Doppler Shift Characterization of Wideband Mobile Radio Channels

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***Abstract*—The prevailing approach for characterizing the Doppler shift (DS) of mobile radio channels assumes the transmission of an unmodulated carrier. This consideration is valid for the analysis of narrowband channels, but its pertinence is questionable in regards to the modeling of wideband channels. In this correspondence, we redefine the DS from a time-frequency analysis perspective that does not depend on the aforementioned assumption. We systematically demonstrate that the DS can be characterized by the instantaneous frequency of the channel transfer function. This generic definition makes evident a fundamental aspect of the DS that is seldom acknowledged, namely, the DS is a frequency-varying quantity. We show that the second-order statistics of wideband mobile radio channels are nonstationary due to the DS's frequency variations. In addition, we present numerical results of a case study showing that such non-stationarities can cause significant system performance degradations.**

***Index Terms*—Doppler shift, instantaneous frequency, nonstationary channels, propagation delay, radio communications.**

I. Introduction

IN the context of terrestrial mobile radio communications, it is customary to characterize the frequency shift experienced by a signal due to the Doppler effect by assuming the transmission of an unmodulated carrier, e.g., see [1], [2]. Building on this assumption, Ossana [3] proposed a constant Doppler shift (DS) model for fixed-to-mobile radio channels which is given as $f_D = \frac{\upsilon}{\lambda}\cos(\phi - \gamma)$, where υ denotes the speed of the mobile station (MS), λ is the carrier signal's wavelength, ϕ indicates the direction of propagation of the received signal, and γ stands for the MS's direction of motion. Ossana's DS model has found widespread acceptance, and it has been revisited several times to incorporate factors that were not considered originally, such as three-dimensional propagation [4], mobile-to-mobile (M2M) links [5], and moving scatterers [6]. However, such revisions are also based on the assumption that an unmodulated carrier is being transmitted. This consideration is valid for the modeling of frequency-nonselective (narrowband) channels, but its pertinence is questionable in regards to the modeling of frequency-selective (wideband) channels. The settlement of this controversy is particularly critical nowadays due to the emergence of a new generation of mobile communications systems having extremely large bandwidths [7].

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In this correspondence, we model the DS from a time-frequency (TF) analysis perspective that is transparent to the transmitted signal's bandwidth. We systematically show that the DS can be characterized in general by the instantaneous frequency (IF) of the channel transfer function (CTF). This generic definition is an extension with respect to (w.r.t.) wideband channels of the idea presented in [8] for characterizing the instantaneous DS of narrowband M2M radio channels under velocity variations of the MSs. Our proposal can also be seen as a generalization of the approach for computing the DS by differentiation of the received carrier's phase [1], [6]. A fundamental aspect of the DS that is seldom known becomes evident from the modeling perspective described in this correspondence, namely: The DS is a frequency-varying quantity. We demonstrate that the DS's frequency variations cause the second-order statistics of wideband channels to be jointly non-stationary in time and frequency, implying that Bello's wide-sense stationary uncorrelated scattering (WSSUS) condition [9] is not fulfilled. We numerically evaluate the impact of such non-stationarities on the bit error rate (BER) of a vehicular communications system based on the IEEE 802.11p Standard [10] by considering four different channel estimation techniques. The obtained results show that the non-stationary effects stemming from the frequency-varying DS cause significant performance degradations when a combined TF interpolation is applied for channel state information tracking.

II. The DS from a TF-Analysis Perspective

A. DS Characterization for Narrowband Signals

To demonstrate that the DS can be characterized in general by the IF of the CTF, let us analyze first the case of an unmodulated carrier of frequency f_c, which is transmitted over a single-path mobile radio channel. For this simple case, the DS can be modeled from a TF-analysis perspective [11] as

$$f_D(t) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\left[\arg\{\widetilde{y}(t)\} - \arg\{\widetilde{x}(t)\}\right] \qquad (1)$$

where $\widetilde{x}(t)$ and $\widetilde{y}(t)$ are the complex pass-band transmitted and received signals, respectively. The previous equation simply states that the DS is given by the difference of the IF of the received signal minus the IF of the transmitted signal. Such IFs should be computed from analytic signals, i.e., from signals which have no negative frequency components [11]. In this regard, we assume that f_c is large enough as to guarantee that the spectra of $\widetilde{x}(t)$ and $\widetilde{y}(t)$ are limited to positive frequencies. The DS is modeled in (1) as a time-varying quantity because the IF of $\widetilde{y}(t)$ changes in time if the transmitter, the receiver, or the scatterers (or a combination thereof) move with a time-varying velocity [8], or if the angular statistics of the received signal vary in time [12], [13]. However, the DS can be characterized as a time-invariant quantity if the channel's angular statistics and the velocities of the aforementioned wandering elements do not change over time, e.g., see [3]–[6].

