR along with *S-Lemma* [16] to express the stochastic constraints by linear matrix inequalities.

A. Semi-Definite Relaxation

It can be noted that the objective and constraint (1d) of (4) involve random channel interference terms. We first introduce slack variables z_C and z_D in order to bring stochastic terms from the objective of (4) to constraints as:

$$\underset{\boldsymbol{p}_{B_{ij}},\boldsymbol{p}_{D_{ji}},\boldsymbol{z}}{\text{maximize}} \quad \log_2(1+z_C) + \log_2(1+z_D) \tag{5a}$$

$$\text{subject to } z_C \le \frac{|\boldsymbol{g}^H_{B_i}\boldsymbol{p}_{B_{ij}}|^2}{N_0 + |\boldsymbol{h}^H_{D_{ji}}\boldsymbol{p}_{D_{ji}}|^2} \tag{5b}$$

$$z_D \le \frac{|\boldsymbol{g}^H_{D_j}\boldsymbol{p}_{D_{ji}}|^2}{N_0 + |\boldsymbol{h}^H_{B_j}\boldsymbol{p}_{B_{ij}}|^2}, \qquad (1c), (1d), (1e) \text{ and } (1f) \tag{5c}$$

Next, substituting the random interference channel vector $\boldsymbol{h}_{D_{ji}} = \tilde{\boldsymbol{h}}_{D_{ji}} + \boldsymbol{e}_{ji}$ and letting $\mathbf{P}_{B_{ij}} := \mathbf{p}_{B_{ij}}\mathbf{p}^H_{B_{ij}}$, $\boldsymbol{P}_{D_{ji}} := \boldsymbol{p}_{D_{ji}}\boldsymbol{p}^H_{D_{ji}}$, respectively, the stochastic inequality (5b), i.e., $z_C \le \frac{|\boldsymbol{g}^H_{B_i}\boldsymbol{p}_{B_{ij}}|^2}{N_0+|\boldsymbol{h}^H_{D_{ji}}\boldsymbol{p}_{D_{ji}}|^2}$ can be re-expressed as:

$$- \boldsymbol{e}_{ji}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{e}_{ji} - \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{e}_{ji} - \boldsymbol{e}_{ji}^H \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} - \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} - N_0 + \frac{1}{z_C} \boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} \geq 0 \tag{6}$$

Note that (6) and $e_{ji}^H \boldsymbol{Q}_{ji} e_{ji} \leq 1$ (in constraint (1f)) are quadratic inequalities for the random error vector e_{ji}. Thus, we exploit the *S-Lemma* [16], to express the stochastic constraints in the from of a linear matrix inequality.

Lemma 1 (S-Lemma). *Let* $\phi_i(\boldsymbol{e}) \triangleq \boldsymbol{e}^H \boldsymbol{A}_i \boldsymbol{e} + \boldsymbol{b}_i^H \boldsymbol{e} + \boldsymbol{e}^H \boldsymbol{b}_i + c_i \;\forall\; i = 0, 1$, *where* $\boldsymbol{A}_i \in \mathbb{H}^{N_{K_D}}$, $\boldsymbol{b}_i \in \mathbb{C}^{N_{K_D}}$ *and* $c_i \in \mathbb{R}$. *Suppose there exists an* $\hat{\boldsymbol{e}} \in \mathbb{C}^{N_{K_D}}$ *such that* $\phi_i(\hat{\boldsymbol{e}}) < 0$, *then the following two conditions are equivalent:*

1) $\phi_0(\boldsymbol{e}) \geq 0$ *for all* $\boldsymbol{e}$ *satisfying* $\phi_1(\boldsymbol{e}) \leq 0$*;*
2) *There exists a* $\zeta \geq 0$ *such that,*
$\begin{bmatrix} \boldsymbol{A}_0 & \boldsymbol{b}_0 \\ \boldsymbol{b}_0^H & c_0 \end{bmatrix} + \zeta \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{b}_1 \\ \boldsymbol{b}_1^H & c_1 \end{bmatrix} \succeq 0$

