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Capturing learning in classroom interaction in mathematics: Methodological considerations

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This paper discusses issues of how to transcribe and analyze video-recordings when studying learning in small group work in mathematics. Since bodily features of interaction and the use of artefacts play important roles in mathematical reasoning, a multimodal approach to transcribing is necessary. Thus, the theoretical grounding for transcriptions has to be in accord with the perspective on learning adopted in the analysis. In the paper, the principles for studying what Radford (2000) refers to as knowledge objectification processes when learning mathematics will be discussed.

Keywords: Analytical approaches, knowledge objectification, multimodality.

INTRODUCTION

This paper discusses ways of doing video analyses that are relevant for understanding mathematics learning. Thus, this is a methodological paper. Our particular focus is on multimodality as a resource for learning but also as a methodological challenge for research. Analytical approaches, selection of episodes and a multimodal transcription will be discussed in light of recent developments in the field.

The background of this study is an international, comparative project called VIDEOMAT (Kilhamn & Røj-Lindberg, 2013), which studies teaching and learning of introductory algebra in four countries: Finland, Norway, Sweden and the USA. Students are between 11 and 13 years old. Data include video-recordings of lessons, written materials from student activity, teacher interviews etc. Five consecutive lessons when algebra was introduced¹ in classrooms in the four countries were documented. A group work session with a pattern task with matchsticks was selected for further investigation. This task resulted in a multitude of problem-solving strategies among students, all the

way from counting to sophisticated forms of mathematical reasoning (multiplicative/generalizing).

In the literature there are many attempts to make use of multimodal analyses to understand learning processes in the context of mathematics. We will comment on some of these below. Considering that we are at an early stage of advancing knowledge through the use of multimodal approaches, we have formulated the following question for this paper: *In what ways can video recordings be transcribed and analyzed in order to study student's learning processes?*

BACKGROUND

The methodical reflections in this paper focus on classroom interaction in a problem-solving, small-group setting. A particular aim is to understand the knowledge objectification process (Radford, 2000, 2002). The object of activity in the classroom, as the students work with the matchstick task, is to develop algebraic thinking; more specifically to perceive the general nature of a pattern, and to use this insight when solving a problem. The ability to generalize is viewed as one of the most important developments in mathematical thinking.

Our analysis will follow a socio-cultural, Vygotskian view on learning and development. A central idea is that learning results from participation in social and interactional processes. Equally important is that this perspective stresses that learning and knowing are cultural phenomena.

Approaching group work in mathematics classrooms with an interest in the contributions of multimodality, the cultural-semiotic theory of learning, developed by Radford (2000), provides a promising route ahead. Radford (2002) suggests that knowledge objectification happens through semiotic activity, that

is through “objects, artifacts, linguistic devices and signs that are intentionally used by individuals in social processes of meaning production, in order to achieve a stable form of awareness, to make apparent their intentions and to carry out their actions” (p. 14). The process of knowledge objectification is understood as the process of placing something at the center of someone’s attention. In this study, knowledge objectification thus refers to the process of perceiving generality; the knowledge of the general nature of the pattern having a genesis and a development, and, as a further step, knowing how to express the generality mathematically and to solve the problem.

METHODOLOGICAL DEVELOPMENT IN THE STUDY OF LEARNING

The methodological deliberations by different researchers have been scrutinized. The studies analyzed all rely on naturalistic data and interpretive approaches to method, and they represent different choices in terms of data collection and analysis.

Bjuland (2002) focused on small group problem-solving in mathematics by student teachers. Data were collected by audio recordings, and the theoretical perspective utilized was dialogical, situated and socio-cognitive. The unit of analysis, referred to as an episode, was “conceived as a sequence of verbalizations focused on a special mathematical topic or idea” (p. 64), relevant to the research questions. These were then categorized according to five features of problem solving processes: sense-making, conjecturing, convincing, reflecting and generalizing.

Carlsen (2009), working in a sociocultural tradition, analyzed the appropriation of mathematical tools by students attending the final year of high school. Video recordings were used. The aim was to trace development of the student’s mathematical reasoning. Relevant parts of the entire audio recorded material were transcribed in detail and subjected to in-depth analysis. The transcripts included multimodal elements in order to investigate the role of inscriptions in the appropriation process.

Radford (2000, 2002, 2012) reported on longitudinal studies involving students’ group work with algebra and more specifically with patterns. This work involves methodological and theoretical developments that are interesting. Radford’s research is based in a

semiotic-cultural perspective on learning building on Vygotsky’s view of signs as linked to and affecting our cognition. In Radford (2012), researchers took part in the process of designing the lesson material and students were organized in small groups. These sections were video recorded and student works were collected.

