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Optimized Portfolios versus Naive Diversification

Could the superior performance of optimized portfolios be attributed to established factor premiums?

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Abstract

Since the publication of the study by DeMiguel, Garlappi & Uppal (2009), where they demonstrate that none of the 14 mean-variance optimization strategies outperform the naive diversification, several studies claim to defend the superiority of portfolio optimization strategies relative to the naive diversification (see e.g. Kritzman, Page & Turkington (2010), Tu & Zhou (2011), Kirby & Ostdiek (2012)). However, in a recent study by Zakamulin (2017), the author states that the superior performance of these optimized strategies appears due to exposures to established factor premiums. Motivated by the study of Zakamulin (2017), this thesis evaluates the out-of-sample performance of four risk-based strategies relative to the naive diversification across 25 empirical datasets provided by Kenneth French. Additionally, we assess whether the (out)performance could be attributed to established factor premiums. We find that three of four risk-based strategies on average deliver superior performance over the naive diversification in terms of Sharpe ratio, although the performance on the individual datasets varies significantly. Each risk-based strategy generates statistically significant alphas in the CAPM, both on average, and in nearly each dataset. In addition, we show that the superior performance of these risk-based strategies compared to the naive diversification, and in terms of CAPM alpha, are mostly generated in bear markets. After controlling for several risk factors through the Fama-French five-factor model, the alphas of any risk-based strategy becomes neither economically nor statistically significant. The main conclusion that we reach in this thesis is that the superior performance of the risk-based strategies is likely to be attributed to established factor premiums.

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1 Introduction

The mean-variance paradigm introduced in the seminal paper by Markowitz (1952) constitutes a theoretical framework to construct efficient portfolios. In a static setting, the mean-variance model represents the optimal way to allocate capital among risky assets. However, the practical usefulness of the model has been overshadowed by the difficulties associated with forecasting the vector of mean returns and the covariance-variance matrix due to estimation error. The estimation error related to the model parameters could lead to extreme portfolio weights and poor out-of-sample performance. Especially with mean returns that are considered notoriously difficult to forecast accurately, compared to the more stable and predictable covariance-variance matrix. Consequently, several researchers have turned their focus to risk-based strategies that optimize the portfolio weights solely based on assets risk, and are therefore less affected by the impact of estimation error. Despite the numerous approaches devoted to mitigate the impact of estimation error, DeMiguel, Garlappi & Uppal (2009) present results that question the value added by optimized portfolios relative to the naive diversification strategy. Specifically, they evaluate 14 mean-variance models across seven empirical datasets and find that none of these optimized portfolios consistently outperform the naive diversification, which allocates capital equally among the assets under consideration.

These findings resulted in numerous studies that claim to defend the superiority of optimized portfolio strategies. Kritzman, Page & Turkington (2010) show that the minimum-variance and mean-variance portfolios provide higher average Sharpe ratio across eight empirical datasets compared to the naive diversification, although the outperformance is not statistically justified. Tu & Zhou (2011) construct optimal combinations of the naive diversification rule and various optimized portfolio rules and show that they outperform the naive diversification strategy across seven empirical datasets. Kirby & Ostdiek (2012) develop two alternative methods of mean-variance portfolio strategies and demonstrate that these outperform the naive diversification strategy with both economically and statistically margins across four empirical datasets. Additionally, several other studies provide compelling results for the optimized portfolios ability to outperform the naive diversification.

More recently, Zakamulin (2017) shows that all recent empirical studies surrounding portfolio optimization use the Sharpe ratio as a performance measure without controlling whether the superior performance of these optimized portfolios appears due to exposures to one or several profitable anomalies. The author constructs three optimal portfolios on 17 empirical datasets and convincingly shows that none of these strategies deliver superior performance after controlling for the low-volatility effect, which Zakamulin (2017) demonstrates is present for nearly all of the datasets provided by Kenneth French. Zakamulin (2017) concludes that portfolio strategies that seem sophisticated in nature can potentially result in rather simple portfolio strategies that only benefit from some profitable market anomalies such as the low-volatility anomaly. This low-volatility anomaly refers to the phenomenon where low-volatility stocks provide superior risk-adjusted returns compared to their riskier peers. The existence of the low-volatility effect has been known for a long period and has been documented in several studies (see e.g. Haugen & Baker (1991), Blitz & Van Vliet (2007), Baker, Bradley & Wurgler (2011)).

Motivated by the study conducted by Zakamulin (2017), this thesis evaluates the performance of four risk-based strategies relative to the naive diversification, and additionally assess whether the (out)performance could be attributed to established factor premiums.[1] We construct four risk-based strategies that are characterized by weighting each asset solely based on the assets risk rather than the mean returns, which is difficult to estimate with precision. Specifically, we use two optimal strategies considered in the literature, the minimum-variance portfolio (MVP), and the volatility-timing strategy (VT) proposed by Kirby & Ostdiek (2012). Additionally, we suggest two ad-hoc strategies that assign weights based on the assets inverse standard deviation and CAPM beta. The goal of these ad-hoc strategies is to demonstrate that one does not need sophisticated optimal strategies to outperform the naive diversification.

Our thesis extends previous literature in several ways. First, existing empirical studies usually evaluate the performance of optimized portfolios relative to the naive diversification using a few arbitrary empirical datasets, chosen among a great number of available datasets in the Kenneth French data library. The performance of a particular portfolio strategy could be affected by the sorting-characteristics to the individual dataset, and those arbitrary datasets could have been selected to substantiate the author's main point. To prevent this "*cherry-picking*" of datasets, this thesis evaluates the performance across 25 empirical datasets formed on portfolios of U.S. stocks provided by Kenneth French. Second, we assess the performance of the risk-based strategies relative to the naive diversification in bull and bear markets, which to the best of our knowledge

[1] Optimized portfolios are henceforth referred to as risk-based strategies. The naive diversification can also be characterized as a risk-based strategy, we wil however only refer to strategies that optimize the weights based on the assets risk.

has never been sufficiently explored. This is done to get a deeper insight into the nature of the performance and assess whether the (out)performance is mostly generated in bull or bear markets. Third, we propose to use a generalized approach to look at the aggregate portfolio performance across the 25 datasets for each of the risk-based strategies. This generalized approach gives us the opportunity to gain insight into the risk factors that could potentially drive the superior performance, and additionally study the risk exposure over time. Fourth, the newly proposed Fama-French five-factor model is used to assess the factor exposure of the risk-based strategies.

We find that on average, three of four risk-based strategies deliver superior performance over the naive diversification in terms of the Sharpe ratio. However, the performance on the individual datasets varies significantly. Each risk-based strategy generates statistically significant alphas in the CAPM, both on average, and in nearly every dataset. When we control for several risk factors, through the Fama-French five-factor model, the positive alpha of any risk-based strategy becomes insignificant both on average and in virtually all datasets. These results are robust to changes in the estimation window, and across different time periods. The results we obtain in the bull and bear markets show that the risk-based strategies superior performance over the naive diversification appears to be mostly generated during bear markets where we observe statistically significantly higher mean returns relative to the naive diversification. Additionally, the risk-based strategies display a higher alpha during bear markets compared to bull markets in the CAPM. Through the aggregated performance, we show that each risk-based strategy, across the 25 datasets, delivers statistically significant alphas in the CAPM, and when we introduce the five-factor model, the significant alphas vanish. The aggregated performance displays that the four risk-based strategies load significantly on the value (HML), profitability (RMW), and investment (CMA) factors. Results that are in line with previous studies (see e.g. Clarke, de Silva & Thorley (2006), Fama & French (2016), Zakamulin (2017)). Additionally, the results we obtain for the two ad-hoc strategies substantiate the point made in Zakamulin (2017), that one can create rather simple portfolio strategies without the need for optimization, though directly exploits profitable anomalies, could result in superior performance over the naive diversification.

The remainder of this thesis is structured as follows: Section 2 provides a review of theory and existing literature. Section 3 presents the data we use in this thesis. Section 4 addresses the research method we use for the empirical analysis. In Section 5 we present the empirical results we obtain from our study. Section 6 covers the discussion, whereas Section 7 draws the main conclusion of our study.

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2 | Theory & Literature Review

2.1 Modern Portfolio Theory and Superiority of Optimized Portfolios

In the seminal paper by Markowitz (1952), the author derived the optimal rule for allocating capital among risky assets to maximize the expected return for a given level of risk or vice versa, minimize the risk for a given level of expected return. The mean-variance framework requires knowledge of mean returns and the covariance-variance matrix to optimize the portfolios, and if one only consider risky assets, then the optimal portfolio will depend on the investor's risk preferences. These mean-variance optimal portfolios create the efficient frontier, which is illustrated graphically as the upper part of a hyperbola in a mean return-standard deviation space. Tobin (1958) extended the paper from Markowitz (1952) and illustrated that the introduction of a risk-free asset shifted the efficient frontier to the Capital Allocation Line, which represents a straight line from the risk-free rate to the tangent of the efficient frontier. This tangent point is known as the tangency portfolio and represents the optimal combination of risky assets in the presence of a risk-free asset.

In theory, the mean-variance model represents the optimal way of allocating capital, though the model has been criticized for its practical usefulness due to the difficulties of forecasting the model parameters. The mean-variance model treats the estimated parameters as true realizations, while they are actually estimated with uncertainty. Therefore, the practical implementation of the mean-variance model tends to generate extreme portfolio weights that are highly time-varying and delivers poor out-of-sample performance. Michaud (1989) refers to the mean-variance optimization model as "*error maximizers*" due to the large errors associated with the estimation of mean returns and the variance-covariance matrix. Several studies suggest that implementation of constraints, shrinkage estimators, and various extensions of the mean-variance strategy could reduce the impact of estimation error (see e.g. Chopra & Ziemba (1993), Jagannathan & Ma (2003), Ledoit & Wolf (2004)). In spite of these studies, DeMiguel et al. (2009) present results that question the value added by optimized portfolios relative to the naive diversification strategy. They thoroughly assess 14 mean-variance optimization strategies across seven empirical datasets and show that none of those strategies produce consistently better out-of-sample performance than the naive diversification strategy. The authors implement the mean-variance model and various extensions designed to reduce the impact of estimation error. Their results indicate that the implementation of various constraints only lead to modest improvement in the performance compared to the naive diversification. DeMiguel et al. (2009) conclude that the estimation error associated with the mean-variance strategies erodes much of the out-of-sample performance. As a result, the naive diversification was consequently proposed as an obvious benchmark strategy when assessing other sophisticated strategies, due to its low implementation costs and relatively good performance.

More recently, several studies claim to defend the superiority of optimized portfolios relative to the naive diversification. Kritzman et al. (2010) find that the minimum-variance and the mean-variance portfolio strategy provide superior performance compared to the naive diversification. Although, they conclude superiority of the mean-variance strategies without relying on any statistical tests of the difference in Sharpe ratio. Tu & Zhou (2011) develop new portfolio strategies that include the weighted combination of the optimized portfolio rules with the naive diversification rule and show that these combinations outperform the naive diversification. Kirby & Ostdiek (2012) develop two alternative mean-variance portfolio strategies based on the earlier work of Kirby, Ostdiek & Fleming (2001, 2003), namely the volatility-timing and the reward-to-risk timing strategy. These two portfolio strategies, which are devoted to mitigate the impact of estimation error, focus only on the assets volatility and returns, and ignores the correlation among the assets to optimize the portfolio weights. Kirby & Ostdiek (2012) evaluate the performance of these two strategies relative to the naive diversification across four preselected datasets and show that they outperform the naive diversification by economically and statistically margins. In line with Kirby & Ostdiek (2012), Stivers & Sun (2016) suggest that these strategies that only focus on the diagonal of the variance-covariance matrix could mitigate the estimation error, and consequently outperform the naive diversification. The authors evaluate the performance of three idiosyncratic-volatility strategies and the volatility-timing strategy proposed by Kirby & Ostdiek (2012) and illustrate that they outperform the naive diversification in terms of Sharpe ratio.

In another recent study conducted by Zakamulin (2017), the author provides a cautionary note regarding the use of Kenneth French datasets while measuring the performance by means of Sharpe ratio. The latter means that the authors do not consider the possibility that some of these optimized strategies provide superior performance simply due to tilting towards one or several profitable anomalies. Zakamulin (2017) first illustrates that the low-volatility effect is present in virtually all 17 datasets obtained from the Kenneth French online data library. Second, the author implements three optimization strategies, the minimum-variance portfolio, volatility-timing, and reward-to-risk timing strategy, and shows that these strategies on average provide higher Sharpe ratio relative to the naive diversification. In addition, Zakamulin (2017) demonstrates that the three strategies generate economically significant annualized CAPM alphas of 1.5-1.8%. When the author controls for an additional risk factor, the Fama-French HML factor, which can be viewed as a proxy for the low-volatility effect (see Blitz (2016)), the augmented 2-factor alphas of these strategies become neither economically nor statistically significant. Zakamulin (2017) concludes that to assess the mean-variance efficiency of optimized portfolios, it must be shown that the superior performance remains when controlling for known factor premiums.

2.2 Asset Pricing Theory

Based on the work by Markowitz (1952), Sharpe (1964), Lintner (1965), and Mossin (1966) developed the Capital Asset Pricing Model (CAPM), which aims to explain the relationship between risk and expected return. According to the CAPM, the portfolio's expected return can be expressed as the sum of the risk-free rate and the portfolio's risk exposure times the expected market risk premium. Thus, investors should be compensated in two ways for buying a portfolio, (i) Time value of money (r_f), (ii) and systematic risk associated with the investment. The CAPM is given by

$$E[r_p] = r_f + \beta_p(E[r_m] - r_f), \tag{2.1}$$

where $E[r_p]$ is the expected return of portfolio p, r_f is the risk-free rate, β_i is the market risk exposure for portfolio p, and $E[r_m]$ is the expected return for the market portfolio.

Several studies have later questioned the adequacy of the model. Early empirical tests of the Security Market Line illustrate that the relationship is flatter than expected by the CAPM (Black, Jensen & Scholes (1972)). In other words, portfolios of low beta stocks deliver higher risk-adjusted returns than predicted by the CAPM, whereas portfolios of high beta stocks provide lower risk-adjusted returns than predicted by the CAPM. A similar conclusion is drawn by Haugen & Heins (1975), where the authors find that the relationship between risk and return was not that straightforward as previously claimed. The criticism was mainly based on the fact that there was no distinct connection that increased risk would give an increased return (will be discussed in Section 2.3). Several other studies reveal other empirical shortcomings with the

CAPM by sorting stocks into portfolios depending on the stocks fundamental characteristics (see e.g. Basu (1977), Reinganum (1981), Banz (1981)). These portfolios provide higher returns than are justified by the CAPM and lead to the discovery of cross-sectional stock return patterns, such as the size and value anomalies.

During the early 1990s, Eugene Fama and Kenneth French published several papers regarding the construction of a multi-factor model that extends the CAPM with two factors, the size and value anomalies (Fama & French (1993)). The size factor, SMB ('Small-Minus-Big'), captures average returns of small-cap stocks relative to large-cap stocks, and value factor, HML ('High-Minus-Low'), captures average returns of value stocks relative to growth stocks. Previous empirical tests indicate that the model provides higher explanatory power in describing the cross-sectional stock returns (Fama & French (1993)). The three-factor model is defined as

$$E[r_p] = r_f + \beta_{p,1}(E[r_m] - r_f) + \beta_{p,2}E[SMB] + \beta_{p,3}E[HML], \tag{2.2}$$

where $E[SMB]$ and $E[HML]$ are the expected return of the size and value factors. The beta coefficients, $\beta_{p,2}$, and $\beta_{p,3}$ denote the exposure to the size and value factors for portfolio p, respectively. The three-factor model improves the ability to explain the cross-sectional stock returns relative to the CAPM, although it fails to describe the cross-sectional variations in portfolios sorted on momentum. Jagadeesh & Titman (1993) find that portfolios ranked on their previous price movements over the past 3-12 months are usually followed by price movements in the same direction. Based on the observations by Jagadeesh & Titman (1993), Carhart (1997) augment the Fama-French three-factor model with an additional factor, the one-year momentum factor (PR1YR).

More recently, Fama & French (2015) include two additional factors to improve the cross-sectional explanatory power of the existing three-factor model. The evidence of Novy-Marx (2013), Titman, Wei & Xie (2004), where they argue that the three-factor model fails to describe the cross-sectional variations related to profitability and investment, led Fama and French to include a profitability and investment factor to their existing three-factor model. The profitability factor, RMW ('Robust-Minus-Weak'), captures average returns of portfolios consisting of robust operating profitability compared to portfolios consisting of weak operating profitability. While, the investment factor, CMA ('Conservative-Minus-Aggressive') captures average returns of portfolios consisting of conservative (low) total asset growths relative to portfolios of aggressive (high) total asset growths. The Fama-French five-factor model is defined as

$$E[r_p] = r_f + \beta_{p,1}(E[r_m] - r_f) + \beta_{p,2}E[SMB] + \beta_{p,3}E[HML] + \beta_{p,4}E[RMW] + \beta_{p,5}E[CMA], \tag{2.3}$$

where $E[RMW]$ and $E[CMA]$ are the expected return of the profitability and investment factors. The beta coefficients, $\beta_{p,4}$, and $\beta_{p,5}$ denote the exposure to the profitability and investment factors for portfolio p, respectively. Fama & French (2016) show that the inclusion of two new factors consistently improve the model performance compared to the three-factor model, and is intended to capture the cross-sectional patterns in average stock returns left unexplained by the three-factor model. Fama & French (2016) further advocate that "*Positive exposures to RMW and CMA go a long way toward capturing the average returns of low-volatility stocks, whether volatility is measured in terms of total returns or residuals*" (Fama & French (2016) p.27).

2.3 Low-Volatility Anomaly

Contrary to a fundamental principle in finance, low-volatility portfolios have historically provided superior risk-adjusted returns compared to their riskier peers. The phenomenon is not new, and empirical studies have for a long period provided compelling evidence for its existence and persistence. Early empirical tests of the CAPM demonstrate that portfolios of low-beta stocks deliver higher risk-adjusted returns compared to portfolios of high-beta stocks (Black et al. (1972), Haugen & Heins (1975)). In more recent time, several studies document the superior performance earned by low-volatility portfolios. Haugen & Baker (1991) construct a minimum-variance portfolio of the 1000 largest U.S. stocks for the period 1972 to 1989 and find that the MVP consistently outperforms the market portfolio in terms of both higher returns and lower volatility. Chan, Karceski & Lakonishok (1999), Jagannathan & Ma (2003) and Clarke et al. (2006) present similar evidence and show the superior performance of a minimum-variance portfolio compared to a value-weighted index. Blitz & Van Vliet (2007) show that there exist a low volatility effect in the U.S., European and Japanese equity markets. The authors suggest a simple methodology approach to exploit the low volatility effect, by sorting stocks into decile portfolios ranked on volatility (beta). They find that there are still significant alphas remaining in low-volatility portfolios after controlling for size, value and momentum factors. In a follow-up article, Blitz, Pang & van Vliet (2013) report a low volatility effect for emerging equity markets, by demonstrating that the empirical relation of the risk-return trade-off is flat and sometimes negative. Frazzini & Pedersen (2014) construct a market neutral Betting-Against-Beta (BAB) factor, which has long exposure in low beta stocks and short exposure in high beta stocks, and demonstrate that this factor produces significant risk-adjusted returns.

Although research studies have provided compelling evidence for the existence of the low-volatility anomaly, there is a disagreement about the explanations behind the low-volatility effect. Some researchers argue from a behavioral standpoint. Baker et al. (2011) apply a similar ranking approach as Blitz & Van Vliet (2007) and find that regardless of the classification of risk, low-volatility portfolios outperform their riskier peers. The authors point to the benchmark hypothesis as to why the low-volatility effect persists. This builds on the idea that portfolio managers who are measured against a particular benchmark will have an incentive to overweight high-volatility stocks and underweight low-volatility stocks in an attempt to beat the benchmark.

Several other studies argue about the possibility that the low-volatility effect is merely a manifestation of other anomalies. Clarke, de Silva & Thorley (2006) show that the MVP provide superior performance over the market portfolio, and demonstrate that the MVP implicitly tend to tilt towards the value and size factors. Similarly, de Carvalho, Lu & Moulin (2012), and Goldberg & Geddes (2014) illustrate that the excess return of a minimum-variance strategy could largely be attributed due to exposures to the value factor. Blitz (2016) constructs a low-volatility risk factor on U.S. stocks, by following the methodology by Fama & French (1993). The author distinguishes between small-cap and large-cap low-volatility strategies, and evaluates the performance by means of the Sharpe ratio and mean excess returns for various subperiods from 1929 to 2014. Blitz, suggests that over half a century the low-volatility effect seemingly can be explained by the HML factor. Yet, the author clarifies that the performance of low-volatility strategies in some periods cannot be explained by exposure to the HML factor.

In an attempt to attribute the superior performance to known risk factors, Scherer (2011) uses the Fama-French three-factor model augmented with two characteristic low-volatility factors. Scherer (2011) shows that 83% of the variation from minimum variance strategies excess returns can be attributed to these five risk factors and concludes that optimized strategies that aim to minimize risk are nothing more than an inefficient way to capture various factor premiums. Chow, Hsu, Kuo & Li (2014) construct the MVP and four heuristic-based portfolios based on the risk-parity strategy. By using the Carhart four-factor model augmented with the Frazzini-Pedersen BAB factor, and a duration factor, Chow et al. (2014) attempt to identify the sources of return premium associated with low-volatility strategies. The authors show that factor analysis of low-volatility portfolios reveals that in excess of market-weighted return, returns are substantially driven by the exposure towards value, BAB, and duration premium. Novy-Marx (2014) shows that the low-volatility effect is explained by the Fama-French three-factor model augmented with a profitability factor, and concludes that "*High profitability is the single most significant predictor of low volatility*" (Novy-Marx (2014) p.2). Further on, Fama & French (2016) verify these results and illustrate that their new five-factor model can explain the returns on both low-beta sorted stocks as well as low-volatility sorted stocks. They argue that positive exposures to the two new factors RMW and CMA absorb the high average returns associated with low-beta and low-volatility portfolios that are left unexplained by the three-factor model. While the negative exposure to RMW and CMA absorb the low average returns associated with high-beta and high-volatility portfolios that are left unexplained by the three-factor model.

Existing literature surrounding portfolio optimization is a subject of conflict. First of, researchers arguably disagree whether portfolio optimization delivers superior performance relative to both the naive diversification, and a value-weighted portfolio, and whether the performance is persistence across time and geographical areas. Second, there exists a disagreement whether the outperformance of optimal portfolios (or risk-based strategies) is attributed to exposure to established factor premiums. Risk-based strategies that solely focus on minimizing risk arguably benefits from the low-volatility effect. Thus, the question arises whether the newly proposed Fama-French five-factor model can *explain* the superior performance generated by risk-based strategies.

3 | Data

3.1 Kenneth French Datasets

The data we use for the empirical analysis consist of 25 empirical datasets, which are obtained from the online data library of Kenneth French.[1] These datasets are similar to those used in previous studies by DeMiguel et al. (2009), Kritzman et al. (2010), Kirby & Ostdiek (2012), and Zakamulin (2017), as well as several other research studies surrounding portfolio optimization. The datasets include portfolios that are formed using different criteria, and contain stocks listed on the NYSE, AMEX, and NASDAQ with available equity data. The portfolios are value weighted and exhibit return series with a monthly frequency. The time periods for the empirical datasets varies, but are adjusted to cover the similar period from July 1963 to December 2016. The choice of the starting point is such that it coincides with the period used in previous studies. However, the length of the period is extended due to newly accessible data in the online data library.

Table 1 reports an overview of the empirical datasets, which includes the dataset number, abbreviation, and the number of portfolios in each dataset. The first 15 empirical datasets contain

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table 1: Kenneth French datasets

The table presents all the empirical datasets that are obtained from the Kenneth French data library. # Denotes the dataset number, and **N** denotes the total number of portfolios included in each dataset.

#	Dataset	Abbreviation	N
1	Portfolios formed on Size	Size	10
2	Portfolios formed on Book-to-Market	BookM	10
3	Portfolios formed on Operating Profitability	OP	10
4	Portfolios formed on Investment	Inv	10
5	Portfolios formed on Momentum	Mom	10
6	Portfolios formed on Short-Term-Reversal	ShortTR	10
7	Portfolios formed on Long-Term-Reversal	LongTR	10
8	Portfolios formed on Accruals	Acc	10
9	Portfolios formed on Market Beta	MktB	10
10	Portfolios formed on Net-Share-Issues	NSI	10
11	Portfolios formed on Variance	Var	10
12	Portfolios formed on Residual Variance	ResVar	10
13	Portfolios formed on Earnings-to-Price	E-P	10
14	Portfolios formed on Cashflow/Price	CF-P	10
15	Portfolios formed on Dividend Yield	Div-Y	10
16	Portfolios formed on Industry	Ind	10
17	Portfolios formed on 30 Industry	30Ind	30
18	Portfolios formed on Size and Book-to-Market	Size-BM	25
19	Portfolios formed on Size and Operating Profitability	Size-OP	25
20	Portfolios formed on Size and Long-Term-Reversal	Size-LTR	25
21	Portfolios formed on Size and Momentum	Size-MOM	25
22	Portfolios formed on Size and Investment	Size-INV	25
23	Portfolios formed on Operating Profitability and Investment	OP-INV	25
24	Portfolios formed on Book-to-Market and Operating Profitability	BM-OP	25
25	Portfolios formed on Book-to-Market and Investment	BM-INV	25

10 portfolios, where all stocks are sorted into decile portfolios based on univariate sorts. The two following empirical datasets include stocks that have been sorted based on industries. The first industry-dataset consists of 10 portfolios, whereas the second industry includes 30 industry portfolios. The rest of the datasets have been constructed by sorting stocks into 25 portfolios based on bivariate sorts.

