Modelling of Non-WSSUS Channels with Time-Variant Doppler and Delay Characteristics

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***Abstract*—This paper deals with the modelling of non-wide-sense stationary uncorrelated scattering (non-WSSUS) channels in which the angles of arrival (AOAs), Doppler frequencies, and propagation delays vary with time. Starting from a geometrical model in which the mobile station (MS) travels along a predefined path with time-variant velocity, it is shown how the parameters of the non-WSSUS model can be computed analytically assuming that the scatterers are fixed. One of the key results of our analysis is that the time-variant Doppler frequencies and the time-variant propagation delays of WSSUS and non-WSSUS channels are connected by a fundamental relationship. Furthermore, the time-variant channel transfer function of the non-WSSUS channel model is derived. In addition, general expressions are presented for the correlation functions in the time and frequency domains. Moreover, it is shown that the proposed non-WSSUS channel model is consistent w.r.t. the mean Doppler shift, Doppler spread, mean propagation delay, and delay spread. The proposed concept is of fundamental importance for the design of physically sound wideband channel models under non-stationary propagation conditions.**

I. Introduction

The mobile radio channel is a highly dispersive transmission medium that induces severe distortions on the signals propagating through it [1]–[3]. One kind of distortion is caused by the dispersion that the received signal experiences in the frequency domain due to the Doppler effect arising from the motion of the transmitter and/or receiver. Another kind of distortion is caused by the dispersion that the received signal experiences in the time domain due to the different propagation delays of the received multipath components.

Significant efforts have been devoted since the early days of the mobile communication systems to characterize the Doppler frequencies and propagation delays of multipath fading channels. Initially, the research work was undertaken assuming that the Doppler frequencies and propagation delays were time-invariant quantities, leading to the development of statistical channel models that fulfill the wide-sense stationary uncorrelated scattering (WSSUS) condition [4]. Such models were proven suitable for the analysis of first, second, and third generation mobile cellular communication systems. However, the emergence of novel highly mobile wireless communication systems, such as railway [5] and vehicular [6], [7] communication systems, calls for the development of new channel models having time-varying parameters, which does not comply with the WSSUS assumption [8]. Moreover, the assumption of time-invariant Doppler frequencies implies that the angles of arrival (AOAs) and angles of departure (AODs) of the received multipath components are themselves time-invariant quantities. This condition can be justified if the multipath components are modeled as electromagnetic plane waves [9]. Nevertheless, for some relevant communication systems (such as those having a very short communication range [10]), a spherical wave propagation model is preferred to account for the temporal variability of the AOAs and AODs of the multipath components.

Recently, several statistical models for non-WSSUS mobile radio channels have been proposed which account for the time-varying nature of the Doppler frequencies, propagation delays, and AOAs/AODs (see, e.g., [11], [12]). However, further research work is needed to thoroughly characterize the temporal dynamics of the aforementioned parameters, and to properly understand the interrelations among them.

To close these gaps, we present in this paper a novel framework for the modelling of non-WSSUS channels in which the AOAs, Doppler frequencies, and propagation delays vary with time. By assuming that the scatterers are fixed and that the MS travels along a predefined path with time-variant velocity, we propose a novel geometrical model for non-WSSUS channels from which general analytical expressions of the time-varying Doppler frequencies and propagation delays can be derived. One of the key results of our analysis is that the time-varying Doppler frequencies and propagation delays are connected by a fundamental relationship. General expressions are also presented for the time-frequency-dependent correlation function (CF), the time-dependent autocorrelation function (ACF), and the time-dependent frequency correlation function (FCF) of the proposed non-WSSUS channel model. In addition, the consistency of the model w.r.t. the mean Doppler shift, Doppler spread, mean propagation delay, and delay spread is demonstrated. The modeling framework presented here is of fundamental importance for the design of physically sound non-stationary wideband channel models.

