The Wage Curve, Once More with Feeling: Bayesian Model Averaging of Heckit Models

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ABSTRACT: The sensitivity of the wage curve to sample-selection and model uncertainty was evaluated with Bayesian methods. More than 8000 Heckit wage curves were estimated using data from the 2017 household survey of Bolivia. After averaging the estimates with the posterior probability of each model being true, the wage curve elasticity in Bolivia is close to −0.01. This result suggests that in this country the wage curve is inelastic and does not follow the international statistical regularity of wage curves.

JEL classification: C11, D00, J30

Keywords: model uncertainty, Bayesian model averaging, wage curve

Introduction

Blanchflower and Oswald (1990) proposed the existence of a micro-econometric association between wage level and local unemployment, portrayed as a downward-sloping "wage curve". Since evidence of a wage curve elasticity of −0.1 has been found in more than 40 countries, Blanchflower and Oswald (1995) and Blanchflower and Oswald (2005) conclude that the wage curve elasticity is an international empirical law of economics.

This paper estimates the wage curve for Bolivia and evaluates whether the elasticity of the wage curve in this country is sensitive to sample-selection and model specification: does the statistical regularity of $\hat{\theta} \approx -0.1$ changes if different control terms are included as

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explanatory variables in wage curve models that adjust for the bias of considering only the working population? Bayesian model averaging was used to account for model uncertainty. Heckit models were used to account for the selection bias of estimating the wage curve with only the subpopulation of individuals that are working.

The contribution of this study to the literature is twofold. First, the estimations of wage curve elasticities have been mainly focused on developed or middle-income countries; less evidence seems to exist about the elasticity in lower and lower-middle income countries such as Bolivia. Thus, an estimation of the elasticity for Bolivia contributes to the accumulation of evidence about the value of the wage curve elasticity in developing economies with lower levels of income. Also, to the best of the author's knowledge, no study has yet considered simultaneously the effects of both sample selection and model uncertainty in estimating the wage curve elasticity with Bayesian methods.[1] Consequently, this study explores a potential methodological improvement in the econometric estimation of wage curve models.

The next section describes the methods used in the study. Section 2 presents the results and Section 3 concludes the study.

1 Bayesian Model Averaging of Heckit Models

Wage curve

The relationship between individual wages (w) and local unemployment (u), which Blanchflower and Oswald named the *wage curve*, is

$$\log w = \theta \log u + \text{controls}. \tag{1}$$

The elasticity coefficient θ has been found to be relatively constant across countries; its estimated value tends to be centered on -0.1. While the Blanchflower-Oswald finding is empirical, it is in line with the theoretical model of Campbell and Orszag (1998), who derived a dynamic efficiency wage model in which the wage elasticity with respect to employment is invariant to changes in productivity, payroll taxes, training subsidies and parameters in the training cost and quitting functions.

The empirical estimation of wage curve elasticity can be sensitive to the inclusion/ exclusion of explanatory terms, i.e., to model uncertainty. This model uncertainty implies that, for example, the elasticity estimated with a regression that takes into account formality of employment may be different from that estimated with a regression that does not take into account this variable. Moreover, some variables may affect the probability of working, but

[1] Hoddinott (1996) used a two-step approach to correct for sample selection bias in the wage equation, with data of Côte d'Ivoire, but does not treat the problem of model uncertainty.

not the wage level – for example, being the household head in the family unit – whereas other variables affect both the probability of working and the wage level – for example, education or professional experience. If different combinations of variables lead to different estimations of the wage curve models, then model uncertainty may be a problem that can affect the proper estimation of the wage dynamics.

Bayesian model averaging (BMA)

The uncertainty about which variables should be included as control terms in the calculation of the wage curve is equivalent to the uncertainty about which model specification is *correct* (in the sense of being suitable for explaining the wage-unemployment relationship) between the $\mathcal{M}_j$ $(j = 1, 2, \ldots, k)$ potential models. Bayesian model averaging (BMA) is an econometric technique that reveals the correct probability of a model specification, given the evidence provided by the data: in this case the probability of a wage curve model being correct given the data of the variables included as control terms in the sample-selection model. BMA was used *inter alia* to select the variables that affect firm default (Traczynski, 2017) or to explain the mixed evidence about the effects of oil rents on growth, which can be explained with model uncertainty according to Arin and Braunfels (2018). See Fragoso et al. (2018) for a survey of the applications of BMA.