3.2 Risk Factors

The return series of the risk factors in the Fama-French five-factor model are also collected from the online data library by Kenneth French. These return series include the MKT (excess return of the market portfolio), SMB (Small-Minus-Big), HML (High-Minus-Low), RMW (Robust-Minus-Weak) and CMA (Conservative-Minus-Aggressive), and are constructed by value-weighted portfolios of stocks listed on the NYSE, AMEX, and NASDAQ. The MKT factor represents the

Table 2: Descriptive statistics of the factor returns

The table provides summary statistics for the risk-factors in the Fama-French five-factor model over the period July 1973 to December 2016. **MKT** listed in column 1 represents the excess return on the market portfolio, **SMB** and **HML** are listed under column 2 and 3, and represent the size and value factors. Column 4 and 5 show **RMW** and **CMA**, which are the profitability and investment factor, respectively.

	MKT	**SMB**	**HML**	**RMW**	**CMA**
Summary statistics					
Mean return	6.86	3.31	4.46	3.23	4.22
Standard deviation	15.85	10.50	10.19	8.20	6.86
Skewness	-0.56	0.38	0.05	-0.39	0.36
Kurtosis	2.01	4.27	2.12	12.67	1.81

excess return on the market portfolio. The one-month Treasury bill rate represents the risk-free rate of return. The size factor SMB reflects the average return of portfolios with small-cap stocks minus the average return of portfolios with large-cap stocks. The value factor HML displays the average return of portfolios with high book-to-market stocks (value stocks) minus the average return of portfolios with low book-to-market stocks (growth stocks). The profitability factor RMW shows the average return of portfolios with robust operating profitability stocks minus the average return of portfolios that contain weak operating profitability stocks. The investment factor CMA reflects the average return of portfolios with conservative investment stocks minus the average return of portfolios with stocks that invest aggressively.

Table 2 presents the descriptive statistics for the various risk factors for the sample period July 1973 to December 2016. It is evident from Table 2 that the MKT and HML factors provide the two largest mean returns of 6.86% and 4.46%, respectively. While the SMB, RMW, and CMA factors provide values of 3.31%, 3.23%, and 4.22%, respectively. Panel B displays the standard deviation, and we note that the MKT present the highest standard deviation of 15.85%. The factors SMB, HML, RMW, and CMA display standard deviations of 10.50%, 10.19%, 8.20%, and 6.86%, respectively. Figure 3.2 plots the logarithmic cumulative return for the market portfolio[2], for the period July 1973 to December 2016. In addition, the gray shaded areas represent the bear periods (we describe the detection of turning points between bull and bear phases in Section 4).

[2]The market portfolio is given by $MKT + r_f$.

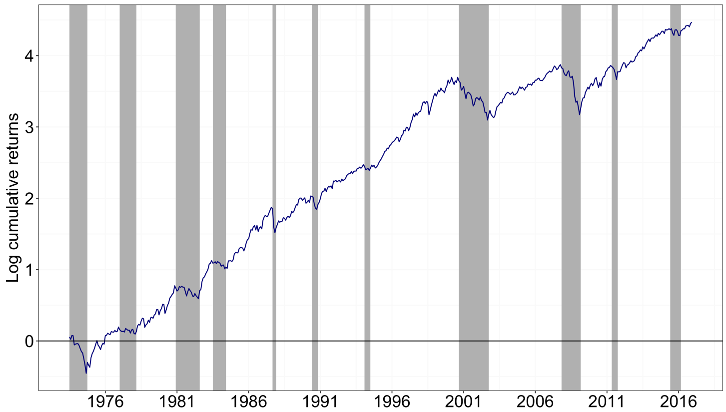

Figure 1: Bull and bear markets in the U.S. stock market
The figure illustrates the cumulative return for the value-weighted market portfolio for the time period July 1973 to December 2016. The detection of turning points between the bull (white areas) and bear phases (grey areas) are obtained using the dating algorithm of Bry & Boschan (1971).

4 Methodology

This section presents a description of the methods we use for the empirical analysis. First, in Subsection 4.1, we describe the various portfolio strategies. Specifically, two optimal portfolio strategies that are considered in the literature, two ad-hoc strategies, and additionally the naive diversification strategy, which we use as a benchmark strategy. In Subsection 4.2, we describe the out-of-sample procedure to estimate the parameter inputs in order to simulate the performance of the risk-based strategies. The description of the various portfolio performance measures is described in Subsection 4.3. In Subsection 4.4, we present the statistical tests that we use for the portfolio performance. Subsection 4.5 includes a description of the methodology to identify turning points between the bull and bear phases. Last, in Subsection 4.6, we present the methodology we use to evaluate the risk exposure across the empirical datasets. The free programming language R have been used to construct, implement and analyze the performance of the various portfolio strategies.[1]

[1] https://www.r-project.org

4.1 Portfolio Strategies

4.1.1 Minimum-Variance Portfolio

The general prescription of the mean-variance optimization model is to minimize the risk for a given level of expected return, or vice versa, maximize the expected return for a given level of risk. While each of the various optimal portfolios located on the efficient frontier requires estimates of mean returns, have MVP the unique property of minimizing the risk without relying on mean returns. Illustrated graphically, is the MVP located at the left-most tip on the efficient frontier. There exist two different approaches that lead to the solution for the optimal MVP weights. The general approach is to derive the solution directly from the optimal weights of the mean-variance portfolio under the assumption that all of the mean returns are equal. The optimal weights for the mean-variance portfolio are then reduced to the optimal solution for the MVP, and will thereby produce the highest Sharpe ratio. This can be justified on the following ground. The Sharpe ratio is given by $SR = \frac{\mu - r_f}{\sigma}$, where μ and σ are the mean return and standard deviation for portfolio p. if μ is equal for all the assets, then minimizing σ will be the only way to increase the Sharpe ratio.

The second approach is to find the weights for asset i, which provides the portfolio with the lowest risk. To find the weights of the respective assets, we solve the following minimization problem

$$\min_{w} \quad \frac{1}{2} w' \Sigma w, \quad \text{s.t.} \;\; w' 1 = 1, \tag{4.1}$$

where w is an $N \times 1$ vector of portfolio weights, Σ is an $N \times N$ covariance matrix, 1 is an $N \times 1$ vector of ones, and $w'1 = 1$ is the budget constraint. Solving this minimization problem leads to the following solution

$$\omega^{mvp} = \frac{\Sigma^{-1} 1}{1' \Sigma^{-1} 1}, \tag{4.2}$$

where ω^{mvp} is a vector of weights for the MVP.

The above-mentioned solution for the MVP weights is a *closed-form* solution. Hence, the solution is in the absence of short-sale restrictions. To obtain the weights for the MVP with short-sale restrictions, we solve the following minimization problem

$$\min_{w} \quad \frac{1}{2} w' \Sigma w, \quad \text{s.t.} \;\; w' 1 = 1, \;\; \text{and} \;\; w_i \geq 0, \tag{4.3}$$

where $w_i \geq 0$ assures non-negativity in the asset weights. To estimate the optimal weights for the

MVP in the presence of short-sale restrictions, we obtain the results through a numerical solution.[2] We impose short-sale restrictions to create our results comparable to the recent literature (DeMiguel et al. (2009), Kirby & Ostdiek (2012), Zakamulin (2017)).[3]

4.1.2 Volatility-Timing Strategy

Kirby & Ostdiek (2012) introduce two new methods of mean-variance portfolio selection, namely the volatility-timing strategy (VT), and the reward-to-risk timing strategy. The VT strategy uses sample information about the assets conditional variance to determine the portfolios weights, while the reward-to-risk timing strategy incorporates information about conditional means. However, since the mean returns are prone to larger estimation error than variances, and we only consider risk-based strategies in this thesis, we will only use the VT strategy. According to Kirby & Ostdiek (2012), there are 4 notable features that characterize the VT strategy: (i) First, it does not require optimization, (ii) Second, it does not require covariance matrix inversion, (iii) Third, it assures non-negative weights, (iv) Fourth, through volatility changes, the sensitivity of the portfolio weights can be adjusted with a tuning parameter.

Kirby & Ostdiek (2012) show that if one assumes that all pair-wise correlations between the assets are 0 (i.e. the covariance matrix becomes a diagonal matrix), then the weights for the MVP is given by

$$\omega_i^{MVP} = \frac{(1/\sigma_i^2)}{\sum_{i=1}^{N}(1/\sigma_i^2)}, \tag{4.4}$$

where σ_i^2 is the estimated conditional variance of the excess return on asset i. Therefore, if the covariance matrix remains diagonal for all t, then the MVP will be equivalent to a very simple volatility-timing strategy. Although Kirby & Ostdiek (2012) do not expect the covariance matrix to be diagonal, they explain that by setting the pair-wise correlations to zero might perform better than using the full covariance matrix. To facilitate the possibility of determining how the portfolio weights respond to volatility changes, they propose the following strategy

$$\omega_i^{VT} = \frac{(1/\sigma_i^2)^\eta}{\sum_{i=1}^{N}(1/\sigma_i^2)^\eta}, \tag{4.5}$$

where ω_i^{VT} is the weight for asset i, σ_i^2 is the estimated conditional variance of the excess return on asset i, and η is a tuning parameter that determines the aggressiveness of rebalancing the portfolio weights due to volatility changes. As $\eta \to 0$, the VT weights will approach the portfolio weights of the naive diversification. While, as $\eta \to \infty$, the weight on the lowest-volatility asset

[2] A quadratic programming solver, *quadprog*, has been used to obtain the weights of the MVP.

[3] Transaction costs and taxation related to monthly rebalancing and capital gains will also be disregarded.

will approach 1. Kirby & Ostdiek (2012) illustrate that these two strategies outperform the naive counterpart with $\eta \in (2, 4)$. Following the approach by Zakamulin (2017), we set $\eta = 4$. Kirby & Ostdiek (2012) argue that the VT strategy outperforms the naive diversification due to the two following features: First, due to its simplicity and long-only weights, the VT strategy is less prone to estimation risk. Second, by increasing the value of the tuning parameter, η, above unity, will decrease the portfolio's turnover and transaction costs.

4.1.3 Ad-Hoc Strategies

To illustrate the point made by Zakamulin (2017), that one can create rather simple portfolio strategies that outperform the naive diversification, we construct two ad-hoc strategies. These two strategies, which aim to exploit the low-volatility effect, are referred to as the Low-Volatility and Low-Beta strategies. In contrast to previous methods (MVP, VT), these two strategies do not optimize the weights for all the available assets. Rather, the principle behind these two strategies is to concentrate the weights on a few assets with the lowest volatility. The first strategy assigns weights based on the assets inverse standard deviation, and the other strategy assigns weights based on the assets inverse CAPM beta. Since low-volatility stocks also tend to be low-beta stocks and vice versa, we will expect these two portfolio strategies to produce relatively similar results.

To illustrate the portfolio weighting procedure, assume we have ten assets in our investment universe. First, we estimate the standard deviation (beta) for each asset and then filter out assets with standard deviation (beta) larger than q, where q is the lower 30% quantile threshold value. Such that the remaining assets with standard deviation (beta) less than q, will be included in the respective portfolio strategies. For this illustration, the lower 30% quantile will consist of 3 assets. These three assets will then be weighted according to their inverse standard deviation (beta). The two portfolios are rebalanced monthly such that the assets that were included one month, will be dropped the next period if the volatility (beta) is above this threshold value. Similarly, with the assets that were excluded one month, will be included the next period if the volatility (beta) is below this threshold value.

To construct the Low-Volatility ($1/\sigma$) strategy, we first estimate the standard deviation of each asset, σ_i, where $i = \{1, 2, \ldots, N\}$, and N denotes the total number of assets in the respective dataset. We then compute the threshold value, q, based on the vector of standard deviations, and filter out assets with $\sigma_i < q$. Let N_τ denotes the remaining assets with $\sigma_i < q$. The

Low-Volatility ($1/\sigma$) strategy is given by

$$\omega_i^{1/\sigma} = \frac{(1/\sigma_i)}{\sum_{i=1}^{N_\tau}(1/\sigma_i)}, \tag{4.6}$$

where $\omega_i^{1/\sigma}$ is the weight for asset i, and σ_i is the estimated standard deviation of asset i.

Similarly, for the Low-Beta ($1/\beta$), we first estimate the CAPM for each asset by using Equation 4.12, and then obtain the beta $\beta_{i,M}$, where $i = \{1, 2, \ldots, N\}$, and N denotes the total number of assets in the respective dataset. We then compute the threshold value q, for the vector of betas, and filter out the assets with $\beta_{i,M} < q$. Let N_τ denote the remaining assets with $\beta_{i,M} < q$. The Low-Beta ($1/\beta$) strategy is then determined by

$$\omega_i^{1/\beta} = \frac{(1/\beta_{i,M})}{\sum_{i=1}^{N_\tau}(1/\beta_{i,M})}, \tag{4.7}$$

where $\omega_i^{1/\beta}$ is the weight for asset i, and $\beta_{i,M}$ is the estimated CAPM beta of asset i.

4.1.4 Naive Diversification

The naive diversification strategy, which is also referred to as the equally-weighted portfolio, is constructed such that each asset is equally weighted. The naive diversification does not require any optimization or parameter estimates and is therefore not affected by the impact of estimation error. Consequently, the naive diversification produces comparable, and often better out-of-sample performance relative to other more complex strategies (DeMiguel et al. (2009)). The naive diversification is mean-variance efficient if the asset returns and the volatility are equal for all assets, and if all the pair-wise correlations are similar. Thus, the idea behind the naive diversification makes sense if one belives that the model parameters can not be forecasted (de Carvalho et al. (2012)).

DeMiguel et al. (2009) suggest two reasons to why the naive strategy is preferred as benchmark: First, it is easy to implement in practice and produce low implementation costs, and Second, despite the huge contribution to the field of portfolio optimization, investors still rely on simple allocation rules to allocate their wealth across assets. The naive strategy will be used as a benchmark to assess the relative performance produced by the various risk-based strategies, and is given by

$$\omega_i^{1/N} = \frac{1}{N}, \tag{4.8}$$

where $\omega_i^{1/N}$ is the weight of asset i, and N represents total number of assets in the respective dataset.

4.2 Estimation Procedure

To calculate the portfolio weights for the respective risk-based strategies, we use a rolling window approach to simulate the out-of-sample performance. This procedure verifies how the various portfolios strategies would have performed during a specific time period. The out-of-sample approach is implemented as follows: Consider a sample with a total number of monthly returns T. The full sample is divided into an in-sample period, and an out-of-sample period. The split between the in-sample and out-of-sample period occurs at time t. The historical period from the start of the sample period 1 to t is used as a look-back period, denoted M, to estimate the model parameters for the portfolio strategy. At the end of time t, we use the input parameters from 1 to t to determine the relevant portfolio weights that are held for the period t to $t+1$. These weights are then used to compute the return in month $t+1$. For the next period, at the end of $t+1$, the new portfolio weights will be adjusted based on the estimation of the model parameters from 2 to $t+1$, and held until the end of $t+2$. This process is then continued by including the next month while discarding the earliest month. This way, the portfolio weights are rebalanced monthly by using the lookback period of length M to estimate the model parameters. This rebalancing process continues until we have a total of $T-M$ out-of-sample returns. Following Zakamulin (2017), DeMiguel et al. (2009), and Kirby & Ostdiek (2012), we set $M = 120$ months as the look-back period to estimate the model parameters. Consequently, the split between the in- and out-of-sample occurs at July 1973. I.e. the first ten years of our sample will be the start of our look-back period.

The estimation error associated with forecasting of the model parameters could impact the out-of-sample performance of the risk-based strategies. Several studies attempt to reduce the impact of estimation error. Chopra & Ziemba (1993) show that misspecification of mean returns could reduce the performance of mean-variance portfolios in a substantial way. The authors argue that the errors obtained from the forecasted mean returns are about ten times as important as the errors obtained from the covariance matrix. They suggest that by removing the mean return input could increase the performance of the mean-variance portfolios, due to the removal of the "*error-in-means*" problem. Jagannathan & Ma (2003) introduce short-selling restrictions to a minimum-variance portfolio and show that this reduces the estimation error. Ledoit & Wolf (2004) propose a shrinkage method to decrease the estimation error of the sample covariance matrix. They suggest that the standard statistical method of estimating the covariance matrix tends to contain errors in the most extreme coefficients. Their shrinkage approach pull extremely high (low) coefficients in the covariance matrix downwards (upwards), and will thereby approach the constant-correlation matrix. The reasoning behind this is that those estimated coefficients in the sample covariance matrix that are extremely high (low) tend to contain a lot of positive (negative) error, and therefore need to be pulled downwards (upwards) to compensate for that. On the contrary, Zakamulin (2015) shows that the shrinkage approach by Ledoit & Wolf (2004) is a computationally intensive method that is unable to reduce the forecasting error nor the tracking error. Therefore, we use the sample covariance-matrix as a predictor for the future covariance-matrix to create comparable results to recent literature (DeMiguel et al. (2009), Kirby & Ostdiek (2012), Zakamulin (2017)).

4.3 Performance Measures

This subsection covers the description of the performance measures we use to evaluate the performance of the various portfolio strategies. This includes a description of the Sharpe ratio, maximum drawdown, factor models, and the Dual Beta Model. We present a description of the statistical tests in the Subsection 4.4.

4.3.1 Sharpe Ratio

The Sharpe ratio introduced in Sharpe (1966) is a risk-adjusted performance measure that is used to evaluate portfolio strategies. Its simplicity and ability to compare the performance of portfolios with different risk exposures are the reason why many favor the technique. However, the Sharpe ratio also has its limitations; it can give a misleading indication if the returns are not normally distributed, weighing the downside risk equally as the upside potential, and it does not control for the risk-based explanations of the performance. Despite its limitations, the Sharpe ratio remains the industry standard risk-adjusted performance measure. The estimated monthly Sharpe ratio is computed according to

$$SR_p^M = \frac{\mu_p - r_f}{\sigma_p}, \tag{4.9}$$

where μ_p and σ_p are the monthly out-of-sample mean return, and standard deviation of portfolio strategy p, respectively. r_f is the risk-free rate of return. The annualized out-of-sample Sharpe ratio is computed based on the monthly Sharpe ratio for portfolio p and can be expressed by

$$S\bar{R}_p = SR_p^M \sqrt{12}. \tag{4.10}$$

4.3.2 Maximum Drawdown

Maximum drawdown reflects the maximum accumulated loss during a specific time period, and provides an indication of the portfolio strategies downside risk. It measures the portfolio strategies largest peak-to-trough decline in value and is quoted as the percentage of the peak value. The maximum drawdown is given by[4]

$$MD = \left[\frac{max_{\tau \in 0,t}(W_\tau - W_t)}{W_\tau} \right], \tag{4.11}$$

where W_t is the value of the portfolio at time $t = \{1, 2, \ldots, T\}$. Note that $\tau \leq t$, which ensures that the peak occurs before the trough during the specific time period.[5]

4.3.3 Factor Models

The CAPM and Fama-French five-factor model is our two performance attribution models. We include the CAPM to show that when one omit known anomalies, the risk-based strategies will generate positive alpha. While the five-factor model is able to control for five known market anomalies that cannot be explained by the CAPM. Various studies show that there remain positive alphas that are unexplained by the exposure to risk factors in the Fama-French three- and Carhart four-factor models. For instance, Frazzini & Pedersen (2014) show that the Carhart four-factor model is not suitable to describe the cross-sectional returns of low-volatility stocks, and Novy-Marx (2014) finds similar results by using the three-factor model. Fama & French (2015) present the new Fama-French five-factor model, which add a profitability (RMW), and an investment (CMA) factor to their three-factor model. The authors test the model on the U.S stock market and find that the two additionally factors improve the explanatory power of the cross-sectional stock returns relative to the three-factor model. The CAPM is given by[6]

$$R_{p,t} = \alpha_p + \beta_{p,1} R_{M,t} + \epsilon_{p,t}, \tag{4.12}$$

where $R_{p,t}$ is the excess return of portfolio p in period t. The intercept α_p, is the pricing error relative to portfolio p's exposure to the market factor. $R_{M,t}$ is the excess market risk premium and $\beta_{p,1}$ is the portfolio's exposure to the systematic risk component, the market factor. Finally, $\epsilon_{p,t}$ denotes the error term and represents the idiosyncratic risk unexplained by the model.

The Fama-French five-factor model allows us to control and further attribute the performance

[4] Definition adopted from Chekhlov & Zabarankin (2005)

[5] The R-code used to obtain the maximum drawdown is provided by Valeriy Zakamuline

[6] More correctly, we use the single-index model, which is a practical implementation of the CAPM. For the sake of simplicity will we only refer to the CAPM, instead of the single-index model.

towards five established factor premiums. The five-factor model is given by

$$R_{p,t} = \alpha_p + \beta_{p,1} R_{M,t} + \beta_{p,2} SMB_t + \beta_{p,3} HML_t + \beta_{p,4} RMW_t + \beta_{p,5} CMA_t + \epsilon_{p,t}, \tag{4.13}$$

where $R_{p,t}$ is the excess return of portfolio p in period t. The intercept α_p, is the pricing error relative to portfolio $p's$ exposure to the systematic risk factors. $R_{M,t}$ is the excess market risk premium, SMB_t and HML_t are the size and value risk factors from Fama & French (1993). RMW_t, CMA_t are the profitability and investment factors introduced by Fama & French (2015), and $\beta_{p,1}$, $\beta_{p,2}$, $\beta_{p,3}$, $\beta_{p,4}$, $\beta_{p,5}$, are the corresponding exposures to the systematic risk factors for portfolio p. Finally, $\epsilon_{p,t}$ denotes the error term and represents the idiosyncratic risk unexplained by the model.

4.3.4 Dual Beta Model

The Dual Beta Model is used to differentiate the model parameters in bull and bear markets and is inspired by the study of Bhardwaj & Brooks (1993). The fundamental principle behind the Dual Beta Model is that there exists a time-varying risk exposure, which is in contrast to the CAPM that assumes constant risk exposure over time. The Dual Beta Model is constructed to statistically test the difference of the parameter estimates in bull and bear markets. i.e. If there exist a time-varying relationship between return and risk across different states of the economy. This way, the Dual Beta Model allow us to get a deeper insight into the nature of the performance, and assess whether the CAPM alpha of the optimal strategies is mostly generated in bear compared to bull markets.

The construction of the Dual Beta Model starts with the dating of bull and bear markets in order to create the dummy variable. We will elaborate on our choice for detecting turning points between bull and bear markets in Subsection 4.5. The Dual Market Beta Model is defined as

$$R_{p,t} = \alpha_1 + \alpha_2 \cdot D_t + \beta_1 \cdot R_{M,t} + \beta_2 \cdot R_{M,t} \cdot D_t + \epsilon_{p,t}, \tag{4.14}$$

which is equivalent to

$$R_{p,t} = \alpha_{p,bull} + (\alpha_{p,bear} - \alpha_{p,bull}) \cdot D_t + \beta_{p,bull} \cdot R_{M,t} + (\beta_{p,bear} - \beta_{p,bull}) \cdot R_{M,t} \cdot D_t + \epsilon_{p,t} \tag{4.15}$$

where $R_{p,t}$ is the excess return of portfolio p, $R_{M,t}$ is the excess market return in month t, and D_t is a dummy variable equal to one for bear months, and zero for bull months. The estimates α_1 and β_1 represent the average risk-adjusted return and the systematic risk exposure for the market in bull periods, respectively. The estimates α_2, and β_2 determine whether the average risk-adjusted return and systematic risk for a given portfolio differ in bull and bear markets.

4.4 Statistical Inference

This subsection describes the testing procedure we use to statistically test the various performance measures presented in the subsection above. Statistical inference usually starts with choosing an appropriate type of test, which is either standard parametric- or non-parametric tests. The former tests, which include tests such as the Student's t-test, are easier to implement, and faster to compute. However, the limitations of using a parametric test procedure surround the strong distributional assumptions of the data. The test statistics might produce inaccurate inference if the data deviate from, e.g. a normally distributed population.

In contrast to parametric tests, non-parametric tests make no assumption of the probability distribution function, and provide an alternative way of obtaining the distribution function of a parameter under investigation and could significantly provide more accurate inference (Brooks (2008)). However, the limitations of non-parametric tests are the lack of statistical power if the normality assumptions of the corresponding parametric method hold. In Appendix 1, we present the normalized moments and the Shapiro-Wilk test for normality. The Shapiro-Wilk test reveals that we reject the null hypothesis of normality for all portfolio strategies on each dataset. Consequently, we use only non-parametric tests to statistically test the various performance measures and the parameters from the regression models.

