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# Applied Mathematics versus Fluid Dynamics: The Catalytic Role of Richard von Mises (1883–1953)

## ABSTRACT

This paper investigates scientific, institutional, and political conflict and collaboration between two different disciplines in the first part of the 20th century: applied mathematics and fluid dynamics. It argues for the catalytic role of Richard von Mises (1883–1953) in this process and analyzes the reasons for von Mises's considerable fame in the former and limited posthumous reputation in the latter field. I argue that von Mises's contributions to fluid dynamics and aerodynamics suffered chiefly from two somewhat interconnected deficiencies compared to the work of his principal competitors. There was, on the one hand, von Mises's methodological preference for applied mathematics as opposed to the reigning hybrid theories of fluid dynamics, which were usually more prone to ad hoc adaptation of theory to experimental data. There was, on the other hand, von Mises's geographical remoteness from the main experimental facilities of fluid dynamics and the data produced there. Additionally, there were external reasons that limited von Mises's influence, among them his fate as a refugee from Nazi Germany. Despite his occasionally polemical mind, von Mises's work as a bridge builder prevailed, as evidenced by the success of his journal *ZAMM*. Indisputably, von Mises was a rare example of an engineer and a mathematician combined.

KEY WORDS: fluid dynamics, applied mathematics, Richard von Mises's biography, turbulence, boundary layer, science under the Nazis

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The following abbreviations are used: GAMM, Gesellschaft für Angewandte Mathematik und Mechanik; *JDMV*, *Jahresbericht der Deutschen Mathematiker-Vereinigung*; LPP, Ludwig Prandtl Papers, Archives of the Max Planck Society in Berlin; RVMP, Richard von Mises Papers, Harvard University Archives, HUG 4574; WAV, War Archives Vienna, Österreichisches Staatsarchiv Wien, Kriegsarchiv; *ZAMM*, *Zeitschrift für Angewandte Mathematik und Mechanik*

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## INTRODUCTION[1]

Well-known historiographic work[2] has discussed the reform of the German system of technical sciences in the decades around 1900. This was part of a broader movement toward higher technical education, scientific engineering, and industrial research in various European countries and the United States. Within this process hybrid disciplines such as aero- and hydrodynamics (today usually listed under fluid dynamics)[3] emerged, and parts of mathematics identified themselves as "applied," seeking at least relative independence from the more traditional "pure" domains. One central figure in this process in Germany was the Göttingen mathematician Felix Klein,[4] whose influential organizational work, starting in the 1880s, led to a new distribution of research and teaching between traditional universities and technical colleges that affected secondary education, too. Klein's reforms first in Prussia and later in Germany were preceded by almost a century of increasing development, institutionalization, and mathematization of these sciences in German-speaking countries, partly following the model of the French École Polytechnique and triggered by changes in research and education at the southern periphery of Prussia (e.g., in Prague, Vienna, Zürich, Munich, Karlsruhe, Dresden).[5]

This paper focuses on fluid dynamics and applied mathematics, assuming—and at the same time providing evidence—that the former was not really part

1. My discussion here goes back to a conference presentation in Rauischholzhausen (near Frankfurt) in October 2006. For a broader biographical approach to von Mises's work without details on his fluid dynamics see Reinhard Siegmund-Schultze, "A Non-Conformist Longing for Unity in the Fractures of Modernity: Towards a Scientific Biography of Richard von Mises (1883–1953)," *Science in Context* 17 (2004): 333–70.

2. One such work, related to Göttingen, which may here stand for many others, is Renate Tobies, "The Development of Göttingen into the Prussian Centre of Mathematics and the Exact Sciences," in *Göttingen and the Development of the Natural Sciences*, ed. Nicolaas Rupke (Göttingen: Wallstein Verlag, 2002), 116–42.

3. For the history of this hybrid discipline, see Olivier Darrigol, *Worlds of Flow: A History of Hydrodynamics from the Bernoullis to Prandtl* (Oxford: Oxford University Press, 2005), and Michael Eckert, *The Dawn of Fluid Dynamics: A Discipline between Science and Technology* (Weinheim: Wiley, 2006). For the current state of the discipline, see Tim J. Pedley, "Current Research in Fluid Dynamics," in *IUTAM: Short History*, 2nd ed., ed. Peter Eberhard and Stephen Juhasz (Switzerland: Springer, 2016), 17–20.

4. Renate Tobies is about to complete a comprehensive biography of Felix Klein. On the institutional aspects related to Göttingen, see Tobies, "Prussian Centre" (ref. 2).

5. It is perhaps not coincidental that all three engineers—von Mises, von Kármán, and Prandtl—received their training at the "southern periphery" with its strong traditions for a rigorous and mathematically sound engineering education.

of the latter. Von Mises's programmatic article, "Tasks and Goals of Applied Mathematics" (1921),[6] leaves no doubt that for him *applied mathematics* (*angewandte Mathematik*) was a part of mathematics. Its delimitation within mathematics was relative and temporary, whereas fields like mechanics and fluid dynamics were persistent objects of application, not parts of applied mathematics. However, the "versus" in the title of my paper has to be understood as a dialectical one, and not as a form of mutual exclusion. Specialists from different disciplines and actors with different social roles have to negotiate their aims and, if necessary, fight for their special interests, but this does not rule out shared interests as well.

In the history of fluid dynamics in Germany and elsewhere, the influence of two theoretical engineers closely connected to Göttingen is undisputed: Ludwig Prandtl of "boundary layer" (1904) fame,[7] and Theodore von Kármán, Prandtl's student, after whom the Kármán "vortex street" (1911) is named.[8] Both men were affected and supported by Felix Klein during their careers. They led the most influential centers for experimental and theoretical research in fluid dynamics in Germany: Prandtl in Göttingen, starting in 1904, with the founding of the Kaiser Wilhelm Institute for Fluid Dynamics (Strömungsforschung) in 1925, and von Kármán in Aachen, beginning in 1913. Later, from 1929 on, von Kármán became the leader of American research in the field in GALCIT at Caltech in Pasadena, California.

The principal aim of this paper is to investigate a different contributor and (temporary) collaborator of Klein's, Richard von Mises (1883–1953), and the cognitive, institutional, and biographical tensions that accompanied Klein's reforms. Von Mises was involved in these reforms on various levels, and his biography is a valuable object of study in this respect. Von Mises had great respect for mathematics in Göttingen, especially Klein's and David Hilbert's work. (Hilbert had been the doctoral advisor of von Mises's superior, Georg Hamel, in Brünn.)[9]

6. Richard von Mises, "Über die Aufgaben und Ziele der angewandten Mathematik," *ZAMM* 1 (1921): 1–15.

7. For a recent and important biography of Prandtl in German, see Michael Eckert, *Ludwig Prandtl—Strömungsforscher und Wissenschaftsmanager: Ein unverstellter Blick auf sein Leben* (Berlin: Springer, 2017).

8. On von Kármán's influence and his transition from Germany to the United States, see Paul A. Hanle, *Bringing Aerodynamics to America* (Cambridge, MA: MIT Press, 1982); and Eckert, *Ludwig Prandtl* (ref. 7).

9. As discussions around 1927 with Klein's successor in Göttingen, Richard Courant, showed, von Mises then saw himself as the true heir to Felix Klein (see below).

The fact that von Mises has largely been excluded from the collective memory of fluid dynamicists might be partly explained by his institutional role and the priority that is usually given to theoretical results when considering posthumous reputation.[10] In contrast, I argue that von Mises's contributions to fluid dynamics and aerodynamics suffered chiefly from two somewhat *interconnected* deficiencies as compared to the work of his principal competitors.

First, there was von Mises's methodological preference for *applied mathematics* as opposed to the reigning hybrid theories of fluid dynamics, which were usually prone to ad hoc adaptation of existing theories to experimental data. Following Epple's analysis,[11] one might argue that von Mises gave preference to the ideal of mathematical *exactness* over precision in relation to the partly theoretical, partly empirical parts of fluid dynamics. Von Mises extended the former ideal as far as possible over the whole applied context.

Second, and related to von Mises's preference for applied mathematics, was his geographical remoteness from the main experimental facilities of fluid dynamics (Aachen, Göttingen) and the data produced there. From 1909, he mainly worked in mathematical institutes at traditional universities (Strassburg, Berlin, Istanbul), and finally, from 1939, at an Engineering Department (Harvard), but always without experimental equipment. There were moments in von Mises's life when he could have gone in the direction of more systematic aerodynamic research, including experimental work, as will be discussed in connection with a failed appointment in Aachen in 1912. These different institutional affiliations of Richard von Mises will be discussed in the course of this paper, with emphasis on his pioneering Institute for Applied Mathematics in Berlin (1920–33) (Fig. 1).

Finally, it should not be forgotten that von Mises's career and posthumous reputation was strongly influenced by his fate as a repeatedly uprooted Jewish scientist. I will go into this point in the last section of this paper.

10. In several respects, Hans Reissner (1874–1967) can be compared to von Mises. Reissner was von Mises's older colleague working at the Technical University in Berlin-Charlottenburg. Haude draws attention to Reissner, whose important contributions have been largely ignored for various personal and political reasons. See Rüdiger Haude, *Grenzflüge: Politische Symbolik der Luftfahrt vor dem Ersten Weltkrieg: Das Beispiel Aachen* (Köln: Böhlau, 2007), 35, 36; and Reissner's biography written by his son, Eric Reissner, "Hans Reissner: Engineer, Physicist and Engineering Scientist," *Engineering Science Perspective* 2, no. 4 (1977): 97–105.

11. Moritz Epple, "Präzision versus Exaktheit: Konfligierende Ideale der angewandten Mathematik," *Berichte zur Wissenschaftsgeschichte* 25 (2002): 171–93.


**FIG. 1.** Identity card of Richard von Mises for the University of Berlin from the 1920s, when he was head of the Institute for Applied Mathematics, (Source: RVMP, HUG 4574.4, http://id.lib.harvard.edu/images/olvwork176805/catalog, accessed 19 Jul 2018. Courtesy Magda Tisza and Harvard University Archives.)

## RETROSPECTIVE FAME OF VON MISES AND OPINIONS BY FELLOW FLUID DYNAMICISTS

Richard von Mises is known and recognized worldwide as one of the foremost applied mathematicians of the early twentieth century. The mathematician Alexander Ostrowski described von Mises's institute in Berlin in the 1920s as the "first mathematically serious school of applied mathematics in Germany." He wrote, "Von Mises was an extraordinarily dynamic person and at the same time, like Runge, amazingly versatile. He was particularly knowledgeable in the realm of technology."[12]

12. Alexander Ostrowski, "Zur Entwicklung der numerischen Analysis," *JDMV* 68 (1966): 97–111, 106. All translations into English are by the author. This quote also reminds us that the earlier school of Carl Runge in Göttingen (from 1904) has to be considered as "serious" as well. But it is important to note, right from the beginning, that von Mises—unlike Runge, the influential student of the Berlin mathematician Karl Weierstrass—was an engineer by training. However, already during his studies at the Technical University in Vienna (1901–05), von Mises had developed a particular taste for mathematics, above all geometry, later to be complemented by analysis and probability theory.

Von Mises worked first in fluid dynamics, particularly turbine theory, and later in aerodynamics.[13] His main competitors in fluid- and aerodynamics, however, did not consider him to be fully their equal. He was, for example, visibly absent as an author in the international six-volume collection on *Aerodynamic Theory* edited by the American engineer William F. Durand between 1934 and 1936.[14] Histories of hydro- and aerodynamics do not reserve much space for von Mises, except for the occasional short reference to the "von Mises transformation" in boundary-layer theory or the two-dimensional "Mises family" of airfoils in aerodynamic theory.[15] Von Kármán, von Mises's long-time friend[16] and competitor, managed not to devote a single word to von Mises in his historical book on aerodynamics (one year after von Mises's death)[17] or in his posthumously published autobiography of 1967.[18]

In contrast, applied mathematician Garrett Birkhoff, who was a colleague of von Mises at Harvard University between 1939 and 1953, wrote in 1983 in his biographical article on the occasion of von Mises's centenary:

> In 1920 he ranked with Ludwig Prandtl, Th. von Kármán and G. I. Taylor as one of the world's half-dozen leading experts on the then nascent science of aeronautics.
>
> Moreover he had a quality of mathematical profundity not shared by the other outstanding scientists just named: he was deeply interested in the philosophy and beauty of mathematics. It was most appropriate that [he] should . . . [become] Professor of Applied Mathematics at the University of Berlin from 1919 [*sic*] to 1933.[19]

13. The most detailed discussion to date of von Mises's contributions to fluid dynamics is in David Bloor, *The Enigma of the Aerofoil: Rival Theories in Aerodynamics, 1909–1930* (Chicago, London: University of Chicago Press, 2011), 241ff. Bloor's book focuses on the differences in the approaches of German engineers and British mathematical physicists.

14. William F. Durand, ed., *Aerodynamic Theory: A General Review of Progress under a Grant of the Guggenheim Fund for the Promotion of Aeronautics*, 6 vols. (Berlin: Springer, 1934–36).

15. John D. Anderson, *A History of Aerodynamics* (Cambridge: Cambridge University Press, 1997), and Darrigol, *Worlds of Flow* (ref. 3), do not mention von Mises at all. Eckert, *The Dawn* (ref. 3), and Eckert, *Ludwig Prandtl* (ref. 7), mention von Mises occasionally; Bloor, *The Enigma* (ref. 13), who examines airfoil theory in particular, discusses his work in more detail.

16. At least as von Mises and his wife saw it; see below Hilda Geiringer's recollections of 1959, quoted in the conclusions.

17. Theodore von Kármán, *Aerodynamics: Selected Topics in the Light of Their Historical Development* (Ithaca, NY: Cornell University Press, 1954).

18. Theodore von Kármán with Lee Edson, *The Wind and Beyond* (Boston, Toronto: Little, Brown and Company, 1967).

19. Garrett Birkhoff, "Richard von Mises' Years at Harvard," *ZAMM* 63 (1983): 283–84.

It is important to stress that Birkhoff does not talk about von Mises's *expertise* in mathematics as distinguishing him from other aerodynamicists (although this would have been true of von Mises, too, when compared to Prandtl). Prandtl's students in Göttingen, such as Heinrich Blasius, Albert Betz, and Max Munk, were equally well trained as von Mises in the types of mathematics required for engineering. If von Mises was broader in his mathematical interests and knowledge, there were indications that von Mises's own horizon in modern mathematical methods was somewhat restricted.[20] It is abundantly clear that von Mises was not interested in a general mathematical theory of fluid dynamics unless it extended the practical applications considerably.