In his early work, Radford (2000) uses concepts from discourse analysis. He follows a three-step analysis of transcripts, a) valuing each utterance as equally important, b) contextualizing utterances, and c) including pauses and hesitations. This approach Radford (2000, p. 244) terms *situated discourse analysis*. The unit of analysis was conceived through a process of refining salient episodes through data managing by indexing and theorizing. Radford emphasizes the importance of natural language in the development of algebraic thinking and the use of algebraic symbols.

Radford, Demers, Guzmán, and Cerulli (2003) introduce the concept of *semiotic node*. This was a response to findings in many studies on the importance of gestures and artifacts in the production of graphs and algebraic expressions. Semiotic nodes are “pieces of the students’ semiotic activity where action, gesture, and words work together to achieve knowledge objectification” (p. 56). The transcripts include description of gestures and the analytical tool of semiotic nodes was applied in the analysis. In Radford, Bardini, and Sabena (2007), the analysis was done in greater detail. A slow-motion, frame-by-frame, fine-grained video micro-analysis was carried out and complemented with a voice-analysis. The same kind of micro-analysis was carried out in Radford (2012), except for the voice analysis, where a multi-semiotic analysis (spoken words, written text, gestures, drawing, and symbols) was done.

Arzarello (2006) outlines a theoretical frame emphasizing the role of multimodality and embodiment in cognition. He argues for a multi-semiotic analysis of objectification processes and claims that the present semiotic frameworks cannot capture didactical processes in a satisfactory manner. Therefore, he introduced the idea of the *semiotic bundle*. In the semiotic bundle, which includes semiotic sets such as gestures, speech, written representations, as well as more formal systems, the distinctions between the sets are only made for analytical purposes while interpreted as a unitary system. The semiotic bun-

dle is dynamic and can shift to include more or less semiotic sets as the event unfolds. The meaning of the mathematical object may not be the same in the different sets. Moreover, even if the transformation from one set to another is accomplished, the meaning the object had in the prior set may linger, and so it can take time before the concept is formalized. By looking at the data synchronically and diachronically, the genesis and evolution of the semiotic objectification process can be traced. The semiotic node introduced by Radford (2003) is similar to looking at the semiotic bundle synchronically.

Arzarello (2006) used the semiotic bundle to analyze the work of one group of five fifth-graders. Video recordings and student work were collected as part of a longitudinal research design. However, the episodes presented were chosen from a 30 minute session on problem solving. The selection process was not commented on, except by saying that four main episodes were chosen. The episodes were subjected to different analytical methods; (episode 1) synchronic analysis; (episode 2) diachronic analysis; (episode 3) synchronic + diachronic analysis; and (episode 4) diachronic analysis. The transcriptions include descriptions of gestures and pictures are presented in the analysis.

Roth and Thom (2009) looked at multimodality and learning from a phenomenological perspective. The aim of the study was to propose a new way of understanding mathematical concepts grounded in a case study. Data were collected in a second grade classroom during group work sessions in geometry. In addition, artifacts used and all work by the teacher and the students were photographed. One episode from a whole class session, lasting 69 seconds, which is called exemplary, was chosen for analysis. The episode is presented in the context of what happened before. The transcript includes details (length of pauses, pitch etc.). The episode is presented over 6 pages and several drawings depicting movements are part of the description. The authors argue that “conceptions can be understood as networks of experiences that indeterminately emerge from lived (rather than intellectual) reorganizations of embodied bodily experiences” (op. cit., p. 188).

The studies presented above are all conducted within the paradigm of interpretivism. They are ethnographic and researchers spend time observing, making field notes, and collecting students’ work; the researchers

are concerned with the context in which the events take place. The video and/or audio recordings are done in classrooms and are naturalistic in the sense that students are in their everyday environment engaging with mathematical activities. The studies also share a common focus on the multimodal aspects of learning, except Bjuland (2002) and Radford (2000).

METHODOLOGICAL CONSIDERATIONS

In spite of the commonalities of the studies presented, the transcripts look very different and include different features of interaction. Bezemer and Mavers (2011), investigating multimodal transcripts in research, point out that “transcripts should be judged in terms of the ‘gains and losses’ involved in remaking video data” (p. 204). The focus should not be on attempting to achieve representational accuracy, rather the approach should be transparent.

