

## **Patient flows through Valhalla**

A survival analysis in the context of short-term care institutions

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## **Abstract**

Individuals experiencing functional decline may often require some form of assistance in order to reassume their activities of daily living. A common form of rehabilitation is a stay at a short-term institution, yet readmissions to such care facilities often occur. Home-based reablement has surfaced in Norway during the past ten years and aims to assist the user in reaching their own activity goals through a self-committed and intensive program, assisted by health care workers. The objective of this study is inquiring into the patient flows between home-nurse areas and short-term institutions in southern Norway over the course of three years. We examine individual characteristics such as gender, age, cohabitation and reablement participation, assessing the differences in risk of admission and readmission based on these variables.

To achieve this, we use multiple-spell discrete-time survival analysis and estimate several logistic regression models. Through our methods, we conclude that males, the elderly and cohabitants all have higher likelihoods of admission and readmission to short-term institutions than their respective counterparts. For reablement, participants are at significantly higher risk of admission, but a marginally lower risk of readmission, compared to non-participants in a similar situation.

**Keywords:** reablement, survival analysis, duration modeling

**JEL classification:** I18, J14, C41

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# 1 Introduction

In this study we have examined the event histories of 4 496 individuals starting out in one of 11 different home-nurse areas. Over the course of approximately three years, some of these individuals have been admitted to, and released from, a short-term institution by the name of Valhalla care center. While some individuals were never admitted at all, others traveled frequently back and forth between the two states of existence.

The transition from one state to another was defined as the target event, and the time elapsing between transitions constitute spells. Using some of the methods that together coin the term survival analysis, we could attempt to recognize whether and when individuals transition between states of existence.

In order to answer the question of why such transitions happen, we needed to know more about the participants whose event histories we were examining. Fortunately, characteristics such as the age, gender, living situation and history of receiving reablement services for each person were available. Therefore, we set course to discern whether we could meaningfully explain the event histories of individuals by examining the relationship between the characteristic variables and the estimated hazard probability of event occurrence.

The investigation of whether, when and why such events occur has been increasingly relevant for policy makers. Reablement services have been at the forefront of this spotlight, and may be defined as time-limited, intensive and goal oriented, home-based rehabilitation targeted towards individuals with, or being at risk of, functional decline.(Forland & Skumsnes, 2016, p. 11) Reablement is often focused around commitment and self-performance from the participant in order to avoid over reliance on help from health care workers.(Forland & Skumsnes, 2016; *Hverdagsmestring og hverdagsrehabilitering*, 2012)

Reablement is a fairly new service in Norway, and the effectiveness of reablement services has been the focus of many practitioners and academics, yet the results have been mixed. While some point to positive results(Tuntland, Aaslund, Espehaug, Førland, & Kjekken, 2015; Burton, Lewin, & Boldy, 2015), others are more skeptical(Faao, Petersen, & Boge, 2016; Glendinning et al., 2010). Still, the general consensus is that more knowledge and experience is needed(Forland & Skumsnes, 2016, pp. 10, 67), which has been a great motivation to



perform this study.

The main aim of this study was inquiring into the differences in risk profiles for gender, age, cohabitation and reablement participation. For many of these characteristics there exists temporal and locational dynamics, and thus the difference in risk depends on time and place. In general, we find that being male, of old age or living in cohabitation are generally associated with higher risks of admission to short-term institutions.

For those that have previously received reablement services, we find that they are generally at higher risk of admission to, and first-time release from, short-term institutions compared to non-participants. In the case of readmission and repeat-visit release, these differentials in risk profiles are opposite of those for initial spells.

There are some individuals that are particularly worthy of our appreciation for their great effort, patience, guidance and support in the fruition of this paper. We would like to thank our supervisor, Jochen Jungeilges, for the constructive and inspiring conversations that we have had together over the course of the past months.

We would also like to thank our assisting supervisor, Tore Bersvendsen, for sharing his idea to write this very paper, his thoughtful suggestions of improvements to our work and for putting us in contact with the municipality of Kristiansand such that we could be given access to the data set we are using.

This would not have been possible without the data set that we were given access to, and therefore we send our deepest regards to the municipality of Kristiansand for allowing us to have the data set of patient flows at our disposal.

Finally, we were inspired and motivated by an article written by John B. Willett and Judith D. Singer named "It's Déjà Vu All Over Again: Using Multiple-Spell Discrete-Time Survival Analysis" in 1995, see (Willett & Singer, 1995). This article has been at the very core of building our empirical approach, and has greatly shaped our outlook to the problem at hand.

The structure of this thesis aims at encompassing all relevant parts of performing a multiple-spell discrete-time survival analysis. We begin with an examination of previous and relevant literature before proceeding to the foundations of survival analysis, the usage of logistic regression and maximum likelihood estimation. Following this, a section about

comparing alternative models and assessing model adequacy is presented.

Going from theory to practical application, we take the reader through our model selection process and the necessary steps in order to prepare for thorough and meaningful interpretation. Before concluding, we endeavor to explain and present the resulting findings from the chosen models, discussing them in the light of previous research and bringing attention to some potential issues with our approach. Finally, we conclude with our principal findings and suggest possibilities for further research efforts.

## 2 Literature review

The literature presented in this review was selected based on relevancy to our own study. It includes research in a similar setting related to our own objectives, and articles that serve as methodological guidelines for performing multiple-spell discrete-time survival analysis.

In “Do hospital Length of stay and staffing ratio affect elderly patients’ risk of readmission? A nation-wide study of Norwegian hospitals”, (Heggstad, 2002) uses Cox proportional hazard regression analysis and Kaplan-Meier survival curves in order to examine the relationship between length of stay and rate of early, defined as within 30 days, readmission using data from 59 hospitals for 113 055 patients. The independent variables used are age, gender, type of admission, type of treatment, discharge destination and a dummy variable for whether the patient lived near the hospital or not.

The research concluded that when average length of stay is relatively short, the rate of early readmission increases significantly but such effects can be compensated for by increased staffing. In addition, it was found that the predictive factors were time dependent as hospital-specific variables were less impactful for late readmissions. When it came down to the main effect of age, the study found that patients older than 80 years had higher odds for readmission compared to those between 67-80. Male patients were found to have higher odds for readmission compared to their female counterparts.

In “Operating conditions of psychiatric hospitals and early readmission – effects of high patient turnover” by (Heggstad, 2001) the objective was to study the relationship between hospital’s operating conditions and the risk of early readmission. She uses Cox regression

analysis adjusting for clustering effects with data from 20 hospitals with 5520 patients. It is concluded that high patient turnover and high bed-occupancy rates are associated with a higher risk of early readmission. Older patients were found to have lower odds for readmission compared to those below the age of 45. The gender and marital status of patients were not found to have statistical significance with regards to readmission.

“Hospital readmissions among elderly patients” by Bjorvatn (Bjorvatn, 2013) investigates determinants of hospital readmission in public Norwegian hospitals from 1999 to 2006. She uses probit and instrumental variable regression models for the analysis to identify the top 30 diagnostic related groups for readmission among elderly, 60 years or older, on a sample of around one million observations. In her data male participants were found to have higher odds for readmission when compared to female patients.

“Hverdagsrehabilitering” by Forland and Skumsnes(Forland & Skumsnes, 2016) is a review article that summarizes the current knowledge about rehabilitation produced by the center for care-research for the Norwegian ministry of health and care services. They define the home-based reablement program as a time-limited, intensive, and optimized rehabilitation program taking place in the participants home and local home-nurse area, targeting elderly with varying degrees of functional decline. Their collection of previous studies in the field of reablement found limited empirical evidence for improvements in activities of daily living (ADL). For the effect of reablement in their collection of studies done with different tests for physical function and activity level, no substantial evidence was found.

“Evidence for the long term cost effectiveness of home care reablement programs” by Lewin, Alfonso and Alan(Lewin, Alfonso, & Alan, 2013) found individuals that received reablement-services in the state of Western Australia were less likely to use home care services over the next three years compared to elderly who had received traditional home care services. They saw a reduction in need for ongoing support and lower costs associated with those elderly that had undergone one of the two reablement-programs in a five-year follow-up study period. Their adjusted prevalence ratio for home care service needs accounted for age, gender, living alone, having a personal assistant, ADL dependency level and having previously received home care service. They found the reablement-programs were more efficient in reducing the need for conventional home care services for: those 78 years and

older compared to those below that age, males compared to females and for those living with family compared to those living alone.

“Predicting nursing home-admissions and length of stay: a duration analysis” by Liu, Coughlin, McBride (Liu, Coughlin, & McBride, 1991) considers the challenge of predicting characteristics of who enters nursing-homes, their length of stay and risk of readmission. Hazard models are developed to examine travel from living in a community to a nursing-home and vice versa, the risk of death while living in the community or at a nursing-home, and adding a wide range of patient characteristics as possible predictors to the models. Some of their results include that being older, living alone, having cognitive impairments, higher levels of ADL impairments, receiving formal or informal care were associated with shorter durations of living in a community before being admitted to a nursing home. Being a homeowner or female were associated with longer durations in the community. As for traveling from nursing-homes to the community they concluded that patients with cognitive impairments are associated with longer stays at a nursing-home before discharge while high availability of home care services were associated with a shorter stay at nursing-homes before discharge.

“It’s Dèjà Vu all over again: using multiple spell discrete-time survival analysis” by John B. Willett and Judith D. Singer (Willett & Singer, 1995) conflicts and solves the problem that vast amounts of the scholarship and technical work regarding survival methods are centered around single non-repeatable events under continuous time with proportional-hazard assumption and time-invariant predictors. As many phenomena experienced through a lifetime occur in the form of spells or episodes, such as unemployment spells, pregnancy or sick leaves and so forth, methods used for single events might not be adequate. Time is often discretely measured, proportional-hazard assumptions are not met, and predictor values are varying with time. All of this leads to the need for multiple-spell discrete-time survival analysis to analyze repeated occurrence of a single event and sequential occurrences of separate types of events. The advantages of this approach are both applicability to many unique problems, the inclusiveness towards to time-invariant and time-varying predictors, no need to invoke proportional-hazard assumptions and the fact that standard logistic regression methods can be applied.

"Methodological Issues in the design of longitudinal research: Principles and recommendations for a Quantitative Study of teachers Careers" by Singer and Willett discusses the methodological features and challenges in longitudinal studies of teachers careers. They thereby create six key principles for research design. The first principle and main focus is that of making sure that one collects truly longitudinal data. Secondly, they consider the importance of viewing time both as an outcome and an explanatory variable. Thirdly, the significance of collecting data on both time-varying and time-invariant measures. Fourth, the principle of collecting data prospectively. Fifth, collecting data from multiple base periods and finally, from all levels of the organizational hierarchy and underlying institutions. In finding an optimal research design when employing longitudinal data, Singer and Willett inform about several possible pitfalls and potential solutions to increase the usefulness and accuracy of the research undertaken. They also outline major methodological issues relevant to the design of large scale longitudinal research on teachers career recidivism.(Singer & Willett, 1996, p. 267-269)

"Discrete-time Methods for the Analysis of Event Histories" by Paul D. Allison (Allison, 1982) concerns and elaborates on the problems surrounding censoring and time-varying explanatory variables in traditional longitudinal data analysis techniques, such as serious bias or loss of information. As praised and argued for by Willett and Singer in their 1995 paper regarding multiple spell discrete-time analysis, due to the limitations of studies in the social sciences, whereby the information about events mostly fall within the study-period in discrete time units that are difficult to measure continuously, one will often conclude that the use of discrete-time methods is optimal. Breaking up everyone's event history into partitions of discrete time units, in which the event either occurred or failed to appear, one can obtain estimators for binary regression models, as for example the ML estimators. This approach is highly transferable to a world where one has repeated events or a vast array of events, as most histories of groups or individuals containing event history have both relevant explanatory variables, interaction terms and competing risks.

## 3 Data

### 3.1 Origins of the data set

The data-set used for this thesis was provided by the municipality of Kristiansand. In agreement with the municipality, this data set was only given temporarily and with strict restrictions against sharing the data set with unauthorized third parties. Therefore, the data set is not available in the appendix and access will not be granted upon request.

### 3.2 The data set and data cleaning procedure

For a complete description of the original data set and the data cleaning procedure, please see Subsection 8.2 and 8.3 in the appendix, respectively. In short, the changes that were performed in order to accommodate the analysis are found in Table 1.

Description	In the data set	
	Before	After
Observations	421 693	320 341
Individuals	5 608	4 496
Locations	6	2

Table 1: *A description of changes in observations, number of individuals and states of existence before and after data cleaning.*

The censoring mechanism in a survival analysis renders the usual descriptive statistics inappropriate, and necessitates that other statistics are utilized. We have therefore moved the section of descriptive statistics to subsection 4.1.5. Prior to this, we present all components that are necessary in order to interpret them.

## 4 Empirical approach

### 4.1 Survival Analysis

#### 4.1.1 What is survival analysis?

The term survival analysis is not always easily explained because it is not by any means contained to a single step-wise procedure, rather it is a branch of statistics for the analysis of a particular type of data which may be done in a wide spectrum of approaches. Survival analysis and the analysis of survival data is a data analytic approach and is, generally, “a collection of statistical procedures for data analysis for which the outcome variable of interest is time until an event occurs.”(Kleinbaum & Klein, 2012, p. 4) Similarly, survival analysis may be known as duration modeling, or modeling of duration data, because we are interested in the time that elapses from the beginning of some event until its end or until the measurement is taken, which may precede termination.(Greene, 2012, p. 902)

Alternatively, one may call such data transition data because what is recorded are the sequences of states that were occupied, the times at which movements between them occurred and we are therefore inherently interested in the transition from one state to another.(Lancaster, 1990, p. 3) Some authors have also called this form of data event history data and defined it as “a record of when events occurred to a sample of individuals.”(Tuma & Hannan, 1979, p. 211) Yet another name for the data which survival analysis utilizes is failure time data. This is because the major applications are often biomedical studies and industrial life testing, and the occurrence of the event the researchers are interested in is often referred to as a failure.(Kalbfleisch & Prentice, 2002, p. 1) In general, the use of survival analysis should be considered whenever one is concerned with describing whether or when events occur.(Singer & Willett, 2003, p. 306)

When deciding if survival analysis is the correct approach to the task at hand one should conduct the “whether” and “when” test. If the interest lies in the question of “whether” events occur and if they do, “when” they occur, then survival analysis is most likely an appropriate approach.(Singer & Willett, 2003, p. 306) When one has decided to conduct a survival analysis, three essential methodological features must be clearly defined before analysis can

begin. These are the target event, the beginning of time and a metric for clocking time.

The target event is the occurrence that we are interested in, representing a transition from one state to another, the requirements being that the states are mutually exclusive and exhaustive. A “state” is simply a position a person exists in, for instance being employed or unemployed are two mutually exclusive states of existence that a person is in at any time by default. The requirement of mutual exclusiveness is that only one state is possible at any given time, and exhaustive implies that the possible states are all covered and show a complete representation of reality, as one cannot be in a third state of employment outside these two broad all-encompassing states of employment.

Some states may only be occupied once, while others may be repeatedly occupied. For instance, death is a non-repeatable event where the states of existence are being alive or dead. On the other hand, marriage is a repeatable event whose states of existence are married or unmarried. For repeatable events, one may find it appropriate to utilize the term spell, which refers to a single transition into, or out of, one of a series of repeatable states.(Singer & Willett, 2003, p. 311) For instance, a person may experience several employment spells during their lifetime, each spell referring to the length of time between being hired and leaving a job.

The beginning of time is the initial starting point where all units in the entire population occupy a single state. No individual has experienced the target event yet, but all are at risk of experiencing it. The term at risk refers to the possibility of event occurrence, and we use the term event time for the duration between the beginning of time and the occurrence of the target event. How one chooses to define the beginning of time depends on the type of study being conducted, but a wide variety of choices are available such as birth, release from hospital or it may be entirely arbitrary when no standard measure is deemed appropriate.

The metric for clocking time must be a form of time scale which allows to count periods in a meaningful way during the analysis. There are many ways of defining this time scale, including but not limiting to, seconds, days, weeks, months, quarters, semesters or years. We usually distinguish between continuous and discrete time based on how short or long the intervals are, with the shortest and most precise intervals being continuous and their longer counterpart being discrete. One should strive to record the data in the most precise and



short intervals attainable with respect to cost and convenience, yet this is highly dependent on the subject matter. For instance, the price of a stock may be recorded every second with little to no cost, while the same cannot be said for recording the current state of health for patients that have undergone surgery at a hospital.

#### 4.1.2 Censoring

Although the term censored refers to any individual with an unknown event time, we distinguish between several types of censoring in survival analysis. First, we differentiate between informative and non-informative censoring and then distinguish types of censoring namely left, right and interval censoring. We begin by separating informative and non-informative censoring. As an example, let us look upon a likely study on recovering addicts where the event is a relapse into old habits. If the recovering addicts do not fail to report to their case-workers during the study up and until the end of said study, the censoring is said to be non-informative. All these participants are representative of those who would have remained in the study had not data collection ended at that pre-planned date and censoring happening does not give us any information or prediction on what happened next with regards to the target event.

However, if the units of observation withdraw or are lost to follow-up one might suspect this to be caused by the units getting the target event, here relapse, rendering our censoring mechanism informative. Participants who are censored are then likely to either have experienced the target event, and the uncensored individuals will then differ greatly from the censored individuals. The validity of survival analysis is largely dependent on the assumption that the censoring mechanisms are non-informative and that attrition and withdrawal is not due to systematic events.(Singer & Willett, 2003, p. 318)

It is usually such that censoring occurs rather than truncation<sup>1</sup>, and the most usual reasons for censoring are the study ending before the unit has the event of interest, that the unit is lost to follow-up during the study or that the unit withdraws from the study before its ends.(Kleinbaum & Klein, 2012, p. 6) There are also a few different versions of

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<sup>1</sup>We will be dealing with censoring, but a full explanation of truncation is found in the appendix, see Subsection 8.5.

censoring depending on the time frame in which information is lacking. The most common form of censoring is right-censoring in which the true survival time is greater to or equal the observed time. If a person does not experience an event, withdraws before the end of the study or is lost to follow-up then that persons survival time is at least the observed one and the data is right-censored. In other words, the units survival time extends beyond the lower bound of our time horizon.

Left-censoring occurs if survival time is less than or equal to the observed survival time and may occur if we have a unit that has already experienced the event, but we are not sure about when. This may be applicable to a situation where we follow pregnant women and the event is delivery of the baby, but at the first check-in one of the women already had their child without knowing exactly when it happened, and the data is thus said to be left-censored. A third form of censoring called interval-censoring incorporates both left and right-censoring. Take again the example with the study of pregnant women and assume that one woman had their child between two check-ins without knowing exactly when it happened. We then know that the event occurred between the last and current check-in, but not exactly when, and the data is thus interval-censored.(Kleinbaum & Klein, 2012, p. 8)

The occurrence of censoring affects statistical analysis and the interpretation of standard descriptive methods of data in several ways which must be dealt with appropriately. For instance, units who were right-censored at the end of the study cannot be assumed to have had the target event at that time. This is because we know for a fact that non-informative censoring tells us about event nonoccurrence rather than occurrence. Statisticians therefore use the term “censored event times” instead of “event times” for these units because they have yet to experience the target event.(Singer & Willett, 2003, p. 321) This unfortunately affects the distribution of the data and renders the usual descriptive statistics such as the mean, standard deviation, skewness and kurtosis inappropriate.

There have been attempts to overcome the issue that censoring has in several different ways. Some researchers have chosen to exclude observational units with censored event times. Others have tried to impute the event time, sometimes assuming that the censored event time is equal to the event time itself. The first approach may be unwanted because it excludes data that may otherwise be of great importance to the analysis. The second

approach may be inappropriate because it wrongly assumes event occurrence when we know for a fact that censored event times state event nonoccurrence.

Some researchers have used a third method of dichotomizing the event histories at a point in time and then asking whether the target event has occurred by that time. A simplification such as dichotomizing the data in such a way unfortunately implies a large cost for the analysis for many reasons, many of which are explained in (Singer & Willett, 2003, p. 323). The alternative approach suggested by the authors of this book is using new methods of describing survival data, namely the use of life tables, the survivor function and the hazard function instead.

### **4.1.3 Descriptive statistics for survival analysis**

The main tool for summarizing the sample distribution of event occurrence is the life table.(Singer & Willett, 2003, p. 326) Such tables will span from the beginning of time when all the units in the sample were at risk of the target event and until the end of the data collection for the study. All life tables will have a few reoccurring columns, in which the first is the column of rows indexing the time intervals for the data set. Please note that each interval includes the beginning value and excludes the ending value such that the first interval may be written as [1,2) because the interval spans from, and including, period one and until, but not including, period two. Thereafter, the columns show the number of individuals that entered the current interval, those who had the target event during the interval and those who were censored at the end of the interval.

Individuals that entered the current interval are known as being eligible to experience the target event and are part of the risk set for that time period. The risk set is an important concept and has the features of being able to decrease due to either event occurrence or censoring and additionally the risk set has the feature of being irreversible. This implies that the risk set for the next period is always the risk set for the previous period after subtracting all units that either had the target event or were censored for various reasons, and these units are never reintroduced back into the risk set. Together with the assumption of non-informative censoring this means the risk set is at any time representative of all individual units who would have been at risk of event occurrence for any given interval even

if there was no censoring. This allows researchers to not only analyze the occurrence of the target event among those in the risk set, yet also generalize the obtained results to the entire population.(Singer & Willett, 2003, p. 329)

The hazard function, survivor function and the median lifetime are three essential ways of statistically summarizing the information conveyed by a life table. The hazard function is a quantity used to assess the unique risk of event occurrence in each discrete time period, also known as the hazard. The survivor function also describes the distribution of event occurrence over time, but unlike the hazard function, the survivor function cumulates the period-by-period risks of event nonoccurrence to assess the probability that a randomly selected will not experience the target event. The median lifetime identifies the center of the distribution, which would have been the mean in the case of no censoring and is therefore the main way to estimate the central tendency in the data.(Singer & Willett, 2003, p. 330-337)

The discrete-time hazard is defined as the “conditional probability that individual  $i$  will experience the event in time period  $k$ , given that he or she did not experience it in any earlier time period.”(Singer & Willett, 2003, p. 330) The hazard is denoted by Equation 1 and the set of the discrete-time hazard probabilities expressed as a function of time, labeled  $h(t_{ik})$ , is known as the population discrete-time hazard function.<sup>2</sup> Note the conditionality portion of the hazard function because it brings forth an essential implication; The conditionality implies that the hazard represents the probability of event occurrence for those units that are eligible to experience the event in that specific interval, or in other words, the units in the risk set. Therefore, one may refer to the hazard as the unique risk of event occurrence for a specific unit in a given period.

$$h(t_{ik}) = \Pr[T_i = k | T_i \geq k] \tag{1}$$

The discrete-time hazard has two important features and may be used for several applications. The main features of the discrete-time hazard is that the interpretation is of a probability. First, this implies that the hazard rate will always be between zero and unity. Second, we interpret hazards closer to unity as having greater risk of occurrence while estimates of hazard closer to zero as unlikely to occur. To show this one may display the

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<sup>2</sup>Note that this is the discrete-time hazard function and that the continuous-time hazard function differs from this one. For the continuous-time hazard function, see (Greene, 2012, p. 904).

estimate of a discrete-time hazard by a function as given in Equation 2 where the estimate of the hazard function is the number of events in period  $k$  divided by the number of units in the risk set in the same period. A useful way of examining the hazard function may be done graphically by plotting the estimated hazard function on the y-axis against time. This allows researchers to identify especially risky time periods and the shape of the hazard function. The shape will be able to determine whether the hazard is increasing, decreasing or staying constant across time.(Singer & Willett, 2003, p. 333)

$$\hat{h}(t_k) = \frac{n \text{ events}_k}{n \text{ at risk}_k} \quad (2)$$

The survivor function, which is sometimes also called the survival function, assesses the probability that a randomly selected individual will not experience the event.(Singer & Willett, 2003, p. 334) By this it is understood that the survivor function focuses solely on event nonoccurrence rather than event occurrence as in the case of the hazard function. The survivor function in discrete-time is given by Equation 3 and is the probability that unit  $i$  will not survive, that is, experience the target event, past time period  $k$ .(Kalbfleisch & Prentice, 2002, p. 6) The same implication follows here as in the conditionality segment of the hazard function in that this individual has been retained in the risk set until period  $k$ , or in other words not experienced the event in any earlier period.<sup>3</sup>

$$S(t_{ik}) = \Pr[T_i > k] \quad (3)$$

The survivor function has the property of always being equal to unity at the beginning of time for the study as all the participants have yet to experience the target event. As units who are censored or experience the event are not reintroduced to the risk set, the survival function is always decreasing over time. The estimated survivor function is then the proportion of those remaining in the risk set at period  $k$  of the study population, shown in Equation 4.

$$\hat{S}(t_k) = \frac{n \text{ who have not experienced the event by the end of time period } k}{n \text{ in the data set}} \quad (4)$$

---

<sup>3</sup>Some authors dealing exclusively with continuous time, for instance (Greene, 2012, p. 903) use a definition for the survival function of “the probability that the spell is of length at least  $t$ ” given by equation  $S(t) = Pr(T \geq t)$ .

