An activity theory perspective on contradictions in flipped mathematics classrooms at the university level

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This study explores contradictions that emerge when utilising a flipped classroom approach to university mathematics education. The work uses Activity Theory and its principle of dialectical contradiction as a theoretical framework to identify and analyse contradictions that arise in flipped mathematics classrooms for engineering students. Data was collected mainly by means of video recording of classroom activities and interviews with two cohorts of first-year engineering students in a Norwegian university over two years. An inductive approach to data analysis based on the interaction between the theoretical framework and the empirical data is used to provide evidence about the contradictions. The results show that contradictions manifest themselves as tensions in flipped mathematics classrooms. They emerge at different activity levels and affect student learning of mathematics. The aim of the study is to address the lack of research on tensions and contradictions in flipped mathematics classrooms at the university level.

Introduction

Students in mathematics education often face tensions that emerge in teaching and learning processes. Tensions occur in the context of interaction with teachers [1], or between learning the subject matter to achieve a grade and learning the course material because of its importance [2]. Tensions also appear when an educational system adopts new elements from the outside, such as an innovative technology, a new pedagogical approach such as flipped classroom, or a new approach to learning, such as student-centred or collaborative learning. The tensions may refer to short term problems that can be resolved, but they may be grounded in contradictions, that is systemic, long term tensions that occur within educational systems [3].

Today, flipped classroom (FC), as a technology-supported pedagogical approach, has gained prominence worldwide in mathematics education. FC is generally characterized by its course structure, which consists of out-of-class activities where videos take the role of direct instruction, and in-class activities where the teacher focuses on key concepts with groups of students [4, 5]. Although FC is recognized as a modern and potentially powerful way of providing student-centred, active learning arenas for students [6], there exists little research addressing what tensions emerge in a mathematics flipped classroom. This article advances knowledge on tensions emerging when this type of pedagogical approach is introduced and employed in university level mathematics courses for engineering students.

The study reported in this paper concerns contradictions that engineering students experience as FC is employed over two years at university level mathematics courses. The participants were two cohorts of computer engineering students over two consecutive years in their first year of study (20 students in the first cohort, 25 in the second cohort). Each cohort attended two individual courses in mathematics over one year, in the autumn and spring terms. A multiple case study is used to identify contradictions that emerge in a flipped mathematics classroom.
The research question we explore in this paper is:

> What types of contradictions emerge in a flipped mathematics classroom, and how do students experience them?

We draw on Engeström's Activity Theory (AT) and the principle of dialectical contradiction [7, 8] to study the characteristics of contradictions that manifest themselves as tensions in a flipped mathematics classroom. Our argument for using this framework is that FC and it impacts on learning cannot be studied without providing deeper insight into the role contradictions has as driving forces of transformation and development in mathematics education.
This article is structured as follows. First, a literature review in the field of flipped mathematics classroom is presented. This is followed by the theoretical framework by outlining Activity Theory and the principle of dialectical contradiction. This is followed by sections: Methodology; Results; Discussion; Limitations, and Conclusion.

Flipped mathematics classroom: Literature review

The FC approach [4] has been used in educational settings for several years. The popularity of the approach, as an alternative to traditional lectures, has grown significantly with the emergence of technologies which allow teachers to produce video lectures and disseminate them online [9]. Although there is no single definition of the term "FC", there is a common understanding of the approach among practitioners, educators and researchers [10, 11]. FC is generally characterized by its structure comprising two stages of instruction: out-of-class and in-class instruction. Firstly, lectures and lessons are delivered as homework prior to in-class activities via online materials in various media formats, such as video clips, podcasts, internet sites, games, simulations, or visualizations. Secondly, in-class instruction is used for collaborative activities, and to actively engage students in challenging problem-solving tasks and conceptual understanding of mathematics [4, 5, 12].
In an attempt to systematize the literature review on flipped mathematics classroom at the university level, we focus on three main research themes.
Firstly, there are a considerable number of studies that have shown positive effects of flipped mathematics classrooms at the university level. For example, Love et al. [13] found that FC students performed better than those taking a traditional course in linear algebra in terms of exams. They also found that the FC students had a more positive perception of the course and performed better in terms of communicating with classmates. Similarly, Sahin et al. [14] indicated that students achieved significantly higher quiz scores in flipped sections than non-flipped ones and most students stated that flipped-taught lessons prepared them better. Likewise, Esperanza et al. [15] described several issues that show positive effects of FC in terms of better understanding of mathematical concepts, students learning at their own pace, and improved students' skills in communicating mathematical ideas. Furthermore, a recent study addressed the affordances and

constraints of FC to explore students' engagement with mathematical learning [16]. The study provided some empirical data on the potential of FC to support mathematics learning.

Secondly, there are also studies reporting on flipped mathematics classrooms in neutral or negative terms. For example, DeSantis et al. [17] found no significant differences in the learning outcomes between flipped and non-flipped classroom; students participating in the traditional model of instruction reported significantly higher satisfaction with their own learning than those involved in the flipped lessons. More importantly, Song et al. [11] indicated that a large number of studies do not present their pedagogical design with strong theoretical frameworks to guide the implementation and evaluation of FC. As a result, there is a lack of clarity on what activities can help students develop their critical skills. Likewise, O'Flaherty and Phillips [18] identified similar problems in their scoping review, for example under-utilization of conceptual frameworks that enable a unified approach to out-of-class and in-class activities, lack of clarity and content focus, lack of engagement with the pre-class activities resulting in variability of student preparedness.

Finally, several studies report on disconnections that arise in flipped mathematics classrooms, but these do not use the term "tension" or "dialectical contradiction" to analyse the characteristics of the disconnections. For example, Tague and Czocher [19] reported on several studies that show disconnections between out-of-class and in-class components. Accordingly, disconnections arise when in-class activities are ineffective in orienting students to the learning tasks, when in-class activities fail to address student misconceptions of mathematics, or when out-of-class activities only require low-level recall. This results in course materials that may not be conceptually coherent, and misunderstanding of the connections between the out-of-class course materials and the in-class problems. Likewise, Esperanza et al. [15] reported on several studies that impede the development of flipped mathematics classrooms. These refer to the insufficient utilization of the ability to support students to follow their own pace through the use of video lectures, students having difficulties in adapting FC, lack of access to a teacher while viewing the videos out-of-class, students feeling excluded from participating in classroom activities when videos are not viewed prior to class, thus requiring more efforts, and the preference of some students of traditional model over the FC approach.