From the theory of linear time-varying (LTV) systems [14], we have $\widetilde{y}(t) = \widetilde{H}(t;f)\exp\{j2\pi f t\}\big|_{f=f_c}$, where $\widetilde{H}(t;f)$ is the TF-varying CTF. Hence, by evaluating (1) for $\widetilde{x}(t) = \exp\{j2\pi f_c t\}$ and $\widetilde{y}(t) = \widetilde{H}(t;f_c)\exp\{j2\pi f_c t\}$, we find $f_D(t) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\arg\{\widetilde{H}(t;f)\}\big|_{f=f_c}$. This result can be rewritten in terms of the complex base-band equivalent CTF $H(t;f)$ as

$$f_D(t) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\arg\{H(t;f)\}\big|_{f=0}. \tag{2}$$

Let us consider now the transmission of $\widetilde{x}(t)$ over a mobile radio channel comprising $\mathcal{L}$ different propagation paths. For this particular case, the complex pass-band received signal can be modeled as $\widetilde{y}(t) = \sum_{\ell=1}^{\mathcal{L}} \widetilde{y}_\ell(t)$, where $\widetilde{y}_\ell(t)$ is the signal that arrives at the receiver through the ℓth propagation path. According to the definition of the IF of a multicomponent signal [11], a single DS has to be computed for each component of $\widetilde{y}(t)$. Thus, by proceeding as we did to obtain (2), we find that the DS of the ℓth component of $\widetilde{y}(t)$ is equal to

$$f_{D,\ell}(t) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\arg\{H_\ell(t;f)\}\big|_{f=0} \tag{3}$$

where $H_\ell(t;f)$ denotes the complex base-band equivalent CTF associated with the ℓth propagation path, for $\ell \in \{1,2,\ldots,\mathcal{L}\}$. Equation (3) shows that the DS of narrowband mobile radio channels is given by the IF of the channel's complex envelope. This DS model is equivalent to that proposed in [8] for the characterization of non-stationary M2M channels under non-uniform motion of the transmitter and the receiver. The narrowband description of $f_{D,\ell}(t)$ given by (3) is also equivalent to the one obtained from differentiation of the received carrier's phase [1], [6].

B. DS Characterization for Wideband Signals

For an arbitrary complex-valued band-limited signal $s(t)$ transmitted over a single-path channel, the DS can be computed also as in (1) by taking $\widetilde{x}(t) = s(t)\exp\{j2\pi f_c t\}$. From the theory of LTV systems [14], we have $\widetilde{x}(t) = \frac{1}{2\pi}\int_{f_c-\frac{B}{2}}^{f_c+\frac{B}{2}} \widetilde{X}(f)\exp\{j2\pi f t\}\,\mathrm{d}f$, and $\widetilde{y}(t) = \frac{1}{2\pi}\int_{f_c-\frac{B}{2}}^{f_c+\frac{B}{2}} \widetilde{H}(t;f)\widetilde{X}(f)\exp\{j2\pi f t\}\,\mathrm{d}f$, where $\widetilde{X}(f)$ is the Fourier transform of $\widetilde{x}(t)$, and B is the signal's bandwidth ($f_c > B/2$). These integrals can be approximated by a Riemann sum consisting of N terms as

$$\widetilde{x}(t) \approx \frac{1}{2\pi}\sum_{n=1}^{N}\widetilde{X}(\xi+n\Delta)\exp\{j2\pi(\xi+n\Delta)t\}\Delta \tag{4}$$

$$\widetilde{y}(t) \approx \frac{1}{2\pi}\sum_{n=1}^{N}\widetilde{H}(t;\xi+n\Delta)\widetilde{X}(\xi+n\Delta) \times \exp\{j2\pi(\xi+n\Delta)t\}\Delta \tag{5}$$

