Definition and Analysis of Quasi-Stationary Intervals of Mobile Radio Channels

Invited Paper

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***Abstract*—This paper presents a non-wide-sense stationary uncorrelated scattering (non-WSSUS) model which takes accurately into account that the Doppler frequencies and propagation delays vary with time if the mobile station (MS) moves. Another feature of the proposed channel model is that the MS can change its velocity along the route. The result is that the Doppler spread and the delay spread deviate from their initial values with increasing time. This effect motivates the definition of quasi-stationary intervals. The lengths of these intervals will be analysed in terms of the velocity parameters. Our analysis shows that real-world channels can be quasi-stationary but not wide-sense stationary (WSS), neither in the time domain nor in the frequency domain. This implies that the generally accepted WSS and WSSUS assumptions are not fulfilled in practice, even not if the travelling distance is short.**

I. Introduction

Mobile radio channels are highly dynamic in time, frequency, and space due to the mobility of the mobile station (MS) [1], [2]. Bello's [3] assumption that mobile radio channels are wide-sense stationary (WSS) combined with the uncorrelated scattering (US) assumption have played a fundamental role in the area of channel modelling for almost half of a century. When the focus was shifted to mobile-to-mobile and vehicular communications, it turned out that the wide-sense stationary uncorrelated scattering (WSSUS) assumption is invalid in vehicular channels [4]–[6].

For the classification of channels in WSSUS and non-WSSUS channels, it is important to have a metric that allows to determine how long the WSSUS conditions are approximately fulfilled. This problem has been addressed in a number of papers. A first definition of stationarity in time and frequency based on the local scattering function and the local correlation function has been introduced in [7]. Statistical tests based on the evolutionary spectrum for determining the validity of the WSS assumption have been proposed in [8], [9]. Experimental contributions to the problem of identifying quasi-stationary regions in vehicle-to-vehicle channels can be found in [10]. An analysis of quasi-stationary regions in urban macrocells has been presented in [11].

In this paper, we define quasi-stationary intervals based on the time-variant Doppler spread and the time-variant delay spread. These characteristic quantities are not only of key importance for the higher-order statistics of the temporal and frequency fading behaviour of mobile radio channels but also for the system performance analysis. Our starting point is a non-stationary geometrical model from which we derive a new non-WSSUS model. The proposed non-WSSUS model includes time-variant angles of arrival (AOA) and captures the effects caused by a change of the speed and the angle of motion (AOM) of the MS. These phenomena result in time-variant Doppler frequencies and time-variant propagation delays for which exact and approximate expressions will be derived. Moreover, we will analyse the temporal behaviour of the spectral moments, including the trend of the mean Doppler shift, Doppler spread, mean delay, and delay spread. In non-WSSUS channels, the time-variant Doppler spread (delay spread) deviates from its initial values with increasing time or, equivalently, with increasing distance from the MS's starting point. This motivates the definition of quasi-stationary intervals based on the Doppler (delay) spread. The analysis of the quasi-stationary intervals in four different propagation scenarios shows that a change of the mobile speed has a greater influence than a change of the AOM if the scatterers are sufficiently far away from the MS. Furthermore, our analysis shows that the quasi-stationary intervals w.r.t. time are much shorter than the quasi-stationary intervals w.r.t. delay. One important result of our study is that non-WSSUS channels can accurately be modelled over large observation intervals by using a second-order Taylor expansion of the time-variant Doppler frequencies, while the time-variant propagation delays call for a third-order Taylor expansion.

The remainder of this paper is divided into six sec-

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tions. Section II describes the non-stationary multipath propagation scenario and the corresponding geometrical channel model. Section III presents the non-WSSUS model. Its spectral moments are analysed in Section IV. Section V introduces the definition of quasi-stationary intervals. The numerical results of the analysis of the quasi-stationary intervals are presented in Section VI. Finally, Section VII draws the conclusion.

