Non-convex Optimization for Resource Allocation in Wireless Device-to-Device Communications

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UiA University of Agder

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Preface

This dissertation presents results of the research I have carried out in my Ph.D. Thesis at the WISENET Center, Department of Information and Communication Technology, Faculty of Engineering and Science, University of Agder, Norway.
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This work has been conducted under the supervision of Professor Baltasar Beferull-Lozano and Associate Professor Daniel Romero.

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I would also like to extend my gratitude to my parents. Thank you for your endless support and patience. I would also like to thank my family for their support. I am also very grateful to all the colleagues that I had before in my previous jobs and schools, specially University of Khartoum, Sudan, you are all part of who I am today. Thank you all.

Mohamed Elnourani
Grimstad, Norway
June 2020

Abstract

Device-to-device (D2D) communication is considered one of the key frameworks to provide suitable solutions for the exponentially increasing data traffic in mobile telecommunications. In this PhD Thesis, we focus on the resource allocation for underlay D2D communications which often results in a non-convex optimization problem that is computationally demanding.

We have also reviewed many of the works on D2D underlay communications and identified some of the limitations that were not handled previously, which has motivated our works in this Thesis.

Our first works focus on the joint power allocation and channel assignment problem in the D2D underlay communication scenario for a unicast single-input and single-output (SISO) cellular network in either uplink or downlink spectrums. These works also consider several degrees of uncertainty in the channel state information (CSI), and propose suitable measures to guarantee the quality of service (QoS) and reliability under those conditions. Moreover, we also present a few algorithms that can be used to jointly assign uplink and downlink spectrum to D2D pairs. We also provide methods to decentralize those algorithms with convergence guarantees and analyze their computational complexity. We also consider both cases with no interference among D2D pairs and cases with interference among D2D pairs. Additionally, we propose the formulation of an optimization objective function that combines the network rate with a penalty function that penalizes unfair channel allocations where most of the channels are assigned to only a few D2D pairs.

The next contributions of this Thesis focus on extending the previous works to cellular networks with multiple-input and multiple-output (MIMO) capabilities and networks with D2D multicast groups. We also present several methods to accommodate various degrees of uncertainty in the CSI and also guarantee different measures of QoS and reliability.

All our algorithms are evaluated extensively through extensive numerical experiments using the Matlab simulation environment. All of these results show favorable performance, as compared to the existing state-of-the-art alternatives.

Publications

The papers listed below are an outcome of the research work carried out by the author of this dissertation, including five conference papers and three journal papers.

Paper A: Mohamed Elnourani, Mohamed Hamid, Daniel Romero, Baltasar Beferull-Lozano, "Underlay Device-to-Device Communications on Multiple Channels", IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), 2018, Calgary, Canada.

Paper B: Mohamed Elnourani, Baltasar Beferull-Lozano, Daniel Romero, Siddharth Deshmukh, "Reliable Underlay Device-to-Device Communications on Multiple Channels", IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), 2019, Cannes, France.

Paper C: Mohamed Elnourani, Siddharth Deshmukh, Baltasar Beferull-Lozano, Daniel Romero, "Robust Underlay Device-to-Device Communications on Multiple Channels", Submitted to IEEE Transactions on Wireless Communications.

Paper D: Mohamed Elnourani, Siddharth Deshmukh, Baltasar Beferull-Lozano, "Resource Allocation for Multiple Underlay Interfering Device-to-Device Communications", Submitted to IEEE Transactions on Communications, 2020.

Paper E: Mohamed Elnourani, Siddharth Deshmukh, Baltasar Beferull-Lozano, Daniel Romero, "Robust Transmit Beamforming for Underlay D2D Communications on Multiple Channels", IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), 2020.

Paper F: Mohamed Elnourani, Siddharth Deshmukh, Baltasar Beferull-Lozano, "Reliable Underlay D2D Communications over Multiple Transmit Antenna Framework", IEEE International Conference on Communications (ICC), 2020.

Paper G: Mohamed Elnourani, Siddharth Deshmukh, Baltasar Beferull-Lozano, "Reliable Multicast D2D Communication Over Multiple Channels in Underlay Cellular Networks", IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), 2020.

Paper H: Mohamed Elnourani, Siddharth Deshmukh, Baltasar Beferull-Lozano, "Distributed Resource Allocation in Underlay Multicast D2D Communications", submitted to IEEE Transactions on Communications, 2020.

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Abbreviations

BS	base station.
CSI	channel state information.
CU	cellular user.
CUs	cellular users.
D2D	device-to-device.
MD2D	multicast device-to-device.
MIMO	multiple-input and multiple-output.
PGD	projected gradient descent.
QoS	quality of service.
SDR	semi-definite relaxation.
SINR	signal to interference plus noise ratio.
SISO	single-input and single-output.

Part I

Chapter 1

Introduction

1.1 D2D Communications

Mobile data traffic has been exponentially increasing in the past few years [1], as shown in Fig. 1.1. This increasing demand in cellular communication networks can no longer be met by only increasing the *spectral* efficiency of point-to-point links through classical techniques, e.g. through improvements in modulation and coding, since existing systems are already approaching the channel capacity [2, 3].

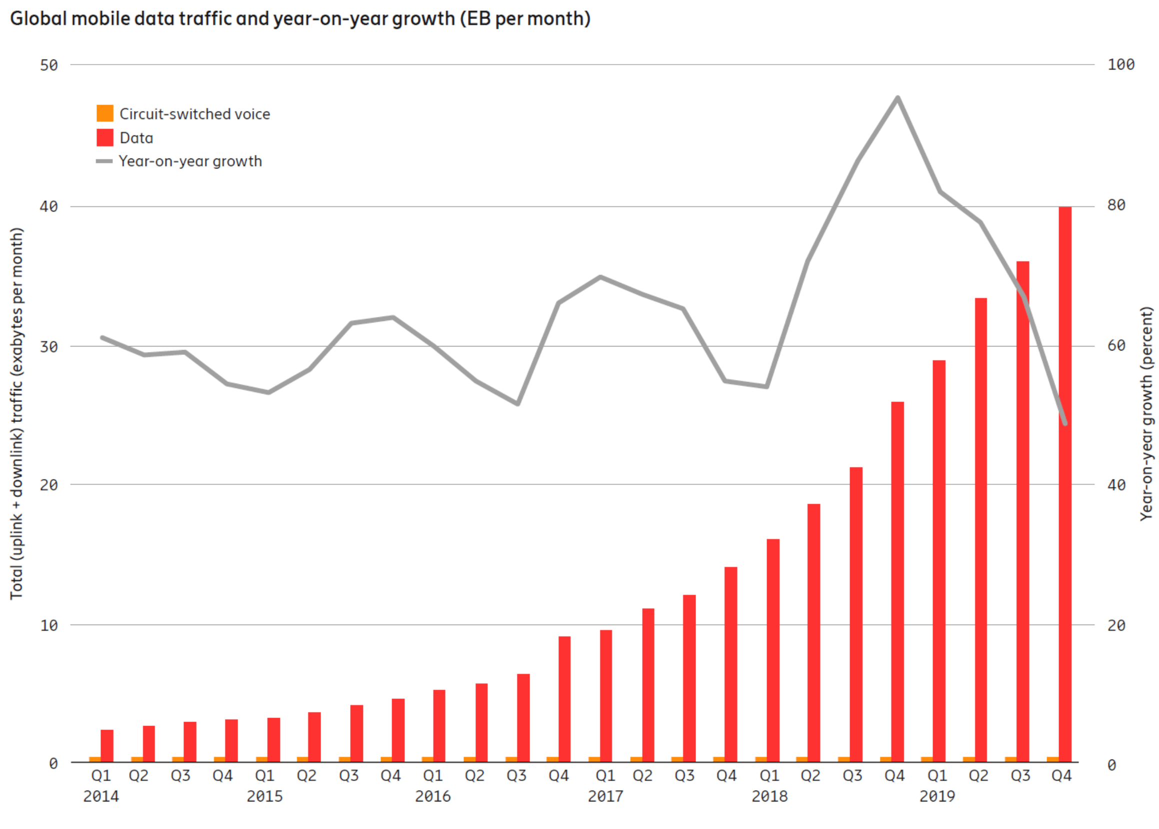

Figure 1.1: Global Monthly Data Consumption from Ericsson Mobility Report [1]

D2D communications constitute a prominent example in improving *spatial* efficiency, where mobile users are allowed to communicate directly with each other without passing their messages through the base station (BS) [4, 5]. Thus, users operating in D2D mode require half of the time resources of those operating in the traditional cellular mode. Moreover, the power consumed for D2D communications is significantly smaller, since D2D users are close to each other in general. D2D communications have been classified into two main types [4]:

1. Overlay: where D2D users use different channels (e.g. frequency bands or time slots) from those used by regular cellular users (CUs). In this case, D2D communications will not interfere with the traditional cellular communications. However, the number of free channels that are assigned to D2D is generally small.

2. Underlay: where channels (e.g. frequency bands or time slots) used by D2D users can be simultaneously used by traditional CUs. However, interference between D2D users and CUs is a major concern in this mode, which require a sophisticated interference management mechanism.

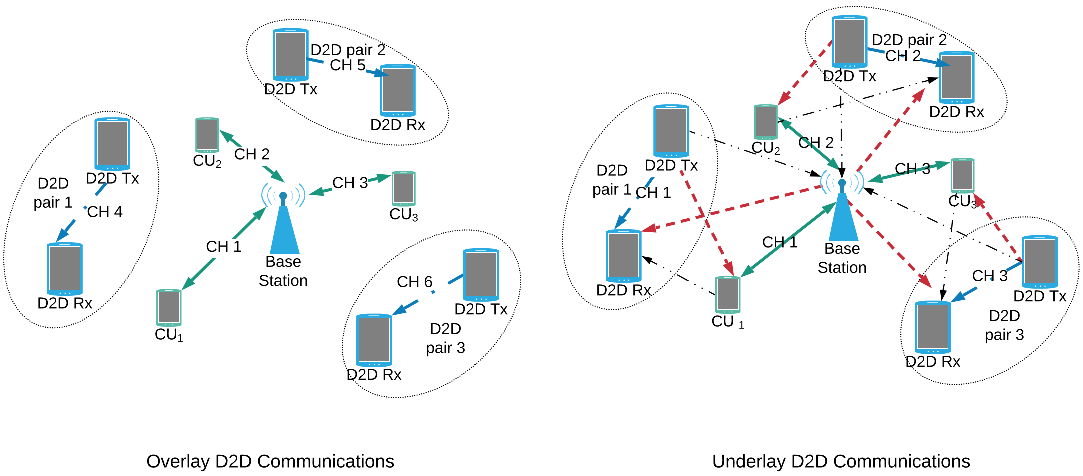

Figure 1.2: Main types of D2D communications

1.2 Motivation

Underlay D2D communication is considered one of the main enablers for dense self-organizing networks. Since the number of devices is much larger than the number of channels, both regular communications and overlay D2D communications can not be efficiently used in those scenarios. However, underlay D2D communications cause interference among all D2D and CU devices that use the same channel.

Thus, it is necessary to devise algorithms that judiciously assign cellular channels to D2D users and prudently control the transmitted power to limit interference to cellular users and guarantee QoS (e.g. SINR, reliability) to all users. In addition, algorithms must be computationally inexpensive and reliable even in the case of imperfect Channel Station Information (CSI) cases.

Moreover, the resource allocation mechanism varies greatly depending on the system assumptions (e.g. perfect CSI or imperfect CSI), communication modes (e.g. unicast or multicast), and devices capabilities (e.g. SISO or MIMO).

This motivates the research investigations presented in this Thesis work, where the goal is to create novel and computationally efficient resource allocation algorithms and strategies that provide near-optimal solutions under all the aforementioned various network assumptions and capabilities.

1.3 Research Questions

The main research questions addressed in this Thesis work are summarized as follows:

Question 1: How to jointly optimize the channel assignment and the power allocation in underlay D2D communications while ensuring the QoS of both CUs and D2D users?
This question is answered in Paper A, PaperB and Paper C.

Question 2: How to jointly optimize the channel assignment and the power allocation in underlay D2D communications in the case of interfering D2D users?
This question is answered in Paper D.

Question 3: How to perform MIMO beamforming while jointly optimizing the channel assignment in underlay D2D communications and ensuring reliability of both CUs and D2D users?
This question is answered in Paper E and Paper F.

Question 4: How to jointly optimize the channel assignment and the power allocation in underlay D2D communications in the case of multicast D2D communications?
This question is answered in Paper G and Paper H.

1.4 Thesis Layout

This dissertation is a compilation Thesis divided into two parts. Part I introduces and summarizes the research carried out throughout the Ph.D. and presents the main contributions. Part II is the collection of eight research papers, numbered A-H, representing the main contribution of this thesis. The remaining Chapters in Part I are as follows:

- Chapter 2 presents a summary of the existing literature and identifies the gaps that this thesis addresses.
- Chapter 3 presents the work in resource allocation for *non-interfering* underlay D2D communications for the SISO/unicast cases developed in Papers A,B,C.
- Chapter 4 presents the work in resource allocation for *interfering* underlay D2D communications for the SISO/unicast case developed in Paper D.

- Chapter 5 presents the work in *MIMO* Beamforming in Underlay D2D Communications developed in Papers E,F.
- Chapter 6 presents the work in *Multicast* resource allocation for Underlay D2D Communications developed in Papers G,H.
- Chapter 7 concludes the dissertation, presents a summary of the work, and points to future research directions.

Chapter 2

Background and Literature Review

2.1 Classical D2D communications

D2D communications allow cellular devices to communicate directly with each other without passing their messages through the BS [4, 6, 5, 7]. This paradigm entails higher throughput and lower latency in the communication for two reasons: first, a traditional cellular communication between two devices requires one time slot in the uplink and one time slot in the downlink, whereas a single time slot suffices in D2D communications. Second, the time slot used by a traditional CU can be simultaneously used by a D2D pair in a sufficiently distant part of the cell. The work in [4] is one of the first works that have formally defined D2D communication. It focuses on exploiting the communication opportunities in the uplink spectrum where D2D communication expects a lower interference from the cellular users (CUs).

Early works on D2D communications rely on simplistic channel assignment schemes, where each D2D pair communicates through a randomly selected cellular sub-channel (hereafter referred to as *channel* for simplicity). This is the case in [8], where the effects of selecting a channel with poor quality are addressed by choosing the best among the following operating modes: underlay mode; overlay mode; and cellular mode (the D2D pair operates as a regular cellular user). In another approach, authors in [9] define two policies for the BS: (i) D2D-unaware spectrum access (DUSA), where BS assigns all resources to the CUs without acknowledging D2D communication; and (ii) D2D-aware spectrum access (DASA), where BS reserves one channels for D2D communications and avoids assigning the reserved channel to CUs until there is availability of free channels. Here authors have also defined two policies for D2D pairs: (i) Cognitive Spectrum Access (CSA), where D2D pairs preform spectrum sensing on the selected sub-channel and opportunistically transmit on the sensed channel; and (ii) no CSA, where D2D pairs immediately attempt to transmit on the selected channel without any consideration of the channel occupancy. These research works suffer from two limitations: (i) random allocation of channels result in a sub-optimal throughput which could be improved by leveraging different degrees of channel-state information; (ii) they do not provide any mechanism to adjust the transmit power of D2D terminals, which generally results in a reduced throughput due to increase in interference.

Few works also consider performing channel assignment to the D2D pairs for underlay communication. In [10, 11], instead of randomly assigning channels, channels are assigned to the D2D pairs using auction games while measuring the fairness in the number of channels assigned to each D2D pair. Similarly, [12] proposes channel assignment to D2D pairs utilizing a coalition-forming game model. Here, millimeter-wave spectrum is also considered as an overlay option for D2D pairs when the CU channels are fully utilized. However, it can be noted that these schemes only perform channel assignment and avoid controlling the transmit power, which limits the achievable throughput of the overall network.

In order to circumvent above limitations, a Stackelberg game based approach is proposed in [13] where each D2D pair simultaneously transmits in all cellular channels and compete non-cooperatively to adjust the transmit power. Here, the BS penalizes the D2D pairs if they generate harmful interference to the cellular communication. The optimization of the transmit power while ensuring minimum signal to interference plus noise ratio (SINR) requirements is also investigated in [14], however, as long as the SINR requirements are satisfied, D2D pairs are allowed to transmit in all channels. In an alternative approach, distributed optimization for power allocation is investigated in [15] for both overlay and underlay scenarios. However, D2D pairs are allowed to transmit in all channels, and power allocation for CUs is also not considered. To summarize, all the works mentioned so far perform either channel assignment or power allocation, but not both.

Other research works consider jointly optimizing channel assignment and power allocation, as they seem to show strong dependency. This joint optimization is considered in [16, 17, 18, 19]. The authors in [16] assumed a model where a single D2D can not be assigned more than one sub-channel, and a sub-channel can not be assigned to more than one D2D pair. They have proposed a solution for both power and channel assignment in an underlay environment by decomposing the problem into power allocation problems that are solved in close-form and a channel assignment problem that is solved using Kuhn-Munkres algorithm for maximum bipartite matching. Similarly, the proposed strategies in [18], rely on the properties of fractional programming and the Dinkelbach method to jointly optimize the channel assignment and the power allocation. [17] also relies on the Kuhn-Munkres algorithm for maximum bipartite matching with some extra scenarios of overlay D2D communications. However, these schemes restrict D2D users to access at most one cellular channel. On the other hand, the work in [20, 21, 22] allow assignment of multiple channels to each D2D pair. Notice that these schemes propose to use either uplink or downlink spectrum for D2D communications. Some recent research works also consider both uplink and downlink spectrum for allocating resources to D2D pairs. In [23, 24, 25], both uplink and downlink spectra are considered in their formulation; however, they limit the assignment to at most one channel to each D2D pair.

Another important point to note is that all of the previously mentioned works assume the availability of perfect CSI. From a practical prospective, obtaining perfect CSI for D2D communications requires a lot of cooperation between all D2D pairs and CUs; thus adds a substantial amount of communication overhead. Some other recent works have also investigated problems that guarantee certain QoS parameters under the scenario of imperfect CSI for underlay D2D communications. In [26, 27, 28], power allocation and channel assignment are considered under imperfect CSI. However, the analysis, once

Works	Multiple channels	Joint UL and DL	CSI uncertainty
[16, 17, 18, 19]			
[20, 21, 22]	X		
[23, 24, 25]		X	
[26, 27, 28]			X

Table 2.1: Selected works that jointly perform channel assignment and power allocation

again, restricts D2D pairs to access at most one cellular channel. Table 2.1 list some of the presented works that jointly perform channel assignment and power allocation compared with our proposed scheme.

In conclusion, no existing work provides a joint channel assignment and power allocation scheme that satisfies all of the following requirements: (i) considers both uplink and downlink spectrum;(ii) accounts for uncertainties in CSI and thus obtains a robust resource allocation solution;and (iii) D2D pairs can simultaneously operate on more than one cellular channel, which is of special interest in areas of high CU density.

2.2 MIMO in D2D Communications

Few works have also considered transmission over multiple antennas. [29] presents a detailed analysis for joint beamforming in D2D underlay cellular networks. However, the analysis is restricted to a single D2D pair scenario under the additional assumption of perfect CSI. Scenarios with multiple D2D pairs are studied in [30], however, prefect CSI is also assumed to be available at the BS. Error in CSI due to quantization is considered in [31], where conventional maximum ratio transmission and interference cancellation techniques are exploited to compute the beamforming vectors. Design of robust beamformers for regular cellular communications has also been investigated in [32]. Under the assumption of Gaussian CSI errors, the authors in [32] propose several convex bounds to approximate the probabilistic rate outage constraints. In recent work, joint beamforming and power control strategies are studied in [33] under both perfect and erroneous CSI scenarios. In this formulation, the objective is to minimize the total transmit power of both BS and D2D pairs while ensuring QoS requirements. In conclusion, none of those previous works considers devising a robust beamforming design while performing resource allocation in underlay D2D communications, which is very relevant for maximizing aggregate network throughput.

Design of robust beamformers for general multiuser communication has also been investigated in past research works (e.g. [32, 34, 35]). Under the assumption of Gaussian CSI uncertainties, analytical methods based on Bernstein-type inequality and decomposition based large deviation inequality are proposed in [32] to approximate the probabilistic rate outage constraints. Similarly, under the assumption of Gaussian channel distribution, the probabilistic rate outage constraint is handled by semi-definite relaxation (SDR) relaxation and sequential convex approximation in [34]. Further, authors in [35] have proposed a decentralized approach to design the robust beamformers considering elliptically bounded CSI errors. In all these works, the objective is either minimization of transmit energy, or sum rate maximization; however, in underlay D2D communication jointly optimizing the power allocation and channel assignment poses additional analytical and computation challenges.

2.3 Muticast D2D Communications

Few works have also considered multicast device-to-device (MD2D). The work in [36] has exploited concepts of stochastic geometry to model and derive analytical expressions for performance metrics under the overlay communication framework. For the underlay framework, a resource allocation problem is formulated in [37] to maximize the sum throughput of multicast groups while keeping the interference to the CUs below a specified threshold. Similarly, a sum throughput maximization problem is formulated in [38] with constraints on minimum SINR requirements. This problem is then approximately solved by generalized Blender decomposition method, followed by proposing a low complexity heuristic solution. Moreover, a channel assignment scheme to maximize the sum effective throughput is proposed in [39] under partial knowledge of the device locations. It can be noted that most of the above works on MD2D communication consider perfect CSI. Furthermore, the optimizations for channel and power allocation are done separately, and in most cases multicast groups are restricted to access at most one channel. In addition, fairness in the allocated resources to the multicast groups is also ignored.

2.4 Summary

In all these scenarios, we were able to identify limitations and research gaps, as shown in previous sections. Those limitations and gaps have directed and motivated our work and publications. Each of our publications is focused on a specific scenario where the limitations and research gaps of high interest are addressed.

Chapter 3

Resource Allocation for Non-interfering Underlay D2D Communications

In this chapter, we summarize the system model, problem formulation and the main results for the work in resource allocation for non-interfering underlay D2D communications. This work was published in Papers A,B, and C.

3.1 System Model

Fig. 3.1 represents the system model that is considered in this Chapter. All communications are assumed to be single-antenna, that is, SISO communications.

Consider an underlay D2D communication scenario in which both uplink and downlink cellular channels are accessible to D2D pairs. In the following description, we describe the considered communication scenario, which is also depicted in Fig. 3.1.

Cellular network configuration: We consider a cell (or sector) of a cellular network in which the serving BS and the associated CUs communicate via $N_C^{(u)}$ uplink and $N_C^{(d)}$ downlink channels, respectively[1]. Considering the worst case underlay scenario, we assume, without loss of generality, a fully loaded cellular communication scenario in which all uplink and downlink channels are assigned to CUs. For notational convenience, the set of CUs communicating in the respective uplink and downlink channels are indexed as $\mathcal{C}^{(u)} = \left\{1, ..., N_C^{(u)}\right\}$ and $\mathcal{C}^{(d)} = \left\{1, ..., N_C^{(d)}\right\}$.

D2D communication configuration: Next, we assume that N_D D2D pairs (indexed by $j \in \mathcal{D} = \{1, ..., N_D\}$) desire to communicate over the aforementioned downlink and uplink channels in an underlay configuration, i.e., simultaneously on the same uplink and downlink channels assigned to the CUs. The assignment of uplink or downlink channels to D2D pairs is represented by the indicator variables $\{\beta_{i,j}^{(u)}\}$ and $\{\beta_{i,j}^{(d)}\}$, respectively, where j denotes D2D pair ($j \in \mathcal{D}$) and i denotes either uplink or downlink channel $\left(i \in \mathcal{C}^{(u)} \textit{ or } \mathcal{C}^{(d)}\right)$. Here $\beta_{i,j}^{(u)} = 1$ or $\beta_{i,j}^{(d)} = 1$ when the j-th D2D pair accesses the i-th uplink or downlink channel. In order to improve throughput of the D2D pairs, we further

[1]Recall that channel in this context may stand for resource blocks, time slots, and so on.

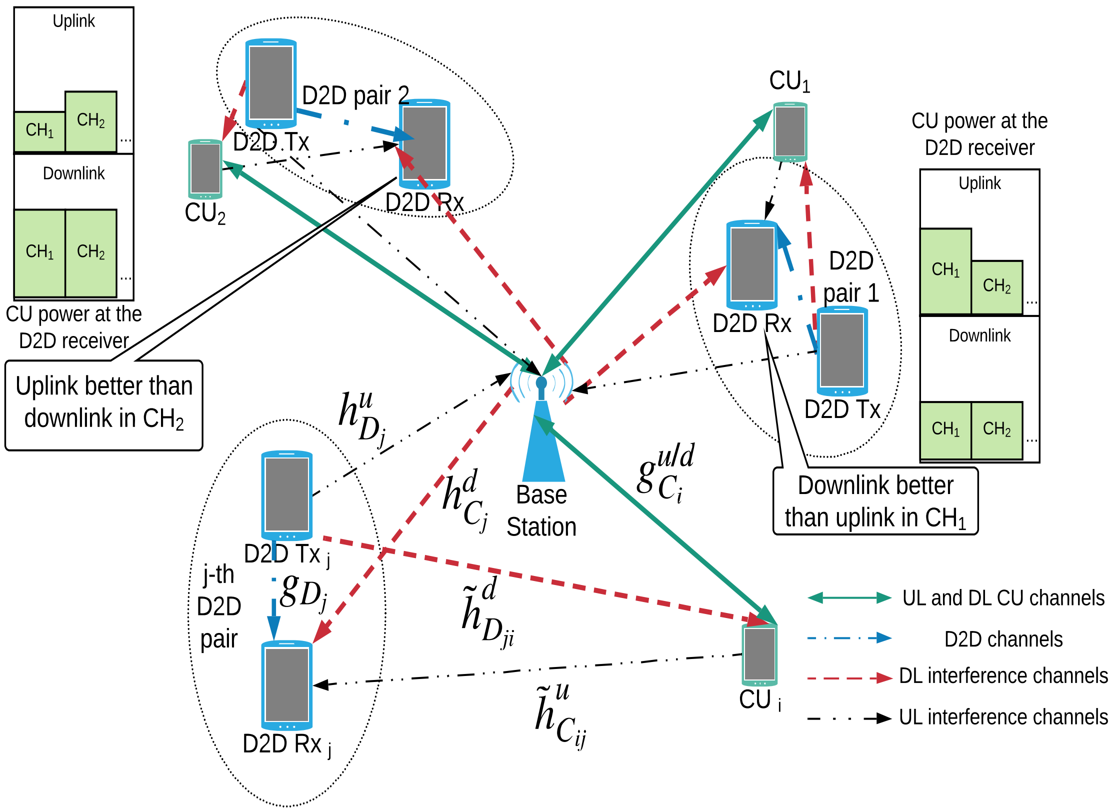

Figure 3.1: System Model (Paper C)

assume that each D2D pair can access multiple channels at the same time. However, in-order to restrict interference among D2D pairs, we assume that each channel can be used by at most one D2D pair, this can be expressed as $\sum_{j=1}^{N_D} \beta_{i,j}^{(u)} \leq 1, \;\sum_{j=1}^{N_D} \beta_{i,j}^{(d)} \leq 1 \;\forall\, i$. In addition, to reduce hardware complexity, we further assume that each D2D pair can have access to multiple channels in either downlink or uplink spectrum band [23, 24], which can be expressed as $\sum_{i=1}^{N_C^{(u)}} \beta_{i,j}^{(u)} \times \sum_{i=1}^{N_C^{(d)}} \beta_{i,j}^{(d)} = 0, \;\forall j$.

Communication channels: First, we define channel gains in the uplink access. Let $g_{C_i}^{(u)}$ denote the channel gain from the i-th CU to the BS and $h_{C_{i,j}}^{(u)}$ denote the channel gain of the interference link from the i-th CU to the j-th D2D pair receiver. Similarly, let g_{D_j} denote the channel gain between transmitter and receiver of the j-th D2D pair and $h_{D_j}^{(u)}$ denote the channel gain of the interference link from the transmitter of the j-th D2D pair to the BS. Next, for downlink access, let $g_{C_i}^{(d)}$ denote the channel gain from the BS to the i-th CU and $h_{C_j}^{(d)}$ denote the channel gain of the interference link from the BS to the j-th D2D pair. Finally, let $h_{D_{j,i}}^{(d)}$ denote the channel gain of the interference link from the transmitter of the j-th D2D pair to the i-th CU. Here we assume that the interference channel gains affecting the CUs are estimated with minimum cooperation from the CUs; thus, gains of these interference links are assumed to be modeled as random variables, denoted respectively by $\tilde{h}^{(u)}_{C_{i,j}}$ and $\tilde{h}^{(d)}_{D_{j,i}}$. Finally, additive noise observed in individual channels is assumed to have a known power N_0. Note that the noise and all channel gains are assume to be frequency flat to simplify the notations; however, the proposed scheme carries over immediately to the frequency selective scenario.

Transmit power constraints: Considering the limited power available at the mobile devices, the transmit power of the j-th D2D pair when assigned to the i-th uplink or downlink channel, denoted as $P^{(u)}_{D_{j,i}}$ and $P^{(d)}_{D_{j,i}}$ is constrained as $0 \leq P^{(u)}_{D_{j,i}} \leq P_{D_{max}}$, $0 \leq P^{(d)}_{D_{j,i}} \leq P_{D_{max}}$. Similarly, the transmit power of the CU on the i-th uplink channel and of the BS on the i-th downlink channels are constrained, respectively, as $0 \leq P^{(u)}_{C_i} \leq P^{(u)}_{C_{max}}$ and $0 \leq P^{(d)}_{C_i} \leq P^{(d)}_{C_{max}}$. Note that $P^{(u)}_{C_{max}}$, $P^{(d)}_{C_{max}}$, and $P_{D_{max}}$ are assumed to be the same for all CUs and D2D pairs to simplify the notations, however, once again the proposed scheme carries over immediately to the scenario where they are different.

This is the system model for Paper C. Paper A is a special case where we allow D2D in either uplink or downlink with the assumption of perfect CSI knowledge. Paper B is also a special case where we allow D2D in either uplink or downlink while keeping the assumption of imperfect CSI knowledge.

3.2 Problem Formulation

In this section, we describe the formulation for Paper C. Since Paper A and B are special cases of Paper C, their problem formulation directly follow for this formulation. *Achievable rates:* Here, we first present the achievable rates for D2D underlay communication on downlink channels and then extend our discussion for underlay on uplink channels. Let $R^{(d)}_{C_{i,j}}$ and $R^{(d)}_{D_{j,i}}$ denote the rate of the i-th CU and of the j-th D2D pair when sharing the downlink channel, which are respectively given as:

$$R^{(d)}_{C_{i,j}} = \log_2\left(1 + \frac{P^{(d)}_{C_i} g^{(d)}_{C_i}}{N_0 + P^{(d)}_{D_{j,i}} \tilde{h}^{(d)}_{D_{j,i}}}\right), \; R^{(d)}_{D_{j,i}} = \log_2\left(1 + \frac{P^{(d)}_{D_{j,i}} g_{D_j}}{N_0 + P^{(d)}_{C_i} h^{(d)}_{C_j}}\right).$$

When the i-th CU does not share the downlink channel, the achievable rate denoted by $R^{(d)}_{C_{i,0}}$ is given as:

$$R^{(d)}_{C_{i,0}} = \log_2\left(1 + \frac{P^{(d)}_{C_{\max}} g^{(d)}_{C_i}}{N_0}\right).$$

Thus, the gain in rate when the i-th CU shares channel with the j-th D2D pair can be stated as, $v^{(d)}_{i,j} = R^{(d)}_{C_{i,j}} + R^{(d)}_{D_{j,i}} - R^{(d)}_{C_{i,0}}$. Finally, the overall network rate in the downlink can be stated as:

$$R^{(d)}(\boldsymbol{B}^{(d)}, \mathbf{P_C}^{(d)}, \boldsymbol{P}^{(d)}_{\boldsymbol{D}}) = \sum_{i \in \mathcal{C}^{(d)}} \left[\sum_{j \in \mathcal{D}} \beta^{(d)}_{i,j} v^{(d)}_{i,j} + R^{(d)}_{C_{i,0}}\right]. \tag{3.1}$$

where $\boldsymbol{B}^{(d)} \triangleq [\beta^{(d)}_{i,j}]$, $\mathbf{P_C}^{(d)} \triangleq [P^{(d)}_{C_1}, P^{(d)}_{C_2}, \ldots, P^{(d)}_{C_{N_C}}]^T$, $\boldsymbol{P}^{(d)}_{\boldsymbol{D}} \triangleq [P^{(d)}_{Dji}]$.
Similarly, the achievable rates in the uplink channels when sharing the i-th CU uplink

channel with the j-th D2D pair can be expressed as:

$$R^{(u)}_{C_{i,j}} = \log_2\left(1 + \frac{P^{(u)}_{C_i} g^{(u)}_{C_i}}{N_0 + P^{(u)}_{D_{j,i}} h^{(u)}_{D_j}}\right), \; R^{(u)}_{D_{j,i}} = \log_2\left(1 + \frac{P^{(u)}_{D_{j,i}} g_{D_j}}{N_0 + P^{(u)}_{C_i} \tilde{h}^{(u)}_{C_{i,j}}}\right).$$

The achievable rates in the uplink channels without sharing the i-th CU uplink channel, the rate gain, as well as the total rate due to underlay uplink communications can be easily expressed by replacing the superscripts (d) by (u) in the above equations.

Quality of Service (QoS) requirements: In order to have a successful communication at a receiver node, a minimum signal to interference plus noise (SINR) ratio requirement is imposed in the problem formulation. Thus, for the i-th CU in the uplink/downlink sharing channel with the j-th D2D pair, the instantaneous SINR $\eta^{(u)}_{C_{i,j}} \geq \eta^{(u)}_{C_{\min}}$ and $\eta^{(d)}_{C_{i,j}} \geq \eta^{(d)}_{C_{\min}}$, where $\eta^{(u)}_{C_{\min}}$ and $\eta^{(d)}_{C_{\min}}$ are the minimum desired SINR for the CU in uplink and donwlink, respectively. Similarly, for the j-th D2D pair, the instantaneous SINR $\eta^{(u)}_{D_{i,j}} \geq \eta_{D_{\min}}$, where $\eta_{D_{\min}}$ is the minimum desired SINR for D2D pairs in both uplink and downlink. Note thatin order to simplify the notation $\eta^{(u)}_{C_{\min}}$, $\eta^{(d)}_{C_{\min}}$, and $\eta_{D_{\min}}$ are also assumed to be the same for all CUs and D2D pairs; however, the scheme carries over immediately to the scenario where they are different.

Since the computations of the SINR for the j-th D2D pair sharing channel with the i-th uplink CU involve the random interference channel gain $\tilde{h}^{(u)}_{Ci,j}$, the minimum SINR requirement can be expressed in terms of a probabilistic constraint as follows:

$$\Pr\{\eta^{(u)}_{D_{i,j}} \geq \eta^{(u)}_{D_{min}}\} \geq 1 - \epsilon \quad \forall i \in \mathcal{C}^{(u)}, \;\; \forall j \in \mathcal{D},$$

where ϵ is the maximum allowed outage probability. Similarly, the minimum SINR requirement for the i-th downlink CU sharing channel with the j-th D2D pair can be expressed in terms of a probabilistic constraint as follows:

$$\Pr\{\eta^{(d)}_{C_{i,j}} \geq \eta^{(d)}_{C_{min}}\} \geq 1 - \epsilon \quad \forall i \in \mathcal{C}^{(d)}, \;\; \forall j \in \mathcal{D}.$$

Fairness in channel assignment to D2D pairs: Let m_j denotes the number of channels assigned to the j-th D2D pair.

$$m_j = \sum_{i_u=1}^{N_C^{(u)}} \sum_{i_d=1}^{N_C^{(d)}} \left(\beta^{(u)}_{i_u,j} + \beta^{(d)}_{i_d,j}\right).$$

Then inspired by the fairness definition in [10], the fairness of a channel allocation can be expressed in terms of a normalized variance from a specified reference assignment m_0 as follows:

$$\delta_J = \frac{\frac{1}{N_D} \sum_{j=1}^{N_D} (m_j - m_0)^2}{m_0^2} \tag{3.2}$$

Problem statement: Given all $g^{(u)}_{C_i}$, $g^{(d)}_{C_i}$, g_{D_j}, $h^{(u)}_{D_j}$, $h^{(d)}_{C_j}$, the statistical distribution of $\tilde{h}^{(u)}_{C_{i,j}}$, and $\tilde{h}^{(d)}_{D_{j,i}}$ $\forall i, j$, as well as N_0, $\eta^C_{\min}$, $\eta^D_{\min}$, $P_{C_{\max}}$, and $P_{D_{\max}}$, the goal is to choose $\beta^{(d)}_{i,j}$, $\beta^{(u)}_{i,j}$,

$P^{(d)}_{D_{ji}}$, $P^{(u)}_{D_{ji}}$, $P^{(d)}_{C_i}$, $P^{(u)}_{C_i}$ to maximize the overall rate of the D2D pairs and CUs while ensuring fairness among the multiple D2D pairs and preventing detrimental interference to CUs.

We then form an optimization problem with a Pareto objective function that combines maximizing the total network rate with minimizing the unfairness measure. The resulting optimization problem is given by:

$$\underset{\boldsymbol{B}^{(u)},\boldsymbol{B}^{(d)},\boldsymbol{P}^{(u)},\boldsymbol{P}^{(d)}}{\text{maximize}} \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}} \left[\beta^{(u)}_{i,j} v^{(u)}_{i,j} + \beta^{(d)}_{i,j} v^{(d)}_{i,j}\right] - \gamma\delta_J(\boldsymbol{B}^{(u)},\boldsymbol{B}^{(d)}), \tag{3.3a}$$

$$\text{s.t. } \beta^{(u)}_{i,j} \in \{0,1\}, \quad \beta^{(d)}_{i,j} \in \{0,1\}, \qquad \forall\, i,j \tag{3.3b}$$

$$\sum_{j\in\mathcal{D}} \beta^{(d)}_{i,j} \le 1, \quad \sum_{j\in\mathcal{D}} \beta^{(u)}_{i,j} \le 1, \qquad \forall\, i \tag{3.3c}$$

$$\left(\sum_{i\in\mathcal{C}^{(d)}} \beta^{(d)}_{i,j}\right) \times \left(\sum_{i\in\mathcal{C}^{(u)}} \beta^{(u)}_{i,j}\right) = 0, \qquad \forall\, j \tag{3.3d}$$

$$0 \le P^{(d)}_{C_{ij}} \le P^{(d)}_{C_{max}}, \quad 0 \le P^{(d)}_{D_{ji}} \le P^{(d)}_{D_{max}}, \ \forall\, i,j, \tag{3.3e}$$

$$\Pr\left\{\frac{P^{(d)}_{C_{ij}} g^{(d)}_{C_i}}{N_0 + P^{(d)}_{D_{ji}} \tilde{h}^{(d)}_{D_{j,i}}} \ge \eta^{(d)}_{C_{min}}\right\} \ge 1-\epsilon, \ \frac{P^{(d)}_{D_{ji}} g^{(d)}_{D_j}}{N_0 + P^{(d)}_{C_{ij}} h^{(d)}_{C_j}} \ge \eta^{(d)}_{D_{min}}, \ \forall\, i,j, \tag{3.3f}$$

$$0 \le P^{(u)}_{C_{ij}} \le P^{(u)}_{C_{max}}, \quad 0 \le P^{(u)}_{D_{ji}} \le P^{(u)}_{D_{max}}, \ \forall\, i,j, \tag{3.3g}$$

$$\frac{P^{(u)}_{C_{ij}} g^{(u)}_{C_i}}{N_0 + P^{(u)}_{D_{ji}} h^{(u)}_{D_j}} \ge \eta^{(u)}_{C_{min}}, \ \Pr\left\{\frac{P^{(u)}_{D_{ji}} g^{(u)}_{D_j}}{N_0 + P^{(u)}_{C_{ij}} \tilde{h}^{(u)}_{C_{ij}}} \ge \eta^{(u)}_{D_{min}}\right\} \ge 1-\epsilon, \ \forall\, i,j, \tag{3.3h}$$

Where $\gamma > 0$ is a user-selected regularization parameter to balance the rate-fairness trade-off. In general, the highest rate is achieved when all channels are assigned only to D2D pairs with good communications conditions. The fairness in the assignment is enforced by adding a term in the objective function that penalizes unfair assignments. As described before, Paper A and B consider special cases of the problem in in (3.3). Both of them consider either uplink or downlink communication. Additionally, Paper A assumes perfect knowledge of CSI while Paper B focuses on the case of uncertainty in CSI.

3.3 Proposed Solutions

In this section, we highlight the solutions presented in Papers A,B and C. Problem (3.3) is mixed integer non-convex optimization problem. Obtaining an optimal solution for such problem is usually computationally demanding. However, we show in our papers that all those problems can be decomposed without loss of optimality into several power allocation subproblems and a single channel assignment subproblem. The power allocation subproblem for the downlink in Paper C is:

$$\begin{aligned}
&\underset{P_{C_{ij}}^{(d)},P_{D_{ji}}^{(d)}}{\text{maximize}} \quad v_{i,j}^{(d)}(P_{C_{ij}}^{(d)}, P_{D_{ji}}^{(d)}) && (3.4a)\\
&\text{s.t. } 0 \leq P_{C_{ij}}^{(d)} \leq P_{C_{max}}^{(d)}, \quad 0 \leq P_{D_{ji}}^{(d)} \leq P_{D_{max}}^{(d)}, && (3.4b)\\
&\Pr\left\{ \frac{P_{C_{ij}}^{(d)} g_{C_i}^{(d)}}{N_0 + P_{D_{ji}}^{(d)} \tilde{h}_{D_{j,i}}^{(d)}} \geq \eta_{C_{min}}^{(d)} \right\} \geq 1 - \epsilon, \quad \frac{P_{D_{ji}}^{(d)} g_{D_j}^{(d)}}{N_0 + P_{C_{ij}}^{(d)} h_{C_j}^{(d)}} \geq \eta_{D_{min}}^{(d)}, && (3.4c)
\end{aligned}$$

with a similar problem for uplink and special cases for Paper A and Paper B. The power allocation subproblems in Paper A are solved in closed-form. However, the power allocation in Paper B is solved by applying a quadratic transformation similar to the one in [40] followed by alternating optimization. Paper C consider both cases and therefore the power allocation subproblems are solved by either a closed-form expression or a quadratic transformation followed by alternating optimization. The channel assignment subproblem in Paper C is written as follows:

$$\begin{aligned}
&\underset{\boldsymbol{B}^{(u)},\boldsymbol{B}^{(d)}}{\text{maximize}} \sum_{i\in\mathcal{C}} \sum_{j\in\mathcal{D}} [\beta_{i,j}^{(u)} U_{i,j}^{(u)^*} + \beta_{i,j}^{(d)} U_{i,j}^{(d)^*}] - \gamma \delta_J(\boldsymbol{B}^{(u)}, \boldsymbol{B}^{(d)}), && (3.5a)\\
&\text{s.t. } \beta_{i,j}^{(u)} \in \{0,1\}, \quad \beta_{i,j}^{(d)} \in \{0,1\}, \quad \forall\, i,j && (3.5b)\\
&\sum_{j=1}^{D} \beta_{i,j}^{(d)} \leq 1, \quad \sum_{j=1}^{D} \beta_{i,j}^{(u)} \leq 1, \quad \forall\, i && (3.5c)\\
&\left(\sum_{i\in\mathcal{C}^{(d)}} \beta_{i,j}^{(d)} \right) \times \left(\sum_{i\in\mathcal{C}^{(u)}} \beta_{i,j}^{(u)} \right) = 0, \quad \forall\, j && (3.5d)
\end{aligned}$$

The channel assignment subproblems in Paper A and B is the same. It is solved by relaxing the binary constraints (3.5b) into continuous and then applying projected gradient descent (PGD). Afterwards, the solution is discretized back into a binary solution. However, the channel allocation problem in Paper C is quite different. It is solved by relaxing the problem and ignoring the constraints (3.5d) and converting the binary constraints into continuous. This problem can also be solved using PGD. Afterwards, the obtained solution needs to be discretized and projected onto to the set defined by (3.5d). Fig. 3.2 highlights the solutions proposed in each paper.

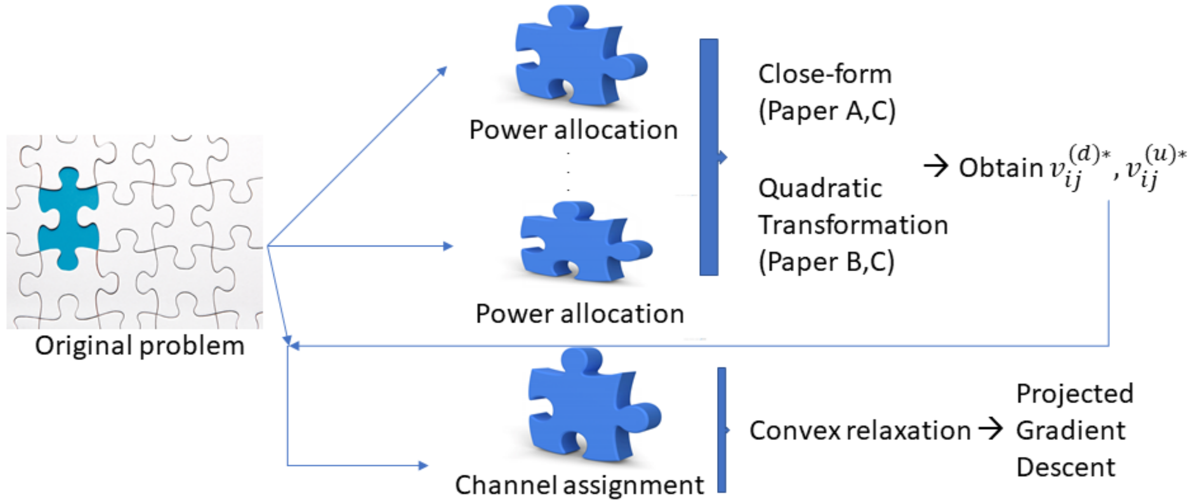

Figure 3.2: Solutions proposed in papers A, B and C

Algorithm 1 Joint Resource Allocation (Paper C)

1: Initialize: $\boldsymbol{B}^{(d)}(0), \boldsymbol{B}^{(u)}(0), \boldsymbol{P}_C^{(d)}(0), \boldsymbol{P}_D^{(d)}(0), \boldsymbol{P}_D^{(u)}(0), \boldsymbol{P}_C^{(u)}(0), k = 0$
2:
3: **for all** $i \in \mathcal{C}$ **do**
4: **for all** $j \in \mathcal{D}$ **do**
5: Perform power assignment in both downlink and uplink
6: to find: $\mathbf{P}_{\mathbf{D_{ji}}}^{(\mathbf{d})^*}, \mathbf{P}_{\mathbf{C_{ij}}}^{(\mathbf{d})^*}, \mathbf{P}_{\mathbf{D_{ji}}}^{(\mathbf{u})^*}, \mathbf{P}_{\mathbf{C_{ij}}}^{(\mathbf{u})^*}$.
7: solve (3.4) Either in close-form or by iteratively alternating between $\mathbf{P}_{\mathbf{D_{ji}}}$ and $\mathbf{P}_{\mathbf{C_{ij}}}$ in both uplink and downlink.
8: Calculate $v_{ij}^{(d)^*}$ and $v_{ij}^{(u)^*}$ from $\mathbf{P}_{\mathbf{D_{ji}}}^{(\mathbf{d})^*}, \mathbf{P}_{\mathbf{C_{ij}}}^{(\mathbf{d})^*}, \mathbf{P}_{\mathbf{D_{ji}}}^{(\mathbf{u})^*}, \mathbf{P}_{\mathbf{C_{ij}}}^{(\mathbf{u})^*}$.
9: **end for**
10: **end for**
11: **repeat**
12: $k = k + 1$
13: BS uses the PGD algorithm to solve (3.5) and calculate: $\boldsymbol{B}^{(d)}(k), \boldsymbol{B}^{(u)}(k)$
14: **until** $\boldsymbol{B}^{(d)}, \boldsymbol{B}^{(u)}$ converges
15: BS discretize $\boldsymbol{B}^{(d)}, \boldsymbol{B}^{(u)}$.

3.4 Convergence Analysis

Paper C provides convergence rate and guarantees for all described cases. The power allocation subproblems in Paper A and in some cases of Paper C are solved entirely in close-form and do not require further convergence analysis. However for Paper B and the rest of the cases in Paper C, they are solved iteratively by alternating between blocks of variable. We show in Paper C that the objective functions in those cases are bounded, real analytic, strongly convex and Lipschitz smooth in each variables block. Furthermore, we show that those power allocation subproblems converge to a stationary point $\bar{\mathbf{p}}$ with

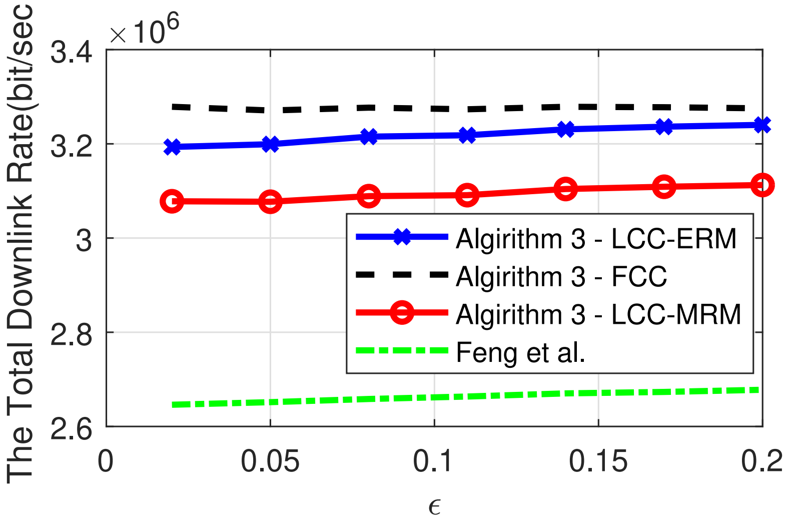

Figure 3.3: Total Rate vs. ϵ (Paper C)

$\|\mathbf{p}^{[k]} - \bar{\mathbf{p}}\|_2 \leq Ck^{-(1-\theta)/(2\theta-1)}$ for some $C > 0$ and $\theta \in [0.5, 1)$. We also show that the relaxed channel assignment subproblem in all cases is quadratic, convex and Lipschitz smooth problem with linear constraints and solving this problem will converge as $\mathcal{O}(1/k)$.

3.5 Numerical Results

In this section we present some of the numerical results obtained in Paper C. In which, we consider a simulation scenario with a single cell of radius 500 m. In this cell, CUs and D2D transmitters are located uniformly at random. The D2D receivers are located uniformly at random in a 5 m radius circle centered at their respective transmitter. A path-loss model with exponent $\alpha = 2$ is used in the calculation of all channel gains. The random channel gains are calculated by applying an exponential random distribution around an average calculated from the path-loss model. $N_C = 10, N_D = 10$ were used in the experiments with Monte-Carlo averages carried over $10,000$ different realizations.

Fig. 3.3 shows comparisons of our proposed methods, compared with a state-of-the-art method in [27]. The achieved rate of all our proposed methods are significantly higher than the method in [27] achieves the lowest rate since it does not allow assigning multiple channels to a D2D pair. The rates of all methods grow with the allowed outage probability ϵ.

3.6 Contributions

In Paper A, we obtain an optimal power allocation and a near optimal channel assignment in an environment with perfect CSI where D2D pairs use either uplink or downlink spectrum.

In Paper B, we obtain a near optimal power allocation and a near optimal channel assignment in an environment with imperfect CSI where D2D pairs use either uplink or downlink spectrum.

In Paper C we consider two different scenarios of D2D pairs communicating over underlay downlink/ uplink channels: (*i*) D2D pairs are pre-organized into uplink and downlink groups based on hardware limitations to communicate in either uplink or downlink channels; (*ii*) The assignment of the D2D pairs to either uplink or downlink channels is also part of the optimization problem. Both the scenarios considers perfect and imperfect CSI cases. We also include convergence proofs for the near optimal power allocation and channel assignment obtained in this case. Furthermore, in order to reduce the computation load on the BS, we also propose decentralized solution for both scenarios. Moreover, Paper C provides convergence guarantees and rate for all cases even for the cases of Paper A and Paper B.

Chapter 4

Resource Allocation for Interfering Underlay D2D Communications

In this chapter we summarized the system model, formulation and the main findings for the work in resource allocation for interfering underlay D2D communications. This work was published in Paper D. Paper D include both SISO and MIMO formulations. However, in this chapter we will only focus on the SISO formulation since the MIMO formulation will be discussed in details in the next chapter.

4.1 System Model

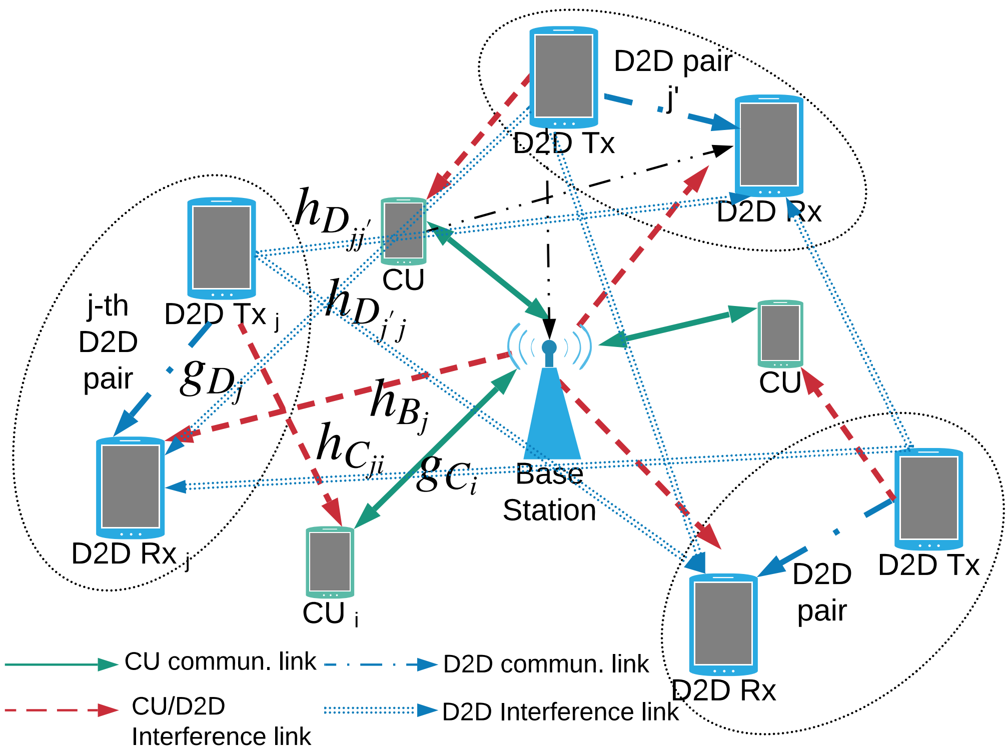

Figure 4.1: System Model (Paper D)

We consider the cellular communication setup shown in Fig. 4.1 where a BS communicates with N_C CUs through N_C downlink channels[1]. The set of CUs (equivalently, channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. In an underlay configuration, N_D D2D pairs, indexed by $\mathcal{D} = \{1, ..., N_D\}$, wish to communicate using the aforementioned N_C downlink channels.

First, let g_{B_i} be the channel gain between the BS and the i-th CU; g_{D_j} be the channel gain of the j-th D2D pair; $h_{c_{j,i}}$ be the channel gain of the interference link from the transmitter of the j-th D2D pair to the i-th CU; h_{B_j} be the channel gain of the interference link from the BS to the receiver of the j-th D2D pair; $h_{D_{j',j}}$ be the channel gain of the interference link from the transmitter of the j'-th D2D pair to the j-th D2D receiver; and N_0 be the noise power on each sub-channel.

The BS assignment of channels to D2D pairs is denoted by the indicator parameters $\{\beta_{i,j}\}_{i\in\mathcal{C},j\in\mathcal{D}}$, where $\beta_{i,j} = 1$ indicates assignment of the i-th channel to the j-th D2D pair and $\beta_{i,j} = 0$ otherwise. For higher throughput, we allow a D2D pair to simultaneous access multiple channels.

The transmit power of the j-th D2D pair when assigned to the i-th channel, denoted as $P_{D_{j,i}}$ is constrained as $\sum_{i\in\mathcal{C}} P_{D_{j,i}} \leq P_{D_{max}},\ \forall\ j$. Similarly, the transmit power of the BS on the i-th channel is constrained as $\sum_{i\in\mathcal{C}} P_{B_i} \leq P_{B_{max}}$. To ensure successful communication, the SINR should also be enforced to be greater than a certain threshold $\eta^D_{\min}$ for the D2D pairs and $\eta^C_{\min}$ for the CUs.

4.2 Problem Formulation

Let us first express the following achievable rates:
R_{c_i} denotes the throughput of the CU user i.

$$R_{c_i} = \log_2\left(1 + \frac{P_{B_i} g_{B_i}}{N_0 + \sum\limits_{j\in\mathcal{D}} \beta_{i,j} P_{D_{ji}} \tilde{h}_{c_{j,i}}}\right)$$

$R_{d_{j,i}}$ denotes the throughput of the D2D pair j when sharing the spectrum with the CU user i.

$$R_{d_{j,i}} = \log_2\left(1 + \frac{\beta_{i,j} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum\limits_{j'\in\mathcal{D}, j'\neq j} \beta_{i,j'} P_{D_{j'i}} h_{D_{j',j}}}\right)$$

R_i denotes the total rate of channel i.

$$R_i = R_{c_i} + \sum_{j\in\mathcal{D}} R_{D_{j,i}}$$

Let $\delta(\boldsymbol{B})$ denotes the fairness function, which measure the fairness of a channel assignment. To define this function, let x_j denotes the number of resources assigned to D2D pair j. $x_j = \sum\limits_{i=1}^{N_C} \beta_{i,j}$, and x_0 denotes the desired resource assignment when every D2D pair

[1]Even though the formulation is done for downlink communications, the same formulation can be directly extended to uplink.

use ($x_0 = rN_C$) channels, where $r \in [0,1]$ is the ratio of channels assigned to each pair on average. $\boldsymbol{B}$ is the channel assignment matrix that combines all values of β as $\boldsymbol{B} = [\beta_{ij}]$. Following [10], the fairness of a channel allocation $\boldsymbol{B}$ is quantified by

$$\delta(\boldsymbol{B}) = \frac{\frac{1}{N_D}\sum_{j=1}^{N_D}(x_j - x_0)^2}{x_0^2} = \frac{1}{(rN_C)^2}\sum_{j=1}^{N_D}\left(\frac{1}{N_D}(\sum_{i=1}^{N_C}\beta_{i,j} - rN_C)^2\right) \tag{4.1}$$

which can be interpreted as a scaled variance of the assignment $\boldsymbol{B}$ from its fairest value x_0.

Next we define an objective function that combines the total network rate and the fairness. We propose using a parameter $\gamma > 0$ to control the trade-off between these two functions. The complete optimization problem is then expressed as:

$$\underset{\boldsymbol{B},\boldsymbol{P_B},\boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\left[R_i(P_{B_i}, P_{D_{ji}}, \boldsymbol{B})\right] - \gamma\delta(\boldsymbol{B}) \tag{4.2a}$$

$$\text{subject to} \quad \beta_{i,j} \in \{0,1\}, \ \forall\, i,j, \tag{4.2b}$$

$$0 \le \sum_{i\in\mathcal{C}} P_{B_i} \le P_{B_{max}} \ \ \forall\, i, \tag{4.2c}$$

$$0 \le \sum_{i\in\mathcal{C}} \beta_{i,j} P_{D_{ji}} \le P_{D_{max}} \ \ \forall\, j, i, \tag{4.2d}$$

$$\frac{P_{B_i} g_{B_i}}{N_0 + \sum_{j\in\mathcal{D}} \beta_{i,j} P_{D_{ji}} h_{c_{j,i}}} \ge \eta_{min}^C, \ \forall\, i, \tag{4.2e}$$

$$\frac{\beta_{i,j} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D}, k\ne j} \beta_{i,k} P_{D_{ki}} h_{d_{k,j}}} \ge \eta_{min}^D, \ \forall j, i. \tag{4.2f}$$

4.3 Proposed Solutions

In this section we highlights the solutions presented in Papers D. Problem (4.2) is mixed integer non-convex optimization problem. As before, obtaining an optimal solution for such problems is usually computationally demanding. However we showed in our paper this problem can be approximately solved by alternating between a power allocation subproblem and a single channel assignment subproblems. The k-th iteration power subproblem for a known $\boldsymbol{B} = \boldsymbol{B}^{(k-1)}$ becomes:

$$
\begin{aligned}
\underset{\boldsymbol{P_B},\boldsymbol{P_D}}{\text{maximize}} \quad & \sum_{i\in\mathcal{C}} \left[R_{C_i} + \sum_{j\in\mathcal{D}} R_{D_{j,i}} \right] && \text{(4.3a)}\\
\text{subject to} \quad & \sum_{i\in\mathcal{C}} P_{B_i} \leq P_{B_{max}} && \text{(4.3b)}\\
& \sum_{i\in\mathcal{C}} \beta_{i,j} P_{D_{ji}} \leq P_{D_{max}} \ \forall\ j && \text{(4.3c)}\\
& \frac{P_{B_i} g_{B_i}}{N_0 + \sum_{j\in\mathcal{D}} \beta_{i,j} P_{D_{ji}} h_{C_{j,i}}} \geq \eta^C_{min}, \ \forall\ i,j, && \text{(4.3d)}\\
& \frac{\beta_{i,j} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} \beta_{i,k} P_{D_{ki}} h_{D_{k,j}}} \geq \eta^D_{min}, \ \forall i,j : \beta_{ij} = 1. && \text{(4.3e)}
\end{aligned}
$$

We propose solving the power allocation subproblem in the following three case: (**C1**) the power allocation subproblem is solved centrally by applying a quadratic transformation and alternating between the power variables; (**C2**) applying dual decomposition to the power allocation subproblem to decouple it across channels and then solving the problem in a partially decentralize manner; (**C3**) constraints (4.2c) and (4.2d) are replaced by tighter constraints con each channel and then the power allocation problem is decoupled across channels, and then solving the problem in a partially decentralize manner.

In all cases, to solve the channel assignment subproblem for the calculated $\boldsymbol{P}^k_B, \boldsymbol{P}^k_D$, we substitute them in (4.2) and solve for $\boldsymbol{B}^{K+1}$. The problem is then expressed as:

$$
\begin{aligned}
\underset{\boldsymbol{B}}{\text{maximize}} \quad & \sum_{i\in\mathcal{C}} \left[\sum_{j\in\mathcal{D}} \beta_{ij} R^k_{D_{j,i}} \right] - \gamma\hat{\delta}(\boldsymbol{B}) && \text{(4.4a)}\\
\text{subject to} \quad & \beta_{i,j} \in \{0,1\}, \ \forall\ i,j. && \text{(4.4b)}
\end{aligned}
$$

This problem is similar to the one in Paper A and is solved as before. Fig. 4.2 highlights the solutions in each case. Algorithm 2 describes the solution for (**C3**) case. Other cases have relatively similar algorithms.

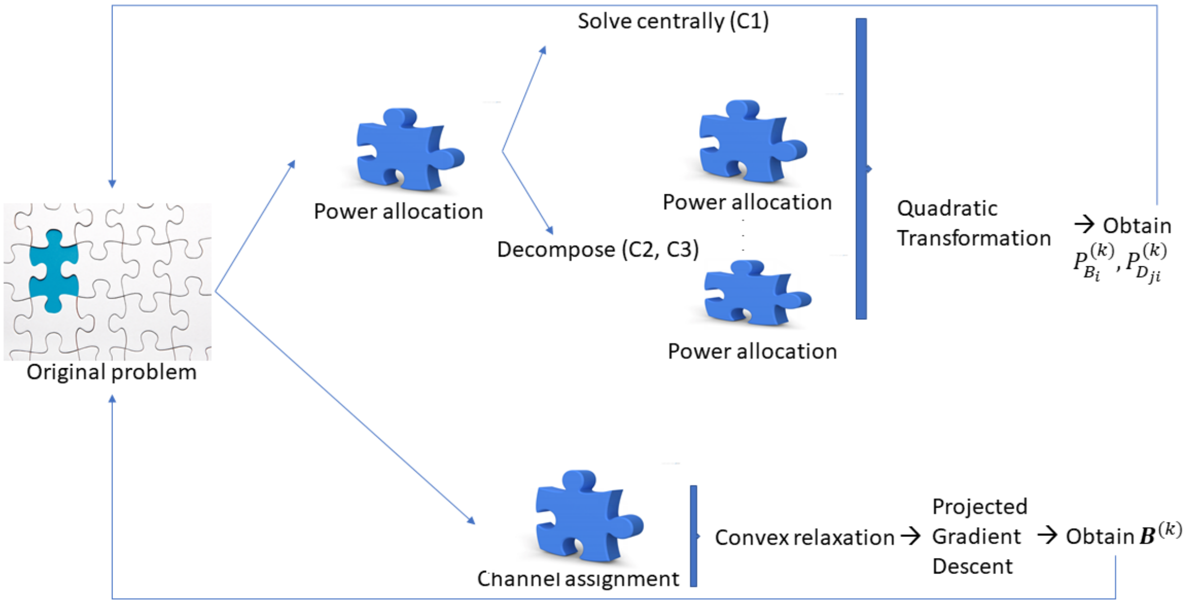

Figure 4.2: Solutions block diagram

Algorithm 2 Resource Allocation (case **C1**)

Initialize: $\boldsymbol{B}^{(0)}, \boldsymbol{P}_B^{(0)}, \boldsymbol{P}_D^{(0)}, k = 0$
repeat
  $k = k + 1$
  for all $i \in \mathcal{C}$ **do**
    Perform power assignment for the previous channel assignment $\boldsymbol{B}^{(k-1)}$ to find: $P_{D_{ji}}^{(k)}\ \forall j, P_{B_i}^{(k)}$.
    By iteratively alternating between all $P_{D_{ji}}$ and P_{B_i}.
  end for
  BS performs PGD algorithm to calculate: $\boldsymbol{B}^{(k)}$ for the calculated $\boldsymbol{P}_D^{(k)}, \boldsymbol{P}_B^{(k)}$.
until $\boldsymbol{B}, \boldsymbol{P}_D, \boldsymbol{P}_B$ converges
BS discretize $\boldsymbol{B}^{(k)}$.