where $\Delta = B/N$ and $\xi = f_c - B/2$. Note that such approximations are defined w.r.t. the same partition of the integration interval, and they converge to $\widetilde{x}(t)$ and $\widetilde{y}(t)$ in the limit if $N \to \infty$. Equations (4) and (5) show that $\widetilde{x}(t)$ and $\widetilde{y}(t)$ are multicomponent signals. The DS should be therefore computed individually for each element of the series in (4) and (5). By analogy with (1), the DS of the nth spectral component of $\widetilde{y}(t)$ is given by

$$f_{D,n}(t) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\left[\arg\{\widetilde{y}_n(t)\} - \arg\{\widetilde{x}_n(t)\}\right] \tag{6}$$

where $\widetilde{x}_n(t) = \frac{\Delta}{2\pi}\widetilde{X}(\xi+n\Delta)\exp\{j2\pi(\xi+n\Delta)t\}$ and $\widetilde{y}_n(t) = \frac{\Delta}{2\pi}\widetilde{H}(t;\xi+n\Delta)\widetilde{X}(\xi+n\Delta)\exp\{j2\pi(\xi+n\Delta)t\}$. Thus, by a direct evaluation of (6), we find $f_{D,n}(t) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\arg\{\widetilde{H}(t;\xi+n\Delta)\}$, for $n = 1,2,\ldots,N$. In the limit as $N \to \infty$ (and $\Delta \to 0$), the countably set of Doppler frequencies $\{f_{D,n}(t)\}_{n=1}^{N}$ can be modeled by a non-countably set $\{f_D(t;f') \in \mathbb{R}\}$, where $f_D(t;f') = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\arg\{\widetilde{H}(t;f)\}\big|_{f=f_c+f'}$, $t \in \mathbb{R}$. Here, the symbol $f' \in \left[-\frac{B}{2},\frac{B}{2}\right]$ stands for the frequency variable of the band-limited signal $s(t)$. Expressing the DS in terms of the complex base-band equivalent CTF, we obtain

$$f_D(t;f) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\arg\{H(t;f)\}, \quad |f| \le B/2 \tag{7}$$

where the operator $|\cdot|$ indicates the absolute value. We note that (7) includes (2) as a particular case for narrowband signals ($B = 0$). The extension of (7) w.r.t. a multipath channel comprising $\mathcal{L}$ propagation paths is straightforward. By proceeding as in Sec. II-A, the DS of the ℓth component of the received wideband multipath signal can be written as

$$f_{D,\ell}(t;f) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\arg\{H_\ell(t;f)\}, \quad |f| \le B/2. \tag{8}$$

From (8), we can conclude that the DS of mobile radio channels is given in general by the IF of the CTF. This relationship was noted previously in [15], but its foundations were not discussed there. Aside from [15], we are not aware of any other work in which the DS of mobile radio channels is modeled as in (8).

C. Remarks on the Frequency-Dependence of the DS

Equation (8) shows that the DS of wideband channels varies in both time and frequency. The time-varying character of $f_D(t;f)$ is only recently acknowledged within the field of land mobile radio communications [8], [12]. The DS's frequency dependence, on the other hand, remains largely unknown, as one can observe from a review of the literature (see, e.g., [1], [2]). The fact that the DS of wideband mobile radio channels is a frequency-varying quantity can be highlighted from the following standard model of the complex base-band equivalent channel impulse response (CIR) at time t due to an impulse applied τ seconds in the past [13], [16]

$$h(t;f) = \sum_{\ell=1}^{\mathcal{L}} g_\ell \exp\left\{-j[\theta_\ell + 2\pi f_c \tau_\ell(t)]\right\}\delta(\tau - \tau_\ell(t)). \quad (9)$$