4.4.1 Statistical Test for the Sharpe Ratio

In terms of Sharpe ratio, we compute the difference in the Sharpe ratio $\Delta SR = SR_p - SR_{1/N}$, where SR_p and $SR_{1/N}$ denote the annualized out-of-sample Sharpe ratio for the risk-based strategy p and the naive diversification, respectively. We then formulate the null hypothesis that this difference, ΔSR, is non-positive

$$H_0 : \Delta SR \leq 0 \quad \text{versus} \quad H_1 : \Delta SR > 0.$$

Lately, Jobson & Korkie (1981) test with Memmel (2003) correction hav been the preferred way of statistically testing the difference in the Sharpe ratios. The test statistic is given by

$$z = \frac{SR_p - SR_{1/N}}{\sqrt{\frac{1}{T} \cdot [2(1-\rho) + \frac{1}{2}(SR_p^2 + SR_{1/N}^2 - 2SR_pSR_{1/N}\rho^2)]}}, \tag{4.16}$$

where z is a standard normally distributed test statistic. SR_p, and $SR_{1/N}$ is the monthly out-of-sample Sharpe ratio for the risk-based strategy p and the naive diversification, respectively.

While ρ is the correlation coefficient between the two portfolios. This parametric test is fairly easy to implement, and several authors use this approach (see e.g. DeMiguel et al. (2009), Stivers & Sun (2016), Zakamulin (2017)). However, this approach assumes normality in both return series, and the test statistic is asymptotically distributed as a standard normal. The model's strong underlying assumptions decreases the power of statistical inference when the data deviate from normality. Therefore, using Jobson & Korkie (1981) test with Memmel (2003) correction could provide inaccurate inference, and thus, it will be more suitable to chose a non-parametric test to evaluate the difference in the Sharpe ratios.

We therefore use the stationary block bootstrap procedure by Politis & Romano (1994) to obtain the distribution of ΔSR. Bootstrapping is used to obtain a description of the model estimators by using the data points themselves, and it involves resampling repeatedly with replacement from the actual data (Brooks (2008)). The block bootstrap procedure is particularly useful when there exists serial dependence or non-normality in the data. The block bootstrap procedure contrary to the standard bootstrap procedure draws random blocks of data instead of one-by-one.[7] This way, the dependence of time series remains intact, while the standard bootstrap destroys the dependence. Opposed to overlapping and non-overlapping block bootstrap procedure which defines a fixed block length, the stationary bootstrap generates blocks of random length, where each block length is generated from a geometric distribution. To choose the appropriate average block length, we use the method proposed by Politis & White (2004) with the correction made in Patton, Politis & White (2009).[8]

Let $X_i = \{R_{p,t}, R_{1/N,t}\}$ denote the observed pairs of excess return for portfolio p and the naive strategy, where $t = \{1, 2, \ldots, T\}$. Let b denote the index for the bootstrap number, and l^b denotes the block length of b. Since the block length is not fixed, the block length l^b is generated from a geometric distribution with probability q. Let $B_b = (X_i, \ldots, X_{i+l^b-1})$, be the block containing l observations starting from X_i, where the Bth block begins from a random index i thas is generated from the discrete uniform distribution on $\{1, \ldots, T\}$. Then the procedure consists of choosing blocks $B_1^*, B_2^*, \ldots, B_b^*$ by randomly resampling with replacement from the available blocks $B_1, B_2, \ldots, B_K$, where K denotes the number of blocks. The process produces a new paired pseudo time-series with the same number of observations as the original sample. In each iteration, we calculate ΔSR, which is the difference between $\{R_{p,t}\}$ and $\{R_{1/N,t}\}$. After N bootstrap simulations, we obtain an approximation of the probability distribution of ΔSR.[9] To test the null hypothesis that the difference is non-positive, let n denote the number of times

[7]The standard bootstrap method first introduced by Efron (1979)

[8]The R-package *np* successfully employs the implementation of the method.

[9]We set $N = 10.000$ iterations to obtain the distribution.

ΔSR is less than zero. The p-value is then calculated as $\frac{n}{N}$. Decision rule: P-value < significance value α, then reject H_0, else fail to reject H_0.[10]

4.4.2 Statistical Test for Regression Models

In terms of the regression output we obtain from the CAPM and the Fama-French five-factor model, let α_p denote the estimated alpha for the risk-based strategy p, and $\beta_{p,j}$ denote the estimated beta of factor j. The null and alternative hypotheses is given by

$$H_0 : \alpha_p = 0 \quad \text{versus} \quad H_1 : \alpha_p \neq 0$$
$$H_0 : \beta_{p,j} = 0 \quad \text{versus} \quad H_1 : \beta_{p,j} \neq 0.$$

Similarly, given the estimated parameters in the Dual-Beta Model, let $\Delta\alpha_p = \alpha_{p,bear} - \alpha_{p,bull}$, where $\alpha_{p,bear}$ and $\alpha_{p,bull}$ provide the estimated alpha in bear and bull periods for portfolio p, respectively. Let $\Delta\beta_p = \beta_{p,bear} - \beta_{p,bull}$, where $\beta_{p,bear}$ and $\beta_{p,bull}$ provide the estimated market exposure in bear and bull periods for portfolio p, and consequently, Δ gives the difference in the respective parameters. We then carry out the following hypothesis test where we test if the model parameters are statistically different from zero in the Dual-Beta Model. The null and alternative hypotheses is given by

$$H_0 : \Delta\alpha_p = 0 \quad \text{versus} \quad H_1 : \Delta\alpha_p \neq 0$$
$$H_0 : \Delta\beta_p = 0 \quad \text{versus} \quad H_1 : \Delta\beta_p \neq 0.$$

We employ a non-parametric residual-resampling bootstrap procedure for accurate statistical inference when we evaluate the respective model parameters. For statistical inference through standard regression models to be accurate, the underlying assumption of normality in the disturbance term ϵ, must be fulfilled. Table 7 in Appendix 2 reveals that we reject the null hypothesis for normality of each regression model considered in this thesis. There exist several reasons as to why the residual resampling approach is the appropriate method for our study. For instance, when there is nonlinearity, non-constant variance or outliers in the underlying data – these properties will not be carried over into the resampled data sets (Fox & Weisberg (2011)). To simplify the residual-resampling bootstrap procedure for the reader, we illustrate the implementation for the CAPM.[11]

First, for the risk-based strategy p, we estimate the CAPM by using Equation 4.12, and obtain the fitted values $\hat{r}_{p,t}$, and the estimated residuals $\hat{\epsilon}_{p,t}$, where $t = \{1, 2, \ldots, T\}$. Next,

[10] We use the R package *boot* to construct the stationary block bootstrap procedure.
[11] This approach is partially adopted from Brooks (2008).

we draw a sample with replacement from the residuals of portfolio p, which we refer to as $\hat{\epsilon}^b_{p,t}$, where b denotes the index for the bootstrap number. Next, we generate a dependent bootstrap variable by adding the fitted values $\hat{r}_{p,t}$, to the bootstrapped residuals, $\hat{r}^b_{p,t}$=$\hat{r}_{p,t} + \hat{\epsilon}^b_{p,t}$. We then regress this new dependent variables on the original data $R_{M,t}$, to obtain pseudo time-series of the monthly excess return of portfolio p

$$\hat{r}^b_{p,t} = \hat{\alpha}_p + \hat{\beta}_{mkt,p} R_{M,t} + \hat{\epsilon}_{p,t}. \tag{4.17}$$

For each iteration, we obtain two bootstrapped coefficients, $\hat{\alpha}^B_p$ and $\hat{\beta}^B_{mkt,p}$, and after N bootstrap simulations we obtain an approximation of the probability distribution of the two estimates.[12] The distribution is then used to estimate the bootstrapped standard error $\hat{SE}^*$, and we compute the $\hat{SE}^*$ as the standard devation of the bootstrapped coefficients. Given the standard error values, we calculate the new respective t-test statistics for each parameter in order to statistically test the null hypothesis

$$t_{\alpha_p} = \frac{\hat{\alpha}^B_p}{\hat{SE}^*(\hat{\alpha}^B_p)}, \quad t_{\beta_{mkt,p}} = \frac{\hat{\beta}^B_{mkt,p}}{\hat{SE}^*(\hat{\beta}^B_{mkt,p})}, \tag{4.18}$$

where t_{α_p} is the new alpha t-statistics, and $t_{\beta_{mkt,p}}$, is the new beta t-statistics for portfolio p. $\hat{SE}^*(\hat{\alpha}_p)$, and $\hat{SE}(^*\hat{\beta}_{mkt,p})$, are the new standard error estimates obtained from the bootstrap procedure. The respective P-values of the t statistics are then obtained. If P-value(t) $<$ significance value α, then reject H_0, else fail to reject H_0.[13]

4.4.3 Wilcoxon Signed-Rank Test

The two sample Wilcoxon test is a non-parametric test that is used to test for differences in the mean of paired observations, and is appropriate to use when the data deviate from normality. Let $\Delta\mu^k_p = \mu^k_p - \mu^k_{1/N}$, denote the difference in mean return, where k represents either bull or bear period, μ^k_p is the mean return for the risk-based strategy p, and $\mu^k_{1/N}$ is the mean return for the naive diversification. We then specify the null hypothesis that the difference in mean return of the risk-based strategy p to the naive diversification is non-positive

$$H_0 : \Delta\mu^k_p \leq 0 \quad \text{versus} \quad H_1 : \Delta\mu^k_p > 0.$$

Let $X_i = \{r_{p,t}, r_{1/N,t}\}$ denote the observed pairs of return for portfolio p and the naive strategy, where $t = \{1, 2, \ldots, T\}$. For each pair X_i, compute the absolute difference $|r_p - r_{1/N}|$, and $sgn(r_p - r_{1/N})$, where sgn is the sign function. Then, exclude pairs with $|r_p - r_{1/N}| = 0$, and

[12] We set $N = 10.000$ iterations to obtain the distribution.
[13] We use the R package *boot* to construct the residual-resampling bootstrap procedure.

let N_r denote the remaining pairs. We then sort the remaining pairs, N_r based on the absolute differences from smallest to largest. Next, we rank the pairs, N_r, and let R_i denote the rank. Finally, compute the test statistic W,

$$W = \sum_{i=1}^{N_r} [sgn(x_p - y_{1/N}) \cdot R_i] \tag{4.19}$$

If P-value(W) $<$ significance value α, then reject H_0, else fail to reject H_0.

4.4.4 Brown's Method to Combine P-values

To ensure a thorough evaluation regardless of the individual datasets, we combine the p-values obtained from the individual statistical tests. The Fisher's Method appears to be the preferred way to combine p-values. However, the Fisher's Method is only appropriate when the statistical tests are independent. Since these statistical tests will be affected by the same underlyings shocks (market events) can we characterize these tests as dependent. As such, we use an extension to Fisher's Method, the Brown's Method.[14] Brown's method is a technique used to combine the p-values from multiple dependent statistical tests to form a single overall test, which bears upon the same null hypothesis. In addition to the p-values we obtain from each dataset, we also present a combined p-value for each statistical tests presented in Table 4, Table 6, and Table 7.

4.5 Classification of Bull-Bear Markets

The detection of turning points between bull and bear phases are obtained from the dating algorithm of Bry & Boschan (1971).[15] Bull markets are commonly understood as a general rise in prices, whereas bear markets are characterized by a general fall in prices. However, a unique definition of the turning points between the two markets has not yet been determined. There exist two main camps when it comes to detecting turning points. The first group believes that in order to qualify for a bull (bear) period, the stock market should rise (fall) substantially, without any consideration of the length of the rise (fall). Whereas the latter group believes that prices should rise (fall) over a substantial period of time.

Designed to detect turning points in the business cycles, the algorithm of Bry & Boschan (1971) consists of two main steps; initial turning points between bull and bear phases, and censoring operations. The detection of the turning points starts with; First, determine the turning points by setting a window of length τ_{window} on either side of the date and then identifying

[14]We use the R package *metap* to compute the combined p-value using Browns Method.

[15]We use the R package *BBdetection* developed by Zakamulin, to employ the algorithm of Bry & Boschan (1971)

a peak or trough to find out if the value is higher or lower than other points in the window. Second, one imposes an alternating sequence of peaks and troughs by selecting the highest maximum and lowest minimum. Next, peaks and troughs are eliminated in the first and last τ_{censor} months. Fourth, phases that last less than τ_{phase} months are eliminated, unless the threshold value θ is below the relative change in value over a single month. Fifth, cycles that last less then τ_{cycle} months are eliminated. Although it is unclear how to appropriately choose the parameters of the censoring operations, we choose to follow the default approach of the R-package, which are the parameters defined in Pagan and Sossounov (2003). The five parameters we use are $\{\tau_{window} = 8, \tau_{censor} = 6, \tau_{phase} = 4, \tau_{cycle} = 16, \theta = 20\}$.

4.6 Aggregate Portfolio Performance

Several studies show that the superior performance of risk-based strategies can be attributed to the exposure to various risk-factors. Clarke et al. (2006) show that the MVP tends to possess both a size and value bias. Scherer (2011), de Carvalho et al. (2012), and Goldberg & Geddes (2014) show that the superior performance of a minimum-variance strategy can be attributed to exposure to the value factor. To evaluate the factor exposure for the risk-based strategies across the 25 empirical datasets, we suggest to use a generalized approach that aggregates the out-of-sample performance generated in each dataset. This will provide us with factor estimates unaffected by the sorting characteristics possessed in each individual dataset, and thereby allow us to get an insight into which risk factors that drive the performance and additionally study the risk exposure over time.

To employ this method, we construct an aggregated return series of the out-of-sample performance across the empirical datasets. Let $r^k_{p,t}$ denote the return vector of portfolio p, and k denotes the respective dataset number, where $t = \{1, 2, \ldots, T\}$. We then compute the aggregated return series across the 25 datasets,

$$R^A_{p,t} = \frac{1}{25} \sum_{k=1}^{25} r^k_{p,t}, \tag{4.20}$$

where $R^A_{p,t}$ is the aggregated return-vector of the risk-based strategy p.

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5 Empirical Results

In this section, we present the results of the empirical study. First, we initiate our analysis by examining the out-of-sample portfolio performance over the 25 datasets for the period July 1973 to December 2016. This includes both summary statistics (Table 3) and statistical tests (Table 4). The statistical tests include; the difference in the Sharpe ratios, the CAPM alphas, and the Fama-French five-factor alphas. Second, we present the performance in bull and bear markets, which include summary statistics that contains mean return and standard deviation (Table 5), and in addition, we present the results obtained from the two sample Wilcoxon test (Table 6), and the Dual Beta Model (Table 7). To the end, we present the results for the aggregated performance across the 25 datasets. This includes the risk factor loadings on each risk-based strategy (Table 8), as well as the time-varying factor risk exposure.(Figure 5, and Figure 6).

5.1 The Performance of Risk-Based Strategies

5.1.1 Summary Statistics

Table 3 summarizes the performance of the four risk-based strategies and the naive strategy, in terms of annualized mean return, annualized standard deviation, annualized Sharpe ratio, and maximum drawdown. Our results indicate that the naive diversification yields on average, across the 25 datasets, the highest mean return of 12.95%. Similarly, the MVP, VT, Low-Vol, Low-Beta present mean returns of 12.21%, 12.62%, 12.61%, and 12.89%, respectively. When we evaluate the risk characteristics of the portfolio strategies, it is evident from Table 3 that the naive diversification also displays the highest standard deviation of 16.69%, on average. The MVP, VT, Low-Vol, and Low-Beta strategies provide values of 14.29%, 14.67%, 14.68%, and 14.78% respectively. Thus, the risk-based strategies provide a substantial reduction in risk relative to the naive strategy. Likewise, the reduction in risk is also reflected by means of the maximum drawdown, MD. The smaller the MD value, the more resilient the portfolio is during market turmoil. The MVP exhibits on average, the lowest maximum drawdown of 46%, whereas the

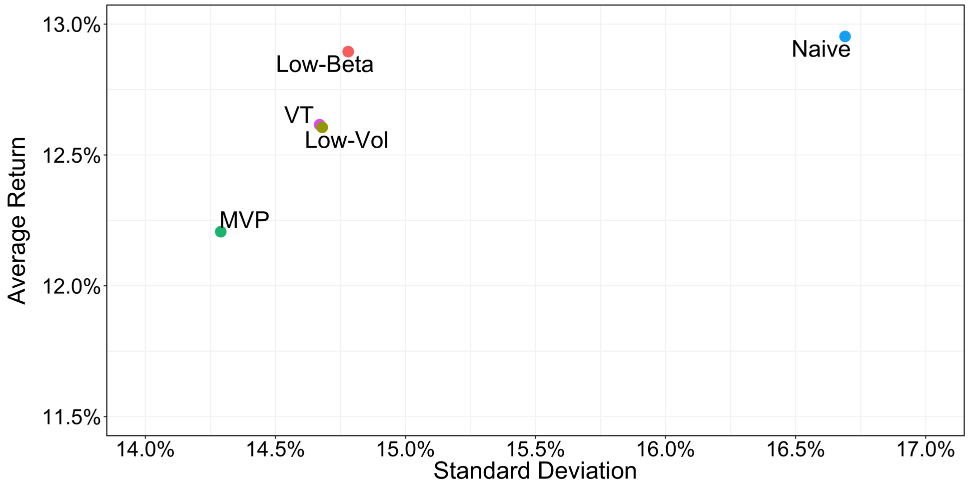

Figure 2: The risk-return tradeoff

This figure provides a graphical illustration of the risk-return tradoeff for the portfolio strategies. The figure is based on average values from Table 3, and cover the period July 1973 to December 2016. The mean returns and standard deviations are annualized and reported in percentage.

VOL, Low-Vol, and Low-Beta strategies provide values of 47%, 47%, and 48%, respectively. The naive diversification is the portfolio strategy that exhibits the largest drop during the sample period, and is reflected by a maximum drawdown of 53%, on average. This indicates that the risk-based strategies earn slightly lower mean returns on average, yet in addition, they provide a substantial reduction of risk relative to the naive diversification on average, across the 25 datasets.

Figure 2 provides a graphical illustration of the risk and return relationship for the risk-based strategies, on average. We observe that the risk-based strategies exhibit slightly lower mean return compared to the naive diversification, but with a significant reduction in risk. The Sharpe ratios from Table 3 show a superior risk-return tradeoff produced by the risk-based strategies relative to the naive diversification. The naive diversification exhibits the lowest Sharpe ratio on average compared to the other risk-based strategies, which all show somewhat similar Sharpe ratios. The naive diversification renders a Sharpe ratio of 0.49, whereas the MVP, VT, Low-Vol, and Low-Beta strategies generate Sharpe ratios of 0.52, 0.54, 0.54, and 0.55 respectively.

5.1.2 Statistical Tests

Table 4 summarizes the statistical tests for the risk-based strategies across the 25 datasets and includes the difference in the Sharpe ratios to the naive diversification, the CAPM alphas, and the alphas in the five-factor model by Fama and French. Table 4 shows that the MVP generates a statistically significant differences in the Sharpe ratios for five datasets, and the difference in the Sharpe ratio is not statistically significant at conventional levels, on average. The VT strategy

Table 3: Summary statistics on 25 datasets

This table provides summary statistics for the risk-based strategies, as well as the naive diversification. # denotes the dataset number, and **Abb.** denotes the abbreviation of the respective dataset. Column 1 and 2, under the respective portfolio strategy display the annualized mean return $\boldsymbol{\mu}$, and standard deviation $\boldsymbol{\sigma}$. Column 3 reports the annualized Sharpe ratio **SR**. Finally, column 4 shows the maximum drawdown of the respective strategy **MD**.

		Naive-Diversification				**Minimum-Variance**				**Volatility-timing**				**Low-Vol ($1/\boldsymbol{\sigma}$)**				**Low-Beta ($1/\boldsymbol{\beta}$)**			
#	Abb.	μ	σ	SR	MD	μ	σ	SR	MD	μ	σ	SR	MD	μ	σ	SR	MD	μ	σ	SR	MD
1	Size	13.82	18.21	0.50	0.53	11.12	15.19	0.42	0.51	12.18	16.18	0.46	0.51	12.42	16.06	0.48	0.52	12.41	16.16	0.47	0.53
2	BM	13.29	15.76	0.54	0.55	12.75	15.10	0.53	0.50	13.14	15.17	0.55	0.52	12.90	15.14	0.54	0.54	13.07	15.31	0.54	0.58
3	OP	11.58	16.16	0.42	0.53	12.50	14.75	0.53	0.43	12.01	15.16	0.48	0.47	12.23	14.93	0.50	0.41	12.42	14.96	0.51	0.41
4	Inv	12.65	15.85	0.50	0.51	12.75	14.19	0.56	0.48	12.77	14.64	0.55	0.49	12.49	14.25	0.54	0.48	12.54	14.31	0.55	0.48
5	Mom	11.68	16.79	0.41	0.56	12.19	15.18	0.49	0.44	12.27	15.20	0.50	0.46	12.08	15.15	0.48	0.43	12.18	15.50	0.48	0.43
6	STR	11.97	16.58	0.44	0.55	11.59	15.18	0.45	0.44	11.79	15.33	0.46	0.46	11.68	15.14	0.46	0.44	11.61	15.23	0.45	0.44
7	LTR	13.46	16.02	0.54	0.50	13.56	15.23	0.58	0.49	13.14	15.14	0.55	0.49	12.82	14.92	0.54	0.48	13.20	15.15	0.56	0.49
8	Acc	11.89	16.21	0.44	0.48	12.41	14.17	0.54	0.43	12.06	14.68	0.50	0.43	11.53	14.29	0.48	0.42	11.52	14.31	0.47	0.42
9	MktB	12.66	16.94	0.47	0.52	11.66	12.36	0.56	0.45	11.96	12.67	0.57	0.45	11.93	12.74	0.57	0.44	11.93	12.74	0.57	0.44
10	NSI	11.32	16.78	0.39	0.54	11.02	15.58	0.40	0.50	11.57	15.73	0.43	0.51	10.75	15.53	0.39	0.50	10.95	15.47	0.40	0.50
11	Var	12.31	18.03	0.42	0.59	10.50	11.68	0.49	0.37	11.44	12.20	0.55	0.39	11.77	12.78	0.55	0.38	11.77	12.78	0.55	0.38
12	ResVar	12.40	18.39	0.42	0.59	11.23	12.82	0.51	0.39	11.99	13.53	0.54	0.43	11.85	13.99	0.51	0.41	12.13	14.00	0.53	0.41
13	E-P	12.97	15.40	0.53	0.50	12.68	15.13	0.53	0.47	12.95	14.98	0.55	0.49	13.02	14.93	0.56	0.47	13.03	15.14	0.55	0.46
14	CF-P	12.90	15.28	0.53	0.51	12.88	14.90	0.55	0.50	13.02	14.93	0.55	0.51	13.07	15.17	0.55	0.50	13.41	14.91	0.58	0.53
15	Div-Y	12.31	14.96	0.51	0.53	11.73	13.61	0.52	0.52	12.31	13.62	0.56	0.49	12.99	13.55	0.61	0.47	12.40	13.95	0.55	0.62
16	10Ind	12.31	14.85	0.51	0.48	12.10	12.44	0.59	0.34	12.32	12.52	0.61	0.37	12.98	12.86	0.64	0.38	14.21	12.72	0.75	0.34
17	30Ind	12.80	16.61	0.48	0.53	11.54	12.52	0.55	0.39	12.11	12.88	0.57	0.41	12.48	13.77	0.56	0.45	13.42	13.70	0.63	0.39
18	Size-BM	14.42	17.76	0.54	0.54	12.63	14.76	0.53	0.51	13.69	15.48	0.58	0.52	13.62	15.55	0.57	0.51	13.85	15.70	0.58	0.56
19	Size-OP	14.04	17.88	0.52	0.53	12.62	14.70	0.54	0.45	12.99	15.63	0.53	0.49	12.76	15.53	0.52	0.50	13.88	15.86	0.58	0.51
20	Size-LTR	15.23	17.81	0.59	0.53	12.70	14.72	0.54	0.49	13.51	15.44	0.57	0.48	13.45	15.57	0.56	0.48	14.01	15.80	0.59	0.49
21	Size-Mom	13.92	18.32	0.50	0.56	12.42	14.89	0.51	0.45	13.36	15.71	0.55	0.47	13.53	15.72	0.56	0.48	13.39	15.93	0.54	0.48
22	Size-Inv	14.47	17.78	0.55	0.52	12.27	14.24	0.53	0.48	13.27	15.13	0.56	0.49	13.85	15.16	0.60	0.48	14.24	15.30	0.62	0.49
23	OP-Inv	12.69	16.11	0.49	0.54	13.71	14.26	0.63	0.43	13.28	14.68	0.58	0.47	12.97	14.51	0.57	0.46	13.09	14.52	0.58	0.48
24	BM-OP	13.15	16.63	0.51	0.55	12.72	14.81	0.54	0.52	13.11	15.10	0.55	0.52	12.94	14.96	0.55	0.54	14.12	15.18	0.62	0.57
25	BM-Inv	13.59	16.04	0.55	0.55	11.88	14.74	0.48	0.53	13.18	15.00	0.56	0.52	13.02	14.85	0.56	0.50	13.58	14.99	0.59	0.54
Average		12.95	16.69	0.49	0.53	12.21	14.29	0.52	0.46	12.62	14.67	0.54	0.47	12.61	14.68	0.54	0.47	12.89	14.78	0.55	0.48

provides a statistically significant difference in the Sharpe ratios for 11 datasets, and the difference is on average, statistically significant at the 1% level. The Low-Vol and Low-Beta strategies deliver statistically significant difference in the Sharpe ratios in 10 and 11 datasets, respectively. While on average, the difference in the Sharpe ratio are for both strategies statistically significant at the 1% level.