Summarizing, the research literature on flipped mathematics classroom reported above motivates the present study in three ways. Firstly, there are a large number of studies that report on positive, negative, or neutral effects of flipped mathematics classrooms but these studies do not provide deep insight into what makes a flipped mathematics classroom work or not. Secondly, the FC approach to mathematics seems to be a source of disconnections and tensions that prevent teachers and researchers from effectively translating the FC concept into practice. Thirdly, there is a lack of robust theoretical frameworks and empirical studies to explore the role of tensions and contradictions characterising flipped mathematics classroom. We argue that Activity Theory (AT) provides an appropriate framework for identifying and analysing contradictions as manifestations of tensions that emerge when employing this innovative approach to mathematics classrooms at the university level.

Theoretical framework: Activity Theory and the principle of contradiction

In this section, we use Engeström's AT and its principle of dialectical contradiction [7, 8] to study mathematical activities that take place in FC at the university level for engineering students. The activities cannot properly be studied at the individual level without considering the broader socio-cultural context where they are situated. From this perspective, the unit of analysis, that is the unit that reflects the whole, is the Activity System (AS) that encompasses the subject, the object-oriented activity that is transformed into outcome, the rules that guide the actions of the AS, for example norms, regulations, and conventions, the community of participants, the division of labour among participants and the instruments used in the AS (Figure 1).

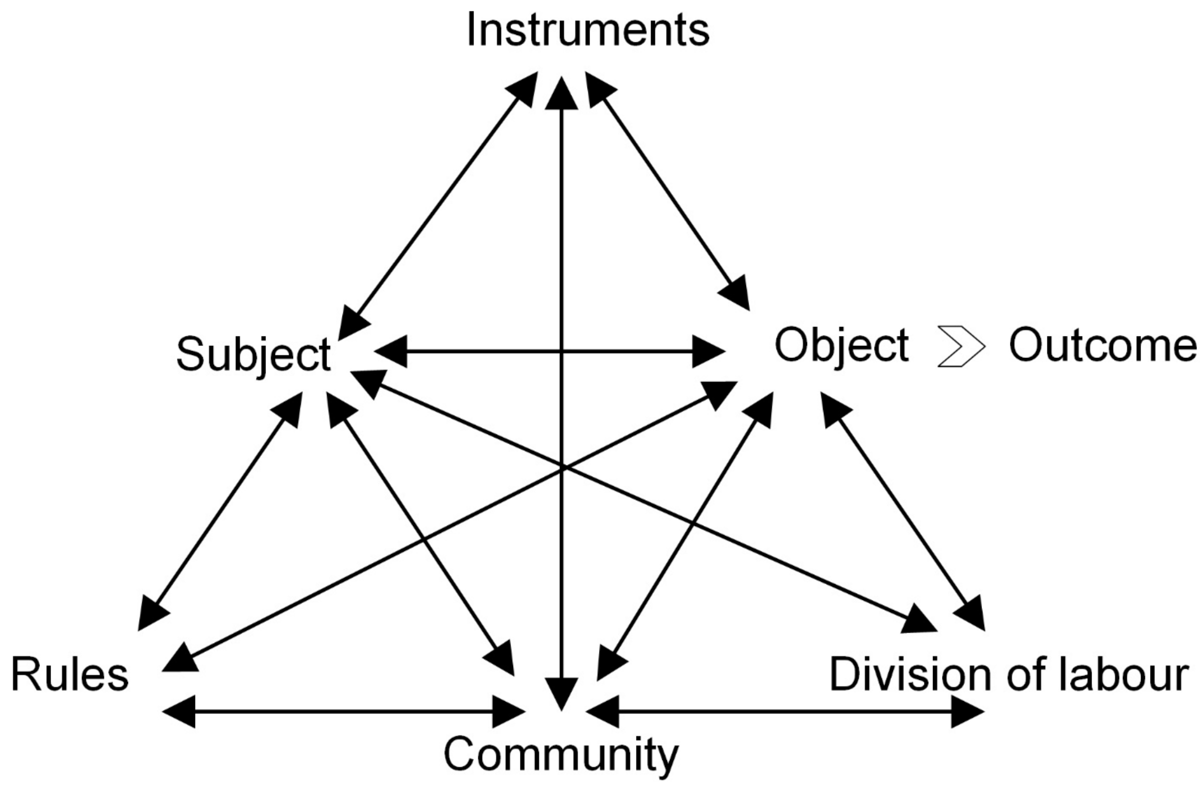

Figure 1: Activity System adapted from Engeström [7]

We now interpret Figure 1 with respect to engineering students engaging in mathematical activities in a flipped classroom setting. The students use instruments, also referred to as tools or mediating artefacts, such as out-of-class videos/quizzes, digital tools, curriculum literature (e.g., textbooks, classroom mathematical tasks and other study documents). Their activities are driven by a motive and are directed towards an object. According to Engeström [8], the object refers to the 'raw material' at which the activity is directed and the "future-oriented purpose" of the activity, which are transformed into outcomes. In the case of flipped mathematics classroom, the object of the activity is solving assigned mathematical tasks, modelling problems, performing mandatory assignments, watching videos, solving quizzes, and doing exams. These are transformed into the desired outcomes (e.g. passing the exams, getting grades for the purpose of obtaining a degree). The activities are regulated by the rules of the AS, for example the university programme, curriculum, assessment, norms, and other commitments such as compulsory exams and assignments, time constraints; the division of labour in terms of teacher's and students' role and position in flipped mathematics classrooms and university; students' roles in group work and collaborative tasks in-

class, and their relation to other participants within the community consisting of fellow students and the teacher, and other stakeholders within the broader university institutional context.
Within the AS, the principle of dialectical contradiction as driving force of transformation and development plays a central role. This principle informs our research.

