Affect and mathematical modeling assessment – A case study on engineering students' experience of challenge and flow during a compulsory mathematical modeling task

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Abstract

This chapter describes a study on engineering students' affect while working on the *Tracker Project Task,* a group assessment task that asks students (1) to use digital tools (the camera in their smart phones and free tracker software) to capture the movement of an object, (2) to mathematically model that movement, and (3) to create a poster reporting on the video analysis of the movement.

We applied an activity-based conceptualization of affect in mathematics ("do you like this activity?"), which differs from a subject-based conceptualization of affect ("do you like mathematics?"). A subject-based conceptualization has two draw-backs: (1) it does not distinguish among different aspects of mathematics, and (2) it draws in students' bias and beliefs from earlier, often bad experiences of poor mathematics teaching. We found an activity-based operationalization of affect by using the concepts of *challenge* and *flow*. Flow is a state of absorption, in which people forget about time and experience feelings of happiness.

We assessed n=346 students through the *Tracker Project Task* . To study affect, we developed an instrument of 10 items (Likert-type) to measure students' experience of challenge and flow. We administered the survey through a web-based platform yielding a high response rate (n=239, 69 %) and good reliability (Cronbach's Alpha: 0,795). The results revealed that three out of five students experienced challenge and flow, which expresses students' positive affect regarding a mathematical assessment activity. This can be ascribed to, on the one hand, the activity and the instrument not clearly being related to mathematics, and thus not being tainted by students' earlier negative experiences with mathematics. On the other hand the *Tracker Project Task* had characteristics that can bring about flow: being open, offering ample time to submit the product, being accessible to all students (low floor), but also enabling the better students to challenge themselves further (high ceiling). Such characteristics may be better feasible within mathematical modeling assessment than canonical mathematics assessment.

1 Introduction

1.1 Introduction to affect, mathematics education and modeling tasks

A recently published book (Pepin & Roesken-Winter, 2015) brings together recent research in the field of mathematics education and affect. In this book, the authors describe affect, values, emotions, beliefs, attitudes, and so forth, which they conceptualize in terms of complex, dynamic systems and participatory environments. However, while distinguishing between many aspects of affect, they hardly differentiate between aspects of mathematics education, whether this is instruction, curriculum or assessment methods. When taking mathematics in such a holistic way, students are asked to give their agreement or disagreement to items such as “mathematics is my favorite subject”, “I enjoy pondering over mathematics tasks” or “in mathematics there is always a reason for everything”. There is no room for how students experience separate phenomena in mathematics education, some of which maybe positive and some maybe negative, some temporary and some more lasting. There is evidence that teachers want the best for their students, but nevertheless knowingly offer inadequate instruction due to time pressure, examination demands, discipline problems, lack of confidence, and so forth (e.g. Nolan, 2012). These limitations can hardly yield positive effects on students’ affect, as repetitive calculation exercises cause boredom, time-restricted tests cause stress, the distance between the teacher’s and the students’ discourses cause alienation, or the seemingly irrelevance and meaninglessness of algebraic expressions cause demotivation. Asking students holistically for their agreement or disagreement on a statement such as “mathematics is my favorite subject” gives little room for nuances and contexts. A student partly agreeing with this item might rather have said: “mathematics with this particular teacher is my favorite subject, but last year it was the opposite” or “mathematics is my favorite subject when we solve problems that I can relate to in my daily life”. Or the students partly agreeing with the statement “in mathematics there is always a reason for everything” might rather have expressed personal reasons: “in mathematics (classes) there is always a reason (for doing it), which is to get a pass”. So, when researchers of affect in mathematics education understand beliefs as “relatively stable, reified mental constructs” (Pepin & Roesken-Winter, 2015, p. 4), the stability of these beliefs can well be related to the stable practices in mathematics classrooms with meaningless tasks, alienating symbols, stressful tests and, often, a friendly teacher, of which few students envy the job.