He did not spare his good friend Leon Lichtenstein in Leipzig, for example, in his (von Mises's) critical review of Lichtenstein's *Foundations of Hydromechanics* (1929) in the journal *Zeitschrift für Angewandte Mathematik und Mechanik* (*ZAMM*). Lichtenstein's former student, Ernst Hölder, wrote: "In his elegant review, which deeply offended Lichtenstein, von Mises wrote: 'We recognize the value of Lichtenstein's book, although it stops at mechanics as of 1870. But its real value lies in having used the mathematics as of 1930.'"[21]

It was von Mises's general methodological, and partly philosophical, approach, and the specific combination of mathematics and engineering in his work, which distinguished him from both engineers such as Prandtl and mathematicians such as Lichtenstein. This is explained in more detail in a 1963 biographical article on von Mises by noted aerodynamicist Sydney Goldstein (1903–89):

> He was distinguished as an engineer, mathematician, philosopher, and authority on the poet Rainer Maria Rilke. . . . Certainly he was pre-eminent as an applied mathematician. . . . To Mises, whether the researches were on

20. In addition, von Mises earned a certain notoriety for making minor mathematical mistakes. Ostrowski even spoke of "occasional heavy blunders" committed by von Mises. But by adding "[o]ne has even forgiven him his theory of probability" (Ostrowski, "Zur Entwicklung," ref. 12, 106), Ostrowski only reproduces the prejudices of the so-called pure mathematicians. However, von Mises's approach to fundamental theorems of probability suffered from a lack of mathematical sophistication. Cf. Reinhard Siegmund-Schultze, "Probability in 1919/20: The von Mises-Pólya-Controversy," *Archive for History of Exact Sciences* 60 (2006): 431–515; and Reinhard Siegmund-Schultze, "Sets versus trial sequences, Hausdorff versus von Mises: 'Pure' mathematics prevails in the foundations of probability around 1920," *Historia Mathematica* 37 (2010): 204–41. Von Mises apparently lacked knowledge or interest in measure theory and in some other modern mathematical theories.

21. Ernst Hölder, review of *Grundlagen der Hydromechanik* by Leon Lichtenstein, *JDMV* 74 (1972–73), part 2: 32–34, 33.

> mechanics or probability, the rules were the same and should be obeyed strictly. There could be no satisfaction with *ad hoc* researches, with rules of thumb, or even with qualitative physical explanations incapable of exact quantitative analysis. There should be a mathematical model of the widest possible generality, where the argument could be made with clarity, elegance and rigor.[22]

Being a younger competitor of von Mises, who starting in the mid-1920s had close affiliations with Göttingen and its experimental facilities under Prandtl, Goldstein mentioned in his necrology von Mises's "two celebrated papers on airfoil theory,"[23] but stressed his results in plasticity and stochastics (probability and statistics) even more. Ignorance about von Mises's contributions to aerodynamics continues to this day; von Mises's equations for the "metacentric parabola" of an airfoil around the "aerodynamic center" are used, but the name of the discoverer, von Mises, is rarely connected to this work.[24]

## VON MISES'S WORK ON HYDRAULICS AND HYDRODYNAMICS BEFORE WORLD WAR I AND HIS RELATION TO FELIX KLEIN

As von Mises's unpublished personal diaries reveal, he was very self-conscious with respect to his choice of research and its possible success, including his job prospects. On January 9, 1905, while still a student in Vienna, he decided to continue working on elasticity and plasticity, which he had been investigating for some time and thought to be very original and theoretical.[25] But, on October 25, 1905, he decided in favor of a dissertation in mechanical engineering.[26]

22. Sydney Goldstein, "Richard von Mises 1883–1953," in Richard von Mises, *Selected Papers of Richard von Mises*, 2 vols. (Providence, RI: American Mathematical Society, 1963–64), vol. 1 (1963): ix–xiv.

23. Goldstein, "Richard von Mises" (ref. 22), xi. This refers to Richard von Mises, "Zur Theorie des Tragflächenauftriebes," *Zeitschrift für Flugtechnik und Motorluftschiffahrt* 8 (1917): 157–63, and 11 (1920): 68–73, 87–89.

24. Anderson's history of aerodynamics (Anderson, *A History*, ref. 15) is chided by one reviewer for not taking into account von Mises's contributions in two-dimensional airfoil theory (Fred E. C. Culick, review of Anderson, *A History of Aerodynamics* [ref.15], *ISIS* 92 (2001): 589–90.)

25. RVMP, von Mises's personal diaries 1903–52, HUG 4574.2, written in Gabelsberger shorthand. By way of contrast, von Mises's scientific diaries, also kept at Harvard, are written in plain German script.

26. Von Mises would return to plasticity later and provide some of the most successful results of his career: his famous *von Mises stress* and the *yield condition* in plasticity of 1913; Richard von

He vacillated between practical and theoretical work. In the summer of 1905, he arranged for voluntary work in a factory in Prague, in order to become familiar with as many practical directions in engineering and craftsmanship (e.g., casting, molding, etc.) as possible.

From early 1906 until fall 1909, von Mises worked as an assistant to Georg Hamel, professor of mechanics at the German Technical University (Deutsche Technische Hochschule) in Brünn (Brno) under the Habsburg monarchy.[27] In March 1908, von Mises, then twenty-five years old, submitted to the faculty in Brünn his habilitation thesis to obtain permission to teach as a *Privatdozent*. Von Mises had chosen to work on turbine theory, which was located at the crossroads between mechanical engineering and hydrodynamics. His habilitation thesis, *The Theory of Water Turbines*, was accepted and published in full (120 pp.) in the *Zeitschrift für Mathematik und Physik* one year later.

It began with the following words:

> This work approaches from the standpoint of *rational hydromechanics* the problems of *technical hydraulics*, which have to do with the investigation of the motion of rotating wheels (turbines, centrifugal pumps, water wheels). Those who know the deep and broad divide between these two directions of research, which historically were once close to each other, will not expect final results. Almost everywhere we could but offer *rudimentary* solutions, in many cases *mathematical hypotheses* have been used, frequently we have merely formulated *conjectures*.[28]

One of the major merits of his habilitation thesis was von Mises's application of the momentum principle to flows in hydraulic machines.[29]

In addition to this substantial work, von Mises submitted two articles published in 1906 and 1907, respectively. The 1907 article was an extended polemical review of Hans Lorenz's book[30] on turbine theory, in which von

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Mises, "Mechanik der festen Körper im plastisch-deformablen Zustand," *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen* (1913): 582–92. See the discussion in Reinhard Siegmund-Schultze, "Zu Richard von Mises' Arbeiten in der Plastizitätstheorie, insbesondere zu seiner Fliessbedingung (1913)," in *Festschrift-Proceedings of the Christoph J. Scriba Memorial Meeting History of Mathematics*, ed. Gudrun Wolfschmidt (Hamburg: Tredition, 2017), 430–51.

27. Pavel Sisma, "Georg Hamel and Richard von Mises in Brno," *Historia Mathematica* 29 (2002): 176–92.

28. Richard von Mises, "Zur Theorie der Wasserräder," *Zeitschrift für Mathematik und Physik* 57 (1909): 1–120, 1–2.

29. This merit has been acknowledged also by Clifford Truesdell in his review of *History of Hydraulics* by Hunter Rouse and Simon Ince, *ISIS* 50 (1959): 69–71, 71.

30. Hans Lorenz, *Neue Theorie und Berechnung der Kreiselräder* (Munich, 1906).

Mises analyzed Leonhard Euler's hydrodynamic equations for ideal fluids and the theory of stream filaments that was based on it. He stressed that such a more general mathematical standpoint is needed and that Lorenz' rather empirical theory did not yet provide conclusions about the optimal choice of the form of turbine wheels. But in addition to Euler's theory, clearly, viscosity effects and vortices originating from turbine vanes played an important role. Von Mises, therefore, generalized the theory of stream filaments to a theory of "stream layers" (*Stromschichte*).

Von Mises wrote:

> In the treatment of the purely mathematical problem of whether arbitrary forms of turbine wheels can be assumed under Euler's equations, I cannot take into consideration any other condition, technically or economically important as they might be, but only the mathematical theorems following from partial differential equations, which I have tried to study before I felt entitled to draw conclusions about the designs of water turbines.[31]

Lorenz replied in the same issue: "Luckily turbine theory is not bound to wait until Mr. von Mises comes to conclusions about the construction of turbines after studying partial differential equations."[32]

In 1900, on the recommendation of Felix Klein, Lorenz had been appointed to a position in Göttingen where both physicists and mathematicians expected much from his technical expertise.[33] However, soon after his appointment began, controversies arose between Lorenz and Klein that were partly related to their different opinions about the value of mathematics in technical disciplines.

In 1902, Lorenz drew on himself criticism by Ludwig Prandtl (then in Hanover) for Lorenz's insufficient theoretical understanding of the flow in turbines,[34] and in 1904, Lorenz left Göttingen, while Klein (through his wide-ranging influence) appointed Prandtl to a professorship for applied mechanics. In the same year, Prandtl presented his seminal talk at the International Congress of Mathematicians (ICM) in Heidelberg, where he introduced his concept of the boundary layer.[35]

31. Richard von Mises, "Über die H. Lorenzsche Theorie der Kreiselräder," *Physikalische Zeitschrift* 8 (1907): 314–18, 509–10, 510.

32. Hans Lorenz, "Zur Theorie der Kreiselräder," *Physikalische Zeitschrift* 8 (1907): 510.

33. Eckert, *Ludwig Prandtl* (ref. 7), 34ff.

34. Ibid., 35.

35. Ludwig Prandtl, "Über Flüssigkeitsbewegung bei sehr kleiner Reibung," in *Verhandlungen des III: Internationalen Mathematiker-Kongresses, Heidelberg 1904* (Leipzig: Teubner, 1905), 484–91.

Given this context, it is highly likely that von Mises's critique of Lorenz' turbine theory in 1907–08 was well received by Göttingen mathematicians, such as Felix Klein. Klein may have considered him an ally in the effort to imbue the technical sciences with mathematics. As early as 1907, Klein invited von Mises to write an article on "Dynamical Problems of Mechanical Engineering"[36] for the famous *Encyclopedia of Mathematics and its Applications*. It was planned as a joint publication with the established applied mathematician Karl Heun in Karlsruhe, who had famously contributed to the Runge-Kutta method in numerical analysis and had recently guided von Mises's supervisor in Brünn, Hamel, toward mechanics. The paper appeared in 1911, however, under von Mises's name alone because Heun had to withdraw due to illness. In preparation, Heun wrote a letter to von Mises on October 3, 1907, in which he agreed with von Mises's plan for the article. Heun wrote:

> We must definitely neglect—though with all necessary caution—the pseudo-authorities in the field. The picture of mechanical engineering in its current state can only gain from such negligence. . . . [For the article in the encyclopaedia] we lack so far almost all methodical points or view. Each progress in mechanics teaching at the Technical Universities will facilitate our work and will gradually instill a scientific mind in the engineer as we see it in England with Osborne Reynolds. . . . We want to provide a picture of the hitherto neglected auxiliary methods for mechanics.[37]

This quote, being internal and private, gives a good idea about the almost subversive tactics that were needed to break the still dominant anti-mathematical mood among engineers. The reference to England and Reynolds is important because English literature was still not sufficiently appreciated.

Von Mises became a close collaborator of Klein in the Encyclopedia project in the years to come and notably contributed his critical approach to other articles, in particular to one by von Kármán.[38] In 1908, Klein publicly called

36. Richard von Mises, "Dynamische Probleme der Maschinenlehre," *Encyclopädie der Mathematischen Wissenschaften* vol. 4, subvol. 2, art. 10 (1911): 153–355.

37. RVMP, HUG 4574.5, box 1, folder 1907.

38. This is clear from extensive correspondence between Klein and von Mises, which is kept in the von Mises Papers at the Harvard University Archives; RVMP, HUA 4574.5, box 11, folder 11. On April 3, 1910, Klein wrote to von Mises: "Dear Colleague, . . . today I can report that page proofs for Kármán are now being produced, after he has reworked stability according to your suggestions." Klein was referring to Theodore von Kármán, "Festigkeitsprobleme im

von Mises "one of the most successful examples so far of a crossbreed between mathematics and technology."[39]

Von Mises's sharp-tongued manner also made him feared, and apparently led to a restriction of his teaching permit, connected to his habilitation thesis in 1908, to (the more mathematically sounding) "mechanics"—excluding mechanical engineering.[40] In 1912, during faculty discussions about a successor to Hamel as *Ordinarius* in mechanics, G. Jaumann, rector of the Technical University, wrote to von Mises, now a professor in Strassburg (Strasbourg): "You have only a few friends here, but I ask you to count me as one of those few."[41]

Von Mises was neither interested in the formal, mathematical treatment of turbines alone, nor did he disregard the technical details of engineering. Both his 1908 habilitation thesis and his 1914 book *Elemente der Technischen Hydromechanik* were filled with empirical details, friction coefficients, pictures of turbine vanes, etc.[42] That knowledge he acquired and enhanced in parallel through his assistantship in mechanics as a constructor at the Technical University and as an engineer in the private company Brand & L'Huillier in Brünn.

Still the practical importance of von Mises's work on turbines was limited, not least due to heavy investments that had already been made in the industry. The engineer Kurt von Sanden (1885–1976), then in Brünn and later successor to Heun in Karlsruhe, wrote to von Mises in Strassburg, on November 30, 1909: "Professor Schultze Pillot who is close to the related industry shares my opinion that the practical importance of your theory lies less in the construction of turbine vanes, with which all bigger factories are already equipped, than in the kinetic operation."[43]

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Maschinenbau" [Strength of materials in mechanical engineering], *Enzyklopädie der Mathematischen Wissenschaften*, vol. 4, subvol. 4 (1907–14), 311–85. There von Kármán quotes von Mises several times, however not under "13. Stabilitätsprobleme," 370ff.

39. "Eines der am besten geglückten Erzeugnisse der Kreuzung zwischen Mathematik u. Technik." RVMP, HUG 4574.5, box 1, folder 1908, Ernst Hellinger (Göttingen) to von Mises, 29 Oct 1908, with an excerpt from the minutes of the meeting of the Göttingen Mathematical Society, Oct 1908.

40. Sisma, "Georg Hamel" (ref. 27).

41. RVMP, G. Jaumann to von Mises, 2 Nov 1912, HUG 4574.5.3, folder "correspondence." As it becomes clear from other sources, academic anti-Semitism played a role in preventing von Mises's appointment to Brünn in 1912.

42. Von Mises, "Wasserräder" (ref. 28); and von Mises, *Elemente der Technischen Hydromechanik* (Leipzig: Teubner, 1914).

43. RVMP, HUG 4574.5, box 1, folder 1909.

Yet, a few months later Hamel wrote to von Mises: “Kaplan has now asked v. Sanden to explain to him your work. He and Musil want to build and test turbine vanes following your theory.”[44]

In 1909, von Mises was appointed extraordinary professor of applied mathematics at the University of Strassburg. While there von Mises prepared his book on technical hydromechanics[45] and continued his work on classical topics of hydrodynamics in addition to new problems in aerodynamics. He corresponded with Aurel Stodola in Zürich, the leading authority on steam and gas turbines.[46] Von Mises’s student, Erich Trefftz, nephew of Carl Runge, also finished his doctorate on circular fluid jets in 1913. This was a typical field of application of Hermann Helmholtz’s classical theory of ideal fluids.[47] It was connected to von Mises’s research, published during the war in 1917, and contained valuable practical conclusions. It made it possible to avoid a wide range of (expensive) tests, and it became the subject of two independent American translations in 1956 and 1964.[48]

44. RVMP, Hamel to von Mises, 16 Feb 1910, HUG 4574.5, box 1, folder 1910. Viktor Kaplan built a water-turbine laboratory at the time in Brünn and became later famous for his type of turbines. Alfred Musil, the father of the novelist Robert Musil, was at the time professor of engineering in Brünn and part of the commission for von Mises’s habilitation in 1908. So far little historical material could be traced about von Mises’s relationship with the two men, Kaplan and Musil.