II. The Multipath Propagation Scenario

Fig. 1 shows a typical multipath propagation scenario, where a fixed and unobstructed base station (BS) transmits electromagnetic waves. A part of the transmitted waves will be redirected towards the MS by N fixed scatterers S_n which are located around the MS at the positions (x_n, y_n) for $n = 1, 2, \ldots, N$. Both the BS and the MS are equipped with monopole antennas having omnidirectional radiation patterns. Furthermore, we assume non-line-of-sight propagation conditions and single-bounce scattering. At time $t = 0$, the MS is located at the origin of the xy-plane. With reference to Fig. 1, the distance from the BS to the origin $(0, 0)$ of the coordinate system is denoted by D, and r_n designates the distance from the nth scatterer S_n to the origin $(0, 0)$. The angle between the propagation direction of the nth incident wave and the x-axis at time $t = 0$ defines the initial AOA $\alpha_n = \alpha_n(0)$. Note that the distance $r_n(t)$ and the associated AOA $\alpha_n(t)$ vary with time t if the MS travels along a predefined route from the origin $(0, 0)$ to the point $(x(t), y(t))$. The route of the MS from $(0, 0)$ to $(x(t), y(t))$ is completely determined by the MS's velocity $\vec{\mathrm{v}}(t)$, which can be presented in the following form

$$\vec{\mathrm{v}}(t) = \mathrm{v}(t) e^{j\alpha_{\mathrm{v}}(t)} \tag{1}$$

where $\mathrm{v}(t)$ and $\alpha_{\mathrm{v}}(t)$ are called the speed and the AOM, respectively. For $t = 0$, we obtain from (1) the initial velocity $\vec{\mathrm{v}}_0 = \vec{\mathrm{v}}(0)$, the initial speed $\mathrm{v}_0 = \mathrm{v}(0)$, and the initial AOM $\alpha_{\mathrm{v}} = \alpha_{\mathrm{v}}(0)$. Of special interest will be the case where the speed $\mathrm{v}(t)$ and the AOM $\alpha_{\mathrm{v}}(t)$ vary linearly with t according to

$$\mathrm{v}(t) = \mathrm{v}_0 + a_0 t \tag{2a}$$

$$\alpha_{\mathrm{v}}(t) = \alpha_{\mathrm{v}} + b_0 t \tag{2b}$$

where a_0 denotes the acceleration or deceleration parameter, depending on whether $a_0 > 0$ or $a_0 < 0$, and b_0 is called the angular speed. At time t, the position $(x(t), y(t))$ of the MS is determined by

$$x(t) = \int_0^t \mathrm{v}(z) \cos(\alpha_{\mathrm{v}}(z))\, dz \tag{3a}$$

$$y(t) = \int_0^t \mathrm{v}(z) \sin(\alpha_{\mathrm{v}}(z))\, dz\,. \tag{3b}$$

From the location (x_n, y_n) of the nth scatterer S_n and the position $(x(t), y(t))$ of the MS, the time-variant AOAs $\alpha_n(t)$ can be obtained as (see Fig. 1)

$$\alpha_n(t) = \mathrm{atan2}(y_n - y(t), x_n - x(t)) \tag{4}$$

for all $n = 1, 2, \ldots, N$, where $\mathrm{atan2}(y, x)$ denotes the four-quadrant inverse tangent function.

In the following, we assume that the MS moves with given velocity $\vec{\mathrm{v}}(t)$ through a propagation area that is characterized by N fixed scatterers S_n located at known positions (x_n, y_n).

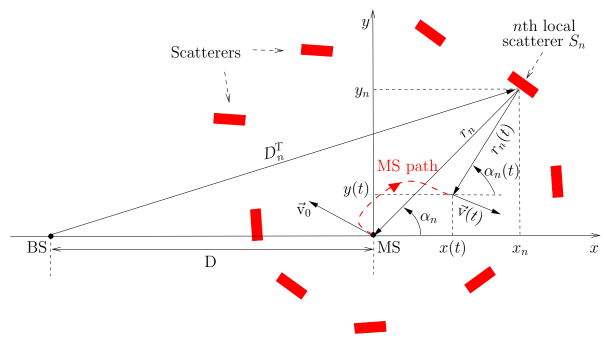

Fig. 1. Geometrical model for a non-stationary multipath propagation channel, in which the MS travels with a time-variant velocity $\vec{\mathrm{v}}(t)$ from the origin $(0, 0)$ along a predefined route (- - -) to the point $(x(t), y(t))$.