The organisation of the paper is as follows. Section II describes the non-stationary propagation scenario, which forms the basic for the derivation of the proposed non-WSSUS channel model. Section III delineates how the AOAs, Doppler frequencies, and propagation delays can be modelled under non-stationary propagation conditions. Section IV establishes the relationship between Doppler frequencies and propagation delays. Section V presents the time-variant impulse response and the corresponding transfer function of the non-WSSUS channel model. The most important statistical properties of the proposed channel model are analysed in Section VI. Finally, Section VII draws the conclusion.

II. The Non-Stationary Propagation Scenario

In this paper, we consider the non-stationary multipath propagation scenario depicted in Fig. 1. In this scenario, a base station (BS) with fixed position is supposed to be the transmitter, while the MS acts as the receiver, which moves with time-variant velocity $\vec{v}(t)$ along a predefined route. It is assumed that the BS and MS are equipped with single omnidirectional antennas and that the line-of-sight component is blocked. The BS antenna is supposed to be unobstructed by objects, whereas the MS antenna is surrounded by N fixed scatterers S_n located at (x_n, y_n) for $n = 1, 2, \ldots, N$. Furthermore, we assume single-bounce scattering. The origin of the coordinate system coincides with the position of the MS at time $t = 0$. With reference to Fig. 1, the initial distance from the scatterer S_n to the MS at $t = 0$ is denoted by r_n. The associated initial AOA α_n is defined as the angle between the propagation direction of the nth incident wave and the x-axis at $t = 0$. If the MS moves from the origin $(0, 0)$ to the point $(x(t), y(t))$, then the distance between the nth scatterer S_n and the position of the MS changes, which implies that both the distance $r_n(t)$ and the AOA $\alpha_n(t)$ are varying over time.

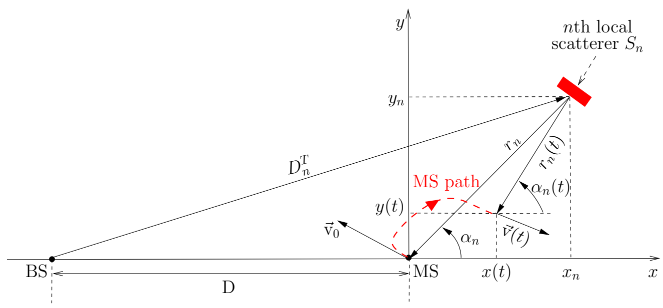

Fig. 1. A non-stationary multipath propagation scenario in which the MS travels along a predefined route (- - -) with time-variant velocity $\vec{v}(t)$.

In the area of channel modelling, one can classify multipath propagation scenarios into random and deterministic scenarios. In random propagation scenarios, the scatterers S_n are randomly distributed and/or the MS moves with random velocity. For randomly distributed scatterers S_n, the initial parameters r_n and α_n are random variables, and their corresponding time-variant quantities $r_n(t)$ and $\alpha_n(t)$ are stochastic processes for all $n = 1, 2, \ldots, N$. This contrasts with deterministic propagation scenarios, in which the scatterers S_n are fixed at known positions (x_n, y_n), and the MS moves with known velocity $\vec{v}(t)$. In this case, the initial parameters r_n and α_n are constants, and the corresponding time-variant quantities $r_n(t)$ and $\alpha_n(t)$ are deterministic processes for all $n = 1, 2, \ldots, N$. Unless mentioned otherwise, we will focus in the following on deterministic propagation scenarios with fixed scatterers S_n located at known positions (x_n, y_n).

