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# **A Comparative Analysis of Alternative Portfolio Insurance Strategies**

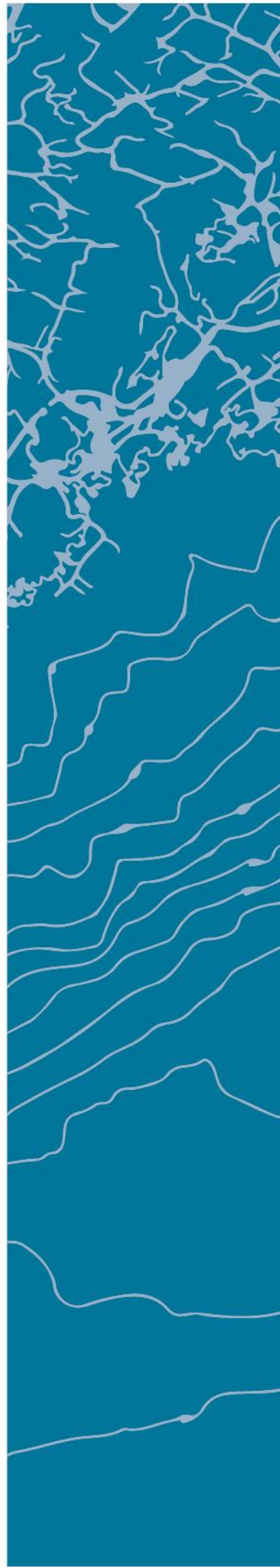
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MASTER THESIS

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## **Abstract**

Literature within the field of Portfolio Insurance (PI) provides no clear evidence for or against the usefulness of implementing PI strategies. The goal of this thesis is to investigate the real-life performance, using out-of-sample testing, of different PI strategies on the S&P 500 index. The strategies we investigate are the Protective Put (PP) strategy, the Cash Call (CC) strategy, the Constant Proportion Portfolio Insurance (CPPI) strategy, the Stop Loss (SL) strategy and a Moving Average (MA) strategy. We compare the performance of the strategies against a passive Buy-and-Hold (BH) strategy. We find that the MA and SL strategy outperforms the BH strategy, but statistical evidence is weak, while the PP, CC, and CPPI strategy underperform the passive strategy. We discuss the results and make suggestions of why option-based strategies and the CPPI strategy perform poorly.

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# 1 Introduction

A common assumption in the financial literature is that investors seek to maximise returns and minimise risk. Investing in financial securities can be profitable during times of increasing prices, but during periods of decline, investors risk to see their portfolio value diminish. The main idea behind Portfolio Insurance (PI) is to profit from increasing prices, but at the same time, limit the risk of losses. Any strategy that possesses these properties is a potential PI strategy.

Throughout the modern financial era, several periods with decreasing prices made investors experience significant losses. Some of these periods are the crash on Wall Street in 1929 which lead to the Great Depression of the 1930s, Black Monday of 1987, the Dot-Com crash in 2000-2001, and the Global Financial Crisis in 2007-2008, to mention a few. A consequence of financial downturns, in addition to losses, is that many investors withdraw their capital from the markets and miss the following upturn. This is also a typical problem with trend-following strategies, like strategies based on Moving Average and Stop Loss. The concept of Portfolio Insurance was introduced by Hayne Leland after a period of decreasing stock markets of 1973-74 (Leland and Rubinstein,1988). He recognised that many pension funds and institutional investors had withdrawn their capital from the markets during the bad period, and thereby missed profits from the following upturn that followed from 1975. He, therefore, recognised the need for strategies that could limit the downside risk of portfolios, but still maintain the upside potential. This way, investors would be able to be invested in the markets without being afraid to experience substantial losses.

Previous literature that analyse the performance of Portfolio Insurance strategies yields mixed conclusions. The outcome from theoretical studies shows that strategies which use options to protect portfolios should work. Similar, theoretical studies for the Constant Proportion Portfolio Insurance strategy also show that the strategy work and Black and Perold (1992), amongst others, demonstrate this. On the other hand, theoretical studies that evaluate trend-following strategies, like the Moving Average and the Stop Loss, yield negative

conclusions with regard to their performances, as Fama (1970) and Kaminski and Lo (2014) conclude. A well-documented fact about PI strategies is that they tend to outperform passive strategies in bear markets but underperform in bull markets. Cesari and Cremonini (2003) show this through Monte Carlo simulations.

As for theoretical, empirical studies also generate mixed conclusions on the performance of different PI strategies. Garcia and Gould (1987) and Do (2002) are examples of researchers which give negative conclusions about the performance of PI strategies. On the other hand, studies by Hakan and Hande (2009) amongst others, find that the implementation of PI strategies increase the performance of a portfolio. Annaert, Van Osselaer, and Verstraete (2009) show that the use of PI strategies may be attractive to some investors, depending on their utility functions. The study by Driessen and Maenhout (2013) concludes that investors optimally hold short positions in options, which means that option-buying strategies do not work. Coval and Shumway (2001) present evidence of weak performance generated by option-buying strategies, which indicates that options contain high risk-premiums, not predicted by option-price theory. In other words, they find that option prices are too high.

Most of the empirical studies that evaluate the performance of PI strategies only include a small number of strategies. This makes it difficult to find concrete conclusions on which strategies that work, and how they compare against each other. Cesari and Cremonini (2003) do implement nine strategies in their study where they compare the performance of these strategies against each other. Some of the strategies that they include are option-based while some are based on Constant Proportion, Moving Average and Stop Loss. Their study and the conclusions that they reach are from a theoretical environment, as they conduct Monte Carlo simulations with artificial data. In other words, this study is theoretical.

Many empirical studies use the Sharpe ratio as the performance measure which considers total variance as the measure of risk. Total variance may not be relevant as a measure of risk in context of Portfolio Insurance. Do (2002), Do and Faff (2004), Hakan and Hande (2009), Ho, Cadle, and Theobald (2011) are examples of studies that implement this risk measurement or similar measures that consider total variance. Finally, several previous studies

fail to implement a valid statistical test for outperformance. This makes it difficult to trust the validity of the results. Examples are Garcia and Gould (1987), Do (2002) and Hakan and Hande (2009). Other studies, like Snorrason and Yusupov (2009) use parametric tests which make strict and unrealistic assumptions concerning the distribution of the variables. Datasets that contain financial data do often violate these assumptions, and it is, therefore, ambitious to rely on the validity of the parametric tests.

The main goal of this thesis is to find out if there are any Portfolio Insurance strategies that manage to outperform a Buy-and-Hold strategy empirically on the S&P 500 index. We evaluate the performance of two option-based strategies; the Protective-Put (PP) and the Cash-Call (CC). We also evaluate the performance of three other strategies; the Constant Proportion Portfolio Insurance (CPPI), the Stop Loss (SL) and a trend-following strategy based on Moving Average (MA). We compare all the strategies against the performance of the Buy-and-Hold (BH) strategy. To the best of our knowledge, there is no other empirical study that includes such a selection of PI strategies and we therefore contribute with new insight to the literature. At first glance, the MA and SL strategies do not look like PI strategies. However, any strategy that seeks to limit downside risk, but still maintains upside potential is a potential PI strategy. The MA and SL strategies attempt to limit the downside risk when they generate sell-signals when prices fall. This means that once the market enters a bear market or a so-called correction of prices, the goal of the strategies is to limit losses. This is the reason for why we include them as PI strategies in our study.

Even though our main focus in this thesis is to study the performance of the PI strategies empirically. However, we first conduct a Monte Carlo simulation for the two option-based strategies. The Monte Carlo simulation is used as motivation to include the PP and CC strategies in the empirical study. In the theoretical investment environment, the underlying stock follows a Geometric Brownian Motion. We conduct 10 000 000 simulations, where we estimate option prices and calculate returns for the strategies. We show that the strategies which use options perform well in the simulation study. The CPPI strategy should also perform well. In other words, they should work according to theory. On the other hand, similar theoretical

studies that evaluate the MA strategy and the SL strategy show that the strategies perform poorly. We also show the different effect dividends have on the two option-based strategies.

In the empirical study, we implement out-of-sample testing, which copes with the data-snooping problem which occurs when one test many strategies in-sample. We test the performance of the strategies on the S&P 500 index and use two different data periods in our study. The first period is from 1990 to 2016, which includes all five strategies. The second period starts in 1975 and ends in 2016. The subsequent test includes the CPPI, MA and SL strategy. We test all the strategies with a wide range of different parameters in-sample and continuously test the optimal parameters of each strategy out-of-sample. This method allows us to generate the real-life performance of the strategies. The out-of-sample testing regime deals with the data-snooping problem which occurs in the in-sample tests.

We also seek to improve on previous studies when we use the Sortino ratio as the performance measure. This measurement deals with weaknesses of other more popularly used measures like the Sharpe ratio. We also implement valid statistical testing for outperformance, which other researchers often fail to do. We use the stationary block bootstrap test to check for statistical outperformance.

Our empirical results show that the MA and SL strategies economically significantly outperform the BH strategy, while the option-buying strategies (PP and CC), and the CPPI strategy underperform. The empirical outperformance of the MA and SL strategies are in contrast with the performance of the strategies in a theoretical environment. The same is true for the underperformance generated by the option-buying strategies. The outperformance that the MA and SL strategies generates is not statistically significant at conventional significance levels in the first out-of-sample test period. The outperformance of the same strategies in the second out-of-sample test period is however statistically significant.

