




Article

Next-Day Bitcoin Price Forecast

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Abstract: This study analyzes forecasts of Bitcoin price using the autoregressive integrated moving average (ARIMA) and neural network autoregression (NNAR) models. Employing the static forecast approach, we forecast next-day Bitcoin price both with and without re-estimation of the forecast model for each step. For cross-validation of forecast results, we consider two different training and test samples. In the first training-sample, NNAR performs better than ARIMA, while ARIMA outperforms NNAR in the second training-sample. Additionally, ARIMA with model re-estimation at each step outperforms NNAR in the two test-sample forecast periods. The Diebold Mariano test confirms the superiority of forecast results of ARIMA model over NNAR in the test-sample periods. Forecast performance of ARIMA models with and without re-estimation are identical for the estimated test-sample periods. Despite the sophistication of NNAR, this paper demonstrates ARIMA enduring power of volatile Bitcoin price prediction.

Keywords: ARIMA; artificial neural network; Bitcoin; cryptocurrency; static forecast

1. Introduction

Bitcoin, the world's first decentralised and currently biggest digital currency, is similar to synthetic commodity money, which shares the attributes of both commodity (e.g., gold) and fiat money (e.g., US dollar) (Selgin 2015). Bitcoin was introduced in 2008 by a group of programmers using the pseudonym 'Satoshi Nakamoto' (Cheah and Fry 2015). Some argue that it has the same finite economic attributes of gold and labelled as digital gold (Popper 2015). Meanwhile, the acceptance of the Bitcoin is still debatable due to its frictionless nature, lack of intrinsic value, and unclear issuing authority (Ciaian et al. 2016). The price volatility of Bitcoin makes it one of the most speculative digital currency, and a poor form of "holder of value". Investors can lose their capital due to high volatility and uncertainty of Bitcoin price.

Media coverage about Bitcoin drew in amateur investors leading to a gambling mentality (Roberts 2017). However, Bitcoin is coming into the mainstream with large institutional investors eyeing its potential. Despite its limitations, Bitcoin is the most valuable and popular cryptocurrency to date (Corbet et al. 2019).

Bitcoin price has been extremely volatile since the inception of the cryptocurrency (Dwyer 2015). Due to concerns with speculative trading, in January 2018, Facebook banned all ads for Bitcoin and other cryptocurrencies (Robertson 2018). Additionally, experts foresee another financial crisis in the near future caused by the cryptocurrency boom (Lam et al. 2018). A major crash of the Bitcoin price can be triggered by a cyber hack and a government crackdown, and can take weeks or months to bounce back (Roberts 2017). Typically, investors predict future Bitcoin price based on past trends. But it is not easy to predict future Bitcoin price with a high level of accuracy. The price of Bitcoin follows a

boom-bust pattern due to its speculative nature (Cheah and Fry 2015). Additionally, Bitcoin investors are speculative and short term oriented (Salisu et al. 2019).

Due to the progressive price change and increase in the market cap, the popularity of investment in Bitcoin has been increasing dramatically. Meanwhile, Urquhart (2016) found that Bitcoin is an inefficient market. Caporale et al. (2018) also observe inefficiency in the cryptocurrency market. In the same vein, due to high price volatility, speculators have a generic question of whether the price of Bitcoin can be forecasted in advance.

Thus far, there have been limited attempts in the literature to forecast the price of Bitcoin. Katsiampa (2017) scrutinised the in-sample goodness-of-fit of GARCH models for Bitcoin price but did not perform out-sample forecast. Kristjanpoller and Minutolo (2018) proposed a hybrid-forecasting model to predict Bitcoin price volatility, integrating artificial neural network (ANN), generalised autoregressive conditional heteroscedasticity (GARCH) and principal components analysis (PCA). They found that the accuracy of the hybrid model increases after incorporation of PCA pre-processing. Similar to Katsiampa (2017), Kristjanpoller and Minutolo (2018) also only examined in-sample forecast performance. Meanwhile, Aalborg et al. (2018) found that Bitcoin returns cannot be predicted ($R^2 \leq 0.01$) using explanatory variables (i.e., google trend, trading volume, transaction volume, VIX index, and unique user addresses).

Given that Bitcoin is becoming more popular, yet still volatile and not well explained, a need exists to study methods to better understand its price fluctuations. Accurately forecasting the daily movements can increase the returns of day traders and consequently make the market more efficient. The choice of forecasting models can have a significant effect on performance (Chen et al. 2019). We contribute to the Bitcoin forecasting literature by testing autoregressive integrated moving average (ARIMA) and neural network autoregression (NNAR). ARIMA is one of the traditional forecasting methods, and NNAR is a rather sophisticated and more modern approach to forecasting (Hyndman and Athanasopoulos 2018). To ensure validation and implementation, as suggested by Adya and Collopy (1998), we use ex-ante Bitcoin forecast performance using a relatively large sample, and multiple training and testing samples to demonstrate the stability of the forecast results. By following this procedure, we differentiate the results of this study with the ones mentioned earlier.

In the next section, we discuss existing literature on Bitcoin price modelling. Section 3 presents daily Bitcoin price data used in this study. Section 4 presents the adopted forecast methodologies and performance measures. The analysis and findings are presented in Section 5, and final remarks are made in Section 6.

2. Literature Review

Bitcoin is the most popular among the cryptocurrencies (Kyriazis 2019). Recent fluctuations in Bitcoin price has captured the attention of academic researchers (Beneki et al. 2019). Given the nascency of this research stream, previous studies on Bitcoin and other digital currencies (for instance, Ethereum, Litecoin, Ripple) mainly explain the concepts, principles and economics of cryptocurrencies (Segendorf 2014; Dwyer 2015; Becker et al. 2011). Among the authors, Dwyer (2015) addressed the principles of Bitcoin and other relevant digital currencies. The author explains the supply and demand of digital currencies, equilibria of Bitcoin, uses of Bitcoin in exchange for goods and services with a rivalry to other currencies (Dwyer 2015). Likewise, Brière et al. (2015) investigated the connection of Bitcoin with other cryptocurrencies.