45. Von Mises, *Elemente* (ref. 42).

46. Six letters by Stodola to von Mises between 1910 and 1931 are in Stodola’s Nachlass at the manuscript division of the ETH Zurich, Hs 496. On December 10, 1910, Stodola asked von Mises for mathematical advice, adding: “We engineers are often struggling with our rudimentary mathematics, where the mathematicians see the difficulty with one glimpse.” Stodola left no doubt about his admiration for von Mises and attributes to him a “harmonic balance” (*harmonische Abgewogenheit*) of talents. Stodola to von Mises, Zurich, 1 Oct 1926. Ibid. There are two additional letters by Stodola (1920 and 1931) in von Mises’s Nachlass at RVMP, HUG 4574.5. In 1924, von Mises used the new edition of Stodola’s book on steam and gas turbines for an impressive discussion of the division of labor between engineers and mathematicians, and to advertise for his new journal *ZAMM*. Richard von Mises, “Maschinenbau und angewandte Mathematik,” *Maschinenbau* 1 (1922): 511–17.

47. I thank Michael Eckert for pointing out this fact, which is also mentioned in von Mises’s programmatic article in *ZAMM*; von Mises, “Über die Aufgaben” (ref. 6), 12.

48. Richard von Mises, “Berechnung von Ausfluss- und Überfallzahlen,” *Zeitschrift Verein Deutscher Ingenieure (VDI)* 61 (1917): 447–52, 469–74, 493–98. Translated as: “Computation of Exit-Flow and Over-Flow Coefficients” (Baltimore: The Johns Hopkins University Applied Physics Laboratory, May 1956), 55 pp., APL/JHU CF-2522; and “Computation of Discharge and Overfall Coefficients” (Vicksburg, MS: U.S. Army Engineer Waterways Experiment Station, Corps of Engineers, Oct 1964), 44 + 6 pp., translation no. 64–8.

Meanwhile, Trefftz became a prominent applied mathematician in his own right, first in Aachen, where he collaborated with von Kármán in wing theory, and later in Dresden. In some respects, his career resembles von Mises's, especially with respect to the unsatisfied need for modern experimental facilities for an engineer working as an aerodynamicist. When Trefftz left Aachen for Dresden in 1922, he turned to elasticity and his work with aerodynamics ended.[49]

## VON MISES'S EARLY, IF RUDIMENTARY, IDEAS ABOUT THE NOTION OF TURBULENCE

Von Mises was an engineer, not someone interested in mathematical theories for their own sake. If anything, it was von Mises's versatility and his mathematical drive combined with intimate technical knowledge that characterized his 1908 habilitation thesis and his 1914 book. His publications, however, leave no doubt that von Mises was also interested in the more general mathematical and physical problems of fluid dynamics, including the new boundary-layer theory introduced by Prandtl in 1904 and the notorious problem of turbulence. The latter problem was in a way the most fundamental, in von Mises's opinion. He had an ambitious tendency to circumvent the work done in Göttingen on viscous boundary layers, to represent it as temporary, and to look for a more general theory that included turbulence.[50]

In his habilitation thesis on turbines, von Mises went from Euler's equations for ideal fluids directly to the notion of turbulence, noting that, although they include viscosity, "Stokes' equations have not been successful so far in solving even the most simple technical cases for the water flow."[51] Von Mises based his view on "the assumption of Reynolds and [Joseph] Boussinesq that each motion to be treated by us, which occurs within fixed guides, is *turbulent*, i.e., is a superposition of a slowly changing velocity

49. See Richard Grammel, "Das wissenschaftliche Werk von Erich Trefftz," *ZAMM* 18 (1938): 1–11. There von Mises's name and contributions are clearly suppressed. One reason might be that von Mises had repeatedly criticized Grammel's work (including the *Handbuch der Physik*, which will be mentioned below). Another reason might have been von Mises's fate as an emigrant at the time.

50. See von Mises's retrospective remark around 1941 in his "Übersicht der Abhandlungen" (ref. 184).

51. Von Mises, "Wasserräder" (ref. 28), 61.

distribution and a distribution of rapid and small pulsations."[52] Von Mises explained, it was

> an idea of Boussinesq to write down from the outset differential equations for averages, i.e., for the quantities of the basic motion. With the help of plausibility arguments, he [Boussinesq] came to the conclusion that the averages of the pressure and the velocity approximately obey equations similar to those of Stokes, if one replaces the constant viscosity coefficient by a special "*coefficient of turbulence*" which has to be determined separately.[53]

However, von Mises admitted that the equations of the French mechanist Boussinesq were even more complicated than those of G. G. Stokes, and therefore "even less amenable to numerical use."[54] Under these conditions von Mises felt compelled to introduce a "decisive auxiliary hypothesis."[55] He wrote,

> The roughly observable averages of the *velocity* obey approximately a system of equations that *results from the Eulerian approach if the pressure is eliminated from it*. These equations I call the Helmholtz equations because they are equivalent to the well-known theorems on vortex motion formulated by Helmholtz.[56] The averages of the *pressure* have to be determined by a separate investigation.[57]

Thus, von Mises was of the opinion that "*the inner friction regulates the pulsations of velocity in such a way that the average is almost equivalent to the average of an ideal fluid*, or to put it briefly, one reaches coincidence by neglecting the viscosity terms in the equations and the pulsations in the observations."[58]

Von Mises did not pretend that he had found—by way of his assumption—the solution to the "turbulence problem," i.e., to find "the real integral of the Stokes equations that correspond . . . to the pulsating motion."[59] However, he hoped to find a confirmation of his approach "once Stokes' theory [was] far

52. Ibid., 62.

53. Richard von Mises, "Über die Probleme der technischen Hydromechanik," *JDMV* 17 (1908): 319–25, 320–21.

54. Ibid., 321. Darrigol, *Worlds of Flow* (ref. 3), 239, stresses the problems that contemporaries had to "follow (the) analytical sophistication" of Boussinesq's theory.

55. Von Mises, "Wasserräder" (ref. 28), 61: "entscheidende *Hilfshypothese*."

56. Darrigol shows how Hermann Helmholtz "studied vortices in ideal fluids as a step toward including internal friction." Darrigol, *Worlds of Flow* (ref. 3), 158.

57. Von Mises, "Über die Probleme" (ref. 53), 321.

58. Ibid., 323.

59. Ibid.

enough developed."[60] He noted: "One will have to show that generally in the Stokes equations one can—by way of some simplifications—separate a basic motion [*Grundbewegung*], which is independent of the viscous stresses."[61]

Von Mises did not develop his theory beyond its use in special cases of turbine construction. He admitted that his approach required, for the time being, "the standpoint of technical hydraulics . . . with a *systematic use of observational and calculated data for the loss of energy in certain simple flows*."[62] He felt the need to defend this semi-empirical approach, at least before the mathematical readership of the *Jahresbericht* (*JDMV*). He wrote,

> One tends condescendingly to call this kind of approach or its misuse "coefficient brokering" [*Koeffizienten-Wirtschaft*] . . . It is self-explanatory that the investigations presented here are somewhat preliminary—compared to *ideal* mechanics. But I believe one has to accept imperfect solutions if one does not wish to leave aside the needs of practice altogether.[63]

As to the Navier-Stokes equations, von Mises doubted in 1908 that Lord Kelvin's approach, which used the "method of small oscillations," would lead any further toward the solution of those equations and thus to the mathematical description of fully developed turbulence .[64] However, for the restricted problem of onset of turbulence, von Mises nevertheless tried Kelvin's method in a contribution to a 1912 *Festschrift* for the mathematician Heinrich Weber.[65] Von Mises treated the so-called plane Couette flow, in which a viscous fluid flows between two parallel planes moving laterally with respect to one another, and investigated solutions to the Orr-Sommerfeld equations, which were derived from the Navier-Stokes equations by eliminating the pressure term.[66] Von Mises's calculation, which used cutting-edge mathematical methods in the eigenvalue theory of integral equations (developed by Ivar Fredholm), resulted in stability as opposed to the onset of turbulence that was observed in reality for higher Reynolds numbers (i.e., lower viscosity). This left von

60. Ibid.

61. Ibid., 324.

62. Ibid., 323.

63. Ibid., 323–25.

64. Ibid., 323–24.

65. Richard von Mises, "Beitrag zum Oszillationsproblem," *Festschrift Heinrich Weber* (Leipzig, Berlin: Teubner, 1912), 252–82.

66. For details of Sommerfeld's approach, see Michael Eckert, "The troublesome birth of hydrodynamic stability theory: Sommerfeld and the turbulence problem," *European Physical Journal, History* 35, no. 1 (2010): 29–51.

Mises with some doubt about whether "the basic equations for the motion of viscous fluids [were] valid at all."[67] As Eckert notes, "with this proof von Mises became convinced that turbulence did not result from an instability within the flow but originated at the walls."[68]

In 1924, von Mises commissioned his student Erich George[69] to use von Mises's mathematical methods from 1912 for checking the results in Werner Heisenberg's thesis, Sommerfeld's exceptional student who had found instability for the plane Poiseuille flow.[70] In his 1938 semi-centennial address to the American Mathematical Society on hydrodynamic stability, which did not discuss empirical work from engineering contexts, the well-known applied mathematician John Lighton Synge said, in clear recognition of that early attempt by von Mises:

> On the whole we may say that the equations of Navier and Stokes have stood the test so far, their conspicuous triumph being in the work of G. I. Taylor [of 1923] . . . On the other hand, the work of R. von Mises and L. Hopf [of 1914] may make us doubtful as to the validity of these equations.[71]

However, Synge suggested that von Mises's and Ludwig Hopf's solutions were not, in fact, mathematically complete and convincing.[72]

67. Richard von Mises, "Kleine Schwingungen und Turbulenz," *JDMV* 21 (1912): 241–48, 248. Von Mises added, however, "I do not believe that we have to go this far." Today it is believed, though not known with certainty, that the Navier-Stokes equations model turbulence properly.

68. Eckert, "Troublesome Birth" (ref. 66), 38.

69. George was born 1901, and took his state exam as a teacher with von Mises in 1926, and his thesis in topology with U. Wegner in Heidelberg in 1939. See Renate Tobies, *Biographisches Lexikon in Mathematik promovierter Personen an deutschen Universitäten und Technischen Hochschulen WS 1907/08 bis WS 1944/45* (Augsburg: Rauner, 2006), 121. The date of George's death is unknown to me.

70. On November 20, 1924, von Mises applied for funds to the German Emergency Fund for Science (Notgemeinschaft Deutscher Wissenschaft). See German Federal Archives: Bundesarchiv, Berlin, R 4901/REM 1447, Institut für Angewandte Mathematik (1920–42), sheets 109–11. The money for George was apparently granted, but nothing seems to have come out of the project. Maybe it was on this occasion that von Mises realized the insufficiency of his 1912 approach (see below).

71. John L. Synge, "Hydrodynamical Stability," in *American Mathematical Society Semicentennial Publications.* (New York: American Mathematical Society) 2 (1938): 227–69, 228. Synge referred in the quote to the two publications by von Mises from 1912; see von Mises, "Oszillationsproblem" (ref. 65), and von Mises, "Kleine Schwingungen" (ref. 67).

72. Synge, "Hydrodynamical Stability" (ref. 71), 262. Synge had asked von Mises before in a letter from Toronto, dated 30 May 1938, whether his solution in von Mises "Oszillationsproblem" (ref. 65) had been complete. Von Mises admitted on June 13, 1938, that his paper had not delivered a "sufficient proof in a mathematical sense." RVMP, HUG 4574.5, box 3, folder 1938.

Von Mises's early work on turbulence was apparently critical—in illustrating the limits of the existing mathematical apparatus—rather than constructive. His results showed, in his opinion, the limits of either the basic equations of the theory themselves or of the theoretical means to solve them.[73]

Von Mises did not present a developed theory of turbulence in his 1914 book *Elemente der Technischen Hydrodynamik*, which appeared two years after his paper in the *Weber Festschrift*. He stressed again that no one had yet developed a "complete theory of turbulence in any sense" and that, in particular, research "is dependent on empirical data to predict the conditions under which laminar motion changes into a turbulent one."[74]

## VON MISES'S TURN TO AVIATION IN STRASSBURG BEFORE WWI

During his time in Strassburg (1909–14), von Mises's research was not restricted to the foundations of fluid dynamics. The most influential of all of his publications in applied mechanics was his 1913 paper on plasticity theory, presented to the Göttingen Academy of Sciences by Carl Runge.[75] It contained his yield condition for the transition from the elastic to the plastic state in ductile materials, today referred to as "von Mises stress," which students of engineering all learn. The 1912 paper also contains tentative equations for plastic flow, which von Mises considered in analogy to the Navier-Stokes equations in fluid dynamics.

Of a more practical nature was von Mises's turn to aviation during this period. As early as May 1909, von Mises planned lectures "on the scientific foundations of the technology of flight" (*wissenschaftliche Grundlagen der Flugtechnik*) in Brünn. Part of his motivation was to cater to a need he felt to not ignore a "modern" field such as aviation. In a letter to his mother on May 15, 1909, he wrote: "At least I cannot be accused of not being modern enough."[76]

73. Interestingly enough, the fluid dynamicists (as opposed to the mathematicians) seem to have considered von Mises's and Hopf's solutions as mathematically complete, though only useable in very restricted cases. See Hermann Schlichting, *Boundary-Layer Theory*, 6th ed. (New York: McGraw-Hill, 1968), 447.

74. Von Mises, *Elemente* (ref. 42), 36.

75. Von Mises, "Mechanik" (ref. 26).

76. RVMP, Letters and postcards to his mother (6 boxes), HUG 4574.5.2, box 2, correspondence 1904–12.

In Strassburg, von Mises would give lectures in "Aeromechanics" (1910) and "Probability Theory" (1911), both very modern topics indeed.[77] Additionally, von Mises gave public lectures on the theory and practice of motor-powered flight. Because Strassburg had been under German rule since the Franco-Prussian war of 1870–71, von Mises's presentations also had a clear political connotation. Von Mises had no military background, but he could not avoid being involved with the German military in this potentially war-relevant discipline. In 1911, he was elected as head of the division for airplanes within the South-West Group of the German Association for Aviation.[78]

The Prince Heinrich Flight Competition drew von Mises fully into the military. He took responsibility for the technical and scientific preparations for the competition, organized annually in the spring under military command from 1911 in the Strassburg environs. From 1913, it was under the protection and name of Prince Heinrich, the younger brother of the emperor, Wilhelm II.

