Chapter 3
Task Contexts in Dutch Mathematics Education

Pauline Vos

Abstract This chapter offers a description of task contexts in mathematics education in the Netherlands. International comparative studies show that the Dutch average percentage of mathematics tasks with real-life connections per lesson exceeds any other country by far. This tradition goes back more than 500 years, when the earliest mathematics textbooks in the Dutch language consisted entirely of tasks set in commercial, naval and building contexts. To analyse and characterise the task contexts, I use the notion of usefulness, which is a perception by students on future practices outside school. A distinction is made between bare tasks (without contexts), tasks with mathematical contexts (e.g., matchstick pattern problems), dressed-up tasks (hiding a mathematical question), tasks with realistic contexts with questions that are useful within the context, and tasks with authentic contexts. The empirical part of this chapter contains an analysis of a mathematics textbook chapter and a sample of examination tasks. This analysis shows that Dutch mathematics education contains many links to real-life, which is not just verbally presented, but also visually with drawings, photos, diagrams and other visualisations. The contexts are drawn from a wide spectrum of areas in real-life, reflecting that mathematics can be found anywhere in society. The examinations contain more authentic aspects than the textbook, and the higher-level examinations have more authentic aspects than the lower-level examinations. Nevertheless, contexts both in the examinations and in the textbook can still be artificial, with questions which would not be asked by actors within the context. Task contexts often come from recreational or professional practices, demonstrating to students the usefulness of mathematics in their future lives beyond school.

P. Vos (✉)
University of Agder, Kristiansand, Norway
e-mail: pauline.vos@uia.no

M. Van den Heuvel-Panhuizen (ed.), *National Reflections on the Netherlands Didactics of Mathematics*, ICME-13 Monographs,
https://doi.org/10.1007/978-3-030-33824-4_3

3.1 The Prevalent Use of Real-Life Contexts in Dutch Mathematics Tasks

In 1999, an international study was carried out in which teaching practices in mathematics classrooms at Grade 8 level in seven countries were analysed: the TIMSS Video Study (Hiebert et al., 2003). The participating countries were Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, Switzerland and the United States. In each country 84 random lessons were video-captured, transcribed and quantitatively analysed. The report offers many tables and bar graphs with descriptive statistics for a variety of mathematics classroom aspects. We can see, for example, the percentages of lessons in each country in which the teacher made a goal statement or summarised the taught content, the amount of time spent on whole-class discussion, the role of homework, the complexity of the taught mathematics content, the type of reasoning asked in the activities, and so forth. The chapter "Instructional Practices: How Mathematics was Worked On" contains findings on the mathematical procedures used to solve tasks, the mathematical representations used, the frequency of students being allowed to select their own methods for solving the mathematical tasks and the number of tasks embedded in real-life situations.

In many of the report's statistics, the Netherlands does not stand out at all. Many of the Dutch frequencies in the tables and bar graphs are similar to those of the other countries, in particular to those from Australia, the Czech Republic or Switzerland. Just like in other countries, the Dutch teacher sits and stands in front of the class, discusses homework, presents new content to the whole class, and the students individually do exercises to practise the mathematics taught.

However, there is one result in which the Netherlands distinguishes itself from the other six countries in the study, see Fig. 3.1. This involves how tasks are presented to the students. In the study, a distinction is made in two categories. The first category contains tasks which are presented by using mathematical language only, such as "Graph the equation: $y = 3x + 7$" or "Find the volume of a cube whose side measures 3.5 cm." The tasks in this category are given in numbers, mathematical operations and symbols, and the verbal expressions relate to mathematical objects only. The second category contains tasks which are presented to students within a real-life context, such as "Estimate the surface area of the frame in the picture below" or "Samantha is collecting data on the time it takes her to walk to school. A table shows her travel times over a two-week period; find the mean." Whether teachers brought in real-life connections at a later stage of the lesson was not included in the statistics. The categorisation only deals with the set-up of the tasks.

In Fig. 3.1, we see that in the 84 randomly selected mathematics lessons in Grade 8 in the Netherlands, there is a relatively smaller percentage of tasks which was set up using mathematical language or symbols only, on average 40% of the tasks per lesson. In the other countries, approximately 70–90% of the tasks was set up only with numbers and symbols. The frequencies for the second category show a reverse picture. In the Netherlands, the percentage of tasks per lesson that started from real-life connections is 42%, while in the other six countries this percentage

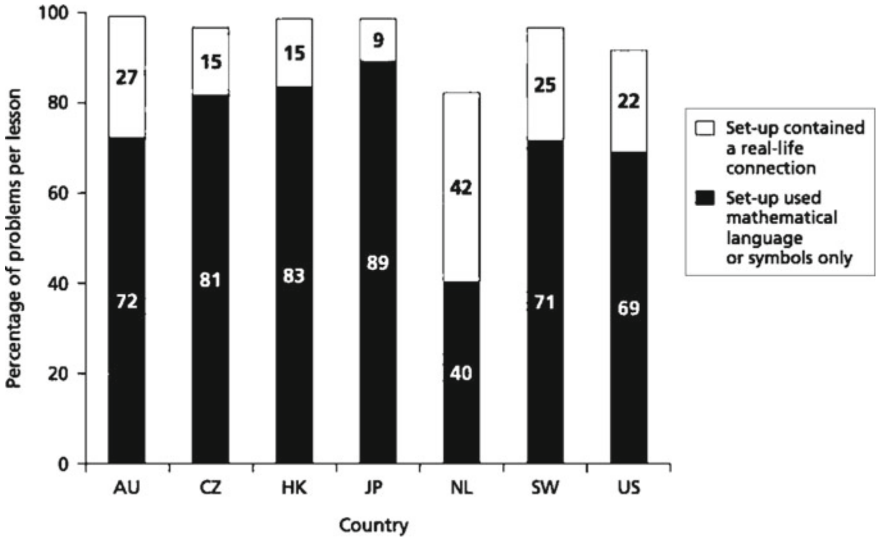

Fig. 3.1 Percentage of problems per mathematics lesson in Grade 8 that was set up with and without the use of a real-life connection (from Hiebert et al., 2003, p. 85); AU = Australia CZ = Czech Republic, HK = Hong Kong, JP = Japan, NL = the Netherlands, SW = Switzerland, US = United States

ranged from 9 to 22%. Another thing that Fig. 3.1 shows is that the percentages for the Netherlands do not add up to 100. The report does not give much explanation for this. There is only a footnote, saying that the researchers were not able to code all tasks. It means that apparently 18% of the tasks in the Dutch classrooms could not be coded as either having mathematical language and symbols only, or having a real-life connection. I will later come back to this issue.

In Fig. 3.1 the Netherlands was compared to only six other countries. When comparing to a larger number of countries we do see a similar phenomenon. The TIMSS 2003 Mathematics report (Mullis, Martin, Gonzalez, & Chrostowski, 2004) contains the results of an analysis of where the emphasis is placed in the intended mathematics curriculum in Grade 8. Educational authorities in all participating countries were asked, for example, whether the focus of mathematics education is on mastering basis skills, on understanding mathematical concepts and principles, on applying mathematics in real-life contexts, on communicating mathematically, on reasoning mathematically, on integrating mathematics with other subjects or on incorporating experiences of ethnic/cultural groups. Again, the Netherlands stands out.

