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IFAC PapersOnLine 50-1 (2017) 3697-3701

### Application of Genetic Algorithm on Parameter Optimization of Three Vehicle Crash Scenarios

Bernard B. Munyazikwiye \* Hamid R. Karimi \*\* Kjell G. Robbersmyr \*

\* Department of Engineering, University of Agder, 4879 Grimstad, Norway

(Emails: bernard.munyazikwiye@uia.no;kjell.g.robbersmyr@uia.no)

\*\* Department of Mechanical Engineering, Politecnico di Milano,
20156 Milan. Italy (Email: hamidreza.karimi@polimi.it)

Abstract: This paper focuses on the development of mathematical models for vehicle frontal crashes. The models under consideration are threefold: a vehicle into barrier, vehicle-occupant and vehicle to vehicle frontal crashes. The first model is represented as a simple spring-mass-damper and the second case consists of a double-spring-mass-damper system, whereby the front mass and the rear mass represent the vehicle chassis and the occupant, respectively. The third model consists of a collision of two vehicles represented by two masses moving in opposite directions. The springs and dampers in the models are nonlinear piecewise functions of displacements and velocities respectively. More specifically, a genetic algorithm (GA) approach is proposed for estimating the parameters of vehicles front structure and restraint system for vehicle-occupant model. Finally, using the existing test-data, it is shown that the obtained models can accurately reproduce the real crash test data.

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Keywords: Modeling, vehicle-occupant, frontal crash, parameters estimation, genetic algorithm.

### 1. INTRODUCTION

Vehicle crashes are one of the major causes of mortality in modern society. To maintain the crash-worthiness, car manufacturers carry out crash tests on a sample of vehicles for checking the effect of the occupant during crash scenarios. Crash-worthiness is the ability of a vehicle to be plastically deformed and still maintains a sufficient survival space for its occupants. However, this process very expensive and time consuming. To minimize the cost associated with the physical crash test, it is better to adopt the simulation of a vehicle crash and validate the model results with the actual crash test. Due to advanced research in simulation tools during the last decades, simulated crash tests can be performed prior to the full-scale crash test. The common approaches are based on Finite element method (FEM) or lumped parameter modeling (LPM). In the literature, much work has been conducted in the field of vehicle crash-worthiness and resulted in several computational models. A brief review is given in this paper. A car crashing into a rigid pole was modeled by a suitable spring-mass-damper arrangement as presented in Pawlus et al. (2011). The response of an occupant during a vehicle crash was investigated in Marzbanrad and Pahlavani (2011) where the author used a 5-DOF lumped parameter model (LPM), while in Ofochebe et al. (2015), using a 4-DOF LPM the authors studied the performance of vehicle front structure. An optimization procedure to assist a multi-body vehicle model was proposed in Sousa et al. (2008) and Carvalho et al. (2011) and in Alnaqi and

Yigit (2011) the author reduced the thoracic injury during a frontal crash by controlling the force on the seat belt restraint system. Klausen et al. (2014), through a firefly optimization approach, estimated the model parameters of vehicle crash into barrier based on a mass-spring-damper model. Diffent methods for modeling vehicle frontal crash scenarios were developed by Munyazikwiye et al. (2013, 2014). In Munyazikwiy et al. (2016), the authors developed a mathematical model for vehicle-Occupant and a vehicle-to-vehicle frontal crash using Genetic Algorithm in Munyazikwiye et al. (2017). Teng et al. (2008), examined the dynamic response of the human body (the head, chest and pelvic injuries of an occupant, respectively) in a crash event. The problem of reconstruction of a piecewise linear model for vehicle crash scenario based on the genetic algorithm has received less attention in the literature and this forms our motivation for the present study.

In this paper, a genetic algorithm is used to estimate and optimize the parameters of different models, namely: a vehicle-to-barrier, a vehicle-occupant and a vehicle-to-vehicle frontal crash models respectively. The structural parameters estimated are spring and damping coefficients. It is observed that the predicted results fit the experimental data very well.