The principle of dialectical contradiction in AT

The principle of contradiction in AT has several characterizations in the research literature. It is associated with terms such as tension, misalignment, dichotomy, opposition, or similar words without defining them theoretically [20]. Contradictions have been viewed as problems, conflicts, clashes, breakdowns, ruptures, or tensions [21,p.3, 22,p.34, 23,p.83]. Barab et al. [2] use the terms "system dualities" and "systemic tensions" and Murphy and Rodriguez-Manzanares [24] refer to a contradiction as opposition between two propositions. Likewise, Engeström and Sannino [20,p.368] use terms such as opposite, dichotomy, paradox, conflict, dilemma, and double bind.
However, Engeström's principle of contradiction should not be equated with paradox, inconsistency, conflict or dilemma [20,p.369]. More specifically, contradictions are not the same as conflicts or problems that can be solved [7]. These differ from contradictions in that they refer to personal and interpersonal crises that affect individual short-time actions, while contradictions instead are systemic tensions that have a much longer life cycle [3]. Moreover, contradictions are "historically accumulating structural tensions within and between activity systems" [7,p.137]. Drawing on Ilyenkov [25,p.185] definition of contradiction as "the concrete unity of mutually exclusive opposites", Roth and Radford [26] use the term of "inner contradiction" to describe "internally contradictory" aspects of the same phenomenon that coexist dialectically. In contrast to logical contradictions, inner contradictions cannot be removed, because they are "characteristic (constitutive) of the thing itself" [27,p.94]. In educational settings, an inner contradiction, hereafter called dialectical contradiction, is the joint teacher-students activity which ties teaching and learning in a symmetric and dialectical manner [26].
Stouraitis et al. [1] provided other instances of dialectical contradictions characterized as dialectical oppositions in mathematics: part-whole, means-goals, static-dynamic, intuition-logic and concrete-abstract, including contradictions related to general pedagogy, such as individual-collective, quality-quantity, and teacher's guidance-student's autonomy.
Dialectical contradictions, that is the concrete unit of opposites, emphasize the dynamic aspect of activity, its movement, and self-development. As such, contradictions are "sources of change and development" [7,p.137], and "driving forces of transformation" [28,p.25]. Contradictions emerge when an Activity System adopts new elements from the outside, such as a new tool (e.g., videos in a flipped classroom setting), or a new rule (e.g., prepare for the lessons using videos), causing a tension within the old elements. Contradictions may emerge within each element of the AS, and between the elements of the AS (e.g., subject-rule, subject-community or subject-division of labour).

Contradictions in mathematics education research

A number of research studies focus on describing contradictions in teachers' professional development [29]. More specifically, Potari [30] described contradictions in two activity systems by studying teachers' actions through the work they presented in the course and discussion. Jaworski et al. [29] suggested to use AT as a framework for the analysis of tensions/contradictions between the ways in which activity is perceived by teachers and students and their differing objects for activity. Page and Clark [31] used AT to study tensions that were created as teachers wrestled with incorporating the affective domain into their mathematics classroom. In these studies, contradictions refer to pedagogical and professional issues. Less attention was devoted to mathematical and technological issues. Finally, drawing on Ilyenkov's understanding of dialectics, Stouraitis et al. [1] use the term "dialectical opposition" to report on contradictions in teaching mathematics. They argued that the possible resolution of the contradictions can provide potentialities for shifts in teachers' practices.

Another research direction is related to the use of contradictions to stimulate expansive learning, a process that is triggered by the emergence of contradictions [32, 33]. This research aims at identifying and overcoming contradictions in attempts to open expansive learning [34, 35].

A more recent work explores how contradictions can explain first-year undergraduate students' experiences of learning advanced mathematics [36]. The authors argue that contradictions between the school and university activity systems, as well as those within the latter, provide explanations of some of the difficulties students can experience when they encounter advanced mathematics at the university level.

Furthermore, Núñez [37] indicated that the identification of contradictions is mostly limited to the top components of the Activity System, the subject, object, and instrument, while the rules, the community, and division of labour are omitted, with the exception of a few more recent studies [29, 30]. Clearly, these limitations restrict the identification of contradictions, by not considering the contradictions that may emerge within and between the bottom three components of the Activity System (rules, community, division of labour), like a contradiction between individual and collective work.

Finally, some research studies have used AT to model flipped mathematics classroom [5, 38], but not as a theoretical framework to explore contradictions.

Tensions as manifestations of contradictions

According to Engeström et al. [39], contradictions cannot be observed directly, they can only be identified through their manifestations. Tensions or similar terms such as disturbances are the visible manifestations of underlying contradictions [40,p.302]. We agree and thus consider that contradictions must be identified through their manifestations. They become recognized when participants express them in words and actions, but they cannot be reduced to subjective experiences or situational articulations [20,p.371]. In other words, contradictions cannot be identified directly without their manifestations that would qualify them as contradictions [20,p.372]. Some indicators for tensions are then necessary to identify contradictions in AS. In this work, the term "tension" is not used in its everyday sense. Instead, tensions are defined and expressed as forces pulling in opposite directions [41], imbalance of participation or divergent objectives among participants [42], disagreement among participants, disagreement between a participant and an external source (e.g., curricular material), or apparent incompatibilities between different utterances made by a single participant [1]. Other similar indicators are proposed by Engeström and Sannino

[20,p.373]: incompatible evaluations (dilemma), conflict in the form of resistance, disagreement, or argument; critical conflict facing contradictory motives, and double bind, that is facing pressing and equally unacceptable alternatives.

Methodology

This section elaborates on the research design and rationale of a flipped mathematics classroom implementation in terms of out-of-class and in-class teaching. Information is also given on data collection and methods used for analysing the data into Tension Categories (TC).

Research design

The work presented in this article consists of multiple case studies over a period of two years. The first case, spanning the study year of 2015/2016, was a pilot study for primary testing of FC for the purpose of providing initial feedback from students which could be used for adjustments towards later implementations. The second case-study was tuned towards a more in-depth design on the study of tensions, but also other phenomena like mathematical discourse in a FC setting.