> Applying mathematics in real-life contexts was given a lot of emphasis in the intended eighth-grade curriculum of 17 participants. Botswana, the Netherlands and South Africa reported placing more emphasis on this approach than on mastering basic skills or understanding mathematics concepts. (Mullis et al., 2004, p. 177)

In most countries, there was more emphasis on mastering basic skills or understanding mathematics concepts, but this was not found for the Netherlands.

The results from the TIMSS Video Study and from TIMSS 2003 do not reflect a new phenomenon. Dutch mathematics education has had an emphasis on the usefulness of mathematics for centuries already. This emphasis distinguished Dutch mathematics education from that of many other nations, where the classical Greek mathematics dominated the curriculum, with topics such as two-dimensional geometry, arithmetic, harmonics (the study of structures in music) and astronomy. Greek philosophers such as Plato considered every kind of skill connected with daily needs as ignoble and vulgar, and they praised mathematics for its purity. The justification for teaching esoteric mathematics was aesthetical: it would raise the learners' spirits (Kline, 1953). In the past two thousand years in Europe, this Greek emphasis was upheld in mainstream mathematics education. The higher classes (nobility and clergy) in the aristocratic European societies had an esoteric activity in studying mathematical deductive reasoning through Euclid's *Elements* (Dunham, 1990). It established mathematics as a deductive science, in which axioms and definitions lead to a hierarchy of theorems, and the truth of theorems was established through proofs.

In the 16th century Netherlands, citizens started a revolt against the Habsburg Monarchy, which led to the proclamation of an independent republic, the Republic of the Seven United Netherlands. The Dutch elite consisted of rich, Calvinistic patricians with a pragmatic mind and an aversion to vanity (Schama, 1991). These powerful, urban merchants needed worldly mathematical applications to organise their society, their businesses and their lives. They needed practical mathematics, which did not come from Greece and which was not written in Latin. Already in the late Middle Ages, many Dutch artisans and merchants, including women, gained mathematical competencies. In an early manuscript in the Dutch language, dating back to 1445 and in which calculations with the Hindu-Arabic numerals were taught, mathematics was presented and practised through tasks with commercial contexts, such as converting measures (weights, lengths) and money (pounds, shillings, pennies) (Kool, 1999). Also, the mathematics taught at the citizen's universities was fairly practical, linking to the training of future civil engineers. The mathematics at 17th century universities contained many applications from navigation and the architecture of fortifications (Van Maanen, 1987).

The Dutch curricular emphasis in which mathematics teaching and learning is connected to real-life contexts can be traced back to a mix of Calvinism (abstaining from esoterica), medieval democracy (accessibility of mathematics education to many), and civil societal needs (engineering, commerce). This utilitarian emphasis was already present centuries before the domain-specific instruction theory of Realistic Mathematics Education (RME) (Van den Heuvel-Panhuizen & Drijvers, 2014) was developed by Hans Freudenthal and his colleagues in the 1970s. In fact, the culture of usefulness of mathematics as a curricular emphasis which already existed for 500 years may have created a fertile ground for RME.

Tasks in mathematics education generally consist of a text (whether with mathematical language or not) and a question, or a sequence of questions. The questions within tasks are meant to make student carry out mathematical activities. As the TIMSS Video Study (Hiebert et al., 2003) and the TIMSS 2003 study (Mullis et al., 2004) demonstrated, many tasks in Dutch mathematics education are presented to students within a real-life context. In this chapter I will describe context tasks in Dutch mathematics education as found in textbooks and examinations, and I will focus on the contexts, the questions, and their relationship to reality. However, in this chapter I will not:

- Discuss students' and teachers' perceptions, appreciation or dissatisfaction of contexts in Dutch tasks
- Analyse instructional settings for context tasks (e.g., whether, how and why used for group work, homework, tests, etc.)
- Describe curricular goals of using contexts within tasks, namely to introduce students to a mathematical concept, allowing them to use out-of-school knowledge and informal procedures, from which they can develop more abstract knowledge and more formal procedures.

In Sects. 3.3 and 3.4, I will qualitatively and quantitatively analyse context tasks in a sample chapter of a textbook and in a sample of national examination tasks. To describe distinguishing features of contexts, I will first explain a framework for categorising task contexts.

3.2 Categories for Mathematical Tasks and Their Relation to Reality

Tasks in mathematics classes may or may not have a link to real-life. The TIMSS Video Study (Hiebert et al., 2003) distinguished two categories: (1) tasks, which are presented by using mathematical language only, and (2) tasks which are presented to students within a real-life context. To describe contexts in Dutch mathematics tasks, I will make a more fine-grained categorisation.

When tasks are presented by using mathematical language only, some researchers speak of abstract tasks. I will call them 'bare tasks', following Van den Heuvel-Panhuizen (2005). For example, the exercise

$$3\frac{1}{2} \div \frac{1}{4}$$

is a bare task; the numbers have no other meaning than being numbers. In many textbooks throughout the world, one may see rows of such tasks. The repetitive nature of such tasks is intended to train students to memorise and practice the rule for the division of a mixed number by a simple fraction: dividing by a fraction means to multiply by its inverse. Rows of such bare tasks are part of mechanistic drill-and-practice.

Drill-and-practice may be useful for instilling automated competencies. However, in this chapter I will use the term 'useful' in the way Freudenthal (1968) meant in his seminal article "Why to Teach Mathematics So As to Be Useful", where usefulness of mathematics means that an individual student manages to flexibly and practically apply the mathematics learned in a rich variety of new situations. Wigfield and Eccles (2000) explained usefulness as a motivator, when students expect and value learned content as something that will help them do things better outside of class. Most students are aware that drill-and-practice tasks are useful to pass tests (and that the skills may be forgotten thereafter). Therefore, Williams (2012) specified usefulness as: (1) having 'exchange' value (relating to the possibility that a mark can be used to enter a next level of learning) and (2) having 'use' value (relating to the competence and understanding required to use and apply mathematics in future practices, as professional or as citizen). Bare tasks clearly have 'exchange' value, but their 'use' value is not easily perceived by students. In this chapter, I will refer to 'use value' when speaking of 'usefulness'.

When tasks for students in mathematics classes are presented within a real-life context, there are many words to describe such tasks: word problems, story problems, context(ual) problems, real-world problems, work-related problems, situated problems, and so forth. In this section I will use the terms 'tasks'[1] and 'contexts'. The term context refers to a situation or event in the task, which often is from real-life or from imaginary situations (e.g., fairy tales). Essentially such contexts look quite unmathematical. Contexts in tasks are also referred to as 'figurative contexts' or 'problem situations'. Below I will discuss sub-categories of context tasks.

Task designers (textbook authors, teachers) can opt to adapt the above division $3½ \div ¼ = \ldots$ into the following task:

> "How many quarters of an hour go into three and a half hours?"

In this reformulation, the fraction exercise is given meaning, with all numbers becoming time chunks and the dimension unit is an hour. This is a contextualisation of the original bare task. A contextualised task has little mathematical language and few symbols. One may observe that the task is connected to an unspecified time situation, as it is not clarified what the quarters of an hour and the three and a half hours are part of, nor is any reason given why the question should be answered.

The bare division exercise $3½ \div ¼ = \ldots$ could also have been contextualised into another unspecified context, for example into a pizza situation:

> "How many quarters of a pizza go into three and a half pizzas?"

Again, this context is unspecified, as it is not clarified what the quarter pizzas are needed for and where the three and a half pizzas come from. Furthermore, the bare division exercise could also have been contextualised into money units:

[1] In this chapter, the term 'problem' is used for non-routine, problem-solving tasks. Therefore, terms such as 'practice problems' and (standard) 'word problems' are avoided.