III. Non-WSSUS Channel Model

From the geometrical model in Fig. 1, one can derive the following time-variant impulse response of a non-WSSUS channel model (without proof)

$$h(\tau', t) = \sum_{n=1}^{N} c_n e^{j[2\pi \int_0^t f_n(t')dt' + \theta_n]} \delta(\tau' - \tau'_n(t)) \tag{5}$$

where the model parameters c_n, $f_n(t)$, θ_n, and $\tau'_n(t)$ are called the path gain, Doppler frequency, phase, and propagation delay of the nth path, respectively. The path gains c_n are supposed to be constant over the observation interval T_{obs}, and the phases θ_n are independent and identically distributed (i.i.d.) random variables, each of which is uniformly distributed over 0 to 2π, i.e., $\theta_n \sim \mathcal{U}(0, 2\pi]$. The Doppler frequencies $f_n(t)$ and propagation delays $\tau'_n(t)$ vary with time t according to

$$f_n(t) = f_{\max}(t) \cos(\alpha_n(t) - \alpha_{\mathrm{v}}(t)) \tag{6}$$

and

$$\tau'_n(t) = \frac{1}{c_0} \Big[\sqrt{D^2 + r_n^2 + 2Dr_n \cos(\alpha_n)} + \sqrt{(r_n \cos(\alpha_n) - x(t))^2 + (r_n \sin(\alpha_n) - y(t))^2} \Big] \tag{7}$$

respectively. In (6), $f_{\max}(t)$ represents the maximum Doppler frequency, which is given by $f_{\max}(t) =$

$\mathrm{v}(t) f_0/c_0$, where f_0 denotes the carrier frequency and c_0 designates the speed of light. Note that (6) and (7) hold for all t and for all routes that do not interfere with scattering objects. For short observation intervals T_{obs} or short routes, the Doppler frequencies $f_n(t)$ in (6) can be approximated by a first-order Taylor expansion around $t = 0$ according to

$$f_n(t) \approx f_n + k_n t \tag{8}$$

where

$$f_n = f_n(0) = f_{\max} \cos(\alpha_n - \alpha_{\mathrm{v}}) \tag{9}$$

$$\begin{aligned} k_n &= \dot{f}_n(0) \\ &= f_{\max} \left[\frac{a_0}{\mathrm{v}_0} \cos(\alpha_n - \alpha_{\mathrm{v}}) + (b_0 - \gamma_n) \sin(\alpha_n - \alpha_{\mathrm{v}}) \right] \end{aligned} \tag{10}$$

$$\gamma_n = \dot{\alpha}_n(0) = \frac{\mathrm{v}_0}{r_n} \sin(\alpha_n - \alpha_{\mathrm{v}}) \tag{11}$$

and $f_{\max} = f_{\max}(0) = f_0 \mathrm{v}_0/c_0$. The overdot in (10) and (11) denotes time derivative. The approximation in (8) reveals the impact of the velocity parameters $\mathrm{v}_0, a_0, a_{\mathrm{v}}$, and b_0 [see (2a,b)] on the temporal behaviour of the time-variant Doppler frequencies $f_n(t)$. Even if the MS moves with constant speed v_0 ($a_0 = 0$) along a straight line ($b_0 = 0$), it follows that in general $\gamma_n = \dot{\alpha}_n(0) \neq 0$, and thus $k_n \neq 0$. For the often assumed case of $\vec{\mathrm{v}}(t) = \text{const.}$, this means that the Doppler frequencies $f_n(t)$ vary with time due to the change of the AOA $\alpha_n(t)$. From the fact that time-variant Doppler frequencies $f_n(t)$ result in multipath channels which are non-WSS in the time domain, we can conclude that all real-world channels are non-WSSUS channels. In other words, the WSS assumption is not satisfied in real-world channels.

In a similar manner, we can approximate the propagation delays $\tau'_n(t)$ by a second-order Taylor expansion as

$$\tau'_n(t) \approx \tau'_n + \kappa'_n t + \frac{\eta'_n}{2} t^2 \tag{12}$$

where

$$\tau'_n = \tau'_n(0) = \frac{1}{c_0} \left[\sqrt{D^2 + r_n^2 + 2 D r_n \cos(\alpha_n)} + r_n \right] \tag{13}$$

$$\kappa'_n = \dot{\tau}'_n(0) = -\frac{f_n}{f_0} \tag{14}$$

and

$$\eta'_n = \ddot{\tau}_n(0) = -\frac{k_n}{f_0} \,. \tag{15}$$

A first look at the approximation in (12) shows that the first two terms of the time-variant propagation delays $\tau'_n(t)$ depend only on the velocity parameters v_0 and α_{v}, but not on a_0 and b_0. A second look at (12) reveals that only the third term captures the effects of the acceleration a_0 and the angular speed b_0, which explains the motivation for using a second-order Taylor expansion for $\tau'_n(t)$. This means that the propagation delays $\tau'_n(t)$ vary approximately in a linear fashion with time t, if the MS moves with constant speed v_0 in a given direction determined by the AOM α_{v}. As the propagation delays $\tau'_n(t)$ vary with time t, we can conclude that multipath channels also do not satisfy the WSS assumption in the frequency domain.