4.4 Convergence Analysis

Paper D also provides convergence analysis for this work. The power allocation subproblems are solved iteratively by alternating between blocks of variable. We show that the objective functions in cases (**C1**) and (**C3**) are bounded, real analytic, strongly convex and Lipschitz smooth in each variables block. Furthermore, we show that those power allocation subproblems converge to a stationary point $\bar{\mathbf{p}}$ with $\|\mathbf{p}^{[k]} - \bar{\mathbf{p}}\|_2 \leq Ck^{-(1-\theta)/(2\theta-1)}$ for some $C > 0$ and $\theta \in [0.5, 1)$. However, the power allocation subproblem for (**C2**) case, is not strongly convex in the dual variable, and therefore, dose not satisfy the same Theorem. In this cases, the convergence of such solution can not be easily guaranteed. We also show that the relaxed channel assignment subproblem in all cases is quadratic, convex and Lipschitz smooth problem with linear constraints and solving this problem

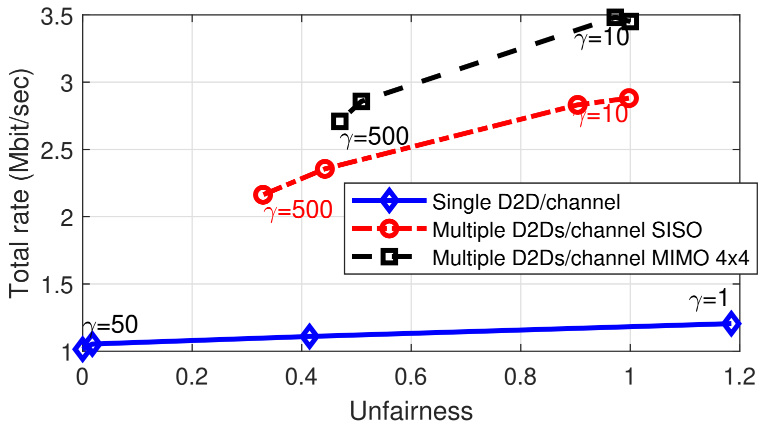

Figure 4.3: Rate vs. Unfairness when changing $\gamma = \{1, ..., 500\} \times \text{BW}$ (Paper D).

will converge as $\mathcal{O}(1/k)$. Moreover, we show that the alternation between the power and channel sub problems will converge to a Nash point.

4.5 Numerical Results

In this section we present some of the numerical results obtained in Paper D. In which, we consider a simulation scenario with a single cell of radius 500 m. In this cell, CUs and D2D transmitters are located uniformly at random. The D2D receivers are located uniformly at random in a 5 m radius circle centered at their respective transmitter. A path-loss model with exponent $\alpha = 2$ is used in the calculation of all channel gains. The random channel gains are calculated by applying an exponential random distribution around an average calculated from the path-loss model. $N_C = 4, N_D = 4$ were used in the experiments with Monte-Carlo averages carried over 1000 different realizations.

Fig. 4.3 shows the results of the proposed method for the SISO (**C**3) case and 4×4 MIMO compared to a case where a channel is assigned to at most one D2D pair. Both SISO and MIMO cases have very high rate compared to the case where a channel is assigned to at most one D2D pair. All similar cases shows similar behaviour where the rate and unfairness decrease when increasing γ, as expected.

4.6 Contributions

In this work, we formulate a resource allocation problem to maximize the aggregate rate of all D2D pairs and CUs with a penalty on unfair channel assignment, under total power constraints and minimum SINR requirements.

The resulting mixed integer non-convex problem is approximately solved by alternating between a power allocation subproblem and a channel assignment subproblem. The power allocation subproblem is solved both centrally and in a distributed manner, while the channel assignment subproblem is solved centrally.

Chapter 5

MIMO Beamforming in Underlay D2D Communications

In this chapter, we summarize the system model, formulation and the main results for the work in resource allocation for MIMO beamforming in underlay wireless D2D communications. This work was published in Papers E,F.

5.1 System Model

The system model that we consider is illustrated in Fig. 5.1.

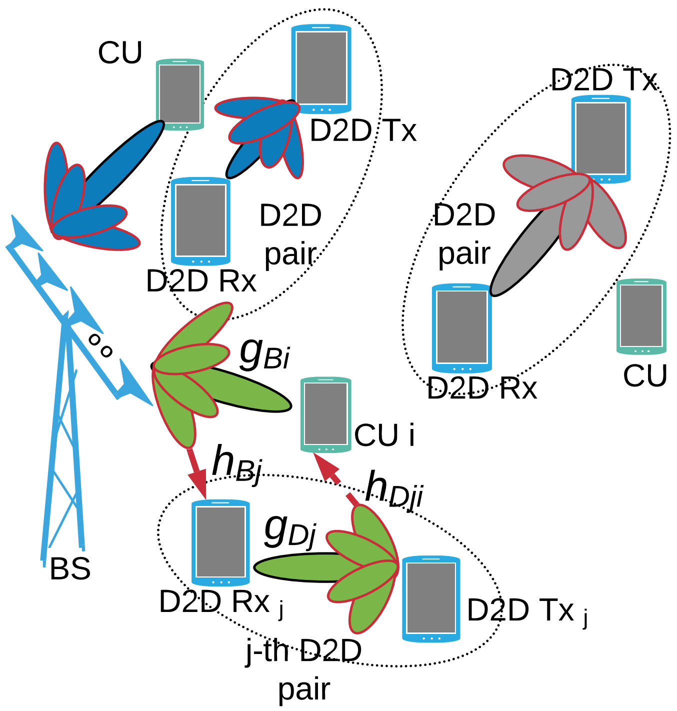

Figure 5.1: System Model (Paper F)

In this framework, we consider an underlay D2D communications scenario under a multiple transmit antenna framework in downlink spectrum[1], as shown in Fig. 5.1. We assume that the BS have K_B transmit antennas to communicate with N_C single antenna CUs through N_C downlink channels. In order to simplify the notation, CUs (equivalently, channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. The D2D pairs wishing to communicate over the aforementioned N_C channels are indexed by $\mathcal{D} = \{1, ..., N_D\}$. Similarly, we assume that the D2D transmitters have K_D transmit antennas to communicate with their respective single antenna D2D receivers. In general, a BS/D2D transmitter with multiple antennas can simultaneously communicate to multiple CUs/D2D receivers on a single channel; however, for simplicity in our analysis, we assume one CUs/D2D pair on every channel. With minor modification, the analysis can be extended to the multi-user case.

The channel between the BS and the i-th CU is denoted by $\boldsymbol{g}_{B_i} \in \mathbb{C}^{K_B \times 1}$. Similarly, the channel between the j-th D2D pair is denoted by $\boldsymbol{g}_{D_j} \in \mathbb{C}^{K_D \times 1}$. The interference channel between the BS and the receiver of the j-th D2D is denoted by[2] $\boldsymbol{h}_{B_j} \in \mathbb{C}^{K_B \times 1}$. Similarly, the interference channel between the transmitter of the j-th D2D pair and the i-th CU is denoted by $\boldsymbol{h}_{D_{j,i}} \in \mathbb{C}^{K_D \times 1}$. Here, we assume that the CUs provide limited cooperation in estimating the gain of the interference channel, as expected in practice. To account for the errors in this estimation, we propose two models for this channel gain:

(M1) statistical description: it is a random vector with complex circular Gaussian distribution, i.e., $\boldsymbol{h}_{D_{ji}} \sim \mathcal{CN}(\tilde{\boldsymbol{h}}_{D_{ji}}, \boldsymbol{M}_{ji})$ where $\tilde{\boldsymbol{h}}_{D_{ji}}$ and $\boldsymbol{M}_{ji}$ are the distribution parameters, that is, mean vector and covariance matrix (Paper E);

(M2) deterministic description: it has an error vector bounded within a specified ellipsoid, thus the correct channel gain can be defined as $\boldsymbol{h}_{D_{j,i}} = \tilde{\boldsymbol{h}}_{D_{j,i}} + \boldsymbol{e}_{ji}$, where $\tilde{\boldsymbol{h}}_{D_{j,i}}$ denotes the estimate of the interference channel gain with error $\boldsymbol{e}_{ji}$ i.e. $\boldsymbol{e}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{e}_{ji} \leq 1$, where $\boldsymbol{Q}_{ji} \in \mathbb{H}^{K_D}$, $\boldsymbol{Q}_{ji} \succeq \boldsymbol{0}$ specifies the size and shape of the ellipsoid, and $\mathbb{H}^{K_D}$ is the space of $K_D \times K_D$ Hermitian matrices (Paper F).

The additive white noise power is denoted by N_0.

We represent the assignment of channels to D2D pairs by the indicators $\{\beta_{i,j}\}_{i \in \mathcal{C}, j \in \mathcal{D}} \in \{0, 1\}$, where $\beta_{i,j} = 1$ when the i-th channel is assigned to the j-th D2D pair and $\beta_{i,j} = 0$ otherwise. In order to provide higher throughput to D2D pairs, we allow simultaneous access of multiple channels to a D2D pair, however, to restrict the interference among D2D pairs, access of more than one D2D pair is not allowed over a particular channel, i.e., $\sum_{j=1}^{N_D} \beta_{i,j} \leq 1$, $\forall i$.

Finally, we denote the beamforming power vector of the BS to communicate with the i-th CU by $\boldsymbol{P}_{B_i} \in \mathbb{C}^{K_D \times 1}$ and for the j-th D2D pair on the i-th channel by $\boldsymbol{P}_{D_{j,i}} \in \mathbb{C}^{K_B \times 1}$. The respective transmit powers are constrained as $||\boldsymbol{P}_{B_i}||_2^2 \leq P_{B,\max}$ and $||\boldsymbol{P}_{D_{j,i}}||_2^2 \leq P_{D,\max}$. To ensure successful communication, the SINR is desired to be greater than

[1]Without loss of generality, the same formulation and algorithm design developed here, can be also applied to the uplink spectrum.

[2]In principle, $\boldsymbol{g}_{D_j}$ and $\boldsymbol{h}_{B_j}$ should also depend on the i-th channel, however, this subscript is dropped as the proposed scheme carries over immediately to accommodate such dependence.

$\eta_{D,\min}$ for D2D pairs and $\eta_{C,\min}$ for CUs. For (**M1**), this SINR bound should be satisfied with probability greater than $1-\epsilon$ where ϵ is the allowed outage probability. For (**M2**), this bound should be satisfied for all channel gains values in the ellipsoid.

5.2 Problem Formulation

In order to take into account the error in the estimate of the interference channels from D2D pairs to CUs, i.e., $\tilde{\boldsymbol{h}}_{D_{ji}}$, we formulate the beamforming design problem for the worst case error in $\tilde{\boldsymbol{h}}_{D_{ji}}$. Let $\Gamma(z) := \text{BW} \times \log_2(1+z)$ denote the rate obtained over channel bandwidth BW for the given SINR z. The total rate that can be achieved over every i-th channel is defined by $R_i := (1-\sum_{j\in\mathcal{D}}\beta_{i,j})R_{C_{i,0}} + \sum_{j\in\mathcal{D}}\beta_{i,j}[R_{D_{j,i}} + R_{C_{i,j}}]$, where:

- $R_{C_{i,0}} := \Gamma\left(\frac{P_{B_{\max}}||\boldsymbol{g}_{B_i}||_2^2}{N_0}\right)$, rate of the i-th CU without assignment of D2D pairs, i.e., $\beta_{ij}=0\ \forall j$.
- $R_{D_{j,i}} := \Gamma\left(\frac{|\boldsymbol{P}_{D_{ji}}^H\boldsymbol{g}_{D_j}|^2}{N_0+|\boldsymbol{P}_{B_i}^H\boldsymbol{h}_{B_j}|^2}\right)$, rate of j-th D2D pair when assigned with i-th CU, i.e., $\beta_{ij}=1$.
- $R_{C_{i,j}} := \Gamma\left(\frac{|\boldsymbol{P}_{B_i}^H\boldsymbol{g}_{B_i}|^2}{N_0+|\boldsymbol{P}_{D_{ji}}^H\boldsymbol{h}_{D_{j,i}}|^2}\right)$, rate achieved by i-th CU when assigned with j-th D2D pair, i.e., $\beta_{ij}=1$. For (**M2**) the value for the channel gain is the worst case channel gain in the bound ellipsoid. For (**M1**) we consider a lower bound SINR that is exceeded with probability $(1-\epsilon)$.

Finally, the aggregate network rate is defined as $R(\mathbb{B},\mathbb{P}_B,\mathbb{P}_D) := \sum_{i\in\mathcal{C}} R_i$, where, $\mathbb{B} := \{\beta_{i,j}\}$, $\mathbb{P}_B := \{\boldsymbol{P}_{B_i}\}$, $\mathbb{P}_D := \{\boldsymbol{P}_{D_{j,i}}\}\ \forall i\in\{1,\cdots N_C\}$ and $j\in\{1,\cdots N_D\}$.

We also define the unfairness measure similar to the one described in earlier chapters $\delta(\mathbb{B}) = 1/(N_Dc^2)\sum_{j=1}^{N_D}(x_j-c)^2$, where $x_j := \sum_{i=1}^{N_C}\beta_{ij}$ is the number of channels assigned to the j-th D2D pair; and where $c := N_C/N_D$ is the fairest assignment. Summing up, the overall problem considering the worst case error in estimation of interference channel, can be formulated as:

$$\underset{\mathbb{B},\mathbb{P}_B,\mathbb{P}_D}{\text{maximize}}\ R(\mathbb{B},\mathbb{P}_B,\mathbb{P}_D) - \gamma\delta(\mathbb{B}) \tag{5.1a}$$

$$\text{subject to } \beta_{i,j}\in\{0,1\},\ \sum_{j=1}^{N_D}\beta_{i,j}\le 1\,\forall i; \tag{5.1b}$$

$$||\boldsymbol{P}_{B_i}||_2^2 \le P_{B,\max}\,\forall i,\ ||\boldsymbol{P}_{D_{j,i}}||_2^2 \le P_{D,\max}\,\forall j,i; \tag{5.1c}$$

$$\frac{|\boldsymbol{P}_{D_{ji}}^H\boldsymbol{g}_{D_j}|^2}{N_0+|\boldsymbol{P}_{B_i}^H\boldsymbol{h}_{B_j}|^2} \ge \eta_{D,\min} \quad \text{if } \beta_{ij}=1,\ \forall i,j \tag{5.1d}$$

$$(\mathbf{M1})\Pr\left\{\frac{|\boldsymbol{P}_{B_i}^H\boldsymbol{g}_{B_i}|^2}{N_0+|\boldsymbol{P}_{D_{ji}}^H\boldsymbol{h}_{D_{j,i}}|^2}\geq\eta_{C,\min}\right\}\geq 1-\epsilon,\quad \text{if } \beta_{ij}=1,\ \forall i,j \tag{5.1e}$$

$$(\mathbf{M2})\frac{|\boldsymbol{P}_{B_i}^H\boldsymbol{g}_{B_i}|^2}{N_0+|\boldsymbol{P}_{D_{ji}}^H\boldsymbol{h}_{D_{j,i}}|^2}\geq\eta_{C,\min},\ \boldsymbol{h}_{D_{j,i}}=\tilde{\boldsymbol{h}}_{D_{j,i}}+\boldsymbol{e}_{ji},\ \boldsymbol{e}_{ji}^H\boldsymbol{Q}_{ji}\boldsymbol{e}_{ji}\leq 1,\ \text{if } \beta_{ij}=1,\ \forall i,j \tag{5.1f}$$

The regularization parameter $\gamma > 0$ is selected to control the trade-off between aggregate rate and fairness in channel assignment. Problem (5.1) is a non-convex mixed-integer program, which is computationally expensive. The next section presents the solvers proposed, respectively, in Paper E and Paper F.

5.3 Proposed Solutions

In this section, we highlight the solutions in Papers E and F. Similar to the work in Chapter 3, we show also in both papers that this problem can be decomposed without loss of optimality into several power allocation subproblems and a single channel assignment subproblem. The power allocation subproblem can be expressed as:

$$\underset{\boldsymbol{P}_{B_{ij}},\boldsymbol{P}_{D_{ji}}}{\text{maximize}}\quad R_{C_{i,j}}+R_{D_{j,i}} \tag{5.2a}$$

$$\text{subject to}\quad ||\boldsymbol{P}_{B_{ij}}||_2^2\leq P_{B,\max}\ \ ||\boldsymbol{P}_{D_{j,i}}||_2^2\leq P_{D,\max} \tag{5.2b}$$

$$\frac{|\boldsymbol{P}_{D_{ji}}^H\boldsymbol{g}_{D_j}|^2}{N_0+|\boldsymbol{P}_{B_{ij}}^H\boldsymbol{h}_{B_j}|^2}\geq\eta_{D,\min}, \tag{5.2c}$$

$$(\mathbf{M1})\Pr\left\{\frac{|\boldsymbol{P}_{B_{ij}}^H\boldsymbol{g}_{B_i}|^2}{N_0+|\boldsymbol{P}_{D_{ji}}^H\boldsymbol{h}_{D_{j,i}}|^2}\geq\eta_{C,\min}\right\}\geq 1-\epsilon, \tag{5.2d}$$

$$(\mathbf{M2})\frac{|\boldsymbol{P}_{B_{ij}}^H\boldsymbol{g}_{B_i}|^2}{N_0+|\boldsymbol{P}_{D_{ji}}^H\boldsymbol{h}_{D_{j,i}}|^2}\geq\eta_{C,\min},\ \boldsymbol{h}_{D_{j,i}}=\tilde{\boldsymbol{h}}_{D_{j,i}}+\boldsymbol{e}_{ji},\ \boldsymbol{e}_{ji}^H\boldsymbol{Q}_{ji}\boldsymbol{e}_{ji}\leq 1. \tag{5.2e}$$

The power allocation subproblems in Paper E is solved by first replacing constraint (5.2d) by a tighter convex alternative using the Bernstein-type inequality in [41]. It is then approximately solved by applying a semidefinite relaxation followed by a quadratic transformation yielding an alternating optimization algorithm. However, the power allocation in Paper F is solved by first applying the S-Lemma [42] to constraint (5.2e). It is then approximately solved by applying a semidefinite relaxation followed by a quadratic transformation yielding a similar alternating optimization algorithm. The channel assignment subproblems in both papers are similar to the one in Chapter 3 and are solved in the same manner. Fig. 5.2 and Algorithm 3 highlight the solutions in each paper.

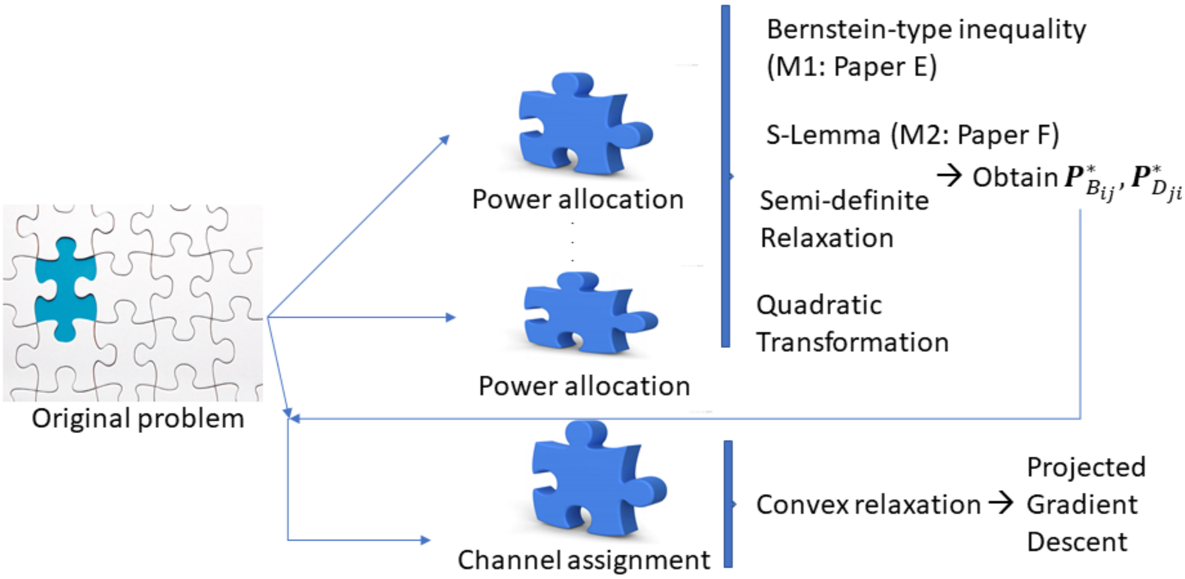

Figure 5.2: Solutions highlights

Algorithm 3 Resource Allocation (Paper E and F)

Initialize: $\boldsymbol{B}^{(0)}, \boldsymbol{P}_B^{(0)}, \boldsymbol{P}_D^{(0)}, k = 0$
for all $i \in \mathcal{C}$ **do**
  for all $j \in \mathcal{D}$ **do**
    Perform power assignment to find: $\boldsymbol{P}^*_{D_{ji}}, \boldsymbol{P}^*_{B_{ij}}$.
    By iteratively alternating between $\boldsymbol{P}_{D_{ji}}$ and $\boldsymbol{P}_{B_{ij}}$.
  end for
end for
BS performs PGD algorithm to calculate: $\boldsymbol{B}^*$ for the calculated $\boldsymbol{P}^*_D, \boldsymbol{P}^*_B$.
BS discretize $\boldsymbol{B}^*$.

5.4 Numerical Results

In this section we present some of the numerical results obtained in Paper F. In which, we consider a simulation scenario comprises of a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. We assume $\tilde{h}_C$ to be exponentially distributed with the mean value obtained from the mentioned path-loss model. Averages over 1,000 independent realizations of the user locations with parameters BW= 15 kHz, $\gamma = 200 \times \text{BW}$, $N_D = 10$, $N_C = 10$, $N_0 = -70$ dBW (γ is scaled with BW to ensure that the unfairness and the achieved rate are of comparable values). The proposed algorithm is tested for the cases where $K_B = K_D = 2$ (2×1 MIMO) and where $K_B = K_D = 4$ (4×1 MIMO). In both cases, we assume that $\boldsymbol{Q} = \epsilon^{-2}\boldsymbol{I}$, which indicates that the error in the channel gains lies in a circle of ϵ radius ($\|e\| \leq \epsilon$).

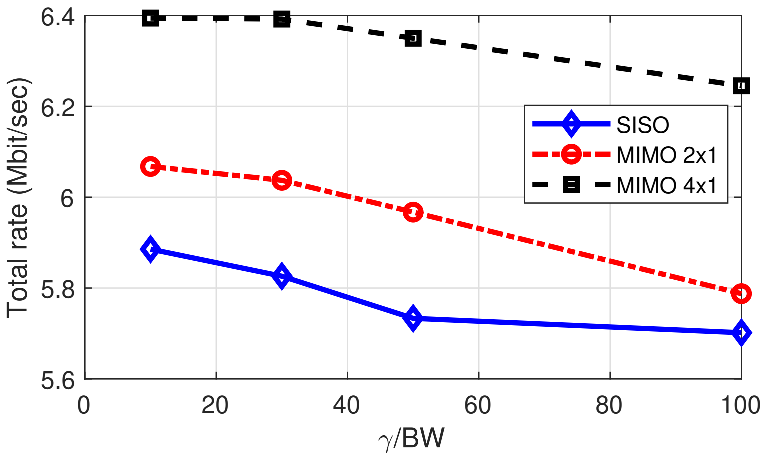

Figure 5.3: Total average rate R vs. γ (Paper F).

In Fig. 5.3, both cases are tested with $\epsilon = 10^{-4}$. It shows that the proposed method achieves higher rates than the SISO method in both cases. When γ increases, the rate decreases in all methods, as expected. The 4×1 MIMO case achieves the highest rates, followed by the 2×1 MIMO case.

5.5 Contributions

The section summarizes the contributions of Paper E and Paper F.

1. We formulate a robust beamforming design problem to maximize the aggregate rate of all D2D pairs and CUs while considering two models for uncertainties in the channel gains; (**M1**) in Paper E and (**M2**) in Paper F.

2. Even though our formulation leads to a mixed integer non-convex problem in both papers, we propose algorithms to compute the power beamforming vectors and channel assignment to D2D pairs in a computationally efficient manner.

3. In order to demonstrate the merits of our proposed formulation and the algorithm to maximize reliably the aggregate rate of the underlay D2D communication network, both papers present extensive Matlab based simulation results where we obtain a better performance than the-state-of-the-art alternatives.

Chapter 6

Multicast Resource Allocation for Underlay D2D Communications

In this chapter, we summarize the system model, problem formulation and the main results for the work in resource allocation for multicast underlay D2D (MD2D) communications. This work was published in Papers G,H.

6.1 System Model

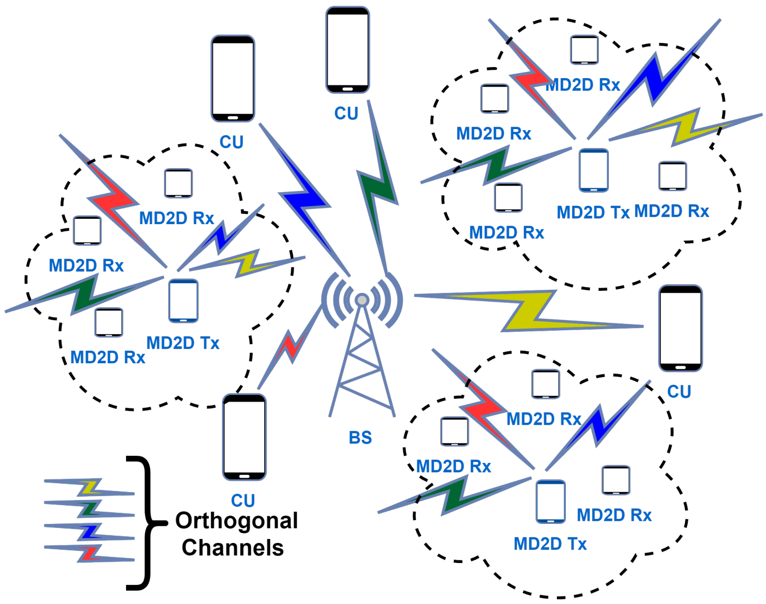

Figure 6.1: System Model (Paper H)

In this framework, we consider a MD2D communications scenario which underlays over the downlink spectrum[1] of a cellular communication network, as shown in Fig. 6.1. We assume

[1]Without loss of generality, the same formulation and algorithm design developed here, can be also applied to the uplink spectrum.

that the BS communicates with the associated CUs over N_C orthogonal downlink channels. Furthermore, we consider a fully loaded network condition with N_C active downlink CUs. In order to avoid confusion in notation, active CUs (equivalently, downlink channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. The MD2D groups are indexed by $\mathcal{D} = \{1, ..., N_D\}$. The j-th MD2D group ($\forall j \in \mathcal{D}$) is assumed to have one transmitter and M_j receivers; the receivers in the j-th MD2D group are indexed by $\mathcal{M}_j = \{1, 2, \cdots, M_j\}$. Furthermore, to provide higher throughput among MD2D groups, we allow simultaneous access of multiple channels to MD2D groups; in addition, we propose two different assumptions: (**A1**) at most one multicast group is allowed to access a particular channel (Paper G); (**A2**) more than one multicast groups are allowed to access a particular channel (Paper H). Since (**A1**) is more restrictive than (**A2**), the rest of the formulations will consider (**A2**) while the key differences will be highlighted.

In this setup, let $b_{i,j}$ denote a binary variable taking value 1 when i-th CU shares channel with j-th multicast group and 0 otherwise; then, the expressions for the respective SINR's observed over the i-th channel by the operating CU and the k-th receiver of the j-th multicast group can be stated as:

$$\Gamma_{C_i} = \frac{g_{C_i} P_{C_i}}{N_0 + I_{C_i}}, \quad \Gamma_{D_{(j:k),i}} = \frac{b_{i,j} g_{D_{(j:k),i}} P_{D_{j,i}}}{N_0 + I_{D_{(j:k),i}}} \tag{6.1}$$

where, g_{C_i}, $g_{D(j:k),i}$ denote the direct channel gains over i-th channel, respectively, between BS and i-th CU and transmitter and k-th receiver in the j-th multicast group; I_{C_i} and $I_{D_{(j:k),i}}$ denote the total interference observed over i-th channel, respectively observed by i-th CU and k-th receiver in the j-th multicast group; and P_{C_i}, $P_{D_{j,i}}$ denote respectively the transmit powers of BS for the i-th CU and transmitter of j-th multicast group over i-th channel.The additive noise is assumed to have one sided power spectral density N_0.

The total observed interference I_{C_i} and $I_{D_{(j:k),i}}$ can be respectively expressed as,

$$I_{C_i} = \sum_{j \in \mathcal{D}} b_{i,j} h_{D_{j,i}} P_{D_{j,i}} \tag{6.2a}$$

$$I_{D_{(j:k),i}} = \sum_{j' \neq j \in \mathcal{D}} b_{i,j'} h_{D_{(j',(j:k)),i}} P_{D_{j',i}} + h_{C_{(j:k),i}} P_{C_i} \tag{6.2b}$$

where, $h_{C_{(j:k),i}}$, $h_{D_{j,i}}$ denotes the interference channel gain over i-th channel, respectively, from BS to k-th receiver of the j-th D2D multicast group and transmitter of j-th multicast group to the operating CU; Similarly, $h_{D_{(j',(j:k)),i}}$ denotes interference channel gain over i-th channel from transmitter of j'-th multicast group to k-th receiver of the j-th D2D multicast group. In the case of (**A2**), the summations in (6.2) disappears since at most one MD2D group is allowed to use the i-th channel (i.e. $\sum_j b_{i,j} \leq 1 \ \forall i$). Those summations reduce to a single term in (6.2a) and no terms in (6.2b).

Next, under the assumption of capacity achieving codes, the achievable capacity for i-th CU can be stated as:

$$R_{C_i} = W \log_2 (1 + \Gamma_{C_i}) \tag{6.3}$$

where W denotes the allocated bandwidth for the downlink channel. For the MD2D group, the maximum achievable rate is determined by the SINR of worst case receiver;

thus, the corresponding achievable rate can be stated as:

$$R_{D_{j,i}} = W \log_2 \left(1 + \min_{k \in \mathcal{M}_j} \Gamma_{D_{(j:k),i}}\right) \tag{6.4}$$

Finally, the sum rate that can be achieved over the whole network, i.e., over all N_C-th channel can be stated as:

$$R = \sum_{i \in \mathcal{C}} \left(R_{C_i} + \sum_{j \in \mathcal{D}} R_{D_{j,i}} \right) \tag{6.5}$$

In the next section, we discuss the problem formulation to maximize the sum rate subjected to several QoS constraints.

6.2 Problem Formulation

Similar to the previous works, the objective in both papers is to maximize the sum rate of all underlay MD2D groups and the CUs. In addition, the optimization formulation is constrained to ensure minimum QoS to all the multicast groups and the CUs. The desired QoS is defined in terms of minimum rate requirement for each user. Furthermore, the objective function is augmented differently in each scenario to either (i) avoid the scenario where most channels are assigned to few multicast groups by enforcing fairness (Paper G (**A1**)); or (ii) avoid the scenario where a multicast group has to communicate over large number of channels by enforcing sparsity (Paper H (**A2**)). Finally, the penalized sum rate maximization problem can be expressed as:

$$\underset{\mathcal{P}_C, \mathcal{P}_D, \mathbf{B}}{\text{maximize}} \quad R - \gamma f(\mathbf{B}) \tag{6.6a}$$

$$\text{subject to:} \quad b_{i,j} \in \{0,1\} \quad \forall i \in \mathcal{C}, \ \ \forall j \in \mathcal{D} \tag{6.6b}$$

$$\sum_{i \in \mathcal{C}} P_{C_i} \leq P_{C_{max,i}} \quad \forall i \in \mathcal{C} \tag{6.6c}$$

$$\sum_{i=1}^{N_C} b_{i,j} P_{D_{j,i}} \leq P_{D_{max,j}} \quad \forall j \in \mathcal{D} \tag{6.6d}$$

$$W \log_2 (1 + \Gamma_{C_i}) \geq R_{C_{min,i}} \ \ \forall i \in \mathcal{C} \tag{6.6e}$$

$$\sum_{i=1}^{N_C} W \log_2 \left(1 + \min_{k \in \mathcal{M}_j} \Gamma_{D_{(j:k),i}}\right) \geq R_{D_{min,j}} \ \ \forall j \in \mathcal{D} \tag{6.6f}$$

$$\min_{k \in \mathcal{M}_j} \Gamma_{D_{(j:k),i}} \geq b_{i,j} \eta_{D_{min}} \quad \forall i \in \mathcal{C}, \forall j \in \mathcal{D} \tag{6.6g}$$

where $\mathcal{P}_C$ and $\mathcal{P}_D$ denote the set of continuous power allocation variables for CUs and MD2D groups, respectively; and $\mathbf{B}$ denotes $N_C \times N_D$ matrix with elements $\mathbf{B}[i,j] = b_{i,j}$. Where $f(\mathbf{B}) = \delta(\mathbf{B})$ in the case of (**A1**) defined in 3.2 or $f(\mathbf{B}) = \sum_{j \in \mathcal{D}} |\mathbf{B}[j]|_1$ in the case of (**A2**). The regularization parameter $\gamma \geq 0$ in the objective (6.6a) is selected to balance the trade-off between sum rate and penalty function $f(.)$. Constraint (6.6b) is an integer constraint.

Constraint (6.6c) and (6.6d) specifies, respective, transmit power limits $P_{C_{max,i}}$ and $P_{D_{max,j}}$ for BS to i-th CU and transmitter of j-th MD2D group. Constraint (6.6e) and (6.6f) specifies the respective minimum rate requirements $R_{C_{min}}$ and $R_{D_{min,j}}$ under sharing of resources between the CU and the MD2D groups. Finally, constraint (6.6g) specifies minimum SINR requirement related to receiver sensitivity for all receivers in the multicast group. Note that similar SINR constraint for CUs is taken care by minimum rate constraint (6.6e). In Paper G (**A1**), constraints (6.6c) and (6.6d) are replaced with tighter constraints that bound the power in each channel. Additionally, constraint (6.6f) is not considered, since constraint (6.6g) also bound the rate in each channel. In Paper H (**A2**), constraint (6.6g) is not considered, since constraint (6.6f) provide an alternative bound on the total rate.

Notice that the optimization problem (6.6) is a non-convex, non-smooth mixed-integer program, which involves exponential complexity. In the next section, we discuss the relaxation techniques to derive a tractable solution of (6.6) with guaranteed polynomial run-time complexity.

6.3 Proposed Solutions

In this section, we highlight the solutions in Paper G and Paper H. In Paper G, we show that this problem can be decomposed without loss of optimality into several power allocation subproblems and a single channel assignment subproblem. The power allocation subproblems are solved by applying a quadratic transformation resulting in an alternating optimization algorithm. The channel assignment subproblem is of similar type to the one in Chapter 3 and can be solved in the same manner. Fig. 6.2 highlights this solution.

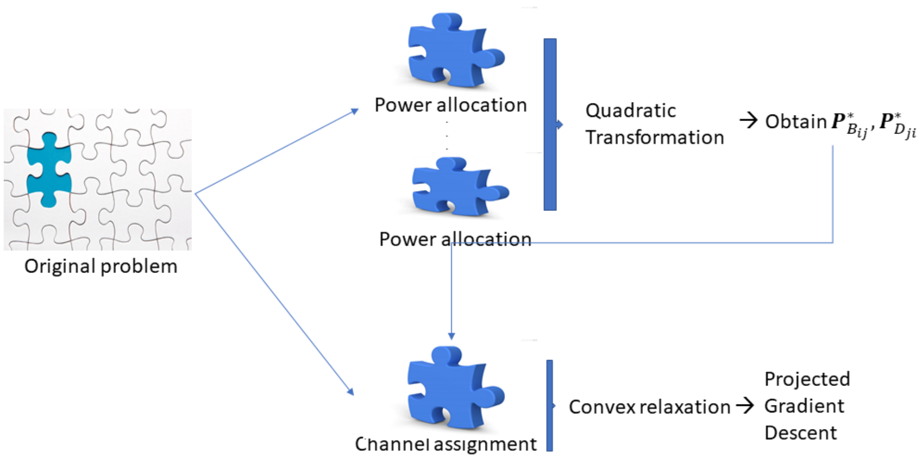

Figure 6.2: Solution proposed in Paper G

In Paper H, we propose using auxiliary variables to bound the received interference power at each CU I_{C_i} and D2D receiver $I_{D_{(j:k),i}}$. The optimization problem in this case is expressed as:

$$\underset{\mathbf{P_C},\mathbf{P_D},\mathbf{B},\mathbf{I_C},\mathbf{I_D}}{\text{maximize}} \quad W\sum_{i\in\mathcal{C}}\left(\log_2\left(1+\frac{g_{C_i}P_{C_i}}{N_0+I_{C_i}}\right)+\sum_{j\in\mathcal{D}}\log_2\left(1+\min_{k\in\mathcal{M}_j}\frac{g_{D_{(j:k),i}}P_{D_{j,i}}}{N_0+I_{D_{(j:k),i}}}\right)\right) - \gamma\sum_{j\in\mathcal{D}}|\mathbf{B}[j]|_1 \tag{6.7a}$$

$$\text{subject to:} \quad b_{i,j}\in\{0,1\} \quad \forall i\in\mathcal{C}, \quad \forall j\in\mathcal{D} \tag{6.7b}$$

$$\sum_{i\in\mathcal{C}}P_{C_i}\le P_{C_{\max}} \tag{6.7c}$$

$$\sum_{i\in\mathcal{C}}P_{D_{j,i}}\le P_{D_{\max,j}} \ \forall j\in\mathcal{D} \tag{6.7d}$$

$$P_{D_{j,i}}\le b_{i,j}P_{D_{\max,j}} \ \forall i\in\mathcal{C} \ \ \forall j\in\mathcal{D} \tag{6.7e}$$

$$W\log_2\left(1+\frac{g_{C_i}P_{C_i}}{N_0+I_{C_i}}\right)\ge R_{C_{\min,i}} \ \forall i\in\mathcal{C} \tag{6.7f}$$

$$\sum_{i\in\mathcal{C}}W\log_2\left(1+\min_{k\in\mathcal{M}_j}\frac{g_{D_{(j:k),i}}P_{D_{j,i}}}{N_0+I_{D_{(j:k),i}}}\right)\ge R_{D_{\min,j}} \ \forall j\in\mathcal{D} \tag{6.7g}$$

$$I_{C_i}\ge\sum_{j\in\mathcal{D}}h_{D_{j,i}}P_{D_{j,i}} \ \forall i\in\mathcal{C} \tag{6.7h}$$

$$I_{D_{(j:k),i}}\ge\sum_{j'\neq j\in\mathcal{D}}h_{D_{(j',(j:k)),i}}P_{D_{j',i}}+h_{C_{(j:k),i}}P_{C_i} \ \forall i\in\mathcal{C}, \quad \forall j\in\mathcal{D}, \quad \forall k\in\mathcal{M}_j \tag{6.7i}$$

This problem is still non-convex with many non-convex constraints. We propose using a quadratic transformation to handles all fractions in both the objective function and the constraints with parameters $\boldsymbol{y}$ for the objective function and $\boldsymbol{u}$ for the constraints. Algorithm 4 highlights the solution for this work.

Algorithm 4 Centralized Resource Allocation (Paper H)

Initialize: feasible $\boldsymbol{B}^{(0)}, \boldsymbol{P}_C^{(0)}, \boldsymbol{P}_D^{(0)}, \alpha^{(r)} = (0,1]$ and $r = 0$
Set $\hat{\boldsymbol{P}}_C^{(0)} = \boldsymbol{P}_C^{(0)}$, $\hat{\boldsymbol{P}}_D^{(0)} = \boldsymbol{P}_D^{(0)}$
Compute $\hat{I}_{C_i}^{(0)}$, $\hat{I}_{D_{(j:k)},i}^{(0)}$ $\forall\ i \ \in \ \mathcal{C}, \forall\ j \ \in \ \mathcal{D}, \forall\ k \ \in \ \mathcal{M}_j$ via (6.2)
repeat
    $r = r + 1$
    for all $i \in \mathcal{C}$ **do**
        Compute Γ_{C_i}, y_{C_i}
        for all $j \in \mathcal{D}$ **do**
            Compute $\Gamma_{D_{j,i}}$, $y_{D_{j,i}}$ and $u_{D_{(j:k)},i}$
        end for
    end for
    Compute $\boldsymbol{B}^{(r)}, P_{C_i}^{(r)}, P_{D_{j.i}}^{(r)}, I_{C_i}^{(0)}, I_{D_{(j:k},i}^{(0)}(\forall i \in \mathcal{C}, j \in \mathcal{D})$.
    Set $\hat{\boldsymbol{P}}_C^{(r+1)} = \hat{\boldsymbol{P}}_C^{(r)} + \alpha^{(r)}\left(\boldsymbol{P}_C^{(r)} - \hat{\boldsymbol{P}}_C^{(r)}\right)$, $\hat{\boldsymbol{P}}_D^{(r+1)} = \hat{\boldsymbol{P}}_D^{(r)} + \alpha^{(r)}\left(\boldsymbol{P}_D^{(r)} - \hat{\boldsymbol{P}}_D^{(r)}\right)$
    Set $\hat{\boldsymbol{I}}_C^{(r+1)} = \hat{\boldsymbol{I}}_C^{(r)} + \alpha^{(r)}\left(\boldsymbol{I}_C^{(r)} - \hat{\boldsymbol{I}}_C^{(r)}\right)$, $\hat{\boldsymbol{I}}_D^{(r+1)} = \hat{\boldsymbol{I}}_D^{(r)} + \alpha^{(r)}\left(\boldsymbol{I}_D^{(r)} - \hat{\boldsymbol{I}}_D^{(r)}\right)$
until $\boldsymbol{B}^{(r)}, P_{C_i}^{(r)}, P_{D_{j.i}}^{(r)}$ converges $(\forall i \in \mathcal{C}, j \in \mathcal{D})$

6.3.1 Convergence Analysis

The solution in Paper G has the same convergence characteristics presented in Chapter 3. Therefore, the power allocation subproblems converge to a stationary point $\bar{\mathbf{p}}$ with $\|\mathbf{p}^{[k]} - \bar{\mathbf{p}}\|_2 \leq Ck^{-(1-\theta)/(2\theta-1)}$ for some $C > 0$ and $\theta \in [0.5, 1)$. Similarly, the relaxed channel assignment subproblem is quadratic, convex and Lipschitz smooth problem with linear constraints and solving this problem will converge as $\mathcal{O}(1/k)$. In paper H, we showed that the proposed sequential parametric convex approximation approach is guaranteed convergence to a stationary point.

6.4 Numerical Results

In this section we present some of the numerical results obtained in Paper G. In which, we consider a simulation scenario with a single cell of radius 500 m. In this cell, CUs and D2D transmitters are located uniformly at random. The D2D receivers are located uniformly at random in a 5 m radius circle centered at their respective transmitter. A path-loss model with exponent $\alpha = 2$ is used in the calculation of all channel gains. The random channel gains are calculated by applying an exponential random distribution around an average calculated from the path-loss model. $N_C = 6, N_D = 3$ were used in the experiments with Monte-Carlo averages carried over 1000 different realizations.

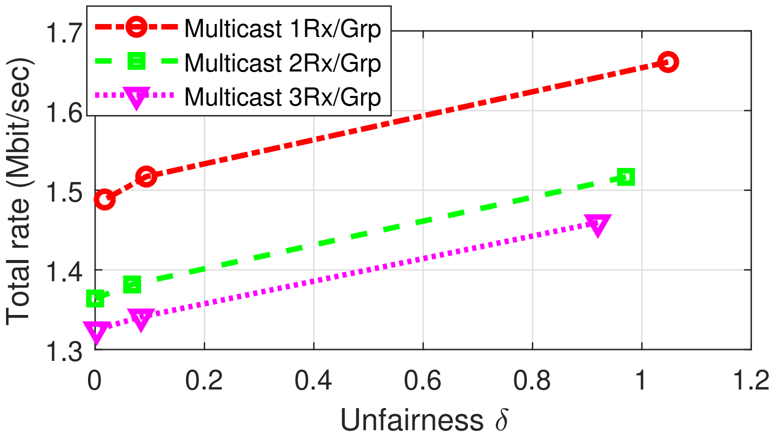

Figure 6.3: Total average rate R vs. Unfairness δ (γ from 10 to 100) (Paper G)

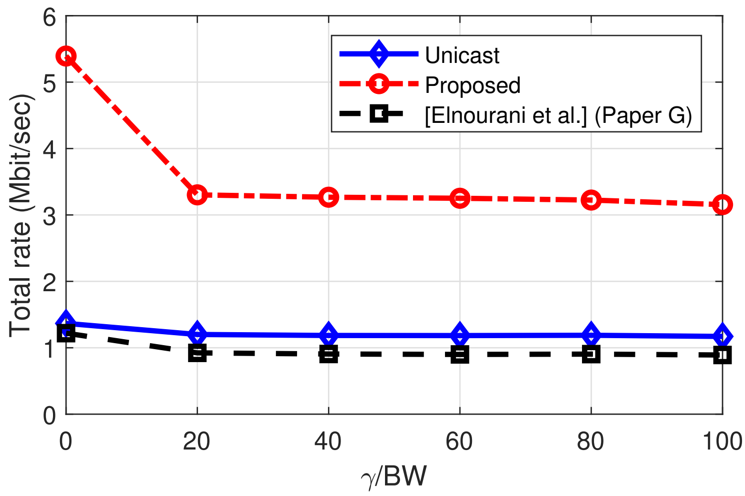

Figure 6.4: Total average rate R vs. γ (Paper H)

Fig. 6.3 shows the performance of the proposed method when changing the number D2D receivers in each multicast group. The total network rate decreases with each additional receiver in the group, since the rate in each group is determined by the receiver with the worst communication conditions. However, these decrements in rate are getting smaller as the number of receivers becomes larger. As expected, increasing the value of γ decreases the rate while decreasing the unfairness.

In Paper H, we simulated a similar environment with $N_C = 4, N_D = 4, |\mathcal{M}_j| = 3$. We compared our proposed solution with the one in Paper G and a unicast solution from Paper A. Fig. 6.4 shows the results of this experiment. It shows that the proposed method achieves significantly higher rate compared to both the unicast method in Paper A and the multicast method in Paper G. When γ increases, the rate decreases in all methods, as expected.

6.5 Contributions

In this work, we present various resource allocation solutions for multicast D2D communication. The main contributions of this work can be summarized as follows:

1. We formulate a joint power allocation and channel assignment problem to maximize the sum rate of all MD2D groups and CUs with a constraints on the minimum rate requirement for both MD2D groups and CUs.

2. We present mathematically efficient algorithm for the formulated problem even though it is a mixed integer non-convex problem. We present both a centralized and a decentralized solution for this problem.

3. We also provide convergence rate guarantees for all algorithms in Paper H.

4. Evaluation of the algorithm is presented on the basis of Matlab simulations to demonstrate the merits of each solution.

Chapter 7

Concluding Remarks and Future Work

This work has formulated and presented several resource allocation algorithms for various non-convex D2D scenarios. The main contributions of this PhD Thesis work are summarized below:

- It studies non-convex optimization problems in various scenarios for D2D communications including SISO, MIMO and multicast frameworks, considering both the cases of non-interference and interference among D2D pairs.
- It analyses these optimizations under both the cases of both perfect CSI and imperfect CSI considering different models, that is, taking into account different qualities of channel gain estimations.
- It proposes several computationally efficient algorithms that can be implemented centrally at the BS.
- It proposes also several computationally efficient alternative algorithms that can be implemented in a decentralized manner in cooperation between the BS, the D2D pairs and the CUs.
- All the proposed algorithms can work in either uplink or downlink spectrum. Moreover, a mechanism to jointly allocate uplink and downlink spectrum is presented.
- In addition, it provides convergence rates and guarantees for most of the presented algorithms.
- It includes several numerical results that show better performance than the existing state-of-the-art algorithms.

7.1 Future works

Wireless D2D communication is an integral part of the current 5G networks and will continue to be part of the next generations for the foreseeable future, i.e. 6G. This motivates even more research work in this field. To this end, this work can be directly extended in the following directions:

- Analyse the dependency of the optimization methods with the estimation quality in channel gain cartography [43] to obtain a complete framework for joint channel gain estimation and resource allocation in D2D networks.
- Formulate Deep Neural Network (DNN) based algorithms, along the lines in [44], for these non-convex optimization problems, so that they can optimize similar objective functions with both supervised and unsupervised learning methods.
- Investigate the use of Graph Neural Networks (GNN) [45] in order to incorporate the knowledge of the network topology in the optimization problem while optimizing the objective function.
- Fully decentralized solution leveraging Game Theory. Previous works have used Game Theory on either channel assignment [10] or power allocation [13]. A complete game-based decentralized method that accommodates both power allocation and channel assignment, can be investigated.
- Implement the algorithms in Software Define Radios to test their performance in real radio environments.

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Part II

Appended Papers

Appendix A

Paper A

Title:	Underlay Device-to-Device Communications on Multiple Channels.
Authors:	Mohamed Elnourani, Mohamed Hamid, Daniel Romero, Baltasar Beferull-Lozano.
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Conference:	IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP 2018, Calgary, Canada.
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PAPER A

Underlay Device-to-Device Communications on Multiple Channels

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Abstract

Since the spectral efficiency of wireless communications is already close to its fundamental bounds, a significant increase in spatial efficiency is required to meet future traffic demands. Device-to-device (D2D) communications provide such an increase by allowing nearby users to communicate directly without passing their packages through the base station. To fully exploit the benefits of this paradigm, proper channel assignment and power allocation algorithms are required. The main limitation of existing schemes, which restrict D2D transmitters to operate on a single channel at a time, is circumvented by the joint channel assignment and power allocation algorithm proposed in this paper. This algorithm relies on convex relaxation to efficiently obtain nearly-optimal solutions to the mixed-integer program arising in this context. Numerical experiments corroborate the merits of the proposed scheme relative to state-of-the art alternatives.

Index Terms

Device-to-device communications, power allocation, channel assignment, convex relaxation.

I. Introduction

The exponentially increasing throughput demands of cellular communications [1], [2] can no longer be met by increasing the *spectral* efficiency of point-to-point links, e.g. through improvements in modulation and coding, since existing systems already approach the channel capacity [3], [4]. Hence, many contemporary research efforts aim at increasing *spatial* efficiency. Device-to-device (D2D) communications constitute a prominent example, where mobile users are allowed to communicate directly with each other without passing their messages through the base station (BS) [5]–[7]. Thus, users operating in D2D mode need half the time slots of those operating in the traditional cellular mode. Moreover, time slots used by D2D users can be simultaneously used by a traditional cellular user if both links do not interfere much, a technique termed *underlay*. To fully unlock the potential of underlay D2D communications, algorithms providing a judicious assignment of cellular sub-channels (e.g. resource blocks or time slots) to D2D users and a prudent power control mechanism that limits interference to cellular users need to be devised.

Early works on D2D communications rely on simplistic channel assignment schemes, where each pair of D2D devices communicate through a cellular sub-channel (hereafter referred to as *channel*) selected uniformly at random. The impact of selecting a channel with poor quality has been counteracted by choosing among different modes of operation [8] or by sensing the selected channel [9]. Unfortunately, these approaches do not provide optimal throughput due to this random channel assignment and because no power control is effected to limit interference. To sidestep these limitations, [10] proposes a scheme where each D2D pair simultaneously transmits in all cellular channels and adjusts the transmit power at each of

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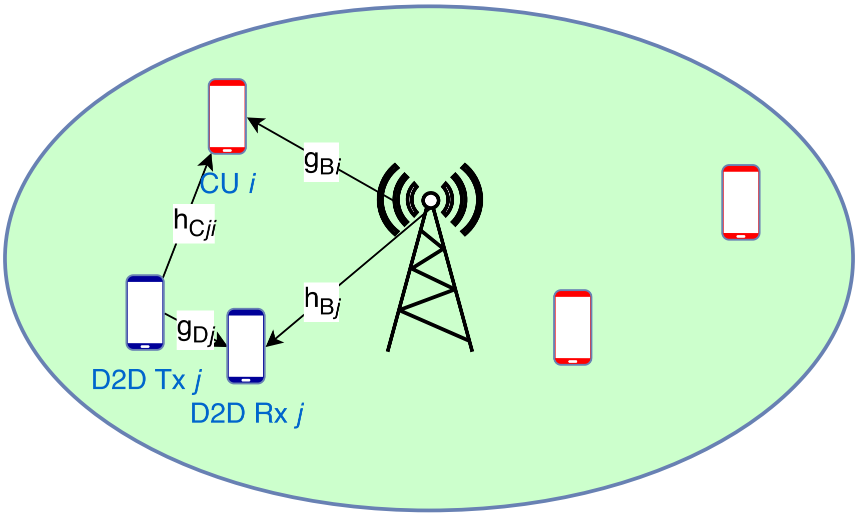

Fig. 1: Illustration of the system model.

them. However, since every D2D pair adjusts power separately, important performance losses are expected when multiple D2D pairs operate in the same cell due to interference. Such a limitation is bypassed in [11], [12], where channels are jointly assigned by the BS to all D2D pairs. However, these works do not implement power control, which renders their channel assignments sub-optimal. This observation motivates joint channel assignment and power allocation as in [13]–[16]. Unfortunately, these schemes restrict D2D users to access at most one cellular channel. To sum up, no existing approach provides joint channel assignment and power allocation for the scenario where D2D users can operate on more than one cellular channel simultaneously, which is of high interest especially in crowded areas.

The present paper fills this gap by developing a joint channel assignment and power allocation scheme that allows each D2D pair to use more than one cellular channel. The adopted objective function involves throughput and promotes fair channel allocations through a regularizer, which is necessary to prevent most channels from being assigned to a small subset of D2D users. An efficient algorithm for approximately solving the resulting mixed-integer optimization problem is developed based on convex relaxation. A simulation study demonstrates the superior performance of the proposed method relative state-of-the-art alternatives.

The rest of this paper is structured as follows. Sec. II describes the system model. Sec. III introduces a novel channel assignment and resource allocation criterion and proposes an efficient solver. Finally, Sec. IV provides the simulations and Sec. V summarizes conclusions.

II. System Model

Consider a cell (or sector) where a BS communicates with N_C cellular users (CUs) through N_C downlink channels.[1] For convenience, the set of CUs (or, equivalently, channels) will be indexed by $\mathcal{C} = \{1, ..., N_C\}$. In this cell, N_D D2D pairs, indexed by $\mathcal{D} = \{1, ..., N_D\}$, wish to communicate using the aforementioned downlink channels at the same time as the BS (underlay). The assignment of channels to D2D pairs will be represented by the indicators $\{\beta_{i,j}\}_{i\in\mathcal{C},j\in\mathcal{D}}$, where $\beta_{i,j} = 1$ when D2D pair j uses channel i and $\beta_{i,j} = 0$ otherwise. It will be assumed that each D2D pair can access multiple channels at the same time, but no channel can be used by multiple D2D pairs, which implies that $\sum_{j=1}^{N_D} \beta_{i,j} \leq 1, \forall i$. The transmission power used by the base station to communicate with the i-th CU is represented by P_{B_i} and

[1]Recall that channel in this context may stand for resource blocks, time slots, and so on.

is constrained to lie in the interval $0 \leq P_{B_i} \leq P_{B_{\max}}$. Similarly, $P_{D_{ji}}$ is the transmission power used by the j-th D2D pair when utilizing the i-th channel and is constrained as $0 \leq P_{D_{ji}} \leq P_{D_{\max}}$. Successful communications require that the signal-to-interference-plus-noise ratio (SINR) be greater than $\eta_{\min}^{C}$ for CUs and $\eta_{\min}^{D}$ for D2D receivers.

Fig. 1 illustrates the notation conventions for channel gains. Specifically, g_{B_i} denotes the gain between the BS and the i-th CU; g_{D_j} the gain of the j-th D2D link; $h_{C_{j,i}}$ the gain of the interference link between the transmitter of the j-th D2D pair and the i-th CU; h_{B_j} the gain of the interference link between the BS and the receiver of the j-th D2D pair; and N_0 the noise power.[2]

Given $g_{B_i},\ g_{D_j},\ h_{C_{ji}},\ h_{B_j}\ \forall i,j$, as well as N_0, $\eta_{\min}^{C}$, $\eta_{\min}^{D}$, $P_{C_{\max}}$, and $P_{D_{\max}}$, the goal is to choose $\beta_{i,j},\ P_{B_i},\ P_{D_{ji}}\ \forall i,j$ to maximize the aggregate throughput of the D2D pairs and CUs while ensuring fairness among multiple D2D pairs and preventing detrimental interference to CUs.

III. Joint channel assignment and power allocation

This section proposes a novel algorithm for channel assignment and power allocation that allows multiple D2D users in each cellular channel. Sec. III-A formulates the optimization problem and Sec. III-B proposes a solver. To simplify notation, collect the requested variables in vector-matrix form as $\boldsymbol{B} = [\beta_{i,j}]_{i,j} \in \mathbb{R}^{N_C \times N_D}$, $\boldsymbol{P_D} = [P_{D_{j,i}}]_{j,i} \in \mathbb{R}^{N_D \times N_C}$, and $\boldsymbol{p_B} = [P_{B_i}]_i \in \mathbb{R}^{N_C}$.

A. Channel Assignment and Power Allocation Criterion

This section formulates the problem of joint channel assignment and power allocation as an optimization problem. The first step is therefore to select a criterion that quantifies how desirable a given channel assignment and power allocation $(\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D})$ is. As described next, the criterion adopted here equals the overall network binary rate plus a term that penalizes unfair channel assignments.

To obtain the overall network rate, let $\Gamma(z) := \log_2(1+z)$ and note that the total rate at channel i is given by $R_i := \sum_{j\in\mathcal{D}} \beta_{i,j}[R_{C_{i,j}} + R_{D_{j,i}}] + (1 - \sum_{j\in\mathcal{D}} \beta_{i,j}) R_{C_{i,0}}$, where $R_{C_{i,j}} = \Gamma(P_{B_i} g_{B_i}/(N_0 + P_{D_{ji}} h_{C_{j,i}}))$ denotes the rate of the i-th CU when sharing the channel with the j-th D2D pair ($\beta_{ij} = 1$); $R_{D_{j,i}} = \Gamma(P_{D_{ji}} g_{D_j}/(N_0 + P_{B_i} h_{B_j}))$ the rate of the j-th D2D pair when sharing the channel with the i-th CU ($\beta_{ij} = 1$); and $R_{C_{i,0}} = \Gamma(P_{B_{max}} g_{B_i}/N_0)$ the rate of the i-th CU when it shares its channel with no D2D pair ($\beta_{ij} = 0\ \forall j$). The overall network rate is therefore $R := \sum_{i\in\mathcal{C}} R_i$.

The second term of the objective penalizes channel assignments where a small fraction of D2D pairs use a large part of the channels. To this end, the *unfairness* measure $\delta(\boldsymbol{B})$ from [11] will be used. It is given by $\delta^2(\boldsymbol{B}) = 1/(N_D x_0^2) \sum_{j=1}^{N_D} (x_j(\boldsymbol{B}) - x_0)^2$, where $x_j := \sum_{i=1}^{N_C} \beta_{i,j}$ is the number of channels assigned to the j-th D2D pair and $x_0 := N_C/N_D$. If N_C is an integer multiple of N_D, then $x_j = x_0\ \forall j$ would be fairest channel assignment possible. $\delta(\boldsymbol{B})$ can be interpreted as the root mean deviation of $\{x_j\}_{j=1}^{N_D}$ from their fairest value x_0 and therefore is larger the more unevenly channels are assigned among D2D pairs.

[2]Note that g_{D_j} and h_{B_j} should in principle depend also on i since the associated gains generally depend on the channel selected by the j-th pair; however, this subscript is dropped for simplicity since the proposed scheme carries over immediately to accommodate such dependence.

The overall problem can then be formulated as:

$$\underset{\boldsymbol{B},\boldsymbol{p_B},\boldsymbol{P_D}}{\text{maximize}} \quad R(\boldsymbol{B},\boldsymbol{p_B},\boldsymbol{P_D}) - \gamma\delta^2(\boldsymbol{B}) \tag{1a}$$

$$\text{subject to} \quad \beta_{i,j} \in \{0,1\}, \ \sum_{j=1}^{N_D} \beta_{i,j} \leq 1\,\forall i \tag{1b}$$

$$0 \leq P_{B_i} \leq P_{B_{max}}\,\forall i \tag{1c}$$

$$0 \leq P_{D_{ji}} \leq P_{D_{max}}\,\forall j,i \tag{1d}$$

$$\forall i,j,\ \frac{P_{B_i} g_{Bi}}{N_0 + P_{D_{ji}} h_{C_{j,i}}} \geq \eta^C_{min} \text{ if } \beta_{ij} = 1 \tag{1e}$$

$$\forall i,j,\ \frac{P_{D_{ji}} g_{Dj}}{N_0 + P_{B_i} h_{B_j}} \geq \eta^D_{min} \text{ if } \beta_{ij} = 1. \tag{1f}$$

Problem (1) is a mixed-integer program. Therefore it is non-convex and difficult to solve since it involves combinatorial complexity. The next section provides an efficient method to find an approximately optimal solution to (1).

B. Optimization via Convex Relaxation

This section presents an efficient method to approximate the solution to (1). Several tricks are applied to decompose (1) into multiple sub-problems of much lower complexity without any loss of optimality. One of these problems is an integer program, whereas the rest are problems that admit a closed-form solution. The proposed algorithm relies on convex relaxation to approximate the solution to the integer program.

The first step is to rewrite R in a simpler form. From the definitions of R and R_i in Sec. III-A, it follows after rearranging terms that

$$R(\boldsymbol{B},\boldsymbol{p_B},\boldsymbol{P_D}) = \sum_{i\in\mathcal{C}} \left[\sum_{j\in\mathcal{D}} \beta_{i,j} v_{i,j}(P_{B_i}, P_{D_{ji}}) + R_{C_{i,0}} \right], \tag{2}$$

where $v_{i,j}(P_{B_i}, P_{D_{ji}}) := R_{C_{i,j}} + R_{D_{j,i}} - R_{C_{i,0}}$ denotes the *rate increment* due to assigning the channel i to D2D pair j relative to the case where the channel i is only used by the CU.

It is next shown that (1) can be solved in two steps without loss of optimality: first, power allocation and, second, channel assignment. The trick is to replicate $\{P_{B_i}\}_i$ as described next. From (2), it follows that the objective of (1) can be written as $\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}} \beta_{i,j} v_{i,j}(P_{B_i}, P_{D_{ji}})$ plus some terms that do not depend on $\{P_{B_i}\}_i$. Clearly, an equivalent problem is obtained if P_{B_i} in each term $\beta_{i,j} v_{i,j}(P_{B_i}, P_{D_{ji}})$ is replaced with $P_{B_{i,j}}$ so long as the constraint $P_{B_{i,1}} = P_{B_{i,2}} = \ldots = P_{B_{i,N_D}}$ is enforced for all i. The resulting objective becomes $\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}} \beta_{i,j}$ $v_{i,j}(P_{B_{i,j}}, P_{D_{ji}})$ plus terms that do not depend on $\{P_{B_{i,j}}\}_{i,j}$. One can similarly replace P_{B_i} with $P_{B_{i,j}}$ in (1e)-(1f) and also replace (1c) with $0 \leq P_{B_{i,j}} \leq P_{B_{max}}\,\forall i,j$ and the resulting problem will be equivalent to (1). Except for the recently introduced equality constraints, the objective and active constraints will only depend on at most one of the $\{P_{B_{i,j}}\}_j$ for each i. Thus, the equality constraint $P_{B_{i,1}} = \ldots = P_{B_{i,N_D}}$ can be dropped without loss of optimality. Similarly, one can also remove the condition "if $\beta_{i,j} = 1$" from (1e)-(1f). The resulting problem reads as

$$\begin{aligned}
&\underset{\boldsymbol{B},\boldsymbol{P_B},\boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\left[\beta_{i,j}v_{i,j}(P_{B_{ij}},P_{D_{ji}})\right]-\gamma\delta^2(\boldsymbol{B})\\
&\text{subject to} \quad \beta_{i,j}\in\{0,1\},\ \sum_{j=1}^{N_D}\beta_{i,j}\le 1\,\forall i \qquad (3)\\
&\forall j,i \quad 0\le P_{B_{ij}}\le P_{B_{max}},\quad 0\le P_{D_{ji}}\le P_{D_{max}}\\
&\forall i,j \quad \frac{P_{B_{ij}}g_{Bi}}{N_0+P_{D_{ji}}h_{C_{j,i}}}\ge\eta^C_{min},\ \frac{P_{D_{ji}}g_{Dj}}{N_0+P_{B_{ij}}h_{B_j}}\ge\eta^D_{min}
\end{aligned}$$

where $\boldsymbol{P_B} := [P_{B_{ij}}]_{i,j}$ and $\gamma > 0$ is a user-selected regularization parameter that balances the fairness-rate trade-off. To recover the optimal $\{P_{B_i}\}_i$ of (1) from the optimal $\{P_{B_{i,j}}\}_{i,j}$ of (3), one just needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $P_{B_i} = P_{B_{i,j}}$. If no such a j exists, i.e. $\beta_{i,j} = 0\ \forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power $P_{B_i} = P_{B_{\max}}$.