The total market cap of Bitcoin is approximately USD237 billion (as of 30 March 2018), which is nearly 42.69% of the entire cryptocurrency market capitalizations (coinmarketcap.com). As such, some studies consider the price dynamics of Bitcoin (Brandvold et al. 2015; Ciaian et al. 2016). Brandvold et al. (2015) investigated the price discovery of Bitcoin exchanges and find that two exchanges—Mt.Gox and BTC-e, are leading the market with the maximum information share. Besides, Ciaian et al. (2016) studied the underlying economics of Bitcoin price by taking into account the traditional determinants of the currency price. Moreover, Shubik (2014) and Rogojuanu and Badea (2014)

studied Bitcoin in the setting of alternative monetary systems by considering the challenges of the economic environment. Meanwhile, Bouoiyour and Selmi (2014); Bouoiyour et al. (2014) and Yermack (2013) described Bitcoin as a speculative investment or speculative bubble. Similarly, according to Yermack (2013), Bitcoin behaves more like a speculative investment rather than currency. It fails to satisfy the features of currency as a medium of exchange, a store of value, and a unit of account. In the same vein, Molnár et al. (2015) studied the exchange rate risk of Bitcoin by comparing with other variables, for instance, gold and Euro and find that Bitcoin is more volatile and riskier than gold and Euro, which restrict the applicability of Bitcoin as a medium of transaction. Furthermore, Bouri et al. (2017) investigated the Bitcoin price and its volatility and found persistence in the Bitcoin price and volatility.

As Bitcoin price volatility is exceptionally high, speculators have a general quest whether future Bitcoin price can be forecasted. Bitcoin price or return forecasting is getting more attention due to its boom-bust nature. Speculators are looking for tools and techniques that can forecast Bitcoin price with higher accuracy, at least better than the naïve forecast to set their investment portfolios in a profit margin. The majority of the studies on Bitcoin either focus on price returns and volatility or consider Bitcoin as a speculative investment or bubble (Bouoiyour and Selmi 2014; Bouoiyour et al. 2014; Yermack 2013). Some studies consider risk, hedge and safe haven attributes of Bitcoin and Ethereum (Beneki et al. 2019, Bouri et al. 2017). However, to the best of the authors' knowledge, there are no studies on the forecasting of test-sample (out-sample) Bitcoin price (Corbet et al. 2019). Thus, this study presents a novel approach to forecasting daily Bitcoin price using both with and without model re-estimation at each step while comparing ARIMA and NNAR models.

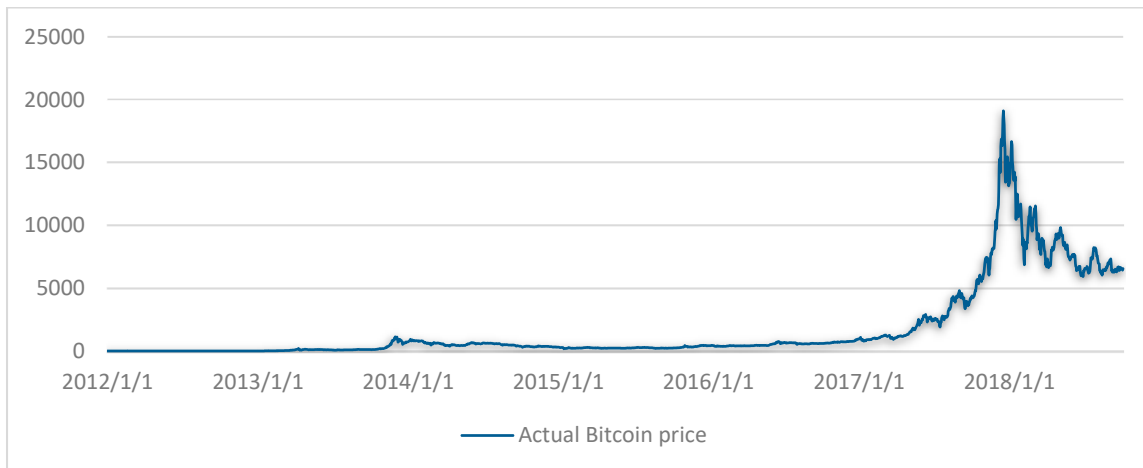
3. Data

Daily Bitcoin exchange rate data (USD per Bitcoin) is collected from the *Quandl*¹ database. Data from the same source has been used by others, too (Chu et al. 2015). We use daily Bitcoin price data from 1 January 2012 to 4 October 2018², daily Bitcoin price of 2466 days. Figure 1 presents the (a) original time series along with (b) log-transformed and (c) first differenced log operator series. For the effectiveness of forecast validation (Adya and Collopy 1998), we divide the dataset into a training-sample (in-sample) and test-sample (out-sample). We consider two training-samples and subsequently two-test samples for cross-validation purposes. The first training sample is from 1 January 2012 until 14 May 2013 (500 days), and the second from 1 January 2012 until 25 June 2017 (2000 days). As a consequence, the first test-sample is from 15 May 2013 to 4 October 2018 (1966 days), and the second from 16 June 2017 to 4 October 2018 (466 days).

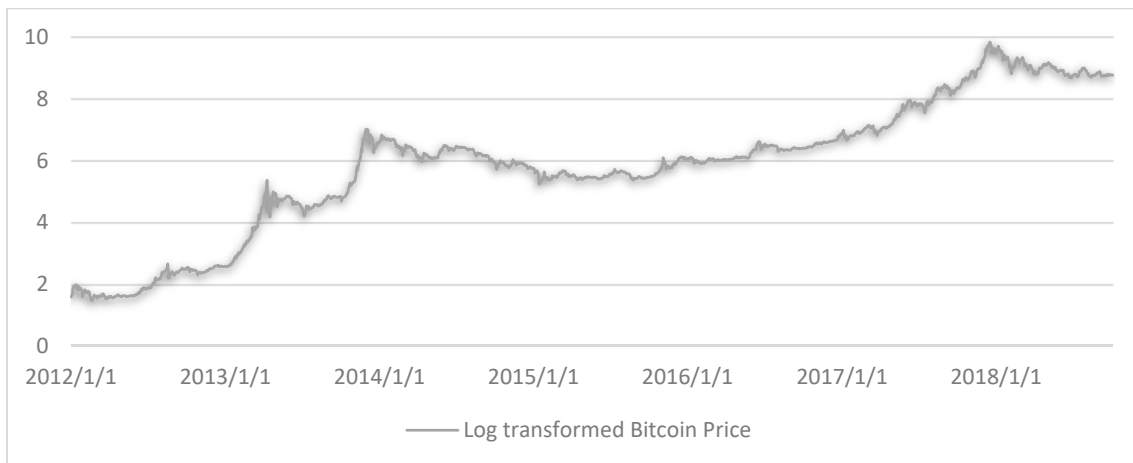
At the end of 2014, the price of Bitcoin dropped significantly to USD 302 (www.coindesk.com). The cause of the price decline was the suspension of trading of Bitcoin by Mt. Gox, one of the leading Bitcoin exchanges, which handled 70% of the Bitcoin exchange worldwide at that time. They reported that around 850,000 Bitcoins were hacked, which belongs to customers and are worth around USD 3.5 billion (Roberts 2017). The incident resulted in a lack of confidence in the security system of Bitcoin; thus, the price decline continued until 2016. At the beginning of 2017, the Bitcoin price increased dramatically, and at the end of 2017 the price of Bitcoin surged at USD 19,661.63, but again after five days from 17 December 2017 it dropped to USD 12,616.64 (www.coindesk.com).