In terms of the CAPM alphas, it is evident from Table 4 that all the risk-based strategies deliver economic and statistical significant alphas for the majority of the datasets. On average, the MVP, VT, Low-Vol, and Low-Beta strategies generate values of 1.768%, 1.826%, 1.878%, and 2.185%, which are statistically significant at the 1% level, respectively. This means that on average, any of the four risk-based strategies allows an investor to generate economically and statistically significant annualized alphas from 1.768% to 2.185%. The MVP produces statistically significant alphas in 19 out of 25 datasets, while the VT, Low-Vol, and Low-Beta produce statistically significant alphas in 22, 21, and 20 datasets, respectively.

When we assess the alphas in the Fama-French five-factor model, we observe that the positive and statistically significant alphas that remain in the CAPM becomes statistically insignificant for the majority of the datasets. Specifically, only two statistical significant alphas remain in the MVP, yet the values are negative. The same applies for VT, and Low-Vol where there exist five and four negative statistically significant alphas. For Low-Beta, there is only one alpha estimate that remains positive and statistically significant, whereas four remain negative and statistically significant. On average, the introduction of the five-factor model results in alpha estimates that are neither economically nor statistically significant for the MVP, VT, and Low-Beta strategies. For the Low-Vol strategy, the five-factor alpha is statistically significant, though its value is negative. These results reveals that the CAPM alphas is most likely generated due to exposure to the five risk factors in the five-factor model.

Table 4: Statistical tests on 25 datasets

The table reports the out-of-sample estimates for the entire period July 1973 to December 2016. The difference in annualized Sharpe ratio of each risk-based strategy versus naive, $SR_p - SR_{1/N}$, is denoted as $\boldsymbol{\Delta SR}$. $\boldsymbol{\alpha_1^{CAPM}}$ denotes the alpha in the CAPM, whereas $\boldsymbol{\alpha_2^{FF5}}$ denotes the alpha in the Fama-French five-factor model. The associated p-values are reported in parentheses. The average p-values are constructed using Brown's method to combine p-values. Note that that alphas are annualized and reportd in percentage. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

	Minimum-Variance			**Volatility-timing**			**Low-Vol ($1/\sigma$)**			**Low-Beta ($1/\beta$)**		
#	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}
1	-0.079	-0.084	0.403	-0.039	0.501	0.084	-0.020	0.783***	0.419	-0.024	0.830	0.594
	(0.87)	(0.85)	(0.32)	(0.86)	(0.14)	(0.77)	(0.69)	(<0.01)	(0.16)	(0.75)	(0.11)	(0.17)
2	-0.012	1.847**	-0.694	0.011	2.097***	-0.589	-0.003	2.105**	-0.975	0.002	2.233**	-1.019
	(0.64)	(0.02)	(0.29)	(0.29)	(<0.01)	(0.19)	(0.54)	(0.02)	(0.15)	(0.47)	(0.01)	(0.13)
3	0.103***	1.559***	0.623	0.056***	0.759***	0.044	0.078***	1.179**	0.210	0.090***	1.352***	0.293
	(<0.01)	(<0.01)	(0.24)	(<0.01)	(<0.01)	(0.87)	(<0.01)	(0.02)	(0.66)	(<0.01)	(<0.01)	(0.54)
4	0.065*	2.153***	-0.254	0.049**	1.831***	-0.380	0.045	1.811***	-0.581	0.046	1.868***	-0.776*
	(0.06)	(<0.01)	(0.62)	(0.01)	(<0.01)	(0.17)	(0.13)	(<0.01)	(0.17)	(0.11)	(<0.01)	(0.08)
5	0.077**	1.222	-0.723	0.082***	1.195*	-0.857	0.071*	1.139	-1.151*	0.067*	1.147	-0.815
	(0.04)	(0.10)	(0.29)	(<0.01)	(0.06)	(0.11)	(0.05)	(0.13)	(0.09)	(0.07)	(0.16)	(0.29)
6	0.015	0.474	-0.111	0.024	0.497	-0.205	0.022	0.573	-0.297	0.015	0.479	-0.277
	(0.34)	(0.39)	(0.84)	(0.18)	(0.17)	(0.56)	(0.28)	(0.29)	(0.58)	(0.34)	(0.39)	(0.62)
7	0.035	2.594***	-0.010	0.010	2.067***	-0.553	-0.002	1.966***	-0.576	0.014	2.241***	-0.245
	(0.15)	(<0.01)	(0.99)	(0.35)	(<0.01)	(0.22)	(0.51)	(<0.01)	(0.35)	(0.37)	(<0.01)	(0.70)
8	0.100**	1.804***	1.020	0.057**	1.059***	0.598	0.034	0.766	0.220	0.033	0.751	0.203
	(0.02)	(<0.01)	(0.12)	(0.01)	(<0.01)	(0.10)	(0.17)	(0.12)	(0.64)	(0.17)	(0.13)	(0.66)
9	0.095	2.557**	0.290	0.104*	2.299***	-0.133	0.098*	2.293***	-0.727	0.098*	2.293***	-0.727
	(0.15)	(0.02)	(0.76)	(0.09)	(<0.01)	(0.84)	(0.08)	(<0.01)	(0.28)	(0.09)	(<0.01)	(0.28)
10	0.011	-0.086	-2.176***	0.042*	0.174	-1.118**	-0.005	-0.446	-2.066***	0.009	-0.227	-1.811***
	(0.42)	(0.91)	(<0.01)	(0.07)	(0.73)	(0.02)	(0.56)	(0.50)	(<0.01)	(0.41)	(0.73)	(<0.01)
11	0.075	1.598	-0.485	0.130*	2.023**	-0.537	0.131**	2.003**	-0.740	0.131**	2.003**	-0.740
	(0.23)	(0.11)	(0.59)	(0.06)	(0.02)	(0.43)	(0.04)	(0.01)	(0.20)	(0.04)	(0.01)	(0.20)
12	0.090	1.484*	0.152	0.120**	1.685***	0.132	0.092*	1.363**	-0.384	0.112**	1.643**	-0.224
	(0.15)	(0.08)	(0.82)	(0.03)	(<0.01)	(0.77)	(0.08)	(0.04)	(0.39)	(0.04)	(0.01)	(0.61)
13	-0.009	1.769**	-0.990	0.014	1.965***	-0.557	0.021	2.212***	-0.457	0.013	2.180***	-0.996
	(0.62)	(0.02)	(0.10)	(0.20)	(<0.01)	(0.19)	(0.27)	(<0.01)	(0.46)	(0.33)	(<0.01)	(0.12)
14	0.012	2.156**	-0.732	0.020	2.099***	-0.801*	0.015	2.261**	-1.155*	0.048	2.772***	-0.722
	(0.36)	(0.01)	(0.29)	(0.16)	(<0.01)	(0.08)	(0.37)	(0.01)	(0.10)	(0.13)	(<0.01)	(0.30)
15	0.009	2.091*	-1.339	0.051	2.306**	-1.301**	0.103**	3.285***	-0.694	0.044	2.638**	-1.471*
	(0.45)	(0.07)	(0.15)	(0.14)	(0.01)	(0.04)	(0.04)	(<0.01)	(0.40)	(0.25)	(0.02)	(0.08)
16	0.083	3.091***	1.251	0.097	3.113***	0.480	0.133**	3.677***	0.870	0.236***	5.151***	2.616**
	(0.16)	(<0.01)	(0.26)	(0.12)	(<0.01)	(0.63)	(0.04)	(<0.01)	(0.40)	(<0.01)	(<0.01)	(0.02)
17	0.060	2.534**	0.450	0.088	2.535***	-0.139	0.078*	2.258***	-0.354	0.149***	3.458***	0.110
	(0.27)	(0.03)	(0.69)	(0.12)	(<0.01)	(0.88)	(0.09)	(<0.01)	(0.61)	(<0.01)	(<0.01)	(0.90)
18	-0.010	1.867**	-0.148	0.033	2.562***	-0.432	0.026	2.551***	-0.801	0.035	2.742***	-0.461
	(0.60)	(0.01)	(0.83)	(0.19)	(<0.01)	(0.38)	(0.30)	(<0.01)	(0.15)	(0.18)	(<0.01)	(0.38)
19	0.016	1.733***	0.700	0.007	1.598***	0.092	-0.004	1.399***	-0.070	0.056*	2.570***	0.247
	(0.42)	(<0.01)	(0.22)	(0.41)	(<0.01)	(0.80)	(0.54)	(<0.01)	(0.84)	(0.07)	(<0.01)	(0.65)
20	-0.048	1.972**	0.016	-0.021	2.349***	-0.323	-0.030	2.266***	-0.718	-0.003	2.769***	0.063
	(0.75)	(0.01)	(0.98)	(0.74)	(<0.01)	(0.50)	(0.78)	(<0.01)	(0.16)	(0.48)	(<0.01)	(0.90)
21	0.014	1.608**	-0.003	0.047*	2.069***	-0.352	0.058*	2.291***	-0.653	0.042	2.119***	-0.427
	(0.41)	(0.04)	(1.00)	(0.07)	(<0.01)	(0.46)	(0.06)	(<0.01)	(0.20)	(0.13)	(<0.01)	(0.45)
22	-0.018	1.634***	-0.498	0.016	2.129***	-0.355	0.054	2.748***	-0.034	0.073**	3.163***	0.360
	(0.61)	(<0.01)	(0.35)	(0.33)	(<0.01)	(0.28)	(0.10)	(<0.01)	(0.93)	(0.02)	(<0.01)	(0.39)
23	0.136***	3.241***	0.405	0.088***	2.357***	-0.165	0.074**	2.176***	-0.569	0.082**	2.343***	-0.740*
	(<0.01)	(<0.01)	(0.56)	(<0.01)	(<0.01)	(0.65)	(0.03)	(<0.01)	(0.18)	(0.01)	(<0.01)	(0.10)
24	0.033	2.105**	-1.055	0.048*	2.155***	-0.886*	0.043	2.109***	-1.078**	0.112***	3.342***	-0.347
	(0.27)	(0.02)	(0.16)	(0.09)	(<0.01)	(0.08)	(0.16)	(<0.01)	(0.05)	(<0.01)	(<0.01)	(0.55)
25	-0.067	1.265	-1.964***	0.011	2.234***	-0.915**	0.006	2.189***	-0.868	0.038	2.757***	-0.484
	(0.92)	(0.15)	(<0.01)	(0.31)	(<0.01)	(0.04)	(0.43)	(<0.01)	(0.10)	(0.15)	(<0.01)	(0.37)
Avg.	0.031	1.768***	-0.235	0.046**	1.826***	-0.367	0.045***	1.878***	-0.529**	0.061***	2.185***	-0.312
	(0.12)	(<0.01)	(0.14)	(0.01)	(<0.01)	(0.12)	(<0.01)	(<0.01)	(0.01)	(<0.01)	(<0.01)	(0.12)

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5.2 Assymetric Performance in Bull-Bear Markets

In this subsection, we present the results for the bull and bear markets. This include summary statistics for the risk-based strategies and the naive diversification, the results of the Wilcoxon test, and the results for the Dual Beta Model.

5.2.1 Summary Statistics

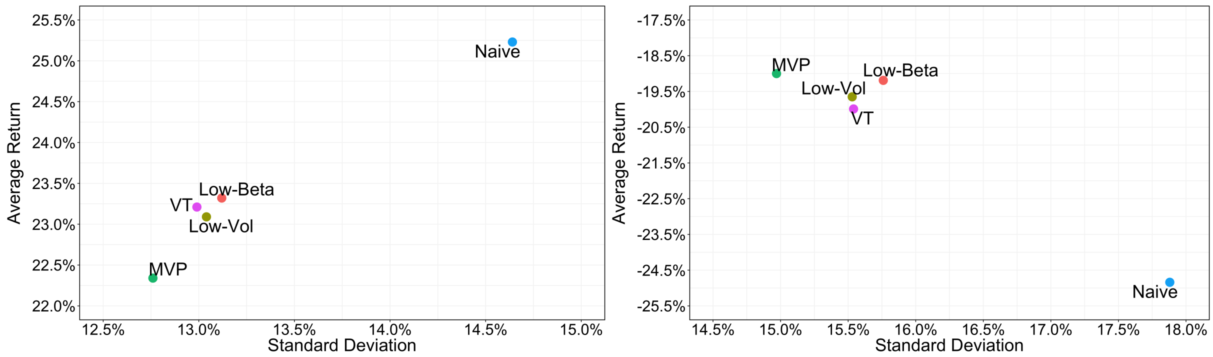
Table 5 reports the annualized mean returns and standard deviations for the risk-based strategies and the naive diversification during bull and bear markets. The naive diversification shows the highest mean return during bull markets with 25.23%, on average. Similarly, the MVP, VT, Low-Vol, Low-Beta generate values of 22.34%, 23.21%, 23.09%, and 23.32%, respectively. Furthermore, the results for mean returns during bear markets shows the resilient feature of the risk-based strategies relative to the naive diversification. The MVP shows the lowest drop in mean return with -19.00 %, on average. The other risk-based strategies illustrate similar tendencies as the MVP with -19.99%, -19.65%, and -19.19% for the VT, Low-Vol, and Low-Beta strategies, respectively. The naive diversification suffers a loss of -24.84%, on average.

Further, we observe during bull markets, that the naive diversification presents a standard deviation of 14.64%, on average. The risk-based portfolio strategies provide a substantial reduction in volatility relative to the naive diversification during bull markets. The MVP, VT, Low-Vol, and Low-Beta strategies provide standard deviations of 12.76%, 12.99%, 13.04%, and 13.12%, respectively. During bear markets, we observe that the volatility increases significantly. The MVP delivers the lowest standard deviation of 14.97%, on average. Similarly, the standard deviations are 17.88%, 15.54%, 15.53%, and 15.76% for the naive, VT, Low-Vol, and Low-Beta strategies, respectively. Figure 3 provides an illustration of the relationship between the mean returns and standard deviations for these four risk-based strategies, on average. In Panel A, we illustrate the relationship during bull markets, while Panel B shows bear markets. We observe that the risk-based strategies appear to be somewhat clustered together whether we look at bull or bear markets. Moreover, the two figures reveal that during bull markets, the riskier strategies generate a higher mean return, while during bear markets, the less risky strategies are more resilient in terms of mean returns. Although, we observe large variations in the performance for each dataset. For instance, the mean return of the Low-Beta during bear markets varies between -25.75% in dataset 1, to -10.54% in dataset 16.

Table 5: Summary statistics in bull and bear markets

This table provides summary statistics of the risk-based strategies, as well as the naive diversification in bull and bear periods. # denotes the dataset number, and **Abb.** denotes the abbreviation of respective dataset. Mean returns $\boldsymbol{\mu}$, and standard deviations $\boldsymbol{\sigma}$, are annualized and reported in percent. Dates on bull and bear periods are obtained using the dating algorithm of Bry and Boschan (1971).

		Naive-diversification				**Minimum-Variance**				**Volatility-timing**				**Low-Vol ($1/\sigma$)**				**Low-Beta ($1/\beta$)**			
		Bull		Bear		Bull		Bear		Bull		Bear		Bull		Bear		Bull		Bear	
#	Abb.	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
1	Size	26.38	16.09	-24.84	19.80	23.30	13.16	-26.38	15.91	24.77	14.04	-26.58	17.25	24.91	13.95	-26.01	17.08	24.80	14.10	-25.75	17.14
2	BM	24.84	13.84	-22.26	16.90	23.68	13.42	-20.89	15.89	24.15	13.37	-20.77	16.22	23.67	13.30	-20.25	16.44	23.74	13.44	-19.77	16.79
3	OP	24.27	13.94	-27.51	17.33	23.47	13.10	-21.25	15.32	23.59	13.27	-23.65	16.03	23.12	13.17	-21.29	15.89	23.40	13.14	-21.38	16.04
4	Inv	24.80	13.96	-24.73	16.50	22.59	12.66	-17.54	15.10	23.56	12.99	-20.41	15.34	22.91	12.61	-19.57	15.07	22.85	12.67	-19.18	15.21
5	Mom	24.46	14.72	-27.68	17.72	22.90	13.88	-20.78	15.11	23.24	13.76	-21.51	15.32	22.74	13.87	-20.74	15.02	22.98	14.17	-21.08	15.53
6	STR	24.85	14.40	-27.67	17.66	23.46	13.42	-24.94	15.50	23.69	13.48	-24.86	15.87	23.45	13.42	-24.55	15.38	23.47	13.52	-24.87	15.41
7	LTR	25.32	14.21	-23.03	16.73	24.36	13.31	-19.69	16.75	24.17	13.34	-20.84	16.17	23.34	13.10	-19.55	16.28	23.91	13.30	-19.79	16.53
8	Acc	24.37	14.01	-26.51	17.50	23.09	12.61	-20.46	14.53	23.43	12.86	-22.94	15.34	22.39	12.54	-21.92	15.04	22.35	12.55	-21.80	15.13
9	MktB	25.50	14.84	-26.89	17.94	19.26	11.25	-11.72	13.19	20.63	11.43	-14.74	13.24	20.84	11.52	-15.50	13.09	20.84	11.52	-15.50	13.09
10	NSI	24.54	14.46	-29.37	17.97	22.41	13.90	-24.02	16.14	23.56	13.90	-25.33	16.31	22.34	13.75	-24.93	16.19	22.58	13.65	-24.87	16.19
11	Var	26.36	15.74	-30.95	18.95	17.60	10.64	-11.34	12.49	19.55	11.06	-13.53	12.73	20.78	11.60	-15.95	12.97	20.78	11.60	-15.95	12.97
12	ResVar	26.72	15.97	-31.68	19.56	20.37	11.65	-16.91	12.88	21.99	12.16	-18.80	13.70	22.39	12.57	-20.60	14.03	22.54	12.60	-19.90	14.08
13	E-P	24.25	13.65	-21.76	16.17	23.60	13.61	-20.91	15.50	23.79	13.33	-20.42	15.70	23.70	13.26	-19.85	15.81	23.46	13.67	-19.09	15.72
14	CF-P	24.22	13.49	-21.95	16.12	23.98	13.17	-21.28	15.64	24.00	13.17	-20.80	15.84	23.95	13.42	-20.42	16.20	24.05	13.18	-19.33	15.99
15	Div-Y	23.44	13.24	-21.97	15.67	20.53	12.15	-15.33	14.87	21.63	12.20	-16.35	14.45	21.43	12.13	-12.98	14.94	21.44	12.12	-15.43	16.03
16	10Ind	23.67	13.00	-22.66	15.68	19.98	11.39	-12.15	12.96	20.19	11.36	-11.88	13.34	20.78	11.82	-11.01	13.49	22.25	11.70	-10.54	13.12
17	30Ind	24.61	14.60	-23.57	18.04	19.27	11.45	-12.24	13.20	20.36	11.54	-13.29	14.04	21.71	12.47	-15.94	14.37	22.02	12.64	-13.03	14.06
18	Size-BM	26.34	15.66	-22.29	19.62	23.33	13.05	-20.31	15.71	24.67	13.54	-20.09	16.99	24.45	13.70	-19.73	16.93	24.94	13.70	-20.28	17.37
19	Size-OP	26.26	15.75	-23.57	19.68	23.54	13.03	-21.00	15.34	24.44	13.67	-22.27	16.91	24.19	13.63	-22.45	16.63	24.66	13.94	-19.31	17.54
20	Size-LTR	26.77	15.79	-20.28	19.74	23.38	12.93	-20.16	15.86	24.39	13.55	-19.96	16.88	24.37	13.66	-20.16	17.09	24.78	13.87	-19.16	17.52
21	Size-Mom	26.46	16.04	-24.69	20.37	23.18	13.53	-20.70	14.87	24.36	13.94	-20.51	16.81	24.45	13.87	-20.07	17.11	24.15	14.06	-19.70	17.52
22	Size-Inv	26.52	15.75	-22.62	19.40	22.65	12.62	-19.69	15.01	24.33	13.34	-20.79	16.10	24.55	13.46	-19.08	16.17	24.73	13.54	-18.03	16.65
23	OP-Inv	25.01	14.06	-25.25	17.11	23.30	12.72	-15.81	15.37	23.84	12.94	-19.25	15.76	23.41	12.84	-19.17	15.44	23.34	12.86	-18.46	15.56
24	BM-OP	25.53	14.52	-24.95	17.90	23.10	13.30	-19.23	15.46	24.00	13.33	-20.42	16.15	23.71	13.24	-20.21	15.91	24.74	13.31	-18.58	16.66
25	BM-Inv	25.29	14.15	-22.41	17.09	22.31	13.06	-20.21	15.75	23.90	13.26	-19.84	16.04	23.57	13.21	-19.43	15.74	24.13	13.26	-18.87	16.13
Average		25.23	14.64	-24.84	17.88	22.34	12.76	-19.00	14.97	23.21	12.99	-19.99	15.54	23.09	13.04	-19.65	15.53	23.32	13.12	-19.19	15.76



(a) Bull markets **(b)** Bear markets

Figure 3: Risk-return tradeoff in bull and bear markets
This figure provides the risk-return relationship between the respective portfolio strategies. Panel A displays the performance in bull markets, whereas Panel B displays the performance in bear markets. The figure is based on average values that we obtain from Table 5. The mean returns and standard deviations are annualized and reported in percentage.

5.2.2 Statistical Tests

The empirical results we obtain from the two sample Wilcoxon test are presented in Table 6. The difference in the mean returns between the risk-based strategies and the naive diversification during bull markets range from -1.91% to -2.89%, on average. During bear markets, we observe a greater dispersion in the mean returns between the risk-based strategies and the naive diversification. The MVP, VT, Low-Vol, and Low-Beta strategies provide statistically significant differences in mean returns over the naive diversification with values of 5.84%, 4.85%, 5.19%, and 5.66%, on average. Interestingly, these results indicate that the four risk-based stragies underperform slightly in bull markets, while outperform during bear markets, relative to the naive diversification.

Table 6 illustrates that there exists a large variation of the difference in mean returns on the individual datasets. For instance, the difference in mean returns for the MVP and naive diversification range from -1.54% to 19.60% in bear phases. In dataset 11, for the MVP, VT, Low-Vol, and Low-Beta, the differences are 19.60%, 17.41%, 15.00%, and 15.00%, respectively. While in the first dataset, the difference in mean returns for the MVP, VT, Low-Vol, and Low-Beta strategies are -1.54%, -1.74%, -1.17%, and -0.91%, respectively. This show that the naive diversification actually perform better during bear markets relative to the risk-based strategies for the first dataset.