To the general public, mathematics has a bad press, being the only subject linguistically associated to negative affect by the terms *math anxienty* (Tobias, 1978) and *mathofobia* (Hodges, 1983). Paulos (1988) observed that incompetence with numbers is socially acceptable and many people have little shame saying “I always hated math”. Also, Brown, Brown, and Bibby (2008) report of students saying that they would rather die than take mathematics. From our own experiences with mathematics education as student, parent, teacher, teacher trainer, researcher and colleague, we have observed that, indeed, many students experience moments of boredom, apathy, stress, alienation or demotivation in mathematics education. This means, in the first place, that there are good reasons for the poor image of mathematics in society and the stable beliefs of students. Second, it means that there is much room for improvement, whether it be in its instruction, curriculum, or assessment methods, all having consequences for students' experiences, their curiosity, their creativity, their self-esteem, their beliefs, and of course not in the least, their knowing and understanding of mathematical concepts and their competencies to use mathematics flexibly for solving non-mathematical problems. Third, it means that even when changing educational practices robustly, this will not immediately or deeply change students' affect towards mathematics at large. The weight of a social perception about mathematics as being hard, meaningless and only for nerds, cannot easily be countered by carefully coordinated reform practices.

The subject of mathematics, being an institutionalized school subject with an elitist tradition of more than two thousand years, cannot easily be changed. However, it is possible to make small steps that break away from canonical mathematics education and design mathematical tasks, that many students experience as pleasant and meaningful challenges, even if these are part of institutionalized assessment. An important presumption for this is that mathematics does not necessarily need to be the context for mathematical activities. There are many non-mathematical areas, in which one needs mathematics to solve problems. One such area is physics, where the use of mathematical models is pertinent to describe phenomena. The study described in this chapter has an inter-disciplinary setting at the cross-road of physics and mathematics, whereby kinematics (the physics of movement) is the context for an assessment task, which requires a significant amount of mathematics. The task asks students to make a translation from the real world in which objects move into the mathematical world of graphs and formula. This translation is known as mathematization (Blum & Leiss, 2005; Niss 2010). The task additionally asks students to reflect on the mathematization (e.g. precision, relation to laws of gravity). The task does not start from a real-world problem that needs to be solved, and students don't make all mathematical modeling steps from the modeling cycle (Blum & Leiss, 2005; Niss 2010). Nevertheless, we perceive the task as a modeling task, as mathematization is an essential activity within mathematical modeling.

Another presumption is, that there exist a variety of task formats that challenge not only the typically gifted students, but also the more average student. Sullivan et al. (2011) and Sullivan and Mornane (2014) describe challenging tasks as requiring students to:

- plan their approach, especially sequencing more than one step;
- process multiple pieces of information, with an expectation that they make connections between

those pieces, and see concepts in new ways;

- choose their own strategies, goals, and level of accessing the task;
- spend time on the task and record their thinking;
- explain their strategies and justify their thinking to the teacher and other students.

The study described in this chapter centers on a task format that fits this description. It is called *project:* a task which cannot be completed within a limited time frame, which has a clear, but not straight-forward goal, and there is variety in the approaches to tackle it (Blomhøj & Kjeldsen, 2006; Kaiser, Schwarz, & Buchholtz, 2011). However, unlike in other studies, in our case the task was an assessment task with a formal evaluation (pass or fail).

Within the context of this situated project task requiring mathematizing, we studied students' affect. To avoid the research being contaminated by students' preconceptions of mathematics, we undertook this study *without the use of the word mathematics* in the instruments that measured students' affect. To use the term mathematics could interfere with students' earlier experiences of canonical mathematics education and their biases about mathematics as an institutionalized subject could interfere with their evaluation of their engagement with the task. The students in our study were from the engineering department; according to Harris et al. (2015) many of these students are disappointed by the mathematical demands in the first year of their studies and some would not have chosen the engineering direction if they had known about these demands. Not many of them have a positive stance towards mathematics, seeing it as a hurdle to get further in their studies. Only later on in their studies, they start to perceive the usefulness of mathematics for their future professional lives. Thus, we expected the participants in our study not to have very positive ideas about mathematics.

1.2 Introduction to the research setting

At the Faculty of Engineering and Science of University of Agder (Norway), we deal with large student numbers (>300). Such numbers are a worldwide phenomenon as more and more students gain access to higher education. A few years ago, the large number of students made the faculty decide to abandon the laboratory training in the first-year Physics courses, because the laboratory facilities and its staff could no longer harbor the students. However, this policy only solved infrastructural problems on the short term. Abandoning lab training could lead to future problems when the graduates from our faculty have become engineers, managers, researchers, and so forth. In their future professional lives, our students will need skills to measure and model phenomena from the real world so they can describe and analyze these, and eventually, make predictions. For their proper training, it is insufficient to offer large-scale lectures, instructional videos or tutoring sessions to train for written examinations. They also need training in relating measurements to theoretical models. They need skills to practically handle instruments, calibrate these, measure as precisely as possible, work with error margins, and so forth. Therefore, we wanted to develop a task, in which lab training was combined with relating measurements to mathematical models, and the preferably outside of laboratory facilities. If such a task would be feasible with large student numbers, then laboratory training can be less dependent of university campuses, and even be feasible in less affluent regions, or within distance education.