The rest of the thesis is organised as follows: Section 2 review relevant literature conducted on the topic of PI strategies. Section 3 describes the methodology used in the thesis. Section 4 presents the data that we have used when conducting the empirical tests and throughout our thesis. Section 5 contains a theoretical simulation of the option-based strategies, where we

demonstrate the effect of dividends on the performance of PI. Section 6 presents the empirical results and section 7 provides a discussion of our empirical results. Section 8 summarise and concludes the thesis.

## **2 Literature Review**

In this section, we review the literature within the field of Portfolio Insurance. We start to list the strategies and advance to look at the early development of different Portfolio Insurance strategies. We continue to review theoretical studies for the strategies, and after that we review some simulation studies. At the end of the section, we review empirical research which studies the performance of PI strategies.

In this thesis we study five different Portfolio Insurance (PI) strategies; Protective-Put, Cash-Call, Constant Proportion Portfolio Insurance, Stop Loss and Moving Average. As an extension to the strategies studied in this thesis, the literature offers additional PI strategies, for example, the Synthetic-Put and other hedging strategies. The objective of Portfolio Insurance is to limit downside risk, but at the same time maintain upside potential. Any strategy that tries to achieve that, can be considered a PI strategy. This is the reason for why we include the trend-following strategy based on Moving Average and a Stop Loss strategy, which one traditionally consider to be strategies in the category of technical analysis.

### **2.1 Early Development**

The first Portfolio Insurance strategy was formulated by Leland and Rubenstein in the late 1970s and the strategy replicates the payoff of options. It is motivated by the paper of Black and Scholes (1973), which contains the famous Black-Scholes formula for option pricing. Leland and Rubinstein (1988) show that in the absence of options on the underlying asset, it is possible to re-create an option payoff using the underlying asset itself and a risk-free asset. The strategy is known as a synthetic option strategy. The strategy was motivated by the lack of liquidity in the option market, which made it hard to use options as insurance. The work by

Leland and Rubinstein motivated further development of the field within Portfolio Insurance. The CPPI strategy was presented by Black and Perold (1992) for fixed income instruments, and by Black and Jones (1987) for equity instruments.

The use of technical analysis dates to the 1600s according to Wong, Manzur, and Chew (2003). Technical analysis has gained a lot of popularity among investors, much due to its simplicity. Both the MA strategy and the SL strategy are normally considered to be in this category. The belief is that future security prices are predictable, based on price history. The idea is that investors can use technical indicators to predict future price movements. Investors predict and take advantage of trends which are thought to exist in the market. The objective of market timing indicators is to increase returns and reduce volatility.

## **2.2 Theoretical Work**

Black and Scholes (1973) derive the Black-Scholes option pricing formula for European options by assuming that no arbitrage is allowed in complete and efficient markets. The formula assumes a continuous trading environment where the underlying asset follows a Geometric Brownian Motion (GBM). Black and Perold (1992) show theoretically that the CPPI strategy is optimal for a piecewise Hyperbolic Absolute Risk Aversion (HARA) utility function with a minimum consumption constraint when trading in efficient markets. Balder, Brandl, and Mahayni (2009) develop a framework for the CPPI in a trading environment with trading restrictions. They point out that with trading restrictions, the statement made by Black and Perold, that the CPPI is optimal for investors with HARA utility is no longer true. Trading restrictions, in this case, are discrete-time trading and transaction costs. Black and Perold (1992) also prove that the CPPI strategy is equivalent to invest in a perpetual American option, also for assets that pay dividends. In addition, Black and Perold (1992) point out that as the multiple used in the CPPI-strategy increases, the payoff of the CPPI strategy approaches the payoff of a Stop Loss strategy.

Fama (1970) argues that in efficient markets, strategies that use previous price information, cannot generate higher returns than the BH strategy. Previous price information is exactly the

information the MA and SL strategies use and they, therefore, should not work according to Fama. Kaminski and Lo (2014) show that the performance of a Stop Loss policy depends on the underlying return-generating process. Specifically, they prove that under the Random Walk Hypothesis, a Stop Loss strategy can never yield positive stopping premium. On the other hand, they argue that under more likely return-generating processes, like momentum or regime-switching models, a Stop Loss strategy can generate positive stopping premium. With a return-generating process which is mean-reverting, a Stop Loss strategy generates negative stopping premium.

### **2.3 Simulation Studies**

Trennepohl, Booth, and Tehranian (1988) create 1500 random portfolios, to study how various option-based strategies perform. They use the stochastic dominance criterium and find that insured portfolios which buy options represent the majority of dominant portfolios. In other words, they show that option-based strategies work based on this performance criteria. Cesari and Cremonini (2003) conduct a Monte Carlo simulation study that compares variations of the strategies that we use in this thesis, but the principle behind the strategies is the same. They study how the different strategies perform in different states of the market. They find that Constant Proportion Portfolios (CPPI variations) perform best in bear and in no-trending market states. In bull markets, the best performing strategy is a constant mix strategy, which is a portfolio that invests a fixed weight in the stock and in the risk-free asset. In the case of ignorance for the market state, the Constant Proportion strategies and the Cash-Call strategy perform the best. They state that the results seem to be independent of the performance measure used. The study implements the Sharpe ratio, the Sortino ratio and the Return at Risk.

Annaert et al. (2009) use empirical data to conduct a bootstrap simulation to compare the performance of PI strategies. The simulation looks at one year of daily returns from empirical data, to not break up the serial dependency in the data. Annaert et al. (2009) find that the CPPI strategy and the SL strategy outperform the BH strategy both in terms of downside protection and risk/return trade-off. The risk-adjusted performance measure used in this study is the



Sharpe ratio. Bertrand and Prigent (2011) implement a similar bootstrap methodology as in Annaert et al. (2009). They, however, use a performance measure that takes a non-normal return distribution into account. They argue that this is important when dealing with strategies that possess non-normal return distributions.

## **2.4 Empirical Studies**

The main goal of this thesis is to test the empirical performance of PI strategies. The most relevant literature is therefore studies that evaluate the empirical performance of PI strategies. Driessen and Maenhout (2007) find that both risk-averse and loss-averse investors optimally hold short positions in options. Thus, it is not optimal to buy put options. They study Out-of-The-Money (OTM) put options and At-The-Money (ATM) straddles on the S&P 500 index in the period from 1987 to 2001. For investors to want to hold long positions in options, they must be loss-averse combined with highly distorted probability assessments. Tian (1996) demonstrates that strategies which buy put-options provide the protection they are supposed to, but the portfolio return is very poor compared to the market in general. Abid, Mroua, and Wong (2007) study the performance of Protective-Put strategies on various stocks in the French stock market. They discover that the PP strategy outperforms the BH strategy, based on the Sharpe ratio. Danis (2007) finds that Portfolio Insurance which uses put options on the S&P 100 index is inefficient. His results show that the historical utility for an insured portfolio is lower than for an uninsured portfolio. This is true for power utility and value functions based on prospect theory. Helmer and Miller (2015) also argue that the Protective-Put strategy performs worse than simply to hold the stock.

Garcia and Gould (1987) argue that PI strategies do not perform better than a static portfolio in the long run. They implement a hedging strategy that rebalances the exposure between a risky asset and a risk-free asset. They find that when transaction costs are considered, the hedging strategy with zero percent floor “costs” 1,7% in annual returns compared to the BH strategy. The value of the insured portfolio is that it protects the portfolio against significant drops in value. The number of floors missed is not reported. Do (2002) studies the perfor-

mance of the CPPI strategy in the Australian market. He concludes that the implementation of the CPPI strategy cannot be justified from a loss minimisation point of view and neither from a gain participation point of view. As Do points out, the results may be time specific, which weakens the conclusion. Hakan and Hande (2009) compare the performance of a CPPI strategy against the performance of a BH strategy in the Turkish market. They find that the CPPI strategy outperforms the BH strategy, both in terms of return and lower standard deviation. It is worth noting that they look at the total variance as the measure of risk. A weakness of the study is that they fail to implement statistical testing for outperformance. Ho et al. (2011) study the performance of PI strategies in the currency market. They find that the CPPI strategy performs well based on the Sharpe ratio, but when looking at Value at Risk and at Expected shortfall, other strategies perform better.

Kaminski and Lo (2014) look at the performance of a Stop Loss strategy, trading in futures. They find evidence of strong performance of SL strategies, evaluated using the Sharpe ratio. Snorrason and Yusupov (2009) find similar evidence in the Swedish stock market. Pätäri and Vilksa (2014) examine the profitability of a Moving Average Crossover strategy in the Finnish market. They discover evidence that suggests that the Moving Average strategy outperforms the Buy-and-Hold strategy, mostly due to stronger performance in bear markets. Kilgallen (2012) and Faber (2007) also demonstrate the superior performance an MA strategy is able to generate in various financial markets. Zakamulin (2017) finds evidence of outperformance in some markets, but when he looks at the performance on the S&P 500 index, the MA strategies generate no statistical evidence of outperformance compared to the BH strategy. Zakamulin (2014) warns that “too good to be true” performances documented by many researchers for MA strategies should raise questions about whether investors can expect similar performances in the future.

## **2.5 Summary**

We divide the literature on PI strategies into three different categories; theoretical literature, Monte Carlo and Bootstrap simulations, and finally empirical studies. For our thesis, the most

relevant studies are the empirical work, while the theoretical work motivates our research. The theoretical studies, suggest that the PP and CC strategies should work (Trennepohl et al.,1988). These studies suggest that the CPPI strategy should work (Annaert et al.,2009). On the other hand, according to theory, the MA and SL strategy should not work in efficient markets, as Fama (1970) and Kaminski and Lo (2014) state.