To assess the risk-adjusted performance of the risk-based strategies, we construct a Dual Beta Model. This model statistically tests the difference in the regression estimates during bear and bull phases.[1] When we assess the risk-adjusted return in bull markets from Table 7, we observe

[1]Note that when we refer to the difference during the two market phases, we refer to $\Delta = Bear - Bull$

Table 6: Two Sample Wilcoxon test in bull and bear markets

This table provides the results from the two sample Wilcoxon test. # denotes the dataset number. The difference between mean returns is denoted as $\boldsymbol{\Delta\mu_p^k}$, where k represents either bull or bear period. Note that $\boldsymbol{\Delta\mu^k}$ is annualized and reported in percentage. The associated p-values are reported in parentheses, and average p-values are constructed using Brown's method to combine p-values. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

	Minimum-Variance		**Volatility-timing**		**Low-Vol ($1/\sigma$)**		**Low-Beta ($1/\beta$)**	
#	$\Delta\mu^{Bull}$	$\Delta\mu^{Bear}$	$\Delta\mu^{Bull}$	$\Delta\mu^{Bear}$	$\Delta\mu^{Bull}$	$\Delta\mu^{Bear}$	$\Delta\mu^{Bull}$	$\Delta\mu^{Bear}$
1	-3.08	-1.54	-1.61	-1.74	-1.47	-1.17	-1.58	-0.91
	(0.98)	(0.66)	(0.96)	(0.74)	(0.92)	(0.72)	(0.97)	(0.70)
2	-1.16	1.37	-0.69	1.49**	-1.17	2.01	-1.10	2.49*
	(0.80)	(0.19)	(0.90)	(0.04)	(0.82)	(0.14)	(0.89)	(0.05)
3	-0.81	6.26***	-0.68	3.86***	-1.16	6.21***	-0.88	6.12***
	(0.93)	(<0.01)	(0.98)	(<0.01)	(0.98)	(<0.01)	(0.95)	(<0.01)
4	-2.21	7.19***	-1.24	4.32***	-1.89	5.16***	-1.95	5.55***
	(1.00)	(<0.01)	(1.00)	(<0.01)	(1.00)	(<0.01)	(1.00)	(<0.01)
5	-1.56	6.90***	-1.22	6.17***	-1.72	6.94***	-1.48	6.60***
	(0.83)	(<0.01)	(0.67)	(<0.01)	(0.89)	(<0.01)	(0.83)	(<0.01)
6	-1.38	2.72**	-1.15	2.81***	-1.39	3.11**	-1.38	2.80**
	(0.98)	(0.05)	(0.98)	(<0.01)	(0.97)	(0.02)	(0.95)	(0.05)
7	-0.96	3.34***	-1.14	2.19***	-1.98	3.48***	-1.41	3.24***
	(0.86)	(<0.01)	(0.98)	(<0.01)	(0.99)	(<0.01)	(0.91)	(<0.01)
8	-1.28	6.06***	-0.94	3.58***	-1.98	4.60***	-2.02	4.71***
	(0.95)	(<0.01)	(0.98)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)
9	-6.24	15.17***	-4.87	12.15***	-4.67	11.39***	-4.67	11.39***
	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)
10	-2.13	5.35**	-0.98	4.04***	-2.20	4.44**	-1.96	4.50**
	(0.99)	(0.01)	(0.99)	(<0.01)	(0.99)	(0.01)	(0.99)	(0.01)
11	-8.76	19.60***	-6.81	17.41***	-5.58	15.00***	-5.58	15.00***
	(0.98)	(<0.01)	(0.97)	(<0.01)	(0.95)	(<0.01)	(0.98)	(<0.01)
12	-6.35	14.76***	-4.73	12.88***	-4.33	11.08***	-4.18	11.77***
	(0.96)	(<0.01)	(0.97)	(<0.01)	(0.96)	(<0.01)	(0.97)	(<0.01)
13	-0.66	0.85	-0.47	1.34***	-0.55	1.91**	-0.79	2.67***
	(0.91)	(0.15)	(0.95)	(<0.01)	(0.90)	(0.02)	(0.94)	(<0.01)
14	-0.25	0.67	-0.22	1.15*	-0.27	1.53	-0.17	2.61*
	(0.77)	(0.26)	(0.80)	(0.07)	(0.67)	(0.28)	(0.41)	(0.07)
15	-2.92	6.64***	-1.82	5.62***	-2.02	8.99***	-2.01	6.54***
	(0.97)	(<0.01)	(0.99)	(<0.01)	(0.97)	(<0.01)	(0.96)	(<0.01)
16	-3.69	10.51***	-3.48	10.78***	-2.89	11.65***	-1.42	12.12***
	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.98)	(<0.01)	(0.74)	(<0.01)
17	-5.34	11.33***	-4.26	10.28***	-2.90	7.64***	-2.60	10.55***
	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.94)	(<0.01)
18	-3.01	1.98	-1.67	2.20*	-1.89	2.56**	-1.40	2.00*
	(0.99)	(0.23)	(0.98)	(0.06)	(0.95)	(0.05)	(0.94)	(0.07)
19	-2.72	2.57	-1.81	1.30*	-2.06	1.12	-1.59	4.27***
	(0.96)	(0.11)	(0.99)	(0.07)	(0.98)	(0.20)	(0.96)	(<0.01)
20	-3.39	0.12	-2.38	0.33	-2.40	0.12	-1.99	1.13*
	(0.97)	(0.35)	(0.99)	(0.21)	(0.98)	(0.32)	(0.99)	(0.09)
21	-3.28	3.99*	-2.10	4.18***	-2.01	4.62***	-2.31	4.99***
	(0.98)	(0.09)	(0.98)	(<0.01)	(0.97)	(<0.01)	(0.99)	(<0.01)
22	-3.88	2.93	-2.19	1.83	-1.97	3.54**	-1.79	4.58***
	(0.99)	(0.15)	(0.99)	(0.11)	(0.99)	(0.02)	(0.99)	(<0.01)
23	-1.72	9.44***	-1.17	6.00***	-1.61	6.08***	-1.67	6.79***
	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)
24	-2.43	5.72***	-1.52	4.52***	-1.82	4.74***	-0.78	6.37***
	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.82)	(<0.01)
25	-2.98	2.20	-1.38	2.57***	-1.72	2.97***	-1.16	3.53***
	(0.99)	(0.18)	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.98)	(<0.01)
Avg.	-2.89	5.84***	-2.02	4.85***	-2.15	5.19***	-1.91	5.66***
	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)	(0.99)	(<0.01)

that the CAPM alphas vary significantly between the respective datasets for all of the risk-based strategies. The MVP delivers statistically significant CAPM alphas in 9 datasets during bull markets. For the VT, Low-Vol, and Low-Beta, the numbers are 15, 8, and 8, respectively. While on average, we observe that each of the four risk-based strategies deliver statistically significant alphas that range from 1.171% to 1.481%. This indicates that even in bull markets, the risk-based strategies do perform significantly well in terms of risk-adjusted return.

When we evaluate the differences in the alphas during bear and bull markets, the MVP generates a statistically significant difference in 3 out of 25 datasets (Table 7). Yet the difference in the first dataset is negative, which indicates that the alpha is higher in bull markets relative to bear markets. For the other two datasets, the MVP shows a higher risk-adjusted return in bear markets relative to bull markets. For the VT, Low-Vol, and Low-Beta, we observe that the three strategies produce statistically significant differences in alpha for 8, 5, and 6 datasets, respectively. While on average, the difference in the alpha during bear and bull markets for the MVP is 0.852%, although the value is not statistically significantly different from zero. Similarly, the VT, Low-Vol, and Low-Beta generate on average, statistically significant difference in the alphas of 1.865%, 1.746%, and 1.929%. Again, we observe large variations for the various datasets. For instance, the Low-Beta delivers a difference in the alpha of 6.346% in dataset 21, while for dataset 9 the difference is -1.642%. The Dual Beta model illustrates that there exists, to some degree, a time-varying relationship in the estimated alpha between bear and bull markets, and in addition, the alphas from the CAPM is generated mostly during bear markets.

To see if any of the four risk-based strategies implicit change the exposure to the market during bear and bull periods, we include a third column which is the difference in the CAPM beta (Table 7). Interestingly, we observe small variation between the factor loading on the CAPM beta in the two markets for each risk-based strategy. The MVP shows 2 datasets where the difference in the CAPM beta is statistically significant. The numbers for the VT, Low-VOL, and Low-Beta are 5, 5, and 6 datasets, respectively. We do observe a reduced exposure towards the market in bear markets for each of the four risk-based strategies during the majority of the datasets. The difference in the market exposure on average, are -0.017, -0.015, -0.021, and -0.018 for the MVP, VT, Low-Vol, and Low-beta, respectively. Although, none of the values are statistically significantly different from zero.

Table 7: Dual Beta Model for bull and bear markets

The table provides the alpha and beta estimates for the Dual Beta Model. # denotes the dataset number. The first column under the respective portfolio strategy reports the estimates of the alpha for bull markets $\boldsymbol{\alpha_{bull}}$. Column two provides the difference in alpha for bear and bull $\boldsymbol{\Delta\alpha}$. Whereas the third column shows the difference in market beta for bear and bull market $\boldsymbol{\Delta\beta}$. The associated p-values are reported in parentheses. Average p-values are constructed using Brown's method to combine p-values. The values for alpha are annualized, and reported in percentage. The rest is presented monthly. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

	Minimum-Variance			**Volatility-timing**			**Low-Vol ($1/\sigma$)**			**Low-Beta ($1/\beta$)**		
#	α_{bull}	$\Delta\alpha$	$\Delta\beta$	α_{bull}	$\Delta\alpha$	$\Delta\beta$	α_{bull}	$\Delta\alpha$	$\Delta\beta$	α_{bull}	$\Delta\alpha$	$\Delta\beta$
1	0.450	-1.865*	0.003	0.492	0.777	0.019	0.695*	0.824	0.013	0.805	0.761	0.017
	(0.39)	(0.10)	(0.85)	(0.24)	(0.39)	(0.20)	(0.06)	(0.29)	(0.32)	(0.22)	(0.58)	(0.45)
2	1.235	0.939	-0.034	1.364*	2.038	-0.018	1.654	1.025	-0.017	1.573	1.997	-0.012
	(0.19)	(0.64)	(0.29)	(0.09)	(0.23)	(0.52)	(0.13)	(0.66)	(0.65)	(0.16)	(0.40)	(0.76)
3	0.947	1.293	-0.025	0.392	1.577**	0.005	0.383	3.045**	0.002	0.679	2.877**	0.009
	(0.15)	(0.35)	(0.26)	(0.25)	(0.03)	(0.66)	(0.52)	(0.02)	(0.93)	(0.26)	(0.02)	(0.66)
4	0.941	3.666**	-0.022	0.913*	2.023*	-0.036*	1.128	1.576	-0.025	1.041	2.035	-0.027
	(0.24)	(0.03)	(0.42)	(0.10)	(0.08)	(0.06)	(0.11)	(0.30)	(0.31)	(0.17)	(0.21)	(0.30)
5	-0.100	1.928	-0.077**	0.042	1.856	-0.062**	-0.309	1.591	-0.097***	-0.130	2.262	-0.064*
	(0.91)	(0.32)	(0.02)	(0.96)	(0.25)	(0.02)	(0.74)	(0.42)	(<0.01)	(0.90)	(0.29)	(0.07)
6	0.535	-1.732	-0.038	0.261	-0.158	-0.026*	0.480	-1.507	-0.047**	0.403	-1.701	-0.050**
	(0.43)	(0.23)	(0.10)	(0.56)	(0.87)	(0.09)	(0.47)	(0.28)	(0.04)	(0.56)	(0.25)	(0.03)
7	2.001**	3.005	0.020	1.256*	2.139	-0.023	1.128	3.074	-0.002	1.443	3.073	0.002
	(0.03)	(0.13)	(0.54)	(0.09)	(0.17)	(0.37)	(0.21)	(0.10)	(0.96)	(0.11)	(0.11)	(0.94)
8	1.651**	-0.328	-0.023	0.977**	0.147	-0.004	0.659	0.581	0.005	0.597	0.941	0.009
	(0.03)	(0.84)	(0.40)	(0.04)	(0.88)	(0.80)	(0.28)	(0.65)	(0.83)	(0.33)	(0.47)	(0.66)
9	2.101	1.105	-0.015	1.720*	0.428	-0.044	1.825*	-1.642	-0.086**	1.825*	-1.642	-0.086**
	(0.12)	(0.70)	(0.74)	(0.10)	(0.85)	(0.22)	(0.10)	(0.48)	(0.02)	(0.10)	(0.48)	(0.02)
10	-0.700	0.362	-0.049	-0.343	0.078	-0.047**	-0.898	0.367	-0.034	-0.530	0.106	-0.026
	(0.47)	(0.86)	(0.14)	(0.58)	(0.95)	(0.03)	(0.27)	(0.83)	(0.24)	(0.51)	(0.95)	(0.35)
11	1.146	2.117	0.011	1.311	0.956	-0.043	1.332	-0.811	-0.084**	1.332	-0.811	-0.084**
	(0.36)	(0.42)	(0.80)	(0.22)	(0.67)	(0.24)	(0.18)	(0.70)	(0.01)	(0.18)	(0.70)	(0.01)
12	1.312	-1.071	-0.044	1.174	-0.376	-0.058**	1.031	-1.541	-0.071**	1.107	-0.902	-0.074***
	(0.21)	(0.63)	(0.22)	(0.13)	(0.82)	(0.03)	(0.21)	(0.37)	(0.01)	(0.17)	(0.60)	(<0.01)
13	0.990	0.836	-0.053	1.058	2.154	-0.031	1.706*	1.881	-0.000	1.021	2.784	-0.039
	(0.29)	(0.68)	(0.11)	(0.16)	(0.17)	(0.22)	(0.07)	(0.35)	(0.99)	(0.32)	(0.20)	(0.27)
14	2.290**	-1.266	-0.019	1.600*	1.019	-0.022	1.941*	0.703	-0.013	2.518**	0.244	-0.018
	(0.03)	(0.57)	(0.59)	(0.05)	(0.56)	(0.45)	(0.08)	(0.76)	(0.74)	(0.03)	(0.92)	(0.65)
15	2.448*	1.604	0.075	1.998*	1.258	0.003	2.816**	3.430	0.042	3.043**	1.476	0.076
	(0.08)	(0.59)	(0.13)	(0.08)	(0.60)	(0.95)	(0.04)	(0.23)	(0.36)	(0.04)	(0.63)	(0.13)
16	3.297**	-1.022	-0.006	2.891**	1.097	0.007	3.009**	1.986	-0.013	5.150***	-2.168	-0.055
	(0.02)	(0.74)	(0.90)	(0.03)	(0.70)	(0.88)	(0.03)	(0.50)	(0.78)	(<0.01)	(0.49)	(0.28)
17	2.758*	0.284	0.028	2.063*	2.855	0.028	1.038	2.905	-0.042	1.805	3.090	-0.079*
	(0.06)	(0.93)	(0.57)	(0.09)	(0.26)	(0.51)	(0.30)	(0.17)	(0.22)	(0.14)	(0.23)	(0.06)
18	1.382	0.995	-0.021	1.721*	3.501*	0.009	1.616	3.463	-0.001	2.166**	2.907	0.019
	(0.14)	(0.61)	(0.52)	(0.05)	(0.06)	(0.76)	(0.11)	(0.10)	(0.98)	(0.04)	(0.19)	(0.60)
19	1.287*	1.154	-0.013	0.785	3.628***	0.015	0.576	3.228***	0.004	0.987	5.997***	0.002
	(0.07)	(0.45)	(0.60)	(0.15)	(<0.01)	(0.43)	(0.27)	(<0.01)	(0.83)	(0.26)	(<0.01)	(0.95)
20	1.716*	1.247	0.007	1.338*	4.445***	0.017	1.271	4.453**	0.019	1.530	5.790***	0.029
	(0.07)	(0.54)	(0.82)	(0.09)	(<0.01)	(0.54)	(0.14)	(0.01)	(0.53)	(0.10)	(<0.01)	(0.36)
21	0.722	0.242	-0.078**	0.819	4.680***	0.000	1.182	5.020***	0.022	0.788	6.346***	0.035
	(0.45)	(0.90)	(0.02)	(0.31)	(<0.01)	(1.00)	(0.18)	(<0.01)	(0.47)	(0.41)	(<0.01)	(0.30)
22	1.013	1.280	-0.026	1.194**	2.643**	-0.022	1.356**	4.019***	-0.030	1.743**	5.280***	-0.001
	(0.19)	(0.44)	(0.33)	(0.05)	(0.04)	(0.29)	(0.05)	(<0.01)	(0.20)	(0.04)	(<0.01)	(0.98)
23	2.060**	3.814*	-0.015	1.341**	3.184**	-0.016	1.202*	2.309	-0.034	1.275	3.026*	-0.025
	(0.04)	(0.07)	(0.66)	(0.03)	(0.02)	(0.47)	(0.10)	(0.13)	(0.18)	(0.11)	(0.07)	(0.36)
24	1.567	1.200	-0.021	1.526*	1.993	-0.009	1.539	1.294	-0.021	2.927***	2.083	0.013
	(0.16)	(0.61)	(0.59)	(0.08)	(0.29)	(0.76)	(0.10)	(0.52)	(0.52)	(<0.01)	(0.38)	(0.73)
25	0.974	1.521	0.011	1.386*	2.676	-0.013	1.317	2.384	-0.022	1.919**	2.372	-0.019
	(0.37)	(0.51)	(0.77)	(0.09)	(0.13)	(0.66)	(0.14)	(0.21)	(0.47)	(0.05)	(0.25)	(0.56)
Avg.	1.361***	0.852	-0.017	1.171***	1.865***	-0.015	1.187***	1.746***	-0.021	1.481***	1.929***	-0.018
	(<0.01)	(0.67)	(0.21)	(<0.01)	(<0.01)	(0.13)	(<0.01)	(<0.01)	(0.12)	(<0.01)	(<0.01)	(0.11)

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5.3 Aggregate Portfolio Performance

This subsection presents the results from the aggregated portfolio performance for each risk-based strategy. This method allows us to study the factor exposure on average across the 25 datasets, and in addition to evaluate the risk exposure over time.

5.3.1 Regression Analysis

Table 8 reports the estimates for the CAPM and the Fama-French five-factor model for the risk-based strategies. We observe that in the CAPM, each risk-based strategy produce on average, positive and highly statistically significant alphas at the 1% level. In Panel A, we observe that the MVP generates an annualized alpha of 1.768%, on average. In Panel B, C, and D, for the VT, Low-Vol, and Low-Beta we see that on average, the annualized alphas are 1.826%, 1.878%, and 2.185%, respectively. Further, Table 8 illustrates that on average, each risk-based strategy present a market beta that range from 0.835 to 0.885, which is not surprising considering that the main idea behind these strategies is to minimize risk. These results coincide with Table 4, where we observe highly statistically significant alphas in the CAPM, on average.

When we examine the regression coefficients in the Fama-French five-factor model, we first observe that the CAPM alphas becomes negative and insignificant (Table 8). This reveals that the positive statistical significant CAPM alphas are likely generated by the exposure to the additional factors in the Fama-French five-factor model. When we assess the factor loadings, which gives an indication of the main drivers behind the performance, the HML, RMW, and CMA factors are all positive and statistically significant for all risk-based strategies at the 1% level. The difference in the alpha from the CAPM to the five-factor model is mainly attributed to exposure towards these three factors (illustrated in Figure 4). The market beta, MKT, in the five-factor model displays more or less similar estimates as in the CAPM model (Table 8). Each of the risk-based strategies display factor loadings below one against the market and are highly statistically significant. The exposure to the SMB factor varies between the risk-based strategies. For the MVP, we observe a statistical factor of -0.056, indicating that to some degree the MVP tends to favor high capitalization stocks. Contrary to the MVP, the VT and Low-Beta render a positive statistical significant exposure towards SMB of 0.021 and 0.038. Further on, the Low-Vol demonstrates an insignificant factor loading of ≈ 0. The factor loading on SMB from each of the risk-based strategies, all display small loadings, which indicate it most likely does not matter whether the stock is small-cap-, or large-cap stocks, as long as it possesses low-volatility.

To further show the relationship between the average excess return for each of the risk-based

Table 8: Regression analysis of the aggregated peformance

The table provides the regression output for the aggregated portfolio peformance for each risk-based strategy. Panel A shows the results for the MVP, and Panel B displays the VT. Panel C and D reports the results for the Low-Volatility and Low-Beta, respectively. The associated p-values are reported in parentheses. Alphas are annualized and reported as percentage. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Model	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	R^2_{adj}
Panel A: Minimum-Variance Portfolio							
CAPM	1.768***	0.835***					0.93
	(<0.01)	(<0.01)					
Fama-French 5 factor	-0.235	0.910***	-0.056***	0.106***	0.147***	0.179***	0.97
	(0.50)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	
Panel B: Volatility-Timing strategy							
CAPM	1.826***	0.885***					0.96
	(<0.01)	(<0.01)					
Fama-French 5 factor	-0.367	0.949***	0.021**	0.117***	0.163***	0.149***	0.98
	(0.23)	(<0.01)	(0.02)	(<0.01)	(<0.01)	(<0.01)	
Panel C: Low Volatility ($1/\sigma$)							
CAPM	1.878***	0.876***					0.94
	(<0.01)	(<0.01)					
Fama-French 5 factor	-0.529	0.952***	-0.002	0.129***	0.179***	0.176***	0.97
	(0.12)	(<0.01)	(0.83)	(<0.01)	(<0.01)	(<0.01)	
Panel D: Low Beta ($1/\beta$)							
CAPM	2.185***	0.874***					0.94
	(<0.01)	(<0.01)					
Fama-French 5 factor	-0.312	0.945***	0.038**	0.15***	0.165***	0.167***	0.97
	(0.35)	(<0.01)	(0.03)	(<0.01)	(<0.01)	(<0.01)	

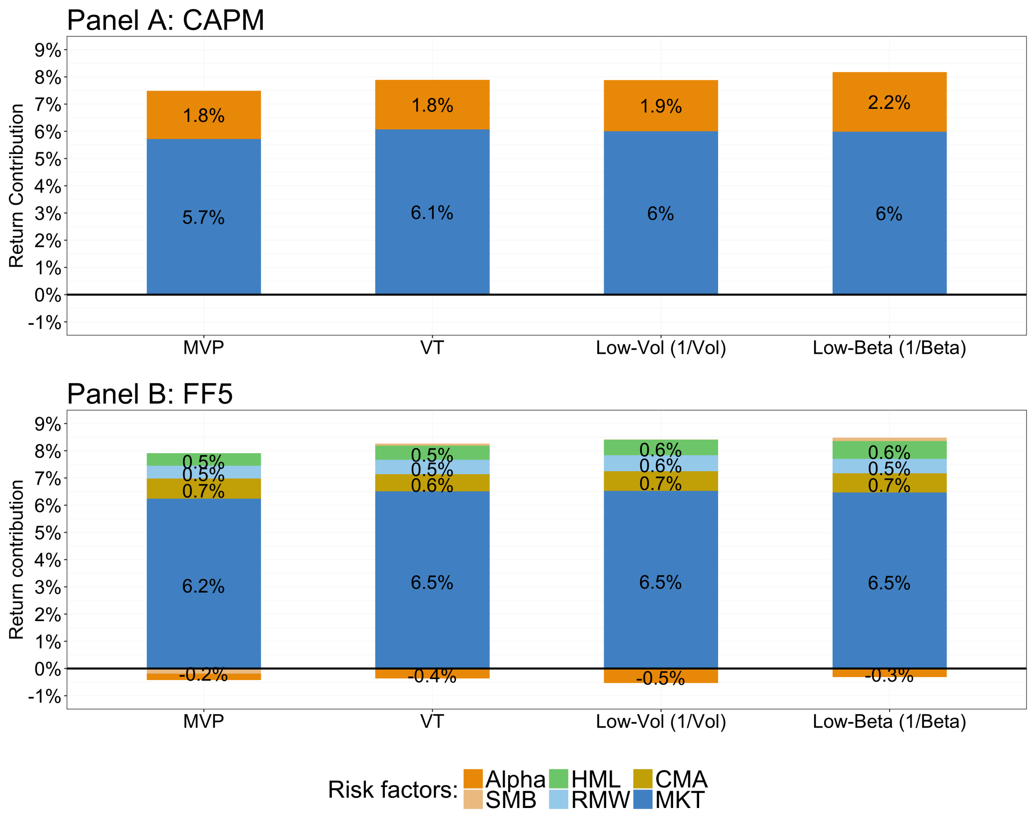

Figure 4: Portfolio Return Decomposition

This figure provides return contribution to various risk factors for the risk-based strategies. Panel A provides the decomposition from the CAPM, whereas Panel B shows the contribution from Fama-French five-factor model. The total return contribution for the respective strategy is annualized and illustrated as excess return over risk-free rate. The sample period is July 1976 to December 2016

strategies and the factor loadings, we graphically illustrates the decomposition of the portfolios excess return in Figure 4. By multiplying the respective factor return with the portfolios factor loading, we obtain the respective factors contribution towards the total excess return. From Figure Figure 4 in Panel A, we present the decomposition of the CAPM, and illustrates the contribution to the market factor MKT, and the CAPM alpha. Further, in Panel B, the positive alphas in the CAPM vanish when the risk factors in the five-factor model are taken into account. Thus, it is clear that a high factor loading on HML, RMW, and CMA combined with the fact that these three factors generate a high mean return over the sample period contribute significantly to the total return of each portfolio (Table 2, Figure 4). Considering that the risk-based strategies are long-only, the exposure to the market is high, and thus the market contributes most to the total return.

5.3.2 Time-Varying Exposure

To evaluate the time-series dimension of each of the risk-based strategies performance, we report estimates from a rolling 3-year alpha (Figure 5). The rolling window approach let us track how the respective portfolio performance varies over time. Panel A illustrates the 3-year rolling alpha for each risk-based strategy in the CAPM, whereas Panel B displays the respective rolling-alpha estimates using the five-factor model. Evidently, from Panel A, the 3-year rolling alpha for each of the risk-based strategies, exhibit time-varying estimates. In the period before the dot-com bubble, we observe a decline in the CAPM alpha estimates, which illustrate that the market portfolio outperforms the risk-based strategies during the bull period. During the bear period from 2001 to 2004 (dot-com bubble), we observe an increase in the rolling alpha estimates for the risk-based strategies. This substantiates our earlier results that these strategies generate superior performance mostly during bear markets. However, we do not observe the same tendency during the global financial crisis, where the rolling alpha estimates remain rather small. Even though the rolling alpha estimates dictate a highly time-varying behavior, is it evident from Panel A that the 3-year alpha remains positive for most of the time. Moreover, Panel B illustrates the 3-year rolling five-factor alpha where we observe a much smaller variability, which is a consequence of the exposure to the five risk factors.

Figure 6 illustrates the time-varying exposure to the five-factor model for each risk-based strategy. It is obvious from Figure 6 that the factor exposures experience a time-varying behavior. The exposure to the SMB factor for each risk-based strategy is for most of the time negative or around zero. We observe a more time-varying behavior on the HML and CMA factors. In fact, during the first 30 years of our analysis until 2006-2007, the rolling factor exposure to the HML factor remained positive. We observe that in bear markets, where the risk-based strategies tend to be more resilient, implicit increase the exposure to the HML factor. We further observe that from the beginning of the global financial crisis the exposure to the HML factor has dropped below zero. From 2010-2011, the implicit exposure towards the RMW and CMA factors have increased substantially, and are the two remaining factors with a positive loading. Fama and French argue that "*low-volatility stocks tends to behave like those of firms that are profitable but conservative in terms of investment*" (Fama & French (2015) p.21), and illustrate that positive exposure to the RMW and CMA factors capture the average returns of low-volatility stocks.