During his time as professor of applied mathematics in Strassburg, there were moments when von Mises could have conducted more systematic research in aerodynamics, including experimental work. In particular, in 1912, von Mises's career could have taken a different path. His former teacher and superior at Brünn, Georg Hamel, was a professor at the Technical University in Aachen. He told von Mises in a letter on November 22, 1912, about the decision by the faculty to propose both von Mises and von Kármán as successor to Hans Reissner, the professor of mechanics, who had recently been appointed to Berlin, *aequo loco*, i.e., with the same priority on the candidacy list. Hamel wrote:

> The conditions are very good here, except for Stark, whose attitude vis-à-vis us is really unpleasant. . . .
>
> As a very important appendix to the position, I have to mention the aerodynamic institute, which is nearly finished. This is, of course, a huge burden, but on the other hand certainly a big attraction for you.[79]

77. See *Verzeichnis der Vorlesungen welche an der Kaiser Wilhelms Universität . . . gehalten werden* (Strassburg: Universitätsdruckerei 1910/11).

78. RVMP, Von Mises to his mother, 4 Nov 1911 (ref. 76). The original German name for the group was Südwestgruppe des Deutschen Luftfahrerverbandes.

79. RVMP, HUG 4574.5, box 1, folder 1912. Hamel alludes to the fact that Johannes Stark, later a Nobel laureate and supporter of National Socialism, opposed the appointment of both von Mises and von Kármán. Anti-Semitic sentiment on Stark's part is very likely part of this context, although Stark's opinion is not attached to Hamel's letter.

One can safely assume that the aerodynamic institute with its experimental facilities would have been "a big attraction" for von Mises. Hamel, however, had been a bit rash. Six weeks later, on New Year's Eve, he wrote von Mises the following:

> The decision has been taken: Kármán is coming. Our proposal went smoothly through the Senate. Waumann asked Prandtl, who had worked more experimentally—in Berlin[80] they seem to focus everything on the aerodynamic institute and thus lose perspective. P. answered, K., and so the latter got the appointment.
>
> Well there is nothing we can do. Let us hope for the next occasion to come together.[81]

Before the war broke out in summer 1914, von Mises's theoretical work in aerodynamics was largely restricted to technical reports for the Prince Heinrich Flight Competition. He was heavily involved in the evaluation of suitable motors for the airplanes. (His qualification for the problem of motor power was documented in his article on mechanical engineering in the Mathematical Encyclopedia.[82]) Von Mises tried, in particular, to find rational criteria to compare the performance of civil airplanes (which usually had weaker and therefore lighter motors) with the performance of military airplanes.

In his article "The Evaluation of Flight Performances during Competitions,"[83] von Mises argued that both the lift and the drag forces produced by the airplane engine were dependent on the square of the velocity $v$, while the power was proportional to $v^3$. This meant that in order to double the velocity (or to halve the time needed to cover a certain distance), the plane had to exert eight times as much power. Based on this assumption von Mises saw sufficient mathematical reason for his proposal to multiply the actual flight time by a factor depending on the third root of the power of the engine, thus requiring planes equipped with stronger machines to complete the flight correspondingly quicker. This third root then appeared in the specifications for the Prince Heinrich Flight Competition.[84]

80. The ministry decided which of the two men was to get the appointment.

81. RVMP, Hamel to von Mises, 31 Dec 1912, HUG 4574.5, box 1, folder 1912.

82. Von Mises, "Dynamische Probleme" (ref. 36).

83. Richard von Mises, "Über die Bewertung von Flugleistungen bei Wettbewerben," *Deutsche Luftfahrer-Zeitschrift* 17 (1913): 59–62.

84. Federal Archives Germany: Military Archives Freiburg, PH 9V/130 "Fliegerwesen: Prinz Heinrich Flug 1913", "Ausschreibung", sheets 23–30a, sheet 26.

Compared to the accumulating knowledge in aerodynamics, e.g., with Nikolay Zhukovsky's[85] and Wilhelm Kutta's two-dimensional theory of airfoils, von Mises's work on the criteria for flight competitions was theoretically rather crude. Nor could the results from flight competitions be compared with the data systematically collected in the fledgling experimental facilities at Göttingen and Aachen.

Parallel to his theoretical work von Mises trained for a pilot license at the Berlin-Johannisthal airfield, which he received in February 1914. Von Mises's training was sponsored by the German National Flying Fund (Nationalflugspende), which stipulated that their grantees offer their services to the German military in the case of a war.[86]

Von Mises would have more practical encounters with aerodynamics in the following years, both as a pilot and as an engineer and mathematician, namely in the Austrian air force during World War I. This involvement led subsequently to von Mises's probably most important contribution to aerodynamics, his two-dimensional wing theory.

## VON MISES IN THE AUSTRIAN AIR FORCE: MILITARY PRACTICE AND HIS THEORY OF STABILITY IN TWO-DIMENSIONAL WING THEORY (1917–20)[87]

When the war broke out in summer 1914, von Mises asked the German authorities to be released from the military duties attached to his funding from the Nationalflugspende, and the request was granted.[88]

Von Mises was trained as a pilot for the Austrian Air Force in Vienna (k.u.k. Luftschiffer-Abteilung) over a four-week period in August 1914. Shortly thereafter he was deployed to supervise the procurement of war-relevant airplane parts. On the order of his commanders, and without compensation being

85. Zhukovsky's name was then mostly spelled Joukowski.

86. This is mentioned, for instance, in a letter by the then-leading German pilot and technical director of the Albatros aircraft company in Berlin, Hellmuth Hirth, to von Mises, dated 5 Mar 1913. RVMP, HUG 4574.5, box 1, folder 1913.

87. The biographical facts are all documented in the Richard von Mises Papers at Harvard University Archives and at WAV, Abteilung 5/L [Luftfahrt] and Bauabteilung der k.u.k. Luftfahrtruppen.

88. Von Mises, Vienna, to Kurator of Strassburg University, 21 Nov 1914, Archives Humboldt University Berlin, Personnel file Richard von Mises, UK M 220, part II, sheet 19. Von Mises's Strassburg personnel file has been incorporated as part II into his Berlin file.

offered to the companies, von Mises confiscated tooling machines in Budapest for the production of airplanes.

Initially conscripted without an officer's diploma, von Mises obtained the rank of second lieutenant in 1915, lieutenant in December 1916, and senior lieutenant in 1918. He returned to the service as a pilot toward the end of the war without, however, actually seeing combat.

Von Mises's main activity in 1915–16 was the construction of a giant 600-horse-power aircraft,[89] which never went into service, mainly due to motor problems. It was primarily planned to serve as a bomber rather than for transport. The Austrians ultimately had to look for German planes to satisfy their needs.

Von Mises also gave courses on Theory of Flight for Austrian officers, which later were published with Springer in Berlin as his successful 1918 book.[90]

In 1937, von Mises traced his ideas about a new kind of air wing with less "travel of the center of pressure" to his design research in the Austrian Air Force:

> In November 1915, I was commissioned by the [imperial] & [royal] air traffic arsenal to design a 600-horse-power giant aircraft [*Grossflugzeug*] and to supervise its construction . . .
>
> Specific attention was directed to the choice of wing profile. As to theoretical results, there was nothing available at that time except for the investigations by Kutta and Joukowski, which dealt with special profiles that could not be immediately used in practice. As to experimental data, one had basically only the results by Eiffel. I connected my work to the Eiffel profile no. 32, but gave it a stronger S-bent form [on the pressure side] in order to reduce the travel of the center of pressure, which I found dangerous given the considerable length of the profile. My later publications in 1917 and 1920, where I gave for the first time lift formulas for arbitrary profiles and where the problem of the travel of the center of pressure was solved theoretically, have proved the correctness of my conjectures in 1915: one can reach profiles with fixed centers of pressure by appropriate bending [*Aufbiegung*] of the profiles.[91]

89. The so-called *Grossflugzeug* is described in detail in M. F. Eacock, "A Twin-engined Bomber for Austria-Hungary: The von Mises-Aviatik G," *Cross and Cockade: Journal of the Society of World War I Aero Historians* 3, no. 4 (1962): 295–310.

90. Richard von Mises, *Fluglehre: Vorträge über Theorie und Berechnung der Flugzeuge in elementarer Darstellung* (Berlin: Springer, 1918). The Austrian printing figured officially as the third edition of von Mises's course for Austrian officers. Apparently, the former editions had been mimeographed versions.

91. Richard von Mises, "Ein 600 PS-Grossflugzeug vom Jahre 1916," in *Beiträge zur Flugtechnik. Denkschrift des Aeromechanischen Laboratoriums der Technischen Hochschule in Wien*, ed.

Interestingly, von Mises referred here to a three-step procedure: previous French experimental data (of Gustave Eiffel!) had inspired him with theoretical ideas, for which the designing during the war offered some confirmation (wings of the *Grossflugzeug*), but which nevertheless had to be verified theoretically afterward (in his paper of 1917/20, mentioned below). The decisive words in von Mises's quotation of 1937 are the "travel of the center of pressure" and the investigation of "profiles with fixed centers of pressure."

Von Mises's main theoretical contribution to two-dimensional airfoil theory[92] was his two-part publication of 1917 and 1920 in the *Zeitschrift für Flugtechnik und Motorluftschiffahrt*.[93] This paper, and particularly its second part in 1920, contains his two main results in two-dimensional wing theory: the von Mises profiles (sometimes called the "von Mises family" of airfoils) and his theory of the focus or "aerodynamic center" of the airfoil, which lies approximately one quarter the chord length from the leading edge of the wing (quarter chord point, see Fig. 2). The focus is a point on the airfoil, in relation to which the moment of lift remains constant for any angle of incidence. It had been experimentally known and used in work on the stability of airplanes during flight by Zhukovsky and the British applied mathematicians E. I. Routh and G. H. Bryan.[94] Von Mises, however, *showed the mathematical existence of the focus* with methods from conformal mappings. He proudly and repeatedly emphasized this in the 1945 English edition of *Theory of Flight*, where he added: "This result has subsequently been confirmed by practically all experiments."[95]

Von Mises took his work one step further and developed a general theory of the "metacentric parabola" related to the focus of the airfoil. By using more

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R. Katzmayer (Vienna: Springer 1937), 9 pp., quoted from von Mises, *Selected Papers* (ref. 22), 1 (1963): 530–40, 530–33.

92. Such theory neglects to first approximate the drag occurring at the wing tips and considers only the two-dimensional section through the wing.

93. Von Mises, "Tragflächenauftrieb" (ref. 23).

94. The first use of the phrase "aerodynamic center" is unknown to me. It is apparently not used in Durand, *Aerodynamic Theory* (ref. 14) of 1934–36. However, one finds "metacentre" in older publications in the 19th century on stability in shipbuilding, the metacenter being different from the "centre of gravity." Bryan discusses the "equation of moments about the metacentre" in his 1911 book, which was well-known to von Mises. Cf. George H. Bryan, *Stability in Aviation: An Introduction to Dynamical Stability as applied to the Motions of Aeroplanes* (London: Macmillan, 1911), 85.

95. Richard von Mises, *Theory of Flight: With the Collaboration of W. Prager and G. Kuerti* (New York: McGraw-Hill, 1945), 147. That von Mises referred to his own result here is clear from pp. 185–86.

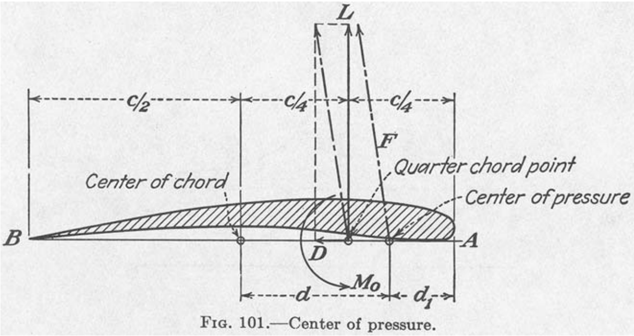


**FIG. 2.** Wing section with aerodynamic center at Quarter Chord Point, according to von Mises, *Theory of Flight* (ref. 95), 145.

general conformal mappings than had hitherto been studied, he was able to derive types of airfoils: e.g., "S-bent" (as in the quote above) that secured greater flight stability for changing angles of incidence of the airfoil due to a fixed center of pressure, that is, the point of intersection between the line of action of the lift and the chord of the airfoil. For these profiles the lift moment for the focus is not just *constant* but *zero*, letting the line of action of the lift always pass through the focus for any angle of incidence. Von Mises felt that the name "aerodynamic center" should be reserved for wing profiles with this type of focal point.[96]

The problem with von Mises airfoils was that they—unlike simpler types, such as the Kármán-Trefftz profiles—basically remained in the realm of mathematics. The theory was complemented neither by systematic numerical and geometrical methods of construction of special types, nor by the production and testing of material prototypes.[97] The metacentric parabola and its focus were in a sense purely theoretical concepts. They lacked the immediate physical, empirical meaning of the "center of pressure," which usually differed from the focus and varied with the angle of incidence. However, by their

96. Ibid., 188. According to von Mises's student in the United States, Geoffrey Ludford, the "... von Mises profiles [are] a beautiful generalization of Joukowski profiles that, for all practical purposes, completed the mapping theory." See Geoffrey S. S. Ludford, "Mechanics in the Applied-Mathematical World of von Mises," *ZAMM* 63 (1983): 281–82, 281.

97. For example, the von Mises profiles were not mentioned in Ira H. Abbott and Albert E. Doenhoff, *Theory of Wing Sections Including a Summary of Airfoil Data* (New York: McGraw-Hill, 1949), and only marginally in Friedrich Wilhelm Riegels, *Aerofoil Sections. Results from Wind-Tunnel Investigations. Theoretical Foundations* (London: Butterworths, 1961), the latter based on a German original of 1958.

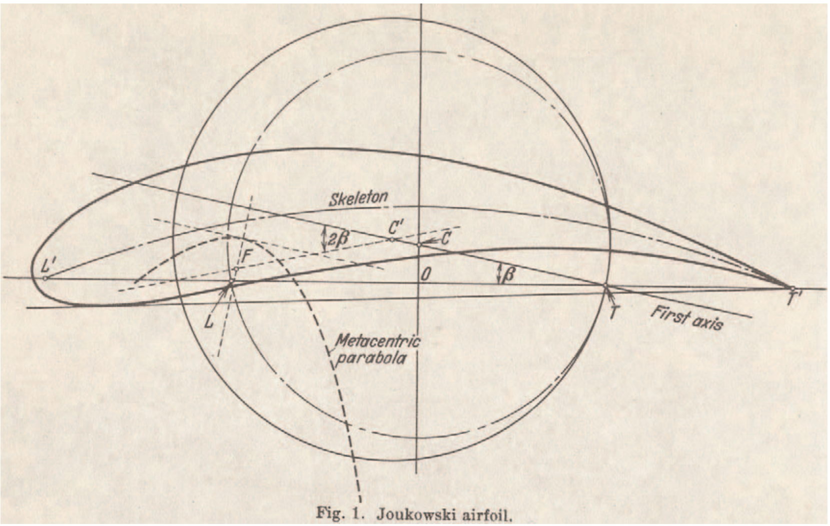


**FIG. 3.** Von Mises's "metacentric parabola" in von Kármán and Jan M. Burgers, "General Aerodynamic Theory" (ref. 98), plate 2, not mentioning von Mises's name.

mathematical properties, the metacentric parabola and focus enabled the mathematical description and analytical location of the lines of lift action to be discerned. The term "aerodynamic center" is still used in airfoil theory; its geometrical properties and existence are recognized, but von Mises's name is rarely connected to these notions. Von Kármán and Burgers, in the second volume of Durand's *Aerodynamic Theory*, called von Mises's methods, without mentioning his name, "a rather elegant presentation of the subject, which explains their popularity, especially with mathematicians."[98] Thus, the generality and elegance of von Mises's approach was neither prioritized nor fully appreciated by his fellow fluid dynamicists.