III. Modelling the Time-Variant Channel Parameters

A. Modelling the Time-Variant AOAs

We consider a non-stationary multipath propagation scenario in which the MS travels with velocity

$$\vec{v}(t) = \mathrm{v}(t) e^{j\alpha_{\mathrm{v}}(t)} \tag{1}$$

where the magnitude $\mathrm{v}(t) = |\vec{v}(t)|$ is called the speed, and the phase $\alpha_{\mathrm{v}}(t)$ is referred to as the angle of motion (AOM). By setting the time t to zero, we obtain the initial velocity $\vec{v}_0 = \mathrm{v}_0 \exp\{j\alpha_{\mathrm{v}}\}$, where $\mathrm{v}_0 = \mathrm{v}(0)$ denotes the initial speed, and $\alpha_{\mathrm{v}} = \alpha_{\mathrm{v}}(0)$ is the initial AOM. Notice that a change of the velocity $\vec{v}(t)$ can be caused by either an increase (or decrease) of the speed $\mathrm{v}(t)$, a change of the AOM $\alpha_{\mathrm{v}}(t)$, or a change in both speed $\mathrm{v}(t)$ and AOM $\alpha_{\mathrm{v}}(t)$. If the speed $\mathrm{v}(t)$ and the AOM $\alpha_{\mathrm{v}}(t)$ are known, then the position $(x(t), y(t))$ of the MS at time t can be computed by means of

$$x(t) = \int_0^t \mathrm{v}(z) \cos(\alpha_{\mathrm{v}}(z))\, dz \tag{2}$$

and

$$y(t) = \int_0^t \mathrm{v}(z) \sin(\alpha_{\mathrm{v}}(z))\, dz\,. \tag{3}$$

Recall that the time-variant AOA $\alpha_n(t)$ is defined as the angle between the propagation direction of the nth incident wave and the x-axis. If the location (x_n, y_n) of the nth scatterer S_n is known, then the corresponding time-variant AOA $\alpha_n(t)$, seen at the position $(x(t), y(t))$ of the MS at time t, can be obtained from

$$\alpha_n(t) = \mathrm{atan2}(y_n - y(t), x_n - x(t)) \tag{4}$$

for $n = 1, 2, \ldots, N$, where the function $\mathrm{atan2}(y, x)$ designates the four-quadrant inverse tangent function, which returns the angle of the vector (x, y) with the positive x-axis in the range from $-\pi$ to $+\pi$.

B. Modelling the Time-Variant Doppler Frequencies

In the general case of time-variant velocities $\vec{v}(t)$ and time-variant AOAs $\alpha_n(t)$, it is obvious that the Doppler frequency $f_n(t)$ of the nth path also varies with time according to

$$f_n(t) = f_{\max}(t) \cos(\alpha_n(t) - \alpha_{\mathrm{v}}(t)) \tag{5}$$

where $f_{\max}(t)$ denotes the maximum Doppler shift. Regarding its computation, we have to distinguish between the following three cases: single-carrier transmission, multi-carrier transmission, and wideband transmission of signals.

1) Single-Carrier Transmission: Let $x(t)$ be an unmodulated carrier signal of the form $x(t) = A\exp\{j2\pi f_0 t\}$, where A denotes a complex-valued constant, and f_0 is the carrier frequency. If we transmit such a signal over a non-stationary multipath fading channel with time-variant speed $\mathrm{v}(t)$, then the maximum Doppler frequency $f_{\max}(t)$ in (5) is given by

$$f_{\max}(t) = \frac{\mathrm{v}(t)}{c_0} f_0 \tag{6}$$

where c_0 designates the speed of light.

2) Multi-Carrier Transmission: Multi-Carrier transmission systems, such as orthogonal frequency-division multiplexing, have $K > 1$ sub-carrier frequencies $f_0^{(k)} (k = 1, 2, \ldots, K)$ that are spaced apart by Δf according to $f_0^{(k)} = f_0 + \Delta f[k - (K+1)/2]$. The maximum Doppler frequency $f_{\max}^{(k)}(t)$, which the kth sub-carrier frequency experiences, can be computed as

$$f_{\max}^{(k)}(t) = \frac{\mathrm{v}(t)}{c_0}\left[f_0 + \Delta f\left(k - \frac{K+1}{2}\right)\right] \tag{7}$$

for $k = 1, 2, \ldots, K$. This means that the sub-carrier frequencies $f_0^{(k)}$ of multi-carrier systems experience different (maximum) Doppler shifts.

