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A New Tool for the Assessment of the Development of Students' Mathematical Competencies

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Abstract

We look at the assessment of mathematical competencies of undergraduate biology students. Mathematical modelling tasks with biological content were introduced to engage students more actively into learning mathematics. Profiles for individual learners were created using five basic families of mathematical competencies. Sixteen mathematical competencies in five families were coded in transcripts of video recordings and students' writings in a reliable unambiguous manner; competencies frequency and intensity were recorded. These data were analysed with a new assessment tool suggested by the authors to monitor students' competencies development.

The construct: mathematical competence

In broad sense, competence is a theoretical construct defined as a "complex ability (…) that (…) [is] closely related to performance in real-life situations" (Hartig, Klieme & Leutner, 2008). Seven facets of competence and its assessment are identified: complexity, performance, standardization, fidelity, level, improvement, and disposition (Shavelson, 2013). Competence cannot be observed directly, but can be inferred from individual's performance on sample tasks; it can be improved through learning and deteriorates through forgetting (Shavelson, 2013). Remarkably, "competence is not the same as academic knowledge and (…) academic competence is not the same as professional competence" (Oser, 2013). We discuss only cognitive (knowledge and skills) component of competence; metacognitive and non-cognitive components are out of the scope of this paper.

Mathematical competence means "the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role. (…) A mathematical competency is a clearly recognizable and distinct, major constituent of mathematical competence" (Niss, 2003). Three mathematical competencies frameworks are often used in the research literature: "Principles and standards for school mathematics" (NCTM, 2000), Danish KOM-project (Niss, 2003), and Mathematical Competencies: A Research Framework (MCFR) (Lithner et al., 2010). One more framework focuses on mathematical giftedness and uses nine component mathematical abilities whose combination can lead to high achievement in mathematics (Krutetskii, 1976).

Research Setting and Data Collection

This research is aimed at increasing biology students' motivation for mathematics using mathematical modelling activities. The first author prepared teaching materials and conducted teaching once a week during one semester complementing regular lectures and seminars in a standard mathematics course MAT101 for the first year students in natural sciences. The topics discussed in the complementary sessions: periodic functions (2 sessions), exponential growth and regression (2 sessions), population dynamics (2 sessions), integrals and modeling (2 sessions). Twelve out of about a hundred biology students enrolled in the course participated in additional sessions providing a purposeful random heterogeneous sampling which mirrors characteristic features of the larger sample to "add credibility to the results of a larger study" (Teddie & Tashakkori, 2009).

An important feature of our approach is the use of "embedded assessment" where "opportunities to assess student progress and performance are integrated into the instructional materials and are virtually indistinguishable from day-to-day instructional activities" (Wilson & Draney, 2013). Becoming an integral part of the teaching and learning process, embedded assessment can be viewed as "assessment for learning" (Black, Harrison, Lee, Marshall, & William, 2003). Using mathematical problems with biological content, we assure that the assessment is curriculum dependent for its full and meaningful embedding into teaching and learning (Wolf & Reardon, 1996).

Data include video recordings of participants, researcher's observation/field notes and students' written material obtained using *Livescribe 3* smart pens and notebooks. One camera was recording the "focus group" and a *GoPro* camera with a panoramic view was used to record all interactions in the classroom. Sessions were taped with minimal disturbance to students so that "the effect of video becomes negligible in most situations after a certain phase of habituation" (Knoblauch, Schnettler & Raab, 2006).

Mathematical Competencies Framework and Scaling

In 2016, the first author conducted a "pilot study" to test the functionality of the KOM mathematical competencies framework (Niss, 2003). Not surprisingly, in several episodes competencies significantly overlapped because "the competencies are closely related - they form a continuum of overlapping clusters - yet they are distinct in the sense that their centres of gravity are clearly delineated and disjoint" (Niss, 2003). To minimise possible complications with coding, we retained only five basic groups of mathematical competencies out of the eight suggested in KOM. All sixteen competencies in five groups are described below. *Thinking/acting mathematically:* pose questions that are characteristic of mathematics; understand and handle the scope and limitations of a given concept; attack mathematical problems. *Mathematical modelling:* assess the range and validity of existing models; interpret and translate elements of a model during the mapping process; interpret mathematical results in an extra-mathematical context and generalize solutions developed for a special task or situation; criticize the model by reviewing, reflecting and questioning results; search for available information differentiating between relevant and irrelevant information; choose appropriate mathematical notation. *Representing and manipulating symbolic forms:* choose a representation; switch between representations; manipulate within a representation. *Reasoning and communicating:* understand others' written, visual or oral information with mathematical content; follow and assess chains of arguments put forward by others; express oneself in oral, visual or written form in mathematical context; provide explanations or justifications to support own results and ideas. *Aids and tools:* know different tools and aids for mathematical activity and their properties; use appropriate aids and tools to develop insight or intuition.

