Adaptive Consensus of Uncertain Nonlinear Systems with Event Triggered Communication and Intermittent Actuator Faults ⋆

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Abstract

This paper investigates distributed consensus tracking problem for uncertain nonlinear systems with event-triggered communication. The common desired trajectory information and each subsystem's state will be broadcast to their linked subsystems only when predefined triggering conditions are satisfied. Compared with the existing related literature, the main features of the results presented in this paper include four folds. *i*) A totally distributed adaptive control scheme is developed for multiple nonlinear systems without Lipschitz condition, while with parametric uncertainties. *ii*) The derivative of desired trajectory function is allowed unknown by all subsystems and directed communication condition is considered. *iii*) The designed event triggering conditions do not require either continuous monitoring of neighboring subsystems' states or global graph information available by all subsystems. *iv*) The results are successfully extended to the case with uncertain intermittent actuator faults by modifying both local control laws and adaptive laws. It is shown that for both fault-free and faulty cases, all closed-loop signals are ensured globally uniformly bounded and the tracking errors of all subsystems states will converge to a compact set. Besides, the tracking performance in the mean square error sense can be improved by appropriately adjusting design parameters.

Key words: Distributed adaptive control; consensus; event triggered communication; intermittent actuator faults; uncertain nonlinear systems.

1 Introduction

The fast few years have witnessed a continuously growing interest in investigating distributed consensus for multiple dynamic subsystems over a shared network. Readers may refer to Ren & Cao (2010); Qin, Ma, Shi, & Wang (2017) for comprehensive review of numerous representative results reported in this area. However, most of currently available distributed consensus control laws are proposed with continuous communication among neighboring subsystems. In practice, since data is usually transmitted in discrete packets through digital communication network, these control laws are implemented based on time-scheduled periodic sampling (Seyboth, Dimarogonas, & Johansson, 2013). Specifically, state information of each subsystem is broadcast to its linked subsystems periodically according to a constant sampling period and distributed controllers are updated synchronously. As a choice to effectively relieve the stress of using limited communication channel bandwidth, event-based consensus has received increasing attention in recent years. The triggering time of subsystem state broadcast is determined by the occurrence of a state dependent function exceeding certain threshold and is not necessarily periodic.

⋆ This paper was not presented at any IFAC meeting. Corresponding author: Jiangshuai Huang.

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In Seyboth et al. (2013), a distributed control strategy for average consensus with event-based broadcasting is presented, where networks of single integrator agents with/without communication delays and networks of double-integrator agents are considered. Continuous monitoring of the neighbors' states as in Dimarogonas, Frazzoli, & Johansson (2012) is successfully avoided, while the communication graph is assumed to be undirected. In Zhu, Jiang, & Feng (2014), event-based consensus for general linear multi-agent systems under directed communication topology condition is investigated. The selected triggering functions depend on not only local states, but also the states of its neighbors. Moreover, it is assumed that each agent knows the overall communication topology which is actually global information. Similar event-based consensus problem is solved in Xing, Wen, Guo, Liu, & Su (2016) by designing simpler triggering protocols, which are determined only by local state changing rates.

It is worth noting that all the aforementioned results on event-based consensus are obtained by assuming the internal dynamics of each subsystem is exactly known. The assumption restricts these results from being applied to the scenarios with uncertain subsystem dynamics involved. Although adaptive control has been shown as a promising tool to handle parametric and structural uncertainties in centralized control of single systems, distributed adaptive event-based consensus results are quite limited. In fact, even for uncertain multi-agent systems with continuous communication strategies, it is still non-trivial to design distributed adaptive consensus controllers especially under directed communication graph condition. As observed from Das and Lewis (2010); Wang, Huang, Wen, & Fan (2014); Huang, Song, Wang, Wen and Li (2017), the main challenge lies in the fact that a directed graph is associated with asymmetric Laplacian matrix. Then if a Lyapunov function is defined in terms of local neighborhood consensus errors, computing its derivative inevitably results in cross-coupling terms which are difficult to be counteracted by designing distributed parameter estimators. In Xie, Xu, Zhang, Li, & Chu (2016), a distributed adaptive consensus protocol is proposed based on event-triggered communication for single-integrator type of multi-agent systems. In You, Hua, & Guan (2018), an event-based distributed adaptive consensus approach is presented for first-order multi-agent systems with uncertain nonlinearities satisfying locally Lipschitz conditions. For each subsystem, the designed triggering condition requires continuous monitoring of its neighbors' states and knowledge of global graph information. Hence, the event-based controllers are not totally distributed.

Motivated by the limitations existed in the related literature, a new distributed adaptive consensus tracking control scheme is developed in this paper for multiple uncertain nonlinear systems with event triggered communication. The main contributions can be summarized as follows.

• A group of N first-order nonlinear subsystems with parametric uncertainties are considered. The subsystems are allowed to have non-identical dynamics, though with similar structures. Different from You et al. (2018), no Lipschitz condition is required for the nonlinear functions of local states involved in each subsystem's dynamics.

• The common desired trajectory $x_0(t)$, which is regarded as the state of a virtual leader, will be broadcast to only a subset of subsystems through event-based communication. However, the exact information of the derivative $\dot{x}_0(t)$ is allowed unknown by all subsystems and the communication condition is described by a directed graph. To tackle with the difficulties as discussed about distributed adaptive consensus control, an estimator is introduced in each subsystem to compensate for the effects of unknown derivative of the desired trajectory signal. Then the Lyapunov functions are defined based on only local estimation errors and the distributed adaptive control laws can be designed in a totally distributed manner.

• Inspired by Xing et al. (2016), the triggering rules for subsystem broadcast are chosen to be dependent only on local state changing rates. Thus continuous monitoring of neighboring subsystems' states, as required in Dimarogonas et al. (2012); You et al. (2018), can be avoided. In addition, the global graph information is not required to be shared by all subsystems, which is in contrast with Das and Lewis (2010); Zhu et al. (2014); You et al. (2018).

• Different from all the cited literatures, the case of unknown actuator faults is also considered in this paper. The leader-follower consensus problem of multi-vehicle systems with actuator fault and discontinuous communication protocols is investigated in Wang et al. (2018), where the fault parameter is assumed to be known. Different from this, the actuator in each subsystem may unawarely experience intermittent partial-loss-of-effectiveness (PLOE) type of faults in this paper. To treat the induced problem of unknown control coefficient with possibly infinite time of jumps, both control laws and parameter update laws need be modified to ensure closed-loop system stability. To the best of the authors' knowledge, this is the first solution of adaptive event-based fault-tolerant consensus control problem. Note that actuator failure compensation with event-based adaptive control is investigated in Xing, Wen, Liu, Su, & Cai (2017), whereas only tracking control of just one single system is considered.

It is shown that in both fault-free and faulty cases, all closed-loop signals are globally uniformly bounded and the tracking errors will converge to a compact set. Besides, Zeno behavior can be excluded and the tracking performance in the mean square error sense could be improved by appropriately adjusting certain design parameters.

An outline of this paper is summarized as follows. The consensus tracking problem for multiple uncertain nonlinear systems is formulated in Section 2. In Section 3, distributed adaptive controllers are designed for fault-free case with event-triggered communication strategies. Analysis of both system stability and consensus tracking performance is provided. In Section 4, the results are extended to the case with intermittent PLOE actuator faults. Simulation studies are given in Section 5 to validate the theoretical results, followed by a conclusion of the paper drawn in Section 6.

2 Problem Formulation

2.1 System Model

In this paper, we consider a group of N nonlinear subsystems modeled as follows.

$$\dot{x}_i = b_i u_i + \varphi_i(x_i)^T \theta_i \tag{1}$$

where $x_i \in \Re$ is the state of the ith subsystem. $\theta_i \in \Re^{p_i}$ is a vector of unknown constants and the control coefficient $b_i \in \Re$ is an unknown non-zero constant. $\varphi_i(\cdot) : \Re^j \to \Re^{p_i}$ is a column vector of known continuous nonlinear functions.