3) Wideband Transmission: Let $x(t)$ be a complex baseband signal with bandwidth B such that the spectrum $X(f')$ of $x(t)$ is confined to the frequency range $[-B/2, B/2]$. If we transmit the signal $x(t)$ at the carrier frequency f_0 over a non-stationary multipath fading channel with time-variant speed $\mathrm{v}(t)$, then the spectral component of $x(t)$ at $f' \in [-B/2, B/2]$ experiences a maximum Doppler shift of

$$f_{\max}(f', t) = \frac{\mathrm{v}(t)}{c_0}\,(f_0 + f')\ . \tag{8}$$

This result shows that the maximum Doppler frequency depends generally on both frequency f' and time t. The frequency dependence of $f_{\max}(f', t)$ can be neglected for narrowband signals characterised by $B/f_0 \ll 1$. However, for wideband signals, where the factor B/f_0 cannot be neglected, the f'-dependence of the maximum Doppler frequency should be an integral part of the designed channel model.

The path gain c_n of the nth propagation path is supposed to be a constant real-valued parameter. If the path gains c_n and the Doppler frequencies $f_n(f', t)$ are known, then the instantaneous mean Doppler shift $B_f^{(1)}(f', t)$ and the instantaneous Doppler spread $B_f^{(2)}(f', t)$ can be computed as follows:

$$B_f^{(1)}(f', t) = \frac{\sum_{n=1}^{N} c_n^2 f_n(f', t)}{\sum_{n=1}^{N} c_n^2} \tag{9}$$

$$B_f^{(2)}(f', t) = \sqrt{\frac{\sum_{n=1}^{N} c_n^2 f_n^2(f', t)}{\sum_{n=1}^{N} c_n^2} - \left(B_f^{(1)}(f', t)\right)^2}\ . \tag{10}$$

The two preceding expressions reveal that the characteristic quantities $B_f^{(1)}(f', t)$ and $B_f^{(2)}(f', t)$ are in general functions of both frequency f' and time t.

C. Modelling the Time-Variant Propagation Delays

With reference to Fig. 1 and by using the speed-time-distance relationship, the time-variant propagation delays $\tau_n'(t)$ of the nth path can be expressed as

$$\begin{aligned}\tau_n'(t) =& \frac{D_n^T + r_n(t)}{c_0}\\ =& \frac{1}{c_0}\Big[\sqrt{D^2 + r_n^2 + 2Dr_n\cos(\alpha_n)}\\ &+ \sqrt{(r_n\cos(\alpha_n) - x(t))^2 + (r_n\sin(\alpha_n) - y(t))^2}\Big]\end{aligned} \tag{11}$$

for $n = 1, 2, \ldots, N$. Note that (11) represents the exact result, which has been obtained without imposing any boundary conditions such as $D \gg r_n(t)$. Note also that the time-variant propagation delay $\tau_n'(t)$ in (11) captures the effect caused by an MS that moves along a given route with velocity $\vec{\mathrm{v}}(t)$.

In analogy to (9) and (10), the instantaneous mean delay $B_{\tau'}^{(1)}(t)$ and the instantaneous delay spread $B_{\tau'}^{(2)}(t)$ can be obtained by

$$B_{\tau'}^{(1)}(t) = \frac{\sum_{n=1}^{N} c_n^2 \tau_n'(t)}{\sum_{n=1}^{N} c_n^2} \tag{12}$$

and

$$B_{\tau'}^{(2)}(t) = \sqrt{\frac{\sum_{n=1}^{N} c_n^2 (\tau_n'(t))^2}{\sum_{n=1}^{N} c_n^2} - \left(B_{\tau'}^{(1)}(t)\right)^2} \tag{13}$$

respectively.