### 2. EXPERIMENTAL SET-UP

Three experimental crash texts were conducted. Data for vehicle into barrier were taken from a calibration test done by Agder Research, Norway. The second and third test data were taken from the National Highway Traffic Safety Administration (NHTSA), open-source database (Database (2016)). The first test was carried out on a typical mid-speed vehicle to pole collision. A test vehicle was subjected to an impact with a vertical, rigid cylinder. During the test, the acceleration was measured in three directions (x - longitudinal, y - lateral, and z - vertical) together with the yaw rate from the center of gravity of the car. The initial velocity of the car was 35 km/h, and the mass of the vehicle (together with the measuring equipment and driver) was 873 kg. Only the measured acceleration in the longitudinal direction was considered in this study because we were interested in the frontal crash. In the second test, a load cell barrier consisting of 36 load cells was impacted by a Volkswagen Scirocco at a velocity of 56.5 km/h. A 50<sup>th</sup> percentile male Anthropomorphic Test Dummy was placed in the car in the driver's seating position. The target vehicle (a 1996 Plymouth Neon) and the bullet vehicle (a 1997 Dodge Caravan) were instrumented with seven longitudinal axis accelerometers, three lateral axis accelerometers, four vertical axis accelerometers. The test weights and velocities of the target(Plymouth Neon) and bullet (a Dodge Caravan) vehicles were 1378.0 kg, 55.9 km/h and 2059.5 kg, 56.5 km/h respectively.

### 3. MODEL DEVELOPMENT

The main objective of this section is to represent dynamic models to capture the vehicle frontal crash phenomena. When the vehicle crashes into a rigid barrier, the two masses will experience an impulsive force during the collision. The second model consists of two masses as shown in Figure 1, where  $m_v$  and  $m_o$  represent the vehicle and the occupant masses, respectively. The third model consists of two masses moving in opposite directions, as shown in Figure 2. In line of the model development to capture the values as mentioned earlier during the crash scenario, the dynamical models proposed in Huang (2002) for the free vibration analysis are adopted for solving the impact responses. Then, the genetic algorithm is used to estimate the model parameters.

#### 3.1 Model 1: Vehicle-to-rigid barrier crash model

Initially a real vehicle crash experiment was conducted on a typical mid-speed vehicle to pole collision. In vehicle into barrier model, the deforming spring and damping forces, developed at time of crash, are piecewise functions in x and  $\dot{x}$  respectively. But for vehicle to barrier cash, the prefix i is dropped. The forces  $F_k$  and  $F_c$  due to spring stiffness and damper constants are defined as follows (Huang (2002)):

$$F_k = kx \tag{1}$$

$$F_c = c\dot{x} \tag{2}$$

$$\ddot{x} = (-F_k - F_c)/m \tag{3}$$

where m, x and  $\dot{x}$  are mass, displacement and velocity of the vehicle, respectively. k and c are spring stiffness and damping coefficients of the vehicle's front structure, respectively.

### 3.2 Model 2: Vehicle-Occupant frontal crash model

Figure 1 represents the vehicle-occupant model with nonlinear spring and dampers that crashes into a fixed barrier.

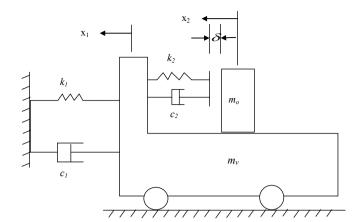


Figure 1. Vehicle - occupant model

Based on the nonlinear characteristics of velocity and displacement of the vehicle and forward movement of the occupant the springs and dampers that simulate such characteristics are modeled as piecewise linear functions. The dynamic equations of the double-mass-spring-damper model are shown in the following:

$$F_{str} = k_1 x_1 + c_1 \dot{x}_1 \tag{4}$$

$$F_{rest} = \begin{cases} k_2(x_2 - x_1) + c_2(\dot{x_2} - \dot{x_1}); & x_1 - x_2 \ge \delta \\ 0; & elsewhere \end{cases}$$
 (5)

where  $k_1, k_2, c_1$  and  $c_2$  are piecewise linear functions defined in Equations (25) - (26).

$$\ddot{x}_1 = (F_{rest} - F_{str})/m_v \tag{6}$$

$$\ddot{x}_2 = (F_{rest})/m_o \tag{7}$$

where  $F_{str}$  and  $F_{rest}$  are the deformation force of the vehicle frontal structure and the restraint system respectively.  $k_1$  and  $c_1$ , are nonlinear spring damper of the front vehicle structure respectively.  $k_2$  and  $c_2$  are spring stiffness and damper coefficients for the restraint system respectively.