The empirical literature presents conflicting conclusions on the performance of PI strategies. Driessen and Maenhout (2007), Helmer and Miller (2015) and Danis (2007) conclude that option-buying strategies perform poorly for investors, while Abid et al. (2007) reach the opposite conclusion for the PP strategy in the French market. Garcia and Gould (1987) and Do (2002) find no evidence of outperformance of the CPPI strategy, while Hakan and Hande (2009) and Ho et al. (2011) demonstrate that the CPPI strategy performs well. Kaminski and Lo (2014) and Snorrason and Yusupov (2009) find evidence of good performance by SL strategies. Finally, the conclusions on the empirical performance of MA strategies are mixed. Zakamulin (2017) discover that MA strategies outperform the BH strategy in some markets. We can conclude that the literature offers no clear conclusion on the performance of PI strategies.

A final note on the literature reviewed is that studies that use risk-adjusted performance, typically use the Sharpe ratio, or return compared to the total standard deviation, which looks similar to the Sharpe ratio.

### **3 Methodology**

In this section, we present the methodology that we use in this thesis. We start to describe the simple Buy-and-Hold strategy and continue to present what options are, the VIX index and the option pricing formula that we use. We go on to present the methodology we use to estimate option prices. Subsequently, a presentation of drawdown and drawup is given. Next, we describe the methodology for each strategy and continue by present how bull and bear markets are detected and then how out-of-sample tests are implemented. Finally, we present

the performance measure that we use, the hypotheses we test in the statistical test and the statistical test for outperformance.

### 3.1 Buy-and-Hold strategy

The BH strategy is a passive strategy where the investor holds the stock for a longer period without managing the portfolio during the investment period. In our study, we buy the S&P 500 index and hold the index over the entire period. The strategy will therefore have the same returns as the index. The return of the strategy is calculated as simple returns for the index. The return for the stock is given below, where  $S_{t-1}$  is the stock price for period  $t - 1$ ,  $S_t$  is the stock price at time  $t$  and  $dy$  is the dividend yield.

$$r_{STOCKt} = \frac{(S_t - S_{t-1})}{S_{t-1}} + dy_t. \quad (1)$$

### 3.2 Options

An option is a contract which gives the buyer the right to buy or sell an asset at a predetermined price. For stock- and index-options, the asset is an underlying stock or index. A put option is the right to sell the underlying security to the issuer of the option at a predetermined price (strike price). Similarly, a call option is the right to buy the underlying security at a predetermined price. The time of when the owner of an option can exercise the option depends on the type of the option. A European option allows the owner of the option to settle the option on the day of expiration only, while an American option allows the owner of the option to settle the option at any time before or at the day of expiration. The options that are traded with the S&P 500 index as the underlying asset is of a European type in the US option market. All the inputs that we need to estimate option prices are easily observable in the market, except for the volatility input (standard deviation). We use the VIX index as input for volatility, and we describe what the VIX index is in the section that follows.

### **3.3 VIX Index**

The VIX index is interpreted as the implied volatility observed in the option market for options on the S&P 500 index. The VIX index starts in 1990, and therefore, the longest possible period on the option-buying strategies is from this year. The Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index (VIX) in 1993 and their intention was that the VIX index could work as a measure of the markets expectation of the 30-day volatility. The calculation is done by re-arranging the option pricing formula, so when option prices are observed, the volatility can be solved for. It was originally calculated using At-The-Money (ATM) option prices from the S&P 100 index, but the calculation of the VIX index was modified in 2003. Instead of using just ATM put and call option prices, both In-The-Money and Out-of-The-Money options was included by using a wide range of strike prices. The modified VIX index is based on the broader S&P 500 index, rather than the S&P 100 index since 2003. CBOE describes this new methodology of calculating the VIX index as a transformation of the VIX index from working as an abstract concept to become a practical standard for trading and hedging volatility. To get reliable results it is important that the data is calculated in a consistent manner. One could think that this would result in problems since adjustments in the calculation was made in 2003 and the VIX data started in 1993. The CBOE has however replicated the new method on past data and created data all the way back to 1990 for the VIX index.

### **3.4 Estimation of Option Prices**

To test the performance of the option-based strategies we need to estimate the empirical option prices. We use the Black-Scholes option pricing formula, where we use the VIX index as input for implied volatility. The rest of the variables we need to estimate the option prices are easily observable in the market. The option prices we compute are estimations of historical option prices. We calculate options with 30 days to expiration and this is consistent with our investment periods, which are one month at the time. For the option-based strategies, we buy

options on the underlying asset, the S&P 500 index. We calculate prices for each investment period (each month), and we use the formula introduced by Black and Scholes (1973). In the Black-Scholes formula,  $C_t$  denotes the call-price and  $P_t$  denotes the put price, at time  $t$ .  $S_t$  is the stock price at the start of the period and  $X$  is the strike price.  $r_{ft}$  is the risk-free rate of return,  $\tau$  is time to expiration,  $q_t$  is the continuously compounded dividend yield and  $\sigma$  is the annual standard deviation.  $N(\cdot)$  is the standard normal cumulative probability distribution.

$$C_t = S_t \times e^{-q_t \tau} \times N(d_1) - X \times e^{-r_{ft} \tau} \times N(d_2), \quad (2)$$

$$P_t = X \times e^{-r_{ft} \tau} \times N(-d_2) - S_t \times e^{-q_t \tau} \times N(-d_1), \quad (3)$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + \tau\left(r_{ft} - q_t + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{\tau}}, \quad (4)$$

$$d_2 = d_1 - \sigma\sqrt{\tau}. \quad (5)$$

### 3.5 Drawdown and Drawup

Drawdown is a measure of the decline from a historical peak in a variable (Burghardt and Walls,2011). It is a popular financial measure of risk, often used in the world of hedge funds. In this thesis, we measure the drawdowns for the cumulative returns for the strategies. Similar, the drawup is a measure of the increase from a historical through. We calculate the drawup from the historical minimum after the previous peak. We use the drawdown and drawup to calculate the trading-signals for the SL strategy. Formally, drawdown is denoted in formula 6, where  $X = (X_t, t \geq 0)$  is a random process where  $X(t)$  is the previous peak. The drawdown at time  $T$  is

$$DD_T = \max(0, X_t - X_T). \quad (6)$$

Drawup is denoted in formula 7, where  $X = (X_t, t \geq 0)$  is a random process where  $X(t)$  is the previous local minimum. The drawup at time  $T$  is

$$DU_T = \max(0, X_T - X_t). \quad (7)$$

In addition to using drawdown and drawup in the SL strategy, we calculate average drawdown and maximum drawdown for each of the strategies. We also plot the drawdowns in graphs in the Empirical Results section. For a deeper explanation and mathematical proves, we refer to Magdon-Ismail, Atiya, Pratap, and Abu-Mostafa (2004).

## 3.6 Protection Strategies

### 3.6.1 Protective-Put strategy

The strategy of insuring the portfolio by a purchase of a put option is the most intuitive one. The investor simply buys a stock (or any risky asset) and at the same time buys a number of put options, which protects the investor against losses. Today there are options available, not only on single stocks, but also on indexes which makes it possible to get a diversified protection. Our strategy is to buy the S&P 500 index (risky asset) and at the same time buy a number of put options on the stock. At the end of each investment period, we gain a profit if the stock increase more than the amount we paid in premium to buy the option. If we buy one option and one stock, we are perfectly protected at the level of the strike price.

We evaluate if the Protective-Put strategy performs better by buying different amounts of put options and if some levels of strike prices are better than others. One put option ATM for each stock (perfectly hedged) is probably not optimal, and we therefore evaluate the performance of buying a different number of options at different strike prices. A put option is the right to sell the stock, but not an obligation and therefore the holder of the option will only exercise if the strike price is higher than the stock price. This means that the minimum value

of a put option at expiration is zero. The value of the put option,  $P_T$ , is illustrated in Formula 8, where  $X$  denotes the strike price and  $S_T$  is the stock price at expiration.

$$P_T = \max(0, X - S_T). \quad (8)$$

The wealth at the end of the period,  $W_T$ , is given in Formula 9, where  $W_t$  is the wealth at the beginning of the period,  $X$  is the strike price,  $B$  is the number of put options bought and  $P_t$  is the put-price at the beginning of the period.  $B \times P_t$  is the amount we pay for the options. At the start of the period, we invest  $W_t - B \times P_t$  in the stock.

$$W_T = (W_t - B \times P_t) \times (1 + r_{STOCK}) + B \times P_T. \quad (9)$$

The return generated by the strategy for the period,  $r_{PP}$  is computed as

$$r_{PP} = \frac{W_T - W_t}{W_t}. \quad (10)$$

### 3.6.2 Cash-Call strategy

In the Cash-Call strategy we buy a number,  $B$ , call options with a strike price,  $X$ , and invest our remaining capital in a risk-free asset. This strategy is somewhat similar to the PP strategy due to the put-call parity. Similar to what we explained about the put option, the option holder has the right, not an obligation to exercise the option and therefore, the lowest value the call option can have on the day at expiration is zero. The value of the call option at expiration,  $C_T$ , is given in Formula 11, where  $X$  is the strike price and  $S_T$  is the stock price at the end of the period:

$$C_T = \max(0, S_T - X). \quad (11)$$

We also have a perfect protection of the portfolio when we use this strategy.  $B \times C_t$  is the amount of capital we use to buy call options, where  $C_t$  is the value of the option at the beginning of the period and  $B$  is the number of options bought. We invest  $W_t - C_t \times B$  in the



risk-free asset, where  $W_t$  is the amount of wealth at the beginning of the period.  $r_{ft}$  is the risk-free return for the period. The portfolio wealth at the end of the investment period is computed as

$$W_T = (W_t - B \times C_t)(1 + r_{ft}) + B \times C_T. \quad (12)$$

The return of the strategy,  $r_{CC}$  is computed as

$$r_{CC} = \frac{W_T - W_t}{W_t}. \quad (13)$$

Note that, at time  $T$  the guaranteed level of wealth is  $(W_t - B \times C_t)(1 + r_{ft})$