Relating $\phi_0(\boldsymbol{e})$ to (6) and $\phi_1(\boldsymbol{e})$ to $e_{ji}^H \boldsymbol{Q}_{ji} e_{ji} - 1 \leq 0$, and applying the S-Lemma, the stochastic constraint can be expressed as:

$$\boldsymbol{\phi}(\zeta_{ji}) \triangleq \begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{z_C} \end{bmatrix} \succeq \boldsymbol{0} \tag{7}$$

where, $f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) := \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} + N_0 + \zeta_{ji}$. Similarly, Performing a SDR and applying the S-Lemma to constraints (1d) of (4), the relaxed semi-definite problem without stochastic constraints can be expressed as:

$$\underset{\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \; \underset{\zeta_{ji}}{\text{minimize}} \quad \log_2(1 + z_C) + \log_2(1 + z_D) \tag{8a}$$

$$\text{subject to} \quad z_D \leq \frac{\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \tag{8b}$$

$$\begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{z_C} \end{bmatrix} \succeq \boldsymbol{0} \tag{8c}$$

$$0 \leq \text{Tr}(\boldsymbol{P}_{B_{ij}}) \leq p_{B,\max}, \qquad 0 \leq \text{Tr}(\boldsymbol{P}_{D_{i,j}}) \leq p_{D,\max} \tag{8d}$$

$$\begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{\eta_{C,\min}} \end{bmatrix} \succeq \boldsymbol{0} \tag{8e}$$

$$\frac{\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \geq \eta_{D,\min}, \quad \zeta_{ji} \geq 0 \quad \boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}} \succeq \boldsymbol{0} \tag{8f}$$

Next, applying the Schur complement on the semi-definite constraint (8c), this constraint in the form of a linear matrix inequality and a general inequality as:

$$- \boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} \succeq \boldsymbol{0} \tag{9a}$$

$$\left(-f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{1}{z_C} \boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} - \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(-\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right) \geq 0 \tag{9b}$$

Thus, the optimal value of ζ_{ji} in (9a) can be computed as:

$$\zeta_{ji}^* = \underset{\zeta}{\text{minimize}} \;\; \zeta \quad \text{subject to} \;\; \zeta \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}} \succ \boldsymbol{0}, \;\; \zeta \geq 0 \tag{10}$$

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Further, rearranging the terms of (9b) and substituting this constraint for a given optimal value of ζ_{ji}^{*}, the relaxed semi definite problem (9) can be restated as:

$$\underset{\boldsymbol{P}_{B_{ij}},\boldsymbol{P}_{D_{ji}},\boldsymbol{z}}{\text{maximize}} \quad \log_2(1+z_C)+\log_2(1+z_D) \tag{11a}$$

$$\text{subject to} \quad z_D \leq \frac{\boldsymbol{g}_{D_j}^{H}\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j}}{N_0+\boldsymbol{h}_{B_j}^{H}\boldsymbol{P}_{B_{ij}}\boldsymbol{h}_{B_j}} \tag{11b}$$

$$z_C \leq \boldsymbol{g}_{B_i}^{H}\boldsymbol{P}_{B_{ij}}\boldsymbol{g}_{B_i} / \Big[f(\boldsymbol{P}_{D_{ji}},\zeta_{ji}^{*}) + \tilde{\boldsymbol{h}}_{D_{ji}}^{H}\boldsymbol{P}_{D_{ji}}\left(\lambda_{ji}^{*}\boldsymbol{Q}_{ji}-\boldsymbol{P}_{D_{ji}}\right)^{-1}\boldsymbol{P}_{D_{ji}}\tilde{\boldsymbol{h}}_{D_{ji}}\Big] \tag{11c}$$

$$0 \leq \mathrm{Tr}(\boldsymbol{P}_{B_{ij}}) \leq p_{B,\max}, \qquad 0 \leq \mathrm{Tr}(\boldsymbol{P}_{D_{i,j}}) \leq p_{D,\max} \tag{11d}$$

$$\begin{bmatrix} -\boldsymbol{P}_{D_{ji}}+\zeta_{ji}^{*}\boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}}\tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^{H}\boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}},\zeta_{ji}^{*})+\frac{\boldsymbol{g}_{B_i}^{H}\boldsymbol{P}_{B_{ij}}\boldsymbol{g}_{B_i}}{\eta_{C,\min}} \end{bmatrix} \succeq \boldsymbol{0} \tag{11e}$$

$$\frac{\boldsymbol{g}_{D_j}^{H}\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j}}{N_0+\boldsymbol{h}_{B_j}^{H}\boldsymbol{P}_{B_{ij}}\boldsymbol{h}_{B_j}} \geq \eta_{D,\min}, \quad \boldsymbol{P}_{B_{ij}},\boldsymbol{P}_{D_{ji}} \succeq \boldsymbol{0} \tag{11f}$$