It shows, however, that von Mises was eager to illustrate the practical usefulness of his formulas for the infinite wing. In fact, in 1922, von Mises

98. Theodore von Kármán and Jan M. Burgers, "General Aerodynamic Theory—Perfect Fluids," which comprises the entire volume 2 of Durand, *Aerodynamic Theory* (ref. 14), 1935, 25. The authors do not connect von Mises's name to the "metacentric parabola," which they mention on pages 64ff., Fig. 3. However, they discuss the "Mises Family of Airfoils" (77–80). Von Mises's contribution is clearly acknowledged in Paul F. Neményi, "The Main Concepts and Ideas of Fluid Mechanics in Their Historical Development," *Archive for History of Exact Sciences* 2 (1962): 52–86, 83. The most detailed and appreciative description of the von Mises theory of airfoils and lift moments I found in Louis M. Milne-Thomson, *Theoretical Aerodynamics* (London: Macmillan, 1948).

considered the data coming from the Göttingen wind tunnel as experimental confirmation of his theoretical results. He published a short article under the heading "Small Communications" in his journal *ZAMM*, which was soon translated into English by the American National Advisory Committee for Aeronautics (NACA).[99]

## VON MISES IN BERLIN (1920–33): INSTITUTIONAL INNOVATION AND COMPETITION, AND INTERNATIONAL COMMUNICATION

After Germany and its allies were defeated, von Mises had to leave Strassburg in late 1918. He spent six months at Frankfort University before becoming professor of mechanics at the Technical University in Dresden. He did not, however, stay long.

On January 19, 1920, von Mises wrote with some pride to his friend and competitor von Kármán: "As you know I am decided to settle in Berlin around Easter and take over a large-scale scientific enterprise [*wissenschaftlicher Grossbetrieb*]."[100] One cannot help feeling that von Mises thought of his future institute for applied mathematics as equivalent to Kármán's aerodynamic institute in Aachen. However, this turned out to be another exaggerated dream of von Mises. His institute was much smaller. Despite training a number of students, he basically had one-and-a-half assistantships. Von Mises also did not have any experimental facilities, and computational equipment was restricted, too.

His word "large-scale scientific enterprise" applied more to his journal *ZAMM*, which he wrote for and edited almost single-handedly. The very first article published in *ZAMM* was the programmatic paper "Tasks and Goals of Applied Mathematics,"[101] which provides an insightful overview of the state of the discipline in the pre-computing age, but has, unfortunately, never been translated into English.

99. Richard von Mises, "Zur Lage der Auftriebsresultierenden von Tragflächen," *ZAMM* 2 (1922): 71–73, 71; and von Mises, "Location of Center of Pressure of Airplane Wings," American National Advisory Committee for Aeronautics (1922), 7 pp.

100. Theodore von Kármán Papers, Archives of California Institute of Technology, Pasadena, 20.35. Apparently, von Mises was alluding to a notion introduced in 1905 by the church historian and president of Kaiser-Wilhelm-Gesellschaft for the Promotion of Science (founded 1911), Adolf von Harnack.

101. Von Mises, "Über die Aufgaben" (ref. 6).

Equally important as *ZAMM* was the foundation of the Society for Applied Mathematics and Mechanics (GAMM) in 1922, with Ludwig Prandtl as chairman and von Mises as managing director (*Geschäftsführer*). Hans Reissner served on the board as well. There were some early tensions between the applied mathematician (von Mises) and the experimental and theoretical aerodynamicist (Prandtl), in part because the latter wanted to avoid a "dominance of mathematics" and preferred a name for the society that pointed more clearly to the engineering context.[102] However, von Mises's proposal prevailed and the society managed a successful integration of the two fields, in so far as it included the mathematics attached to engineering problems.

In addition to *ZAMM* and GAMM, and von Mises's institute, another important institutional innovation in the 1920s has to be mentioned. International Congresses for Applied Mechanics were held at frequent intervals from 1924 onward.[103] However, the process of internationalization, partly represented by the congresses, did not remain undisturbed by political problems in the aftermath of WWI, as France and Belgium enforced a scientific boycott against Germany and its former wartime allies. This in turn provoked nationalistic feelings among participants, including von Mises. Although he had not been as directly involved in the war as von Mises, Prandtl also remained skeptical about the resumption of scientific work with scholars from Western allies during WWI, particularly with the French and the Belgians.[104] This affected the preparation of the 1924 international congress for mechanics in Delft, so much so that the French withdrew from participating.[105]

As we have seen, Prandtl and von Mises held some of the same political resentments. However, this agreement ended when Prandtl's institutional interests, and his solidarity with his colleagues in Göttingen, was affected. In 1928, when von Mises—together with several mathematicians from Berlin, such as Ludwig Bieberbach and Erhard Schmidt, and supported by the Dutch mathematician L.E.J. Brouwer—opposed the participation of German mathematicians at the International Congress of (pure) Mathematicians

102. Eckert, *Ludwig Prandtl* (ref. 7), 146.

103. Eberhard and Juhasz, *IUTAM* (ref. 3). After World War II, the Congresses added "Theoretical and" to "Applied Mechanics" in their name.

104. Eckert, *Ludwig Prandtl* (ref. 7), 128ff. Prandtl received much financial support from the Americans in exchange for scientific advice throughout the 1920s.

105. More on Prandtl's and von Mises's concerns related to reconciliation with former war enemies can be found in their correspondence in the Max Planck Society Archives in Berlin. LPP, div. III, rep. 61, no. 1081.

(ICM) in Bologna, Prandtl initially agreed with their position. But, he was persuaded by the arguments of his closer colleagues in Göttingen, in particular Richard Courant and David Hilbert, that Bologna was an opportunity for Germans to return with honor to the international scene.[106]

Clearly, Prandtl's institutional solidarity with Göttingen prevailed in this instance. In a similar vein Prandtl would react against von Mises's claims about an alleged decline of applied mathematics at Göttingen, to which Ostrowski alluded in hindsight, in his 1966 article.[107] After Runge's death in 1927, Richard Courant in Göttingen had written an obituary in the widely read *Die Naturwissenschaften* in which he argued that addressing the special concerns of applied mathematics had become obsolete, and the institutional divide between pure and applied mathematics was no longer necessary.[108]

Personal jealousy on von Mises's part was also involved in his opposition to Göttingen. In his 1928 review of Volume VII, Mechanics of Fluid and Gaseous Bodies, of the German *Handbuch der Physik* (Handbook of Physics), von Mises criticized the article by the Göttingen aerodynamicist Albert Betz, Prandtl's deputy in the Kaiser-Wilhelm Institute, on airfoils and hydraulic machines.[109] Alluding to the neglect in this article of his contribution to fluid dynamics, von Mises wrote:

> One is meanwhile used to the fact that original works from Göttingen (not just in mechanics) present everything as though no science worth of any note is being done "extra Gottingiam" [outside Göttingen].[110]

Von Mises used other occasions to speak out in his journal about what he felt went wrong in Göttingen. In a short note in the first issue of 1930, von Mises reiterated his criticism of Courant in *Die Naturwissenschaften* and alluded to

106. For details and documentation, see again correspondence with von Mises in Prandtl's Nachlass at LPP, div. III, rep. 61, no. 1081. Since Prandtl's famous 1904 talk on the boundary layer, "Über Flüssigkeitsbewegung" (ref. 35), had been presented at the ICM in Heidelberg, Prandtl's participation in Bologna would not have been unthinkable.

107. Ostrowski, "Zur Entwicklung" (ref. 12).

108. Richard von Mises, "Pflege der angewandten Mathematik in Deutschland," *Die Naturwissenschaften* 15 (1927), 473. One consequence was that Gustav Herglotz became Runge's successor. With some justification, von Mises did not consider Herglotz an applied mathematician.

109. Those were, of course, von Mises's two foremost fields of expertise within fluid dynamics: turbine and wing theory.

110. Richard von Mises, review of *Handbuch der Physik*, vols. VII and V, ed. Richard Grammel, *ZAMM* 8 (1928): 76–77.

"the regretful development in Göttingen, where the heritage of Felix Klein is quickly wasted."[111]

In a letter to von Mises on March 21, 1930, Prandtl showed, once again, institutional solidarity with Courant and protested against this and similar remarks by von Mises in *ZAMM* about the Göttingen mathematical institute. He wrote:

> If you would have seen the good rooms for practicing and the nice collection of mathematical instruments in the Institute, you would probably dropped the idea that Klein's traditions have been neglected here.[112]

Prandtl's allusion to the rather lavish equipment in Göttingen was not well received by von Mises, who worked under more modest conditions. Von Mises replied:

> It is beyond me what beautiful rooms for practicing have to do with the fact that applied mathematics is taught by complete beginners. As far as I can judge, the impression that applied mathematics is forced out—spearheaded by Göttingen—from German universities once again, is generally shared.[113]

In reality, of course, von Mises's criticism was much more directed toward *pure mathematicians* and their ideology than against Courant in Göttingen, who after all had an open mind for applied mathematics and fought for it on many levels. In his article in *Die Naturwissenschaften*, von Mises gave the following description:

> The overwhelming majority of our university teachers declare with more or less pride—at least, however, with full justification—that they are unable to perform the smallest numerical calculation or geometrical construction.[114]

This seems an apt description of the low esteem in which applied mathematics was held at the time in the camp of pure mathematicians, and shows that von Mises always had to fight in different directions, and not only against neglect of mathematics among engineers. I have shown elsewhere how von Mises, at about the same time, had to struggle to secure the habilitation

111. Richard von Mises, "Hochschultagung Dresden 1928," *ZAMM* 10 (1930), 103–104, 104.
112. Prandtl's letter and the following by von Mises are in LPP, div. III, rep. 61, no. 1082.
113. Ibid., von Mises to Prandtl, 24 Mar 1930.
114. Von Mises, "Pflege" (ref. 108), 473.

(permission to teach) for his assistant and future wife, Hilda Geiringer.[115] Other publications have shown how strongly many pure mathematicians opposed von Mises's work on the foundations of probability theory, which was strongly inspired by applied mathematics.[116] This leads to one aspect of von Mises work in applied mathematics, which is connected to fluid dynamics as well, though probably less directly, and will be dealt with in the following section.

## VON MISES'S PROBABILISTIC AND PHILOSOPHICAL VIEWS WITH RESPECT TO FLUID DYNAMICS[117]

There are two reasons for reflecting on the relationship between von Mises's probabilistic and philosophical convictions and his work on fluid dynamics at this point. The first reason is that von Mises's research on stochastics (i.e., probability and statistics) in the 1920s occupied about half of his research capacity and clearly eclipsed his work in fluid dynamics proper, at least temporarily until his emigration to the United States in 1939.

The second reason for going into von Mises's philosophical positions is his very public appearance in discussions about causality and determinism in physics during his Berlin years (1920–33), which culminated in his semi-popular 1928 book, *Probability, Statistics, and Truth*,[118] reaching beyond a scientific audience. As is well known to historians of science, this discussion plays a central role in Paul Forman's 1971 influential paper. Although von Mises receives attention in Forman's study, I do not consider von Mises as a typical example for "conversion to a-causality" under the influence of Weimar Culture, as Forman argues.[119]

115. Reinhard Siegmund-Schultze, "Hilda Geiringer-von Mises, Charlier Series, Ideology, and the Human Side of the Emancipation of Applied Mathematics at the University of Berlin during the 1920s," *Historia Mathematica* 20 (1993), 364–81.

116. As alluded to in Ostrowski's 1966 article, "Zur Entwicklung" (ref. 12), quoted above in ref. 20. See also Siegmund-Schultze, "Sets" (ref. 20), and Siegmund-Schultze, "Probability" (ref. 20).

117. Bloor, *The Enigma* (ref. 13) discusses von Mises, "Über die Probleme" (ref. 53) of 1908, predominantly from a philosophical point of view.

118. Richard von Mises, *Wahrscheinlichkeit, Statistik, und Wahrheit* (Wien: Springer, 1928), from 1939 (in English). It has been published in many German, English, and other editions and is in print even today.

119. Paul Forman, "Weimar Culture, Causality, and Quantum Theory, 1918–1927: Adaptation by German Physicists and Mathematicians to a Hostile Intellectual Environment," *Historical Studies in the Physical Sciences* 3 (1971): 1–115, 55, 82. Forman's paper is still stimulating, but for the concrete case of von Mises, his interpretation has been debated. See, for example,

Generally, one has to be aware of von Mises's deep-rooted and multifaceted scientific, philosophical, and emotional attachment to his home country, Austria, and above all his reverence for the physicist and philosopher Ernst Mach.[120] Early entries in von Mises's personal diaries, around 1904, show a permanent occupation with the work of Mach and H. Poincaré. It remains unclear how much of von Mises's interest then was related to philosophy and, in particular, to positivism and conventionalism. In contrast, in the 1920s, von Mises's occasional participation in discussions of the Vienna Circle of empiricism philosophy is documented.[121] In 1938, von Mises published a long article on Mach. He also wrote the first textbook on logical positivism in 1939, which was translated into English in 1951.[122]

Of course—as von Mises said repeatedly himself—Mach had not been particularly interested in probability theory and reflected only a little on technical mathematics in his philosophical work. It was von Mises's broader positivistic epistemological convictions and discussions with his friend, Austrian physicist and philosopher of physics Philipp Frank, that determined

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Thomas Hochkirchen, *Die Axiomatisierung der Wahrscheinlichkeitsrechnung und ihre Kontexte.* (Göttingen: Vandenhoeck & Ruprecht, 1999); Reinhard Siegmund-Schultze, "Indeterminismus vor der Quantenmechanik: Richard von Mises's wahrscheinlichkeitstheoretischer Purismus in der Theorie physikalischer Prozesse," in *Mathematics Meets Physics: A Contribution to Their Interaction in the 19th and the First Half of the 20th Century*, ed. Karl-Heinz Schlote and Martina Schneider (Frankfurt a.M: Harri Deutsch, 2011), 241–70; and Michael Stöltzner in an unpublished paper in Bielefeld 2001, available at http://archiv.ub.uni-bielefeld.de/wissensgesellschaft (accessed 17 Feb 2018).

120. This rational and emotional attachment includes not only Mach, but also von Mises's reverence to the Austrian philosopher and pioneer of the theory of aviation, Josef Popper-Lynkeus (1838–1921), a friend of Mach. Von Mises published repeatedly on Popper-Lynkeus and Mach. Even Forman admits that von Mises was a "loyal scion of Austrian positivism"; Forman, "Weimar culture" (ref. 119), 80. One should also not ignore von Mises's strong devotion to the Austrian poet Rainer Maria Rilke. Von Mises was an acknowledged authority on the early work of Rilke and published widely on him.