IV. Relationship Between Doppler Frequencies and Propagation Delays

A comparison of the Doppler frequencies $f_n(f', t)$ in (5) and the propagation delays $\tau_n'(t)$ in (11) show that these two quantities are seemingly independent of each other, but this is not true. In fact, it is shown in the Appendix that there exists a fundamental relationship between $f_n(f', t)$ and $\tau_n'(t)$, which can be expressed as

$$f_n(f', t) = -(f_0 + f')\,\dot{\tau}_n'(t) \tag{14}$$

where the overdot denotes the differentiation with respect to time t. This result indicates that the Doppler frequencies are negatively proportional to the slope of the propagation delays. We can also say that the faster the reduction (extension) of the propagation delays is, the larger are the positive (negative) Doppler shifts. A practical implication of (14) is that the estimation of the Doppler frequencies can be reduced to the estimation of the trend of the propagation delays.

Another fundamental relationship is obtained by substituting (14) in (9) and (10), which leads to

$$B_f^{(i)}(f',t) = -(1)^i (f_0 + f') B_{\tau'}^{(i)}(t) \tag{15}$$

for $i = 1, 2$. In other words, the instantaneous mean Doppler shift $B_f^{(1)}(f',t)$ and the instantaneous Doppler spread $B_f^{(2)}(f',t)$ are determined by the mean and spread of the slope $\dot{\tau}_n'(t)$ of the propagation delays $\tau_n'(t)$. Notice that $B_{\dot{\tau}'}^{(1)}(t) = \dot{B}_{\tau'}^{(1)}(t)$, and thus $B_f^{(1)}(f',t) = -(f_0 + f')\dot{B}_{\tau'}^{(1)}(t)$, i.e., the mean Doppler shift is determined by the temporal derivative of the mean propagation delay.

In the following, we will briefly discuss some special cases. In the first case, we assume that the distance between BS and MS is small compared to the distance from the scatterer S_n to the origin, i.e., $D \ll r_n$. Furthermore, we assume that the MS travels only a short distance along a straight path. Under these assumptions, the propagation delays $\tau_n'(t)$ in (11) can be approximated as

$$\tau_n'(t) \approx \tau_n'(0) - \frac{\mathrm{v}_0 t}{c_0} \cos(\alpha_n - \alpha_\mathrm{v}) \tag{16}$$

where $\tau_n'(0) = \{D + r_n[1 + \cos(\alpha_n)]\}c_0^{-1}$. Substituting (16) in (14) gives

$$f_n(f') \approx (f_0 + f') \frac{\mathrm{v}_0}{c_0} \cos(\alpha_n - \alpha_\mathrm{v}) \,. \tag{17}$$

Hence, if $D \ll r_n$ and the MS travels only a short distance, it turns out that the Doppler frequency is independent of time. Finally, for the cases of single carrier transmission and narrowband transmission ($f' \ll f_0$), the result in (17) reduces further to the well-known standard expression of the Doppler shift

$$\begin{aligned} f_n &\approx f_0 \frac{\mathrm{v}_0}{c_0} \cos(\alpha_n - \alpha_\mathrm{v}) \\ &= f_{\max} \cos(\alpha_n - \alpha_\mathrm{v}) \end{aligned} \tag{18}$$

which can be found in many textbooks [1], [3], [13].

V. The Non-WSSUS Channel Model

It has been shown in [3, Eq. (3.4)] that the time-variant impulse response $h(\tau',t)$ of a non-stationary multipath channel can be written in the following general form

$$h(\tau,t) = \sum_{n=1}^{N(t)} c_n(t) e^{j[\theta_n(t) - 2\pi f_0 \tau_n'(t)]} \delta(\tau' - \tau_n'(t)) \,. \tag{19}$$

In the equation above, $N(t)$ denotes the number of propagation paths at time t, $c_n(t)$ represents the time-variant path gain, and $\theta_n(t)$ is the corresponding phase shift, which is caused by the interaction of the transmitted signal and the nth scatterer S_n. Next, we simplify the channel model described by (19) by restricting its validity to sufficiently short observation time intervals T_0, i.e., $t \in [0, T_0]$. Then, we can consider the number of propagation paths $N(t)$, the path gains $c_n(t)$, and the phases $\theta_n(t)$ as independent of time t, i.e. $N(t) = N$, $c_n(t) = c_n$, and $\theta_n(t) = \theta_n$. Hence, the time-variant impulse response $h(\tau',t)$ in (19) can be written as

$$h(\tau,t) = \sum_{n=1}^{N} c_n e^{j[\theta_n - 2\pi f_0 \tau_n'(t)]} \delta(\tau' - \tau_n'(t)) \,. \tag{20}$$