Notice that constraints (11b) involve a ratio between two convex functions and (11c) involve a ratio of a convex and a non-convex function. Hence in the next subsection we use fractional programming [17] to relax the non convexity due to these ratios.

B. Fractional Programming by Quadratic Transformation

It is easy to note that the optimal values of the slack variables z_C^{*} and z_D^{*} are:

$$\begin{aligned} z_C^{*} =& \frac{\boldsymbol{g}_{B_i}^{H}\boldsymbol{P}_{B_{ij}}\boldsymbol{g}_{B_i}}{f(\boldsymbol{P}_{D_{ji}},\zeta_{ji}^{*})+\tilde{\boldsymbol{h}}_{D_{ji}}^{H}\boldsymbol{P}_{D_{ji}}\left(\zeta_{ji}^{*}\boldsymbol{Q}_{ji}-\boldsymbol{P}_{D_{ji}}\right)^{-1}\boldsymbol{P}_{D_{ji}}\tilde{\boldsymbol{h}}_{D_{ji}}} \\ z_D^{*} =& \frac{\boldsymbol{g}_{D_j}^{H}\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j}}{N_0+\boldsymbol{h}_{B_j}^{H}\boldsymbol{P}_{B_{ij}}\boldsymbol{h}_{B_j}} \end{aligned} \tag{12}$$

Taking a partial Lagrangian of (11) by considering only the constraints related to the slack variables z_C and z_D in (11b) and (11c) respectively, we obtain:

$$\begin{aligned} L(\boldsymbol{P},\boldsymbol{z},\boldsymbol{\lambda}) =& \log_2(1+z_C)+\log_2(1+z_D) \\ & -\lambda_C\left(z_C - \frac{\boldsymbol{g}_{B_i}^{H}\boldsymbol{P}_{B_{ij}}\boldsymbol{g}_{B_i}}{\big(f(\boldsymbol{P}_{D_{ji}},\zeta_{ji}^{*}) + \tilde{\boldsymbol{h}}_{D_{ji}}^{H}\boldsymbol{P}_{D_{ji}}\left(\zeta_{ji}^{*}\boldsymbol{Q}_{ji}-\boldsymbol{P}_{D_{ji}}\right)^{-1}\boldsymbol{P}_{D_{ji}}\tilde{\boldsymbol{h}}_{D_{ji}}\big)} \right) \\ & -\lambda_D\left(z_D - \frac{\boldsymbol{g}_{D_j}^{H}\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j}}{N_0+\boldsymbol{h}_{B_j}^{H}\boldsymbol{P}_{B_{ij}}\boldsymbol{h}_{B_j}} \right) \end{aligned} \tag{13}$$

At the stationary point, $\frac{\partial L}{\partial \boldsymbol{z}} = 0$, and since the optimal value of $\boldsymbol{z}^{*}$ is known, the optimal values of Lagrange variables are related to the optimal values of the slack variables and can be computed as follows:

$$\lambda_C^* = \frac{1}{1+z_C^*} = \frac{f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}}{\left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right)}$$

$$\lambda_D^* = \frac{1}{1+z_D^*} = \frac{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}}{N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \tag{14}$$

Substituting only the optimal values of the Lagrange variables λ_C^* and λ_D^* in (11), we obtain:

$$\begin{aligned} \underset{\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \quad & \log_2(1+z_C) + \log_2(1+z_D) - z_C - z_D \\ & + \frac{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{\left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right)} \\ & + \frac{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \\ \text{subject to} \quad & \text{(11d), (11e) and (11f)} \end{aligned} \tag{15}$$