¹ www.quandl.com/data/BCHARTS/BITSTAMPUSD-Bitcoin-Markets-bitstampUSD.

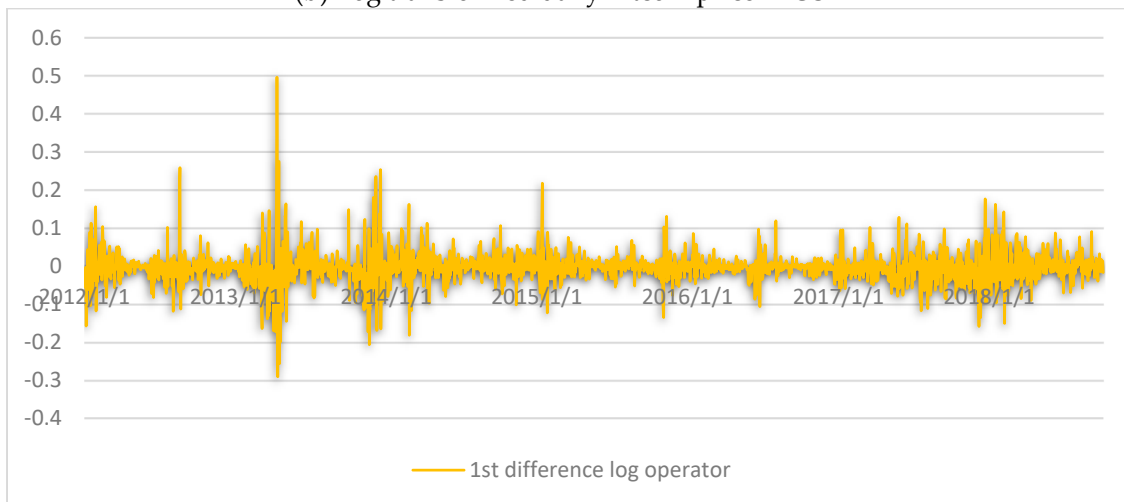
² Bitcoin price data for three days, that is, 6–8 January 2015 was not available.



(a) Original daily time series of Bitcoin price in USD



(b) Log transformed daily Bitcoin price in USD



(c) First difference log operator of daily Bitcoin price in USD

Figure 1. (a) Original, (b) log-transformed and (c) first difference log operator bitcoin price in USD.

Stationarity of data is a prerequisite for predictive modelling, particularly when using autoregressive time series models such as ARIMA. Table 1 shows results of the stationary test of the training-data samples using the Augmented Dicky-Fuller test (ADF) (Dickey and Fuller 1979) and Phillips-Perron test (PP) (Phillips and Perron 1988). Data, both in levels and log-transformed series, are not stationary but become stationary at first difference log operator; thus, the ARIMA modelling

approach is feasible. It might be noted that the stationarity of data is not essential for neural network models (Hyndman and Athanasopoulos 2018).

Table 1. Stationary test of data.

Data	Training Sample	ADF Test	PP Test
<i>First in-sample window (500 days)</i>			
Original data	01/01/2012~14/05/2013	-1.725 (0.695)	-10.597 (0.519)
Log transformed data	01/01/2012~14/05/2013	-1.478 (0.800)	-3.333 (0.919)
1st difference log operator	01/01/2012~14/05/2013	-9.593 (0.01)	-338.72 (0.01)
<i>Second in-sample window (2000 days)</i>			
Original data	01/01/2012~25/06/2017	1.021 (0.99)	6.109 (0.99)
Log transformed data	01/01/2012~25/06/2017	-1.327 (0.86)	-3.321 (0.92)
1st difference log operator	01/01/2012~25/06/2017	-11.19 (0.01)	-1513.50 (0.01)

ADF. Augmented Dicky-Fuller test; PP. Phillips-Perron test. *p*-values in parenthesis, *p*-value less than 0.05 confirms stationarity.

4. Methodology

4.1. Forecast Methods

Association of Bitcoin prices with other micro and macro-economic indicators, such as oil price and gold price, are still not clear (Aalborg et al. 2018). Thus, the univariate modelling approach, where data speaks for itself (Gujarati and Porter 2003), becomes an appropriate forecasting tool. Additionally, a positive association between past and future values of Bitcoin price is evident in the literature (Caporale et al. 2018). However, the degree of association varies over time (Caporale et al. 2018); thus, re-estimating the forecast model every time for the one-step forecast with each additional daily Bitcoin price becomes relevant. Additionally, this signifies the essence for investigating non-linear approach. Thus, we employ two univariate time series models—ARIMA and NNAR. Application of ARIMA can be found in many fields of studies such as in finance (Ariyo et al. 2014), shipping (Munim and Schramm 2017), logistics (Miller 2018), and electric power (Contreras et al. 2003). Meanwhile, NNAR models are also used to forecast global solar radiation (Benmouiza and Chekane 2013), river flow (Abrahart and See 2000), tourism demand (Álvarez-Díaz et al. 2018). For both ARIMA and NNAR models, we scrutinize forecasting next-day Bitcoin price with and without re-estimating the forecast model for each step. For the computational purpose, we used the Forecast package (Hyndman and Khandakar 2007) in the R software.

4.1.1. ARIMA

ARIMA is probably the most popular method when it comes to time series forecasting, initially developed by Box and Jenkins (1976). Typically, an ARIMA model has two components: an autoregressive (AR) component and a moving average (MA) component. The AR component models association between the value of a variable at a specified time with its value in previous time(s), and the MA component models association between values of error term of a variable at a specified time with its error term value in previous time(s). The integrated (I) component comes into consideration when the time series becomes stationary after the first (or second) difference. An ARIMA (*p,d,q*) model can be represented by Equation (1).

$$\Delta z_t = \sum_{i=1}^p \varphi_i \Delta z_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \varepsilon_t \tag{1}$$

Here, $\Delta z_t = z_t - z_{t-1}$; z_t is the Bitcoin price in USD at time t , z_{t-i} is the Bitcoin price in USD of all previous periods until lag p , φ_i is the parameter for z_{t-i} , ε_t is the error term in time t , ε_{t-i} is the error term of all previous periods until lag q and θ_i is the parameter for ε_{t-i} .