By substituting (8) and (12) in (5), we can approximate the time-variant impulse response $h(\tau', t)$ of non-WSSUS channels by

$$h(\tau', t) \approx \sum_{n=1}^{N} c_n e^{j\theta(t)} \delta(\tau' - \tau'_n - \kappa'_n t - \frac{\eta'_n}{2} t^2) \tag{16}$$

where $\theta(t) = 2\pi \left(f_n t + \frac{k_n}{2} t^2 \right) + \theta_n$. If the propagation delays $\tau'_n(t)$ are almost constant and equal to a common fixed delay τ'_0 within the observation interval, i.e., if $\tau'_n(t) \approx \tau'_n - \kappa'_n t \approx \tau'_0$ for $t \in [0, T_{\mathrm{obs}}]$ and $n = 1, 2, \ldots, N$, then the multipath channel described by (16) can be modeled by a non-stationary complex channel gain of the form

$$\mu(t) = \sum_{n=1}^{N} c_n e^{j\left[2\pi\left(f_n t + \frac{k_n}{2} t^2\right) + \theta_n\right]} \,. \tag{17}$$

The expression above is known as a *sum of chirps (SOC_h)* process [12] which captures here the effects of both velocity variations and AOA variations through the influence of the parameters a_0, b_0, and γ_n.

IV. Time-Variant Spectral Moments

A. Time-Variant Mean Doppler Shift and Doppler Spread

By integrating the nth component of the time-variant impulse response $h(\tau', t)$ in (5) over τ', we obtain the complex channel gain

$$\mu_n(t) = c_n e^{j\left[2\pi \int_0^t f_n(t')\, dt' + \theta_n\right]} \tag{18}$$

of the nth multipath component. The mean power σ_n^2 of this multipath component can be obtained as

$$\sigma_n^2 = E\left\{ |\mu_n(t)|^2 \right\} = c_n^2 \,. \tag{19}$$

Hence, from the assumption that the path gain c_n does not vary with time t over sufficiently short observation intervals T_{obs} or the position $(x(t), y(t))$ of the MS, it follows that the mean path power σ_n^2 of the nth received multipath component $\mu_n(t)$ equals the constant c_n^2. With this preliminary note in mind, we can refer to [13] and define the time-variant average Doppler shift $B_f^{(1)}(t)$ as the ratio of the sum of the power-weighted instantaneous Doppler shifts and the total received path power, i.e.,

$$B_f^{(1)}(t) = \frac{\sum_{n=1}^{N} c_n^2 f_n(t)}{\sum_{n=1}^{N} c_n^2} \,. \tag{20}$$

Analogously, the time-variant Doppler spread $B_f^{(2)}(t)$ of non-WSSUS channels with constant path gains and time-variant Doppler frequencies $f_n(t)$ can be defined as [13]

$$B_f^{(2)}(t) = \sqrt{\frac{\sum_{n=1}^{N} c_n^2 f_n^2(t)}{\sum_{n=1}^{N} c_n^2} - \left(B_f^{(1)}(t)\right)^2}. \quad (21)$$

Note that the evaluation of (20) and (21) by using the expression of $f_n(t)$ in (6), we can obtain the exact solution of $B_f^{(1)}(t)$ and $B_f^{(2)}(t)$, respectively. Otherwise, by using the first-order Taylor expansion in (8), we obtain the corresponding approximations of $B_f^{(1)}(t)$ and $B_f^{(2)}(t)$.