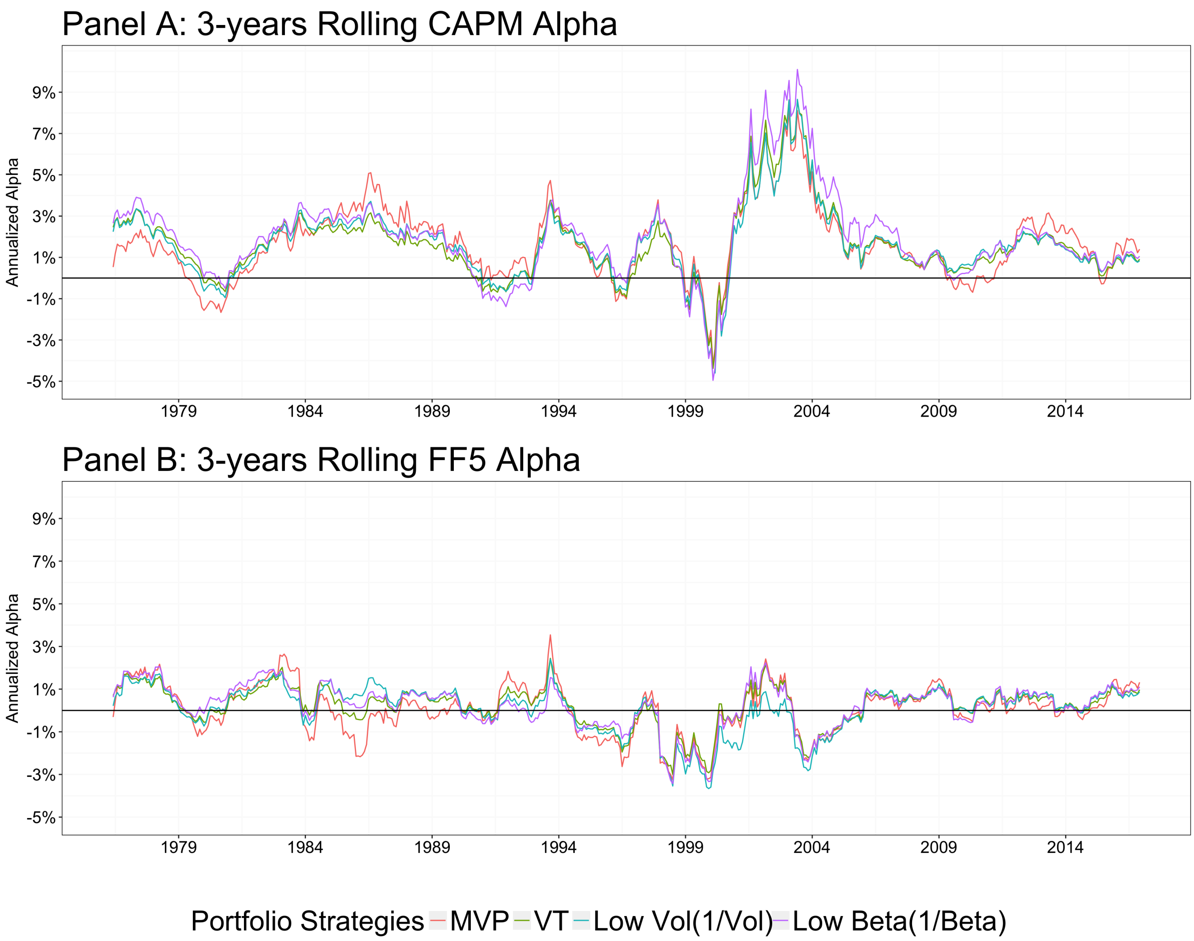

Figure 5: Rolling 3-year alpha

This figure provides the 3-year rolling alpha estimates for the risk-based strategies. The first 3-year alpha estimate is from July 1973 to July 1976. The next 3-year alpha estimate will be rolled on month forward, while we discard the first month. Panel A illustrates the CAPM alpha, whereas Panel B depicts the 3-year rolling Fama-French five-factor alpha estimates. Note that the alpha estimates are annualized and presented in percentage.

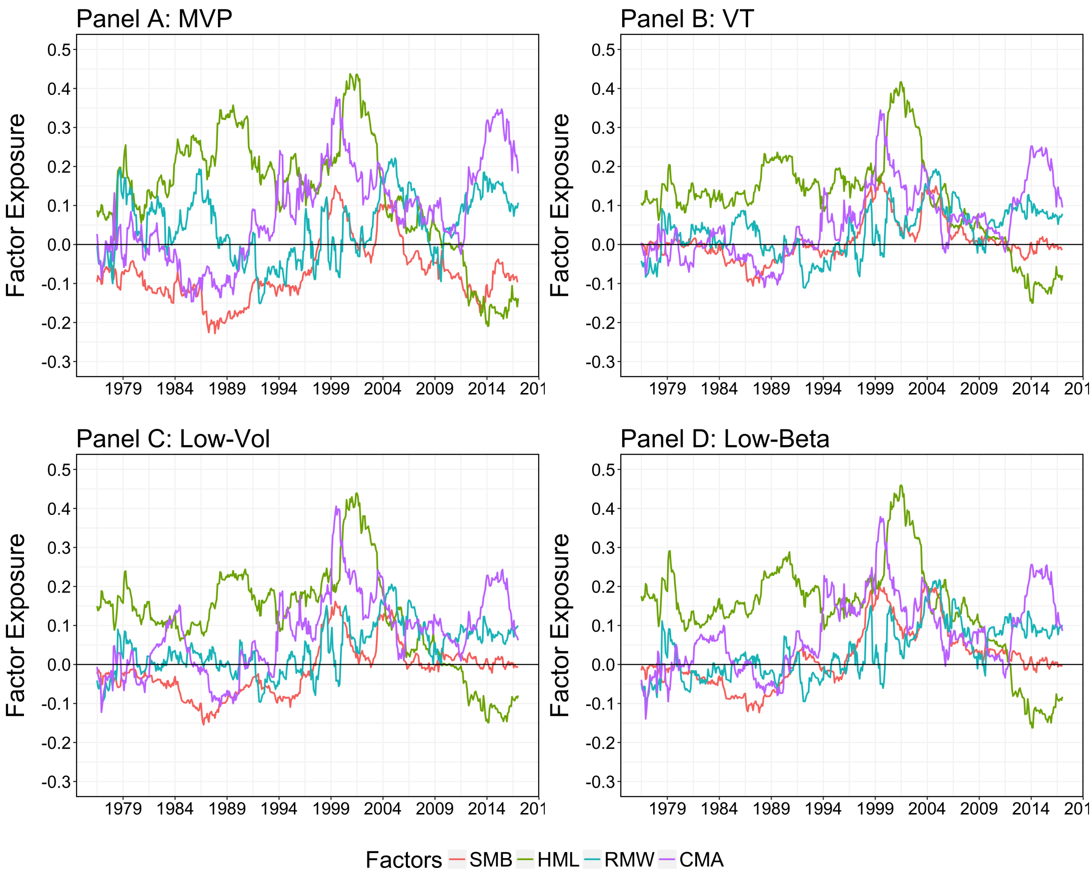

Figure 6: Rolling 3-year factor exposure

This figure provides a 3-year rolling factor exposure. The first 3-year factor estimate is from July 1973 to July 1976. The next 3-year factor estimate will be rolled on month forward, while we discard the first month. Panel A1, A2, B1, B2 shows MVP, VOL, Low-Vol, and Low-Beta, respectively.

5.4 Robustness Checks

In this subsection we briefly discuss the various changes we undertake in the design setup to check for robustness of our main results. First, we present the robustness check from dividing the dataset into two equal sample sizes. The first period is from July 1973 to March 1995, whereas the second period is from April 1995 to December 2016. Second, we change the setup by using a 5-year lookback period instead of using 10-years. The results are further presented in Appendix 5, 6, and 7.

The general conclusion across the robustness tests are that the risk-based strategies still provide a statistically significant difference in the Sharpe ratio compared to the naive diversification. The results vary slightly across the two subperiods, and the outperformance appears to be stronger during the second period, from July 1995 to Dec 2016. Further, in terms of CAPM alpha, the risk-based strategies still deliver statistically significant values on average, regardless of whether one looks at the first or second subperiod, or change the lookback period. Similarly, when we controll for several risk factors through the Fama-French five-factor model, the statistically significant alphas from the CAPM vanish. Thus, our main results remain intact regardless of changes in the setup.

6 Discussion

Although risk-based strategies merely focus on minimizing risk, it turns out that these strategies also deliver good performance. However, we do observe large variations in the portfolio performance when we assess the results for each datasets. The various tables that present statistical tests, all show large variations regardless of the various performance measures we use (see Table 4, Table 6, and Table 7). There exists no particular consistency in the risk-based strategies ability to outperform the naive diversification in each respective dataset. Already Kirby & Ostdiek (2012) point out this fact in their paper, where they show that their timing strategies are influenced by the datasets characteristics. Measuring the performance of various optimal portfolios compared to the naive diversification on a few arbitrary datasets provided by Kenneth French could create a misleading impression of superiority of optimized portfolios over naive diversification (or vice versa), while in fact the various portfolio strategies simply could benefit from the respective sorting characteristics possessed in each dataset. Several studies claim superiority over the naive diversification while they simply evaluate the performance over a few arbitrary datasets among a large number of available datasets on the Kenneth French library. These datasets could consequently have been "*cherry-picked*" to best suit the author's conclusion. Yet our reason for including 25 empirical datasets was to prevent this bias, and more thoroughly evaluate the relative performance of four risk-based strategies over the naive diversification.

When we assess the difference in the Sharpe ratio between the risk-based strategies and the naive diversification, we observe that three of four risk-based strategies statistically significantly outperform the naive diversification on average, across the 25 datasets. The MVP is the only risk-based strategy that does not outperform the naive diversification on average, by conventional levels. Similarly, Zakamulin (2017) shows that the MVP does not outperform the naive diversification by statistically significant margins, on average. Also, DeMiguel et al. (2009) found no evidence indicating that the MVP consistently outperform the naive diversification. The authors suggest that the estimation error associated with forecasting the model parameters leads to poor out-of-sample performance. Stivers & Sun (2016) argue that the use of full-matrix strategies generally can lead to larger estimation error compared to portfolio strategies that only uses the diagonal of the covariance matrix. Indeed, the VT, and Low-Vol strategies, which simply use the diagonal of the covariance-matrix to assign the portfolio weights, deliver a statistically significant difference in the Sharpe ratio over the naive diversification, on average. The Low-Beta strategy that uses the assets inverse beta to assign weights also delivers statistical significant differences in the Sharpe ratio over the naive diversification, on average. Kirby & Ostdiek (2012) document superior performance for the VT strategy relative to the naive diversification across four empirical datasets, and Stivers & Sun (2016) present similar results as Kirby & Ostdiek (2012), by demonstrating that the VT outperform the naive diversification across five empirical datasets. The results obtained for the two ad-hoc strategies, the Low-Vol, and Low-Beta show how one can implement simple portfolio strategies on Kenneth French datasets, measure the performance by means of Sharpe ratio and statistically significantly outperform the naive diversification. Although the performance varies over each dataset, these two strategies do however deliver the highest number of statistically significant differences across the 25 datasets, relative to the two other risk-based strategies. Though the outperformance is most likely attributed to the low-volatility effect, which Zakamulin (2017) illustrates is present in virtually all datasets provided by Kenneth French.

All the risk-based strategies deliver statistically significant CAPM alphas for almost all of the empirical datasets, and on average, the CAPM alphas are statistically significant at the 1% level. Zakamulin (2017) demonstrates that both the MVP and VT produce economically significant annualized alphas of 1.5% and 1.68% in the CAPM. When Zakamulin (2017) augments the CAPM with the HML factor, the alpha of any optimized portfolios strategy becomes neither economically nor statistically significant. When we assess the performance based on the five-factor model of Fama-French, almost all of the statistically significant positive alphas in the CAPM vanish. Even though some alphas are statistically significant, those values are negative. The empirical results we obtain for the five-factor model may be interpreted as if the remaining CAPM alpha is fully explained by exposure to the five risk factors. In line with Fama & French (2016), who show that their new five-factor model fully captures the remaining significant alphas that are left unexplained by the three-factor model. For the sake of thoroughness, we present the results from the three- and four-factor models in Appendix 3. The results show that there remain positive and statistically significant alphas on average, and for the majority of the datasets. Consistent with Frazzini & Pedersen (2014), and Novy-Marx (2014) who find that there remain significant positive alphas of low-volatility portfolios from the Fama-French three- and Carhart four-factor model.

When we assess the performance of the risk-based strategies during bull and bear markets, we observe that the superior performance over the naive diversification is mainly due to the performance during bear markets. The risk-based strategies arguably benefit from their resilient feature during market downturns. The difference in the mean returns illustrates that the risk-based strategies compared to the naive diversification only slightly underperform during bull markets, while outperform during bear markets (see Table 6). These results indicate that the risk-based strategies experience a much less drawdown than the naive diversification during bear markets, while simultaneously capturing the upside potential during bull markets. This is further confirmed by our results obtained from the Dual Beta Model, where we find that each risk-based strategies deliver statistically significant alphas in bull markets. In addition, the VT, Low-Vol, and Low-Beta strategies deliver a statistically significant difference in the alpha during bear and bull markets. Implying that the three strategies statistically deliver higher risk-adjusted return in bear markets. Though, we have not found any studies that statistically test the performance of risk-based strategies during bull and bear markets, a few papers do however mention the outperformance of risk-based strategies. Scherer (2011) evaluates the MVP during different regimes, without statistically testing the performance, and shows that in bull markets, the MVP tends to underperform relative to the market portfolio, while in bear markets, it tends to outperform.

Through the aggregated portfolio performance we show that over all datasets, each of the four risk-based strategies produce statistical significant CAPM alphas. This indicates that the market is not able to explain all the excess return of these strategies and that one or several risk factors are necessary to explain the excess return. Further, we report statistically insignificant alphas in the Fama-French five-factor model. The factor loading of value (HML), profitability (RMW), and investment(CMA) are highly statistically significant for all the risk-based strategies, which indicate that the three factors contribute economically to the overall portfolio returns (see Figure 4). Value stocks also tend to be low-risk stocks: so minimizing the volatility would naturally create a positive factor loading on value factor. Despite the fact that we use an approach that to the best of our knowledge has not been used before, our results are comparable to previous studies. Clarke et al. (2006) show that the MVP tend to have both a value- and a size bias. Similarly, Scherer (2011) points to the value exposure in the MVP and suggests that strategies aimed at minimizing risk are nothing more than an inefficient way to capture factor risk premium. Several other studies (de Carvalho et al. (2012), Goldberg & Geddes (2014), Chow et al. (2014)) also illustrate that the returns of risk-based strategies are substantially driven by the exposure to the value factor. Zakamulin (2017) shows that the CAPM augmented with the HML factor, which can be viewed as a proxy for the low-volatility effect, erodes every positive significant alpha in the CAPM. The obtained results from these authors are in line with what we find, and further strengthen our results.

Figure 6 illustrates how the behavior of factor exposure is highly time-varying. After the global financial crisis, the exposure towards HML has dropped below zero, indicating a shift from value to more growth stocks. This may be explained by the increased popularity and huge cash-inflows into low-volatility strategies (see Goldberg & Geddes (2014)). Similarly to our results, Goldberg & Geddes (2014), illustrate that over the period 1973 to December 2012 much of the excess return of a minimum variance strategy could be largely attributed to tilts towards value, and further argue that the boosted interest in low-volatility portfolios largely explains why the value factor has shifted from cheap to more expensive stocks post global financial crisis.

7 | Conclusion

Since the publication of the study by DeMiguel et al. (2009), where they demonstrate that none of the 14 mean-variance optimization strategies outperform the naive diversification, several empirical studies claim to defend the role of portfolio optimization (see e.g. Kritzman et al. (2010), Tu & Zhou (2011), Kirby & Ostdiek (2012)). However, in a recent study conducted by Zakamulin (2017), the author states that the superior performance of these optimized portfolio strategies appears due to exposure to one or several profitable market anomalies, and not as a result of better mean-variance efficiency.

This thesis evaluates the performance of four risk-based strategies relative to the naive diversification, and additionally assess whether this (out)performance could be attributed to established factor premiums. We use two optimal mean-variance portfolio strategies considered in the literature, namely the minimum-variance portfolio (MVP) and the volatility-timing strategy (VT), and two ad-hoc portfolio strategies that assign weights based on the assets inverse standard deviation and the CAPM beta. These two ad-hoc strategies directly exploit the low-volatility effect, which Zakamulin (2017) illustrates is present in virtually all datasets provided by Kenneth French. We extend previous studies by using 25 empirical datasets provided by Kenneth French, where the time period is extended due to newly accessible data. The performance of risk-based strategies relative to the naive diversification is measured across both bull and bear markets to differentiate the performance during the two phases. To gain insight into the risk factors that drives the performance, and additionally study the risk exposure over time, we construct a generalized approach where we look at the aggregate portfolio performance across 25 datasets. The newly proposed Fama-French five-factor model is used to assess the factor exposures.

Our results indicate on average, that the VT, Low-Vol, and Low-Beta strategies outperform the naive diversification in terms of Sharpe ratio. Although, the results vary significantly over the individual datasets. The two simple ad-hoc strategies constructed to exploit the low-volatility effect delivers superior performance over the naive diversification and thus substantiating the point that one does not necessarily need sophisticated portfolio strategies to outperform the naive diversification. Moreover, each risk-based strategy generates statistically significant alphas in the CAPM, both on average, and in nearly each dataset. The superior performance of the risk-based strategies compared to the naive diversification, and in terms of CAPM alpha, are mostly generated in bear markets. When we control for several established factor premiums through the Fama-French five-factor model, the alpha of any risk-based strategy becomes neither economically nor statistically significant. The positive factor loadings on HML, RMW, and CMA indicate that these factors significantly contribute to the total return for each risk-based strategy. These findings are in line with several other studies that advocate that the performance of risk-based strategies could be attributed to factor exposures. We reach the same general conclusion as Zakamulin (2017) that the superior performance of risk-based strategies is likely to be attributed to exposure towards established factor premiums rather than better mean-variance efficiency.

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Appendix 1: Normality Test for the Risk-based strategies

Table 9: Normality test for the risk-based strategies

This table reports the normalized moments of each risk-based strategy across 25 datasets, as well as a normality test. Shapiro-Wilk test for normality tests if the distribution under scrutiny follows a normal distribution. Formally represented as: $H_0 : \epsilon \sim \mathcal{N}(\mu, \sigma^2), H_1 : \epsilon \neg \sim \mathcal{N}(\mu, \sigma^2)$.

	Minimum-Variance			**Volatility-timing**			**Low-Vol (1/σ)**			**Low-Beta (1/β)**		
#	Kurt.	Skew.	Shapiro-W.	Kurt.	Skew.	Shapiro-W.	Kurt.	Skew.	Shapiro-W.	Kurt.	Skew.	Shapiro-W.
1	1.97	-0.43	<0.01	2.33	-0.6	<0.01	2.06	-0.51	<0.01	2.15	-0.54	<0.01
2	2.41	-0.46	<0.01	2.49	-0.52	<0.01	2.69	-0.51	<0.01	2.54	-0.47	<0.01
3	2.4	-0.5	<0.01	2.34	-0.52	<0.01	2.32	-0.49	<0.01	2.69	-0.56	<0.01
4	2.08	-0.47	<0.01	2.26	-0.5	<0.01	2.36	-0.51	<0.01	2.08	-0.45	<0.01
5	1.97	-0.35	<0.01	1.94	-0.4	<0.01	1.97	-0.39	<0.01	1.76	-0.37	<0.01
6	1.97	-0.51	<0.01	2.12	-0.52	<0.01	1.96	-0.49	<0.01	1.94	-0.5	<0.01
7	3.42	-0.54	<0.01	3.21	-0.51	<0.01	3.49	-0.57	<0.01	3.29	-0.53	<0.01
8	1.56	-0.35	<0.01	1.96	-0.46	<0.01	1.8	-0.42	<0.01	1.84	-0.44	<0.01
9	2.36	-0.35	<0.01	2.55	-0.43	<0.01	2.35	-0.43	<0.01	2.33	-0.42	<0.01
10	2.26	-0.38	<0.01	2.36	-0.45	<0.01	2.26	-0.39	<0.01	2.36	-0.41	<0.01
11	1.52	-0.34	<0.01	2.04	-0.4	<0.01	2.04	-0.38	<0.01	2.02	-0.37	<0.01
12	1.43	-0.28	<0.01	1.83	-0.36	<0.01	1.92	-0.35	<0.01	1.91	-0.35	<0.01
13	2.09	-0.27	<0.01	2.35	-0.41	<0.01	2.31	-0.38	<0.01	2.43	-0.3	<0.01
14	1.7	-0.28	<0.01	2.22	-0.44	<0.01	2.19	-0.31	<0.01	2.14	-0.26	<0.01
15	3	0.02	<0.01	2.08	-0.25	<0.01	3.06	-0.21	<0.01	3.59	-0.37	<0.01
16	1.12	-0.08	<0.01	1.32	-0.26	<0.01	1.43	-0.28	<0.01	1.05	-0.26	<0.01
17	1.36	-0.14	<0.01	2.05	-0.38	<0.01	2.56	-0.36	<0.01	2.63	-0.27	<0.01
18	1.88	-0.28	<0.01	2.8	-0.61	<0.01	2.47	-0.54	<0.01	2.91	-0.65	<0.01
19	2.32	-0.3	<0.01	2.71	-0.6	<0.01	2.37	-0.53	<0.01	2.59	-0.59	<0.01
20	3.74	-0.51	<0.01	3.45	-0.62	<0.01	3.43	-0.62	<0.01	3.65	-0.67	<0.01
21	1.72	-0.15	<0.01	2.46	-0.55	<0.01	2.81	-0.58	<0.01	2.41	-0.57	<0.01
22	1.9	-0.37	<0.01	2.69	-0.59	<0.01	2.58	-0.51	<0.01	2.8	-0.63	<0.01
23	2.68	-0.42	<0.01	2.6	-0.51	<0.01	2.35	-0.48	<0.01	2.47	-0.5	<0.01
24	2.43	-0.27	<0.01	2.44	-0.45	<0.01	2.3	-0.4	<0.01	2.26	-0.46	<0.01
25	1.88	-0.33	<0.01	2.45	-0.48	<0.01	2.48	-0.39	<0.01	2.82	-0.46	<0.01

Appendix 2: Normality Test for Regression Models

Table 10: Normality tests for regression models

This table reports the aggregated portfolios normalized moments, as well as a normality test for the residuals obtained from CAPM and Fama-French five factor. Shapiro-Wilk test for normality tests if the distribution under scrutiny follows a normal distribution. Formally represented as: $H_0 : \epsilon \sim \mathcal{N}(\mu, \sigma^2), H_1 : \epsilon \neg \sim \mathcal{N}(\mu, \sigma^2)$.