The studies use different analytical approaches to dialogue. Bjuland and Carlsen use the *dialogical approach* elaborated by Linell (1998), while Radford uses *situated discourse analysis*. Consequently, the process of analysis is different. Radford’s first step is to look at each utterance in its own right and categorize it. As a second step, he contextualizes them. In contrast, sequentiality is central to the dialogical approach as each contribution in a dialogue gets its meaning from both prior and subsequent turns. Arzarello (2006) and Roth and Thom (2009) do not fully reveal their approach for analyzing dialogue.

An important aspect is the selection of salient episodes. Bjuland (2002) transcribed all verbalizations and then identified relevant episodes according to the analytical interest. Carlsen (2009) worked with video recordings. After several viewings, he chose 14 sessions which were roughly transcribed. Following this, relevant episodes were identified and transcribed in detail. From this sample salient episodes were chosen. Radford (2000) used *situated discourse analyses* as a first approach to the data set, which was transcribed in its entirety. The studies by Arzarello (2006) and Roth and Thom (2009) do not fully comment on the selection process.

These studies show that multimodality is an essential part of understanding how students learn mathematics. Thus, it becomes important for this branch of re-

search to enter into a discussion on how to advance the use of multimodal methods of analysis.

CAPTURING LEARNING: SUGGESTED METHODS

The aim of this paper is to describe ways of doing video analysis that focus specifically on learning processes and which include attention to multimodality. The approach will be discussed in three sections: *video analysis*; *multimodal transcription*; *learning processes and video analysis*. Our discussion will be twofold as it a) provides arguments for the methodological choices and b) is practically oriented in that an excerpt of a multimodal transcript is included and analytical approaches are briefly exemplified.

Video analysis

The video analysis follows an interpretivist paradigm. The aim is to understand learning processes by closely following how participants engage in meaning making. Applying the notion of knowledge objectification through semiotic activity implies analyzing multimodal aspects of interaction. According to Knoblauch (2012), one has to apply two types of interpretation in order to preserve the essence of multimodal elements of interaction. The first is to interpret what is seen and heard as it appears from an everyday understanding and from the actors' point of view. The second level is the professional interpretation of the interaction.

The ethnographical aspect of this research is important in terms of the validity of interpretations. Observation of lessons, interviews with the teachers and the written materials collected improve the ability to interpret the situation. The validity of the interpretations will depend on the assumption that "people are existent and, that they have been conducting (acting) in ways that are open for reconstruction (capture) by video data" (Knoblauch, 2012, p. 73). This allows *subjective adequacy*, which means that there is a correspondence between what the researchers say and the statements by the participants. Psychological studies have shown that people often "see events similarly in terms of causal, behavioral, and thematic structures" (Derry et al., 2010, p. 7), which supports the validity of an *everyday* interpretation of interaction.

The empirical material in this study is considered to be *naturally occurring data*. We recognize that the presence of three cameras, two professionals operating them, and one to three researchers observing

exert some influence on the situation. However, students today are familiar with cameras, and in consultations with the teachers after lessons they expressed that students behaved as usual.

In order to approach the complexity of the interaction in the groups, the discourse is separated into two main parts: dialogue and multimodal elements. However the two parts are interpreted as belonging to a unified system of communication and therefore seen as integral parts of meaning making. Two methodological concepts will be considered when analyzing the multimodal elements (Knoblauch, 2012, pp. 74–75): *sequentiality*, considering any action as motivated by prior actions and motivating future actions; *reflexivity*, actors do not only act but also indicate, frame or contextualize how their action is to be understood and how they have interpreted a prior action to which they are responding. These concepts correspond well with the dialogical approach which also emphasizes sequentiality.

The issue of multimodal transcription

In attempting to transcribe visual data of video recordings there is a challenge in doing adequate data reduction. The focus of our research is the interaction in the groups. Luckmann (2012, p. 32) argues that "the elements of the interaction which the analyst, based on his knowledge of social life, must assume were relevant to the participants in the original interaction, must be noted in the transcript". Knoblauch (2012, p. 75) argues that video analysis is a hermeneutic activity. "[T]he task set is not to only describe and explain non-verbal behavior". As a researcher one has to decide what knowledge is needed to make sense of a situation and to identify visible conducts constituting the situation. Therefore, multimodal transcribing is not only a preliminary stage to the analysis; the activity forms an essential part of the analysis.