"How many quarters of a dollar go into three and a half dollars?"

Again, this is an unspecified context, as it is not clarified what the quarter coins and the three and a half dollars are used for. Moreover, one may notice that the exercise cannot be contextualised well in money units for some countries; for example, the Euro has no quarter coins.

With the above contextualisations, the bare division of fractions acquires a certain meaning, because the numbers become concrete. Amongst others, Clausen-May and Vappula (2005) and Palm (2002) have convincingly demonstrated that such contextualisations change the cognitive demand of bare tasks, for a variety of reasons. First, the adapted task requires students to read words instead of symbols. Second, many students are discouraged by symbolical tasks and more motivated for contextualised tasks. Also, most students are able to mobilise knowledge acquired outside school and use it for solving the task. For example, they may use the idea that a quarter of an hour equals 15 min, and then use the fact that four times 15 min make an hour. Or, in the pizza situation, they may use the idea that four quarter pizzas make one pizza; or, in the dollar coins situation they may use the idea that four quarter dollars make one dollar. In this way, the divisor is no longer a simple fraction, and the fraction task loses one of the fractions. Thus, contextualisation may increase the task's accessibility and support students' understanding that a division by ¼ can be translated into a multiplication by 4. Such contextualisations could be used in the introduction to a teaching sequence to assist students in understanding mathematical rules for fraction operations, allowing them to use their out-of-school knowledge to first develop informal procedures, from which they can later develop more formal procedures.

By contextualising a task, the numbers get a meaning (in units and dimensions). However, this does not mean that the exercise becomes useful (meaningful or interesting) to all students. Why should anyone calculate the number of quarters of an hour that go into three and a half hours? Why should anyone calculate the number of quarter dollars that go into three and a half dollars? Why should anyone calculate the number of quarter pizzas that go into three and a half pizza? What is the justification for the calculation? In particular, if there are no credible actors described within the context: people or institutions with a problem that needs to be solved. Thus, a context does not imply that the question posed to the students has justification. Therefore, it is important to consider whether the posed question would be asked within the context. If there is no clear need to perform the mathematical activities, other than a didactical need to get a correct answer within the discourse of mathematics learning, the contextualised task is as a 'dressed-up' task, hiding a mathematical task (Blum & Niss, 1991). Many so-called 'word problems' are dressed-up tasks with pointless questions. The task in which is asked "How many quarters of an hour go into three and a half hours?" is a dressed-up mathematical task.

To improve dressed-up tasks, a task designer can make the context more realistic. I use the term 'realistic' here as being related to real. The relationship between real and realistic is considered parallel to the relationships absolute-absolutistic, central-centralistic, dual-dualistic, ideal-idealistic, material-materialistic, natural-naturalistic, and so forth. Realistic means: as if from real-life, close to reality, or could be imagined as real. The term ‘realistic’ is used in this chapter for contexts in tasks only, and it needs to be distinguished from the meaning of ‘realistic’ in RME, which refers to the use of a certain sequence of activities, starting from more concrete tasks, for which students use common knowledge and after a carefully designed sequence of activities, the students are guided towards more formal mathematical thinking. Thus, in RME the curriculum may contain tasks with realistic contexts, but there may also be bare tasks. The adjective ‘realistic’ in RME is not the adjective for all tasks within that approach. Moreover, I would like to emphasise that in this chapter I only describe contexts for tasks in Dutch mathematics education, and not the philosophy for including or sequencing different sorts of tasks.

The dressed-up task “How many quarters of an hour go into three and a half hours?” can be contextualised with a realistic context. A first example is:

> A doctor in a health centre has consultations in the morning from 8.30 to 12.00 h. The patients have consultation visits of a quarter of an hour. How many patients can the doctor see?

In this task, again the students have to calculate how many quarters of an hour go into three-and-a-half hours. However, in this doctor context the justification for the calculation is to know the maximum number of patients, excluding the options for coffee breaks or speedy five-minute consultations. The question in the task is a question that may be asked of a medical doctor in a real situation. An answer to the question is useful for planning purposes within a professional practice. Additionally, assuming that most students know the system of medical consultations by appointment, the division exercise becomes experientially real (Gravemeijer, Cobb, Bowers, & Whitenack, 2000). A second example of a realistic context for this task is:

> A team of whale watchers (biologists) has a boat in a coastal area of a deep ocean. One day they pursue one animal for observations. Their boat has fuel that will last for a trip of three and a half hours. In between breathing the animal plunges into the deep water and then it cannot be observed. The animal plunges for a quarter of an hour before it needs to breathe air again. How many times can the whale watchers see the mammal?

The question in the task is a question that might be asked in a real whale watching situation, and the answer to the division gives an estimate for the maximum number of sightings. Thus, in this context such questions are asked, or in other words, the context of whale watching justifies the division calculation making the mathematical calculation useful. The whale watching context is an example of a context that the students may never have experienced in their lives, unlike the context of the medical doctor and his consultation slots. However, many students may have heard of the experience of whale watching, or may have seen it on television. This makes the task imaginable for students, without being experientially real. With this whale watching context, one may also observe, that adding realism implies adding complexity. It is a realistic context, and not a real, authentic context. In a truly authentic situation, the whale watchers need extra fuel for returning home, for possible bad weather, and whales do not surface exactly at the beginning of a trip. In real life, the question may require a more complex calculation than a mere fraction division.

In the above text, I have described a designer's hypothetical road, thereby distinguishing between possible contexts for one and the same bare task. This creates a more fine-grained categorisation of the category of tasks which are presented to students within a real-life context that has been distinguished in the TIMSS Video study (Hiebert et al., 2003). I now have the following sub-categories for mathematical tasks and their relation to reality:

- 'Bare tasks', which are presented in mathematical language and symbols
- 'Dressed-up tasks', which hide a mathematical task; they have a certain context and a pointless question; this category includes tasks with realistic contexts, in which the need for answering the question is not justified through the context
- 'Tasks with a realistic context' (experientially real or imaginable), in which the question makes sense within the context, and an answer to this question has use value within the context.

In addition to these categories, I will introduce two more categories of mathematical tasks. The bare task on the division of a mixed number by a simple fraction $3\frac{1}{2} \div \frac{1}{4} = \ldots$ can be embedded into a mathematical context, by showing a bar, which consists of three-and-a-half units (see Fig. 3.2).

Without using (much) mathematical language, a task designer can ask for the number of small units that would fit into the larger, or can ask how many of the small units would make the same area as the larger one. This yields another category:

- 'Tasks with mathematical contexts'.

Tasks with mathematical contexts do not contain (much) mathematical language, but they are about mathematical objects and their properties. Such tasks can be found in geometry and are often visual. They can be, for example, about tiling. Also, tasks on matchstick patterns or growing patterns of triangular shapes, as used in early algebra (see, for example, Radford, 2006) have mathematical contexts, which are not encountered in real-life. The need to answer the question is never justified by the context, because mathematical contexts do not have actors who need solutions. For students, it will often be hard to perceive any 'use value' to an answer. Tasks with

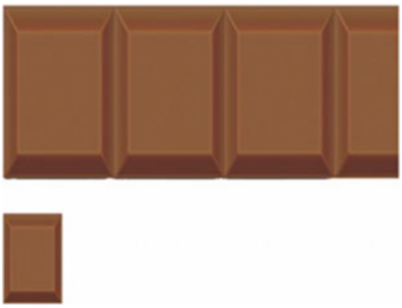

Fig. 3.2 Bar visualisation for the fractions 3½ and ¼

a mathematical context can be distinguished from a bare task through the language used. Bare tasks contain mainly mathematical language and symbols, while tasks with a mathematical context have more informal language. A task with a mathematical context contains descriptions that give a certain meaning to mathematical concepts. In some cases, the mathematical context can even be associated to real-life objects. In the example above, the rectangular bar can be associated to a chocolate bar, without this explicitly being mentioned in the task.