	Skewness	Kurtosis	Shapiro-Wilk	Skewness	Kurtosis	Shapiro-Wilk
		CAPM			Fama-French 5 factor	
Minimum-variance portfolio	0.19	7.07	<0.001	0.34	2.85	<0.001
Volatility targeting	0.79	12.23	<0.001	0.69	5.8	<0.001
Low Volatility (1/σ)	0.82	14.09	<0.001	0.75	7.07	<0.001
Low Beta (1/β)	1.13	12.13	<0.001	0.87	6.86	<0.001

Appendix 3: Results from Fama-French Three-Factor Model

Table 11: Performance tests on 25 datasets for FF3

The table reports the out-of-sample estimates for the entire period 1973:07 - 2016:12. The difference in annualized Sharpe ratio of each risk-based strategy versus naive, $SR_p - SR_{1/N}$, is denoted as $\boldsymbol{\Delta SR}$. $\boldsymbol{\alpha_1^{CAPM}}$ denotes the alpha in the CAPM, whereas $\boldsymbol{\alpha_2^{FF3}}$ denotes the alpha in the Fama-French three-factor model. The associated p-values are reported in parentheses. The average p-values are constructed using Brown's method to combine p-values. Note that that alphas are annualized and reportd in percentage. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

	Minimum-Variance			**Volatility-timing**			**Low-Vol ($1/\sigma$)**			**Low-Beta ($1/\beta$)**		
#	ΔSR	α_1^{CAPM}	α_2^{FF3}	ΔSR	α_1^{CAPM}	α_2^{FF3}	ΔSR	α_1^{CAPM}	α_2^{FF3}	ΔSR	α_1^{CAPM}	α_2^{FF3}
1	-0.079	-0.084	0.387	-0.039	0.501	0.109	-0.020	0.783***	0.445	-0.024	0.830	0.323
	(0.88)	(0.85)	(0.32)	(0.85)	(0.14)	(0.69)	(0.67)	(<0.01)	(0.12)	(0.74)	(0.11)	(0.46)
2	-0.012	1.847**	0.305	0.011	2.097***	0.303	-0.003	2.105**	-0.064	0.002	2.233**	-0.153
	(0.63)	(0.02)	(0.64)	(0.31)	(<0.01)	(0.51)	(0.56)	(0.02)	(0.93)	(0.43)	(0.01)	(0.82)
3	0.103***	1.559***	1.182**	0.056***	0.759***	0.472*	0.078**	1.179**	0.838*	0.090***	1.352***	0.929*
	(<0.01)	(<0.01)	(0.02)	(<0.01)	(<0.01)	(0.08)	(0.01)	(0.02)	(0.08)	(<0.01)	(<0.01)	(0.05)
4	0.065*	2.153***	1.058*	0.049**	1.831***	0.924***	0.045	1.811***	0.834*	0.046	1.868***	0.800
	(0.07)	(<0.01)	(0.05)	(0.02)	(<0.01)	(<0.01)	(0.12)	(<0.01)	(0.08)	(0.11)	(<0.01)	(0.12)
5	0.077**	1.222	0.547	0.082***	1.195*	0.461	0.071*	1.139	0.344	0.067*	1.147	0.450
	(0.03)	(0.10)	(0.45)	(<0.01)	(0.06)	(0.44)	(0.05)	(0.13)	(0.64)	(0.08)	(0.16)	(0.58)
6	0.015	0.474	0.402	0.024	0.497	0.255	0.022	0.573	0.313	0.015	0.479	0.246
	(0.36)	(0.39)	(0.46)	(0.18)	(0.17)	(0.47)	(0.28)	(0.29)	(0.55)	(0.36)	(0.39)	(0.66)
7	0.035	2.594***	1.424**	0.010	2.067***	0.886*	-0.002	1.966***	0.879	0.014	2.241***	1.025
	(0.17)	(<0.01)	(0.04)	(0.32)	(<0.01)	(0.08)	(0.53)	(<0.01)	(0.18)	(0.37)	(<0.01)	(0.12)
8	0.100**	1.804***	2.037***	0.057**	1.059***	1.339***	0.034	0.766	1.007**	0.033	0.751	0.963**
	(0.01)	(<0.01)	(<0.01)	(0.01)	(<0.01)	(<0.01)	(0.16)	(0.12)	(0.03)	(0.16)	(0.13)	(0.04)
9	0.095	2.557**	1.767*	0.104*	2.299***	1.561**	0.098*	2.293***	1.560**	0.098	2.293***	1.560**
	(0.17)	(0.02)	(0.06)	(0.07)	(<0.01)	(0.03)	(0.07)	(<0.01)	(0.04)	(0.10)	(<0.01)	(0.04)
10	0.011	-0.086	-0.568	0.042*	0.174	-0.119	-0.005	-0.446	-0.778	0.009	-0.227	-0.610
	(0.40)	(0.91)	(0.45)	(0.07)	(0.73)	(0.81)	(0.53)	(0.50)	(0.23)	(0.40)	(0.73)	(0.34)
11	0.075	1.598	0.593	0.130*	2.023**	1.103	0.131**	2.003**	1.128*	0.131**	2.003**	1.128*
	(0.21)	(0.11)	(0.50)	(0.07)	(0.02)	(0.13)	(0.05)	(0.01)	(0.09)	(0.04)	(0.01)	(0.09)
12	0.090	1.484*	1.096	0.120**	1.685***	1.221**	0.092*	1.363**	0.990*	0.112**	1.643**	1.222**
	(0.14)	(0.08)	(0.10)	(0.03)	(<0.01)	(0.01)	(0.07)	(0.04)	(0.05)	(0.04)	(0.01)	(0.02)
13	-0.009	1.769**	0.643	0.014	1.965***	0.776	0.021	2.212***	0.799	0.013	2.180***	0.460
	(0.60)	(0.02)	(0.34)	(0.20)	(<0.01)	(0.12)	(0.28)	(<0.01)	(0.22)	(0.34)	(<0.01)	(0.51)
14	0.012	2.156**	0.746	0.020	2.099***	0.633	0.015	2.261**	0.518	0.048	2.772***	0.619
	(0.36)	(0.01)	(0.31)	(0.14)	(<0.01)	(0.23)	(0.37)	(0.01)	(0.49)	(0.14)	(<0.01)	(0.40)
15	0.009	2.091*	-0.347	0.051	2.306**	0.234	0.103**	3.285***	0.828	0.044	2.638**	-0.208
	(0.45)	(0.07)	(0.70)	(0.13)	(0.01)	(0.73)	(0.04)	(<0.01)	(0.32)	(0.25)	(0.02)	(0.81)
16	0.083	3.091***	2.304**	0.097	3.113***	2.021**	0.133**	3.677***	2.455**	0.236***	5.151***	4.329***
	(0.16)	(<0.01)	(0.03)	(0.10)	(<0.01)	(0.04)	(0.05)	(<0.01)	(0.02)	(<0.01)	(<0.01)	(<0.01)
17	0.060	2.534**	1.614	0.088	2.535***	1.487	0.078*	2.258***	1.738**	0.149***	3.458***	2.886***
	(0.29)	(0.03)	(0.14)	(0.11)	(<0.01)	(0.10)	(0.09)	(<0.01)	(0.02)	(<0.01)	(<0.01)	(<0.01)
18	-0.010	1.867**	0.556	0.033	2.562***	0.490	0.026	2.551***	0.171	0.035	2.742***	0.092
	(0.57)	(0.01)	(0.41)	(0.21)	(<0.01)	(0.33)	(0.27)	(<0.01)	(0.76)	(0.19)	(<0.01)	(0.86)
19	0.016	1.733***	1.473***	0.007	1.598***	0.681*	-0.004	1.399***	0.429	0.056*	2.570***	0.971*
	(0.40)	(<0.01)	(<0.01)	(0.42)	(<0.01)	(0.07)	(0.51)	(<0.01)	(0.22)	(0.07)	(<0.01)	(0.07)
20	-0.048	1.972**	1.091	-0.021	2.349***	0.752	-0.030	2.266***	0.526	-0.003	2.769***	0.911*
	(0.79)	(0.01)	(0.14)	(0.73)	(<0.01)	(0.13)	(0.78)	(<0.01)	(0.33)	(0.52)	(<0.01)	(0.08)
21	0.014	1.608**	1.101	0.047*	2.069***	0.703	0.058*	2.291***	0.578	0.042*	2.119***	0.438
	(0.43)	(0.04)	(0.16)	(0.08)	(<0.01)	(0.18)	(0.06)	(<0.01)	(0.30)	(0.10)	(<0.01)	(0.45)
22	-0.018	1.634***	0.878	0.016	2.129***	0.907**	0.054	2.748***	1.352***	0.073**	3.163***	1.458***
	(0.63)	(<0.01)	(0.13)	(0.33)	(<0.01)	(0.02)	(0.11)	(<0.01)	(<0.01)	(0.01)	(<0.01)	(<0.01)
23	0.136***	3.241***	2.091***	0.088***	2.357***	1.253***	0.074**	2.176***	1.130**	0.082**	2.343***	0.977*
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(0.03)	(<0.01)	(0.02)	(0.03)	(<0.01)	(0.06)
24	0.033	2.105**	0.394	0.048*	2.155***	0.364	0.043	2.109***	0.175	0.112***	3.342***	0.792
	(0.25)	(0.02)	(0.61)	(0.06)	(<0.01)	(0.50)	(0.16)	(<0.01)	(0.76)	(<0.01)	(<0.01)	(0.20)
25	-0.067	1.265	-0.254	0.011	2.234***	0.559	0.006	2.189***	0.602	0.038	2.757***	0.657
	(0.94)	(0.15)	(0.74)	(0.31)	(<0.01)	(0.27)	(0.45)	(<0.01)	(0.29)	(0.12)	(<0.01)	(0.24)
Avg.	0.031	1.768***	0.901***	0.046**	1.826***	0.775***	0.045***	1.878***	0.752***	0.061***	2.185***	0.891***
	(0.14)	(<0.01)	(<0.01)	(0.04)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)

Appendix 4: Results from Carhart Four-Factor Model

Table 12: Performance tests on 25 datasets for Carhart four-factor model
The table reports the out-of-sample estimates for the entire period 1973:07 - 2016:12. The difference in annualized Sharpe ratio of each risk-based strategy versus naive, $SR_p - SR_{1/N}$, is denoted as $\boldsymbol{\Delta SR}$. $\boldsymbol{\alpha_1^{CAPM}}$ denotes the alpha in the CAPM, whereas $\boldsymbol{\alpha_2^{Carhart}}$ denotes the alpha in the Carhart four-factor model. The associated p-values are reported in parentheses. The average p-values are constructed using Brown's method to combine p-values. Note that that alphas are annualized and reportd in percentage. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

	Minimum-Variance			**Volatility-timing**			**Low-Vol ($1/\sigma$)**			**Low-Beta ($1/\beta$)**		
#	ΔSR	α_1^{CAPM}	$\alpha_2^{Carhart}$	ΔSR	α_1^{CAPM}	$\alpha_2^{Carhart}$	ΔSR	α_1^{CAPM}	$\alpha_2^{Carhart}$	ΔSR	α_1^{CAPM}	$\alpha_2^{Carhart}$
1	-0.085	-0.191	0.052	-0.035	0.567	0.050	-0.020	0.785**	0.444	-0.033	0.738	0.324
	(0.92)	(0.65)	(0.89)	(0.78)	(0.11)	(0.86)	(0.70)	(0.02)	(0.14)	(0.94)	(0.22)	(0.50)
2	0.002	2.110***	0.033	0.034*	2.429***	0.452	0.060*	2.969***	0.545	0.086**	3.405***	0.779
	(0.54)	(<0.01)	(0.96)	(0.06)	(<0.01)	(0.33)	(0.08)	(<0.01)	(0.40)	(0.04)	(<0.01)	(0.25)
3	0.092***	1.377**	1.069**	0.062***	0.840***	0.546*	0.060***	0.902*	0.496	0.057**	0.839*	0.511
	(<0.01)	(0.01)	(0.04)	(<0.01)	(<0.01)	(0.05)	(<0.01)	(0.06)	(0.30)	(0.04)	(0.07)	(0.26)
4	0.022	1.572**	0.319	0.042*	1.727***	0.704*	0.039	1.729***	0.791	0.036	1.747***	0.563
	(0.38)	(0.02)	(0.58)	(0.06)	(<0.01)	(0.06)	(0.16)	(<0.01)	(0.11)	(0.16)	(<0.01)	(0.30)
5	0.059*	1.017	0.226	0.070***	1.056	0.292	0.056**	0.964	-0.126	0.042	0.882	0.738
	(0.08)	(0.23)	(0.78)	(<0.01)	(0.12)	(0.65)	(0.04)	(0.24)	(0.87)	(0.20)	(0.35)	(0.40)
6	0.002	0.294	-0.114	0.019	0.433	0.034	0.019	0.534	-0.022	0.010	0.416	-0.173
	(0.48)	(0.60)	(0.85)	(0.14)	(0.26)	(0.93)	(0.32)	(0.35)	(0.97)	(0.34)	(0.48)	(0.77)
7	0.038	2.561***	0.948	0.022	2.197***	0.716	0.043	2.575***	1.075*	0.024	2.327***	0.696
	(0.18)	(<0.01)	(0.14)	(0.18)	(<0.01)	(0.13)	(0.18)	(<0.01)	(0.08)	(0.26)	(<0.01)	(0.26)
8	0.106**	1.893***	1.771***	0.064**	1.161***	1.317***	0.046*	0.931**	1.058**	0.050*	1.000**	1.094**
	(0.02)	(<0.01)	(<0.01)	(0.02)	(<0.01)	(<0.01)	(0.08)	(0.05)	(0.03)	(0.06)	(0.04)	(0.02)
9	0.089	2.409**	1.199	0.099*	2.232***	0.972	0.098*	2.292**	0.869	0.100*	2.318***	0.893
	(0.18)	(0.02)	(0.19)	(0.08)	(<0.01)	(0.17)	(0.08)	(0.01)	(0.26)	(0.06)	(<0.01)	(0.24)
10	0.009	-0.108	-0.442	0.048*	0.282	-0.157	0.043	0.304	-0.091	0.042	0.271	-0.029
	(0.50)	(0.89)	(0.56)	(0.06)	(0.59)	(0.76)	(0.20)	(0.64)	(0.89)	(0.20)	(0.67)	(0.96)
11	0.082	1.651*	0.380	0.129*	2.007**	0.673	0.142***	2.099***	0.782	0.143***	2.133***	0.887
	(0.30)	(0.09)	(0.67)	(0.06)	(0.02)	(0.36)	(<0.01)	(<0.01)	(0.22)	(<0.01)	(<0.01)	(0.17)
12	0.085	1.349*	0.743	0.114**	1.596***	0.835*	0.108**	1.546**	0.770	0.117*	1.717**	0.850
	(0.20)	(0.08)	(0.25)	(0.04)	(<0.01)	(0.09)	(0.04)	(0.01)	(0.10)	(0.06)	(0.01)	(0.10)
13	-0.035	1.423*	0.034	0.013	1.965***	0.625	0.008	1.957***	0.648	0.027	2.333***	0.506
	(0.82)	(0.07)	(0.96)	(0.26)	(<0.01)	(0.22)	(0.38)	(<0.01)	(0.30)	(0.32)	(<0.01)	(0.46)
14	0.002	1.998**	0.475	0.014	2.006***	0.491	0.005	2.078**	0.239	0.045	2.712***	0.358
	(0.56)	(0.02)	(0.52)	(0.22)	(<0.01)	(0.36)	(0.46)	(0.02)	(0.75)	(0.22)	(<0.01)	(0.62)
15	0.082	2.966***	1.045	0.116**	3.185***	1.339*	0.146**	3.772***	1.599*	0.099	3.246***	1.079
	(0.12)	(<0.01)	(0.24)	(0.02)	(<0.01)	(0.05)	(0.04)	(<0.01)	(0.05)	(0.10)	(<0.01)	(0.20)
16	0.061	2.763**	1.837*	0.078	2.834***	1.818*	0.053	2.546**	1.421	0.174***	4.233***	2.750**
	(0.26)	(0.01)	(0.09)	(0.16)	(<0.01)	(0.07)	(0.14)	(0.02)	(0.16)	(<0.01)	(<0.01)	(0.01)
17	0.031	2.142*	1.234	0.089	2.537***	1.613*	0.082	2.346***	1.474*	0.118**	3.108***	1.780*
	(0.38)	(0.06)	(0.27)	(0.22)	(<0.01)	(0.08)	(0.10)	(<0.01)	(0.07)	(0.04)	(<0.01)	(0.08)
18	0.009	2.182***	0.602	0.046	2.756***	0.651	0.031	2.627***	0.360	0.046	2.953***	0.242
	(0.52)	(<0.01)	(0.41)	(0.22)	(<0.01)	(0.22)	(0.16)	(<0.01)	(0.53)	(0.14)	(<0.01)	(0.66)
19	0.042	2.103***	1.618***	0.018	1.770***	0.713*	0.011	1.653***	0.622	0.032	2.180***	0.415
	(0.34)	(<0.01)	(<0.01)	(0.32)	(<0.01)	(0.07)	(0.34)	(<0.01)	(0.12)	(0.16)	(<0.01)	(0.46)
20	-0.072	1.545**	0.428	-0.023	2.289***	0.761	-0.022	2.349***	0.592	-0.015	2.553***	0.714
	(0.76)	(0.04)	(0.54)	(0.68)	(<0.01)	(0.11)	(0.72)	(<0.01)	(0.25)	(0.56)	(<0.01)	(0.18)
21	-0.007	1.327	0.581	0.023	1.732**	0.568	0.057	2.346***	0.878	0.019	1.873**	0.589
	(0.64)	(0.10)	(0.47)	(0.28)	(0.01)	(0.32)	(0.10)	(<0.01)	(0.15)	(0.32)	(0.03)	(0.36)
22	-0.033	1.460**	0.304	0.006	1.997***	0.592	0.028	2.337***	0.991**	0.047*	2.768***	1.029**
	(0.74)	(0.03)	(0.63)	(0.48)	(<0.01)	(0.15)	(0.26)	(<0.01)	(0.03)	(0.08)	(<0.01)	(0.03)
23	0.110**	2.852***	1.660**	0.083***	2.285***	1.006**	0.081***	2.294***	0.986**	0.066***	2.106***	0.685
	(0.04)	(<0.01)	(0.03)	(<0.01)	(<0.01)	(0.02)	(<0.01)	(<0.01)	(0.05)	(<0.01)	(<0.01)	(0.19)
24	0.087*	2.947***	0.979	0.079***	2.589***	0.671	0.092***	2.847***	0.721	0.159***	4.050***	1.335**
	(0.06)	(<0.01)	(0.24)	(<0.01)	(<0.01)	(0.22)	(<0.01)	(<0.01)	(0.21)	(<0.01)	(<0.01)	(0.04)
25	0.002	2.262***	0.580	0.035**	2.560***	0.766	0.035**	2.560***	0.971*	0.059**	3.024***	0.689
	(0.48)	(<0.01)	(0.44)	(0.04)	(<0.01)	(0.12)	(0.04)	(<0.01)	(0.08)	(0.04)	(<0.01)	(0.24)
Avg.	0.031*	1.756***	0.702**	0.050***	1.881***	0.722***	0.052***	1.974***	0.724***	0.062***	2.197***	0.772***
	(0.10)	(<0.01)	(0.03)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)

Appendix 5: Robustness Tests: Subperiod 1973:07 - 1995:03

Table 13: Robustness checks 1: Statistical tests for subperiod 1

The table shows out-of-sample estimates for the subperiod 1973:07 - 1995:03. The difference in annualized Sharpe ratio of each risk-based strategy versus naive, $SR_p - SR_{1/N}$, is denoted as $\boldsymbol{\Delta SR}$. $\boldsymbol{\alpha_1^{CAPM}}$ denotes the alpha in the CAPM, whereas $\boldsymbol{\alpha_2^{FF5}}$ denotes the alpha in the Fama-French five-factor model. The associated p-values are reported in parentheses. The average p-values are constructed using Brown's method to combine p-values. Note that that alphas are annualized and reportd in percentage. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

	Minimum-Variance			**Volatility-timing**			**Low-Vol ($1/\sigma$)**			**Low-Beta ($1/\beta$)**		
#	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}
1.00	-0.167	-1.056	-0.938*	-0.093	-0.002	0.190	-0.064	0.478	0.776**	-0.070	0.409	0.472
	(0.95)	(0.13)	(0.07)	(0.97)	(1.00)	(0.57)	(0.83)	(0.15)	(0.02)	(0.92)	(0.41)	(0.36)
2.00	-0.032	1.755**	0.423	-0.005	2.096***	0.418	0.006	2.407**	0.508	0.006	2.428**	0.016
	(0.75)	(0.04)	(0.61)	(0.59)	(<0.01)	(0.37)	(0.45)	(0.02)	(0.53)	(0.47)	(0.02)	(0.98)
3.00	0.066*	1.328*	0.930	0.026**	0.615**	0.486*	0.027	0.675	0.488	0.046*	0.986*	0.741
	(0.06)	(0.07)	(0.25)	(0.04)	(0.03)	(0.07)	(0.21)	(0.22)	(0.40)	(0.07)	(0.08)	(0.21)
4.00	0.092*	2.352***	1.061	0.049**	1.665***	0.467	0.047	1.651**	0.477	0.063	1.896***	0.564
	(0.05)	(<0.01)	(0.15)	(0.05)	(<0.01)	(0.14)	(0.14)	(0.01)	(0.39)	(0.10)	(<0.01)	(0.32)
5.00	0.055	0.700	1.250	0.062***	0.734	0.818	0.028	0.280	0.426	0.072	1.039	1.505
	(0.10)	(0.40)	(0.18)	(<0.01)	(0.22)	(0.21)	(0.26)	(0.74)	(0.66)	(0.10)	(0.27)	(0.14)
6.00	-0.015	0.240	-0.040	-0.009	0.260	-0.023	-0.016	0.237	-0.317	-0.018	0.214	-0.116
	(0.60)	(0.72)	(0.96)	(0.64)	(0.47)	(0.95)	(0.65)	(0.73)	(0.67)	(0.67)	(0.77)	(0.89)
7.00	-0.028	1.137	0.359	-0.016	1.262**	0.232	-0.020	1.245	0.402	-0.027	1.162	0.298
	(0.70)	(0.18)	(0.69)	(0.69)	(0.05)	(0.71)	(0.58)	(0.14)	(0.65)	(0.71)	(0.18)	(0.74)
8.00	0.038	0.920	0.177	0.032	0.726*	0.295	0.031	0.750	0.119	0.023	0.637	-0.068
	(0.29)	(0.25)	(0.82)	(0.16)	(0.09)	(0.43)	(0.27)	(0.23)	(0.83)	(0.30)	(0.32)	(0.90)
9.00	0.106	2.502*	1.121	0.098	2.066**	0.577	0.082	1.801**	0.237	0.082	1.801**	0.237
	(0.18)	(0.10)	(0.45)	(0.14)	(0.04)	(0.55)	(0.17)	(0.05)	(0.77)	(0.16)	(0.05)	(0.77)
10.00	-0.068	-1.861**	-1.419	-0.013	-1.077***	-0.455	-0.053	-1.648**	-1.278*	-0.076	-2.030***	-1.744**
	(0.90)	(0.03)	(0.12)	(0.78)	(<0.01)	(0.30)	(0.88)	(0.02)	(0.09)	(0.99)	(<0.01)	(0.02)
11.00	0.074	1.456	0.404	0.096	1.475	0.378	0.072	1.093	0.122	0.072	1.093	0.122
	(0.26)	(0.28)	(0.75)	(0.15)	(0.13)	(0.67)	(0.19)	(0.17)	(0.87)	(0.20)	(0.17)	(0.87)
12.00	0.032	0.748	0.034	0.069	1.097	0.651	0.048	0.810	0.479	0.048	0.810	0.479
	(0.39)	(0.48)	(0.97)	(0.19)	(0.11)	(0.26)	(0.28)	(0.18)	(0.30)	(0.25)	(0.18)	(0.30)
13.00	-0.001	1.500*	0.560	0.018	1.725***	0.725	0.058	2.446***	1.089	0.005	1.677*	-0.305
	(0.49)	(0.05)	(0.48)	(0.15)	(<0.01)	(0.13)	(0.12)	(<0.01)	(0.20)	(0.44)	(0.08)	(0.73)
14.00	-0.017	1.143	0.279	0.012	1.486***	0.286	-0.018	1.145	-0.438	0.057	2.392**	0.285
	(0.64)	(0.18)	(0.75)	(0.28)	(<0.01)	(0.55)	(0.67)	(0.18)	(0.61)	(0.13)	(0.02)	(0.75)
15.00	0.065	2.512*	-0.141	0.081	2.400**	0.227	0.123*	3.116**	0.047	0.130*	3.259**	0.043
	(0.32)	(0.09)	(0.91)	(0.10)	(0.02)	(0.78)	(0.06)	(0.02)	(0.96)	(0.07)	(0.02)	(0.97)
16.00	0.090	2.804*	1.967	0.084	2.528*	1.412	0.178**	3.801***	1.932	0.206**	4.201***	3.246**
	(0.27)	(0.09)	(0.25)	(0.24)	(0.08)	(0.35)	(0.04)	(<0.01)	(0.17)	(0.05)	(<0.01)	(0.03)
17.00	0.117	3.104*	1.906	0.088	2.463**	1.141	0.063	2.033**	1.388	0.092	2.493**	0.990
	(0.25)	(0.05)	(0.25)	(0.20)	(0.05)	(0.38)	(0.22)	(0.02)	(0.11)	(0.11)	(0.01)	(0.32)
18.00	-0.116	0.824	-0.554	-0.022	2.183***	0.517	-0.023	2.196***	0.174	-0.007	2.469***	0.374
	(0.90)	(0.38)	(0.53)	(0.67)	(<0.01)	(0.29)	(0.64)	(<0.01)	(0.72)	(0.58)	(<0.01)	(0.52)
19.00	-0.070	0.711	1.075	-0.071	0.616*	0.795**	-0.062	0.739**	0.718**	-0.057	0.837**	0.888**
	(0.81)	(0.35)	(0.13)	(0.91)	(0.09)	(0.02)	(0.86)	(0.04)	(0.03)	(0.84)	(0.05)	(0.02)
20.00	-0.193	0.100	-0.679	-0.097	1.507**	0.520	-0.121	1.118	0.291	-0.096	1.606*	0.361
	(0.98)	(0.93)	(0.57)	(0.96)	(0.04)	(0.42)	(0.99)	(0.14)	(0.66)	(0.98)	(0.07)	(0.61)
21.00	-0.097	0.163	1.034	-0.027	1.175*	0.794	-0.035	1.051	0.292	-0.054	0.806	0.044
	(0.87)	(0.87)	(0.36)	(0.74)	(0.09)	(0.15)	(0.81)	(0.15)	(0.63)	(0.88)	(0.36)	(0.95)
22.00	-0.065	1.288	-0.134	-0.029	1.784***	0.583	0.002	2.295***	1.160***	0.017	2.559***	0.879*
	(0.72)	(0.12)	(0.84)	(0.72)	(<0.01)	(0.11)	(0.46)	(<0.01)	(<0.01)	(0.30)	(<0.01)	(0.05)
23.00	0.072	2.438**	0.440	0.037	1.724***	0.398	0.021	1.490**	-0.121	0.045	1.863***	0.344
	(0.13)	(0.02)	(0.68)	(0.11)	(<0.01)	(0.33)	(0.33)	(0.01)	(0.82)	(0.12)	(<0.01)	(0.52)
24.00	0.056	2.489**	0.428	0.034	2.024***	0.537	0.054	2.316***	0.535	0.113***	3.295***	0.378
	(0.19)	(0.02)	(0.67)	(0.13)	(<0.01)	(0.31)	(0.10)	(<0.01)	(0.39)	(<0.01)	(<0.01)	(0.57)
25.00	-0.038	1.717*	-0.116	0.002	2.248***	0.470	-0.006	2.166***	0.247	0.032	2.747***	0.413
	(0.73)	(0.07)	(0.90)	(0.44)	(<0.01)	(0.30)	(0.56)	(<0.01)	(0.70)	(0.24)	(<0.01)	(0.47)
Avg.	-0.002	1.241***	0.377	0.016**	1.391***	0.498	0.017	1.428***	0.390	0.028**	1.626***	0.418
	(0.52)	(<0.01)	(0.76)	(0.03)	(<0.01)	(0.12)	(0.23)	(<0.01)	(0.20)	(0.02)	(<0.01)	(0.31)