In the proposed channel model, the number of propagation paths N is equal to the number of scatterers; the path gains c_n are constant; the phases θ_n are modelled as independent and identically distributed (i.i.d.) random variables, each has a uniform distribution over the interval $(0, 2\pi]$, i.e., $\theta_n \sim \mathcal{U}(0, 2\pi]$; and the time-variant propagation delays $\tau_n'(t)$ are given by (11).

By taking the Fourier transform of $h(\tau',t)$ w.r.t. τ', we obtain the time-variant channel transfer function

$$H(f',t) = \sum_{n=1}^{N} c_n e^{j[\theta_n - 2\pi(f_0 + f')\tau_n'(t)]} \,. \tag{21}$$

From the phase $\phi_n(f',t) = \theta_n - 2\pi(f_0 + f')\tau_n'(t)$ of $H(f',t)$, we can compute the instantaneous Doppler frequency $f_n(f',t)$ by invoking the phase-frequency relationship [14, Eq. (1.3.40)]

$$f_n(f',t) = \frac{1}{2\pi} \frac{d\phi_n(f',t)}{dt} \tag{22}$$

This results in the instantaneous Doppler frequency in the form of $f_n(f',t) = -(f_0 + f')\dot{\tau}_n'(t)$, which is already known to us from (14).

VI. Analysis of the Non-WSSUS Channel Model

A. Time-Frequency-Dependent CF

The time-frequency-dependent CF $\mathcal{R}_H(\upsilon',\tau;f',t)$ of the time-variant channel transfer function $H(f',t)$ is defined as

$$\begin{aligned} \mathcal{R}_H(\upsilon',\tau;f',t) = E\Big\{ & H\left(f' + \frac{\upsilon'}{2}, t + \frac{\tau}{2}\right) \\ & \cdot H^*\left(f' - \frac{\upsilon'}{2}, t - \frac{\tau}{2}\right)\Big\} \end{aligned} \tag{23}$$

where $E\{\cdot\}$ and $(\cdot)^*$ represent the expectation operator and the complex conjugate operator, respectively. By substituting (21) in (23) and using (14), it can be shown that the time-frequency-dependent CF $\mathcal{R}_H(\upsilon',\tau;f',t)$ can be brought into the form

$$\begin{aligned} \mathcal{R}_H(\upsilon',\tau;f',t) = & \sum_{n=1}^{N} c_n^2 e^{j2\pi \int_{t-\tau/2}^{t+\tau/2} f_n(f',x)dx} \\ & \cdot e^{-j\pi\upsilon'\left[\tau_n'\left(t+\frac{\tau}{2}\right) + \tau_n'\left(t-\frac{\tau}{2}\right)\right]} \,. \end{aligned} \tag{24}$$

B. Time-Dependent ACF

The time-dependent ACF $\mathcal{R}_H(\tau,t)$ of $H(f',t)$ is obtained from $\mathcal{R}_H(\upsilon',\tau;f',t)$ by setting $\upsilon' = 0$, i.e.,

$$\begin{aligned} \mathcal{R}_H(\tau,t) &= \mathcal{R}_H(0,\tau;f',t) \\ &= \sum_{n=1}^{N} c_n^2 e^{j2\pi \int_{t-\tau/2}^{t+\tau/2} f_n(f',x)dx} \,. \end{aligned} \tag{25}$$

For the special case that $f_n(f',t)$ varies linear with time t, i.e., $f_n(t) = f_n + k_n t$, where k_n is a real-valued constant, we obtain [15]

$$\mathcal{R}_H(\tau,t) = \sum_{n=1}^{N} c_n^2 e^{j2\pi f_n(t)\tau} . \quad (26)$$

