Distributed Adaptive Consensus Tracking Control of Uncertain High-order Nonlinear Systems under Directed Graph Condition

Jiang Long, Wei Wang, Jing Zhou and Hao Wu

Abstract— **In this paper, we investigate the output consensus tracking problem for a class of high-order nonlinear systems subjected to unknown parameters and uncertain external disturbances. A novel backstepping based distributed adaptive control scheme is presented under the directed communication status. For the subsystems without direct access to time-varying desired trajectory, local estimators are introduced and the corresponding adaptive laws are designed in a totally distributed fashion. With the presented scheme, the assumption on linearly parameterized reference signal and the information exchange operation of subsystem inputs in the existing results are no longer needed. It is shown that all the closed-loop signals are globally uniformly bounded and desired output consensus tracking can be achieved.**

I. INTRODUCTION

Multi-agent systems have received significant attention over the past decades due to its wide application in various areas such as mobile robot networks, intelligent transportation managements, wireless sensor networks and power networks [1]–[6]. Compared with traditional single system, distributed consensus control aims at achieving an agreement on the states or outputs of the subsystems, by designing a local controller for each subsystem using information only collected within its neighboring area. Leader-following consensus control is one of the most typical distributed consensus problems, which has been studied extensively [4]–[9], [13], [16]. In most available results, the desired reference trajectory is generated by a prescribed agent with the similar dynamics to the followers and zero/known inputs. For more general cases, the common desired trajectory can be represented by a time-varying function, such issues are termed consensus tracking control [7]–[9]. It is worth noting that the main challenge in consensus tracking control is that only a small portion of subsystems in the networks can access the full knowledge of desired trajectory directly.

Some distributed control schemes are presented to achieve adaptive output consensus tracking control; see for examples [5], [6], [9]–[11], [13], [16], [17]. In [5], [10] and [6] partial information of reference trajectories is assumed to be known by all of the subsystems. Based on this, distributed observers are designed to account for the remaining uncertainties for the subsystems without direct access to the desired trajectory. In [11], local controllers are designed based on their neighbors' control inputs and states. Thus, satisfactory tracking performance can be achieved though the reference trajectory is totally unknown by partial subsystems. However, the mutual dependence of control inputs without a prescribed priority will bring new problems during implementation [13]. In [9], based on an assumption that the reference signal is linearly parameterized with basis functions known by all the subsystems, asymptotic output consensus tracking of uncertain nonlinear subsystems is achieved under directed graph condition. Moreover, in the design of virtual control input at the first step, a stabilizing term is included with a global graph parameter adopted.

To address the aforementioned issues, some effective solutions are presented in [16], [17]. In [16], for those subsystems without direct access to the desired trajectory $y_r(t)$, the only available information of y_r is that $\dot{y}_r$ is upper bounded by an unknown positive constant. By introducing compensating terms in a smooth function form of consensus errors and certain positive integrable functions in each step of virtual control design, asymptotic output consensus tracking is achieved. However, the proposed scheme is only applicable to the case with undirected communication topology. The results are extended to the case with balanced and weakly connected digraph in [17], whereas the considered system is limited to second-order Euler Lagrange system.

In this paper, we shall revisit the distributed adaptive control problem for nonlinear systems with unknown parameters and uncertain external disturbances. Compared with the existing results, the main features of the presented scheme can be summarized as follows. i) Backstepping technique is adopted, hence all the subsystems share the same but arbitrarily high relative degree. ii) The topology condition is a digraph containing a directed spanning tree. iii) The linear parametrization assumption on the desired trajectory is removed and extra information exchange of subsystem inputs or parameter estimates can be avoided. iv) For the subsystems without full information of the reference trajectory, local estimators are designed to estimate the time-varying reference in a distributed fashion. It is shown that all closed-loop signals are globally uniformly bounded and desired output consensus tracking performance of all subsystem outputs can be achieved with adjustable tracking errors.

The remaining part of this paper is organized as follows. In Section II, the considered MAS model and related graph theory are introduced and the control objective is stated. The distributed controller design and closed-loop system stability analysis are provided in Section III and IV, respectively. Simulation results are given in Section V followed by a

J. Long and W. Wang are with School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China. jlong@buaa.edu.cn, w.wang@buaa.edu.cn

J. Zhou is with The Department of Engineering Sciences, University of Agder, Grimstad, Norway 4898, Norway. jing.zhou@uia.no

H. Wu is with The Laboratory, CNCERT/CC, Beijing, 100094, China. whman@isc.org.cn

conclusion drawn in Section VI.

II. Problem formulation

A. System Model

In this paper, a group of N nonlinear subsystems modeled as follows [12] are considered.

$$y_i^{(n)}(t) - \sum_{l=1}^{p_i} \theta_{il}\varphi_{il}(y_i, \dot{y}_i, \ldots, y_i^{(n-1)}) = b_i u_i + d_i(t) \qquad (1)$$

where $y_i \in R$ and $u_i \in R$ are the output and control input of subsystem i for $i = 1, \ldots, N$, respectively. $b_i \in R$, $\theta_{il} \in R$ are unknown constant parameters and b_i is nonzero. $\varphi_{il} : R^n \to R$ is a known smooth nonlinear function. $d_i(t) \in R$ denotes uncertain external disturbance.