In the foregoing equation, the CIR is modeled by the superposition of $\mathcal{L}$ electromagnetic waves, each arriving at the receiver antenna with an amplitude g_ℓ, a phase θ_ℓ, and a time-varying propagation delay $\tau_\ell(t)$, where $\delta(\cdot)$ stands for the Dirac delta function (see [13] for a detailed description of the model). For the channel model defined by (9), the corresponding complex base-band equivalent CTF is given as

$$H(t;f) = \sum_{\ell=1}^{\mathcal{L}} g_\ell \exp\left\{-j[\theta_\ell + 2\pi \tau_\ell(t)(f_c + f)]\right\} \quad (10)$$

for $|f| \leq B/2$ [13]. By substituting (10) into (8), we find

$$f_{D,\ell}(t;f) = -(f_c + f)\frac{\mathrm{d}\tau_\ell(t)}{\mathrm{d}t} \quad (11)$$

for $f \in [-B/2, B/2]$. This latter equation is noteworthy, as it not only shows that the DS is a frequency-dependent quantity, it also states that the DS and the propagation delay are related to each other in such a way that $f_{D,\ell}(t;f)$ can be determined directly from $\tau_\ell(t)$. In turn, $\tau_\ell(t)$ can be computed from $f_{D,\ell}(t;f)$ provided that the initial conditions of the system are known, such that

$$\tau_\ell(t) = \tau_\ell^0 - \frac{1}{f_c + f}\int_0^t f_{D,\ell}(x;f)\,\mathrm{d}x \quad (12)$$

where τ_ℓ^0 is the initial propagation delay of the ℓth multipath component (MPC) of $H(t;f)$. These two fundamental relationships between $f_D(t;f)$ and $\tau_\ell(t)$ are often left aside assuming that the channel's Doppler and delay statistics are separable, or that the DSs are different from zero and the propagation delays are time invariant. The relationship defined by (11) has been noted by several authors, but only for the particular case in which $f = 0$, i.e., for the particular case of an unmodulated carrier. For example, the DS is modeled in [2] by a time-invariant quantity equal to $f_D = -f_c\frac{\mathrm{d}\tau(t)}{\mathrm{d}t}$, whereas the DS is characterized in [12] by a time-varying quantity given as

$$f_D(t) = -f_c\frac{\mathrm{d}\tau(t)}{\mathrm{d}t}. \quad (13)$$

Equation (11) can be seen as an extension with respect to wideband channels of these narrowband representations of the DS. However, the extended DS relationship in (11) shows that different spectral components of transmitted wideband signals experience different DSs, while this property cannot be deduced from (13). Aside from laying the groundwork for this extension, the generic definition of $f_{D,\ell}(t;f)$ given by (8) places the results in (11) and (12) within the TF analysis framework. One can therefore leverage on the mathematical tools of such a framework to explore the intrinsic relationships established by (11) and (12) between the DS and the propagation delays. Some preliminary results in this direction are presented in [15].

III. Influence of the Frequency-Varying DS on the Channel's Non-Stationary Characteristics

Even though the DS's frequency dependence has passed nearly unnoticed in the field of land mobile radio communications, this feature of $f_D(t;f)$ is widely acknowledged in other areas of the telecommunications, such as in underwater acoustic communications [17]. However, several questions remain open, such as those pertaining to the effects that the frequency-varying character of $f_D(t;f)$ has on the channel's statistical properties.

The origin of the DS's frequency-varying character is explained by observing that the maximum DS (MDS) $f_{\max}$ of a monochromatic signal is proportional to the signal's frequency f according to the well-known relation $f_{\max} = f\upsilon/c_0$, where c_0 is the speed of light [1]. Hence, for a wideband signal having spectral components whose frequencies span an interval $f_c - B/2 < f < f_c + B/2$, the MDS is a monotonically increasing linear function of f given as

$$f_{\max}(f) = f_{\max}^0 \gamma(f) \quad (14)$$

where $f_{\max}^0 = f_c\upsilon/c_0$ is the carrier signal's MDS, and $\gamma(f)$ is a proportionality factor equal to

$$\gamma(f) = \frac{f_c + f}{f_c}, \qquad f \in [-B/2, B/2]. \quad (15)$$

The frequency dependence of $f_{\max}(f)$ produces faster fades for the signal's spectral components of higher frequencies. Such effects are illustrated in Fig. 1, where we show the absolute value of the TF-varying CTF $\widetilde{H}(t;f) = H(t;f)\exp\{-j2\pi f_c t\}$. This figure was generated by simulating the complex base-band equivalent CTF defined in (10) for $\mathcal{L} = 20$ and the edge frequencies, $f_1 = f_c - B/2$ and $f_2 = f_c + B/2$, of a band-limited signal of bandwidth B. We assumed that the gains g_ℓ are modeled by independent and identically distributed (i.i.d.) random variables characterized by a Rayleigh distribution, whereas the phases θ_ℓ are modeled by i.i.d. random variables uniformly distributed in $[-\pi, \pi)$. In addition, we modeled the time-varying propagation delays $\tau_\ell(t)$ by considering an M2M channel based on the geometrical one-ring scattering model described in [13, Sec. VI]. Specifically, following [13, Eq. (22)], we computed $\tau_\ell(t)$ as