Optimizing (3) with respect to $\boldsymbol{P_B}$ and $\boldsymbol{P_D}$ decouples across i and j into the N_CN_D subproblems

$$\begin{aligned}
&\underset{P_{B_{ij}},P_{D_{ji}}}{\text{maximize}} \quad v_{i,j}(P_{B_{ij}},P_{D_{ji}}) \qquad (4)\\
&\text{subject to} \quad 0\le P_{B_{ij}}\le P_{B_{max}},\ 0\le P_{D_{ji}}\le P_{D_{max}}\forall i,j\\
&\frac{P_{B_{ij}}g_{Bi}}{N_0+P_{D_{ji}}h_{C_{j,i}}}\ge\eta^C_{min},\ \frac{P_{D_{ji}}g_{Dj}}{N_0+P_{B_{ij}}h_Bj}\ge\eta^D_{min},\ \forall i,j,
\end{aligned}$$

which should be solved $\forall i\in\mathcal{C},\forall j\in\mathcal{D}$. This power allocation subproblem coincides with the one arising in [13], which can be solved in closed-form as described therein.

Once (4) has been solved $\forall i\in\mathcal{C},\forall j\in\mathcal{D}$, it remains to substitute the optimal values of $v_{i,j}$ into (3) and minimize with respect to $\boldsymbol{B}$. If (4) is infeasible for a given (i,j), then set its optimal value to $v_{i,j} = -\infty$. The resulting channel assignment subproblem becomes:

$$\begin{aligned}
&\underset{\boldsymbol{B}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\beta_{i,j}v_{i,j}-\gamma\delta^2(\boldsymbol{B}), \qquad (5)\\
&\text{subject to} \quad \beta_{i,j}\in\{0,1\}\ \forall i,j,\quad \sum_{j\in\mathcal{D}}\beta_{i,j}\le 1\ \forall i.
\end{aligned}$$

Problem (5) is an integer program of combinatorial complexity. Finding an exact solution is too computationally expensive and time consuming for sufficiently large N_CN_D, and therefore not suitable for real-time implementation as required by the application at hand. For this reason it is preferable to sacrifice some optimality if an approximately optimal solution can be found with a low computational complexity and therefore short processing time. To this end, one can leverage the notion of *convex relaxation* as described next.

The idea is that the source of non-convexity of (5) is the integer constraint $\beta_{i,j}\in\{0,1\}$. Replacing such a constraint with $\beta_{i,j}\in[0,1]$ will render (5) convex.[3] The resulting convexified problem can be efficiently solved e.g. through projected gradient descent [17]. Discretizing the solution $\{\tilde{\beta}_{i,j}\}_{i,j}$ to such a problem is expected to yield an approximately optimal optimum of (5). To this end, this paper considers two approaches: (A1) For every i, set $\beta_{i,j} = 1$ if $j = \arg\max_j \tilde{\beta}_{i,j}$. (A2) For each i, consider a random variable J_i taking values $1,\ldots,N_D$ with probabilities $P(J_i = j) = \tilde{\beta}_{i,j}$ (normalize $\{\tilde{\beta}_{i,j}\}_j$ to sum 1 if necessary). Then generate multiple realizations of $\{J_i\}_i$ and form the matrix $\boldsymbol{B}$, whose (i,j)-th entry is 1 if $J_i = j$ and 0 otherwise. Now evaluate the objective of (5) for all these realizations and select the realization with the highest objective value.

[3]Strictly speaking, minimizing the negative of the objective of (5).

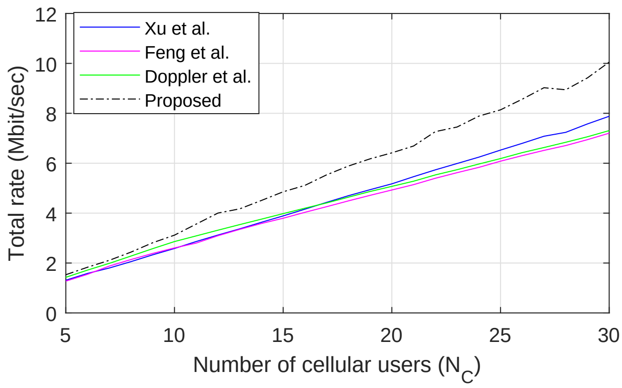

Fig. 2: Total rate R vs. N_C ($\gamma = 20,\ N_D = 10,\ N_0 = -70$ dBW, discretization via A2).

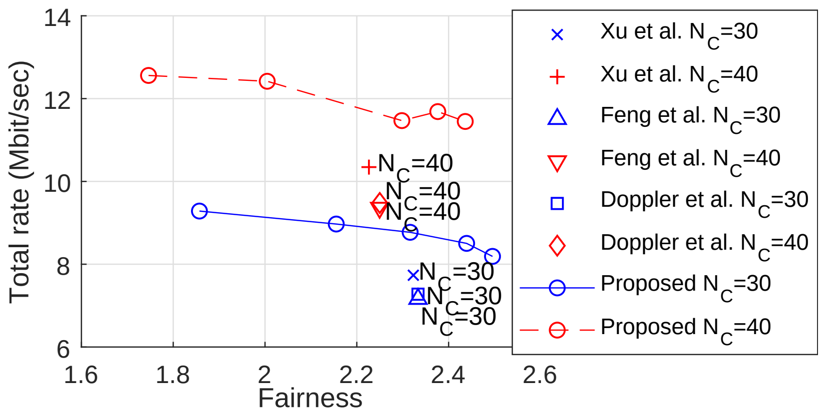

Fig. 3: Total rate R vs. fairness $\bar{\delta}$ for different values of N_C ($N_D = 10,\ N_0 = -70$ dBW, discretization via A2).

IV. Simulations

This section compares the algorithm developed in Sec. III with state-of-the-art alternatives. The simulation setup comprises a circular cell with 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. Figures display averages over 100 independent realizations of the user locations with channels of 15 kHz. The proposed algorithm is compared with (i) the method by Xu et al. [11], which uses a price auction game for channel assignment without any power control, yet it allows D2D users to use multiple channels at the same time; (ii) the method by Doppler et al. [8], which randomly assigns a single channel to each D2D pair and selects among three modes of operation; and (iii), the method by Feng et al. [13], which jointly assigns a channel to each D2D pair and allocates power to maximize the total rate.

Fig. 2 depicts the total rate R of all four compared methods as a function of the number of cellular channels N_C. It is observed that the proposed method uniformly achieves the highest rate among all compared schemes; in particular, for $N_C = 30$, the rate of the proposed algorithm is approximately 25% more than the nearest competing alternative. In contrast to the methods by Feng et al. and Doppler et al., whose rates increments saturate for sufficiently large N_C since each D2D pair is only allowed to use at most one cellular channel, the rate of the proposed method steadily increases with N_C.

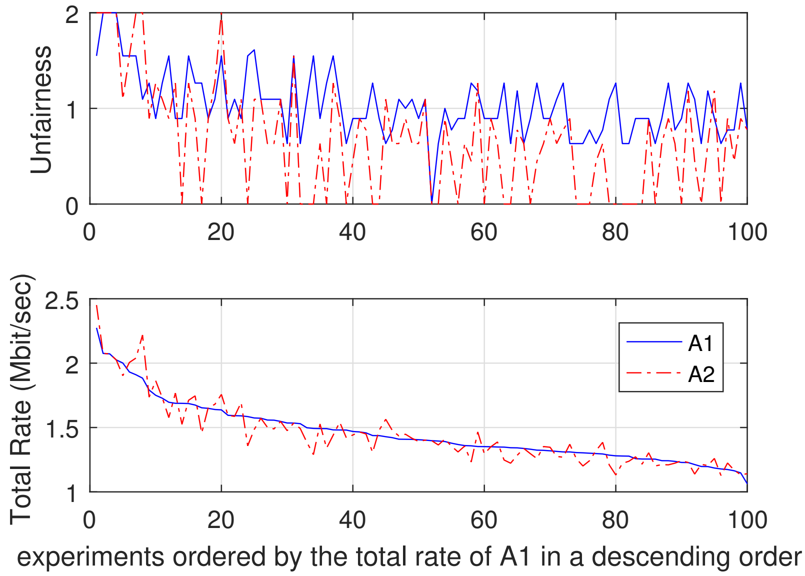

Fig. 4: Unfairness and rate for the discussed discretization approaches ($N_C = 5,\ N_D = 5$, $N_0 = -70$ dBW, $\gamma = 20$).

Fig. 3 represents the total rate vs. *fairness*, which is defined as $\bar{\delta}(\boldsymbol{B}) := \sqrt{N_D - 1} - \delta(\boldsymbol{B}) \in [0, \sqrt{N_D - 1}]$. Multiple points are obtained for the proposed method by varying γ between 40 and 200. In Fig. 3, the flexibility of the proposed method to adjust the desired point of the rate-fairness trade-off is manifest. Competing methods lack such flexibility. Moreover, over 10% increment in the total rate with respect to the nearest competing method is achieved with roughly the same fairness. This relative advantage increases further with N_C.

Finally, Fig. 4 compares the two discretization approaches provided at the end of Sec. III to recover the solution of (5) from the solution to its relaxed counterpart. Approach A2 is seen to yield nearly the same rate as A1 and an improved fairness. However, the computational complexity of A2 is significantly higher than that of A1.

V. Conclusions

This paper presented an algorithm for joint channel assignment and power allocation in underlay D2D cellular networks. The major novelty is to allow D2D pairs to operate on multiple cellular channels at the same time, which greatly increases throughput. After adopting a criterion that promotes high throughput and fairness, the resulting mixed-integer program is decomposed into multiple subproblems that are efficiently solved. Future research will develop distributed implementations, accommodate uncertainty in the channel gains, and incorporate user behavior models.

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Appendix B

Paper B

Title:	Reliable Underlay Device-to-Device Communications on Multiple Channels.
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PAPER B

Reliable Underlay Device-to-Device Communications on Multiple Channels

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B

Abstract

Device-to-device (D2D) communications provide a substantial increase in spectrum usage and efficiency by allowing nearby users to communicate directly without passing their packets through the base station (BS). In previous works, proper channel assignment and power allocation algorithms for sharing of channels between cellular users and D2D pairs, usually require exact knowledge of the channel-state-information (CSI). However, due to the non-stationary wireless environment and the need to limit the communication and computation overheads, obtaining perfect CSI in the D2D communication scenario is generally not possible. In this work, we propose a joint channel assignment and power allocation strategy for D2D pairs and cellular users to maximize the overall aggregate throughput, under imperfect knowledge of CSI, while guaranteeing the outage probability for all users and encouraging fairness among D2D pairs. The proposed solution does not restrict the D2D transmitters to operate on a single channel, allowing each D2D pair to simultaneously access multiple channels and increase the overall throughput. We propose both a centralized and a decentralized method to solve our problem, where the computation load of the BS is alleviated by decomposing our problem into several subproblems, each of them being solved iteratively at the individual D2D pairs. Numerical experiments corroborate the merits of the proposed schemes when compared with state-of-the-art alternatives.

Index Terms

D2D communications, power allocation, channel assignment, reliability, convex relaxation.

I. Introduction

The exponentially increasing throughput demand in cellular communication networks [1] can no longer be met by increasing the *spectral* efficiency of point-to-point links, since existing systems are already approaching the channel capacity [2]. D2D communications constitute a prominent example in improving *spatial* efficiency, where mobile users are allowed to communicate directly with each other without passing their messages through the BS [3], [4]. Thus, users operating in D2D mode need half of resources of those operating in the traditional cellular mode. Moreover, channels (e.g. frequency bands or time slots) used by D2D users can be simultaneously used by a traditional cellular user under restricted interference configuration, a framework termed *underlay*. It is necessary to devise algorithms that judiciously assign cellular channels to D2D users and prudently control the power to limit interference to cellular users and guarantee quality of service (QoS) (e.g. SINR, reliability) to all users. In addition, algorithms must be computationally inexpensive and reliable in imperfect CSI cases.

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Early works on D2D communications typically rely on simplistic channel assignment schemes, where each D2D pair communicate through a cellular sub-channel (hereafter referred to as *channel*) selected uniformly at random by the BS [5]. However, these approaches control interference in a simplistic way. To overcome these limitations, [6] proposes a scheme where each D2D pair simultaneously transmits in all cellular channels. However, every pair control its own power independently. In [7], channels are judiciously assigned by the BS. However, this work does not incorporate power control. In [8]–[10], joint channel assignment and power allocation was proposed. These schemes restrict D2D users to access at most one channel. In addition, all the aforementioned works have adopted a model with perfect CSI. There are few works that consider reliability by guaranteeing the desired outage probabilities for cellular users (CUs) under imperfect CSI. In [11]–[13], powers and channels are assigned while restricting D2D users to access at most one cellular channel. To sum up, non of the existing approaches provide a reliable joint channel assignment and power allocation for the scenario where D2D users can simultaneously operate on multiple cellular channels, which is very relevant for maximizing throughput among D2D pairs. In addition, considering energy and infrastructure cost at BS, investigation on reducing the computational load at BS also needs appropriate attention.

This paper considers the above challenges by proposing two *reliable* joint channel assignment and power allocation solutions (centralized and decentralized) that allows each D2D pair to use more than one cellular channel while guaranteeing certain SINR and outage probability under *imperfect CSI* scenario. Here, we consider the downlink scenario, however, with minor modifications, the proposed algorithms can easily be adapted to the uplink scenario. The key research contributions of this work can be summarized as:

- We formulate an optimization problem to jointly assign channels and allocate power to D2D pairs in a downlink cellular environment. We also consider guaranteeing a certain outage probability to address imperfect CSI. In addition, we include an unfairness measure which penalizes assigning most channels to a small fraction of D2D pairs. The resulting optimisation problem is a mixed integer non-convex problem.
- We show that our overall problem can be decomposed into several power allocation subproblems and a channel assignment problem without loss of optimality, and propose an efficient centralized algorithm performed at the BS in order to solve our problem.
- We also propose a decentralized algorithm that reduces the computation load at the BS by performing alternating maximization over each of the power allocation subproblems associated to each of the D2D pairs. Moreover, some of the computations for the channel assignment problem are performed by the D2D pairs.

Our simulations show good performance relative to the state-of-the-art alternatives. The rest of this paper is structured as follows. Sec. II describes the system model. Sec. III introduces the joint channel assignment and resource allocation problem and proposes two efficient algorithms to solve it. Finally, Sec. IV provides the simulations.

II. System Model

Consider a cell (or sector) where a BS communicates with N_C CUs through N_C downlink channels[1][2]. For convenience, the set of CUs (or, equivalently, channels) will be indexed by $\mathcal{C}$ = $\{1, ..., N_C\}$. In this cell, N_D D2D pairs, indexed by $\mathcal{D}$ = $\{1, ..., N_D\}$ (typically $N_D < N_C$), wish to communicate using the aforementioned downlink channels at the same time as the BS (underlay communications). The assignment of channels to D2D pairs will be represented

[1]Recall that a channel here may stand for resource blocks, or time slots.

[2]In general, a CU can use multiple channels simultaneously. The same model can be used by putting similar CUs in every channel that was assigned to the same user.

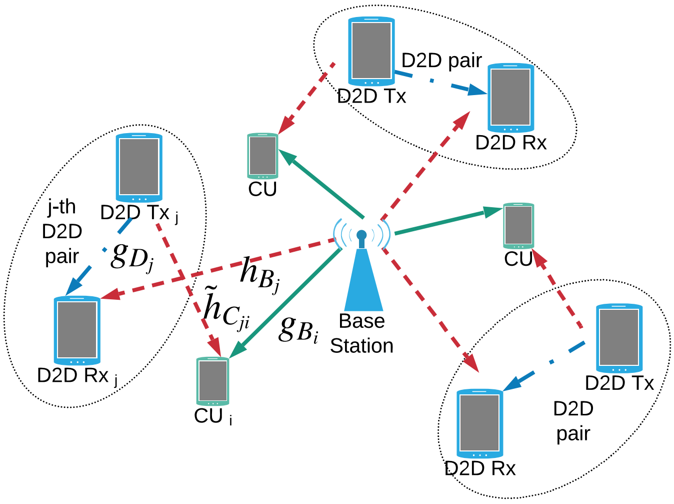

Fig. 1: Illustration of the system model.

by the indicators $\{\beta_{i,j}\}_{i\in\mathcal{C},j\in\mathcal{D}}$, where $\beta_{i,j} = 1$ when the D2D pair j uses channel i and $\beta_{i,j} = 0$ otherwise. It is assumed that each D2D pair can access multiple channels at the same time, but no channel can be used by multiple D2D pairs simultaneously, which implies that $\sum_{j=1}^{N_D} \beta_{i,j} \leq 1, \forall i$. The transmission power used by the BS to communicate with the i-th CU is represented by P_{B_i} and is constrained to lie in the interval $0 \leq P_{B_i} \leq P_{B_{\max}}$. Similarly, $P_{D_{ji}}$ is the transmission power used by the j-th D2D pair when utilizing the i-th channel and is constrained to $0 \leq P_{D_{ji}} \leq P_{D_{\max}}$. Successful communications require the signal-to-interference-plus-noise ratio (SINR) to be greater than $\eta^C_{\min}$ for CUs and $\eta^D_{\min}$ for D2D receivers, and cellular users have a maximum allowed outage ratio of ϵ.

Fig. 1 illustrates the system model. Specifically, g_{B_i} denotes the gain between the BS and the i-th CU; g_{D_j} denotes the gain[3] of the j-th D2D link; $\tilde{h}_{C_{j,i}}$ denotes the gain of the interference link from the transmitter of the j-th D2D pair to the i-th CU, which is modeled as a random variable since it is usually estimated at the receiver with a minimum cooperation with the CU (as opposed to the model in [14]); h_{B_j} denotes the gain of the interference link between the BS and the receiver of the j-th D2D pair; and N_0 the noise power.

Given g_{B_i}, g_{D_j}, h_{B_j}, the distribution of $\tilde{h}_{C_{ji}}$ $\forall i, j$, as well as N_0, $\eta^C_{\min}$, $\eta^D_{\min}$, ϵ, $P_{C_{\max}}$, and $P_{D_{\max}}$, the goal is to choose $\beta_{i,j}$, P_{B_i}, $P_{D_{ji}}$ $\forall i, j$ to maximize the aggregate throughput of the D2D pairs and CUs while ensuring fairness among D2D pairs, by discouraging assigning channels to D2D pairs unequally, and preventing detrimental interference to CUs by guaranteeing the desired outage probability.

III. Joint Channel Assignment and Power Allocation

We next formulate the optimization problem. Then in Sec. III-B, we propose two efficient algorithms. To simplify the notation, let us collect the requested variables in vector-matrix form as $\boldsymbol{B} = [\beta_{i,j}] \in \mathbb{R}^{N_C \times N_D}$, $\boldsymbol{P_D} = [P_{D_{j,i}}] \in \mathbb{R}^{N_D \times N_C}$, and $\boldsymbol{p_B} = [P_{B_i}] \in \mathbb{R}^{N_C}$.

A. Problem formulation

The first step is to select a criterion that quantifies the desirability of a given channel assignment and power allocation $(\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D})$. To guarantee a desired outage probability ϵ,

[3]Note that g_{D_j} and h_{B_j} should in principle depend also on i since the associated gains generally depend on the channel selected by the j-th pair; however, this subscript is dropped for simplicity since the proposed scheme carries over immediately to accommodate such dependence.

we adopt a criterion to maximize the minimum network rate, which must be at least achieved for a $(1-\epsilon)$ portion of the overall time. Additionally, an unfairness term that penalizes unfair channel assignments is included in the objective function. Let us define the rate $\Gamma(z) := \text{BW} \times \log_2(1+z)$, where BW is the channel bandwidth and z is the SINR. The minimum network rate can be considered by analyzing the lower bound of the total rate at channel i, which is defined as: $R_i^{LB} := (1-\sum_{j\in\mathcal{D}} \beta_{i,j}) R_{C_{i,0}} + \sum_{j\in\mathcal{D}} \beta_{i,j} [R_{D_{j,i}} + R_{C_{i,j}}^{LB}]$, where:

- $R_{C_{i,0}} = \Gamma(P_{B_{max}} g_{B_i}/N_0)$ is the rate of the i-th CU when it does not share its channel with D2D pairs $i.e.$ $\beta_{ij} = 0\ \forall j$,
- $R_{D_{j,i}} = \Gamma(P_{D_{ji}} g_{D_j} /(N_0 + P_{B_i} h_{B_j}))$ is the rate of the j-th D2D pair when sharing the channel with the i-th CU $i.e.$ $\beta_{ij} = 1$,
- $R_{C_{i,j}}^{\text{LB}}$ denotes the lower bound (which must be at least achieved $(1-\epsilon)$ portion of the time) of the rate of the i-th CU when sharing the channel with the j-th D2D pair ($\beta_{ij} = 1$). Since $\tilde{h}_{C_{j,i}}$ is random, we can compute $R_{C_{i,j}}^{LB} = \Gamma(z_{C_{i,j}}^{LB})$ where $z_{C_{i,j}}^{LB} : \Pr\{z_{C_{i,j}}^{LB} \leq P_{B_i} g_{B_i}/(N_0 + P_{D_{ji}} \tilde{h}_{C_{j,i}})\} = 1-\epsilon$.
- The minimum network rate is therefore $R(\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D}) := \sum_{i\in\mathcal{C}} R_i^{LB}$.

We consider a second term of the objective that penalizes channel assignments where a small fraction of D2D pairs use a large part of the channels. To this end, the *unfairness* measure $\delta(\boldsymbol{B})$ from [7], [14] is considered in this work. It is given by $\delta^2(\boldsymbol{B}) = 1/(N_D x_0^2) \sum_{j=1}^{N_D} (x_j(\boldsymbol{B}) - x_0)^2$, where $x_j := \sum_{i=1}^{N_C} \beta_{i,j}$ is the number of channels assigned to the j-th D2D pair and $x_0 := N_C/N_D$. If N_C is an integer multiple of N_D, then $x_j = x_0\ \forall j$ would be fairest channel assignment possible. $\delta(\boldsymbol{B})$ can be interpreted as the root mean deviation of $\{x_j\}_{j=1}^{N_D}$ from their fairest value x_0 and thus the more unevenly channels are assigned among D2D pairs, the larger it is.

The overall problem can then be formulated as:

$$\underset{\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D}}{\text{maximize}} \quad R(\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D}) - \gamma\delta^2(\boldsymbol{B}) \tag{1a}$$

$$\text{subject to } \beta_{i,j} \in \{0,1\},\ \sum_{j=1}^{N_D} \beta_{i,j} \leq 1\, \forall i \tag{1b}$$

$$0 \leq P_{B_i} \leq P_{B_{max}}\, \forall i,\ 0 \leq P_{D_{ji}} \leq P_{D_{max}}\, \forall j, i \tag{1c}$$

$$\Pr\left\{\frac{P_{B_i} g_{B_i}}{N_0 + P_{D_{ji}} \tilde{h}_{C_{j,i}}} \geq \eta_{min}^C\right\} \geq (1-\epsilon) \text{ if } \beta_{ij} = 1,\ \forall i, j, \tag{1d}$$

$$\frac{P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j}} \geq \eta_{min}^D \text{ if } \beta_{ij} = 1,\ \forall i, j. \tag{1e}$$

The parameter $\gamma > 0$ is a regularization parameter that balances the rate-fairness trade-off, which is selected in the scale of BW to ensure that the rate and the fairness are of comparable values. Problem (1) is a non-convex mixed-integer program, which involves combinatorial complexity. The next subsection provides two efficient methods to find a solution of (1), namely centralized and decentralized methods.

B. Proposed Optimization Algorithms

Several approaches can be applied to decompose (1) into multiple sub-problems of lower complexity without loss of optimality.

Given the statistical distribution of $\tilde{h}_{C_{j,i}}$, the probabilistic constraint in (1d) can be rewritten as follows:

$$\Pr\left\{\tilde{h}_{Cj,i} \leq \frac{P_{B_{ij}} g_{B_i} - \eta_{min}^C N_0}{P_{D_{ji}} \eta_{min}^C}\right\} \geq 1-\epsilon$$

$$\frac{P_{B_{ij}} g_{Bi}}{N_0 + P_{D_{ji}} F_{\tilde{h}_{C_{j,i}}}^{-1}(1-\epsilon)} \geq \eta_{min}^C,$$

B

where $F^{-1}_{\tilde{h}_{C_{j,i}}}$ is the inverse CDF of $\tilde{h}_{C_{j,i}}$. Similarly, $R^{LB}_{C_{i,j}}$ can be written as: $R^{LB}_{C_{i,j}} = \Gamma(P_{B_i}g_{B_i}/(N_0 + P_{D_{ji}}F^{-1}_{\tilde{h}_{C_{j,i}}}(1-\epsilon)))$.

Next, we rewrite R in a simpler form to facilitate the decomposition of (1) into subproblems of lower complexity. From the definitions of R and R_i in Sec. III-A,

$$R(\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D}) = \sum_{i\in\mathcal{C}}\left[\sum_{j\in\mathcal{D}}\beta_{i,j}v_{i,j}(P_{B_i}, P_{D_{ji}}) + R_{C_{i,0}}\right], \tag{2}$$

where $v_{i,j}(P_{B_i}, P_{D_{ji}}) := R^{LB}_{C_{i,j}} + R_{D_{j,i}} - R_{C_{i,0}}$ represents the minimum *rate increment* due to the assignment of channel i to D2D pair j relative to the case where the channel i is only used by the CU.

Next, we show that problem (1) can be decomposed without loss of optimality into several power allocation problems and a channel assignment problem. Notice that (1) can be equivalently expressed by replicating $\{P_{B_i}\}$ to multiple $\{P_{B_{ij}}\}$ and removing the constant terms from the objective function. The resulting problem can be stated as:

$$\begin{aligned}
&\underset{\boldsymbol{B},\boldsymbol{P_B},\boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\left[\beta_{i,j}v_{i,j}(P_{B_{ij}}, P_{D_{ji}})\right] - \gamma\delta^2(\boldsymbol{B})\\
&\text{subject to} \quad \text{(1b), (1c), (1e)} \\
&\frac{P_{B_{ij}}g_{B_i}}{N_0 + P_{D_{ji}}F^{-1}_{\tilde{h}_{C_{j,i}}}(1-\epsilon)} \geq \eta^C_{min}, \;\; \forall i, j,
\end{aligned} \tag{3}$$

where $\boldsymbol{P_B} := [P_{B_{ij}}]_{i,j}$. To recover the optimal $\{P^*_{B_i}\}$ of (1) from the optimal $\{P^*_{B_{i,j}}\}$ of (3), one just needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $P^*_{B_i} = P^*_{B_{i,j}}$. If no such a j exists, i.e. $\beta_{i,j} = 0 \;\forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power $P^*_{B_i} = P_{B_{\max}}$.

In addition, it can be be shown that optimizing (3) with respect to $\boldsymbol{P_B}$ and $\boldsymbol{P_D}$ decouples across i and j into the following $N_C \times N_D$ subproblems:

$$\begin{aligned}
&\underset{P_{B_{ij}},P_{D_{ji}}}{\text{maximize}} \quad v_{i,j}(P_{B_{ij}}, P_{D_{ji}}) \\
&\text{subject to} \quad 0 \leq P_{B_{ij}} \leq P_{B_{max}}, \;\; 0 \leq P_{D_{ji}} \leq P_{D_{max}} \\
&\frac{P_{B_{ij}}g_{B_i}}{N_0 + P_{D_{ji}}F^{-1}_{\tilde{h}_{C_{j,i}}}(1-\epsilon)} \geq \eta^C_{min}, \;\; \frac{P_{D_{ji}}g_{D_j}}{N_0 + P_{B_{ij}}h_{B_j}} \geq \eta^D_{min},
\end{aligned} \tag{4}$$

which should be solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$. We propose two methods for solving each of the power allocation subproblems in parallel.

a) Centralized Resource Allocation Algorithm

In this case, each power allocation subporblem is solved at the BS by parallelly executing closed-form solutions. The closed-form solutions are obtained based on the fact that optimal power assignment lie on the border of the feasibility region and the objective is convex on those regions [14].

Once (4) has been solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$, it remains to substitute the optimal values $v^*_{i,j}$ into (3) and minimize with respect to $\boldsymbol{B}$. If (4) is infeasible for a given (i, j), then we can set its optimal value to $v^*_{i,j} = -\infty$. Thus, the resulting channel assignment subproblem can be stated as:

$$\begin{aligned}
&\underset{\boldsymbol{B}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\beta_{i,j}v^*_{i,j} - \gamma\delta^2(\boldsymbol{B}), \\
&\text{subject to} \quad \beta_{i,j} \in \{0,1\} \;\forall i, j, \quad \sum_{j\in\mathcal{D}}\beta_{i,j} \leq 1 \;\forall i.
\end{aligned} \tag{5}$$

Problem (5) is an integer program of combinatorial nature and exhibits prohibitive computational complexity even for reasonable values of $N_C,\ N_D$. However, by relaxing the integer constraints to $\beta_{i,j} \in [0,1]\ \forall i,j$, we have a differentiable strongly convex objective function with linear constraints. The resulting problem can be solved efficiently using the projected gradient descent algorithm. The obtained solution should be finally discretized back to satisfy the original constraints $\beta_{i,j} \in \{0,1\}\ \forall i,j$. This is done by setting the highest positive value in every row of $\boldsymbol{B}$ to 1 while setting other values in the same row to 0. The resulting algorithm is summarized in Algorithm 1.

B

Algorithm 1 Centralized Resource Allocation

```
Initialize: B^0, α^0, k = 0
for all j ∈ D, i ∈ C do
    BS calculates: P_{B_{i,j}}, P_{D_{i,j}}, v_{i,j}
end for
repeat
    k = k + 1
    BS uses projected gradient descent algorithm to find B^k
until B converges
BS discretize B.
```

b) Decentralized Resource Allocation Algorithm

Next, we describe a decentralized method to solve problem (4). First, to convert each of the $N_C N_D$ subproblems to a form suitable for fractional programming, we introduce two auxiliary variables z_1, z_2, which represents lower bounds of the SINR of the i-th CU and the j-th D2D pair, as follows:

$$z_1 \leq \frac{P_{B_{ij}} g_{Bi}}{N_0 + P_{D_{ji}} F^{-1}_{\tilde{h}_{C_{j,i}}}(1-\epsilon)} \text{ and } z_2 \leq \frac{P_{D_{ji}} g_{Dj}}{N_0 + P_{B_{ij}} h_{B_j}}.$$

The resulting objective function culminates to $\log_2(1+z_1)+\log_2(1+z_2)$, with two additional constraints for the SINR bounds defined by z_1 and z_2 above. Next, we find the Lagrangian of this objective function with respect to those two constraints and solve it with respect to the dual variable. We obtain the following objective function by substituting the optimal values of the dual variables in the Lagrangian:

$$\log_2(1+z_1) + log_2(1+z_2) + \frac{(1+z_2) P_{D_{ji}} g_{D_j}}{P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j}} + \frac{(1+z_1) P_{B_i} g_{B_i}}{P_{B_i} g_{B_i} + N_0 + P_{D_{ji}} F^{-1}_{\tilde{h}_{C_{j,i}}}(1-\epsilon)} - z_2 - z_1.$$

This objective function includes a sum of concave over convex fractions. The quadratic transformation proposed in [15] is identified as suitable for such extensions of fractional programming. However, this transformation will lead to a convex problem with an optima corresponding to a local optima of the original problem[4]. Moreover, the resulting function have a closed-form solution for every group of variables when fixing the values of other groups of variables. This makes it a suitable candidate for alternating optimization. The following iterative closed-form solution can be obtained:

$$z_1^{(k)} = \frac{P_{B_{ij}}^{(k-1)} g_{B_i}}{N_0 + P_{D_{ji}}^{(k-1)} F^{-1}_{\tilde{h}_{C_{j,i}}}(1-\epsilon)}, \quad z_2^{(k)} = \frac{P_{D_{ji}}^{(k-1)} g_{D_j}}{N_0 + P_{B_{ij}}^{(k-1)} h_{B_j}},$$

$$y_1^{(k)} = \frac{\sqrt{(1+z_1^{(k)}) P_{B_{ij}}^{(k-1)} g_{B_i}}}{P_{B_{ij}}^{(k-1)} g_{B_i} + N_0 + P_{D_{ji}}^{(k-1)} F^{-1}_{\tilde{h}_{C_{j,i}}}(1-\epsilon)},$$

$$y_2^{(k)} = \frac{\sqrt{(1+z_2^{(k)}) P_{D_{ji}}^{(k-1)} g_{D_j}}}{P_{D_{ji}}^{(k-1)} g_{D_j} + N_0 + P_{B_{ij}}^{(k-1)} h_{B_j}},$$

[4]We omit the full details of the proof here due to lack of space.

$$P_{B_{ij}}^{(k)} = \text{proj}_S \left(\frac{(y_1^{(k)})^2 (1 + z_1^{(k)}) g_{B_i}}{((y_1^{(k)})^2 g_{B_i} + (y_2^{(k)})^2 h_{B_j})^2} \right),$$

$$P_{D_{ji}}^{(k)} = \text{proj}_S \left(\frac{(y_2^{(k)})^2 (1 + z_2^{(k)}) g_{D_j}}{((y_2^{(k)})^2 g_{D_j} + (y_1^{(k)})^2 F^{-1}_{\tilde{h}_{C_{j,i}}}(1-\epsilon))^2} \right),$$

B

where y_1, y_2 are generated from the quadratic transformation, and $\text{proj}_S(.)$ denotes the projection on the set S defined by feasible set of (4) which has linear constraints.

Once (4) is solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$, substituting the optimal values $v_{i,j}^*$ into (3), leads to (5). Furthermore, the same relaxation, algorithm, and discretization can also be used. Due to the iterative nature of the proposed power allocation algorithm and the projected gradient descent, the problem can be solved in a partially decentralized fashion. However, the fairness part of the object function contains a quadratic term that can not be easily decoupled across D2D pairs. Thus, classical decomposition methods can not be directly used. Nevertheless, the gradient of the object function can be decoupled across D2D pairs. Thus, by having each D2D pair perform a gradient descent on the corresponding column of $\boldsymbol{B}$, part of the computations can be done by the pairs. The projection and the discretizations must be done centrally at the BS. Each iteration will lead to a feasible solution that can be used instantaneously by the BS and the D2D pairs for communications even before converging to the final solution as shown in Algorithm 2.

Algorithm 2 Decentralized Resource Allocation

```
Initialize: B^(0), P_B^(0), P_D^(0), k = 0
for all j ∈ D do
    BS send to D2D pair j: B_j^0, P_{B_j}^0, P_{D_j}^0
end for
repeat
    k = k + 1
    for all j ∈ D do
        D2D pair j calculates z_2^(k), y_2^(k), P_{D_j}^(k) and send them to the BS
        D2D pair uses the gradient decent algorithm and sends to BS: B_j^(k)
        BS calculates z_1^(k), y_1^(k), P_{B_j}^(k) and send them to pair j.
    end for
    BS projects B and sends each column to the corresponding D2D pair.
until B, P_B, P_D converges
```

IV. SIMULATIONS

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. We assume $\tilde{h}_C$ to be exponentially distributed with the mean value obtained from the mentioned path-loss model. Averages over 100,000 independent realizations of the user locations with parameters $BW = 15$ kHz, $\gamma = 50 \times BW$, $N_D = 10$, $N_C = 10$, $N_0 = -70$ dBW (γ is scaled with BW to ensure that the unfairness and the achieved rate are of comparable values). The two proposed algorithms are compared with; (i) the method by Feng et al. [12], which to the best of our knowledge is the best existing alternative; and (ii)

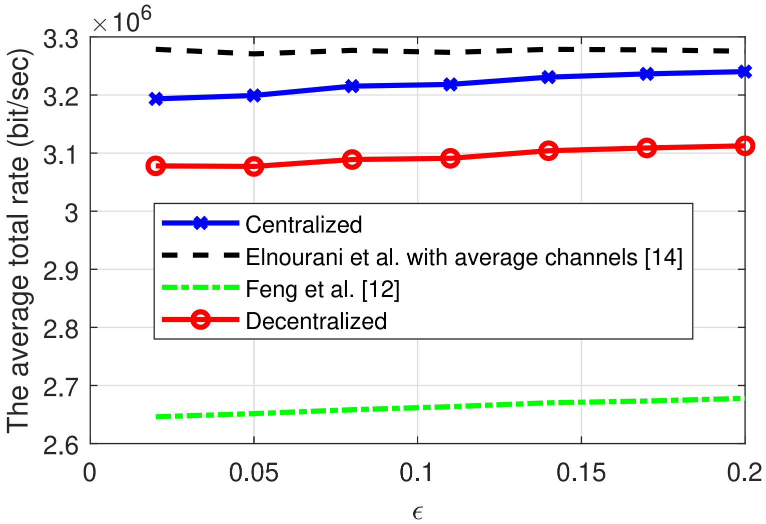

Fig. 2: Total average rate R vs. ϵ.

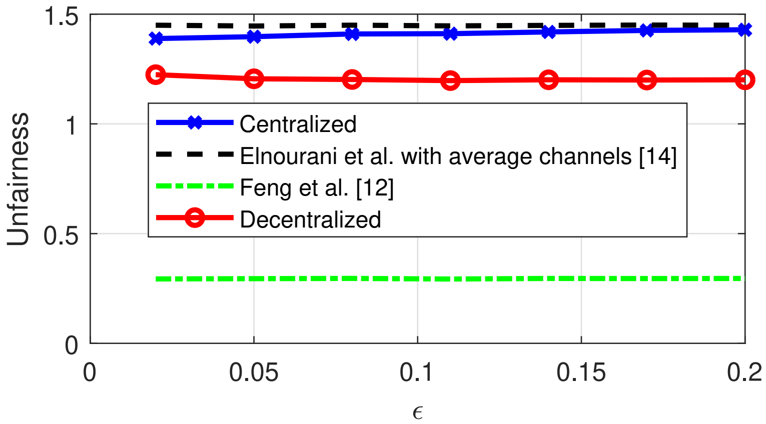

Fig. 3: Unfairness vs. ϵ .

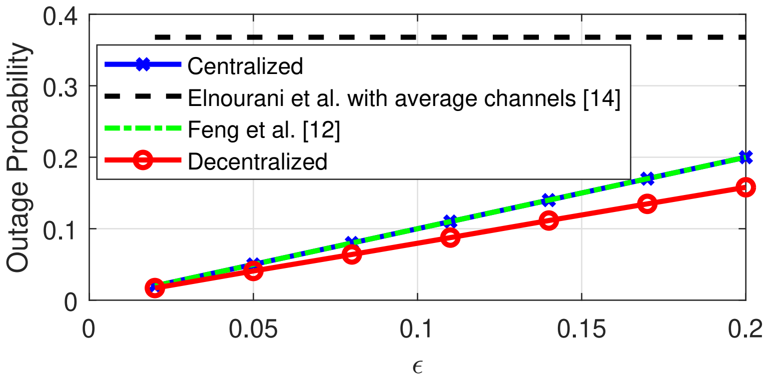

Fig. 4: Outage probability vs. ϵ.

the closed-form method in [14], where the average of $\tilde{h}_C$ is an estimate of the instantaneous channel gain values.

Fig. 2 shows that the proposed methods achieve high average rates, which are also very close to the case where the probabilistic outage constraint is ignored. Notice that the gap decreases with increasing the desired outage probability ϵ. This is in contrast to the method in [12] whose achieved rates are considerably lower than the proposed methods.

Fig. 3 shows that the method in [12] achieves the best fairness as expected, since a D2D pair can not use more than one channel at a time. The proposed methods achieve similar *fairness* level where the decentralized method achieves slightly better fairness. Moreover, our

B

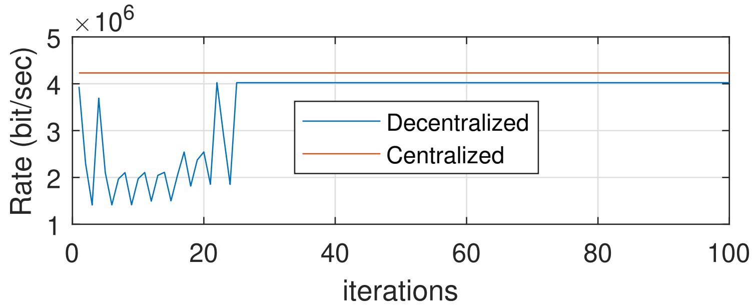

Fig. 5: The convergence rate of a single realization.

proposed methods provides a rate vs fairness trade-off flexibility by changing the scaling parameter γ.

Fig. 4 shows that the proposed centralized method and the method in [12] achieve similar outage probabilities which are exactly equivalent to the desired outage values. On the other hand, the decentralized method achieves slightly lower outage probability, since (4) converges to a local optima where the rate and risk are lower than the optima.

Finally, Fig. 5 shows that the decentralized method converges to a rate close to the one achieved by the centralized method in relatively few iterations.

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Appendix C

Paper C

Title:	Robust Underlay Device-to-Device Communications on Multiple Channels.
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Robust Underlay Device-to-Device Communications on Multiple Channels

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C

Abstract

Most recent works in device-to-device (D2D) underlay communications focus on the optimization of either power or channel allocation to improve the spectral efficiency, and typically consider uplink and downlink separately. Further, several of them also assume perfect knowledge of channel-state-information (CSI). In this paper, we formulate a joint uplink and downlink resource allocation scheme, which assigns both power and channel resources to D2D pairs and cellular users in an underlay network scenario. The objective is to maximize the overall network rate while maintaining fairness among the D2D pairs. In addition, we also consider imperfect CSI, where we guarantee a certain outage probability to maintain the desired quality-of-service (QoS). The resulting problem is a mixed integer non-convex optimization problem and we propose both centralized and decentralized algorithms to solve it, using convex relaxation, fractional programming, and alternating optimization. In the decentralized setting, the computational load is distributed among the D2D pairs and the base station, keeping also a low communication overhead. Moreover, we also provide a theoretical convergence analysis, including also the rate of convergence to stationary points. The proposed algorithms have been experimentally tested in a simulation environment, showing their favorable performance, as compared with the state-of-the-art alternatives.

Index Terms

Device-to-device communication, channel assignment, power allocation, non-convex optimization, convergence guarantees, quality of qervice, decentralized.

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I. Introduction

Throughput demands of cellular communications have been exponentially increasing over the last few years [1], [2]. Classical techniques to enhance the *spectral* efficiency of point-to-point links can not satisfy this demand, since existing systems already achieve rates close to the single-user channel capacity [3]. For this reason, recent research efforts target to improve the *spatial* efficiency. D2D communications constitute a prominent example, where cellular devices are allowed to communicate directly with each other without passing their messages through the Base station (BS) [4]–[7]. This paradigm entails higher throughput and lower latency in the communication for two reasons: first, a traditional cellular communication between two devices requires one time slot in the uplink and one time slot in the downlink, whereas a single time slot suffices in D2D communications. Second, the time slot used by a traditional cellular user (CU) can be simultaneously used by a D2D pair in a sufficiently distant part of the cell, a technique termed *underlay*. In order to fully exploit the potential of underlay D2D communications, algorithms that provide a judicious assignment of cellular sub-channels (e.g. resource blocks in LTE or time slots at each frequency in general) to D2D users as well as a prudent power control mechanism that precludes detrimental interference to cellular users (CUs) are necessary. These research challenges constitute the main motivation of this paper.

Early works on D2D communications rely on simplistic channel assignment schemes, where each D2D pair communicates through a randomly selected cellular sub-channel (hereafter referred to as *channel* for simplicity). This is the case in [8], where the effects of selecting a channel with poor quality are addressed by choosing the best among the following operating modes: underlay mode; overlay mode (the D2D pair is assigned a channel that is unused by the CUs); and cellular mode (the D2D pair operates as a regular cellular user). Alternatively, a scheme is proposed in [9] which allows D2D pairs to perform spectrum sensing and opportunistically communicate over a single channel randomly selected by the BS for all the D2D communications. These research works suffer from two limitations: (i) random allocation of channels result in a sub-optimal throughput which could be improved by leveraging different degrees of channel-state information; (ii) they do not provide any mechanism to adjust the transmit power of D2D terminals, which generally results in a reduced throughput due to increase in interference.

Few works also consider performing channel assignment to the D2D pairs for underlay communication. In [10], [11], instead of randomly assigning channels, channels are assigned to the D2D pairs using auction games while maintaining the fairness in the number of channels assigned to each D2D pair. Similarly, [12] proposes channel assignment to D2D pairs utilizing a coalition-forming game model. Here, millimeter-wave spectrum is also considered as an overlay option for D2D pairs. However, it can be noted that these schemes only perform channel assignment and avoid controlling the transmit power, which limits the achievable throughput of the overall network.

In order to circumvent above limitations, a Stackelberg game based approach is proposed in [13] where each D2D pair simultaneously transmits in all cellular channels and compete non-cooperatively to adjust the transmit power. Here, the BS penalizes the D2D pairs if they generate harmful interference to the cellular communication. The optimization of the transmit power while ensuring minimum signal to interference plus noise ratio (SINR) requirements is also investigated in [14], however, as long as the SINR requirements are satisfied, D2D pairs are allowed to transmit in all channels. In an alternative approach, distributed optimization for power allocation is investigated in [15] for both overlay and underlay scenarios. To summarize, all the works mentioned so far perform either channel assignment or power allocation, but not both.

Few research works consider jointly optimizing channel assignment and power allocation, as they seem to show strong dependency. This joint optimization is considered in [16]–[19] but they restrict D2D users to access at most one cellular channel. However, the work in [20]–[22] allow assignment of multiple channels to each D2D pair. Notice that these schemes propose to use either uplink or downlink spectrum for D2D communications. Some recent research works also consider both uplink and downlink spectrum for allocating resources to D2D pairs. In [23]–[25], both uplink and downlink spectra are considered in their formulation; however, they limit the assignment to at most one channel to each D2D pair.

Another important point to note is that all of the previously mentioned works assume the availability of perfect channel state information (CSI). From a practical prospective, obtaining perfect CSI for D2D communications requires a lot of cooperation between all D2D pairs and CUs; thus adds a substantial amount of communication overhead. Some other recent works have also investigated problems that guarantee certain QoS parameters under the scenario of imperfect CSI for underlay D2D communications. In [26]–[28], power allocation and channel assignment

Works	Multiple channels	Joint UL and DL	CSI uncertainty
[16]–[19]			
[20]–[22]	X		
[23]–[25]		X	
[26]–[28]			X
Proposed	X	X	X

TABLE I: Selected works that jointly perform channel assignment and power allocation

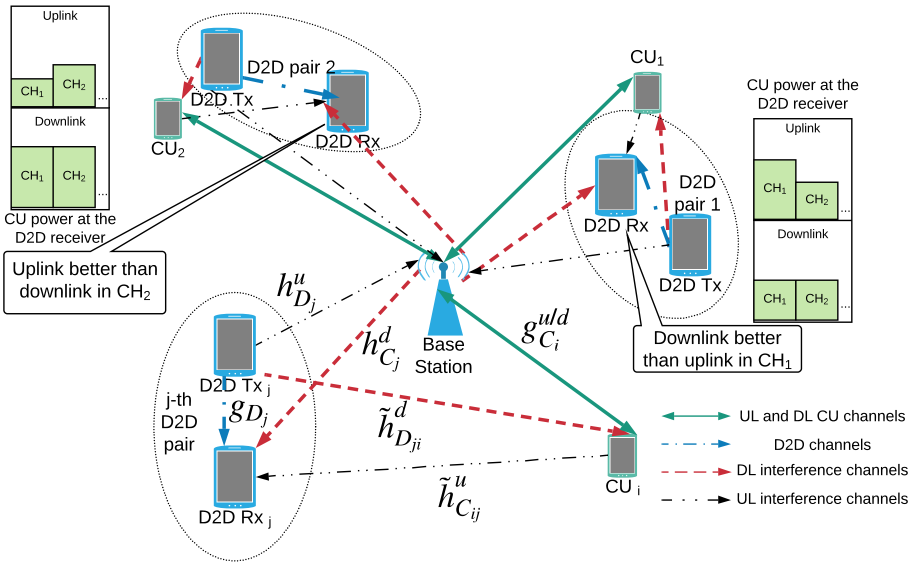

Fig. 1: Overall Network model

are considered under imperfect CSI. However, the analysis, once again, restricts D2D pairs to access at most one cellular channel. Table I list some of the presented works that jointly perform channel assignment and power allocation compared with our proposed scheme.

In this paper, we consider the resource allocation involving both uplink and downlink. Fig. 1 illustrates the potential of such an approach. Here for instance, if channel CH_1 is assigned to D2D pair 1, it is better to use downlink spectrum since it observes less cellular interference. Similarly, if channel CH_2 is assigned to D2D pair 2, it is better to use uplink spectrum. Further, one can notice it is better to assign channel CH_2 to D2D pair 1 and channel CH_1 to D2D pair 2 if only uplink spectrum is available for underlay communication. Furthermore, with the possibility of assigning multiple channels to each D2D pair, the number of potential choices increases substantially, allowing a more favourable channel assignment and power allocation. In conclusion, no existing work provides a joint channel assignment and power allocation scheme that satisfies all of the following requirements: (i) considers both uplink and downlink spectrum;(ii) accounts for uncertainties in CSI and thus obtains a robust resource allocation solution;and (iii) D2D pairs can simultaneously operate on more than one cellular channel, which is of special interest in areas of high CU density. This paper addresses the above mentioned limitations and provides the following research contributions:

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- We propose a joint uplink and downlink resource allocation scheme, which assigns both power and channel resources to D2D pairs and CUs. The objective of this scheme is to maximize the total network rate while maintaining fairness in the channel assignment among the D2D pairs. In addition, the scheme also allows assigning multiple channels to each D2D pair. Moreover, the proposed scheme also accounts for uncertainties in the channels by introducing probabilistic constraints that guarantee the desired outage probabilities.
- We propose a computationally efficient solution for the resulting problem, even though it is a mixed integer non-convex optimization problem, which involves exponential complexity to compute the optimal solution. We first show that without loss of optimality, the overall problem can be decomposed into several power allocation subproblems and a channel assignment problem. The solution of the power allocation sub-problems in the case of perfect CSI is obtained in closed form, whereas in the scenario of imperfect CSI, a quadratic transformation and alternating optimization methods are proposed. The proposed algorithms can be implemented centrally at the BS.
- We also propose decentralized algorithms that reduce the computational load at the BS by solving each power allocation sub-problem in parallel at the corresponding D2D pair. Moreover, some of the computations for the channel assignment problem are also performed by the D2D pairs. Furthermore, the communication can start immediately after the first iteration without waiting for the algorithms convergence.
- We provide convergence guarantees to stationary points for all of our algorithms and we show linear convergence rate for some of the considered cases and sub-linear convergence rates for other cases.
- Extensive simulations are also presented to demonstrate the advantages of the proposed method as compared to current state-of-the-art alternatives.

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II. SYSTEM MODEL AND PROBLEM STATEMENT

Consider an underlay D2D communication scenario in which both uplink and downlink cellular channels are accessible to D2D pairs. In the following description, we describe the considered communication scenario, which is also depicted in Fig. 1.

Cellular network configuration: We consider a cell (or sector) of a cellular network in which the serving BS and the associated CUs communicate via $N_C^{(u)}$ uplink and $N_C^{(d)}$ downlink channels, respectively[1]. Considering the worst case underlay scenario, we assume, without loss of generality, a fully loaded cellular communication scenario in which all uplink and downlink channels are assigned to CUs. For notational convenience, the set of CUs communicating in respective uplink and downlink channels are indexed as $\mathcal{C}^{(u)} = \left\{1, ..., N_C^{(u)}\right\}$ and $\mathcal{C}^{(d)} = \left\{1, ..., N_C^{(d)}\right\}$.

D2D communication configuration: Next, we assume that N_D D2D pairs $\left(\text{indexed by } j \in \mathcal{D} = \{1, ..., N_D\}\right)$ desire to communicate over the aforementioned downlink and uplink channels in an underlay configuration, i.e., simultaneously on the same uplink and downlink channels assigned to the CUs. The assignment of uplink or downlink channels to D2D pairs is represented by the indicator variables $\{\beta_{i,j}^{(u)}\}$ and $\{\beta_{i,j}^{(d)}\}$, respectively, where j denotes D2D pair ($j \in \mathcal{D}$) and i denotes either uplink or downlink channel $\left(i \in \mathcal{C}^{(u)} \ or \ \mathcal{C}^{(d)}\right)$. Here $\beta_{i,j}^{(u)} = 1$ or $\beta_{i,j}^{(d)} = 1$ when the j-th D2D pair accesses the i-th uplink or downlink channel. In order to improve throughput of the D2D pairs, we further assume that each D2D pair can access multiple channels at the same time. However, in-order to restrict interference among D2D pairs, we assume that each channel can be used by at most one D2D pair, this can be expressed as $\sum_{j=1}^{N_D} \beta_{i,j}^{(u)} \leq 1, \ \sum_{j=1}^{N_D} \beta_{i,j}^{(d)} \leq 1 \ \forall \ i$. In addition, to reduce hardware complexity, we further assume that each D2D pair can have access to multiple channels in either downlink or uplink spectrum band [23], [24], which can be expressed as $\sum_{i=1}^{N_C^{(u)}} \beta_{i,j}^{(u)} \times \sum_{i=1}^{N_C^{(d)}} \beta_{i,j}^{(d)} = 0, \ \forall j$.

Communication channels: First, we define channel gains in the uplink access. Let $g_{C_i}^{(u)}$ denote the channel gain from the i-th CU to the BS and $h_{C_{i,j}}^{(u)}$ denote the channel gain of the interference link from the i-th CU to the j-th D2D pair receiver. Similarly, let g_{D_j} denote the channel gain between transmitter and receiver of the j-th D2D pair and $h_{D_j}^{(u)}$ denote the channel gain of the

[1]Recall that channel in this context may stand for resource blocks, time slots, and so on.

interference link from the transmitter of the j-th D2D pair to the BS. Next, for downlink access, let $g_{C_i}^{(d)}$ denote the channel gain from the BS to the i-th CU and $h_{C_j}^{(d)}$ denote the channel gain of the interference link from the BS to the j-th D2D pair. Finally, let $h_{D_{j,i}}^{(d)}$ denote the channel gain of the interference link from the transmitter of the j-th D2D pair to the i-th CU. Here we assume that the interference channel gains affecting the CUs are estimated with minimum cooperation from the CUs; thus, gains of these interference links are assumed to be modeled as random variables, denoted respectively by $\tilde{h}_{C_{i,j}}^{(u)}$ and $\tilde{h}_{D_{j,i}}^{(d)}$. Finally, additive noise observed in individual channels is assumed to have a known power N_0. Note that the noise and all channel gains are assume to be frequency flat to simplify the notations; however, the proposed scheme carries over immediately to the frequency selective scenario.

Transmit power constraints: Considering the limited power available at the mobile devices, the transmit power of the j-th D2D pair when assigned to the i-th uplink or downlink channel, denoted as $P_{D_{j,i}}^{(u)}$ and $P_{D_{j,i}}^{(d)}$ is constrained as $0 \leq P_{D_{j,i}}^{(u)} \leq P_{D_{max}}$, $0 \leq P_{D_{j,i}}^{(d)} \leq P_{D_{max}}$. Similarly, the transmit power of the CU on the i-th uplink channel and of the BS on the i-th downlink channels are constrained, respectively, as $0 \leq P_{C_i}^{(u)} \leq P_{C_{max}}^{(u)}$ and $0 \leq P_{C_i}^{(d)} \leq P_{C_{max}}^{(d)}$. Note that $P_{C_{max}}^{(u)}$, $P_{C_{max}}^{(d)}$, and $P_{D_{max}}$ are assumed to be the same for all CUs and D2D pairs to simplify the notations, however, once again the proposed scheme carries over immediately to the scenario where they are different.

Achievable rates: Here, we first present the achievable rates for D2D underlay communication on downlink channels and then extend our discussion for underlay on uplink channels. Let $R_{C_{i,j}}^{(d)}$ and $R_{D_{j,i}}^{(d)}$ denote the rate of the i-th CU and of the j-th D2D pair when sharing the downlink channel, which are respectively given as:

$$R_{C_{i,j}}^{(d)} = \log_2\left(1 + \frac{P_{C_i}^{(d)} g_{C_i}^{(d)}}{N_0 + P_{D_{j,i}}^{(d)} \tilde{h}_{D_{j,i}}^{(d)}}\right), \; R_{D_{j,i}}^{(d)} = \log_2\left(1 + \frac{P_{D_{j,i}}^{(d)} g_{D_j}}{N_0 + P_{C_i}^{(d)} h_{C_j}^{(d)}}\right).$$

When the i-th CU does not share the downlink channel, the achievable rate denoted by $R_{C_{i,0}}^{(d)}$ is given as:

$$R_{C_{i,0}}^{(d)} = \log_2\left(1 + \frac{P_{C_{\max}}^{(d)} g_{C_i}^{(d)}}{N_0}\right).$$

Thus, the gain in rate when the i-th CU shares channel with the j-th D2D pair can be stated as,

$v_{i,j}^{(d)} = R_{C_{i,j}}^{(d)} + R_{D_{j,i}}^{(d)} - R_{C_{i,0}}^{(d)}$. Finally, the overall network rate in the downlink can be stated as:

$$R^{(d)}(\boldsymbol{B}^{(d)}, \mathbf{p_C}^{(d)}, \boldsymbol{p}_{\boldsymbol{D}}^{(d)}) = \sum_{i \in \mathcal{C}^{(d)}} \left[\sum_{j \in \mathcal{D}} \beta_{i,j}^{(d)} v_{i,j}^{(d)} + R_{C_{i,0}}^{(d)} \right]. \tag{1}$$

where $\boldsymbol{B}^{(d)} \triangleq [\beta_{i,j}^{(d)}],\ \mathbf{p_C}^{(d)} \triangleq [P_{C_1}^{(d)}, P_{C_2}^{(d)}, \ldots, P_{C_{N_C}}^{(d)}]^T,\ \boldsymbol{P}_{\boldsymbol{D}}^{(d)} \triangleq [P_{Dji}^{(d)}]$.

Similarly, the achievable rates in the uplink channels when sharing the i-th CU uplink channel with the j-th D2D pair can be expressed as:

$$R_{C_{i,j}}^{(u)} = \log_2 \left(1 + \frac{P_{C_i}^{(u)} g_{C_i}^{(u)}}{N_0 + P_{D_{j,i}}^{(u)} h_{D_j}^{(u)}} \right),\ R_{D_{j,i}}^{(u)} = \log_2 \left(1 + \frac{P_{D_{j,i}}^{(u)} g_{D_j}}{N_0 + P_{C_i}^{(u)} \tilde{h}_{C_{i,j}}^{(u)}} \right).$$

The achievable rates in the uplink channels without sharing the i-th CU uplink channel, the rate gain, as well as the total rate due to underlay uplink communications can be easily expressed by replacing the superscripts (d) by (u) in the above equations.

Quality of Service (QoS) requirements: In order to have a successful communication at a receiver node, a minimum signal to interference plus noise (SINR) ratio requirement is imposed in the problem formulation. Thus, for the i-th CU in the uplink/downlink sharing channel with the j-th D2D pair, the instantaneous SINR $\eta_{C_{i,j}}^{(u)} \geq \eta_{C_{\min}}^{(u)}$ and $\eta_{C_{i,j}}^{(d)} \geq \eta_{C_{\min}}^{(d)}$, where $\eta_{C_{\min}}^{(u)}$ and $\eta_{C_{\min}}^{(d)}$ are the minimum desired SINR for the CU in uplink and donwlink, respectively. Similarly, for the j-th D2D pair, the instantaneous SINR $\eta_{D_{i,j}}^{(u)} \geq \eta_{D_{\min}}$, where $\eta_{D_{\min}}$ is the minimum desired SINR for D2D pairs in both uplink and downlink. Note thatin order to simplify the notation $\eta_{C_{\min}}^{(u)}$, $\eta_{C_{\min}}^{(d)}$, and $\eta_{D_{\min}}$ are also assumed to be the same for all CUs and D2D pairs; however, the scheme carries over immediately to the scenario where they are different.

Since the computations of the SINR for the j-th D2D pair sharing channel with the i-th uplink CU involve the random interference channel gain $\tilde{h}_{Ci,j}^{(u)}$, the minimum SINR requirement can be expressed in terms of a probabilistic constraint as follows:

$$Pr\{\eta_{D_{i,j}}^{(u)} \geq \eta_{D_{min}}^{(u)}\} \geq 1 - \epsilon \quad \forall i \in \mathcal{C}^{(u)},\ \ \forall j \in \mathcal{D},$$

where ϵ is the maximum allowed outage probability. Similarly, the minimum SINR requirement for the i-th downlink CU sharing channel with the j-th D2D pair can be expressed in terms of a probabilistic constraint as follows:

$$Pr\{\eta_{C_{i,j}}^{(d)} \geq \eta_{C_{min}}^{(d)}\} \geq 1 - \epsilon \quad \forall i \in \mathcal{C}^{(d)},\ \ \forall j \in \mathcal{D}.$$

Fairness in channel assignment to D2D pairs: Let m_j denotes the number of channels assigned to the j-th D2D pair.

$$m_j = \sum_{i_u=1}^{N_C^{(u)}} \sum_{i_d=1}^{N_C^{(d)}} \left(\beta_{i_u,j}^{(u)} + \beta_{i_d,j}^{(d)} \right).$$

Then inspired by the fairness definition in [10], the fairness of a channel allocation can be expressed in terms of a normalized variance from a specified reference assignment m_0 as follows:

$$\delta = \frac{\frac{1}{N_D} \sum_{j=1}^{N_D} (m_j - m_0)^2}{m_0^2} \tag{2}$$

Problem statement: Given all $g_{C_i}^{(u)}$, $g_{C_i}^{(d)}$, g_{D_j}, $h_{D_j}^{(u)}$, $h_{C_j}^{(d)}$, the statistical distribution of $\tilde{h}_{C_{i,j}}^{(u)}$, and $\tilde{h}_{D_{j,i}}^{(d)}$ $\forall i, j$, as well as N_0, $\eta_{\min}^C$, $\eta_{\min}^D$, $P_{C_{\max}}$, and $P_{D_{\max}}$, the goal is to choose $\beta_{i,j}^{(d)}$, $\beta_{i,j}^{(u)}$, $P_{D_{ji}}^{(d)}$, $P_{D_{ji}}^{(u)}$, $P_{C_i}^{(d)}$, $P_{C_i}^{(u)}$ to maximize the overall rate of the D2D pairs and CUs while ensuring fairness among the multiple D2D pairs and preventing detrimental interference to CUs.

We consider two different scenarios of D2D pairs communicating over underlay downlink/ uplink channels: (**S1**) D2D pairs are pre-organized into uplink and downlink groups based on hardware limitations to communicate in either uplink or downlink channels; (**S2**) The assignment of the D2D pairs to either uplink or downlink channels is also part of the optimization problem. Furthermore, in order to reduce the computation load on the BS, we also propose decentralized solution for both the scenarios. In both the scenarios, we analyze both perfect and imperfect CSI cases.

III. Separate Downlink and Uplink Resource Allocation

Recall from Sec. II that each D2D pair is allowed to operate either in the uplink or in the downlink, but not in both simultaneously. For the sake of the exposition, this section assumes that the assignment of D2D pairs to either the uplink or downlink is given. Sec. IV will extend the approach presented in this section to the scenario where such an assignment is not given and therefore becomes part of the resource allocation task. Thus, there are two pre-organized sets of D2D pairs, namely $\mathcal{D}^{(d)}$ and $\mathcal{D}^{(u)}$, intending to communicate in downlink and uplink channels, respectively. Since the D2D pairs are already pre-organized in two separate sets, the joint resource allocation problems simplifies to solving two separate but similar problems: (i) allocating downlink resources to the D2D pairs in the set $\mathcal{D}^{(d)}$; (ii) and allocating uplink resources to the D2D pairs in the set $\mathcal{D}^{(u)}$. Thus, due to the similarity of the two problems, we only discuss the donwlink resource allocation in this section. Since it introduces no ambiguity and simplifies the notation, in this section, we will drop the superscript denoting uplink and downlink.