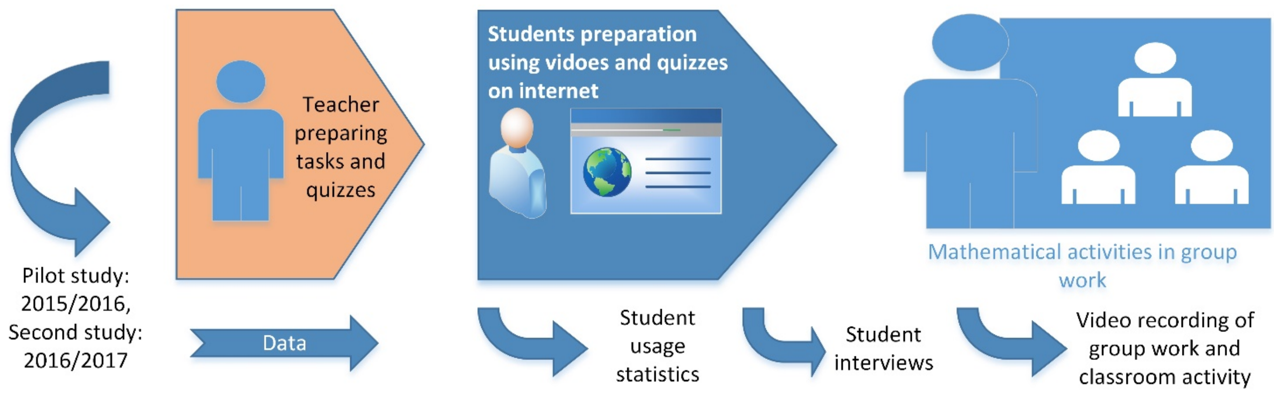

Figure 2: Multiple case study research design, showing the use of out-of-class and in-class data collection

Figure 2 gives an overview of the main structure of the data collection and implementation of the two case-studies; further elaboration is given below.

Data collection and analysis methods

Data were collected through filming of students' activity in groups in addition to individual semi-structured interviews. The pilot study also contained an anonymous survey, where student opinions on the FC interventions were elicited. Interviews and filmed sessions were transcribed verbatim and coded according to an inductive coding strategy based on the interplay between our theoretical framework and the empirical data [43,p.465]. Thus, the method of coding was somewhat inspired by grounded theory, but not to the extent that we expect to be fully liberated from previous

assumptions about the world [44]. We believe that an acceptance of one's own role as a researcher should rather enrich the inquiry, by placing specific events into a fuller, more meaningful context [12, 45].
The first author had the dual role of being both the researcher and the teacher in this context, and as such taking an active role in the object of study. An ethnographic approach allowed us to take both an insider (being a teacher) and an outsider (being a researcher) role as an participating observer of the culture [45]. The inside perspective gives the researcher the opportunity to fully experience phenomena as they occur. However, the researcher must be aware that "blind spots" could occur due to the participative role, making it important to step into the outsider role when analysing the data, most importantly here the filming of classroom activity.
Data from students' interviews may be coloured by students' unwillingness to express their real opinions, being afraid of answering "incorrect" to expected outcomes. Observing the phenomena through authentic situations in filmed classroom activity should enhance the trustworthiness of statements made in interviews [46].
Our approach to data analysis is inspired by Stouraitis et al. [1]. We use indicators to analyse students' views, discussions, decisions, actions and choices to reveal tensions. An indicator of a tension could be based on similar criteria described in the theoretical framework on identifying contradictions, such as: a) Disagreements among participants; b) Disagreement between a participant and a rule (e.g., preparing through out-of-class video-watching); c) Imbalance of student participation or divergent objectives [42]; d) Degree of consciousness and motivation during group work; and e) Students' disagreement on pedagogical structures and imposed mathematical learning objectives. As an example for the last of the indicators is the tension "Individualistic problem solving, students not willing to adhere to the rule of collaborative learning in-class" was observed during classroom filming, but was also uttered during interviews with the students.

We use codes to describe each identified tension relating to its content (e.g. students' difficulties/ disagreements, task), and characteristics of the tension (e.g. a tension between a student and a rule of the FC). In the process of data analysis, similar tensions were classified in tension categories (TC). Accordingly, a TC is understood as a set of concrete detected tensions similar to such a degree that they can be abstracted as the same phenomena.

The coding of TCs was performed in several stages of the multiple case study. Firstly, through the analysis of the data from the pilot study six (6) TCs were identified through an inductive open coding approach [47]. Secondly, based on our own experiences, and grounded in memo-writing, we considered the presence of other possible tensions in the activity system of FC. From this analysis, another eleven (11) TCs emerged as candidates for investigation in the second case study.

In the first stage of the second case study, we used two instruments. The first one was the interview guide, which was developed with the basis of shedding light on these altogether seventeen (17) TCs. The second instrument was a design for how group compositions could be synthesized to best spawn TCs related to collaborative work in-class. In the second stage, another data collection phase was initiated, where the planned interviews and filming of collaborative work were guided by these instruments. The data collected were coded according to the 17 established TCs but remained open for new codes/TCs to emerge from the data. In the final stage, we used our theoretical framework to analyse and interpret the TCs, which led to the identification of dialectical contradictions.

Summarizing, while the classification of TC's is the result of our theoretically informed approach based on elements of grounded theory, the identification of dialectical contradictions was a result of our efforts to interpret the TC's as dialectical contradictions on the basis of our theoretical framework.

Context of the two case studies

Both case studies were conducted on the same University campus in Norway, each spanning a whole year of study. As mentioned, there were 20 students in the first case and 25 in the second. These were computer engineering students in their first year of study. The students attended two individual courses, Mathematics-1 in the autumn where the majority of topics were calculus-based, followed by Mathematics-2 in the spring semester, where series, Fourier series, Laplace transform, recursion equations, proofs, logic and optimization on functions of two variables were taught. Both courses were 10 ECTS (European Credits), and were mandatory to get a bachelor degree.

The students were informed about the format of the teaching and the research they were going to be subject to during the opening session of the course, to ensure that the students were fully aware of the out-of-class preparatory component that they were expected to comply to. They were also given the right to withdraw as a participant in the study, but still act as a member of the class community.

Two FC sessions were usually performed each week, both consisting of an out-of-class and an in-class component. The preparatory out-of-class part, consisting of 3-5 videos, each 5-15 minutes in length, was usually provided one working day before the in-class part. Most videos were quite procedural in their content, since the short, to-the-point format that is recognized as an important feature of FC videos [48], left little room for in-depth mathematical reasoning. In-between the videos, quizzes were provided in approximately a fifth (20%) of the out-of-class sessions that related directly to the understanding of the content of the preceding video.