$$\tau_\ell(t) = \tau_\ell^0 - t\frac{f_{D,\ell}^0}{f_c} \quad (16)$$

where $f_{D,\ell}^0$ is the carrier signal's DS for the ℓth MPC of $H(t;f)$ (see [13, Eq. (23)]). We computed τ_ℓ^0 and $f_{D,\ell}^0$ according to [13, Eqs. (22), (23), and (29)] by considering the parameters summarized in Table I. The values of f_c and B can be chosen arbitrarily, but B should be large, say $B > f_c/10$, to make the effects discussed in the lines below easy to observe on a graph. For the example presented in Fig. 1, we chose $f_c = 5.9$ GHz and $B = 2$ GHz (note that such a large bandwidth is considered only for the purposes of illustration; the practical implications of having a comparatively small

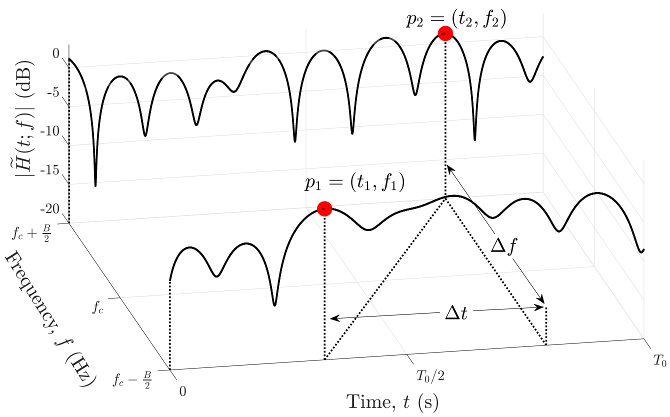


Fig. 1. Absolute value of the CTF at the edges of a wideband signal's bandwidth for an observation time interval of length T_0.

TABLE I
VALUES OF THE RELEVANT PARAMETERS FOR THE COMPUTATION OF THE INITIAL PROPAGATION DELAYS τ_ℓ^0 AND THE CARRIER SIGNAL'S DS $f_{D,\ell}^0$.

Parameter	**Value**
Initial distance between the transmitting and receiving MSs	500 m
Initial distance between the receiving MS and the scatterers	30 m
Direction of motion of the transmitting MS	60°
Speed of the transmitting MS	45 km/h
Direction of motion of the receiving MS	120°
Speed of the receiving MS	45 km/h
Distribution of the angle of arrival of the received MPCs of $H(t;f)$	Uniform on the circle

bandwidth $B \ll f_c$ are analyzed in Section IV). We conducted our simulations for an observation time interval of length $T_0 = 192$ ms.

Figure 1 shows that the waveform of high frequency $f_2 = f_c + B/2$ varies faster than the waveform of low frequency $f_1 = f_c - B/2$. These differences in fading rate cause the second-order statistics of $H(t;f)$ to be jointly non-stationary in time and frequency, because the waveforms of higher frequencies decorrelate faster than those of lower frequencies. This implies that the correlation function of $H(t;f)$ evaluated at two different observation points $p_1 = (t_1; f_1)$ and $p_2 = (t_2; f_2)$ does not only depend on the time and frequency lags $\Delta t = t_2 - t_1$ and $\Delta f = f_2 - f_1$, but it also depends on the particular values of p_1 and p_2, meaning that the WSSUS condition [9] is not fulfilled.