A second reason to develop a new task was to improve students' motivation. The students in our engineering courses are no different from those described in Harris et al. (2015), who found that many first-year engineering students have a negative stance towards mathematics. Thus, we wanted a task, in which mathematics would serve engineering aims, for example by being related to technology and moving objects.

We were inspired by Domínguez et al. (2015), who carried out research at a university in Mexico. They asked their students in an interdisciplinary mathematics/physics course: *a child is throwing a candy to another. Make a mathematical model of this movement.* With such an open-ended, inquiry-based modeling task, students need to consider the what, how, and why themselves. Research in science education has demonstrated the advantages of such inquiry-based tasks over traditional lectures or teacher demonstrations (De Jong, Linn, & Zacharia, 2013; Minner, Levy, & Century, 2010).

We adapted this open-ended, inquire-based, kinematical modeling task from Dominguez et al. (2015) and added the use of smart phones for filming. Many students now have smart phones that contain cameras with the quality to film motion sufficiently precise for video analysis. To use equipment from students' extra-institutional, daily lives gave students more ownership over the task. Additionally, we added the use of free software that can capture the motion from videos. This software is based on pattern recognition through contrasts and is known as *tracker software*. Such democratic availability of digital equipment, both smart phone cameras and free software, opens new possibilities for inquiry-based laboratory training for which expensive laboratories are no longer needed.

1.3 The task

With support from the faculty administration, we gave the students an obligatory, inquiry-based laboratory task, which they had to fulfill to get access to the written examination.

The task asked students to select a movement of an object; they could choose whatever: throwing a ball, jumping their skate board, driving a car. They had to film this movement with their phones. Thereafter, they had to use free tracker software (http://physlets.org/tracker/) on their laptops to transform the movement into measurements, approximate the movement with a mathematical model, and then present their findings on a poster. The poster had to contain an introduction, observations (measurements), a mathematical model of the moving object's trajectory, and a discussion of the accuracy of the model in comparison to the measurements. The task had to be completed within the first two weeks of the course and to be done in groups of two or three. Collaboration was convenient, because one student alone cannot easily create and film a movement simultaneously. The delivered posters were assessed on their quality (pass or fail), whereby students had to obtain a pass to get access to the written Physics examinations at the end of the course. In our communication with the students, we indicated the task by the name "Tracker Task". We limited the word mathematics, and used it only in sentences such as "you must find a mathematical model that describes the position of the object as a function of time".

It was our first time to implement such an open, practical task. Therefore, we did not want to focus on students' learning effects in the first place. We considered it a pilot study with the aim to find out whether such an obligatory assessment was feasible with large numbers, without expensive laboratory equipment, and with students who have little experience with open-ended tasks. We felt that we - as lecturers and researchers - should first take the opportunity to learn and see whether the task activated students in a positive way, to the extent that the obligation and the assessment weren't the instigators, but that the task in itself activated the students. Our research question was: to what extent does an open assessment task about video analysis of motion with smart phones and free tracker software challenge and activate the students?

2 An activity-based conceptualization of affect

If one wants to study affect in mathematics education, there are many conceptualizations and instruments, but most of these address mathematics holistically. Instead, we wanted to study students' affect when they engage in an activity, which is different from standard activities of canonical mathematics education. Thus, we sought an activity-based, and not a subject-based conceptualization.

Looking at mathematics from an activity-based perspective goes back to, among others, Freudenthal (1973). He distinguished between mathematics as (1) a well-organized deductive system, or as (2) a human activity which consists of organizing mathematical patterns when solving problems. The two perspectives are also known as "mathematics as a noun" and "mathematics as a verb". The first perspective relates to mathematics as a holistic academic discipline, with its own symbols, and its procedures for establishing truth (by creating proofs), and so forth. The second perspective relates to mathematical activities, such as solving non-routine problems and using mathematics to solve problems within non-mathematical contexts, for example within kinematics. Within the first perspective, humans and their activities are less visible than within the second.