The Bayesian posterior probability of a wage curve model $\mathcal{M}_j$ being true, conditional on a information set $\mathcal{D}$, is,

$$\mathbb{P}\left(\mathcal{M}_j|\mathcal{D}\right) = \frac{\mathscr{L}\left(\mathcal{D}|\mathcal{M}_j\right)\mathbb{P}(M_j)}{\sum_{i=1}^{K}\mathscr{L}\left(\mathcal{D}|\mathcal{M}_j\right)\mathbb{P}(M_j)} \tag{2}$$

where $\mathscr{L}\left(\mathcal{D}|\mathcal{M}_j\right)$ is the marginal likelihood for each model. Because the denominator of the previous equation is hard to calculate directly, it is common to compare two models, i and j, using the ratio of their posterior model probabilities,

$$\mathcal{P}_{ij} = \frac{\mathbb{P}\left(\mathcal{M}_i|\mathcal{D}\right)}{\mathbb{P}\left(\mathcal{M}_j|\mathcal{D}\right)} = \frac{\mathscr{L}\left(\mathcal{D}|\mathcal{M}_i\right)\mathbb{P}\left(\mathcal{M}_i\right)}{\mathscr{L}\left(\mathcal{D}|\mathcal{M}_j\right)\mathbb{P}\left(\mathcal{M}_j\right)}. \tag{3}$$

When equal prior weight is attached to each model, $\mathbb{P}\left(\mathcal{M}_i\right) = \mathbb{P}\left(\mathcal{M}_j\right)$, the posterior odds ratio $\mathcal{P}_{ij}$ becomes a ratio of marginal likelihoods, that is, the Bayes factor $\mathcal{B}_{ij}$,

$$\mathcal{B}_{ij} = \frac{\mathscr{L}\left(\mathcal{D}|\mathcal{M}_i\right)}{\mathscr{L}\left(\mathcal{D}|\mathcal{M}_j\right)}. \tag{4}$$

See, among others, Hoeting et al. (1999) or Koop (2003).

Asymptotic approximation to Bayes factors

Raftery (1995) proposed an asymptotic approximation to a Bayes factor $\mathcal{B}_{ij}$ through the Bayesian information criterion (BIC, see Schwarz (1978)),

$$
\begin{aligned}
\mathcal{B}_{ij} &= \frac{\mathscr{L}(\mathcal{D}|\mathcal{M}_i)}{\mathscr{L}(\mathcal{D}|\mathcal{M}_j)}, && (5)\\
&= \left[\frac{\mathscr{L}(\mathcal{D}|\mathcal{M}_S)}{\mathscr{L}(\mathcal{D}|\mathcal{M}_i)}\right] / \left[\frac{\mathscr{L}(\mathcal{D}|\mathcal{M}_S)}{\mathscr{L}(\mathcal{D}|\mathcal{M}_j)}\right], && (6)\\
&= \mathcal{B}_{Si}/\mathcal{B}_{Sj}, && (7)\\
2\log\mathcal{B}_{ij} &= 2\log\mathcal{B}_{Si} - 2\log\mathcal{B}_{Sj} && (8)\\
&\approx BIC_i - BIC_j. && (9)
\end{aligned}
$$

Thus, two models can be compared by taking the difference of their BIC values ($\mathcal{M}_S$ is a saturated model in which each data point is fit exactly). When the baseline model is the null model $\mathcal{M}_0$, with no independent variables, then $2\log\mathcal{B}_{ij} = 2\log\mathcal{B}_{0j}$, for $\mathcal{B}_{0j}$ the Bayes factor for the null model $\mathcal{M}_0$ against the model of interest $\mathcal{M}_j$.