As for the relationship and interaction between the hazard and survivor function, there is “a clearly defined relationship between the two”.(Kleinbaum & Klein, 2012, p. 15) Whenever the hazard is high, the survivor function will decrease quickly in that time period and vice versa. One may derive the survivor function for any period by using the survivor function for the previous period multiplied by one minus the hazard probability for that period as shown in Equation 5. By repeated substitution of this equation it is possible to show that “each years estimated survival probability is the successive product of the complement of the estimated hazard probabilities across the current and all previous periods.”(Singer & Willett, 2003, p. 337)

$$\hat{S}(t_k) = \hat{S}(t_{k-1})[1 - \hat{h}(t_k)] \quad (5)$$

The median lifetime takes the place of what would normally be the sample mean in a study with no censoring. Formally, the estimated median lifetime “identifies that value of  $T$  for which the value of the estimated survivor function is 0.5”.(Singer & Willett, 2003, p. 337) In other words, that is the time we estimate that half of the study sample have experienced the target event. The equation for the estimated median lifetime is given in Equation 6 where  $t_m$  is the time interval in which the survivor function is just above 0.5,  $\hat{S}(t_m)$  the value of the estimated survival function in that time period and  $\hat{S}(t_{m+1})$  the value of the survivor function in the next interval in which it has fallen just below 0.5. In some cases, the survivor function may not reach 0.5 even by the end of the study, which simply means that less than half of the population is expected to experience the event by the last period in the life table. One may present cumulative survival rates instead, which are values of the estimated survivor function after pre-specified lengths of time.(Singer & Willett, 2003, p. 338)

$$t_m + \left[ \frac{\hat{S}(t_m) - 0.5}{\hat{S}(t_m) - \hat{S}(t_{m+1})} \right] ((t_m + 1) - t_m) \quad (6)$$

#### 4.1.4 Shortfall and empty cells

There is a structural and sampling shortfall when dealing with data on event occurrence. (Willett & Singer, 1995, p. 52) This implies that as spells and periods increase, there is less data that can be used for analysis. The structural shortfall arises because the life time of the study is fixed, and there is therefore increasingly limited time to describe spells. For

instance, in a study with a total of 10 periods, there is a maximum of 10 periods to describe a first spell, a maximum of 9 periods of describe a second spell, and so on. The analytic impact of structural shortfall is like that of structural zeros in contingency table analysis.(Willett & Singer, 1995, p. 52)

The sampling shortfall becomes evident from the fact that the risk set shrinks because of event occurrence and censoring, see Figure 1 and Table 15. Fewer individuals experience later periods of any given spell, and fewer experience later spells as well.(Willett & Singer, 1995, p. 52) By looking at the first five spells situated in a HNA we understand that, as the spells increase, the periods within each spell decrease. This is suggested by the horizontal axis denoting time, which shortens drastically between the first and ninth spell. These shortfalls imply that one should be wary of empty cells when fitting statistical models of hazard and showing care in interpreting estimated hazard plots as observed differences may simply be due to sampling variation.(Willett & Singer, 1995, p. 53)

Tables usually contain some cells where  $n_i = 0$ , which are referred to as empty cells, and these are categorized into two types, namely structural and sampling zeros. The difference between a structural and sampling zero or an empty cell lies in its expected value. First, a sampling zero is an empty cell with expected value greater than zero. As the number of observations in the sample increases, we expect this cell to eventually be non-zero. On the other hand, a structural zero is an empty cell that will have expected value equal to zero, regardless of the sample size. Structural zeros are not really part of the data, and therefore do not contribute to the likelihood function or model fitting and any contingency tables containing structural zeros are known as incomplete tables.(Agresti, 2013, p. 405) The sampling zeros are part of the data and are often the cause of the nonexistence of the maximum likelihood estimator. This is because certain patterns of zero counts in a table make it impossible to maximize the log likelihood function by any vector of finite form.(Fienberg & Rinaldo, 2012, p. 997)

#### **4.1.5 Descriptive statistics applied to data**

Now that we have introduced the hazard function, survivor function and the median lifetime, we are ready to apply them to our data. Due to the large number of spells, we only plot

the hazard and survivor curves for the ten first spells in our sample. The sample hazard functions for the first 10 spells are shown in Figure 1, the sample survival functions in Figure 2 and the estimated median lifetimes for each of the first ten spells are presented in Table 2. After each figure we shortly describe the main characteristics.

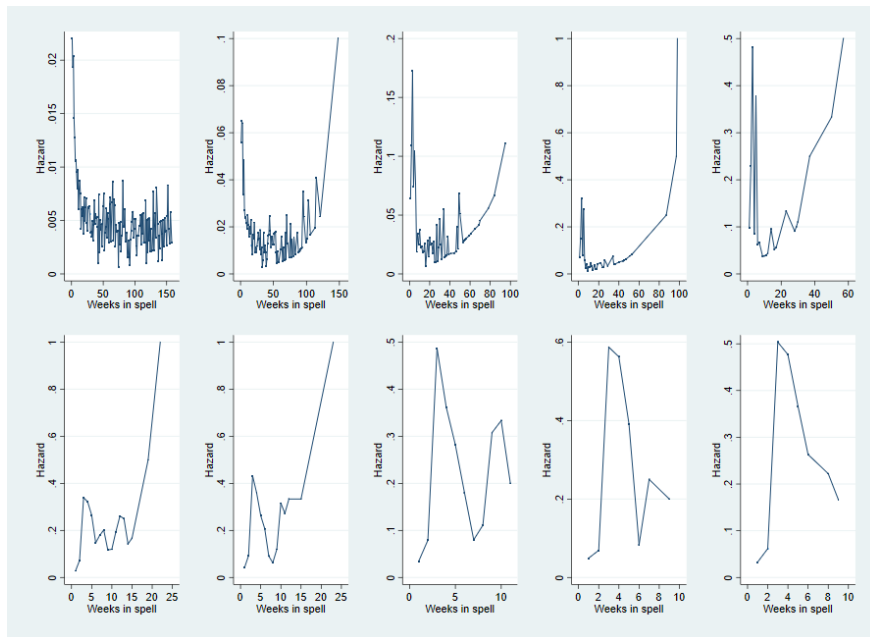


Figure 1: *Sample hazard functions for the first 10 spells.*

*The top row represents spells in home-nurse areas (1, 3, 5, 7, 9).*

*The bottom row represents spells at short-term institutions (2, 4, 6, 8, 10).*

For spells in a HNA (top row), the hazard is large initially and decreases over time. This indicates that the risk of admission, or readmission, is largest in the first few periods of any given spell. For spells at a STI (bottom row), the hazard peaks around the third period. This indicates that the likelihood of being released after three weeks at a short-term institution is comparatively high to other periods. For some spells, hazard increases drastically towards the end of the spell. This is clear for all spells in the top row except the first, and for the two first spells in the bottom row.<sup>4</sup> It is also possible to examine the ranges of the x-axes to get an impression of the maximum length of each spell. For instance, the length of stay for the first five spells at a short-term institution is rarely much longer than 25 weeks.

<sup>4</sup>A possible explanation is the sampling shortfall, which causes any event occurrence in a small risk set to heavily influence the estimated hazard, as implicated by Equation 2.



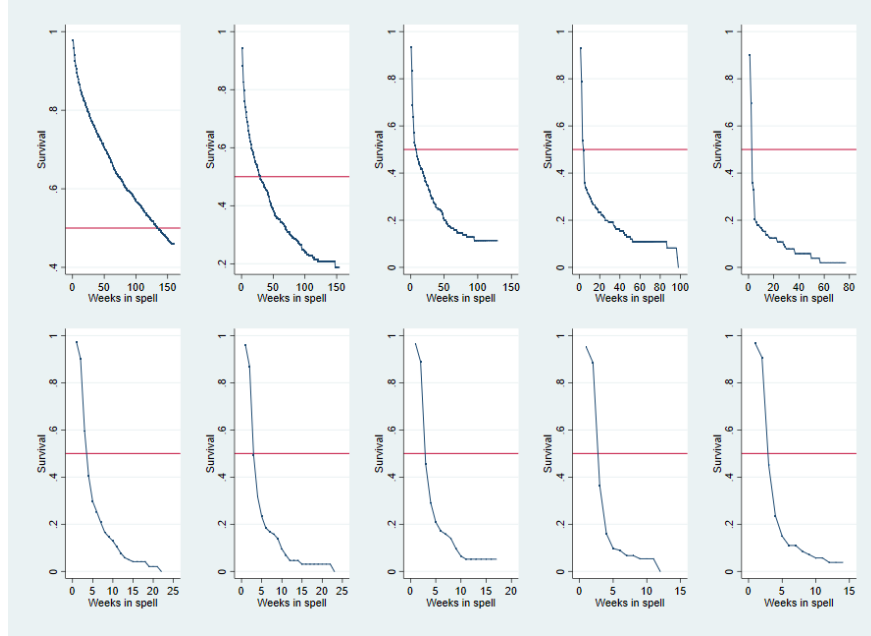


Figure 2: *Sample survivor functions for the first 10 spells.*

*Spells in home-nurse areas (1, 3, 5, 7, 9) are in the top row.*

*Spells at short-term institutions (2, 4, 6, 8, 10) are in the bottom row.*

*The red line represents the median lifetime.*

For the top row, the sample survivor function drops off slowly in the first spell, reaching the median lifetime by period 134. In the subsequent spell at a HNA, the median lifetime is reduced drastically before beginning to stabilize around three to four weeks for spells 7 and 9. On the other hand, the sample survivor functions for the bottom row are nearly indiscernible in shape. An interesting observation is that the median lifetime in- and out-of-institution spells seem to converge. If this pattern continues beyond the first ten spells, it indicates that individuals with many readmissions tend to spend approximately as much time inside as outside of the institution.

Spell	Median
1	133.7
2	3.5
3	29.2
4	3.0
5	8.0
6	2.9
7	3.9
8	2.7
9	2.6
10	2.9

Table 2: *Sample median lifetime for the first ten spells. These values are where the red line and sample survivor functions intersect in Figure 2.*

## 4.2 Logistic regression

Any data analysis that concerns itself with describing the relationship between a dependent variable and one or several explanatory variables, might consider using a logistic regression model. Contrary to linear regression, the logistic regression model has an dependent variable that is binary or dichotomous.(Hosmer, Lemeshow, & Sturdivant, 2013, p. 1) Both the logit and probit model approaches uses functions that transforms the regression models to a bound range between zero and one for a snakelike, S-shaped appearance of the logistic function rather than the more unbelievable straight line from a linear model, see Equation 7.(Brooks, 2008, p. 514) The asymptotes of 0 and 1 in the logistic model means that even though the x-axis values might fall to infinitesimally small or large values, they will stay bounded in the range. The logistic model is due to its non-linear nature not estimable by ordinary least squares and thus require something like maximum likelihood.(Brooks, 2008, p. 514)

$$f(z) = \frac{1}{1 + e^{-z}}, \quad f(-\infty) = 0 \quad f(+\infty) = 1 \quad (7)$$

Many distribution functions have been suggested for usage when dealing with dichotomous outcome variables. Two advantages when choosing the logistic distribution are the mathematical flexibility which gives great ease of usage and secondly and most importantly, the

model parameters provides meaningful estimates.(Hosmer et al., 2013, p. 7) With a dichotomous outcome variable, the conditional mean must be greater than or equal to zero and also less than or equal to one  $0 \leq E(Y | x) \leq 1$ . When the logistic distribution is used, one can use the quantity  $\pi(x) = E(Y | x)$  to represent the conditional mean of  $Y$  given  $x$ .(Hosmer et al., 2013, p. 7)

There are several approaches to the distribution of error term in linear and logistic regressions. This section follows the arguments made in Hosmer and Lemeshow’s book “Applied Logistic Regression”.(Hosmer et al., 2013, p. 7) An important distinction between linear and logistic regression models is the conditional distribution of the dependent variable. In a linear model, the observation of the dependent variable can be expressed as  $Y = E(Y | x) + \varepsilon$ . The term  $\varepsilon$  is called the error term and represents an observations conditional mean deviation. Usually, when dealing with non-binary dependent variables  $\varepsilon$  follows a normal distribution with mean zero and a variance that is constant across different values of the independent variables. When we live in models with dichotomous dependent variables, this harmony is broken. We can express said dependent variable as  $Y = \pi(x) + \varepsilon$ . The error term  $\varepsilon$  can here have two possible values, for  $Y = 1$  the corresponding error term is given by  $1 - \pi(x)$  with the probability  $\pi(x)$ . If  $Y = 0$ , the error term is  $-\pi(x)$  with probability  $1 - \pi(x)$ . This means that the error term has a distribution with a mean of zero and a variance of  $\pi(x)[1 - \pi(x)]$ , from this follows the conclusion that the dependent variable has a binomial distribution with a probability  $\pi(x)$ . We can surmise that the binomial and not the normal distribution describes the distribution of errors, therefore being the distribution to use when living in the logistic regression model analysis universe.

#### 4.2.1 The Logistic Model

The framework for the logistic model approach consists of the observed independent variables on the subject that we can use in the exponent in the logistic model to denote the conditional probability statement for the event occurring during the given time period, as shown in Equation 8, and Definition 1.(Kleinbaum & Klein, 2010, p. 8)

**Definition 1 (Logistic model)** *The model is defined as logistic if the expression for the probability of developing the disease, given the  $X$ s, is 1 over 1 plus the  $e$  to minus the quantity*

$\alpha$  plus the sum from  $i$  equals 1 to  $I$  of the  $\beta_i$  times  $X_i$ .

$$P(\mathbf{X}) = \frac{1}{1 + e^{-(\alpha + \sum \beta_i X_i)}} \quad (8)$$

#### 4.2.2 Logistic link function

Specifying a model for a discrete-time hazard based on the logistic link function with the dichotomous event indicator can be a good choice. This recommendation as formulated in Equation 9 is according to the advice of Cox and Snell as Willett and Singer pointed out. (Willett & Singer, 1995, p. 51)

$$E(Y_{ijk} = 1) = h_{ij}(k) = \frac{1}{1 + e^{-(\text{COVARIATES}, \text{SPELL}, \text{PERIOD})}} \quad (9)$$

#### 4.2.3 Odds ratio

The odds for something happening to an individual is the likelihood for it happening divided by the likelihood of it not occurring. The formula for the odds will therefore simply be the probability divided by 1 minus the probability as seen in Equation 10. (Kleinbaum & Klein, 2010, p. 18)

$$\text{odds} = \frac{P}{1 - P} \quad (10)$$

Following this line of thought the odds ratio is the ratio of two odds compared to each other. This indicates that the odds of one group is compared to the odds of another group as seen in Equation 11. (Kleinbaum & Klein, 2010, p. 22)

$$\text{OR}_{\mathbf{X}_1, \mathbf{X}_0} = \frac{\text{odds for } \mathbf{X}_1}{\text{odds for } \mathbf{X}_0} \quad (11)$$

#### 4.2.4 Logit transformation

To find an expression of  $P(\mathbf{X})$  that has many familiar properties to linear regression models, one can perform a logit transformation as seen in Equation 12 to find the log odds and a linear representation of the quantities. (Kleinbaum & Klein, 2010, p. 19)

$$\begin{aligned} \text{logit } P(\mathbf{X}) &= \ln_e \left[ \frac{P(\mathbf{X})}{1 - P(\mathbf{X})} \right] \\ &= \log \text{ odds for } \mathbf{X} \\ &= \alpha + \sum \beta_i X_i \end{aligned} \quad (12)$$

When we insert the previous regression model from Equation 8 into the transformation we get the expression for the independent variables on a linear form:  $\alpha + \sum \beta_i X_i$ . Thus, the logit  $P(\mathbf{X})$  is linear in its expression, the x-values can be continuous, ranging from minus to plus infinity if necessary.(Hosmer et al., 2013, p. 7)

#### 4.2.5 Risk odds ratio

We can take the odds-ratio for a group one and two, describe the risk in the logistic model and get the comparative risk odds ratio from the same logistic model. To use an appropriate formula for the risk odds ratio, we substitute the probability of  $\mathbf{X}$  happening with the equally valid logit form linear sum into the risk odds formula. Algebraic theory states that in cases where one has exponents on both sides in a fraction the statement can be rewritten as the exponential with the numerator exponent minus the denominator exponent. Doing this we find that the risk odds ratio is the exponential of the difference between the two linear sums.

The intercepts ( $\alpha$ ) in the linear sums will cancel each other out, giving us a general exponential formula for the risk odds ratio generated from the logistic model framework for the comparison of two groups. If the odds for an event are 1.35, taken from antilogging the coefficient 0.3 from the explanatory variable MALE, then the odds for the event happening for a male participant (MALE=1) are 35% higher than for a female(MALE=0).(Willett & Singer, 1995, p. 58) One can use the Equation 13 for the same risk odds ratio from 1 to  $I$ , giving us a representation on how each variable in the logistic model contributes jointly to the odds ratio.(Kleinbaum & Klein, 2010, p. 25) To further understand the techniques for estimation in a discrete-time model, one can look at the maximum likelihood approach.

$$\begin{aligned}
 ROR &= e^{(\alpha + \sum \beta_i X_{1i}) - (\alpha + \sum \beta_i X_{0i})} \\
 &= e^{(\alpha - \alpha + \sum \beta_i (X_{1i} - X_{0i}))} \\
 &= e^{\sum \beta_i (X_{1i} - X_{0i})} \\
 ROR_{\mathbf{X}_1, \mathbf{X}_0} &= e^{\sum_{i=1}^I \beta_i (X_{1i} - X_{0i})} \tag{13}
 \end{aligned}$$

## 4.3 Maximum Likelihood

### 4.3.1 Motivation and drawbacks

There exists a variety of techniques for the generation of estimators in a setting of discrete models, and some of these are the maximum likelihood, method of moments and least squares. We focus on the technique of maximum likelihood as it is the method which we will utilize throughout this paper. The estimators produced by the maximum likelihood method are guaranteed to have optimal properties, which makes it very important for researchers to be in control of.

The main drawback of this method is that closed form solutions are often not available and thus requires numerical tools, but this is something that technological advancements have made considerably easier to accommodate for. The underlying idea of maximum likelihood estimation is based upon the assumption that the events which we observe in real life are the most likely ones to occur. In other words, a realization of a sample was the most likely outcome at the time which it was recorded. If we agree this this train of thought, then it would be reasonable to accept parameter estimates as those estimators which maximize the likelihood of the observed sample.(Jungeilges, 2017, p. 2)

### 4.3.2 Unconditional ML approach

Within the maximum likelihood estimation approach there is the unconditional and the conditional method. In this section we focus on the unconditional one, but a short description of the conditional method is found in the appendix, see Subsection 8.6. The formula for the unconditional method is in Equation 14, and it describes the joint probability of the sample data as the product of joint probability for the cases and the joint probability for the non-cases. By cases and non-cases, it is here meant as subjects that have gotten the event and those that were censored in the study. When the logistic model formula involving the parameters of interest, Equation 8, is substituted into Equation 14, we obtain Equation 15.(Kleinbaum & Klein, 2010, p. 114)

$$L_U = \prod_{l=1}^y \mathbf{P}(\mathbf{X}_l) \prod_{l=y+1}^n [1 - \mathbf{P}(\mathbf{X}_l)] \quad (14)$$

$$L_U = \frac{\prod_{l=1}^n \exp(\alpha + \sum_{i=1}^I \beta_i X_{il})}{\prod_{l=1}^n [1 + \exp(\alpha + \sum_{i=1}^I \beta_i X_{il})]} \quad (15)$$

### 4.3.3 Likelihood function

The maximum likelihood estimation approach can be best described by investigating the likelihood function ( $L(\boldsymbol{\theta})$ ), which represents the joint probability or likelihood of observing the data that we have collected. (Kleinbaum & Klein, 2010, p. 112) By joint probability we simply mean it is a probability that takes into consideration the contributions of all the subjects in the study. The  $\boldsymbol{\theta}$  in the likelihood function is a vector of all the parameters in the model including the intercept and contains the values that we wish to estimate. The method of maximum likelihood estimation will choose the estimator of the set of parameters for vector  $\boldsymbol{\theta}$  which maximizes the likelihood function, and this estimator is denoted as a vector,  $\hat{\boldsymbol{\theta}}$ . The components of  $\boldsymbol{\theta}$  are found by solving the equations of partial derivatives of the likelihood, shown in Equation 16, for each individual parameter ( $p$ ) in the model. Note that maximizing the likelihood function is equivalent to maximizing the log likelihood function, which is an easier task computationally, and is often used instead. If there are  $q$  parameters in total, then there is a  $q$  set of equations in  $q$  number of unknowns, and the equations must be solved in iterations by the program. (Kleinbaum & Klein, 2010, p. 113)

$$\frac{\partial \ln L(\boldsymbol{\theta})}{\partial \theta_p} = 0, p = 1, 2, \dots, q \quad (16)$$

The estimation procedure can then be separated into four steps for a parameter ( $\theta$ ) as follows: (Jungeilges, 2018b, p. 12)

1. Given: Random sample  $X_1, X_2, \dots, X_n$  from  $f_x(X, \theta)$
2. Determine the probability of the occurrence of the sample  $x_1, x_2, \dots, x_n$ , the likelihood of the sample as  $L(\theta|x_1, x_2, \dots, x_n) = \prod_{i=1}^n f_x(x_i; \theta)$ .
3. Find that value of  $\theta$  which maximizes the likelihood  $\max_{\theta \in \Theta} L(\theta|x_1, x_2, \dots, x_n)$ .
4. Any value  $\hat{\theta}$  such that  $L(\hat{\theta}) \geq L(\theta)$  for  $\theta \neq \hat{\theta} \in \Theta$  constitutes a maximum likelihood estimator of  $\theta$ .

#### 4.3.4 Sample likelihood function for survival analysis

In the dataset, the entire population can be viewed as mutually exclusive subsample sets, where a subsample  $j$  contains  $n_j$  participants, who have had spell 1 to  $j$ . Subsample 1 thus contains only those that have had spell 1 and none of the other succeeding spells. Likewise subsample 2 contains those participants that have had both spell 1 and 2, but not any of the later spells. So, if  $j_i = 1$  the last spell participant  $i$  experienced was spell 1 and thus belongs in subsample 1. (Willett & Singer, 1995, p. 44) The highest number of spells achieved in the population,  $J$ , therefore determines the overall number  $n$  of subsamples.

In the general likelihood function, each sample individuals contribution can be represented through the subsequent density function for the observed event occurrence of the terminating event. This contributions representation will subsequently be 1 minus the cumulative distribution of the hazard if the participant is censored in the given period  $k$  where the event  $y$  may happen later, after  $k$ . (Allison, 1982, p. 69)

Those that belong to subsample 1 only have the first spell occur in the time-span of the study, yet it can be interesting to look at the censored members of said subsample. We denote  $K_{ij}$  as the last period individual  $i$  was observed in spell  $j$ , and  $T_j$  as the time period of event occurrence for spell  $j$ . Their contribution to the sample likelihood can be given by the probability that their first event will happen after period  $K_{i1}$ , presented in Equation 17,  $S_{i1}(K_{i1})$  being the value of the survivor function of  $T_1$  in period  $K_{i1}$ . (Willett & Singer, 1995, p. 45)

$$L_i^{(1)}(\text{Censored}) = \Pr_i\{T_1 > K_{i1}\} = S_{i1}(K_{i1}) \quad (17)$$

The uncensored members of subsample 1 contribute to the sample likelihood when they have the event in period  $K_{i1}$ . Their contribution to the sample likelihood can be expressed as in Equation 18,  $f_{i1}(K_{i1})$  being value of the probability mass function of  $T_1$  in period  $K_{i1}$ . (Willett & Singer, 1995, p. 44)

$$L_i^{(1)}(\text{uncensored}) = \Pr_i\{T_1 = K_{i1}\} = f_{i1}(K_{i1}) \quad (18)$$

We define a dichotomous censoring indicator  $c_{ij}$ , equal to 0 if the  $j^{th}$  spell of the participant  $i$  is uncensored and 1 if the individual is censored. In combining the additions of the uncensored and censored members to the subsample likelihood we can use the censoring



indicator, the probability mass function and the survivor function to express the participants contribution as presented in Equation 19.(Willett & Singer, 1995, p. 45)

$$L_i^1 = [f_{i1}(K_{i1})]^{1-c_{i1}} [S_{i1}(K_{i1})]^{c_{i1}} \quad (19)$$

Individuals in subsample 2 experience only the first and second spell. In order to qualify for the second spell, one must not be censored during the first. If the terminating event does not reoccur, they are censored in their second spell. Still, one needs to account for their contribution from the first event occurrence. The contribution to the sample likelihood must then be a product of the probability of event occurrence in  $K_{i1}$  and the probability of event nonoccurrence prior to  $K_{i2}$ . For uncensored individuals, the latter probability is that of terminating the second spell in  $K_{i2}$  instead. Proceeding to do this for all spells, one achieves the net sample likelihood presented in Equation 20, which is a product of all the contributions for all  $J$  subgroups, up to and including the last observed spell number  $J_i$  for participant  $i$ .(Willett & Singer, 1995, p. 46)

$$L = \prod_{i=1}^n \prod_{j=1}^{J_i} ([f_{ij}(K_{ij})]^{1-c_{ij}} [S_{ij}(K_{ij})]^{c_{ij}}) \quad (20)$$

The net sample likelihood can be expressed using the product of the hazard probabilities, given by the values for the probability mass function and the survivor function, respectively, see Equation 21. For the computation of the value of probability mass function in the last time period of the  $j^{th}$  spell for the  $i^{th}$  participant, expressed as hazard probabilities, one can summarize the function as a product of terms for each period within the spell. This describes the conditional probability for event occurrence in the specified period, but not in any period leading up to that event occurrence.(Willett & Singer, 1995, p. 47)

Using the same idea in expressing the survivor function values for the  $i^{th}$  participant in the last time period of the  $j^{th}$  spell, into hazard probabilities ( $h_{ij}$ ), we take the approach of expressing the conditional probabilities for event occurrence not taking place in any of the periods observed up to that period  $k_{ij}$ . To replace the censoring indicator, one can employ a dichotomous event indicator ( $y_{ijk}$ ). It is zero in all periods during the spell if the event did not occur and the participant was censored, or one in the last period of the spell if there was no censoring and the terminating event occurred in the last period of said spell.(Willett

& Singer, 1995, p. 48)

$$L = \prod_{i=1}^n \prod_{j=1}^{J_i} \prod_{k=1}^{K_{ij}} (h_{ij}(k))^{y_{ijk}} (1 - h_{ij}(k))^{1-y_{ijk}} \quad (21)$$

From the dichotomous event indicator ( $y_{ijk}$ ) we can draw some further remarks. The expectation of the indicator is the probability that the terminating event occurs to the  $i^{th}$  participant in period  $k$  of spell  $j$ . We can further surmise that  $y_{ijk}$  only equals 1 in period  $k$  if the participant did not have the event in all the previous periods  $k$  of the spell  $j$ . The values of the dichotomous event indicator are observed realizations of the hazard probability. (Willett & Singer, 1995, p. 48) Having presented the sample likelihood function for survival analysis, we proceed to the assumptions of maximum likelihood estimation.