An activity-based conceptualization of mathematics also aligns with a socio-cultural perspective. One of its advocates, Lerman (2000), describes mathematics as a socio-cultural practice embedded within a community. If embedded within a school institution, mathematics is a practice embedded in a community of a teacher and a group of students. The activities consist, among others, of explanations by the teacher, and work on tasks by the students. This practice differs markedly from mathematics as a practice embedded within a research community, whereby the actors work on problems to which nobody knows an answer. Describing mathematics socio-culturally as a practice embedded within a community entails focusing on the activities undertaken by the actors, which are mediated by language, tools, and so forth. Using an activity-based conceptualization of mathematics enables us to (1) relate affect to activities and not to mathematics as a holistic entity, and (2) distinguish between different kind of activities within different contexts of mathematics education (doing repetitive exercises or an inquiry-based project).

To study students' affect while they engaged in an activity, one can still focus on different aspects, such as their emotions, their perception of the relevance or meaningfulness of a task, their boredom, their apathy, and so forth. We decided to focus on their perception of being challenged and activated by the activities.
According to the Cambridge dictionary a *challenge* is: "(the situation of being faced with) something that needs great mental or physical effort in order to be done successfully and therefore tests a person's ability" (http://dictionary.cambridge.org/dictionary/english/challenge). Thus, in itself, a challenge is not necessarily activating, for example, because it is perceived as 'too challenging'. However, a challenge can be related to activation through the use of the concept of *flow*, which is "a state in which people are so involved in an activity that nothing else seems to matter; the experience is so enjoyable that people will continue to do it even at great cost, for the sheer sake of doing it" (Csíkszentmihályi, 1990, p.4). Flow is typically an activity-based concept. Further on in this paragraph, we will explain how this concept links challenge and activation to affect. However, we first present more background information on the concept of flow.

Flow is a term connected to the Hungarian psychologist Mihaly Csikszentmihalyi, a researcher in the area of *positive psychology,* an area of psychology that studies causes for human's happiness. In an overview of their studies, Csikszentmihalyi and Csikszentmihalyi (1988) describe how they observed artists, rock climbers, gamers and scientific researchers during their challenge, and how these people got fully absorbed in their activities and forgot about time, about basic needs such as eating and resting, or about simple responsibilities such as collecting the kids from kindergarten. However, once the activity came to an end and a product was finished (the paint was dry, so to speak), the observed people completely lost interest in the product. This implied that the process was considered more important than the end product. The state of this process was initially described as an *autotelic* experience, that is: as having a purpose in itself. Later, the autotelic experience was coined as *flow*, and nowadays it is also described as *being in the zone*.

Csikszentmihalyi and his group at the University of Chicago studied many different groups, amongst which also adolescents. They observed that of all the places students hang out, the school is the one place they least wish to be were extended to other target groups (Csikszentmihalyi & Hunter, 2003; Csikszentmihalyi & McCormack, 1986). When they were in school, the classroom was the place they most strongly wished to avoid. They rather were in the cafeteria, the library, or the hallways. Interestingly, in secondary education, these researchers discovered that the people most likely to experience flow were teachers. According to the findings, teachers can experience flow if they have a sense of competency in their own work and a supportive work environment.

Flow has also been studied in mathematics education (a.o. Armstrong, 2008, Drakes, 2012, Liljedahl, 2006). However, the typical situation of most students in canonical mathematics classes is not to experience flow at all.

Important aspects for flow to occur are: clear goals, during the activity one often gets feedback on the progress made, and one has a feeling of being in control. During the experience of flow, a person loses awareness of the self, of the larger environment, and loses awareness of time. Then, the activity is intrinsically rewarding irrespective of the outcome, or as Csikszentmihalyi and Csikszentmihalyi (1988, p. 33) write: "the mountaineer does not climb in order to reach the top of the mountain, but tries to reach the summit in order to climb".

According to Csikszentmihalyi and Csikszentmihalyi (1988), a precondition for experiencing flow is that a person should perceive that he/she is capable of doing it, that is: that one has sufficient skills for the activity, but that this activity is not perceived as easy. At this point, we see that skills bring challenge and flow together: a certain tension between skills and challenge brings dynamics into the activity, which can result in the actor experiencing flow. Flow forces people to stretch themselves in an activity, and improve on their abilities. However, if the skills are becoming better, and the activity does not become more challenging, the person will become bored. On the other hand, if the skills cannot meet the challenge, the person will become discouraged, alienated or, in the worst case, anxious. Thus, in relation to affect, flow is a technical term in the fields of intrinsic motivation and interest, describing "an optimal state of experience" (Csikszentmihalyi & Csikszentmihalyi , 1988, p.3). Flow is an activity-based concept (without activity, there cannot be an experience of flow), and it can only occur if there is a certain tension between challenge and skills.