The in-class sessions, lasting one and a half hour each, were usually structured as follows:

- A short introductory talk reviewing major points from the videos. This lasted for about 10 minutes if a majority of the class was prepared, but it could last up to 30 minutes if few had prepared, or there were many questions arising on the topic. Usually, this talk was interactive, with the teacher prompting students about issues, or students asking questions.
- Then students would attend to tailored tasks related to the videos in the out-of-class session. These were either from the curricula textbook or produced by the teacher. Some of the topics reviewed mathematics from earlier courses and were suitable for working with modelling tasks on a more conceptual level. Other topics, like the Laplace-transform, were totally new to the students and were deemed suitable for working with tasks on a more procedural level.
- Students would spend the rest of the session working in groups. These groups were predominantly assembled by the students themselves, except on some occasions where it was reasonable for pedagogical or research purposes to place persons in alternative group configurations. The group work would temporally be interrupted by a whole-class walk-through if this was part of the planned teaching scenario, or if many groups struggled with

similar issues. An end of session summing up was sometimes conducted if the teacher found it necessary.

The pilot study was performed as an initial attempt of implementing flipped mathematics classroom during the study year of 2015/2016 [49]. This cohort was subject to a traditional lecture-based teaching during the first semester, while we attempted FC teaching on two occasions during the second semester, each intervention lasting two weeks.
Informed by this pilot study, we set about to do a more elaborate study with the 2016/17 cohort of students. This time, the initial number of students had increased from 20 to 25, and FC teaching was conducted throughout the whole year. Consequently, all lecturing was dropped in favour of the FC model except for short introductions at the start of lessons. While performing this flipped teaching, experiences from the teaching were noted in memos.
The interviews took place during the end of the second semester, where nine students were invited to express their experience with the FC teaching. These students were selected on the basis of getting a representative sample according to performance (using exam results from Mathematics-1), participation in class, and a sense of how able the person was to make critical remarks. The interviews were semi-structured, allowing focus on the specifics of the tension-related questions, while at the same time being sensitive to participants' own experiences. To minimize bias in student responses by interviewer being the same person as the teacher, an independent researcher not affiliated with the course conducted the interviews. The interviewed students were informed that recordings would not be subject for investigation before final exam in the course. This strategy is similar to the one followed by Strayer [12] and Tawfik and Lilly [50] in their qualitative studies on FC in statistics courses in tertiary education.
In addition to the interviews, 15 individual filmed sessions of group work were obtained throughout this case study. Three of these sessions were planned especially for producing data about the potential tensions at play. These sessions were performed at the end of the study-year during the Mathematics-2 course, providing opportunity to obtain experience with the student personalities and profiles according to these categories:

- Preference towards working individually or in groups
- Discursive vs. silent person in group work situations
- High achiever vs. low achiever in the mathematical solution processes based on the exam results from Mathematics-1.
- Prepared students vs. non-prepared students

The three sessions consisted of two groups being filmed about half of the session each. Based on previous categorization of possible tensions related to group work, the groups were setup to consist of members with a mix of profiles that would, we hoped, reveal information about these group-work related tensions.
These filmed sessions and the nine interviews were then transcribed verbatim. As a basis for the coding, the 17 already identified TCs were used. However, as an inductive coding strategy was followed, the emergence of new codes/tensions as we analysed the data were allowed. All in all, nine (9) new TCs were identified in this process.

In the process of coding the transcripts, an inter-coding reliability test [51] was performed with two fellow researchers. The two tests were conducted on independent samples of transcripts, and the ratio match:mismatch in these two tests was 9:3 in the first and 9:4 in the second.

Results

The results from the analysis of the empirical data are given in this section. In the subsection "Identification of dialectical contradictions", we present the three dialectical contradictions that emerged from the analysis of the TCs. Then, all TCs, and how they relate to these three contradictions, are presented in the subsection "Classification of the tension categories". Finally, some samples from the analysis of the transcripts are presented in the subsection "Analysis of group sessions and interview data" to give in-depth understanding on the coding process.

Identification of dialectical contradictions

During the coding process of the interviews and the three filmed sessions 158 concrete tensions were coded into 26 tension categories. These categories relate to various issues on the implementation of FC, ranging from themes of a more psychological nature like motivation, towards ideas of mathematical language and the nature of mathematics learning.
According to AT, a further analysis of these tension categories towards their inherent dialectical nature is undertaken. However, not all tensions can be classified as dialectic. As pointed out by Engeström and Sannino [20], many conflicts, inconsistencies and dilemmas are *not* materializations of contradictions, but rather expressions of personal dilemmas and attributes, or interpersonal disagreements giving rise to short-time actions and disputes among members in the activity system. Thus, we differ between tensions which are dialectical in nature, called contradictions, and those which relate to personal and interpersonal crises and affect individual short-time actions [7]. Below we present three contradictions emerging from the TCs:

a) Conceptual-procedural: This contradiction relates to a key feature of flipped mathematics classroom to support conceptual understanding on the topics covered in a more procedural fashion in the videos. According to Bergsten et al. [52,p.981-982], a definition of this contradiction specifically adopted to the context of engineering mathematics is formulated as the following: A *conceptual approach* includes translations between verbal, visual (graphical), numerical and formal/algebraic mathematical expressions (representations); linking relationships; and interpretation and applications of concepts (for example, by way of diagrams) to mathematical situations. A *procedural approach* includes (symbolic and numerical) calculations, employing (given) rules, algorithms, formulae and symbols. This contradiction relates to an epistemological dimension of mathematics. Sfard [53] pointed to the dual nature of conceptual-procedural, and related contradictions such as abstract-algorithmic and operational-structural, as two manifestations of the same mathematical activity. The dual nature of mathematics is also expressed in similar dialectical contradictions, such as object-process [53] cited in [1].
b) Teacher guidance-student autonomy: The contradiction between the expectation that the teacher will control most learning activities, contrasted with enhanced student autonomy found in more active learning environments is a key contradiction in flipped mathematics

classroom. In this context, the teacher takes a new role as orchestrator and a guide on the side in-class instead of being a lecturer. In addition to this, the out-of-class component requires students' active engagement in the preparation process. This contradiction relates to the teaching and learning of mathematics, and expresses two manifestations of joint teacher-students activity. It connects two mutually exclusive opposites (teacher guidance and students autonomy) in a dialectical manner, and cannot be separated and thought of independently [26, 27,p.95].