#### 4.3.5 Assumptions

The assumptions for the maximum likelihood estimation method are:

(Jungeilges, 2018b, p. 13)

1. The probabilistic law  $f_x(x, \theta)$  is known.
2. Each sample point  $X_a$  is generated by the same underlying process  $f_x(x, \theta)$ .
3.  $X_a$  and  $X_b$ ,  $a \neq b$  are pairwise stochastically independent. ( $\Leftrightarrow f(X_a X_b) = f(X_a) f(X_b)$ )
4. The function  $L(\theta|x_1, x_2, \dots, x_n)$  has a global maximum.
5. There exists a method to locate the value in the parameter space  $\Theta$  for which  $L(\theta|x_1, x_2, \dots, x_n)$  assumes its global maximum.

The first assumption about the probabilistic law being known is the same as stating that we have information concerning the distribution function which has generated each observation in the sample. The second assumptions states that the data are identically distributed where identically refers to coming from the same distribution. The third assumption implies that the realization of any two sample points is viewed as the occurrence of two stochastically independent events. The fourth assumption states that the function has a global maximum which for instance excludes the possibility of it having several global maxima. The fifth and

final assumption states there must exist a method to ensure that the likelihood function assumes its global maximum. The assumptions outlined are important to keep in mind as one executes data analysis as many challenges often hinge on the violation of these assumptions.

When the assumptions of the maximum likelihood technique are met, the maximum likelihood estimators have two important properties, namely that of invariance and optimality. (Jungeilges, 2017, p. 6)

**Property 1 (Invariance)** *Let  $g$  denote a continuous function. If  $\hat{\theta}$  is a MLE for  $\theta$  then  $g(\hat{\theta})$  is a MLE for  $g(\theta)$ .*

This invariance property states that if we are interested in a function of the parameter  $\theta$ , then we can apply that function to our estimate of the parameter ( $\hat{\theta}$ ), and if our estimate is a maximum likelihood estimator for the true underlying parameter then the function applied to our estimated parameter will be a maximum likelihood estimator for the same function of the true but unknown  $\theta$  as well. It is important to note that for small, or fixed, sample sizes some maximum likelihood estimators are unbiased while others are not, and the same being true for the case of the estimators being minimum variance unbiased estimators as well. In large samples we have the second property of the maximum likelihood technique, which is optimality. (Jungeilges, 2017, p. 7)

**Property 2 (Optimality)** *Let  $\hat{\theta}_n$  denote a ML estimator of the parameter  $\theta$  based on a random sample of size  $n$  from  $f_x(X, \theta)$ . Then*

1.  $\hat{\theta}_n \sim N(\theta, \xi_n)$  where

$$\xi = \frac{1}{-nE\left[\frac{\partial^2 \ln f_X(X; \theta)}{\partial \theta^2}\right]} \quad (22)$$

2. *The sequence of ML estimators  $\hat{\theta}_1, \dots, \hat{\theta}_n, \dots$  is best asymptotically normal (BAN).*

Here it is stated that the sequence of maximum likelihood estimators will be best asymptotically normal with the variance of those estimators being determined by the Rao-Cramer lower bound. In simple terms this property can be explained by the notions of the estimator being optimal in large samples, or that the maximum likelihood estimator is “as good an estimator there is”. (Jungeilges, 2017, p. 7)

## 4.4 Comparing alternative models

### 4.4.1 Deviance statistic

When evaluating alternative models, one needs statistical tools to decide which one has a better fit and which predictors are significant and should be kept in the model. We first need to inquire into log likelihood statistics and deviance statistics. Following this we present an explanation of three classical test procedures based on maximum likelihood estimators: the likelihood ratio test, the Wald test and the Lagrange multiplier test<sup>5</sup>. Firstly, the log likelihood statistic is closely related to the log likelihood function. It is the numerical value of the log likelihood function when the maximum likelihood estimates are substituted for their corresponding parameter values.(Kleinbaum & Klein, 2010, p. 132) The log likelihood statistic is a summary statistic, in many cases given as part of the output when running a regression that uses a maximum likelihood procedure. A single log likelihood statistic is not entirely informative, but is valuable for comparing similar models because larger log likelihood statistics indicate a better fit.(Singer & Willett, 2003, p. 397)

As the statistical tests used to compare alternative models use deviance statistics and not log likelihood statistics, we need to transform these. The idea of deviance is that it quantifies how the model at hand is compared to the best model possible, also known as the saturated model. The deviance statistic for a saturated model must be zero, and we reach the deviance statistic from the log likelihood statistic by applying Equation 23. When faced with two deviance statistics one may be tempted to simply pick the model with the smallest one, but this is an inefficient approach. For instance, one model may be parsimonious and a better choice for the problem at hand in contrast to an unnecessarily complex alternative model with a smaller deviance statistic. We therefore introduce the likelihood ratio test, the Wald test and the Lagrange multiplier test which may be utilized when one is dealing with such challenges.

$$\text{Deviance} = -2 \times \ln(\text{likelihood})_{\text{current model}} \quad (23)$$

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<sup>5</sup>We have not used the LM test in this thesis, and have therefore moved the description to the Appendix, see Subsection 8.7.

#### 4.4.2 Likelihood ratio test

The likelihood ratio test can be used to test hypotheses about one or several parameters when dealing with nested models. Two models are required for the test, where one is called the full model, containing all parameters of interest. The second model is known as the reduced model which does not contain the parameters whose significance, we are interested in evaluating. We emphasize that the reduced model is only a subset of the full model, or equivalently, that the reduced model is nested in the full model. Therefore, the reduced model may only be achieved by setting one or several parameters in the full model equal to zero. The idea behind this test is if the restrictions that are made are valid, imposing those restrictions will not lead to a large reduction in the log likelihood value.(Greene, 2012, p. 565)

The test statistic is the difference of the deviance statistic between the full and reduced model, which is equal to Equation 24 (Kleinbaum & Klein, 2010, p. 134). The null hypothesis is  $H_0 : \beta_{W_1} = \beta_{W_2} = \dots = \beta_{W_I} = 0$  (Singer & Willett, 2003, p. 399) and states that the coefficients of the parameters being tested,  $W_1$  through  $W_I$ , are zero, or equivalently, the odds ratios of said parameters are equal to unity.(Kleinbaum & Klein, 2010, p. 135) The test statistic is known as a likelihood ratio statistic and is approximately chi-square ( $\chi^2$ ), distributed in large samples under the null hypothesis, with degrees of freedom equal to the difference in the number of parameters between the two models that are being tested.

$$LR = -2 \times \ln L_1 - (-2 \times \ln L_2) \quad (24)$$

The decision rule is (Jungeilges, 2017):

$$\underline{\text{If}} LR \geq \chi_{1-\alpha}^2(r) \underline{\text{then}} \text{ reject } H_0 \underline{\text{else}} \text{ fail to reject } H_0$$

LR denotes the value of the test statistic and  $\chi_{1-\alpha}^2$  that level of a chi-squared random variable with  $r$  degrees of freedom which is exceeded with probability  $\alpha$ . If we reject the null hypothesis, this means we have witnessed a realization of the test statistic that is very unlikely under the null hypothesis, and the coefficient of at least one of the parameters being tested is not equal to zero. One of the unfortunate downsides of the likelihood ratio test is that one must estimate two models, both the full and reduced one, and to circumvent this issue one may use the Wald test.(Greene, 2012, p. 567)

### 4.4.3 Wald test

The Wald test is an alternative to the likelihood ratio test for testing the significance of parameters in a model, usually used when the interest lies in the significance of only one parameter.(Kleinbaum & Klein, 2010, p.138) The test for a single restriction is based on the idea of comparing a parameter estimate to its asymptotic standard error, whose test statistic is shown in Equation 25 (Singer & Willett, 2003; Greene, 2012, p. 403; p. 568).<sup>6</sup> This is known as the Wald chi-square statistic because the ratio is squared and is therefore chi-square distributed with one degree of freedom. Note that some authors also use a Wald test statistic that is not squared where it instead is approximately normally distributed in large samples under the null hypothesis.

$$Wald \chi^2 = \left[ \frac{\hat{\beta}_W}{ase(\hat{\beta}_W)} \right]^2 \sim \chi^2 \text{ on } 1 \text{ d.f.} \quad (25)$$

The null hypothesis when testing a single coefficient is that the coefficient of the parameter being tested is zero, or equivalently written as  $\beta_W = 0$  (Singer & Willett, 2003, p. 403) and the decision rule for the chi-square version of the Wald test is:

$$\underline{\text{If}} W \geq \chi^2_{1-\alpha}(r) \underline{\text{then}} \text{ reject } H_0 \underline{\text{else}} \text{ fail to reject } H_0$$

It is equivalent to that of the likelihood ratio test.(Jungeilges, 2017, p. 6) When rejecting the null the researcher states that the realization of the test statistic is highly unlikely under the null hypothesis, and that the coefficient of the parameter of interest is significantly different from zero given that all other parameters are kept in the model. The Wald test is more convenient than the LR test because only one model is required to be fitted instead of two, and both tests give equal test statistics in large samples. However, when dealing with small to moderate samples, the LR test should be preferred to the Wald test.(Kleinbaum & Klein, 2010; Singer & Willett, 2003, p. 139; p. 403)

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<sup>6</sup>The test statistic for the Wald test when testing multiple restrictions at once can be found, for instance, in (Greene, 2012, p. 568).

#### 4.4.4 Information Criteria

One might be interesting in comparing the goodness of fit of alternative models that are not nested. If so, using information criteria can be a useful endeavor.(Singer & Willett, 2003, p. 401) Measures of the goodness of fit that are very popular as model selection criteria are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).(Brooks, 2008, p. 232) Both penalizes the log-likelihood statistic for the number of parameters present in the model and in addition the BIC statistic also considers the total sample size.(Singer & Willett, 2003, p. 402) Both criteria represent a trade off between fit measured by the log likelihood value, and parsimony measured by the number of free parameters. The model with the smallest AIC and BIC value is mostly preferred, however one can choose to deviate from this if the criterion value differences are small for the subset of the models.(Verbeek, 2004, p. 285) For a model that has  $p$  parameters,  $AIC = Deviance + (2 \times p)$ . Similarly, we can compute BIC as  $BIC = Deviance + (\ln(N) \times p)$ .(Singer & Willett, 2003, p. 402)

#### 4.4.5 Goodness of link test procedure

The goodness of link test, sometimes known as the link test, is a specification test that considers whether the link function is appropriate, or whether we may have a link error.(Baum, 2013, p. 11) The test is due to (Pregibon, 1980, p. 16), and here we present a slightly different but very intuitive approach of the test procedure, originally suggested in his PhD dissertation in 1979.(StataCorp, 2017, p. 1318) The link test is performed by estimating the model under the hypothesized link, and then regressing the dependent variable on the the predicted values and their squares, including an intercept. If the link function is correctly specified, then the squares of the predicted values should be insignificant.(Baum, 2013, p. 11)

### 4.5 Assessing model adequacy

Once a logistic regression model has been fitted to our given data, the next question is whether the model explains the underlying data well, which is often known as examining the model adequacy. More specifically, we wish to know whether the probabilities given by

our model reflect the true outcome from the data, which is referred to as goodness of fit. We begin by denoting the observed values of the dependent variable ( $y$ ) in vector form as  $\mathbf{y}$  where  $\mathbf{y} = (y_1, y_2, y_3, \dots, y_n)$ . We then denote the values estimated by our model, known as fitted values, as  $\hat{\mathbf{y}}$  where  $\hat{\mathbf{y}}' = (\hat{y}_1, \hat{y}_2, \hat{y}_3, \dots, \hat{y}_n)$ . If the model fits well, we expect that the summary measures of distance between the two is small, and that contribution of each pair  $(y_i, \hat{y}_i), i = 1, 2, 3, \dots, n$ , to these summary measures is unsystematic and small relative to the error structure of the model.(Hosmer et al., 2013, p. 153)

In a general manner, the main approach in assessing the fit of the model consists of three steps. The first step is computing and evaluating of overall measures of fit, the second is examination of the individual components of the summary statistics, which is often done graphically and, finally, examination of other measures of the difference between observed and fitted values.(Hosmer et al., 2013, p. 154) More specifically, the three main methods for examining the adequacy of a logistic regression model are overall goodness of fit tests, examining the area under the receiver operating characteristic curve and examination of influential observations. The first step for assessing model adequacy in logistic regression is the use of overall goodness of fit tests. These are usually general tests that compare the observed and fitted values, testing for the fitted models general departure from the observed data.(Lemeshow & Archer, 2006, p. 97)

#### 4.5.1 Covariate patterns

First, we introduce the term covariate patterns, which are the different configurations of the covariates that are possible within the given model. For instance, a model has four covariate patterns if the model contains race and gender only, where each variable is coded at two levels.(Hosmer et al., 2013, p. 154) Our motivation for the explanation of covariate pattern follows from the fact that there is a divergence from the testing of models, as in Subsection 4.4, with the tests of overall goodness of fit. The tests at the model development stage are concerned with differences in parameters for the degrees of freedom and not the number of covariate patterns. The tests for overall goodness of fit may on the other hand be affected by the number of covariate patterns, whose number often increases drastically when continuous variables are included in the model.(Hosmer et al., 2013, p. 155)



To explain this divergence, consider a fitted model with  $p$  independent variables,  $\mathbf{x}' = (x_1, x_2, \dots, x_p)$ , and let  $R$  denote the number of distinct values of  $\mathbf{x}$  observed. (Hosmer et al., 2013, pp. 154-155) From this we understand that  $R$  represents the number of unique covariate patterns. (Lemeshow & Archer, 2006, p. 98) Some subjects may have the same values for  $\mathbf{x}$ , and then  $R < n$ . We can therefore denote the number of participants with  $\mathbf{x} = \mathbf{x}_r$  by  $m_r$ ,  $r = 1, 2, 3, \dots, R$ . From this follows that we may, for any fitted value and a given covariate pattern  $r$ , separate it into the number of subjects of with the same covariate pattern, denoted  $m_r$ , and the estimated logistic probability, denoted  $\hat{\pi}_r$ , which is equal for all subjects in the same covariate pattern. (Lemeshow & Archer, 2006, p. 98) The fitted value for the  $r^{\text{th}}$  covariate pattern may then be denoted by  $\hat{y}_r$ , shown in Equation 26, where  $\hat{g}(\mathbf{x}_r) = \hat{\beta}_0 + \hat{\beta}_1 x_{r1} + \hat{\beta}_2 x_{r2} + \dots + \hat{\beta}_p x_{rp}$  is the estimated logit. (Hosmer et al., 2013, p. 155)

$$\hat{y}_r = m_r \hat{\pi}_r = m_r \left\{ \frac{e^{\hat{g}(\mathbf{x}_r)}}{1 + e^{\hat{g}(\mathbf{x}_r)}} \right\} \quad (26)$$

#### 4.5.2 Three types of residuals

Before proceeding to the first test, we introduce three initial measures of the difference between the observed and fitted values. These are the Pearson residual, the deviance residual and the residual used in linear regression, following (Hosmer et al., 2013, pp. 155-156). The Pearson residual ( $Res$ ) is given for some particular covariate pattern  $r$  in Equation 27 and the summary statistic based on these residuals is the Pearson chi-square statistic shown in Equation 28. This residual has the difference between the observed and fitted value,  $y$  and  $\hat{y}$  respectively, in the numerator. The denominator contains the square root of the fitted value multiplied by one subtracted by the estimated probability for the participants in covariate pattern  $m_r$ .

$$Res(y_r, \hat{\pi}_r) = \frac{(y_r - m_r \hat{\pi}_r)}{\sqrt{m_r \hat{\pi}_r (1 - \hat{\pi}_r)}} \quad (27)$$

$$\chi^2 = \sum_{r=1}^R [Res(y_r, \hat{\pi}_r)]^2 \quad (28)$$

The deviance residual is given in Equation 29, the sign  $\pm$  being the same as for  $y_r - m_r \hat{\pi}_r$ , with the summary statistic based on these given in Equation 30.

$$d(y_r, \hat{\pi}_r) = \pm \left\{ 2 \left[ y_r \ln \left( \frac{y_r}{m_r \hat{\pi}_r} \right) + (m_r - y_r) \ln \left( \frac{(m_r - y_r)}{m_r (1 - \hat{\pi}_r)} \right) \right] \right\}^{1/2} \quad (29)$$

$$D = \sum_{r=1}^R d(y_r, \hat{\pi}_r)^2 \quad (30)$$

Finally, the linear regression-like residual is the difference between the observed and predicted values, shown in Equation 31 where the accompanying fit statistic is the sum-of-squares, given in Equation 32.(Hosmer et al., 2013, pp. 155-156)

$$s(y_r, \hat{\pi}_r) = (y_r - m_r \hat{\pi}_r) \quad (31)$$

$$S = \sum_{r=1}^R s(y_r, \hat{\pi}_r)^2 \quad (32)$$

### 4.5.3 Overall goodness of fit tests

The first test for the overall goodness of fit presented in this section is the Pearson chi-squared test. While this test may be used for several different goals, such as goodness of fit, homogeneity or independence, we focus on the first application. The test statistic is given in Equation 28, which is approximately chi-square distributed with  $R - (p + 1)$  degrees of freedom when  $m_r \hat{\pi}_r$  is large for every  $r$ , where  $R$  is the total number of covariate patterns and  $p$  is the number of independent parameters.(Lemeshow & Archer, 2006, p. 98) The null hypothesis is that there are no significant differences between the observed and expected values. If the test statistic exceeds the critical value, we reject the null and conclude that the difference is significant and we therefore do not have a well fitting model. The main issue with the test is the inclusion of continuous variables in the model, which severely increases the number of covariate patterns and renders the test ineffective as  $m_r \hat{\pi}_r$  may be small for every  $r$  when  $R \sim n$ (Lemeshow & Archer, 2006, p. 99), which leads us to a set of overall goodness of fit tests developed to deal with this issue.

The Hosmer-Lemeshow goodness of fit tests were developed in (Hosmer & Lemeshow, 1980) and (Lemeshow & Hosmer, 1982), where they suggested grouping based on the values of the estimated probabilities.(Hosmer et al., 2013, p. 157) The most common suggestion is to group the estimated probabilities into deciles of risk, by setting the number of groups, denoted  $G$ , equal to ten. The test statistic is estimated by partitioning the observations into these ten groups by their ordered estimated probability,  $\hat{\pi}_i$ , and a chi-squared test is then calculated with the test statistic shown in Equation 33.(Lemeshow & Archer, 2006; Hosmer

et al., 2013, p. 99; p. 158)

$$\hat{C} = \sum_{u=1}^G \frac{(o_{1u} - n'_u \bar{\pi}_u)^2}{n'_u \bar{\pi}_u (1 - \bar{\pi}_u)} \quad (33)$$

To decompose the test statistic, we note that it is a sum from one to the number of groups ( $G$ ). The numerator is given by a sum of observed cases,  $y_r$ , from one to the number of covariates in the  $r^{th}$  group,  $o_{1u} = \sum_{r=1}^{c_u} y_r$ ,  $c_u$  being the number of covariate patterns in the  $u^{th}$  group. This sum of cases ( $o_{1u}$ ) is subtracted by the number of subjects in the  $u^{th}$  group, multiplied by the average estimated probability in the  $u^{th}$  group which is equal to the average of the sum from one to the number of covariate patterns for the fitted values,  $\hat{y} = m_r \hat{\pi}_r$ . The test statistic is approximately chi-square distributed when the number of covariate patterns is equal to, or approximately equal to, the total sample size. (Hosmer et al., 2013, p. 158) The null hypothesis and decision criteria are the same as for the Pearson goodness of fit test outlined above.

The Hosmer-Lemeshow goodness of fit tests have also been criticized by academics as potentially having serious issues. The main problem is that the results may vary wildly depending on the number of groups that are used, with no theory to guide researchers to the most appropriate number of groups to be used. Adding substantive predictors with highly significant p-values may be expected to add to the goodness of fit of a model, but the HL goodness of fit test sometimes suggests that the addition of highly significant terms such as interaction terms is inappropriate. Lastly, the opposite of this result may also be encountered, where adding a non-significant interaction or non-linearity and performing the HL goodness of fit test may lead to the wrongful belief that the addition has improved the fit of the model, when it is the contrary that is actually correct. (*Hosmer-Lemeshow Test for Logistic Regression* | *Statistical Horizons*, 2019, p. 1)

#### 4.5.4 Accuracy: Sensitivity and specificity

Before moving on to the second step of assessing model adequacy, which is examination of the individual components of the summary statistics, we take a quick aside to explain the terms sensitivity and specificity, and their relation to a statistical understanding of the term accuracy. Accuracy is defined as the number of correct decisions divided by the total number

of cases, and the number of correct decisions may be separated into the number of true positive and true negative decisions.(Metz, 1978, p. 284) Decisions in this case refer to what we have predicted will happen, while cases refer to what we observed to happen. A positive simply means target event occurrence, and a negative means target event nonoccurrence. Lastly, a true positive means that our prediction of event occurrence was correct, and a false positive that our prediction of event occurrence was incorrect. The same applies for true and false negatives only that it is for event nonoccurrence instead.

Sensitivity and specificity are ways to represent two kinds of accuracy, namely for true positive and true negative cases, respectively. Sensitivity, which may be called a true positive fraction (TPF), is defined as the number of true positive decisions divided by the number of actually positive cases. Specificity, often referred to as the true negative fraction (TNF), is then the number of true negative decisions divided by the number of actually negative cases.(Metz, 1978, p. 285) The counterpart to the TPF and TNF are then the false positive fraction (FPF) and the false negative fraction (FNF), and for each category the fractions adding up to unity. Having explained these measures for accuracy, we turn our focus to how they may be summarized in a simple table, known as a classification table, or graphically by what is known as the receiver characteristic curve.

#### **4.5.5 Classification table**

The classification table shows a comparison between observed and expected values of the outcome variable in a binary way rather than as a probability of event occurrence.(Hosmer et al., 2013, p. 170) It is usually presented in the form shown in Table 3, where each cell contains each of the fractions described in Subsection 4.5.4. Since we wish to dichotomize outcomes from estimated logistic probabilities, a cutoff point is required, in which 0.5 is most used. A disadvantage is that the classification is sensitive to the cutoff point used. Classification is also sensitive to the relative sizes of the two component groups and always favors classification in the larger one, and this is independent from the actual fit of the model.(Hosmer et al., 2013, p. 171) Thus, accurate or inaccurate classification does not address criteria for goodness of fit and should only supplement a more rigorous assessment using methods described in Subsection 4.5.3.(Hosmer et al., 2013, pp. 169-170) Since the

classification table depends on a single cutoff value, a better and more complete description of classification accuracy is usually presented by the area under the receiver characteristic curve (Hosmer et al., 2013, p. 173), which is the focus of our next subsection.