Bayes factor approximation to posterior model probabilities

If $\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{M}_k$ models are being considered and each of $\mathcal{M}_1, \ldots, \mathcal{M}_k$ is compared in turn with $\mathcal{M}_0$, yielding Bayes factors $\mathcal{B}_{10}, \ldots, \mathcal{B}_{k0}$, then the posterior probability of $\mathcal{M}_j$, $j = 1, 2, \ldots, k$, is,

$$\mathbb{P}(\mathcal{M}_j|\mathcal{D}) = \frac{\alpha_j \mathcal{B}_{j0}}{\sum_{r=0}^{k} \alpha_r \mathcal{B}_{r0}}, \tag{10}$$

where $\alpha_j = \mathbb{P}(\mathcal{M}_j)/\mathbb{P}(\mathcal{M}_0)$ is the prior odds for $\mathcal{M}_j$ against $\mathcal{M}_0$. A natural choice is taking all the prior odds α_j equal to 1. See Kass and Raftery (1995).

BMA estimator of the wage curve elasticity

The BMA estimator of the wage curve elasticity $\hat{\theta}_{BMA}$ is obtained by adding the estimates of θ_j in each j-model, weighted by the posterior probability $\mathbb{P}(\mathcal{M}_j|\mathcal{D})$ of each model being correct,

$$
\begin{aligned}
\hat{\theta}_{BMA} &:= \mathbb{E}(\theta|\mathcal{D}, \theta \neq 0), \\
&\propto \sum\sum\nolimits_{\mathcal{A}_\theta} \mathbb{E}(\theta|\mathcal{D}, \mathcal{M}_j)\, \mathbb{P}(\mathcal{M}_j|\mathcal{D}).
\end{aligned} \tag{11}
$$

Heckman correction

Arango et al. (2010) used the Heckman (1979) correction to account for the selection bias due to estimating the wage determinants from the subpopulation of individuals who work, who are selected non-randomly and may differ from the subpopulation who do not work. The Heckman correction implies estimating a bi-equational sample selection model (henceforth, Heckit model)

$$\begin{cases} z_i^* := \mathbb{P}\left(working = 1|\mathbf{x}_j\right) = \Phi(\mathbf{x}_j'\beta_j), \\ \log w_i|z_i^* = \theta \log u + \mathbf{x}_j'\beta_j + \beta_\lambda \hat{\lambda}_i(\mathbf{x}_j'\beta_j|\sigma_2) + \nu_i, \end{cases} \tag{12}$$

where the first equation is a selection equation (a probit model) to estimate the probability of working and the second equation is the traditional wage curve, plus a correction term from the first equation, i.e., the inverse Mills ratio $\lambda_i(\mathbf{x}_j'\beta_j|\sigma_2) = \phi(-(\mathbf{x}_j'\beta_j|\sigma_2))/(1-\Phi(-(\mathbf{x}_j'\beta_j|\sigma_2)))$. See, *inter alia*, Puhani (2000) or Cameron and Trivedi (2005). In the term $\mathbf{x}_j'\beta_j$, $\mathbf{x}_j'$ is a vector containing the $1, 2, ..., j$-control covariates and β_j are the parameters that measured the effect of each covariate on the probability of working and/or the wage level.

Model space of Heckit wage curves

The possible combinations of k-explanatory variables in a model is defined by the power set $\wp(k)$,

$$\wp(k) = \{\{\emptyset\}, \{x_1\}, \{x_2\}, \ldots, \{x_k\}, \{x_1, x_2\}, \ldots, \{x_1, \ldots, x_k\}\}. \tag{13}$$

The cardinality of $\wp(k)$ gives the total combinations of models that have to be estimated,

$$|\wp(k)| = \sum_{p=0}^{k} \binom{k}{p} = 2^k, \tag{14}$$

with $\binom{k}{p}$ the Binominal coefficient $\binom{k}{p} = \frac{k!}{p!(k-p)!}$. In the case of Heckit models, the $x_1, \ldots, x_k$ control variables can enter as explanatory terms in the selection equation, the outcome equation (the wage curve), or both. If the same variables are considered in the selection equation and the outcome equation, a total of $2^k\left(2^k-1\right)$ Heckit wage curves need to be estimated.