121. See Friedrich Stadler, *The Vienna Circle: Studies in the Origins, Development, and Influence of Logical Empiricism* (Cham, Switzerland: Springer, 2015), 457–66. For a general cultural approach to the probabilistic tradition in Vienna, see Deborah Coen, *Vienna in the Age of Uncertainty: Science, Liberalism & Private Life* (Chicago: University of Chicago Press, 2007), esp. 255–98, and on a more scientific level, Paul A. Hanle, "Indeterminacy before Heisenberg: The Case of Franz Exner and Erwin Schrödinger," *Historical Studies in the Physical Sciences* 10 (1979), 225–69. For the philosophy of the Vienna Circle and its relation to politics, see Nancy Cartwright, Jordi Cat, Lola Fleck, and Thomas E. Uebel, *Otto Neurath: Philosophy between Science and Politics* (Cambridge: Cambridge University Press, 1996).

122. Richard von Mises, *Positivism: A Study in Human Understanding* (Cambridge, MA: Harvard University Press, 1951), 180.

von Mises's use and interpretation of modern mathematical theories such as probability theory.

Von Mises's efforts to generalize the traditional notions of determinism and causality were above all connected to his conviction of the power of mathematics to make old and rigid physical models superfluous and replace them with more general and flexible ones, which allowed for a mathematical description of observed quantities. This was part of a broader trend toward mathematical modeling and not restricted to a probabilistic view.[123] Work in statistical mechanics provoked von Mises to write in a paper on Brownian motion in 1920:

> Our standpoint with respect to the determinacy postulate of mechanics is therefore the following: only if the mechanical system is defined with all relevant side-appearances, irregularities etc. . . . the differential equations of motion allow to calculate from the initial state the consecutive states. For the *idealized* system—and this is the one we have exclusively to deal with in statistical mechanics—that determinacy is not secured: here the mechanical approach fails and only the theory of probability leads to some assertion—if of a different kind—about the form of the motion.[124]

This also characterizes von Mises's skepticism vis-à-vis the model of the Navier-Stokes equations in fluid mechanics. In fact, in contrast to this measured opinion, von Mises was quite explicit one year later in his 1921 presentation before the German Mathematicians' Association in Jena, when he said that the phenomena of turbulence "point categorically toward a completely different kind of reasoning . . . mechanical statistics."[125]

Forman has emphasized that the title of von Mises's talk—"On the Present Crisis in Mechanics"—shows that under the conditions of Weimar Culture, "a conversion to acausality carried with it significant social approbation, social rewards so substantial that von Mises could not bear to let the atomic physicists monopolize them."[126] Forman may be right that von Mises enjoyed the

123. Bloor, *The Enigma* (ref. 13, 180) comes with respect to von Mises, "Über die Probleme" (ref. 53), to the following more general conclusion: "Von Mises adopted an empiricist or 'positivist' stance toward the equations of fluid dynamics and treated both the Euler and the Stokes equations as abstractions."

124. Richard von Mises, "Ausschaltung der Ergodenhypothese in der physikalischen Statistik," *Physikalische Zeitschrift* 21 (1920): 225–32, 256–62, 231.

125. Richard von Mises, "Über die gegenwärtige Krise der Mechanik," *ZAMM* 1 (1921): 425–31, 428, reprinted in *Die Naturwissenschaften* 10 (1922), 25–29.

126. Forman, "Weimar Culture" (ref. 119), 82.

"rewards," and some parts of his talk seem to indicate that he found this new statistical view even more revolutionary than general relativity.[127] But to count von Mises among the "mathematical physicists who went so far in assimilating the values and mood of their intellectual milieu as to effectively repudiate their own discipline"[128] can only be seen as a misunderstanding. Von Mises was not a *mathematical physicist* but an *applied mathematician*, and his "own discipline" crucially included probability and statistics and applications to classical mechanics. He certainly jumped on the occasion in his talk "On the Present Crisis in Mechanics" to propagandize his views.

Although von Mises never explicitly says so, it seems evident that his early engagement in turbine theory and fluid dynamics, in which he stressed the averages of physical quantities, was a major stimulus for his systematic approach to the foundations of probability and statistics. He published on these foundations for the first time in 1912 and gave a detailed discussion of them in 1919. These works informed all of his applications and campaigns for his studies of probability in the following decades. His notoriously controversial theory of "collectives"[129] was—for all its mathematical technicality—a kind of philosophical stance, which von Mises took above all vis-à-vis "pure" mathematicians. By his own admission the notion of "collective" (random sequence) was positioned between traditional physical and mathematical notions. Basing the notion of probability on the limit of relative frequency of occurrence of an event connected immediately to observations and thus kept the middle ground between hypothetical physical models and abstractly defined mathematical entities (such as sets and their measure), which cannot be "observed."[130]

A passage from von Mises's obituary, written by his friend, Philipp Frank, may serve to summarize von Mises's methodology and philosophy:

> The *problem of connection* between sense observations and abstract principles has always been the critical point in the philosophy of science. As we see the problem, it is tackled most precisely by the methods of applied mathematics, and it is in this sense that v. Mises dealt with the tasks of "Applied Mathematics and Mechanics," building upon the ideas of the great Austrian

127. Von Mises, "Gegenwärtige Krise" (ref. 125), 427.

128. Forman, "Weimar Culture" (ref. 119), 55.

129. Richard von Mises, "Grundlagen der Wahrscheinlichkeitsrechnung," *Mathematische Zeitschrift* 5 (1919): 52–99. See, for example, the discussion in Thomas Hochkirchen, *Die Axiomatisierung* (ref. 119).

130. Siegmund-Schultze, "A Non-conformist" (ref. 1), 354.

scientist and philosopher Ernst Mach, who regarded both science and its philosophy as theories of sensations.[131]

On various occasions von Mises explicitly polemicized against mixing statistical and traditional mechanical approaches.[132] Such mixed approaches had been used for decades by physicists and engineers alike and would be used in the years to come (by, e.g., Boltzmann, Einstein on Brownian motion, Smoluchowski). In turbulence theory, Prandtl's "mixing length" approach or Kármán's logarithmic theory[133] relied on empirical data that came in at each step of their calculations, creating new ad hoc hypotheses. Another mixed approach to turbulence was what was later called the "statistical kinetic approach,"[134] proposed by the English engineer Geoffrey I. Taylor, who developed it in more detail in 1935.[135]

One may of course speculate as to why von Mises did not go into more general (or abstract) stochastic work on turbulence, similar to the "statistical probabilistic approach"[136] cultivated in work by Kolmogorov, Heisenberg, and others. One tentative answer might be that his particular form of probability theory, based on "collectives," was not adapted or appropriate to the task, a point that has been made with respect to its failure to treat stochastic processes. However, von Mises, like many others, may have found the turbulence problem too hard to solve by *any* mathematical means. I am aware of only one major example of a follower of von Mises, the engineer Hans Gebelein (1907–85),[137] who tried—without much success—to apply von Mises's methods in probability and statistics to turbulence.

131. Philipp Frank, "The Work of Richard von Mises: 1883–1953," *Science* 119 (1954): 823–24.

132. Von Mises, "Ausschaltung" (ref. 124). He expressed his methodological convictions more systematically in von Mises, "Gegenwärtige Krise" (ref. 125). On von Mises's "probabilistic purism" in the theory of physical processes, see Siegmund-Schultze, "Indeterminismus" (ref. 119).

133. Eckert, *The Dawn* (ref. 3), 121, and Giovanni Battimelli, "The Mathematician and the Engineer: Statistical Theories of Turbulence in the 20s," *Rivista di storia scienze* 1 (1984): 73–94.

134. Marie Farge and Etienne Guyon, "A Philosophical and Historical Journey through Mixing and Fully-Developed Turbulence," in *Mixing: Chaos and Turbulence*, ed. Hugues Chaté, Emmanuel Villermaux, and Jean-Marc Chomaz (New York, Boston, Dordrecht: Kluwer, 1999), 11–36, 19–21.

135. Commentary by von Mises on Taylor's statistical theory, whom he otherwise held in high regard, is lacking. It is revealing in this context that Taylor, for his part, was not interested in the purely statistical approach by Kolmogorov developed in the 1930s, as noted by Farge and Guyon, "Turbulence" (ref. 134), 21.

136. Ibid., 21–23.

137. Hans Gebelein, *Turbulenz* (Berlin: Springer, 1935). According to Farge and Gyon, "Turbulence" (ref.134), 21, "the probabilist approach was initiated by Gebelein in 1935." However,

## FLUID DYNAMICS IN *ZAMM* AND THE DISCUSSION ON THE "VON MISES TRANSFORMATION" IN BOUNDARY-LAYER THEORY

Fluid dynamics and turbulence in particular were major topics in von Mises's journal. Important contributions by Theodore von Kármán, Fritz Noether, Johann Nikuradse, Karl Pohlhausen, Ludwig Prandtl, Otto Tietjens, and Walter Tollmien appeared in *ZAMM* between 1921 and 1933, when von Mises finally had to leave Berlin. Already in his introductory 1921 programmatic paper in *ZAMM*, von Mises had connected recent research by Prandtl on three-dimensional airfoils (which took into account the drag originating from vortices at the wing tips) to the long-standing turbulence problem. On this occasion von Mises's revived his old conviction about the dominance of the Euler model of frictionless flow: "One realizes with surprise that the basic flow within the turbulent flow by and large follows the laws of ideal fluids."[138]

Von Mises himself published little on fluid dynamics during the years before his emigration. Nor did von Mises revive his old topic during his years in Turkey before he emigrated to the United States in 1939. He focused on other fields of research in mechanics (e.g., frameworks, strength of materials, plasticity, including vector methods), applied mathematics (e.g., numerical methods), and stochastics (largely published outside *ZAMM*). There were basically only three publications on fluid dynamics by von Mises between 1921 and 1933: the short, above-mentioned note in 1922 published in *ZAMM* under "Kleine Mitteilungen" on airfoil theory;[139] his 1927 "Remarks on Hydrodynamics," also published in *ZAMM*; and his 1931 Supplements or *Zusätze* to the German translation of the fifth English edition (1924) of Horace Lamb's *Hydrodynamics*.[140] In the forty-six pages of the *Zusätze*, von Mises deliberately and expressly abstained from adding recent results in hydrodynamics by researchers, such as Tullio Levi-Civita and Carl Oseen. He confined his commentary to his own contributions on turbines (1907–14), discharge coefficients (1917), airfoil theory (1917–20), and boundary-layer theory (1927).

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the authors give no explanation or hint to the book's impact. I have not found an analysis of Gebelein's book, and it is rarely quoted, but rather harshly criticized by von Mises and A. N. Kolmogorov in reviews around 1935.

138. Von Mises, "Über die Aufgaben" (ref. 6), 12.

139. Von Mises, "Zur Lage" (ref. 99).

140. Richard von Mises, "Zusätze zu Lambs Hydrodynamik," in *Lehrbuch der Hydrodynamik*, Horace Lamb, German translation by Elise Helly of the 5th English ed. (Leipzig, Berlin: Teubner, 1931), 817–62.

In spite of several, partly skeptical allusions to the then-new boundary-layer theory by Ludwig Prandtl, von Mises would finally contribute to boundary-layer[141] in his "Remarks on Hydrodynamics" published in *ZAMM*. In this paper, von Mises derived first the Prandtl equations from the Navier-Stokes equations, starting from a streamline inside the flow and not in the boundary layer. Von Mises connected this approach, as he did in 1908, to his convictions about the boundary layer and, in particular, to the "assumption . . . that there is no significant contrast between the boundary layer and the inner flow," such that the Navier-Stokes equations "above a certain *R* [Reynolds number] no longer express reality."[142] Von Mises also stressed in this context that "this can once become of great practical importance while currently applications are mostly restricted to the boundary."[143] He argued that his "rigorous form of derivation abolished any doubt whether the influence of the curvature had been fully taken into account,"[144] which von Mises did not see as guaranteed in Prandtl's more intuitive 1904 approach.

Then, von Mises proposed to simplify further the Prandtl equations by treating the stream function $\psi$ as one of the independent variables. He found that the following expression within the main Prandtl equation—rewritten for stream functions—depends only on the $x$ variable:

$$\frac{\delta\psi}{\delta y}\frac{\delta^2\psi}{\delta x\delta y} - \frac{\delta\psi}{\delta x}\frac{\delta^2\psi}{\delta y^2} - \frac{\delta^3\psi}{\delta y^3} = f(x)$$

Von Mises then replaced this by the following second-order equation for the "flow energy" $z$, a type of equation which, as von Mises emphasizes, is known from heat conduction:[145]

$$\frac{\delta z}{\delta x} = k\frac{\delta^2 z}{\delta\psi^2}$$

141. As von Kármán admitted, the boundary-layer theory had generally not found much attention in the first years after Prandtl's 1904 introduction of it. Theodore Kármán, "Mathematische Probleme der modernen Aerodynamik," reprinted in *Collected Works of Theodore von Kármán*, 4 vols. (London: Butterworths, 1956), vol. 2, 277–89, 285. On the slow recognition of this theory and, moreover, of the circulation theory of lift in the United Kingdom, see Bloor, *The Enigma* (ref. 13).

142. Richard von Mises, "Bemerkungen zur Hydrodynamik," *ZAMM* 7 (1927): 425–31, 426.

143. Ibid., 427.

144. Ibid.

145. Ibid., 427–28.

The initial reaction both by Prandtl and von Kármán to Mises's 1927 remarks was, however, rather reserved. An exchange[146] between von Mises and Prandtl in early 1928 raised the question of the superior degree of rigor that von Mises claimed for his paper. On January 25, 1928, Prandtl submitted a critical note on von Mises's 1927 paper to *ZAMM*—i.e., to its managing editor von Mises—acknowledging in the cover letter that it was "embarrassing for an editor to publish something against himself in his own journal."[147] Prandtl admitted in his critique that von Mises's "more general formulation in deriving the basic formulas [*Grundformeln*] indeed constitute[d] progress, because it [was] not burdened by intuitive ideas, which sometimes cause embarrassment for people who think more formally."[148]

As to the factual content and priority, however, Prandtl insisted that he had now found among his old manuscripts a calculation from May 1914 that was exactly in the same form as von Mises's basic formulas. Principally, Prandtl did not see much difference in their positions, except that von Mises, unlike Prandtl and his students, was looking for mathematical generality. Prandtl wrote:

> I myself know that my insufficient mathematical education frequently does not enable me to express myself clearly enough to prevent a clever mathematician from outwitting me with an artificial example. But I believe the choice of admissible functions is here a question of physical tact, and your example is not admissible.[149]

The claims in the discussion were clearly defined: an engineer with superior "physical tact" (Prandtl) opposed another engineer (von Mises) with superior mathematical education. Von Mises did not, however, have the benefit of a scientific environment with a stream of students and access to experimental data, such as the Göttingen facilities could offer.

This unpublished correspondence resulted in an article by Prandtl in *ZAMM* and a response by von Mises. Prandtl argued in a somewhat patronizing manner: "First I want to express my pleasure about the fact that now even

146. The correspondence between the two men on this topic is located in LPP, div. III, rep. 61, no. 1080. The discussion, in which an alleged mathematical "mistake" on von Mises's part played a minor role, is briefly summarized in Eckert's Prandtl biography. See Eckert, *Ludwig Prandtl* (ref. 7), 175–76.