2.2 Communication Condition Among the N Subsystems

Suppose that the communications among the N subsystems can be represented by a fixed directed graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, \ldots, N\}$ denotes the set of indexes (or vertices) corresponding to each subsystem, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges between two distinct subsystems. An edge $(i, j) \in \mathcal{E}$ indicates that subsystem j can obtain information from subsystem i, but not necessarily vice versa (Ren and Cao, 2010). In this case, subsystem i is called a neighbor of subsystem j. We denote the set of neighbors for subsystem i as $\mathcal{N}_i$. Self edges (i, i) is not allowed in this paper, thus $(i, i) \notin \mathcal{E}$ and $i \notin \mathcal{N}_i$. The connectivity matrix $A = [a_{ij}] \in \Re^{N\times N}$ is defined such that $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. Clearly, the diagonal elements $a_{ii} = 0$. We introduce an in-degree matrix $\triangle$ such that $\triangle = \mathrm{diag}(\triangle_i) \in \Re^{N\times N}$ with $\triangle_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the ith row sum of A. Then, the Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L} = \triangle - A$.

2.3 Event based Broadcast Mechanism

The desired trajectory is characterized by a bounded time varying function $x_0(t)$, which is also regarded as the state of a virtual leader 0. We now use $\mu_i = 1$ to indicate the case that the desired trajectory information $x_0(t)$ is accessible directly to subsystem i through event-based communication; otherwise, μ_i is set as $\mu_i = 0$.

Besides, notations $t_0^j, t_1^j, \ldots, t_k^j, \ldots$ with $0 \le t_0^j < t_1^j < t_2^j \ldots < t_k^j < t_{k+1}^j < \ldots < \infty, j \in \{0, \mathcal{V}\}$ are adopted in this paper to denote the sequence of event times for subsystem j (or the leader node if $j = 0$) to broadcast its state information. Then for each subsystem i, the instantaneous state information from its neighboring subsystems is updated only at the time instants t_k^j for $j \in \mathcal{N}_i$.

The *control objective* in this paper is to design event-based distributed adaptive controllers $u_i(t)$ with appropriate triggering condition for each subsystem by utilizing the continuous local state $(x_i(t))$ and the discrete-time states collected from its neighboring subsystem through event-based communication $(x_j(t_k^j))$ such that
$i)$ all the signals in the closed-loop system are globally uniformly bounded;
$ii)$ the states of the overall system can still track the desired trajectory $x_0(t)$ as closely as possible, though $\mu_i = 1$ only for a small fraction of the subsystems and $\dot{x}_0(t)$ is unavailable to all subsystems.

To achieve the objective, the following assumptions are imposed.

Assumption 1 *The directed graph $\mathcal{G}$ contains a spanning tree and the root node i_l has direct access to $x_0(t)$, i.e. $\mu_{i_l} = 1$.*

Assumption 2 *For all subsystems, the only available information about $x_0(t)$ is that $|\dot{x}_0(t)| \le F$ where F is an unknown positive constant.*

Assumption 3 *The sign of b_i is available in constructing u_i for each subsystem i.*

Remark 1 *Note that compared to traditional centralized tracking problem of one single system, the main challenge of solving the distributed consensus tracking problem lies in the constraint that only part of the subsystems with $\mu_i = 1$ can acquire the desired trajectory information directly. Besides, Assumption 2 indicates that the exact information of $x_0(t)$ is allowed to be unknown by all subsystems. This is more general than the assumptions required in the existing results on consensus tracking control including Wang et al. (2014); Yu and Xia (2012) that the reference signals are linearly parameterized and the basis function vectors are known by all subsystems. Besides, Assumption 3 is standard in adaptive control results; see for examples Krstic et al. (1995); Wang et al. (2014, 2017).*

The following lemma brought from Das and Lewis (2010) is then introduced, which will be useful in our design and analysis of distributed adaptive controllers.

Lemma 1 *Based on Assumption 1, the matrix $(\mathcal{L} + \mathcal{B})$ is nonsingular where $\mathcal{B} = diag\{\mu_1, \ldots, \mu_N\}$. Define*

$$\begin{aligned}
\bar{q} &= [\bar{q}_1, \ldots, \bar{q}_N]^T = (\mathcal{L} + \mathcal{B})^{-1}[1, \ldots, 1]^T \\
P &= diag\{P_1, \ldots, P_N\} = diag\left\{\frac{1}{\bar{q}_1}, \ldots, \frac{1}{\bar{q}_N}\right\} \\
Q &= P(\mathcal{L} + \mathcal{B}) + (\mathcal{L} + \mathcal{B})^T P,
\end{aligned} \tag{2}$$

then $\bar{q}_i > 0$ for $i = 1, \ldots, N$ and Q is positive definite.

3 The Case without Actuator Faults

3.1 Design of Distributed Adaptive Controllers

• **Step 1.** Design of Triggering Condition

In this paper, we adopt $\bar{x}_j(t)$ to denote the state information of subsystem j or the leader node with $j = 0$, which is broadcast to its connected subsystems through event-based communication strategy. Therefore, $\bar{x}_j(t) = x_j(t_k^j)$, $j \in \{0, \mathcal{V}\}$, $t \in [t_k^j, t_{k+1}^j)$. As defined in Section 2, t_k^j is the kth event time for subsystem j broadcasting its state information. This indicates that for time $t \in [t_k^j, t_{k+1}^j)$, the neighbour's states available for subsystem i are kept unchanged as $\bar{x}_j(t) = x_j(t_k^j)$, $j \in \mathcal{N}_i$.

The triggering condition is then designed as

$$t_{k+1}^j = \inf\{t > t_k^j, |x_j(t) - \bar{x}_j(t)| > m_j\}, \quad j \in \{0, \mathcal{V}\} \quad (3)$$

where m_j is a positive constant to be determined by the designer. t_0^j is the first instant when (3) is fulfilled for subsystem j (or the leader node with $j = 0$).

• **Step 2.** Design of Control Law

In each subsystem, we introduce $\hat{x}_{i,0}(t) \in \Re$ to estimate the unknown desired trajectory information $x_0(t)$. Then the following error variables are defined, which will be adopted in the design of adaptive control laws.

$$z_i(t) = x_i(t) - \hat{x}_{i,0}(t) \quad (4)$$

$$\epsilon_i(t) = \sum_{j=1}^{N} a_{ij}[x_i(t) - \bar{x}_j(t)] + \mu_i[x_i(t) - \bar{x}_0(t)] \quad (5)$$

From (4), we have

$$z_i = x_i - x_0 + (x_0 - \hat{x}_{i,0}) = \delta_i + \tilde{x}_{i,0} \quad (6)$$

where $\tilde{x}_{i,0}$ denotes the estimation errors such that $\tilde{x}_{i,0} = x_0 - \hat{x}_{i,0}$.

The derivative z_i is computed as

$$\dot{z}_i = b_i u_i + \varphi_i^T \theta_i - \dot{\hat{x}}_{i,0} \quad (7)$$

The control signal is designed as

$$u_i = \hat{\varrho}_i \alpha_i, \quad (8)$$

$\hat{\varrho}_i$ is the estimate of $\varrho_i = \frac{1}{b_i}$ and α_i is a virtual control signal to be chosen. Substituting (8) into (7), we have

$$\dot{z}_i = b_i \hat{\varrho}_i \alpha_i + \varphi_i^T \theta_i - \dot{\hat{x}}_{i,0} \quad (9)$$

The virtual control signal α_i is chosen as

$$\alpha_i = -k\hat{P}_i \epsilon_i - c_i z_i - \varphi_i^T \hat{\theta}_i + \dot{\hat{x}}_{i,0} \quad (10)$$

where k and c_i are positive constants. $\hat{P}_i$ and $\hat{\theta}_i$ are parameter estimates of P_i in Lemma 1, θ_i, respectively. Substituting (10) into (9) yields that

$$\begin{aligned} \dot{z}_i &= \alpha_i - b_i \tilde{\varrho}_i \alpha_i + \varphi_i^T \theta_i - \dot{\hat{x}}_{i,0} \\ &= -k\hat{P}_i \epsilon_i - c_i z_i - b_i \tilde{\varrho}_i \alpha_i + \varphi_i^T \tilde{\theta}_i \end{aligned} \quad (11)$$

where $\tilde{\varrho}_i$ and $\tilde{\theta}_i$ are defined as $\tilde{\varrho}_i = \frac{1}{b_i} - \hat{\varrho}_i$ and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, respectively.