A. Resource Allocation Under Perfect CSI (PCSI)

Here, we first analyze the ideal scenario in which perfect CSI can be exploited to maximize the aggregate throughput of both the D2D pairs and CUs while ensuring fairness among D2D pairs. To this end, our problem formulation is as follows:

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$$\underset{\boldsymbol{B},\mathbf{p_C},\boldsymbol{P_D}}{\text{maximize}} \quad R(\boldsymbol{B},\mathbf{p_C},\boldsymbol{P_D}) - \gamma\delta(\boldsymbol{B}) \tag{3a}$$

$$\text{subject to} \quad \beta_{i,j} \in \{0,1\},\ \forall\ i,j,\ \sum_{j=1}^{N_D}\beta_{i,j} \leq 1\ \forall\ i,, \tag{3b}$$

$$0 \leq P_{C_i} \leq P_{C_{max}},\ \forall\ i,\ 0 \leq P_{D_{ji}} \leq P_{D_{max}},\ \forall\ i,j, \tag{3c}$$

$$\frac{P_{C_i} g_{C_i}}{N_0 + P_{D_{ji}} h_{D_{j,i}}} \geq \eta^C_{min} \text{ if } \beta_{ij} = 1,\ \forall\ i,j, \tag{3d}$$

$$\frac{P_{D_{ji}} g_{Dj}}{N_0 + P_{C_i} h_{C_j}} \geq \eta^D_{min} \text{ if } \beta_{ij} = 1,\ \forall\ i,j, \tag{3e}$$

where the total rate $R(\boldsymbol{B},\boldsymbol{p_C},\boldsymbol{P_D})$ is given by (1). The fairest resource assignment in this framework corresponds to uniformly distributing the N_C available channels equally over the D2D pairs ($m_0 := N_C/N_D$). Substituting m_0 in (2), the fairness in channel allocation $\delta(\boldsymbol{B})$ can be expressed as:

$$\delta(\boldsymbol{B}) = \frac{N_D}{N_C^2}\sum_{j=1}^{N_D}\left(\sum_{i=1}^{N_C}\beta_{i,j} - N_C/N_D\right)^2 \tag{4}$$

We consider a user-selected regularization parameter $\gamma > 0$ in (3a) to balance the rate-fairness trade-off. In general, the highest rate is achieved when all channels are assigned only to D2D pairs with good communications conditions. The fairness in the assignment needs to be enforced by adding a term in the objective function that penalizes unfair assignments.

The optimization problem in (3) is a non-convex mixed-integer problem and obtaining the optimal solution of such a combinatorial problem will incur an exponential complexity. Next, we show that problem (3) can be decomposed into two steps without loosing optimality: (S1) power allocation; and (S2) channel assignment.

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First, consider solving (3a) w.r.t. $\mathbf{p}_C$ for fixed $\boldsymbol{P}_D$ and $\boldsymbol{B}$. It can be seen from (1) that the objective of (3) can be written as $\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\beta_{i,j}\ v_{i,j}(P_{C_i}, P_{D_{ji}})$ plus some terms that do not depend on $[P_{C_i}]$. Notice that an equivalent problem can be obtained by replacing P_{C_i} with an artificial auxiliary variable $P_{C_{i,j}}$ in each term $\beta_{i,j}v_{i,j}(P_{C_i}, P_{D_{ji}})$ and further enforcing the constraint $P_{C_{i,1}} = P_{C_{i,2}} = \ldots = P_{C_{i,N_D}}$ for each i. Then, the modified objective can be expressed as $\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\beta_{i,j}v_{i,j}(P_{C_{i,j}}, P_{D_{ji}})$ plus terms that do not depend on $[P_{C_{i,j}}]$. Similarly, we can replace P_{C_i} with $P_{C_{i,j}}$ in (3d)-(3e) and also in (3c) with $0 \le P_{C_{i,j}} \le P_{C_{max}}\,\forall i,j$ and the resulting problem will be equivalent to (3). Thus, except for the recently introduced equality constraints, the objective and the constraints will only depend on at most one of the $P_{C_{i,j}}$ for each i, specifically the one with $\beta_{i,j} = 1$. Hence, the equality constraint $P_{C_{i,1}} = \ldots = P_{C_{i,N_D}}$ can be dropped without loss of optimality. To recover the optimal $[P^*_{C_i}]$ in (3) from the optimal $[P^*_{C_{i,j}}]$, one just needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $P^*_{C_i} = P^*_{C_{i,j}}$. If no such j exists, i.e. $\beta_{i,j} = 0\ \ \forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power $P_{C_i} = P_{C_{\max}}$. Similarly, without loss of optimality, we can also remove the condition "if $\beta_{i,j} = 1$" from (3d)-(3e). Thus, the resulting problem can be expressed as:

$$\underset{\boldsymbol{B},\boldsymbol{P_C},\boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\left[\beta_{i,j}v_{i,j}(P_{C_{ij}}, P_{D_{ji}})\right] - \gamma\delta(\boldsymbol{B}) \tag{5a}$$

$$\text{subject to} \quad \beta_{i,j} \in \{0,1\},\ \forall\ i,j,\ \sum_{j=1}^{N_D}\beta_{i,j} \le 1\ \ \forall\ i, \tag{5b}$$

$$0 \le P_{C_{ij}} \le P_{C_{max}},\quad 0 \le P_{D_{ji}} \le P_{D_{max}},\ \forall\ i,j, \tag{5c}$$

$$\frac{P_{C_{ij}}g_{C_i}}{N_0 + P_{D_{ji}}h_{D_{j,i}}} \ge \eta^C_{min},\ \frac{P_{D_{ji}}g_{D_j}}{N_0 + P_{C_{ij}}h_{C_j}} \ge \eta^D_{min},\ \forall\ i,j \tag{5d}$$

where $\boldsymbol{P_C} \triangleq [P_{C_{ij}}]$.

Since $\boldsymbol{B}$ is binary, (5) can now be decoupled without loss of optimality into a power allocation problem and a channel allocation problem. Furthermore, the optimization of (5) with respect to $\boldsymbol{P_C}$ and $\boldsymbol{P_D}$ (power allocation problem) decouples across i and j into the $N_C N_D$ sub-problems

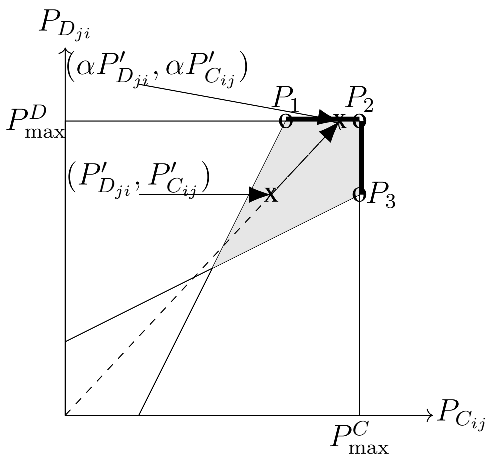

Fig. 2: Power feasibility region

of the form:

$$\begin{aligned} \underset{P_{C_{ij}}, P_{D_{ji}}}{\text{maximize}} \quad & v_{i,j}(P_{C_{ij}}, P_{D_{ji}}) \\ \text{subject to} \quad & 0 \leq P_{C_{ij}} \leq P_{C_{max}}, \quad 0 \leq P_{D_{ji}} \leq P_{D_{max}}, \\ & \frac{P_{C_{ij}} g_{C_i}}{N_0 + P_{D_{ji}} h_{D_{j,i}}} \geq \eta^C_{min}, \quad \frac{P_{D_{ji}} g_{D_j}}{N_0 + P_{C_{ij}} h_{C_j}} \geq \eta^D_{min}, \end{aligned} \tag{6}$$

which should be solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$. This power allocation subproblem coincides with the one arising in [16], [29], [30], which can be solved in closed-form, since the solution should be on the borders of the feasibility region (defined by the constraints in (6)). More specifically, as illustrated in Fig. 2, it can be shown that for any point $(P'_{C_{ij}}, P'_{D_{ji}})$ in the interior of the feasibility region, there exist a point $(\alpha P'_{C_{ij}}, \alpha P'_{D_{ji}})$ at the border segments that has a higher objective value. Moreover, since the objective function is convex on the border segments, and therefore the optimal point is one of the intersection points of the border segments (the maximum of a convex function is achieved at the borders of the feasibility region). Once (6) has been solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$, it remains to substitute the optimal values $[v^*_{i,j}]_{i,j}$ into (5) and then minimize with respect to $\boldsymbol{B}$. If (6) is infeasible for a given (i, j), then we set its optimal value to $v^*_{i,j} = -\infty$. The resulting channel assignment subproblem can be expressed as follows,

$$\begin{aligned} \underset{\boldsymbol{B}}{\text{maximize}} \quad & \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \beta_{i,j} v^*_{i,j} - \gamma \delta(\boldsymbol{B}), \\ \text{subject to} \quad & \beta_{i,j} \in \{0, 1\} \ \forall i, j, \quad \sum_{j \in \mathcal{D}} \beta_{i,j} \leq 1 \ \forall i. \end{aligned} \tag{7}$$

Notice that problem (7) is an integer program of combinatorial nature. Finding an exact solution using exhaustive search would be computationally unaffordable and time consuming for a sufficiently large $N_C N_D$. Thus, considering the practicality of implementation, we compute a sub-optimal solution with a smaller computational complexity by relaxing the integer constraint $\beta_{i,j} \in \{0,1\}$ of (7) with $\beta_{i,j} \in [0,1]$.

The resulting problem is convex and the resulting solutions $[\tilde{\beta}_{i,j}]$ can be efficiently obtained e.g. through projected gradient descent (PDG) [31]. Discretizing the solution $[\tilde{\beta}_{i,j}]$ to such a problem is expected to yield an approximately optimal optimum of (7). For this discretization we consider two approaches: (i) for every i, set $\beta_{i,j} = 1$ if $j = \arg\max_j \tilde{\beta}_{i,j}$. (ii) for each i, consider a random variable J_i taking values $1, \ldots, N_D$ with probabilities $P(J_i = j) = \tilde{\beta}_{i,j} / \sum_{j \in \mathcal{D}} \tilde{\beta}_{i,j}$. Then, we generate a certain set of realizations of $\{J_i\}$ and form the corresponding set of matrices $\{\boldsymbol{B}\}$, whose (i,j)-th entry is 1 if $J_i = j$ and 0 otherwise. Finally, we evaluate the objective of (7) for all these realizations and select the realization with the highest objective value.

B. Resource Allocation Under Imperfect CSI (ICSI)

In this scenario, we assume having infrequent and limited measurements from the CUs and the D2D pairs that are used in estimating the channel gain from the D2D pairs to the CUs. This will create uncertainty in the available CSI. Thus, in this case, the objective function and the SINR constraints in (5) involve a random channel gain $\tilde{h}_{D_{j,i}}$ for the interference link from the j-th D2D pair to the i-th CU.

First, the SINR constraint (5d) can be replaced with a probabilistic constraint to guarantee a maximum outage probability ϵ which can be expressed as:

$$\Pr\left\{\eta_{ij}^{c} \triangleq \frac{P_{C_{ij}} g_{C_i}}{N_0 + P_{D_{ji}} \tilde{h}_{D_{j,i}}} \geq \eta_{min}^{c}\right\} \geq 1 - \epsilon. \tag{8}$$

The probabilistic constraint in (8) can be expressed in closed form for a given statistical distribution of $\tilde{h}_{D_{ji}}$. Generally, (8) is equivalent to: $\Pr\left\{\tilde{h}_{D_{j,i}} \leq \frac{P_{C_{ij}} g_{C_i} - \eta_{min}^{c} N_0}{P_{D_{ji}} \eta_{min}^{c}}\right\} \geq 1 - \epsilon$, or, equivalently

$$\frac{P_{C_{ij}} g_{C_i}}{N_0 + P_{D_{ji}} \mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1 - \epsilon)} \geq \eta_{min}^{c}, \tag{9}$$

where $\mathrm{F}^{-1}_{\tilde{h}}(1 - \epsilon)$ is the inverse cumulative distribution function (CDF) function for $\tilde{h}$ evaluated at $1 - \epsilon$. We will consider exponential, Gaussian, Chi-squared, and log-normal distributions in the following sections, since they are the most common in wireless communication environment.

Next, focussing on the objective function, we consider two approaches: (i) expected network rate maximization; and (ii) minimum network rate maximization.

Calculation method	Deviation $\lvert\hat{v}_{i,j} - \bar{v}_{i,j}\rvert / \bar{v}_{i,j}$ between the approximation $\hat{v}_{i,j}$ and the Monte-Carlo average $\bar{v}_{i,j}$ for 10^6 samples	
Approximation / Distributions	First order	Second order
Exponential ($\bar{h}_{C_{ij}} = 0.2$)	0.6499%	0.1392%
Gaussian ($\bar{h}_{C_{ij}} = 0.2,\ \text{Var}\left\{h_{C_{ij}}\right\} = 0.01$)	0.8934%	0.1062%
Chi-squared ($\bar{h}_{C_{ij}} = 0.2,\ \text{Var}\left\{h_{C_{ij}}\right\} = 0.01$)	0.8930%	0.1058%
Log-normal ($\bar{h}_{C_{ij}} = 0.2,\ \text{Var}\left\{h_{C_{ij}}\right\} = 0.01$)	0.6898%	0.0991%

TABLE II: Expectation approximations

1) Expected Network Rate Maximization (ERM): One possibility is to replace the objective of (3) with its expectation. To this end, notice that $\mathbb{E}_{\tilde{\mathbf{h}}}\{R\} = \sum_{i\in\mathcal{C}} \left[\sum_{j\in\mathcal{D}} \beta_{i,j} \mathbb{E}_{\tilde{h}_{D_{j,i}}}\{v_{i,j}\} + R_{C_{i,0}}\right]$, and from the definition of v_{ij}:

$$\mathbb{E}_{\tilde{h}_{D_{j,i}}}\{v_{i,j}(P_{C_{ij}}, P_{D_{ji}})\} = \mathbb{E}_{\tilde{h}_{D_{j,i}}}\{R_{C_{i,j}}(P_{C_{ij}}, P_{D_{ji}})\} + R_{D_{j,i}}(P_{C_{ij}}, P_{D_{ji}}) - R_{C_{i,0}}.$$

Since the expectation of $R_{C_{i,j}}(P_{C_{ij}}, P_{D_{ji}})$ is not tractable analytically for the aforementioned distributions, one can replace the expectation by the first-order or the second-order Taylor series approximations around the mean of $\tilde{h}_{D_{j,i}}$. Table II shows the comparison between first-order and second-order approximations in the computation of $\mathbb{E}_{\tilde{h}_{D_{j,i}}}\{v_{ij}\}$, where we can note that both approximations are very close to the Monte-Carlo averages in all the tested distributions. Besides, the first-order approximation results in an error comparable to the second-order approximation. Because of this reason and the higher simplicity, we consider the first-order approximation. Moreover, the resulting expectation is the so-called certainty equivalence approximation, which is an extensively adopted approximation in stochastic optimization [31]. Using the expectation of the first-order Taylor approximation in the objective function along with aforementioned constraints in (9) leads to a problem similar to (6), which can be solved in closed form as before.

2) Minimum guaranteed rate maximization (MRM): In this approach, the criterion to maximize is the network rate exceeded for a (1-ϵ) portion of the time. First, we define the $(1-\epsilon)$-guaranteed SINR for the i-th CU when sharing the channel with the j-th D2D pair as $\eta^{\epsilon}_{C_{i,j}}$ such that Pr$\{\eta_{C_{i,j}} > \eta^{\epsilon}_{C_{i,j}}\} = 1-\epsilon$. Next, we define $v^{\epsilon}_{ij} = \log(1+\eta^{\epsilon}_{C_{i,j}}) + \log(1+\eta_{D_{i,j}}) - R_{C_{i,0}}$ and $R^{\epsilon} = \sum_{i\in\mathcal{C}} \left[\sum_{j\in\mathcal{D}} \beta_{i,j} v^{\epsilon}_{i,j} + R_{C_{i,0}}\right]$, siilar to the work in [32]. The resource allocation problem can be formulated as (3) with R replaced by R^{ϵ} and the SINR constraints replaced by (9). Proceeding as in Sec. III-A, such a problem is equivalent to (5) with v_{ij} replaced with v^{ϵ}_{ij} and the SINR constraints replaced by (9). Similar steps can also be followed to decouple the problem into power assignment and channel allocation subproblems. Up to a constant term, the objective of the power allocation sub-problems can be expressed as:

$$F_0 \triangleq \log_2\left(1+\frac{P_{C_{ij}}g_{C_i}}{N_0+P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}\right)+\log_2\left(1+\frac{P_{D_{ji}}g_{D_j}}{N_0+P_{C_{ij}}h_{C_j}}\right) \tag{10}$$

where $\eta^{\epsilon}_{C_{i,j}}$ is expressed in closed-form similar to (8) for a given statistical distribution of $\tilde{h}_{D_{j,i}}$. The rest of this section proposes a method to solve this power allocation subproblem.

C

This objective function is non-convex. However, it can be seen as a sum of log-functions of "concave-over-convex" fractions. Given this structure, fractional programming techniques [33], [34] constitute a natural fit.To take the fractions outside the log-functions, we introduce the slack variables $\mathbf{z} \triangleq [z_1,\ z_2]^T$. The resulting power assignment problem can be rewritten as follows:

$$\underset{P_{C_{ij}},P_{D_{ji}},\boldsymbol{z}}{\text{maximize}} \quad \log_2(1+z_1)+\log_2(1+z_2) \tag{11a}$$

$$\text{subject to} \quad z_1 \le \frac{P_{C_{ij}}g_{C_i}}{N_0+P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}, \quad z_2 \le \frac{P_{D_{ji}}g_{D_j}}{N_0+P_{C_{ij}}h_{C_j}} \tag{11b}$$

$$0 \le P_{C_{ij}} \le P_{C_{max}}, \quad 0 \le P_{D_{ji}} \le P_{D_{max}}, \tag{11c}$$

$$\frac{P_{C_{ij}}g_{C_i}}{N_0+P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)} \ge \eta^{C}_{min}, \quad \frac{P_{D_{ji}}g_{D_j}}{N_0+P_{C_{ij}}h_{C_j}} \ge \eta^{D}_{min}. \tag{11d}$$

The optimal values of the auxiliary variables occur when the inequalities hold with equality ($z_1^* = P_{C_{ij}}g_{C_i}/N_0+P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon),\ z_2^* = P_{D_{ji}}g_{D_j}/N_0+P_{C_{ij}}h_{C_j}$). Let us consider the Lagrangian of (11) with respect to the first two inequalities:

$$L(\boldsymbol{p},\boldsymbol{z},\boldsymbol{\lambda}) = \log_2(1+z_1)+\log_2(1+z_2)-\lambda_1\left(z_1-\frac{P_{C_{ij}}g_{C_i}}{N_0+P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}\right) - \lambda_2\left(z_2-\frac{P_{D_{ji}}g_{D_j}}{N_0+P_{C_{ij}}h_{C_j}}\right) \tag{12}$$

A stationary point of L with respect to $\boldsymbol{z}$ is achieved when $\partial L/\partial \boldsymbol{z} = \mathbf{0}$. This leads to $\lambda_1 = 1/(1+z_1),\ \lambda_2 = 1/(1+z_2)$. Substituting $\boldsymbol{z}^*$ in these equations yields:

$$\lambda_1^* = \frac{N_0+P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}{P_{C_{ij}}g_{C_i}+N_0+P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}, \quad \lambda_2^* = \frac{N_0+P_{C_{ij}}h_{C_j}}{P_{D_{ji}}g_{D_j}+N_0+P_{C_{ij}}h_{C_j}} \tag{13}$$

Substituting λ^* in (12), we obtain:

$$\begin{aligned} \underset{P_{C_{ij}},P_{D_{ji}},\boldsymbol{z}}{\text{maximize}} \quad & F_1 \triangleq L(\boldsymbol{p},\boldsymbol{z},\boldsymbol{\lambda}^*) = \log_2(1+z_1) + \log_2(1+z_2) - z_1 \\ & + \frac{(1+z_1)P_{C_{ij}}g_{C_i}}{P_{C_{ij}}g_{C_i} + N_0 + P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)} - z_2 + \frac{(1+z_2)P_{D_{ji}}g_{D_j}}{P_{D_{ji}}g_{D_j} + N_0 + P_{C_{ij}}h_{C_j}} \\ \text{subject to} \quad & \text{(11c)},\ \text{(11d)} \end{aligned} \tag{14}$$

Finally, to handle the fractions in the objective function, we use the quadratic transformation in [33], [34], to transform each fraction into a substitute concave expression. Then, we obtain:

$$\begin{aligned} \underset{P_{C_{ij}},P_{D_{ji}},\boldsymbol{z},\boldsymbol{y}}{\text{maximize}} \quad & F_2 \triangleq \log_2(1+z_1) + \log_2(1+z_2) - z_1 + 2y_1\sqrt{(1+z_1)P_{C_{ij}}g_{C_i}} \\ & - y_1^2(P_{C_{ij}}g_{C_i} + N_0 + P_{D_{ji}}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)) - z_2 + 2y_2\sqrt{(1+z_2)P_{D_{ji}}g_{D_j}} \\ & - y_2^2(P_{D_{ji}}g_{D_j} + N_0 + P_{C_{ij}}h_{C_j}) \\ \text{subject to} \quad & \text{(11c)},\ \text{(11d)}, \end{aligned} \tag{15}$$

where $\mathbf{y} \triangleq [y_1\ y_2]^T$ are the auxiliary variables given by the quadratic transformation.

This problem is then solved by alternating maximization with respect to the individual y_1, y_2, $P_{C_{ij}}, P_{D_{ji}}$ variables. At each step, all iterates can be obtained in closed form by taking the partial derivative with respect to each variable and setting it to 0, and projecting the solution onto the feasible set. The overall iteration can be expressed as:

$$z_1^{[k+1]} = \frac{P_{C_{ij}}^{[k]}g_{C_i}}{N_0 + P_{D_{ji}}^{[k]}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}, \quad z_2^{[k+1]} = \frac{P_{D_{ji}}^{[k]}g_{D_j}}{N_0 + P_{C_{ij}}^{[k]}h_{C_j}} \tag{16a}$$

$$y_1^{[k+1]} = \frac{\sqrt{(1+z_1^{[k+1]})P_{C_{ij}}^{[k]}g_{C_i}}}{P_{C_{ij}}^{[k]}g_{C_i} + N_0 + P_{D_{ji}}^{[k]}\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}, \ y_2^{[k+1]} = \frac{\sqrt{(1+z_2^{[k+1]})P_{D_{ji}}^{[k]}g_{D_j}}}{P_{D_{ji}}^{[k]}g_{D_j} + N_0 + P_{C_{ij}}^{[k]}h_{C_j}} \tag{16b}$$

$$P_{C_{ij}}^{[k+1]} = \text{Proj}_{\mathcal{S}_1^{[k]}}\left(\frac{(y_1^{[k+1]})^2(1+z_1^{[k+1]})g_{C_i}}{((y_1^{[k+1]})^2 g_{C_i} + (y_2^{[k+1]})^2 h_{C_j})^2}\right) \tag{16c}$$

$$P_{D_{ji}}^{[k+1]} = \text{Proj}_{\mathcal{S}_2^{[k+1]}}\left(\frac{(y_2^{[k+1]})^2(1+z_2^{[k+1]})g_{D_j}}{((y_2^{[k+1]})^2 g_{D_j} + (y_1^{[k+1]})^2\mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon))^2}\right) \tag{16d}$$

where k is the iteration index, $\text{Proj}_{\mathcal{A}}(*)$ is a projection of $*$ onto the set $\mathcal{A}$; $S_1^{[k]} \triangleq \{P_{C_{ij}} : (P_{C_{ij}}, P_{D_{ji}}^{[k]})$ satisfy (11c) and (11d)$\}$, and $S_2^{[k+1]} \triangleq \{P_{D_{ji}} : (P_{C_{ij}}^{[k+1]}, P_{D_{ji}})$ satisfy (11c) and (11d)$\}$. Next, we show that, with this alternating optimization solution, $|F_0(P_{C_{ij}}^{[k]}, P_{D_{ji}}^{[k]}) - F_0(P_{C_{ij}}^*, P_{D_{ji}}^*)|$ converges in the order $\mathcal{O}(k^{-\alpha})$, for some $\alpha > 0$).

Algorithm 1 Centralized Separate Resource Allocation

Initialize: $\boldsymbol{B}^{(d)}(0), \boldsymbol{B}^{(u)}(0), \boldsymbol{P}_C^{(d)}(0), \boldsymbol{P}_D^{(d)}(0), \boldsymbol{P}_D^{(u)}(0), \boldsymbol{P}_C^{(u)}(0), k = 0$
for all $j \in \mathcal{D}^{(d)}$ **do** ▷ Power Assignment in downlink to find: $\mathbf{P}_{\mathbf{D_j}}^{(\mathbf{d})^*}, \mathbf{P}_{\mathbf{C_j}}^{(\mathbf{d})^*}$.
    if PCSI mode **OR** ICSI-ERM mode **then**
        BS uses the closed-form power allocation in Sec. III-A.
    else ▷ *ICSI-MRM mode*
        BS applies (16) iteratively until convergence.
    end if
end for
repeat
    $k = k + 1$
    BS uses the PGD algorithm to calculate: $\boldsymbol{B}^{(d)}(k)$
until $\boldsymbol{B}^{(d)}$ converges
$\ldots$ ▷ The same for uplink
BS discretize $\boldsymbol{B}^{(d)}, \boldsymbol{B}^{(u)}$.

Theorem 1. *Let* $\{\mathbf{p}^{[k]}\}_{k \in \mathbb{N}_+}$ *be the sequence generated by* (16) *with* $\mathbf{p}^{[k]} \triangleq [P_{D_{ji}}^{[k]}\ P_{C_{ij}}^{[k]}]^T$. *Then, (i)* $\lim\limits_{k \to \infty} \mathbf{p}^{[k]} = \bar{\mathbf{p}}$ *where* $\bar{\mathbf{p}}$ *is a stationary point of* (15)*, and (ii)* $|\mathbf{p}^{[k]} - \bar{\mathbf{p}}| \leq C k^{-(1-\theta)/(2\theta-1)}$ *for some* $C > 0$.

Proof: see Appendix A ■

After solving the power allocation subproblems in both the ERM or MRM cases, a channel allocation problem similar to (7) will arise, and a similar solution based on integer relaxation can be used. Algorithm 1 highlights the operation of the separate resource allocation method with all the previously discussed cases, with $\boldsymbol{B}^{(d)}(k), \boldsymbol{B}^{(u)}(k), \boldsymbol{P}_C^{(d)}(k), \boldsymbol{P}_D^{(d)}(k), \boldsymbol{P}_D^{(u)}(k), \boldsymbol{P}_C^{(u)}(k)$ are the values of each variable at the k-th iteration, and $\mathbf{P}_{\mathbf{D_j}}^{(\mathbf{d})}, \mathbf{P}_{\mathbf{C_j}}^{(\mathbf{d})}$ are the j-th columns of $\mathbf{P}_{\mathbf{D}}^{(\mathbf{d})}, \mathbf{P}_{\mathbf{C}}^{(\mathbf{d})}$ matrices. Since the objective function of the channel allocation problem is Lipschitz smooth, this algorithm will converge as $\mathcal{O}(1/k)$ (as shown in Theorem 3.7 in [35]) in the case of PCSI and ICSI-ERM with $\mathcal{O}(N_C^2 N_D)$ computational operations per iteration. Similarly, in the case of ICSI-MRM, the algorithm will converge as $\mathcal{O}(1/k + k^{-\alpha})$ for some $\alpha > 0$, with similar computations per iteration.

IV. Joint Uplink and Downlink Resource Allocation

In this section, we analyze the scenario in which D2D pairs are assigned uplink or downlink channels on the basis of instantaneous channel conditions, i.e., the algorithm itself generates a decision on the set of D2D pairs communicating in the uplink or the downlink while maximizing the aggregate network throughput. Problem (5) can be extended to the joint uplink and downlink resource allocation case by considering the following modified objective function:

$$\underset{\boldsymbol{B},\boldsymbol{P_C},\boldsymbol{P_D}}{\text{maximize}} \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}[\beta_{i,j}^{(u)}U_{i,j}^{(u)}(P_{C_{ij}}^{(u)},P_{D_{ji}}^{(u)})+\beta_{i,j}^{(d)}U_{i,j}^{(d)}(P_{C_{ij}}^{(d)},P_{D_{ji}}^{(d)})]-\gamma\delta_J(\boldsymbol{B}^{(u)},\boldsymbol{B}^{(d)}), \tag{17}$$

where $U_{i,j}^{(u)}(P_{C_{ij}}^{(u)},P_{D_{ji}}^{(u)})$ and $U_{i,j}^{(d)}(P_{C_{ij}}^{(d)},P_{D_{ji}}^{(d)})$ are general utility functions for the uplink and downlink selected depending on the working conditions (PCSI, ICSI-ERM, or ICSI-MRM), which are set to either the rate gain or the expected rate gain or the minimum rate gain defined in Sec. III. In addition, the constraints must also be extended to take into account both up-link and down-link communications.

Here, we redefine a joint unfairness metric δ_J for joint resource allocation in uplink and downlink. Let N_D be the number of D2D pairs and let $N_C^{(u)}$ and $N_C^{(d)}$ be the total number of channels available in the uplink and downlink respectively. A D2D pair is allowed to communicate in either the downlink or uplink ($\sum_{i\in\mathcal{C}^{(d)}}\beta_{i,j}^{(d)}\times\sum_{i\in\mathcal{C}^{(u)}}\beta_{i,j}^{(u)}=0\ \forall j$). The fairest possible assignment is the one assigning $m_0=(N_C^{(u)}+N_C^{(d)})/N_D=N_C^{(u)}/N_D+N_C^{(d)}/N_D\triangleq m_0^{(u)}+m_0^{(d)}$ to each D2D pair. Similarly to Sec. II, we can adopt the following fairness metric:

$$\begin{aligned}\delta_J(\boldsymbol{B}^{(u)},\boldsymbol{B}^{(d)})&=\frac{\sum_{j\in\mathcal{D}}(m_j-m_0)^2}{m_0^2\ N_D}=\frac{1}{m_0^2\ N_D}\sum_{j\in\mathcal{D}}\left(\sum_{i\in\mathcal{C}^{(u)}}\beta_{i,j}^{(u)}+\sum_{i\in\mathcal{C}^{(d)}}\beta_{i,j}^{(d)}-m_0^{(u)}-m_0^{(d)}\right)^2\\&=\frac{(m_0^{(u)})^2}{m_0^2}\delta(\boldsymbol{B}^{(u)})-\frac{2}{m_0^2\ N_D}\sum_{j\in\mathcal{D}}\left(m_0^{(d)}\sum_{i\in\mathcal{C}^{(u)}}\beta_{i,j}^{(u)}\right)+\frac{(m_0^{(d)})^2}{m_0^2}\delta(\boldsymbol{B}^{(d)})\\&-\frac{2}{m_0^2\ N_D}\sum_{j\in\mathcal{D}}\left(m_0^{(u)}\sum_{i\in\mathcal{C}^{(d)}}\beta_{i,j}^{(d)}\right)+\frac{2m_0^{(u)}m_0^{(d)}}{m_0^2}\end{aligned} \tag{18}$$

The resulting optimization problem is now given by:

$$\underset{\boldsymbol{B}^{(u)},\boldsymbol{B}^{(d)},\boldsymbol{P}^{(u)},\boldsymbol{P}^{(d)}}{\text{maximize}} \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}} \Big[\beta_{i,j}^{(u)} U_{i,j}^{(u)}(P_{C_{ij}}^{(u)}, P_{D_{ji}}^{(u)}) + \beta_{i,j}^{(d)} U_{i,j}^{(d)}(P_{C_{ij}}^{(d)}, P_{D_{ji}}^{(d)})\Big] - \gamma\delta_J(\boldsymbol{B}^{(u)}, \boldsymbol{B}^{(d)}), \tag{19a}$$

$$\text{s.t. } \beta_{i,j}^{(u)} \in \{0,1\}, \quad \beta_{i,j}^{(d)} \in \{0,1\}, \quad \forall\, i,j \tag{19b}$$

$$\sum_{j\in\mathcal{D}} \beta_{i,j}^{(d)} \leq 1, \quad \sum_{j\in\mathcal{D}} \beta_{i,j}^{(u)} \leq 1, \quad \forall\, i \tag{19c}$$

$$\left(\sum_{i\in\mathcal{C}^{(d)}} \beta_{i,j}^{(d)}\right) \times \left(\sum_{i\in\mathcal{C}^{(u)}} \beta_{i,j}^{(u)}\right) = 0, \quad \forall\, j \tag{19d}$$

$$\text{UL/DL power and SINR constraints similar to (5)} \tag{19e}$$

Similar to (5), this problem can further be decomposed into power and channel problems as before without loss of optimality. The power allocation problems are of the form of (6) or (11) depending on the available CSI (PCSI or ICSI) and the selected criteria (ERM or MRM).

A. Resource Allocation under Perfect CSI (PCSI)

In this case, the objective function in (19) becomes deterministic and the decomposition of the problem leads to a similar independent power allocation problem for each pair (i, j) as in III-A, which can be solved in closed form and obtain the optimal $U_{i,j}^{(u)^*}$, $U_{i,j}^{(d)^*}$.
The channel allocation problem becomes:

$$\underset{\boldsymbol{B}^{(u)},\boldsymbol{B}^{(d)}}{\text{maximize}} \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}} [\beta_{i,j}^{(u)} U_{i,j}^{(u)^*} + \beta_{i,j}^{(d)} U_{i,j}^{(d)^*}] - \gamma\delta_2(\boldsymbol{B}^{(u)}, \boldsymbol{B}^{(d)}), \tag{20a}$$

$$\text{s.t. (19b), (19c), and (19d).} \tag{20b}$$

Relaxing the problem by ignoring the constraints (19d) and converting the binary constraints in (19b) to linear constraints as in sec. III-A, leads to a convex problem with linear constraints. This problem can also be solved using PGD, since it is differentiable with linear constraints. Finally, the obtained solution needs to be discretized and projected onto to the set defined by (19d). We propose obtaining a binary solution in the same way used for discretizing (7). Afterwards, for each pair, we evaluate the objective function with the pair assigned to either uplink or downlink; we then select the one which has a higher objective function. After that, we then repeat the channel assignment with the pair removed from the deselected spectrum. The whole process is then repeated until all pairs are assigned. In general, there are many ways to project a solution into the constraints in (19d), however, one cannot guarantee optimallity since (20) has been relaxed.

Algorithm 2 Centralized Joint Resource Allocation

Initialize: $\boldsymbol{B}^{(d)}(0), \boldsymbol{B}^{(u)}(0), \boldsymbol{P}_C^{(d)}(0), \boldsymbol{P}_D^{(d)}(0), \boldsymbol{P}_D^{(u)}(0),\ \boldsymbol{P}_C^{(u)}(0), k = 0$

for all $j \in \mathcal{D}$ **do** ▷ Power assignment in both downlink and uplink to find: $\mathbf{P}_{\mathbf{D_j}}^{\mathbf{(d)}^*}, \mathbf{P}_{\mathbf{C_j}}^{\mathbf{(d)}^*}, \mathbf{P}_{\mathbf{D_j}}^{\mathbf{(u)}^*}, \mathbf{P}_{\mathbf{C_j}}^{\mathbf{(u)}^*}$.
    if PCSI mode **OR** ICSI-ERM mode **then**
        BS uses the closed-form power allocation in Sec. III-A.
    else ▷ ICSI-MRM mode
        BS applies (16) iteratively until convergence.
    end if
end for
repeat
    $k = k + 1$
    BS uses the PGD algorithm to calculate: $\boldsymbol{B}^{(d)}(k), \boldsymbol{B}^{(u)}(k)$
until $\boldsymbol{B}^{(d)}, \boldsymbol{B}^{(u)}$ converges
BS discretize $\boldsymbol{B}^{(d)}, \boldsymbol{B}^{(u)}$.

B. Resource Allocation under Imperfect CSI (ICSI)

The power allocation subpoblems here will be similar to Sec. III-B and will adhere to similar solutions. The channel allocation problem is similar to Sec. IV-A and will follow the same solutions. Algorithm 2 describes the operation of the joint resource allocation methods. As shown in Sec. III, this algorithm will converge as $\mathcal{O}(1/k)$ in the case of PCSI and ICSI-ERM, and in the case of ICSI-MRM, the algorithm will converge as $\mathcal{O}(1/k + k^{-\alpha})$ for some $\alpha > 0$, with $\mathcal{O}(N_C^2 N_D)$ computational operations per iteration in all cases.

V. Decentralized Algorithms

In order to limit dependence of D2D communication on BS together with reducing BS's computational load, we also consider decentralizing the resource allocation algorithms. Furthermore, our aim is to start the communication immediately after the first iteration without waiting for convergence of the algorithm. Since the power assignment subproblems are independent, they can be solved entirely by the D2D pairs. To decompose the channel allocation problem, let G be the objective function of (7). G and its gradient can be express as:

$$G = \mathrm{vec}(\boldsymbol{V})^T \mathrm{vec}(\boldsymbol{B}) - k_1 \|\boldsymbol{B}^T \mathbf{1} - k_2 \mathbf{1}\|^2, \tag{21a}$$

$$\nabla G = \boldsymbol{V} - 2k_1(\mathbf{1}\mathbf{1}^T \boldsymbol{B} - k_2 \mathbf{1}\mathbf{1}^T), \tag{21b}$$

where $\boldsymbol{V} \triangleq [v_{ij}]$, and k_1 and k_2 are constants. Notice that, the channel allocation problem can not be directly decomposed into disjoint subproblems for each D2D pair, due to the quadratic term in (21). Nevertheless, the gradient is linear in $\boldsymbol{B}$, thus, the descent part of the channel allocation algorithm can be directly decomposed and each D2D pair can perform an optimization step over its corresponding part, without loss of optimality. Only the projection and the discretization have to be performed centrally at the BS.

C

A. *Separate Uplink and Downlink Resource Allocation*

Algorithm 3 below describes how this scenario can be solved in a decentralized manner. The BS initializes the power assignment vectors and the channel allocation matrices, and broadcasts them. Each D2D pair perform a step of the power allocation algorithm suitable for the network operation scenario (i.e. closed form for PCSI or ICSI-ERM or the alternating minimization in (16) for ICSI-MRM). Then, each D2D pair updates its vectors of the channel allocations $(\boldsymbol{B}_j^{(d)}, \boldsymbol{B}_j^{(u)})$ by performing a gradient step. Each D2D pair then sends its channel allocation vectors along with the calculated power values to the BS. Then, the BS assembles all the vectors of the channel allocation matrices and projects them into a feasible solution and resends them to all D2D pairs. The BS and D2D pairs uses these calculated powers and channel assignments for communications. These steps are then repeated until all variables converge. Algorithm 3 will also converge as $\mathcal{O}(1/k)$ in the case of PCSI and ICSI-ERM, and as $\mathcal{O}(1/k+k^{-\alpha})$ for some $\alpha > 0$ for the case of ICSI-MRM. However, $\mathcal{O}(N_C^2)$ computational operations per iteration are performed by each D2D pair, and $\mathcal{O}(N_C N_D)$ computational operations per iteration are performed by the BS with $2N_C N_D$ variables exchanged between the D2D pairs and the BS in every iteration.

B. *Joint Uplink and Downlink Resource Allocation*

Algorithm 4 below describes how this scenario can be solved in a decentralized manner. The BS initializes the power assignment vectors and the channel allocation matrices, and broadcasts them. Each D2D pair performs a step of the power allocation method suitable for the operation scenario followed by updating the vectors of the channel allocation $(\boldsymbol{B}_j^{(d)}, \boldsymbol{B}_j^{(u)})$ using a gradient

Algorithm 3 Decentralized Separate Resource Allocation

Initialize: $\boldsymbol{B}^{(d)}(0), \boldsymbol{B}^{(u)}(0), \boldsymbol{P}_C^{(d)}(0), \boldsymbol{P}_D^{(d)}(0), \boldsymbol{P}_D^{(u)}(0),\ \boldsymbol{P}_C^{(u)}(0), k = 0$
for all $j \in \mathcal{D}^{(d)}$ **do**
  BS sends $\mathbf{B_j^{(d)}(0)}, \mathbf{P_{C_j}^{(d)}(0)}, \mathbf{P_{D_j}^{(d)}(0)}$ to D2D pair j.
end for
for all $j \in \mathcal{D}^{(u)}$ **do**
  BS sends $\mathbf{B_j^{(u)}(0)}, \mathbf{P_{C_j}^{(u)}(0)}, \mathbf{P_{D_j}^{(u)}(0)}$ to D2D pair j.
end for
repeat
  $k = k + 1$
  for all $j \in \mathcal{D}^{(d)}$ **do** ▷ Find: $\mathbf{P_{D_j}^{(d)}(k)}, \mathbf{P_{C_j}^{(d)}(k)}, \mathbf{B_j^{(d)}(k)}$.
    if PCSI mode **OR** ICSI-ERM mode **then**
      D2D pair j uses the closed-form power allocation in Sec. III-A.
    else ▷ ICSI-MRM mode
      D2D pair j applies (16) for a single iteration.
    end if
    D2D pair j sends $\mathbf{P_{C_j}^{(d)}(k)}$ to the BS.
    D2D pair j applies a single iteration of the PGD algorithm and sends $\mathbf{B_j^{(d)}(k)}$ to the BS.
  end for
  ... ▷ The same for uplink
  BS projects $\boldsymbol{B}^{(d)}(k)$ and $\boldsymbol{B}^{(u)}(k)$ and sends each column to the corresponding D2D pair.
until $\boldsymbol{B}^{(d)}, B^{(u)}, \boldsymbol{P}_C^{(d)}, \boldsymbol{P}_D^{(d)}, \boldsymbol{P}_D^{(u)}, \boldsymbol{P}_C^{(u)}$ converges

step. Then, the BS assembles all the vectors of the channel allocation matrices and projects them into a feasible solution and broadcasts them to all D2D pairs. The BS and D2D pairs uses these calculated powers and channel assignments for communications. These steps are then repeated until all variables converge. Algorithm 4 has the same convergence and computational behaviour as Algorithm 3.

VI. Simulations

We consider a simulation scenario with a single cell of radius 500 m. In this cell, CUs and D2D transmitters are located uniformly at random. The D2D receivers are located uniformly

Algorithm 4 Decentralized Joint Resource Allocation

Initialize: $\boldsymbol{B}^{(d)}(0), \boldsymbol{B}^{(u)}(0), \boldsymbol{P}_C^{(d)}(0), \boldsymbol{P}_D^{(d)}(0), \boldsymbol{P}_D^{(u)}(0),\ \boldsymbol{P}_C^{(u)}(0), k = 0$
for all $j \in \mathcal{D}$ **do**
    BS sends $\mathbf{B_j^{(d)}(0), B_j^{(u)}(0), P_{C_j}^{(d)}(0), P_{D_j}^{(d)}(0), P_{D_j}^{(u)}(0), P_{C_j}^{(u)}(0)}$ to D2D pair j.
end for
repeat
    $k = k + 1$
    for all $j \in \mathcal{D}$ **do** ▷ Find: $\mathbf{P_{D_j}^{(d)}(k), P_{C_j}^{(d)}(k), P_{D_j}^{(u)}(k), P_{C_j}^{(u)}(k), B_j^{(d)}(k), B_j^{(u)}(k)}$.
        if PCSI mode **OR** ICSI-ERM mode **then**
            D2D pair j uses the closed-form power allocation in Sec. III-A.
        else ▷ ICSI-MRM mode
            D2D pair j applies (16) for a single iteration.
        end if
        D2D pair sends $P_{C_j}^{(d)}(k), P_{C_j}^{(u)}(k)$ to the BS.
        D2D pair applies a single iteration of the PGD algorithm and sends $\mathbf{B_j^{(d)}(k), B_j^{(u)}(k)}$ to the BS.
    end for
    BS projects $\boldsymbol{B}^{(d)}(k), \boldsymbol{B}^{(u)}(k)$ and sends each column to the corresponding D2D pair.
until $\boldsymbol{B}^{(d)}, \boldsymbol{B}^{(u)}, \boldsymbol{P}_C^{(d)}, \boldsymbol{P}_D^{(d)}, \boldsymbol{P}_D^{(u)}, \boldsymbol{P}_C^{(u)}$ converges

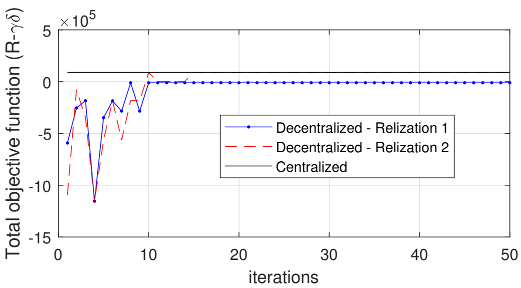

Fig. 3: Rate of convergence

at random in a 5 m radius circle centered at their respective transmitter. A path-loss model with exponent $\alpha = 2$ is used in the calculation of all channel gains. The random channel gains are calculated by applying an exponential random distribution around an average calculated from the path-loss model. $N_C = 10, N_D = 10$ were used in the experiments with Monte-Carlo averages carried over $10,000$ different realizations. Fig. 3 shows the convergence results for the

Decentralized Algorithm 4 with imperfect CSI (ICSI)-minimum guaranteed rate maximization (MRM) compared to the Centralized Algorithm 2 with ICSI-MRM of a simulation scenario with two realizations for $\gamma = 50$. It shows that the decentralized algorithm converges in a relatively small number of iterations. Similar behaviour is also observed when comparing the the Decentralized Algorithm 3 compared to the Centralized Algorithm 1. The obtained decentralized solutions, in general, are not identical to the centralized solution but it are very close. This is as expected because the alternating optimization for the power allocation and the binary channel allocation problem might have different solutions based on the initialization and the projection, since it is not convex.

Figs. 4, 5, and 6 shows comparisons of Algorithm 3 in the cases of perfect CSI (PCSI), ICSI-expected rate maximization (ERM) and ICSI-MRM, compared with the previous state-of-the-art methods in [27]. Additionally, we assumed all D2D pairs will use only downlink spectrum.The achieved rate of the PCSI case is the highest, as expected, since it ignores the probabilistic constraints and only uses the average channel gains. The cases of ICSI-ERM and ICSI-MRM achieve the second and third rate respectively. The method in [27] achieves the lowest rate since it does not allow assigning multiple channels to a D2D pair. The rates of all methods, except the PCSI case, grow with the allowed outage probability ϵ. However, the fairness of the method in [27] is the best for the same reason (D2D pair can not access multiple channels). All cases of Algorithm 3 achieve relatively similar fairness, with the order of ICSI-MRM, ICSI-ERM, and PCSI from the second best to the forth respectively. The achieved outage probabilities of [27] and case ICSI-ERM are exactly equal to the allowed outage probability ϵ, since the achieved optimal power assignment lies in the border of the feasibility region in both methods. Case ICSI-MRM achieves a better outage probability than the desired ϵ with the corresponding gap increasing when ϵ increases; this can be caused by the fact that the power allocation algorithm converges to a local optima rather than the global one, and the number of feasible local optimums increases when expanding the feasibility set. The PCSI case achieves a very high outage probability, which is fixed, regardless of the value of ϵ.

Fig. 7 shows comparisons between algorithms 3 and 4 in a ICSI-ERM case and ICSI-MRM case. Algorithm 3 is tested in the following scenarios; all D2D pairs in downlink, all D2D pairs in uplink, half D2D pairs in downlink and half in uplink. The results shows that uplink is generally better than downlink, as expected, due to the lower interference in uplink caused by the lower

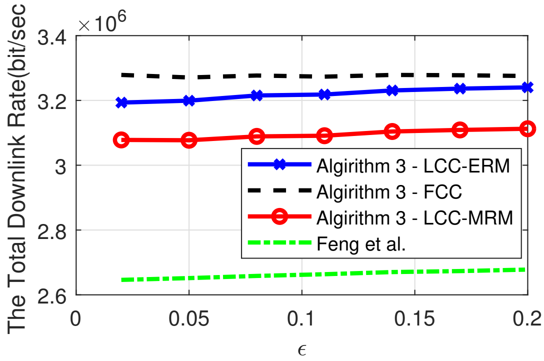

Fig. 4: Total Rate vs. ϵ

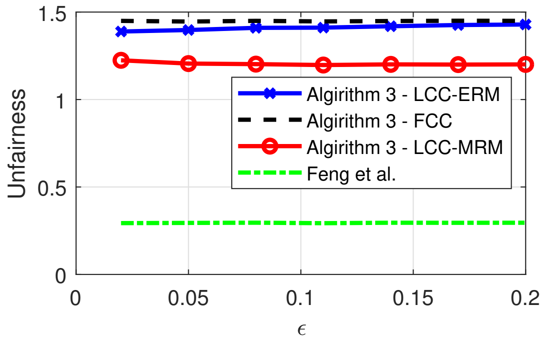

Fig. 5: Fairness vs. ϵ

maximum transmitting power and the possibly longer distances between the D2D pairs and the CUs. Moreover, distributing users among both uplink and downlink achieves significantly higher data rates, with Algorithm 4 achieves the highest rates, since the distribution of users is also optimized.

VII. Conclusion

This paper formulates a joint channel allocation and power assignment problem in underlay D2D communications. This problem aims at maximizing the total network rate while keeping the fairness among the D2D pairs. It also allows assigning multiple channels to each D2D

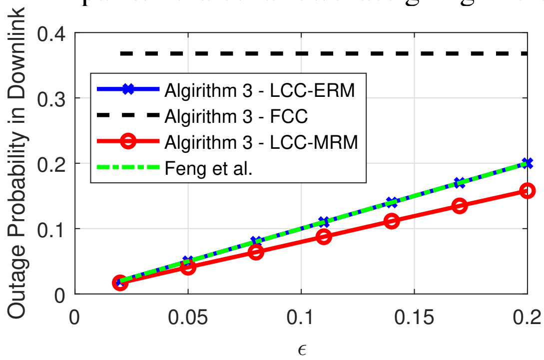

Fig. 6: Outage Probability vs. ϵ

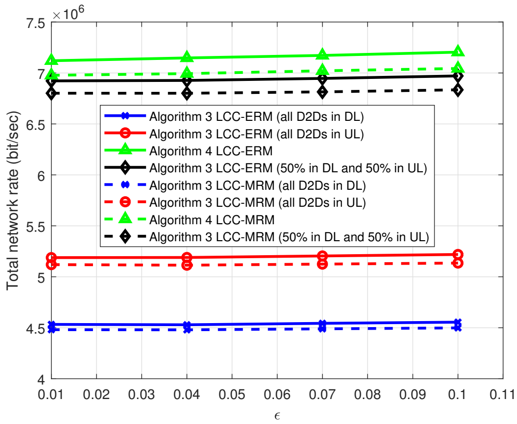

Fig. 7: The total network rate (UL+DL) vs. ϵ

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pair. Furthermore, it assigns both downlink and uplink resources either jointly or separately. Moreover, it considers uncertainties in the CSI by including probabilistic SINR constraints to guarantee the desired outage probability. Although this problem is a non-convex mixed-integer problem, we solve it in a computationally efficient manner by convex relaxation, quadratic transformation and alternating optimization techniques. Additionally, decentralized algorithms to solve this problem are also presented in this paper. Numerical experiment show that our algorithms achieve substantial performance improvements as compared to the state-of-the-art.

Appendix A
Proof of Theorem 1

First, let us define an equivalent problem to (15) as follows:

$$\underset{\boldsymbol{p},\boldsymbol{z},\boldsymbol{y}}{\text{maximize}} \quad F_2(\boldsymbol{p},\boldsymbol{z},\boldsymbol{y}) \tag{22a}$$

$$\text{subject to} \quad (11c),\ (11d),$$

$$z_1 = \frac{P_{C_{ij}} g_{C_i}}{N_0 + P_{D_{ji}} \mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}, \qquad z_2 = \frac{P_{D_{ji}} g_{D_j}}{N_0 + P_{C_{ij}} h_{C_j}}, \tag{22b}$$

$$y_1 = \frac{\sqrt{(1+z_1) P_{C_{ij}} g_{C_i}}}{P_{C_{ij}} g_{C_i} + N_0 + P_{D_{ji}} \mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon)}, \; y_2 = \frac{\sqrt{(1+z_2) P_{D_{ji}} g_{D_j}}}{P_{D_{ji}} g_{D_j} + N_0 + P_{C_{ij}} h_{C_j}}, \tag{22c}$$

where the additional constraints (22b) and (22c) are obtained from the solutions of (15) in (16a) and (16b) respectively. Since the solution of (15) lies in the feasible set of (22a), both problems are equivalent.

Next, we prove that the limit point $\bar{\mathbf{p}}$ of $\{\mathbf{p}^{[k]}\}_{k\in\mathbb{N}_+}$ is a stationary point of (22a). It is shown in Theorem 2.8 in [36] that, for any bounded continuous function that (i) is *locally Lipschitz smooth and strongly convex* for each block in the feasibility set, (ii) has a Nash point, and (iii)

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satisfies the Kurdyka Lojasiewicz (KL) property in a neighborhood around a stationary point, the sequence generated by an alternation optimization algorithm with a fixed update scheme initialized in that neighborhood will converge to that stationary point. We will next show that $-F_2$ satisfies (i), (ii), and (iii).

A Nash point $\bar{\mathbf{x}}$ for a function $f(\mathbf{x})$ is defined as a block-wise minimizer where $f(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_i, \ldots, \bar{\mathbf{x}}_s) \leq f(\bar{\mathbf{x}}_1, \ldots, \mathbf{x}_i, \ldots, \bar{\mathbf{x}}_s)$ $\forall i$ and $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_s)$ [36]. Since F_2 is continuous and the feasible set of (22a) is compact, F_2 attains a locally optimal point as stated by Weierstrass' Theorem described in (A.2.7) in [37]. Thus, the function F_2 has a Nash point. The function $-F_2(\mathbf{p}, \mathbf{z}, \tilde{\mathbf{y}})$ can be shown to be strongly convex and Lipschitz smooth in each variable separately in the bounded feasible set of (22a) ($\partial^2 F_2/\partial z_1^2 = -(1+z_1)^{-2} - y_1\sqrt{P_{C_{ij}} g_{C_i}}(1+z_1)^{-3/2}/2$, $\partial^2 F_2/\partial z_2^2 = -(1+z_2)^{-2} - y_2\sqrt{P_{D_{ji}} g_{D_i}}(1+z_2)^{-3/2}/2$, $\partial^2 F_2/\partial y_1^2 = -2(P_{C_{ij}} g_{C_i} + N_0 + P_{D_{ji}} \mathrm{F}^{-1}_{\tilde{h}_{D_{j,i}}}(1-\epsilon))$, $\partial^2 F_2/\partial y_2^2 = -2(P_{D_{ji}} g_{D_j} + N_0 + P_{C_{ij}} h_{C_j})$, $\partial^2 F_2/\partial P_{C_{ij}}^2 = -y_1\sqrt{(1+z_1) g_{C_i}}(P_{C_{ij}})^{-3/2}/2$, $\partial^2 F_2/\partial P_{D_{ji}}^2 = -y_2\sqrt{(1+z_2) g_{D_j}}(P_{D_{ji}})^{-3/2}/2$).

To see that (iii) holds, we use the following definition of the Kurdyka Lojasiewicz (KL) property [36]:

Definition 1. Kurdyka Lojasiewicz (KL) property: A function $f(\mathbf{x})$ satisfies the KL property at point $\bar{\mathbf{x}} \in \mathrm{dom}(\partial f)$ if $\eta = \dfrac{|f(\mathbf{x}) - f(\bar{\mathbf{x}})|^\theta}{dist(\mathbf{0}, \partial f(\mathbf{x}))}$ is bounded for $0 \leq \theta < 1$ $\forall \mathbf{x}$ in some neighborhood U of $\bar{\mathbf{x}}$.

We then introduce a lemma as follows:

Lemma 1. *The function $F_2(\mathbf{p}, \mathbf{z}, \mathbf{y})$ satisfies the KL property at any point $\mathbf{p} \in \mathcal{P}$, $\mathbf{z} \in \mathbb{R}_+^2$ and $\mathbf{y} \in \mathbb{R}_+^2$, for some $\theta \in [1/2, 1)$.*

Proof: see Appendix B

Since F_2 is analytic everywhere, it is also analytic around a stationary point $[\bar{\mathbf{p}},\ \bar{\mathbf{z}},\ \bar{\mathbf{y}}]^T$ and, consequently, satisfies (iii). Thus the alternation sequence in (16) initialized at any feasible point $\mathbf{p}^0$ will converge to the nearest stationary point of F_2, since $\mathbf{p}^0$ is in the neighborhood of the nearest stationary point $\bar{\mathbf{p}}$. Moreover, any stationary point of F_2 is a stationary point of F_0 [33], [34]. Thus the sequence $\{\mathbf{p}^{[k]}\}_{k \in N_+}$ converges to a stationary point of F_0.

Next, we prove that $|\mathbf{p}^{[k]} - \bar{\mathbf{p}}| \leq C k^{-(1-\theta)/(2\theta-1)}$ for some $C > 0$. It is shown in theorem 2.9 in [36], for a function $f(\mathbf{x})$ that satisfies Theorem 2.8 in [36] and the KL property for some $\theta \in (1/2, 1)$, the update sequence $\mathbf{x}^{[k]}$ converges to a stationary point $\bar{\mathbf{x}}$ as $|\mathbf{x}^{[k]} - \bar{\mathbf{x}}| \leq C k^{-(1-\theta)/(2\theta-1)}$ with a certain $C > 0$. Since $F_2(\mathbf{p}, \mathbf{z}, \mathbf{y})$ satisfy the KL property for some $\theta \in (1/2, 1)$, the update sequence $\{\mathbf{p}^k\}_{k\in N_+}$ converges as $|\mathbf{p}^{[k]} - \bar{\mathbf{p}}| \leq Ck^{-(1-\theta)/(2\theta-1)}$ to a stationary point $\bar{\mathbf{p}}$.

APPENDIX B
PROOF OF LEMMA 1

It can be shown that any real analytic function satisfies the KL property for some $\theta \in [1/2, 1)$ [36, sec. 2.2]. Next, we need to show that $F_2(\mathbf{p}, \mathbf{z}, \mathbf{y})$ is a real analytic function. A function $f(\mathbf{x})$ is a real analytic function if it is infinitely differentiable and its Taylor series around a point $\mathbf{x}_0$ converges to $f(\mathbf{x})$ for $\mathbf{x}$ in some neighbourhood of $\mathbf{x}_0$ [38]. To simplify our problem, we first need to consider the following properties of real analytic functions [38]: (i) The sum and product of real analytic functions is a real analytic function. (ii) Any polynomial is a real analytic function. (iii) The composition of real analytic functions is a real analytic function. Exploiting, the first property, it suffices to show that all individual terms in the expression of $F_2(\mathbf{p}, \mathbf{z}, \mathbf{y})$ in (15) are real analytic.

We first show that $\log(.)$ is real analytic on a positive real argument, i.e., $x \in (0, \infty)$. This can be formally proved by showing that the remainder of the order-n Taylor series expansion of $\log(x)$ centered around a point c goes to zero as n goes to infinity. The Taylor series expansion of $\log(x)$ centered at $c > 0$, can be expressed as: $\sum_{n=0}^{\infty} \frac{(-1)^n (n-1)!}{n! c^n} (x-c)^n$.Our objective is to show that above expansion converges to $\log(x)\ \forall\ x \in (c/2, 3c/2)$. The Lagrange reminder of the Taylor's series expansion of function $f(x)$ can be expressed as: $R_n(x) = \frac{f^{(n+1)}(\zeta)}{(n+1)!}(x-c)^{n+1}$, where, $f^{(n+1)}(.)$ is the $(n+1)$-th derivative of f. Substituting $f(\zeta) = \log(\zeta)$, we have $R_n(x) = \frac{(-1)^n n!}{(n+1)! \zeta^{n+1}}(x-c)^{n+1}$, where, $x, \zeta \in (c/2, 3c/2)$. Simplifying further, $|R_n(x)| = \frac{1}{(n+1)!} \frac{|x-c|^{n+1}}{|\zeta|^{n+1}} \leq \frac{1}{n+1}$. Thus, $\lim_{n\to\infty}$, $|R_n(x)| \to 0$. Hence, Taylor's series expansion of $\log(x)$ centered at c converges to $\log(x)$ on $(c/2, 3c/2)$. Further, if $c \to \infty$, $\log(x)$ is real analytic for $x \in (0, \infty)$. Thus, $\log(1 + z_1)$ and $\log(1 + z_2)$ are real analytic functions for $z_1, z_2 > 0$.

Next, we consider the following terms of $F_2(\mathbf{p}, \mathbf{z}, \mathbf{y})$: $z_1; z_2; y_1^2(p_B g_D + N_0 + P_D h_D); y_2^2(p_D g_D + N_0 + P_B h_B)$. It can be noted that all these terms are positive polynomials. Thus, by the second property, all of these terms are real analytic functions. Finally, for the terms $2y_1\sqrt{(1+z_1)p_B g_B}$ and $2y_2\sqrt{(1+z_1)p_B g_B}$, we exploit the first and the third properties. Note that y_1, y_2, $(1+z_1)p_B g_B$ and $(1+z_1)p_B g_B$ are positive polynomials; hence, they are real analytic functions. Thus, we just need to show that the square root is a real analytic function. Let $f(x) : \mathbb{R} \to \mathbb{R}$ be a real analytic function. Then, $\sqrt{f(x)} = e^{\log\left(\sqrt{f(x)}\right)} = e^{\frac{\log(f(x))}{2}}$. Since the composition of real analytic functions is real analytic; given that $e^{(\cdot)}$ is real analytic and $\log(f(x))$ is real analytic for $f(x) > 0$; then, we can conclude that $\sqrt{(f(x))}$ is real analytic for $f(x) > 0$.

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Appendix D

Paper D

Title:	Resource Allocation for Multiple Underlay Interfering Device-to-Device Communications.
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PAPER D

Resource Allocation for Multiple Underlay Interfering Device-to-Device Communications

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D

Abstract

Underlay device-to-device (D2D) communications improve the spectral efficiency by simultaneously allowing direct communication between the users on the same channels as traditional cellular users. However, most works in resource allocation for D2D communication have considered single antenna transmission and restrict the assignment of multiple channels to D2D pairs. This work formulates an optimization problem for maximizing the aggregate rate of all D2D pairs and cellular users (CUs) in both SISO and MIMO scenarios. This formulation guarantees an signal to interference plus noise ratio (SINR) above a specified threshold. In addition, it also ensures fairness in channel allocation to D2D pairs. The resulting problem is a mixed integer non-convex problem, and we propose to approximately solve it by alternating between power allocation and channel assignment subproblems. The power allocation subproblem is approximately solved by exploiting a quadratic transformation, which leads us to an alternating optimization method. The channel assignment subproblem is solved by relaxing it into a convex problem and using the project gradient descend algorithm. We also propose two computationally efficient algorithms that approximately solve this problem in a partially decentralized manner. Simulation results corroborate the merits of the proposed approach by illustrating higher network throughput and more reliable communication.

Index Terms

D2D communications, resource allocation, channel assignment, power allocation, beamforming.

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I. Introduction

Underlay D2D communications improve the spectral efficiency by simultaneously allowing multiple transmissions of D2D pairs and traditional cellular network in the same spectrum [1]–[4]. However, simultaneous communications increase interference when using the same bands, which must be carefully handled by devising algorithms for the assignment of channels to D2D pairs and the control of transmission powers. The allocation of resources must also be fair while guaranteeing the desired Quality of service (QoS).

Few works also consider performing channel assignment to the D2D pairs for underlay communication. In [5], [6], channels are allocated to the D2D pairs using auction games while measuring the fairness in the number of channels assigned to each D2D pair. [7] proposes channel assignment to D2D pairs utilizing a coalition-forming game model. Additionally, millimeter-wave spectrum is also used as an overlay option for D2D pairs. However, it can be noted that these schemes only perform channel assignment and avoid controlling the transmit power, which limits the achievable throughput of the overall network.

In order to solve the above limitations, a Stackelberg game based approach is proposed in [8] where each D2D pair simultaneously transmits in all cellular channels and adjust the power used for transmission. Here, the BS penalizes the D2D pairs if they generate disruptive interference to the cellular communication. The optimization of the transmit power while ensuring minimum SINR requirements is also investigated in [9], however, as long as the SINR requirements are satisfied, D2D pairs are allowed to transmit in all channels. In an alternative approach, distributed optimization for power allocation is studied in [10] for both overlay and underlay scenarios. To summarize, all the works mentioned so far perform either channel assignment or power allocation, but not both.

Few research works consider jointly optimizing channel assignment and power allocation, as they seem to show strong dependency. This joint optimization is considered in [11]–[14] but they restrict D2D users to access at most one cellular channel. However, the work in [15]–[17] allow assignment of multiple channels to each D2D pair. Notice that these schemes propose to use either uplink or downlink spectrum for D2D communications. Some recent research works also consider both uplink and downlink spectrum for allocating resources to D2D pairs. In [18]–[20], both uplink and downlink spectra are considered in their formulation; however, they limit the assignment to at most one channel to each D2D pair.

Most works on resource allocation problems have considered jointly allocation power and channel resources for underlay D2D communication while assuming a single antenna transmission framework [11], [13]. These schemes also restrict D2D pairs from accessing more than one channel and are also assumed to have prefect channel state information (CSI). However, the work in [15]–[17] allow assignment of multiple channels to each D2D pair. Scenarios of imperfect CSI for single antenna transmission are considered in [21], [22].

Under the multi antenna transmission framework, [23] provides a comprehensive analysis for joint beamforming in D2D underlay cellular networks. However, this work is restricted to a single D2D pair. In [24] beamforming with multiple D2D pairs are considered.

Design of robust beamformers for general multiuser communication has also been investigated in past research works [25]–[27]. Under the assumption of Gaussian CSI uncertainties, analytical methods based on Bernstein-type inequality and decomposition techniques are proposed in [25] to approximate the probabilistic rate outage constraints.