2 Results

The 2017 household survey of Bolivia was used to estimate the country's wage curve elasticity. Bolivia was used as a case study because estimates of the wage curve elasticity were needed for the Computable General Equilibrium Model MAMS (*Maquette for Millennium Development Goals Simulation*) of the United Nations Development Program and the United Nations

Department of Economic and Social Affairs. See Lofgren and Diaz-Bonilla (2010) for details.[2] Geographical clustering was performed at the municipal level, thus, local unemployment is equal to municipal unemployment rates. The selection equation and the outcome equation were jointly estimated with pseudo-maximum likelihood, using expansion factors, because inconsistent estimates will result from two-stage least squares applied to the survey data.

Six control terms were considered as possible wage determinants in the outcome equation, and seven covariates were considered as possible explanatory terms of the probability of working in the selection equation of the Heckit models. In the case of the outcome equation, the possible wage determinants considered were: (1) education (measured with years of schooling), (2) experience (measured as the squared years of the individual's years of tenure in a business), (3) age, (4) gender, (5) a dummy variable equal to one if the person works in a formal business and (6) a dummy variable equal to one if the person is indigenous. For the selection equation (the probability of working), the following variables were considered as potential control covariates: (1) education, (2) age, (3) gender, (4) a dummy variable equal to one for formal businesses, (5) a dummy variable equal to one if the person is indigenous, (6) the number of family members and (7) a dummy variable equal to one if the person is a household head. These variables were selected based on data availability and also following previous studies such as Baltagi et al. (2017), who estimated the wage curve for Brazil and considered as control covariates the age of the individual, gender, race, education, the individual's years of tenure at a firm and formality of employment, among other variables available for Brazil. In Bolivia, the level of education and experience have a direct effect on the salary and on the probability of working; also, it is expected that differences in salary may exist for different age categories and between males and females. Age and gender also may affect the probability of working. Due to the high proportion of informal businesses in Bolivia, a binary variable equal to one if the worker's employment is a formal business was included as a potential control covariate for the outcome and selection equation. Also, to account for possible ethnic discrimination in salary and/or job allocation, a dummy variable equal to one was included for indigenous individuals in both equations of the Heckit models. Finally, the number of family members variable and an indicator variable for household-head individuals were included in the selection equation because higher incentives for working may exist if a person is a household-head and/or is responsible for a greater number of people in a family unit.[3] A total of $2^6(2^7-1) = 8128$ Heckit models were estimated with the

[2] The estimates for the MAMS model were originally obtained with data from the 2006 household survey of Bolivia, because the base year of the MAMS model was 2006. Based on the reviewer's suggestion, we updated the estimation with data from the more recent survey and, thus, this section presents results based on data from the 2017 household survey of Bolivia. In the results for 2006, the estimate of the wage curve elasticity for Bolivia was equal to 7%.

[3] A similar variable was included by Baltagi et al. (2017) as a control covariate, household type, where the response categories were couples without children, couples with children, single mother with children, and

combinations of these control terms.

Figures 1 and 2 show the results of the estimation. In Bolivia, when sample selection and uncertainty in control covariates are considered, the estimation of the wage curve elasticity is bi-modal in a range between −0.05 and 0.06. The posterior probability of the Heckit models being true is higher for negative wage curve elasticities (Figure 1), and the concentration of the Bayes factors (i.e., the model evidence) suggests that more support exists for estimations of the wage curve elasticity between −0.02 and −0.04 (Figure 2). This result is theoretically expected if higher unemployment leads to a fall in the level of wages.

Adding the estimates of θ_j in each j-model, weighted by the posterior probability of each model being correct, allows us to obtain a point estimate of the wage curve elasticity ($\hat{\theta}_{BMA}$) equal to −0.0104. This value is closer to zero than the expected statistical regularity of −0.1 reported for developing countries and suggests that the wage curve in Bolivia is inelastic.