147. LPP, Prandtl to von Mises, 25 Jan 1928, div. III, rep. 61, no. 1080, sheet 3.

148. Ibid., sheet 5.

149. LPP, Prandtl to von Mises, 21 Feb 1928, div. III, rep. 61, no. 1080, sheet 41.

outside the Göttingen circle—in which I take the liberty to count the Aachen Aerodynamic Institute—the theory of laminar friction layers or 'boundary layers' has been approached as well."[150] Prandtl did not admit von Mises's priority, instead he referred to his own (Prandtl's) earlier manuscript.

In 1932, in the sixth edition of *Hydrodynamics*, the senior English hydrodynamicist Horace Lamb called von Mises's 1927 remarks "an interesting independent treatment"[151] of the boundary-layer theory. In 1938, in a general article titled "On the Calculation of Boundary Layers," Prandtl found worth mentioning what later would be called the "von Mises transformation."[152] Prandtl also admitted that von Mises "had 'priority' in the usual sense, because [Prandtl] had not published [his] earlier result."[153] This concession appeared in an issue of *ZAMM* devoted to Erich Trefftz, the recently deceased former student of von Mises. Because von Mises was a Jew and then in Turkey, one may find in Prandtl's late admission something of an honorable gesture. In 1955, in the official *Festschrift* celebrating fifty years of boundary-layer theory, however, von Mises was not mentioned in the lead article by Walter Tollmien or in the bibliography, which contained forty-seven references. It was Betz, criticized by von Mises in his 1928 review of the *Handbuch der Physik* (see previous section), who mentioned von Mises's transformation as a kind of tribute to the recently (1953) deceased man in the *Festschrift*.[154]

Since then, the "von Mises transformation" from Cartesian coordinates $x$, $y$ to $x$, $\psi$ has been frequently used to represent two-dimensional stationary movement and incompressible media. Von Mises's method has also attained

150. Ludwig Prandtl, "Bemerkungen zur Hydrodynamik," *ZAMM* 8 (1928): 249–51, 249. This quote is, of course, justified in the broader sense by the fact that there were almost no publications outside of Göttingen on the boundary-layer theory in the first two decades of its existence. Cf. Itirô Tani, "History of Boundary Layer Theory," *Annual Review of Fluid Mechanics* 9 (1977): 87–111, 87.

151. Horace Lamb, *Hydrodynamics*, 6th ed. (Cambridge: Cambridge University Press, 1932), 685.

152. Karl Nickel, "Die Prandtlsche Grenzschichttheorie vom Blickpunkt eines Mathematikers aus gesehen," *Interner Bericht*, no. 1 (1972): 42 pp., 6 (Karlsruhe: University of Karlsruhe).

153. Ludwig Prandtl, "Zur Berechnung von Grenzschichten," *ZAMM* 18 (1938): 77–82, 79.

154. See Walter Tollmien, "50 Jahre Grenzschichttheorie: Ihre Entwicklung und Problematik," in *50 Jahre Grenzschichtforschung. Eine Festschrift in Originalbeiträgen*, ed. Henry Görtler and Walter Tollmien (Braunschweig: Vieweg, 1955), 1–12, which is rather self-advertising. Cf. also Albert Betz, "Zur Berechnung des Überganges laminarer Grenzschichten in die Aussenströmung" in the same *Festschrift*, 63–70, where the von Mises transformation is discussed on 63–64.

importance in the theory of nonlinear partial differential equations in more general engineering contexts.[155] In recent publications, particularly by Canadian mathematicians and engineers, the usefulness of the von Mises transformation for computational aspects (e.g., its convenience for finite difference methods)[156] of airfoil theory in transonic flows has been repeatedly stressed. One publication of 1995 acknowledges certain problems with the implementation of the "von Mises transformation" because of the occurrence of a singularity, but gives a method to overcome the latter, saying generally:

> Historically, the von Mises transformation received some fame in theoretical boundary-layer analysis, but recently it has been demonstrated that the transformation is also well suited for the numerical simulation of viscous and potential fluid flows in curvilinear domains and over bodies of arbitrary shapes.[157]

## VON MISES IN TURKISH AND AMERICAN EMIGRATION[158]

On January 30, 1933, Hitler became Reichskanzler with all the known disastrous historical consequences, among the first being the mass expulsions of Jewish scholars. These consequences, however, only gradually became visible. On von Mises's fiftieth birthday, on April 19, 1933, his colleagues from GAMM and from the broader VDI (Verein Deutscher Ingenieure) devoted an entire issue of *ZAMM* to von Mises and solicited many contributions from engineers and mathematicians both from Germany and abroad.[159]

155. In William F. Ames, *Nonlinear Partial Differential Equations in Engineering*, 2 vols. (New York: Academic Press, 1965, 1972), vol. 2, 153, it is discussed under the label "von Mises linearization."

156. R. K. Naeem and Ronald M. Barron, "Lifting Airfoil Calculations Using von Mises Variables," *Communications in Applied Numerical Methods* 5 (1989): 203–10, 203.

157. Robert Ford and Mohammad H. Hamdan, "Analysis of the Polar Form of the von Mises Transformation," *Applied Mathematics and Computation* 72 (1995): 205–17, 205.

158. More details on the emigration from Germany of von Mises and his future wife, Hilda Geiringer, may be found in Reinhard Siegmund-Schultze, *Mathematicians fleeing from Nazi Germany: Individual Fates and Global Impact* (Princeton, NJ, Oxford: Princeton University Press, 2009). The details presented in this paper are for the most part not included in that source, which describes the broader picture.

159. Another special issue of *ZAMM* devoted to von Mises was no. 7 of 1983, on the centenary of his birth, including biographical articles on von Mises by Birkhoff, "Harvard" (ref. 19), the famous numerical analyst Lothar Collatz, and Ludford, "Mechanics" (ref. 96).

In the congratulatory address, signed by the two organizations, and drafted by Hans Reissner,[160] one finds, along with kind words for von Mises, the following:

> This issue demonstrates that applied mathematics and mechanics, which work on less virginal ground than for instance the theories of atoms and relativity in neighboring fields, do not have a comparable public appeal as these revolutionary [*umstürzende*] theories. But the issue also shows that a more quiet but nevertheless international [*weltumspannende*] scientific community is progressing on this classical ground as well, and achieves new important results and possibilities.[161]

This passage emphasized, once again, the shared interests of disciplines such as applied mathematics and mechanics, here in opposition to theoretical physics, but often also felt in controversies with pure mathematicians.

Both von Mises and Reissner would soon be ousted by the Nazis as "Jews." In June 1933, both men left GAMM quietly. Writing from Berlin, on October 2, 1933, von Mises thanked Prandtl for a friendly letter that he had just received. He compared the reaction by GAMM to Nazi policies favorably with preemptive political submissiveness of other organizations,[162] when he stressed: "I appreciate, in particular if I compare it with procedures in other associations, in which dignified form our society drew the consequences of the current situation for which after all nobody who belongs to our circles has to take responsibility."[163]

Due to a clause in the Nazi law pertaining to former WWI frontline soldiers, von Mises was temporarily protected from dismissal in his job at Berlin University. But he was farsighted enough to see the impending discrimination and accepted a professorship at the new university in Istanbul (Turkey) in late 1933.

160. Reissner was the author of the untitled address, as Prandtl told von Mises in a letter dated 27 Apr 1933. LPP, div. III, rep. 61, no. 1082, sheet 62.

161. *ZAMM* 13 (1933), 65.

162. Von Mises must have thought here of the "Reichsverband" of Mathematical Societies under Hamel, once in Brünn and now in Berlin, and of the German Mathematicians' Association (DMV), in which the Nazi Ludwig Bieberbach was influential. Both associations showed much willingness to adapt to the Nazis in order to secure the position and funding of mathematics particularly at schools and universities. One has to, however, consider that the Nazis were more interested in engineers than in math teachers, which made the pure mathematicians more prone to submissiveness in order to secure their field. Cf. Herbert Mehrtens, "The 'Gleichschaltung' of Mathematical Societies in Nazi Germany," *The Mathematical Intelligencer* 11, no. 3 (1989), 48–60. From 1933, von Mises broke off his connections with his former superior Hamel and his former good friend Bieberbach.

163. LPP, von Mises to Prandtl, 2 Oct 1933, folder GAMM.

Arriving there in January 1934, he wrote a personal letter to Reissner in Berlin on his (Reissner's) sixtieth birthday:

> I hope that the occasional, somewhat excited discussions in the past have not left in you the impression that it would be quite nice not to have me at too close a distance. For my part I always liked being with you, in the house that you and your wife ran with so much hospitality.[164]

Hilda Geiringer—von Mises's former assistant who would later join him in Istanbul, but was for now a refugee in Brussels—also wrote to Reissner and alluded to the "well-known cool-headedness of our friend"[165] in Istanbul.

While aeronautical research in Germany reached unprecedented heights during Nazi re-armament, von Mises was increasingly cut off from these developments. In fact, during his exile in Turkey between 1933 and 1939, von Mises worked primarily on probability theory and on statistics, especially on the influential notion of "statistical functions."

When von Mises was approached by Springer Verlag in Berlin to republish his 1918 *Theory of Flight* (the 1933 fourth edition had sold out due to increasing demand in Germany), he wrote to his mother in Vienna on March 16, 1935, with a frustrated undertone:

> Meanwhile I am looking for somebody whom I can hand over the entire matter for the future and who would then—as it is usual after the death of an author—publish further editions. I am not interested at all to appear in front of the international public as a supporter of German armament.[166]

When the book appeared in Berlin in 1936, it was still under von Mises's name but had been revised by Kurt Hohenemser, who would survive in Germany as a so-called "half-Jew." The preface, which was signed by both von Mises and Hohenemser, states that von Mises had been hindered "by other work from staying in contact with progress in aviation."[167]

Von Mises was, indeed, losing "contact with aviation" in a deeper sense, which also prevented his participation in the monumental international six-volume report, *Aerodynamic Theory*. It was edited between 1934 and 1936 by the

164. Von Mises to H. Reissner, 14 Jan 1934. Hans Reissner Papers (6 boxes), box 1, folder 28, within Eric Reissner Papers MSS 0416, Mandeville Special Collections Library University of California, San Diego.

165. Ibid, Geiringer to H. Reissner, 16 Jan 1934, folder 32.

166. RVMP, HUG 4574.5.2, box 5 (ref. 76).

167. Von Mises, *Fluglehre* (ref. 90), 5th ed. (1936) v.

American mechanical engineer William Frederick Durand and published by Springer Verlag in Berlin.[168] The twenty-one articles were written by twenty-one different authors: eight were from the U.S. (at least two, von Kármán and M. Munk, were German immigrants). The other authors were German, British, French, Italian, Dutch, and Polish, all apparently in secure professional positions. It is not surprising that no Jewish scholar from Germany contributed to the report (e.g., L. Hopf, F. Noether, P. Nemenyi, Hohenemser, H. Reissner, von Mises); most of them were in insecure positions of transition or fleeing from Germany. Lively conversation between Durand and the Nazi aviation functionaries as late as 1938[169] did not provide a climate amenable to von Mises's or other emigrants' participation in the volume. This may well have contributed to the gradual obliteration of their names from the literature, although some of their work was quoted in the Durand volumes.

Von Mises kept a friendly relationship with Prandtl and congratulated him on his sixtieth birthday in 1935, calling him a "classic of our science" and assuming that "perhaps an inner factual relation still exists between us."[170] Prandtl remained in his position in Göttingen, which was hugely expanded as the Nazis recognized the importance of aeronautics to future war.[171] There is no doubt that the growth of his research facilities could not fail to influence and strengthen Prandtl's loyalties to the German political system—which explains, in addition to his traditional conservative political beliefs, some very embarrassing letters that he wrote to the English aerodynamicist Geoffrey I. Taylor and to Taylor's wife in defense of Nazi policies and of Hitler personally.[172] But at the same time, Prandtl experienced the damaging influence of anti-Semitic Nazi policies on his field of study and on international collaboration.

In February 1938, von Mises contributed to a memorial issue of *ZAMM*, dedicated to his student Erich Trefftz, the former editor of the journal, who had prematurely died in 1937. In a letter to Prandtl dated June 9, 1938, Dr. Dames from the Nazi Education Ministry complained that among the *ZAMM* authors were several "non-Aryans," and he asked whether Prandtl had been informed before about this fact. Prandtl replied on June 15 in a three-page letter

168. Durand, *Aerodynamic Theory* (ref. 14).

169. Eckert, *Ludwig Prandtl* (ref. 7), 251.

170. LPP, von Mises to Prandtl, 1 Feb 1935, div. III, rep. 61, no. 1082, sheet 67.

171. Herbert Mehrtens, "Mathematics and War: Germany, 1900–1945," in *National Military Establishments and the Advancement of Science and Technology: Studies in the 20th Century History*, ed. Paul Forman and J. M. Sánchez-Ron (Dordrecht, Boston, London: Kluwer, 1996), 87–134.

172. Eckert, *Ludwig Prandtl* (ref. 7), 261.

in which he insisted that foreign scientists had no understanding of the exclusion of Jews in Germany. In particular, he wrote:

> As to the non-Aryans mentioned by you, von Mises was Trefftz's doctoral advisor and, in addition, predecessor in his [Dresden] chair and could hardly be excluded on such an occasion. . . . Von Kármán, however, is due to his absolutely fundamental achievements so much above any critique, that his exclusion could not come to the mind of anybody who knows the facts.[173]

Apparently in order not to provoke the Nazi functionary further, Prandtl refrained from mentioning von Mises's other merits, such as the founding of the very journal *ZAMM* in 1921 in which this special issue was published.

This affair about the Trefftz memorial issue has to be understood in the context of Prandtl's effort to draw the International Congress for Applied Mechanics to Germany in 1942. D. Hoffmann and Eckert have described how this effort was doomed to failure because Prandtl, at the 1938 Congress at Harvard University, could not guarantee that Jewish scholars from Germany would be allowed to participate.[174] With the war imminent and foreign scientists and politicians increasingly shocked by Nazi atrocities, the prospects for continued international communication with German scientists began to wane. The Congress at Harvard chose Paris as the next venue, leaving von Mises, who took part in the Harvard Congress, with mixed feelings;[175] the outbreak of the war in 1939 would soon make discussions about impending congresses pointless.

Although von Mises served as the official representative from Turkey at the Harvard Congress, one gets the impression that he was primarily there to check for opportunities for future employment for himself. In fact, he made the long journey to the United States without presenting anything at the congress, whereas Prandtl, von Kármán, and also von Mises's colleague in

173. LPP, folder GAMM.

174. D. Hoffmann, cited in Ronald E. Doel, Dieter Hoffmann, and Nikolai Krementsov, "National States and International Science," *Osiris* (2) 20 (2005): 49–76. Eckert, *Ludwig Prandtl* (ref. 7), 191.

175. In his personal diaries, von Mises confesses that he favored Paris for scientific reasons but was disappointed that his proposal for Istanbul fell through. RVMP (ref. 25), 13 Sep 1938. This divided loyalty says something about the challenges a refugee may face when wanting to have a leading role in his discipline.

Istanbul, William Prager, were very visible.[176] This was of course partly a reflection of the fact that mechanics had not been von Mises's primary research area for some time.