By defining the state variables as $x_{i,q} = y_i^{(q-1)}$, $q = 1, \ldots, n$, system (1) can be described by the following state space representation

$$\begin{aligned} \dot{x}_{i,q} &= x_{i,q+1}, \quad q = 1, 2, \ldots, n-1; \\ \dot{x}_{i,n} &= b_i u_i + \varphi_i^{\mathrm{T}}\theta_i + d_i(t) \\ y_i &= x_{i,1} \end{aligned} \qquad (2)$$

where $\varphi_i = [\varphi_{i1}, \varphi_{i2}, \ldots, \varphi_{ip_i}]^T$ and $\theta_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{ip_i}]^T$.

B. Information Transmission Condition among the N Subsystems

Suppose that the information transmission condition among the N linked subsystems can be described by a fixed directed graph $\mathcal{G} \triangleq (\mathcal{V}, \varepsilon)$, where $\mathcal{V} = \{1, \ldots, N\}$ denotes the set of indexes corresponding to the N linked subsystems and $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges between two distinct subsystems. The agent j can access the information from agent i if $(i, j) \in \varepsilon$, but not necessarily vice versa [14]. In this case, the agent i is called the neighbor of agent j and we use $\mathcal{N}_j$ to denote the set of neighbors of agent j, that is, $\mathcal{N}_j \triangleq \{j \in \mathcal{V} : (i, j) \in \varepsilon\}$. Note that the self edges (i, i) are not allowed, thus $(i, i) \notin \varepsilon$ and $i \notin \mathcal{N}_i$. The adjacency matrix $A = [a_{ij}] \in R^{N\times N}$ with nonnegative elements is defined such that $a_{ij} = 1$ if $(j, i) \in \varepsilon$, otherwise $a_{ij} = 0$. It is clear that the diagonal elements are all 0 due to $(i, i) \notin \varepsilon$. We introduce an in-degree matrix defined as $\triangle = diag(\triangle_i) \in R^{N\times N}$ with the diagonal elements $\triangle_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the ith row sum of A. The Laplacian matrix of A is defined as $L = \triangle - A$. In a directed graph, a directed path from node i to j is defined if there exists a sequence of successive edges $\{(i, k), (k, m), (m, l), (l, j)\} \subseteq \varepsilon$. A directed graph contains a directed spanning tree if there exists at least a root node i such that all the remaining nodes in the graph can be reached from i through a directed path. In this paper, we use $\mu_i = 1$ to denote the case that the subsystem i can directly access the full information of reference trajectory, otherwise $\mu_i = 0$.

C. Control Objective

The control objective of this paper is to design a local adaptive controller u_i in a distributed fashion for each subsystem i under a directed graph condition such that:

- all the closed-loop signals are globally uniformly bounded;
- all subsystem outputs can track a desired time-varying trajectory $y_r(t)$ as closely as possible, though $y_r(t)$ is exactly known by only a small fraction of the subsystems.

In order to achieve the control objective, some necessary assumptions are imposed.

Assumption 1: The directed graph $\mathcal{G}$ contains a spanning tree and the the full knowledge of $y_r(t)$ is directly available to at least one root node i, that is, $\mu_i = 1$.

Assumption 2: The first nth-order derivatives of $y_r(t)$ are bounded and piecewise continuous. For subsystem i with $\mu_i = 0$, the only known trajectory information is that $|\dot{y}_r(t)| \leq F$ with F being an unknown positive constant.

Assumption 3: The sign of b_i is known in each subsystem i.

Assumption 4: $|d_i(t)| \leq D_i$, where D_i is an unknown positive constant.

Remark 1: Compared with currently available results in [5], [6], [9] and [20], the assumption that $y_r(t)$ is linearly parameterized and basis functions known by all subsystems is relaxed. Although an effective distributed adaptive consensus solution is provided in [16] with similar relaxed condition, only undirected topology is considered.

Before we proceed with designing distributed adaptive controllers, the following lemmas are also introduced, which will take important roles in controller design and stability analysis.

Lemma 1: [15] Based on Assumption 1, the matrix $L + B$ is nonsingular where $B = diag\{\mu_1, \ldots, \mu_N\}$. Define $\bar{q} = [\bar{q}_1, \ldots, \bar{q}_N]^T = (L + B)^{-1}[1, \ldots, 1]^T$

$$P = diag\{P_1, \ldots, P_N\} = diag\{\frac{1}{\bar{q}_1}, \ldots, \frac{1}{\bar{q}_N}\}$$

$$Q = P(L + B) + (L + B)^T P$$

then $\bar{q}_i > 0$ for $i = 1, \ldots, N$ and P and Q are positive definite.

Lemma 2: [18] The following inequality holds

$$0 \leq |z| - z \cdot sg(z) \leq \eta$$

for any scalars $z \in R$, $\eta > 0$ and $sg(z) = \frac{z}{\sqrt{z^2+\eta^2}}$.

III. Design of distributed adaptive controllers

To generate distributed adaptive control laws, backstepping technique [19] is adopted and the detailed design procedure will be provided recursively.

Introduce the change of coordinates as

$$\begin{aligned} e_i =& y_i - \mu_i y_r - (1 - \mu_i)\hat{y}_{r,i} \\ =& y_i - y_r + (1 - \mu_i)(y_r - \hat{y}_{r,i}) \\ =& \delta_i + (1 - \mu_i)\tilde{y}_{r,i} \end{aligned} \qquad (3)$$

$$z_{i,1} = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + \mu_i(y_i - y_r) \qquad (4)$$

$$z_{i,k} = x_{i,k} - \alpha_{i,k-1}, k = 2, \ldots, n \qquad (5)$$

where $\delta_i = y_i - y_r$ denotes the actual tracking error. $\hat{y}_{r,i}$ is an estimate introduced to account for unknown $y_r(t)$ in each subsystem i with $\mu_i = 0$. $\tilde{y}_{r,i} := y_r - \hat{y}_{r,i}$ is the corresponding estimation error. $z_{i,1}$ is often known as the local neighborhood consensus error. $\alpha_{i,k-1}$ is the virtual control to be determined in each recursive step.