### 3.3 Model 3: Vehicle-to-Vehicle crash model

An impact between two masses can be represented schematically, as in Figure 2. Each of the two masses having a contact with the Kelvin element, a set of spring and damper in parallel. If the connection between the mass and the element is a rigid contact, the element may undergo tension and compression. If not, due to separation between the mass and element, the element can only be subjected to compression. To simplify the analysis, the two sets of Kelvin elements can be combined into one resultant Kelvin element. The parametric relationship between the two individual Kelvin elements and the resultant Kelvin element can be obtained as in the following. The spring force  $(F_k)$  and damping force  $(F_c)$  relationships can then be established as follows:

$$\alpha = x_1 + x_2 \tag{8}$$

$$\frac{F_k}{k} = \frac{F_k}{k_1} + \frac{F_k}{k_2} \tag{9}$$

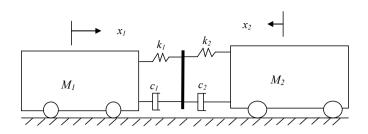


Figure 2. Vehicle to vehicle impact model - two Kelvin elements in series

$$\dot{\alpha} = \dot{x_1} + \dot{x_2} \tag{10}$$

$$\frac{F_c}{c} = \frac{F_c}{c_1} + \frac{F_c}{c_2} \tag{11}$$

$$k = \frac{k_1 k_2}{k_1 + k_2} \tag{12}$$

$$c = \frac{c_1 c_2}{c_1 + c_2} \tag{13}$$

In a two-mass system shown in Figure 2, the mass  $M_2$  is impacted by  $M_1$  at an initial relative speed (or closing speed) of  $v_{12}$  where  $v_{12}=v_1+v_2=v_0$ . If one of the masses in the two-mass system is infinite, the system becomes a vehicle-to-barrier (VTB) model. The only mass moving in this system is referred to as the effective mass,  $M_e$ . The relative motion of the mass with respect to the fixed barrier is the same as the absolute motion of the mass with respect to a fixed reference frame. In a system where there are multiple masses involved in an impact, the analysis can be simplified by using the relative motion and effective mass approaches. The relative displacement of the effective mass,  $M_e$ , is  $\alpha$ . The dynamic responses of the two-mass system and one effective mass system are summarized as follows:

$$\ddot{x}_1 = \gamma_1 \ddot{\alpha} \qquad \qquad \ddot{x}_2 = \gamma_2 \ddot{\alpha} \tag{14}$$

where

$$\ddot{\alpha} = -v_{12}\omega_e sin(\omega_e t) \tag{15}$$

$$\omega_e = \sqrt{\frac{k}{M_e}} \tag{16}$$

$$\gamma_1 = \frac{M_2}{M_1 + M_2} \tag{17}$$

$$\gamma_2 = \frac{M_1}{M_1 + M_2} \tag{18}$$

$$M_e = \frac{M_1 M_2}{M_1 + M_2} \tag{19}$$

where  $\omega_e$  is the natural frequency,  $\gamma_1$  and  $\gamma_2$  denote mass reduction factors and  $M_e$  is the effective mass. The dynamic motion of the effective mass system can be expressed as:

$$M_e \ddot{\alpha} = -c\dot{\alpha} - k\alpha \tag{20}$$

$$\ddot{\alpha} = (-c\dot{\alpha} - k\alpha)/M_e \tag{21}$$

substituting (8) and (10) into (21), we get:

$$\ddot{\alpha} = (-c(\dot{x_1} + \dot{x_2}) - k(x_1 + x_2))/M_e \tag{22}$$

Therefore, the dynamic responses of the two-mass system in Equation (14) can be presented as follows:

$$\ddot{x}_1 = \gamma_1(-c(\dot{x}_1 + \dot{x}_2) - k(x_1 + x_2))/M_e \tag{23}$$

$$\ddot{x}_2 = -\gamma_2(-c(\dot{x}_1 + \dot{x}_2) - k(x_1 + x_2))/M_e \qquad (24)$$