### 3.6.3 Constant Proportion Portfolio Insurance strategy

The Constant Proportion Portfolio Insurance strategy is a dynamic strategy which rebalances the invested capital between a risky asset and a risk-free asset with the objective to protect a guaranteed amount  $G_t$ . The guaranteed amount can be interpreted as a minimum acceptable value we allow the portfolio to decrease to. In other words, the investor will not allow the value of the portfolio to fall below  $G_t$ . We start to invest an amount,  $E_t$ , in the risky asset, equal to a multiplier,  $m$ , times a cushion value,  $C_t$ . The cushion is computed as the difference between the value of the portfolio at the beginning of the period,  $W_t$ , and the  $G_t$  discounted by the risk-free rate  $r_{ft}$ . Formally, the cushion is computed as follows:

$$C_t = W_t - \frac{G_t}{1 + r_{ft}}. \quad (14)$$

The exposure in the risky asset is computed as

$$E_t = m \times C_t, \quad (15)$$

and we invest the remaining in the risk-free asset, denoted as

$$W_t - E_t. \quad (16)$$

In a continuous trading environment, it is possible to maintain the floor value in wealth, such that it will never fall below the guaranteed amount  $G_t$ . In the same environment with no risk of price-jumps, the CPPI provide a guarantee such that the value does not fall below  $G_t$ . In our empirical test, we have an investment period of one month. During the investment period, we do not rebalance the portfolio. This means that there exists a risk that the portfolio falls below the guaranteed amount. Therefore, in contrast to the strategies which involves options, we are not perfectly insured. We investigate the performance of the CPPI strategy and test different initial floor values and different multipliers. The multiplier,  $m$ , can be considered a leverage variable. As mentioned, in a continuous trading environment, the multiplier does not increase the risk of falling below the guaranteed amount, but it does in a discrete trading environment, like ours. For each period we adjust the floor so that

$$G_{t+1} = G_t \times (1 + r_{ft}). \quad (17)$$

This is not a standard feature in the CPPI strategy. We make this adjustment because of the long investment period in our study. Without the adjustment, the CPPI strategy will begin to behave in the same way as the BH after a certain increase in the portfolio value. The end of the period wealth,  $W_T$ , is given by Formula 16, where  $r_{STOCK}$  is the return for the stock during the period and  $r_{ft}$  is the risk-free return for the period.

$$W_T = E_t \times (1 + r_{STOCK}) + (W_t - E_t) \times (1 + r_{ft}) \quad (18)$$

The return for the strategy,  $r_{CPPI}$  is calculated as

$$r_{CPPI} = \frac{W_T - W_t}{W_t}. \quad (19)$$

### 3.6.4 Stop Loss strategy

The Stop Loss strategy is a simple and popular strategy to protect the portfolio against losses. We invest our capital in the risky asset and we stay invested in the risky asset as long as the drawdown does not exceed a predetermined value  $v$ .  $v$  is the maximum acceptable loss of the portfolio. When the drawdown exceeds  $v$  we close the position in the risky asset and invest the capital in the risk-free asset.

We remain invested in the risk-free asset until the drawup for the stock exceeds  $v$  from the lowest minimum since the last time we exit the risky asset. An illustration of the portfolio weight invested in the stock,  $\alpha_t$  is given in Formula 20, where  $DD$  is the drawdown of the stock,  $DU$  is the drawup for the stock and  $alpha_{t-1}$  is the portfolio weight in the stock in the previous period. The portfolio weight in the risky asset is denoted by

$$\alpha_t = \begin{cases} 0 & \text{if } DD \geq v \text{ and } \alpha_{t-1} = 1 \text{ (exit)} \\ 1 & \text{if } DU \geq v \text{ and } \alpha_{t-1} = 0 \text{ (re-enter)} \\ 1 & \text{if } DD \leq v \text{ and } \alpha_{t-1} = 1 \text{ (stay in)} \\ 0 & \text{if } DU \leq v \text{ and } \alpha_{t-1} = 0 \text{ (stay out).} \end{cases} \quad (20)$$

The return of the strategy,  $r_{SL}$  is given in Formula 21, where  $r_{STOCK}$  is the return of the stock during the period,  $alpha_t$  is the weight of the stock in the portfolio and  $r_{ft}$  is the risk-free return:

$$r_{SL} = r_{STOCK} \times \alpha_t + r_{ft} \times (1 - \alpha_t) \quad (21)$$

The standard SL strategy consists of allocating the capital in the risk-free asset for the remainder of the investment period if the sell-signal is triggered. We make a practical adjustment and include a buy signal. This adjustment makes it possible for the portfolio value to decrease every time we invest in the risky asset. In practice, an investor will most likely re-enter the market of risky assets, if the SL has been triggered. Our adjustment allows for this to occur.

### 3.6.5 Market-timing strategy

We include a market timing strategy in our analysis. More specifically we simulate the return of a strategy and use the Simple Moving Average (SMA) to determine buy- and sell signals. The market timing strategies attempt to take advantage of the presumed trends in the stock price. We use historical prices when we try to predict future movements in the stock price. The objective is to buy before the price increase and sell when the stock price declines. The SMA is calculated by adding the prices of the stock  $n$  periods back and then divide it by the number of periods. The simple moving average is illustrated in Formula 22, where  $t$  denotes the period,  $n$  is the number of periods included in the moving average and  $P$  is the price of the stock.

$$SMA_t(n) = \frac{1}{n} \times \sum_{i=0}^{n-1} P_{t-i} \quad (22)$$

The trading rules, which decides when we buy and sell the stock is defined below. We use the moving averages to calculate technical trading indicators, which generates the trading signals. A positive (negative) indicator means that a buy (sell) signal is generated. The technical indicator we use in this study is the Simple Moving Average Crossover (SMAC). The moving average crossover uses two moving averages to calculate the value of the indicator. We have a moving average with a short window size  $s$  and a moving average with a longer window size  $l$ . We impose a restriction so that  $l > s$ . Formally, we calculate the moving average crossover indicator by:

$$SMAC(S, L)_t = SMA_t(S) - SMA_t(L) \quad (23)$$

When the indicator is positive we are holding the stock. Vice versa, when the indicator is negative, we hold the risk-free asset. A trading is generated when the indicator switches from either positive to negative, or from negative to positive. When the indicator changes from positive to negative we have a sell signal. When the indicator goes the other way, from negative to positive, we have a buy signal. The indicator is calculated as

$$\alpha = \begin{cases} 1 & \text{if } MAC(S, L)_t > 0 \\ 0 & \text{if } MAC(S, L)_t \leq 0 \end{cases} \quad (24)$$

We also evaluate the performance of a SMAC strategy where the SMA has a window size is equal to 1. This is the same as the stock price. The strategy is illustrated as

$$P - SMA(1). \quad (25)$$

The return of the strategy,  $r_{MA}$  is given in Formula 26, where  $r_{STOCK}$  is the return of the stock during the period,  $alpha_t$  is the weight of stock in the portfolio and  $r_{ft}$  is the risk-free return:

$$r_{MA} = \alpha_t \times r_{STOCK} + (1 - \alpha_t) \times r_{ft}. \quad (26)$$

### 3.7 Out-of-Sample Testing

We test the performance of various strategies against the performance of a Buy-and-Hold strategy. We use a testing methodology known as out-of-sample testing. Out-of-sample testing is conducted by splitting the data-sample into two different subsamples. The first subsample is called the in-sample period and the second subsample is called the out-of-sample period. We conduct two out-of-sample tests which mean that we have two in-sample periods (1990-2003 and 1975-1994) and two out-of-sample periods (2004-2016 and 1995-2016).

We start the out-of-sample test to conduct an in-sample test (also known as a back-test) and continue to use the optimal parameters for each strategy in the second data-sample to “verify” the performance. To only implement a back-test is a popular way of testing performance, due to its simplicity. Even though a back-test is a popular way of testing performance, it is well known to be a flawed testing method. When we test the performance of the various strategies, with multiple parameters which are optimised, we are really testing a vast number of strategies. This process of finding the best performing strategy is called data-mining (also

known as data-snooping) according to Agapitos, O'Neill, and Brabazon (2010). Zakamulin (2017) describes how the performance of the best performing strategy in back-tests are systematically overstated due to the random nature of asset returns. The phenomenon is called the "data-mining-bias". Zakamulin (2017) explain how the observed performance of a strategy is comprised of two components; the true performance of the strategy and a random component. When we test many strategies, some of the strategies are likely to overperform its true performance due to luck and some are going to underperform due to bad luck. When evaluating the performance of the strategies we tend to choose the strategies which benefit the most from luck. For a mathematical illustration, we refer to Zakamulin (2017). White (2000) argues that data-snooping is a dangerous practice which should be avoided, but it is often impossible because there is no alternative time series of interest to study. As mentioned, we choose to use out-of-sample testing, which copes with the data-snooping problem we observe in in-sample tests (back-tests).

We start the test to find the optimised parameters for each of the strategies in the in-sample period. In other words, we start to conduct an in-sample test and then test the best performing strategies in the second subsample, the out-of-sample period. With this practice, we reduce the biasness of the performance and validate the result. Zakamulin (2017) describes it, as dividing the dataset into a training set and a validation set.