4.1.2. Neural Network Autoregression (NNAR)

Artificial neural network (ANN) methods rely on mathematical models in a similar pattern as ‘neurons’ in the brain. ANN models help design complex non-linear associations between the dependent variable and its predictors (Adya and Collopy 1998; Hyndman and Athanasopoulos 2018). The simplest ANN models would only have predictors (independent variables or inputs) in the bottom layer and the dependent variable (output) in the top layer, which would be equivalent to a linear regression model. After adding the hidden layer(s) in-between bottom and top layers, the ANN structure becomes non-linear. A sample ANN model is depicted in Figure 2. This type of ANN is called multi-layered feed-forward network, where each layer of neurons (nodes) receive inputs from the previous layer. The inputs to each node are estimated using a weighted linear combination, as in Equation (2):

$$z_j = \beta_j + \sum_{i=1}^n W_{i,j} X_i \tag{2}$$

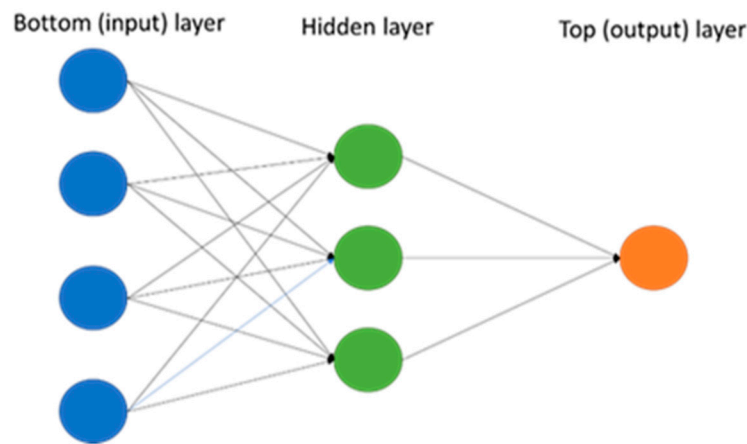


Figure 2. An ANN model with four inputs and one hidden layer with three neurons.

Here, z_j is the value of output node j , β_j is the constant for node j , $W_{i,j}$ is the weight from the input node i to output node j , X_i represents the inputs, and n is number of input variables. In the hidden layer, Equation (2) is transformed into non-linear function using sigmoid, as shown in Equation (3).

$$s(z) = \frac{1}{1 + e^{-z}} \tag{3}$$

The parameters $\beta_1, \beta_2, \beta_3, \dots, \beta_n$ and $W_{1,1}, \dots, W_{4,3}$ are “learned” from the training data. To prevent the weights from becoming too large, usually, the values of the weights are restricted. Decay parameter—the parameter that restricts the weights is typically set to be equal to 0.1 (Hyndman and Athanasopoulos 2018). With time series data such as daily Bitcoin price, lagged values of the time series can be used as inputs in an ANN structure, which is known as neural network autoregression (NNAR). A non-seasonal feed-forward network model with one hidden layer is usually denoted as NNAR (p,k), where p represents the number of lags and k represents the number of nodes in the hidden layer.

4.2. Forecast Accuracy Measures

Forecasting models are evaluated based on their accuracy of the forecast. Typical forecast accuracy measures such as RMSE (root mean square error) and MAPE (mean absolute percent error) are criticised for their instability with varying number of test-sample forecast periods. Thus, we adopt three indices to measure the accuracy of forecast results: RMSE, MAPE, and MASE (mean absolute scaled error). MASE was proposed by Hyndman and Koehler (2006) as a remedy to overcome the drawbacks of

RMSE and MAPE when dealing with a varying number of test-sample periods. The three adopted accuracy measures can be expressed as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (d_t - z_t)^2} \tag{4}$$

$$MAPE = \frac{100}{n} \sum_{t=1}^n \left| \frac{(d_t - z_t)}{d_t} \right| \tag{5}$$

$$MASE = \text{mean} \left| \frac{e_t}{\frac{1}{n-1} \sum_{t=2}^n |z_t - z_{t-1}|} \right| \tag{6}$$

Here, e_t is the forecast error calculated as $(d_t - z_t)$, d_t is the actual Bitcoin price at time t , z_t is the forecasted price at time t , n is the total number of observations and $z_t - z_{t-1}$ is the forecast error of the naïve forecast.

5. Empirical Results

First, the appropriate ARIMA and NNAR models are to be selected to forecast next-day Bitcoin price for the test-sample. ARIMA models are chosen based on the lowest AIC, while considering the PP test for stationarity using the `auto.arima` function provided by the Forecast package in R. However, it is challenging to select the appropriate NNAR model. For the first training-sample period (500 days), 14 different NNAR(p,k) specifications are estimated and evaluated for the forecast (without re-estimation) performance of the first test-sample period (1966 days). The results are presented in Figure A1 in Appendix A. Interestingly, training-sample forecast performance gets better with increasing the numbers of lags and hidden layers (see Figure A1a) but NNAR (2,1) performs best for test-sample forecast (see Figure A1b). Therefore, NNAR (2,1) is selected for the estimation of the first training and test samples. The same 14 models are estimated and compared for the second training and test samples (see Figure A2), and NNAR (1,2) is selected based on test-sample forecast performance. In the employed NNAR framework, it is noteworthy that test-sample forecast performance is always better with a lower number of lags and nodes in contrast to the training-sample forecast performance.

For next-day Bitcoin price forecast without re-estimation of the model for next step, the two selected models for first training and test samples are ARIMA (4,1,0) and NNAR (2,1), and for the second training and test samples are ARIMA (4,1,1) and NNAR (1,2). We adopt the static forecast approach, as depicted in Figure 3. When using an autoregressive model in the static forecast approach, the actual value of the dependent variable in previous periods is used to estimate each step forecast for the training sample. On the contrary, when forecasting multiple periods, dynamic forecast approach uses the previously forecasted value (out-sample period) of the dependent variable to compute a forecast. In Table 2, first, we present the training-sample forecast performance of ARIMA and NNAR models by means of RMSE, MAPE and MASE. Then, in Table 3, we present the test-sample forecast performance of the employed models. According to Table 2, NNAR models perform better than ARIMA in the first training-sample period, but ARIMA is better in the second training-sample. According to Table 3, for both cases, without and with re-estimation of forecast models for next-day Bitcoin price forecasting, ARIMA models outperform NNAR in the test-sample forecast. Log-transformed Bitcoin price series and its forecasted values using ARIMA and NNAR under different estimation approaches are presented in Figure 4.