Furthermore, if $f_n(f',t)$ is independent of frequency f' and time t, i.e., $f_n(f',t) = f_n$, then (25) reduces to the expression

$$\mathcal{R}_H(\tau) = \mathcal{R}_H(\tau,t) = \sum_{n=1}^{N} c_n^2 e^{j2\pi f_n \tau} \quad (27)$$

which describes the ACF of sum-of-cisoids channel models [16]. Finally, if $\alpha_n \sim \mathcal{U}(0,2\pi]$, then the expression above reduces to the ACF $\mathcal{R}_H(\tau) = 2\sigma_0^2 J_0(2\pi f_{\max}\tau)$, where $2\sigma_0^2 = \sum_{n=1}^{N} c_n^2$ and $J_0(\cdot)$ designates the zeroth-order Bessel function of the first kind [17, Eq. (9.1.18)]. Thus, the temporal characteristics of the proposed non-WSSUS channel model include those of the classical Jakes/Clarke model [1], [18] as a special case.

C. Time-Dependent FCF

The time-dependent FCF $\mathcal{R}_H(\upsilon',t)$ of $H(f',t)$ is obtained from $\mathcal{R}_H(\upsilon',\tau;f',t)$ by setting $\tau = 0$, i.e.,

$$\mathcal{R}_H(\upsilon',t) = \mathcal{R}_H(\upsilon',0;f',t) = \sum_{n=1}^{N} c_n^2 e^{-j2\pi\tau_n'(t)\upsilon'} . \quad (28)$$

D. Consistency

From the time-dependent ACF $\mathcal{R}_H(\tau,t)$ in (25), we can obtain the instantaneous mean Doppler shift

$$B_{H_f}^{(1)}(t) = \frac{1}{2\pi j} \frac{\dot{\mathcal{R}}_H(\tau,t)}{\mathcal{R}_H(\tau,t)}\bigg|_{\tau=0} \quad (29)$$

and the instantaneous Doppler spread

$$B_{H_f}^{(2)}(t) = \frac{1}{2\pi}\sqrt{\left(\frac{\dot{\mathcal{R}}_H(\tau,t)}{\mathcal{R}_H(\tau,t)}\right)^2 - \frac{\ddot{\mathcal{R}}_H(\tau,t)}{\mathcal{R}_H(\tau,t)}}\Bigg|_{\tau=0} \quad (30)$$

where $\dot{\mathcal{R}}_H(\tau,t)$ ($\ddot{\mathcal{R}}_H(\tau,t)$) denotes the first (second) order derivative of $\mathcal{R}_H(\tau,t)$ w.r.t. the time-lag variable τ.

Analogously, from the time-dependent FCF $\mathcal{R}_H(\upsilon',t)$ in (28), we can compute the instantaneous mean delay

$$B_{H_{\tau'}}^{(1)}(t) = \frac{1}{2\pi j} \frac{\dot{\mathcal{R}}_H(\upsilon',t)}{\mathcal{R}_H(\upsilon',t)}\bigg|_{\upsilon'=0} \quad (31)$$

and the instantaneous delay spread

$$B_{H_{\tau'}}^{(2)}(t) = \frac{1}{2\pi}\sqrt{\left(\frac{\dot{\mathcal{R}}_H(\upsilon',t)}{\mathcal{R}_H(\upsilon',t)}\right)^2 - \frac{\ddot{\mathcal{R}}_H(\upsilon',t)}{\mathcal{R}_H(\upsilon',t)}}\Bigg|_{\upsilon'=0} \quad (32)$$

where $\dot{\mathcal{R}}_H(\upsilon',t)$ ($\ddot{\mathcal{R}}_H(\upsilon',t)$) denotes the first (second) order derivative of $\mathcal{R}_H(\upsilon',t)$ w.r.t. the frequency-lag variable υ'.