The fact that the WSSUS condition cannot be fulfilled due to the DS's frequency variations can be demonstrated analytically on the grounds of the TF correlation function (TFCF) [18]

$$R_H(t,f;\Delta t,\Delta f) = \mathcal{E}\{H^*(t-\Delta t,f)H(t,f+\Delta f)\} \quad (17)$$

where the operators $\mathcal{E}\{\cdot\}$ and $(\cdot)^*$ denote the statistical expectation and the complex conjugate, respectively. If the channel's angular statistics and the velocity vectors of the transmitting and receiving MSs remain constant in time, then the DS of the ℓth MPC of $H(t;f)$ can be modeled by a time-invariant but frequency-varying quantity, $f_{D,\ell}(f)$ (see Sec. II-A). Under these conditions, and based on (12), we have

$$\tau_\ell(t) \quad = \quad \tau_\ell^0 - t\frac{f_{D,\ell}(f)}{f_c+f}. \quad (18)$$

For the computation of $R_H(t,f;\Delta t,\Delta f)$, we characterize the gains g_ℓ and the phases θ_ℓ as discussed previously in this section for generating the graph in Fig. 1. We also assume that the time-varying propagation delays $\tau_\ell(t)$ are mutually independent but statistically equivalent random processes. Under these conditions, we find $\mathcal{E}\{H(t;f)\} = 0$, and we obtain the following result by evaluating (17) using (10) and (18):

$$R_H(t,f,\Delta t,\Delta f) = \sigma_0^2\mathcal{E}\left\{\exp\left\{j2\pi\left[\Delta t f_D(f) - \Delta f \tau(t)\right]\right\}\right\} \quad (19)$$

where σ_0^2 stands for the channel's average power, whereas $f_D(f)$ and $\tau(t)$ are arbitrary random processes in the sets $\{f_{D,\ell}(f)\}_{\ell=1}^{\mathcal{L}}$ and $\{\tau_\ell(t)\}_{\ell=1}^{\mathcal{L}}$, respectively.

Equation (19) shows that $H(t;f)$ characterizes a non-WSSUS channel, because its TFCF depends on the time variable t and also on the frequency variable f. Furthermore, from (18) and (19), it is evident that the second-order statistics of $H(t;f)$ are non-stationary jointly in time and frequency due to the frequency-varying nature of the DS. If the DS's frequency variations are neglected, e.g., by invoking the uncorrelated scattering (US) assumption [9] (such that $f_D(f) = f_D^0$ and $f_c + f = f_c$ for $|f| \leq B/2$), then the TFCF in (19) is equal to a frequency-independent but time-dependent function

$$R_H(t,\Delta t,\Delta f) = \sigma_0^2\mathcal{E}\Big\{\exp\big\{j2\pi\big[\Delta t f_D^0 - \Delta f\tau(t)\big]\big\}\Big\} \quad (20)$$

where $\tau(t) = \tau_0 - tf_D^0/f_c$. For this particular case, $H(t;f)$ is wide-sense stationary (WSS) in the frequency domain, but non-WSS in the time domain. Furthermore, from (18) and (19), we note that under the assumption that the WSSUS condition holds, both the DS's frequency variations and the time-varying component of the propagation delays should be neglected, such that $f_D(f) = f_D^0$ for all $|f| \leq B/2$ and $\tau(t) = \tau_0$ for all t. While these two conditions are in conflict with the fundamental relationship between the DS and the propagation delays, as discussed in Sec. II-C, they can be met as approximations within a finite region $\mathcal{Q}$ of the TF plane, such that $f_D(f) \approx f_D^0$ and $\tau(t) \approx \tau_0$ for $(t,f) \in \mathcal{Q}$. Thereby, $\mathcal{Q}$ defines a quasi-stationary region of non-WSSUS channels, because $\mathcal{E}\big\{\exp\big\{j2\pi\big[\Delta t f_D(f) - \Delta f\tau(t)\big]\big\}\big\} \approx \mathcal{E}\big\{\exp\big\{j2\pi\big[\Delta t f_D^0 - \Delta f\tau_0\big]\big\}\big\}$ for $(t,f) \in \mathcal{Q}$.

The result presented in (19) is similar to the TFCF presented in other papers. For example, (19) is equivalent to the TFCF obtained in [13, Eq. (36)] for non-WSSUS M2M Rayleigh fading channels. There, it was found that[1]

$$R_H(t,f,\Delta t,\Delta f) \quad = \quad \sigma_0^2\mathcal{E}\Bigg\{\exp\Bigg\{-j2\pi\Bigg[\Delta t f_D^0\left(\frac{f_c-f}{f_c}\right) + \Delta f\left(\tau_0 - t\frac{f_D^0}{f_c}\right)\Bigg]\Bigg\}\Bigg\}. \quad (21)$$

[1] In [13], the proportionality factor $(f_c+f)/f_c$ is written as $(f_c-f)/f_c$, because the CIR is modeled in that paper for a positive argument of the complex exponentials, such that $H(t;f) = \sum_{\ell=1}^{\mathcal{L}} g_\ell \exp\left\{j[\theta_\ell + 2\pi\tau_\ell(t)(f_c - f)]\right\}$.