The video material and the written works of students have been examined in order to understand the problem solving process through the dialogue and the semiotic actions that appear both by each individual student and as part of the joint group activities. Several multimodal elements of the interaction have been identified. These fall into three categories of use of mediating resources: *inscriptions* such as drawings, tables, texts, numbers, arithmetic, algebraic (including variable/s); *concretes* i.e. matchsticks; *gestures*

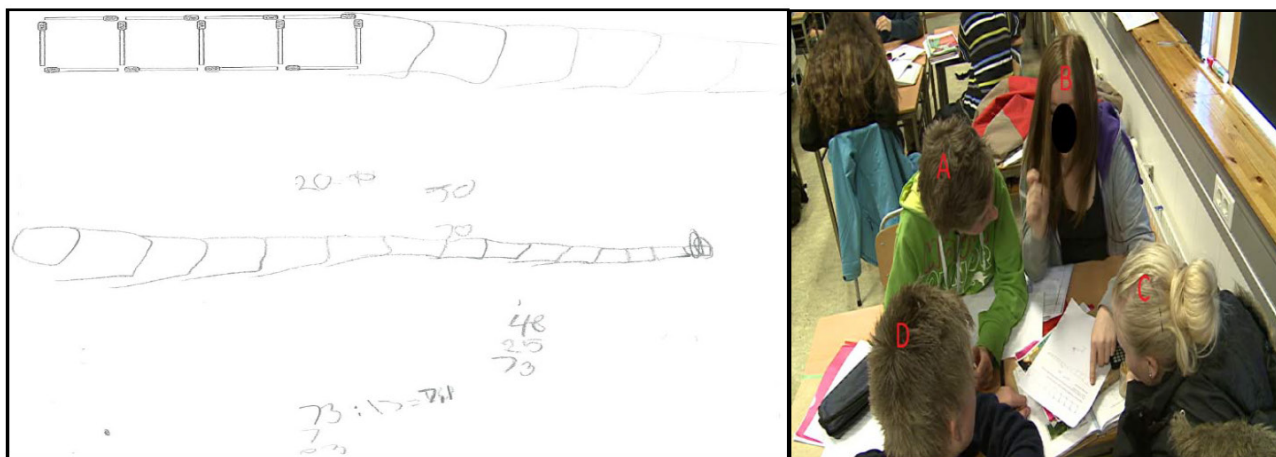


Figure 1: Representation of part of the multimodal transcription

such as pointing, tracing in air/figure/table, glance, rhythmic hand movement, raising hand.

The learning processes and video analysis

Derry and colleagues (2010) stress the importance of being systematic when selecting salient episodes. Schoenfeld (1985) parceled the dialogue according to the mode of reasoning (i.e. planning, exploration) as he expected strategic decisions to be located at the junctures between such episodes. In this study, the dialogue will also be parceled according to the mathematical strategy the students are working with. This is done in order to explore how the students’ discourse on the problem evolves during the problem solving process and to reveal mechanisms which drives it. In light of these explorations, fragments of the text which show the first traceable step and its successors in the objectification process will be identified.

An excerpt from a multimodal transcription of a Norwegian group is presented below. A group of 8th grade students, Ben (A), Ann (B), Trish (C) and Sam (D), are given an algebra task (adapted from TIMSS 2007) involving matchsticks and patterns. The teacher hands out toothpicks as a material to use in order to solve the problem. Only Ann writes on the task paper. Marks indicating if the students are in the process of conjecturing (Cj) or convincing (Co) and also specifying the mathematical strategy used such as additive (A) or multiplicative (M) have been inserted into the transcript in order to show the analytical approaches to the text.

8 Trish: We can make them [squares] on the table. But should we just use these or? [Trish shakes the can of toothpicks she is holding in her hand].

9 Ann: But see, we get 7.1 [Ann points to the division, 73 divided by 13, she has been working on], then if you have taken () then you get 7.1 squares. 1, 2, 3, 4, 5, 6, 7 [Ann points at the squares in the task paper as she counts them and continues by pointing at imaginary squares until she reaches 7]. So then you get less than sev...then we get, if we make 7 squares. Ok, 4.

The girls try to add a square to the figure using the toothpicks. They give it up quickly as they notice that the dimensions are different.