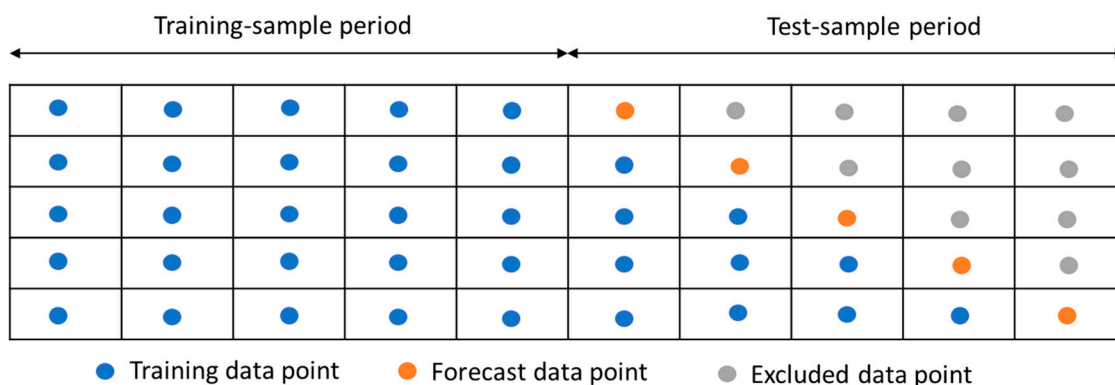


Figure 3. Illustration of static forecast approach.

Table 2. Training-sample forecast performance.

Forecast Model	Training Sample	RMSE	MAPE	MASE
<i>First training-sample window (500 days)</i>				
ARIMA (4,1,0)	01/01/2012~14/05/2013	0.053	1.225	0.987
NNAR (2,1)	01/01/2012~14/05/2013	0.055	1.172	0.963
<i>Second training-sample window (2000 days)</i>				
ARIMA (4,1,1)	01/01/2012~25/06/2017	0.040	0.565	0.970
NNAR (1,2)	01/01/2012~25/06/2017	0.042	0.567	0.983

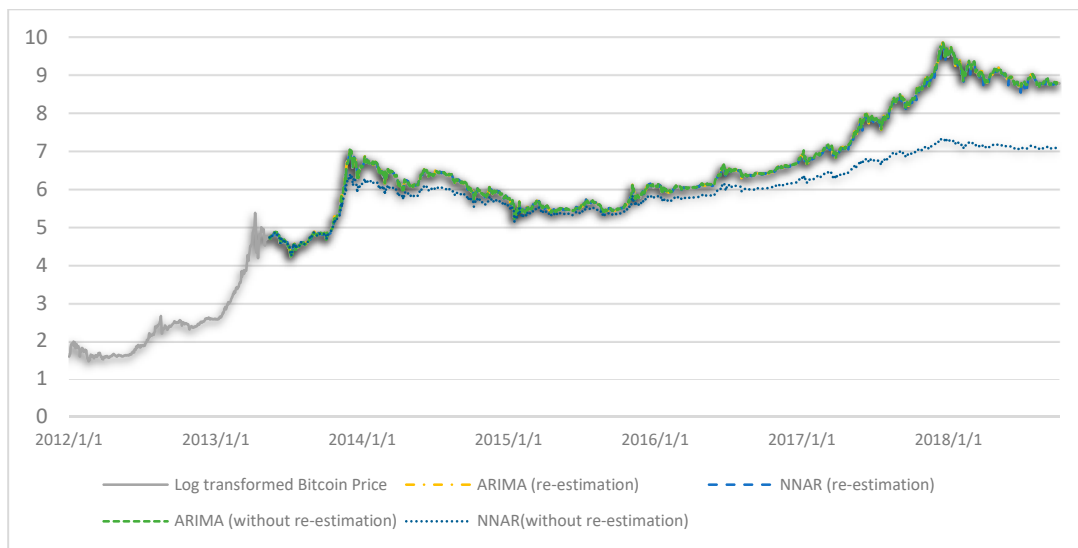
Bold numbers indicate best performance.

Table 3. Test-sample static forecast performance.

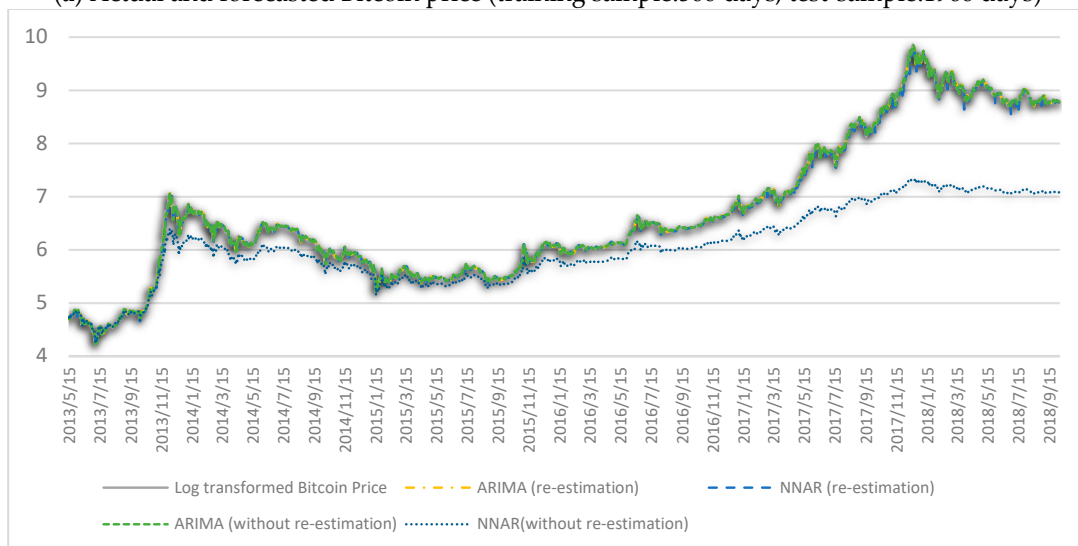
Forecast Model	Test Sample	RMSE	MAPE	MASE
<i>First test-sample window (1966 days)</i>				
<i>Forecast without re-estimation at each step</i>				
ARIMA (4,1,0)	15/05/2013~04/10/2018	0.038	0.379	0.969
NNAR (2,1)	15/05/2013~04/10/2018	0.924	8.289	24.990
<i>Forecast with re-estimation at each step</i>				
ARIMA	15/05/2013~04/10/2018	0.037	0.365	0.935
NNAR	15/05/2013~04/10/2018	0.048	0.453	1.186
<i>Second test-sample window (466 days)</i>				
<i>Forecast without re-estimation at each step</i>				
ARIMA (4,1,1)	26/06/2017~04/10/2018	0.042	0.354	1.207
NNAR (1,2)	26/06/2017~04/10/2018	0.093	0.837	2.911
<i>Forecast with re-estimation at each step</i>				
ARIMA	26/06/2017~04/10/2018	0.042	0.354	1.209
NNAR	26/06/2017~04/10/2018	0.069	0.537	1.828

Bold numbers indicate best performance.