Classified	Observed		Total
	<u>P</u> ositive	<u>N</u> egative	
<u>P</u> ositive	True P	False P	Predicted P
<u>N</u> egative	False N	True N	Predicted N
Total	Actual P	Actual N	TOTAL

Table 3: *The components of a classification table.*

*Sensitivity = (True P / Actual P), Specificity = (True N / Actual N)*

#### 4.5.6 Receiver Operating Characteristic (ROC) Curve

The ROC curve plots the true positive fraction, sensitivity, against the false negative fraction, 1 - specificity, for an entire range of possible cutoff points. The area under the ROC, AUROC for short, ranges from 0.5 to 1 and is a measure of the model's ability to discriminate between participants who experience the target event versus those that do not. (Hosmer et al., 2013, p. 174) In its graphical representation, the ROC curve will have values approaching unity when it is further towards the top left and approaching 0.5 when it lies close to the 45-degree line. As for deciding what signifies a good discrimination as given by the AUROC curve,

there is a rule of thumb given in the literature(Hosmer et al., 2013, p. 177):

$$\text{If} = \begin{cases} ROC = 0.5 & \text{This suggests no discrimination, so we might} \\ & \text{as well flip a coin.} \\ 0.5 < ROC < 0.7 & \text{We consider this poor discrimination, not much} \\ & \text{better than a coin toss.} \\ 0.7 \leq ROC < 0.8 & \text{We consider this acceptable discrimination.} \\ 0.8 \leq ROC < 0.9 & \text{We consider this excellent discrimination.} \\ ROC \geq 0.9 & \text{We consider this outstanding discrimination.} \end{cases}$$

#### 4.5.7 Examining influential observations

The third and final step of assessing model adequacy is by examination of influential observations. The idea is to see if the model fit is supported over all the sets of covariance patterns ( $R$ ), which can be done through a series of specialized measures, known as regression diagnostics, on all  $p$  covariates.(Hosmer et al., 2013, p. 186) This is different from the summary statistics considered in Subsection 4.5.3, as they provide with a single number to base conclusions on. To provide intuitive understanding of regression diagnostics for logistic regression, one may draw comparisons to the procedures of regression diagnostics in the framework of linear regression. Logistic regression diagnostics differ to those in a linear regression analysis due to the fact that in linear regression, it is assumed that the error variance does not depend on the conditional mean. In contrast, as mentioned in Subsection 4.2, the error variance in a logistic regression is a function of the conditional mean due to the nature of binomial errors, as shown in Equation 34.(Hosmer et al., 2013, p. 168)

$$\begin{aligned} \text{var}(Y_r|\mathbf{x}_r) &= m_r E(Y_r|\mathbf{x}_r) \times [1 - E(Y_r|\mathbf{x}_r)] \\ &= m_r \pi(\mathbf{x}_r)[1 - \pi(\mathbf{x}_r)] \end{aligned} \tag{34}$$

Continuing where we left off in Subsection 4.5.1, we introduce crucial elements that will aid our understanding as we go further along. Let  $Res_r$  and  $d_r$  denote the values of the expressions for the Pearson residual from Equation 27 and the deviance residual from Equation 23, respectively, for a covariate pattern  $\mathbf{x}_r$ . Examining equations 27 and 23, each

residual has been divided by an approximate estimate of its standard error, and if the logistic model is correct, we can expect the values to have a mean around zero and a variance close to one.(Hosmer et al., 2013, p. 187)

A crucial element in the diagnostics of linear regression is what is known as the "hat" matrix,  $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ ,  $\mathbf{X}$  being a  $R \times (p + 1)$  data matrix, or design matrix, where the first column equals one which represents the intercept of the model. The "hat" matrix provides the fitted values of the dependent variable, in the form of  $\hat{y} = \mathbf{H}y$ , which is why it is also known as the projection matrix.(Jungeilges, 2018a, p. 19) As for a "hat" matrix for a logistic regression, this was approximated by (Pregibon, 1981), and is shown in Equation 35, where  $\mathbf{V}_{R,R}$  is a diagonal matrix with general element equal to Equation 36.(Hosmer et al., 2013, 187)

$$\mathbf{H} = \mathbf{V}^{1/2}\mathbf{X}(\mathbf{X}'\mathbf{V}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{1/2} \quad (35)$$

$$v_r = m_r\hat{\pi}(\mathbf{x}_r)[1 - \hat{\pi}(\mathbf{x}_r)] \quad (36)$$

In the setting of linear regression, the diagonal elements of the "hat" matrix are called leverage values,  $h_{ir}$ <sup>7</sup> and enable the interpretation of the amount of influence the observed value  $y_r$  has on the fitted value  $\hat{y}_i$ .(Hoaglin & Welsch, 1978, p. 17) The leverage values are proportional to the distance of  $\mathbf{x}_r$  to the mean of the data,  $\bar{\mathbf{x}}$ . Large values of leverage then indicate points far away from the mean of the data, or the center of gravity (Jungeilges, 2018c, p. 30), and as such may have a large effect on the values of the estimated parameters.(Hosmer et al., 2013, p. 187).

In the setting of logistic regression, we denote  $h_r$  as the  $r^{th}$  diagonal element from  $\mathbf{H}$  from Equation 35, and state the possibility of showing the relationship in Equation 37.  $v_r = m_r\hat{\pi}(\mathbf{x}_r)[1 - \hat{\pi}(\mathbf{x}_r)]$  is the model based estimator of the variance of  $y_r$  and  $b_r = \mathbf{x}'_r(\mathbf{X}'\mathbf{V}\mathbf{X})^{-1}\mathbf{x}_r$  is the weighted distance of  $\mathbf{x}_r$ , the vector of covariate values for the  $r^{th}$  covariate pattern, from  $\bar{\mathbf{x}}$ , the vector of means.(Hosmer et al., 2013, p. 187) Some care should be taken in interpretation of the magnitude of the the leverage as it is a combined effect of  $y_r$  and  $b_r$ .(Hosmer et al., 2013, p. 188) Finally, while leverage values are useful for

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<sup>7</sup>Note these should not be confused with hazard probabilities  $h_{ij}$ .

detecting extreme points, they are not as viable for assessing their impact on the various aspects of the fit.(Pregibon, 1981, p. 706) We therefore introduce some additional diagnostic statistics, building upon the ideas introduced in Subsection 4.5.2.

$$\begin{aligned} h_r &= m_r \hat{\pi}(\mathbf{x}_r)[1 - \hat{\pi}(\mathbf{x}_r)]\mathbf{x}'_r(\mathbf{X}'\mathbf{V}\mathbf{X})^{-1}\mathbf{x}_r \\ &= v_r \times b_r \end{aligned} \tag{37}$$

One might be interested in the effect that deleting all subject with a particular covariate pattern has on the values of the estimated coefficients and on summary statistics such as  $\chi^2$  and  $D$ .(Hosmer et al., 2013, p. 191) In order to motivate the following statistics, we use the Pearson residual ( $Res_r$ ), from Equation 27, and standardize for covariate pattern  $\mathbf{x}_r$  as shown in Equation 38. It was approximated in Equation 31 that  $[y_r - m_r \hat{\pi}_r] \approx (1 - h_r)y_r$  which would make the estimator of the variance of the residual approximately  $y_r - m_r \hat{\pi}_r[1 - \hat{\pi}_r](1 - h_r)$ .(Hosmer et al., 2013, p. 190) Comparing to the Pearson residual in Equation 27, this implies that  $Res_r$  does not have variance equal to one unless we standardize it further.

$$Res_{sr} = \frac{Res_r}{\sqrt{1 - h_r}} \tag{38}$$

The effects of deleting all participants with a particular covariate pattern may mainly be presented in three different ways in a setting of logistic regression. Following the work of (Pregibon, 1981, pp. 716-720), he used linear approximations to derive the three diagnostic statistics  $\Delta\hat{\beta}_r$ ,  $\Delta\chi_r^2$  and  $\Delta D_r$ , which are shown in Equations 39, 40 and 41, respectively. The first statistic reveals those covariate patterns that have a great influence on the values of the estimated parameters(Hosmer et al., 2013, p. 192), and is obtained as a standardized difference between  $\hat{\beta}$  and  $\hat{\beta}_{(-r)}$ . The former represents the maximum likelihood estimates for all  $R$  covariate patterns and the latter excludes  $m_r$  subjects with pattern  $\mathbf{x}_r$ , and finally standardizing using the estimated covariance matrix of  $\hat{\beta}$ .

$$\begin{aligned} \Delta\hat{\beta}_r &= (\hat{\beta} - \hat{\beta}_{(-r)})'(\mathbf{X}'\mathbf{V}\mathbf{X})(\hat{\beta} - \hat{\beta}_{(-r)}) \\ &= \frac{Res_r^2 h_r}{(1 - h_r)^2} \\ &= \frac{Res_{sr}^2 h_r}{(1 - h_r)} \end{aligned} \tag{39}$$



$$\Delta\chi_r^2 = \frac{Res^2}{(1 - h_r)} \quad (40)$$

$$= Res_{sr}^2$$

$$\Delta D_r = \frac{d_r^2}{(1 - h_r)} \quad (41)$$

Derived similarly, the decrease in the value of the Pearson chi-square statistic ( $\Delta\chi_r^2$ ), and the change in the deviance ( $\Delta D_r$ ) may reveal covariate patterns that are poorly fit by large values for either or both diagnostic statistics. (Hosmer et al., 2013, p. 191) Finally, we include a list of what is considered the four most essential plots for an analysis of diagnostics from (Hosmer et al., 2013, pp. 193-194) below. This list consist of all four diagnostic statistics that we have described, and they have all been plotted versus  $\hat{\pi}_r$ , which is the estimated logistic probability from the regression. Having introduced the necessary tools for model assessment, we proceed to the section of the model building approach.

1. Plot  $h_r$  versus  $\hat{\pi}_r$ .
2. Plot  $\Delta\chi_r^2$  versus  $\hat{\pi}_r$ .
3. Plot  $\Delta D_r$  versus  $\hat{\pi}_r$ .
4. Plot  $\Delta\hat{\beta}_r$  versus  $\hat{\pi}_r$ .

## 4.6 Model building approach

### 4.6.1 Initial models for hazard

The three, initial discrete-time hazard models (A, B and C) include the main effect of period, the effect of period and the main of effect of spell and the effect of period, main effect of spell and their two-way interaction, respectively. The first model, denoted A, includes only the main effect of period and constrains the population logit-hazard profiles to be identical across spells, and is shown in Equation 42.

$$\text{logit}(h_{ij}(k)) = [\alpha_1 P_1 + \alpha_2 P_2 + \dots + \alpha_{159} P_{159}] \quad (42)$$

The model includes all the period indicators yet does not include an intercept to avoid complete linear dependency. The estimated parameters then describe the size and importance

of the period-by-period constrained hazard probabilities.(Willett & Singer, 1995, p. 53) The second model, denoted B, adds the main effect of spell by including all but one of the spell indicators, and is shown in Equation 43. This allows the logit-hazard profiles to differ in level across spell, but not in shape. The coefficients of the spell indicators describe the vertical displacement in hazard profile between the first spell and each subsequent spell, respectively.(Willett & Singer, 1995, p. 54)

$$\begin{aligned} \text{logit}(h_{ij}(k)) = & [\alpha_1 P_1 + \alpha_2 P_2 + \dots + \alpha_{159} P_K] \\ & + [\beta_2 S_2 + \beta_3 S_3 + \dots + \beta_J S_J] \end{aligned} \quad (43)$$

The third model, denoted C, adds cross product terms which represent the two-way interaction between periods and spells, and is shown in Equation 44. While model B allowed the logit-hazard profiles to differ in levels across spells, model C also allows for them to differ in shape from spell to spell. The number of spell-by-period cross product terms is case specific, but one should constrict them appropriately, here shown for the second and third spell using 10 and 9 periods, respectively. This is to avoid linear redundancy and to account for the presence of structural zeros caused by the data shortfalls described in Subsection 4.1.4. Still, there is a fine balance between model complexity and parsimony. A fully interactive model such as model C may encounter several sampling zeros in later spells and periods, which may be detrimental to the analysis. One should therefore explore possible smooth functions to represent spells and periods instead by using appropriate substitutes.(Willett & Singer, 1995, p. 56)

$$\begin{aligned} \text{logit}(h_{ij}(k)) = & \sum_{m=1}^K \alpha_m P_m + \sum_{m=2}^J \beta_m S_m \\ & + \sum_{m=1}^{10} \gamma_m (S_2 \times P_m) + \sum_{m=1}^9 \gamma_{m+10} (S_3 \times P_m) \end{aligned} \quad (44)$$

#### 4.6.2 Reparameterization of spell and period

Reparameterizations of spells and periods should be done in line with observations made during investigations of the sample hazard plots, such as those presented in Figure 1. We now present common themes that may be encountered for the spell indicator, before moving on to possible reparameterizations of the period counter in the following section. The first

observation that may be made is the commonalities of the hazard plots, for instance they are all either generally increasing or decreasing across spells.

The second observation that is often found is that the hazard plots for spells of odd and even numbers act differently, but that they are similar within their respective groups. This simply means that hazard plots for spells in the states of existence one and two act differently when compared, but that within one state of existence we observe similar patterns. Lastly, it is possible to observe differences within states, for instance the initial spell significantly differing from repeat spells. Earlier literature that have encountered such patterns have chosen two reparameterizations of the spell counter by using two dichotomous predictors as well as their two-way interaction. One predictor is used to distinguish even and odd numbered spells, and another predictor is used for distinguishing the initial from repeat spells.

The parameterization of periods follows the same guidelines as for spells; by the inspection of hazard sample plots. Since the parameterization of period dummies is usually retained because of the jaggedness of sample hazard plots, interactions between periods and other predictors are investigated instead. This is done by inspecting the interactions terms in model C and applying an appropriate linear, quadratic or logarithmic transformation in interaction with other predictors in the model.(Willett & Singer, 1995, p. 56) Examples of such transformations include, but are not limited to, linear transformation by dividing one by the period indicator, squaring the period counter or taking the natural logarithm of the period indicator variable. These transformations are then used in interactions with predictors such as those outlined for the reparameterization of the spell indicator variable or other substantive predictors, such as age or gender, in the fully extended version of the model.

### 4.6.3 Standard Errors

Standard errors give a measure of the degree of uncertainty in the estimated values of the coefficients.(Brooks, 2008, p. 46) Period to period fluctuations are to be expected in the population with sampling variation. To estimate the standard error for an estimated hazard probability,  $\hat{h}(t_k)$ , we begin by considering the population hazard probability  $h(t)$  in the  $k^{th}$

time period  $h(t_k)$ . We then estimate the parameter using Equation 2 as the proportion of the population that have experienced the target event out of the risk set for period  $k^{th}$ . One can then estimate the corresponding standard error using the formula used when estimating the standard error of a proportion, shown in Equation 45.(Singer & Willett, 2003, p. 348)

$$se(\hat{h}(t_k)) = \sqrt{\frac{\hat{h}(t_k)(1 - \hat{h}(t_k))}{n \text{ at risk}_k}} \quad (45)$$

Two general ideas can be interpreted from the equation and accompanying results. First, the numerator in Equation 45 is at its maximum when the estimated hazard probability is 0.5. From this we can interpret that when  $\hat{h}(t_k)$  is close to the extremes of 0 and 1, the estimate will be more precise. Since there are more than two periods in most spells, leading to the estimated hazard often staying below 0.5, we can say that a smaller estimated hazard probability will often be measured more precisely. Second, the denominator in Equation 45 is the size of the risk set in period  $k$ . Thus, in a larger risk set, we have a more precise hazard probability estimate. Additionally, in a spell where the risk set decreases due to the individuals getting the target event, standard errors for the successive periods will tend to increase, and thus the precision of the hazard estimate may be expected to decrease in later periods.(Singer & Willett, 2003, p. 349)

Estimating the standard errors of the estimated survival probabilities is unfortunately more complicated compared to the estimation of standard errors for the hazard probabilities. The main reason for this is that the survival probability is a product of the probability for survival in this time period and all previous periods, as implied by Equation 5. The idea of an estimate based on products of former estimates is quite difficult to execute statistically, and therefore one might find it useful to rely on Greenwood's approximation from his seminal paper on life tables, see (Greenwood, 1926). Greenwood shows that the standard error of the estimated survival probability in a time period  $k$  can be approximated by Equation 46.<sup>8</sup> This equation tells us that the standard error is calculated by the estimated survival probability for that period multiplied with a square root whose content involves the estimated hazard probabilities of all periods up to and including the time unit of interest. This is advantageous

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<sup>8</sup>As suggested by (Fayers, Harris, & Albert, 1992), one should be careful to trust Greenwood's approximation of standard errors in risk sets with fewer than 20 participants.

as one one does not have to consider the estimation of the standard error using all of the survival probabilities that have been estimated.(Singer & Willett, 2003, p. 350)

$$se(\hat{S}(t_k)) = \hat{S}(t_k) \sqrt{\frac{\hat{h}(t_1)}{n_1(1 - \hat{h}(t_1))} + \frac{\hat{h}(t_2)}{n_2(1 - \hat{h}(t_2))} + \dots + \frac{\hat{h}(t_k)}{n_k(1 - \hat{h}(t_k))}} \quad (46)$$

#### 4.6.4 Clustered sandwich estimator of variance

By using the robust estimator of variance, one can relax the independence of observations assumption in the conventional estimator.(StataCorp, 2017, p. 323) The equation for the robust estimate of variance is presented in Equation 47, where  $\hat{\mathbf{V}}$  is the conventional estimator of variance and  $\mathbf{u}_n$  is a row vector of the contribution from the  $n^{th}$  observation.(StataCorp, 2017, p. 325)

$$\hat{\mathbf{V}} = \hat{\mathbf{V}} \left( \sum_{n=1}^N \rho'_n \rho_n \right) \hat{\mathbf{V}} \quad (47)$$

For both linear and logistic regression models we have an asymptotic co-variance matrix given by Equation 48.(Greene, 2012, p. 586)

$$\text{Asy.Var}[\mathbf{b}] = (\mathbf{X}'\mathbf{X})^{-1} [\mathbf{X}'(\sigma^2\Omega)\mathbf{X}] (\mathbf{X}'\mathbf{X})^{-1} \quad (48)$$

The center matrix in the sandwich, as seen in Equation 49, motivates the robust estimator. (Greene, 2012, p. 586)

$$\mathbf{X}'(\sigma^2\Omega)\mathbf{X} = \sum_{n=1}^N \mathbf{X}'_n \Sigma \mathbf{X}_n \quad (49)$$

The asymptotic variance in this context is called the sandwich estimator, the picture here being that the center is a piece of ham between two pieces of bread. Data on individuals or other micro-level cases are often grouped in "clusters", a useful feature of panel data when observations cannot be considered independent.(Greene, 2012, p. 586) Each cluster is drawn from a joint density  $f_n(\mathbf{y}_n|\mathbf{X}_n, \boldsymbol{\theta})$ , with the data-set consisting of  $n$  multivariate observations  $[y_{n1}, \dots, y_{nT_n}]$  where  $n$  goes from 1 to  $N$ . By these limitations, Greene suggests a general case where the panel data-set consists of  $n$  multivariate observations dependent within cluster and where each independent cluster across the data-set of observations is drawn from a density. Greene suggests using the maximized pseudo-log-likelihood under the assumption that the

same parameter vector  $\theta$  enters the pseudo-likelihood, seen in Equation 50, as enters the correct and appropriate sample log-likelihood. (Greene, 2012, p. 587)

$$\ln L_P = \sum_{n=1}^N \sum_{t=1}^{T_n} \ln g(\mathbf{y}_n | \mathbf{X}_n, \theta) \quad (50)$$

When robust cluster variance estimators are employed in our software, standard errors are based upon the inverse of the negative hessian matrix. Under such circumstances the usage of the likelihood-ratio test is not appropriate. (StataCorp, 2013, p. 4) Having gained the necessary knowledge for the model building approach, we proceed to the next section where we introduce our initial model for time.

## 5 Empirical analysis and findings

### 5.1 Towards an initial model for time

#### 5.1.1 Choosing an initial model for time

Following the illustrative approach from earlier articles that concern themselves with multiple-spell discrete-time survival analysis, such as (Willett & Singer, 1995, p. 55), we introduce a figure which summarizes the main characteristics of all the preliminary hazard models fitted to the data set. This figure, shown in Figure 3, includes all the models discussed in Subsection 4.6.1 as well as their reparametrized versions. For each model we have presented the variables included, its deviance statistic and the number of parameters in the model. Between each of the nested models, we have supplied the value of the difference in the deviance statistic, which is the basis for the likelihood ratio test introduced in Subsection 4.4.2, in accordance with (Willett & Singer, 1995, p. 54).

After the difference in deviance statistics we show the difference in parameters between the models and the conclusion of the likelihood ratio test.<sup>9</sup> Since most of the coefficients

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<sup>9</sup>Models B, B2 and C drop 3 observations from the perfectly predicted event non-occurrences for spell 53. Comparing models with those that have a different number of observations is a violation of an assumption for the likelihood-ratio test and these tests are therefore marked F as we have applied a force option in our statistical program to perform the test.

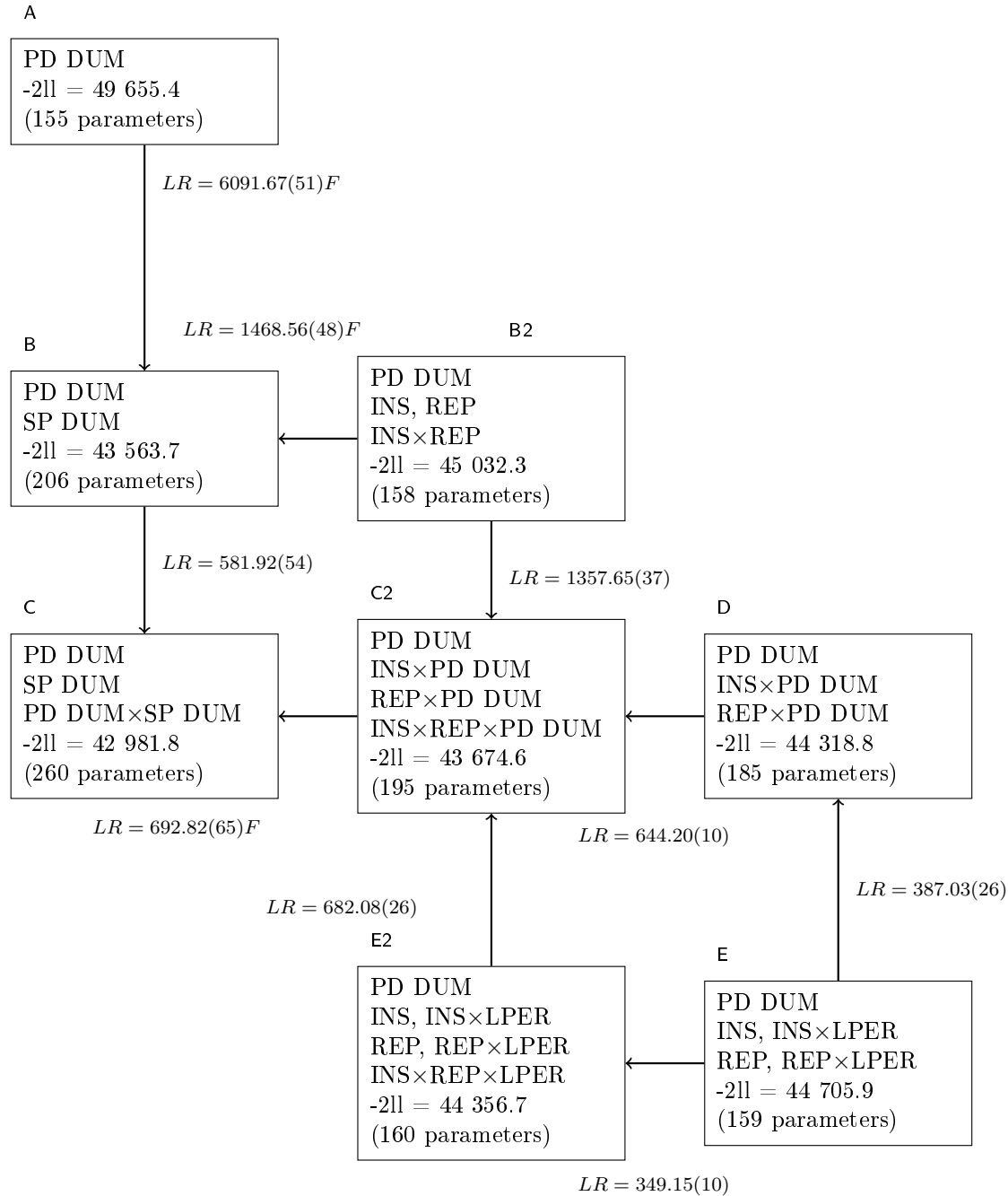


Figure 3: A taxonomy of multiple-spell discrete-time hazard models fitted to the data-set. Boxes contain the predictors for each model, the deviance statistic and number of parameters. Arrows between boxes indicate nested models, and their direction the model with the better fit as indicated by a likelihood ratio test. Text beside each arrow displays the difference in the deviance statistic. PD DUM = Period dummies, SP DUM = Spell dummies  
INS = Institution spells ( $STI = 1$ ), REP = Repeat spells ( $SP > 2$ ), LPER =  $\ln(\text{Period})$

are significant on a 1% level, we refrain from using stars to represent significance levels of coefficients and tests. Instead, we use ~~strike-through~~ to represent insignificance ( $p\text{-value} \geq 0.10$ ), *italic* to represent significance on a 10% level, **bold** to represent significance on a 5% level and all other values presented in plain text represent significance on a 1% level unless stated otherwise by including the corresponding p-value.

The arrows between the models signify which model was concluded to fit better by the means of the likelihood-ratio test alone. Whenever it was deemed necessary to investigate the models further, used values for the AIC, BIC, the conclusions from a Hosmer-Lemeshow goodness of fit test and a goodness of link test, found in Table 4. For readers interested in the complete process of model selection we have included this in the appendix, see Subsection 8.10. Our final choice for the initial model was model E, and the reasoning for this choice is given below.

Model	AIC	BIC	$\hat{C}$	Link
A	49 965.4	51 618.9	<del>0.00</del>	Y
B	43 975.7	46 173.3	69.62	N
B <sub>2</sub>	45 348.3	47 033.8	134.61	N
C	43 501.8	46 275.5	77.75	N
C <sub>2</sub>	44 064.6	46 144.9	<del>0.11</del>	Y
D	44 688.8	46 662.4	<del>0.79</del>	N
E	45 023.9	46 720.1	<del>8.85</del>	Y
E <sub>2</sub>	44 676.7	46 383.6	45.67	N

Table 4: Values for AIC, BIC, Hosmer-Lemeshow goodness of fit test ( $G=10$ ) and goodness of link test ( $Y = Pass, N = No\ pass$ ) for each of the model during the model selection process.

Model E cannot be said to have the best fit of all the initial models for time that we have investigated up to this point, but it is parsimonious in its specification and is concluded to be well specified. Compared to model D, it has a significantly higher deviance statistic, ( $LR = 398.00$ ), but in contrast passes both specification tests. It is also more parsimonious in respect to the number of parameters involved, having 26 and 36 fewer parameters than model D and C<sub>2</sub>, respectively. Finally, by using this model we have a sound way of interpreting its coefficients as it has been used in the earlier literature, as in (Willett & Singer, 1995,



p. 58). While model C2 may be superior in some aspects, we consider model E completely appropriate for the purpose of this analysis and leave the use of the more complex model as a suggestion for further research. Having chosen our initial model for time and concluded that the model fits well as per the result of the Hosmer-Lemeshow goodness of fit test, we now take a closer look at other summary measures for goodness of fit and perform some model diagnostics before interpreting the estimated parameters.