Despite the above research efforts, none of the existing approaches provide a joint channel assignment and power allocation to the D2D pairs and CUs while guaranteeing the desired QoS and maintaining fairness while allowing multiple D2D pairs to access the same channel. The main contributions of this work are:

- We formulate a resource allocation problem under the assumption of both SISO and MIMO communications. Our objective is to maximize the aggregate rate of all D2D pairs and CUs with a penalty on unfair channel assignment, under a constraint on the minimum SINR requirement.
- Since the resulting problem is a mixed integer non-convex problem, we propose alternating between multiple power allocation subproblems and a channel assignment subproblem. The power allocation subproblems are solved by alternating optimization obtained after applying fractional programming via a quadratic transformation. The channel assignment subproblem is solved by integer relaxation.
- We propose two algorithm were those problems can be solved in a decentralized manner with applying either dual decomposition or replacing the coupling constraints with tighter separable ones.
- We also include convergence guarantees and short proofs for the main algorithms proposed here.
- Finally, numerical experiments are performed to corroborate the merits of the proposed approach by illustrating a higher throughput while maintaining the desired fairness and QoS.

II. System Model

Consider the cellular communication setup shown in Fig. 1 where a BS with N_B transmit

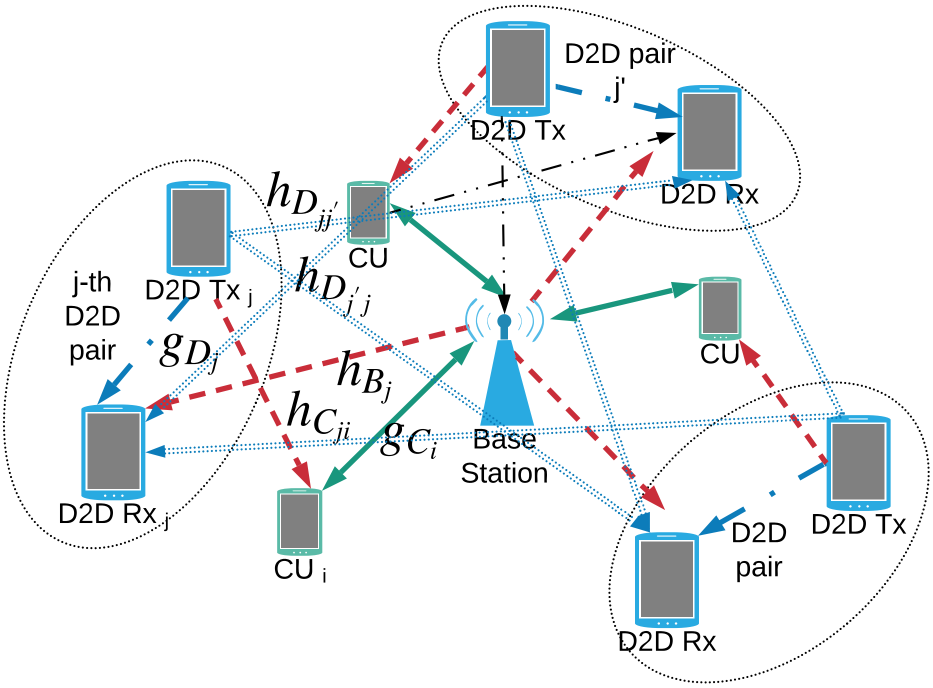

Fig. 1: Illustration of the overall system model.

antennas communicates with N_C CUs through N_C downlink channels[1]. The set of CUs (equivalently, channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. In an underlay configuration, N_D D2D pairs, indexed by $\mathcal{D} = \{1, ..., N_D\}$, wish to communicate using the aforementioned N_C downlink channels. Each CU and D2D terminal has N_T antennas.

First, let $\boldsymbol{G}_{B_i} \in \mathcal{C}^{N_B \times N_T}$ be the channel gain between the base station (BS) and the i-th CU; $\boldsymbol{G}_{D_j} \in \mathcal{C}^{N_T \times N_T}$ be the channel gain of the j-th D2D pair; $\boldsymbol{H}_{C_{j,i}} \in \mathcal{C}^{N_T \times N_T}$ be the channel gain of the interference link from the transmitter of the j-th D2D pair to the i-th CU; $\boldsymbol{H}_{D_{k,j}} \in \mathcal{C}^{N_T \times N_T}$ be the channel gain of the interference link from the transmitter of the k-th D2D pair to the receiver of the j-th D2D pair; $\boldsymbol{H}_{B_j} \in \mathcal{C}^{N_B \times N_T}$ be the channel gain of the interference link from the BS to the receiver of the j-th D2D pair; and N_0 be the noise power on each sub-channel.

The BS assignment of channels to D2D pairs is denoted by the indicator parameters $\{\beta_{i,j}\}_{i \in \mathcal{C}, j \in \mathcal{D}}$, where $\beta_{i,j} = 1$ indicates assignment of the i-th channel to the j-th D2D pair and $\beta_{i,j} = 0$ otherwise. For higher throughput, we allow a D2D pair to simultaneous access multiple channels. We denote the transmit precoder vector for the BS to communicate with

[1]Even though the formulation is done for downlink communications, the same formulation can be directly extended to uplink.

the i-th CU as $\boldsymbol{p}_{B_i} \in \mathbb{C}^{N_B \times 1}$ and as $\boldsymbol{p}_{D_{j,i}} \in \mathbb{C}^{N_T \times 1}$ for the j-th D2D transmitter on the i-th channel. The precoders are constrained to a maximum total power as $\sum_{i \in \mathcal{C}} ||\boldsymbol{p}_{B_i}||_2^2 \leq p_{B,\max}$ and $\sum_{i \in \mathcal{C}} ||\boldsymbol{p}_{D_{j,i}}||_2^2 \leq p_{D,\max}$. To ensure successful communication, the SINR should also be enforced to be greater than a certain threshold $\eta_{\min}^D$ for the D2D pairs and $\eta_{\min}^C$ for the CUs.

III. SISO Scenario

For simplicity, we will start the formulation with a single antenna case at the BS and at each CU and D2D terminals in this section. The MIMO case will be presented in the next section. Let us first express the following achievable rates:
R_{C_i} denotes the throughput of the CU user i.

$$R_{C_i} = \log_2 \left(1 + \frac{P_{B_i} g_{B_i}}{N_0 + \sum\limits_{j \in \mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \right)$$

$R_{D_{j,i}}$ denotes the throughput of the D2D pair j when sharing the spectrum with the CU user i.

$$R_{D_{j,i}} = \log_2 \left(1 + \frac{P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum\limits_{j' \in \mathcal{D}, j' \neq j} P_{D_{j'i}} h_{D_{j',j}}} \right)$$

R_i denotes the total rate of channel i.

$$R_i = R_{C_i} + \sum_{j \in \mathcal{D}} \beta_{ij} R_{D_{j,i}}$$

Let $\delta(\boldsymbol{B})$ denotes the fairness function, which measures the fairness of a channel assignment. To define this function, let x_j denotes the number of resources assigned to D2D pair j, namely $x_j = \sum\limits_{i=1}^{N_C} \beta_{i,j}$, and x_0 denotes the desired resource assignment when every D2D pair uses ($x_0 = rN_C$) channels, where $r \in [0, 1]$ is the ratio of channels assigned to each pair on average. $\boldsymbol{B}$ is the channel assignment matrix that combines all values of β as $\boldsymbol{B} = [\beta_{ij}]$. Following [5], the fairness of a channel allocation $\boldsymbol{B}$ can be quantified by:

$$\begin{aligned} \delta(\boldsymbol{B}) &= \frac{\frac{1}{N_D} \sum\limits_{j=1}^{N_D} (x_j - x_0)^2}{x_0^2} \\ &= \frac{1}{(rN_C)^2} \sum_{j=1}^{N_D} \left(\frac{1}{N_D} \left(\sum_{i=1}^{N_C} \beta_{i,j} - rN_C \right)^2 \right) \end{aligned} \tag{1}$$

which can be interpreted as a scaled variance of the assignment $\boldsymbol{B}$ from its fairest value x_0.

Next, we define an objective function that combines the total network rate and the fairness. We propose using a parameter $\gamma > 0$ to control the trade-off between these two functions. The complete optimization problem can then be expressed as:

$$\underset{\boldsymbol{B},\boldsymbol{p_B},\boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}} \left[R_i(P_{B_i}, P_{D_{ji}}, \boldsymbol{B})\right] - \gamma\delta(\boldsymbol{B}) \tag{2a}$$

$$\text{subject to} \quad \beta_{i,j} \in \{0,1\},\ \forall\, i, j, \tag{2b}$$

$$\sum_{i\in\mathcal{C}} P_{B_i} \leq P_{B_{max}}\ \ \forall\, i, \tag{2c}$$

$$\sum_{i\in\mathcal{C}} P_{D_{ji}} \leq P_{D_{max}}\ \ \forall\, j, i, \tag{2d}$$

$$\frac{P_{B_i} g_{B_i}}{N_0 + \sum\limits_{j\in\mathcal{D}} \beta_{i,j} P_{D_{ji}} h_{C_{j,i}}} \geq \eta^C_{min},\ \forall\, i, \tag{2e}$$

$$\frac{P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum\limits_{k\in\mathcal{D},k\neq j} \beta_{i,k} P_{D_{ki}} h_{D_{k,j}}} \geq \eta^D_{min},\ \forall j, i. \tag{2f}$$

Problem (2) is a mixed integer non convex problem and finding the optimal solution to such problem is computationally expensive. We propose to use alternating optimization between the power and channel allocation problems, thus a computationally efficient approximate solution can be found.

D

A. Power Allocation

For a given $\boldsymbol{B}^{(k-1)}$, we should now solve (2) for $\boldsymbol{p_B}, \boldsymbol{P_D}$. However, this problem is still non-convex with many variables. Moreover, constraints (2c) and (2d) are coupling across channels preventing the problem from being decomposed and solved in parallel. To approximately solve solve this problems we propose three different methods:

(**M1**) Solve the whole power allocation subproblem centrally without decomposition.

(**M2**) Include constraints (2c) and (2d) in the objective function of (2). This will make the problem decomposable.

(**M3**) Replacing constraints (2c) and (2d) by tighter constraints on the power in each channel, i.e. $P_{B_i} \leq P_{B_{max}}/N_C$ and $P_{D_{ji}} \leq P_{D_{max}}/N_C$. This will also make the problem decomposable.

*1) Case (*M1*):* In this case we solve problem (2) without decomposing it across channels. When alternating between the channel and power subproblems, the k-th iteration power subproblem for a known $\boldsymbol{B} = \boldsymbol{B}^{(k-1)}$ becomes:

$$\underset{\boldsymbol{p_B}, \boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i \in \mathcal{C}} \left[R_{C_i} + \sum_{j \in \mathcal{D}} R_{D_{j,i}} \right] \tag{3a}$$

$$\text{subject to} \quad \sum_{i \in \mathcal{C}} P_{B_i} \leq P_{B_{max}} \tag{3b}$$

$$\sum_{i \in \mathcal{C}} \beta_{i,j} P_{D_{ji}} \leq P_{D_{max}} \ \ \forall \ j \tag{3c}$$

$$\frac{P_{B_i} g_{B_i}}{N_0 + \sum\limits_{j \in \mathcal{D}} \beta_{i,j} P_{D_{ji}} h_{C_{j,i}}} \geq \eta_{min}^{C}, \ \ \forall \ i, j, \tag{3d}$$

$$\frac{\beta_{i,j} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum\limits_{k \in \mathcal{D}, k \neq j} \beta_{i,k} P_{D_{ki}} h_{D_{k,j}}} \geq \eta_{min}^{D}, \ \ \forall i, j : \beta_{ij} = 1. \tag{3e}$$

To address the non-convexity in objective function, we introduce the auxiliary variables z_{C_i} and $z_{D_{j,i}}$ as follows:

$$\underset{P_B, \mathbf{P_D}, \mathbf{z}}{\text{maximize}} \quad F_1 \triangleq \sum_{i \in \mathcal{C}} \left(\log\left(1 + z_{C_i}\right) + \sum_{j \in \mathcal{D}} \log\left(1 + z_{D_{j,i}}\right) \right) \tag{4a}$$

$$\text{subject to} \quad z_{C_i} \leq \frac{P_{B_i} g_{B_i}}{N_0 + \sum\limits_{j \in \mathcal{D}} \beta_{ij} P_{D_{ji}} \tilde{h}_{C_{j,i}}} \ \forall i \tag{4b}$$

$$z_{D_{j,i}} \leq \frac{\beta_{ij} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum\limits_{k \in \mathcal{D}, k \neq j} \beta_{ik} P_{D_{ki}} h_{D_{k,j}}} \ \forall i, j \tag{4c}$$

$$\text{(10b), (10c), (10d), and (3e)} \tag{4d}$$

Where $z_{C_i}^{*} = \dfrac{P_{B_i} g_{B_i}}{N_0 + \sum\limits_{j \in \mathcal{D}} P_{D_{ji}} \tilde{h}_{C_{j,i}}}$ and $z_{D_{j,i}}^{*} = \dfrac{\beta_{ij} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum\limits_{k \in \mathcal{D}, k \neq j} \beta_{ik} P_{D_{ki}} h_{D_{k,j}}}$.

From hereafter, let $P_{D_{ji}} \triangleq \beta_{ij} P_{D_{ji}}$ to absorb the value of the binary channel assignment variables into the power allocation variables. The partial Lagrangian of F_1 with respect to (4b) and (4c) is:

$$L(\boldsymbol{p}, \boldsymbol{z}, \boldsymbol{\lambda}) = \sum_{i \in \mathcal{C}} \Bigg(\log\left(1 + z_{C_i}\right) + \sum_{j \in \mathcal{D}} \log\left(1 + z_{D_{j,i}}\right) \Bigg)$$
$$- \sum_{i \in \mathcal{C}} \Bigg(\lambda_{C_i} \left(z_{C_i} - \frac{P_{B_i} g_{B_i}}{N_0 + \sum\limits_{j \in \mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \right)$$
$$- \sum_{j \in \mathcal{D}} \lambda_{D_{j,i}} \left(z_{D_{j,i}} - \frac{P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum\limits_{k \in \mathcal{D}, k \neq j} P_{D_{ki}} h_{D_{k,j}}} \right) \Bigg)$$

A stationary point of L, $(\boldsymbol{z}*, \boldsymbol{\lambda}^{*})$, must satisfy $\dfrac{\partial L}{\partial \boldsymbol{z}} = \mathbf{0}$. This leads to:

$$\lambda^*_{C_i} = \frac{1}{1+z^*_{C_i}} = \frac{N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}} h_{C_{j,i}}}{P_{B_i} g_{B_i} + N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \tag{5}$$

$$\lambda^*_{D_{j,i}} = \frac{1}{1+z^*_{D_{j,i}}} = \frac{N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}} h_{D_{k,j}}}{P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}} h_{D_{k,j}}} \tag{6}$$

An equivalent optimization problem to (3) can be expressed as:

$$\begin{aligned}
\underset{P_B,P_D,\boldsymbol{z}}{\text{maximize}} \quad & F_2 \triangleq L(\boldsymbol{p},\boldsymbol{z},\boldsymbol{\lambda}^*) = \sum_{i\in\mathcal{C}} \Bigg(\log\left(1+z_{C_i}\right) - z_{C_i} + \\
& \sum_{j\in\mathcal{D}} \left(\log\left(1+z_{D_{j,i}}\right) - z_{D_{j,i}}\right) + \frac{(1+z_{C_i}) P_{B_i} g_{B_i}}{P_{B_i} g_{B_i} + N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \\
& + \sum_{j\in\mathcal{D}} \frac{(1+z_{D_{j,i}}) P_{D_{ji}} g_{D_j}}{P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}} h_{D_{k,j}}} \Bigg) \\
\text{subject to} \quad & \text{(10b), (10c), (10d), and (10e).}
\end{aligned} \tag{7}$$

Then using the quadratic transformation proposed in [28], [29]:

$$\begin{aligned}
\underset{P_B,P_D,\boldsymbol{z},\boldsymbol{y}}{\text{maximize}} \quad & F_3 \triangleq \sum_{i\in\mathcal{C}} \Bigg(\log\left(1+z_{C_i}\right) + \sum_{j\in\mathcal{D}} \log\left(1+z_{D_{j,i}}\right) \\
& - z_{C_i} + 2y_{C_i}\sqrt{(1+z_{D_{j,i}}) P_{B_i} g_{B_i}} - y^2_{C_i}(P_{B_i} g_{B_i} + N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}} h_{C_{j,i}}) \\
& + \sum_{j\in\mathcal{D}} -z_{D_{j,i}} + 2y_{D_{j,i}}\sqrt{(1+z_{D_{j,i}}) P_{D_{ji}} g_{D_j}} \\
& - y^2_{D_{j,i}}(P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}} h_{D_{k,j}}) \Bigg) \\
\text{subject to} \quad & \text{(3b), (3c), (3d), and (3e).}
\end{aligned} \tag{8}$$

D

Iteratively solving this problem by alternating the variables will lead to:

$$y_{C_i}^{(t)} = \frac{\sqrt{(1+z_{C_i}^{(t)})P_{B_i}^{(t-1)} g_{B_i}}}{P_{B_i}^{(t-1)} g_{B_i} + N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}}^{(t-1)} h_{C_{j,i}}}, \tag{9a}$$

$$y_{D_{ji}}^{(t)} = \frac{\sqrt{(1+z_{D_{ji}}^{(t)})P_{D_{ji}}^{(t-1)} g_{D_j}}}{P_{D_{ji}}^{(t-1)} g_{D_j} + N_0 + P_{B_i}^{(t-1)} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}}^{(t-1)} h_{D_{k,j}}}, \tag{9b}$$

$$P_{B_i}^{(t)} = \text{Proj}_{\mathcal{S}_1}\left(\frac{(y_{C_i}^{(t)})^2(1+z_{C_i}^{(t)})g_{B_i}}{((y_{C_i}^{(t)})^2 g_{B_i} + \sum_{j\in\mathcal{D}}(y_{D_{ji}}^{(t)})^2 h_{B_j})^2}\right), \tag{9c}$$

$$P_{D_{ji}}^{(t)} = \text{Proj}_{\mathcal{S}_1}\left(\frac{y_{D_{ji}}^{(t)})^2(1+z_{D_{ji}}^{(t)})g_{D_j}}{[(y_{D_{ji}}^{(t)})^2 g_{D_j} + (y_{C_i}^{(t)})^2 h_{C_{j,i}} + \sum_{k\in\mathcal{D},k\neq j}(y_{D_{ki}}^{(t)})^2 h_{D_{k,j}}]^2}\right), \tag{9d}$$

where $\text{Proj}_{\mathcal{S}_1}(.)$ is a projection into the set $\mathcal{S}_1$ defined by the constraints (3b), (3c), (3d) and (3e), and t is the internal iteration number[2].

2) Case ($\mathbf{M2}$*):* When alternating between the channel and power subproblems, the k-th iteration power subproblem for a known $\boldsymbol{B}^{(k-1)}$ becomes:

$$\underset{\boldsymbol{p_B},\boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\left[R_{c_i} + \sum_{j\in\mathcal{D}} R_{D_{j,i}}\right] \tag{10a}$$

$$\text{subject to} \quad \sum_{i\in\mathcal{C}} P_{B_i} \leq P_{B_{max}} \tag{10b}$$

$$\sum_{i\in\mathcal{C}} \beta_{i,j} P_{D_{ji}} \leq P_{D_{max}} \ \ \forall\, j \tag{10c}$$

$$\frac{P_{B_i} g_{B_i}}{N_0 + \sum_{j\in\mathcal{D}} \beta_{i,j} P_{D_{ji}} h_{C_{j,i}}} \geq \eta_{min}^{C}, \ \ \forall\, i, j, \tag{10d}$$

$$\frac{\beta_{i,j} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} \beta_{i,k} P_{D_{ki}} h_{D_{k,j}}} \geq \eta_{min}^{D}, \ \ \forall i, j : \beta_{ij} = 1. \tag{10e}$$

[2] k is the outer loop iteration number for the alternation between the power and channel allocation subproblems, while t is the inner loop iteration number inside the power allocation subproblem.

To address the non-convexity in objective function, we introduce the auxiliary variables z_{C_i} and $z_{D_{j,i}}$ as follows:

$$\underset{P_B, \mathbf{P_D}, \mathbf{z}}{\text{maximize}} \quad F_1 \triangleq \sum_{i \in \mathcal{C}} \Big(\log\left(1 + z_{C_i}\right) + \sum_{j \in \mathcal{D}} \log\left(1 + z_{D_{j,i}}\right) \Big) \tag{11a}$$

$$\text{subject to} \quad z_{C_i} \leq \frac{P_{B_i} g_{B_i}}{N_0 + \sum_{j \in \mathcal{D}} \beta_{ij} P_{D_{ji}} \tilde{h}_{C_{j,i}}} \ \forall i \tag{11b}$$

$$z_{D_{j,i}} \leq \frac{\beta_{ij} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum_{k \in \mathcal{D}, k \neq j} \beta_{ik} P_{D_{ki}} h_{D_{k,j}}} \ \forall i, j \tag{11c}$$

$$\text{(10b), (10c), (10d), and (10e)} \tag{11d}$$

Where $z^*_{C_i} = \dfrac{P_{B_i} g_{B_i}}{N_0 + \sum_{j \in \mathcal{D}} P_{D_{ji}} \tilde{h}_{C_{j,i}}}$ and $z^*_{D_{j,i}} = \dfrac{\beta_{ij} P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum_{k \in \mathcal{D}, k \neq j} \beta_{ik} P_{D_{ki}} h_{D_{k,j}}}$.
From hereafter, let $P_{D_{ji}} \triangleq \beta_{ij} P_{D_{ji}}$ to absorb the value of the binary channel assignment variables into the power allocation variables. The partial Lagrangian of F_1 with respect to (11b) and (11c) is:

$$L(\boldsymbol{p}, \boldsymbol{z}, \boldsymbol{\lambda}) = \sum_{i \in \mathcal{C}} \Big(\log\left(1 + z_{C_i}\right) + \sum_{j \in \mathcal{D}} \log\left(1 + z_{D_{j,i}}\right) \Big)$$
$$- \sum_{i \in \mathcal{C}} \Bigg(\lambda_{C_i} \left(z_{C_i} - \frac{P_{B_i} g_{B_i}}{N_0 + \sum_{j \in \mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \right)$$
$$- \sum_{j \in \mathcal{D}} \lambda_{D_{j,i}} \left(z_{D_{j,i}} - \frac{P_{D_{ji}} g_{D_j}}{N_0 + P_{B_i} h_{B_j} + \sum_{k \in \mathcal{D}, k \neq j} P_{D_{ki}} h_{D_{k,j}}} \right) \Bigg)$$

A stationary point of L, $(\boldsymbol{z}*, \boldsymbol{\lambda}^*)$, must satisfy $\dfrac{\partial L}{\partial \boldsymbol{z}} = \mathbf{0}$. This leads to:

$$\lambda^*_{C_i} = \frac{1}{1 + z^*_{C_i}} = \frac{N_0 + \sum_{j \in \mathcal{D}} P_{D_{ji}} h_{c_{j,i}}}{P_{B_i} g_{B_i} + N_0 + \sum_{j \in \mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \tag{12a}$$

$$\lambda^*_{D_{j,i}} = \frac{1}{1 + z^*_{D_{j,i}}} = \frac{N_0 + P_{B_i} h_{B_j} + \sum_{k \in \mathcal{D}, k \neq j} P_{D_{ki}} h_{D_{k,j}}}{P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum_{k \in \mathcal{D}, k \neq j} P_{D_{ki}} h_{D_{k,j}}} \tag{12b}$$

D

An equivalent optimization problem to (10) can be expressed as:

$$\underset{P_B, P_D, \boldsymbol{z}}{\text{maximize}} \quad F_2 \triangleq L(\boldsymbol{p}, \boldsymbol{z}, \boldsymbol{\lambda}^*) = \sum_{i \in \mathcal{C}} \Bigg(\log\left(1 + z_{C_i}\right) - z_{C_i} + \tag{13}$$

$$\sum_{j \in \mathcal{D}} \left(\log\left(1 + z_{D_{j,i}}\right) - z_{D_{j,i}} \right) + \frac{(1 + z_{C_i}) P_{B_i} g_{B_i}}{P_{B_i} g_{B_i} + N_0 + \sum\limits_{j \in \mathcal{D}} P_{D_{ji}} h_{C_{j,i}}}$$

$$+ \sum_{j \in \mathcal{D}} \frac{(1 + z_{D_{j,i}}) P_{D_{ji}} g_{D_j}}{P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum\limits_{k \in \mathcal{D}, k \neq j} P_{D_{ki}} h_{D_{k,j}}} \Bigg)$$

$$\text{subject to} \quad \text{(10b), (10c), (10d), and (10e).}$$

Then using the quadratic transformation proposed in [28], [29]:

$$\underset{P_B, P_D, \boldsymbol{z}, \boldsymbol{y}}{\text{maximize}} \quad F_3 \triangleq \sum_{i \in \mathcal{C}} \Bigg(\log\left(1 + z_{C_i}\right) + \sum_{j \in \mathcal{D}} \log\left(1 + z_{D_{j,i}}\right) \tag{14}$$

$$- z_{C_i} + 2 y_{C_i} \sqrt{(1 + z_{D_{j,i}}) P_{B_i} g_{B_i}} - y_{C_i}^2 (P_{B_i} g_{B_i} + N_0 + \sum_{j \in \mathcal{D}} P_{D_{ji}} h_{C_{j,i}})$$

$$+ \sum_{j \in \mathcal{D}} - z_{D_{j,i}} + 2 y_{D_{j,i}} \sqrt{(1 + z_{D_{j,i}}) P_{D_{ji}} g_{D_j}}$$

$$- y_{D_{j,i}}^2 (P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum_{k \in \mathcal{D}, k \neq j} P_{D_{ki}} h_{D_{k,j}}) \Bigg)$$

$$\text{subject to} \quad \text{(10b), (10c), (10d), and (10e).}$$

The auxiliary variable y_{C_i} and $y_{D_{j,i}}$ can be updated as

$$y_{C_i} = \frac{\sqrt{(1 + z_{C_i}) P_{B_i} g_{B_i}}}{P_{B_i} g_{B_i} + N_0 + \sum\limits_{j \in \mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \tag{15a}$$

$$y_{D_{j,i}} = \frac{\sqrt{(1 + z_{D_{j,i}}) P_{D_{ji}} g_{D_j}}}{P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum\limits_{k \in \mathcal{D}, k \neq j} P_{D_{ki}} h_{D_{k,j}}}, \tag{15b}$$

In order to reduce computation complexity of (14), we decouple the optimization problem across i-th downlink channel. Notice that constraints (10b) and (10c) form the coupling constraints. Thus, we exploit dual decomposition for which the Lagrangian of (14) with respect to the coupling constraints (10b) and (10c) can be expressed as:

$$\underset{P_B,P_D,\boldsymbol{z},\boldsymbol{y}}{\text{maximize}} \quad F_3 \triangleq \sum_{i\in\mathcal{C}}\Bigg(\log\left(1+z_{C_i}\right)+\sum_{j\in\mathcal{D}}\log\left(1+z_{D_{j,i}}\right) \tag{16}$$

$$-z_{C_i}+2y_{C_i}\sqrt{(1+z_{D_{j,i}})P_{B_i}g_{B_i}}-y_{C_i}^2(P_{B_i}g_{B_i}+N_0+\sum_{j\in\mathcal{D}}P_{D_{ji}}h_{C_{j,i}})$$

$$+\sum_{j\in\mathcal{D}}-z_{D_{j,i}}+2y_{D_{j,i}}\sqrt{(1+z_{D_{j,i}})P_{D_{ji}}g_{D_j}}$$

$$-y_{D_{j,i}}^2(P_{D_{ji}}g_{D_j}+N_0+P_{B_i}h_{B_j}+\sum_{k\in\mathcal{D},k\neq j}P_{D_{ki}}h_{D_{k,j}})\Bigg)$$

$$-\mu_C\Big(\sum_{i\in\mathcal{C}}P_{B_i}-P_{B_{max}}\Big)-\sum_{j\in\mathcal{D}}\mu_{D_j}\Big(\sum_{i\in\mathcal{C}}\beta_{i,j}P_{D_{ji}}-P_{D_{max}}\Big)$$

The decoupled optimization problem across i-th downlink channel can be expressed as,

$$\underset{P_{B_i},P_{D_i},\boldsymbol{z},\boldsymbol{y}}{\text{maximize}} \quad F_3 \triangleq \log\left(1+z_{C_i}\right)+\sum_{j\in\mathcal{D}}\log\left(1+z_{D_{j,i}}\right) \tag{17}$$

$$-z_{C_i}+2y_{C_i}\sqrt{(1+z_{D_{j,i}})P_{B_i}g_{B_i}}-y_{C_i}^2(P_{B_i}g_{B_i}+N_0+\sum_{j\in\mathcal{D}}P_{D_{ji}}h_{C_{j,i}})$$

$$+\sum_{j\in\mathcal{D}}-z_{D_{j,i}}+2y_{D_{j,i}}\sqrt{(1+z_{D_{j,i}})P_{D_{ji}}g_{D_j}}$$

$$-y_{D_{j,i}}^2(P_{D_{ji}}g_{D_j}+N_0+P_{B_i}h_{B_j}+\sum_{k\in\mathcal{D},k\neq j}P_{D_{ki}}h_{D_{k,j}})$$

$$-\mu_C P_{B_i}-\sum_{j\in\mathcal{D}}\mu_{D_j}P_{D_{ji}}$$

subject to (10d), and (10e)

The closed form solution for updating power variables is

$$P_{B_i}=\text{Proj}_{\mathcal{S}_2}\left(\frac{y_{C_i}^2(1+z_{C_i})g_{B_i}}{(y_{C_i}^2g_{B_i}+\sum_{j\in\mathcal{D}}y_{D_{j,i}}^2h_{B_j}+\mu_C)^2}\right) \tag{18a}$$

$$P_{D_{j,i}}=\text{Proj}_{\mathcal{S}_2}\left(\frac{y_{D_{j,i}}^2(1+z_{D_{j,i}})g_{D_j}}{[y_{D_{j,i}}^2g_{D_j}+y_{D_{j,i}}^2h_{C_{j,i}}+\sum_{k\in\mathcal{D},k\neq j}y_{D_{k,i}}^2h_{D_{k,j}}+\mu_{D_j}]^2}\right) \tag{18b}$$

Where $\text{Proj}_{\mathcal{S}_2}(.)$ is a projection into the set $\mathcal{S}_2$ defined by the constraints (10d) and (10e). The dual variables can be updated as:

$$\mu_C=\left(\mu_C+\alpha\Big(\sum_{i\in\mathcal{C}}P_{B_i}-P_{B_{max}}\Big)\right)_+ \tag{19a}$$

$$\mu_{D_j}=\left(\mu_{D_j}+\alpha\Big(\sum_{i\in\mathcal{C}}\beta_{i,j}P_{D_{ji}}-P_{D_{max}}\Big)\right)_+ \tag{19b}$$

D

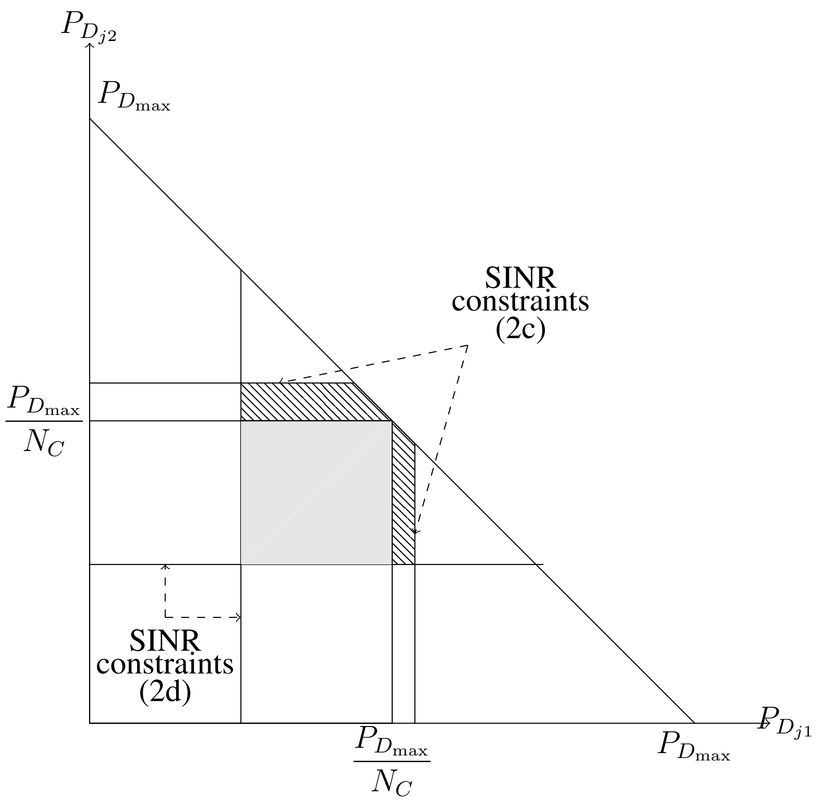

Fig. 2: Feasibility region of a single D2D pair for fixed $\boldsymbol{B},\ \boldsymbol{P}_B,\ P_{D_{ki}}\ \forall\ k \neq j$

Note that since (14) is a convex optimization problem, updating power variables by (18a), (18b) followed by updating dual variables (19a) and (19b) will converge to optimal solution of (14).

3) Case ($\mathbf{M3}$*):* In this case we replace the coupling constraints (2c) and (2d) in problem (2) with a tighter power bound in each channel. The difference between the feasibility sets of case ($\mathbf{M1}$) and case ($\mathbf{M3}$) is shown in Fig. 2 for two channels with a single D2D pair. We believe the difference in the feasibility region in this case is quite small, this has been verified numerically in the following sections.

The k-th iteration of the resulting problem for a fixed $\boldsymbol{B}^{(k-1)}$ can be expressed as follows:

$$\underset{\boldsymbol{p_B},\boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\left[R_{c_i}+\sum_{j\in\mathcal{D}}\beta_{ij}^{k-1}R_{D_{j,i}}\right] \tag{20a}$$

$$\text{subject to} \quad 0\le P_{B_i}\le P_{B_{max}} \ \ \forall\ i, \tag{20b}$$

$$0\le P_{D_{ji}}\le P_{D_{max}} \ \ \forall\ j,i, \tag{20c}$$

$$\frac{P_{B_i}g_{B_i}}{N_0+\sum_{j\in\mathcal{D}}\beta_{i,j}P_{D_{ji}}h_{C_{j,i}}}\ge\eta_{min}^{C}, \ \ \forall\ i,j, \tag{20d}$$

$$\frac{P_{D_{ji}}g_{D_j}}{N_0+P_{B_i}h_{B_j}+\sum_{k\in\mathcal{D},k\neq j}\beta_{i,k}P_{D_{ki}}h_{D_{k,j}}}\ge\eta_{min}^{D}, \ \ \forall i,j:\beta_{ij}=1. \tag{20e}$$

This problem decouples across i. For every i, using auxiliary variables $\mathbf{z}$, the problem can be rewritten as:

$$\underset{P_{B_i},\mathbf{P_{D_i}},\mathbf{z}}{\text{maximize}} \quad F_1\triangleq\log\left(1+z_{C_i}\right)+\sum_{j\in\mathcal{D}}\log\left(1+z_{D_{ji}}\right) \tag{21a}$$

$$\text{subject to} \quad z_{C_i}\le\frac{P_{B_i}g_{B_i}}{N_0+\sum_{j\in\mathcal{D}}\beta_{ij}P_{D_{ji}}h_{C_{j,i}}}, \tag{21b}$$

$$z_{D_{ji}}\le\frac{\beta_{ij}P_{D_{ji}}g_{D_j}}{N_0+P_{B_i}h_{B_j}+\sum_{k\in\mathcal{D},k\neq j}\beta_{ik}P_{D_{ki}}h_{D_{k,j}}}, \quad \forall j, \tag{21c}$$

$$\text{(20b), (20c), (20d), and (20e),} \tag{21d}$$

where

$$z_{C_i}^{*}=\frac{P_{B_i}g_{B_i}}{N_0+\sum_{j\in\mathcal{D}}P_{D_{ji}}\tilde{h}_{C_{j,i}}}, \tag{22a}$$

$$z_{D_{ji}}^{*}=\frac{\beta_{ij}P_{D_{ji}}g_{D_j}}{N_0+P_{B_i}h_{B_j}+\sum_{k\in\mathcal{D},k\neq j}\beta_{ik}P_{D_{ki}}h_{D_{k,j}}}. \tag{22b}$$

From hereafter, let $P_{D_{ji}}\triangleq\beta_{ij}P_{D_{ji}}$ absorb the value of the binary channel assignment variables into the power allocation variables. The partial Lagrangian of F_1 with respect to (21b) and (21c) is:

$$L(\boldsymbol{p},\boldsymbol{z},\boldsymbol{\lambda})=\log\left(1+z_{C_i}\right)+\sum_{j\in\mathcal{D}}\log\left(1+z_{D_{ji}}\right)$$
$$-\lambda_{C_i}\left(z_{C_i}-\frac{P_{B_i}g_{B_i}}{N_0+\sum_{j\in\mathcal{D}}P_{D_{ji}}h_{C_{j,i}}}\right)-\sum_{j\in\mathcal{D}}\lambda_{D_{ji}}\left(z_{D_{ji}}-\frac{P_{D_{ji}}g_{D_j}}{N_0+P_{B_i}h_{B_j}+\sum_{k\in\mathcal{D},k\neq j}P_{D_{ki}}h_{D_{k,j}}}\right)$$

D

A stationary point of L, $(\boldsymbol{z}*, \boldsymbol{\lambda}^*)$, must satisfy $\dfrac{\partial L}{\partial \boldsymbol{z}} = \mathbf{0}$. This leads to:

$$\lambda^*_{C_i} = \frac{1}{1+z^*_{C_i}} = \frac{N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}} h_{C_{j,i}}}{P_{B_i} g_{B_i} + N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \tag{23}$$

$$\lambda^*_{D_{ji}} = \frac{1}{1+z^*_{D_{ji}}} = \frac{N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}} h_{D_{k,j}}}{P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}} h_{D_{k,j}}} \tag{24}$$

An equivalent optimization problem to (21a) can be expressed as:

$$\begin{aligned} \underset{P_{B_{ij}}, P_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \quad & F_2 \triangleq L(\boldsymbol{p}, \boldsymbol{z}, \boldsymbol{\lambda}^*) = \log\left(1 + z_{C_i}\right) - z_{C_i} + \\ & \sum_{j\in\mathcal{D}} \left(\log\left(1+z_{D_{ji}}\right) - z_{D_{ji}}\right) + \frac{(1+z_{C_i}) P_{B_i} g_{B_i}}{P_{B_i} g_{B_i} + N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}} h_{C_{j,i}}} \\ & + \sum_{j\in\mathcal{D}} \frac{(1+z_{D_{ji}}) P_{D_{ji}} g_{D_j}}{P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}} h_{D_{k,j}}} \\ \text{subject to} \quad & \text{(20b), (20c), (20d), and (20e).} \end{aligned} \tag{25}$$

Then using a quadratic transformation [28], [29], this results in:

$$\begin{aligned} \underset{P_{B_{ij}}, P_{D_{ji}}, \boldsymbol{z}, \boldsymbol{y}}{\text{maximize}} \quad & F_3 \triangleq \log\left(1 + z_{C_i}\right) + \sum_{j\in\mathcal{D}} \log\left(1 + z_{D_{ji}}\right) \\ & - z_{C_i} + 2y_{C_i}\sqrt{(1+z_{C_i}) P_{B_i} g_{B_i}} \\ & - (y_{C_i})^2 (P_{B_i} g_{B_i} + N_0 + \sum_{j\in\mathcal{D}} P_{D_{ji}} h_{C_{j,i}}) \\ & + \sum_{j\in\mathcal{D}} -z^d_j + 2y_{D_{ji}}\sqrt{(1+z_{D_{ji}}) P_{D_{ji}} g_{D_j}} \\ & - (y_{D_{ji}})^2 (P_{D_{ji}} g_{D_j} + N_0 + P_{B_i} h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P_{D_{ki}} h_{D_{k,j}}) \\ \text{subject to} \quad & \text{(20b), (20c), (20d), and (20e)} \end{aligned} \tag{26}$$

D

Iteratively solving this problem by alternating the variables will lead to:

$$y^*_{C_i}[t] = \frac{\sqrt{(1+z^*_{C_i}[t])P^*_{B_i}[t-1]g_{B_i}}}{P^*_{B_i}[t-1]g_{B_i} + N_0 + \sum_{j\in\mathcal{D}} P^*_{D_{ji}}[t-1]h_{C_{j,i}}} \tag{27a}$$

$$y^*_{D_{ji}}[t] = \sqrt{(1+z^*_{D_{ji}}[t])P^*_{D_{ji}}[t-1]g_{D_j}}/(P^*_{D_{ji}}[t-1]g_{D_j} + N_0 + P^*_{B_i}[t-1]h_{B_j} + \sum_{k\in\mathcal{D},k\neq j} P^*_{D_{ki}}[t-1]h_{D_{k,j}}), \tag{27b}$$

$$P^*_{B_i}[t] = \text{Proj}_{\mathcal{S}_3}\left(\frac{(y^*_{C_i}[t])^2(1+z^*_{C_i}[t])g_{B_i}}{((y^*_{C_i}[t])^2 g_{B_i} + \sum_{j\in\mathcal{D}}(y^*_{D_{ji}}[t])^2 h_{B_j})^2}\right), \tag{27c}$$

$$P^*_{D_{ji}}[t] = \text{Proj}_{\mathcal{S}_3}\left(\frac{y^*_{D_{ji}}[t])^2(1+z^*_{D_{ji}}[t])g_{D_j}}{[(y^*_{D_{ji}}[t])^2 g_{D_j} + (y^*_{C_i}[t])^2 h_{C_{j,i}} + \sum_{k\in\mathcal{D},k\neq j}(y^*_{D_{ki}}[t-1])^2 h_{D_{k,j}}]^2}\right), \tag{27d}$$

where $\text{Proj}_{\mathcal{S}_3}(.)$ is a projection into the set $\mathcal{S}_3$ defined by the constraints (20b), (20c), (20d) and (20e).

D

B. Channel Assignment

After finding $\boldsymbol{P}^k_B, \boldsymbol{P}^k_D$, we substitute them in the objective function and solve for $\boldsymbol{B}^K$. The objective function is then:

$$\underset{\boldsymbol{B}}{\text{maximize}} \quad \sum_{i\in\mathcal{C}}\left[\sum_{j\in\mathcal{D}}\beta_{ij}R^k_{D_{j,i}}\right] - \gamma\hat{\delta}(\boldsymbol{B}) \tag{28a}$$

$$\text{subject to} \quad \beta_{i,j}\in\{0,1\},\ \forall\, i,j. \tag{28b}$$

By relaxing the binary constraints to $0 \leq \beta_{i,j} \leq 1$, we obtain a convex problem. We can solve it by the projected gradient descent algorithm. We can re-discretize the $\boldsymbol{B}$ by selecting an appropriate threshold.

We keep alternating between channel and power allocation until convergence.

IV. Algorithms

In this section we presents the algorithms for all cases and highlights the operation performed by each element in the network.

A. Case (M1)

In this case, we propose solving the problem centrally at the BS. Algorithm 1 highlights the operation of this solution. The projection needed by the power allocation subproblems is computationally demanding.

Algorithm 1 Centralized Resource Allocation for $(\mathbf{M1})$ case

Initialize: $\boldsymbol{B}^{(0)}, \mathbf{P_B^{(0)}}, \mathbf{P_D^{(0)}}, \mathbf{k}=\mathbf{0}$
repeat
  $k = k+1$
  Power Allocation
    For the previous channel assignment $(B^{(k-1)})$
    BS applies (9) iteratively until convergence
    to find $\mathbf{P_B^{(k)}},\ \mathbf{P_D^{(k)}}$.
  Channel Assignment
    For the new power allocation $(\mathbf{P_B^{(k)}},\ \mathbf{P_D^{(k)}})$
    BS uses the PGD algorithm to calculate: $\boldsymbol{B}^{(k)}$
until $\boldsymbol{B},\ \mathbf{P_B},\ \mathbf{P_D}$ converges

B. Case $(\mathbf{M2})$

In this case the power allocation subproblem can be decoupled across channels. We propose solving this problem in decentralized manner as described in Algorithm 2. The dual variables update is done centrally at the BS in addition to the channel assignment.

C. Case $(\mathbf{M3})$

In this case the power allocation subproblem can be decoupled across channels. We propose solving this problem in decentralized manner as described in Algorithm 3. Only the channel assignment needs to be done centrally. Moreover, each iteration of the power allocation and the channel assignment problems provide a feasible solution and can be directly used for communication.

V. Convergence Analysis

Problem (8) for $(\mathbf{M1})$ case and problem (26) for $(\mathbf{M3})$ case are bounded, real analytic, strongly convex and Lipschitz smooth in each variables block. Thus, they both satisfy Theorem 1. in [30]. Therefore, the power allocation subproblems, in both cases, can be shown to converge to a stationary point $\bar{\mathbf{p}}$ with $|\mathbf{p}^{[k]} - \bar{\mathbf{p}}| \leq Ck^{-(1-\theta)/(2\theta-1)}$ for some $C > 0$ and $\theta \in [0.5, 1)$.

However, Problem (14) for $(\mathbf{M2})$ case, is not strongly convex in the dual variable, and therefore, dose not satisfy the same theorem. In this cases, the convergence of such solution

Algorithm 2 Decentralized Resource Allocation for ($\mathbf{M2}$) case

```
Initialize and broadcast: B^(0), P_B^(0), P_D^(0), μ_c^(0), μ_d^(0), k = 0
repeat
    k = k + 1
    Power Allocation
      For the previous channel assignment (B^(k−1))
    repeat
        BS and every D2D pair apply (12) and exchange the resulting z_{C_i}, z_{D_ji} ∀ i, j.
        BS and every D2D pair apply (15a) and exchange the resulting y_{C_i}, y_{D_ji} ∀ i, j.
        BS and every D2D pair apply (18) and exchange the resulting P_{C_i}, P_{D_ji} ∀ i, j.
        BS applies (19) centrally and update μ_C, μ_{D_j} ∀ j.
    until P_B and P_D converge resulting in P_B^(k) and P_D^(k).
    Channel Assignment
      For the new power allocation (P_B^(k), P_D^(k))
      BS uses the PGD algorithm to calculate: B^(k)
      BS discretize B^(k)
until B, P_B, P_D converge.
```


can not be easily guaranteed. However, some works such as [31] suggest adding additional quadratic smoothing term to make the problem strongly convex even in the dual variables. However, adding this term to our solution will not allow us to obtain closed-form expressions for each iteration and thus adding extra computational cost.

The relaxed channel assignment subproblem in all cases is quadratic-convex Lipschitz smooth problem with linear constraints. Solving this problem will converge as $\mathcal{O}(1/k)$ (as shown in Theorem 3.7 in [32]).

The alternation between the power and channel sub problems follows Theorem 2.3 in [33] and thus will converge to a Nash point. Numerical experiment that shows the convergence in a single realization with $N_C = N_D = 5,\ \gamma = 300 \times \text{BW}$ are shown in Fig. 3.

Fig. 3 shows the convergence of the power allocation subproblem in all cases. It shows that case ($\mathbf{M1}$) converges in the lowest number of iterations, however it has the highest computational cost per iteration. Case ($\mathbf{M3}$) has the second fastest convergence rate with a very close objective function to ($\mathbf{M1}$) and, more importantly, has the lowest computational cost per iteration. Case ($\mathbf{M2}$) has the slowest convergence among all three cases, moreover, in the

Algorithm 3 Decentralized Resource Allocation for (**M3**) case

Initialize and broadcast: $\boldsymbol{B}^{(0)},\ \mathrm{\mathbf{P}}_B^{(0)},\ \boldsymbol{P}_D^{(0)},\ k = 0$

repeat

  $k = k + 1$

  Power Allocation (single iteration)

    For the previous channel assignment ($B^{(k-1)}$)

    BS and every D2D pair apply (22) and exchange the resulting $z_{C_i},\ z_{D_{ji}}\ \forall\ i, j$.

    BS and every D2D pair apply (27a) and (27b) and exchange the resulting $y_{C_i},\ y_{D_{ji}}\ \forall\ i, j$.

    BS and every D2D pair apply (27c) and (27d) and exchange the resulting $P_{C_i}^{(k)},\ P_{D_{ji}}^{(k)}\ \forall\ i, j$.

  Channel Assignment (single iteration)

    For the new power allocation ($\mathrm{\mathbf{P}}_B^{(k)},\ \boldsymbol{P}_D^{(k)}$)

    BS performs a single iteration of the PGD algorithm to calculate: $\boldsymbol{B}^{(k)}$

    BS discretize $\boldsymbol{B}^{(k)}$

  use the current values of $\boldsymbol{B}^{(k)},\ \mathrm{\mathbf{P}}_B^{(k)},\ \boldsymbol{P}_D^{(k)}$ for communication.

until $\boldsymbol{B},\ \mathrm{\mathbf{P}}_B,\ \boldsymbol{P}_D$ converge.

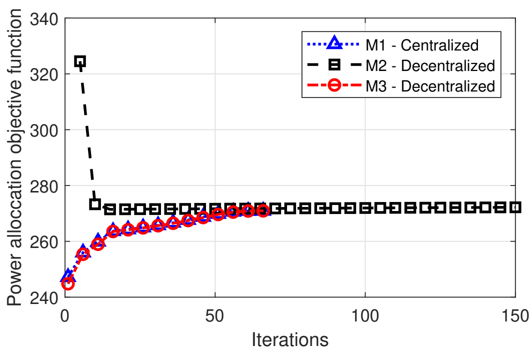

Fig. 3: Convergence of the power allocation subproblem in the different cases.

first few iterations it has primal-infeasible solutions as feasibility is achieved asymptotically in such solutions.

The convergence of the overall problem when alternating between the power allocation and the channel assignment problems. With just two iterations the objective function reaches a very good value that is very close to the maximum. Moreover, the problem satisfy all the selected convergence conditions in around 10 iteration.

VI. MIMO CASE

In the MIMO case, adopting different cases for handling the constants as we did in the SISO case is impractical due to the huge computational load it requires. So we propose only adopting case (M3), where the total power constraints are replaced by a bound on the maximum power allowed in each channel (tighter constraints). The achievable rates can be expressed as: $R_{c_{i,j}}$ denotes the throughput of the CU user i when sharing the spectrum with the D2D pair j.

$$R_{c_{i,j}} = \log_2 \left(1 + \boldsymbol{P}_{B_i}^H \boldsymbol{G}_{B_i}^H \left(\sum_{j \in \mathcal{D}} \beta_{ij} \boldsymbol{H}_{c_{j,i}} \boldsymbol{P}_{D_{ji}} \boldsymbol{P}_{D_{ji}}^H \boldsymbol{H}_{c_{j,i}}^H + N_0 \boldsymbol{I} \right)^{-1} \boldsymbol{G}_{B_i} \boldsymbol{P}_{B_i} \right)$$

$R_{d_{j,i}}$ denotes the throughput of the D2D pair j when sharing the spectrum with the CU user i.

$$\begin{aligned} R_{d_{j,i}} = \log_2 \Bigg(1 + \boldsymbol{P}_{D_{ji}}^H \boldsymbol{G}_{D_j}^H \Bigg(& N_0 \boldsymbol{I} + \boldsymbol{H}_{B_j} \boldsymbol{P}_{B_i} \boldsymbol{P}_{B_i}^H \boldsymbol{H}_{B_j}^H \\ & + \sum_{j' \in \mathcal{D}, j' \neq j} \beta_{ij'} \boldsymbol{H}_{d_{j',j}} \boldsymbol{P}_{D_{j'i}} \boldsymbol{P}_{D_{j'i}}^H \boldsymbol{H}_{d_{j',j}}^H \Bigg)^{-1} \boldsymbol{G}_{D_j} \boldsymbol{P}_{D_{ji}} \Bigg) \end{aligned}$$

$R_{c_{i,0}}$ denotes the throughput of CU user i without sharing the resources (no D2D is transmitting in the same sub-channel).

$$R_{c_{i,0}} = \log_2 \left(1 + \frac{P_{B_{\max}} \max[\mathrm{diag}\left\{ \boldsymbol{G}_{B_i}^H \boldsymbol{G}_{B_i} \right\}]}{N_0} \right)$$

$v_{i,j}$ denotes the throughput gain (increment) when assigning the channel of CU i to D2D pair j.

$$v_{i,j} = R_{c_{i,j}} + R_{d_{j,i}} - R_{c_{i,0}}$$

Similar to the SISO case, the objective now is to optimize a similar problem to (2) with modified constraints. This problem can be approximately solved by alternating between two separate power assignment and channel allocation problems.

D

A. Power Assignment

The power allocation problem is decomposed across channels. The resulting power assignment subproblem for the i-th channel can be expressed as:

$$\underset{\boldsymbol{P}_{B_{ij}},\boldsymbol{P}_{D_{ji}},\boldsymbol{\gamma}}{\text{maximize}} \quad \log\left(1+\gamma_1\right)+\sum_{j\in\mathcal{D}}\beta_{ij}\log\left(1+\gamma_j^d\right) \tag{29}$$

subject to

$$\gamma_1 \leq \boldsymbol{P}_{B_i}^H\boldsymbol{G}_{B_i}^H(N_0\boldsymbol{I}+\sum_{j\in\mathcal{D}}\beta_{ij}\boldsymbol{H}_{c_{j,i}}\boldsymbol{P}_{D_{ji}}\boldsymbol{P}_{D_{ji}}^H\boldsymbol{H}_{c_{j,i}}^H)^{-1}\boldsymbol{G}_{B_i}\boldsymbol{P}_{B_i},$$

$$\gamma_j^d \leq \boldsymbol{P}_{D_{ji}}^H\boldsymbol{G}_{D_j}^H(N_0\boldsymbol{I}+\boldsymbol{H}_{B_j}\boldsymbol{P}_{B_i}\boldsymbol{P}_{B_i}^H\boldsymbol{H}_{B_j}^H+\sum_{j'\in\mathcal{D},j'\neq j}\beta_{ij'}\boldsymbol{H}_{d_{j',j}}\boldsymbol{P}_{D_{j'i}}\boldsymbol{P}_{D_{j'i}}^H\boldsymbol{H}_{d_{j',j}}^H)^{-1}\boldsymbol{G}_{D_j}\boldsymbol{P}_{D_{ji}},$$

$$\|\boldsymbol{P}_{B_i}\|_2^2 \leq P_{B_{max}}/N_C,\ \ \|\boldsymbol{P}_{D_{ji}}\|^2 \leq P_{D_{max}}/N_C,$$

$$\boldsymbol{P}_{B_i}^H\boldsymbol{G}_{B_i}^H(N_0\boldsymbol{I}+\sum_{j\in\mathcal{D}}\beta_{ij}\boldsymbol{H}_{c_{j,i}}\boldsymbol{P}_{D_{ji}}\boldsymbol{P}_{D_{ji}}^H\boldsymbol{H}_{c_{j,i}}^H)^{-1}\boldsymbol{G}_{B_i}\boldsymbol{P}_{B_i} \geq \eta_{min}^C,$$

$$\boldsymbol{P}_{D_{ji}}^H\boldsymbol{G}_{D_j}^H(N_0\boldsymbol{I}+\boldsymbol{H}_{B_j}\boldsymbol{P}_{B_i}\boldsymbol{P}_{B_i}^H\boldsymbol{H}_{B_j}^H+\sum_{j'\in\mathcal{D},j'\neq j}\beta_{ij'}\boldsymbol{H}_{d_{j',j}}\boldsymbol{P}_{D_{j'i}}\boldsymbol{P}_{D_{j'i}}^H\boldsymbol{H}_{d_{j',j}}^H)^{-1}\boldsymbol{G}_{D_j}\boldsymbol{P}_{D_{ji}} \geq \eta_{min}^D.$$

Absorbing β into the beamforming vectors and applying a quadratic transformation, this can be solved iteratively. For fixed $\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}$, we can solve (29) for $\boldsymbol{\gamma}$ and $\boldsymbol{y}$ in close form as follows:

$$\begin{aligned}\gamma_1^{(k)} &= \boldsymbol{P}_{B_i}^H\boldsymbol{G}_{B_i}^H(N_0\boldsymbol{I}+\sum_{j\in\mathcal{D}}\boldsymbol{H}_{c_{j,i}}\boldsymbol{P}_{D_{ji}}\boldsymbol{P}_{D_{ji}}^H\boldsymbol{H}_{c_{j,i}}^H)^{-1}\boldsymbol{G}_{B_i}\boldsymbol{P}_{B_i},\\ \gamma_j^{d(k)} &= \boldsymbol{P}_{D_{ji}}^H\boldsymbol{G}_{D_j}^H(N_0\boldsymbol{I}+\boldsymbol{H}_{B_j}\boldsymbol{P}_{B_i}\boldsymbol{P}_{B_i}^H\boldsymbol{H}_{B_j}^H)^{-1}\boldsymbol{G}_{D_j}\boldsymbol{P}_{D_{ji}},\end{aligned}$$

$$\begin{aligned}y_1^{(k)} =&(N_0\boldsymbol{I}+\tilde{\boldsymbol{H}}_{c_{j,i}}\boldsymbol{P}_{D_{ji}}\boldsymbol{P}_{D_{ji}}^H\tilde{\boldsymbol{H}}_{c_{j,i}}^H+\boldsymbol{G}_{B_i}\boldsymbol{P}_{B_i}\boldsymbol{P}_{B_i}^H\boldsymbol{G}_{B_i}^H)^{-1}\times\sqrt{(1+\gamma_1^{(k)})}\boldsymbol{G}_{B_i}\boldsymbol{P}_{B_i},\\ y_j^{d(k)} =&(N_0\boldsymbol{I}+\boldsymbol{H}_{B_j}\boldsymbol{P}_{B_i}\boldsymbol{P}_{B_i}^H\boldsymbol{H}_{B_j}^H+\boldsymbol{G}_{D_j}\boldsymbol{P}_{D_{ji}}\boldsymbol{P}_{D_{ji}}^H\boldsymbol{G}_{D_j}^H\\ &+\sum_{j'\in\mathcal{D},j'\neq j}\boldsymbol{H}_{d_{j',j}}\boldsymbol{P}_{D_{j'i}}\boldsymbol{P}_{D_{j'i}}^H\boldsymbol{H}_{d_{j',j}}^H)^{-1}\times\sqrt{(1+\gamma_j^{d(k)})}\boldsymbol{G}_{D_j}\boldsymbol{P}_{D_{ji}},\end{aligned}$$

where $\boldsymbol{y}$ are the quadratic transformation parameters.

Then solving (29) for $\boldsymbol{P}_{B_{ij}}$ and $\boldsymbol{P}_{D_{ji}}$ for fixed $\boldsymbol{\gamma}$ and $\boldsymbol{y}$, we obtain a convex optimization problem as follows:

$$\underset{\boldsymbol{P}_{B_{ij}},\boldsymbol{P}_{D_{ji}}}{\text{maximize}} \quad 2\sqrt{(1+\gamma_1^{(k)})}\text{Re}\{P_{B_{ij}}^H G_{B_i}^H y_1^{(k)}\} - (y_1^{(k)})^H (N_0\boldsymbol{I} + \sum_{j\in\mathcal{D}} \beta_{ij}\boldsymbol{H}_{c_{j,i}}\boldsymbol{P}_{D_{ji}}\boldsymbol{P}_{D_{ji}}^H\boldsymbol{H}_{c_{j,i}}^H) y_1^k$$
$$+\sum_{j\in\mathcal{D}}\Bigg(2\sqrt{(1+\gamma_j^{d(k)})}\text{Re}\{P_{D_{ji}}^H G_{D_j}^H y_j d^{(k)}\} - (y_j^{d(k)})^H (N_0\boldsymbol{I} + \boldsymbol{H}_{B_j}\boldsymbol{P}_{B_i}\boldsymbol{P}_{B_i}^H\boldsymbol{H}_{B_j}^H +$$
$$\sum_{j'\in\mathcal{D},j'\neq j}\boldsymbol{H}_{d_{j',j}}\boldsymbol{P}_{D_{j'i}}\boldsymbol{P}_{D_{j'i}}^H\boldsymbol{H}_{d_{j',j}}^H) y_j^{d(k)}\Bigg) \tag{30}$$

subject to

$$\|\boldsymbol{P}_{B_i}\|_2^2 \leq P_{B_{max}},\ \|\boldsymbol{P}_{D_{ji}}\|^2 \leq P_{D_{max}},$$
$$2\text{Re}\{(y_1^{(k)})^H G_{B_i}\boldsymbol{P}_{B_i}\} - (y_1^{(k)})^H (N_0\boldsymbol{I} + \sum_{j\in\mathcal{D}}\boldsymbol{H}_{c_{j,i}}\boldsymbol{P}_{D_{ji}}\boldsymbol{P}_{D_{ji}}^H\boldsymbol{H}_{c_{j,i}}^H) y_1^{(k)} \geq \eta_{min}^C,$$
$$2\text{Re}\{(y_j^{d(k)})^H \boldsymbol{G}_{D_j}\boldsymbol{P}_{D_{ji}}\} - (y_j^{d(k)})^H (N_0\boldsymbol{I} + \boldsymbol{H}_{B_j}\boldsymbol{P}_{B_i}\boldsymbol{P}_{B_i}^H\boldsymbol{H}_{B_j}^H$$
$$+\sum_{j'\in\mathcal{D},j'\neq j}\boldsymbol{H}_{d_{j',j}}\boldsymbol{P}_{D_{j'i}}\boldsymbol{P}_{D_{j'i}}^H\boldsymbol{H}_{d_{j',j}}^H) y_j^{d(k)} \geq \eta_{min}^D.$$

D

B. Channel Assignment

After finding $\boldsymbol{P}_B^{(k)}, \boldsymbol{P}_D^{(k)}$, we substitute them in the objective function and solve for $\boldsymbol{B}^{(K)}$. We then obtain an optimization problem identical to (28), which we propose to solve similarly as described in section III-B.

VII. Simulations

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius $5m$ centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of $1m$. Averages over 400 independent realizations of the user locations with parameters BW = 15 kHz, $N_0 = -70$ dBW.The proposed method is compared with the method by Elnourani et al. [34], which to the best of our knowledge is the best existing method for the SISO case with no interference among D2D pairs.

In the first experiments we tested our proposed SISO solution for all threes cases. Fig. 4 shows the results of those experiments. Eventhough our problem is not convex, all three different approaches have very close results. As shown in the figure, case $(\mathbf{M1})$ is the most fair and shows slightly better reaction to changes in γ. In general, case $(\mathbf{M1})$ is the most

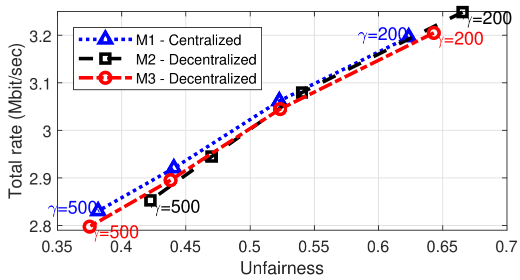

Fig. 4: Rate vs. Unfairness when changing $\gamma = \{1, ..., 500\} \times$ BW ($N_C = 5,\ N_D = 5$).

accurate, however, the high computational load render it impractical. Case ($\mathbf{M3}$) has the lowest computational demand and results in a very close performance to both ($\mathbf{M1}$) and ($\mathbf{M2}$). Thus, we used ($\mathbf{M3}$) for the rest of the experiments.

Figs. 5 and 6 shows a network with a comparison between the method of [34] and the proposed SISO method in a network with $N_C = 10$ and $\gamma = 500 \times$ BW. Fig. 5 shows the rate and the unfairness in both methods and how they change when changing the number of D2D pairs. In the method in [34], the rate is almost constant when increasing the number of D2D pairs, since the total number of channels is fixed ($N_C = 10$) and assigning the same channel to multiple D2D pairs is not allowed. On the other hand, the rate keeps increasing in the proposed method when increasing the number of D2D pairs. However, in both cases, the unfairness increases when increasing the number of D2D pairs since more D2D pairs will not be assigned any resources in [34] or it will be more difficult to assign equal number of channels to all those D2D pairs while ensuring relatively high rate. Fig. 6 shows the average achieved rate per D2D pair. The method in [34] consistently shows a decrease in the rate when increasing the number of D2D pairs. However, the proposed method shows an increase in the average rate until $N_D = N_C$. When N_D exceeds N_C, the average rate starts decreasing. This can is expected because the interference is increasing with increasing N_D and when $N_D > N_C$ the interference can not be avoided. In general the average rate for each D2D pair is significantly higher in the proposed method, unless N_D is too small, where the method in [34] can have almost optimal solution without allowing interference between D2D pairs.

Fig. 7 shows the case where $N_D = N_C = 10$ and γ was changed from 10 to 1000. The proposed method in the SISO case shows huge improvement in the rate compared to the method in [34], however the unfairness is slightly higher in the proposed method. Both methods have similar behaviour where the rate and unfairness decrease when increasing γ.

A 4×4 MIMO case was tested with 4 D2D pairs and 4 CUs. The results are shown in

D

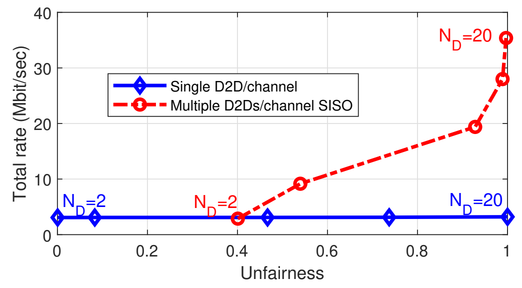

Fig. 5: Rates vs. Unfairness when changing $N_D = 2, 5, 10, 15, 20$ ($N_C = 10,\ \gamma = 500 \times$ BW).