• **Step 3.** Design of Adaptive Laws

The state estimator generating $\hat{x}_{i,0}$ and parameter update laws for $\hat{P}_i$, $\hat{\varrho}_i$, $\hat{\theta}_i$ are designed as

$$\dot{\hat{x}}_{i,0} = -\gamma_{x_{i0}} \epsilon_i - \gamma_{x_{i0}} \kappa_{x_{i0}} (\hat{x}_{i,0} - x_{i,0}) \quad (12)$$

$$\dot{\hat{P}}_i = \gamma_{P_i} \epsilon_i z_i - \gamma_{P_i} \kappa_{P_i} \left(\hat{P}_i - P_{i,0}\right) \quad (13)$$

$$\dot{\hat{\varrho}}_i = -\gamma_{\varrho_i} \mathrm{sgn}(b_i) \alpha_i z_i - \gamma_{\varrho_i} \kappa_{\varrho_i} (\hat{\varrho}_i - \varrho_{i,0}) \quad (14)$$

$$\dot{\hat{\theta}}_i = \Gamma_{\theta_i} \varphi_i z_i - \Gamma_{\theta_i} \kappa_{\theta_i} \left(\hat{\theta}_i - \theta_{i,0}\right) \quad (15)$$

where $\gamma_\bullet$ and $\kappa_\bullet$ ('$\bullet$' denotes arbitrary subscript including x_{i0}, P_i, ϱ_i and θ_i) are positive design parameters. $x_{i,0}$, $P_{i,0}$, $\varrho_{i,0}$ and $\theta_{i,0}$ respectively in the last brackets of (12)-(15) are constant bias parameters, whose functions will be explained in detail in Remark 4 and Remark 8.

Remark 2 *Different from most of the related results including Das and Lewis (2010); Mei et al. (2011); Yu and Xia (2012); Yoo (2013); Wang et al. (2014, 2017); Huang et al. (2017), we investigate the adaptive consensus tracking problem with event-based state broadcasting strategy, which can effectively reduce the communication burden. However, it can be seen that rather than the continuous local neighborhood consensus errors e_i in (16) which are normally used in the above cited references, only event-based consensus errors ϵ_i in (5) can be adopted in control design. Hence new techniques such as introducing the last terms in (12)-(15) need be applied to handle the terms resulted from the mismatches between e_i and ϵ_i.*

Remark 3 *Observing (3), it indicates that the triggering condition for each subsystem j is only determined by its local state changing rate, which is similar to Xing et al. (2016). Thus continuous monitoring of the neighboring subsystems' states as in Dimarogonas et al. (2012); You et al. (2018) can be avoided. Furthermore, (10) only involves local estimates of P_i in Lemma 1. Hence, in contrast to Zhu et al. (2014) and Das and Lewis (2010), no global communication graph information is needed in either event triggering rule or distributed adaptive control laws. On the other hand, different from Xing et al. (2016) where linear system with exactly known subsystem dynamics is considered, the system here is nonlinear with unknown parameters.*

Remark 4 *Let $\hat{\Theta}_i$ be a generalized notation to express the estimate of a parameter vector Θ_i and $\tilde{\Theta}_i$ be the estimation error vector, i.e. $\tilde{\Theta} = \Theta - \hat{\Theta}$. It will be observed from (19) and (20) in subsequent stability analysis that by adopting the terms in the form of $-\gamma_{\Theta_{i0}} \kappa_{\Theta_{i0}} (\hat{\Theta}_i - \Theta_{i,0})$*

in (12)-(15), the property $\tilde{\Theta}_i^T(\hat{\Theta}_i - \Theta_{i,0}) = \tilde{\Theta}_i^T(-\tilde{\Theta}_i + \Theta_i - \Theta_{i,0}) \leq -\frac{1}{2}\|\tilde{\Theta}\|^2 + \frac{1}{2}\|\Theta_i - \Theta_{i,0}\|^2$ can be obtained. This property is crucial to establish the terms $-\sigma V + M^*$ in the last row of (20), hence is crucial to guarantee the closed-loop system stability.

3.2 System Stability and Consensus Analysis

The main results in this section are formally stated in the following theorem.

Theorem 1 *Consider a group of N uncertain subsystems as modeled in (1) with a desired trajectory $x_0(t)$ under Assumptions 1-3. By applying the distributed adaptive controllers in (8) and (10), event-triggered communication rules (3), distributed estimators (12), parameter update laws (13)-(15) to (1), the following results can be guaranteed.*
1) All the closed-loop signals are globally uniformly bounded.
2) The tracking error signals $\delta = [\delta_1, \delta_2, ..., \delta_N]^T$ will converge to a compact set.
3) The upper bound of $\|\delta(t)\|^2_{[0,T]} = \frac{1}{T}\int_0^T \|\delta(t)\|^2 dt$ can be decreased by choosing suitable design parameters.
4) Zeno behavior is excluded.

Proof. Firstly, the actual distributed synchronization error is defined as

$$e_i(t) = \sum_{j=1}^{N} a_{ij}[x_i(t) - x_j(t)] + \mu_i[x_i(t) - x_0(t)] \quad (16)$$

Then we choose the Lyapunov function as

$$V = \sum_{i=1}^{N} V_i \quad (17)$$

where

$$V_i = \frac{1}{2}z_i^2 + \frac{k}{2\gamma_{P_i}}\tilde{P}_i^2 + \frac{|b_i|}{2\gamma_{\varrho_i}}\tilde{\varrho}_i^2 + \frac{1}{2}\tilde{\theta}_i^T\Gamma_{\theta_i}^{-1}\tilde{\theta}_i + \frac{kP_i}{2\gamma_{x_{i0}}}\tilde{x}_{i,0}^2 \quad (18)$$

where $\tilde{P}_i = P_i - \hat{P}_i$.