To illustrate the adaptive backstepping design procedures, only the first step is elaborated in detail.

• Step 1: From (3) and (5), the derivative of e_i is computed as

$$\dot{e}_i = z_{i,2} + \alpha_{i,1} - \mu_i \dot{y}_r - (1-\mu_i)\dot{\hat{y}}_{r,i} \tag{6}$$

The virtual control $\alpha_{i,1}$ is chosen as

$$\alpha_{i,1} = -c_1 e_i - k\hat{P}_i z_{i,1} + \mu_i \dot{y}_r + (1-\mu_i)\dot{\hat{y}}_{r,i} \tag{7}$$

where c_1 and k are positive design parameters. $\hat{P}_i$ is the local estimate of P_i, which is defined in Lemma 1. Substituting (7) into (6) gives

$$\dot{e}_i = z_{i,2} - c_1 e_i - k\hat{P}_i z_{i,1} \tag{8}$$

The trajectory and parameter update laws are designed as

$$\dot{\hat{y}}_{r,i} = -\gamma_{y_{r,i}}[z_{i,1} + \kappa_{y_{r,i}}(\hat{y}_{r,i} - y_{r,i0})] \tag{9}$$

and

$$\dot{\hat{P}}_i = -\gamma_{P_i}[e_i z_{i,1} - \kappa_{P_i}(\hat{P}_i - P_{i0})] \tag{10}$$

where $\gamma_{y_{r,i}}$, γ_{P_i}, $\kappa_{y_{r,i}}$, κ_{P_i}, $y_{r,i0}$,P_{i0} are all positive design parameters.

The Lyapunov function candidate is chosen as

$$V_1 = \frac{1}{2}\sum_{i=1}^{N} {e_i}^2 + \frac{1}{2}\sum_{i=1}^{N}\frac{k}{\gamma_{P_i}}\tilde{P}_i^2 + \frac{1}{2}\sum_{i=1}^{N}\frac{kP_i(1-\mu_i)}{\gamma_{y_{r,i}}}\tilde{y}_{r,i}^2 \tag{11}$$

where $\tilde{P}_i = P_i - \hat{P}_i$ and $\tilde{y}_{r,i} = y_{r,i} - \hat{y}_{r,i}$. From (8), the derivative of V_1 is computed as

$$\begin{aligned}\dot{V}_1 =& \sum_{i=1}^{N} e_i z_{i,2} - c_1\sum_{i=1}^{N} {e_i}^2 - k\delta^T PH\delta + \sum_{i=1}^{N}\frac{k}{\gamma_{P_i}}\tilde{P}_i(\gamma_{P_i} e_i z_{i,1} - \dot{\hat{P}}_i)\\ &+ \sum_{i=1}^{N}\frac{kP_i(1-\mu_i)}{\gamma_{y_{r,i}}}\tilde{y}_{r,i}(\dot{y}_r - \dot{\hat{y}}_{r,i} - \gamma_{y_{r,i}} z_{i,1})\end{aligned} \tag{12}$$

where $\delta = [\delta_1, \ldots, \delta_N]$. From (9), (10) and Lemma 1, (12) can be further derived as

$$\begin{aligned}\dot{V}_1 =& \sum_{i=1}^{N} e_i z_{i,2} - c_1\sum_{i=1}^{N} {e_i}^2 - k\delta^T PH\delta + \sum_{i=1}^{N} k\kappa_{P_i}\tilde{P}_i(\hat{P}_i - P_{i0})\\ &+ \sum_{i=1}^{N}\frac{kP_i(1-\mu_i)}{\gamma_{y_{r,i}}}\tilde{y}_{r,i}\dot{y}_r + \sum_{i=1}^{N} kP_i\kappa_{y_{r,i}}(1-\mu_i)\tilde{y}_{r,i}(\hat{y}_{r,i} - y_{r,i0})\\ \le& \sum_{i=1}^{N} e_i z_{i,2} - c_1\sum_{i=1}^{N} {e_i}^2 - \frac{k}{2}\lambda_{\min}(Q)\|\delta\|^2\\ &+ \sum_{i=1}^{N}\frac{kP_i(1-\mu_i)}{\gamma_{y_{r,i}}}\left|\tilde{y}_{r,i}\right| F + \sum_{i=1}^{N} k\kappa_{P_i}\tilde{P}_i(\hat{P}_i - P_{i0})\\ &+ \sum_{i=1}^{N} kP_i\kappa_{y_{r,i}}(1-\mu_i)\tilde{y}_{r,i}(\hat{y}_{r,i} - y_{r,i0})\\ \le& \sum_{i=1}^{N} e_i z_{i,2} - c_1\sum_{i=1}^{N} {e_i}^2 - \frac{k}{2}\lambda_{\min}(Q)\|\delta\|^2\\ &+ \sum_{i=1}^{N}\frac{kP_i(1-\mu_i)}{\gamma_{y_{r,i}}}[\frac{1}{4}\gamma_{y_{r,i}}\kappa_{y_{r,i}}\tilde{y}_{r,i}^2 + \frac{F^2}{\gamma_{y_{r,i}}\kappa_{y_{r,i}}}]\\ &+ \sum_{i=1}^{N} k\kappa_{P_i}\tilde{P}_i(\hat{P}_i - P_{i0}) + \sum_{i=1}^{N} kP_i\kappa_{y_{r,i}}(1-\mu_i)\tilde{y}_{r,i}(\hat{y}_{r,i} - y_{r,i0})\\ \le& \sum_{i=1}^{N} e_i z_{i,2} - c_1\sum_{i=1}^{N} {e_i}^2 - \frac{k}{2}\lambda_{\min}(Q)\|\delta\|^2 - \sum_{i=1}^{N}\frac{k\kappa_{P_i}}{2}\tilde{P}_i^2\\ &- \sum_{i=1}^{N}\frac{kP_i\kappa_{y_{r,i}}(1-\mu_i)}{4}\tilde{y}_{r,i}^2 + \sum_{i=1}^{N}\frac{kP_i(1-\mu_i)F^2}{{\gamma_{y_{r,i}}}^2\kappa_{y_{r,i}}}\\ &+ \sum_{i=1}^{N}\frac{k\kappa_{P_i}}{2}(P_i - P_{i0})^2 + \sum_{i=1}^{N}\frac{kP_i\kappa_{y_{r,i}}(1-\mu_i)}{2}(y_r - y_{r,i0})^2\end{aligned} \tag{13}$$