It remains important to note, that flow is an experience of a person, and that not all activities result in flow because of the above described tensions between skills and challenge. A task designer, thus, needs to consider this tension: if a task is too easy, the more gifted students will be bored, and if the task is too hard, the less talented students will not be able to start. Therefore, in our research we used a task with a low entry level to understanding the overall aims. In fact, being a group task, it was accessible to all students, irrespective of their initial skills. However, being accessible did not imply it was an easy task. As the task was open-ended and allowed for a variety of approaches, it invited the more gifted students to challenge themselves. This task characteristic is also known as *low floor - high ceiling.* Because of this type of task, we ignored the aspect of skills, and operationalized students' affect in terms of students' experience of challenge and flow.

3 Methods

In the Spring of 2017 we presented the task described in Paragraph 1.3 to the students of the engineering department (Mechatronics, Electrical Engineering, Renewable Energy, Data Engineering, ICT, and others), as part of the first-year physics course. There were 346 students for whom the task was obligatory.
The research design for studying students' challenge and flow was a survey, whereby data were collected through a digital questionnaire (described below) within the university's Virtual Learning System. Participation in the survey was voluntary and encouraged with prizes of NOK 500 (approx $60) for three randomly drawn participants. We removed irregular participation (e.g. participants who chose constantly a 3 as answer; second-year students for whom the task wasn't obligatory) and remained with n=239 students. The response rate of 69% can be considered high for a web-based survey (Bryman, 2015).

We developed the instrument, because a literature review did not yield any existing instrument that matched with the aims of our study. Reasons for discarding them were: too long questionnaires, or unsuitability to our Tracker Task. Therefore, we adapted items from instruments from earlier research and developed these in alignment with our needs. The items asked for (dis-)agreement to statements on a 5-point Likert scale. Ten items were designed to measure students' perception of Challenge and Flow, see Table 1. It should be noted that the word mathematics does not appear in the instrument.
While developing the items, we asked a few colleagues to review the items, and we organized a small pilot with a few second-year students. The questionnaire contained six further questions about students' collaboration and the ease to use the equipment or to find the mathematical formula. Those items were

included to inform us about practical and technical issues, of which the results are irrelevant to the study presented here.

Table 1: The ten Flow and Challenge items in the questionnaire, with Cronbach's Alpha = 0.795

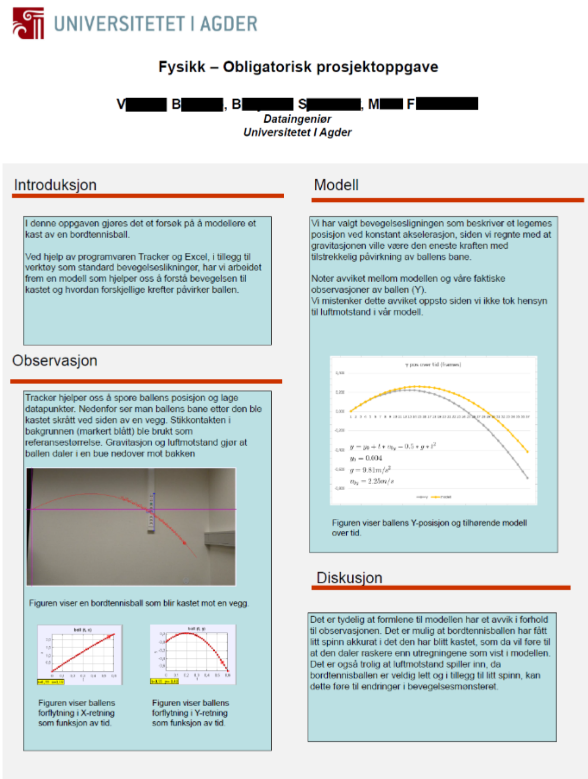
	Statements	Cronbach's Alpha if item deleted
q1	The "Modeling with Tracker Task" made me curious.	0.765
q2	(Inv.) This Tracker Task took too much of my time.	0.801
q6	Making a poster made me feel like a 'real scientist'.	0.785
q7	Time was flying when we worked in this task.	0.768
q8	(Inv.) This task is more suitable for Secondary Schools.	0.775
q9	This task helped me to better understand the theory.	0.768
q11	(Inv.) I was easily distracted when we worked on this task.	0.771
q13	During this task I started thinking about other movements (what if…?)	0.794
q14	I would do this task even if it wasn't obligatory.	0.778
q16	I would like to have more of such practical tasks.	0.771