Finally, Table 1 shows the posterior inclusion probability of the control covariates included in the Heckit models. The estimates suggest that education and gender are particularly relevant for determining the wage level, whereas age, formal employment, and being a household-head affect the probability of working.

3 Conclusion

Taking into account selection bias and model uncertainty, the estimated wage curve elasticity for Bolivia in 2017 was equal to −0.0104. This estimate (without regard to its sign) is below the absolute value of −0.1 reported as a statistical regularity of the wage curve for developing countries, and is also below the unbiased estimation of Nijkamp and Poot (2005), who performed a meta-analysis of 208 wage/unemployment elasticities from the literature to correct for publication bias and found a value of −0.07 for the wage curve elasticity.[4]

The results of the wage curve estimation for Bolivia do not provide empirical support for the burgeoning literature about non-competitive features of the job market and further suggest that the wage curve in Bolivia is inelastic. An explanation for this finding may be related to the exponential increases in the minimum wage of Bolivia since 2006, which are coupled with low levels of formal unemployment (Figure 3). In 2006, the minimum monthly wage of Bolivia was USD 62, while in 2017 the minimum wage reached USD 287, which corresponds to an increase of 363%. The increase in the minimum wage in Bolivia could have led to a reduction in the amount of labor that firms demand, increasing unemployment

other.

[4]The estimate for Bolivia in 2017 is also different from those obtained for other Latin American development countries by Galiani (2000), Garcia and Granados (2005), Lugo (2006), Arango et al. (2010), Cruces and Ham (2010), Ines Terra et al. (2010), Ramos et al. (2010), Carvalho Filho and Estevão (2012), Martinez (2012) or Anaya and Rodríguez-Villamil (2012).

as labor becomes more expensive. At the same time, the higher minimum wage may have stimulated the supply of labor, thereby decreasing unemployment. This is in line with Gregg et al. (2014), who suggest that in a case of wage inflexibility, real wages do not seem to respond to the varying local labor market conditions. A similar result was also put forward recently by Daouli et al. (2017), who found only a short-lived wage curve elasticity in Greece during a time of reduction in the national minimum wage and a restructuring of the collective bargaining regime in the labor market.

Future studies can employ dynamic wage curves to explore if the inflexibility of the labor market of Bolivia was indeed caused by the changes in the minimum wage. Because in Bolivia the changes in the national minimum wage were related to a change in the government, a quasi-experimental design can be used to compare models before and after the change of government in 2006. If model uncertainty is considered during the quasi-experimental estimation, Bayesian methods can be employed to provide a comprehensive and coherent framework for comparing non-nested models (Hepple, 2004).

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Figure 1: Histogram of the wage curve estimations and posterior probability $\mathbb{P}(\mathcal{M}_j|\mathcal{D})$ of a Heckit wage curve being true