### 5.1.2 Model diagnostics

Having chosen our initial model, we proceed to examining the classification table and the AUROC curve. We have already performed the Hosmer-Lemeshow goodness of fit test and the goodness of link test, which is shown in Figure 3, in which the model was concluded to be well specified by both tests. The classification table, with the default cutoff point of 0.5, is shown in Table 5 together with the sensitivity and specificity shown beneath the table. The overall rate of correct classification is estimated as 97.97%, with 99.7% correct classification of event nonoccurrence (specificity) but only 13.71% correct classification for event occurrence (sensitivity). To answer why we get such a low value for the sensitivity, we focus our attention to Figure 4, which plots the sensitivity and specificity for all possible cutoff points for the study.

Classified	Observed		Total
	Y = 1	Y = 0	
Y = 1	876	918	1 794
Y = 0	5 514	310 109	315 623
Total	6 390	311 027	317 417

Sensitivity =  $876/6\ 390 = 13.71\%$ ;  
Specificity =  $310\ 109/311\ 027 = 99.70\%$ .

Table 5: *Expected classification table based on the logistic regression model E using a cutoff point equal to 0.5.*

We can then see that the two curves cross somewhere much closer to zero than 0.5, which would be a more appropriate choice for a cutoff point if our goal was to choose a cutoff point

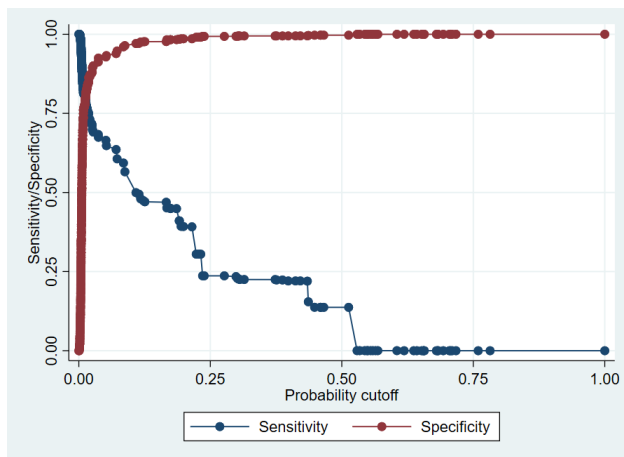


Figure 4: *Plot of sensitivity and specificity versus all possible cutoff points based on the logistic regression model E.*

to maximize both sensitivity and specificity. After all, the ability of a model to discriminate between event occurrence and nonoccurrence can be said to be more of a function of the difference between the groups than the logistic model itself (Hosmer et al., 2013, p. 174), and we are already fully aware that most people do not experience many spells, as evident in Table 15, see Subsection 8.4 in the appendix. To complete this discussion of sensitivity and specificity, we present the AUROC for the model in Figure 5, which is 0.867, considered “excellent” discrimination by the guidelines in Subsection (ROC). Examining the AUROC curve for the models considered, we find it is never outside the realm of what might be considered “excellent” discrimination. We now move on towards the third and final step of assessing model adequacy; examination of influential observations.

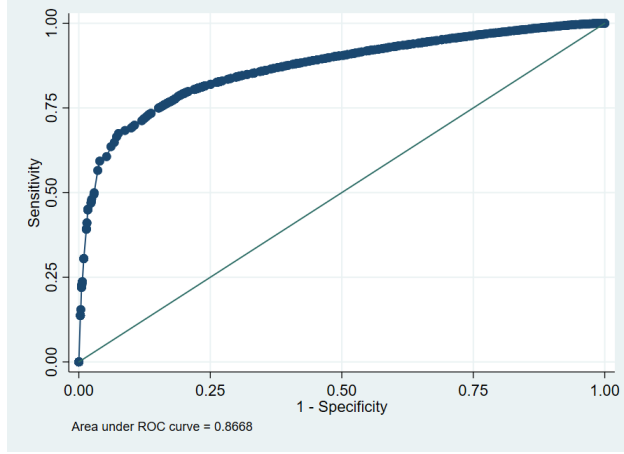


Figure 5: *Plot of sensitivity versus 1-specificity for all possible cutoff points based on the logistic regression model E.*

### 5.1.3 Influential observations

For our discussion of influential observations, we have included the four plots discussed in Subsection 4.5.7 in Figure 6. All plots have the estimated logistic probabilities ( $\hat{\pi}$ ) on the x-axis, and for the y-axis the top left plot has the leverage values ( $h$ ), the top right has  $\Delta\chi^2$ , the bottom left has  $\Delta D$  and bottom right has  $\Delta\hat{\beta}$ . For the points in each plot we have added the number of the covariate pattern, ranging from the somewhat arbitrary numbers 1 to 375, as they vary with how one aggregates the data. (Hosmer et al., 2013, p. 198) Our process will be to consider each plot, discuss its general appearance and take note of significant outliers that can be observed. Before we proceed, we will summarize our knowledge of the covariate pattern numbers.

For each covariate pattern, all values of the independent variables and estimated probabilities are equal. For instance, covariate pattern number one only has participants in their first spell ( $REP = 0$  &  $INS = 0$ ), a stay at the HNA, in the 158th period ( $PD\ NUM = 158$ ), and estimated probability of event occurrence is  $\sim 0.003$ . For the final covariate pattern, 375, all participants within this covariate pattern are in the first period ( $PD\ NUM = 1$ ) of a stay in the STI ( $INS = 1$ ) which is not their first stay ( $REP = 1$ ) and the estimated probability of event occurrence is  $\sim 0.052$ . Whenever we observe covariate patterns deemed as appropriate candidates for closer investigation, we describe the characteristics of said

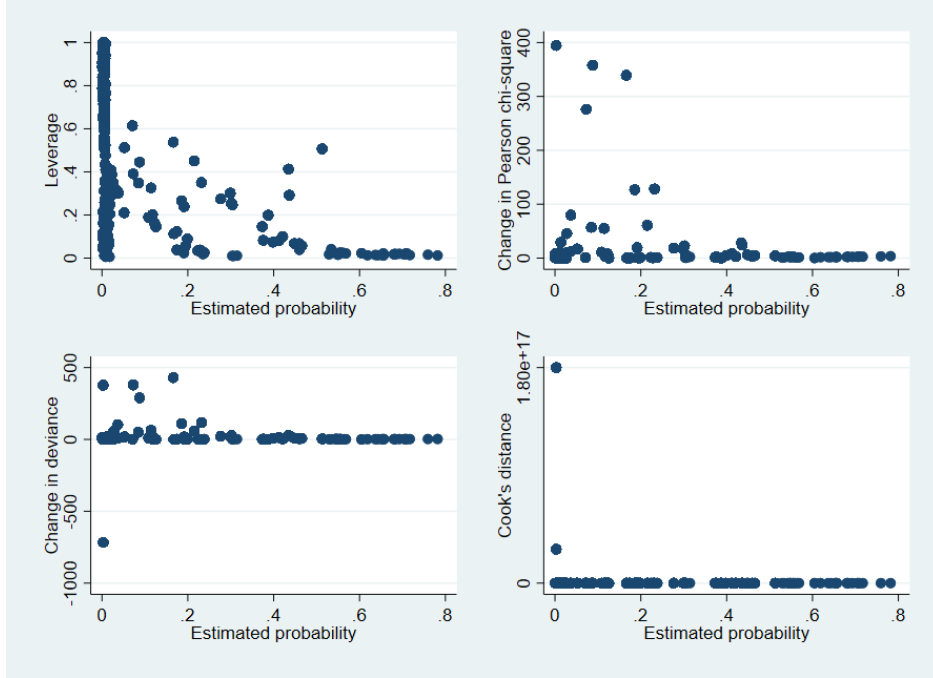


Figure 6: Plots of four diagnostic statistics ( $h$ ,  $\Delta\chi^2$ ,  $\Delta D$  and  $\Delta\hat{\beta}$ ) versus estimated hazard probability ( $\hat{\pi}$ ) based on the logistic regression model  $E$ .

covariate pattern in the examination that follows.

For the plot of leverage values against estimated probabilities, we can make a few statements on its general appearance and take notes on covariate patterns of interest. First, observe that many estimated probabilities are approximately zero, ranging widely from a leverage value nearly zero to unity. Examining what the covariate patterns represent, the covariate patterns with high leverage ( $h > 0.9$ ) values are those in later periods ( $PD\ NUM = 114 - 159$ ), and for early spells at the HNA ( $SP\ NUM = 1, 3$ ). The covariate patterns that do not have estimated probabilities close to zero, and leverage values not approximately zero, usually have covariate pattern numbers larger than 300. This indicates that these covariate patterns are for the earlier periods, not depending on the type of spell. Lastly, a good amount of covariate patterns in the range (200 – 250) have high estimated probabilities ( $\hat{\pi} = 0.6 - 0.8$ ), and after closer examination it turns out all these covariate patterns are for repeat visitors staying at the STI in periods ranging from 20 to 50.

For the plot of  $\Delta\chi^2$  against estimated probabilities, we find four covariate patterns (3, 364, 369 and 371) especially standing out from the rest, requiring closer examination. Covariate

patterns 3 and 364 are both in the first spell and have low estimated probabilities ( $\hat{\pi} < 0.1$ ). They are in period 155 and 3 and have leverage values equal to 1 and 0.4, respectively. For covariate patterns 369 and 371, both are in the second period of repeat spells, 369 being at the HNA and 371 at the STI, with leverage values of 0.45 and 0.54, respectively. Thus, for the first spell, period 155 has a great effect on the fit as given by  $\Delta\chi^2$ . It also has a large effect on the coefficients, given by the leverage value. Additionally, period 3 has a big impact on the fit as well, and it might not be coincidental that said period is also the peak of the sample hazard profiles as shown in Figure 1. As for repeat spells of either type, the second period seems to be of great affect to the overall fit of the model from examination of this plot.

For the plot of  $\Delta D$  against estimated probabilities, in addition to the four points already covered, we see a remarkably negative value for the first covariate pattern. For this covariate pattern, participants are in their first spell, and in the final period, 158, and this covariate pattern has a leverage value equal to 1. The leverage value indicates that this covariate pattern greatly affects the estimated coefficients, and the highly negative value of  $\Delta D$  leads us to the expectation that removing it will increase the deviance of the model significantly.

For the final plot we have  $\Delta\hat{\beta}$  against estimated probabilities, and we again observe extremely large values for covariate pattern 1 and 3. To summarize this section, we include a table shown in Table 6 where we state the covariate pattern numbers, the corresponding values of the independent variables and the values for each of the diagnostic statistics.

When adjusting for clustering effects, we have included plots of the four diagnostic statistics against estimated hazard probabilities in Figure 8 and a summary table describing the most ill-fitted covariate patterns in Table 17 in the appendix. Adjusting for clustering effects, covariate patterns 1 and 3 are no longer observed as being highly influential, yet we still experience issues with covariate pattern numbers above 300. Additionally, some of the leverage values exceed unity for reasons unknown. Overall, the observed values of the diagnostic statistics are lower, especially for  $\Delta\hat{\beta}$ . Having examined our chosen model, we are now ready to interpret the initial model for time in the next section.

Covar.No.	1	3	364	369	371
INS	0	0	0	0	1
REP	0	0	0	0	1
PD NUM	158	155	3	2	2
$y_r$	2	2	84	421	115
$\hat{\pi}$	0.0030	0.0029	0.0728	0.0876	0.1662
$\Delta\chi^2$	-	394.7	276.2	357.9	339.3
$\Delta D$	-716.8	377.0	379.1	289.6	430.1
$h$	1.000	1.000	0.391	0.445	0.538
$\Delta\hat{\beta}$	1.8e+17	2.82e+16	177.1	286.9	394.9

Table 6: *Covariate values, number of event occurrences ( $y_r$ ), estimated logistic probability ( $\hat{\pi}$ ) and the value of four diagnostic statistics  $\Delta\chi^2$ ,  $\Delta D$ ,  $\Delta\hat{\beta}$  and leverage ( $h$ ) for five of the most influential covariate pattern numbers. Based on the logistic regression model E, not adjusted for clustering effects.*

## 5.2 Interpreting the initial model for time

For our chosen initial model for time, model E, we plot the fitted hazard functions for the first 10 spells in Figure 7. The full model output is shown in Table 19 in the appendix, and odds ratios with corresponding 95% confidence intervals adjusted for clustering effects are in Table 7. We got a recommendation that adjusting for clustering effects might be appropriate, which has been done in previous literature such as (Heggstad, 2001, 2002). This is because some of the observations in the data set might not be independent, while adjusting for clustering effects negates this requirement, as outlined in Subsection 4.6.4. Therefore, all models and corresponding tables and figures from this point onward have been adjusted for clustering effects by clustering on the identification variable, ID.

The coefficients for  $Period_1, Period_2, \dots, Period_{159}$  describe the logit-hazard profile for the first spell at the HNA, when INS and REP are zero. Their magnitudes suggest the logit-hazard is initially quite high in the first periods before decreasing and converging to a constant level, reflected in the fitted first-spell hazard profile in Figure 7. The fitted hazard functions were attained by substituting estimated parameters into Equation 61 and solving for  $logit(h_{ij}(k))$ , which for the first spell is simply antilogging each parameter estimate for each period. To consider how the risk profiles differ across spells in different states,

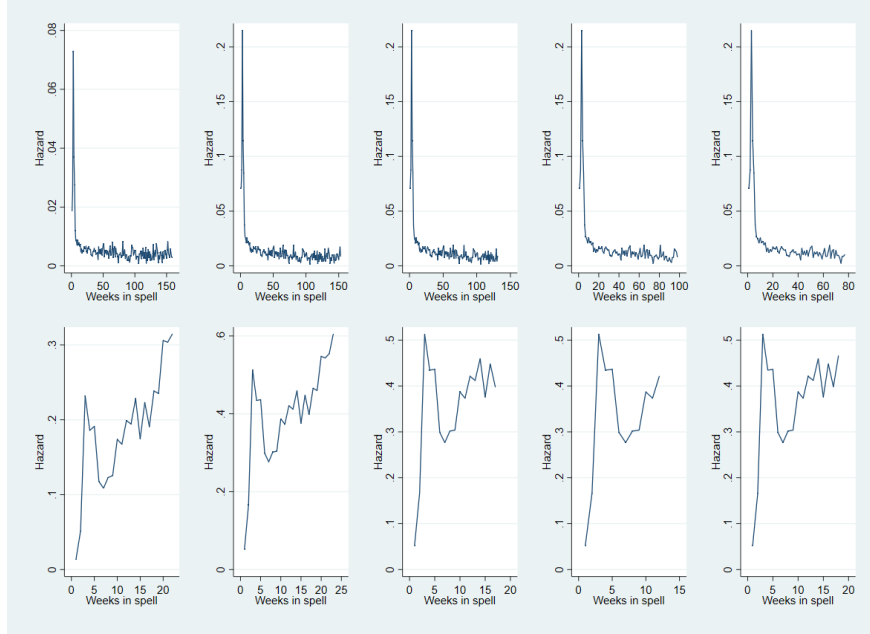


Figure 7: *Fitted hazard functions for the first 10 spells based on the logistic regression model E. The top row represents spells at a home-nurse area (Spell 1, 3, 5, 7, 9). The bottom row represents spells at short-term institutions (Spell 2, 4, 6, 8, 10).*

we investigate the coefficient for the INS and the interaction term with LPER. Since the logarithm of unity is zero, the coefficient for INS,  $-0.32$ , estimates the difference between fitted in- and out-of-STI logit-hazard probabilities in Period 1. (Willett & Singer, 1995, p. 57) After antilogging, the estimated odds ratio of 0.72 means that the estimated odds of

Variable	Odds ratio	95% CI
INS	0.72	(0.502, 1.047)
INS $\times$ LPER	4.57	(3.416, 6.106)
REP	4.01	(3.478, 4.620)
REP $\times$ LPER	0.88	(0.829, 0.938)

Table 7: *Estimated odds ratios and 95% Confidence Intervals for the initial model for time, Model E, adjusted for clustering effects.*

returning to a HNA after one period are 0.72 times that of leaving a HNA after one period. If we divide one by the estimated odds ratio, we get 1.38, and we can say that the estimated odds of leaving the HNA after one period are 38% higher than leaving the STI after one

period.<sup>10</sup>

Since the estimated coefficient on the interaction term is opposite to the main effect, and very large, this means that the differential will reverse, and rather quickly. For instance, by the second period the differential has already reversed, and after the second period a person at a STI is more than twice as likely to leave the institution than someone at a HNA is likely to enter, the procedure that led to this interpretation being shown in Equation 51.

$$\frac{e^{(\hat{\beta}_{P_2} + \hat{\beta}_{INS} + (\ln(2) \times \hat{\beta}_{(INS \times LPER)}))}}{e^{\hat{\beta}_{P_2}}} = \frac{e^{(-3.645 + (-0.322) + (\ln(2) \times 1.519))}}{e^{-3.645}} \approx \frac{0.05476}{0.02613} \approx 2.08 \quad (51)$$

The effects of initial and repeat spells, whether the participants are at a HNA or STI, are described by the coefficients on the predictors REP and  $REP \times LPER$ . The antilog of 1.39 (4.01) indicates that the odds a repeat spell of either type<sup>11</sup> will end in the first period are about 4 times larger than the odds for an initial spell ending in the same period. (Willett & Singer, 1995, p. 58) Since the sign of the coefficient on the interaction term is opposite to that of the main effect, this suggests that this differential will also reverse over time. Because the magnitude of the interaction term is not as large as in the previous case, it will take significantly longer before these differential reverses. By the 10<sup>th</sup> period, the odds ratio has decreased to 3, to 2.5 by the 43<sup>rd</sup> period. However, it does not decrease below 2 during the first 159 periods. Thus, in the first three years of a spell, returnees from a STI to a HNA are at higher risk, at least twice as likely, of readmission than someone that has not been admitted earlier. Similarly, for the same periods in a spell, those having been to the institution previously are at higher risk of leaving than those there for the first time. Before we proceed to interpreting other predictors of interest, we will describe how they are defined and how we go about adding them to the original model.

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<sup>10</sup>When adjusting for clustering effects, the coefficient for INS is only significant on a 10% level as opposed to 1% without adjustment.

<sup>11</sup>Assuming no significant difference in risk for in- or out-of-institution spells.



### 5.3 Specifying the demographic variables

The predictors we will be using are characteristics representing the gender, cohabitation, age and reablement program participation for each participant. The gender variable, MALE, is a dichotomous variable equal to 1 if the participant is a male, and zero otherwise. The cohabitation variable, ALONE, is equal to 1 if the participant lives alone, and zero otherwise.<sup>12</sup> The variable for age, AGE, is a categorical variable with 3 values, 0 to 2, and the age that has been used is the participants age in the second year during the study. The variable is equal to zero if the participant is younger than 67, equal to one if he or she is between 67 and 80, and equal to two if older than 80. We will be using dummy variables in order to compare different age groups, using group 1 as the baseline group. The reablement program participation variable, REHV1, is a dichotomous variable equal to 1 from the week the participant enters the reablement program and for the rest of their time in the study. We have also added REHV2, a dichotomous variable similar, but not equal to, the original REHV1. The difference between them is that REHV2 does not include the time spent in the program, but instead only records the time after reablement program participation. We now summarize our estimation procedure for each variable involved.

We investigate the relationship between predictors by adding them to the initial model E and comparing model goodness of fit with a likelihood ratio test<sup>13</sup>, (Willett & Singer, 1995, p. 59) and run a HL goodness of fit test before interpreting the odds ratio of the main effect and interaction effects. For each variable we estimate separate models and, in addition to the main effect, add two-way interaction terms whenever meaningful and significant. The main effect describes the differential in hazard probabilities between values 0 and 1. If the two-way interaction between the main effect and log-period is included and significant, then the coefficient for the main effect explains the first period in the first spell only. If not, the differential of the main effect does not change across time within spell. The interaction term between the main effect and INS describes how hazard probabilities differs in spells at the

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<sup>12</sup>More precisely, this variable is equal to one if the participant is unmarried, or not married/in a partnership, or a widow(er), or separated/divorced, or married/partner is at an institution.

<sup>13</sup>The likelihood-ratio test is invalid when adjusting for clustering effects, and the value given is for the corresponding unadjusted models.

HNA and the STI. Finally, the interaction term between the main effect and REP describes how hazard probabilities differ between initial and repeat spells.

## 5.4 Separate models for demographic variables

We will begin this section by estimating separate models for each characteristic predictor by adding them to the initial model. After this is complete, we will combine the separate models into a fully integrated one. Following the conventions in literature such as (Hosmer et al., 2013, p. 77), we provide the odds ratios and confidence intervals of interest for each model, adjusted for clustering effects, indicating the significance level using the previously established system. For completeness we have included tables with all coefficients, standard errors, p-values and confidence intervals for each model in the appendix, see Subsection 8.13.

### 5.4.1 Gender

Model Gender includes the main effects of the gender variable, MALE, with odds ratio of the main effect in Table 8 with corresponding 95% confidence intervals adjusted for clustering effects. The three two-way interactions with MALE were not significant when adjusting for clustering effects,  $W = 4.6, p = 0.2034$ , and were therefore omitted. The hazard profile for men differs significantly from that of women,  $LR = 14.96, p = 0.0001$ , and the model fits reasonably well,  $\hat{C} = 5.12, p = 0.7446$ . The antilog of the coefficient for MALE, 1.11, implies that men have 11% higher odds compared to women of event occurrence, a differential that does not change across time or depending on the type of spell considered.

Variable	Odds ratio	95% CI
MALE	1.115	(1.026, 1.211)

Table 8: *Estimated odds ratio and 95% Confidence Interval for the Gender model, adjusted for clustering effects.*

### 5.4.2 Living alone

Model Living alone includes the main and interaction effects of living arrangements, ALONE, with odds ratios and corresponding 95% confidence intervals in Table 9, all adjusted for clustering effects. The two-way interaction for the main effect with REP was not significant when adjusting for clustering effects,  $W = 1.87, p = 0.1716$ , and was therefore omitted. The fit improved significantly by adding the additional parameters,  $LR = 256.75$ , and it was concluded to be well specified,  $\hat{C} = 8.48, p = 0.3879$ . The odds ratio of the main effect (0.41) is the differential for someone living alone in the first period of a spell at a HNA.<sup>14</sup> This means that someone that someone not living alone is more than twice as likely ( $2.46 = \frac{1}{0.41}$ ) of being admitted to the STI in the first period of any odd numbered spell.

The interaction with log-period is opposite to that of the main effect which means the differential reverses across time within spell, and by the 131st week in a spell at a HNA, it has fully reversed. For the first period of spells at a STI, someone living alone has 0.78 ( $e^{-0.9+0.65}$ ) times the odds of someone not living alone of release, which reverses by the 4th period. Thus, while someone not living alone has higher odds of leaving, after four periods the odds are turned and the differential increases from the 4th period and onward.

Variable	Odds ratio	95% CI
ALONE	0.407	(0.317, 0.523)
ALONE×LPER	1.202	(1.106, 1.307)
ALONE×INS	1.925	(1.505, 2.462)

Table 9: *Estimated odds ratios and 95% Confidence Intervals for the cohabitation model, adjusted for clustering effects.*

### 5.4.3 Age

Model Age considers the effect aging has on hazard probabilities for event occurrence, with odds ratios and their confidence intervals presented in Table 10, all adjusted for clustering effects. We have added two interactions terms, one for each age group, for each of the three variables LPER, INS and REP. While some of these terms were significant for one age group

<sup>14</sup>If the interaction with REP was significant, the differential would only be for initial spells at a HNA.

but not the other, all were kept in the model in order to preserve the complete structure of the categorical variable. The model gives the largest decrease in the deviance statistic of all models considered,  $LR = 482.74$ . The model passes both of the specification tests,  $\hat{C} = 5.52, p = 0.7005$ .

For the younger group, the main effect and all but one two-way interaction term is significant, which is the one between the main effect and LPER. Antilogging, the odds ratio describing the differential in hazard due to age is 0.31 in any period of the first spell, indicating that the reference group (age 67-80) is more than 3 times as likely of event occurrence compared to those younger than 67 in any period during an initial spell at a HNA. If the spell is not initial, but rather a repeat spell at a HNA, this odds ratio is 0.56 ( $e^{-1.18+0.59}$ ) instead.

In any period of an initial spell at a STI, the younger cohort has an odds ratio of 0.81 compared to the reference group, meaning the reference group has 23% higher odds of leaving a STI in any period of an initial spell. If it is a repeat spell at a STI, the odds ratio is 1.47 in any period, with the younger group having 47% higher odds of leaving in any period. This indicates persons in the reference group are more likely to leave the STI than the younger cohort, but only if it is their first time at the STI.

For the group older than the reference group, anyone above the age of 80, the main effect is not significant while the two-way interactions term between AGE and LPER is significant. This means there are no significant differences due to age in the first period of any spell, but a differential occurs across time within spell, not depending on whether the spell is at a HNA or a STI, nor initial or repeat. The interaction with LPER is positive and significant, meaning this differential escalates. For instance, by the 19th period the odds ratio is 1.51, indicating that those older than 80 have 50% higher odds of being admitted to, or released from, a STI than the reference group.

Variable	Odds ratio	95% CI
AGE		
Group 0	0.309	(0.210, 0.454)
Group 2	<del>0.858</del>	(0.662, 1.112)
AGE×LPER		
Group 0	<del>1.037</del>	(0.915, 1.175)
Group 2	1.150	(1.057, 1.252)
AGE×INS		
Group 0	2.636	(1.686, 4.121)
Group 2	<del>0.998</del>	(0.765, 1.302)
AGE×REP		
Group 0	1.806	(1.260, 2.588)
Group 2	<i>0.854</i>	(0.712, 1.026)

Table 10: *Estimated odds ratios and 95% Confidence Intervals for the age model, adjusted for clustering effects.*

#### 5.4.4 Reablement

This section considers two models for the effects of reablement program participation in two different ways, one including (REHV1) and the other excluding (REHV2) the time spent in the program. Model REHV1 includes the time spent in the program, with odds ratios and confidence intervals adjusted for clustering effects shown in Table 11.