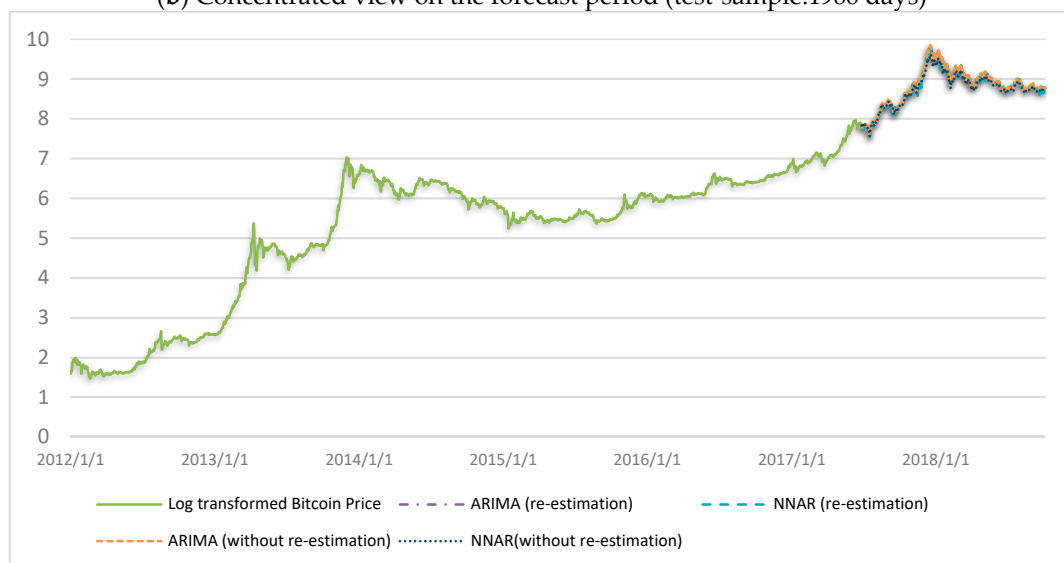
To confirm the validity of forecast models, diagnostic checks are conducted. p -values of the Box-Ljung (BL) test (Ljung and Box 1978) suggest that residuals of all employed models are free from autocorrelation (p -values > 0.05 considering eight lags). The BL test result of squared residuals of ARIMA models indicates the presence of conditional heteroscedasticity (p -values < 0.05); thus, future research on Bitcoin price forecast should consider nested ARIMA models combining ARCH and GARCH. The Jarque-Bera test (Jarque and Bera 1980) results suggest that residuals are not normally distributed (p -values < 0.05). Normality of residuals should not be an issue for the NNAR model as the error series in such models are assumed to be homoscedastic (and normally distributed) when training the model based on the training-sample (Hyndman and Athanasopoulos 2018).



(a) Actual and forecasted Bitcoin price (training sample:500 days, test-sample:1966 days)

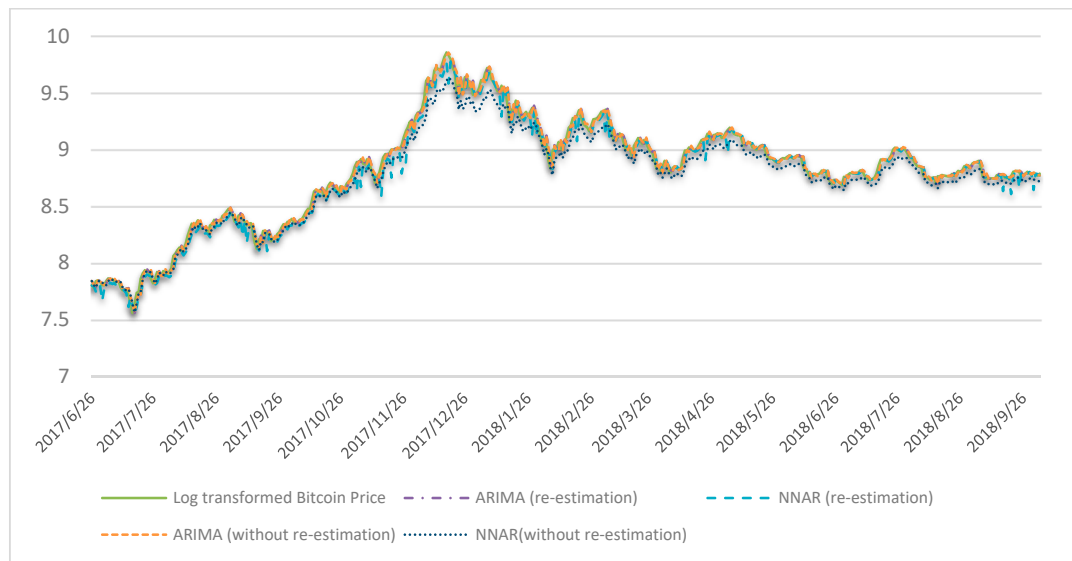


(b) Concentrated view on the forecast period (test-sample:1966 days)



(c) Actual and forecasted Bitcoin price (training sample:2000 days, test-sample:466 days)

Figure 4. Cont.



(d) Concentrated view on the forecast period (test-sample:466 days)

Figure 4. Bitcoin price forecast. (a,b) refer to the first training and test samples forecast in comprehensive and concentrated view, respectively, and (c,d) refer to the second training and test samples forecast in comprehensive and concentrated view, respectively.