where the property that $\tilde{\Theta}^T(\hat{\Theta} - \Theta_0) = \tilde{\Theta}^T(-\tilde{\Theta} + \Theta - \Theta_0) \le -\frac{1}{2}\left\|\tilde{\Theta}\right\|^2 + \frac{1}{2}\|\Theta - \Theta_0\|^2$ has been applied.

Remark 2: In this step, we introduce the term $-k\hat{P}_i z_{i,1}$ in designing the first virtual control signal for the purpose of generating negative definite quadratic term of δ in the derivative of Lyapunov function defined later for the entire closed-loop system. Note that similar technique was initiated in [9], where P_i is adopted directly. However, P_i can only be obtained with full knowledge of the overall graph. Motivated by this fact, a modification is made here hence the presented adaptive consensus control laws are ensured to be totally distributed.

• Step $k (k = 2, \ldots, n-1)$: The virtual control $\alpha_{i,k}$ is chosen as

$$\alpha_{i,k} = -z_{i,k-1} - c_k z_{i,k} + \dot{\alpha}_{i,k-1} \tag{14}$$

with

$$\begin{aligned}\dot{\alpha}_{i,k-1} =& \sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial x_{i,l}}x_{i,l+1} + \frac{\partial\alpha_{i,k-1}}{\partial\hat{P}_i}\dot{\hat{P}}_i + \sum_{j=1}^{N}\sum_{l=1}^{k-1} a_{ij}\frac{\partial\alpha_{i,k-1}}{\partial x_{j,l}}x_{j,l+1}\\ &+ \mu_i\sum_{l=1}^{k}\frac{\partial\alpha_{i,k-1}}{\partial {y_r}^{(l-1)}}{y_r}^{(l)} + (1-\mu_i)\frac{\partial\alpha_{i,k-1}}{\partial\hat{y}_{r,i}}\dot{\hat{y}}_{r,i}\end{aligned} \tag{15}$$

where c_k is a positive constant. Define the Lyapunov function as

$$V_k = V_1 + \frac{1}{2}\sum_{i=1}^{N}\sum_{l=2}^{k} {z_{i,l}}^2 \tag{16}$$

whose derivative can be computed as

$$\begin{aligned}\dot{V}_k \le& \sum_{i=1}^{N} z_{i,k}z_{i,k+1} - c_1\sum_{i=1}^{N} {e_i}^2 - \frac{k}{2}\lambda_{\min}(Q)\|\delta\|^2 - \sum_{i=1}^{N}\sum_{l=2}^{k} c_l {z_{i,l}}^2\\ &- \sum_{i=1}^{N}\frac{k\kappa_{P_i}}{2}\tilde{P}_i^2 - \sum_{i=1}^{N}\frac{kP_i\kappa_{y_{r,i}}(1-\mu_i)}{4}\tilde{y}_{r,i}^2\\ &+ \sum_{i=1}^{N}\frac{kP_i(1-\mu_i)F^2}{{\gamma_{y_{r,i}}}^2\kappa_{y_{r,i}}} + \sum_{i=1}^{N}\frac{k\kappa_{P_i}}{2}(P_i - P_{i0})^2\\ &+ \sum_{i=1}^{N}\frac{kP_i\kappa_{y_{r,i}}(1-\mu_i)}{2}(y_r - y_{r,i0})^2\end{aligned} \tag{17}$$

• Step n: From (2) and (5), there is

$$\dot{z}_{i,n} = b_i u_i + {\varphi_i}^T\theta_i + d_i(t) - \dot{\alpha}_{i,n-1} \tag{18}$$

Design the control input as

$$u_i = \hat{\varrho}_i[\alpha_{i,n} - \varphi_i{}^T\hat{\theta}_i - sg(z_{i,n})\hat{D}_i] \tag{19}$$

where $\hat{\varrho}_i$, $\hat{\theta}_i$ and $\hat{D}_i$ are the parameter estimates of $\frac{1}{b_i}$, θ_i and the upper bound D_i of external disturbance, respectively. $\alpha_{i,n}$ is defined in (14) for $k = n$.