Variable	Odds ratio	95% CI
REHV1	1.519	(1.297, 1.780)
REHV1×REP	0.520	(0.408, 0.663)

Table 11: *Estimated odds ratios and 95% Confidence Intervals for the previous or current reablement program participation model, adjusted for clustering effects.*

The two-way interaction between the main effect and LPER or INS are not significant,  $W = 4.65, p = 0.0976$ , and have therefore been omitted. The fit is improved when adding the extra parameters compared to model E,  $LR = 45.94$ , and passes both specification tests,  $\hat{C} = 9.94, p = 0.2694$ . Antilogging the coefficient of the main effect, we get an odds ratio of 1.52, indicating that participants currently receiving or those that previously received

reablement services have 52% higher odds of event occurrence in initial spells of either type. Considering repeat spells of either type, the odds ratio is 0.79 in any period. This means that in initial spells, having received or receiving reablement services brings forth higher risk of either entering or leaving a STI compared to non-participants, while the opposite is true for repeat spells of any type.

The odds ratios and confidence intervals adjusted for clustering effects for the model considering only previous program participation, REHV2, is shown in Table 12. The same two-way interaction terms were insignificant as for model REHV1,  $W = 4.7, p = 0.0954$ , and were therefore omitted. The model has a significantly better fit than model E,  $LR = 48.25$ , and passes both specifications tests,  $\hat{C} = 7.81, p = 0.4522$ . Considering only the period after program participation, the odds ratios are similar to that of REHV1. The odds ratio of the main effect is 1.86, indicating that previous reablement program participators have 86% higher odds of being admitted to, or released from, a STI in any given period of an initial spell. In repeat spells, the odds ratio is 0.94 for any period. Summarizing, the odds are significantly higher for being admitted to a STI for the first time, and also higher leaving than for non-participants. The odds are somewhat lower for being readmitted compared to non-participants, but also lower for being released once readmission has occurred.

Variable	Odds ratio	95% CI
REHV2	1.862	(1.560, 2.223)
REHV2×REP	0.506	(0.392, 0.652)

Table 12: *Estimated odds ratios and 95% Confidence Intervals for the previous reablement program participation model, adjusted for clustering effects.*

## 5.5 Combined models for demographic variables

We are interested in a model which incorporates several of the variables that we have considered thus far. The main goal is to investigate whether the conclusions made about reablement program participation hold when we control for gender and age.<sup>15</sup> Therefore, we combine

<sup>15</sup>The effects of living alone have not been included, the reasoning behind this discussed in Subsection 6.2.2.

the models for reablement program participation with the models for effects of gender and age and add two-way interaction terms for each of the main effects of reablement, gender and age. Since we have two versions of the reablement participation variable, one including the time spent in the program and one excluding it, we will have two versions of the combined model.

We find that the most intuitive approach is the comparison of the coefficients for the combined models against the coefficients from the separate models, which we have presented in Table 13. The first combined model, COMB1, fits significantly better than model E,  $LR = 569.48$ , and passes both specification tests,  $\hat{C} = 9.2, p = 0.3254$ . The second combined model, COMB2, has very similar statistics with the same properties,  $LR = 565.27$  for the likelihood ratio test and  $\hat{C} = 11.09, p = 0.1967$  for the HL goodness of fit test.

Combining the separate models have modified the coefficients involved, both for the effect of time and characteristic predictors. The differential in odds ratio for  $INS^{16}$  has increased from 0.72 in model E to 0.63 and 0.62 in model COMB1 and COMB2, respectively, while being approximately equal for the interaction with log-period. This indicates the odds of event occurrence in the first period of an initial spell at a STI are even lower when accounting for the other predictors involved, but the reversal across time within spell is the same. The odds ratio for REP is approximately equal for model E and COMB1, but lower for COMB2, while the interaction with log-period is the same for all three models. This indicates controlling for REHV2 as opposed to REHV1 has the effect of reducing the differential when considering the first period in a repeat spell of any type, while the reversal effect remains unchanged.

For the predictors from the separate models, all coefficients are still significant in the combined models, yet their magnitude has in some cases been modified. The main effect of gender is of larger magnitude for the combined models, odds ratios equal to 1.3 and 1.32 for COMB1 and COMB2, respectively, as opposed to 1.11 for the separate model. This indicates the differential in odds ratio due to gender is larger than we previously estimated when we account for age and reablement program participation. The odds ratio for the youngest cohort is slightly higher for the combined models, indicating a smaller differential due to

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<sup>16</sup>Note that  $INS$  is now significant at 1%, while it previously was only significant at a 10% level.

age when accounting for gender and reablement program participation. For the two-way interaction terms accounting for gender and reablement had little to no effect, indicating that previous expectations for differential behavior in repeat spells of any kind still hold.

Comparing the odds ratio for reablement in the combined models against their corresponding separate models, the magnitude of the main effect decreases while the two-way interaction with REP increases in both cases. The odds ratio of the main effect for REHV1 is 1.52 and 1.86 for REHV2, while it is 1.44 and 1.82 in COMB1 and COMB2, respectively. In other words, controlling for age and gender decreased the likelihood of event occurrence in initial spells due to reablement program participation. The odds ratio for the interaction with REP increased, meaning there is a decrease in difference between program participants and non-participants in repeat spells. For instance, recall that the odds ratio in repeat spells was 0.94 for program participants in model REHV2, while now it is  $0.99^{17}$  ( $e^{0.6-0.61}$ ). This means accounting for gender and age has not only decreased the differential in initial spells, but also in repeat spells.

Finally, none of the two-way interactions between the characteristic predictors were significant at our chosen confidence level. Even if the interaction between age and gender was close to being significant, p-values equal to 0.054 and 0.05 for COMB1 and COMB2, respectively, we do not put any weight on this result. Our greatest interest was in finding meaningful interactions between age or gender and reablement participation, none of which turned out to be of any significance. In the next section we compare our findings to that of previous literature in similar settings and assess the potential impact of our choices.

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<sup>17</sup>We tried adding the interactions terms REHV2×LPER and REHV2×INS back into model COMB2 to see if this would have an effect. Both interactions terms were insignificant,  $W = 2.24, p = 0.3268$ , yet the odds ratio for REHV2 increased to 2.33 and the interaction with REP decreased to 0.508. Thus, adding the interaction terms back in, while themselves being insignificant, had the effect of making the main effect much larger in magnitude and the odds ratio in repeat spell increased from 0.99 to 1.186.



Variable	Odds ratio (95% CI)		
	Separate	COMB1	COMB2
INS	<i>0.725</i>	0.628 (0.443, 0.891)	0.621 (0.440, 0.878)
INS×LPER	4.567	4.574 (3.496, 5.984)	4.591 (3.521, 5.986)
REP	4.008	4.035 (3.353, 4.857)	3.891 (3.235, 4.679)
REP×LPER	0.882	0.879 (0.826, 0.935)	0.880 (0.827, 0.936)
MALE	1.115	1.303 (1.109, 1.530)	1.317 (1.121, 1.547)
AGE			
Group 0	0.309	0.364 (0.238, 0.559)	0.364 (0.237, 0.558)
Group 2	<del>0.858</del>	0.946 (0.725, 1.234)	<del>0.940</del> (0.722, 1.225)
AGE×LPER			
Group 0	<del>1.037</del>	1.032 (0.912, 1.167)	<del>1.034</del> (0.913, 1.171)
Group 2	1.150	1.144 (1.053, 1.243)	1.145 (1.054, 1.244)
AGE×INS			
Group 0	2.636	2.628 (1.683, 4.105)	2.640 (1.698, 4.105)
Group 2	<del>0.998</del>	1.000 (0.767, 1.302)	<del>1.007</del> (0.777, 1.306)
AGE×REP			
Group 0	1.806	1.821 (1.265, 2.621)	1.862 (1.293, 2.679)
Group 2	<i>0.854</i>	<del>0.885</del> (0.735, 1.066)	<del>0.893</del> (0.741, 1.076)
REHV1	1.519	1.436 (1.076, 1.916)	
REHV1×REP	0.520	0.556 (0.438, 0.705)	
REHV2	1.862	1.824 (1.349, 2.467)	
REHV2×REP	0.506	0.542 (0.422, 0.697)	
AGE×MALE			
Group 0		<i>0.727</i> (0.526, 1.005)	<i>0.721</i> (0.520, 1.000)
Group 2		<del>0.864</del> (0.713, 1.041)	<del>0.862</del> (0.713, 1.042)
MALE×REHV		<del>1.164</del> (0.892, 1.518)	<del>1.104</del> (0.845, 1.433)
AGE×REHV			
Group 0		<del>0.899</del> (0.567, 1.425)	<del>0.864</del> (0.516, 1.446)
Group 2		<del>0.874</del> (0.652, 1.173)	<del>0.853</del> (0.638, 1.142)

Table 13: *Estimated odds ratios and 95% Confidence Intervals for all separate and combined models, adjusted for clustering effects.*

## 6 Discussion

Here we discuss our findings in the light of previous literature, the choices we have made during the work process and the potential effects our choices had on the results. In many cases our findings are consistent with the conclusions of previous studies. The main themes of choices and issues concerning these choices will be for the data cleaning approach and

variable specification.

## 6.1 Results in the light of previous studies

On the topic of gender, it has previously been shown that men have a hazard ratio of 1.18 compared to women concerning unplanned readmissions to Norwegian hospitals within 30 days, adjusted for clustering effects.(Heggestad, 2002, p. 655) Our results show that men have an odds ratio of 1.12<sup>18</sup> compared to women in a setting of traveling between home-nurse areas and the short-term institutions in questions when adjusting for clustering effects. On the other hand, Heggestad found no such relationship for later<sup>19</sup> readmissions, while our findings indicate that this differential is constant across time within spell.

On the topic of living alone, previous literature has considered the effect that living alone and marriage has on the risk of being admitted to, or released from, a nursing-home. Their results indicate living alone is associated with spending a shorter time living in a community before being admitted to a nursing-home, and being married is associated with spending longer time in the community before being admitted to a nursing-home.(Liu et al., 1991, p. 132) Our findings indicates those living alone<sup>20</sup> have lower odds of being admitted to a STI<sup>21</sup> and also lower odds of being released compared to those cohabiting, but that the odds differential reverses within four weeks in a spell at a STI.

For the effect of age it has been found previously that individuals over the age of 80 have a hazard ratio of 1.09 compared to individuals in the age group 67-80 for early readmissions, and 1.21 for late readmissions to Norwegian hospitals.(Heggestad, 2002, p. 658) In a study investigating nursing-home admission, it was found higher age was associated with shorter duration in the community before admission, but no relationship between age and nursing-home dismissal.(Liu et al., 1991, p. p 132) Our findings show the youngest cohort (AGE

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<sup>18</sup>This odds ratio is for the separate model, while the combined models gave significantly higher odds ratios of 1.30 and 1.32.

<sup>19</sup>90-180 days after discharge.

<sup>20</sup>Recall that ALONE is made up of many variables, one of those representing whether a candidate is married or not. Those that are unmarried have (ALONE=1) and as such are assumed to live alone.

<sup>21</sup>This finding does not coincide with (Liu et al., 1991), but they considered nursing-homes, a form of long-term institution. We have only covered admission to short-term institutions and not long-term alternatives.

<67) are much less likely of being admitted to a STI, in all periods for both initial and repeat spells. For the release from a STI, we find the youngest cohort much more likely to be released than the reference group (ages 67-80), but only in repeat spells at a STI. For the relationship between the reference group and the oldest cohort (80+), we find no initial difference, but instead a differential appears and grows over time within spell. Additionally, this escalation is not significantly altered by which state or spell the participants are currently in.

In previous studies investigating the effect of reablement program participation, the results have been mixed. One study concluded individuals who had received a reablement service were less likely to use any type of home care over the three following years.(Lewin et al., 2013, p. 1273) Another study concluded reablement was associated with a significant decrease in the use of social care service.(Glendinning et al., 2010, p. 117) On the other hand, it was noted in a large review article on the effectiveness of reablement that two out of three randomized controlled trials considered did not find significantly improved physical functionality among recipients of reablement compared to standard home-care services.(Forland & Skumsnes, 2016, p. 33) Another study also noted that a common weakness of reablement studies was that they were lacking in quality by using small samples and being subject to various types of bias.(Burton et al., 2015, p. 468)

Our findings on the topic of reablement are twofold and mainly depend on whether we consider initial or repeat spells in a home-nurse area or in a short-term institution. We found that for all models considered, individuals who are receiving or have received home-based reablement services have higher odds of event occurrence in initial spells compared to those not receiving such services. We believe this might be due to individuals receiving reablement services often being in need of rehabilitation services because they have, or are at risk of, some form of functional decline.(Forland & Skumsnes, 2016; Hjelle, Tuntland, Forland, & Alvsvaag, 2017)

Considering this, all models also indicate that in repeat spells, current or previous reablement participants have lower odds of event occurrence compared to non-participants. Still, this effect is rather small (OR=0.989) when only considering the time after ending the program and controlling for age and gender. Here it is important to note that this not only

means lower odds of being readmitted but also lower odds of being released from the institution compared to non-participants in repeat spells. Finally, we did not find any significant interactions between reablement and gender or age in this study, and neither did we find any meaningful rate of decay for the effects of reablement when adjusting for clustering effects. Keeping this in mind we now move on to some choices that we have made during the study process that could potentially affect the findings that we presented here.

## 6.2 Issues

### 6.2.1 Data

We begin by discussing the data set and the data cleaning procedure. The data set includes considerable observational gaps and some missing values for different variables. In our case, we chose to ignore such gaps, while observational gaps in the data set may include critical information that we have simply assumed to be unimportant.

As for the data cleaning procedure, manual changes were made, observations were omitted and all other infringements on the original data set all constitute choices that may, and reasonably should, be met with skepticism when interpreting the results given.

One particular issue is that defining the first spell as the first observable period a participant is at the HNA is a questionable choice. We do not know when the person moved into the HNA to begin with, and we therefore do not know their event time at the time of event occurrence. Neither do we know if the person has been admitted to a STI before entering the study, and then the second spell would not be his or her first time at the STI after all, something which clearly compromises our distinction of initial and repeat spells for the analysis.

Finally, we wish to bring attention to the point that stays at a STI are usually short, and people are in most cases expected to return to their HNA, which deviates from previous literature. Taking the study of (Willett & Singer, 1995, p. 42) as an example, a person that is in the teaching profession and leaves is not entirely expected to return to teaching, and therefore dividing spells into sets of two seems appropriate. Under other conditions, when considering short stays at an institution in which those being admitted are expected

to recover and be able to return to their previous location, this might not be the case. This could easily influence the analysis and suggest other, more appropriate choices for structuring the variables than we have done in this article.

### **6.2.2 Variables**

The next topic is that of variable specification. The way in which the variables have been defined may influence their interpretation in unforeseeable ways. First, the variable representing whether the participant lives alone has been defined by combining several other variables into a single, dichotomous variable. We were warned that the quality of each single indicator is not great, and some values are missing, which makes the quality of the combined variable questionable as well.

Second, our variable for the short-term institutions, STI, is a combination of two locations. The first location is the short-term institution itself, and the second is a rehabilitation institution. We were informed that this rehabilitation institution is also a kind of short-term institution, and we added it to our analysis as suggested.

By accounting for the issues in this section, one might expect changes in the hazard functions, survivor functions and the estimated parameters in the estimated models. For instance, the patient flows of the two institutions may differ if one institution is designed for different sub-populations. Thus, the estimated hazard functions may differ when considering the two institutions separately.

## **7 Conclusion**

### **7.1 Summary and concluding remarks**

In choosing an initial model, we based our decision on the ease of use, interpretation, and a requirement of being well specified. Suitable reparametrizations of the period and spell indicators were found through visual inspection of the sample hazard functions for the first 10 spells. The initial model had high discrimination and a high overall rate of correct classification, but we experienced problematic covariance patterns as we were checking for

potentially influential observations.

By comparing Figures 7 and 1, we could conclude that the initial model did a decent job of describing the dependence of risk across periods and spells. For instance, the hazard in spells at a home-nurse area was decreasing across time within spell, and the peak of the hazard function was in the third period in spells at short-term institutions for both the sample and fitted functions. We created separate models for the effect of time and characteristic predictors before estimating two combined models for the effect of time, gender, age and reablement. In the first combined model, the time spent in the reablement program was included, while for the second combined model, it was excluded. We will be using the odds ratios from the combined models for the concluding statements of differences in risk.

For initial spells, we find that individuals in their first week at a short-term institution have odds approximately 37% lower for leaving than someone at a home-nurse area entering in the same week. This differential reverses by the second week, when it is approximately 80% more likely that someone at a STI will leave than someone in a HNA in a similar situation will enter. For repeat spells of any type, we find it approximately four times more likely that a repeat spell of either type will end in the first week than an initial spell ending in the same week. This differential is expected to slowly reverse over the course of more than three years.

We find males have odds approximately 30% higher to leave or enter a short-term institution compared to women in a similar situation, not depending on how many times they have been admitted previously.

For those living alone, we find they are much less likely, at an odds ratio equal to 0.41, to be admitted or readmitted to a short-term institution compared to those not living alone in the first period of a spell. This difference in risk slowly reverses across time within spell, and has completely reversed after around two and a half years. For the release from a short-term institution, those cohabiting have approximately 28% higher odds of being released in the first period. Interestingly, this differential reverses quicker, and has fully reversed by the fourth week in such a spell.

Concerning differences in risk due to age in initial spells, our results indicate those younger than 67 years of age have odds approximately 64% lower for being admitted to a STI com-

pared to those between 67 and 80 years of age. Concerning the risk of readmission for the same age groups, the odds for the younger group are only around 33% lower instead. Concerning release after initial admission, those in the younger cohort have approximately 4% lower odds of being released compared to their older counterparts. For release after readmission, the younger cohort has approximately 75% higher odds for being released in any period compared to the reference group.

For individuals over the age of 80, we only find a significant escalation of a difference in risk profiles across time within spell. This means those older than 80 are more likely to experience event occurrence compared to the reference group as more time is spent in a spell of any type.

Reablement program participation is associated with higher risk of event occurrence in initial spells. If we include the time spent in the program, those receiving and those who previously received reablement services have 44% higher odds of being admitted to, or released from, a short-term institution compared to non-participants in any period. Excluding the time in reablement, those that have previously received reablement services have 82% higher odds of being admitted to, or released from, a short-term institution in any period.

For repeat spells, reablement program participation is associated with a lower risk of readmission or release compared to non-participants. Including the time in the program, those that have had reablement have odds approximately 20% lower than non-participants for either readmission or release after readmission in any period. Excluding the time during the program, previous program participators have odds that are approximately 1% lower of being readmitted or released compared to non-participants. For either of the combined models, we find no two-way interactions between the variables for age, gender or reablement participation for our chosen level of significance.

During the making of this paper, there were many ideas that could simply not come to fruition due to constraints. Therefore, we finalize this section with an outlook to the future, and present some suggestions for further research.

## 7.2 Suggestions for further research

There are many interesting possibilities for further research efforts including, but not limited to, adding frailty components, removing the most troubling covariate patterns, trying different approaches to model estimation, defining variables differently and accounting for other, interesting parameters.

While we have adjusted for clustering effects in this paper, it could prove useful to account for heterogeneity in the model by adding what is known as frailty components.(Willett & Singer, 1995, p. 61) During examination of several diagnostic statistics, we encountered some highly influential and poorly fitted covariate patterns, and their removal could add an interesting layer to later research efforts, see (Hosmer et al., 2013, p. 199).

It could be interesting to use approaches which differ from those we have put to work so far, to see whether our results hold, i.e. are robust. For instance, using the complimentary log-log link function(Willett & Singer, 1995, p. 51), or perhaps the conditional instead of the unconditional maximum likelihood estimation approach.(Kleinbaum & Klein, 2010, p. 122)

One should perhaps attempt to deal with observational gaps differently than we have and see if the yielded results change significantly. The same goes for defining a different beginning of time, and dividing the dynamics of periods and spells different than simply distinguishing between spells in-and-out of the institutions and whether the spells are initial or repeat.

While we find living alone is associated with much lower risk of admission to short-term institutions, we do not know the risk-profiles for admission to long-term institutions. We therefore highly recommend a study which considers such risks, as they might reveal important features in risk-profiles which we were unable to account for.

We also observed some interesting results by explicitly distinguishing between persons that had been in the study from the start and late entrants. Finally, an interesting outlook would be to delve into the effects that living in a particular home-nurse area might have, and the effect on patient flows with varying degrees of capacity constraints at the short-term institutions.



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# 8 Appendix

## 8.1 List of symbols and acronyms

Symbol	Representation	Acronym	Meaning
$N()$	Normally distributed	-2LL	Deviance statistic
$c$	Dichotomous censoring indicator	ADL	Activities of daily living
$\hat{C}$	HL goodness of fit test statistic	AGE	Age group
d.f.	Degrees of freedom	AIC	Akaike information criterion
$\xi$	Rao-Cramer lower bound	ALONE	Living alone indicator
$\mathbf{E}$	Expected value of	AUROC	Area under ROC
$e$	Error term	BIC	Bayesian information criterion
$F$	Forced	COMB1	Combined model 1
$G$	Number of groups	COMB2	Combined model 2
$g$	Continuous function	FNF	False negative fraction
$\Gamma$	Conditional mean of Y given x	FPF	False positive fraction
$\Omega$	Co-variance	HL	Hosmer Lemeshow
$\Theta$	Parameter space	HL GOF	HL goodness of fit
$\theta$	Vector of parameters	HNA	Home-nurse Area
$\sigma^2$	Variance	ID	Patient identification number
$h()$	Hazard function	INS	Institution
$\mathbf{H}$	Hat matrix	Link	Goodness of link-test
$H_0$	Null-hypothesis	LL	Log likelihood
$h_{ir}$	Leverage	LPER	Natural logarithm of period number
$h(t_{ik})$	Hazard	LR	Likelihood ratio
$i$	Individual	MALE	Gender indicator variable
$I$	Highest number of individuals	ML	Maximum likelihood
$j$	Spell	MLE	Maximum likelihood estimator
$J$	Highest number of spells within study	OR	Odds ratio
$k$	Period	PD	Parental divorce
$K$	Highest number of periods within spell	PD DUM	Period dummy variable
$L$	Likelihood function	REHV1	Reablement member since start
med	Median value	REHV2	Reablement member since end
$n$	Numerator	REP	Repeat spell
$N$	Highest number	Res	Pearson residual
$\alpha_{1u}$	Sum of observed cases	ROC	Receiver operating curve
Pr	Probability	ROR	Risk odds ratio
$\pi$	Estimated hazard probability	SP DUM	Spell dummy variable
$q$	Number of parameters	STI	Short-term institution
$r$	Covariate pattern	TNF	True negative fraction
$R$	Highest number of cov.patterns	TPF	True positive fraction
$S()$	Survivor function	VAR	Predictor placeholder
$t$	Time		
$T$	Time at target event		
$\mathbf{V}$	Diagonal matrix		
$W$	Wald test statistic		
$x,y,z$	Unknowns		
$\chi^2$	Chi-square		
$y$	Event occurrence indicator		

Table 14: *Table of acronyms and abbreviations used in the paper.*

## 8.2 Description of original data set

Our original data set has a total of 421 693 observations of 35 variables and a total number of units in the study equal to 5608. There are 6 dichotomous variables indicating the location or state of existence for a unit at any time, whether it is a home-nurse area (HJSY / HJSY), one of the available institutions or in a residential care home (BOLOM). These variables equal to one whenever a unit is at the specified locations and zero otherwise. The institutions available for residence includes a stay at a rehabilitation (REHA) institution, special patient care (STERK) institution, short-term (KORT) institution or long-term (LANG) institution. If units are not at an institution, either short or long term, they are in a home-nurse area or in a residential care home.

Three variables denote time, the identity of an unit and lastly a variable for easily identifying the location of a unit in any given time interval. The variable ID identifies every unit in the data set by giving each a single identification number, from 1 to 5608. The data set therefore in total contains observations from 5608 different persons. The variable Uke denotes which week the observation concerns and is a number from one and no larger than 159, a total period slightly longer than three years. Some of the units entered the study later than the first week and some left before the study ended after three years, and therefore not all units have observations for all the time periods. The variable Ansvar is an identification code used to determine who has the responsibility for the units stay at any given point in time and can take the form of either zero, a three-digit or a four-digit number. If it is zero then the unit is at an institution in that period, if it has three digits then the unit is at a residential care home and if it has four digits then the unit is in a home-nurse area in the given period.

Additionally, there are some gaps in the data for some of the units in the set, not necessarily being restricted to one gap per unit. This entails that there are observations for the unit when he or she enters the study but then some missing observations for one or more periods before observations are again recorded for that unit. There is no guarantee that the unit will be at the same location after the gap as before it, and unfortunately no way to examine whether that unit experienced the event during the data gap. There are 10 variables

used to calculate the average number of hours used per week for each of the variables.

There are 10 variables covering gender, marital status and the age of the unit in any given period. The variable Mann is a time invariant dichotomous variable equal to one if the unit is male and zero otherwise. There are 6 variables that can be used to examine the marital status of the unit. These variables are all dichotomous variables equal to one if the statement is true and they can change over time. The variables can be used to see if the unit was married, in a partnership, divorced, a widow or widower, if the partner was at an institution or if the social status was unknown in the given period. A note of caution was given on the quality of these variables and that they perhaps could be restructured into a dummy variable for whether the unit was living alone in the given period or not.