Further, we perform the Diebold Mariano (DM) test (Diebold and Mariano 1995) to compare test-sample forecast results obtained from the two models used, ARIMA and NNAR. DM test results are presented in Table 4. In this case, the alternative hypothesis is that the forecast results of the second method are less accurate than the first method. Thus, a p -value of less than 0.05 indicates better accuracy of the first method. Result of the DM test is similar to as revealed in Table 3—the ARIMA model is more accurate than NNAR in forecasting the test-sample Bitcoin price. It is noteworthy that, forecast of ARIMA models, with or without model re-estimation in each step, are identical. Meanwhile, the NNAR model with re-estimation in each step performs considerably better than the without re-estimation approach.

Table 4. DM test of forecast results.

Models Compared	DM Statistics	p -Value
<i>First test-sample window (1966 days)</i>		
ARIMA vs. NNAR (re-estimation)	-7.054	1.20×10^{-12}
ARIMA vs. NNAR (without re-estimation)	-27.061	2.20×10^{-16}
ARIMA (re-estimation) vs. ARIMA (without re-estimation)	-4.502	3.56×10^{-6}
NNAR (re-estimation) vs. NNAR (without re-estimation)	-27.052	2.20×10^{-16}
<i>Second test-sample window (466 days)</i>		
ARIMA vs. NNAR (re-estimation)	-6.006	1.92×10^{-9}
ARIMA vs. NNAR (without re-estimation)	-12.348	2.20×10^{-16}
ARIMA (re-estimation) vs. ARIMA (without re-estimation)	-0.785	0.217
NNAR (re-estimation) vs. NNAR (without re-estimation)	-6.172	7.34×10^{-10}

$p < 0.05$ indicates that forecast results of the first method is better than the second method.

6. Discussion and Conclusions

This study forecasts the next-day Bitcoin price using two univariate models—ARIMA and NNAR. Based on the employed forecast accuracy measures (RMSE, MAPE and MASE), while NNAR models perform better than ARIMA in the first training-sample (500 days) Bitcoin price forecasts, ARIMA models outperform NNAR models in both the test-samples. In line with this, from Figure 4, one could argue that NNAR models perform better than ARIMA (see Table 2) in times of less volatility, but not during extremely volatile test-sample periods of Bitcoin price, particularly in the year 2018. Furthermore, the DM test suggests the same, that is, ARIMA forecast results are more accurate than the NNAR forecasts in the test-sample forecasts.

Meanwhile, existing studies offer interesting insights. In a review of neural network models in forecasting, [Adya and Collopy \(1998\)](#) find that neural networks are not necessarily the best modelling approach for all types of data. [Abrahart and See \(2000\)](#) and [Álvarez-Díaz et al. \(2018\)](#) find that ARIMA and NNAR perform similarly. On the other hand, similar to this study, [Alon et al. \(2001\)](#) and [Munim and Schramm \(2018\)](#) also find that neural networks outperform ARIMA in some training-sample, but the opposite holds for test-sample. The reason for better accuracy of ARIMA models could be that we employ the feed-forward NNAR model, which is found to be inferior by [Ho et al. \(2002\)](#) as well when comparing with ARIMA and recurrent neural network (RNN) models. Thus, future study should attempt the RNN approach to Bitcoin price forecast. Furthermore, according to the DM test results, the forecast of ARIMA models are similar for with or without model re-estimation in each step. However, the NNAR model with re-estimation in each step performs better than without re-estimation. Thus, this unique approach of model re-estimation at each step can be adopted in inter-day forecasts, such as in next-hour and next-minute Bitcoin price (also stock price) forecasts. However, the model re-estimation approach to forecast next-day price increases computational duration slightly. To this end, with the growing market-cap of cryptocurrencies and extreme volatility of cryptocurrency prices, further attention should be paid to modelling their returns.

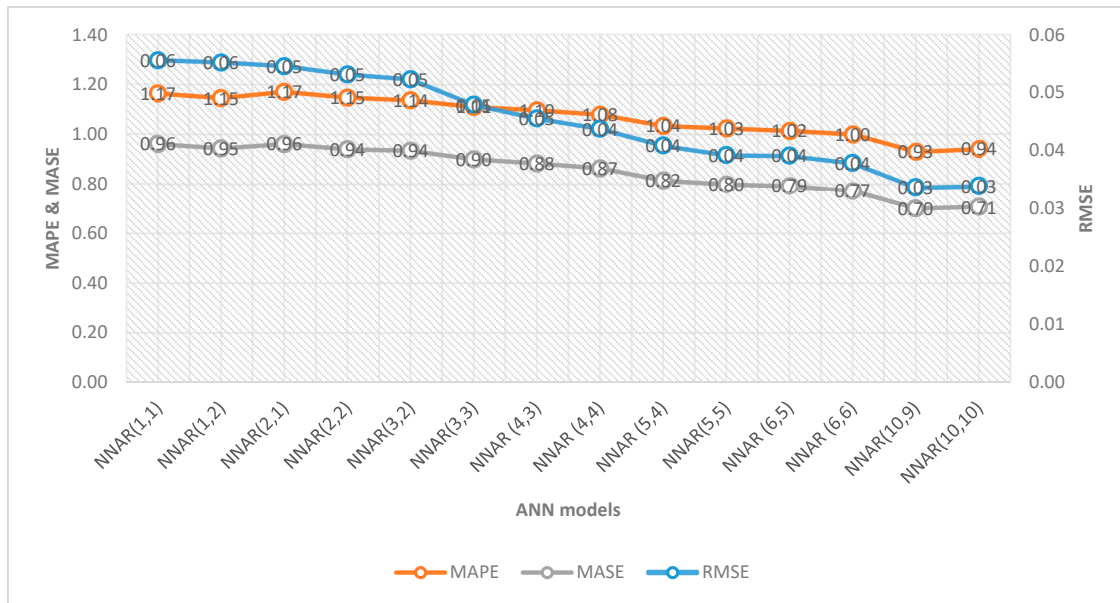
Author Contributions: Conceptualization Z.H.M. and I.A., Data curation Z.H.M., Methodology Z.H.M., Visualization Z.H.M., Introduction M.H.S., Literature review M.H.S. and I.A., Writing–review & editing Z.H.M., M.H.S. and I.A.