Appendix 6: Robustness Tests: Subperiod 1995:04 - 2016:12

Table 14: Robustness checks 2: Statistical tests for subperiod 2

The table shows out-of-sample estimates for the subperiod 1995:04 - 2016:12. The difference in annualized Sharpe ratio of each risk-based strategy versus naive, $SR_p - SR_{1/N}$, is denoted as $\boldsymbol{\Delta SR}$. $\boldsymbol{\alpha_1^{CAPM}}$ denotes the alpha in the CAPM, whereas $\boldsymbol{\alpha_2^{FF5}}$ denotes the alpha in the Fama-French five-factor model. The associated p-values are reported in parentheses. The average p-values are constructed using Brown's method to combine p-values. Note that that alphas are annualized and reportd in percentage. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

	Minimum-Variance			**Volatility-timing**			**Low-Vol ($1/\sigma$)**			**Low-Beta ($1/\beta$)**		
#	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}
1.00	0.016	0.855*	1.070**	0.019	0.988	0.341	0.029	1.089**	0.444	0.024	1.242	1.004
	(0.44)	(0.08)	(0.04)	(0.33)	(0.10)	(0.40)	(0.32)	(0.03)	(0.33)	(0.32)	(0.18)	(0.10)
2.00	0.010	2.003	-0.968	0.029	2.155*	-0.641	-0.012	1.872	-1.294	-0.003	2.068	-1.364
	(0.46)	(0.11)	(0.35)	(0.20)	(0.06)	(0.40)	(0.62)	(0.19)	(0.24)	(0.45)	(0.17)	(0.22)
3.00	0.147***	1.824**	0.410	0.095***	0.962**	0.021	0.142***	1.772**	0.411	0.145***	1.807**	0.426
	(<0.01)	(0.02)	(0.57)	(<0.01)	(0.04)	(0.96)	(<0.01)	(0.03)	(0.60)	(<0.01)	(0.02)	(0.57)
4.00	0.032	1.976*	-1.021	0.048	2.061***	-0.503	0.040	2.031**	-0.846	0.024	1.891*	-1.335**
	(0.32)	(0.05)	(0.15)	(0.14)	(<0.01)	(0.25)	(0.23)	(0.04)	(0.17)	(0.34)	(0.07)	(0.04)
5.00	0.115*	1.914	-0.793	0.119**	1.833*	-0.775	0.136**	2.193*	-0.717	0.072	1.425	-1.234
	(0.05)	(0.11)	(0.43)	(0.02)	(0.09)	(0.35)	(0.02)	(0.07)	(0.46)	(0.17)	(0.28)	(0.28)
6.00	0.053	0.772	-0.144	0.064*	0.794	-0.109	0.070	0.984	-0.147	0.057	0.811	-0.267
	(0.21)	(0.37)	(0.87)	(0.07)	(0.20)	(0.85)	(0.11)	(0.23)	(0.85)	(0.18)	(0.34)	(0.75)
7.00	0.120**	4.282***	1.520	0.049	3.072***	0.235	0.029	2.903***	0.069	0.069	3.519***	0.788
	(0.04)	(<0.01)	(0.11)	(0.11)	(<0.01)	(0.70)	(0.29)	(<0.01)	(0.93)	(0.11)	(<0.01)	(0.38)
8.00	0.171***	2.768***	1.564	0.087**	1.462**	0.591	0.037	0.825	-0.168	0.043	0.904	-0.036
	(<0.01)	(<0.01)	(0.10)	(0.02)	(0.02)	(0.34)	(0.23)	(0.28)	(0.82)	(0.20)	(0.23)	(0.96)
9.00	0.082	2.697*	-0.566	0.115	2.697**	-0.499	0.125	3.033**	-0.740	0.125	3.033**	-0.740
	(0.28)	(0.08)	(0.65)	(0.15)	(0.04)	(0.60)	(0.15)	(0.04)	(0.49)	(0.16)	(0.04)	(0.49)
10.00	0.119*	1.884	-1.096	0.118**	1.570*	-0.372	0.063	0.918	-1.435	0.121**	1.745	-0.412
	(0.08)	(0.14)	(0.30)	(0.02)	(0.08)	(0.64)	(0.20)	(0.40)	(0.14)	(0.05)	(0.11)	(0.67)
11.00	0.085	1.876	-0.774	0.185*	2.796**	-0.432	0.209**	3.120**	-0.524	0.209**	3.120**	-0.524
	(0.31)	(0.20)	(0.53)	(0.06)	(0.04)	(0.67)	(0.04)	(0.02)	(0.56)	(0.04)	(0.02)	(0.56)
12.00	0.153	2.273*	0.257	0.181*	2.370**	0.175	0.148*	2.060*	-0.340	0.190**	2.619**	0.005
	(0.10)	(0.08)	(0.80)	(0.06)	(0.03)	(0.80)	(0.07)	(0.08)	(0.66)	(0.03)	(0.02)	(0.99)
13.00	-0.016	2.208*	-1.346	0.011	2.357**	-0.670	-0.022	2.095*	-1.073	0.023	2.830**	-0.846
	(0.59)	(0.09)	(0.14)	(0.36)	(0.03)	(0.30)	(0.63)	(0.10)	(0.23)	(0.33)	(0.04)	(0.36)
14.00	0.046	3.348**	-0.151	0.031	2.867**	-0.450	0.052	3.568**	-0.302	0.037	3.314**	-0.442
	(0.25)	(0.02)	(0.89)	(0.18)	(0.02)	(0.55)	(0.21)	(0.02)	(0.78)	(0.31)	(0.03)	(0.68)
15.00	-0.056	1.696	-2.078	0.013	2.309	-1.778*	0.074	3.509**	-1.023	-0.050	2.014	-2.326*
	(0.75)	(0.33)	(0.12)	(0.40)	(0.13)	(0.07)	(0.16)	(0.05)	(0.42)	(0.72)	(0.30)	(0.08)
16.00	0.079	3.474**	0.981	0.120	3.866**	0.560	0.086	3.747**	0.306	0.276**	6.344***	2.983*
	(0.24)	(0.03)	(0.51)	(0.15)	(0.01)	(0.68)	(0.27)	(0.03)	(0.84)	(0.02)	(<0.01)	(0.09)
17.00	-0.002	2.073	-0.617	0.098	2.819*	-0.392	0.118*	2.790**	-0.718	0.241**	4.781***	0.460
	(0.50)	(0.22)	(0.70)	(0.17)	(0.05)	(0.75)	(0.08)	(0.03)	(0.48)	(0.02)	(<0.01)	(0.73)
18.00	0.103*	2.949**	0.503	0.090*	2.951**	-0.499	0.073	2.890**	-1.115	0.074*	2.957**	-0.555
	(0.09)	(0.01)	(0.62)	(0.06)	(0.02)	(0.54)	(0.14)	(0.04)	(0.24)	(0.07)	(0.04)	(0.48)
19.00	0.122*	2.869***	1.089	0.094**	2.641***	0.515	0.060	2.091***	0.100	0.169***	4.352***	1.130*
	(0.08)	(<0.01)	(0.18)	(0.02)	(<0.01)	(0.31)	(0.10)	(<0.01)	(0.85)	(<0.01)	(<0.01)	(0.08)
20.00	0.139**	4.022***	1.892**	0.071*	3.323***	0.420	0.080**	3.564***	0.167	0.105***	4.057***	1.259*
	(0.04)	(<0.01)	(0.04)	(0.06)	(<0.01)	(0.52)	(0.05)	(<0.01)	(0.81)	(<0.01)	(<0.01)	(0.07)
21.00	0.153**	3.195***	0.735	0.141***	3.107***	0.258	0.167***	3.656***	0.194	0.153***	3.548***	0.623
	(0.03)	(<0.01)	(0.47)	(<0.01)	(<0.01)	(0.72)	(<0.01)	(<0.01)	(0.80)	(<0.01)	(<0.01)	(0.44)
22.00	0.037	2.051**	-0.433	0.069	2.555***	-0.232	0.114**	3.293***	0.045	0.135***	3.845***	0.900
	(0.32)	(0.03)	(0.55)	(0.10)	(<0.01)	(0.63)	(0.04)	(<0.01)	(0.94)	(<0.01)	(<0.01)	(0.12)
23.00	0.213***	4.157***	1.043	0.152***	3.112***	0.360	0.138***	2.986***	-0.312	0.129**	2.962***	-0.575
	(<0.01)	(<0.01)	(0.28)	(<0.01)	(<0.01)	(0.53)	(<0.01)	(<0.01)	(0.62)	(0.03)	(<0.01)	(0.40)
24.00	0.014	1.872	-1.697	0.069	2.416*	-0.881	0.034	2.001	-1.359	0.112*	3.453**	-0.443
	(0.44)	(0.20)	(0.13)	(0.13)	(0.05)	(0.29)	(0.31)	(0.13)	(0.12)	(0.08)	(0.03)	(0.65)
25.00	-0.096	0.960	-2.792**	0.022	2.337**	-1.073	0.021	2.336*	-1.040	0.044	2.838**	-0.617
	(0.90)	(0.51)	(0.01)	(0.32)	(0.05)	(0.16)	(0.30)	(0.05)	(0.22)	(0.17)	(0.03)	(0.49)
Avg.	0.074***	2.400***	-0.136**	0.084***	2.377***	-0.233	0.080***	2.453***	-0.457	0.101***	2.859***	-0.086
	(<0.01)	(<0.01)	(0.04)	(<0.01)	(<0.01)	(0.86)	(<0.01)	(<0.01)	(0.88)	(<0.01)	(<0.01)	(0.20)

Appendix 7: Robustness Tests: 5 Year Look-Back Period

Table 15: Robustness checks 3: Statistical test 5-year look-back period
The table shows out-of-sample estimates for the entire period 1973:07 - 2016:12 with a look-back period of 5 years instead of 10 years. The difference in annualized Sharpe ratio of each risk-based strategy versus naive, $SR_p - SR_{1/N}$, is denoted as $\boldsymbol{\Delta SR}$. $\boldsymbol{\alpha_1^{CAPM}}$ denotes the alpha in the CAPM, whereas $\boldsymbol{\alpha_2^{FF5}}$ denotes the alpha in the Fama-French five-factor model. The associated p-values are reported in parentheses. The average p-values are constructed using Brown's method to combine p-values. Note that that alphas are annualized and reportd in percentage. Note that that alphas are annualized and reportd in percentage. Significance values: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

	Minimum-Variance			**Volatility-timing**			**Low-Vol ($1/\sigma$)**			**Low-Beta ($1/\beta$)**		
#	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}	ΔSR	α_1^{CAPM}	α_2^{FF5}
1.00	-0.078	-0.074	0.059	-0.035	0.567	0.029	-0.024	0.715**	0.349	-0.037	0.689	0.505
	(0.86)	(0.87)	(0.86)	(0.83)	(0.11)	(0.92)	(0.70)	(0.01)	(0.22)	(0.85)	(0.26)	(0.30)
2.00	0.038	2.675***	-0.194	0.034*	2.429***	-0.146	0.059*	2.962***	0.201	0.084**	3.384***	0.202
	(0.20)	(<0.01)	(0.78)	(0.08)	(<0.01)	(0.74)	(0.09)	(<0.01)	(0.76)	(0.02)	(<0.01)	(0.77)
3.00	0.099***	1.505***	0.583	0.062***	0.840***	0.052	0.062**	0.924*	-0.144	0.058*	0.851*	-0.074
	(<0.01)	(<0.01)	(0.28)	(<0.01)	(<0.01)	(0.85)	(0.05)	(0.06)	(0.76)	(0.05)	(0.06)	(0.87)
4.00	0.030	1.676**	-0.897*	0.042*	1.727***	-0.506*	0.039	1.728***	-0.510	0.036	1.748***	-1.035**
	(0.27)	(0.01)	(0.08)	(0.07)	(<0.01)	(0.08)	(0.15)	(<0.01)	(0.25)	(0.18)	(<0.01)	(0.03)
5.00	0.054	0.926	-1.079	0.070**	1.056	-1.021*	0.056	0.953	-1.050	0.043	0.898	-1.014
	(0.14)	(0.25)	(0.15)	(0.04)	(0.12)	(0.09)	(0.12)	(0.24)	(0.16)	(0.20)	(0.34)	(0.25)
6.00	0.003	0.278	-0.590	0.019	0.433	-0.349	0.018	0.524	-0.529	0.009	0.410	-0.742
	(0.48)	(0.61)	(0.27)	(0.26)	(0.26)	(0.35)	(0.32)	(0.35)	(0.33)	(0.41)	(0.49)	(0.21)
7.00	0.029	2.376***	-0.004	0.022	2.197***	-0.251	0.043	2.567***	0.225	0.024	2.326***	-0.142
	(0.25)	(<0.01)	(0.99)	(0.20)	(<0.01)	(0.56)	(0.14)	(<0.01)	(0.70)	(0.28)	(<0.01)	(0.81)
8.00	0.098**	1.769***	1.010*	0.064***	1.161***	0.567	0.049*	0.978**	0.464	0.055*	1.058**	0.544
	(0.02)	(<0.01)	(0.09)	(<0.01)	(<0.01)	(0.13)	(0.08)	(0.04)	(0.31)	(0.06)	(0.03)	(0.25)
9.00	0.094	2.546**	0.143	0.099	2.232***	-0.014	0.100	2.327***	-0.651	0.103*	2.373***	-0.604
	(0.18)	(0.02)	(0.88)	(0.11)	(<0.01)	(0.98)	(0.11)	(<0.01)	(0.34)	(0.09)	(<0.01)	(0.39)
10.00	0.012	-0.039	-1.943***	0.048*	0.282	-1.067**	0.042	0.291	-1.319**	0.043	0.299	-1.362**
	(0.41)	(0.96)	(<0.01)	(0.07)	(0.59)	(0.03)	(0.19)	(0.66)	(0.03)	(0.17)	(0.64)	(0.02)
11.00	0.102	1.964*	-0.442	0.129*	2.007**	-0.426	0.142**	2.109***	-0.444	0.144**	2.155***	-0.537
	(0.19)	(0.06)	(0.62)	(0.09)	(0.02)	(0.54)	(0.03)	(<0.01)	(0.45)	(0.03)	(<0.01)	(0.37)
12.00	0.097	1.537*	0.225	0.114*	1.596**	0.170	0.111*	1.578**	-0.034	0.121**	1.775***	-0.179
	(0.13)	(0.06)	(0.72)	(0.05)	(0.01)	(0.71)	(0.05)	(0.01)	(0.94)	(0.03)	(<0.01)	(0.69)
13.00	0.003	2.001**	-0.868	0.013	1.965***	-0.635	0.008	1.962***	-0.577	0.029	2.366***	-0.624
	(0.48)	(0.02)	(0.21)	(0.27)	(<0.01)	(0.15)	(0.41)	(<0.01)	(0.33)	(0.24)	(<0.01)	(0.32)
14.00	0.031	2.508***	-0.566	0.014	2.006***	-0.881*	0.006	2.083**	-1.266*	0.046	2.724***	-0.802
	(0.26)	(<0.01)	(0.45)	(0.23)	(<0.01)	(0.06)	(0.45)	(0.02)	(0.06)	(0.15)	(<0.01)	(0.24)
15.00	0.105	3.453***	-0.188	0.116**	3.185***	-0.304	0.148**	3.812***	-0.303	0.117*	3.655***	-0.813
	(0.11)	(<0.01)	(0.84)	(0.02)	(<0.01)	(0.64)	(0.02)	(<0.01)	(0.70)	(0.06)	(<0.01)	(0.36)
16.00	0.052	2.783**	0.703	0.078	2.834***	0.458	0.054	2.576**	-0.154	0.130*	4.034***	0.766
	(0.30)	(0.02)	(0.53)	(0.17)	(<0.01)	(0.64)	(0.26)	(0.02)	(0.87)	(0.09)	(<0.01)	(0.52)
17.00	0.052	2.528**	0.226	0.089	2.537***	0.133	0.086*	2.391***	-0.420	0.071	2.966**	-0.925
	(0.32)	(0.04)	(0.85)	(0.15)	(<0.01)	(0.88)	(0.09)	(<0.01)	(0.56)	(0.21)	(0.02)	(0.44)
18.00	0.013	2.378***	0.008	0.046	2.756***	-0.253	0.031	2.605***	-0.662	0.046	2.920***	-0.303
	(0.43)	(<0.01)	(0.99)	(0.15)	(<0.01)	(0.62)	(0.26)	(<0.01)	(0.23)	(0.15)	(<0.01)	(0.58)
19.00	0.027	1.945***	1.178**	0.018	1.770***	0.125	0.010	1.615***	0.033	0.034	2.205***	-0.088
	(0.36)	(<0.01)	(0.05)	(0.31)	(<0.01)	(0.74)	(0.41)	(<0.01)	(0.93)	(0.21)	(<0.01)	(0.87)
20.00	-0.099	1.200	-0.913	-0.023	2.289***	-0.354	-0.025	2.274***	-0.675	-0.019	2.479***	-0.291
	(0.95)	(0.11)	(0.20)	(0.77)	(<0.01)	(0.43)	(0.78)	(<0.01)	(0.15)	(0.72)	(<0.01)	(0.58)
21.00	-0.026	1.101	-0.693	0.023	1.732**	-0.732	0.053*	2.257***	-0.615	0.017	1.835**	-0.678
	(0.67)	(0.20)	(0.40)	(0.29)	(0.01)	(0.18)	(0.09)	(<0.01)	(0.29)	(0.34)	(0.04)	(0.30)
22.00	-0.059	1.094	-0.987*	0.006	1.997***	-0.485	0.023	2.248***	-0.374	0.045	2.717***	0.155
	(0.78)	(0.11)	(0.09)	(0.43)	(<0.01)	(0.17)	(0.29)	(<0.01)	(0.33)	(0.12)	(<0.01)	(0.73)
23.00	0.121**	2.972***	0.553	0.083**	2.285***	-0.253	0.083**	2.308***	-0.506	0.069**	2.146***	-0.879**
	(0.02)	(<0.01)	(0.43)	(0.01)	(<0.01)	(0.50)	(0.02)	(<0.01)	(0.23)	(0.05)	(<0.01)	(0.05)
24.00	0.101**	3.134***	0.044	0.079***	2.589***	-0.360	0.095**	2.880***	-0.424	0.162***	4.090***	0.451
	(0.04)	(<0.01)	(0.96)	(<0.01)	(<0.01)	(0.48)	(0.01)	(<0.01)	(0.43)	(<0.01)	(<0.01)	(0.47)
25.00	0.021	2.620***	-0.534	0.035*	2.560***	-0.374	0.036	2.565***	-0.111	0.063**	3.086***	-0.162
	(0.34)	(<0.01)	(0.48)	(0.09)	(<0.01)	(0.41)	(0.17)	(<0.01)	(0.83)	(0.04)	(<0.01)	(0.77)
Avg.	0.037***	1.874***	-0.207	0.050***	1.881***	-0.275	0.052***	1.969***	-0.380	0.060***	2.208***	-0.345
	(<0.01)	(<0.01)	(0.24)	(<0.01)	(<0.01)	(0.32)	(<0.01)	(<0.01)	(0.36)	(<0.01)	(<0.01)	(0.27)

Reflection Notes

The School of Business and Law at University of Agder request us to write a reflection note that includes a discussion of how the topic of this thesis relates to internationalisation, innovation and accountability, and will start this reflective note by briefly discuss the thesis main theme, and then our findings and conclusion.

In this thesis have we evaluated the performance of four risk-based strategies relative to the naive diversification strategy, and additionally assessed whether the (out)performance could be attributed to established factor premiums. The four risk-based strategies are characterized by weighting each asset solely based on the assets risk. We found that three of four risk-based strategies, on average, delivered superior performance in terms of Sharpe ratio over the naive diversification. Further, we evaluated the outperformance during bull and bear markets to get a deeper insight whether the outperformance of risk-based strategies over the naive is mostly generated during bear markets contra bull markets. The results obtained in the bull- and bear phases illustrated that risk-based strategies perform significantly better during bear markets compared to the naive diversification. Finally, we evaluated whether the outperformance could be attributed to established factor premiums. The results obtained from the Fama-French five-factor model indicated that all risk-based strategies tilt towards known market anomalies. We suggest in line with Zakamulin (2017) that the superior performance of risk-based strategies is likely to be attributed to exposure towards established factor premiums rather than better mean-variance efficiency. Our thesis extends previous literature in several ways. First, existing empirical studies usually evaluate the performance of optimized portfolios relative to the naive diversification using a few arbitrary empirical datasets, chosen among a great number of available datasets in the Kenneth French library. The performance of a particular portfolio strategy could be affected by the sorting-characteristics to the individual dataset, and those arbitrary datasets could have been selected to substantiate the authorâĂŹs main point. To prevent this "*cherry-picking*" of datasets, this thesis evaluates the performance across 25 empirical datasets formed on portfolios of U.S. stocks provided by Kenneth French. Second, we assess the performance of the risk-based strategies relative to the naive diversification in bull and bear markets, which to the best of our knowledge has never been sufficiently explored. This is done to get a deeper insight into the nature of the performance and assess whether the outperformance is mostly generated in bull or bear markets. Third, we propose to use a generalized approach to look at the aggregate portfolio performance across 25 datasets for each of the risk-based strategies. This generalized approach gives us the opportunity to gain insight into the risk factors that drive the superior performance, and additionally study the risk exposure over time. Fourth, the newly proposed Fama-French five-factor model is used to assess the factor exposure of the risk-based strategies. Last, to demonstrate that one does not need complex optimal strategies to beat the naive diversification, we introduce two new ad-hoc portfolio strategies that directly exploit the low volatility effect.

Reflection Note 1

The quantitative courses, especially Advanced Econometrics and the Computational Finance courses at the university undoubtedly equipped us with the correct knowledge to implement such a quantitative thesis. In the context of internationalization, our thesis is highly international with a theme surrounding portfolio optimization. There exists a heated debate in the academic community in regards to whether optimized portfolios add value, which our study contributes to. We use datasets formed on U.S stocks provided by Kenneth French, but our study is still adaptable to other countries. In the context of innovation, the methodology we use could not have been done without the superior computer power that exists today.

Further, as of today, several investors have never heard of optimized portfolios, and several investors only invest in a few risky stocks, rather than gain from the possible diversification benefit that follows from optimized portfolios. Recently though several global investment funds have gained interest into risk-based strategies, due to their resilient feature during market downturns. In the aftermath of the 2007-2008 global financial crisis, MSCI Barra, S&P 500, including other index providers constructed various risk-based strategy indices by means of optimized- and ranking-based (heuristic) approaches to suit its characteristics. Their motivation includes; increased downside protection during recessions, attractive substitute over other asset classes such as bonds and cash, which lately has offered poor return, and proven high performance during bull markets as well as bear markets. Several other providers have started with factor investing, which directly exploit the market anomalies that exist. These factor premiums, which we argue is the main part of the outperformance of risk-based strategies. We believe that our study is highly relevant. Pension funds, among others, are responsible to manage and allocate large amount of money and the need for diversification and optimally balance the tradeoff between risk and return is important. These funds do not have the possibility to "gamble" with stocks, and more importantly they must protect their investment during market downturns. Therefore, risk-based strategies that simply focus on risk is without a doubt a subject that is especially interesting nowadays.

Reflection Note 2

I will now address the following three factors: international, innovation, and responsibility. The School of Business and Law at the University of Agder highlights these factors as important for productive professionals within the field of business and administration. During the master program with major in Finance, we have gained some knowledge about economics, business administration, and finance, throughout the different courses that have been presented by the School of Business and Law at the University of Agder. All the specific courses have been taught in English. This is very positive in the way that we become more international through using English as the common research/international language.

In the context of internationalization, the management industry has become more complex, due to the increasing development of computer technology. Quantitative portfolio strategies and portfolio optimization have gradually been a more widespread theme within the financial industry. The Modern Portfolio Theory and other related financial theories focus on quantitative models, but these models have been difficult to implement in practice earlier due to the difficulties associated with the extensively number crunching. However, as the computer technology has been developed, the portfolio rules and procedures have become more easily adaptable. Additionally, the financial industry and markets have become increasingly advanced, interconnected, and globalized. The increasing advances of the Internet and the computer technology have to lead to an increasing pace of news and information flow, and the availability of international financial assets, which was before only restricted to domestic markets. As a consequence, the increasing dependencies between countries have emerged. For example, the massive collapse of the 2007-2008 global financial crises led to a global downturn for countries worldwide, since the financial markets are so interconnected. As a result, it has been much more important with computer algorithms that handle portfolios that optimize the weights based on risk. These algorithms change portfolio weights depending on different market phases that should benefit the overall portfolio.

The innovation related to the portfolio management industry and its portfolio strategies are continuously under progress. Briefly stated earlier, the advances in the computer technology have also led to an increasing number of quantitative funds that use computer-based models and quantitative portfolio strategies to exploit market abnormalities. For example, the AQR Capital that uses statistical methods and research-based consistent approach for portfolio construction. These processes of construction of new models will continue as the computer technology develops and innovation associated with exploiting stock characteristics. During the specialization in finance, we have obtained a much deeper insight into financial theories and technical expertise in financial and econometric problems. Additionally, we have been introduced to statistical programs such as STATA and R.

Next, we will concentrate on the responsibility issues associated with portfolio optimization and the financial industry. Portfolio optimization represents the process where one optimizes a portfolio depending on a set of respective assets. As such, there exist a vast of ethical challenges associated with these portfolio investment strategies. For example, a portfolio manager could only use return-series of firms to optimize a portfolio without looking at firm characteristics. Thus, if there exist some financial assets that prevail ethical issues, then these portfolios do indirectly support their views. This is a potential problem for portfolio managers that need to be aware of such issues before they implement various algorithms on stocks and financial assets. Additionally, the portfolio manager could potentially benefit the portfolio and the sustainability by addressing such issues.