# 3.4 Piecewise linear approximations for springs and dampers

The springs and damping coefficients in the types of models in the previous sections, are defined by the piecewise functions. The predefined spring and damper characteristics are chosen based on the shapes of the displacement and velocity responses from the crash test. The predefined spring and damper are defined by Equations (25) and (26).

$$k(x_i) = \begin{cases} k_{i1} + \frac{k_{i2} - k_{i1}}{x_{i1}} x_i & x_i \le x_{i1} \\ k_{i2} + \frac{k_{i3} - k_{i2}}{x_{i2} - x_{i1}} (x_i - x_{i1}) & x_{i1} \le x_i \le x_{i2} \\ k_{i3} + \frac{k_{i4} - k_{i3}}{C_i - x_{i2}} (x_i - x_{i2}) & x_{i2} \le x_i \le C_i \end{cases}$$

$$(25)$$

$$c(\dot{x}_i) = \begin{cases} c_{i1} - \frac{c_{i1} - c_{i2}}{\dot{x}_{i1}} \dot{x}_i & \dot{x}_i \leq \dot{x}_{i1} \\ c_{i2} - \frac{c_{i2} - c_{i3}}{\dot{x}_{i2} - \dot{x}_{i1}} (\dot{x}_i - \dot{x}_{i1}) & \dot{x}_{i1} \leq \dot{x}_i \leq \dot{x}_{i2} \\ c_{i3} - \frac{c_{i3} - c_{i4}}{v_0 - \dot{x}_{i2}} (\dot{x}_i - \dot{x}_{i2}) & \dot{x}_{i2} \leq \dot{x}_i \leq v_0 \end{cases}$$
(26)

where the index i=1,2 stand for  $1^{st}$  and  $2^{nd}$  mass respectively.  $C_i$  is the dynamic crash of the vehicle or occupant.  $v_0$  is the initial impact velocity. The index i designates the models with two masses such as vehicle-to-vehicle and vehicle-occupant models respectively. The same piecewise functions, without the index i, are used to model the vehicle into barrier crash. At the maximum crash, the spring stiffness is assumed to be high, but the damper coefficient is small for maintaining the shape of displacements and velocities respectively.

# 4. OPTIMIZATION SCHEME OF THE GENETIC ALGORITHM

Genetic Algorithm (GA) is an adaptive heuristic search based on the evolutionary ideas of nature selection and genetics. It represents an intelligent exploitation of a random search used to solve optimization problems. This Evolutionary Algorithm holds a population of individuals (chromosomes), which evolve by means of selection and other operators like crossover and mutation. Given a clearly defined problem to be solved and a bit string representation for candidate solutions, a simple GA works as follows in Melanie (1999):

- (1) Start with a randomly generated population of n lbit chromosomes (candidate solutions to a problem).
- (2) Calculate the cost function f(x) of each chromosome x in the population.
- (3) Repeat the following steps until n offspring have been created:

- (4) Replace the current population with the new population.
- (5) Go to Step 2

Each iteration of this process is called a generation. A GA is typically iterated for anywhere from 50 to 500 or more generations. The proposed algorithm seeks to find the minimum function between several variables as can be stated in a general form minf(x), where x denotes the unknown variables, which are the damping and stiffness constants in the model. The cost function f(x) is the objective function which should be optimized. The cost function to be minimized is the norm of the absolute error between the displacement of the simulated cash and the experimental crash data and is defined as:

$$[Error] = sum(|Est - Exp|^T \times |Est - Exp|)$$
 (27) where  $Est$  and  $Exp$  are the model and experimental variables (displacements, velocity and acceleration) respectively. A Genetic Algorithm shown is developed to solve the problems defined by Equations (3), (6), (7), (23) and (24).