However, the DS's frequency-varying character was overlooked in that paper, and it was concluded from (21) that the channel's non-WSSUS characteristics stem from the time-varying nature of the propagation delays. This is not an accurate conclusion, because for some particular cases, the channel's second-order statistics are stationary in the frequency domain even if the propagation delays vary in time, as shown in (20). The statistical expectation in (21) can be expressed in a form similar to (19) by noting that the DS of the wideband M2M channel model presented in [13] is equal to $f_D(f) = f_D^0(f_c - f)/f_c$. This can be verified easily by evaluating (8) using the complex conjugate of the CTF defined in [13, Eq. (31)].[2] The fact that the DS of the channel model presented in [13] is a frequency-varying quantity conveys a physical meaning to the results obtained in that paper concerning the channel's TF-dependent Doppler spectrum $\mathcal{D}(t, f; \upsilon) = \int_{-\infty}^{\infty} R_H(t, f; \Delta t, 0) \exp\{-j2\pi\upsilon\Delta t\} \, \mathrm{d}\Delta t$ and its variations over the frequency variable f. The reader is referred to [13] for graphical examples of such variations.

The TFCF presented in (19) bears similarities also with the TFCF computed in [12] for M2M channels. In that paper, the DS of the carrier signal is modeled by a time-varying but frequency independent quantity $f_D^0(t)$ given as in (13). Invoking the US assumption, it is shown in [12, Sec. II-C] that the channel's TFCF is equal to the expectation in (20) with f_D^0 replaced by $f_D^0(t)$. The generalization of (19) and its particular case in (20) with respect to a time-varying DS is not discussed in this paper, as our interest is in highlighting the DS's frequency-varying character. However, the similarities of (20) with the results obtained in [12] reinforce the statement that the channel's statistics are WSS in the frequency domain but non-WSS in the time domain if the DS is a frequency-invariant quantity and the propagation delays vary in time.

IV. Influence of the Frequency-Varying DS on the Performance of a Vehicular Communication System

In practice, the frequency dependence of the DS could be neglected stating that the bandwidth B of the transmitted information signal is much smaller than the carrier frequency. Thereby, the proportionality factor defined in (15) can be approximated as $\gamma(f) \approx 1$ for $f \in [-B/2, B/2]$, and the MDS given in (14) becomes a frequency-invariant quantity, which motivates the assumption that $H(t; f)$ fulfills the WSSUS condition. However, such a simplification should not be done thoughtlessly, as the channel's non-stationary characteristics that result from the frequency-varying DS could degrade the system's performance even if $f_c \gg B$.

To demonstrate that such non-stationarities are relevant in practice, we present in this section a BER performance analysis of a vehicular communications system based on the IEEE 802.11p Standard [10]. For that purpose, we have simulated the transmission of 10,000 physical layer convergence procedure frames. Each frame comprised 2 orthogonal frequency division multiplexing (OFDM) training symbols, and 32 OFDM data symbols. The OFDM symbols were simulated in accordance to the specifications of the IEEE 802.11p Standard [10] with the parameters listed in Table II. At the receiver side, we considered the zero-forcing equalizer and four different channel estimation techniques, namely: The Least Squares (LS) [19], Constructed Data Pilots (CDP) [20], Spectral-Temporal Averaging (STA) [19], and Frequency Linear-Averaged Data Pilot (FLDP) [21] estimation techniques. The former two techniques apply a time domain interpolation for the channel state information tracking and noise cancellation, whereas the latter two apply a joint TF interpolation. The parameters α and β of the STA channel estimator [19] were chosen equal to $\alpha = \beta = 2$. We computed the BER of the four channel estimators for the case that the propagation channel fulfills the WSSUS condition, and also for the case that this condition is not met. In both cases, the channel was simulated on the grounds of (10) following the procedure described in Sec. III for generating the graph shown in Fig. 1. We note that a WSSUS channel model is obtained from (10) if the factor $\tau_\ell(t)(f_c + f)$ is replaced by $f_D^0 t + f\tau_0$, as this is equivalent to assume that $f_{D\ell}(f) = f_D^0$ for all $|f| \leq B/2$ and $\tau_\ell(t) = \tau_\ell^0$ for all t. The reader is referred to [22] for a detailed description of the four channel estimation techniques and our simulation set up.