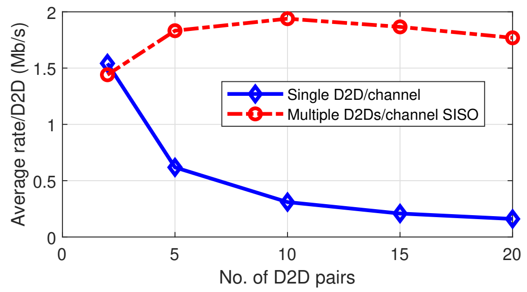

Fig. 6: Average rate per D2D pair when changing N_D ($N_C = 10,\ \gamma = 500 \times$ BW).

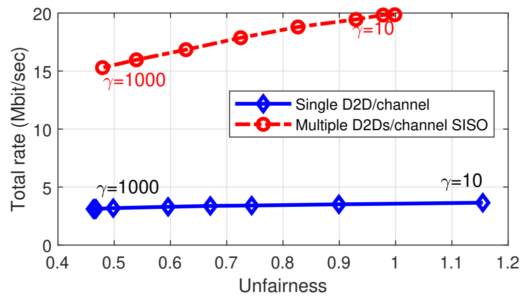

Fig. 7: Rates vs. Unfairness when changing $\gamma = \{10, ..., 1000\} \times$ BW ($N_C = 10,\ N_D = 10$).

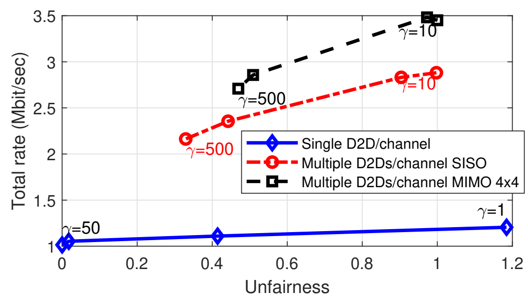

Fig. 8: Rate vs. Unfairness with 4×4 MIMO when changing $\gamma = \{1, ..., 500\} \times$ BW ($N_C = 4,\ N_D = 4$).

Fig. 8 in comparison to the SISO case and the work in [34]. The MIMO case has the highest rate, as expected, and shows similar behaviour to the SISO case.

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Appendix E

Paper E

Title:	Robust Transmit Beamforming for Underlay D2D Communications on Multiple Channels.
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PAPER E

Robust Transmit Beamforming for Underlay D2D Communications on Multiple Channels

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Abstract

Underlay device-to-device (D2D) communications lead to improvement in spectral efficiency by simultaneously allowing direct communication between the users and the existing cellular transmission. However, most works in resource allocation for D2D communication have considered single antenna transmission and with a focus on perfect channel state information (CSI). This work formulates a robust transmit beamforming design problem for maximizing the aggregate rate of all D2D pairs and cellular users (CUs). Assuming complex Gaussian distributed CSI error, our formulation guarantees probabilistically a signal to interference plus noise ratio (SINR) above a specified threshold. In addition, we also ensure fairness in allocation of resources to D2D pairs. We accommodate the probabilistic SINR constraint by exploiting a Bernstein-type inequality.The resulting problem is a mixed integer non-convex problem, and we propose to approximately solve it by exploiting a semi-definite relaxation (SDR) and a quadratic transformation, which leads us to an alternating optimization method. Simulation results corroborate the merits of the proposed approach by illustrating higher network throughput and more reliable communication.

Index Terms

D2D communications, resource allocation, robust beamforming, semi-definite relaxation.

E

I. Introduction

D2D communications in the underlay framework, improve the spectral efficiency by simultaneously allowing transmissions of D2D pairs and traditional cellular network in the same spectrum [1]. However, simultaneous transmissions in the same spectrum bands increase interference, which must be deliberately handled by devising judicious algorithms for the assignment of channels to D2D pairs and the control of transmission powers. The allocation of resources must also be fair while guaranteeing the desired Quality of service (QoS) and also robust to errors in CSI.

Most works on resource allocation problems for underlay D2D communication have assumed single antenna transmission [2], [3]. These schemes also restrict D2D pairs from accessing more than one channel and are also assumed to have prefect CSI. Scenarios of imperfect CSI for single antenna transmission are considered in [4], [5].

Under the multi antenna transmission framework, [6] provides a comprehensive analysis for joint beamforming in D2D underlay cellular networks. However, this work is restricted to a single D2D pair with the further assumption of perfect CSI. In [7] multiple D2D pairs are considered; however, once again knowledge of perfect CSI is assumed at the base station (BS). Quantization error in CSI due to limited feedback is assumed in [8] to study the

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conventional maximum ratio transmission and interference cancellation based beamforming techniques. Recently, a joint beamforming and power control strategy is presented in [9] under the assumption of both perfect and imperfect CSI. Here the objective is to minimize the total transmit power of BS and D2D pairs while ensuring QoS (SINR) requirements.

Design of robust beamformers for general multiuser communication has also been investigated in past research works [10]–[12]. Under the assumption of Gaussian CSI uncertainties, analytical methods based on Bernstein-type inequality and decomposition based large deviation inequality are proposed in [10] to approximate the probabilistic rate outage constraints. Similarly, under the assumption of Gaussian channel distribution, the probabilistic rate outage constraint is handled by SDR relaxation and sequential convex approximation in [11]. Further, authors in [12] have proposed a decentralized approach to design the robust beamformers considering elliptically bounded CSI errors. In all these works the objective is either minimization of transmit energy, or sum rate maximization; however, in underlay D2D communication jointly optimizing the power allocation and channel assignment poses additional analytical and computation challenges.

Despite the above research efforts, none of the existing approaches provide a robust beamforming design while performing joint channel assignment and power allocation to the D2D pairs and CUs. The main contributions of this work are:

- We formulate a robust beamforming design problem under the assumption of complex Gaussian distributed CSI error in the channel gain vector. Our objective is to maximize the aggregate rate of all D2D pairs and CUs with a penalty on unfair channel assignment, under a constraint on the minimum SINR requirement to guarantee a specified outage probability. The probabilistic constraints are handled by exploiting the Bernstein type inequality [10], [13] for a quadratic form of Gaussian random variables.
- Since the resulting problem is a mixed integer non-convex problem, with aid of auxiliary variables and with no loss in optimality, we decompose the problem into multiple power allocation subproblems and a channel assignment subproblem. The power allocation subproblems are solved by alternating optimization obtained after applying semi-definite relaxation [14] and fractional programming via a quadratic transformation [15]. The channel assignment subproblem is solved by integer relaxation.
- Finally, numerical experiments are performed to corroborate the merits of the proposed approach by illustrating a higher throughput and more robust communication.

E

II. System Model

Consider the cellular communication setup shown in Fig. 1 where a BS with K_B transmit antennas communicates with N_C single antenna CUs through N_C downlink channels[1]. The set of CUs (equivalently, channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. In an underlay configuration, N_D D2D pairs, indexed by $\mathcal{D} = \{1, ..., N_D\}$, wish to communicate using the aforementioned N_C downlink channels. The D2D transmitters are assumed to have K_D antennas communicating with respective single antenna D2D receivers. Furthermore, we denote the channel gain between the BS and the i-th CU by $\boldsymbol{g}_{B_i} \in \mathbb{C}^{K_B \times 1}$; and between the j-th D2D pair by $\boldsymbol{g}_{D_j} \in \mathbb{C}^{K_D \times 1}$. Similarly, the interference channel gain[2] between the BS and the receiver of the j-th D2D by $\boldsymbol{h}_{B_j} \in \mathbb{C}^{K_B \times 1}$; and between the transmitter of the j-th D2D pair and the i-th CU by $\tilde{\boldsymbol{h}}_{D_{j,i}} \in \mathbb{C}^{K_D \times 1}$. We assume minimum cooperation from CUs in estimating the gain of the interference channel; thus, $\tilde{\boldsymbol{h}}_{D_{j,i}}$ is modeled as a random vector with complex circular Gaussian distribution, i.e., $\tilde{\boldsymbol{h}}_{D_{ji}} \sim \mathcal{CN}(\boldsymbol{h}_{D_{ji}}, \boldsymbol{M}_{ji})$ where $\boldsymbol{h}_{D_{ji}}$ and $\boldsymbol{M}_{ji}$ are assumed to be known or learned in advance. The additive white noise power is denoted by N_0.

[1]Even though the formulation is done for downlink, the same formulation can be directly extended to uplink

[2]In principle, $\boldsymbol{g}_{D_j}$ and $\boldsymbol{h}_{B_j}$ should also depend on the i-th channel; however, this subscript is dropped as the proposed scheme carries over immediately to accommodate such dependence.

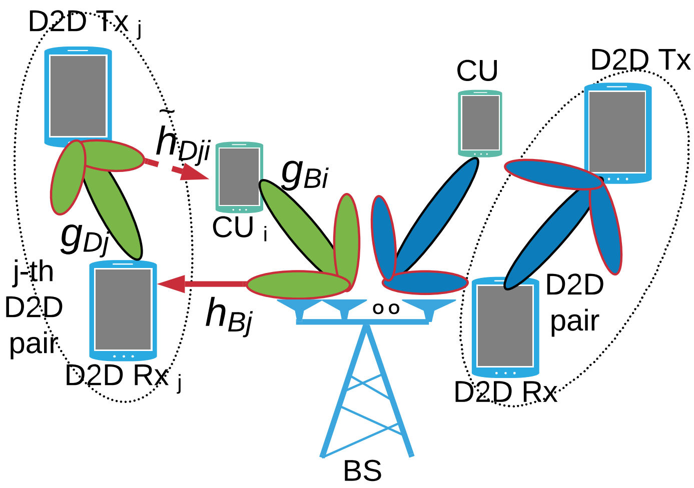

Fig. 1: Illustration of the overall system model.

The BS assignment of channels to D2D pairs is denoted by the indicator parameters $\{\beta_{i,j}\}_{i\in\mathcal{C},j\in\mathcal{D}}$, where $\beta_{i,j}=1$ indicates assignment of the i-th channel to the j-th D2D pair and $\beta_{i,j}=0$ otherwise. For higher throughput, we allow a D2D pair to simultaneous access multiple channels. However, to restrict interference, no more than one D2D pair can access each channel, i.e., $\sum_{j=1}^{N_D}\beta_{i,j}\leq 1$, $\forall i$. We denote the transmit precoder vector for the BS to communicate with the i-th CU as $\boldsymbol{p}_{B_i}\in\mathbb{C}^{K_B\times 1}$ and as $\boldsymbol{p}_{D_{j,i}}\in\mathbb{C}^{K_D\times 1}$ for the j-th D2D transmitter on the i-th channel. The precoders are constrained as $||\boldsymbol{p}_{B_i}||_2^2\leq p_{B,\max}$ and $||\boldsymbol{p}_{D_{j,i}}||_2^2\leq p_{D,\max}$. To ensure successful communication, the SINR should also be enforced to be greater than a certain threshold $\eta_{D,\min}$ for the D2D pairs and $\eta_{C,\min}$ for the CUs with a maximum allowed outage ratio of ϵ.

III. Problem Formulation

Due to limited cooperation of CUs in estimating the interference channel $\tilde{\boldsymbol{h}}_{D_{ji}}$, the objective of this work is to guarantee a maximum outage probability ϵ to the CUs, i.e., we maximize the minimum total network rate that is achieved at least a $(1-\epsilon)$ portion of the time. This can be realized by defining a lower bound on the total rate that can be achieved over every channel. To this end, let $\Gamma(z):=\text{BW}\log_2(1+z)$, where BW is the channel bandwidth. For the i-th channel, this rate can be expressed as $R_i^{LB}:=(1-\sum_{j\in\mathcal{D}}\beta_{i,j})R_{C_{i,0}}+\sum_{j\in\mathcal{D}}\beta_{i,j}[R_{D_{j,i}}+R_{C_{i,j}}^{LB}]$, where:

- $R_{C_{i,0}}:=\Gamma(p_{B_{max}}||\boldsymbol{g}_{B_i}||_2^2/N_0)$, rate of the i-th CU without assignment of D2D pairs, i.e., $\beta_{ij}=0\ \forall j$.
- $R_{D_{j,i}}:=\Gamma(|\boldsymbol{p}_{D_{ji}}^H\boldsymbol{g}_{D_j}|^2\ /(N_0+|\boldsymbol{p}_{B_i}^H\boldsymbol{h}_{B_j}|^2))$, rate of the j-th D2D pair when assigned with the i-th CU, i.e., $\beta_{ij}=1$.
- $R_{C_{i,j}}^{LB}:=\Gamma(z_{C_{i,j}}^{LB})$, where $z_{C_{i,j}}^{LB}$ is such that $\Pr\{z_{C_{i,j}}^{LB}\leq|\boldsymbol{p}_{B_i}^H\boldsymbol{g}_{B_i}|^2/(N_0+|\boldsymbol{p}_{D_{ji}}^H\tilde{\boldsymbol{h}}_{D_{j,i}}|^2)\}=1-\epsilon$, rate that must be exceeded a $(1-\epsilon)$ portion of the time by the i-th CU when assigned with the j-th D2D pair, i.e., $\beta_{ij}=1$.

Finally, the minimum total network rate is defined as $R(\mathbb{B},\mathbb{P}_B,\mathbb{P}_D):=\sum_{i\in\mathcal{C}}R_i^{LB}$, where, $\mathbb{B}:=[\beta_{i,j}]$, $\mathbb{P}_B:=[\boldsymbol{p}_{B_i}]$, $\mathbb{P}_D:=[\boldsymbol{p}_{D_{j,i}}]\ \forall i\in\mathcal{C}$ and $j\in\mathcal{D}$.

In order to have fairness in the channel assignment, we introduce a secondary objective that penalizes greedy channel assignments to the D2D pairs. We consider an unfairness measure $\delta(\mathbb{B})=1/(N_Dc^2)\sum_{j=1}^{N_D}(x_j-x_0)^2$ along the lines in [16], [17], where $x_j:=\sum_{i=1}^{N_C}\beta_{ij}$ is the number of channels assigned to the j-th D2D pair, and $x_0 := N_C/N_D$ is the fairest assignment. Summing up, the overall problem can then be formulated as:

$$\underset{\mathbb{B},\mathbb{P}_B,\mathbb{P}_D}{\text{maximize}} \; R(\mathbb{B},\mathbb{P}_B,\mathbb{P}_D) - \gamma\delta(\mathbb{B}) \tag{1a}$$

$$\text{subject to } \beta_{i,j} \in \{0,1\}, \; \sum_{j=1}^{N_D} \beta_{i,j} \leq 1 \, \forall \, i; \tag{1b}$$

$$||\boldsymbol{p}_{B_i}||_2^2 \leq p_{B,\max} \;\; \forall \, i, \quad ||\boldsymbol{p}_{D_{j,i}}||_2^2 \leq p_{D,\max} \;\; \forall \, j, i; \tag{1c}$$

$$\Pr\left\{ \frac{|\boldsymbol{p}_{B_i}^H \boldsymbol{g}_{B_i}|^2}{N_0 + |\boldsymbol{p}_{D_{ji}}^H \tilde{\boldsymbol{h}}_{D_{j,i}}|^2} \geq \eta_{C,\min} \right\} \geq 1 - \epsilon, \tag{1d}$$

$$\frac{|\boldsymbol{p}_{D_{ji}}^H \boldsymbol{g}_{D_j}|^2}{N_0 + |\boldsymbol{p}_{B_i}^H \boldsymbol{h}_{B_j}|^2} \geq \eta_{D,\min} \quad \text{if } \beta_{ij} = 1, \; \forall \, i, j. \tag{1e}$$

The regularization parameter $\gamma > 0$ is selected to balance the trade-off between the minimum total rate and the fairness in channel assignment. Problem (1) is a non-convex mixed-integer stochastic program, which involves exponential complexity. In the next section, we propose efficiently solving problem (1) by exploiting semi-definite relaxation and a quadratic transformation.

IV. Proposed Optimization Algorithm

The complexity to obtain the solution of (1) can be reduced by decomposing the problem into multiple sub-problems of lower complexity. Thus, we first rewrite the sum rate as:

$$R(\mathbb{B},\mathbb{P}_B,\mathbb{P}_D) = \sum_{i\in\mathcal{C}} \left[\sum_{j\in\mathcal{D}} \beta_{i,j} v_{i,j}(\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}) + R_{C_{i,0}} \right], \tag{2}$$

where $v_{i,j}(P_{B_i}, P_{D_{ji}}) := R_{C_{i,j}}^{LB} + R_{D_{j,i}} - R_{C_{i,0}}$ represents the minimum rate increment due to the assignment of channel i to the D2D pair j, relative to the case where the channel i is only used by the CU. Next, notice that the objective of (1), with substitution of (2), can be equivalently expressed by replicating $\{\boldsymbol{p}_{B_i}\}$ with multiple auxiliary variables $\{\boldsymbol{p}_{B_{ij}}\}$ and removing the constant terms from the objective function. The resulting equivalent problem can be stated as:

$$\begin{aligned} \underset{\mathbb{B},\mathbb{P}_B,\mathbb{P}_D}{\text{maximize}} \quad & \sum_{i\in\mathcal{C}} \sum_{j\in\mathcal{D}} \left[\beta_{i,j} v_{i,j}(\boldsymbol{p}_{B_{ij}}, \boldsymbol{p}_{D_{ji}})\right] - \gamma\delta(\mathbb{B}) \\ \text{subject to} \quad & \text{(1b), (1c), (1d), (1e).} \end{aligned} \tag{3}$$

To recover the optimal $\{\boldsymbol{p}_{B_i}^*\}$ of (1) from the optimal $\{\boldsymbol{p}_{B_{i,j}}^*\}$ of (3), one just needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $\boldsymbol{p}_{B_i}^* = \boldsymbol{p}_{B_{i,j}}^*$. If no such j exists, i.e. $\beta_{i,j} = 0 \; \forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power.

In addition, similar to [17], it can be shown that (3) decouples across i and j into $N_C \times N_D$ power allocation sub-problems and a final channel assignment problem. The power allocation sub-problem can be stated as:

$$\begin{aligned} \underset{\boldsymbol{p}_{B_{ij}}, \, \boldsymbol{p}_{D_{ji}}}{\text{maximize}} \quad & R_{C_{i,j}}^{LB} + R_{D_{j,i}} \\ \text{subject to} \quad & \text{(1c), (1d) and (1e),} \end{aligned} \tag{4}$$

which should be solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$. The subsequent channel assignment problem is discussed in subsection IV-D. We can notice that problem (4) is still a non-convex stochastic problem. Hence, we derive next closed-form expressions for the stochastic terms.

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A. Closed-form stochastic constraints

In order to bring the stochastic terms from the objective (4) to the constraints, we introduce slack variables $\boldsymbol{z} \triangleq [z_C,\ z_D]^T$ as follows:

$$\underset{\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \quad \log_2(1+z_C) + \log_2(1+z_D) \tag{5a}$$

$$\text{subject to } \Pr\left(z_C \leq \frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{N_0 + |\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2} \right) \geq 1-\epsilon \tag{5b}$$

$$z_D \leq \frac{|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}{N_0 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2}, \quad \text{(1c), (1d) and (1e)} \tag{5c}$$

Representing the random interference channel vector $\tilde{\boldsymbol{h}}_{D_{ji}} = \boldsymbol{h}_{D_{ji}} + \boldsymbol{e}_{ji}$, where $\boldsymbol{e}_{ji} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{M}_{ji})$, the stochastic inequality, $z_C \leq \frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{N_0 + |\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2}$ can be equivalently expressed as $N_0 + |\boldsymbol{h}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2 + \boldsymbol{e}_{ji}^H \boldsymbol{p}_{D_{ji}} \boldsymbol{p}_{D_{ji}}^H \boldsymbol{e}_{ji} + 2\,\mathrm{Re}\left\{\boldsymbol{e}_{ji}^H \boldsymbol{p}_{D_{ji}} \boldsymbol{p}_{D_{ji}}^H \boldsymbol{h}_{D_{ji}}\right\} \leq \frac{1}{z_C} |\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2$. Further, letting $\boldsymbol{e}_{ji} := \boldsymbol{M}_{ji}^{1/2} \boldsymbol{v}_{ji}$ where, $\boldsymbol{v}_{ji} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$, this stochastic inequality can equivalently stated as:

$$N_0 + |\boldsymbol{h}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2 + \boldsymbol{v}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{v}_{ji} + 2\,\mathrm{Re}\left\{\boldsymbol{v}_{ji}^H \boldsymbol{u}_{ji}\right\} \leq \frac{1}{z_C} |\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 \tag{6}$$

where, $\boldsymbol{Q}_{ji} := \boldsymbol{M}_{ji}^{1/2} \boldsymbol{p}_{D_{ji}} \boldsymbol{p}_{D_{ji}}^H \boldsymbol{M}_{ji}^{1/2}$ and $\boldsymbol{u}_{ji} := \boldsymbol{M}_{ji}^{1/2} \boldsymbol{p}_{D_{ji}} \boldsymbol{p}_{D_{ji}}^H \boldsymbol{h}_{D_{ji}}$. Thus, the stochastic constraint (5b) can be re-stated as:

$$\Pr(\boldsymbol{v}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{v}_{ji} + 2\,\mathrm{Re}\left\{\boldsymbol{v}_{ji}^H \boldsymbol{u}_{ji}\right\} \leq c_{ji}) \geq 1-\epsilon, \tag{7}$$

where $c_{ji} = \frac{1}{z_C} |\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 - N_0 - |\boldsymbol{h}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2$. Next, in order to obtain a closed from expression for (7), we use a Bernstein-type inequality for the quadratic form of Gaussian vectors [13].

Lemma 1. *Let $G = \boldsymbol{v}^H \boldsymbol{Q} \boldsymbol{v} + 2\,\mathrm{Re}\left\{\boldsymbol{v}^H \boldsymbol{u}\right\}$ where $\boldsymbol{Q} \in \mathbb{H}^{K_D}$ is a complex Hermitian matrix, $\boldsymbol{u} \in \mathbb{C}^{K_B}$ and $\boldsymbol{v} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$. Then for any $\delta > 0$, we have:*
$Pr\{G \leq Tr(\boldsymbol{Q}) + \sqrt{\delta}\sqrt{||\boldsymbol{Q}||_F^2 + 2||\boldsymbol{u}||_2^2} + 2\delta s^+(\boldsymbol{Q})\} \geq 1 - e^{-\delta}$,
where $s^+(\boldsymbol{Q}) = \max\{\lambda_{\max}(\boldsymbol{Q}), 0\}$, $\lambda_{\max}(\boldsymbol{Q})$ denotes the maximum eigenvalue of $\boldsymbol{Q}$, and $||\cdot||_F$ denotes the matrix Frobenius norm.

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Considering Lemma 1 and setting $\delta = -\ln(\epsilon)$, equation (7) holds if the following inequality is satisfied:

$$\mathrm{Tr}(\boldsymbol{Q}_{ji}) + \sqrt{\delta}\sqrt{||\boldsymbol{Q}_{ji}||_F^2 + 2||\boldsymbol{u}_{ji}||_2^2} + 2\delta s^+(\boldsymbol{Q}_{ji}) \leq c_{ji}. \tag{8}$$

Rearranging the terms in (8) and exploiting Lemma 1 for constraint (1d), problem (5) can be stated as,

$$\underset{\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \quad \log_2(1+z_C) + \log_2(1+z_D) \tag{9a}$$

$$\text{subject to } z_C \leq \frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji})}, \quad z_D \leq \frac{|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}{N_0 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2} \tag{9b}$$

$$\frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji})} \geq \eta_{C,\min}, \quad \text{(1c) and (1e)} \tag{9c}$$

where, $f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji}) := N_0 + |\boldsymbol{h}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2 + \mathrm{Tr}(\boldsymbol{Q}_{ji}) + \sqrt{\delta}\sqrt{||\boldsymbol{Q}_{ji}||_F^2 + 2||\boldsymbol{u}_{ji}||_2^2} + 2\delta s^+(\boldsymbol{Q}_{ji})$. Notice that the constraints (9b) involve (i) a ratio of a convex and a non-convex function, and (ii) a ratio between two convex functions, which are non-convex. In the next subsection, we use fractional programming to relax the non-convexity due to these ratios.

B. Fractional Programming by Quadratic Transformation

Taking a partial Lagrangian by considering only the constraints related to the slack variables z_C and z_D (9b), we have:

$$L(\boldsymbol{p},\boldsymbol{z},\boldsymbol{\lambda}) = \log_2(1+z_C) + \log_2(1+z_D) - \\ \lambda_C\left(z_C - \frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{f(\boldsymbol{p}_{D_{ji}},\boldsymbol{Q}_{ji},\boldsymbol{u}_{ji})}\right) - \lambda_D\left(z_D - \frac{|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}{N_0 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2}\right)$$

At a stationary point, $\frac{\partial L}{\partial \boldsymbol{z}} = 0$; thus, the optimal values of the Lagrange variables can be computed as $\lambda_C = \frac{1}{1+z_C}$ and $\lambda_D = \frac{1}{1+z_C}$. Furthermore, the optimal values of the slack variables are achieved when the inequality constraints (9b) are satisfied with equality. Thus, by calculating λ_C^* and λ_D^* and substituting them in problem (9a) we obtain:

$$\begin{aligned} \underset{\boldsymbol{p}_{B_i},\boldsymbol{p}_{D_{ji}},\boldsymbol{z}}{\text{maximize}} \quad & \log_2(1+z_C) + \log_2(1+z_D) \\ & - z_C + \frac{(1+z_C)|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 + f(\boldsymbol{p}_{D_{ji}},\boldsymbol{Q}_{ji},\boldsymbol{u}_{ji})} \\ & - z_D + \frac{(1+z_D)|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}{N_0 + |\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2} \\ \text{subject to (9c)} & \end{aligned} \tag{10}$$

Next, we absorb the fractions in the objective by introducing two auxiliary variables y_C and y_D through a quadratic transformation [15], obtaining:

$$\begin{aligned} & \underset{\boldsymbol{p}_{B_i},\boldsymbol{p}_{D_{ji}},\boldsymbol{z},\boldsymbol{y}}{\text{maximize}} \quad \log_2(1+z_C) + \log_2(1+z_D) - z_C - z_D \\ & + 2y_C\sqrt{(1+z_C)|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2} + 2y_D\sqrt{(1+z_D)|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2} \\ & - y_C^2\left(|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 + f(\boldsymbol{p}_{D_{ji}},\boldsymbol{Q}_{ji},\boldsymbol{u}_{ji})\right) \\ & - y_D^2\left(N_0 + |\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2\right) \\ & \text{subject to} \quad (9c) \end{aligned} \tag{11}$$

The optimal values of y_C and y_D can be readily obtained as:

$$\begin{aligned} y_C^* &= \sqrt{(1+z_C)|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}/(|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 + f(\boldsymbol{p}_{D_{ji}},\boldsymbol{Q}_{ji},\boldsymbol{u}_{ji})), \\ y_D^* &= \sqrt{(1+z_D)|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}/(N_0 + |\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2) \end{aligned} \tag{12}$$

Notice that for the updated values of the auxiliary variables z_C, z_D, y_C and y_D, optimization problem (11) is still jointly non-convex in $\boldsymbol{p}_{B_i}$ and $\boldsymbol{p}_{D_{ji}}$. We propose performing a semi-definite relaxation in (11) on the variables $\boldsymbol{p}_{B_i}$ and $\boldsymbol{p}_{D_{ji}}$ as shown in the following subsection.

C. Semi-definite Relaxation

In order to obtain convex sub-problems from (11), let us denote $\boldsymbol{P}_{B_i} := \boldsymbol{p}_{B_i}\boldsymbol{p}_{B_i}^H$ and $\boldsymbol{P}_{D_{ji}} := \boldsymbol{p}_{D_{ji}}\boldsymbol{p}_{D_{ji}}^H$. We consider the SDR sub-problem of (11) to optimize $\boldsymbol{P}_{B_i}$ and $\boldsymbol{P}_{D_{ji}}$

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for given values of z_C, z_D, y_C and y_D, which can be stated as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_{B_i},\boldsymbol{P}_{D_{ji}},w_1,w2}{\text{maximize}} \quad & 2y_C\sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H\boldsymbol{P}_{B_i}\boldsymbol{g}_{B_i}} - y_C^2\boldsymbol{g}_{B_i}^H\boldsymbol{P}_{B_i}\boldsymbol{g}_{B_i} \\
& - y_D^2\boldsymbol{h}_{B_j}^H\boldsymbol{P}_{B_i}\boldsymbol{h}_{B_j} + 2y_D\sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j}} \\
& - y_D^2\boldsymbol{g}_{D_j}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j} - y_C^2\left(\boldsymbol{h}_{D_{ji}}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{h}_{D_{ji}} + w_1 + 2\delta w_2\right) \\
\text{subject to} \quad & 0 \leq \text{Tr}(\boldsymbol{P}_{B_i}) \leq p_{C,\max}, \quad \boldsymbol{P}_{B_i} \succeq 0, \\
& 0 \leq \text{Tr}(\boldsymbol{P}_{D_{ji}}) \leq p_{D,\max}, \quad \boldsymbol{P}_{D_{ji}} \succeq 0, \\
& \eta_{C,\min}\left(N_0 + \boldsymbol{h}_{D_{ji}}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{h}_{D_{ji}} + w_1 + 2\delta w_2\right) - \boldsymbol{g}_{B_i}^H\boldsymbol{P}_{B_i}\boldsymbol{g}_{B_i} \leq 0, \\
& \eta_{D,\min}\left(N_0 + \boldsymbol{h}_{B_j}^H\boldsymbol{P}_{B_i}\boldsymbol{h}_{B_j}\right) - \boldsymbol{g}_{D_j}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j} \leq 0, \\
& \sqrt{\delta}\left\|\begin{bmatrix}\text{vec}(\boldsymbol{M}_{ji}^{1/2}\boldsymbol{P}_{D_{ji}}\boldsymbol{M}_{ji}^{1/2}) \\ \sqrt{2}(\boldsymbol{M}_{ji}^{1/2}\boldsymbol{P}_{D_{ji}}\bar{\boldsymbol{h}}_{D_{ji}})\end{bmatrix}\right\| \leq w_1 - \text{Tr}(\boldsymbol{M}_{ji}^{1/2}\boldsymbol{P}_{D_{ji}}\boldsymbol{M}_{ji}^{1/2}), \\
& \boldsymbol{M}_{ji}^{1/2}\boldsymbol{P}_{D_{ji}}\boldsymbol{M}_{ji}^{1/2} - w_2\boldsymbol{I} \preceq 0,
\end{aligned} \tag{13}$$

where, $w_1, w_2 \in \mathbb{R}$ are slack variables. Note that the obtained optimal solution for the relaxed problem in (13) may not be rank one; thus, additional rank one approximation procedures may be needed to obtain the $\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}$ vectors from the respective $\boldsymbol{P}_{B_i}^*, \boldsymbol{P}_{D_{ji}}^*$ matrices. We propose solving this by scaling the eigen-vector $\boldsymbol{v}_{\max}$ corresponding to the highest eigenvalue $\lambda_{\max}$ with the square root of this eigenvalue. In case that the obtained vectors are not feasible with respect to the original constraints in (9c), we propose using the following equation to obtain a feasible solution:

$$\begin{aligned}
\boldsymbol{p}_{B_i} &= \alpha\sqrt{\lambda_{\max}(\boldsymbol{P}_{B_i})}\boldsymbol{v}_{\max}(\boldsymbol{P}_{B_i}) + (1-\alpha)\sqrt{p_{B,\max}}\frac{\boldsymbol{g}_{B_i}}{\|\boldsymbol{g}_{B_i}\|}, \\
\boldsymbol{p}_{D_{ji}} &= \alpha\sqrt{\lambda_{\max}(\boldsymbol{P}_{D_{ji}})}\boldsymbol{v}_{\max}(\boldsymbol{P}_{D_{ji}}),
\end{aligned}$$

where α is the highest number $\in [0,1]$ such that $\boldsymbol{p}_{B_i}$ and $\boldsymbol{p}_{D_{ji}}$ are feasible, and $\alpha = 0$ will lead to a solution where the D2D pair is not transmitting while other constraints are satisfied. To sum up, our power optimization problem in (4) is solved by iteratively updating the auxiliary variables z_C, z_D, y_C, y_D followed by solving the relaxed convex sub-problem (13) for updating the values of $\boldsymbol{p}_{B_i}$ and $\boldsymbol{p}_{D_{ji}}$ until convergence. Once (4) is solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$, the next step is to perform channel assignment to D2D pairs, as explained next.

D. Channel Assignment via Integer Relaxation

For the channel assignment to D2D pairs, sub-optimal values $\tilde{v}_{i,j}$ (obtained after solving (4) $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$) are substituted into (3) and then we need to maximize with respect to $\mathbb{B}$.The resulting channel assignment sub-problem can be stated as:

$$\begin{aligned}
\underset{\mathbb{B}}{\text{maximize}} \quad & \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\beta_{i,j}v_{i,j}^* - \gamma\delta(\mathbb{B}), \\
\text{subject to} \quad & \beta_{i,j} \in \{0,1\}\ \forall\ i,j, \quad \sum_{j\in\mathcal{D}}\beta_{i,j} \leq 1\ \forall\ i.
\end{aligned} \tag{14}$$

Due to the integer constraints, solving (14) involves prohibitive computational complexity even for reasonable values of N_C, N_D. Thus, similar to [17], we relax the integer constraints to $\beta_{i,j} \in [0,1]\ \forall i,j$ to obtain a differentiable strongly convex objective function with linear constraints which can be efficiently solved using the Projected Gradient Descent algorithm. The obtained solution is finally discretized back to satisfy the original constraints $\beta_{i,j} \in \{0,1\}\ \forall i,j$. This is done by setting the highest positive value in every row of $\mathbb{B}$ to 1 while setting other values in the same row to 0.

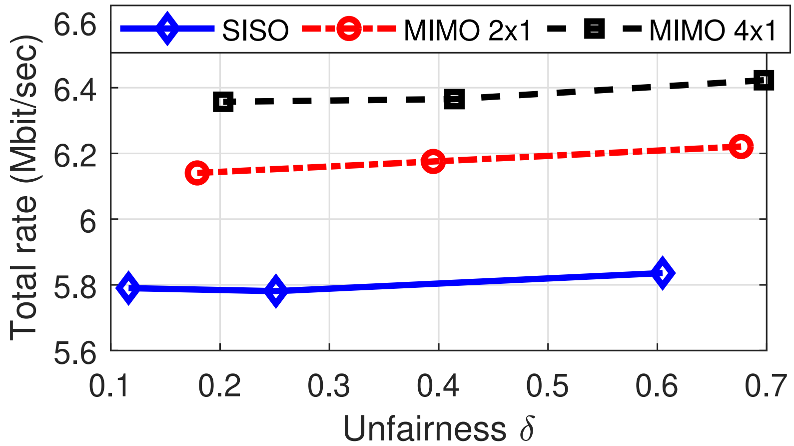

Fig. 2: Total average rate R vs. Unfairness δ

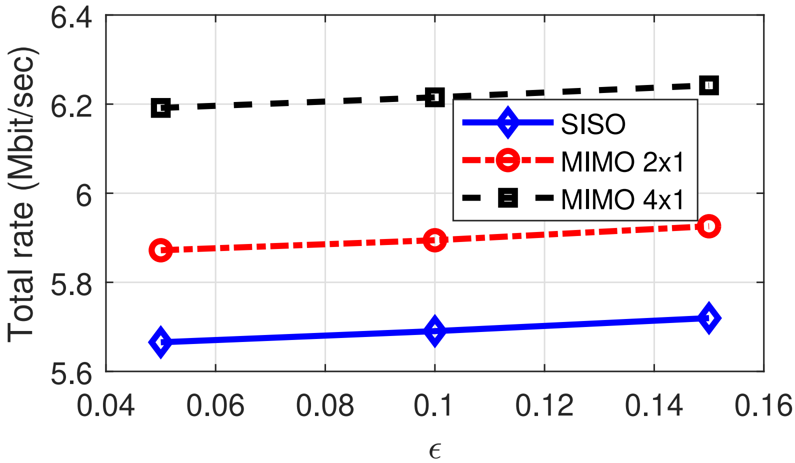

Fig. 3: Total average rate R vs. ϵ

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V. SIMULATIONS

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius $5m$ centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of $1m$. Averages over 1,000 independent realizations with parameters BW $=$ 15 kHz, $\gamma = 50 \times$ BW, $N_D = 5$, $N_C = 5$, $N_0 = -70$ dBW. Two values of the number of transmit antennas were tested $K_B = K_D = 2$ and $K_B = K_D = 4$. The proposed method is compared with the method by Elnourani et al. [18], which to the best of our knowledge is the best existing method for the SISO case.

Fig. 2 shows that the proposed methods achieve higher rate than the method by [18] for $\epsilon = 0.1$ and for different values of γ between 10 and 30. The increment in the total rate is around 3% for all values of γ for the 2×1 MIMO case and around 9% for the 4×1 MIMO case. In general, all rates decrease when γ increases.

Fig. 3 shows that the proposed methods achieve similar increment in rate for different values of ϵ with $\gamma = 100$. In general, all rates increase when ϵ increases. The unfairness values for all the tested cases are between 0 and 0.035, which are very small and very close. This indicates that the selected methods were able to achieve a good rate-fairness trade-off for the specified values of γ and ϵ.

Fig. 4 and Fig. 5 show a comparison between the proposed method and an unreliable beamforming method in [15]. The unreliable method achieves better rate compared to the

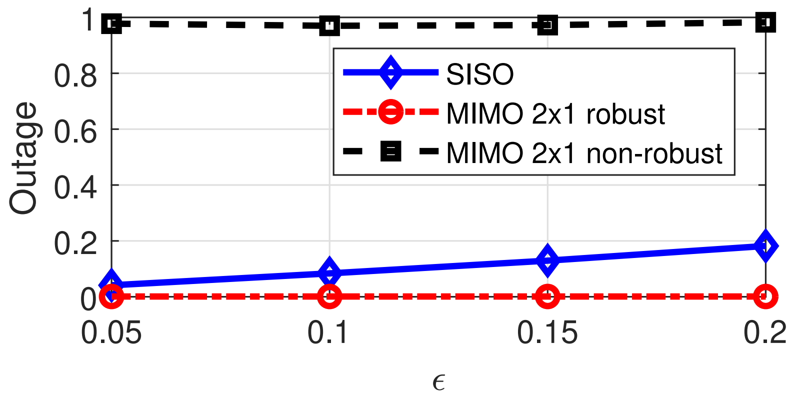

Fig. 4: Outage probability vs. ϵ.

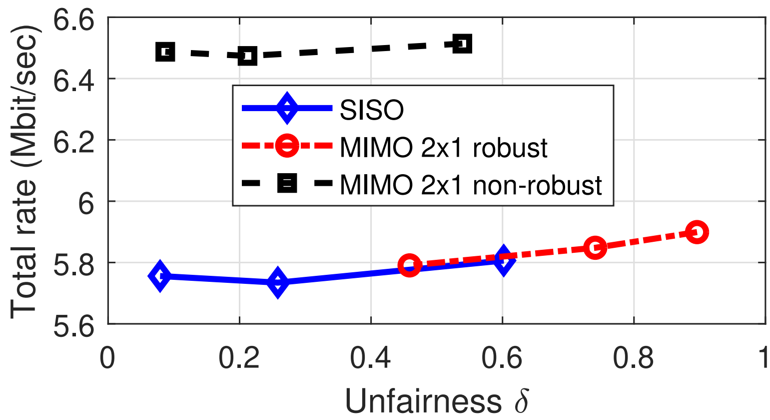

Fig. 5: Total average rate R vs. Unfairness δ ($\epsilon = 0.1$ and γ from 10 to 30)

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proposed method, however the outage probability is close to 100%. On the other hand, our proposed method achieves almost 0 outage probability, since the Bernstein-type inequality is a very conservative approximation for the reliability constraint.

In general, MIMO beamforming can be considered as adding more degrees of freedom to the system. In the cases where a D2D pair is considered inadmissible by [18], due to infeasiblility in the power allocation, beamforming might render the power allocation problems feasible, and thus, resulting in better fairness. Moreover, it will also results in a higher total rate, since it usually generates lower interference.

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E

Appendix F

Paper F

Title:	Reliable Underlay D2D Communications over Multiple Transmit Antenna Framework.
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PAPER F

Reliable Underlay D2D Communications over Multiple Transmit Antenna Framework

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Abstract

Robust beamforming is an efficient technique to guarantee the desired receiver performance in the presence of erroneous channel state information (CSI). However, the application of robust beamforming in underlay device-to-device (D2D) communication still requires further investigation. In this paper, we investigate resource allocation problem for underlay D2D communications by considering multiple antennas at the base station (BS) and at the transmitters of D2D pairs. The proposed design problem aims at maximizing the aggregate rate of all D2D pairs and cellular users (CUs) in downlink spectrum. In addition, our objective is augmented to achieve a fair allocation of resources across the D2D pairs. Further, assuming elliptically bounded CSI errors, the formulation ensures maintaining signal to interference plus noise ratio (SINR) above a specified threshold. The derived optimization problem results in a mixed integer non-convex problem and requires exponential complexity to obtain the optimal solution. We perform a semi-definite relaxation (SDR) to handle the stochastic SINR constraints by using the S-Lemma, obtaining a number of linear matrix inequalities. The non-convexity is addressed by introducing slack variables and performing a quadratic transformation to obtain sub-optimal beamformers via alternating optimization. The solution for channel assignments to D2D pairs is obtained by convex relaxation of the integer constraints. Finally, we demonstrate the merit of the proposed approach by simulations in which we observe higher and more robust network throughput, as compared to previous state-of-the-art.

Index Terms

D2D communications, resource allocation, robust beamforming, semi-definite relaxation.

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I. Introduction

Robust transmit beamforming is recognized as a powerful technique to provide significant throughput gains in comparison to single antenna design [1]. However, most works in underlay D2D communications have considered single-antenna transmission, thus creating an opportunity for further investigation in a multi-antenna framework. The D2D communications in underlay configuration is a promising approach to improve efficiency in spectrum utilization by allowing simultaneous transmissions of existing cellular network and D2D pairs in the same spectrum [2], [3]. On the other hand, simultaneous transmissions in the same spectrum bands increase interference at the respective receivers which must be appropriately handled by devising judicious resource (power, channel) allocation algorithms. Introducing multiple antennas to transmit (beamforming) can further limit the interference and can act as an additional degree of freedom in devising resource allocation algorithms.

Resource allocation problems for underlay D2D communications have been extensively investigated under single antenna transmission in [2], [4], [5]. Considering simplicity in design,

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algorithms proposed in these works, restrict D2D pairs to access more than one channel. In contrast, improving throughput of D2D pairs by allowing access over multiple channels is studied in [6], [7]. It also important to note that these works assume perfect CSI in their problem formulation. Consideration of error in CSI while forming the resource allocation problem is considered in [8]–[10]. However, once again, these works limit investigation to single antenna transmission, leaving the scope for multi-antenna transmission which can be exploited to control the interference and improve the overall throughput of the network.

Considering transmission over multiple antennas, [6] presents a detailed analysis for joint beamforming in D2D underlay cellular networks. However, the analysis is restricted to a single D2D pair scenario under the additional assumption of perfect CSI. Scenarios with multiple D2D pairs are studied in [7], however, prefect CSI is also assumed to be available at the BS. Error in CSI due to quantization is considered in [11], where conventional maximum ratio transmission and interference cancellation techniques are exploited to compute the beamforming vectors. Design of robust beamformers for regular cellular communications has also been investigated in [12]. Under the assumption of Gaussian CSI errors, they propose several convex bounds to approximate the probabilistic rate outage constraints. In recent work, joint beamforming and power control strategies are studied in [13] under both perfect and erroneous CSI scenarios. In their formulation, the objective is to minimize the total transmit power of both BS and D2D pairs while ensuring quality of service (QoS) requirements. In conclusion, none of those previous works considers devising a robust beamforming design while performing resource allocation in underlay D2D communications, which is very relevant for maximizing aggregate network throughput.

In this work, we investigate the robust beamforming design problem in underlay D2D communications configuration under an erroneous (imperfect) CSI scenario. The main research contributions of this work are summarized as follows:

1) We formulate a robust beamforming design problem to maximize the aggregate rate of all D2D pairs and CUs while satisfying SINR to be above a specified threshold for both D2D and CUs. Under the assumption of CSI errors to be bounded within a specified ellipsoid, the proposed formulation maximizes the aggregate rate of the network in the worst case scenario of error in CSI. The objective of the design is also augmented to include the unfairness in channel assignment to D2D pairs. Further, our proposed formulation ensures higher throughput to D2D pairs by allowing simultaneous access of multiple channels to respective D2D pairs.
2) Our formulation leads to a mixed integer non-convex problem, for which we propose an algorithm to compute the power beamforming vectors and channel assignment to D2D pairs in a computationally efficient manner by exploiting SDR aided with a quadratic transformation. The power beamforming vectors and channel assignment are obtained by alternating optimization and convex relation of integer constraints, respectively.
3) In order to demonstrate the merits of our proposed formulation and the algorithm in reliably maximizing the aggregate rate of the underlay D2D communications network, we present Matlab based simulation results where we obtain a better performance than the-state-of-the-art alternatives.

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II. System Model

The underlay D2D communications scenario under a multiple transmit antenna framework in downlink spectrum[1] is shown in Fig. 1. We assume that the BS have K_B transmit antennas to communicate with N_C single antenna CUs through N_C downlink channels. In order to avoid confusion in notation, CUs (equivalently, channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. The

[1]Without loss of generality, the same formulation and algorithm design developed here, can be also applied to the uplink spectrum.

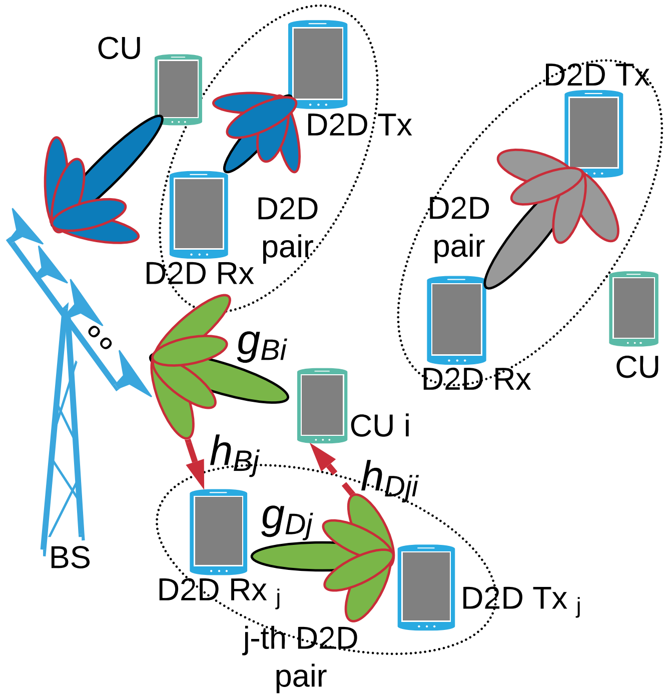

Fig. 1: Illustration of the overall system model.

D2D pairs wishing to communicate over the aforementioned N_C channels are indexed by $\mathcal{D}$ = $\{1, ..., N_D\}$. Similarly, we assume that the D2D transmitters have K_D transmit antennas to communicate with their respective single antenna D2D receivers[2].

The channel between the BS and the i-th cellular user (CU) is denoted by $\boldsymbol{g}_{B_i} \in \mathbb{C}^{K_B \times 1}$. Similarly, the channel between the j-th D2D pair is denoted by $\boldsymbol{g}_{D_j} \in \mathbb{C}^{K_D \times 1}$. The interference channel between the BS and the receiver of the j-th D2D is denoted by[3] $\boldsymbol{h}_{B_j} \in \mathbb{C}^{K_B \times 1}$. Similarly, the interference channel between the transmitter of the j-th D2D pair and the i-th CU is denoted by $\boldsymbol{h}_{D_{j,i}} \in \mathbb{C}^{K_D \times 1}$. Here, we assume that the CUs provide limited cooperation in estimating the gain of the interference channel (as expected in practice). Thus, if $\tilde{\boldsymbol{h}}_{D_{j,i}}$ denotes the estimate of the interference channel gain with error $\boldsymbol{e}_{ji}$, then the correct channel gain can be defined as $\boldsymbol{h}_{D_{j,i}} = \tilde{\boldsymbol{h}}_{D_{j,i}} + \boldsymbol{e}_{ji}$. This error vector is assumed to bounded within a specified ellipsoid, i.e., $\boldsymbol{e}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{e}_{ji} \leq 1$ where, $\boldsymbol{Q}_{ji} \in \mathbb{H}^{K_D}$, $\boldsymbol{Q}_{ji} \succeq \boldsymbol{0}$ specifies the size and shape of ellipsoid, and $\mathbb{H}^{K_D}$ is the space of $K_D \times K_D$ Hermitian matrices. The additive white noise power is denoted by N_0.

We represent the assignment of channels to D2D pairs by the indicators $\{\beta_{i,j}\}_{i \in \mathcal{C}, j \in \mathcal{D}} \in \{0, 1\}$, where $\beta_{i,j} = 1$ when the i-th channel is assigned to the j-th D2D pair and $\beta_{i,j} = 0$ otherwise. In order to provide higher throughput to D2D pairs, we allow simultaneous access of multiple channels to a D2D pair, however, to restrict the interference among D2D pairs, access of more than one D2D pair is not allowed over a particular channel, i.e., $\sum_{j=1}^{N_D} \beta_{i,j} \leq 1, \forall i$.

[2]In general, a BS/D2D transmitter with multiple antennas can simultaneously communicate to multiple CUs/D2D receivers on a single channel; however, for simplicity in our analysis, we assume one CUs/D2D pair on every channel. With minor modification, the analysis can be extended to the multi-user case.

[3]In principle, $\boldsymbol{g}_{D_j}$ and $\boldsymbol{h}_{B_j}$ should also depend on the i-th channel, however, this subscript is dropped as the proposed scheme carries over immediately to accommodate such dependence.

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Finally, we denote the beamforming power vector of the BS to communicate with the i-th CU by $\boldsymbol{p}_{B_i} \in \mathbb{C}^{K_D \times 1}$ and for the j-th D2D pair on the i-th channel by $\boldsymbol{p}_{D_{j,i}} \in \mathbb{C}^{K_B \times 1}$. The respective transmit powers are constrained as $||\boldsymbol{p}_{B_i}||_2^2 \leq p_{B,\max}$ and $||\boldsymbol{p}_{D_{j,i}}||_2^2 \leq p_{D,\max}$. To ensure successful communication, the SINR is desired to be greater than $\eta_{D,\min}$ for D2D pairs and $\eta_{C,\min}$ for CUs.

III. Problem Formulation

In order to take into account the error in the estimate of the interference channels from D2D pairs to CUs, i.e., $\tilde{\boldsymbol{h}}_{D_{ji}}$, we formulate the beamforming design problem for the worst case error in $\tilde{\boldsymbol{h}}_{D_{ji}}$. Let $\Gamma(z) := \text{BW} \times \log_2(1+z)$ denote the rate obtained over channel bandwidth BW for the given SINR z. The total rate that can be achieved over every i-th channel is defined by $R_i := (1 - \sum_{j\in\mathcal{D}} \beta_{i,j}) R_{C_{i,0}} + \sum_{j\in\mathcal{D}} \beta_{i,j}[R_{D_{j,i}} + R_{C_{i,j}}]$, where:

- $R_{C_{i,0}} := \Gamma(p_{B_{\max}} ||\boldsymbol{g}_{B_i}||_2^2 / N_0)$, rate of the i-th CU without assignment of D2D pairs, i.e., $\beta_{ij} = 0\ \forall j$.
- $R_{D_{j,i}} := \Gamma(|\boldsymbol{p}_{D_{ji}}^H \boldsymbol{g}_{D_j}|^2\ /(N_0 + |\boldsymbol{p}_{B_i}^H \boldsymbol{h}_{B_j}|^2))$, rate of j-th D2D pair when assigned with i-th CU, i.e., $\beta_{ij} = 1$.
- $R_{C_{i,j}} := \Gamma(|\boldsymbol{p}_{B_i}^H \boldsymbol{g}_{B_i}|^2 / (N_0 + |\boldsymbol{p}_{D_{ji}}^H \boldsymbol{h}_{D_{j,i}}|^2))$, rate achieved by i-th CU when assigned with j-th D2D pair, i.e., $\beta_{ij} = 1$.

Finally, the aggregate network rate is defined as $R(\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D) := \sum_{i\in\mathcal{C}} R_i$, where, $\mathbb{B} := \{\beta_{i,j}\}$, $\mathbb{P}_B := \{\boldsymbol{p}_{B_i}\}$, $\mathbb{P}_D := \{\boldsymbol{p}_{D_{j,i}}\}\ \forall i \in \{1, \cdots N_C\}$ and $j \in \{1, \cdots N_D\}$.

In order to have fairness in channel assignment, we introduce a secondary objective that penalizes greedy channel assignments to the D2D pairs. We also define the unfairness measure $\delta(\mathbb{B}) = 1/(N_D c^2) \sum_{j=1}^{N_D} (x_j - c)^2$ along similar lines to [14], [15], where $x_j := \sum_{i=1}^{N_C} \beta_{ij}$ is the number of channels assigned to the j-th D2D pair; and where $c := N_C/N_D$ is the fairest assignment. Summing up, the overall problem considering the worst case error in estimation of interference channel, can be formulated as:

$$\underset{\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D}{\text{maximize}}\ R(\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D) - \gamma\delta(\mathbb{B}) \tag{1a}$$

$$\text{subject to } \beta_{i,j} \in \{0,1\},\ \sum_{j=1}^{N_D} \beta_{i,j} \leq 1\, \forall i; \tag{1b}$$

$$||\boldsymbol{p}_{B_i}||_2^2 \leq p_{B,\max}\, \forall i,\ ||\boldsymbol{p}_{D_{j,i}}||_2^2 \leq p_{D,\max}\, \forall j, i; \tag{1c}$$

$$\frac{|\boldsymbol{p}_{B_i}^H \boldsymbol{g}_{B_i}|^2}{N_0 + |\boldsymbol{p}_{D_{ji}}^H \boldsymbol{h}_{D_{j,i}}|^2} \geq \eta_{C,\min} \quad \text{if } \beta_{ij} = 1,\ \forall i, j \tag{1d}$$

$$\frac{|\boldsymbol{p}_{D_{ji}}^H \boldsymbol{g}_{D_j}|^2}{N_0 + |\boldsymbol{p}_{B_i}^H \boldsymbol{h}_{B_j}|^2} \geq \eta_{D,\min} \quad \text{if } \beta_{ij} = 1,\ \forall i, j \tag{1e}$$

$$\boldsymbol{h}_{D_{j,i}} = \tilde{\boldsymbol{h}}_{D_{j,i}} + \boldsymbol{e}_{ji}, \quad \boldsymbol{e}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{e}_{ji} \leq 1, \quad \forall i, j \tag{1f}$$

The regularization parameter $\gamma > 0$ is selected to balance the trade-off between aggregate rate and fairness in channel assignment. Problem (1) is a non-convex mixed-integer program, which involves exponential complexity. In next section, we discuss the proposed strategy by exploiting semi-definite relaxation and quadratic transformation.

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IV. Proposed Optimization Algorithm

The complexity to obtain the solution of (1) can be reduced by decomposing the problem into multiple sub-problems of lower complexity. We first re-express the aggregate throughput as:

$$R(\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D) = \sum_{i\in\mathcal{C}} \left[\sum_{j\in\mathcal{D}} \beta_{i,j} v_{i,j}(\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}) + R_{C_{i,0}} \right] \tag{2}$$

where $v_{i,j}(P_{B_i}, P_{D_{ji}}) := R_{C_{i,j}} + R_{D_{j,i}} - R_{C_{i,0}}$ represents the rate increment due to the assignment of channel i to the D2D pair j relative to the case where the channel i is only used by the CU. Next, notice that the objective of (1) with the substitution of (2) can be equivalently expressed by replicating $\{\boldsymbol{p}_{B_i}\}$ with multiple auxiliary variables $\{\boldsymbol{p}_{B_{ij}}\}$ and removing the constant terms from the objective function. The resulting problem can be stated as:

$$\begin{aligned} \underset{\mathbb{B}, \mathbb{P}_B, \mathbb{P}_D}{\text{maximize}} \quad & \sum_{i\in\mathcal{C}} \sum_{j\in\mathcal{D}} \left[\beta_{i,j} v_{i,j}(\boldsymbol{p}_{B_{ij}}, \boldsymbol{p}_{D_{ji}}) \right] - \gamma\delta(\mathbb{B}) \\ \text{subject to} \quad & (1\text{b}), (1\text{c}), (1\text{d}), (1\text{e}) \text{ and } (1\text{f}) \end{aligned} \tag{3}$$

To recover the optimal $\{\boldsymbol{p}^*_{B_i}\}$ of (1) from the optimal $\{\boldsymbol{p}^*_{B_{i,j}}\}$ of (3), one only needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $\boldsymbol{p}^*_{B_i} = \boldsymbol{p}^*_{B_{i,j}}$. If no such a j exists, i.e. $\beta_{i,j} = 0 \; \forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power $\boldsymbol{p}^*_{B_i} = p_{B,\max}$.

In addition, we can also notice that (3) decouples across i and j into $N_C \times N_D$ power allocation sub-problems and a final channel assignment problem. Then, for each i, j, the power allocation sub-problem can be stated as:

$$\begin{aligned} \underset{\boldsymbol{p}_{B_{ij}}, \boldsymbol{p}_{D_{ji}}}{\text{maximize}} \quad & R_{C_{i,j}} + R_{D_{j,i}} \\ \text{subject to} \quad & (1\text{c}), (1\text{d}), (1\text{e}) \text{ and } (1\text{f}) \end{aligned} \tag{4}$$

which should be solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$. The subsequent channel assignment problem is discussed in section IV-C. We can notice that problem (4) is still a non-convex stochastic problem. In the next section, we perform SDR along with *S-Lemma* [16] to express the stochastic constraints by linear matrix inequalities.

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A. Semi-Definite Relaxation

It can be noted that the objective and constraint (1d) of (4) involve random channel interference terms. We first introduce slack variables z_C and z_D in order to bring stochastic terms from the objective of (4) to constraints as:

$$\underset{\boldsymbol{p}_{B_{ij}}, \boldsymbol{p}_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \quad \log_2(1 + z_C) + \log_2(1 + z_D) \tag{5a}$$

$$\text{subject to } z_C \le \frac{|\boldsymbol{g}^H_{B_i} \boldsymbol{p}_{B_{ij}}|^2}{N_0 + |\boldsymbol{h}^H_{D_{ji}} \boldsymbol{p}_{D_{ji}}|^2} \tag{5b}$$

$$z_D \le \frac{|\boldsymbol{g}^H_{D_j} \boldsymbol{p}_{D_{ji}}|^2}{N_0 + |\boldsymbol{h}^H_{B_j} \boldsymbol{p}_{B_{ij}}|^2}, \qquad (1\text{c}), (1\text{d}), (1\text{e}) \text{ and } (1\text{f}) \tag{5c}$$

Next, substituting the random interference channel vector $\boldsymbol{h}_{D_{ji}} = \tilde{\boldsymbol{h}}_{D_{ji}} + \boldsymbol{e}_{ji}$ and letting $\mathbf{P}_{B_{ij}} := \mathbf{p}_{B_{ij}} \mathbf{p}^H_{B_{ij}}$, $\boldsymbol{P}_{D_{ji}} := \boldsymbol{p}_{D_{ji}} \boldsymbol{p}^H_{D_{ji}}$, respectively, the stochastic inequality (5b), i.e., $z_C \le \frac{|\boldsymbol{g}^H_{B_i} \boldsymbol{p}_{B_{ij}}|^2}{N_0 + |\boldsymbol{h}^H_{D_{ji}} \boldsymbol{p}_{D_{ji}}|^2}$ can be re-expressed as:

$$- \boldsymbol{e}_{ji}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{e}_{ji} - \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{e}_{ji} - \boldsymbol{e}_{ji}^H \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} - \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} - N_0 + \frac{1}{z_C} \boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} \geq 0 \tag{6}$$

Note that (6) and $e_{ji}^H \boldsymbol{Q}_{ji} e_{ji} \leq 1$ (in constraint (1f)) are quadratic inequalities for the random error vector e_{ji}. Thus, we exploit the *S-Lemma* [16], to express the stochastic constraints in the from of a linear matrix inequality.

Lemma 1 (S-Lemma). *Let* $\phi_i(\boldsymbol{e}) \triangleq \boldsymbol{e}^H \boldsymbol{A}_i \boldsymbol{e} + \boldsymbol{b}_i^H \boldsymbol{e} + \boldsymbol{e}^H \boldsymbol{b}_i + c_i \ \forall \ i = 0, 1,$ *where* $\boldsymbol{A}_i \in \mathbb{H}^{N_{K_D}}$, $\boldsymbol{b}_i \in \mathbb{C}^{N_{K_D}}$ *and* $c_i \in \mathbb{R}$. *Suppose there exists an* $\hat{\boldsymbol{e}} \in \mathbb{C}^{N_{K_D}}$ *such that* $\phi_i(\hat{\boldsymbol{e}}) < 0$, *then the following two conditions are equivalent:*

1) $\phi_0(\boldsymbol{e}) \geq 0$ *for all* $\boldsymbol{e}$ *satisfying* $\phi_1(\boldsymbol{e}) \leq 0$;
2) *There exists a* $\zeta \geq 0$ *such that,*
$\begin{bmatrix} \boldsymbol{A}_0 & \boldsymbol{b}_0 \\ \boldsymbol{b}_0^H & c_0 \end{bmatrix} + \zeta \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{b}_1 \\ \boldsymbol{b}_1^H & c_1 \end{bmatrix} \succeq 0$

Relating $\phi_0(\boldsymbol{e})$ to (6) and $\phi_1(\boldsymbol{e})$ to $e_{ji}^H \boldsymbol{Q}_{ji} e_{ji} - 1 \leq 0$, and applying the S-Lemma, the stochastic constraint can be expressed as:

$$\boldsymbol{\phi}(\zeta_{ji}) \triangleq \begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{z_C} \end{bmatrix} \succeq \boldsymbol{0} \tag{7}$$

where, $f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) := \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} + N_0 + \zeta_{ji}$. Similarly, Performing a SDR and applying the S-Lemma to constraints (1d) of (4), the relaxed semi-definite problem without stochastic constraints can be expressed as:

$$\underset{\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \ \underset{\zeta_{ji}}{\text{minimize}} \quad \log_2(1 + z_C) + \log_2(1 + z_D) \tag{8a}$$

$$\text{subject to} \quad z_D \leq \frac{\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \tag{8b}$$

$$\begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{z_C} \end{bmatrix} \succeq \boldsymbol{0} \tag{8c}$$

$$0 \leq \text{Tr}(\boldsymbol{P}_{B_{ij}}) \leq p_{B,\max}, \qquad 0 \leq \text{Tr}(\boldsymbol{P}_{D_{i,j}}) \leq p_{D,\max} \tag{8d}$$

$$\begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{\eta_{C,\min}} \end{bmatrix} \succeq \boldsymbol{0} \tag{8e}$$

$$\frac{\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \geq \eta_{D,\min}, \quad \zeta_{ji} \geq 0 \quad \boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}} \succeq \boldsymbol{0} \tag{8f}$$

Next, applying the Schur complement on the semi-definite constraint (8c), this constraint in the form of a linear matrix inequality and a general inequality as:

$$- \boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} \succeq \boldsymbol{0} \tag{9a}$$

$$(-f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{1}{z_C} \boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} - \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(-\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}) \geq 0 \tag{9b}$$

Thus, the optimal value of ζ_{ji} in (9a) can be computed as:

$$\zeta_{ji}^* = \underset{\zeta}{\text{minimize}} \quad \zeta \qquad \text{subject to} \quad \zeta \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}} \succ \boldsymbol{0}, \quad \zeta \geq 0 \tag{10}$$

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Further, rearranging the terms of (9b) and substituting this constraint for a given optimal value of ζ_{ji}^*, the relaxed semi definite problem (9) can be restated as:

$$\underset{\boldsymbol{P}_{B_{ij}},\boldsymbol{P}_{D_{ji}},\boldsymbol{z}}{\text{maximize}} \quad \log_2(1+z_C)+\log_2(1+z_D) \tag{11a}$$

$$\text{subject to} \quad z_D \leq \frac{\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \tag{11b}$$

$$z_C \leq \boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} / \Big[f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\lambda_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \Big] \tag{11c}$$

$$0 \leq \mathrm{Tr}(\boldsymbol{P}_{B_{ij}}) \leq p_{B,\max}, \qquad 0 \leq \mathrm{Tr}(\boldsymbol{P}_{D_{i,j}}) \leq p_{D,\max} \tag{11d}$$

$$\begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji}^* \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{\eta_{C,\min}} \end{bmatrix} \succeq \boldsymbol{0} \tag{11e}$$

$$\frac{\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \geq \eta_{D,\min}, \quad \boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}} \succeq \boldsymbol{0} \tag{11f}$$

Notice that constraints (11b) involve a ratio between two convex functions and (11c) involve a ratio of a convex and a non-convex function. Hence in the next subsection we use fractional programming [17] to relax the non convexity due to these ratios.