The parameter update laws are designed as

$$\begin{aligned}\dot{\hat{\varrho}}_i =& \gamma_{\varrho_i} z_{i,n}\mathrm{sgn}(b_i)[\varphi_i{}^T\hat{\theta}_i + sg(z_{i,n})\hat{D}_i - \alpha_{i,n}] \\ & - \gamma_{\varrho_i}\kappa_{\varrho_i}(\hat{\varrho}_i - \varrho_{i,0}) \end{aligned}\tag{20}$$

$$\dot{\hat{\theta}}_i = z_{i,n}\Gamma_i\varphi_i - \Gamma_i\kappa_{\theta_i}(\hat{\theta}_i - \theta_{i,0}) \tag{21}$$

$$\dot{\hat{D}}_i = \gamma_{D_i} z_{i,n} sg(z_{i,n}) - \gamma_{D_i}\kappa_{D_i}(\hat{D}_i - D_{i,0}) \tag{22}$$

where γ_{κ_i}, γ_{D_i}, κ_{ϱ_i}, κ_{D_i}, Γ_i, κ_{θ_i}, $\kappa_{i,0}$, $\theta_{i,0}$ and $D_{i,0}$ are all positive design parameters with appropriate dimension.

IV. Stability analysis

The main results of this paper are now formally stated in the following theorem.

Theorem 3: Consider the closed-loop system consisting of N uncertain nonlinear subsystems (2) satisfying Assumptions 1-4. With the local controllers (19) and parameter update laws (9), (10), (20)-(22), the following conclusions can be drawn.

- All the signals in the closed-loop system are globally uniformly bounded;
- The tracking errors for all subsystems will converge to a compact set and the upper bound of overall tracking errors in the mean square sense is adjustable.

Proof: Define a Lyapunov function for the entire system as

$$\begin{aligned} V_n =& V_{n-1} + \frac{1}{2}\sum_{i=1}^{N} z_{i,n}{}^2 + \sum_{i=1}^{N}\frac{|b_i|}{2\gamma_{\kappa_i}}\tilde{\varrho}_i^2 + \frac{1}{2}\sum_{i=1}^{N}\tilde{\theta}_i^T\Gamma_i{}^{-1}\tilde{\theta}_i \\ & + \sum_{i=1}^{N}\frac{1}{2\gamma_{D_i}}\tilde{D}_i^2 \end{aligned}\tag{23}$$

where $\tilde{\varrho}_i = \varrho_i - \hat{\varrho}_i$, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{D}_i = D_i - \hat{D}_i$.

Differentiating V_n with respect to time t yields

$$\begin{aligned} \dot{V}_n \leq& \dot{V}_{n-1} + \sum_{i=1}^{N} z_{i,n}(\alpha_{i,n} - \dot{\alpha}_{i,n-1}) + \sum_{i=1}^{N}\tilde{\theta}_i^T\Gamma_i{}^{-1}(z_{i,n}\Gamma_i\varphi_i - \dot{\hat{\theta}}_i) \\ & + \sum_{i=1}^{N}\frac{|b_i|}{\gamma_{\varrho_i}}\tilde{\varrho}_i(\gamma_{\varrho_i}z_{i,n}\mathrm{sgn}(b_i)(\varphi_i{}^T\hat{\theta}_i + \mathrm{sg}(z_{i,n})\hat{D}_i - \alpha_{i,n}) - \dot{\hat{\varrho}}_i) \\ & + \sum_{i=1}^{N}\frac{1}{\gamma_{D_i}}\tilde{D}_i(\gamma_{D_i}z_{i,n}\mathrm{sg}(z_{i,n}) - \dot{\hat{D}}_i) + \sum_{i=1}^{N}\eta_i D_i \end{aligned}\tag{24}$$

where the Lemma 2 has been utilized to handle the effects of external disturbances.

From (14), (15), (17) and (20)-(22), we have

$$\begin{aligned} \dot{V}_n \leq& -c_1\sum_{i=1}^{N} e_i{}^2 - \frac{k}{2}\lambda_{\min}(Q)\|\delta\|^2 - \sum_{i=1}^{N}\sum_{l=2}^{k} c_l z_{i,l}{}^2 - \sum_{i=1}^{N}\frac{k\kappa_{P_i}}{2}\tilde{P}_i^2 \\ & - \sum_{i=1}^{N}\frac{kP_i\kappa_{y_{r,i}}(1-\mu_i)}{4}\tilde{y}_{r,i}^2 - \sum_{i=1}^{N}\frac{|b_i|\,\kappa_{\varrho_i}}{2}\tilde{\varrho}_i^2 - \sum_{i=1}^{N}\frac{\kappa_{\theta_i}}{2}\left\|\tilde{\theta}_i\right\|^2 \\ & - \sum_{i=1}^{N}\frac{\kappa_{D_i}}{2}\tilde{D}_i^2 + M^* \\ \leq& -\frac{k}{2}\lambda_{\min}(Q)\|\delta\|^2 - \sigma V + M^* \end{aligned}\tag{25}$$

where

$$\sigma = \min\{2c_i, \gamma_{P_i}\kappa_{P_i}, \frac{1}{2}\gamma_{y_{r,i}}\kappa_{y_{r,i}}, \gamma_{\varrho_i}\kappa_{\varrho_i}, \frac{\kappa_{\theta_i}}{\lambda_{\max}(\Gamma_i{}^{-1})}, \gamma_{D_i}\kappa_{D_i}\} \tag{26}$$