I will use one more category to describe the context in mathematical tasks. Among others, Dierdorp, Bakker, Eijkelhof, and Van Maanen (2011), Palm (2002), Vos (2011, 2015) and Wijers, Jonker, and Kemme (2004) have used the term 'authenticity' when describing a context of a task. This term refers to being a genuine (true, honest) context, not being a copy or a simulation. Such a context may be related to practices outside school (e.g., the workplace). Authenticity is a characteristic that requires clear evidence, for example, through photos (as opposed to drawings), or when governmental datasets are used in a statistical task. Thus, I add another category of tasks and their contexts:

– 'Tasks with authentic contexts', in which the origin of the context is explained through convincing resources. In this category, also, the context justifies the question, and an answer is useful within the described context.

It remains to be noted that not all tasks with an authentic context contain meaningful questions. For example, the context of the 'Big foot' task (Blum, 2011), in which a giant shoe is depicted by a photo, is authentic. The photo is the proof of its existence in real-life. The question here is to calculate the height of a person who fits this shoe. This task matches the curricular concept of mathematical similarity and proportionality, which makes the task relevant to mathematics teachers. However, depending on one's background, one may raise other questions. A shoemaker may ask how much leather is needed for such a giant shoe. A thief may ask how much the statue weighs. An art student may ask into what artistic tradition the statue fits. In other words, the task resources may be authentic, but the mathematical question that is posed in the task is only useful to practise mathematical operations. It is not a question that would emerge from people working with statues, nor from people admiring art. Blum's (2011) classroom experiment also showed that the question did not make sense to students. They just took the numbers out of the text and performed erratic operations. Therefore, the task is a dressed-up mathematical task on similar triangles, only distinguished from an ordinary word problem by the authenticity of its context, but not of its question. In an interesting alternative to the 'Big foot' task, Biccard and Wessels (2011) designed a task to assist the police in relating foot prints found at crime scenes to the possible size (height and weight) of suspects. In this way, this task did not merely ask for finding a number, but became realistic (not fully authentic, but imaginable) and the posed question was a useful component of crime scene investigation.

In sum, the above categorisation contains five categories: the first is bare tasks and then I listed four categories for context tasks, including the category of tasks with mathematical contexts. This categorisation will assist in describing tasks and their contexts in Dutch mathematics education.

3.3 Tasks Contexts in a Dutch Secondary Education Mathematics Textbook

In this section I will provide the results of an analysis of tasks in a mathematics textbook following the previously outlined categorisation. For this analysis, I used the textbook series *Getal & Ruimte* (Reichard et al., 2006) which is most widely used in the Netherlands. The analysis is based on the textbook for students in Grade 10 in HAVO (general secondary education)[2] and within this textbook I chose the chapter "Working with Formulae", which I consider as representative for this textbook in particular, and for textbooks used in mathematics education in the Netherlands in general. This chapter has four sections, each of which can be covered within approximately two to three hours. The chapter is introduced with a page-wide photograph of four students doing a physics experiment and a text stating that it takes a number of measurements to create a formula, which can be used to predict where a moving object will halt. This introduction (without task) has a realistic context, which is experientially real, as many students have done experiments in physics classes. It explains the usefulness of mathematical models for making predictions. The photograph showing four students doing a speed experiment in a laboratory creates an aspect of authenticity.

The first section is about creating and working with formulae in two variables. In all tasks and worked examples[3] the variables are x and y. Out of the 18 tasks, there are 14 bare tasks and 4 context tasks (22%). Out of the six worked examples there is one worked context task (17%). To give an idea of the contexts, the first context task is about a school class going on a weekend camping trip and Peter (an unspecified boy) organises the shopping to the bakery, buying only loaves and buns, each of which has a unit price. The question is to create a formula for the total costs in two variables (x for loaves of bread, y for buns). The worked example is about a concert hall, which has two price levels with tickets being €12 or €15, and the total income will be $12x + 15y$. The created formulae are simplified versions of more complex price models, which are used for economic decisions. However, the contexts of a certain Peter buying bread or a certain concert hall selling tickets do not provide any evidence whatsoever that there is a need for creating such formulae within such simplified contexts. Also, the formulae in two variables are not used for any further problem solving related to the context. Thus, the tasks are dressed-up tasks offering students training to find formulae.

The second section is about using given equations in two variables. It starts with five bare tasks and a bare worked example on finding intersection points of two

[2] Grants admission to higher vocational education.

[3] A worked example is a task with a complete explanation showing how to solve the task.

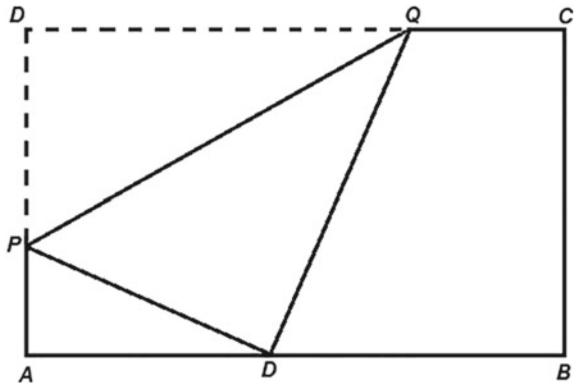

Fig. 3.3 Folding task on the position of *P* to make *AP* equal to *AD*

graphs, or on determining the parameters of a parabola $y = ax^2 + bx + c$ passing through three given points. Then, there are five tasks with mathematical contexts. The first is on paper folding (see Fig. 3.3). The context is a rectangular piece of paper *ABCD* of size 20 cm by 30 cm and point *D* is folded onto side *AB*. The question is to find the position of point *P* on *AD*, which will make *AP* equal to *AD*. The students are invited to try the folding physically first. Through some scaffolding, the students are guided to take $AP = x$, determine the quadratic equation $x^2 + x^2 = (20 - x)^2$ and from there find point *P*. The mathematical context is described in limited mathematical language.

The worked example also has a mathematical context and asks: "How can a letter T be drawn inside a circle with radius 6, with the restriction that the vertical bar of the T must be equally long to its horizontal bar?" Thus, out of 10 tasks in the second section, there are five tasks with mathematical contexts (50%). Out of the two worked examples there is one with a mathematical context (50%). This section does not have a single realistic context.

The third section contains 12 tasks, all of which are tasks with contexts. A sample of four tasks is shown in Fig. 3.4, and these tasks illustrate the others in this section. The tasks have a repetitive format. First a context is described and a formula with parameters *a* and *b* is given, then two data points are given that need to be fitted into the formula, and from there the parameters can be calculated. All tasks end with the same small sentence: "Calculate *a* and *b*."