B. Time-Variant Mean Delay and Delay Spread

By invoking similar arguments as in the previous subsection, we can define the time-variant mean delay $B_{\tau'}^{(1)}(t)$ and the time-variant delay spread $B_{\tau'}^{(2)}(t)$ of non-WSSUS channels with constant gains c_n and time-variant propagation delays $\tau_n'(t)$ as

$$B_{\tau'}^{(1)}(t) = \frac{\sum_{n=1}^{N} c_n^2 \tau_n'(t)}{\sum_{n=1}^{N} c_n^2} \quad (22)$$

and

$$B_{\tau'}^{(2)}(t) = \sqrt{\frac{\sum_{n=1}^{N} c_n^2 (\tau_n'(t))^2}{\sum_{n=1}^{N} c_n^2} - \left(B_{\tau'}^{(1)}(t)\right)^2} \quad (23)$$

respectively. Notice that the exact solutions of $B_{\tau'}^{(1)}(t)$ and $B_{\tau'}^{(2)}(t)$ are obtained, if we evaluate (22) and (23), respectively, by using the expression for $\tau_n'(t)$ according to (7). On the other hand, the corresponding approximations of $B_{\tau'}^{(1)}(t)$ and $B_{\tau'}^{(2)}(t)$ can be obtained by using the second-order Taylor expansion of $\tau_n'(t)$ as provided in (12).

V. Quasi-Stationary Mobile Radio Channels

Stochastic processes are usually classified into strict-sense, WSS, and non-WSS processes, where the latter is often used as a synonym for non-stationary processes. In the following, we will introduce quasi-stationary processes, which can be grouped between WSS processes and non-stationary processes, as illustrated in Fig. 2.

A stochastic process is called WSS if its mean is constant and its autocorrelation function depends only on the time difference $\tau = t_1 - t_2$ [14]. If the autocorrelation function of a stochastic process is independent of time t and only a function of the time difference τ, then all kth-order moments are constant.

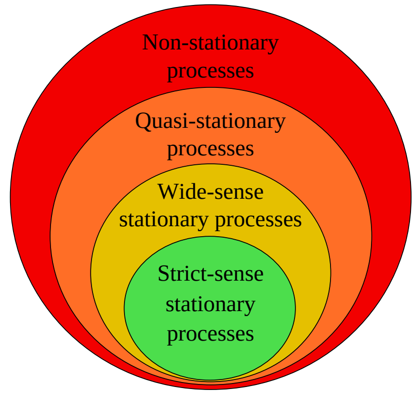

Fig. 2. Classification of stochastic processes.

A WSS channel must therefore have a constant mean Doppler shift and a constant Doppler spread. In Section IV, we have seen that the mean Doppler shift $B_f^{(1)}(t)$ and the Doppler spread $B_f^{(2)}(t)$ vary with time t, meaning that the considered channel model is non-WSS w.r.t. time t. A measure for the deviation of the time-variant Doppler spread $B_f^{(2)}(t)$ from the stationary case $B_f^{(2)}(t) = \text{const.}$, is given by the relative error $|B_f^{(2)}(t) - B_f^{(2)}(0)|/B_f^{(2)}(0)$. This motivates the following definition of quasi-stationary intervals T_q.

Definition 5.1: The shortest time interval T_q for which the absolute value of the relative error of the Doppler spread

$$\epsilon(T_q) = \frac{|B_f^{(2)}(T_q) - B_f^{(2)}(0)|}{B_f^{(2)}(0)} = \frac{q}{100\,\%} \quad (24)$$

equals q percent is called the *quasi-stationary interval w.r.t. time* t.

Quasi-stationary intervals T_q' w.r.t. delay τ' can be defined analogously.

Definition 5.2: The shortest time interval T_q' for which the absolute value of the relative error of the delay spread

$$\epsilon(T_q') = \frac{|B_{\tau'}^{(2)}(T_q') - B_f^{(2)}(0)|}{B_{\tau'}^{(2)}(0)} = \frac{q}{100\,\%} \quad (25)$$

equals q percent is called the *quasi-stationary interval w.r.t. delay* τ'.

Note that the values of T_q and T_q' are infinite if the channel model fulfills the WSSUS condition. A mobile radio channel is considered as quasi-stationary w.r.t. time over the finite interval $(0, T_q]$ up to $q = 10\,\%$. Analogously, the channel is said to be quasi-stationary w.r.t. delay τ' over the finite interval $(0, T_q']$ up to $q = 10\,\%$. Finally, we mention that the mobile radio channel behaves like a non-stationary channel if the observation duration T_{obs} exceeds T_q (T'_q).