c) Individual-collective: We identified a contradiction in the preference for individual versus collective learning in-class. Collaboration and discussion among students about the mathematics introduced in the videos is considered vital for engagement and the facilitation of conceptual understanding in a flipped mathematics classroom [50]. However, a fundamental contradiction about different forms of student engagement in mathematical activities in this field can be observed in several of the TCs. We connect this contradiction to individual vs. collaborative learning in-class. The term "collaborative learning" is used as a synonym for collective learning. This involves a group of students working together to share ideas, solve a problem, or accomplish a common goal. Research reveals that learning is enhanced when students work collaboratively on well-structured tasks that are carefully implemented so that they can engage in active learning that allows for problem solving and understanding beyond the capability of individual students [54]. Yet, there is a dialectical relationship between individual and collective learning. They exert a reciprocal influence on each other through interactions among diverse participants in classroom [55].

Classification of the tension categories

In all 158 concrete coded tensions were found in the transcripts. We present here the TCs, numbered 1 – 26 in four different tables (Table 1, 2, 3, and 4). The first three tables contains TCs classified according to the mentioned contradictions, while the last contains TCs found not to signify contradictions. Some examples from the analysis of the data are provided throughout the classification. The last column marked # gives the number of occurrences found in the transcripts.

Table 1: Conceptual-procedural contradiction

	Tension category	#
1)	Students' inability to cope with problem-based learning or modelling This tension was noted in group work transcripts where the teacher either had to intervene to a large extent for the group work to progress, or students were not able to use digital tools like Geogebra necessary to do the modelling activity. During one of the sessions, one of the students was noted to be very inactive in the collaboration process, an impression confirmed in the interview by the statement "You get the discussion on how to proceed with the task. However, when the calculations take place in the group afterwards, one tends to be put behind the others"	8
2)	The necessity of group work progress leading to acceptance of routines, avoiding conceptual discussion of the origin of the routines	9

	As an example illustrating this TC, we might consider this statement from one of the students working with formalities on the mathematical solution process during group work: "It's correct the way we have done it (referring to notes taken from the videos), but I don't have a clue on how this formula was created in the first place". This was a concrete tension that students mentioned about one of the videos being too procedural, where they expressed frustration about not knowing how it was derived. However, instead of consulting the teacher for more conceptual understanding of the origin of the formula, they preferred moving on towards finding a solution, applying the formula.	
3)	Students wish to focus on procedural learning for exam preparation, contrary to teacher desire to elicit conceptual learning A student expressed this tension during the interviews through the following statement: "Well, it's how I'm used to work in my job, so it's great in that sense, but I don't think I learn more from it. I learn from concrete examples, as when we work our way through the task the way it was supposed to be done". Another was concerned about being properly prepared for the exam, worrying that it would be better to stick with "drill tasks".	4

Table 2: Teacher guidance-student autonomy contradiction

	Tension category	#
4)	Students' inability to meet higher demands on self-discipline and structure to prepare for in-class active learning	17
5)	Students' expectation of being taught "directly" in-class by a teacher This TC was expressed during interview by some students, referring to a desire to have the teacher show them more about "how to do the task" on the whiteboard. One of the students claimed that he needed the structure of a teacher-centred session to be able to focus: "As I said, I am a person that needs things to be gone through on the whiteboard by the teacher in a careful manner, followed by a part of the lesson which is dedicated to task solving, much the way that the pre-calculus teaching was setup"	8
6)	Teacher imposition of new group work structures	15
7)	Video preparation considered to be too time consuming by the students	10
8)	New rule of using most of the class-time for solving problems viewed as problematic. Considered inappropriate for achieving mathematical knowledge	5
9)	Risk that whole-class discussions might have poor quality with little gain in mathematics learning	4
10)	Students group work suffering from individuals not preparing watching videos	18
11)	Unprepared students watching the videos in-class or reading the textbook to catch up with the rest of the group	3

Table 3: Individual-collective contradiction

	Tension category	#
12)	Not attending group work due to anxiety about not being able to participate in the collaboration, or for other personal reasons	5

13)	Preference for working in solitude, not willing to adhere to the rule of collaborative learning in groups	8
14)	Students failing to keep up with the others in group work, needing more time to "think" In some interviews, students expressed anxiety for group work, especially in situations where they had not completed the out-of-class preparation. Filmed group-work activity showed that certain students worked separately from the rest of the group using alternative solution techniques than the collaborating part of the group. On several occasions, we also observed students with apparently strong mathematical problem solving skills, becoming a leader in the group work, solving the task singlehandedly without discussing the solution process. The other members would then follow the process from the side line, however, seemingly learning from the process.	5

Table 4: Tension categories considered not to signify contradictions

	Tension category	#
15)	Students experience a lack of mathematics fluency	3
16)	Students questioning the purpose of learning mathematics as a topic in their education	5
17)	Communication breach in lecture form. Unable to interact with lecturer through the one-way video medium	5
18)	Unable to stay focused on the mathematics learning throughout an in-class session, drifting into non-mathematics activities	6
19)	Learning mathematics is very resource intensive in general	6
20)	Quality of the videos were not adequate	5
21)	Difficult to learn through watching videos	3
22)	Focus problems during video watching. For example: too easy to pause them and do other activities	2
23)	It is problematic to use English as a first language	1
24)	Group work has "stable" problems, some in the regular group setups are not able to adopt to the working habits of the others in the groups	3
25)	Textbook tasks not properly aligned with the rules of in-class active learning	0
26)	Utilizing videos as a means of lecturing leads to meagre whole-class discussion about the topics, since it is an individualistic learning arena	0

We observe that the two last TCs had no occurrences in the transcripts. These TCs were constructed during the analytical phase described in the methodology section, but did not appear empirically in our data.