Next, we transform the fractions in the objective by introducing two auxiliary variables y_C and y_D through a quadratic transformation [17], obtaining:

$$\begin{aligned} \underset{\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}, \boldsymbol{z}, \boldsymbol{y}}{\text{maximize}} \quad & \log_2(1+z_C) + \log_2(1+z_D) - z_C - z_D \\ & + 2y_C \sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}} \\ & - y_C^2 \left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right) \\ & + 2y_D \sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}} \\ & - y_D^2 \left(N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}\right) \\ \text{subject to} \quad & \text{(11d), (11e) and (11f)} \end{aligned} \tag{16}$$

The optimal values of the auxiliary variables y_C and y_D can be readily computed as:

$$y_C^* = \frac{\sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}}{\left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji} \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right)}$$

$$y_D^* = \frac{\sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}}{N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \tag{17}$$

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Finally, introducing a slack variable $s_{ji} \geq f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \bar{\boldsymbol{h}}_{D_{ji}}$, to facilitate alternating optimization between the power variables, and rearranging once again by taking the Schur compliment, the optimization problem (16), can be restated as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}, \boldsymbol{z}, \boldsymbol{y}}{\text{maximize}} \quad & \log_2(1+z_C) + \log_2(1+z_D) - z_C - z_D \\
& + 2y_C\sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}} - y_C^2 \left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + s_{ji}\right) \\
& + 2y_D\sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}} \\
& - y_D^2 \left(N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}\right) \\
\text{subject to} \quad & \begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji}^* \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + s_{ji} \end{bmatrix} \succeq \boldsymbol{0} \\
& \text{(11d), (11e) and (11f)}
\end{aligned} \tag{18}$$

Notice that for given values of slack variables z_C and z_D and auxiliary variables y_C and y_D, the optimization problem (18) is still jointly non-convex in $\boldsymbol{P}_{B_{ij}}$ and $\boldsymbol{P}_{D_{ji}}$. Hence, we propose to perform alternating optimization in (18) between $\boldsymbol{P}_{B_{ij}}$ and $\boldsymbol{P}_{D_{ji}}$.

The SDR [18] sub-problem of (18) to optimize $\boldsymbol{P}_{B_{ij}}$ for given updated values of ζ_{ji}, z_C, z_D, y_C, y_D and $\boldsymbol{P}_{D_{ji}}$, can be stated as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_{B_{ij}}}{\text{maximize}} \quad & 2y_C\sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}} \\
& - y_C^2 \left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}\right) - y_D^2 \left(\boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}\right) \\
\text{subject to} \quad & 0 \leq \operatorname{Tr}\left(\boldsymbol{P}_{B_{ij}}\right) \leq p_{B,\max}, \ \boldsymbol{P}_{B_{ij}} \succeq \boldsymbol{0} \\
& \boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} \geq \\
& \eta_{C,\min}\left(f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji} \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right) \\
& \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j} \leq \frac{1}{\eta_{D,\min}}\left(\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}\right) - N_0
\end{aligned} \tag{19}$$

Similarly, the SDR sub-problem of (18) to optimize $\boldsymbol{P}_{D_{ji}}$ for given values of ζ_{ji}, z_C, z_D, y_C, y_D and $\boldsymbol{P}_{B_{ij}}$, can be stated as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_{D_{ji}}, s_{ji}}{\text{maximize}} \quad & 2y_D\sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}} \\
& - y_D^2 \left(\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}\right) - y_C^2 s_{ji} \\
\text{subject to} \quad & 0 \leq \operatorname{Tr}(\boldsymbol{P}_{D_{i,j}}) \leq p_{D,max}, \ \boldsymbol{P}_{D_{ji}} \succeq \boldsymbol{0} \\
& \begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + s_{ji} \end{bmatrix} \succeq \boldsymbol{0} \\
& \begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{\eta_{C,min}} \end{bmatrix} \succeq \boldsymbol{0} \\
& \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} \geq \eta_{D,min}\left(N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}}^{(k+1)} \boldsymbol{h}_{B_j}\right)
\end{aligned} \tag{20}$$

Note that the obtained optimal solution for the relaxed problems (19) and (20) may not be rank one; thus, additional rank one approximation procedures such as (i) eigen vector corresponding to maximum eigen value, or (ii) randomization [18]; are needed to obtain the power beamforming vectors $\boldsymbol{p}_{B_{ij}}$ and $\boldsymbol{p}_{D_{ji}}$ from the respective $\boldsymbol{P}_{B_{ij}}^*$ and $\boldsymbol{P}_{D_{ji}}^*$ matrices.