To understand how we conduct the out-of-sample test, we provide an example; we want to test the real-life performance of the Stop Loss strategy. The strategy has one parameter,  $v$  which can be optimised. We start the test and split our data sample  $[1, T]$  into two subsets; the in-sample set  $[1, s]$  and the out-of-sample set  $[s, T]$ .  $s$  is the split point we choose, and  $T$  is the final observation in our dataset. We then use the in-sample period  $[1, s]$  to find the parameter,  $v^*$  which performs best in that sample period. We then continue to test the performance of the  $SL(v^*)$  in the out-of-sample period  $[s, T]$ . The idea is to behave like a practicing investor will in real-life, if he tests a strategy in-sample. The out-of-sample period is equivalent to the investors unknown future in the way that we do not know how the strategy performs in this subsample.

An important part when we conduct an out-of-sample test is to choose the split point. The choice we make with regards to the split point may impact the results we obtain substantially. The advantage of having a long in-sample subset is that the data-mining-bias decrease and we reduce our chance of finding false results. On the other hand, when we expand the in-sample period, we shorten our out-of-sample period. This means that the statistical power of the test for outperformance is reduced. This increases our chance to not reject a false null-hypothesis. In other words, that we do not find true outperformance to be true. Zakamulin (2017) suggests to choose a split point somewhere close to the middle of the full data sample. Zakamulin (2014) finds that to reduce the unbiasedness of the performance of the strategies, it is important that the in-sample period contains both bull and bear-markets. Zakamulin (2017) argues that the outperformance of a trend-following strategy is non-uniform. Generally, a trend-following strategy will outperform a passive Buy-and-Hold strategy in a bear market and underperform in a bull market. We believe that the performance of Portfolio Insurance strategies also may be non-uniform and therefore we believe it is important to use an in-sample period which contains both bull and bear markets. This is used as motivation to analyse and identify bull and bear markets.

### **3.8 Bull and Bear Markets**

Our motivation to identify bull and bear markets is closely related to those issues described regarding in- and out-of-sample periods. To make sure that the results we find in the in-sample-test are reliable for the out-of-sample-test we want to see if the periods are comparable. By this, we mean that we want to include both bull and bear market states in both of the in-sample and out-of-sample periods. This is consistent with Zakamulin (2014) and when we consider the strategies that we test, we recognise the importance that both of the periods include alternating market states. The result of this practice is that we can be sure that we test the performance of the strategies in market conditions where investors are supposed to benefit from portfolio protection.

Bull and bear markets are commonly used expressions in the financial language. There

is a lot of research on the area, but a clear and commonly agreed upon definition is yet to be derived. There are two different ways to identify bull and bear periods, that stand out as the most popular. The first is based on the length of a specific period of market movements. The other method is based on a percentage change in the market. Bry and Boschan (1971) and later research by Pagan and Sossounov (2003), amongst others, base their research from the first of the mentioned definitions and consider bull and bear markets to be defined by market movements in a specific period. Lunde and Timmermann (2004), amongst others, represent a different line of research based on the second definition. This research leans on a definition of bull and bear markets as market movements of a specific size, e.g. a percentage change of 20%. Zakamulin (2017) explains that both definitions often lead to results with corresponding conclusions but that the first is the most commonly used in financial literature and that it does not consider market movements from the shortest periods of time. We choose our definition based on these arguments and use the definition from Chauvet and Potter (2000) (p.90). “In stock market terminology, bull (bear) market corresponds to periods of generally increasing (decreasing) market prices”. This is the same definition as Pagan and Sossounov (2003) use in their research. They also use the same algorithm as we do, which Bry and Boschan (1971) derived, to identify bull and bear markets.

When we carry out the process and identify bull and bear markets, we use the same parameters in the algorithm as Pagan and Sossounov (2003). The identification of changes in market movements is required to identify in what periods the market is in bull and bear states. This is done in a three-step-procedure; it starts to look at all the observed data points, eight months prior and eight months subsequent to a specific date. Then all the peaks and troughs are found. A turning point is considered to be a peak (trough) in which it is the highest (lowest) point on both sides of the 8 months window. The process also includes a censoring operation with a parameter set to six months which eliminates cycles and phases which are shorter than 16 months and 4 months respectively. That is if the price has not increased or decreased by more than 20% in one month. In figures 1 and 2 respectively, we present our results regarding bull and bear markets in the periods 1990-2016 and 1975-2016. We can observe when we split



the data sample into in-sample and out-of-sample periods, with split points in 2004 (the 1990-2016 period) and 1995 (the 1975-2016 period), that both periods include alternating market states for all subsamples. We present some information about the bull and bear phases in both of the two data samples in Tables 1 and 2.



Figure 1: Figure showing bull and bear markets on the S&P 500 index between 1990 and 2016. White areas show periods considered as bull markets whereas shaded areas show bear markets. The x-axis show the year and the y-axis show the cumulative return of the S&P 500 index.

Bull and bear periods 1990-2016			Bull and bear periods 1975-2016		
Statistics	Bull	Bear	Statistics	Bull	Bear
Number of phases	6.00	6.00	Number of phases	11.00	10.00
Minimum duration	26.00	5.00	Minimum duration	11.00	3.00
Average duration	51.25	12.75	Average duration	39.67	11.25
Median duration	52.50	10.50	Median duration	39.00	8.00
Maximum duration	74.00	25.00	Maximum duration	74.00	25.00
Minimum amplitude	85.50	-7.75	Minimum amplitude	56.54	-7.75
Average amplitude	125.85	-30.90	Average amplitude	95.08	-25.45
Median amplitude	88.15	-31.65	Median amplitude	85.50	-20.41
Maximum amplitude	241.61	-52.56	Maximum amplitude	241.61	-52.56

Table 1: Presents descriptive statistics about the bull and bear periods from 1990-2016

Table 2: Presents descriptive statistics about the bull and bear periods from 1975-2016

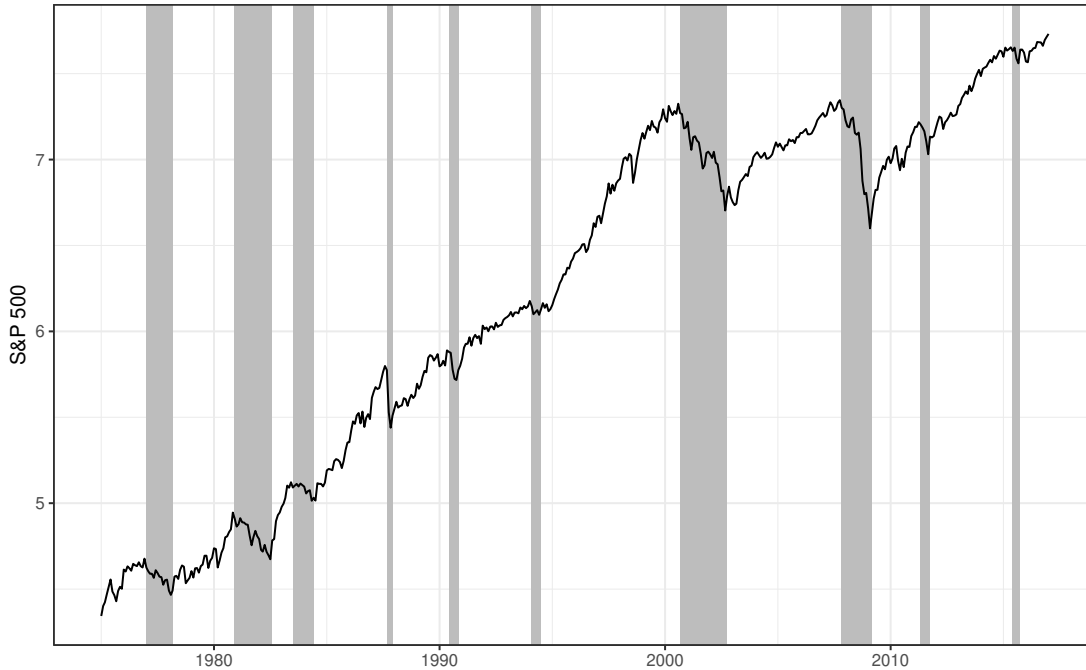


Figure 2: Figure showing bull and bear markets on the S&P 500 index between 1975 and 2016. White areas show periods considered as bull markets whereas shaded areas show bear markets. The x-axis show the year and the y-axis show the cumulative return of the S&P 500 index.

### 3.9 Performance Measurement

To evaluate the performance of the various strategies we need a risk-adjusted performance measure. One of the most used performance measures is the Sharpe ratio, by Sharpe (1966). It bases on the Modern Portfolio Theory, by Markowitz (1952). In Markowitzs theory, we find the efficient portfolio when we look at expected return compared to standard deviation. Markowitz considers standard deviation as the risk. The Sharpe ratio is a reward to risk measure that looks at the ratio between the excess return and the standard deviation. The performance measure uses total variance to evaluate the risk associated with the excess return and therefore punishes both upside and downside variation. The Sharpe ratio depends on the normality of the distribution of returns, which our strategies do not generate. The idea behind PI is to limit the downside risk, but maintain upside potential, which makes the distribution non-normal. Because of the presumed non-normality distribution of the returns generated by PI strategies, the Sharpe ratio is not an appropriate performance measure. The ratio is presented in Formula 27, where  $E[r_p - r_f]$  is the expected excess return of the portfolio,  $r_p$  is the return of the portfolio,  $r_f$  is the risk-free rate of return and  $\sigma_p$  is the standard deviation of the

portfolio.

$$\text{Sharpe ratio} = \frac{E[r_p - r_f]}{\sigma_p}. \quad (27)$$

Some studies on Portfolio Insurance such as Ho et al. (2011) and Hakan and Hande (2009) use the Sharpe ratio as the performance measure when they evaluate PI strategies. In a Portfolio Insurance environment, we think about risk as losses and not as the total variation, which explains why the Sharpe ratio is not a good measurement to use. Investors who find PI strategies attractive is looking to reduce the downside risk.