B. Fractional Programming by Quadratic Transformation

It is easy to note that the optimal values of the slack variables z_C^* and z_D^* are:

$$\begin{aligned} z_C^* =& \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}} \\ z_D^* =& \frac{\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \end{aligned} \tag{12}$$

Taking a partial Lagrangian of (11) by considering only the constraints related to the slack variables z_C and z_D in (11b) and (11c) respectively, we obtain:

$$\begin{aligned} L(\boldsymbol{P}, \boldsymbol{z}, \boldsymbol{\lambda}) &= \log_2(1+z_C) + \log_2(1+z_D) \\ &- \lambda_C \left(z_C - \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{\left(f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \right)} \right) \\ &- \lambda_D \left(z_D - \frac{\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \right) \end{aligned} \tag{13}$$

At the stationary point, $\frac{\partial L}{\partial \boldsymbol{z}} = 0$, and since the optimal value of $\boldsymbol{z}^*$ is known, the optimal values of Lagrange variables are related to the optimal values of the slack variables and can be computed as follows:

$$\lambda_C^* = \frac{1}{1+z_C^*} = \frac{f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}}{\left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right)}$$

$$\lambda_D^* = \frac{1}{1+z_D^*} = \frac{N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}}{N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \tag{14}$$

Substituting only the optimal values of the Lagrange variables λ_C^* and λ_D^* in (11), we obtain:

$$\begin{aligned} \underset{\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \quad & \log_2(1+z_C) + \log_2(1+z_D) - z_C - z_D \\ & + \frac{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{\left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right)} \\ & + \frac{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}{N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \\ \text{subject to} \quad & \text{(11d), (11e) and (11f)} \end{aligned} \tag{15}$$

Next, we transform the fractions in the objective by introducing two auxiliary variables y_C and y_D through a quadratic transformation [17], obtaining:

$$\begin{aligned} \underset{\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}, \boldsymbol{z}, \boldsymbol{y}}{\text{maximize}} \quad & \log_2(1+z_C) + \log_2(1+z_D) - z_C - z_D \\ & + 2y_C \sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}} \\ & - y_C^2 \left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right) \\ & + 2y_D \sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}} \\ & - y_D^2 \left(N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}\right) \\ \text{subject to} \quad & \text{(11d), (11e) and (11f)} \end{aligned} \tag{16}$$

The optimal values of the auxiliary variables y_C and y_D can be readily computed as:

$$y_C^* = \frac{\sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}}{\left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji} \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right)}$$

$$y_D^* = \frac{\sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}}}{N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}} \tag{17}$$

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Finally, introducing a slack variable $s_{ji} \geq f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji}^* \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \bar{\boldsymbol{h}}_{D_{ji}}$, to facilitate alternating optimization between the power variables, and rearranging once again by taking the Schur compliment, the optimization problem (16), can be restated as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_{B_{ij}}, \boldsymbol{P}_{D_{ji}}, \boldsymbol{z}, \boldsymbol{y}}{\text{maximize}} \quad & \log_2(1+z_C) + \log_2(1+z_D) - z_C - z_D \\
& + 2y_C\sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}} - y_C^2 \left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} + s_{ji}\right) \\
& + 2y_D\sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}} \\
& - y_D^2 \left(N_0 + \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}\right) \\
\text{subject to} \quad & \begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji}^* \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}^*) + s_{ji} \end{bmatrix} \succeq \boldsymbol{0} \\
& \text{(11d), (11e) and (11f)}
\end{aligned} \tag{18}$$

Notice that for given values of slack variables z_C and z_D and auxiliary variables y_C and y_D, the optimization problem (18) is still jointly non-convex in $\boldsymbol{P}_{B_{ij}}$ and $\boldsymbol{P}_{D_{ji}}$. Hence, we propose to perform alternating optimization in (18) between $\boldsymbol{P}_{B_{ij}}$ and $\boldsymbol{P}_{D_{ji}}$.

The SDR [18] sub-problem of (18) to optimize $\boldsymbol{P}_{B_{ij}}$ for given updated values of ζ_{ji}, z_C, z_D, y_C, y_D and $\boldsymbol{P}_{D_{ji}}$, can be stated as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_{B_{ij}}}{\text{maximize}} \quad & 2y_C\sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}} \\
& - y_C^2 \left(\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}\right) - y_D^2 \left(\boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j}\right) \\
\text{subject to} \quad & 0 \leq \text{Tr}\left(\boldsymbol{P}_{B_{ij}}\right) \leq p_{B,\max}, \ \boldsymbol{P}_{B_{ij}} \succeq \boldsymbol{0} \\
& \boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i} \geq \\
& \eta_{C,\min} \left(f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} \left(\zeta_{ji} \boldsymbol{Q}_{ji} - \boldsymbol{P}_{D_{ji}}\right)^{-1} \boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}}\right) \\
& \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{h}_{B_j} \leq \frac{1}{\eta_{D,\min}} \left(\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}\right) - N_0
\end{aligned} \tag{19}$$

Similarly, the SDR sub-problem of (18) to optimize $\boldsymbol{P}_{D_{ji}}$ for given values of ζ_{ji}, z_C, z_D, y_C, y_D and $\boldsymbol{P}_{B_{ij}}$, can be stated as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_{D_{ji}}, s_{ji}}{\text{maximize}} \quad & 2y_D\sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}} \\
& - y_D^2 \left(\boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j}\right) - y_C^2 s_{ji} \\
\text{subject to} \quad & 0 \leq \text{Tr}(\boldsymbol{P}_{D_{i,j}}) \leq p_{D,max}, \ \boldsymbol{P}_{D_{ji}} \succeq \boldsymbol{0} \\
& \begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + s_{ji} \end{bmatrix} \succeq \boldsymbol{0} \\
& \begin{bmatrix} -\boldsymbol{P}_{D_{ji}} + \zeta_{ji} \boldsymbol{Q}_{ji} & -\boldsymbol{P}_{D_{ji}} \tilde{\boldsymbol{h}}_{D_{ji}} \\ -\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{P}_{D_{ji}} & -f(\boldsymbol{P}_{D_{ji}}, \zeta_{ji}) + \frac{\boldsymbol{g}_{B_i}^H \boldsymbol{P}_{B_{ij}} \boldsymbol{g}_{B_i}}{\eta_{C,min}} \end{bmatrix} \succeq \boldsymbol{0} \\
& \boldsymbol{g}_{D_j}^H \boldsymbol{P}_{D_{ji}} \boldsymbol{g}_{D_j} \geq \eta_{D,min} \left(N_0 + \boldsymbol{h}_{B_j}^H \boldsymbol{P}_{B_{ij}}^{(k+1)} \boldsymbol{h}_{B_j}\right)
\end{aligned} \tag{20}$$

Note that the obtained optimal solution for the relaxed problems (19) and (20) may not be rank one; thus, additional rank one approximation procedures such as (i) eigen vector corresponding to maximum eigen value, or (ii) randomization [18]; are needed to obtain the power beamforming vectors $\boldsymbol{p}_{B_{ij}}$ and $\boldsymbol{p}_{D_{ji}}$ from the respective $\boldsymbol{P}_{B_{ij}}^*$ and $\boldsymbol{P}_{D_{ji}}^*$ matrices.

To sum up, the power optimization subproblem (4) is solved by iteratively updating the parameter ζ_{ji} in the S-Lemma (through (10)); the slack variables z_C and z_D (through (12)) ; the auxiliary variables y_C and y_D (through (17)); and the power vectors $\boldsymbol{p}_{B_{ij}}$ and $\boldsymbol{p}_{D_{ji}}$ through (19) and (20). Once (4) is solved $\forall i \in \mathcal{C}$ and $\forall j \in \mathcal{D}$, the next step is to perform channel assignment to D2D pairs, as explained in next subsection.

C. Channel Assignment via Integer Relaxation

For the channel assignment to D2D pairs, the resulting values $\tilde{v}_{i,j}$ (obtained after solving (4) $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$) are substituted into (3) and then we need to maximize the objective of (3) with respect to $\mathbb{B}$. The resulting channel assignment sub-problem can be stated as:

$$\begin{aligned} \underset{\mathbb{B}}{\text{maximize}} \quad & \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \beta_{i,j} v_{i,j}^* - \gamma \delta(\mathbb{B}), \\ \text{subject to} \quad & \beta_{i,j} \in \{0,1\} \;\; \forall i,j, \quad \sum_{j \in \mathcal{D}} \beta_{i,j} \leq 1 \;\; \forall i. \end{aligned} \tag{21}$$

Due to the integer constraints, solving (21) involves prohibitive computational complexity even for reasonable values of $N_C,\ N_D$. Thus, we relax the integer constraints to $\beta_{i,j} \in [0,1]\ \forall i,j$ to obtain a differentiable Lipschitz smooth objective function with linear constraints which can be efficiently solved using the Projected Gradient Descent algorithm. The obtained solution is finally discretized back to satisfy the original constraints $\beta_{i,j} \in \{0,1\}\ \forall i,j$. In our approach, this is done by setting the highest positive value in every row of $\boldsymbol{B}$ to 1 while setting other values in the same row to 0. Other solutions were investigated in [15], our selected approach is the one with the lowest computational complexity, nevertheless, it achieves very close results to the most computationally complex one.

V. Simulations

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. We assume $\tilde{h}_C$ to be exponentially distributed with the mean value obtained from the mentioned path-loss model. Averages over 1,000 independent realizations of the user locations with parameters BW= 15 kHz, $\gamma = 200 \times$ BW, $N_D = 10,\ N_C = 10$, $N_0 = -70$ dBW (γ is scaled with BW to ensure that the unfairness and the achieved rate are of comparable values). The proposed algorithm is tested for the cases where $K_B = K_D = 2$ (2×1 MIMO) and where $K_B = K_D = 4$ (4×1 MIMO). In both cases, we assume that $\boldsymbol{Q} = \epsilon^{-2}\boldsymbol{I}$, which indicates that the error in the channel gains lies in a circle of ϵ radius ($\|e\| \leq \epsilon$). These cases are further compared with the method by Elnourani et al. [19], which to the best of our knowledge is the best existing method is the SISO case, with exponential channel gains and an allowed outage probability of 0.1.

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In Fig. 2, both cases are tested with $\epsilon = 10^{-4}$. It shows that the proposed method achieves higher rates than the SISO method in both cases. When γ increases, the rate decreases in all methods, as expected. The 4×1 MIMO case achieves the highest rates, followed by the 2×1 MIMO case. Moreover, the differences in rates between all methods are almost constant.

Fig. 3 shows that the proposed methods achieve very small *unfairness* and that is very close to the SISO case. When γ increases, the unfairness decreases in all methods, as expected. Fig. 4 shows that the proposed methods achieve high average rates in both cases for different values of ϵ. The 4×1 MIMO case achieves the highest rate, followed by the case of 2×1 MIMO. The SISO case achieves the lowest rate, as expected. The rates for both MIMO cases decreases when ϵ increases, which indicates that having a larger error will cause our solution

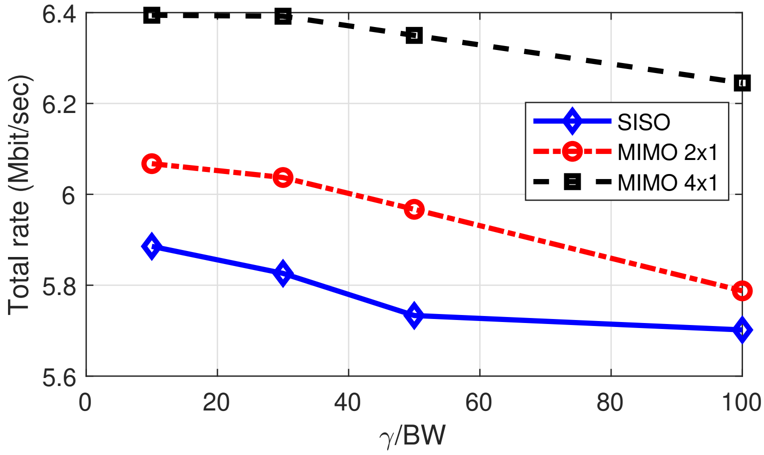

Fig. 2: Total average rate R vs. γ.

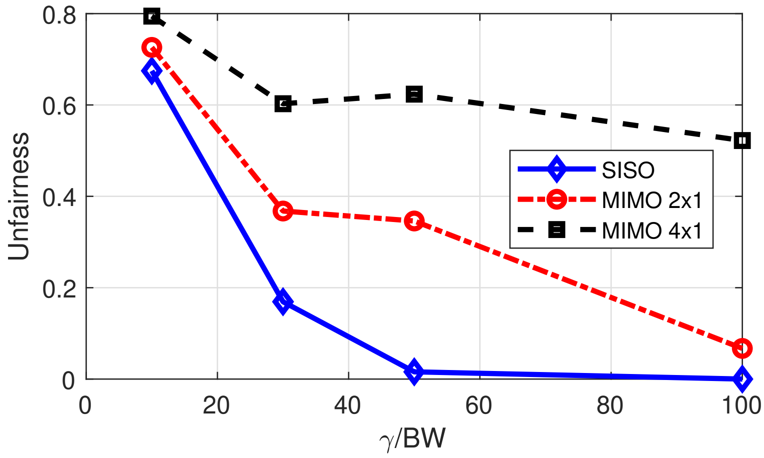

Fig. 3: Unfairness δ vs. γ .

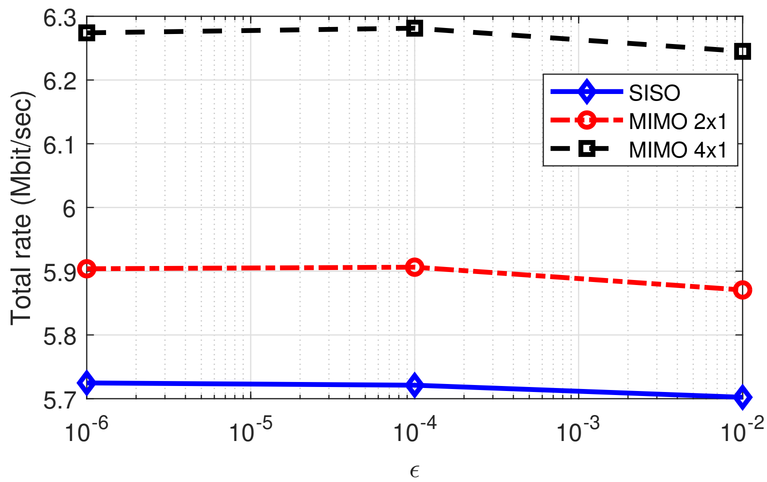

Fig. 4: Total average rate R vs. ϵ.

to be more conservative, resulting in lower rates. The rate in the SISO case is almost constant, since this method does not depend on ϵ, and the resulting achieved rates are considerably lower than the proposed methods. The fairness when changing ϵ is observed to be almost constant for all methods (in the selected range).

In general, the proposed method converges to a stationary solution. This solution is always better than the optimal one achieved by the SISO method, in both cases of 2×1 MIMO and

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4×1 MIMO, for all the tested values of ϵ and γ. It is observed that increasing the number of antennas leads always to higher rates as expected, due to the additional degrees of freedom available.

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Appendix G

Paper G

Title:	Reliable Multicast D2D Communication Over Multiple Channels in Underlay Cellular Networks.
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PAPER G

Reliable Multicast D2D Communication Over Multiple Channels in Underlay Cellular Networks

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Abstract

Multicast device-to-device (D2D) communications operating underlay with cellular networks is a spectral efficient technique for disseminating data to the nearby receivers. However, due to critical challenges such as, mitigating mutual interference and unavailability of perfect channel state information (CSI), the resource allocation to multicast groups needs significant attention. In this work, we present a framework for joint channel assignment and power allocation strategy to maximize the sum rate of the combined network. The proposed framework allows access of multiple channels to the multicast groups, thus improving the achievable rate of the individual groups. Furthermore, fairness in allocating resources to the multicast groups is also ensured by augmenting the objective with a penalty function. In addition, considering imperfect CSI, the framework guarantees to provide rate above a specified outage for all the users. The formulated problem is a mixed integer nonconvex program which requires exponential complexity to obtain the optimal solution. To tackle this, we first introduce auxiliary variables to decouple the original problem into smaller power allocation problems and a channel assignment problem. Next, with the aid of fractional programming via a quadratic transformation, we obtain an efficient power allocation solution by alternating optimization. The solution for channel assignment is obtained by convex relaxation of integer constraints. Finally, we demonstrate the merit of the proposed approach by simulations, showing a higher and a more robust network throughput.

Index Terms

D2D multicast communications, resource allocation, imperfect CSI, fractional programming.

I. INTRODUCTION

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Multicast D2D communication represents the operation of directly disseminating the data to nearby devices without passing the packets through the base station (BS). Some important applications include: (i) dissemination of marketing/advertisement data in the commercial networks; (ii) device discovery, clustering, co-ordination in self organizing networks; (iii) dissemination of critical information such as police, fire, ambulance, etc. in the public safety networks [1]. In these scenarios, D2D multicast in underlay configuration is a promising approach to improve spectrum utilization as it allows simultaneous transmissions of existing cellular network and multicast groups in the same spectrum [2]. However, unlike the unicast D2D communication, multicast D2D communication has its own challenges in terms of heterogeneous channel conditions for individual receivers in the multicast group, thus, achievable performance of the multicast group is generally limited by the receiver with the worst channel conditions. In addition, similar to underlay unicast D2D communications, simultaneous

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transmissions in the same spectrum bands increases interference at the respective receivers and may adversely reduce the overall network performance. Further, acquiring perfect CSI for optimizing network performance poses critical challenges in practical networks. Thus, it is necessary to devise a judicious and reliable resource allocation algorithm which can maximize the overall network performance.

Resource allocation problems for underlay unicast D2D communications have been extensively investigated in [3]–[5]. In D2D multicast settings, previous work in [1] has exploited concepts of stochastic geometry to model and derive the analytical expressions for performance metrics under the overlay communication framework. For the underlay framework, a resource allocation problem is formulated in [6] to maximize the sum throughput of multicast groups while restricting interference to cellular users (CUs) below a certain specified threshold. Similarly, a sum throughput maximization problem is formulated in [2] with constraints on minimum signal to interference plus noise ratio (SINR) requirements. Moreover, a channel assignment scheme to maximize the sum effective throughput is proposed in [7] under partial information of the device location. It can be noted that most of the above work on multicast D2D communication consider perfect CSI. Further, the optimizations for channel and power allocation are done separately and most of the times also limiting multicast groups to access more than one channel. In addition, fairness in allocation resources to the multicast groups is also ignored.

In this work, we investigate the sum rate maximization problem for underlay multicast D2D communication under the assumption of imperfect CSI. The main contributions of this work can be summarized as follows:

1) We formulate a joint power allocation and channel assignment problem to maximize the sum rate of all D2D multicast groups and CUs with a probabilistic constraint on the minimum SINR for both receivers in multicast groups and CUs. The objective function is also augmented to include penalty on the unfairness in channel assignment to D2D multicast groups. Further, the formulation ensures higher throughput to multicast groups by allowing simultaneous access of multiple channels to the respective groups.
2) The formulation is a mixed integer non-convex problem, for which we first introduce auxiliary variables to decouple without loosing optimality, the original problem into multiple power allocation subproblems and a channel assignment subproblem. The non-convex power allocation subproblems are handled by fractional programming via quadratic transformation followed by alternating optimization. The channel assignment subproblem is solved by integer relaxation.
3) Evaluation of the algorithm is presented on the basis of Matlab simulations to demonstrate the merits, showing a superior and more robust and performance.

The rest of this paper is structured as follows. Sec. II describes the system model. Sec. III introduces the joint channel assignment and resource allocation problem. Sec. IV proposes an efficient algorithm to solve it. Finally, Sec. V provides the simulations and Sec. VI summarizes conclusions.

II. System Model

Consider a multicast D2D communications scenario which underlays over the downlink spectrum[1] of cellular communication as shown in Fig. 1. We assume that the BS communicates with the associated CUs over N_C orthogonal downlink channels. Further, we consider a fully loaded network condition with N_C active downlink CUs. In order to avoid confusion in notation, active CUs (equivalently, downlink channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. The

[1] Without loss of generality, the same formulation and algorithm design developed here, can be also applied to the uplink spectrum.

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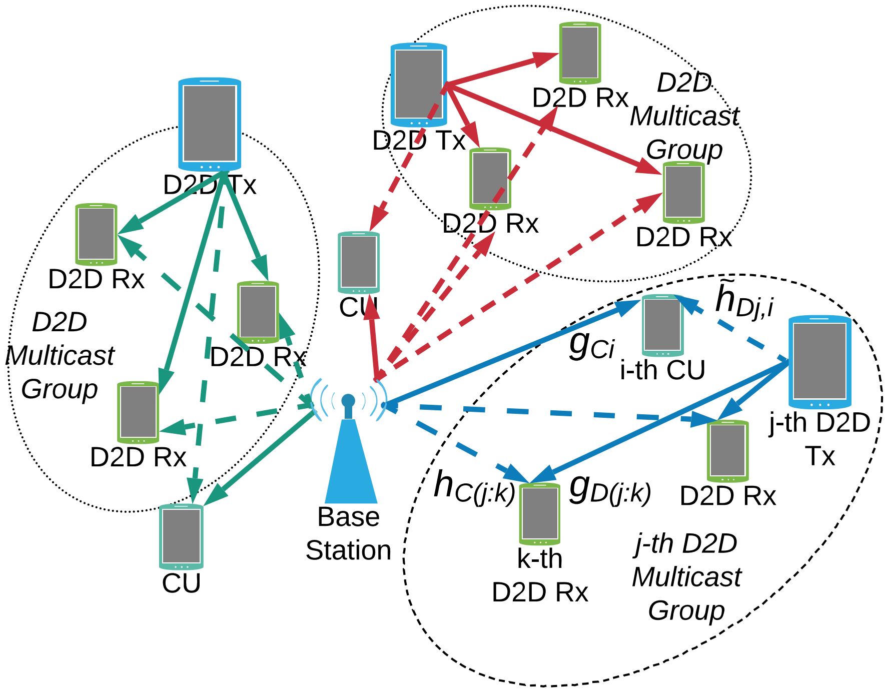

Fig. 1: Illustration of the overall system model.

D2D multicast groups wishing to communicate over the aforementioned N_C channels are indexed by $\mathcal{D} = \{1, ..., N_D\}$. The j-th D2D muticast group ($\forall j \in \mathcal{D}$) is assumed to have one transmitter and M_j receivers; the receivers in the j-th D2D muticast group are indexed by $\mathcal{M}_j = \{1, 2, \cdots, M_j\}$. Further, to provide higher throughput among D2D multicast groups, we allow simultaneous access of multiple channels to D2D multicast groups; however, to restrict interference among the D2D multicast groups, access of more than one multicast group is not allowed over a particular channel.

In this setup, consider the generic scenario where the i-th cellular user (CU) shares the channel resource with j-th D2D multicast group. Then, the expressions for the respective SINR's observed by i-th CU and k-th receiver of j-th D2D multicast group can be stated as:

$$\Gamma_{C_{i,j}} = \frac{g_{C_i} p_{C_i}}{N_0 + \tilde{h}_{D_{j,i}} p_{D_{j,i}}}, \quad \Gamma_{D_{(j:k),i}} = \frac{g_{D_{(j:k)}} p_{D_{j,i}}}{N_0 + h_{C_{(j:k)}} p_{C_i}} \tag{1}$$

where, g_{C_i}, $g_{D(j:k)}$ denote[2] the channel gains, respectively, between BS and i-th CU and transmitter and k-th receiver in the j-th multicast group; $h_{C_{(j:k)}}$, $h_{D_{j,i}}$ denotes the interference channel gain between BS and k-th receiver of the j-th D2D multicast group and transmitter of j-th multicast group and the i-th CU; and p_{C_i}, $p_{D_{j,i}}$ denote respectively the transmit powers of BS for the i-th CU and transmitter of j-th multicast group over i-th channel.The additive noise is assumed to have one sided power spectral density N_0.

In this analysis, channel gains g_{C_i}, $g_{D(j:k)}$ and $h_{C_{(j:k)}}$ are assumed to be perfectly known during the computation of the resource allocation. However, as expected in practice, we consider limited cooperation from CUs in estimating the interference channel gain. Thus, we assume that the statistical characterization of $h_{D_{j,i}}$ (based on channel-gain maps, pilot signal

[2]In principle, $g_{D_{(j:k)}}$ and $h_{C_{(j:k)}}$ should also depend on the operated channel i, however, this subscript is dropped as the proposed scheme carries over immediately to accommodate such dependence.

transmission, etc.) is known during the computation resource allocation. The imperfect CSI nature of $h_{D_{j,i}}$ is denoted by $\tilde{h}_{D_{j,i}}$.

Let $R_{C_{i,j}}^{LB}$ denote the lower bound on the rate of the i-th CU, which must be achieved $(1-\epsilon)$ portion of the time, and can be expressed as:

$$R_{C_{i,j}}^{LB} = W \log_2 \left(1 + \Gamma_{C_{i,j}}^{LB}\right)$$
$$\text{where,} \quad \Gamma_{C_{i,j}}^{LB} : \Pr \left\{ \Gamma_{C_{i,j}}^{LB} \leq \frac{g_{C_i} p_{C_i}}{N_0 + \tilde{h}_{D_{j,i}} p_{D_{j,i}}} \right\} = 1 - \epsilon \tag{2}$$

Here W denotes the allocated bandwidth for the downlink channel. For the D2D multicast group, the maximum achievable rate is determined by the SINR of worst case receiver; thus, the corresponding achievable rate can be stated as:

$$R_{D_{j,i}} = W \log_2 \left(1 + \min_{k \in \mathcal{M}_j} \Gamma_{D_{(j:k),i}}\right) \tag{3}$$

For the case where the i-th CU does not share any resource with D2D multicast groups, the maximum achievable rate for the i-th CU is given by:

$$R_{C_{i,0}} = W \log_2 \left(1 + \Gamma_{C_{i,0}}\right), \quad \text{where} \quad \Gamma_{C_{i,0}} = \frac{g_{C_i} p_{C_{max}}}{N_0} \tag{4}$$

Here $p_{C_{max}}$ is the maximum transmit power of the BS. Denoting $\beta_{i,j}$ as the binary variable which takes value 1 when the i-th CU shares channel with the j-th multicast group and 0 otherwise; the minimum sum rate that can be achieved over the i-th downlink channel (under the assumption of restricted D2D interference, i.e., $\sum_{j \in \mathcal{D}} \beta_{i,j} \leq 1$), can be expressed as:

$$R_i = \left(1 - \sum_{j \in \mathcal{D}} \beta_{i,j}\right) R_{C_{i,0}} + \sum_{j \in \mathcal{D}} \beta_{i,j} (R_{D_{j,i}} + R_{C_{i,j}}^{LB}) \tag{5}$$

The minimum sum rate of the whole multicast D2D network underlayed over the cellular downlink channels is $R = \sum_{i \in \mathcal{C}} R_i$. In the next section, we discuss the problem formulation to maximize the sum rate subjected to several quality of service (QoS) constraints.

III. Problem Formulation

The objective of this work is to maximize the minimum sum rate of all underlay D2D multicast groups and the CUs. In addition, our objective is also to ensure fairness in channel assignment to the D2D multicast groups. Thus, we define the unfairness measure $\delta(\mathbb{B}) = 1/(N_D c^2) \sum_{j=1}^{N_D} (x_j - c)^2$ along similar lines to [8], [9], where $x_j := \sum_{i=1}^{N_C} \beta_{i,j}$ is the number of channels assigned to the j-th D2D multicast group; $c := N_C/N_D$ is the fairest assignment; and $\mathbb{B}$ denotes the discrete channel assignment matrix. Finally, the sum rate maximization problem with fairness in the channel assignment can be expressed as:

$$\underset{\mathcal{P}_C, \mathcal{P}_D, \mathbb{B}}{\text{maximize}} \quad R - \gamma \delta(\mathbb{B}) \tag{6a}$$
$$\text{subject to:} \quad \beta_{i,j} \in \{0, 1\}, \qquad \sum_{j \in \mathcal{D}} \beta_{i,j} \leq 1 \tag{6b}$$
$$p_{C_i} \leq p_{C_{max}}, \qquad p_{D_{j,i}} \leq p_{D_{max}} \tag{6c}$$
$$W \log_2 \left(1 + \Gamma_{C_{i,j}}^{LB}\right) \geq \beta_{i,j} \eta_{C_{min}} \tag{6d}$$
$$W \log_2 \left(1 + \min_{k \in \mathcal{M}_j} \Gamma_{D_{(j:k),i}}\right) \geq \beta_{i,j} \eta_{D_{min}} \tag{6e}$$
$$\forall j \in \mathcal{D}, \; i \in \mathcal{C}$$

where $\mathcal{P}_C$ and $\mathcal{P}_D$ denote the set of continuous power allocation variables for CUs and D2D multicast groups, respectively. The regularization parameter $\gamma \geq 0$ in the objective (6a)

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is selected to balance the trade-off between sum rate and fairness in channel assignment. Constraint (6b) is an integer constraint, restricting interference among D2D multicast groups. Constraint (6c) specifies, respective, transmit power limits $p_{C_{max}}$ and $p_{D_{max}}$ for BS and transmitters of D2D multicast groups[3]. Constraint (6d) and (6e) specifies the respective minimum rate requirements $\eta_{C_{min}}$ and $\eta_{D_{min}}$ under sharing of resources between the CU and the D2D multicast group.

Note that the optimization problem (6a) is a non-convex mixed-integer program, which involves exponential complexity. In addition, due to imperfect CSI, objective (6a) and constraint (6d) involve stochastic terms. In the next section, we discuss the relaxation techniques to derive a tractable solution of (6a) with guaranteed polynomial run-time complexity.

IV. Proposed Convex Relaxation Approach

The first challenge to obtain a tractable solution of (6a) is the joint optimization over integer variables ($\mathbb{B}$) and continuous variables ($\mathcal{P}_C$ and $\mathcal{P}_D$). Thus, in the next subsection, we decouple without loss of optimality of the problem (6a) to separate power allocation and channel assignment sub-problems.

A. Decoupling Resource Allocation Problem

We first re-express the sum rate R in (6a) as:

$$R(\mathbb{B}, \mathcal{P}_C, \mathcal{P}_D) = \sum_{i \in \mathcal{C}} \left[\sum_{j \in \mathcal{D}} \beta_{i,j} v_{i,j}(p_{C_i}, p_{D_{ji}}) + R_{C_{i,0}} \right] \tag{7}$$

where $v_{i,j}(p_{C_i}, p_{D_{ji}}) := R^{LB}_{C_{i,j}} + R_{D_{j,i}} - R_{C_{i,0}}$ represents the rate increment due to the assignment of channel i to the D2D pair j relative to the case where the channel i is only used by the CU. Next, notice that the objective of (6a) with the substitution of (7) can be equivalently expressed by replicating $\{p_{C_i}\}$ with multiple auxiliary variables $\{p_{C_{i,j}}\}$ and removing the constant terms from the objective function. The resulting problem can be stated as:

$$\begin{aligned} &\underset{\mathbb{B}, \mathcal{P}_C, \mathcal{P}_D}{\text{maximize}} \quad \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \left[\beta_{i,j} v_{i,j}(p_{C_{i,j}}, p_{D_{j,i}}) \right] - \gamma \delta(\mathbb{B}) \\ &\text{subject to:} \quad (6b), (6c), (6d), \text{ and } (6e) \end{aligned} \tag{8}$$

To recover the optimal $\{p^*_{C_i}\}$ of (6a) from the optimal $\{p^*_{C_{i,j}}\}$ of (8), one only needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $p^*_{C_i} = p^*_{C_{i,j}}$. If no such a j exists, i.e. $\beta_{i,j} = 0 \; \forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power $p^*_{C_i} = p_{C,\max}$.

In addition, we can also notice that (8) decouples across i and j into $N_C \times N_D$ power allocation sub-problems and a final channel assignment problem. Then, for each i, j, the power allocation sub-problem can be stated as:

$$\begin{aligned} &\underset{p_{C_{i,j}}, p_{D_{j,i}}}{\text{maximize}} \quad R^{LB}_{C_{i,j}} + R_{D_{j,i}} \\ &\text{subject to:} \quad 0 \le p_{C_{i,j}} \le p_{C_{max}}, \qquad 0 \le p_{D_{j,i}} \le p_{D_{max}} \\ &W \log_2 \left(1 + \Gamma^{LB}_{C_{i,j}} \right) \ge \eta_{C_{min}} \\ &W \log_2 \left(1 + \min_{k \in \mathcal{M}_j} \Gamma_{D_{(j:k),i}} \right) \ge \eta_{D_{min}} \end{aligned} \tag{9}$$

[3]In general, constraining the transmit power in each band is more restrictive than restricting the total sum of transmitting power over all bands. Moreover, this allows a more balanced transmit power among different channels.

Denoting $\Gamma_{C_{min}} := 2^{\frac{\eta_{C_{min}}}{W}} - 1$ and $\Gamma_{D_{min}} := 2^{\frac{\eta_{D_{min}}}{W}} - 1$, the optimization problem (9) can be re-expressed in-terms of optimization variables $p_{C_{i,j}}$ and $p_{D_{j,i}}$ as follows:

$$\underset{p_{C_{i,j}}, p_{D_{j,i}}}{\text{maximize}} \quad \log_2\left(1 + \Gamma^{LB}_{C_{i,j}}\right) + \log_2\left(1 + \min_{k \in \mathcal{M}_j} \frac{g_{D_{(j:k)}} p_{D_{j,i}}}{\sigma^2 + h_{C_{(j:k)}} p_{C_{i,j}}}\right) \tag{10a}$$

$$\text{subject to:} \quad \Pr\left\{\Gamma^{LB}_{C_{i,j}} \leq \frac{g_{C_i} p_{C_i}}{N_0 + \tilde{h}_{D_{j,i}} p_{D_{j,i}}}\right\} = 1 - \epsilon \tag{10b}$$

$$0 \leq p_{C_{i,j}} \leq p_{C_{max}} \qquad 0 \leq p_{D_{j,i}} \leq p_{D_{max}} \tag{10c}$$

$$\Pr\left\{\frac{g_{C_i} p_{C_{i,j}}}{\sigma^2 + \tilde{h}_{D_{j,i}} p_{D_{j,i}}} \geq \Gamma_{C_{min}}\right\} \geq 1 - \epsilon \tag{10d}$$

$$\frac{g_{D_{(j:k)}} p_{D_{j,i}}}{\sigma^2 + h_{C_{(j:k)}} p_{C_{i,j}}} \geq \Gamma_{D_{min}} \quad \forall k \in \mathcal{M}_j \tag{10e}$$

Next, under the assumption that the statistical distribution of interference channel gain $h_{D_{j,i}}$ is pre-specified, the probabilistic constraint (10b) can be restated as:

$$\Pr\left\{\tilde{h}_{D_{j,i}} \leq \frac{g_{C_i} p_{C_i} - \Gamma^{LB}_{C_{i,j}} N_0}{p_{D_{j,i}} \Gamma^{LB}_{C_{i,j}}}\right\} \geq 1 - \epsilon$$
$$\implies \Gamma^{LB}_{C_{i,j}} \leq \frac{g_{C_i} p_{C_i}}{N_0 + F^{-1}_{h_{D_{j,i}}}(1 - \epsilon) p_{D_{j,i}}} \tag{11}$$

where $F^{-1}_{h_{D_{j,i}}}(\cdot)$ is the inverse cumulative distribution function of $h_{D_{j,i}}$. Similarly, constraint (10d) can be expressed as,

$$\frac{g_{C_i} p_{C_i}}{N_0 + F^{-1}_{h_{D_{j,i}}}(1 - \epsilon) p_{D_{j,i}}} \geq \Gamma_{C_{min}} \tag{12}$$

It can be noted that the objective (10a) and the modified constraint (11) involve ratio between two convex functions which is not convex in general. Hence, in the next subsection, we use fractional programming [10] to relax the non convexity due to these ratios.

B. Fractional Programming via Quadratic Transformation

Introducing the auxiliary variable $\Gamma^{LB}_{D_{j,i}}$ as the lower bound on achievable SINR over all receivers in the j-th multicast group, the power allocation problem (10a) can be re-stated as:

$$\underset{p_{C_i}, p_{D_{j,i}}}{\text{maximize}} \quad \log_2\left(1 + \Gamma^{LB}_{C_{i,j}}\right) + \log_2\left(1 + \Gamma^{LB}_{D_{j,i}}\right) \tag{13a}$$

$$\text{subject to:} \quad \Gamma^{LB}_{D_{j,i}} \leq \frac{g_{D_{(j:k)}} p_{D_{j,i}}}{N_0 + h_{C_{(j:k)}} p_{C_{i,j}}} \quad \forall k \in \mathcal{M}_j \tag{13b}$$

$$(11), (10c), (12) \text{ and } (10e) \tag{13c}$$

Taking a partial Lagrangian of (13a) by considering only the constraints related to the auxiliary variables $\Gamma^{LB} := \{\Gamma^{LB}_{C_{i,j}}, \Gamma^{LB}_{D_{j,i}}\}$ in (11) and (13b), respectively, we obtain

$$L(\mathbf{p}, \Gamma^{LB}, \lambda) = \log_2\left(1 + \Gamma^{LB}_{C_{i,j}}\right) + \log_2\left(1 + \Gamma^{LB}_{D_{j,i}}\right) - \lambda_C\left(\Gamma^{LB}_{C_{i,j}} - \frac{g_{C_i} p_{C_{i,j}}}{N_0 + F^{-1}_{h_{D_{j,i}}}(1 - \epsilon) p_{D_{j,i}}}\right) - \sum_{k \in \mathcal{M}_j}\left(\lambda_{D_k}\left(\Gamma^{LB}_{D_{j,i}} - \frac{g_{D_{(j:k)}} p_{D_{j,i}}}{N_0 + h_{C_{(j:k)}} p_{C_{i,j}}}\right)\right) \tag{14}$$

At a stationary point, $\frac{\partial L}{\partial \Gamma^{LB}} = 0$; thus, the optimal values of the Lagrange variables can be computed as $\lambda_C = \frac{1}{1+\Gamma^{LB}_{C_{i,j}}}$ and $\sum_{k \in \mathcal{M}_j} \lambda_{D_k} = \frac{1}{1+\Gamma^{LB}_{D_{j,i}}}$. Note that the optimal value of the Lagrange variable λ_C is achieved when the inequality constraints (11) is satisfied with equality. Furthermore, by complementary slackness at optimality, $\lambda_{D_k} = 0$ for all relaxed constraints and $\lambda_{D_k} \geq 0$ for tight constraint in (13b). Here, a tight constraint applies to the receiver with *lowest* SINR; thus, if $(j:l)$ denotes the receiver in the multicast group j which observes the *lowest* SINR, then the optimal value is $\lambda_{D_l} = \frac{1}{1+\Gamma^{LB}_{D_{j,i}}}$. Next, by calculating λ^*_C and $\lambda^*_{D_l}$ and substituting them in problem (13a), we obtain:

$$
\begin{aligned}
\underset{p_{C_i}, p_{D_{j,i}}}{\text{maximize}} \quad & \log_2\left(1+\Gamma^{LB}_{C_{i,j}}\right) + \log_2\left(1+\Gamma^{LB}_{D_{j,i}}\right) \\
& - \Gamma^{LB}_{C_{i,j}} + \frac{(1+\Gamma^{LB}_{C_{i,j}}) g_{C_i} p_{C_{i,j}}}{g_{C_i} p_{C_{i,j}} + N_0 + F^{-1}_{h_{D_{j,i}}}(1-\epsilon) p_{D_{j,i}}} \\
& - \Gamma^{LB}_{D_{j,i}} + \frac{(1+\Gamma^{LB}_{D_{j,i}}) g_{D_{j,l}} p_{D_{j,i}}}{g_{D_{j,l}} p_{D_{j,i}} + N_0 + h_{C_{(j:l)}} p_{C_{i,j}}} \\
\text{subject to:} \quad & (10c), (12) \text{ and } (10e)
\end{aligned} \tag{15}
$$

Next, we transform the fractions in the objective by introducing auxiliary variables y_C and y_D through a quadratic transformation [10], obtaining:

$$
\begin{aligned}
\underset{p_{C_{i,j}}, p_{D_{j,i}}}{\text{maximize}} \quad & \log_2\left(1+\Gamma^{LB}_{C_{i,j}}\right) + \log_2\left(1+\Gamma^{LB}_{D_{j,i}}\right) \\
& - \Gamma^{LB}_{C_{i,j}} + 2y_C \sqrt{(1+\Gamma^{LB}_{C_{i,j}}) g_{C_i} p_{C_{i,j}}} \\
& - y_C^2 \left(g_{C_i} p_{C_{i,j}} + N_0 + F^{-1}_{h_{D_{j,i}}}(1-\epsilon) p_{D_{j,i}} \right) \\
& - \Gamma^{LB}_{D_{j,i}} + 2y_D \sqrt{(1+\Gamma^{LB}_{D_{j,i}}) g_{D_{j,l}} p_{D_{j,i}}} \\
& - y_D^2 \left(g_{D_{j,l}} p_{D_{j,i}} + N_0 + h_{C_{(j:l)}} p_{C_{i,j}} \right) \\
\text{subject to:} \quad & (10c), (12) \text{ and } (10e)
\end{aligned} \tag{16}
$$

The optimal values of the auxiliary variables y_C and y_D can be readily computed as:

$$
\begin{aligned}
y_C^* &= \frac{\sqrt{(1+\Gamma^{LB}_{C_{i,j}}) g_{C_i} p_{C_{i,j}}}}{g_{C_i} p_{C_{i,j}} + N_0 + F^{-1}_{h_{D_{j,i}}}(1-\epsilon) p_{D_{j,i}}} \\
y_D^* &= \frac{\sqrt{(1+\Gamma^{LB}_{D_{j,i}}) g_{D_{j,l}} p_{D_{j,i}}}}{g_{D_{j,l}} p_{D_{j,i}} + N_0 + h_{C_{(j:l)}} p_{C_{i,j}}}
\end{aligned} \tag{17}
$$

Notice that for the given values of slack variables $\Gamma^{LB}_{C_{i,j}}$ and $\Gamma^{LB}_{D_{j,i}}$ and auxiliary variables y_C and y_D, the optimization problem (16) is jointly convex in $p_{C_{i,j}}$ and $p_{D_{j,i}}$. Hence, in the next subsection, we propose to perform alternating optimization in (16) between $p_{C_{i,j}}$ and $p_{D_{j,i}}$.

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C. Alternating Optimization

Optimization problem (16) is solved by alternating maximization with respect to the individual $\Gamma^{LB}_{C_{i,j}}$, $\Gamma^{LB}_{D_{j,i}}$, y_C, y_D. $p_{C_{i,j}}$ and $p_{D_{j,i}}$ variables. At each step, all iterates can be obtained in closed form by taking the partial derivative with respect to each variable and setting it to 0, and projecting the solution onto the feasible set. The overall iteration can be expressed as:

- Compute $\Gamma^{LB}_{C_{i,j}}$ following tight constraint (11). Compute $\Gamma^{LB}_{D_{j,i}}$ following tight constraint for receiver $(j:l)$ with *lowest* SINR in (13b).
- Compute auxiliary variables y_C and y_D from equation (17).

- Updates for $p_{C_{i,j}}$ and $p_{D_{j,i}}$ can be computed as:

$$p_{C_{i,j}} = \text{Proj}_{\mathcal{S}_1}\left(\frac{y_C^2(1+\Gamma_{C_{i,j}}^{LB})g_{C_i}}{(y_C^2 g_{C_i} + y_D^2 h_{C_{(j:l)}})^2}\right)$$
$$p_{D_{j,i}} = \text{Proj}_{\mathcal{S}_2}\left(\frac{y_D^2(1+\Gamma_{D_{j,i}}^{LB})g_{D_{j,l}}}{(y_D^2 g_{D_{j,l}} + y_C^2 F^{-1}_{h_{D_{j,i}}}(1-\epsilon))^2}\right) \tag{18}$$

  where, $\text{Proj}_{\mathcal{A}}(*)$ is a projection of $*$ onto the set $\mathcal{A}$; $S_1 \triangleq \{p_{C_{i,j}} : (p_{C_{i,j}}, p_{D_{ji}})\}$ satisfy (10c), (12), and (10e) for specified last update of $p_{D_{j,i}}$. $S_2 \triangleq \{p_{D_{j,i}} : (p_{C_{i,j}}, p_{D_{j,i}})\}$ satisfy (10c), (12), and (10e) for specified last update of $p_{C_{i,j}}$.

The convergence analysis of above alternating optimization is omitted due to lack of space; however, the analysis can be easily performed by following the extensive discussion in our previous work [11]. Once (9) is solved $\forall i \in \mathcal{C}$ and $\forall j \in \mathcal{D}$, the next step is to perform channel assignment to D2D pairs, as explained in the next section.

D. Channel Assignment via Integer Relaxation

For the channel assignment to D2D pairs, the resulting values $\tilde{v}_{i,j}$ (solution obtained after solving (9) $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$) are substituted into (8) and then we need to maximize the objective of (8) with respect to $\mathbb{B}$. The resulting channel assignment sub-problem can be stated as:

$$\begin{aligned} &\underset{\mathbb{B}}{\text{maximize}} && \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}} \beta_{i,j}\tilde{v}_{i,j} - \gamma\delta(\mathbb{B}), \\ &\text{subject to} && \beta_{i,j} \in \{0,1\} \ \ \forall i,j, \quad \sum_{j\in\mathcal{D}} \beta_{i,j} \le 1 \ \ \forall i. \end{aligned} \tag{19}$$

Due to the integer constraints, solving (19) involves prohibitive computational complexity even for reasonable values of $N_C,\ N_D$. Similar to [9], we relax the integer constraints to $\beta_{i,j} \in [0,1]\ \forall i,j$ to obtain a differentiable Lipschitz smooth objective function with linear constraints, which can be efficiently solved using the Projected Gradient Descent algorithm. The obtained solution is finally discretized back to satisfy the original constraints $\beta_{i,j} \in \{0,1\}\ \forall i,j$. In our approach, this is done by setting the highest positive value in every row of $\mathbb{B}$ to 1 while setting other values in the same row to 0. This relaxation yields good solutions with low computational complexity (as compared to other types of relaxations [9]) and the performance of this relaxation has been extensively discussed in [9].

V. Simulations

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. We assume $\tilde{h}_D$ to be exponentially distributed with the mean value obtained from the mentioned path-loss model. Averages are calculated over 400 independent realizations of the user locations with parameters BW= 15 kHz, $\epsilon = 0.1,\ N_D = 3,\ N_C = 6,\ M_j = 3,$ $N_0 = -70$ dBW. The proposed algorithm is compared with the unicast method in [11] when each D2D group is considered as M_J D2D pairs. Other works that focus on multicast D2D communications have very different network assumptions (e.g. perfect CSI in the case of [2] and network assisted transmission in the case of [1]) and can not be directly compared to our proposed method.

Fig. 2 shows that the proposed method achieves slightly lower rate compared to the unicast method in [11]. However, in the multicast case, the number of transmitted signals is much

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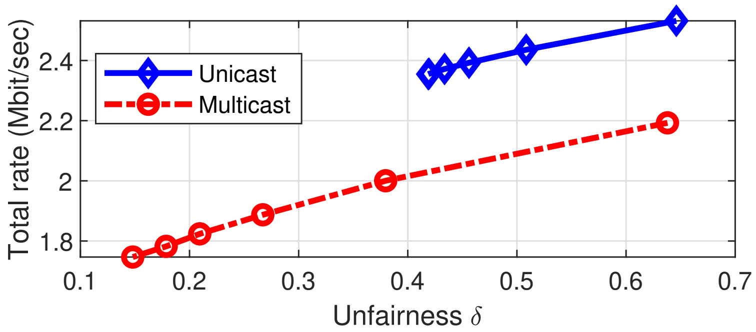

Fig. 2: Total average rate R vs. Unfairness δ (γ from 10 to 100)

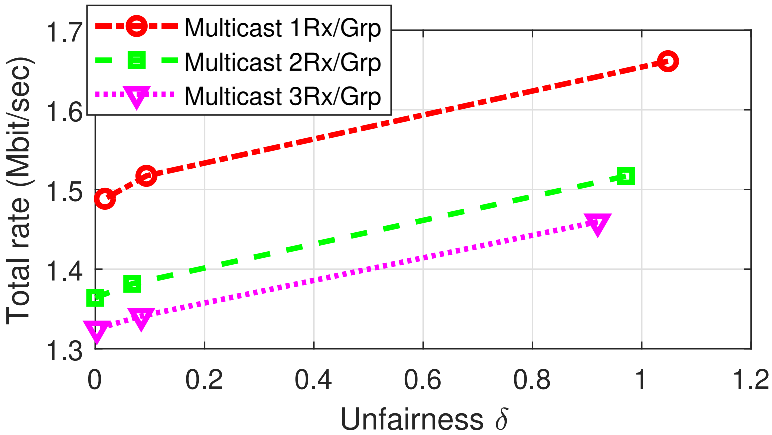

Fig. 3: Total average rate R vs. Unfairness δ (γ from 10 to 100)

smaller than the unicast case. When γ increases, the rate decreases in all methods while the unfairness decreases. This is expected because γ controls the trade-off between the rate and fairness, high γ will force the solution to have better fairness (lower unfairness) on the expense of achieving lower rate.

Fig. 3 shows the performance of the proposed method when changing the number D2D receivers in each multicast group. The total network rate decreases with each additional receiver in the group, since the rate in each group is determined by the receiver with the worst communication conditions. As before, increasing the value of γ decreases the rate while decreasing the unfairness.

Fig. 4 shows the achieved outage probability of the multicast case compared to the unicast case for different values of ϵ and $\gamma = 100$. It can be seen that the proposed multicast algorithm is very conservative and achieves very small outage probabilities compared to the unicast method which achieves outage probability that is very close to the desired outage ϵ.

VI. Conclusion

This paper presented a reliable algorithm for joint channel assignment and power allocation in multicast underlay D2D cellular networks that ensures (i) reliability by probabilistically constraining the SINR for both CUs and D2D to guarantee the desired outage probability, (ii) the fairness among D2D pairs by penalizing unfair assignments.

In general, multicast communications allow sending the same information to several receivers with the same network resources. Our proposed algorithm achieves this goal while ensuring the reliability of cellular communication. Moreover, our algorithm provides an additional freedom by selecting a trade-off parameter to balance between rate and fairness.

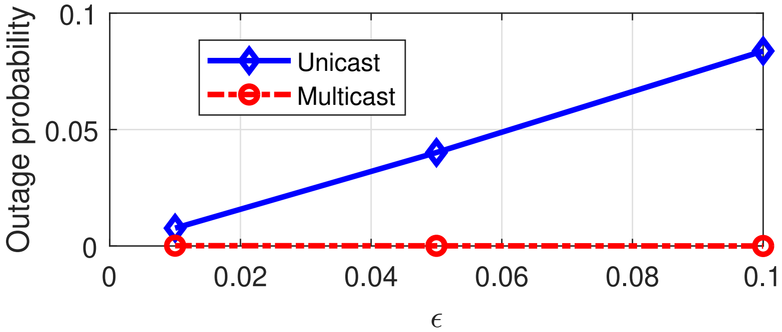

Fig. 4: Outage probability vs. ϵ.

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Appendix H

Paper H

Title:	Distributed Resource Allocation in Underlay Multicast D2D Communications.
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Distributed Resource Allocation in Underlay Multicast D2D Communications

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Abstract

Multicast device-to-device communications operating underlay with cellular networks is a spectral efficient technique for disseminating data to nearby receivers. However, due to the critical challenge of having an intelligent interference coordination between multicast groups along with the cellular network, it is necessary to judiciously perform resource allocation for the combined network. In this work, we present a framework for a joint channel and power allocation strategy to maximize the sum rate of the combined network while guaranteeing minimum rate to individual groups and cellular users. The objective function is augmented by an austerity function that penalizes excessive assignment of low rate channels. The formulated problem is a mixed-integer-non-convex program, which requires exponential complexity to obtain the optimal solution. To tackle this, we exploit fractional programming and integer relaxation to obtain a parametric convex approximation. Based on sequential convex approximation approach, we first propose a centralized algorithm that ensures convergence to a limit point. Next, we propose a distributed algorithm in which via dual decomposition, separable sub-problems are formulated to be solved at the respective groups in cooperation with the base station. We provide convergence guarantees of the proposed solutions and demonstrate their merits by simulations, showing improvement in network throughput.

Index Terms

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D2D-multicast communications, resource allocation, successive-convex-approximation.

I. Introduction

Multicast Device to Device (Multicast device-to-device (MD2D)) communication is a spectral efficient operation of directly disseminating the data to nearby devices without passing the packets through the base station (base station (BS)). Such operations also avoid wasting unicast device-to-device (D2D) transmission opportunities when common data is intended for multiple receivers. Thus, MD2D communication offers higher prospects of spectral as well as energy saving [1]. Some important applications of MD2D communication include: (i) dissemination of social content/marketing/advertisement data in the commercial networks; (ii) device discovery, clustering, co-ordination in self organizing networks; (iii) dissemination of critical information such as police, fire, ambulance, etc. in the public safety networks [2]–[4]. In these scenarios, allowing simultaneous transmission of MD2D groups and existing cellular network in the same spectrum (also termed ad underlay configuration) is a promising approach to improve spectrum utilization [5]. However, unlike the unicast D2D communication, MD2D communication has its own challenges in terms of heterogeneous channel conditions for individual receivers in the multicast group, thus, the achievable performance of the multicast group is generally limited by the receiver with the worst channel conditions. In addition, similar to underlay unicast D2D communications, simultaneous transmissions in the same spectrum bands increases interference at the respective receivers and may adversely reduce the overall network performance. Thus, it is necessary to devise a judicious and reliable resource allocation algorithm which can maximize the overall network performance. Further, a resource allocation solution which operates in distributed manner across the cellular network and MD2D network will make solution more scalable, will substantially reduce the overhead communication and will also reduce the computation load at the BS.

Resource allocation problems for underlay unicast D2D communications have been extensively investigated in [6]–[8]. Addressing requirements of efficient computation and minimum overhead communication among cellular users (CUs) and D2D transmitters, distributed resource allocation for underlay D2D communication has also been investigated in [9]–[12].

These research efforts address the challenges limited to unicast communication. However, besides interference management, MD2D communication poses its own challenges such as the selection of the head cluster, or the strategy for forming the groups [13], [14]. In this work, we address the challenge of joint power allocation and channel assignment in underlay MD2D communication with the objective to maximize the sum rate of overall network while meeting the power constraints and desired quality of service (QoS) requirements.

A. Related Work

Power allocation and channel assignment for MD2D communication is investigated by several research works in the existing literature. In order to obtain a statistical model of MD2D communication in the overlay framework, concepts of stochastic geometry are exploited in [2] to derive analytical expressions of several performance metrics. In the underlay framework, concepts of stochastic geometry are exploited once again in [15] to define exclusion zones around CUs where receivers of MD2D groups are prohibited. An alternate strategy to avoid interference from MD2D operation is presented in [16] for single frequency network systems. Here MD2D groups avoids sharing frequencies with CUs in the same single frequency network and based on a specified signal to interference threshold form a safe frequency reuse region. It can be noted that making interference zero between cellular network and MD2D network is not the best strategy for utilizing the available resources. An intelligent power allocation can limit the interference while maximizing the overall network performance.

Motivated to control the interference between the cellular network and the MD2D network by judiciously adjusting the corresponding transmit powers, an interference coordination scheme is proposed in [17]. Here, the power allocation scheme provides an upper bound on the MD2D transmit power, which is used later to devise a resource block assignment strategy. A sum throughput optimization problem with signal to interference plus noise ratio (SINR) constraints is formulated in [18], where MD2D groups are allowed to reuse at-most one channel and respective CUs are also allowed to share the channel with at-most one MD2D group. Similarly, a frequency reuse based resource allocation scheme is proposed in [19] where each MD2D group is allowed to reuse only one frequency at a time. Alternatively, particle swarm optimization based power control strategy is studied in [20], where MD2D groups are allowed to reuse frequencies of multiple CUs. Notice that in order to obtain a tractable and less computationally complex resource allocation solution, the above research work imposes restrictions on the channel access to MD2D groups. However, once again, some research works have shown higher network performance for scenarios where interference between MD2D groups is allowed.

An alternate approach of improving the network efficiency is formulation of optimization problem to maximize the overall energy efficiency of MD2D groups [21]–[23]. For improving network efficiency by maximizing sum throughput, [24] formulates a resource allocation problem where feasible solution is first obtained by performing channel assignment subject and later transmit power is optimized to maximize the throughput. Similarly, a resource allocation problem is formulated in [25] to maximize the sum throughput of MD2D groups while restricting interference to CUs below a certain specified threshold. A similar throughput maximization problem is also formulated in [5] with constraints on minimum SINR requirements. In the massive MIMO setup, precoder design at BS and power allocation to MD2D is investigated in [26] to maximize the achievable data rates of MD2D groups while maintaining the QoS for CUs. The trade-off between spectral efficiency and energy efficiency is studied in [27], where multiple MD2D groups can access multiple channels. Moreover, a channel assignment scheme to maximize the sum effective throughput is proposed in [28] under partial information of the device location. It can be noted that most of the above works propose resource allocation solutions by separately optimizing over channel assignment and power allocation. Further, meeting specific QoS constraints such as desired rate by MD2D groups is under addressed in above formulations. This is of vital importance as different MD2D groups might operate different applications, such as dissemination of text, video, real time gaming etc. which have separate minimum rate requirements.

Finally, few research works have also addressed distributed resource allocation as most of the above discussed works assume centralized schemes, where all the processing takes place at the BS, making it less convenient for scalability. In [29], a two stage semi-distributed resource allocation solution to maximize the overall system energy efficiency is proposed. Similarly, a self organizing resource allocation scheme is proposed in [30] using stochastic optimization for MD2D communication.

B. Contributions

In this work, we investigate both centralized and distributed resource allocation solutions for sum rate maximization in underlay MD2D communication. The main contributions of this work can be summarized as follows:

1) We formulate a joint power allocation and channel assignment problem to maximize the sum rate of all MD2D groups and CUs with a constraint on the minimum rate requirement for both MD2D groups and CUs. The objective function is also augmented to include a penalty on having a large number of low rate channels accessed by MD2D groups. This can be seen as an austerity measure in channel assignment to MD2D groups, which essentially limits the interference and power consumption in RF chains [31]. The proposed formulation ensures improved throughput to MD2D groups by allowing simultaneous access of multiple channels to the respective groups. Further, the formulation also allows for controlled interference between MD2D groups by sharing common channels.
2) Our formulation results in a mixed integer non-convex optimization program, for which we first propose a centralized resource allocation algorithm. Here, in order to obtain a tractable solution, we first address the non-convexity due to product of integer and continuous optimization variables by formulating equivalent constraints with affine combination of integer and power variables. Later, we perform integer relaxations to approximate integer variables to continuous variables. In order to handle non-convexity in our objective function and minimum rate requirement constraints, we exploit fractional programming via a quadratic transformation to obtain a parametric convex approximation. Finally, by sequential parametric convex approximation, we obtain a tractable and less computationally complex joint power allocation and channel assignment solution.
3) A distributed resource allocation algorithm is proposed by exploiting dual decomposition

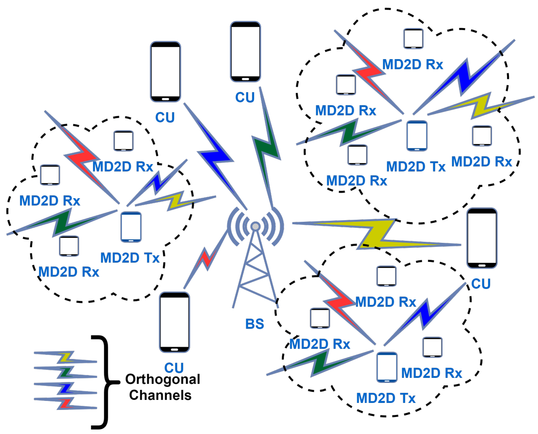

Fig. 1: Illustration of MD2D communication in underlay with cellular network.

of the centralized solution. The decoupled formulation can be solved in a distributed manner across the respective MD2D groups in cooperation with the BS. Moreover, we also discuss the communication overhead requirements for coordination in the proposed distributed solution.

4) We provide convergence guarantees for all of our algorithms and our experiments in the simulations indicate fast convergence.
5) Evaluation of the algorithms are presented based on Matlab simulations to demonstrate the merits, showing improved sum-rate of combined network.

The reset of this paper is structured as follows: Section II presents system model followed by problem formulation in Section III. The relaxations to formulate the parametric convex approximation and its sequential solution in form of centralized Algorithm is presented Section IV. Section V present distributed Algorithm and Section VI presents the simulation analysis. Finally, conclusion is presented in Section VII.

II. System Model

Consider a MD2D communications scenario which underlays over the downlink spectrum of a cellular communication network, as shown in Fig. 1. We assume that the BS communicates with the associated CUs over N_C orthogonal downlink channels[1] and under fully loaded

[1]Downlink channels can also be considered resource blocks in time, frequency etc.

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network condition has N_C active downlink CUs. In order to avoid confusion in notation, active CUs (equivalently, downlink channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. The MD2D groups wishing to communicate over the aforementioned N_C channels are indexed by $\mathcal{D} = \{1, ..., N_D\}$. The j-th MD2D group ($\forall j \in \mathcal{D}$) is assumed to have one transmitter and M_j receivers; the receivers in the j-th MD2D group are indexed by $\mathcal{M}_j = \{1, 2, \cdots, M_j\}$. Further, to provide higher throughput among the MD2D groups, we allow simultaneous access of multiple channels to MD2D groups; in addition, more than one multicast group are also allowed to access a particular channel.

In this setup, let $b_{i,j}$ denote a binary variable taking value 1 when the i-th cellular user (CU) shares channel with the j-th multicast group and 0 otherwise; then, the expressions for the respective SINR's observed over the i-th channel by the operating CU and the k-th receiver of j-th multicast group can be stated as,

$$\Gamma_{C_i} = \frac{g_{C_i} P_{C_i}}{N_0 + I_{C_i}}, \qquad \Gamma_{D_{(j:k),i}} = \frac{b_{i,j} g_{D_{(j:k),i}} P_{D_{j,i}}}{N_0 + I_{D_{(j:k),i}}} \tag{1}$$

where, over the i-th channel: g_{C_i}, $g_{D(j:k),i}$ denote, respectively the direct channel gains between the BS and i-th CU and between the transmitter and k-th receiver in the j-th multicast group; I_{C_i} and $I_{D_{(j:k),i}}$ denote, respectively the total interference observed at the i-th CU and at the k-th receiver in the j-th multicast group; and P_{C_i}, $P_{D_{j,i}}$ denote the respective transmit powers of the BS and of the j-th multicast group transmitter. The additive noise is assumed to have one sided power spectral density N_0.

The total observed interference I_{C_i} and $I_{D_{(j:k),i}}$ can be respectively expressed as:

$$I_{C_i} = \sum_{j \in \mathcal{D}} b_{i,j} h_{D_{j,i}} P_{D_{j,i}}, \qquad I_{D_{(j:k),i}} = \sum_{j' \neq j \in \mathcal{D}} b_{i,j'} h_{D_{(j',(j:k)),i}} P_{D_{j',i}} + h_{C_{(j:k),i}} P_{C_i} \tag{2}$$

where, once again over the i-th channel: $h_{C_{(j:k),i}}$, $h_{D_{j,i}}$ denote, respectively, the interference channel gain from the BS to the k-th receiver of the j-th MD2D group and from the transmitter of the j-th multicast group to the operating CU; $h_{D_{(j',(j:k)),i}}$ denotes the interference channel gain from the transmitter of another j'-th multicast group $(j' \neq j)$ to the k-th receiver of the j-th multicast group. Next, under the assumption of capacity achieving codes, the achievable capacity over the i-th channel for the CU and j-th MD2D grup can be stated as:

$$R_{C_i} = W \log_2 (1 + \Gamma_{C_i}) \qquad R_{D_{j,i}} = W \log_2 \left(1 + \min_{k \in \mathcal{M}_j} \Gamma_{D_{(j:k),i}}\right) \tag{3}$$

where, W denotes the allocated bandwidth for the i-th downlink channel. Here for the MD2D group, the maximum achievable rate is determined by the SINR of the worst case receiver. Finally, the sum rate that can be achieved over the whole network, i.e., over all the N_C channels, can be stated as:

$$R = \sum_{i \in \mathcal{C}} \left(R_{C_i} + \sum_{j \in \mathcal{D}} R_{D_{j,i}} \right) \tag{4}$$

In the next section, we discuss the problem formulation to maximize the sum rate subject to the QoS and maximum transmit power constraints.