#### 5. RESULTS AND DISCUSSION

This sections is a summary of major findings observed on the three vehicle crash models. Namely: vehicle into barrier, vehicle-occupant into barrier and vehicle-to-vehicle models respectively. The label symbols s,v and s in Figure 3 to Figure 5 stand for displacement, velocity and acceleration respectively. Exp and Mod stand for Experimental and Model. Figure 3 shows the comparison between the model response and the experimental test results for a vehicle into a barrier crash. It is noted that the dynamic crush from the model is exactly equal to that obtained from the test. The maximum dynamic crash, the time of crash and the rebounce velocity for both, the model and test results are summarized in Table 1. Using the same

Table 1. Estimated Parameters for vehicle into barrier model

| Spring | Value            | Damper | Value              |
|--------|------------------|--------|--------------------|
| $k_1$  | 3.9880e+03  N/m  | $c_1$  | 8.7727e+04  Ns/m   |
| $k_2$  | 2.8403e+04  N/m  | $c_2$  | 6.6938e + 04  Ns/m |
| $k_3$  | 0.44386e+01  N/m | $c_3$  | 3.0115e+04  Ns/m   |
| $k_4$  | 2.2337e+05  N/m  | $c_4$  | 5.9893e+04  Ns/m   |

algorithm as in vehicle into barrier, a comparison between the crash test from vehicle-occupant crash and the model shown in Figure 1 is shown in Figure 4. The results show that the model is very accurate.

Table 2. Estimated Parameters for vehicle-Occupant model

| Vehicle  | Value             | Occupant | Value              |
|----------|-------------------|----------|--------------------|
| $k_{11}$ | 7.6665e+04  N/m   | $k_{21}$ | 6.8536e+03  N/m    |
| $k_{12}$ | 7.9498e+04  N/m   | $k_{21}$ | 2.5529e+04  N/m    |
| $k_{13}$ | 9.6887e + 03  N/m | $k_{23}$ | 9.9998e+04  N/m    |
| $k_{14}$ | 9.9998e+04  N/m   | $k_{24}$ | 7.2212e+04  N/m    |
| $c_{11}$ | 8.4895e+04  Ns/m  | $c_{21}$ | 4.8212e+03  Ns/m   |
| $c_{12}$ | 2.8460e+03  Ns/m  | $c_{22}$ | 1.3677e + 03  Ns/m |
| $c_{13}$ | 3.3299e+03  Ns/m  | $c_{23}$ | 3.2491e+03  Ns/m   |
| $c_{14}$ | 1.4046e+04  Ns/m  | $c_{24}$ | 2.2323e+03  Ns/m   |

From Figure 4, the model accuracy is obtained by using force elements with two break point piecewise functions.

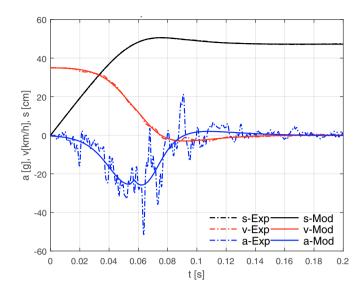


Figure 3. Model vs Experimental results for Vehicle into barrier frontal crash

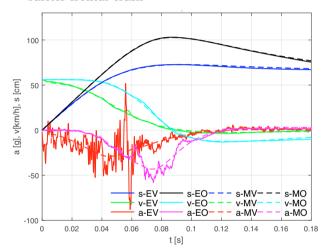


Figure 4. Model vs Experimental results for vehicleoccupant frontal crash

The maximum dynamic crash of the vehicle model is 0.05% less than that in the real crash test. The displacement of the occupant is 0.09% larger than that from crash test. Improvement of the model accuracy is also observed from the time at maximum displacement and the rebound velocities for both the vehicle and occupant. The optimized estimated parameters are shown in Table 2. The main results for vehicle-to-vehicle crash modeling are presented in Figs. 5. It is noted that the model results are much closer to the experimental results for crash test. The maximum dynamic crash of 70.24cm is observed on the target from the test, while the dynamic crash from the model is 69.92 cm. At maximum dynamic crash, the bullet vehicle keeps on moving in the same direction as before crash but the target vehicle re-bounces. The rebounce velocities are -19.6 m/s and -18.3 m/s from the test and the model respectively. This is observed by the velocity curves of the two vehicles, where a negative velocity is noted for the target vehicle and a positive velocity is noted for the bullet vehicle after maximum crash. The accuracy of the model is also observed on the time of maximum crash,  $t_m$ . The time of maximum crash,  $t_m$  is 0.06568 s from the test and