The contexts in these four tasks present problem situations in which phenomena have to be modelled mathematically: the growth of bacteria, the density of traffic related to the tariffs of toll roads, the productivity of timber production depending on the growth time of trees before chopping them, and the effectiveness of TV-commercials. The other tasks in the section additionally have contexts of balls in sports, packaging of tin cans and wooden boxes, and farmers enclosing paddocks. The contexts are realistic and imaginable, assuming that most Grade 10 students have little personal experience with these areas, but a certain notion that such areas could exist. The mathematical models of the problem situations resonate with the text on laboratory research at the beginning of the chapter, in which it was explained that

30. For the number of bacteria N in millions within a closed space one assumes the formula $N = at^3 + bt + 200$. Here t in the time in days, with $0 \leq t \leq 10$.
At $t = 5$ there are 285 million bacteria and at $t = 8$ there are 648 million. Calculate a and b.

31. For the number of passenger cars per day A that uses a new part of a toll road a consultancy company uses the model $A = aT^2 + bT + 6000$. Here T is the fare in Euros, in which T can maximally be €7,00.
Research shows that at a fare of €2,50 there will be 40 000 passenger cars using the road. This figure decreases to 25 000 with a fare of €5,00.
Calculate a and b.

32. On a forest plot one plants 750 young trees per ha. After t years, with $15 \leq t \leq 40$, the trees are logged. For the timber production P per ha one uses the model $P = at^2 + bt - 2200$. Here P is the timber production in m^3 per ha.
When logging after 20 years one expects an average timber production per year of 80 m^3 per ha and when logging after 25 years one assumes an average timber production per year of 89,5 m^3 per ha.
Calculate a and b.

33. Research has shown that the effect of a tv-commercial first increases and eventually decreases. In this, the term effectivity E is used. E is a number between 0 and 10.
A marketing agency uses the formula $E = an^2 + bn$, in which n is the number of times that a commercial is broadcasted.
For a commercial the effectivity is 7,2 at 20 broadcastings and 9,0 at 30 broadcastings.
Calculate a and b.

Fig. 3.4 Tasks from the chapter "Working with Formulae" in the textbook series *Getal & Ruimte, Wiskunde HAVO B, Part 2* (Reichard et al., 2006, p. 22) (translated from Dutch by the author)

mathematical models are used for making predictions. However, the usefulness of calculating a and b is nowhere explained.

The textbook authors have kept the contexts vague. There is no evidence of an authentic resource of the contexts. They describe the contexts in impersonal terms by using the pronoun 'one', like in 'one can assume' or 'one uses a model' or by referring to unspecified consultancy companies and marketing agencies. In the other tasks a photo was added (the packaging task shows a tin can) and people's names were invented (the farmer who encloses a paddock is a certain farmer Wunderink and the tennis player is a certain Richard) to improve the realism. Also, it is highly unlikely that the mathematical formulae are authentic. Instead they are quadratic or cubic polynomials to fit the cognitive level of Grade 10. If the formulae had originated from real research, this could have been mentioned.

In the above analysed section of the chapter, students encounter twelve different real-world situations, in which mathematical formula are used to model phenomena. However, these contexts are not to be considered for answering the posed question to calculate the parameters. What students need to do is lift the formula from the text, identify the two variables and, in the text, find the two appropriate data points that should be inserted into the formula. This results in two equations with two unknowns, which can be solved for *a* and *b* respectively. The answers that the student will give are either correct or incorrect, and these answers have nothing to do with the contexts. In not one task, the created formula is put to use and given meaning within the described context. These tasks can also be termed as ‘reproductive mathematising’ (Vos, 2013). They are dressed-up tasks for solving two variables from two equations. In fact, the sequence of twelve tasks in this section is a dressed-up drill-and-practise activity.

The fourth section of this textbook chapter contains ten tasks and two worked examples, all of which are contextualised. The contexts are on the trajectory of a football, a physics experiment with a cart moving over a rough surface, water running out of a bath tub, a biologist counting crane-flies, labour productivity, and a meteorologist measuring weather temperatures. In these tasks, a context is described realistically (imaginable and not authentically), a formula is given with two parameters, and two data points are offered so students can determine the parameters *a* and *b* (just like in Fig. 3.4). The difference with the previous section is that there are additional questions to put the formulae to use to make predictions or to find maximal values. In all tasks, the questions are useful to imaginable actors in the context. Therefore, I evaluate these tasks as having realistic contexts, and the questions make sense within the context.

Summing up, in this average Dutch mathematics textbook chapter there were 50 tasks, out of which 19 (38%) were bare tasks posed in mathematical language only, 5 (10%) had mathematical contexts, and a little more than half of the tasks were related to real-life. Of these tasks 16 (32%) were dressed-up tasks and there were 10 (20%) tasks with realistic contexts and questions that made sense within the context. There were no authentic contexts. As a whole, this textbook chapter confirms the high level of contextualised activities in Dutch mathematics classrooms.

The analysis of this exemplary chapter may also offer a possible explanation for the tasks that the TIMSS Video Study (Hiebert et al., 2003) could not code, which means that according to this study 18% of the tasks in the Dutch classrooms had neither mathematical symbols only nor a real-life connection. It is possible that such tasks had informal language only for describing a mathematical context. As we have seen, such tasks can be found to a small extent in Dutch mathematics education (10% of the tasks in the analysed textbook chapter). Therefore, the TIMSS Video Study found tasks in the Netherlands that did not match either of the categories of (1) having mathematical symbols only, or (2) having a real-life connection.

Another observation that has to be made is that judging tasks individually within a chapter has its limitations. The tasks in the third section, which all asked: “Calculate a and b” were judged as dressed-up, because the answers for the parameters *a* and *b* were not useful at the very moment of doing the tasks. However, this judgement should be nuanced when the chapter is observed as a whole. The chapter starts with an explanation that gives a connecting thread until the end of the chapter. The introduction to the chapter highlights the importance of mathematical models for making predictions, and in the final section this is practised through scaffolded tasks, in which formulae are created to make predictions. Thus, the practise-and drill questions “Calculate a and b” become useful as an intermediate step for follow-up activities. So, what in the short term may look as dressed-up may be a stepping stone towards a realistic task where the context justifies the question, and where an answer is useful within the described context.

3.4 Contexts in Dutch Secondary Education National Mathematics Examinations

In this section I discuss the results from analysing a sample of mathematics tasks from the Dutch national examinations at the end of secondary education. The Netherlands has a system of exit examinations, which implies that all students who want to enter tertiary education (higher vocational education or university) have to pass one of the national examinations. Since teachers at secondary schools have to prepare students for these examinations, the national examinations have quite an impact on mathematics classroom practice, including the role of contexts in mathematics tasks. The purpose of my analysis of the examination tasks was: (1) to verify earlier claims on the high frequency of mathematics tasks with a real-life connection in Dutch education, and (2) to characterise the tasks.

For this analysis, I selected the examinations from 2010 for all available secondary school levels: pre-vocational (VMBO, examination at the end of Grade 10), general (HAVO, examination at the end of Grade 11), and pre-university (VWO, examination at the end of Grade 12). Each of these secondary school levels (pre-vocational, general and pre-university) has several examinations, depending on the track that students follow. At the pre-vocational level, there are three different examinations (for the tracks KB, BB, and GLTL), which differ mainly on cognitive demand (KB for the lowest achievers, and GLTL for the highest achievers in pre-vocational education). At general and pre-university level, there are two different examinations, for Mathematics A and for Mathematics B. The subject of Mathematics A is meant for students who are more interested in the social and economic sciences, while Mathematics B is for students interested in natural sciences and technology. In the analysis, I did not include the experimental computer-based examinations and the re-examinations, because the characterisation of contexts in these was not expected to differ from the regular examinations.