In our research, we make a difference between challenge and flow as concepts (described in the previous paragraph), and the measurement scale of Challenge and Flow (with capital letters). The concepts of challenge and flow are subjective experiences of a person, and therefore these cannot be measured. However, we assume that they can be approximated by a score, which results from answering to the ten Challenge and Flow items from our questionnaire. A participant's score on the Challenge and Flow items then is indicator of the extent to which he/she had positively experienced being challenged and/or activated.

The score on the scale is calculated by adding the scores on the ten questions. As the score on one item ranges from 1-5, the score on the Challenge and Flow scale ranges from 10-50. To increase reliability, three items were inversely posed, and the scoring on these items was inverted, too. As measure of consistency between items (internal reliability), we calculated Cronbach's Alpha for the ten items. If lower than 0.6, the consistency of a group of items is considered poor and unacceptable (Bryman, 2015). It turned out that the ten items of the Challenge and Flow scale together had a good reliability, with Cronbach Alpha being 0.8. Additionally, we tested whether the consistency would improve if one of the ten items were deleted (it would mean that the item is inconsistent with the others). This analysis showed that nine items contributed positively to the scale and deleting them would lower the consistency. Only one item ("(Inv.) This Tracker Task took too much of my time") did not show this, but deleting it would not significantly increase the consistency either, see Table 1.

4 Results

As lecturers, we observed informally how enthusiastic students were everywhere on campus, throwing apples or golf balls, and even a cat was thrown (and fell on its feet). Students analyzed the flight of their skateboard, the turning of cars in the parking garages and the fall of a small parachute. Also, in the working groups where students come to practice examination tasks, we heard them discuss lively about the Tracker Task. We had offered office hours in case the students wanted clarification on the task, but not one student appeared. After two weeks we received more than 100 posters in our Virtual Learning System, of which we show two samples in Figure 1 to give the reader an impression of students' products. The format of the poster asked students to write an Introduction, give their Observation, give a Model, and write a Discussion. The poster on the left in Figure was made by students who threw a table tennis ball and mathematized its trajectory with a quadratic equation. On the right, students filmed the bouncing of a rubber ball and used MatLab to mathematize a sequence of parabolas, of which the height and width reduces with each bounce.



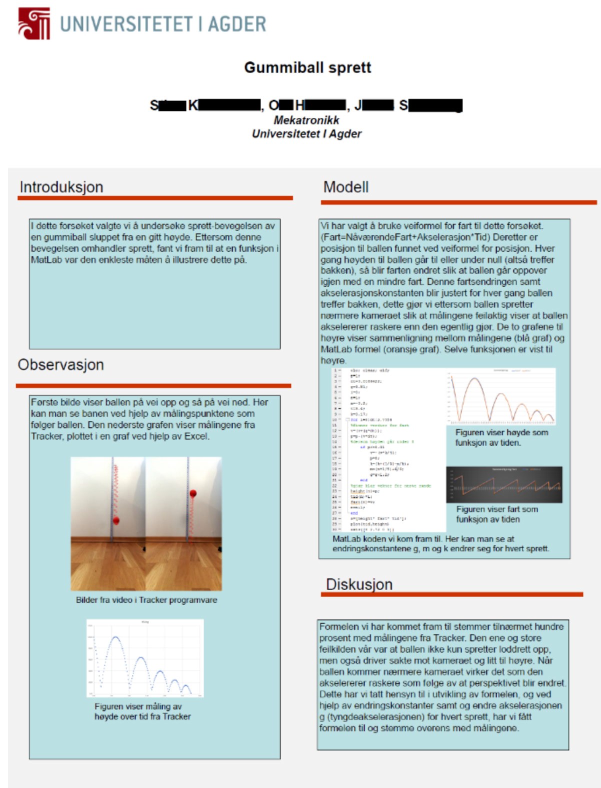


Figure 1. Two examples of products by the students

In this chapter we don't analyze the cognitive performance of the students in this assessment, such as their understanding of modeling, the depth of their analysis, the discussion of their measurements in relation to kinematical laws of gravitation, and so forth. Instead, we focus on students' affect, which we operationalized in terms of challenge and flow and which we measured through the questionnaire. Table 2 shows the mean scores on each item (1=low, 3= middle, 5=high).