### 0.06824 s from the model respectively. The deformation

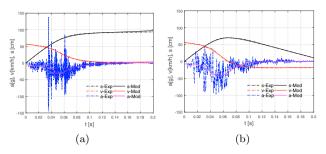


Figure 5. Model vs Experimental results for vehicle-tovehicle frontal crash (a) Bullet (b) Target

Table 3. Estimated Parameters for vehicle-tovehicle model

| Bullet   | Value              | Target   | Value              |
|----------|--------------------|----------|--------------------|
| $k_{11}$ | 1.2843e+05 N/m     | $k_{21}$ | 1.5030e+04 N/m     |
| $k_{12}$ | 2.5142e+05  N/m    | $k_{21}$ | 1.5030e+04  N/m    |
| $k_{13}$ | 1.4932e+05  N/m    | $k_{23}$ | 4.1930e+05  N/m    |
| $k_{14}$ | 6.5159e+05  N/m    | $k_{24}$ | 5.1878e + 04  N/m  |
| $c_{11}$ | 4.1688e+04  Ns/m   | $c_{21}$ | 6.3884e+05  Ns/m   |
| $c_{12}$ | 1.7727e + 04  Ns/m | $c_{22}$ | 7.3768e + 04  Ns/m |
| $c_{13}$ | 3.0696e+03  Ns/m   | $c_{23}$ | 3.3250e+03  Ns/m   |
| $c_{14}$ | 2.6614e+03  Ns/m   | $c_{24}$ | 0.8603e+03  Ns/m   |

of the target vehicle is due to the compressive force at dynamic crash. A summary of kinematics results from all models studied is tabulated in Table 4, where VTB, V-Occ and VTV stand for Vehicle to Barrier, Behicle-Occupant and Vehicle-to-Vehicle models respectively. T and M stand for Test and Model results, respectively.

Table 4. A summary of kinematics results from tests (T) and the models (M)

|                  | VTB    | V-Occ  |       | VTV    |        |
|------------------|--------|--------|-------|--------|--------|
| Results          |        | Veh    | Occ   | Bulet  | Target |
| $C_m[m]$ $T$     | 0.5063 | 0.7269 | 1.03  | 0.9359 | 0.7024 |
| $C_m[m]$ $M$     | 0.5061 | 0.7274 | 1.028 | 0.9613 | 0.6992 |
| $t_m[s]$ T       | 0.0749 | 0.0894 | 0.086 | 0.1984 | 0.065  |
| $t_m[s]$ M       | 0.0748 | 0.093  | 0.087 | 0.1981 | 0.068  |
| $V_{reb}[m/s]$ T | -3.3   | -3.7   | -13   | 0.9    | -19.6  |
| $V_{reb}[m/s]$ M | -2.96  | -2.4   | -12.6 | 2.7    | -18.3  |

### 6. CONCLUSION

In this paper, a mathematical-based method is presented to estimate the parameters of three different vehicle crashes. It is observed that the developed mathematical model results in responses in all vehicle crash models are closer to the experimental crash tests. Therefore, the overall behavior of the models matches the real vehicle's crush well. Two of the main parameters characterizing the collision are the maximum dynamic crush - which describes the highest car's deformation and the time at which it occurs-  $t_m$ . They are pertinent to the occupant crash-worthiness since they help to assess the maximum intrusion into the passenger's compartment.

### ACKNOWLEDGEMENTS

The authors would like to thank the Dynamic Research Group members at the University of Agder for their constructive comments to improve this research work.

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