Each analysed examination paper is 11–13 pages long and contains much text, in which contexts are described, often accompanied by illustrations, diagrams or photographs. Students' reading time for these examinations must be considerable. All questions are grouped under a theme, which is indicated by a clear title. For example, the following titles are used:

- At pre-vocational level (VMBO GLTL 2010) there are 25 tasks grouped under the following headers: 'Pita bread', 'Quetelet index', 'From Betancuria to Antigua', 'Magnetic', 'Façade flag', 'Thunder and lightning'
- At general level (HAVO Mathematics A 2010) there are 23 tasks grouped under the following headers: 'A game of tennis', 'China's defence budget', 'Gas transport', 'Bullet proof vests', 'Fuel consumption by airplanes'

– At pre-university level (VWO Mathematics B 2010) there are 18 tasks grouped under the following headers: ‘Equal surfaces’, ‘Trivet’, ‘Rectangles touching a circle’, ‘Condensators’, ‘A rectangle in pieces’, ‘Logarithm and 4th power’, ‘A geo triangle’.

The titles indicate a wide variety of areas where mathematics can be used. Under each title a context is described, which serves as a context for several questions. In this way, each question does not have its own context, which reduces the reading time. Questions belonging to a title are independent of one another, that is, if students cannot answer one question, they can still complete ensuing questions that have the same context.

For example, the examination at the pre-vocational level includes questions grouped under the title ‘Pita bread’. The context is an event in the city of Eindhoven on 24 December 2004 where a huge pita bread was baked. The questions are about the diameter and the area of the baking tray, the required amount of flour, and the number of normal-size sandwiches that could be cut from it. The context is clearly authentic, as testified by a given date and an existing Dutch city (allowing for verification of the event), and the questions make sense within the context. The task shows clearly how mathematics can be useful outside school within recreational domains.

In this examination at the pre-vocational level, I also found a number of questions grouped under the title ‘Façade flags’, which contains the illustrations shown in Fig. 3.5. The text explains that there are three possible models. The students are asked to make a drawing in which Model 1 is mirrored, to calculate the lengths of sides c (in Model 1) and d (in Model 2), and to calculate the area of Model 3. As

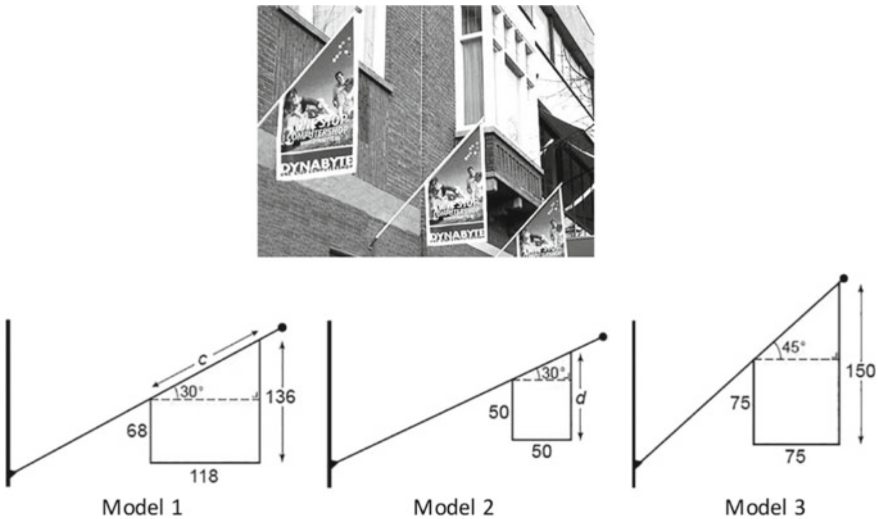

Fig. 3.5 Illustrations that go with the ‘Façade flags’ task (VMBO GLTL Examination, 2010)

there is no clear reason given why the drawing and calculations are needed, I coded the questions as 'dressed-up'.

The examination for Mathematics A at the general level (HAVO) starts with two groups of questions clustered by authentic contexts. The context of the first group is a tennis match between Roger Federer and Fernando Gonzalez at the Australian Open Championships of 2007. The statistics of the match are given (points played, points won on first service, points on second service, and so on) and the questions are about the probabilities of winning points, which are meaningful for sports fans (and betting companies). The context of the second group of questions is China's defence budget according to the Pentagon and according to the Chinese government information. The questions are about trends in the data, which are meaningful for critical observers of political information.

The third group of questions is clustered under a context of a company that transports gas, which is a context related to the fact that the Netherlands has a natural gas reserve and exports gas. The text explains that in the Netherlands there is a network of gas pipes bringing gas to families and businesses for heating and cooking. If it is very cold, then customers will need more gas and the maximum capacity of the network is reached. A certain unidentified company for gas transport uses the formula $P = 5.5 + \frac{18-T}{30} \cdot 94.5$, where P is the percentage of the capacity used, and T is the temperature. First, the students are asked about the properties of the formula (the range for T). Then some data are given on the occurrence of temperatures below -12 °C over the past 100 years, and the students are asked for a probability that such low temperatures occur on a day within a three-month winter season. Finally, the students are told that the above stated formula can be re-written in the shape $P = aT + b$ and do they have to calculate a and b.

The above questions are all set within an industrial context, implying that companies use mathematical formulae for their planning. However, the context is artificial, the given formula lacks credibility for real-life use, the probability question is not used within the context, and the final question to calculate parameters is pointless. All questions were therefore coded as dressed-up tasks.

In the Mathematics B examination at pre-university level I found a number of bare mathematics tasks, mainly on calculus. There is a task in which a trivet is shown (see Fig. 3.6), which consists of bars that can hinge. The text explains that this trivet has 19 equal rhombuses, and that the thickness of the bars will be ignored for creating a mathematical model for this trivet. The leftmost hinging point is indicated with P, the midpoint of the middle rhombus with O. The inner angle at P is α (in radians), and for the side of a rhombus length 1 is taken. Length l and width w of the model are functions of α, whereby $0 \leq \alpha \leq \pi$, and it is given that $l = 10\cos(½\alpha)$ and $w = 6\sin(½\alpha)$. The question then is: "Show that the formula for l and w are correct." In the following questions these formulae have to be used for calculating angle α, at which w increases with the same rate as l decreases, for reconstructing a given formula for distance OQ, and for calculating angle α at which the trivet fits within a circle. I coded the task as a task with a mathematical context.

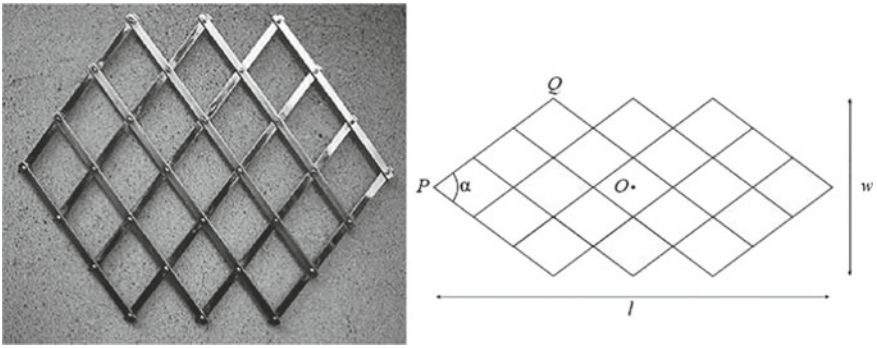

Fig. 3.6 Illustrations that go with the 'Trivet' task (Pre-university secondary education, Mathematics B Examination, 2010)

To give an overview of the context characteristics of the tasks in the Dutch mathematics examinations, Table 3.1 displays the types of tasks involved at each school level and the proportion of points that can be earned for each type of task.