From (6), (11)-(15), the derivative of V in (17) is computed as

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{N}\Big\{-kP_iz_i\epsilon_i - c_iz_i^2 + \frac{k}{\gamma_{P_i}}\tilde{P}_i\left[-\dot{\hat{P}}_i + \gamma_{P_i}\epsilon_iz_i\right] \\
&\quad + \frac{|b_i|}{\gamma_{\varrho_i}}\tilde{\varrho}_i\left[-\dot{\hat{\varrho}}_i - \gamma_{\varrho i}\mathrm{sgn}(b_i)\alpha_iz_i\right] + \tilde{\theta}_i^T\Gamma_{\theta_i}^{-1} \\
&\quad \times\left[-\dot{\hat{\theta}}_i + \Gamma_{\theta_i}\varphi_iz_i\right] + \frac{kP_i}{\gamma_{x_{i0}}}\tilde{x}_{i,0}\left(\dot{x}_0 - \dot{\hat{x}}_{i,0}\right)\Big\} \\
&= \sum_{i=1}^{N}\Big\{ - kP_i(\delta_i + \tilde{x}_{i,0})\epsilon_i - c_iz_i^2 \\
&\quad + k\kappa_{P_i}\tilde{P}_i\left(\hat{P}_i - P_{i,0}\right) + |b_i|\kappa_{\varrho_i}\tilde{\varrho}_i\left(\hat{\varrho}_i - \varrho_{i,0}\right) \\
&\quad + \kappa_{\theta_i}\tilde{\theta}_i^T\left(\hat{\theta}_i - \theta_{i,0}\right) + \frac{kP_i}{\gamma_{x_{i0}}}\tilde{x}_{i,0}\left(\dot{x}_0 - \dot{\hat{x}}_{i,0}\right)\Big\} \\
&= \sum_{i=1}^{N}\Big\{ - kP_i\delta_i\Big[e_i + \sum_{j=1}^{N}a_{ij}(x_j - \bar{x}_j) + \mu_i(x_0 - \bar{x}_0)\Big] \\
&\quad - c_iz_i^2 + k\kappa_{P_i}\tilde{P}_i\left(\hat{P}_i - P_{i,0}\right) + |b_i|\kappa_{\varrho_i}\tilde{\varrho}_i\left(\hat{\varrho}_i - \varrho_{i,0}\right) \\
&\quad + \kappa_{\theta_i}\tilde{\theta}_i^T\left(\hat{\theta}_i - \theta_{i,0}\right) + \frac{kP_i}{\gamma_{x_{i0}}}\tilde{x}_{i,0}\left(\dot{x}_0 - \dot{\hat{x}}_{i,0} - \gamma_{x_{i0}}\epsilon_i\right)\Big\} \\
&\leq \sum_{i=1}^{N}\Big\{ - kP_i\delta_ie_i + |\delta_i|kP_i(\Delta_i + \mu_i)m - c_iz_i^2 \\
&\quad + k\kappa_{P_i}\tilde{P}_i\left(\hat{P}_i - P_{i,0}\right) + |b_i|\kappa_{\varrho_i}\tilde{\varrho}_i\left(\hat{\varrho}_i - \varrho_{i,0}\right) \\
&\quad + \kappa_{\theta_i}\tilde{\theta}_i^T\left(\hat{\theta}_i - \theta_{i,0}\right) + \frac{kP_i}{\gamma_{x_{i0}}}|\tilde{x}_{i,0}|F \\
&\quad + k\kappa_{x_{i0}}P_i\tilde{x}_{i,0}(\hat{x}_{i,0} - x_{i,0})\Big\}
\end{aligned} \quad (19)$$

where $m = \max\{m_j\}$ for $j \in \{0, \mathcal{V}\}$.

$$\begin{aligned}
\dot{V} &\leq -k\delta^TP(\mathcal{L} + \mathcal{B})\delta + \sum_{i=1}^{N}\Big\{k\Big[\frac{1}{4}\lambda_{\min}(Q)\delta_i^2 \\
&\quad + \frac{P_i^2(\Delta_i + \mu_i)^2m^2}{\lambda_{\min}(Q)}\Big] - c_iz_i^2\Big\} \\
&\quad + \sum_{i=1}^{N}\Big\{\frac{k\kappa_{P_i}}{2}\Big[-\tilde{P}_i^2 + (P_i - P_{i,0})^2\Big]\Big\} \\
&\quad + \sum_{i=1}^{N}\Big\{\frac{|b_i|\kappa_{\varrho_i}}{2}\Big[-\tilde{\varrho}_i^2 + (\varrho_i - \varrho_{i,0})^2\Big]\Big\} \\
&\quad + \sum_{i=1}^{N}\Big\{\frac{\kappa_{\theta_i}}{2}\Big[-\|\tilde{\theta}_i\|^2 + \|\theta_i - \theta_{i,0}\|^2\Big]\Big\} \\
&\quad + \sum_{i=1}^{N}\Big\{\frac{kP_i\kappa_{x_{i0}}}{2}\Big[- \tilde{x}_{i,0}^2 + (x_0 - x_{i,0})^2\Big]\Big\} \\
&\quad + \sum_{i=1}^{N}\Big\{\frac{kP_i}{\gamma_{x_{i0}}}\Big[\frac{1}{4}\gamma_{x_{i0}}\kappa_{x_{i0}}\tilde{x}_{i,0}^2 + \frac{F^2}{\gamma_{x_{i0}}\kappa_{x_{i0}}}\Big]\Big\} \\
&\leq -\frac{k}{4}\lambda_{\min}(Q)\|\delta\|^2 - \sum_{i=1}^{N}\left(c_iz_i^2 + \frac{k\kappa_{P_i}}{2}\tilde{P}_i^2\right) \\
&\quad - \sum_{i=1}^{N}\left(\frac{|b_i|\kappa_{\varrho_i}}{2}\tilde{\varrho}_i^2 + \frac{\kappa_{\theta_i}}{2}\|\tilde{\theta}_i\|^2 + \frac{kP_i\kappa_{x_{i0}}}{4}\tilde{x}_{i,0}^2\right) + M^* \\
&\leq -\frac{k}{4}\lambda_{\min}(Q)\|\delta\|^2 - \sigma V + M^*
\end{aligned} \quad (20)$$

where

$$\sigma = \min\left\{2c_i, \kappa_{P_i}\gamma_{P_i}, \kappa_{\varrho_i}\gamma_{\varrho_i}, \frac{\kappa_{\theta_i}}{\lambda_{\max}(\Gamma_{\theta_i}^{-1})}, \frac{1}{2}\kappa_{x_{i0}}\gamma_{x_{i0}}\right\} \tag{21}$$

$$\begin{aligned} M^* = \sum_{i=1}^{N}\bigg\{ & \frac{kP_i^2(\Delta_i+\mu_i)^2 m^2}{\lambda_{\min}(Q)} + \frac{k\kappa_{P_i}}{2}(P_i - P_{i,0})^2 \\ & + \frac{|b_i|\kappa_{\varrho_i}}{2}(\varrho_i - \varrho_{i,0})^2 + \frac{\kappa_{\theta_i}}{2}\|\theta_i - \theta_{i,0}\|^2 + \frac{kP_i\kappa_{x_{i0}}}{2} \\ & \times(x_0 - x_{i,0})^2 + \frac{kP_iF^2}{\gamma_{x_{i0}}^2\kappa_{x_{i0}}}\bigg\} \end{aligned} \tag{22}$$

We now establish the results in Theorem 1 one by one.

1) By direct integrations of the following inequality

$$\dot{V} \le -\sigma V + M^*, \tag{23}$$

we have

$$V(t) \le V(0)e^{-\sigma t} + \frac{M^*}{\sigma}\left(1 - e^{-\sigma t}\right) \le V(0) + \frac{M^*}{\sigma} \tag{24}$$

which shows that V is uniformly bounded. Thus the signals z_i, $\hat{P}_i$, $\hat{\varrho}_i$, $\hat{\theta}_i$ and $\hat{x}_{i,0}$ are bounded. From (6), δ_i is bounded. From (8) and (10), u_i is bounded. Therefore all the closed-loop signals are globally uniformly bounded.

2) From (4), the definitions of V_i in (18) and V in (17), we have

$$\begin{aligned} \|\delta(t)\|^2 &\le \sum_{i=1}^{N}\frac{1}{2}\left[z_i(t)^2 + \tilde{x}_{i,0}(t)^2\right] \\ &\le \sum_{i=1}^{N}\max\left\{1, \frac{\gamma_{x_{i0}}}{kP_i}\right\}V_i(t) \\ &\le \xi V(t) \end{aligned} \tag{25}$$

where $\xi = \max\{1, \frac{\gamma_{x_{10}}}{kP_1}, \ldots, \frac{\gamma_{x_{N0}}}{kP_N}\}$. From (24), it follows that

$$\|\delta(t)\|^2 \le \xi\left[V(0)e^{-\sigma t} + \frac{M^*}{\sigma}(1 - e^{-\sigma t})\right]. \tag{26}$$

This implies that the tracking errors in Euclidean norm will converge to a compact set $E_r = \{\delta \mid \|\delta\|^2 \le \xi(M^* + \varsigma)/\sigma\}$ for $t \ge (1/\sigma)\ln(|V(0)\sigma - M^*|/\varsigma)$ with ς an arbitrarily small positive constant. It is worthy to point that the compact set E_r can actually be made as small as desired by increasing the control gain c_i and adaptive gains γ_{P_i}, $\gamma_{\varrho_i}, \Gamma_{\theta_i}, \gamma_{xi0}$, while fixing all the remaining design parameters. However, such small E_r is obtained at the expense of increasing control amplitude. Thus determining the design parameters is a trade-off issue.