There are in fact several alternative performance measures to the Sharpe ratio. One alternative is the Sortino ratio which does not assume normally distributed returns as opposed to the Sharpe ratio. According to Kaplan and Knowles (2004), the Sortino measure focuses on the likelihood of not meeting some target return. Annaert et al. (2009) explain and show that high volatility can be due to positive return outliers, which can be attractive to investors. In the Sharpe ratio, all variance, both upside, and downside, can have a negative effect on the performance, as mentioned above. We show that the Sharpe ratio is not the appropriate measurement, but the Sortino ratio has, however, many similarities with and is a variation of the Sharpe ratio. It does, make adjustments when it considers the standard deviation and the risk-free rate of return. While the Sharpe ratio considers the total variance in a risky asset, the Sortino ratio only looks at the downside deviation, which is a necessary adjustment considering Portfolio Insurance as discussed above.

Volatility is often looked negatively at, but investors are not necessarily afraid of the upside deviation. Downside deviation is relevant in relation to loss-aversion, where loss-aversion is explained to be the case where investors utility is more affected by a loss than a gain (Barberis and Huang,2001). On the other hand, risk-aversion is explained to be the case where an investor will choose an investment with less risk (standard deviation), given the same expected return. We assume that investors that use Portfolio Insurance strategies desire upside volatility and a measure that only “punishes” downside deviation such as the Sortino ratio is therefore a better tool than the Sharpe ratio. In other words, we consider investors who find PI strategies

attractive to be loss-averse. The Sortino ratio uses a minimum acceptable return or a target return instead of the risk-free rate of return. This target return is however often set to equal the risk-free rate of return, which we also do, and this makes the numerator in the Sortino ratio identical to the one in the Sharpe ratio in practice.

We present the Sortino ratio in Formula 28. Recall that  $E[r_p - r_f]$  is the expected excess return of the portfolio and  $\sigma_D$  is the downside deviation. We present the downside deviation in Formula 29 where  $N$  is the number of returns. The calculation of the downside deviation considers observed returns under the target return and all other returns are set to zero.

$$\text{Sortino ratio} = \frac{E[r_p - r_f]}{\sigma_D}, \quad (28)$$

where

$$\sigma_D = \sqrt{\frac{1}{N} \times \sum_{n=1}^N \min(0, r_i - r_f)^2}. \quad (29)$$

## 3.10 Statistical Test

### 3.10.1 Hypotheses

In this section, we present the methodology we use to test whether the active strategies manage to outperform the passive Buy-and-Hold strategy. We denote the value of the performance measure, the Sortino ratio, as  $SOR_{PI}$  and  $SOR_{BH}$  for the PI strategies and the BH strategy respectively.

To only look at the performance of the strategies generated from the data-sample is not enough. The returns generated by the strategies are considered random variables and this means that any outperformance may be the result of “luck”. We formulate the null and alternative hypothesis we use in the statistical test for outperformance. The null- and the alternative hypothesis is presented as:

$$H_0 = SOR_{PI} \leq SOR_{BH} \quad (30)$$

and

$$H_A = SOR_{PI} > SOR_{BH}. \quad (31)$$

From the test of outperformance, we get a p-value which states at what significance level we can reject the null-hypothesis. A p-value at  $X$  percent means that we can say with  $100 - X$  percent certainty that the PI strategy truly generates a higher performance than the BH strategy. Another way of saying it is that with a p-value at  $X$  percent, there is an  $X$  percent probability that the PI strategy is not outperforming the BH strategy.

### 3.10.2 Non-Parametric Test

The objective of a statistical test is to test whether we can reject the null hypothesis. Statistical tests of significance are divided into two main categories; parametric tests and non-parametric tests as pointed out by Zakamulin (2017).

A parametric test is a test based on strict assumptions with regards the underlying probability distributions of the random variables. The most common approach according to Zakamulin (2017) is to assume that the random variables follow a bivariate normal distribution. This means that the two random variables both follow a normal distribution. The “parametric” test gets its name from the assumption that both random variables follow the same distribution. The distribution is defined by mean and variance. In a parametric test, a test-statistic is calculated based on the data sample. One of the advantages of a parametric test is that the test-statistic and p-value is easily calculated. We use the Sortino ratio as the performance measure and it does not exist any test-statistic for this performance measure. We, therefore, need to use a non-parametric test.

An advantage with the use of a non-parametric test is that it does not require the same strict assumptions as a parametric test does. When the assumption regarding the probability distribution and independence of the data-sample cannot be met, it is more appropriate to use a non-parametric test. This is often true when one conduct empirical research, especially in finance, where the data tend to be non-normal and serial dependent. Another advantage

of non-parametric tests is that they reduce the risk of drawing wrong conclusions, but at the same time, a non-parametric test has lower statistical power than a parametric test. This is a direct consequence of having to make fewer assumptions about the characteristics of the data (Nahm,2016).

According to Nahm (2016), one of the main drawbacks from using a non-parametric test compared to a parametric test is that large data samples make computation more complicated and that it requires more computer power. As the technology has improved, this is however no longer an issue, as most computers nowadays have the power to compute such tests and to perform large resamples (Zakamulin,2017). On the other hand, non-parametric tests have some advantages that make them very popular and easy to implement. In addition to the advantages already mentioned, there is no need for large data-samples and they are often easier to understand and implement for those with limited knowledge of statistics.

A popular non-parametric method is called “bootstrapping”. The general idea behind bootstrapping is to create new sets of data by resampling from the original data-sample. The method was first suggested by Efron (1979). The standard bootstrap method draws random resamples from the original data to create new pseudo time-series. In our case, we draw paired resamples from observations of excess returns for two strategies we want to statistically evaluate. The resamples are paired so that  $(X_t^{PI}, X_t^{BH})$ . Because the paired resample represents two excess returns, two pseudo time-series are created, and the historical correlation between the strategies are maintained.

Because we deal with financial data we choose to use a method called stationary block bootstrapping. This method avoids breaking up the serial dependency which financial data often contains. If we use a standard bootstrap method, we would break up the dependency of the data when we resample one random draw at the time. The block method resamples “blocks” of data and thereby avoids breaking the serial dependency. The stationary block bootstrap allows for blocks to overlap each other, which is preferable when the data sample is small relative to the block length. Also, the stationary block bootstrap method resamples blocks of data with mean length equal to the optimal block length. As in the standard bootstrap, we pair the

resampled returns so that  $(X_t^{PI}, X_t^{BH})$  creates two new pseudo time-series. For every random resample the Sortino ratio is computed for every time-series of excess return for the PI and BH strategy. We then calculate the difference between  $\Delta = SOR_{PI} - SOR_{BH}$ . The p-value is calculated as  $\frac{n}{N}$ , where  $n$  is the number of positive  $\Delta$ , and  $N$  is the number of resamples. We use 10 000 resamples. We calculate the optimal block length using the formula suggested by Hall, Horowitz, and Jing (1995) and this is consistent with later literature, such as Zakamulin (2017). The formula  $T^{\frac{1}{h}}$  where  $h = 4$  and  $T$  is the number of observations, gives an estimate for optimal block length.  $h = 4$  is consistent with testing a one-sided hypothesis from a block bootstrap resample, which is what we do. The optimal block-length is estimated to be equal to 4 for all our testing periods.

## 4 Data

In this section, we present and describe the data that we use in the analysis and throughout this thesis. All data are monthly data from January 1975 to January 2017 or from January 1990 to January 2017. January 2017 is included in the dataset to get returns for a round number of years in the out-of-sample periods. We do however refer to the out-of-sample periods as they last to the end of 2016 from now on. We test all five strategies in the period from 1990 to 2016, but only CPPI, SL, and MA in the period from 1975 to 2016 because of restrictions of the VIX index as we described earlier.

We choose to study the American stock market. Specifically, we study the S&P 500 index, as the security of which we conduct our analysis. The S&P 500 index is a broad and diversified index containing 500 large American companies. It is, therefore, considered a good proxy of the stock market in general. We obtain the dataset consisting of prices of the S&P 500 index from a dataset provided by Amit Goyal <sup>1</sup> called “PredictorData2016. We use data for the period from 1975 to December 2016. We extend our dataset to include January 2017 and download data for S&P 500 index from Yahoo Finance <sup>2</sup>. In figure 3 an illustration of the

<sup>1</sup>The data from Amit Goyal is downloaded from <http://www.hec.unil.ch/agoyal/>

<sup>2</sup>Data from Yahoo Finance is downloaded and can be found from <https://finance.yahoo.com/>

logarithmic returns on the S&P 500 index is given.