From Table 3.1, we see that many of the tasks in the examinations at the vocational level and for Mathematics A at the general secondary education level, were set in contexts, in particular in realistic contexts. There were quite some dressed-up tasks (realistic descriptions, but questions that were not justified through the context), such as the 'Façade flags' task or the 'Gas company' task. All tasks contained some authentic contexts and questions that were relevant in such contexts, such as the 'Pita bread', the 'Game of tennis' and the 'China's defence budget' tasks. The authentic contexts were mostly used in Mathematics A in general and pre-university secondary education and not that much in vocational education, meant for students who generally have a lower level of learning. Obviously, authentic contexts are more complex and require mathematics with higher demands. The examination papers for

Table 3.1 Context characteristics of the tasks in the Dutch secondary education mathematics examinations in 2010

Secondary school type (total amount of points)	Proportion of points that can be earned in				
	Bare tasks (%)	Mathematical context tasks (%)	Dressed-up tasks (%)	Realistic context tasks (%)	Authentic context tasks (%)
Vocational	0	14	16	57	13
Mathematics A					
General	0	0	18	37	45
Pre-university	0	9	38	0	52
Mathematics B					
General	29	5	56	0	10
Pre-university	25	57	18	0	0

Mathematics B (for students aspiring natural sciences and technology) contain more bare tasks than any other examination paper. This can be explained, because Mathematics A is a subject that aims more at modelling competencies and the practical use of mathematics, while Mathematics B aims more at conceptual understanding of mathematical concepts, such as the derivative or trigonometric functions. When we take the Mathematics A and B examinations together and compare the examinations at the general secondary education level with those at the pre-university secondary education level we see that the former has more dressed-up tasks, while the latter has more mathematical contexts (such as the 'Trivet ask').

Overall, the Dutch mathematics examinations of 2010 contain many context tasks, whether dressed-up, realistic or authentic, confirming the Dutch emphasis on connecting mathematics to real-life contexts. The contexts in the examinations were primarily from recreational practices (sports and leisure) or professional practices (commerce, research). As a driving force in classroom practice, the examinations clearly set out that mathematics is useful in many real-life situations, and that students can expect to encounter unexpected areas of mathematics application in the examinations.

3.5 Conclusion on Contexts in Dutch Mathematics Education

In this chapter I have described characteristics of contexts in mathematics tasks in the Netherlands. The underlying frame was the notion of usefulness as a subjective perception by students on future practices outside school. In analysing the tasks used in Dutch mathematics education, I made a distinction between bare tasks (without contexts), tasks with mathematical contexts (e.g., matchstick pattern tasks), dressed-up tasks (a context with a pointless question that hides a mathematical question), tasks with realistic or authentic contexts with questions that are useful within the context. I analysed a chapter of a mathematics textbook and a sample of examination tasks, confirming that, indeed, Dutch mathematics education contains many links to real-life, which are not just presented verbally, but also visually with drawings, photos, diagrams and other visualisations. The contexts are drawn from a wide spectrum of areas in real-life, reflecting that mathematics can be found anywhere in society. Most task contexts come from recreational or professional practices (economy, research), demonstrating to students the usefulness of mathematics in their future lives beyond school.

Many contexts can be said to be realistic (imaginable or experientially real), without being authentic. It was observed that the analysed examinations contained more authentic aspects than the textbook chapter, and the higher-level examinations have more authentic aspects than the lower level examinations. Nevertheless, there were also many artificial contexts in which the posed questions would not be asked by possible actors in these contexts.

Finally, a consequence of the typical Dutch feature of offering mathematics tasks with a relation to real-life is that the attribution of tasks to subjects is not always clear. This means that a task which in other countries is considered a task belonging to science education, can in the Netherlands be considered a mathematics task. This is what I experienced when I offered one of the physics tasks of TIMSS 1999 (see Fig. 3.7) to a number of mathematics teachers, and all of them said that they con-

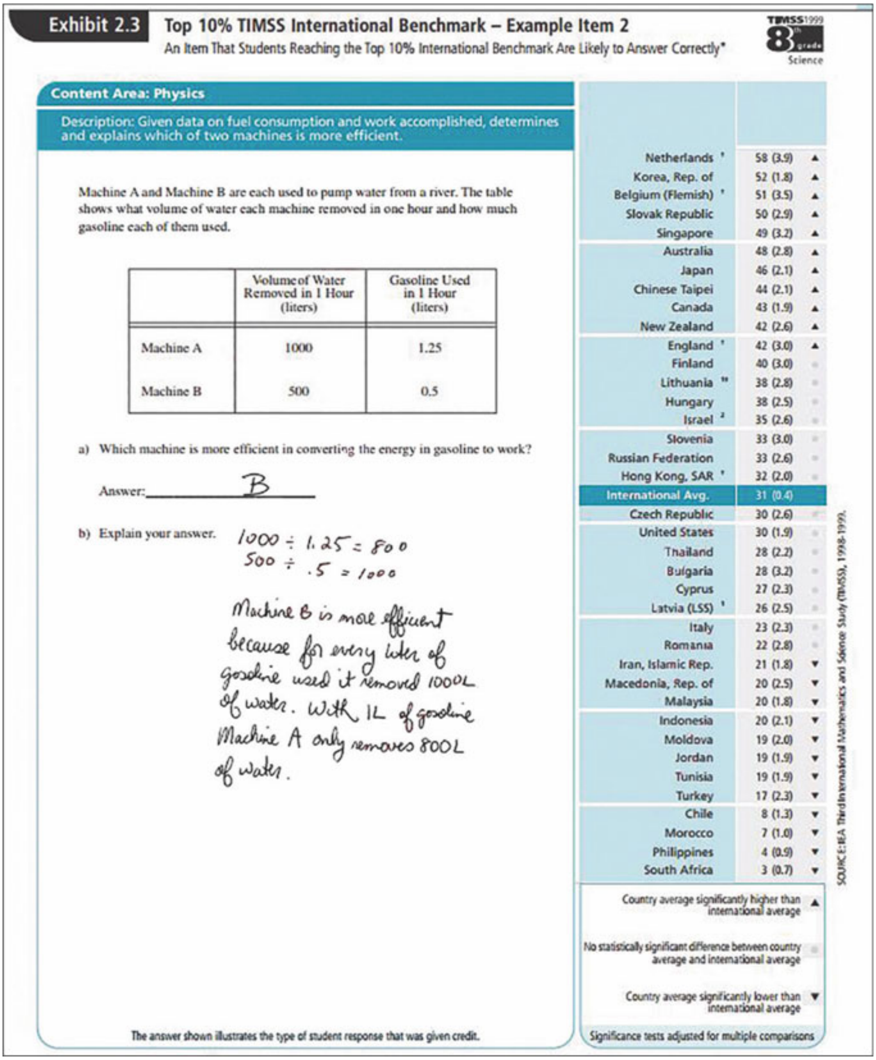

Exhibit 2.3 Top 10% TIMSS International Benchmark – Example Item 2

An Item That Students Reaching the Top 10% International Benchmark Are Likely to Answer Correctly*

TIMSS 1999 8th grade Science

Content Area: Physics

Description: Given data on fuel consumption and work accomplished, determines and explains which of two machines is more efficient.