3) From (20), we have

$$\dot{V} \le -\frac{k}{4}\lambda_{\min}(Q)\|\delta\|^2 + M^* \tag{27}$$

Integrating both sides of (27) yields that

$$\begin{aligned} \|\delta(t)\|_{[0,T]}^2 &= \frac{1}{T}\int_0^T \|\delta(t)\|^2 dt \\ &\le \frac{4}{k\lambda_{\min}(Q)}\left[\frac{V(0) - V(T)}{T} + M^*\right] \\ &\le \frac{4}{k\lambda_{\min}(Q)}\left[\frac{V(0)}{T} + M^*\right] \end{aligned} \tag{28}$$

From (17), (18) and (22), it follows that the upper bound of the overall tracking errors in the mean square sense of (28) can be decreased by decreasing m_i, κ_{P_i}, κ_{ϱ_i}, κ_{θ_i} and increasing k, γ_{P_i}, γ_{ϱ_i}, Γ_{θ_i}, $\gamma_{x_{i0}}$.

4) To show the exclusion of Zeno behavior, we shall show that the inter-execution intervals $(t_{k+1}^j - t_k^j)$ for $j \in \bar{\mathcal{V}}$, $\forall k \in Z^+$ are lower-bounded by a positive constant. Define $\eta_k^j(t) = x_j(t) - \bar{x}_j(t)$ for $t \in [t_k^j, t_{k+1}^j)$, whose derivative is computed as

$$\frac{d\left|\eta_k^j\right|}{dt} = \frac{d(\eta_k^j \times \eta_k^j)^{\frac{1}{2}}}{dt} = \mathrm{sgn}(\eta_k^j)\dot{\eta}_k^j \le \left|\dot{\eta}_k^j\right|. \tag{29}$$

Since $\bar{x}_j(t)$ keeps unchanged for $t \in [t_k^j, t_{k+1}^j)$, we have

$$\left|\dot{\eta}_k^j(t)\right| = \left|b_j u_j + \varphi_j^T\theta_j\right|, \quad \text{for} \quad j \in \mathcal{V} \tag{30}$$

and $\left|\dot{\eta}_k^0(t)\right| = |\dot{x}_0(t)|$. From the boundedness of $u_j, x_j, \dot{x}_0$ and the assumption that ϕ_j is continuous, it is concluded that there exist a positive constant ι_j such that $\left|\dot{\eta}_k^j(t)\right| \le \iota_j$ for $j \in \bar{\mathcal{V}}$. Then the inter-execution intervals must satisfy that $t_{k+1}^j - t_k^j \ge m_j/\iota_j$, i.e. Zeno behavior is excluded. □

4 The Case with Intermittent Actuator Faults

4.1 Intermittent Actuator Fault Model

Suppose that the internal dynamics in actuators is negligible. For system i, $i \in \mathcal{V}$, we denote u_{ci} as the input of its actuator, which is to be designed. An actuator with its input equal to its output, i.e. $u_i = u_{ci}$, is regarded as fault free. The actuator fault of interest is modeled as follows,

$$u_i(t) = \rho_i(t)u_{ci}(t), \tag{31}$$

where

$$\rho_i(t) = \rho_{ih}, \qquad \begin{aligned} & t \in [t_{ih,s}, t_{ih,e}), \\ & h \in Z^+ \end{aligned} \tag{32}$$

$$0 < \underline{\rho}_i \le \rho_{ih} \le 1 \tag{33}$$

where ρ_{ih}, $t_{ih,s}, t_{ih,e}$ and $\underline{\rho}_i$ are all unknown constants. $0 \le t_{i1,s} < t_{i1,e} \le t_{i2,s} < \cdots < t_{jh,e} \le t_{j(h+1),s} < t_{j(h+1),e}$ and so forth. Equations (31) and (32) indicate that the actuator of system i loses $(1 - \rho_{ih}) \times 100\%$ of its effectiveness from time $t_{ih,s}$ till $t_{ih,e}$. $t_{i1,s}$ denotes the time instant when the first PLOE fault takes place on the actuator of system i.

Remark 5 *As explained in Wang & Wen (2011), the considered fault model includes the possibility of actuators unawarely changing from a faulty mode to a normally working mode or another different faulty mode infinitely many times. Thus it is more general than the fault models with just one single occurrence which are commonly seen in fault tolerant control results.*

4.2 Design of Distributed Adaptive Controllers

- **Step 1.** Design of Triggering Condition

The triggering condition is designed the same as (3) in Section 3.1.

- **Step 2.** Design of Control Law

Similar to Section 3, the error variables $z_i(t)$ and $\epsilon_i(t)$ are designed in the same forms as in (4) and (5), respectively. The control signal is designed as

$$u_{ci} = \text{sgn}(b_i)\alpha_i \tag{34}$$

where α_i will be chosen in (37). The derivative of z_i is computed as

$$\dot{z}_i = d_i\alpha_i + \varphi_i^T\theta_i - \dot{\hat{x}}_{i,0} \tag{35}$$

where $0 < \underline{d}_i \leq d_i(t) = |b_i|\rho_i(t) \leq |b_i|$, $\underline{d}_i = |b_i|\underline{\rho}_i$.

We define ω_i as

$$\omega_i = c_i + \frac{1}{2\varepsilon_i}\left(k\hat{P}_i\epsilon_i^2 + \|\varphi_i\|^2\hat{\bar{\theta}}_i + \dot{\hat{x}}_{i,0}^2\right) \tag{36}$$

where k, c_i and ε_i are positive constants. $\hat{\bar{\theta}}_i$ is the estimate of $\bar{\theta}_i = \theta_i^2$. Then α_i is designed as

$$\alpha_i = \hat{\varrho}_i\beta_i \tag{37}$$

where

$$\beta_i = -\omega_i z_i \tag{38}$$

and $\hat{\varrho}_i$ is the estimate of $\varrho_i = \frac{1}{\underline{d}_i}$.

- **Step 3.** Design of Adaptive Laws

The parameter update laws are designed as

$$\dot{\hat{x}}_{i,0} = -\gamma_{x_{i0}}\epsilon_i - \gamma_{x_{i0}}\kappa_{x_{i0}}\left(\hat{x}_{i,0} - x_{i,0}\right) \tag{39}$$

$$\dot{\hat{P}}_i = \frac{\gamma_{P_i}}{\varepsilon_i}\epsilon_i^2 z_i^2 - \gamma_{P_i}\kappa_{P_i}\left(\hat{P}_i - P_{i,0}\right) \tag{40}$$

$$\dot{\hat{\varrho}}_i = \gamma_{\varrho_i}\omega_i z_i^2 - \gamma_{\varrho_i}\kappa_{\varrho_i}\left(\hat{\varrho}_i - \varrho_{i,0}\right) \tag{41}$$

$$\dot{\hat{\bar{\theta}}}_i = \frac{\gamma_{\bar{\theta}_i}}{\varepsilon_i}\|\varphi_i\|^2 z_i^2 - \gamma_{\bar{\theta}_i}\kappa_{\bar{\theta}_i}\left(\hat{\bar{\theta}}_i - \bar{\theta}_{i,0}\right) \tag{42}$$

where $\gamma_\bullet$ and $\kappa_\bullet$ ('•' denotes arbitrary subscript including x_{i0}, P_i, ϱ_i and $\bar{\theta}_i$) are positive constants. $x_{i,0}$, $P_{i,0}$, $\varrho_{i,0}$ and $\bar{\theta}_{i,0}$ in the last brackets of (39)-(42) are constant bias parameters whose functions are the same as those in (12)-(15). Besides, the initial states $\hat{P}_i(0)$, $\hat{\varrho}_i(0)$, $\hat{\bar{\theta}}_i(0)$ and bias parameters $P_{i,0}$, $\varrho_{i,0}$, $\bar{\theta}_{i,0}$ are all chosen to be non-negative.

By doing so, $\hat{P}_i(t)$, $\hat{\varrho}_i(t)$ and $\hat{\bar{\theta}}_i(t)$ are rendered non-negative for all $t \geq 0$ due to the fact given in Lemma 2 below.