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To sum up, the power optimization subproblem (4) is solved by iteratively updating the parameter ζ_{ji} in the S-Lemma (through (10)); the slack variables z_C and z_D (through (12)) ; the auxiliary variables y_C and y_D (through (17)); and the power vectors $\boldsymbol{p}_{B_{ij}}$ and $\boldsymbol{p}_{D_{ji}}$ through (19) and (20). Once (4) is solved $\forall i \in \mathcal{C}$ and $\forall j \in \mathcal{D}$, the next step is to perform channel assignment to D2D pairs, as explained in next subsection.

C. Channel Assignment via Integer Relaxation

For the channel assignment to D2D pairs, the resulting values $\tilde{v}_{i,j}$ (obtained after solving (4) $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$) are substituted into (3) and then we need to maximize the objective of (3) with respect to $\mathbb{B}$. The resulting channel assignment sub-problem can be stated as:

$$\begin{aligned} \underset{\mathbb{B}}{\text{maximize}} \quad & \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \beta_{i,j} v^*_{i,j} - \gamma \delta(\mathbb{B}), \\ \text{subject to} \quad & \beta_{i,j} \in \{0,1\} \;\; \forall i,j, \quad \sum_{j \in \mathcal{D}} \beta_{i,j} \leq 1 \;\; \forall i. \end{aligned} \tag{21}$$

Due to the integer constraints, solving (21) involves prohibitive computational complexity even for reasonable values of N_C, N_D. Thus, we relax the integer constraints to $\beta_{i,j} \in [0,1]$ $\forall i,j$ to obtain a differentiable Lipschitz smooth objective function with linear constraints which can be efficiently solved using the Projected Gradient Descent algorithm. The obtained solution is finally discretized back to satisfy the original constraints $\beta_{i,j} \in \{0,1\}$ $\forall i,j$. In our approach, this is done by setting the highest positive value in every row of $\boldsymbol{B}$ to 1 while setting other values in the same row to 0. Other solutions were investigated in [15], our selected approach is the one with the lowest computational complexity, nevertheless, it achieves very close results to the most computationally complex one.

V. SIMULATIONS

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. We assume $\tilde{h}_C$ to be exponentially distributed with the mean value obtained from the mentioned path-loss model. Averages over 1,000 independent realizations of the user locations with parameters BW= 15 kHz, $\gamma = 200 \times$ BW, $N_D = 10$, $N_C = 10$, $N_0 = -70$ dBW (γ is scaled with BW to ensure that the unfairness and the achieved rate are of comparable values). The proposed algorithm is tested for the cases where $K_B = K_D = 2$ (2×1 MIMO) and where $K_B = K_D = 4$ (4×1 MIMO). In both cases, we assume that $\boldsymbol{Q} = \epsilon^{-2}\boldsymbol{I}$, which indicates that the error in the channel gains lies in a circle of ϵ radius ($\|e\| \leq \epsilon$). These cases are further compared with the method by Elnourani et al. [19], which to the best of our knowledge is the best existing method is the SISO case, with exponential channel gains and an allowed outage probability of 0.1.

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In Fig. 2, both cases are tested with $\epsilon = 10^{-4}$. It shows that the proposed method achieves higher rates than the SISO method in both cases. When γ increases, the rate decreases in all methods, as expected. The 4×1 MIMO case achieves the highest rates, followed by the 2×1 MIMO case. Moreover, the differences in rates between all methods are almost constant.

Fig. 3 shows that the proposed methods achieve very small *unfairness* and that is very close to the SISO case. When γ increases, the unfairness decreases in all methods, as expected. Fig. 4 shows that the proposed methods achieve high average rates in both cases for different values of ϵ. The 4×1 MIMO case achieves the highest rate, followed by the case of 2×1 MIMO. The SISO case achieves the lowest rate, as expected. The rates for both MIMO cases decreases when ϵ increases, which indicates that having a larger error will cause our solution