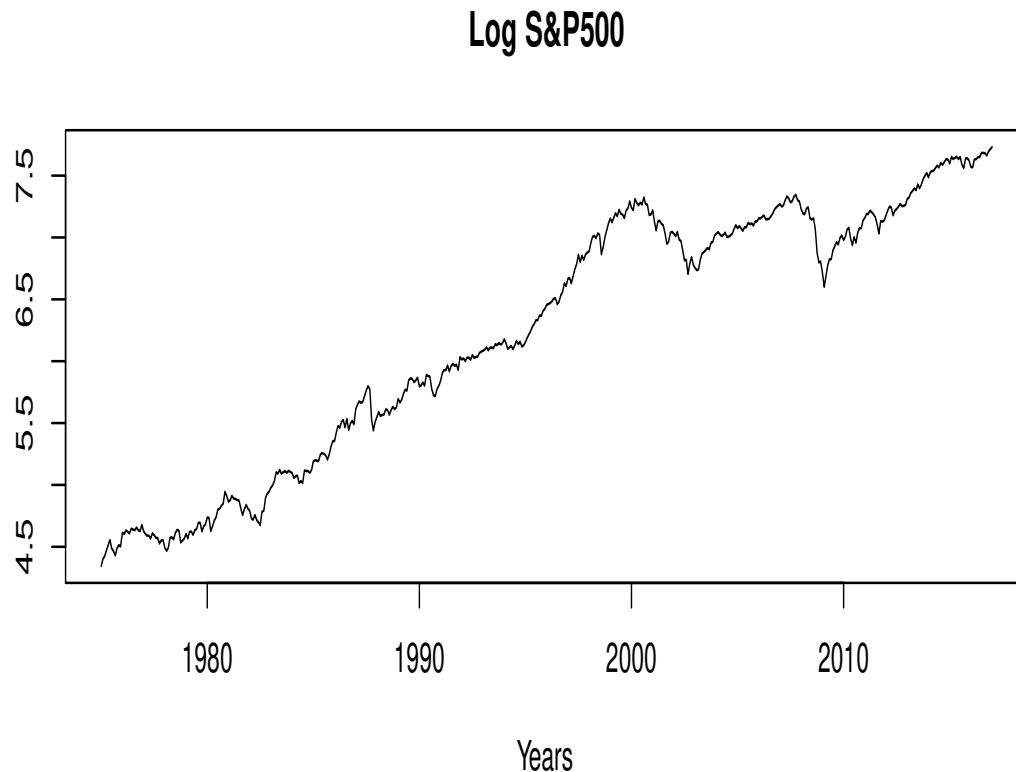


Figure 3: Presents the logarithmic returns for the S&P 500 index in the period from January 1975 to January 2017.

Data for dividends and variance on the S&P 500 index is also downloaded from the dataset (PredictorData2016) provided by Amit Goyal and for the same period. Goyal describes dividend in the dataset as the twelve-month moving sum of dividends paid on the S&P 500 index. We also extend our dataset here, with the dividend for January 2017<sup>3</sup>. The dataset also contains the variance for the S&P 500 index. The variance is calculated as sums of squared daily returns on the index. We use stock variance to show that the realised volatility on the S&P 500 index is systematically lower than implied volatility (VIX index).

We use data for the VIX index as an estimate for volatility when we estimate the option prices. The VIX index is downloaded from Yahoo Finance<sup>4</sup>. In figure 4 a presentation of the

<sup>3</sup>Dividend for January 2017 is downloaded from <http://www.multip1.com/s-p-500-dividend-yield/table?f=m>

<sup>4</sup>Data for the VIX index is downloaded and can be found from <https://finance.yahoo.com>



VIX index is given.

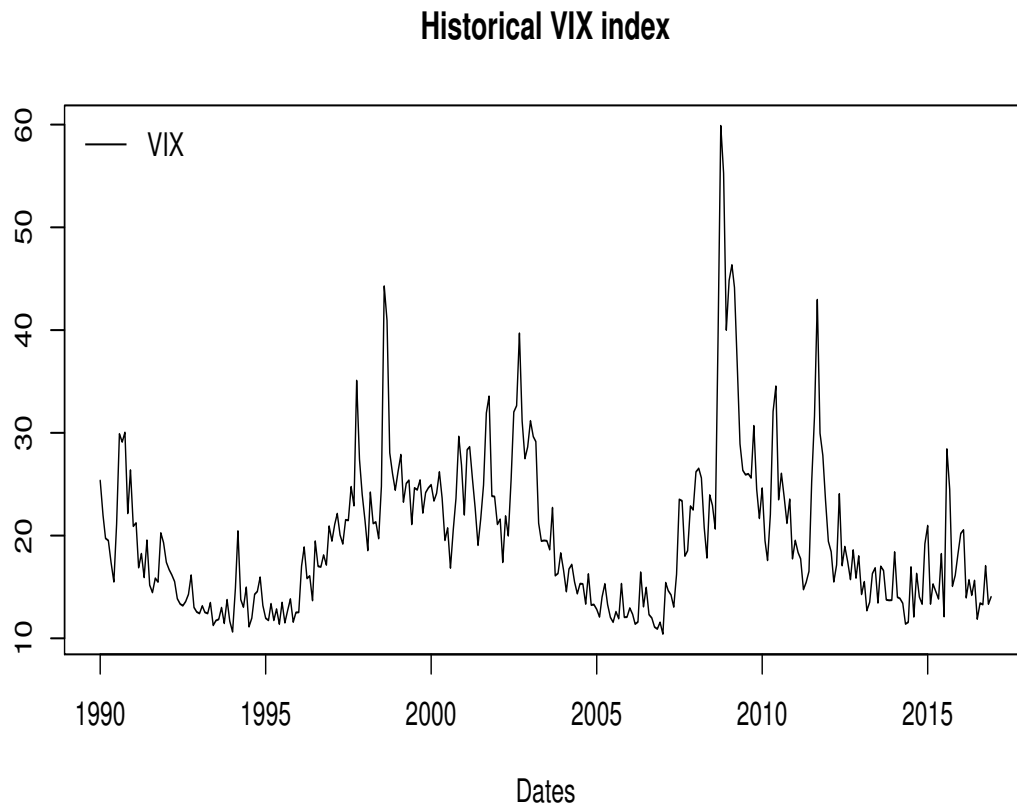


Figure 4: Illustrates the VIX index from January 1990 to January 2017. The VIX index present the one-month implied volatility on the S&P 500 index.

The data for the risk-free rate of return is provided by Kenneth R. French<sup>5</sup>. This is the one-month Treasury bill rate and is originating from Ibbotson and Associates, Inc. In table 3 we provide some descriptive statistics of the datasets. The statistics are for the full period, for both the long and short data sample.

## 5 Monte Carlo Simulation Study

Before we present the empirical results, we conduct a simulation study on the performance of the two option strategies. These findings serve as a motivation for the empirical study in this

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<sup>5</sup>Data is downloaded from [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

Descriptive data for returns on the S&P 500 index		
Statistics	BH 1975-2017	BH 1990-2017
Min return	-21.54	-16.74
Average return	11.11	10.31
Max return	16.78	11.43
Std. dev	15.16	14.36
Skewness	-0.44	-0.59
Kurtosis	1.92	1.26
Lower Std.dev	2.98	2.83
Sortino ratio	0.62	0.77

Table 3: Descriptive data of the returns on the S&P 500 index in the periods from January 1975 to January 2017 and January 1990 to January 2017. The statistics are calculated using monthly data.

thesis. The study looks at the theoretical performance of the option-based strategies and how dividends affect the performance of the option strategies.

The simulation study is a Monte Carlo simulation and we make some assumptions about the return-generating process and the trading environment of the stock. The first assumption is that the underlying asset follows a Geometric Brownian Motion. This is a process that researchers commonly assume in financial literature. The Geometric Brownian Motion relates to a so-called random walk of the asset price. That is, the observed prices are incorporated, and the next price is conditionally independent of what has happened before. For further information about the properties of the Brownian motion, we refer to the paper by Ermogenous (2006) and Reddy and Clinton (2016). We make assumptions about the parameters in the Black-Scholes-model. These assumptions are realistic and try to mimic the parameters of the S&P 500 index. For this simulation, we set the mean return to 10%, the volatility (standard deviation) to 20%, the risk-free rate of return to 3% and time to maturity equal to 1. We conduct a large number of simulations (10 million). Consistent with what we do in the empirical research, we measure the performance with the use the Sortino ratio.

In the simulation of the Cash-Call strategy, we set the strike price to At-The-Money ( $M=1$ ) and the number of call-options (upside potential) to 1,5 ( $B=1,5$ ). The latter is done as this multiplier gives an average return of the protection strategy close to the average return of the Buy-and-Hold strategy. In the simulation study for the Protective-Put strategy, we also set the strike price to At-The-Money but set  $B=0,8$ . With this amount of put options (protection), we

generate a performance for the two strategies that are quite similar. This is illustrated in Tables 4 and 5. We see that both of the PI strategies outperform the BH strategy.

Cash-Call			Protective-Put		
Statistics	BH	CC	Statistics	BH	PP
Min return	-64.43	-11.50	Min return	-63.64	-14.61
Average return	10.52	10.49	Average return	10.51	8.12
Max return	200.73	289.59	Max return	205.74	189.95
Std. dev	22.32	26.70	Std. dev	22.33	17.42
Skewness	0.61	1.49	Skewness	0.62	1.38
Kurtosis	0.68	2.46	Kurtosis	0.69	2.12
Sortino ratio	0.73	0.80	Sortino ratio	0.73	0.82

Table 4: Simulation of Cash-Call strategy without dividends. Table 5: Simulation of the Protective-Put strategy without dividends. Strike price is set to at-the-money and protection to 150% of the portfolio. The table shows statistics from the Monte-carlo simulation of the portfolio. The table shows statistics from the Monte-carlo simulation.