Lemma 2 *Consider a first-order differential equation*

$$\dot{s} = \gamma_s f_s - \gamma_s\kappa_s(s - s_0) \tag{43}$$

where γ_s, κ_s are both positive constants, $f_s \geq 0$ and $s_0 \geq 0$. The solution of (43) is rendered non-negative for all $t \geq 0$ such that

$$s(t) = e^{-\gamma_s\kappa_s t}s(0) + \int_0^t e^{-\gamma_s\kappa_s(t-\tau)}(\gamma_s f_s + \gamma_s\kappa_s s_0)d\tau \\ \geq 0 \tag{44}$$

Define

$$V_{i,1} = \frac{1}{2}z_i^2 \tag{45}$$

The derivative of $V_{i,1}$ is shown to satisfy that

$$\begin{aligned}\dot{V}_{i,1} =& z_i\left(d_i\hat{\varrho}_i\beta_i + \varphi_i^T\theta_i - \dot{\hat{x}}_{i,0}\right) \\ \leq& -\underline{d}_i(\varrho_i - \tilde{\varrho}_i)\omega_i z_i^2 + \frac{z_i^2\|\varphi_i\|^2}{2\varepsilon_i}\theta_i^2 + \frac{z_i^2\dot{\hat{x}}_{i,0}^2}{2\varepsilon_i} + \varepsilon_i \\ \leq& -c_i z_i^2 - \frac{k}{2\varepsilon_i}\left(P_i - \tilde{P}_i\right)\epsilon_i^2 z_i^2 + \frac{z_i^2\|\varphi_i\|^2}{2\varepsilon_i}\tilde{\bar{\theta}}_i \\ &+ \underline{d}_i\tilde{\varrho}_i\omega_i z_i^2 + \varepsilon_i\end{aligned} \tag{46}$$

where $z_i\varphi_i^T\theta_i \leq \frac{z_i^2\|\varphi_i\|^2\theta_i^2}{2\varepsilon_i} + \frac{\varepsilon_i}{2}$, $z_i\dot{\hat{x}}_{i,0} \leq \frac{z_i^2\dot{\hat{x}}_{i,0}^2}{2\varepsilon_i} + \frac{\varepsilon_i}{2}$ have been used to obtain the first inequality and $\tilde{P}_i = P_i - \hat{P}_i$, $\tilde{\varrho}_i = \varrho_i - \hat{\varrho}_i$, $\tilde{\bar{\theta}}_i = \bar{\theta}_i - \hat{\bar{\theta}}_i$.

Remark 6 *It should be emphasized that the control method presented in Section 3 cannot be applied directly in this section. This is because the intermittent faults will cause the unknown virtual control coefficient $d_i(t)$ in (35) to be a time-varying parameter which may experience infinite number of sudden changes, resulting in the time derivative $\dot{d}_i(t)$ unbounded. If previous control design is adopted here without modification, $\tilde{\varrho}_i = \varrho_i - \hat{\varrho}_i = 1/d_i - \hat{\varrho}_i$ as indicated by the definition of $\tilde{\varrho}_i$ below (11). Then computing the derivative of V_i in (18) will result in an unbounded term $\frac{|b_i|}{\gamma_{\varrho_i}}\tilde{\varrho}_i\left(\dot{\varrho}_i - \dot{\hat{\varrho}}_i\right)$ such that the proof of Theorem 1 is no longer valid. To handle this issue, the design of adaptive controllers is modified by adopting the Young's inequality several times and introducing the nonlinear damping terms in the quadratic form as in (36) and (38). The parameter update laws are also modified accordingly as in (39)-(42).*

4.3 System Stability and Consensus Analysis

The main results in this section are formally stated in the following theorem.

Theorem 2 *Consider a group of N uncertain subsystems as modeled in (1) with a desired trajectory $x_0(t)$ under Assumptions 1-3 and possibly intermittent actuator faults as modeled in (31)-(33). By applying the distributed adaptive fault tolerant controllers in (34) and (37)-(38), event-triggered communication rules (3), distributed estimators (39), parameter update laws (40)-(42) to (1), the following results can be guaranteed.*
1) All the closed-loop signals are globally uniformly bounded.
2) The tracking error signals $\delta = [\delta_1, \delta_2, ..., \delta_N]^T$ will converge to a compact set.
3) The upper bound of $\|\delta(t)\|^2_{[0,T]} = \frac{1}{T}\int_0^T \|\delta(t)\|^2 dt$ can be decreased by choosing suitable design parameters.
4) Zeno behavior is excluded.

Proof. Similar to the Proof of Theorem 1, an error variable $e_i(t)$ is firstly defined in the same form as in (16). Then we choose the Lyapunov function as

$$V = \sum_{i=1}^{N} V_i \tag{47}$$

$$V_i = V_{i,1} + \frac{k}{4\gamma_{P_i}}\tilde{P}_i^2 + \frac{d_i}{2\gamma_{\varrho i}}\tilde{\varrho}_i^2 + \frac{1}{4\gamma_{\bar{\theta}_i}}\tilde{\bar{\theta}}_i^2 + \frac{kP_i}{2\gamma_{x_{i0}}}\tilde{x}_{i,0}^2 \tag{48}$$

From (39)-(42) and (46), we have

$$\begin{aligned}
\dot{V} \le& \sum_{i=1}^{N}\left(-c_i z_i^2 - \frac{k}{2\varepsilon_i}P_i\epsilon_i^2 z_i^2 + \varepsilon_i\right) + \sum_{i=1}^{N}\Bigg\{\frac{k}{2\gamma_{P_i}}\tilde{P}_i \\
&\times\left(-\dot{\hat{P}}_i + \frac{\gamma_{P_i}}{\varepsilon_i}\epsilon_i^2 z_i^2\right) + \frac{d_i}{\gamma_{\varrho_i}}\tilde{\varrho}_i\left(-\dot{\hat{\varrho}}_i + \gamma_{\varrho i}\omega_i z_i^2\right) \\
&+ \frac{1}{2\gamma_{\bar{\theta}_i}}\tilde{\bar{\theta}}_i\left(-\dot{\hat{\bar{\theta}}}_i + \frac{\gamma_{\bar{\theta}_i}}{\varepsilon_i}\|\varphi_i\|^2 z_i^2\right) \\
&+ \frac{kP_i}{\gamma_{x_{i0}}}\tilde{x}_{i,0}\left(\dot{x}_0 - \dot{\hat{x}}_{i,0}\right)\Bigg\} \\
\le& \sum_{i=1}^{N}\left[-kP_i(\delta_i + \tilde{x}_{i,0})\epsilon_i - c_i z_i^2 + \left(\frac{kP_i}{2}+1\right)\varepsilon_i\right] \\
&+ \sum_{i=1}^{N}\Bigg[-\frac{k\kappa_{P_i}}{4}\tilde{P}_i^2 + \frac{k\kappa_{P_i}}{4}(P_i - P_{i,0})^2 - \frac{d_i\kappa_{\varrho i}}{2}\tilde{\varrho}_i^2 \\
&+ \frac{d_i\kappa_{\varrho i}}{2}(\varrho_i - \varrho_{i,0})^2 - \frac{\kappa_{\bar{\theta}_i}}{4}\tilde{\bar{\theta}}_i^2 + \frac{\kappa_{\bar{\theta}_i}}{4}\left(\bar{\theta}_i - \bar{\theta}_{i,0}\right)^2 \\
&+ \frac{kP_i}{\gamma_{x_{i0}}}|\tilde{x}_{i,0}|F - \frac{kP_i}{\gamma_{x_{i0}}}\tilde{x}_{i,0}\dot{\hat{x}}_{i,0}\Bigg] \\
\le& \sum_{i=1}^{N}\Bigg\{-kP_i\delta_i\left[e_i + \sum_{j=1}^{N}a_{ij}(x_j - \bar{x}_j) + \mu_i(x_0 - \bar{x}_0)\right] \\
&- c_i z_i^2 - \frac{k\kappa_{P_i}}{4}\tilde{P}_i^2 - \frac{d_i\kappa_{\varrho i}}{2}\tilde{\varrho}_i^2 - \frac{\kappa_{\bar{\theta}_i}}{4}\tilde{\bar{\theta}}_i^2 - \frac{kP_i}{\gamma_{x_{i0}}}\tilde{x}_{i,0} \\
&\times\left(\dot{\hat{x}}_{i,0} + \gamma_{x_{i0}}\epsilon_i\right) + \frac{kP_i}{\gamma_{x_{i0}}}|\tilde{x}_{i,0}|F + \left(\frac{kP_i}{2}+1\right)\varepsilon_i \\
&+ \frac{k\kappa_{P_i}}{4}(P_i - P_{i,0})^2 + \frac{d_i\kappa_{\varrho i}}{2}(\varrho_i - \varrho_{i,0})^2 \\
&+ \frac{\kappa_{\bar{\theta}_i}}{4}(\bar{\theta}_i - \bar{\theta}_{i,0})^2\Bigg\} \\
\le& -\frac{k}{4}\lambda_{\min}(Q)\|\delta\|^2 - \sigma V + M^*
\end{aligned} \tag{49}$$