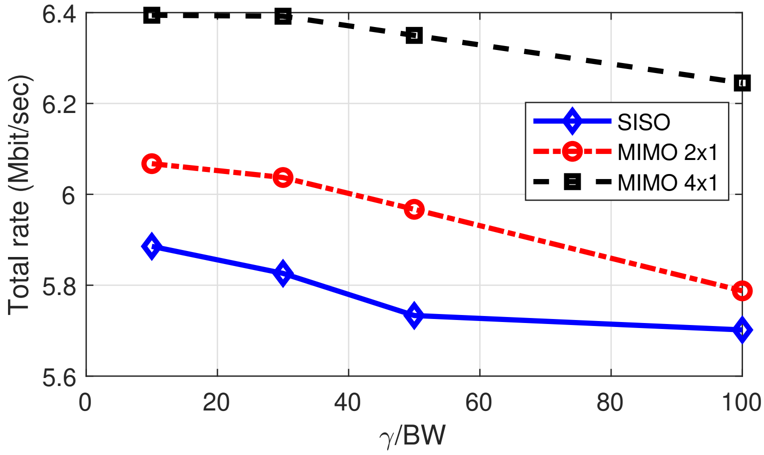

Fig. 2: Total average rate R vs. γ.

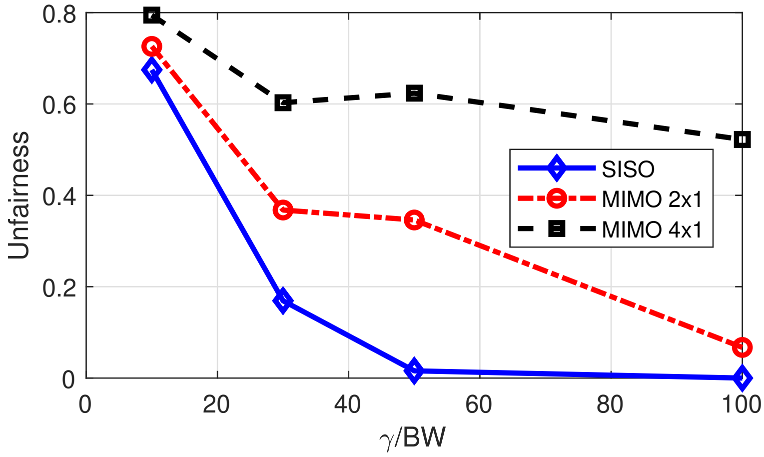

Fig. 3: Unfairness δ vs. γ .

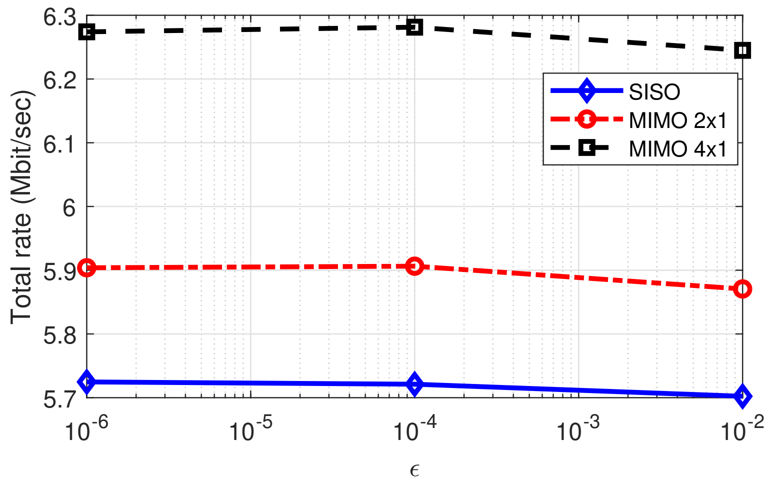

Fig. 4: Total average rate R vs. ϵ.

to be more conservative, resulting in lower rates. The rate in the SISO case is almost constant, since this method does not depend on ϵ, and the resulting achieved rates are considerably lower than the proposed methods. The fairness when changing ϵ is observed to be almost constant for all methods (in the selected range).

In general, the proposed method converges to a stationary solution. This solution is always better than the optimal one achieved by the SISO method, in both cases of 2×1 MIMO and

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4×1 MIMO, for all the tested values of ϵ and γ. It is observed that increasing the number of antennas leads always to higher rates as expected, due to the additional degrees of freedom available.

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