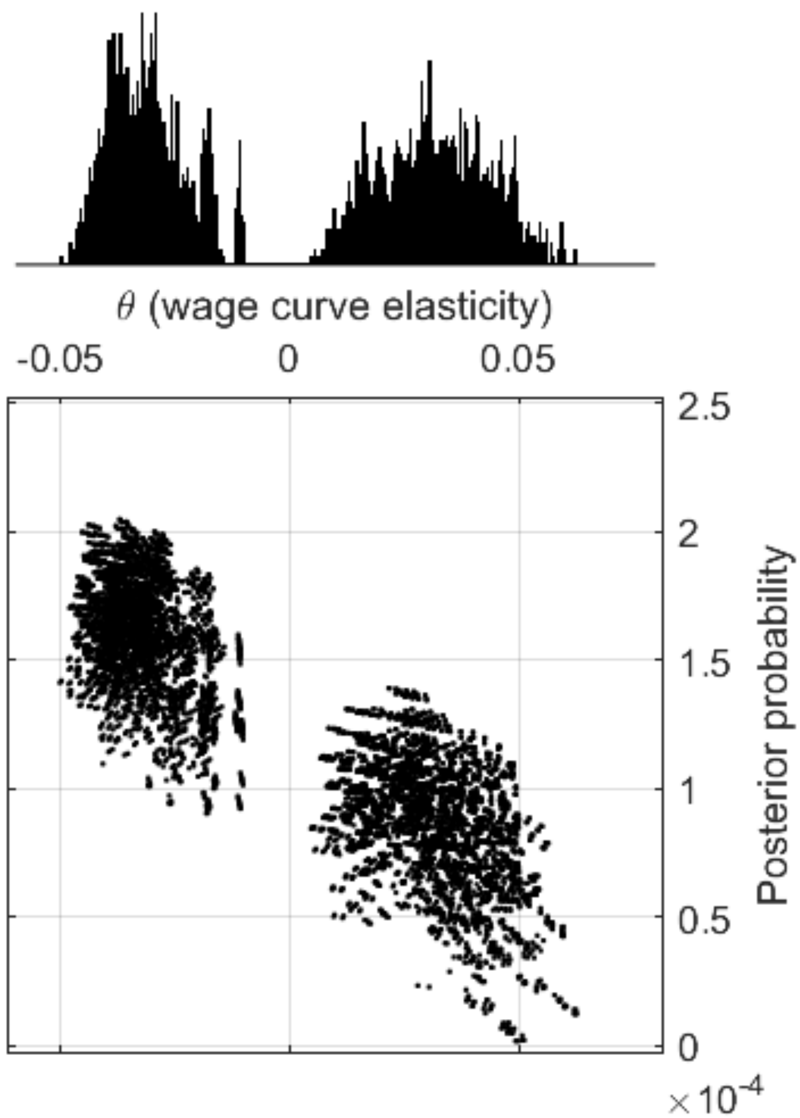

Figure 2: Bayes factors and wage curve elasticity

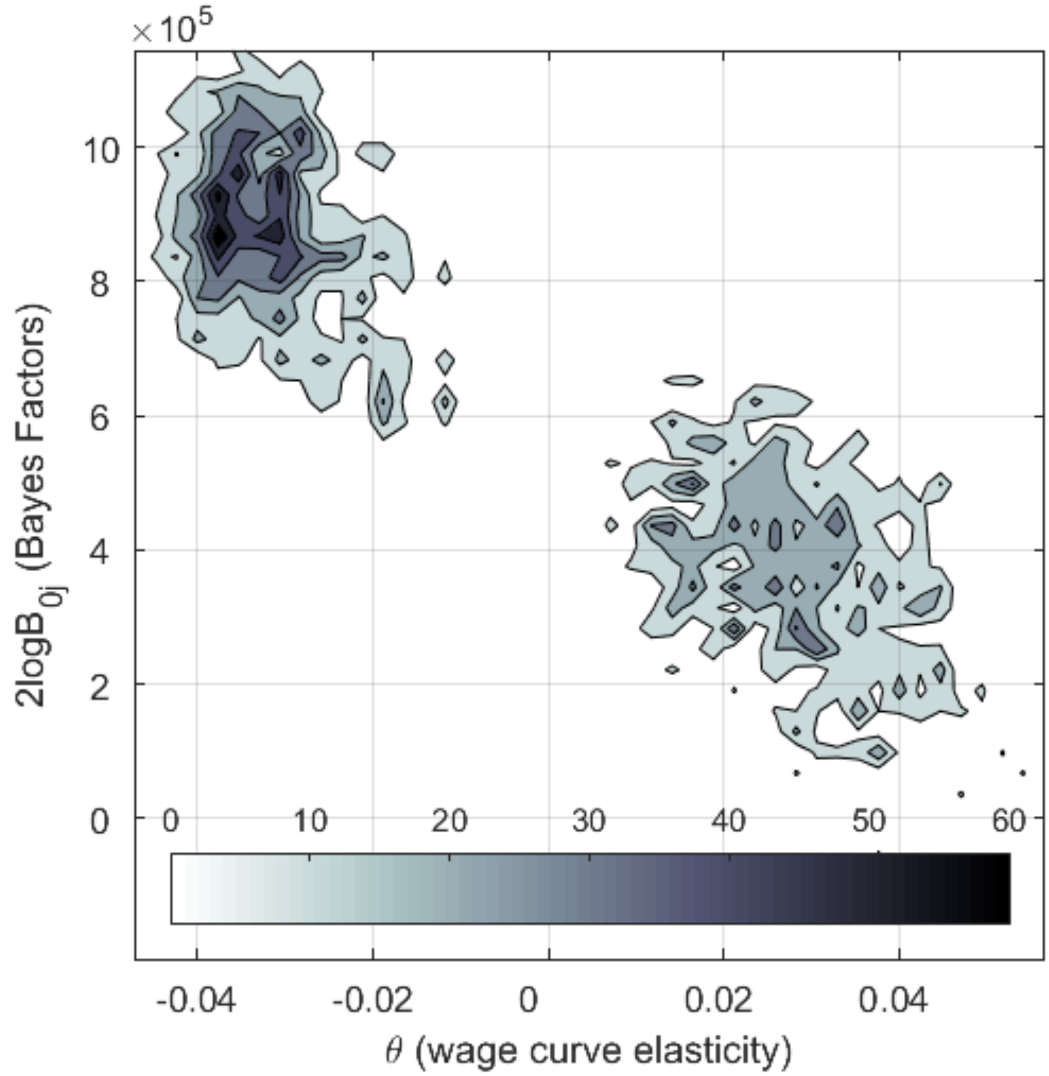

Figure 3: Minimum wage and unemployment rate in Bolivia
Source: World Bank and UDAPE

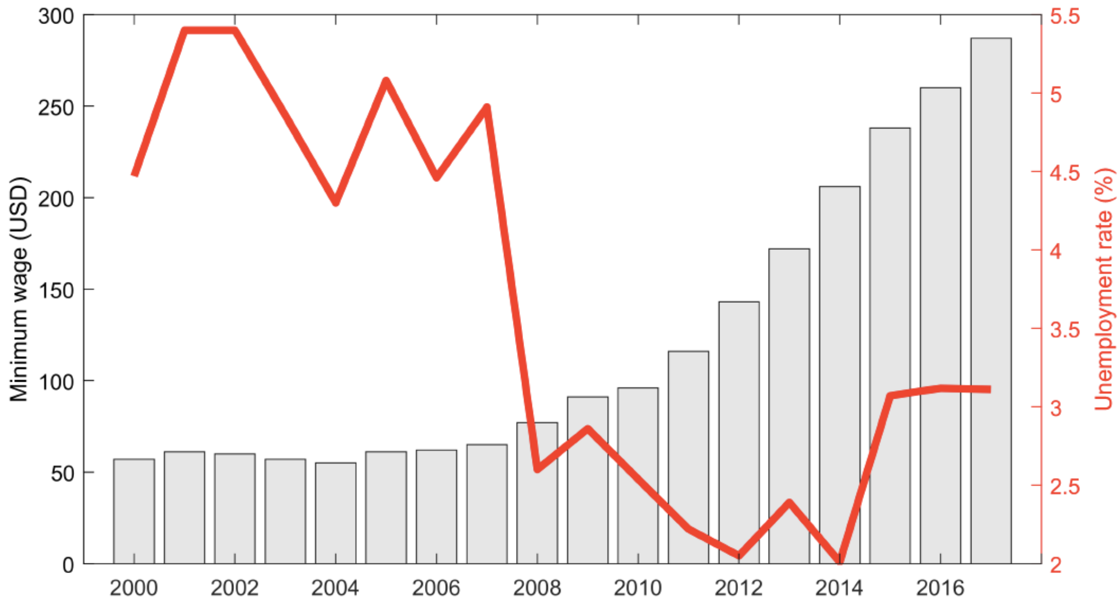

Table 1: Posterior inclusion probability of control covariates

Control	Wage curve equation	Selection equation
Education	0.649	0.506
Experience	0.511	–
Age	0.506	0.550
Gender	0.546	0.506
Fomal employment	0.523	0.537
Indigenous	0.511	0.506
Family members	–	0.506
Household-head	–	0.547