Table 2: Mean scores on Challenge and Flow questions (n=239)

	Statements	Mean (sd.)
q1	The “Modeling with Tracker Task” made me curious.	3.61 (0.75)
q2	(Inv.) This Tracker Task took too much of my time.	3.67 (0.88)
q6	Making a poster made me feel like a ‘real scientist’.	3.96 (1.03)
q7	Time was flying when we worked in this task.	3.40 (0.92)
q8	(Inv.) This task is more suitable for Secondary Schools.	2.58 (0.95)
q9	This task helped me to better understand the theory.	3.39 (0.88)
q11	(Inv.) I was easily distracted when we worked on this task.	3.55 (0.91)
q13	During this task I started thinking about other movements (what if…?)	3.31 (1.12)
q14	I would do this task even if it wasn’t obligatory.	2.60 (1.13)
q16	I would like to have more of such practical tasks.	3.70 (1.02)

The mean score on five of the ten items is higher than 3.5, being well on the positive affect side. This indicates that a majority of the students experienced challenge and flow to quite an extent. They largely agreed that the task made them curious (item 1), did not take too much of their time (item 2), or that they would like to have more of these tasks (item 16). The remaining five items obtained a score in the middle range (between 2.5 and 3.5). Not one item was answered below 2.5.

When adding the students’ scores on the ten items, we obtain their total score on the scale Challenge and Flow. On this scale, the minimal score is 10 (not attained by any student), the middle score is 30 and the maximal score is 50 (not attained by any student). Figure 2 shows a histogram of the frequencies of the Challenge and Flow scores, with the bar for the middle score 30 in red.

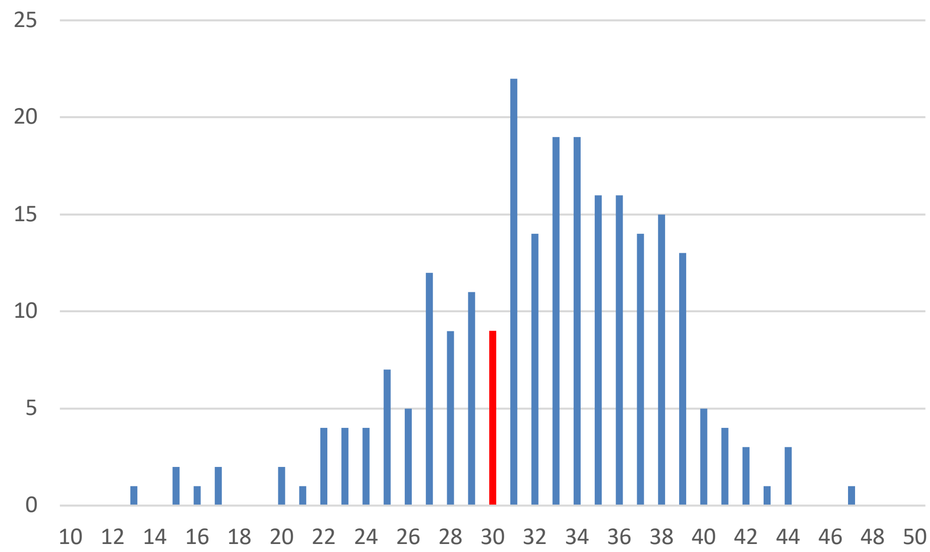


Figure 2. Frequencies of Challenge and Flow scores (middle score 30 in red).

The histogram shows that the scores on the Challenge and Flow survey are skewed to the right, which means that students' scores are, on average, on the high side of the scale. Out of 239 students, 54 students (22.6%) scored 28 points or lower, 42 students (17.6%) scored in the middle range of 29-31 points, and 143 students (59.8%) scored 32 points or higher. If we take 32 as a threshold score, it would mean that approximately three out of five students experienced a certain positive challenge and flow. Of course, the threshold is arbitrary; if we rather take 33 as a threshold score, then 129 students (54.0%) experienced challenge and flow.

5 Discussion and Conclusion

Our research question was: to what extent does an open assessment task about video analysis of motion with smart phones and free tracker software challenge and activate the students? Based on the results from the survey, we find that a clear majority of the students (59%) experienced challenge and flow. They indicate that they forgot about time and wanted more of such activities. This result is anecdotally supported by our observations of buzzing students on campus, their discussions during workgroup sessions, and the high response rate to the survey. We searched the literature, but didn't find earlier research in assessment of mathematics in which students expressed to want more of such assessment tasks.