Machine A and Machine B are each used to pump water from a river. The table shows what volume of water each machine removed in one hour and how much gasoline each of them used.

	Volume of Water Removed in 1 Hour (liters)	Gasoline Used in 1 Hour (liters)
Machine A	1000	1.25
Machine B	500	0.5

a) Which machine is more efficient in converting the energy in gasoline to work?

Answer: B

b) Explain your answer.

$1000 \div 1.25 = 800$
$500 \div .5 = 1000$

Machine B is more efficient because for every liter of gasoline used it removed 1000L of water. With 1L of gasoline Machine A only removes 800L of water.

The answer shown illustrates the type of student response that was given credit.

Country	Percent correct	
Netherlands †	58 (3.9)	▲
Korea, Rep. of	52 (1.8)	▲
Belgium (Flemish) †	51 (3.5)	▲
Slovak Republic	50 (2.9)	▲
Singapore	49 (3.2)	▲
Australia	48 (2.8)	▲
Japan	46 (2.1)	▲
Chinese Taipei	44 (2.1)	▲
Canada	43 (1.9)	▲
New Zealand	42 (2.6)	▲
England †	42 (3.0)	▲
Finland	40 (3.0)	
Lithuania **	38 (2.8)	
Hungary	38 (2.5)	
Israel ²	35 (2.6)	
Slovenia	33 (3.0)	
Russian Federation	33 (2.6)	
Hong Kong, SAR †	32 (2.0)	
International Avg.	31 (0.4)	
Czech Republic	30 (2.6)	
United States	30 (1.9)	
Thailand	28 (2.2)	
Bulgaria	28 (3.2)	
Cyprus	27 (2.3)	
Latvia (LSS) †	26 (2.5)	
Italy	23 (2.3)	
Romania	22 (2.8)	
Iran, Islamic Rep.	21 (1.8)	▼
Macedonia, Rep. of	20 (2.5)	▼
Malaysia	20 (1.8)	▼
Indonesia	20 (2.1)	▼
Moldova	19 (2.0)	▼
Jordan	19 (1.9)	▼
Tunisia	19 (1.9)	▼
Turkey	17 (2.3)	▼
Chile	8 (1.3)	▼
Morocco	7 (1.0)	▼
Philippines	4 (0.9)	▼
South Africa	3 (0.7)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Fig. 3.7 'Fuel consumption of pumping machines' task from TIMSS 1999 (Martin et al., 2000, p. 65; copyright 2000 by International Association for the Evaluation of Educational Achievement (IEA), reprinted with permission) sidered it a normal mathematics task. The task was about two machines, which have a different fuel consumption and a different pumping capacity. The question was: "Which one is more efficient?" Such a question within the context of pumping water makes particular sense in a low-lying country which needs to stay dry, and which has a commercial culture in which effectivity and productivity are frequently used concepts. No wonder that on this task the Dutch students had the highest average score.

References

Biccard, P., & Wessels, D.C.J. (2011). Documenting the development of modelling competencies of grade 7 mathematics students. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 375 − 384). New York, NY: Springer.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15 − 30). New York, NY: Springer.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects—State, trends and issues in mathematics instruction. *Educational Studies in Mathematics, 22*(1), 37–68.

Clausen-May, T., & Vappula, H. (2005). Context in maths test questions—Does it make a difference? In D. Hewitt & A. Noyes (Eds.), *Proceedings of the 6th British Congress of Mathematics Education* (pp. 41 − 48), Warwick, UK: BCME.

Dierdorp, A., Bakker, A., Eijkelhof, H., & Van Maanen, J. (2011). Authentic practices as contexts for learning to draw inferences beyond correlated data. *Mathematical Thinking and Learning, 13*(1–2), 132–151.

Dunham, J. (1990). *Journey through genius—The great theorems of mathematics*. New York, NY: Wiley.

Freudenthal, H. (1968). Why to teach mathematics so as to be useful. *Educational Studies in Mathematics, 1,* 3–8.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, modeling, and instructional design. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 225–274). Mahwah, NJ: Lawrence Erlbaum Associates.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K., Hollingsworth, H., Jacobs, J., et al. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*. Washington, DC: National Center for Educational Statistics.

Kline, M. (1953). *Mathematics in western culture*. Oxford, UK: University Press.

Kool, M. (1999). *Die conste vanden getale. Een studie over Nederlandstalige rekenboeken uit de vijftiende en zestiende eeuw, met een glossarium van rekenkundige termen* [The art of numbers. A study on Dutch mathematics textbooks from the 15th and 16th century]. Doctoral dissertation. Hilversum, The Netherlands: Uitgeverij Verloren.

Martin, M. O., Mullis, I. V., Gonzalez, E. J., Gregory, K. D., Smith, T.A., Chrostowski, S. J., Garden, R. A., & O'Connor, K. M. (2000). *TIMSS 1999 international science report. Findings from IEA's Repeat of the Third International Mathematics and Science Study at the eighth grade*. Chestnut Hill, MA: Boston College.

Mullis, I. V. A., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). *TIMSS 2003 international mathematics report. Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Chestnut Hill, MA: Boston College.

Palm, T. (2002). *The realism of mathematical school tasks: Features and consequences*. Doctoral dissertation. Umeå, Sweden: Umeå University.
Radford, L. (2006). Algebraic thinking and the generalization of patterns: A semiotic perspective. In S. Alatorre, et al. (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, North American Chapter* (Vol. 1, pp. 2–21). Mérida, Mexico: Universidad Pedagógica Nacional.
Reichard, L., Rozemond, S., Dijkhuis, J. H., Admiraal, C. J., Te Vaarwerk, G. J., Verbeek, J. A., et al. (2006). *Getal & ruimte, wiskunde HAVO B, deel 2* [Number & space, mathematics HAVO B, part 2]. Houten, The Netherlands: EPN.
Schama, S. (1991). *The embarrassment of riches: An interpretation of Dutch culture in the Golden Age*. London, UK: Fontana Press.
Van den Heuvel-Panhuizen, M. (2005). The role of contexts in assessment problems in mathematics. *For the Learning of Mathematics, 25*(2), 2–23.
Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 521–525). Dordrecht, Heidelberg, New York, London: Springer.
Van Maanen, J. A. (1987). *Facets of seventeenth century mathematics in the Netherlands. Doctoral dissertation*. Zwolle, the Netherlands: De Boer.
Vos, P. (2011). What is 'authentic' in the teaching and learning of mathematical modelling? In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 713 − 722). New York, NY: Springer.
Vos, P. (2013). Assessment of modelling in mathematics examination papers: Ready-made models and reproductive mathematising. In G. A. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 479–488). New York, NY: Springer.
Vos, P. (2015). Authenticity in extra-curricular mathematics activities; researching authenticity as a social construct. In G. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice: Cultural, social and cognitive influences* (pp. 105–114). New York, NY: Springer.
Wigfield, A., & Eccles, J. S. (2000). Expectancy–value theory of achievement motivation. *Contemporary Educational Psychology, 25*(1), 68–81.
Wijers, M., Jonker, V., & Kemme, S. (2004). Authentieke contexten in wiskundemethoden in het vmbo [Authentic contexts in mathematics textbooks]. *Tijdschrift voor Didactiek der Bètawetenschappen, 22*(1), 1–19.
Williams, J. (2012). Use and exchange value in mathematics education: Contemporary CHAT meets Bourdieu's sociology. *Educational Studies in Mathematics, 80*(1–2), 57–72.