TABLE II
Parameters of the simulated OFDM symbols.

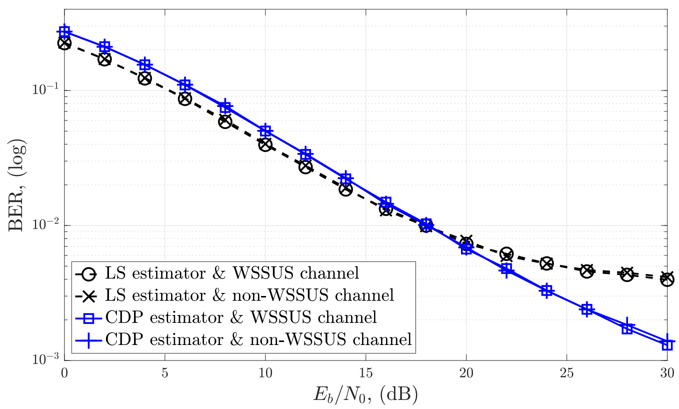
Parameter	Value
Carrier frequency	5.9 GHz
Bandwidth	10 MHz
Number of total subcarriers	64
Number of data subcarriers	48
Number of pilot subcarriers	4
Modulation	Binary phase shift keying
Coding rate	None

The results of our simulations are presented in Fig. 2. Figure 2(a) shows that the BER performance of the channel estimators that only apply a time-domain interpolation (LS and CDP) is nearly the same for the WSSUS and non-WSSUS channels. However, Fig. 2(b) shows that the performance of the channels estimators that apply a joint TF interpolation (STA and FLDP) is severely affected by the channel's non-stationarities. Such a performance degradation is explained by the fact that both the STA and the FLDP estimators rely on the stationarity of the channel's second-order statistics for noise cancellation and channel state information tracking, but such statistics are non-stationary due to the DS's frequency-varying character.

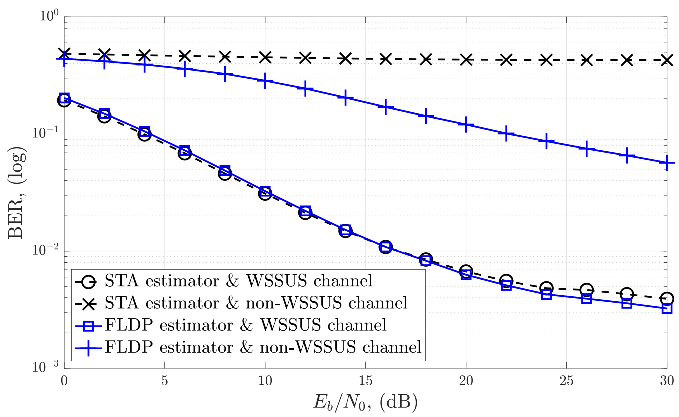
V. Conclusions

In this correspondence, we have shown that the DS of wideband mobile radio channels can be characterized in general by the IF of the CTF. Based on this generic definition, we demonstrated that the DS is a frequency-varying quantity.

[2]In the light of the concepts discussed in Section II, it follows that the frequency-invariant quantities defined in [13, Eqs. (13) and (23)] characterize only the carrier signal's DS.



(a)



(b)

Fig. 2. BER of a vehicular communication system based on the IEEE 802.11p Standard [10] considering the LS, STA, CDP, and FLDP channel estimation techniques for WSSUS and non-WSSUS channels.

Even though the DS varies in the frequency domain at a slow rate, this feature of the DS cannot be discarded thoughtlessly, as it causes the channel's second-order statistics to be non-stationary jointly in time and frequency. We presented a numerical BER performance analysis of a vehicular communication system based on the IEEE 802.11p Standard. The obtained results show that the channel's non-WSSUS characteristics stemming from the DS's frequency variations degrade significantly the system's performance when a combined TF interpolation strategy is applied for channel estimation and noise cancellation.

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