After substituting (25) in (29) and (30), it can be shown (without proof) that $B_{H_f}^{(i)}(t) = B_f^{(i)}(t)$ holds for $i = 1,2$. Similarly, it can be shown after substituting (28) in (31) and (32) that $B_{H_{\tau'}}^{(i)}(t) = B_{\tau'}^{(i)}(t)$ holds for $i = 1,2$. This means that the proposed non-WSSUS model is consistent w.r.t. the mean Doppler shift (mean propagation delay) and the Doppler spread (delay spread).

VII. Conclusion

In this paper, we have presented a procedure for the modelling of non-WSSUS channels with time-variant model parameters (AOAs, Doppler frequencies, and propagation delays). Our starting point was a generic geometrical model for non-stationary multipath propagation scenarios comprising a fixed transmitter (BS), M stationary scatterers, and a mobile receiver (MS) that moves along a predefined route with time-variant velocity. From the geometrical model, we have derived general expressions for the AOAs, Doppler frequencies, and propagation delays. Our analysis has revealed the existence of a fundamental relationship between the Doppler frequencies and the propagation delays. More precisely, it turned out that the Doppler frequencies are negatively proportional to the slope of the propagation delays. Moreover, we have analysed the temporal and frequency correlation properties of the time-variant transfer function of the derived non-WSSUS model. Finally, we have shown that the model is consistent w.r.t. the mean Doppler shift, Doppler spread, mean delay, and delay spread. The main conclusion of the paper is that the Doppler characteristics are completely determined by the delay characteristics. The secondary conclusion is that the proposed modelling approach leads to a physically sound and consistent non-WSSUS model.

Appendix

Proof of Equation (14). By using $x_n = r_n\cos(\alpha_n)$ and $y_n = r_n\sin(\alpha_n)$, the time-variant propagation delay $\tau_n'(t)$ in (11) can be written as

$$\tau_n'(t) = \frac{1}{c_0}\Big[\sqrt{D^2 + r_n^2 + 2Dx_n} + \sqrt{(x_n - x(t))^2 + (y_n - y(t))^2}\Big] . \quad (A.1)$$

The derivative of $\tau_n'(t)$ w.r.t. time t results in

$$\dot{\tau}_n'(t) = -\frac{(x_n - x(t))\dot{x}(t) + (y_n - y(t))\dot{y}(t)}{c_0\sqrt{(x_n - x(t))^2 + (y_n - y(t))^2}} . \quad (A.2)$$

From (2) and (3), it follows $\dot{x}(t) = \mathrm{v}(t)\cos(\alpha_{\mathrm{v}}(t))$ and $\dot{y}(t) = \mathrm{v}(t)\sin(\alpha_{\mathrm{v}}(t))$, respectively, which allows us to express (A.2) as

$$\dot{\tau}_n'(t) = -\frac{\mathrm{v}(t)}{c_0} \cdot \frac{(x_n - x(t))\cos(\alpha_{\mathrm{v}}(t)) + (y_n - y(t))\sin(\alpha_{\mathrm{v}}(t))}{\sqrt{(x_n - x(t))^2 + (y_n - y(t))^2}} . \quad (A.3)$$

From Fig. 1, we know that the relation

$$\tan(\alpha_n(t)) = \frac{y_n - y(t)}{x_n - x(t)} = \frac{\sin(\alpha_n(t))}{\cos(\alpha_n(t))} \quad (A.4)$$

holds. By means of (A.4) and by using [17, Eqs. (4.3.10) and (4.3.17)], we can express $\dot{\tau}_n(t)$ as

$$\dot{\tau}_n(t) = -\frac{\mathrm{v}(t)}{c_0}\cos(\alpha_n(t) - \alpha_{\mathrm{v}}(t))\,. \tag{A.5}$$

Finally, after inserting (8) in (5) and using (A.5), we obtain

$$f_n(f', t) = -(f_0 + f')\,\dot{\tau}'_n(t)\,. \tag{A.6}$$

□

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