Data analysis of three sessions suggested that not only the frequency of activation of a competency should be recorded but also its intensity at each activation instance. Furthermore, for each competency, three perspectives are considered: task solving vision (T.S.V.), use of mathematical language/vocabulary (M.L.V.), and independent thinking (Ind.T.). The first perspective relates activation of a competence with the depth of student's understanding of the steps towards solution. The second puts in the spotlight the use of appropriate mathematical language needed to activate competencies in written and oral communication since insufficiently developed mathematical language can be the reason for the overall deceleration of mathematics learning (van der Walt, Maree, & Ellis, 2008). The third perspective monitors student's independent thinking measured by the extent of instructor's prompting. Bernstein (1967) emphasised the importance of reducing alternative actions during skill acquisition; less scaffolding means more stimulation for the independent work and a higher competency intensity.

We rate competencies intensity by the evidence of understanding of mathematical content: C1 - little or no evidence, C2 - occasional, B1- limited, B2 - basic, B3 - substantial, A1 - full, A2 - in-depth, A+ - exceptional. The hierarchy of qualitatively distinct levels of performance with a clear description of students' abilities/skills is needed for the construct validity of our assessment tool (Kane, 2001). Our scaling relates two facets of mathematical competence: "performance – a capacity not just to "know" but also to be able to perform" and "level – the performance must be at a "good enough" level to show competence" Shavelson (2013). The evidence for cognitive validity is achieved through the analysis of the "logical link between the interpretive claim and the nature of the assessment – the characteristics of the task, response demands, and scoring system" (Ruiz-Primo, Shavelson, Li, & Schultz, 2001).

Assessment Tool

To design an assessment tool for students' mathematical competencies development within and across semester cohort, we adopt operationalist view defining measurement as "a procedure for the assignment of numbers to specified properties of experimental units in such a way as to characterize and preserve specified relationships in the behavioural domain" (Lord & Novick, 1968). Transcripts and students writings were coded and rated each time the competency was activated resulting in a large number of sets of heterogeneous data for each competencies frequency and intensity, for all students and sessions. We converted data into quantitative (the value 1 is assigned to C1, 2 to C2, 3 to B1, etc.) to spot the trends in competencies development. The intensity of one competency, reasoning, is presented in Figure 1 for two students, M and J. We see that the competency has been activated for M 26 times in all seven sessions and its intensity never dropped below the green "developing" competency intensity strip and often jumped above it, even to the highest level 8 (A+). For J, the reasoning competency has been activated 23 times in all sessions but the last one, the intensity was dropping below the green strip but sometimes jumped up, at most to the level 7 (A).