and

$$\begin{aligned} M^* =& \sum_{i=1}^{N}\frac{kP_i(1-\mu_i)F^2}{\gamma_{y_{r,i}}{}^2\kappa_{y_{r,i}}} + \sum_{i=1}^{N}\frac{k\kappa_{P_i}}{2}(P_i - P_{i0})^2 \\ & + \sum_{i=1}^{N}\frac{kP_i\kappa_{y_{r,i}}(1-\mu_i)}{2}(y_r - y_{r,i0})^2 + \sum_{i=1}^{N}\frac{|b_i|\,\kappa_{\varrho_i}}{2}(\varrho_i - \varrho_{i0})^2 \\ & + \sum_{i=1}^{N}\frac{\kappa_{\theta_i}}{2}\left\|\theta_i - \theta_{i,0}\right\|^2 + \sum_{i=1}^{N}\frac{\kappa_{D_i}}{2}(D_i - D_{i0})^2 + \sum_{i=1}^{N}\eta_i D_i \end{aligned}\tag{27}$$

By neglecting the first term in the right hand side of inequality (25) and then directly integrating it, we obtain

$$V_n(t) \leq V_n(0)e^{-\sigma t} + \frac{M_1^*}{\sigma}(1 - e^{-\sigma t}) \leq V_n(0) + \frac{M_1^*}{\sigma} \tag{28}$$

which implies that V_n is globally uniformly bounded. Thus, the signals e_i, $z_{i,k}$ for $k = 2,\ldots,n$, $\hat{y}_{r,i}$, $\hat{P}_i$, $\hat{\varrho}_i$, $\hat{\theta}_i$ and $\hat{D}_i$ are all bounded. From (3) and (4), $x_{i,1}$ and $z_{i,1}$ are bounded. From (7), it implies $\alpha_{i,1}$ is bounded. Similarly, from (5), (14) and (15) for $k = 2$, $x_{i,2}$ and $\alpha_{i,2}$ are bounded. Through recursive analysis along this line, it concludes that $x_{i,k}$ and $\alpha_{i,k}$ for $k = 2,\ldots,n$ are bounded. Finally, from (19), the distributed control inputs u_i are bounded. Therefore, all the signals in the closed-loop system are bounded.

From (3) and (23), we have

$$\|\delta\|^2 \leq \sum_{i=1}^{N} 2(e_i{}^2 + (1-\mu_i)^2\tilde{y}_{r,i}^2) \leq \zeta V_n(t) \tag{29}$$

where $\zeta = \max\left\{4, \frac{4(1-\mu_i)\gamma_{y_{r,i}}}{kP_i}\right\}$ for $i = 1,\ldots,N$. Further, combining (28) yields

$$\|\delta\|^2 \leq \zeta[V_n(0)e^{-\sigma t} + \frac{M_1^*}{\sigma}(1 - e^{-\sigma t})] \tag{30}$$

which shows that the tracking errors in Euclidean norm will converge to a compact set $E_r = \{\delta | \|\delta\|^2 \leq \zeta(M_1^* + \tau)/\sigma\}$ with convergence time $t \geq (1/\sigma)ln(\left|V_n(0)\sigma - M_1^*\right|/\tau)$, in which τ is an arbitrary small positive constant.

In terms of (25), we have

$$\dot{V}_n \leq -\frac{k}{2}\lambda_{\min}(Q)\|\delta\|^2 + M^* \tag{31}$$

Integrating both sides of the foregoing equation yields

$$\begin{aligned} \|\delta\|^2{}_{[0,T]} =& \frac{1}{T}\int_0^T \|\delta\|^2 dt \\ \leq& \frac{2}{k\lambda_{\min}(Q)}\left[\frac{V_n(0) - V_n(t)}{T} + M_2{}^*\right] \end{aligned}$$

$$\leq \frac{2}{k\lambda_{\min}(Q)}\left[\frac{V_n(0)}{T}+M_2{}^{*}\right] \tag{32}$$

where $M_2{}^{*}$ is a positive constant. From (25) and (27), it follows that the upper bound of the overall tracking errors in the mean square sense can be decreased by decreasing $\kappa_{P_i}, \kappa_{y_{r,i}}, \kappa_{\varrho_i}, \kappa_{\theta_i}, \kappa_{D_i}$ and increasing $k, \gamma_{P_i}, \gamma_{y_{r,i}}, \gamma_{\kappa_i}, \Gamma_i, \gamma_{D_i}$. ■

Remark 3: From (9), (10), (20)-(22) and (19), it can be seen that information exchange of local control inputs or parameter estimates in [11], [13], [17] is not needed in this paper to implement the designed distributed adaptive controllers.

V. Simulation Results

To verify the effectiveness of our proposed distributed adaptive control scheme, we consider a group of 4 second order subsystems modeled as follows,