where

$$\sigma = \min\left\{2c_i, \kappa_{P_i}\gamma_{P_i}, \kappa_{\varrho_i}\gamma_{\varrho_i}, \kappa_{\bar{\theta}_i}\gamma_{\bar{\theta}_i}, \frac{1}{2}\kappa_{x_{i0}}\gamma_{x_{i0}}\right\} \tag{50}$$

$$\begin{aligned}
M^* =& \sum_{i=1}^{N}\Bigg\{\frac{kP_i^2(\Delta_i + \mu_i)^2 m^2}{\lambda_{\min}(Q)} + \frac{k\kappa_{P_i}}{4}(P_i - P_{i,0})^2 \\
&+ \frac{d_i\kappa_{\varrho i}}{2}(\varrho_i - \varrho_{i,0})^2 + \frac{\kappa_{\bar{\theta}_i}}{4}(\bar{\theta}_i - \bar{\theta}_{i,0})^2 \\
&+ \frac{kP_i\kappa_{x_{i0}}(x_0 - x_{i,0})^2}{2} + \frac{kP_i F^2}{\gamma_{x_{i0}}^2\kappa_{x_{i0}}} \\
&+ \left(\frac{kP_i}{2}+1\right)\varepsilon_i\Bigg\}
\end{aligned} \tag{51}$$

The remaining analysis is similar to the proof of Theorem 1. □

Remark 7 *In this paper, a local estimator $\hat{x}_{i,0}(t)$ is introduced in each subsystem for uncertain desired trajectory signal $x_0(t)$. Then the Lyapunov functions V_i in (18) (or (48) with (45) for faulty case) is defined based on only local estimation errors including z_i, $\tilde{P}_i$, $\tilde{\varrho}_i$, $\tilde{\theta}_i$ ($\tilde{\bar{\theta}}_i$ for faulty case) and $\tilde{x}_{i,0}$. Such way of constructing Lyapunov function is fundamentally different from most of the existing distributed consensus control results, where the Lyapunov functions are defined based on local neighborhood consensus errors in the form of (16). It should be noted that if a local parameter estimator $\hat{\theta}_i$ is designed in each subsystem, then computing the derivative of the latter form of Lyapunov functions will result in cross-coupling terms related to local consensus errors e_i and neighboring subsystems' parameter estimation errors $\tilde{\theta}_j$ with $j \in \mathcal{N}_i$. And these terms are difficult to be canceled by designing distributed adaptive laws under directed graph condition.*

Remark 8 *The mismatches between e_i and ϵ_i as mentioned in Remark 2 and relaxed assumption on $\dot{x}_0(t)$ lead to the terms $-kP_i\delta_i\left[\sum_{j=1}^{N}a_{ij}(x_j - \bar{x}_j) + \mu_i(x_0 - \bar{x}_0)\right]$ and $\frac{kP_i}{\gamma_{x_{i0}}}\tilde{x}_{i,0}\dot{x}_0$ in the computed $\dot{V}$ in (19) for the fault-free case. One important technique to handle these terms is to* *introduce the terms in the form of* $-\gamma_{\Theta_{i0}}\kappa_{\Theta_{i0}}(\hat{\Theta}_i - \Theta_{i,0})$ *in (12)-(15) as discussed in Remark 4.*

Apart from the design parameters m_i, $\kappa_\bullet$, k, $\gamma_\bullet$ *and* $\Gamma_\bullet$ *in the proof of Theorem 1, the function of constant bias parameters* $\bullet_{i,0}$ *in the distributed adaptive laws (12)-(15), (39)-(42) is to provide more possibilities of improving the consensus tracking performance. More specifically, if some a priori knowledge of the true values of* $x_0(t)$, P_i, ϱ_i, θ_i *(or* $\bar{\theta}_i$ *in the faulty case) is available,* M^* *in (22) (or (51) for the faulty case) can be effectively reduced if* $x_{i,0}$, $P_{i,0}$, $\varrho_{i,0}$, $\theta_{i,0}$ *and* $\bar{\theta}_{i,0}$ *are chosen to be sufficiently close to* x_0, P_i, ϱ_i, θ_i *and* $\bar{\theta}_i$*, respectively.*

Remark 9 *Clearly, only first-order system is considered in this paper. Extending the results to a more general case of higher-order multi-agent system with mismatched parametric uncertainties is an open and interesting issue, which is left for future study. Backstepping technique (Krstic et al., 1995) may be a useful tool. However, the main difficulty is to compute the derivatives of virtual control signals, which actually do not exist as they involve piecewise constant state information of neighboring subsystems due to event-based communication. Besides, improving consensus tracking performance with modified event triggering conditions is also an interesting topic, which is worthy of further investigation.*

5 Simulation Studies

We consider a numerical example to verify the established theoretical results. Suppose that there are a group of 4 nonlinear subsystems with the following dynamics

$$\dot{x}_i = u_i + x_i^2\theta_i \tag{52}$$

where $\theta_1 = 1$, $\theta_2 = 2$, $\theta_3 = 3$, $\theta_4 = 4$ are unknown system parameters. The communication topology for the 4 subsystems is given in Fig. 1.

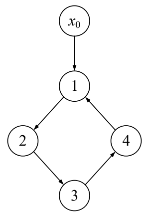

Fig. 1. Communication graph for the 4 nonlinear subsystems.

- **Fault-free Case**: The desired trajectory is given by $x_0(t) = 1 + \sin(0.1t) + \sin(0.05t)$.

The state initials including $x_i(0)$, $\hat{x}_{i,0}(0)$, $\hat{P}_i(0)$, $\hat{\varrho}_i(0)$ and $\hat{\theta}_i(0)$ are set to zeros for all $i \in \{1,2,3,4\}$. The design parameters are chosen as $k = 5$, $c_i = 5$, $m_i = 0.3$, $\gamma_{x_{i0}} = \gamma_{P_i} = \gamma_{\varrho_i} = \Gamma_{\theta_i} = 5$, $\kappa_{x_{i0}} = \kappa_{P_i} = \kappa_{\varrho_i} = \kappa_{\theta_i} = 0.005$. The tracking performance of $x_i(t)$ with compared to $x_0(t)$ is provided in Fig. 2. Fig. 3-Fig. 7 exhibit the triggering time, control $u_i(t)$, local estimates $\hat{x}_{i,0}(t)$, and parameter estimates for subsystems 1-2, respectively. Desired tracking performance of all subsystem states to $x_0(t)$ is seen while all the observed signals are bounded.

To demonstrate the effect of adjusting m_i, we change m_i from $m_i = 0.4$ to $m_i = 0.2$ while keeping other parameters unchanged. The comparisons of errors δ_i and triggering time is given in Fig. 8. It can be seen that the tracking performance can be improved by reducing m_i, however the triggering frequency will be increased.