III. Problem Formulation

The objective of this work is to maximize the sum rate of all underlay MD2D groups and the CUs. In addition, the optimization formulation is constrained to ensure minimum QoS to all the multicast groups and the CUs. Here, we defined desired QoS in terms of minimum rate requirement of the users. This is of significance as individual multicast groups might run different applications such as, texting, video streaming, etc., which have separate minimum rate requirements. Further, objective function is augmented by an austerity function which penalizes the scenarios where the multicast group has to communicate over a large number of low rate channels to achieve the desired minimum sum rate over the assigned channels. Finally, the optimization problem can be expressed as:

$$\underset{\boldsymbol{P}_C, \boldsymbol{P}_D, \boldsymbol{B}}{\text{maximize}} \quad R - \gamma \sum_{j \in \mathcal{D}} |\boldsymbol{B}_{[j]}|_1 \tag{5a}$$

$$\text{subject to:} \quad b_{i,j} \in \{0,1\} \qquad \forall i \in \mathcal{C}, \quad \forall j \in \mathcal{D} \tag{5b}$$

$$\sum_{i \in \mathcal{C}} P_{C_i} \leq P_{C_{\max}} \tag{5c}$$

$$\sum_{i \in \mathcal{C}} b_{i,j} P_{D_{j,i}} \leq P_{D_{\max,j}} \qquad \forall j \in \mathcal{D} \tag{5d}$$

$$W \log_2 (1 + \Gamma_{C_i}) \geq R_{C_{\min,i}} \qquad \forall i \in \mathcal{C} \tag{5e}$$

$$\sum_{i \in \mathcal{C}} W \log_2 \left(1 + \min_{k \in \mathcal{M}_j} \Gamma_{D_{(j:k),i}} \right) \geq R_{D_{\min,j}} \qquad \forall j \in \mathcal{D} \tag{5f}$$

where, $\boldsymbol{P}_C$ and $\boldsymbol{P}_D$ denote the BS and MD2D group power optimization variables respectively stacked in form of a vector and a matrix, i.e., $\boldsymbol{P}_C[i] = P_{C_i}$ and $\boldsymbol{P}_D[j,i] = P_{D_{j,i}}$. Similarly, $\boldsymbol{B}$ denotes the channel assignment matrix with elements $\boldsymbol{B}[i,j] = b_{i,j}$; $\boldsymbol{B}_{[j]}$ denotes channels assignment column vector for the j-th multicast group. Subsequently, augmentation of the

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objective function by subtracting ℓ_1 norm of the column vector $\boldsymbol{B}_{[j]}$, minimizes the number of low rate channel assignment to the j-th muticast group. The regularization parameter $\gamma \geq 0$ in the objective function (5a) is selected to balance the trade-off between the sum-rate and the austerity in the channel assignment.

Constraint (5b) is an integer constraint for the channel assignment taking 0 or 1 value. Constraints (5c) and (5d) specify, respectively, the transmit power limits $P_{C_{\max}}$ and $P_{D_{\max,j}}$ for the BS and the transmitter of the j-th MD2D group. Constraint (5e) and (5f) specifies the respective minimum rate requirements $R_{C_{\min,i}}$ and $R_{D_{\min,j}}$ under sharing of the resources between the CU and the MD2D groups. Notice that the optimization problem (5) is a non-convex (nonconvex objective function (5a) and nonconvex constraint (5f)), non-smooth mixed-integer program, which involves exponential complexity to obtain the optimal solution. In the next section, we propose relaxation techniques to derive a tractable solution for the problem (5) with guarantee of convergence.

IV. Proposed Convex Relaxation Approach

Our optimization problem (5) has several non-convexities. First, notice the terms involving product of integer and continuous optimization variables, i.e., $b_{i,j}P_{D_{j,i}}$ in objective (5a) (through the terms in (2)), and constraints (5d)-(5f). One way to address the non-linearity due to these product terms is by transforming to a linear combination of $b_{i,j}$ and $P_{D_{j,i}}$ terms. This can be perceived as initial step towards reducing the computation complexity in obtaining a tractable solution. Here we replace the product term $b_{i,j}P_{D_{j,i}}$ by $P_{D_{j,i}}$ with additional linear constraint $P_{D_{j,i}} \leq b_{i,j}P_{D_{max,j}}$. Notice that with this transformation, when $b_{i,j} = 0$, $P_{D_{j,i}} \leq 0$ implying $P_{D_{j,i}} = 0$. Thus, with the above transformation and by substituting the expressions for rate and corresponding SINR terms, the equivalent optimization problem (5) can be restated

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as,

$$\underset{\boldsymbol{P}_C,\boldsymbol{P}_D,\boldsymbol{B},\boldsymbol{I}_C,\boldsymbol{I}_D}{\text{maximize}} \quad W\sum_{i\in\mathcal{C}}\left(\log_2\left(1+\frac{g_{C_i}P_{C_i}}{N_0+I_{C_i}}\right)+\sum_{j\in\mathcal{D}}\log_2\left(1+\min_{k\in\mathcal{M}_j}\frac{g_{D_{(j:k),i}}P_{D_{j,i}}}{N_0+I_{D_{(j:k),i}}}\right)\right)-\gamma\sum_{j\in\mathcal{D}}|\boldsymbol{B}_{[j]}|_1 \quad (6a)$$

$$\text{subject to:} \quad b_{i,j}\in\{0,1\} \qquad \forall i\in\mathcal{C},\quad \forall j\in\mathcal{D} \quad (6b)$$

$$\sum_{i\in\mathcal{C}}P_{C_i}\le P_{C_{\max}} \quad (6c)$$

$$\sum_{i\in\mathcal{C}}P_{D_{j,i}}\le P_{D_{\max,j}} \qquad \forall j\in\mathcal{D} \quad (6d)$$

$$P_{D_{j,i}}\le b_{i,j}P_{D_{\max,j}} \qquad \forall i\in\mathcal{C}\ \ \forall j\in\mathcal{D} \quad (6e)$$

$$W\log_2\left(1+\frac{g_{C_i}P_{C_i}}{N_0+I_{C_i}}\right)\ge R_{C_{\min,i}} \qquad \forall i\in\mathcal{C} \quad (6f)$$

$$\sum_{i\in\mathcal{C}}W\log_2\left(1+\min_{k\in\mathcal{M}_j}\frac{g_{D_{(j:k),i}}P_{D_{j,i}}}{N_0+I_{D_{(j:k),i}}}\right)\ge R_{D_{\min,j}} \qquad \forall j\in\mathcal{D} \quad (6g)$$

$$I_{C_i}\ge\sum_{j\in\mathcal{D}}h_{D_{j,i}}P_{D_{j,i}} \qquad \forall i\in\mathcal{C} \quad (6h)$$

$$I_{D_{(j:k),i}}\ge\sum_{j'\ne j\in\mathcal{D}}h_{D_{(j',(j:k)),i}}P_{D_{j',i}}+h_{C_{(j:k),i}}P_{C_i} \qquad \forall i\in\mathcal{C},\quad \forall j\in\mathcal{D},\quad \forall k\in\mathcal{M}_j \quad (6i)$$

where, $\boldsymbol{I}_C$ and $\boldsymbol{I}_D$ denote, respectively the interference slack variables related to BS and MD2D groups stacked in form of a matrix and a tensors. Note that the interference terms I_{C_i} and $I_{D_{(j:k),i}}$ are relaxed with new inequality constraints (6h) and (6i). Further, to address the combinatorial complexity due to integer programming, we relax the integer constraint (6b) to continuous box constant as:

$$b_{i,j}\in[0,1] \qquad \forall i\in\mathcal{C}\ j\in\mathcal{D} \quad (7)$$

so that our objective function and feasibility set is defined over continuous optimization variables.

Remark. *The inclusion of the austerity function in the objective in the form if* ℓ_1 *minimization ensures sparsity in channel assignment to MD2D groups. Thus, with selection of suitable regularization parameter* γ*, only a few channel assignment variables in the optimized solution are expected to be non-zero, which can be easily discretized to* 1*. This discretization maintains feasibility in constraint* (6e)*; thus, abides to feasibility in original problem* (4).

Next note that problem (6) comprises of non-smooth non-convex objective function (6a) and non-convex constraints (6g). In the following subsections, we formulate a lower bound convex

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approximation of the objective and an upper bound convex approximation of the non-convex constraints. The motivation for these parmetrized approximations is to obtain a sub-optimal solution of problem (6) by solving a sequence of parametric convex approximation problems.

A. Parametric Approximation of Non-convex Constraints

In this subsection, we focus on the non-convex constraint (6g) which makes feasibility set non-convex. Here, we first introduce a bounding slack variable $x_{D_{j,i}}$ such that constraint (6g) is represented as:

$$\sum_{i\in\mathcal{C}} W\log_2\Big(1+x_{D_{j,i}}\Big) \geq R_{D_{min,j}} \qquad \forall j\in\mathcal{D} \tag{8a}$$

$$x_{D_{j,i}} - \frac{g_{D_{(j:k),i}} P_{D_{j,i}}}{N_0 + I_{D_{(j:k),i}}} \leq 0 \qquad \forall k\in\mathcal{M}_j \tag{8b}$$

Next, we relax constraint (8b) by a parametric approximation of the ratio to a concave expression. Notice that the ratio involves affine functions, and a quadratic transformation (presented in [32]) can be applied so that the parametric approximated convex inequality can be stated as:

$$x_{D_{j,i}} - 2u_{D_{(j:k),i}}\sqrt{g_{D_{(j:k),i}} P_{D_{j,i}}} + u^2_{D_{(j:k),i}}\Big(N_0 + I_{D_{(j:k),i}}\Big) \leq 0 \qquad \forall k\in\mathcal{M}_j \tag{9}$$

where, $u_{D_{(j:k),i}} \in \mathbb{R}_+$ is specified parameter. For sequential parametric optimization, the specified parameter $u_{D_{(j:k),i}}$ is updated on the basis of the solution obtained from the previous iteration.

Lemma 1. *The parametric convex approximation expressed in* (9) *is upper bound to non-convex constraint* (8b) $\forall\ u_{D_{(j:k),i}} \in \mathbb{R}_+$*; thus, substituting* (8b) *with* (9) *ensures feasibility of original optimization* (6a) *in every iteration.*

Proof: See Appendix A.

The parameters $\{u_{D_{(j:k),i}}\}_{\forall k\in\mathcal{M}_j,\forall j\in\mathcal{D},\forall i\in\mathcal{C}}$ are update in a manner to ensure convergence of sequential convex approximations to a sub-optimal solution. The iterative process is elaborated further in subsection IV-C (Centralized Solution).

B. Parametric Approximation of the Non-convex Objective Function

Next, we focus on the non-convexity in the objective function, which involves a sum of logarithms of quasi convex functions. Here, we first introduce auxiliary variables $\Gamma^{LB}_{C_i}$ and

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$\Gamma^{LB}_{D_{j,i}}$ as lower bounds on the achievable SINRs that can observed at the i-th CU and over all the receivers of the j-th multicast group. The introduction of these auxiliary variables facilitates moving he ratios (SINR terms) from logarithms in the objective function to inequality constraints. Thus, the objective of optimization problem (6) with the additional new constraints, can be stated as:

$$\underset{\boldsymbol{P}_C,\boldsymbol{P}_D,\boldsymbol{B},\boldsymbol{I}_C,\boldsymbol{I}_D,\Gamma^{LB}}{\text{maximize}} W\sum_{i\in\mathcal{C}}\left(\log_2\left(1+\Gamma^{LB}_{C_i}\right)+\sum_{j\in\mathcal{D}}\log_2\left(1+\Gamma^{LB}_{D_{j,i}}\right)\right)-\gamma\sum_{j\in\mathcal{D}}|\boldsymbol{B}_{[j]}|_1 \tag{10a}$$

$$\text{subject to: } \Gamma^{LB}_{C_i}\leq\frac{g_{C_i}P_{C_i}}{N_0+I_{C_i}} \qquad \forall i\in\mathcal{C} \tag{10b}$$

$$\Gamma^{LB}_{D_{j,i}}\leq\frac{g_{D_{(j:k),i}}P_{D_{j,i}}}{N_0+I_{D_{(j:k),i}}} \qquad \forall k\in\mathcal{M}_j,\forall i\in\mathcal{C},\forall j\in\mathcal{D} \tag{10c}$$

Taking a partial Lagrangian of (10a) by considering only the constraints related to auxiliary variables $\Gamma^{LB}:=\{\Gamma^{LB}_{C_i},\Gamma^{LB}_{D_{j,i}}\}_{\forall i\in\mathcal{C},\forall j\in\mathcal{D}}$, we obtain:

$$\begin{aligned}L(\boldsymbol{P}_C,\boldsymbol{P}_D,\boldsymbol{I}_C,\boldsymbol{I}_D,\Gamma^{LB},\lambda)=&\,W\sum_{i\in\mathcal{C}}\left(\log_2\left(1+\Gamma^{LB}_{C_i}\right)+\sum_{j\in\mathcal{D}}\log_2\left(1+\Gamma^{LB}_{D_{j,i}}\right)\right)-\gamma\sum_{j\in\mathcal{D}}|\boldsymbol{B}[:,j]|_1\\&-\sum_{i=1}^{N_C}\lambda_{C_i}\left(\Gamma^{LB}_{C_i}-\frac{g_{C_i}P_{C_i}}{N_0+I_{C_i}}\right)-\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\sum_{k\in\mathcal{M}_j}\lambda_{D_{(j:k),i}}\left(\frac{g_{D_{(j:k),i}}P_{D_{j,i}}}{N_0+I_{D_{(j:k),i}}}\right)\end{aligned} \tag{11}$$

where, $\lambda:=\{\lambda_{C_i},\lambda_{D_{(j:k),i}}\}_{\forall i\in\mathcal{C},j\in\mathcal{D},\forall k\in\mathcal{M}_j}$. At a stationary point, $\frac{\partial L}{\partial\Gamma^{LB}}=0$; thus, the optimal values of the Lagrange variables can be computed as $\lambda_{C_i}=\frac{1}{1+\Gamma^{LB}_{C_i}}$ and $\sum_{k\in\mathcal{M}_j}\lambda_{D_{(j:k),i}}=\frac{1}{1+\Gamma^{LB}_{D_{j,i}}}$. Note that the optimal values of the Lagrangian variable λ_{C_i} is archived when the inequality (10b) is achieved with equality. Furthermore, by complementary slackness at optimality, $\lambda_{D_{(j:k),i}}=0$ for all the relaxed constraints and $\lambda_{D_{(j:k),i}}\geq 0$ for the tight constraint (10c). Here, the tight constraint applies to the receiver with lowest SINR; thus, if $\{(j:l),i\}$ denotes the receiver in the muticast group j which observes lowest SINR, then the optimal value is $\lambda_{D_{(j:l),i}}=\frac{1}{1+\Gamma^{LB}_{D_{(j,i)}}}$. Next, by computing $\lambda^*_{C_i}$ and $\lambda^*_{D_{(j:l),i}}$ and substituting them in problem (10a), objective function (6a) can be restated as:

$$\begin{aligned}\mathbb{F}_1\triangleq\;& W\sum_{i\in\mathcal{C}}\left(\log_2\left(1+\Gamma^{LB}_{C_i}\right)+\sum_{j\in\mathcal{D}}\log_2\left(1+\Gamma^{LB}_{D_{j,i}}\right)\right)-\gamma\sum_{j\in\mathcal{D}}|\boldsymbol{B}[:,j]|_1\\&+\sum_{i\in\mathcal{C}}\left(-\Gamma^{LB}_{C_i}+\frac{(1+\Gamma^{LB}_{C_i})g_{C_i}P_{C_i}}{g_{C_i}P_{C_i}+N_0+I_{C_i}}\right)+\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\left(-\Gamma_{D^{LB}_{j,i}}+\frac{(1+\Gamma_{D^{LB}_{j,i}})g_{D_{(j:l),i}}P_{D_{j,i}}}{g_{D_{(j:l),i}}P_{D_{j,i}}+N_0+I_{D_{(j:l),i}}}\right)\end{aligned} \tag{12a}$$

Notice that expression $\mathbb{F}_1$ defined in (12a) is marginally concave in Γ^{LB}; substituting optimal values of Γ^{LB}, i.e.:

$$\Gamma^{LB}_{C_i}=\frac{g_{C_i}P_{C_i}}{N_0+I_{C_i}} \qquad \Gamma^{LB}_{D_{j,i}}=\frac{g_{D_{(j:l),i}}P_{D_{j,i}}}{N_0+I_{D_{(j:l),i}}} \tag{13a}$$

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in (12a), expression $\mathbb{F}_1$ is equivalent to objective function (6a) but even-though it is still non-convex as there are ratio of affine functions. Thus, we can apply quadratic transformation (as presented in [32]) to obtain a parametric convex approximation of (12a), i.e.:

$$\begin{aligned}\mathbb{F}_2 \triangleq\ & W\sum_{i\in\mathcal{C}}\left(\log_2\left(1+\Gamma_{C_i}^{LB}\right)+\sum_{j\in\mathcal{D}}\log_2\left(1+\Gamma_{D_{j,i}}^{LB}\right)\right)-\gamma\sum_{j\in\mathcal{D}}|\boldsymbol{B}_{[j]}|_1\\ &+\sum_{i=1}^{N_C}\left(-\Gamma_{C_i}^{LB}+2y_{C_i}\sqrt{(1+\Gamma_{C_i}^{LB})g_{C_i}P_{C_i}}-y_{C_i}^2\left(g_{C_i}P_{C_i}+N_0+I_{C_i}\right)\right)\\ &+\sum_{i=1}^{N_C}\sum_{j=1}^{N_D}\left(-\Gamma_{D_{j,i}^{LB}}+2y_{D_{j,i}}\sqrt{(1+\Gamma_{D_{j,i}^{LB}})g_{D_{(j:l),i}}P_{D_{j,i}}}-y_{D_{j,i}}^2\left(g_{D_{(j:l),i}}P_{D_{j,i}}+N_0+I_{D_{(j:l),i}}\right)\right)\end{aligned} \tag{14}$$

where, $y_{C_i}\in\mathbb{R}_+$ and $y_{D_{j,i}}\in\mathbb{R}_+$ are the specified parameters.

Lemma 2. *The parametric convex approximation expressed in* (14) *is a lower bound for the non-convex objective function* (6a) $\forall\ y_{C_i}\ , y_{D_{j,i}}\ \in\ \mathbb{R}_+$*; thus, maximizing* $\mathbb{F}_2$*, maximizes the original objective function* (6a).

Proof: See Appendix A.

Similarly, the specified parameters y_{C_i} and $y_{D_{j,i}}$, are updated in a manner to ensure convergence of the sequential convex approximations to a sub-optimal solution. In the next subsection, we further elaborated the iterative process and discuss the centralized solution to the optimization formulation (6).

C. Centralized Solution

The parametric convex approximation of the original optimization problem (6) on substituting the objective with surrogate (14) and the constraint (8b) with (9), is jointly convex in P_{C_i} $P_{D_{j,i}}$ and $b_{j,i}$, and can be sated as:

$$\begin{aligned}&\underset{\boldsymbol{P}_C,\boldsymbol{P}_D,\boldsymbol{B},\boldsymbol{I},\boldsymbol{X}}{\text{maximize}}\ (14)\\ &\text{subject to:}\quad (7),(6c),(6d),(6e),(6f),(8a),(9),(6h),\text{and }(6i)\end{aligned} \tag{15}$$

where $\boldsymbol{X}\triangleq[x_{D_{j,i}}]$. Due to the ℓ_1 norm term in the objective function, (15) is a non-smooth convex optimization problem which can be solved by proximal methods. The parametric convex approximation (15) is solved sequentially to obtain updated values of the parameters at every iteration. Further, to update the parameters in every iteration, we follow the iNner

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cOnVex Approximation (NOVA) algorithm proposed in [33]. Let $\{\hat{\boldsymbol{P}}_C^{(r)}, \hat{\boldsymbol{P}}_D^{(r)}, \hat{\boldsymbol{I}}_C^{(r)}, \hat{\boldsymbol{I}}_D^{(r)}\}$ denote the point of approximation at iteration (r), then, we define the corresponding updates for the parameters as following:

$$u_{D_{(j:k),i}} = \frac{\sqrt{g_{D_{(j:k),i}} \hat{P}^{(r)}_{D_{j,i}}}}{N_0 + \hat{I}^{(r)}_{D_{(j:k),i}}} \tag{16}$$

$$y_{C_i} = \frac{\sqrt{(1+\Gamma^{LB}_{C_i}) g_{C_i} \hat{P}^{(r)}_{C_i}}}{g_{C_i}\hat{P}^{(r)}_{C_i} + N_0 + \hat{I}^{(r)}_{C_i}} \tag{17}$$

$$y_{D_{j,i}} = \frac{\sqrt{(1+\Gamma_{D^{LB}_{j,i}}) g_{D_{(j:l),i}} \hat{P}^{(r)}_{D_{j,i}}}}{g_{D_{(j:l),i}}\hat{P}^{(r)}_{D_{j,i}} + N_0 + \hat{I}^{(r)}_{D_{(j:l),i}}}. \tag{18}$$

The overall centralized algorithm can be summarized in Algorithm 1. Note here that the approximating point $\{\hat{\boldsymbol{P}}_C^{(r)}, \hat{\boldsymbol{P}}_D^{(r)}, \hat{\boldsymbol{I}}_C^{(r)}, \hat{\boldsymbol{I}}_D^{(r)}\}$ for the surrogate objective and constraint function is obtained by ascending in the direction of the optimal solution obtained from previous iterate, i.e., $\{\boldsymbol{P}_C^{(r-1)}, \boldsymbol{P}_D^{(r-1)}, \boldsymbol{I}_C^{(r-1)}, \boldsymbol{I}_D^{(r-1)}\}$.

Algorithm 1 Centralized Resource Allocation

Initialize: feasible $\boldsymbol{B}^{(0)}, \boldsymbol{P}_C^{(0)}, \boldsymbol{P}_D^{(0)}, \alpha^{(r)} = (0,1]$ and $r = 0$

Set $\hat{\boldsymbol{P}}_C^{(0)} = \boldsymbol{P}_C^{(0)}$, $\hat{\boldsymbol{P}}_D^{(0)} = \boldsymbol{P}_D^{(0)}$

Compute $\hat{I}^{(0)}_{C_i}$, $\hat{I}^{(0)}_{D_{(j:k),i}}$ $\forall\, i \in \mathcal{C}$, $\forall\, j \in \mathcal{D}$, $\forall\, k \in \mathcal{M}_j$ via (2)

repeat

$r = r + 1$

for all $i \in \mathcal{C}$ **do**

Compute $\Gamma^{LB}_{C_i}$ via (13a), y_{C_i} via (17)

for all $j \in \mathcal{D}$ **do**

Compute $\Gamma^{LB}_{D_{j,i}}$ via (13a), $y_{D_{j,i}}$ via (18) and $u_{D_{(j:k),i}}$ via (16)

end for

end for

Compute $\boldsymbol{B}^{(r)}, P^{(r)}_{C_i}, P^{(r)}_{D_{j.i}}, I^{(0)}_{C_i}, I^{(0)}_{D_{(j:k),i}} (\forall i \in \mathcal{C}, j \in \mathcal{D})$ via (15).

Set $\hat{\boldsymbol{P}}_C^{(r+1)} = \hat{\boldsymbol{P}}_C^{(r)} + \alpha^{(r)}\Big(\boldsymbol{P}_C^{(r)} - \hat{\boldsymbol{P}}_C^{(r)}\Big)$, $\hat{\boldsymbol{P}}_D^{(r+1)} = \hat{\boldsymbol{P}}_D^{(r)} + \alpha^{(r)}\Big(\boldsymbol{P}_D^{(r)} - \hat{\boldsymbol{P}}_D^{(r)}\Big)$

Set $\hat{\boldsymbol{I}}_C^{(r+1)} = \hat{\boldsymbol{I}}_C^{(r)} + \alpha^{(r)}\Big(\boldsymbol{I}_C^{(r)} - \hat{\boldsymbol{I}}_C^{(r)}\Big)$, $\hat{\boldsymbol{I}}_D^{(r+1)} = \hat{\boldsymbol{I}}_D^{(r)} + \alpha^{(r)}\Big(\boldsymbol{I}_D^{(r)} - \hat{\boldsymbol{I}}_D^{(r)}\Big)$

until $\boldsymbol{B}^{(r)}, P^{(r)}_{C_i}, P^{(r)}_{D_{j.i}}$ converges $(\forall i \in \mathcal{C}, j \in \mathcal{D})$

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D. Convergence Analysis

The primary virtues of the proposed sequential parametric convex approximation approach include: (i) preserving the feasibility at every iterate; (ii) guaranteed convergence to a lmit point; and (iii) immediate formulation for a distributed implementation across BS and MD2D groups. The preservation of feasibility at every iteration is shown in **Lemma** 1, which essentially ensures that if the resource allocation algorithm is interrupted before the convergence, the obtained feasible solution from previous iterate still usable. Convergence to a limit point of proposed **Algorithm** 1 based on sequence of parametric convex approximations is shown in following **Theorem**.

Theorem 1. *Let* $\{\boldsymbol{P}^{(r)}\}_{r\in\mathbb{N}_+}$ *be the sequence generated by the optimal solution of the parametric convex approximation* (15) *with* $\boldsymbol{P}^{(r)} \triangleq [\boldsymbol{P}_C^{(r)}\ \boldsymbol{P}_D^{(r)}]$. *Then,* $\lim_{r\to\infty} \boldsymbol{P}^{(r)} = \bar{\boldsymbol{P}}$ *is a limit point of problem* (6) *and at-least one of the limit point is a stationary point.*

Proof: See Appendix A.

V. Distributed Algorithm

In this section, we decouple the centralized resource allocation problem into sub-problems that can be solved across MD2D groups in cooperation with the BS. Firstly, notice that the objective function in the parametric convex approximation formulation (15), expressed in (14) can be decoupled in the form,

$$f_C(\boldsymbol{P}_C, \boldsymbol{I}_C, \Gamma_C^{LB}, \boldsymbol{y}_C) + \sum_{j\in\mathcal{D}} f_{D_j}(\boldsymbol{P}_{D_j}, \boldsymbol{B}, \boldsymbol{I}_{D_j}, \Gamma_{D_j}^{LB}, \boldsymbol{y}_{D_j}) \tag{19}$$

where,

$$\begin{aligned} f_C(\boldsymbol{P}_C, \boldsymbol{I}_C, \Gamma_C^{LB}, \boldsymbol{y}_C) := \sum_{i\in\mathcal{C}} \Bigg[& W \log_2\left(1+\Gamma_{C_i}^{LB}\right) - \Gamma_{C_i}^{LB} + 2y_{C_i}\sqrt{(1+\Gamma_{C_i}^{LB}) g_{C_i} P_{C_i}} \\ & - y_{C_i}^2\left(g_{C_i}P_{C_i} + N_0 + I_{C_i}\right)\Bigg] \end{aligned} \tag{20a}$$

$$\begin{aligned} f_{D_j}(\boldsymbol{P}_{D_j}, \boldsymbol{B}, \boldsymbol{I}_{D_j}, \Gamma_{D_j}^{LB}, \boldsymbol{y}_{D_j}) := \sum_{i\in\mathcal{C}} \Bigg[& W \log_2\left(1+\Gamma_{D_{j,i}}^{LB}\right) - \Gamma_{D_{j,i}^{LB}} + 2y_{D_{j,i}}\sqrt{(1+\Gamma_{D_{j,i}^{LB}}) g_{D_{(j:l),i}} P_{D_{j,i}}} \\ & - y_{D_{j,i}}^2\left(g_{D_{(j:l),i}}P_{D_{j,i}} + N_0 + I_{D_{(j:l),i}}\right)\Bigg] - \gamma\sum_{j\in\mathcal{D}} |\boldsymbol{B}_{[j]}|_1 \end{aligned} \tag{20b}$$

Next, notice that in the parametrized feasibility set of (15), constraints (6h) and (6i) are affine coupling constraints; thus, we exploit the dual decomposition technique to decouple the constraints. The partial Lagrangian with coupling constraints (6h) and (6i), can be expressed as:

$$L(\boldsymbol{P}_C, \boldsymbol{P}_D, \boldsymbol{B}, \boldsymbol{I}, \Gamma^{LB}, \boldsymbol{D}_C, \boldsymbol{D}_D) :=$$
$$f_C(\boldsymbol{P}_C, \boldsymbol{I}_C, \Gamma_C^{LB}, \boldsymbol{y}_C) + \sum_{j\in\mathcal{D}} f_{D_j}(\boldsymbol{P}_{D_j}, \boldsymbol{B}, \boldsymbol{I}_{D_j}, \Gamma_{D_j}^{LB}, \boldsymbol{y}_{D_j}) - \sum_{i\in\mathcal{C}} d_{C_i}\Big(\sum_{j\in\mathcal{D}} h_{D_{j,i}} P_{D_{j,i}} - I_{C_i}\Big)$$
$$-\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\sum_{k\in\mathcal{M}_j}\Big(d_{D_{(j:k),i}}\Big(\sum_{j'\neq j\in\mathcal{D}} h_{D_{(j',(j:k)),i}} P_{D_{j',i}} + h_{C_{(j:k),i}} P_{C_i} - I_{D_{(j:k),i}}\Big)\Big) \tag{21a}$$

where, $\{d_{C_i},\ d_{D_{(j:k),i}}\}_{i\in\mathcal{C},j\in\mathcal{D},k\in\mathcal{M}_j}$ are the associated dual variables. Further, we split the feasibility set formed by all the constraints (7), (6c), (6d), (6e), and (6f), (8a), (9) to feasibly sets $\mathcal{S}_C$ and $\{\mathcal{S}_{D_j}\}_{j\in\mathcal{D}}$ which will be associated to the respective sub-problems to be solved at the BS and j-th MD2D group as follows:

$$\mathcal{S}_C := \{\text{(6c) and (6f)}\} \qquad \forall i \in \mathcal{C} \tag{22}$$

$$\mathcal{S}_{D_j} := \{\text{(7), (6d), (6e), (8a) and (9)}\} \qquad \forall k \in \mathcal{M}_j,\ \forall i \in \mathcal{C} \tag{23}$$

Notice that maximization of the partial Lagrangian (21a) for given values of dual variables, can be easily decoupled across sub-problems to be maximized at BS and MD2D groups. In addition, for the minimization of the corresponding dual function, the dual variables can be updated by gradient descent as:

$$d_{C_i}^{(r+1)} = \Big[d_{C_i}^{(r)} + \beta\Big(\sum_{j\in\mathcal{D}} h_{D_{j,i}} P_{D_{j,i}}^{(r)} - I_{C_i}^{(r)}\Big)\Big]_+ \tag{24}$$

$$d_{D_{(j:k),i}}^{(r+1)} = \Big[d_{D_{(j:k),i}}^{(r)} + \beta\Big(\sum_{j'\neq j\in\mathcal{D}} h_{D_{(j',(j:k)),i}} P_{D_{j',i}}^{(r)} + h_{C_{(j:k),i}} P_{C_i}^{(r)} - I_{D_{(j:k),i}}^{(r)}\Big)\Big]_+ \tag{25}$$

where, (r) denotes the iteration index.

A. Decoupled Sub-problem at the BS

The maximization sub-problem to be solved at the BS for a given update of the dual variables can be expressed as,

$$\underset{\boldsymbol{P}_C,\boldsymbol{I}_C}{\text{maximize}}\ f_C(\boldsymbol{P}_C, \boldsymbol{I}_C, \Gamma_C^{LB}, \boldsymbol{y}_C) + \sum_{i\in\mathcal{C}} d_{C_i} I_{C_i} - \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\sum_{k\in\mathcal{M}_j}\Big(d_{D_{(j:k),i}} h_{C_{(j:k),i}} P_{C_i}\Big)$$
$$\text{subject to:} \{P_{C_i}, I_{C_i}\}_{i\in\mathcal{C}} \in \mathcal{S}_C \tag{26}$$

The objective of this maximization sub-problem can be further decoupled across the individual i-th channels and we can easily compute the closed-form solution for the unconstrained

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problem as,

$$P_{C_i} = \frac{y_{C_i}^2 g_{C_i}\left(1+\Gamma_{C_i}^{LB}\right)}{\left(y_{C_i}^2 g_{C_i} + \sum_{j\in\mathcal{D}}\sum_{k\in\mathcal{M}_j} d_{D_{(j:k),i}} h_{C_{(j:k),i}}\right)^2} \tag{27a}$$

$$I_{C_i} = \sum_{j\in\mathcal{D}} h_{D_{j,i}} P_{D_{j,i}} \quad \text{under the condition} \quad y_{C_i}^2 \geq d_{C_i} \tag{27b}$$

Notice that the feasibility can be easily satisfied by projecting the power values computed from (27a) for all N_C channels to the BS feasibility set defined in (22). Further, the condition for computing I_{C_i} in (27b) is derived from the observation that under $y_{C_i}^2 \leq d_{C_i}$, the objective becomes unbounded. Moreover, at given iterate, the BS, after computing the optimal values $\{P_{C_i}, I_{C_i}\}_{i\in\mathcal{C}}$, can easily update the following variables: (i) $\Gamma_{C_i}^{LB}$ (via (13a)); (ii) obtain the next point of parametric approximation approximation; and (iii) update the parameter y_{C_i} (via (17)). For preforming the next iteration, BS needs to follow the updates from the MD2D groups: (i) $\{P_{D_{j,i}}\}_{i\in\mathcal{C},j\in\mathcal{D}}$; (ii) $\{\sum_{j\in\mathcal{D}}\sum_{k\in\mathcal{M}_j} d_{D_{(j:k),i}} h_{C_{(j,k),i}}\}_{i\in\mathcal{C}}$. In addition, based on the update variables received from MD2D groups, the dual variables $\{d_{C_i}\}_{i\in\mathcal{C}}$, can be be computed via (24) (projected on the condition $y_{C_i}^2 \leq d_{C_i}$) at the BS. The overall procedure that runs at the BS in cooperation with MD2D groups is summarized in **Algorithm** 2.

B. Decoupled Sub-problem at MD2D groups

Similarly, the sub-problem solved at the j-th multicast group for a given iteration update of the dual variables is given by:

$$\begin{aligned} \underset{\boldsymbol{P}_{D_j},\boldsymbol{B}_j,\boldsymbol{I}_{D_j}}{\text{maximize}} \quad & f_{D_j}(\boldsymbol{P}_{D_j}, \boldsymbol{B}_{[j]}, \boldsymbol{I}_{D_j}, \Gamma_{D_j}^{LB}, \boldsymbol{y}_{D_i}) - \sum_{i\in\mathcal{C}} d_{C_i} h_{D_{j,i}} P_{D_{j,i}} \\ & - \sum_{i\in\mathcal{C}} \sum_{j'\neq j,\ j'\in\mathcal{D}} \sum_{k_{j'}\in\mathcal{M}_{j'}} d_{D_{(j':k_{j'}),i}} h_{D_{(j,(j':k_{j'})),i}} P_{D_{j,i}} + \sum_{i\in\mathcal{C}} \sum_{k\in\mathcal{M}_j} d_{D_{(j:k),i}} I_{D_{(j:k),i}} \\ \text{subject to:} \quad & \{\boldsymbol{B}_{[j]}, P_{D_{j,i}}, I_{D_{(j:k),i}}\}_{k\in\mathcal{M}_j, i\in\mathcal{C}} \in \mathcal{S}_{D_j} \end{aligned} \tag{28}$$

Here, at a given iterate, each j-th MD2D group, after computing the optimal values $\{P_{D_{j,i}}$ and $I_{D_{(j:k),i}}\}_{i\in\mathcal{C},k\in\mathcal{M}_j}$, make the following updates: (i) $\Gamma_{D_{j,i}}^{LB}$ (via (13a)); (ii) obtain the next point of parametric approximation; and (iii) update the parameters $y_{D_{j,i}}$ (via (18)), and $u_{D_{j,i}}$ (via (16)). For performing the next iteration, the j-th MD2D group needs the following update from the BS: (i) $\{P_{C_i}\}_{i\in\mathcal{C}}$; (ii) $\{d_{C_i} h_{D_{j,i}}\}_{i\in\mathcal{C}}$. Moreover, each j-th MD2D is assumed to share

Algorithm 2 Decoupled Resource Allocation sub-problem at BS

Initialize: feasible $\boldsymbol{P}_C^{(0)}, \boldsymbol{D}_C^{(0)}, \alpha^{(0)} = (0, 1]$ and $r = 0$

Broadcast $\boldsymbol{P}_C^{(0)}$ & communicate $d_{C_i}^{(0)} h_{D_{j,i}}$ to j-th MD2D group

Obtain from MD2D groups $\{P_{D_{j,i}}^{(0)}\}_{i\in\mathcal{C},j\in\mathcal{D}}$ and $\{\sum_{j\in\mathcal{D}}\sum_{k\in\mathcal{M}_j} d_{D_{(j:k),i}}^{[0]} h_{C_{(j,k),i}}\}_{i\in\mathcal{C}}$

Set $\hat{\boldsymbol{P}}_C^{(0)} = \boldsymbol{P}_C^{(0)}$; Compute $\{\hat{I}_{C_i}^{(0)}\}_{i\in\mathcal{C}}$ via (2)

repeat

$r = r + 1$

Compute $\{\Gamma_{C_i}^{LB}, y_{C_i}\}_{i\in\mathcal{C}}$ via (13a),(17)

Compute $\{P_{C_i}^{(r)}, I_{C_i}^{(r)}\}_{i\in\mathcal{C}}$ via (27a),(27b). Project $\{P_{C_i}^{(r)}\}_{i\in\mathcal{C}}$ onto constraint (6c)

Broadcast $\boldsymbol{P}_C^{(r)}$ & communicate $d_{C_i}^{(r)} h_{D_{j,i}}$ to j-th MD2D group

Obtain from each j-th MD2D groups $\{P_{D_{j,i}}^{(r)}\}_{i\in\mathcal{C},j\in\mathcal{D}}$ and $\{\sum_{j\in\mathcal{D}}\sum_{k\in\mathcal{M}_j} d_{D_{(j:k),i}}^{(r)} h_{C_{(j,k),i}}\}_{i\in\mathcal{C}}$

Compute $\boldsymbol{d}_C^{(r)}$ via (24) with projection on $y_{C_i}^2 \le d_{C_i}$

Set $\hat{\boldsymbol{P}}_C^{(r+1)} = \hat{\boldsymbol{P}}_C^{(r)} + \alpha^{(r)}\left(\boldsymbol{P}_C^{(r)} - \hat{\boldsymbol{P}}_C^{(r)}\right)$; $\hat{\boldsymbol{I}}_C^{(r+1)} = \hat{\boldsymbol{I}}_C^{[r]} + \alpha^{(r)}\left(\boldsymbol{I}_C^{(r)} - \hat{\boldsymbol{I}}_C^{(r)}\right)$

until $P_{C_i}^{(r)}$ converges $\forall i \in \mathcal{C}$

interference channel gains with its neighboring MD2D groups. In addition, based on the updates received from the BS, the dual variables $\{I_{D_{(j:k),i}}\}_{i\in\mathcal{C},k\in\mathcal{M}_j}$, are computed via (25) at each j-th MD2D group. The overall procedure at each j-th MD2D group is summarized in **Algorithm** 3.

C. Overhead Communication Requirements

The overhead communication between BS and MD2D groups for distributed implementation of the resource allocation algorithm is illustrated in Figure 2. Here, we first discuss the local message passing within the cellular network or MD2D group. It can be noted that in a typical cellular network, the i-th CU communicates the estimated direct channel gain g_{C_i} to the BS. For the proposed distributed architecture, the i-th CU has to additionally communicate the interference channel gain $h_{D_{j,i}}$ to the BS. Further, due to path loss, CUs only needs to communicate a certain limited number of significant interference channel gains from

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Algorithm 3 Decoupled Resource Allocation Sub-problem at j-th MD2D group

Initialize: feasible $\boldsymbol{B}_{[j]}^{(0)}, \boldsymbol{P}_{D_j}^{(0)}, \boldsymbol{I}_{D_j}^{(0)}, \alpha^{(r)} = (0, 1]$ and $r = 0$

Broadcast $\boldsymbol{P}_{D_j}^{(0)}$, $\{\sum_{k\in\mathcal{M}_j} d_{D_{(j:k),i}}^{(0)} h_{C_{(j,k),i}}\}_{i\in\mathcal{C}}$

Obtain $\boldsymbol{P}_C^{(0)}$ & $\{d_{C_i}^{(0)} h_{D_{j,i}}\}_{i\in\mathcal{C}}$ from the BS

Set $\hat{\boldsymbol{P}}_D^{(0)} = \boldsymbol{P}_D^{(0)}$; Compute $\{\hat{I}_{D_{(j:k),i}}^{(0)}\}_{i\in\mathcal{C},j\in\mathcal{D},k\in\mathcal{M}_j}$ via (2)

repeat

  $r = r + 1$

  Compute $\{\Gamma_{D_{j,i}}^{LB}\}_{i\in\mathcal{C}}$ via (13a), $\{y_{D_{j,i}}\}_{i\in\mathcal{C}}$ via (18) and $u_{D_{j,i}}$ via (16)

  Compute $\boldsymbol{B}_{[j]}^{[r]}, \{P_{D_{j.i}}^{(r)}, I_{D_{(j:k),i}}^{(r)}\}_{i\in\mathcal{C}}$ via (28)

  Broadcast $\boldsymbol{P}_{D_j}^{(r)}$, $\{\sum_{k\in\mathcal{M}_j} d_{D_{(j:k),i}}^{(r)} h_{C_{(j,k),i}}\}_{i\in\mathcal{C}}$

  Obtain $\boldsymbol{P}_C^{(r)}$ & $\{d_{C_i}^{(r)} h_{D_{j,i}}\}_{i\in\mathcal{C}}$ from the BS

  Compute $\boldsymbol{d}_{D_j}^{(r)}$ via (25)

  Set $\hat{\boldsymbol{P}}_D^{(r+1)} = \hat{\boldsymbol{P}}_D^{(r)} + \alpha^{(r)}\left(\boldsymbol{P}_D^{(r)} - \hat{\boldsymbol{P}}_D^{(r)}\right)$; $\hat{\boldsymbol{I}}_D^{(r+1)} = \hat{\boldsymbol{I}}_D^{(r)} + \alpha^{(r)}\left(\boldsymbol{I}_D^{(r)} - \hat{\boldsymbol{I}}_D^{(r)}\right)$

until $\boldsymbol{B}_{[j]}^{(r)}, P_{D_{j.i}}^{(r)}$ converges $(\forall i \in \mathcal{C})$

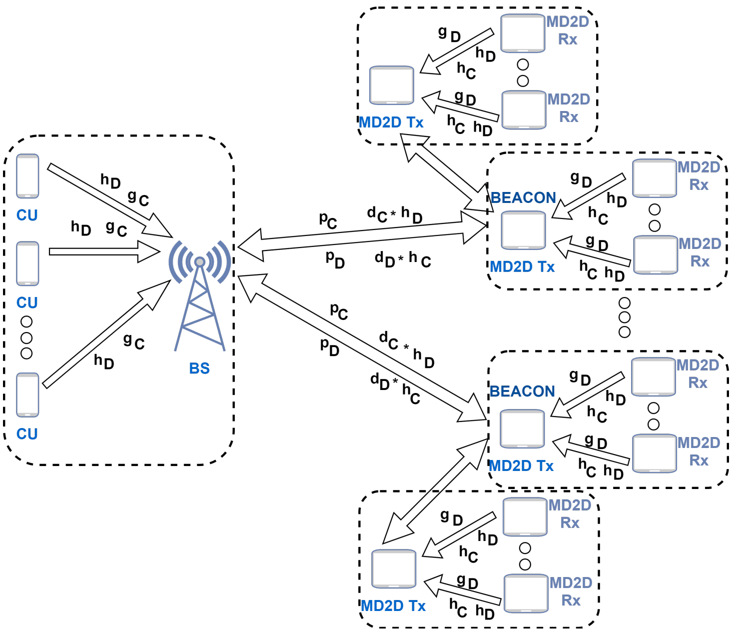

Fig. 2: Illustration of overhead communication requirements for diistributed resource allocation.

neighbouring MD2D groups only. Similarly, each j-th MD2D group will typically require direct channel gains $g_{D_{(j:k),i}}$ from all its $k \in \mathcal{M}_j$ receivers. Here the k-th receiver in a given MD2D group additionally needs to communicate a certain limited number of significant interference channel gains from neighboring CUs and MD2D transmitters. Thus, it can be concluded that the additional overhead communication for the proposed distributed resource allocation solution is limited and local within the cellular network or MD2D groups.

Next, we discuss the message passing required between the BS and MD2D groups. In order to reduce the communication overhead, we propose to have some spatially distributed beacon MD2D groups which essentially act as message aggregating and disseminating nodes. These beacon nodes receive $\{P_{C_i}\}_{i\in\mathcal{C}}$ and $\{d_{C_i}d_{D_{ji}}\}_{i\in\mathcal{C},j\in\mathcal{D}}$ from the BS and relay the messages to the neighboring $MDtD$ groups. Similarly, these nodes collect $\{P_{D_{j,i}}\}_{i\in\mathcal{C},j\in\mathcal{D}}$ and $\{\sum_{k\in\mathcal{M}_j} d_{D_{(j:k),i}} h_{C_{(j:k),i}}\}_{i\in\mathcal{C},j\in\mathcal{D}}$ from the neighboring MD2D groups and relay it to BS.

D. Convergence Analysis

In order to establish the convergence of the proposed distributed resource allocation solution, we state the following Theorem.

Theorem 2. *Let* $\{\boldsymbol{P}_C^{(r)}\}_{r\in\mathbb{N}_+}$, $\{\boldsymbol{P}_D^{(r)}\}_{r\in\mathbb{N}_+}$ *be the respective sequence of optimal solutions generated by the decoupled resource allocation methds provided by* ***Algorithm*** *2 (at BS) and* ***Algorithm*** *3 (at each j-th MD2D group, $\forall j \in \mathcal{D}$). Then,* $\lim_{r\to\infty}[\boldsymbol{P}_C^{(r)},\ \boldsymbol{P}_D^{(r)}] = \bar{\boldsymbol{P}}$ *where* $\bar{\boldsymbol{P}}$ *is a limit point of* (6) *and at-least one of the limit points is a stationary point.*

Proof: See Appendix A.

VI. Simulations

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. locations with parameters BW= 15 kHz, $N_D = 4$, $N_C = 4$, $M_j = 3$, $N_0 = -70$ dBW. The proposed algorithm is compared with the unicast method in [34] when

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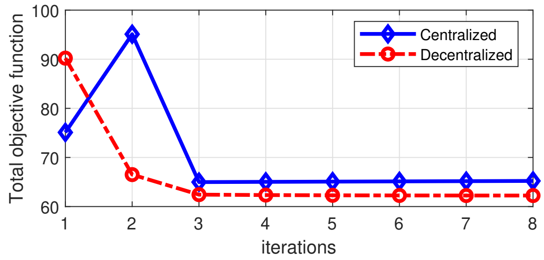

Fig. 3: The convergence of the proposed centralized and distributed algorithms

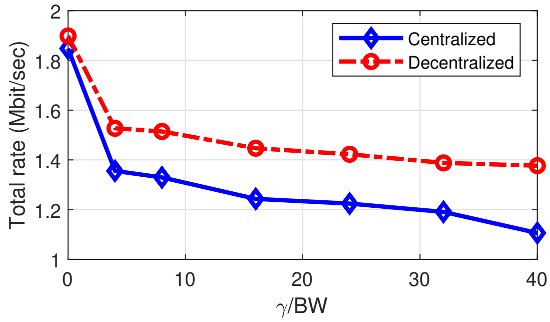

Fig. 4: Total average rate R vs. tradeoff/regularization parameter γ

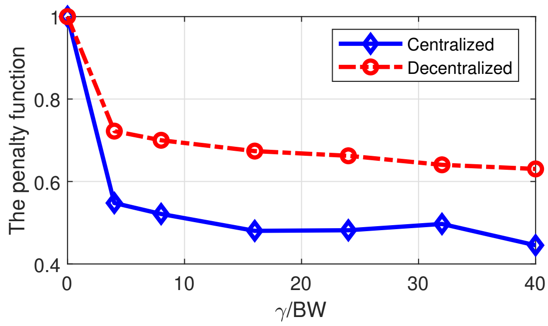

Fig. 5: The penalty function R vs. tradeoff/regularization parameter γ

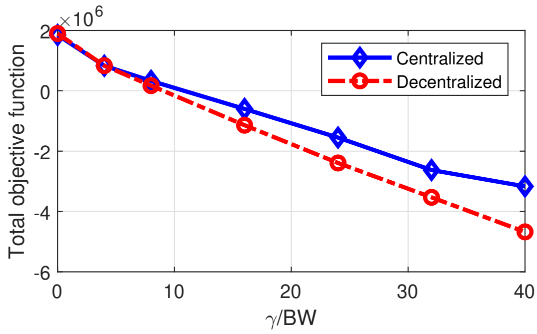

Fig. 6: Total objective function R vs. tradeoff/regularization parameterγ

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each D2D group is considered as M_J D2D pairs, and a multicast method in [35] where each channel is assigned to at most one D2D group.

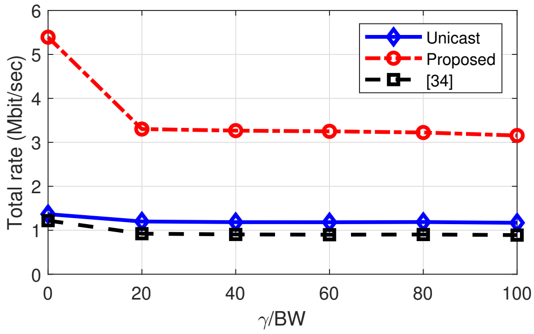

Fig. 7: Total average rate R vs. tradeoff/regularization parameter γ

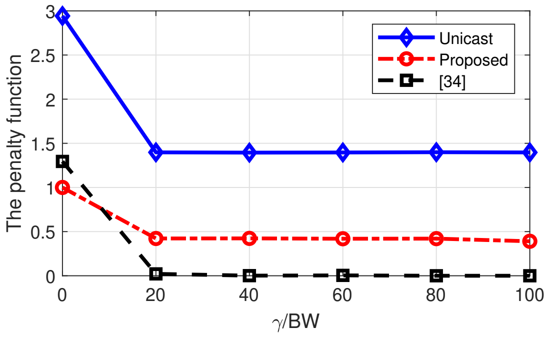

Fig. 8: The penalty function vs. tradeoff/regularization parameter γ

Comparison of convergence of the centralized and the decentralized methods is shown in fig. 3. Notice that both the methods take same number of iterations to converge; however, implementation of decentralized method requires additional communication overhead. Figs. 4, 5, and 6 show the performance of the proposed centralized and the decentralized methods when changing γ in the problem (4).The results show that the both methods have similar performance. Here, in both cases the solution converges to a limit point as it is not possible to guarantee that it is the optimal solution. Both centralized and decentralized have similar performance, thus, for simplicity, we only show the results of the centralized solution in the next experiments. Fig. 7 shows that the proposed method achieves significantly higher rate compared to both the unicast method in [34] and the multicast method in [35]. When γ increases, the rate decreases in all methods, as expected. Fig. 8 shows the penalty function in all cases. While the penalty function in the proposed method measures sparsity of the channel assignment ($||.||_1$), in both [34], [35] it measures the fairness in the channel assignment. Note that the penalty function is normalized by dividing over $N_C N_D$. The results

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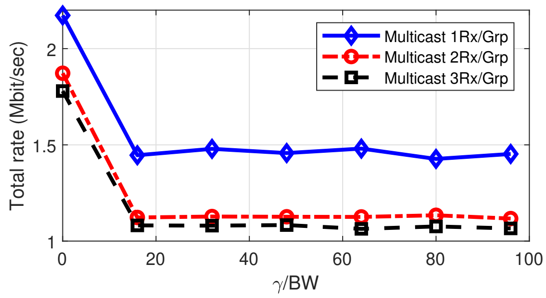

Fig. 9: Total average rate R vs. parameter γ ($M_j = 1,\ 2,\ 3$)

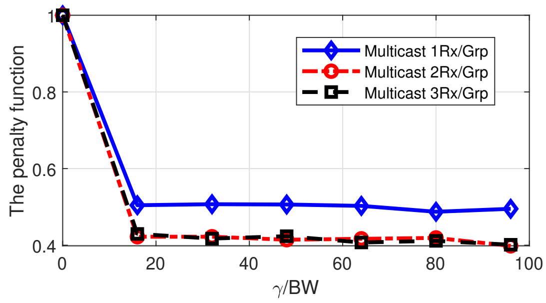

Fig. 10: The penalty function vs. parameter γ ($M_j = 1,\ 2,\ 3$)

show that the penalty function decreases when γ increases in all cases. In the unicast case, the penalty function does not reach 0 since the number channels here $N_C = 4$ is smaller than the number D2D receivers $N_D M_J = 12$. Thus, in this case, some D2D pairs will not be allowed to communicate. However, in the multicast case in [35], the penalty function reaches 0 since the number of channels here $N_C = 4$ is equal to the number D2D groups $N_D = 4$. In the proposed method, obtaining 0 in the penalty function means not assigning channels to any of the D2D groups, which is not desirable. Figs. 9 and 10 show the performance of the proposed method when changing the number of D2D receivers in each multicast group. The total network rate decreases with each additional receiver in the group, since the rate in each group is determined by the receiver with the worst communication conditions. In general, multicast communications allow sending the same information to several receivers with the same network resources. Our proposed algorithm achieves this goal while outperforming other alternatives and maintaining sparsity in the channel assignment.

VII. Conclusion

In this work, we present resource allocation algorithms, both centralized and distributed for MD2D communication operating in underlay with a cellular network. The algorithms allow MD2D groups to access multiple channels and also does not limit multiple MD2D groups to operate on a typical channel. The algorithms are analytically shown to converge to a limit point and simulations demonstrate improvement in network throughput.

Appendix A

A. *Proof of **Lemma** 1*

Proof: The parametric approximation of the ratio $\frac{g_{D_{(j:k),i}} P_{D_{j,i}}}{N_0 + I_{D_{(j:k),i}}}$, expressed by $2u_{D_{j,i}}\sqrt{g_{D_{(j:k),i}} P_{D_{j,i}}} - u^2_{D_{j,i}}\left(N_0 + I_{D_{(j:k),i}}\right)$ is concave over the parameter $u_{D_{j,i}}$ and its maximum is achieved at $u^*_{D_{j,i}} = \frac{g_{D_{(j:k),i}} P_{D_{j,i}}}{N_0 + I_{D_{(j:k),i}}}$. Thus, the adopted parametric approximation is a lower bound on the ratio, which makes the parametric convex approximated constraint (9) be an upper bound to the non-convex constraint (8b) $\forall u_{D_{j,i}} \in \mathbb{R}_+$. ■

B. *Proof of **Lemma** 2*

Proof: Similar to the arguments presented in the proof of Lemma 1, the ratios $\frac{(1+\Gamma^{LB}_{C_i}) g_{C_i} P_{C_i}}{g_{C_i} P_{C_i} + N_0 + I_{C_i}}$, $\frac{(1+\Gamma_{D^{LB}_{j,i}}) g_{D_{(j:l),i}} P_{D_{j,i}}}{g_{D_{(j:l),i}} P_{D_{j,i}} + N_0 + I_{D_{(j:l),i}}}$ in $\mathbb{F}_1$ (12a), are approximated by the respective parametric lower bound convex expressions $2y_{C_i}\sqrt{(1+\Gamma^{LB}_{C_i}) g_{C_i} P_{C_i}} - y^2_{C_i}\left(g_{C_i} P_{C_i} + N_0 + I_{C_i}\right)$ and $2y_{D_{j,i}}\sqrt{(1+\Gamma_{D^{LB}_{j,i}}) g_{D_{(j:l),i}} P_{D_{j,i}}} - y^2_{D_{j,i}}\left(g_{D_{(j:l),i}} P_{D_{j,i}} + N_0 + I_{D_{(j:l),i}}\right)$, $\forall y_{C_i} \in \mathbb{R}_+, y_{d_{j,i}} \in \mathbb{R}_+$ in $\mathbb{F}_2$ (14). Hence maximizing $\mathbb{F}_2$ leads to maximizing $\mathbb{F}_1$. ■

C. *Proof of **Theorem 1***

Proof: Denoting non-convex optimization problem as $\mathcal{P}$ (with non-convex objective $\boldsymbol{U}$ and non-convex feasible set $\mathcal{X}$), we first state the conditions that guarantee the convergence to a limit solution $\bar{\boldsymbol{x}} \in \mathcal{X}$ obtained by successive parametric convex approximation with parameters updated by iNner cOnVex Approximation (NOVA) method [33].

(Z1) On non-convex problem $\mathcal{P}$: (i) The feasibility set excluding non-convex constraints, denoted by $\mathcal{K}$ is closed and convex; (ii) The non-convex objective function $\boldsymbol{U}$ and non-convex constraints g_i (for some index i) are continuously differentiable over $\mathcal{K}$; (iii) $\nabla_{\boldsymbol{x}}\boldsymbol{U}$ is

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Lipschitz continuous on $\mathcal{K}$. (iv) $\boldsymbol{U}$ is coercive on $\mathcal{K}$; (v) All feasible points in $\mathcal{X}$ are regular.

(Z2) On parametric approximation of $\boldsymbol{U}$ *denoted by* $\tilde{\boldsymbol{U}}(\boldsymbol{x}, y_U(\hat{\boldsymbol{x}}))$ *for parameter* $y_U(\hat{\boldsymbol{x}})$ *and* $\hat{\boldsymbol{x}} \in \mathcal{X}$: (i) $\tilde{\boldsymbol{U}}(\cdot, y_U(\hat{\boldsymbol{x}}))$ is uniformly strongly convex on $\mathcal{K}$; (ii) $\nabla_{\boldsymbol{x}}\boldsymbol{U}(\boldsymbol{x}) = \nabla_{\boldsymbol{x}}\tilde{\boldsymbol{U}}(\boldsymbol{x}, y_U(\boldsymbol{x}))$ $\forall \boldsymbol{x} \in \mathcal{X}$; (iii) $\nabla_{\boldsymbol{x}}\tilde{\boldsymbol{U}}(\cdot,\cdot)$ is continuous over $\mathcal{K} \times \mathcal{X}$;

(Z3) On parametric approximation of constraint g_i *denoted* $\tilde{g}_i(\boldsymbol{x}, y_{g_i}(\hat{\boldsymbol{x}}))$ *for parameter* $y_{g_i}(\hat{\boldsymbol{x}})$ *and* $\hat{\boldsymbol{x}} \in \mathcal{X}$: (i) $\tilde{g}_i(\cdot, y_{g_i}(\hat{\boldsymbol{x}}))$ is convex on $\mathcal{K}$; (ii) $g_i(\boldsymbol{x}) = \tilde{g}_i(\boldsymbol{x}, y_{g_i}(\boldsymbol{x}))$; (iii) $g_i(\boldsymbol{x}) \leq \tilde{g}_i(\boldsymbol{x}, y_{g_i}(\hat{\boldsymbol{x}}))$; (iv) $\tilde{g}_i(\cdot,\cdot)$ is continuous on $\mathcal{K} \times \mathcal{X}$; (v) $\nabla_{\boldsymbol{x}} g_i(\boldsymbol{x}) = \nabla_{\boldsymbol{x}}\tilde{g}_i(\boldsymbol{x}, y_{g_i}(\boldsymbol{x}))$; (vi) $\nabla_{\boldsymbol{x}}\tilde{g}_i(\cdot,\cdot)$ is continuous on $\mathcal{K} \times \mathcal{X}$

First focusing on conditions in *(Z1)* for: (i) the non-convex objective function U is defined by $\mathbb{F}_1$ (12a); (ii) the non-convex constraints g_i are defined by (8b); and (iii) the feasibility set excluding non convex constrains $\mathcal{K}$ is defined by (7), (6c), (6d), (6e), (6f), (6h), and (6i). Notice that the feasibility set excluding the non-convex constraints is closed and convex. However, due to augmentation of objective by the austerity function $\sum_{j\in\mathcal{D}} ||\boldsymbol{B}_{[j]}||_1$, the overall objective function is not continuously differentiable over $\mathcal{K}$. Nonetheless, note that the non-smooth component $\sum_{j\in\mathcal{D}} ||\boldsymbol{B}_{[j]}||_1$ is similar to a convex regularizer term, making $\mathbb{F}_1$ a structured non-smooth problem (equation (24) of [33]) for which convergence to a limit point is still guaranteed. Thus, it is sufficient to show that the gradient of objective function excluding non-smooth part is Lipschitz continuous over $\mathcal{K}$. Here, notice that the interference terms are the slack variables and we can substitute them by equations in (2). The second order derivatives of $\mathbb{F}_1$ with respect to the power variables, P_{C_i} and $P_{D_{j,i}}$ are given by:

$$\begin{aligned}
\frac{\partial^2 \mathbb{F}_1}{\partial P_{C_i}^2} &= -\frac{2g_{C_i}^2(1+\Gamma_{C_i}^{LB})(N_0 + \sum_{j\in\mathcal{D}} b_{i,j} h_{D_{j,i}} P_{D_{j,i}})}{(N_0 + \sum_{j\in\mathcal{D}} b_{i,j} h_{D_{j,i}} P_{D_{j,i}} + g_{C_i} P_{C_i})^3} \\
\frac{\partial^2 \mathbb{F}_1}{\partial P_{D_{j,i}}^2} &= -\frac{2g_{D_{(j:l),i}}^2(1+\Gamma_{D_{j,i}}^{LB})(N_0 + \sum_{j'\neq j\in\mathcal{D}} b_{i,j'} h_{D_{(j',(j:k)),i}} P_{D_{j',i}} + h_{C_{(j:k),i}} P_{C_i})}{(N_0 + \sum_{j'\neq j\in\mathcal{D}} b_{i,j'} h_{D_{(j',(j:k)),i}} P_{D_{j',i}} + h_{C_{(j:k),i}} P_{C_i} + g_{D_{(j:l),i}} P_{D_{j,i}})^3}
\end{aligned} \tag{29}$$

Notice that all the terms in (29) are bounded from above, thus, $\nabla_{\boldsymbol{x}}\boldsymbol{U}$ is Lipschitz continuous on $\mathcal{K}$. Moreover, notice that the objective function $\mathbb{F}_1$ is coercive in the power variables. Next, we can observe that all points in the feasible region $\mathcal{X}$ are regular, as it is easy to note that gradient of all the constraints over the feasible region are linearly independent (also termed as linearly independent constraint qualification).

Next, we consider conditions *(Z2)* for the parametric convex approximation of the objective function expressed by $\mathbb{F}_2$ (14). The second order derivative of $\mathbb{F}_2$ with respect to power and interference terms are given by,

$$\frac{\partial \mathbb{F}_2}{\partial P_{C_i}^2} = -0.5 y_{C_i} \sqrt{(1+\Gamma_{C_i}^{LB}) g_{C_i}} P_{C_i}^{\frac{-3}{2}} \qquad \frac{\partial \mathbb{F}_2}{\partial P_{D_{j,i}}^2} = -0.5 y_{D_{j,i}} \sqrt{(1+\Gamma_{D_{j,i}}^{LB}) g_{D_{(j:l),i}}} P_{D_{j,i}}^{\frac{-3}{2}} \tag{30}$$

Notice once again that all the terms in (30) are bounded from below, thus $\mathbb{F}_2$ is strongly convex in the power variables. We can observe also that the terms in (30) are bounded from above, the gradient of $\mathbb{F}_2$ is also continuous in the parametrized feasible set. In addition, on the basis of observations in **Lemma 2**, the assumption $\nabla_{\boldsymbol{x}} \boldsymbol{U}(\boldsymbol{x}) = \nabla_{\boldsymbol{x}} \tilde{\boldsymbol{U}}(\boldsymbol{x}, y_U(\boldsymbol{x}))\ \forall \boldsymbol{x} \in \mathcal{X}$ is also satisfied.

Finally, consider conditions in *(Z3)*. Here the parametrized constraint $\tilde{g}_i(\cdot, y_{g_i}(\hat{\boldsymbol{x}}))$, corresponds to (9). Note that based on **Lemma 1**, we can confirm that (i) $\tilde{g}_i(\cdot, y_{g_i}(\hat{\boldsymbol{x}}))$ is convex on $\mathcal{K}$; (ii) $g_i(\boldsymbol{x}) = \tilde{g}_i(\boldsymbol{x}, y_{g_i}(\boldsymbol{x}))$; and (iii) $g_i(\boldsymbol{x}) \leq \tilde{g}_i(\boldsymbol{x}, y_{g_i}(\hat{\boldsymbol{x}}))$ hold true. Further, we can also observe that the derivative of (9) with respect to the power variables and parametrization variables exists, thus, $\tilde{g}_i(\cdot, \cdot)$ is continuous on $\mathcal{K} \times \mathcal{X}$. In addition, as $g_i(\boldsymbol{x}) \leq \tilde{g}_i(\boldsymbol{x}, y_{g_i}(\hat{\boldsymbol{x}}))$, $\nabla_{\boldsymbol{x}} g_i(\boldsymbol{x}) = \nabla_{\boldsymbol{x}} \tilde{g}_i(\boldsymbol{x}, y_{g_i}(\boldsymbol{x}))$. Moreover, it can be easily seenc that $\nabla_{\boldsymbol{x}} \tilde{g}_i(\cdot, \cdot)$ is continuous on $\mathcal{K} \times \mathcal{X}$. ■

D. Proof of Theorem 2

Proof: For the convergence of distributed solution to a limit point with parameters updated by NOVA method [33], conditions in addition to *Z1*, *Z2*, and *Z3* are as following:

Z4: On decomposability: (i) The feasibility set $\tilde{\mathcal{X}}$ excluding coupling constraints, has a Cartesian structure, i.e., $\tilde{\mathcal{X}} = \mathcal{X}_1 \times \mathcal{X}_2 \cdots \times \mathcal{X}_I$; (ii) The parametrized objective function must be decomposable, i.e., $\tilde{\boldsymbol{U}}(\boldsymbol{x}, y_U(\hat{\boldsymbol{x}})) = \sum_{i=1}^{I} \tilde{\boldsymbol{U}}_i(\boldsymbol{x}_i, y_U(\hat{\boldsymbol{x}}))$; and (iii) The coupling constraints must be block separable.

It is easy to observe that decomposing the feasibility set excluding coupling constraints into $\mathcal{S}_C$ and $\mathcal{S}_{D_j}\ \forall j \in \mathcal{D}$ in (22) and (23), ensures the Cartesian structure assumption. Further, decomposability of the parametrized objective function is evident from (19). Finally, we can also note that the coupling constraints (6h) and (6i) are block separable across BS and MD2D groups. Therefore, all conditions are satisfied. ■

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