One year later, von Mises—as ever, farsighted about political developments—felt that his position in Istanbul was no longer secure. In March 1939, he wrote to his old friend and competitor von Kármán: "The risk of being captured by the Third Reich is becoming too great."[177]

When von Mises arrived at Harvard University in September 1939, he changed his research topic again and returned to his interest in mechanics. This had to do with the fact that he had found an (originally unpaid!) position in Harvard's engineering department. But this change was also related to the fact that Andrej N. Kolmogorov's new and competing paradigm in probability theory had meanwhile convinced many immigrants to the United States, such as Willy Feller, and also Americans, such as Joseph L. Doob.[178]

One of von Mises's first activities in America was to oversee the first English publication of his results in air-wing theory, dating back to 1920. In his 1940 paper in in the *Journal of the Aeronautical Sciences*,[179] von Mises was not shy about the importance of his theory of lift, connected to the concept of the aerodynamic center. It must have been a disappointment for von Mises when, three years later, in an article with a similar topic in an American journal for mechanical engineers, his old colleague Hans Reissner—now also in American exile—did not mention von Mises's results of 1917 and 1920, not a single word.[180]

In 1945, von Mises republished his 1918 *Lectures on Aviation and Aircraft*[181] in an extended English edition titled *Theory of Flight*.[182] Although the book focused on subsonic flight and incompressible fluids, it is available in print to this day. Von Mises worked on compressible fluids as well, however, which led to the posthumous publication of the 1958 book *Mathematical Theory of*

176. See *Proceedings of the Fifth International Congress for Applied Mechanics*, ed. J. P. Den Hartog and H. Peters (New York: Wiley, 1939).

177. Siegmund-Schultze, "A Non-conformist" (ref. 1), 342.

178. Von Mises was quoted by his wife as saying, "There is here a combination of ignorance and racket with respect to probability. Here I can only do mechanics." Ibid., 362.

179. Richard von Mises, "New Developments in the Theory of Airfoils of Infinite Span," *Journal of the Aeronautical Sciences* 7 (1940): 290–94.

180. Hans Reissner, "Aerodynamic Center, Control and Stability of Airplanes," *Transactions of the A.S.M.E.*, Aug 1943, 625–28.

181. Von Mises, *Fluglehre* (ref. 90).

182. Richard von Mises, *Theory of Flight* (ref. 95).

*Compressible Fluid Flow*.[183] Together with German immigrant Kurt Friedrichs, and within the U.S. war-preparation campaign, von Mises gave lectures at the Brown Summer School for Applied Mechanics in 1941. Their lectures were mimeographed and published together much later by Springer as *Fluid Dynamics* (1971). The published lectures show that von Mises continued to reflect on old unsolved problems, such as turbulence, and was also willing to reconsider earlier positions that he had held, for example, with respect to ideal fluids.

In 1941, von Mises also wrote an unfinished German manuscript titled *Overview of Publications* (*Übersicht der Abhandlungen*), which was published much later in his *Selected Papers*. The *Übersicht* includes commentaries on four of the ten main areas of publication by von Mises (crucially not including stochastics): geometry, dynamics, elasticity and strength of materials, and hydromechanics. On the last-mentioned topic von Mises remarked:

> My occupation with hydromechanics began 1904, shortly after my first geometric work. My starting points were: the very low level of the hydraulic lectures at the time, the problem (in which I was exclusively interested then) to find the paths of flow through rotating turbines, the insufficiency of the existing hydrodynamic theory to solve that problem, [and] finally, a certain reservation against the Göttingen theories which were propagandized as the universal remedy [*Allheilmittel*]. I saw that Prandtl's boundary layer (which only much later was called the laminar boundary layer) had nothing to do with turbulence and that one could explain the so-called separation with the help of the theory of ideal fluids as a Helmholtz Surface of Discontinuity. However, I found it impossible to do anything with the theory without introducing a decisive hypothesis. This was the assumption that the basic flow of a turbulent motion obeys to a certain degree the laws of an ideal fluid.[184]

Although von Mises here summarizes some of the results of his 1908 habilitation thesis on turbines, the quotation remains open to detailed interpretation, which cannot be provided here. In particular, it seems von Mises pre-dates

183. Richard von Mises, *Mathematical Theory of Compressible Fluid Flow, Completed by Hilda Geiringer and G. S. S. Ludford.* (New York: Academic Press, 1958). The book was based on mimeographed lectures at Harvard University in 1949.

184. Richard von Mises, "Übersicht der Abhandlungen," in von Mises, *Selected Papers* (ref. 22), vol. 1, xv–xxiii, xxii. The real (if not official) editor of the *Selected Papers*, his widow Hilda Geiringer, added bibliographical references to von Mises's other areas of activity, without providing commentary.

both his own work and his conflict with the Göttingen fluid dynamicists. It is certainly true that the term “laminar boundary layer” came in use rather late. One should not misjudge the quotation, either, by assuming that von Mises was not aware of the transition from laminar to turbulent flow within the boundary layer. As has been argued above, von Mises tried to create a more general theory that embraced the boundary-layer theory by generalizing the theory of ideal fluids to include turbulence. One might assume, though, that von Mises had, as late as 1940, an over-optimistic expectation with respect to the heuristic power of classical notions such as Hermann Helmholtz's “Surfaces of Discontinuity.”[185]

After the war, the entire international communication structure in applied mathematics and fluid dynamics changed. Von Mises—not least due to old age and illness, but also in view of the new tendencies toward computing[186]—could not and did not return as a major player, in spite of his contributions to compressible fluids mentioned above. One consolation was that Istanbul was chosen at the London Congress in 1948 as venue for the Applied Mechanics Congress in 1952, which von Mises had suggested in 1938. When the congress took place in Istanbul, it bore witness to one of the last appearances of von Mises in public. He was critically ill with cancer when he received an honorary doctorate in his former country of exile, together with John von Neumann and George Taylor (Fig. 4).

## CONCLUSIONS AND PERSPECTIVES

Von Mises's position and his contributions as a pioneer of applied mathematics can probably be characterized best by his efforts to stress the peculiarities of his field compared to more established (and materially better equipped) disciplines such as mechanical engineering, pure mathematics, and theoretical physics. Several examples for these debates have been given in this paper.

However, the paper has focused more on one of these debates, between applied mathematics and engineering, and in particular, on von Mises's

185. See, for instance, Anderson, *A History* (ref. 15), “Surfaces of Discontinuity: A Blind Alley for Drag Predictions,” 101ff.

186. In fluid dynamics the older function theoretic methods became gradually obsolete at the time and computational fluid dynamics (CFD) was on the rise. Cf. Pedley, “Current Research” (ref. 3).

**FIG. 4.** Honorary doctoral degrees by the University of Istanbul to John von Neumann, Richard von Mises, and Geoffrey Taylor (front row from left to right), during the Eighth International Congress for Theoretical and Applied Mechanics at Istanbul, Turkey, August 20 to 28, 1952. (Source: RVMP, HUG 4574.90P, Courtesy Magda Tisza and Harvard University Archives.)

relationship with engineers who did not have his particular and strong taste for mathematics or his "quality of mathematical profundity not shared by the other outstanding scientists," as Garrett Birkhoff described it in 1983,[187] thirty years after von Mises's death.

Starting with his habilitation thesis on water turbines (1908), von Mises became increasingly interested in the general problems of fluid dynamics, among them viscosity and turbulence. He stressed his mathematical standpoint in comparison to engineers, such as H. Lorenz. In his tentative approach to the turbulence problem, von Mises tried to give priority to a the more general modified form of Euler equations over the Navier-Stokes equations, replacing viscosity with "turbulence factors" and considering "averages" of physical quantities, which may have triggered his interest in probabilistic methods. Although this conjecture could not fully be substantiated in this paper, if it is true, von Mises's probabilistic tools, which were important in statistics, remained less successful in the field for which they had apparently been designed, fluid dynamics.

187. Birkhoff, "Harvard" (ref. 19).

In any case, it appears that von Mises hoped for a while to solve the turbulence problem with "pen and paper," although he never had a successful breakthrough in this area. In the special case of the Couette flow (1912), von Mises and others believed to have found arguments not based on probability but on very recent mathematics (eigenvalues), limiting the traditional model of the Navier-Stokes equation. But von Mises had to admit much later (1938) that his mathematical arguments were not complete nor convincing.

In his theory of airfoil stability (1917–20), von Mises spent much effort on the idealized two-dimensional case of the "infinite" wing and used non-physical notions such as the "metacentric parabola" based on methods from complex function theory. Some engineers found this development to be of lesser importance. To some it seemed to have gone a step too far in the mathematization of engineering, beyond what was already empirically known and accepted. The "Mises airfoils," which were mathematically described, never went into testing and production.

In boundary layer theory (1927) von Mises tried, once again, to use a more general mathematical approach not restricted to the flow in the boundary, suggesting the possible future "importance for applications" of the "von Mises transformation." Most of his contemporaries, with exception of more mathematically minded scholars such as the old Horace Lamb, were not impressed. Engineers stressed "physical tact" (Prandtl) over von Mises's "more formal" mind. However, similar to some of his efforts in probability theory, von Mises's approach had a renaissance in modern computational fluid dynamics, and even as a method in general theories of differential equations.

Speaking occasionally in this paper about von Mises's *deficiencies* and a *lack of success* in fluid dynamics should not conceal the fact that, historically, the establishment and institutionalization of applied mathematics as a separate field of respected academic research and teaching owes much to a clarification of its goals within the whole of mathematics and to its partial delimitation from even more applied contexts, such as fluid dynamics. In this respect von Mises's activity was an undisputed success.

On the one hand, the *importance* of mathematics within engineering and mechanics, especially fluid dynamics, was felt and recognized by all participants. Some mathematical models of fluid mechanics have reached a high cognitive status even within pure mathematical research, as the nomination of the Navier-Stokes equations as one of the One-Million-Dollar "millennium problems" by the Clay Institute in Boston in 2000 has shown. However, this example also reveals the persistence of disciplinary divides between mathematics and engineering, given that modern computational fluid dynamics has cut parts of its former connections to mathematics (conformal mappings) and has connected to other parts, in particular to computation and numerical analysis. The latter is in the tradition of other work by von Mises's as well.

On the other hand, both the leading figures in the field of fluid dynamics, such as Prandtl and von Kármán, and the less influential H. Reissner and von Mises realized the *indispensability of experimental facilities* to suggesting and checking fundamental results—a need that is as vividly felt in modern fluid dynamics as it was one hundred years ago. However, in this respect both Reissner and von Mises were comparatively disadvantaged. Whereas Prandtl and von Kármán became the leaders of the aerodynamical research centers in Germany and the United States, respectively, Reissner and von Mises fell behind.

An anecdote about Reissner in von Kármán's posthumously published autobiography of 1967 is illuminating, despite all of the just criticism that has been directed against this rather unreliable historical source. The anecdote reflects on the influence of institutional aspects on the reputation and posthumous fame of scientists and engineers. In fact, von Kármán (through the voice of the assisting journalist Edson) hit on something interesting with respect to Reissner, something that has value for my argument regardless its actual authenticity:

> I remember that he once said to me sadly: "People attribute to Prandtl, and to you, discoveries which neither of you ever made. But they don't even credit me for the little things in aerodynamics which I actually contributed." I told him that he could be consoled by the philosophy of Felix Klein. "It is unfortunate, but most people don't quote the researcher who provided the new results," Klein once said. "They quote the teacher from whose writing or lecture they first understood them."[188]

It seems to me, one can apply this equally to the case of von Mises, who did not have many students in fluid dynamics at Berlin University,[189] and who, like

188. Kármán and Edson, *The Wind* (ref. 18), 76. Strong criticism of the many factual errors and lacunae of this biography is in Charles Süsskind, Review of Kármán and Edson, *The Wind and Beyond*, *Technology and Culture* 9, no. 3 (Jul 1968), 507–09.

189. This more classical university was not an institution for educating engineers. Von Mises gave occasional "special lectures" in aero- and hydrodynamics between 1927 and 1933. See Hannelore Bernhardt, "Zur Institutionalisierung der angewandten Mathematik an der Berliner Universität," *NTM-Schriftenreihe* 17, no. 1 (1980): 23–31.

Reissner, did not publish in the leading handbooks for aerodynamics, such as Durand's six volumes in 1934–36. Von Kármán's autobiography also did not mention von Mises, not a single word.

Last but not least, the problems of anti-Semitism and Nazi rule after 1933 were among the external political factors that affected the future development of German science and engineering generally, and the biographies of Reissner and von Mises in particular. As Haude has sensitively remarked,[190] the historiography of German aerodynamics in the decades following von Mises's emigration has—to a large degree—been written by Prandtl's students, but also by the Nazi Aviation Ministry (under Hermann Göring), which provided the money for publications (both specialist and historical) and for careers. Three of the four people in the focus of this paper were of Jewish origin and were therefore affected by the Nazi's seizure of power in 1933: von Kármán, Reissner, and von Mises himself.

Still, their fates were very different. In 1929—parallel to his work in Aachen—von Kármán secured a foothold in the United States (Caltech) to where he would withdraw in 1933 and acquire fame for his contribution to aerodynamics, not least due to the American war effort from 1941.[191] Von Mises had to leave for a less influential institution in Istanbul, where there was little interest or equipment for aerodynamics. Even more precarious was the situation for Reissner, who was about ten years older than the two other Jewish refugees. With his regular academic retirement being imminent, he hesitated over emigrating. When he finally did emigrate to the United States in 1939, it saved his life but it did not secure him an influential position.[192]

Von Mises's difficult relationship with the Prandtl and von Kármán schools might be summarized by the following indirect quotation, given by von Mises's widow Hilda Geiringer in 1959:

> He said that he knew that he had not as much scientific (mechanics) influence as Kármán and Prandtl. He thought very highly of both of them, particularly Prandtl. Kármán was a lifelong friend. He spoke of Prandtl's sleepwalking instinct for mechanics. He had a theory that a certain amount of "*Verworrenheit*" [befuddlement] (Prandtl) was attractive. He

190. Haude, *Grenzflüge* (ref. 10), 35.

191. In my book of 2009, I count him among the emigrants from Germany because his connections to Aachen, among them regular teaching assignments, and several projects were curtailed by the Nazis. Siegmund-Schultze, *Mathematicians Fleeing* (ref. 158).

192. Reissner, "Hans Reissner" (ref. 10).

> said, “Ich bin zu klar.” [I am too clear.] On the other hand, he had almost contempt for the “activity” in Göttingen, the conscious effort to quote each other, etc.[193]

Without fully discussing von Mises’s broader impact on applied mathematics as compared to fluid dynamics, this paper has shown him personifying scientific, institutional, and political conflicts and collaboration during the emergence of fluid dynamics. For all of the occasional manifestations of his polemical mind, von Mises’s role as a bridge builder between applied mathematics and fluid dynamics was nevertheless dominating, not least because he was one of the few scholars who were both engineers and mathematicians at the same time. Von Mises was thus able to play a catalytic role in the important historical process of the development of fluid dynamics and applied mathematics.

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193. Siegmund-Schultze, “A Non-conformist” (ref. 1), 364.