The age of the unit during the study is found in three variables named Alder14, Alder15 and Alder16. These give the age of the unit during the year 2014, 2015 or 2016, respectively. Finally, there are 3 dummy variables used to denote in which year the observation was made. These three variables are y2014, y2015 and y2016 and denote if the given observation was made in 2014, 2015 or 2016, respectively.

### **8.3 Data cleaning procedure**

The procedure of data cleaning is to accommodate for the upcoming analysis of the data set. We begin by removing unnecessary variables, restructuring existing variables and doing thorough visual inspection. The goal is to make the data easier to work with given our research questions and uncover potential flaws and mistakes made during the recording of the data set. We wish to decrease the available states of existence into a dichotomy where only two states are possible. This section of data cleaning will go through the structuring of the variable which addresses state existence and several important predictors used in the final model. In addition, we work our way through the other possible states of existence in order to incorporate them into the already defined variables in our data set.

We begin by explaining our restructuring of the variables in the data set. The variable home-nurse area (HNA) is a dichotomous variable equal to unity whenever the unit is in the state of being in a home-nurse area in the given period and zero otherwise. This variable has been defined as being equal to one whenever the variable Ansvar is larger than 999,

but less than 10000. Second, the variable short-term institution (STI) is a dichotomous variable equal to unity whenever KORT is equal to unity or REHA is equal to unity and zero otherwise.<sup>22</sup> Thirdly, the variable long-term institution (LTI) is a dichotomous variable equal to unity whenever a unit is in either STERK or LANG in the current period and zero otherwise. Finally, BOLOM is a dichotomous variable equal to unity whenever the unit is in residential home-care area and zero otherwise. The last two variables, BOLOM and LTI, will be worked with in order to finally exclude them from the data set by the time data cleaning is complete.

As we have defined a stay at the home-nurse area as the first spell, no units may begin their first spell until observed in a home-nurse area in the data set. We may therefore safely delete all units that never stay in a home-nurse area during the study. We create a variable, called HNAuser, which is equal to unity if a unit has been in a HNA at least once during the study and zero otherwise. We proceed to delete all units for which this variable is zero, and thus deleting 68 245 observations equal to 960 unique IDs and we are then left with 4648 unique IDs in the data set. Before we move on to the variable of BOLOM we check if there are any participants with only a single observation, of which there are 135. Since a single observation for any unit does not constitute longitudinal data but a mere cross-section instead(Singer & Willett, 2003, 1996, p. Preamble V; p. 267), we delete those participants with only a single observation during the entire study.

We proceed to deal with the variable of residential home-care (BOLOM). We wish to eliminate this variable, but before this can be attempted one must investigate the potential flaws this will bring. Possible issues could occur if the stays at BOLOM were in the middle of the observations for any given unit because deleting such observations would distort the reality of event occurrences. We therefore use several conditional statements in our data program to identify those with stays at BOLOM in the middle of their observational period. We then perform visual inspection of each identified unit in order to decide whether deleting the observation for BOLOM would have significant negative consequences to the nature of the data at hand. We conclude that in all cases except for one, stays at BOLOM occur either

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<sup>22</sup>This has been done after discussion with our supervisor where we were informed that the rehabilitation institution could be part of a short-term stay



at the beginning or at the end of the observations for any one study participant. For the participant with a short one-week stay at BOLOM during the middle of their contribution to the sample both preceded and followed by stays at a HNA, we simply assumed this stay to be a part of the stay at the HNA. For the rest of the cases, the variable does not interfere and may thus safely be deleted without also deleting the entire event history of those that either began or ended at BOLOM during the time they were participating in the study. We delete BOLOM which removes 3316 observations from the data set, but does not remove any participants from the data set.

Next in line is the variable for stays at the long-term institution, LTI. Like BOLOM, we wish to delete this variable by the end of the data cleaning process in order to only consider two possible states of existence for data analysis. Our concerns are the same as previously in that stays at an LTI in the middle of study for any given unit will create gaps in the data among other possible negative consequences. Instead of removing the entire ID whenever they have been at LTI during the study, we decide that units that are either at the LTI at the end or at the beginning need not be eliminated completely from the sample. Instead, only the observations themselves that concern these stays should be omitted in order to preserve the highest amount of potentially useful information. We create a variable identifying any participant that has been to an LTI and is then sent to either STI or HNA in the next period. Similarly, we create a variable which identifies any unit that has been to either a HNA or STI in any period with stays at LTI both before and after said period.

Although most participants that are sent to the LTI never return during the sample, we notice that many participants tend to have what we term a “trial period” in which they are sent from the LTI to either a HNA or STI for a short period, usually one week, before ultimately being sent back to the LTI. We decide this trial period may be considered a part of the LTI stay. On the other hand, some participants do return from an extended stay at the LTI which complicates the data analysis. This is because it creates large structural gaps in the middle of the observations for those units if we were to delete those observations. We therefore made some manual changes to make sure our censoring mechanism would work correctly in the future. For instance, one of these changes was to delete three observations for a unit that returned to HNA after an extended stay at the LTI. To summarize this part of

the data cleaning we made 12 changes to single observations to account for trial periods and deleted a total of 169 single observations for seven unique IDs to make sure future censoring works as intended. We were then ready to delete the variable LTI which removed 27 055 observations, and we were left with 4513 unique IDs in the sample.

The final stage of the data cleaning process was to define an initial state of existence for all participants involved in the study. We define this as the first period a person is observed in a HNA in sample. As some participants were not located at a HNA at their first observation, we delete those entries which lead up to the defined beginning of time for each individual. We therefore delete 2415 observations assuring all the participants occupy the same initial state. Finally, we check again for any participants left with a single observation as a consequence of the data cleaning. There are 17 such cases we promptly omit from the sample and we are thus left with 4496 unique participants and a total of 320 358 observations in the finalized sample.

## 8.4 Distribution of spells (Subsamples)

Table 15 shows the maximum number of spells for all 4 496 participants of the study. The sampling shortfall is clear here as 65% percent of the individuals only experience a first spell, 11% experience a first and a second spell and 10% experience a first, second and third. While some experience 53 spells in total, nearly 90% have only experienced at most four spells, which leaves little data for the explanation of event occurrence for later spells.

Subsample	Freq.	Percent	Cum.	Subsample	Freq.	Percent	Cum.
1	2,938	65.35	65.35	24	4	0.09	99.07
2	487	10.83	76.18	25	3	0.07	99.13
3	445	9.9	86.08	26	2	0.04	99.18
4	159	3.54	89.61	27	2	0.04	99.22
5	147	3.27	92.88	28	4	0.09	99.31
6	68	1.51	94.4	29	2	0.04	99.35
7	49	1.09	95.48	30	4	0.09	99.44
8	29	0.65	96.13	32	1	0.02	99.47
9	22	0.49	96.62	33	1	0.02	99.49
10	17	0.38	97	34	3	0.07	99.56
11	13	0.29	97.29	35	3	0.07	99.62
12	14	0.31	97.6	36	3	0.07	99.69
13	11	0.24	97.84	38	2	0.04	99.73
14	8	0.18	98.02	40	2	0.04	99.78
15	4	0.09	98.11	41	1	0.02	99.8
16	10	0.22	98.33	42	1	0.02	99.82
17	5	0.11	98.44	44	1	0.02	99.84
18	8	0.18	98.62	46	1	0.02	99.87
19	5	0.11	98.73	47	1	0.02	99.89
20	6	0.13	98.87	48	1	0.02	99.91
22	3	0.07	98.93	49	2	0.04	99.96
23	2	0.04	98.98	53	2	0.04	100
				Total	4,496	100	

Table 15: *Summary of the distribution of spells. Includes frequencies of participants in each subsample, percentage of frequencies in each subsample and cumulative percent frequencies. Subsamples are defined in accordance with Subsection 4.3.4.*

## 8.5 The difference between censoring and truncation

Terms such as censoring and truncation are similar yet different ideas one must become familiar with to perform survival analysis. Censored data occurs if the data for the dependent variable is unobserved outside a certain range. Truncated data occurs if the data for both the dependent and independent variables are unobserved for a certain range.(Brooks, 2008, p. 535) To demonstrate this difference, consider that we are interested in the time for a person to complete a test, which is the event, and they are required to fill in personal information such as age either before or after the test, which will be used as independent variables later.

Now consider a person that withdraws from the test in the middle of it. If that person filled out his personal information at the beginning of the test the researchers still have access to it and only the dependent variable, the time at which the test was completed, is

unobserved. Then we may say that the data for this unit was censored. If, on the other hand, the person was to fill out his personal at the end of the test and left during, then the researchers would not have access to those independent variables and the data for that unit would then be truncated instead.

## 8.6 Conditional maximum likelihood

The selection criteria for whether one should use the conditional or unconditional method is the number of parameters relative to the total number of units in the study. If this number is small then the unconditional is preferred, while if it is large one should choose the conditional method. Additionally, one should use the conditional method whenever there is matching in the model and if there is any doubt on which should be used. That is because the conditional method is always unbiased while the unconditional method may overestimate the parameters of interest if it is inappropriate in the given case.

The formula for the conditional method is in Equation 52 and it may be observed that the numerator of this equation is the same as the formula for the unconditional method with the exception of the intercept ( $\alpha$ ). It is different only in the denominator which sums the joint probability of all possible configurations of the given data. Substituting the formula for the logistic model into the equation for the conditional methods, we obtain Equation 53. The main difference between the equations for the conditional and unconditional approach is that the estimate of the intercept  $\alpha$  is not included in the equation for the conditional method. As the measure of effect in a logistic model is the odds ratio which does not include  $\alpha$ , it may be interpreted as a nuisance parameter. (Kleinbaum & Klein, 2010, p. 116)

$$L_C = \frac{\prod_{l=1}^y \mathbf{P}(\mathbf{X}_l) \prod_{l=y+1}^n [1 - \mathbf{P}(\mathbf{X}_l)]}{\sum_u \left\{ \prod_{l=1}^y \mathbf{P}(\mathbf{X}_{ul}) \prod_{l=y+1}^n [1 - \mathbf{P}(\mathbf{X}_{ul})] \right\}} \quad (52)$$

$$L_C = \frac{\prod_{l=1}^y \exp(\sum_{i=1}^k \beta_i X_{li})}{\sum_u \left[ \prod_{l=1}^y \exp(\sum_{i=1}^k \beta_i X_{lui}) \right]} \quad (53)$$

## 8.7 Lagrange Multiplier test

The Lagrange multiplier test is an alternative to the likelihood ratio test and allows one to test restrictions that are imposed in estimation. Unlike the Wald test which requires that

the model without the restriction is estimated, the Lagrange multiplier test requires that the model under the null hypothesis is estimated instead. (Verbeek, 2004, p. 172) The idea of the test is that if the restriction imposed is valid, then the restricted estimator should be near the point that maximizes the log likelihood. If correct, the slope of log likelihood function should be close to zero at the restricted estimator. (Greene, 2012, p. 566) As the slopes, the first derivatives, are referred to as scores, the Lagrange multiplier test is often known as the score test. (Verbeek, 2004, p. 174) It is important to note that there are several different versions of the Lagrange multiplier test statistic as well, as it may be based either on the gradient or on the Lagrangian multiplier. (Jungeilges, 2017, p. 8)

The motivation of the test statistic can be made by performing the calculus of a constrained optimization problem, which we will quickly show for a case of scalar magnitudes only. The constrained optimization problem is shown in Equation 54 and if  $\theta$  solves this problem it fulfills the associated first order condition, shown in Equation 55. (Jungeilges, 2017, pp. 7-8) We may display the Lagrangian multiplier version of the test statistic in Equation 56, where the first and third term on the right-hand side are 55 and its transpose, and the middle term denoting the inverse of the information matrix. Please note all the terms are evaluated at the restriction given in the null hypothesis. Under the null hypothesis, the test statistic has a limiting chi squared distribution with degrees of freedom equal to the number of restrictions. (Greene, 2012, p. 570) The decision criteria are then equivalent to the other tests above:

If  $LM \geq \chi_{1-\alpha}^2(r)$  then reject  $H_0$  else fail to reject  $H_0$ .

$$\max_{\theta \in \Theta} \mathcal{L}(\theta) = l(\theta) - \lambda(\theta - \theta^W) \quad (54)$$

$$\frac{\partial \mathcal{L}(\theta)}{\partial \theta} = \frac{\partial l(\theta)}{\partial \theta} - \lambda = 0 \Leftrightarrow \frac{\partial l(\theta)}{\partial \theta} = \lambda \quad (55)$$

$$LM = \left( \frac{\partial \ln L(\hat{\theta}_W)}{\partial \hat{\theta}_W} \right)' [\mathbf{I}(\hat{\theta}_W)]^{-1} \left( \frac{\partial \ln L(\hat{\theta}_W)}{\partial \hat{\theta}_W} \right) \quad (56)$$

## 8.8 Collinearity and multicollinearity

Collinearity concerns the extent to which one or more explanatory variables in a model can be predicted from other predictors in the model. (Kleinbaum & Klein, 2010, p. 270) If there

is no relationship between these explanatory variables, we can say they are orthogonal to each other.(Brooks, 2008, p. 170) However, living in the real world, we rarely expect there to be zero correlation between variables, and as such, a small degree of collinearity is to be expected. There is something rotten in the state of Denmark, a dark age of Shakespearian villainy, when the explanatory variables are highly correlated with each other, in what we can call multicollinearity.(Brooks, 2008, p. 171) If there exist highly correlated relationships among some or all the predictors, the fitted model could yield some strange regression coefficients, with high variances. The model will in such a case struggle with the collinearity problems.(Kleinbaum & Klein, 2010, p. 271)

We must differentiate between perfect multicollinearity and near multicollinearity as they are two classes of the problem. If one is to use the same predictor twice in the model by accident, one will be getting perfect multicollinearity as there is a wrongfully inserted, but real exact relationship between two variables in the model. Two variables that are perfectly related to one and another, for example one being a multiple of the other, contain only enough information to estimate one parameter. Going deeper into this situation, one would have a problem inverting the variance-covariance matrix since the matrix would not be of full rank, due to two of the columns being linearly dependent on each other.(Brooks, 2008, p. 171) Note that the Fisher information matrix is used to calculate covariance matrices, in cases with non-linear models where maximum likelihood estimation is used, and thus the estimated variance-covariance matrix is called the inverse of the information matrix in such situations.(Kleinbaum & Klein, 2010, p. 271)

Near multicollinearity refers to the cases where there are strong relationships between explanatory variables that should not be ignored.(Brooks, 2008, p. 171) This is a common occurrence when two explanatory variables are highly correlated, such as the explanatory variables number of hours of sunshine and mean temperature during a summer is highly correlated in explaining the yearly sale of ice cream. Note however that a high correlation between the dependent variable, sale of ice cream, and one of the independent variables, mean summer temperature, is not multicollinearity, rather it is hopefully a successful significant explanation of the dependent variable. In searching for collinearity, one should have two objectives. First, to determine the reliability of the fitted model and, second, to determine

if the estimated variances with corresponding standard errors are large enough to indicate problems in the models explanatory variables.(Kleinbaum & Klein, 2010, p. 271) Solutions to the problem of multicollinearity ranges from ignoring it if the model is otherwise adequate, transforming variables into ratios and to dropping the collinear variables. Note that when one drops a correlated variable one must be vary of omitted variable bias if the removed variable was relevant in the data generating process.(Brooks, 2008, p. 173)

## 8.9 Time invariant and time varying variables

Time-invariant predictors such as gender, geographical area and their relationship with the hazard probability are interesting tools for greater clarity in a model. One can investigate these relationships by adding them to the full multiple-spell discrete-time model and draw comparisons on the goodness-of-fit, before and after the addition. An interaction coefficient might indicate whether a difference increases or decreases over all the observations when a time-invariant predictor is at play.(Willett & Singer, 1995, p. 59)

Time-varying predictors such as outside and inside spell states can be analyzed by similar or identical methods, yet one must account for the strong possibility for there being time-varying predictors operating only in some of the spells and not in other spells. Due to this complication the effects must be regarded and studied with the utmost of care as the time spent in the state depends on exogenous covariates that are likely to change over the period.(Petersen, 1986, p. 287) Time-varying predictors are a challenge as the interpretation of the hazard functions, due to a vast amount of time-based combinations possible from the different values on the timeline of the time-varying predictor.(Willett & Singer, 1995, p. 60)

By adding a time-varying predictor we can compare different groups of participants at different times, such as comparing children with or without divorced parents growing up. Over time children with the group membership “no parent divorce” might sadly loose said membership, transferring to the  $PD = 1$  since their parents broke up.(Singer & Willett, 2003, p. 430) To examine the sample hazard functions based upon separate temporal patterns of time-varying explanatory variables is rarely done. Firstly, due to the multitude of different temporal patterns, there is often too small samples of unique patterns to say anything of significance. An additional complication is when the model does not specify a relationship

between the explanatory variable's patterns and the logit hazard, giving us little insight into what contributions each type of pattern has.(Singer & Willett, 2003, p. 431)

A time-varying predictor can be state dependent if the explanatory variable value is affected by the participants event status, whether the event has occurred or not. Thus, one should look out if the predictors values at the time  $t_j$  are affected by the state of the event occurrence at that time.(Singer & Willett, 2003, p. 440)

## 8.10 The model selection process

There are a few commonalities for all the models involved, namely the exclusion of an intercept and the exclusion of several period indicator variables. First, exclusion of the intercept is to accommodate the inclusion of all the period indicator variables without complete linear dependency.(Willett & Singer, 1995, p. 53) Second, something all the models have in common, is the exclusion of four period indicator variables, namely the period indicator variables for period 135, 154, 157 and 159.<sup>23</sup> As the period indicators are excluded, so are the observations tied to these periods, the number of observations being 875, 706, 678 and 663, respectively. This is also why the parameters in model A is shown as being 155 instead of the total number of period indicators, which is 159.

First, we consider the three initials models that do not include any reparameterizations at all, namely models A, B and C. We reject the null hypothesis of there being no difference in risk across spells for the likelihood ratio test between models A and B, ( $LR = 6045.96$ ). (Willett & Singer, 1995, p. 54) Wishing to avoid adding too many interactions yet still covering the first ten spells, our application of model C has been defined as shown in Equation 57, and adds 54 cross-product terms to model B. Comparing models B and C with a likelihood ratio test, ( $LR = 630.53$ ), we reject the null hypothesis of the effect of the period indicator variables on logit-hazard being constant across spells.(Willett & Singer, 1995, p. 54) It is worthwhile to mention that while most of the interactions in

---

<sup>23</sup>This is a mechanism of the statistical software that has been utilized and is caused by the reason that there are no target event occurrences in any of said periods. If these periods were to be included, target occurrence, or rather non-occurrence, for these periods would be predicted perfectly and thus the coefficient for the period indicators would be infinite.



model C are not significant, there are a few observations to be made. For the first spells, 2-4, it is the initial periods, 1-3, that are significant while for the last spells, especially 7, 9 and 10, most of the interactions are significant. While this information may be invaluable to the experienced econometric modeler, we can only content ourselves with simply stating these observations for now and move on to the reparametrized models.

$$\begin{aligned} \text{logit}(h_{ij}(k)) = & \sum_{m=1}^{159} \alpha_m P_m + \sum_{m=2}^{53} \beta_m S_m + \sum_{m=1}^{10} \gamma_m (S_2 P_m) + \sum_{m=1}^9 \gamma_{(m+10)} (S_3 P_m) + \\ & \sum_{m=1}^8 \gamma_{(m+19)} (S_4 P_m) + \sum_{m=1}^7 \gamma_{(m+27)} (S_5 P_m) + \dots + \sum_{m=1}^2 \gamma_{(m+52)} (S_{10} P_m) \quad (57) \end{aligned}$$

The reparameterizations of the spell and period indicators must accommodate for the data at hand and the research questions involved. In our case, from investigating the sample hazard plots, see Figure 1 in Subsection 4.1.5, we can conclude there is a clear difference between odd and even numbered spells, in other words a difference between spells at the HNA and the STI, respectively. Additionally, we find it useful to distinguish between initial and repeat visits to the short-term institution, as Table 15 in the appendix indicates most participants never experience more than a single or at most two spells. For distinguishing the first and second state, we introduce the dummy variable named INS for institution, which is equal to one whenever a participant is at the STI, and zero otherwise. For distinguishing the initial from recurring spells, we create a dummy variable named REP for repeat, which is equal to one whenever a participant is not in their first or second spell. These variables are deemed appropriate counterparts for those that were used in studies performed with different subpopulations and fewer numbers of spells in their data set, as in (Willett & Singer, 1995, p. 56). Having covered our reparameterizations of the spell indicator, we will introduce the models that utilize them and save our discussion of period indicator for the next subsection, see 4.6.2.

The reparametrized versions of the models B and C, denoted B2 and C2 in Figure 3, are shown in Equation 58 and 59, respectively, and have significantly different deviance statistics compared to their original counterparts. Compared to B, B2 has a higher deviance statistic and higher values for both AIC and BIC, both the models failing the specifications tests that have been applied. Model B can be concluded to be preferred over B2. Compared to C, C2

has a higher deviance statistic but it passes both specification tests contrary to its original parameterization, has a lower value for BIC, but not AIC. As it is clear from the summary in Figure 3, model C2 is perhaps the fit of all the initial models for time as well as having the advantage of passing both specification tests. Its greatest merits are encompassed in its three-way interaction between INS, REP and the period dummies, which we will now demonstrate with our next model.

$$\begin{aligned} \text{logit}(h_{ij}(k)) = \sum_{m=1}^{159} \alpha_m P_m + \beta_1 INS \\ + \beta_2 REP + \beta_3 (INS \times REP) \end{aligned} \quad (58)$$

$$\begin{aligned} \text{logit}(h_{ij}(k)) = \sum_{m=1}^{159} \alpha_m P_m + \sum_{m=1}^{15} \beta_m (INS \times P_m) \\ + \sum_{m=1}^{15} \beta_{(m+15)} (REP \times P_m) + \sum_{m=1}^{10} \beta_{(m+30)} (REP \times INS \times P_m) \end{aligned} \quad (59)$$

In model D, the last term from model C2 has been eliminated, and the model is given by Equation 60. This last term represents the two-way interaction between INS and REP, denoted  $INS \times REP$ , and the three-way interaction between INS, REP and the period dummies. Comparing model C2 and D, we reject the hypothesis that the logit-hazard profile differs by whether a spell is initial or repeat and, independently, whether it is at a HNA or STI, given by the difference in the deviance statistics, ( $LR = 634.18$ ). (Willett & Singer, 1995, p. 56) It is also implied model D is lacking as it fails the goodness of link, while its predecessor does not.<sup>24</sup> It should be noted that even if we reintroduce the two-way interaction  $INS \times REP$  to model D, we still reject the null hypothesis of no three-way interaction, ( $LR = 89.87$ ), but the main contribution comes from the two-way interaction itself. This is found by testing the two-way interaction using a Wald test, ( $\chi^2 = 520.14, df = 1$ ). We shall keep this in mind as we introduce the next model which utilizes reparameterizations of the period indicator as well as reparameterizations of the spell indicator.

$$\text{logit}(h_{ij}(k)) = \sum_{m=1}^{159} \alpha_m P_m + \sum_{m=1}^{15} \beta_m (INS \times P_m) + \sum_{m=1}^{15} \beta_{(m+15)} (REP \times P_m) \quad (60)$$

---

<sup>24</sup>We still reject the null hypothesis of the goodness of link test at a 1% significance level if we replace the logit-link function with the complimentary log-log link function.

We shall explore different forms of transformations for the period indicator, namely a linear, quadratic or logarithmic transformation, for which a summary is found in Table 16. The idea is to replace the period dummy variables used in two-way interactions between INS and REP used for model D. We have tried different approaches to transforming the period indicator. A linear transformation in which we take 1 divided by the period indicator,  $\frac{1}{PDNUM}$ , a quadratic where we square the period indicator,  $(PDNUM)^2$ , and lastly, a logarithmic transformation by taking the natural logarithm of the period indicator,  $\ln(PDNUM)$ . The quadratic transformation gives the worst fit and the linear the best in terms of deviance statistics, but the linear transformation fails the HL goodness of fit test while the logarithmic transformation does not. As the model utilizing a logarithmic transformation is the one that passes both specifications tests, this is the one we will be using going forward, and it is shown in Equation 61, and will be denoted as model E.