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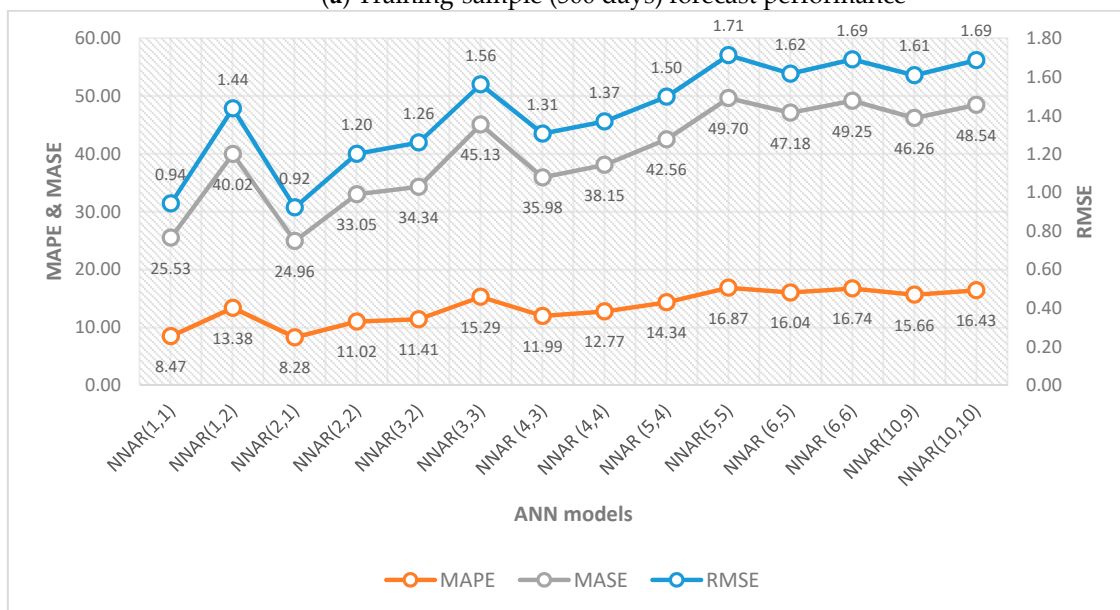
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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. The 14 Estimated NNAR(p,k) Models

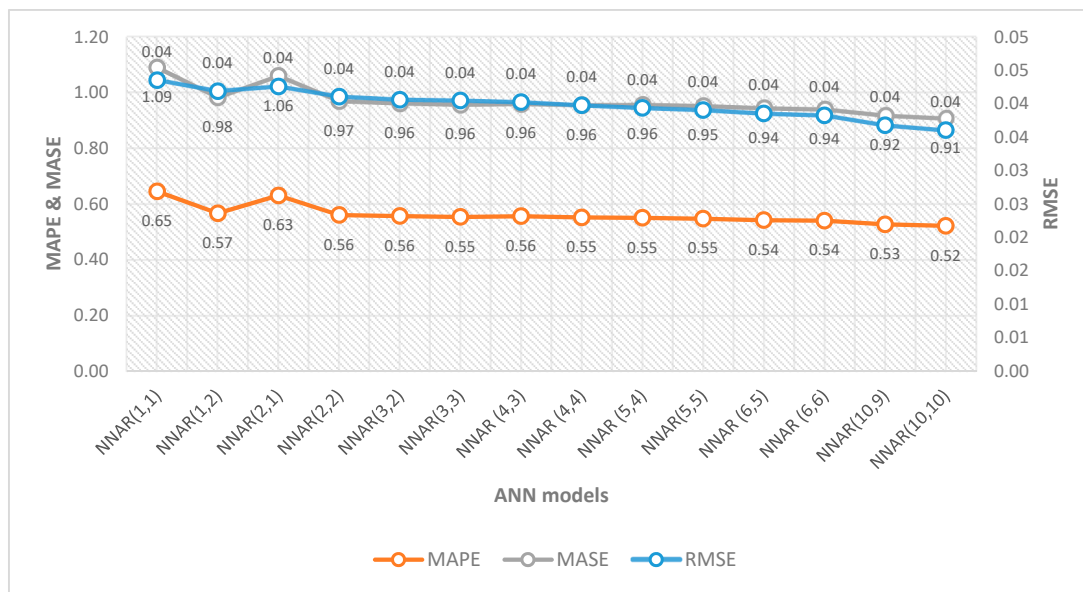


(a) Training-sample (500 days) forecast performance

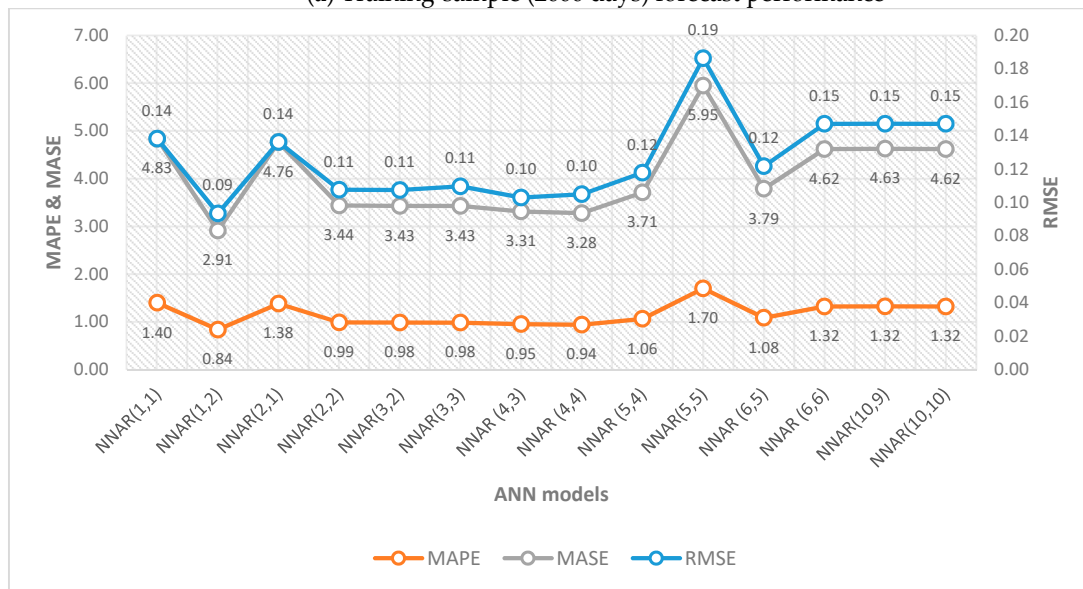


(b) Test-sample (1966 days) forecast performance

Figure A1. Examined NNAR model performance. (a,b) The first training and test-sample forecast performance, respectively.



(a) Training-sample (2000 days) forecast performance



(b) Test-sample (466 days) forecast performance

Figure A2. Examined NNAR model performance. (a,b) The second training and test-sample forecast performance, respectively.

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