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2};\\ \dot{x}_{i,2} &= b_i u_i + \varphi_i{}^{\mathrm{T}}\theta_i + d_i(t), i = 1,\ldots,4.\end{aligned} \tag{33}$$

where $b_1 = 2$, $b_2 = -1$, $b_3 = 0.5$, $b_4 = 1$, $\theta_1 = 1$, $\theta_2 = 0.5$, $\theta_3 = -2$, $\theta_4 = -3$ are all unknown system parameters and $\varphi_1 = x_{1,1}x_{1,2}$, $\varphi_2 = x_{2,2}^2$, $\varphi_3 = x_{3,2}$, $\varphi_4 = x_{4,1}x_{4,2}$, $d_1(t) = \sin(t)$, $d_2(t) = \sin(t)^2$, $d_3(t) = 2\sin(t)$ and $d_4(t) = \sin(t)$. The communication status among the 4 subsystems and $y_r(t)$ are represented by a directed graph, as shown in Fig. 1. The reference trajectory is given by $y_r(t) = \cos(0.1t)$ whose information is directly available for subsystem 1 shown in Fig. 1. In simulation, all the initials including $x_{i,1}(0), x_{i,2}(0), \hat{F}_i(0), \hat{y}_{r,i}(0), \hat{\theta}_i(0), \hat{\kappa}_i(0), \hat{D}_i(0)$ are set as zero. The design parameters are chosen as $c_1 = c_2 = 1$, $k = 5$, $\gamma_{P_i} = \gamma_{\kappa_i} = \gamma_{\theta_i} = \gamma_{D_i} = 1$, $\gamma_{y_{r,i}} = 5$, $\kappa_{P_i} = \kappa_{\varrho_i} = \kappa_{\theta_i} = \kappa_{D_i} = 0.005$, $\kappa_{y_{r,i}} = 0.01$, $P_{i,0} = \kappa_{i,0} = \theta_{i,0} = D_{i,0} = y_{ri,0} = 0.01, \eta = 0.2e^{-0.03t}$. The performance of all the 4 subsystem outputs and tracking errors is provided in Fig. 2- 3. Control inputs (19) and all the parameter estimates are shown in Fig. 4- 9, respectively. It can be seen that satisfied output consensus tracking for each subsystem is achieved and all the closed-loop signals are ensured bounded.

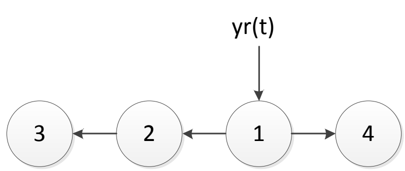

Fig. 1. Communication topology for 4 subsystems.

VI. Conclusion

In this paper, the output consensus tracking problem for a class of high-order nonlinear systems with unknown parameters and uncertain external disturbances is investigated under the directed communication status. A novel backstepping based distributed adaptive consensus control scheme is presented. The assumption on linearly parameterized reference trajectories and known basis function existing in currently available results are relaxed. Besides, extra information exchange of local control inputs is not needed. It is shown

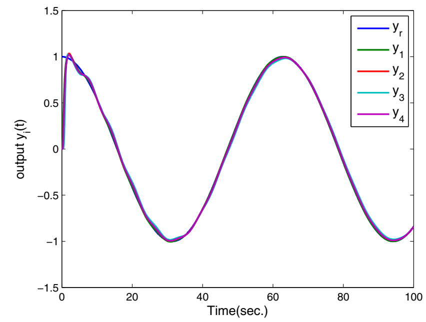

Fig. 2. The outputs $y_i(t)$, $i = 1,\ldots,4$.

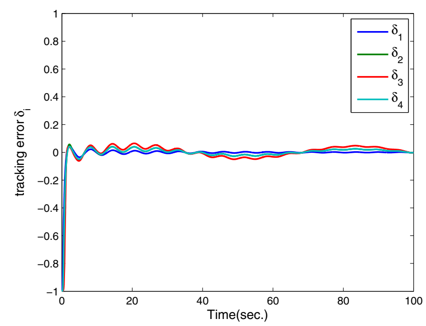

Fig. 3. Tracking errors $\delta_i(t) = y_r(t) - \hat{y}_{r,i}$, $i = 1,\ldots,4$.

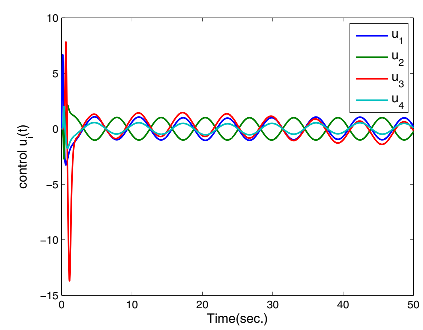

Fig. 4. Control inputs $u_i(t)$, $i = 1,\ldots,4$.

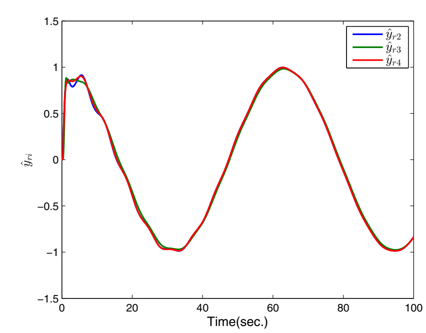

Fig. 5. Parameter estimates $\hat{y}_{r,i}$, $i = 2,\ldots,4$.

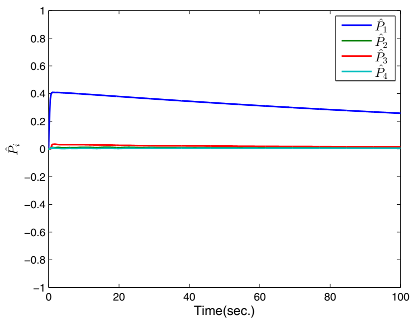

Fig. 6. Parameter estimates $\hat{P}_i$, $i = 1, \ldots, 4$.