Model	Deviance	AIC	BIC	HL GOF	Link
Quadratic	45 460.14	45 778.14	47 474.35	<del>6.36</del>	N
Linear	44 665.66	44 983.66	46 679.86	14.41	Y
Logarithmic	44 705.86	45 023.86	46 720.07	<del>8.85</del>	Y

Table 16: Values for AIC, BIC, Hosmer-Lemeshow goodness of fit test ( $G=10$ ) and goodness of link test for each of the model during the period reparameterization process.

$$\begin{aligned}
 \text{logit}(h_{ij}(k)) = & [\alpha_1 P_1 + \alpha_2 P_2 + \dots + \alpha_{159} P_{159}] \\
 & + \beta_1 INS + \beta_2 (INS \times LPER) \\
 & + \beta_3 REP + \beta_4 (REP \times LPER) \quad (61)
 \end{aligned}$$

## 8.11 Diagnostics adjusted for clustering effects

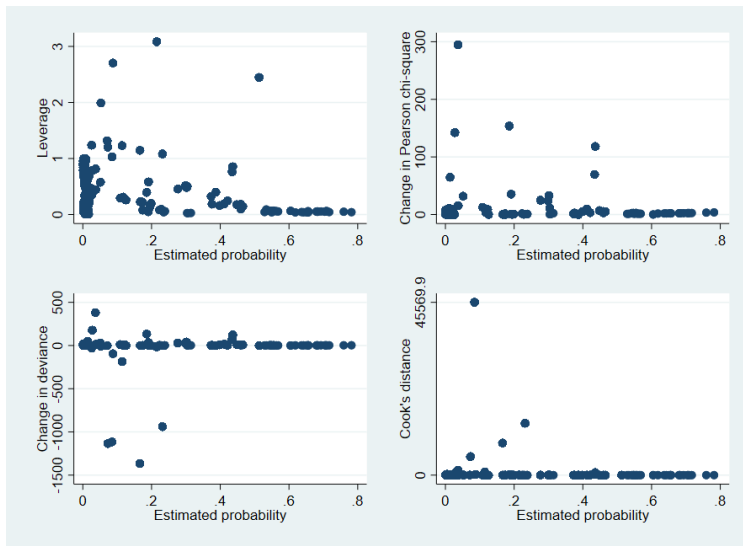


Figure 8: *Plots of four diagnostic statistics ( $h$ ,  $\Delta\chi^2$ ,  $\Delta D$  and  $\Delta\hat{\beta}$ ) versus estimated hazard probability ( $\hat{\pi}$ ) based on the logistic regression model  $E$ , adjusted for clustering effects.*

Covar.No.	357	360	364	365	371
INS	0	0	0	0	1
REP	1	0	0	1	1
PERIOD	5	4	3	3	2
$y_r$	177	57	84	552	115
$\hat{\pi}$	0.0848	0.0369	0.0728	0.2151	0.1662
$\Delta\chi^2$	-	294.92	-	-	-
$\Delta D$	-115.13	380.51	-1134.51	-15.19	-1364.45
Leverage	1.029	0.813	1.204	3.084	1.146
$\Delta\hat{\beta}$	45 569.9	1 283.219	4 887.362	27.708	8464.287

Table 17: *Covariate values, number of event occurrences ( $y_r$ ), estimated logistic probability ( $\hat{\pi}$ ) and the value of four diagnostic statistics  $\Delta\chi^2$ ,  $\Delta D$ ,  $\Delta\hat{\beta}$  and leverage ( $h$ ) for five of the most influential covariate pattern numbers. Based on the logistic regression model  $E$ , adjusted for clustering effects.*

## 8.12 Differences in admission among Home Nurse Areas

The odds ratios and 95% confidence intervals for the model investigating differences in admission among the 11 home-nurse areas is shown in Table 18. The dummy-variables for the different home nurse areas are dichotomous variables with value 1 if the observation is in

that specific area and 0 if elsewhere. Each HNA dummy variable compares to the arbitrary null-baseline district. For the different HNA dummy variables in the model, we can see there are some significant differences between some areas in terms of odds ratios for getting the target event compared to the randomly chosen baseline area.

The model considers the main effects of each home-nurse area has on the odds for target event occurrence. The deviance statistic decreases compared to the plain E model (78.46,  $p = 0.000$  and the model is well specified ( $\hat{C} = 7.85, p = 0.447$ ). We can see from the model that 3 areas (and 1 on a 10% level) have a significantly higher odds ratios at a 5% level for getting the target event compared to the baseline area. These areas have therefore higher odds for sending their patients to the short-term institution compared to the baseline area. As one cannot be at the institution and the home nurse area at the same time in the data-set, these main effects are only impacting the odds of being sent from the specified area to the institution and not the other way.<sup>25</sup>

Variable	Odds ratio	95% CI
HNA1	<del>1.21</del>	(0.85, 1.71)
HNA2	1.29	(0.97, 1.72)
HNA3	<del>1.03</del>	(0.75, 1.42)
HNA4	<del>1.21</del>	(0.91, 1.60)
HNA5	0.87	(0.64, 1.18)
HNA6	<b>1.41</b>	(1.07, 1.88)
HNA7	<b>1.45</b>	(1.09, 1.93)
HNA8	<b>1.35</b>	(1.03, 1.78)
HNA9	<del>1.06</del>	(0.80, 1.40)
HNA10	<del>1.11</del>	(0.84, 1.48)

Table 18: *Estimated odds ratio and 95% Confidence Interval for the model of HNA affiliation, adjusted for clustering effects.*

---

<sup>25</sup>This can be shown by including a two-way interaction term between the home-nurse area dummies and INS, which will all be omitted due to multicollinearity.



















## 8.14 Stata do-file(s)

```
// Import
use "C:\Users\Panel_master_students2.dta", clear
// Restructure home-nurse area variable
gen HNA = 0
replace HNA = 1 if (Ansvar > 999) & (Ansvar < 10000)
drop HJNA HJHJ HJSY
// Restructure cohabitation
gen boralene = 0
replace boralene = 1 if (Ugift == 1 | Enke_enkem == 1 | Sep_skilt == 1 | Gift_sam_inst == 1 | Gift_sam ==
0)
drop Ugift Gift_sam Sep_skilt Enke_enkem Gift_sam_inst Ikke_oppgit
// Generate STI variable (KORT or REHA)
gen STI = 0
replace STI = 1 if (KORT == 1 | REHA == 1)
drop KORT REHA
// Generate LTI variable (STERK, LANG)
gen LTI = 0
replace LTI = 1 if (STERK == 1 | LANG == 1) // 80 989
drop LANG STERK
// HNA user
egen hnauser = max(HNA), by(ID)
// Drop if never been to HNA, no spell 1
sort ID
by ID: drop if hnauser == 0 // 68 245
drop hnauser
// Drop if only 1 obs
by ID: egen nobs = count(Uke)
drop if nobs == 1 // 135
drop nobs
// Tackle BOLOM
tab ID if BOLOM == 1 & (HNA[_n+1] == 1 | STI[_n+1] == 1) & (HNA[_n-1] == 1 | STI[_n-1] == 1)
replace HNA = 1 if BOLOM == 1 & ID == 3153
drop if BOLOM == 1 // 3 316
drop BOLOM
// Tacle LTI
by ID: gen trial = 1 if (HNA == 1 | STI == 1) & (LTI[_n -1] == 1) & (LTI[_n +1] == 1)
// Set trial STI == LTI so that we can delete all LTI after
replace LTI = 1 if trial == 1 // 11
drop trial
// LTIuser
//egen ltiuser = max(LTI), by(ID)
//drop if ltiuser == 0
// Manual checks of any users with more than 2 spells
//tsspell LTI, fcond( (LTI != LTI[_n-1]) )
//tab _spell
//tab ID if _spell == 3
// ID 302 447 650 1088 1350 1463 1466 1470 2993 3321 3759 4138 4326 4875
// Manual changes and deleting
// Single deletions
by ID: drop if ID == 1466 & (_n == _N)
by ID: drop if ID == 2993 & (_n == _N)
replace STI = 1 if (ID == 4238 & Uke == 107) // This is also trial period
// More than single changes
by ID: gen backlti = 1 if LTI == 1 & (HNA[_n+1] == 1 | STI[_n+1] == 1)
by ID: replace backlti = 1 if (ID == 1088) & (backlti[_n-1] == 1) // 24
by ID: replace backlti = 1 if (ID == 1470) & (backlti[_n-1] == 1) // 9
by ID: replace backlti = 1 if (ID == 3759) & (backlti[_n-1] == 1) // 3
by ID: replace backlti = 1 if (ID == 4138) & (backlti[_n-1] == 1) // 15
by ID: replace backlti = 1 if (ID == 4326) & (backlti[_n-1] == 1) // 91
```

```

// Drop observations
by ID: drop if backlti == 1 // 167
by ID: drop if LTI == 1 // 27 055
// Now tsspell the data
// Begin by identifying spell 1, first time HNA
by ID (Uke), sort: gen noccur = sum(HNA)
by ID: gen byte hnastart = noccur == 1 & noccur[_n - 1] != noccur
by ID: replace hnastart = 1 if hnastart[_n-1] == 1
drop if hnastart == 0 // 2 415
drop hnastart noccur backlti
// Units only have 1 observation in total, delete these (147)
by ID: egen nobs = count(Uke)
drop if nobs == 1 // 17
drop nobs
// How many unique units are we left with?
by ID, sort: gen nvals = _n == 1
count if nvals // 4 496
drop nvals
replace HNA = 1 if (Ansvar == . | Ansvar == 2) // 140
// Ready to tsspell
tsspell HNA, fcond( (HNA != HNA[_n-1]) ) // Ignores gaps
by ID : gen byte censoredleft = _seq == 1 & _n == 1
by ID : gen byte censoredright = _end == 1 & _n == _N
gen event = _end - censoredright // 6 390 events
// 320 341 observations
drop LTI Alder14 KORT_d_u REHA_d_u HJNA_t_u HJHJ_t_u HJSY_t_u REHV_t_u y2016 HNAdummy11

// Set up for analysis
forvalues j = 1/159 {
gen _period 'j'=0
replace _period 'j' = 1 if _seq == ['j']}
forvalues j = 1/53 {
gen _spell 'j'=0
replace _spell 'j' = 1 if _spell == ['j']}
gen OUTSIDE = 0 // This is institution (INS)
replace OUTSIDE = 1 if (STI == 1)
gen LPER = ln(_seq)
by ID (Uke), sort: gen noccur = sum(REHV)
by ID: gen byte REHVMEM = noccur == 1 & noccur[_n - 1] != noccur
by ID: replace REHVMEM = 1 if REHVMEM[_n-1] == 1 // This is REHV1
drop noccur
gen REHVMEM2 = REHVMEM
replace REHVMEM2 = 0 if (REHV == 1) // This is REHV2
gen LATE = 1
by ID, sort: gen firstobs = _n == 1
by ID: replace LATE = 0 if (Uke == 1 & _seq == 1 & firstobs == 1)
by ID: replace LATE = 0 if LATE[_n-1] == 0
gen NOTFIRST = 1 // This is REP
replace NOTFIRST = 0 if (_spell == 1) | (_spell == 2)
egen ALDER3 = cut(Alder15), at(0,67,81,120) icodes // This is age
gen HNADUM = 0
replace HNADUM = 0 if Ansvar == 1116
replace HNADUM = 1 if Ansvar == 1542
replace HNADUM = 2 if Ansvar == 1667
replace HNADUM = 3 if Ansvar == 2051
replace HNADUM = 4 if Ansvar == 2142
replace HNADUM = 5 if Ansvar == 2730
replace HNADUM = 6 if Ansvar == 3424
replace HNADUM = 7 if Ansvar == 3449
replace HNADUM = 8 if Ansvar == 3789
replace HNADUM = 9 if Ansvar == 4413

```

```

replace HNADUM = 10 if Ansvar == 4446
sort ID Uke

// Run taxonomy of models
// Model A LL -24827.7 (155)
logit event _period1-_period159, nocons
est store mA
estat gof, group(10) // 0.00 1.000
linktest // Y
// Repeat for all models below
// M B -21781.86 (206)
logit event _period1-_period159 _spell2-_spell53, nocons
// M B2 -22516.14 (158)
logit event _period1-_period159 OUTSIDE NOTFIRST c.OUTSIDE#c.NOTFIRST, nocons
// M C -21490.9 (260)
logit event _period1-_period159 _spell2-_spell53 c.(_period1-_period10)#c.(_spell2) c.(_period1-_period9)#
c.(_spell3) c.(_period1-_period8)#c.(_spell4) c.(_period1-_period7)#c.(_spell5) c.(_period1-_period6)#
c.(_spell6) c.(_period1-_period5)#c.(_spell7) c.(_period1-_period4)#c.(_spell8) c.(_period1-_period3)#
c.(_spell9) c.(_period1-_period2)#c.(_spell10), nocons
// M C2 -21837.31 (195)
logit event _period1-_period159 c.(_period1-_period15)#c.OUTSIDE c.(_period1-_period15)#c.NOTFIRST c.(
_period1-_period10)#c.(c.OUTSIDE#c.NOTFIRST), nocons
// M D -22159.41 (185)
logit event _period1-_period159 c.(_period1-_period15)#c.OUTSIDE c.(_period1-_period15)#c.NOTFIRST, nocons
// M E -22352.93 (159)
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.LPER#c.NOTFIRST, nocons
// M E2 -22178.35 (160)
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.LPER#c.NOTFIRST c.LPER#c.OUTSIDE#c.
NOTFIRST, nocons

// OTHER REPARAM OF PERIOD
gen SQPER = _seq^2
gen LINPER = 1/_seq
// MSQ -22730.07 (159)
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.SQPER NOTFIRST c.SQPER#c.NOTFIRST, nocons
// MDIV -22332.83 (159)
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LINPER NOTFIRST c.LINPER#c.NOTFIRST, nocons
// Do lr-tests for all nested models
lrtest M1 M2, stats // force if necessary

// Creating sample plots
stset _seq if _spell == 1, failure(event) id(ID)
drop s // or h for hazard
sts gen s = s // or h
sort _seq
tway line s _seq if _seq < 160, ytitle(Survival) xtitle(Weeks in spell) yline(0.5) // or Hazard
graph save Graph "C:\Users\Stata\Sample_hazard_may\spell1.gph"
cd "C:\Users\Stata\"
gr combine "spell1" "spell3" "spell5" "spell7" "spell9" "spell12" "spell4" "spell6" "spell8" "spell10", col
(5)
// Creating fitted hazard plots
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.NOTFIRST#c.LPER, nocons vce(cluster ID
)
predict p
graph twoway line p _seq if _spell == 1, sort xtitle(Weeks in spell) ytitle(Hazard)
graph save Graph "C:\Users\Stata\spell1.gph"

// Classification tables and ROC curves after model E
lroc
lsens
// Influential observations

```



```

predict p // Probability of a positive outcome (pi hat)
predict covno, number // Gen covariate pattern numbers
predict db, dbeta // Pregibon delta beta hat influence stat
predict dx, dx2 // HL delta x^2 influence stat
predict ddev, ddeviance // HL delta D influence stat
predict hat, hat // Pregibon leverage
scatter hat p, mlab(covno)
scatter dx p, mlab(covno)
scatter ddev p, mlab(covno)
scatter db p, mlab(covno)
// Check details, e.g.
tab _seq if covno == 3
tab _spell if covno == 3
tab p if covno == 3
tab hat if covno == 3

// Models for gender, age, cohabitation, rehv1, rehv2, comb1, comb2 w/clustering
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.NOTFIRST#c.LPER Mann c.Mann#c.LPER c.
Mann#c.OUTSIDE c.Mann#c.NOTFIRST, nocons vce(cluster ID)
test c.Mann#c.LPER c.Mann#c.OUTSIDE c.Mann#c.NOTFIRST // 4.6 0.2034
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.NOTFIRST#c.LPER Mann, nocons vce(
cluster ID)
// -22 345.451 GOF 5.12 0.7446 link OK
parmest, saving("C:\Users\Stata\Gendervce.dta", replace)
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.NOTFIRST#c.LPER boralene c.boralene#c.
LPER c.boralene#c.OUTSIDE c.boralene#c.NOTFIRST, nocons vce(cluster ID)
test c.boralene#c.NOTFIRST // 1.87 0.1716
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.NOTFIRST#c.LPER boralene c.boralene#c.
LPER c.boralene#c.OUTSIDE, nocons vce(cluster ID)
// -22 226.385 GOF 8.48 0.3879 link OK
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.LPER#c.NOTFIRST b1.ALDER3 i0.ALDER3#c.
LPER i2.ALDER3#c.LPER b1.ALDER3#c.OUTSIDE b1.ALDER3#c.NOTFIRST, nocons vce(cluster ID)
// GOF 5.52 0.7005 link OK
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.LPER#c.NOTFIRST REHVMEM c.REHVMEM#c.
LPER c.REHVMEM#c.OUTSIDE c.REHVMEM#c.NOTFIRST, nocons vce(cluster ID)
test c.REHVMEM#c.OUTSIDE c.REHVMEM#c.LPER // 4.65 0.0976
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.LPER#c.NOTFIRST REHVMEM c.REHVMEM#c.
NOTFIRST, nocons vce(cluster ID)
// -22 329.958 GOF 9.94 0.2694 link OK
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.LPER#c.NOTFIRST REHVMEM2 c.REHVMEM2#c.
LPER c.REHVMEM2#c.OUTSIDE c.REHVMEM2#c.NOTFIRST, nocons vce(cluster ID)
test c.REHVMEM2#c.OUTSIDE c.REHVMEM2#c.LPER // 4.7 0.0954
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.LPER#c.NOTFIRST REHVMEM2 c.REHVMEM2#c.
NOTFIRST, nocons vce(cluster ID)
// -22 328.802 GOF 7.81 0.4522
// COMBINED1
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.NOTFIRST#c.LPER Mann b1.ALDER3 i0.
ALDER3#c.LPER i2.ALDER3#c.LPER b1.ALDER3#c.OUTSIDE b1.ALDER3#c.NOTFIRST REHVMEM c.REHVMEM#c.NOTFIRST
b1.ALDER3#c.Mann c.Mann#c.REHVMEM b1.ALDER3#c.REHVMEM, nocons vce(cluster ID)
// -22 073.413 GOF 9.2 0.3254 link OK
// COMBINED2
logit event _period1-_period159 OUTSIDE c.OUTSIDE#c.LPER NOTFIRST c.NOTFIRST#c.LPER Mann b1.ALDER3 i0.
ALDER3#c.LPER i2.ALDER3#c.LPER b1.ALDER3#c.OUTSIDE b1.ALDER3#c.NOTFIRST REHVMEM2 c.REHVMEM2#c.NOTFIRST
b1.ALDER3#c.Mann c.Mann#c.REHVMEM2 b1.ALDER3#c.REHVMEM2, nocons vce(cluster ID)
// 22 075.078 GOF 11.09 0.1967 link OK
// Re-run all models without clustering and do LR-tests

// Some other analysis tools

// Subsamples
// Make last obs for each unit

```

```

by ID, sort: egen firsttime = min(cond(_spell == 1, Uke, .))
by ID: egen lasttime = max(cond(_spell == 1, Uke, .))
gen byte first = Uke == firsttime
gen byte last = Uke == lasttime
// Make max spell
by ID: egen spellmax = max(_spell)
tab spellmax if last == 1 // Shows last spell
tab _seq if last == 1 & _spell == 1 // Shows last period

// Gives max number of Uke for each unit
by ID: egen nob = count(Uke)
// Shows maximum number of periods for each ID
by ID : egen seqmax = max(_seq)
// Subsample 1 = Spellmax(1)
tab ID if spellmax == 1
// (Un)censored members of subsample 1
by ID : egen evmax = max(event) // Had event? If yes == 1
tab ID if spellmax == 1 & evmax == 1 // Uncensored, as expected 0
tab ID if spellmax == 1 & evmax == 0 // Censored
// Like Assign variable, REHV is time-variant and missing when unit in STI
tab ID if REHV == 1 & STI == 1
replace REHV = . if (STI == 1)
// How many late entrants?
sum ID if (Uke == 1 & _seq == 1 & firstobs == 1) // 1887

```

## 8.15 Reflection note

This reflection note serves the purpose of discussing the topics internationalization, innovation and responsibility in the light of the theme and findings for the thesis. The main theme of this paper is health care economics. Specifically, we have examined patient flows between home-nurse areas and short-term institutions in Norway. The main objective has been investigating whether, when and which individuals are admitted to short-term institutions. The writing of this thesis has been a journey filled with learning new concepts and overcoming many challenges.

We did not know much about neither reablement nor survival analysis when we began our journey approximately six months ago, and a deep delve into the academic literature was needed. We spent many hours findings articles, some of which turned out to be of critical importance to our work, while others did not bear any fruit with respect to the formation of our thesis. Still, it was very exciting to learn about many of the fine nuances within the topic that we had chosen. The data cleaning procedure was a challenging affair, and we spent a lot of time creating event history paths and assessing ways we could approach the data cleaning in an appropriate manner. We consulted many books, performed recommended exercises and attempted to recreate examples in order to understand the way survival analysis was

conducted and the inner mechanics of the methods applied.

Whenever we had specific issues with no simple solution in our textbooks, we consulted various places for guidance. For instance, it was very helpful to us how much Dr. Nicholas J. Cox from Durham University has contributed to Statalist, a forum for our main software of choice. When we encountered an interpretation in an article that we could not understand, we consulted MatRIC for assistance. When even further inquiry into the interpretations was required, we consulted the respective authors for clarification. We were always met with respect and understanding, and the people we met did their best in order to lead us onto the right path. In short, the journey of writing our thesis has been an overall great experience full of learning, sometimes requiring considerable effort in order to overcome challenges that we met.

Summarizing the findings, we find that being male, of old age or living in cohabitation are all generally associated with higher risk of admission to short-term institutions. The last variable we investigated was reablement, and we found that admission was much more likely among previous participants in the reablement program, but readmission was slightly less likely, compared to non-participants in similar situations. Since reablement is a recent phenomenon in Norway and has gained much interest with policy makers, it will be the largest focus throughout this section.

The unit of analysis may depend on the perspective one considers. In other words, one may focus on either the product or the process. For the first case, the units of analysis are the individuals situated in home-nurse areas and are at risk of being admitted to a short-term institution. Otherwise, one may consider the short-term institutions as the units of analysis because they constitute the operating environment the patients are processed by. The question remaining is whether we should focus on the people or the institution, and it may be argued the former is more relevant in this case. This is because home-based reablement is a service given to the individuals in order to increase self-sufficiency and reduce the likelihood of admission to an institution, and is not intervening with the process at the institutions in question.

An important point is that many parts of the world are facing what is known as the age wave, which refers to a population shift resulting in a larger subpopulation of elderly

people. A natural consequence of age is functional decline in a variety of forms and may in some cases inhibit the ability to efficiently perform activities of daily living. If this occurs, some rehabilitation is often needed, where short-term institutions are one of many possible alternatives. Unfortunately, the institutions only have a certain amount of capacity and operating costs are significant once all factors such as medication and wages for health care workers are considered. In the light of our results, we find elderly are at higher risk of admission to short-term institutions compared to the younger population. It is therefore necessary that reablement services are further improved because of the population shift expected to occur. Reablement services aim to increase the performance of those receiving it so they can stay in the community with a reduced need for stays at such institutions.

This brings us to the international perspectives of reablement programs, where close cooperation and sharing of knowledge is required in order to increase the efficiency and quality of those services. Throughout the thesis we have investigated the findings of research in accordance to reablement services in other countries such as Canada, Australia and the United Kingdom. Reablement programs differ with respect to the target audience, the strategy and the level of commitment from both the user and the surrounding community. Some span over just a few weeks in a person's own home, while others are performed at institutions over a longer period. With all the different nuances the reablement services inhibit, there has been a large variety in their results. It is therefore essential for the experiences of both users and administrators of the services are recorded and shared, such that they may be improved upon and learned from, especially because reablement was quite recently introduced as an alternative to other rehabilitation services. Summarizing, it is through the sharing of knowledge and experiences among health care workers and reablement participants one may improve the already existing services that are provided today.

For the theme of innovation, many possibilities for improvements of reablement services exist. It has been mentioned that reablement services have many different modifications with respect to length, location and services provided. One possibility is that, as new knowledge and experience is gained, it is found that different forms of reablement are more effective for some people than others. Since reablement is often suggested for those with challenges in activities of daily living, different forms of reablement programs may be organized around

those activities that groups of users require the most. One reablement program may be targeted towards those with physical health challenges, and another for those with mental health issues. The argument for this is those who require assistance with personal hygiene, cooking or getting dressed from a purely physical perspective may warrant other types of rehabilitation than people struggling with anxiety, forgetfulness or loneliness.

Specialized reablement programs can therefore be administrated by different specialists for each program in their own field of expertise, and services will be better suited to the needs of individuals. In the light of our study, we find being male is associated with a higher risk of admission. One should examine the difficulties males experience, and customize the reablement services males receive to address the challenges they are facing. Furthermore, we find living alone has much lower risk of admission compared to cohabitants. Therefore, one should examine why people living alone have lower risk of admission and, if possible, apply principles that take advantage of this knowledge when mapping out reablement services. Summarizing, innovations for reablement services can be made by examining the patterns in needs and risk profiles of the participants and customizing programs targeted towards those with higher risk of admission, addressing specific challenges they are experiencing.

For the topic of responsibility, it is important to remember, in a health care setting, we are dealing with people. Reablement services are targeted towards those having, or being at risk of, functional decline, often elderly people. This means one must show extreme care so personally identifying information is not compromised, and is stored with outmost security. Furthermore, when dealing with individuals in vulnerable populations it is important to show compassion and empathy in dealing with their situation. Finally, the work required in recording the process and progress of participants should be upon those that administer the services rather than the participants themselves. Vulnerable populations are receiving reablement services for a reason and requiring them to fill out large amounts of paperwork may be detrimental to their performance and motivation to continue participating and making progress according to their own goals.

Finally, we must not forget to consider the people in our own life that are possibly experiencing challenges of their own. One should take time out of their day in order to take care of their loved ones and thereby contributing to an increase in their overall health. The

theme of responsibility is relevant not only for health care workers, but for everyone that surround those that are effected by various degrees of functional decline. Summarizing, one must show great care when dealing with patients and patient data, and not forget to make contributions themselves. Not only should one treat the person in a way which shows respect for their situation in a professional setting, but in a personal setting as well, and personal information should always be handled in accordance with the policies and regulations that are required by law.

In short, reablement services are a new and exciting form of treatment with many interesting qualities and room for improvement, and it has been a pleasure to delve into the finer details of this topic. First, one should attempt to share newfound knowledge and experience in order to learn more, and improve the services provided. Second, improving reablement services can possibly be achieved by customized programs specializing in different goals patients set for themselves and require the most assistance with. Third, ones that administer the services must know they are dealing with people who are often part of a vulnerable sub-population. Fourth, when assessing the efficiency of the services offered through analysis, one should be especially careful in the storage and use of the personal data for the participants it concerns. Finally, we must all work together and be responsible in order to secure the health of our loved ones and the community at large.

- Daniil Evgenjevich Rudsengen