Analysis of interview data and group sessions

The interview guide for the investigation of the TCs in the second case was divided into the following themes: group work; the tasks that were worked on in-class; student discussions; and short lectures that were performed in the plenum, and general questions about the FC method. Although most students expressed a positive attitude towards FC in general, two of the eight interviewed students were critical about FC due to the missing lecturing part. They reported a lack of motivation for watching a video about mathematics in their leisure time, or claimed that there was too little time to spend on this due to a part-time job. We coded these worries of time constraint as TC 7: "Video preparation considered too time consuming by the students", which again is an expression for contradiction teacher guidance-student autonomy. One may question if time constraint among students really relates to a contradiction in a FC activity system, considering its strong ties to priorities in their private lives. However, we consider this to express a contradiction due to the redistribution of the in-class/out-of-class time that FC imposes (through teacher guidance/authority) on students. Since the FC method does not require more time compared to lecture-based teaching per se, we consider TC 7 as an expression of students not being conscious on the necessary redistribution of lecture/task time compared to more traditional settings. One of the critical students explained it like this:

> "Well, I think it's good that we work on tasks, but it's a bit awkward if you don't have time to prepare, then you are put far behind, and it's like that with the videos. I'm bad at prioritizing my time, because I work a lot in the evenings, and I don't want to watch the videos at 11 in the evening. I think it's much better to have a decent lecture to attend to."

In the last sentence we also coded TC 5: "Students expectation of being taught directly in-class by a teacher". Claiming to be 'put far behind' is furthermore an expression for TC 14: "Students failing to keep up with the others in group work, needing more time to think", a TC considered to express the individual-collective contradiction.

As we seek to understand contradictions in flipped mathematics classroom as not being limited to individuals' understandings (sampled through interviews), but rather through a collective whole, it is of crucial importance to consider the social aspects of its implementation. We studied this insight through the analysis of in-class group work, focusing on how contradictions can emerge as an explaining phenomena in the AT [28]. An episode taken from students work with moments and centre of mass of two-dimensional structures is illustrative for the appearance of the conceptual-procedural contradictions. The task given to the students was to find and solve two definite integrals expressing moments in x- and y-direction which could be used to locate the centre of mass. The two students Ivan and Marcus lead the discussion. Marcus had just solved the task, and told Ivan that he wanted to move on to the next task

286 Marcus: But did you finish the task?
287 Ivan: No, but I sort of understood how to do it
288 Marcus: But you have to do…
289 Ivan: No, well, I just ended up getting a lot of calculation errors, it was awfully boring to do that task.
… (Marcus has left the room for a moment) …
308 Hassan: What do you say?
309 Ivan: I'm just trying to understand the meaning of this formula (looking in the book)
310 Hassan: It's the centre of mass
311 Ivan: Yeah, the mass gives meaning I guess, but the moment, what is the purpose of that? What do M_y really means?
…

Before Marcus returns to the group again, Ivan tried to express experimentally and verbally to the others in the group what is meant by the centre of mass by balancing a pencil on his finger

329 Ivan: Do you know what the moment in the y-direction means?

… (some more elaboration on this question by the others in the group)…

337 Marcus: You just need to accept this and move on (pointing on Ivan)

338 Hassan: Yes, that's what I too think. Look it up in urban dictionary.

After this, the discussion ended. What we observe is two students pulling in the opposite directions when it comes to the idea of exploring what the physical properties the mathematics they were dealing with really expressed. Ivan, feeling that ordinary blind calculations of integrals were boring, tried to 'hijack' the groups discussion after Marcus, who was leading the group into getting through with the task, left the room for a moment. After returning, Marcus more or less immediately terminated the discussion on understanding the concept of moment. We labelled this episode with TC 2: "The necessity of group work progress leading to acceptance of processual work, avoiding discussion of the origin of the routines", which is an expression of the conceptual-procedural contradiction.

Discussion

The research question of the paper aims at exploring the types of contradictions that emerged in a flipped mathematics classroom, and how students experience them. To explore this question, AT and its principle of dialectical contradiction is used as a theoretical framework.

In line with the framework, we discovered 158 tensions that were coded into 26 tension categories (TCs). The majority of these tensions are manifestations of dialectical contradictions emerging within elements of the AS of the flipped mathematics classroom. However, we do not exclude that some of the tensions may be interpreted as manifestations of contradictions between elements of the AS. In this paper, we discuss those within the elements of the FC activity system. Based on the results, we have seen that most of the tensions are manifestations of three types of dialectical contradictions: Conceptual-procedural, teacher guidance-student autonomy, and individual-collective. In terms of AT, these are structural tensions within the AS of the flipped mathematics classroom. As such, the contradictions are sources of change and development.

In this section, we discuss the implications of these results in light of the literature and our own observations, and provide some strategies for balancing these dialectical contradictions to promote mathematical learning in a flipped mathematics classroom.

Firstly, the most prominent of these, according to the number of occurrences of TCs found in transcripts ($N=75$), is the dialectical contradiction teacher guidance-student autonomy. We interpret this as a specific form of student-centred learning vs. teacher-led instruction. In terms of AT, we relate this contradiction to the division of labour between teachers and students, in that both need to constantly balance their practices in the teaching-learning interaction process in a dialectical and reciprocal manner [26].

Within the FC setting, the students consider teaching as something that should be led by the teacher in the classroom, and find it hard to spend an hour in the afternoon for mathematical video-learning for various reasons. Also, this contradiction is related to ways in which the FC controls their work habits in-class. Group work, especially in the case of teacher setup of group members, is considered to restrict their autonomy related to work habits. Also, some students have the impression that the flipped mathematics classroom is more time consuming compared to traditional teaching. The empirical evidence that FC is actually imposing less autonomy on the students is in stark contrast to other research, for example Abeysekera and Dawson [56,p.5] which argue that “Learning environments created by the flipped classroom approach are likely to satisfy students’ need for autonomy and, thus, entice greater levels of extrinsic motivation”. At the opposing end of the scale, some students seem to seek *less* autonomy, where teacher-centric lecturing is considered more appropriate for learning mathematics than student-centred group work. Other researchers point to similar findings, where the lack of “control” is found by some students to hamper learning [12, 57]. However, this contradiction seems to relate to only a third of the students interviewed. The other two thirds expressed a positive attitude towards the greater flexibility the videos gave them in terms of when to prepare and how they could be utilized. Observing that a large minority of the students seem to have problems exercising the enhanced autonomy afforded by FC, this group is denied the preferred option of learning through direct in-class lecturing. For this group of students, the relationship between autonomy and teacher guidance remains a contradiction with mutually exclusive opposites that are difficult to reconcile. On the other hand, some students enjoy the enhanced autonomy of FC, while others see it as an extra burden being put on them. To ease the transition towards greater autonomy, some students might need more guidance in-class in order to experience the gains of FC and become more motivated for video preparation [57]. In a flipped mathematics classroom setting, certain tools for the distribution of videos might offer user statistics and feedback mechanisms from students on problematic mathematical areas that the instructor can access in the preparation phase for the in-class session. As such, the contradiction can somewhat be remedied by the repeating central themes or letting the students perform a task during the start of the in-class session.