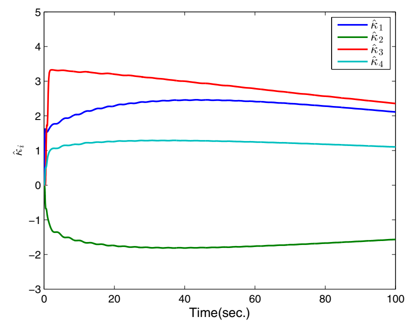

Fig. 7. Parameter estimates $\hat{\varrho}_i$, $i = 1, \ldots, 4$.

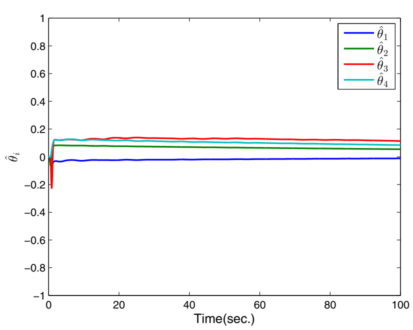

Fig. 8. Parameter estimates $\hat{\theta}_i$, $i = 1, \ldots, 4$.

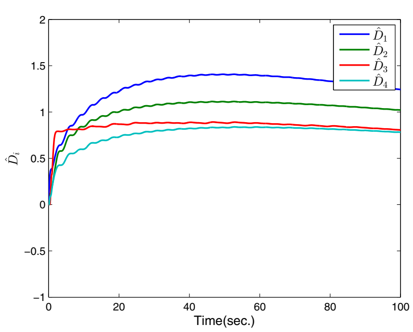

Fig. 9. Parameter estimates $\hat{D}_i$, $i = 1, \ldots, 4$.

that all the signals in the closed-loop system are globally uniformly bounded and desired output consensus tracking can be achieved.

VII. Acknowledgments

This work was partly supported by National Natural Science Foundation of China under Grants 61673035, 61203068, and the National Key Research and Development Program of China under grant no. 2017YFC0602000.

References

[1] R. Olfati-Saber and R.M. Murray, “Consensus Problems in Networks of Agents With Switching Topology and Time-Delays“, *IEEE Transactions on Automatic Control*, vol. 49, 2004, pp. 1520-1533.

[2] L. Moreau, “Stability of multiagent systems with time-dependent communication links“, *IEEE Transactions on Automatic Control*, vol. 50, 2005, pp: 169-182.

[3] W. Ren and R.W. Beard, “Consensus seeking in multiagent systems under dynamically changing interaction topologies“, *IEEE Transactions on Automatic Control*, vol. 50, 2005, pp: 655-661.

[4] M. Arcak. “Passivity as a design tool for group coordination“, *IEEE Transactions on Automatic Control*, vol. 52, 2007, pp: 1380-1390.

[5] H. Bai, M. Arcak and J. Wen, “Adaptive design for reference velocity recovery in motion coordination“, *Systems & Control Letters*, vol. 57, 2008, pp: 602-610.

[6] H. Bai, M. Arcak and J. Wen, “Adaptive motion coordination:using relative velocity feedback to track a reference velocity“,*Automatica*, vol. 46, 2009. pp: 1020-1025.

[7] H. Zhang and F.L. Lewis, “Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics“, *Automatica*, vol. 48, 2012, pp: 1432-1439.

[8] S.J. Yoo, “Distributed consensus tracking for multiple uncertain nonlinear strict-deedback systems under a directed graph“, *IEEE Transactions on Automatic Control*, vol. 24, 2013, pp: 666-672.

[9] W. Wang, J. Huang, C. Wen and H. Fang, “Distributed adaptive control for consensus tracking with application to formation control of nonholonomic mobile robots“, *Automatica*, vol. 50, 2014, pp: 1254-1263.

[10] Y. Hong, J. Hu and L. Cao, “Tracking control for multi-agent consensus with an active leader and variable topology“, *Automatica*, vol. 42, 2006, pp:1177-1182.

[11] W. Ren, “Multi-vehicle xonsensus with a time-varying reference state“, *Systems & Control Letters*, vol. 56, 2007, pp: 474-483.

[12] J. Zhou, C. Wen and Y. Zhang, “Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis“, *IEEE transactions on Automatic Control*, vol. 49, 2004, pp: 1751-1759.

[13] W. Wang, C. Wen, J. Huang and Z. Li, “Hierarchical decomposition based consensus tracking for uncertain interconnected systems via distributed adaptive output feedback control“, *IEEE Transactions on Automatic Control*, vol. 61, 2016, pp: 1938-1945.

[14] W.Ren and Y.Cao, *Distruibuted coordination of multi-agent networks:emergent problems, models and issues*, London: Springer-Verlag, 2010.

[15] A. Das and F.L. Lewis, “Distributed adaptive control for synchronization of unknown nonlinear networked systems“, *Autonatica*, vol. 46, 2010, pp: 2014-2021.

[16] W. Wang, C. Wen and J. Huang, “Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances“, *Automatica*, vol. 77, 2017, pp: 133-142.

[17] W. Wang, C. Wen, J. Huang and H. Fan, “Distributed adaptive asymptotically consensus tracking control of uncertain Euler-Lagrange systems under directed graph condition“, *ISA transactions*, vol. 71, 2017, pp: 121-129.

[18] Z. Zuo and C. Wang, “Adaptive trajectory tracking control of output constrained multi-rotors systems“, *IET Control Theory & Applications*, vol. 8, 2014, pp: 1163-1174.

[19] M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic. *Nonlinear and adaptive control design*, Wiley, 1995.

[20] J. Hu, and X. Wei, “Adaptive tracking control of leader-follower systems with unknown dynamics and partial measurements“, *Automatica*, vol. 50, 2014, pp: 1416-1423.