VI. Numerical Results

In the following, we consider a multipath propagation scenario consisting of $N = 10$ scatterers. The extended method of exact Doppler spread (EMEDS) [15] has been applied to compute the path gains c_n and the initial AOA $\alpha_n = \alpha_n(0)$ according to

$$c_n = \sigma_0 \sqrt{\frac{2}{N}} \quad \text{and} \quad \alpha_n = \frac{2\pi}{N}\left(n - \frac{1}{4}\right) \tag{26}$$

where $\sigma_0 = 1$. The phases θ_n have been obtained from N realizations of a random generator having a uniform distribution over $(0, 2\pi]$. The positions (x_n, y_n) of the N fixed scatterers S_n have been computed by using $x_n = r_n \cos(\alpha_n)$ and $y_n = r_n \sin(\alpha_n)$, where α_n is given by (26) and r_n was set to 50 m for all $n = 1, 2, \ldots, N$. A carrier frequency f_0 of 5.9 GHz has been chosen, and the observation duration T_{obs} was set to 5 s. The velocity parameters $\text{v}_0, a_0, \alpha_{\text{v}}$, and b_0 are listed in Table I for four different non-stationary propagation scenarios. Scenario I considers the standard case that the MS moves with a constant speed v_0 along the x-axis. This scenario allows to study the isolated effect of time-variant AOAs $\alpha_n(t)$. Scenario II takes in addition into account the effect caused by an angular speed b_0 of $\pi/10$ rad/s. Finally, the Scenarios III and IV are aimed to obtain insight into the influence of the acceleration parameter a_0. For these scenarios, the initial speed $\text{v}_0 = \text{v}(0)$ was set to 3 km/h and the finishing speed $\text{v}(T_{\text{obs}})$ equals 16.5 km/h for Scenario III and 30 km/h for Scenario IV. This chosen parameter constellation results in an initial maximum Doppler frequency $f_{\max} = f_{\max}(0)$ of 16.4 Hz and finishing maximum Doppler frequencies $f_{\max}(T_{\text{obs}})$ of 90.14 Hz (Scenario III) and 164 Hz (Scenario IV).

TABLE I
List of velocity parameters for four non-stationary propagation scenarios.

	v_0 (km/h)	a_0 (m/s^2)	α_{v} (rad)	b_0 (rad/s)
Scenario I	3	0	0	0
Scenario II	3	0	0	$\pi/10$
Scenario III	3	0.75	0	$\pi/10$
Scenario IV	3	1.5	0	$\pi/10$

Fig. 3 shows the behaviour of the time-variant Doppler spread $B_f^{(2)}(t)$ over the observation interval $[0, T_{\text{obs}}]$. One objective of this figure is to visualise the deviations of $B_f^{(2)}(t)$ obtained by using the exact expression of $f_n(t)$ according to (6) and the approximate solution presented in (8). Obviously, the approximate solution is quite accurate over the observation interval $[0, T_{\text{obs}}]$. Another conclusion that can be drawn from the results in Fig. 3 is that the acceleration parameter a_0 has a much larger effect than the angular speed b_0.

Fig. 4 demonstrates the corresponding results for the time-variant delay spread $B_{\tau}'^{(2)}(t)$. The presented graphs have been obtained by evaluating $B_{\tau}'^{(2)}(t)$ in (23) in combination with the exact expression of $\tau'_n(t)$ according to (7) and the approximate solution presented in (12). Similar conclusions as drawn from Fig. 3 also pertain to this figure. In addition, we can conclude from a comparison of the results in Figs. 3 and 4 that the Doppler spread $B_f^{(2)}(t)$ reacts much more sensitive to the velocity parameters listed in Table I than the delay spread $B_{\tau}'^{(2)}(t)$. This must be seen in perspective that typical channel sounders have a Doppler resolution of about 1 Hz and a delay resolution in the order of 10 ns.

Figs. 5 and 6 present the obtained quasi-stationary intervals T_q and T'_q, respectively. The results have been obtained by solving (24) and (25) numerically for each of the four non-stationary propagation scenarios specified in Table I. A comparison of the graphs in Figs. 5 and 6 reveals clearly that the quasi-stationary intervals T_q are much shorter than T'_q.

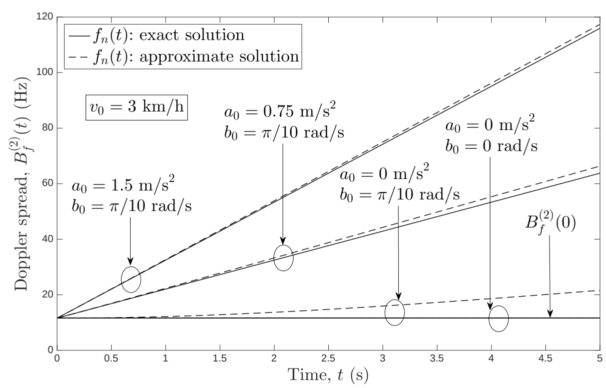

Fig. 3. Time-variant Doppler spread $B_f^{(2)}(t)$ of non-WSSUS channels for the four non-stationary propagation scenarios specified in Table I.