Secondly, an important feature with flipped mathematics classroom is to free up time for facilitating learning on the topics covered in a more procedural fashion in the videos. This concerns the contradiction conceptual-procedural, which in AT terms, relates to the mathematical aspects of the object of the FC activity system. More specifically, the object encompasses both mathematical tasks/problems given to the students and epistemological issues of mathematics, like the importance of abstraction and concepts, conceptual understanding versus procedural fluency, procedures and algorithms [1]. It is unlikely that the contradiction conceptual-procedural will merge fully to a unity of the opposites due to certain constraints in the FC activity system, like institutional and time constraints, compulsory exams, division of labour between participants, new rules of flipped mathematics classroom, like watching videos prior to in-class activities, and the type of instruments/tools being used. Nevertheless, it is possible to reconcile conceptual understanding with procedural fluency using various pedagogical approaches. One strategy to balance the opposing elements of the contradiction is to consider in-class task designs that spur discussions and collaboration in the groups on conceptual issues. This can be achieved through active learning strategies like problem-based learning [50], inquiry based teaching [58], or Realistic Mathematics Education [59]. Throughout our study we implemented several sessions where tasks were designed in this direction. Although most students endorsed the idea that mathematical modelling elicited conceptual understanding of mathematics, there were certain students considering such activities as inappropriate for exam preparation, where the procedural capacity of students predominantly are being tested. We noted instances where certain students were in favour of “getting the job done” to such a degree that little mathematical reasoning appeared to take place. In other settings, we noticed that certain students seemed to have difficulty handling the added complexity of mathematical modelling. These observations express the contradicting view among many students about the value of facilitating a more conceptual strategy of learning. Nowadays strong voices oppose the contradiction of procedural-conceptual thinking in mathematics [60], claiming that a certain competence in procedures is necessary to be able to advance on the conceptual level of mathematics. However, it is evident from this study that these are indeed opposing forces that a teacher needs to balance when designing activities in flipped mathematics classrooms. In this regard, the idea of in-class group work and collaborative learning, which play a fundamental role in FC, could help to address the contradiction. Certainly, a teacher could think of sessions where students practice a procedural drill on an individual basis, but this should be used with care. An important motivational factor for attending flipped mathematics classrooms should be the added value of conceptual understanding. However, final assessment in calculus courses at the beginner level are usually written exams, focusing on procedural task solutions on an individual basis. Not having a specific focus on this skill attainment can lead to students worrying about not being properly prepared for the exam situation. As such, a mix of sessions focusing on Problem Based Learning/modelling, while in others utilizing textbook tasks, can be considered. This would avoid the sessions having too little variation, while at the same time promoting necessary procedural skills development.

Finally, phrased in the terminology of AT, the contradiction individual-collective relates to the community of students working collaboratively in groups toward a common goal in a flipped mathematics classroom. Most students seemed to enjoy collaborative work, a finding obtained through an analysis of filming and interviews. However, as we saw, several students struggled in various ways to adapt to the collaborative way of working with mathematics, especially in cases where the teacher found it necessary to break traditional group configurations due to certain groups of students being unprepared. Again, referring to the dialectical nature of the contradiction individual-collective, we need to take into consideration that individuals learn differently according to how active they are as participants in group work situations. Some students may prefer working in solitude with the occasional help of the teacher, while others prefer constant collaborations with others. These are fundamental differences that relate to variations in students' ways of learning, that the teacher in a flipped mathematics classroom needs to consider in the setup of learning environments in-class. Hence, the contradiction individual-collective can be somehow reconciled when individual and collaborative learning in-class take place in various forms.

Limitations

The findings of this study are based on analysis of data from two cohorts of students on one single campus. Other implementations of flipped mathematics classroom in other settings will probably yield similar and different results in terms of contradictions from those drawn here, both within and between the elements of the FC activity system. Hence, one needs to be careful to claim anything final about general students' experiences in mathematics education. These are indeed difficult to reproduce, because it is "nearly impossible to replicate the original conditions under which the data were collected" [61,p.266]. Furthermore, the study of flipped mathematics classroom could result in different findings depending on the design and implementation of FC approach, which is beyond the scope of this research work to discuss. However, as stated earlier, in-depth qualitative case studies like these provide ideas on central issues likely to be found in other settings as well, but maybe expressed differently. Also, the findings point towards fundamental dialectical contradictions and awareness of these should be central to any implementation of FC. Nevertheless, further investigations should be conducted, preferably in other learning environments for mathematics. One could for instance picture having larger classes, another type of FC design and implementation, different out-of-class videos/artefacts and mathematical topics and other educational and cultural settings with their specific curriculum.

Conclusions

As the FC instructional approach at the university level for mathematics continues to evolve, it is important to be aware of the inherent contradictions in this model of teaching and learning. We have seen from our empirical study that the majority of tensions emerging in the practice of a FC can be considered as materializations of dialectical contradictions that are not possible to fully resolve as they coexist in a mutual exclusive wholeness [27]. As stated in the literature review, current studies do not provide deep insight into what makes a flipped mathematics classroom work or not, simply reporting on positive and negative effects of FC in comparison to traditional non-flipped courses, neglecting the role of dialectical contradictions as source of development and transformation. Hence, acknowledging these contradictions provides opportunities for designing and implementing mathematics flipped classrooms that promote learning, development, and change as the teacher can customize it to students' learning styles, knowledge levels, and other differences. Our study provides three opportunities for mathematical learning in a FC environment. The most important ones seem to be the degree of student autonomy preferred, the extent to which the students are willing to collaborate in-class, and what motivates their learning according to conceptual understanding.

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