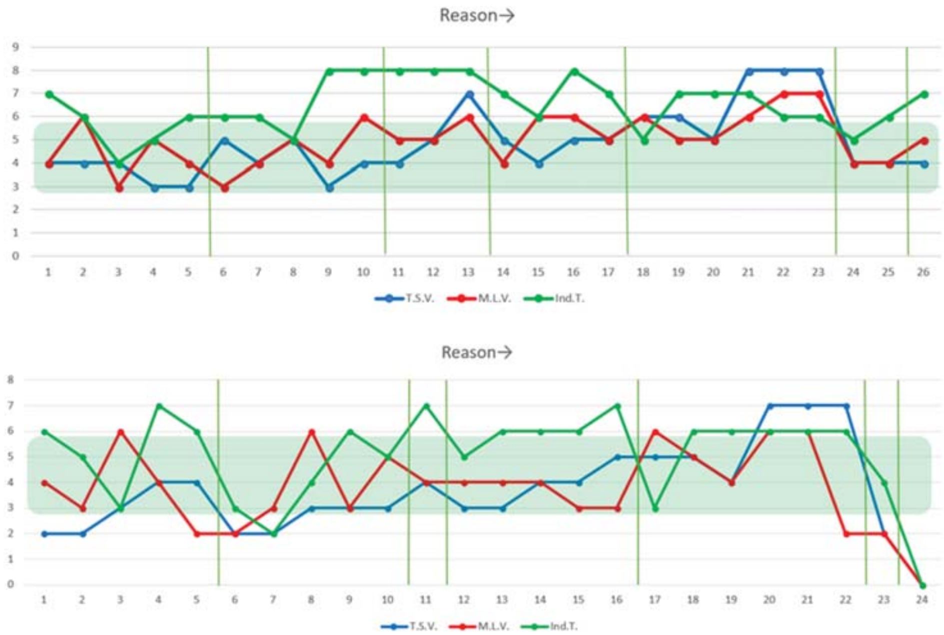

Figure 1 Development of the reasoning competency for students M (upper diagram) and J

To classify the learning progress, we introduce four key indicators: *total progress indicator* (TPI), the difference between the final and initial intensity values, *winding number* (WN), the total number of intensity value changes (slope changes) between two successive instants, *intensity spans 1 and 2* (IS1 and IS2), the difference between the highest (respectively, lowest) and the final intensity values. Using key indicators, we define five learning types: progressive, persistent, unsteady, alternating and transient in Table 1.

Learning type	TPI value	WN value	IS1 value	IS2 value
Progressive (Pr)	Large	Any	Small	Large
Persistent (Pe)	Small	Any	Small	Small
Unsteady (U)	Small	Low	Small	Large
Alternating (A)	Small	Any	Large	Large
Transient (T)	Small	Low	Large	Small

Table 1 Classification of learning types

Not surprisingly, learning type changes for teaching blocks with different topics (separated in Figure 1 with vertical lines), see Table 2. Classification of learning types allows to compare the individual competency development through the topic/semester or compare learning types of two students. For example, we observe from Table 2 that M performs better with mostly persistent or progressive type of learning whereas J's learning type is mainly unsteady and is not even classified in the last block.

TPI, WN, IS1, IS2 and learning type for student M in four teaching blocks (1, 2&3,				
Ind. T.	-1/2/1/-2 Pe	2/2/0/-3 U	3/6/1/-1 Pe	1/0/0/-2 Pr
T.S.V.	-1/1/1/0 Pe	2/4/0/-4 Pr	¾/0/-3 Pr	0/0/0/0 Pe
M.L.V.	0/4/2/-1 Pe	3/5/0/-3 Pr	3/5/0/-2 Pr	1/1/1/-1 Pr
TPI, WN, IS1, IS2 and learning type for student J in four teaching blocks (1, 2&3, 4&5,				
Ind. T.	0/3/1/-3 U	4/3/1/-5 Pr	2/0/0/-2 U	-4/1/4/0 n/c
T.S.V.	2/1/0/-2 Pr	1/0/0/-1 Pe	2/0/0/-2 U	-2/0/2/0 n/c
M.L.V.	-2/3/4/0 T	¾/1/-3 U	-1/2/1/0 Pe	-2/1/2/0 n/c

Table 2 Learning types for M and J

Changes in the learning type between teaching blocks indicate that activation of a competency in a new learning environment may follow a different pattern; tasks should be carefully designed to avoid dramatic drops in competencies intensity up to a complete failure to activate a competency.

Conclusions and Discussion

Undoubtedly, our assessment tool has certain limitations since "it is never a question of whether the models are true; it is a question of whether they provide adequate approximations of performance, which fit our current understanding of learning in the domain and prove useful for their intended purposes" (Mislevy, 2016). Good performance tasks are not easy to design, they should be rated by experienced assessors, and the scores should be correctly interpreted to make competence claims that are then used to make decisions. We suggested the tool and a novel classification of learning types and look for the feedback. We believe that our tool allows to follow students' competencies development fairly well and hope to improve it through further testing.

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