The data from the survey do not allow us to compare students' appreciation of the Tracker Task to experiences of challenge and flow on other activities in the course (attending lectures, working on textbook problems). However, our first year engineering students aren't in any way different from those described in Harris et al. (2015), avoiding additional mathematical tasks and seeking to minimize mathematical activities. In light of that, the high score on item q16 ("I would like to have more of such practical tasks") can be interpreted as a comparison, whereby students express to favor the Tracker Task over other mathematics tasks.

The results can be ascribed to a number of components. In the first place, we asked the student about an activity for which they could use their own smart phones. Thus, they had the laboratory equipment in their pockets. Being students from the engineering department, it could be expected that they liked using technological devices. These gave them ownership over the activity, and it made the activity different from prior experiences in mathematics education. Also, we were careful not to connect the activity to mathematics. In the questionnaire the word mathematics was not used once, and in the task the word mathematics was only used once when asking the students to create a mathematical formula. We did this, so the students would evaluate the activity in itself and not connect it to earlier, often negative experiences in mathematics classes. Harris et al. (2015) have clearly established that students in engineering, like the ones in our study, generally perceive mathematics not as their favorite.

In the second place, the results can be explained in light of the task characteristics. Although the students were assessed on their product (the poster), the task was open, the students had ample time to submit the product, and the task was accessible to all students (low floor), yet enabling the better students to challenge themselves further (high ceiling). Such task characteristics may be better feasible within mathematical modeling education than canonical mathematics education. Also, the assessment being for groups may have added to students' positive affect: unlike the large lectures that the students attended, the small groups offered them companions, informality and possibly even safety.

We would like to highlight that we used an activity-based conceptualization for both mathematics and affect. Thus, we did not study affect in relation to mathematics holistically, but in relation to a certain mathematical activity. Central in the activity was the mathematization of the trajectory of a moving object, that is: the creation of a mathematical formula describing position of a moving object as a function of time, and additionally discussing to what extent that formula deviated from the actual measurements. We anticipated that students had not often been given such a task, in particular not in assessment. This

newness enabled us to detach the task from mathematics education at large, which is dominated by explaining teachers and students practicing exercises (Nolan, 2012).
Also, we did not study affect holistically, but studied affect through an activity-based perspective. We studied whether the students were challenged and activated by a task, to the extent that they possibly got fully absorbed into the activity. In that case, the task was motivating in itself. For an activity-based conceptualization of challenge, we were able to build on Sullivan et al. (2011) and Sullivan and Mornane (2014). For an activity-based conceptualization of activation, we used the concept of flow, which is a state of happiness caused by an activity that is sufficiently challenging in relation to someone's skills. In our research we contend that flow is an important aspect of affect, and recommend more research into students' experiences of flow in mathematics classrooms.

The concept of flow brings a new perspective on affect, not only being activity-based, but also carrying the possibility that flow can be experienced by students in mathematics classrooms, and even in assessment. How often do students experience flow in canonical mathematics classrooms, if at all? With the existence of *math anxienty* (Tobias, 1978) and *mathofobia* (Hodges, 1983), we contend that in many mathematics classrooms many students will hardly ever experience flow. However, our study demonstrates that mathematical activities can result in flow among a large group of students, even if these students are first year engineering students who, as reported by Harris et al (2015), are not extremely good in mathematics.

Of course, we do not know how the students' responses would have been, if we had repeatedly included the word mathematics into the task or into the questionnaire. However, in general with modeling tasks, or in particular with a task to mathematize motion in the real-world, such tasks don't look like the tasks from canonical mathematics education. Nevertheless, such tasks make students engage in mathematical activities. Thus, it remains a question whether affect research in mathematics education is tainted by the term mathematics, the looks of repetitive tasks in canonical mathematics education, and the lack of challenging tasks that intrinsically motivate students so they experience flow.

In our study, we made first-year engineering students engage in mathematical activities, that made many of them experience challenge and flow. This was expressed by the high Challenge and Flow score, or expressed by the 3.7 score on item q16 ("I would like to have more of such practical tasks"). Therefore, we recommend mathematics education to include more open, easily accessible and inquiry-based tasks, in particular modeling tasks, and also we recommend more affect research into students' experiences of challenge and flow, and into the lack thereof.

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