10 Trish: Ha..ha
 11 Ann: You, this didn’t work
 12 Trish: We’ll draw it.
 13 Ann: [She adds a square to the figure by drawing three sides in one motion, she then points at each square as she counts them] 1, 2, 3, 4...[adds another square in the same manner], 5. [starts counting the matchsticks making up the squares] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14... 17, 18. Ok, but see...ah...I got a good idea...look [Now she only counts the horizontal matchsticks] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12...[adds more squares using the same motion] 13, 14...15, 16...17, 18...19, 20 [There are now 10 squares altogether]. So if we take [She now counts the squares] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. When we have 20 rows we have [writes 20 and then counts the vertical matchsticks silently]...((then we have...then we have)) =
 14 Sam: ((But what are we going to do with them...Ann?)).
 15 Ann: = When we have 20 we have 50 pieces [writes 50]. Or, when we have 20, when

- we have 20 such things... [*she points or taps repeatedly at the figure*].
- 16 Sam: It is those [*Sam holds up a toothpick*].
- 17 Ann: Yes, matchsticks, then we have 50 altogether [*points to the number written*], used 50 such matchsticks [*points back at the figure*] and we are going to use 73, right? =
- 18 Sam: Just make...
- 19 Ann: = So then...
- 20 Trish: ((really one more will be 53 and then 56))
- 21 Ben: ((We are going to use...))
- 22 Ann: No, if we have one more with 10 in it, then it becomes... =
- 23 Sam: ((Yes because it is four in one)).
- 24 Ann: = So, then we get 20 more and it becomes 70 [*writes 70*]. ((It is 1, 2, 3...so then we get 70... No, now there is too much here)) =
- 25 Ben: [*looks at Sam and responds to his comment*] ((No, it is 3, it is 4 in one and 3...1, 2, 3, 4, 5, 6, 7, 8, 9))
- 26 Ann: = I think I sort of lost count of it.
- 27 Trish: No, 70, and then you should have 1 thing more and then it becomes exactly 73.
- 28 Ann: Ah, but see, oh yes because 20...
- 29 Trish: It is really only three in each, it is only the first there is four in, and then there is only three in each the whole time [*points at the figure while she explains*].
- 30 Ann: But see...
- 31 Trish: If you do like that then...4 [*she holds her finger over the first square*]
- 32 Ann: 1, 2, 3. [*counts three matchsticks in the first square, then pushes away Trish's finger and starts counting in the pattern she has developed, horizontal matchsticks first and then the vertical ones*] Ok, 1, 2, 3, 4, 5, 6, 7, 8, 9 () 18, 19. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20. 20. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. 11.

While Ann is counting, Ben and Sam start paying attention to something that is going on in the classroom which is not relevant for the mathematical discussion. When the teacher approaches the group, the boys attend to solving the task again.

- 33 Trish: [*traces the matchsticks in the squares using the same motion as Ann used earlier when drawing new squares*] Oh, you!

73 divided by 3 and then just add 1! [*she picks up her calculator*]

- 34 Ann: There you said one. [While Trish is working on the calculator, Ann traces first the four matchsticks in the first square and then the 3 matchsticks in each of the following squares. She is using the same motion as earlier when drawing the squares].
- 35 Trish: No. [*The teacher comes over to the group, but Trish only looks at the calculator while she speaks*] 73 divided ((by 3, plus 1, 25)).
- 36 Ann: [*Ann looks at the teacher*] ((divided by...3. Is that right?))

In turn (9) Ann suggests a solution to the task based on a multiplicative strategy. In order to make sense of the answer she found, she turns back to the task paper and applies an additive strategy.

The marks in the text indicate important events in the problem solving process. If we focus the attention on the objectification process, we see in (20) the first verbalization of the 3+3 pattern, which is discussed and developed by Sam (23), Ben (25) and Trish (29), and finally expressed as 4+3+3. However, in (33) we see that Trish traces the matchsticks with the same motion used by Ann that appears early in the text (13), immediately before she expresses a new conjecture for how to solve the task (33). Ann is not taking part in the discussion of the 4+3+3 pattern but seems to drive it with the gestures and the drawing she is making.

CONCLUSION

The video recordings available of 16 groups working with the same task offer an opportunity to study the role of features of thinking in the objectification process. These features, as elaborated through the empirical materials and the theoretical perspective, have been identified as: *elements of reasoning* (sense-making, conjecturing, convincing, reflecting, generalizing), *mathematical strategies* (additive, multiplicative, equations, functional), *semiotic resources* (use of language, inscriptions, concretes, gestures) and indicators of the *culture of collaboration*.

The analytical methods described are developed in order to understand how these different features of thinking are incorporated in learning processes. The ambition is to shed light on a) what role mediating tools play as students decide on mathematical strate-

gies, b) what features of the knowledge objectification process that can be discerned, and c) what are the differences, if any, between classrooms and cultures of work in the different countries.

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ENDNOTE

1. Defined in the project as when letters are introduced as variables in order to collect similar data in the four countries.