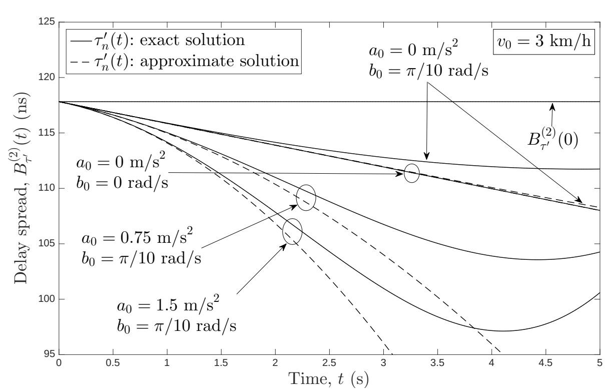

Fig. 4. Time-variant delay spread $B_{\tau'}^{(2)}(t)$ of non-WSSUS channels for the four non-stationary propagation scenarios specified in Table I.

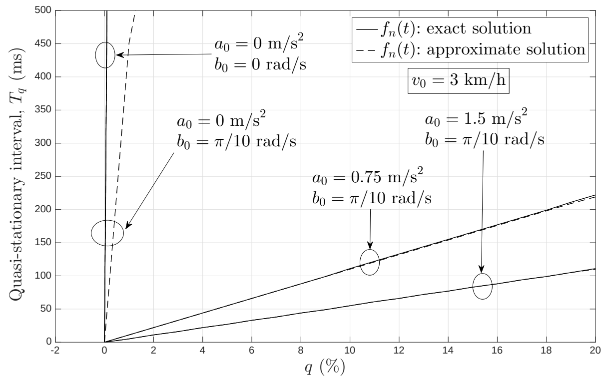

Fig. 5. Quasi-stationary intervals T_q of non-WSSUS channels for the four non-stationary propagation scenarios specified in Table I.

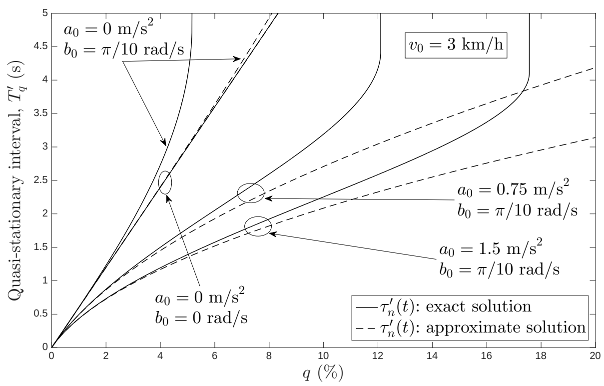

Fig. 6. Quasi-stationary intervals T'_q of non-WSSUS channels for the four non-stationary propagation scenarios specified in Table I.

VII. Conclusion

In this paper, we have presented a non-WSSUS model with time-variant Doppler frequencies and time-variant propagation delays. For the time-variant Doppler frequencies and the propagation delays, exact and approximate expressions have been derived. Moreover, we have analysed the temporal behaviour of the spectral moments with emphasis on the Doppler spread and the delay spread. The deviation of the time-variant Doppler and delay spreads with increasing values of time from their initial values motivated the definition of quasi-stationary intervals. Under the assumption of isotropic scattering and the considered velocity scenarios, our analysis of the quasi-stationary intervals has revealed that a change of the mobile speed has a greater effect than a change of the AOM. Another finding of our analysis was that the quasi-stationary intervals w.r.t. time are much shorter than the quasi-stationary intervals w.r.t. delay. From our study, we can finally conclude that non-WSSUS channels can accurately be modelled over large observation intervals by using a second-order Taylor expansion of the time-variant Doppler frequencies and a third-order expansion of the time-variant propagation delays. In this case, the resulting non-WSSUS model can be interpreted as an extension of the sum-of-chirps model w.r.t. frequency selectivity. The analysis of the time-variant correlation and spectral properties of such non-stationary processes is a topic for future research.

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