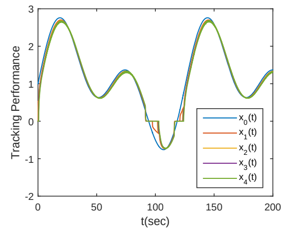

Fig. 2. Tracking performance of $x_i(t)$ for $1 \le i \le 4$ with compared to $x_0(t)$.

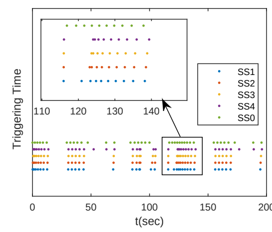

Fig. 3. Triggering time of $x_0(t)$ (SS0) and all the 4 subsystems (SS1–4).

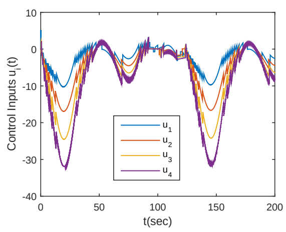

Fig. 4. Control inputs $u_i(t)$ for $1 \le i \le 4$.

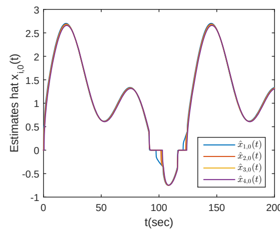

Fig. 5. The estimates $\hat{x}_{i,0}(t)$ for $1 \le i \le 4$.

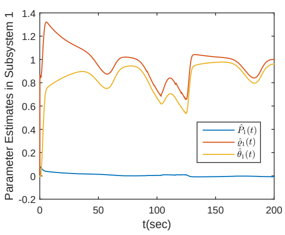

Fig. 6. Parameter estimates $\hat{P}_1(t)$, $\hat{\varrho}_1(t)$ and $\hat{\theta}_1(t)$ for subsystem 1.

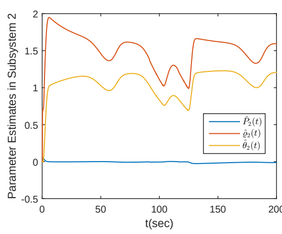

Fig. 7. Parameter estimates $\hat{P}_2(t)$, $\hat{\varrho}_2(t)$ and $\hat{\theta}_2(t)$ for subsystem 2.

- **Faulty Case**: The desired trajectory is given by $x_0(t) = 1 + 0.5\sin(0.1t) + 0.5\sin(0.05t)$.

In the faulty case, the local controllers are designed as in (34) with (37), event triggering condition (3) and estimators (39)-(42). The faulty case considered in simulation is modeled as

$$u_2(t) = \rho_{2h}u_{c2}(t), \quad t \in [hT^*, (h+1)T^*), \quad h = 1, 3, \ldots, \tag{53}$$

where $\rho_{2h} = 30\%$ and $T^* = 50$ seconds, which are both unknown in the designs.

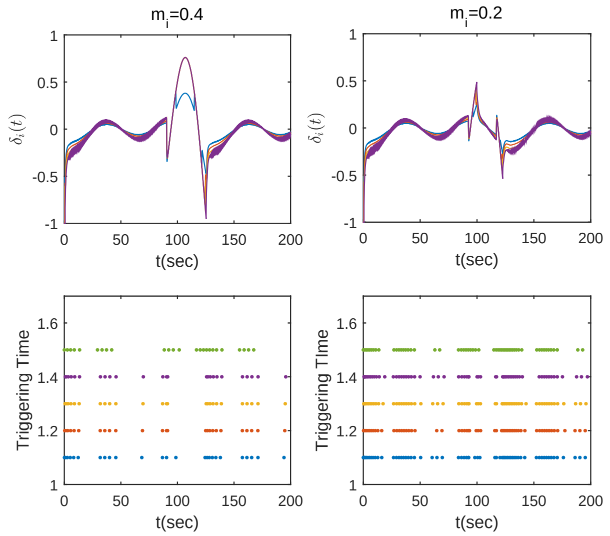

Fig. 8. The comparison of $\delta_i(t)$ and triggering time for $x_0(t)$ and the 4 subsystems with different m_i.

The state initials including $x_i(0)$, $\hat{x}_{i,0}(0)$, $\hat{P}_i(0)$, $\hat{\varrho}_i(0)$ and $\hat{\theta}_i(0)$ are set to zeros for all $i \in \{1, 2, 3, 4\}$. The design parameters are chosen as $k = 3$, $c_i = 3$, $m_i = 0.3$, $\gamma_{x_{i0}} = \gamma_{P_i} = \gamma_{\varrho_i} = \gamma_{\bar{\theta}_i} = 3$, $\kappa_{x_{i0}} = \kappa_{P_i} = \kappa_{\varrho_i} = \kappa_{\bar{\theta}_i} = 0.005$, $\varepsilon_i = 10$. The tracking performance of $x_i(t)$ with compared to $x_0(t)$ is provided in Fig. 9. Fig. 10-Fig. 14 exhibit the triggering time, control $u_i(t)$, local estimates $\hat{x}_{i,0}(t)$, and parameter estimates for subsystems 1-2, respectively. Satisfactory tracking performance of all subsystem states to $x_0(t)$ in the considered faulty case can be observed while all the observed signals are ensured bounded.

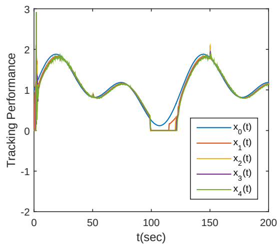

Fig. 9. Tracking performance of $x_i(t)$ for $1 \leq i \leq 4$ with compared to $x_0(t)$.

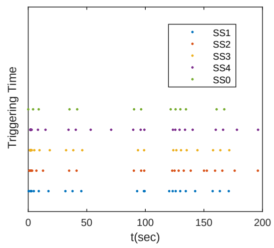

Fig. 10. Triggering time of $x_0(t)$ (SS0) and all the 4 subsystems (SS1–4).

6 Conclusion

In this paper, a distributed adaptive control scheme is presented for a group of uncertain nonlinear systems under directed communication condition with event-triggered communication strategy. For desired trajectory regarded as virtual leader and the remaining subsystems, triggering conditions are all designed based on local state changing rates. Compared with the existing related references, neither continuous monitoring of neighbors' states nor global communication graph information available by all subsystems is required. The results are then successfully extended to the case with uncertain intermittent actuator faults. Simulation studies are provided to demonstrate the theoretical results.

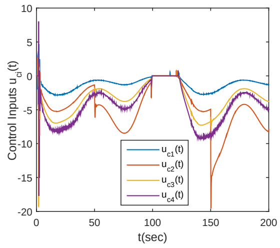

Fig. 11. Control inputs $u_{ci}(t)$ for $1 \leq i \leq 4$.

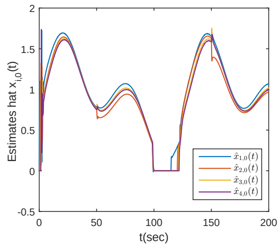

Fig. 12. The estimates $\hat{x}_{i,0}(t)$ for $1 \leq i \leq 4$.

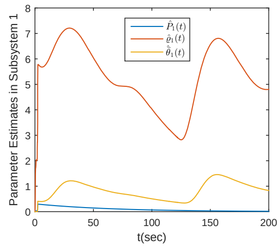

Fig. 13. Parameter estimates $\hat{P}_1(t)$, $\hat{\varrho}_1(t)$ and $\hat{\theta}_1(t)$ for subsystem 1.

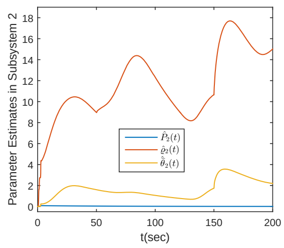

Fig. 14. Parameter estimates $\hat{P}_2(t)$, $\hat{\varrho}_2(t)$ and $\hat{\theta}_2(t)$ for subsystem 2.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grants No. 61973017, 61673035 and 61703061.

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