In the next part of our simulation study, we introduce dividends to study the effect it has on the performance. We do this because the S&P 500 index pays dividends. We set dividend to 2% and re-run the simulations again. The results are reported in Table 6 and 7 and from these, we can observe a change in the results. There is still a difference between the average return on the strategies, and the difference has increased. The main change is however in the Sortino ratios. Whilst Tables 4 and 5 show a similar performance, the performance in Tables 6 and 7 is not the same. The CC strategy now outperforms the PP strategy. This can be explained by the nature of how option prices depend on dividends. A call option will decrease its price when the dividends increase while put option prices will increase with increased dividends. In other words, the put prices become more expensive and therefore decrease the returns of the protective put strategy. On the other hand, the call options become less expensive, and the reduction in premium paid, increase the performance of the cash-call strategy.

The Monte Carlo simulation illustrates that dividend affects the strategies. Both the Protective-Put and the Cash-Call outperform the Buy-and-Hold strategy in a theoretical environment without dividends. The dividends do however have a different impact on the strategies. The performance of the Cash-Call strategy improves with dividends while the performance of the Protective-Put strategy becomes weaker. The PP strategy no longer outperforms the BH strategy when dividends are implemented. Fama (1970) and Kaminski and Lo (2014) argue that

Cash-Call			Protective-Put		
Statistics	BH	CC	Statistics	BH	PP
Min return	-53.64	-9.73	Min return	-53.60	-8.40
Average return	12.82	14.84	Average return	12.82	8.61
Max return	150.42	215.90	Max return	156.13	139.32
Std. dev	22.80	28.12	Std. dev	22.82	17.66
Skewness	0.61	1.39	Skewness	0.62	1.36
Kurtosis	0.68	2.12	Kurtosis	0.65	1.97
Sortino ratio	1.04	1.54	Sortino ratio	1.04	0.92

Table 6: Simulation of Cash-Call strategy with dividends of 2%. Strike price is set to at-the-money and protection to 150% of the portfolio. The table show statistics from the Monte-carlo simulation. Table 7: Simulation of the Protective-Put strategy with dividends of 2%. Strike price is set to at-the-money and protection to 80% of the portfolio. The table show statistics from the Monte-carlo simulation.

the MA and SL strategies do not work when the stock follows a Random Walk process. Agić-Šabeta (2016) finds that the CPPI strategy underperforms a BH strategy in bull and no-trending markets.

We show the Probability Density Functions with no dividends and with dividends in Figures 5 and 6. We see that in both cases, the distributions are highly non-normal.

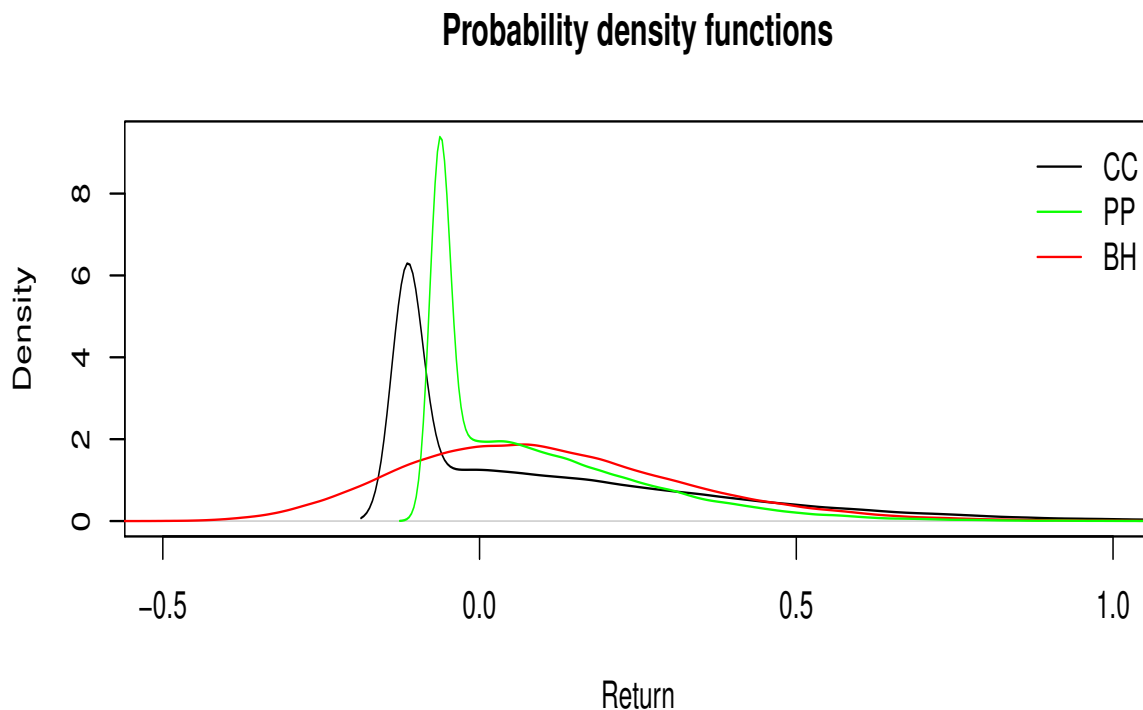


Figure 5: Presents the distributions of the option-based strategies and the Buy-and-Hold strategy, theoretically without dividends. Recall that CC is the Cash-Call strategy, PP is the Protective-Put strategy and BH is the passive Buy-and-Hold strategy.

## Probability density functions

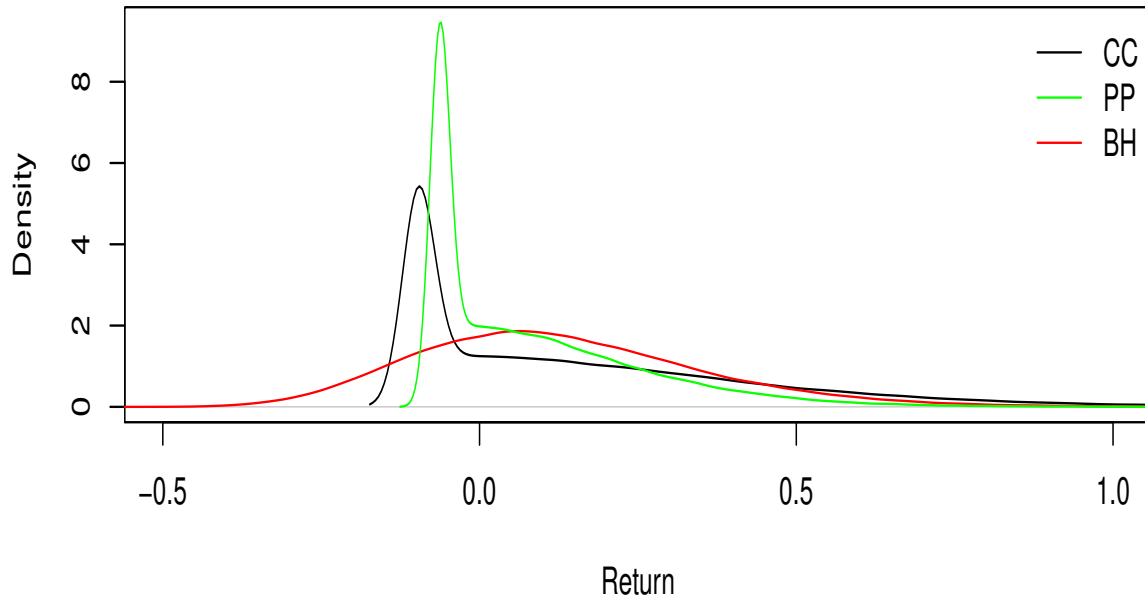


Figure 6: Presents the distributions of the option-based strategies and the Buy-and-Hold strategy, theoretically with dividends. Recall that CC is the Cash-Call strategy, PP is the Protective-Put strategy and BH is the passive Buy-and-Hold strategy.

## 6 Empirical Results

In this section, we present our results from the empirical study. We begin to present the results of the first in-sample test which starts at the beginning of 1990 and concludes at the end of 2003. For the option-buying strategies, we cannot go further back than to the start of 1990 due to the VIX index. We then present the out-of-sample results from the period from 2004 to 2016. We proceed by presenting the results from the in-sample period from 1975 to 1994 and the out-of-sample period from 1995 to 2016.

### 6.1 In-Sample 1990-2003

Tables 8 and 9 present the performance of the option-buying strategies with different parameters for the strike price and the number of options. The two option-buying strategies both

underperform if we compare them to the passive BH strategy for all the combinations of parameters. When we optimise the parameters for the option-based strategies in-sample, we allow for B to go from 0.75 to 1.25, by a 0.05 change and we allow for M to go from 0.9 to 1.1, by a 0.05 change. Our results indicate that the parameters which perform the best in-sample are to buy 75% of an option (B=0.75) with a strike price at 90% of the stock price (M=0.9). The optimal parameters for both strategies consist of the lowest possible value which we allow for in the test. The optimal parameters for the Cash-Call strategy and the Protective-Put strategy are therefore to buy 75% of an option with a strike price of 90% of the stock price. One can observe from Table 8 that the Sortino ratios for different amounts of options used in the Cash-Call strategy look identical for most strike prices. In Table 20 in the appendix, we present the same table with more digits to show that there is a difference in these ratios, though a very small.

M \ B	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25
0.90	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68
0.95	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
1.00	-0.54	-0.54	-0.54	-0.54	-0.54	-0.54	-0.54	-0.54	-0.54	-0.55	-0.55
1.05	-2.03	-2.04	-2.04	-2.04	-2.04	-2.04	-2.04	-2.04	-2.04	-2.04	-2.04
1.10	-2.05	-2.05	-2.05	-2.06	-2.06	-2.06	-2.06	-2.06	-2.06	-2.06	-2.06
	Sortino ratio BH = 0.78										

Table 8: Presents the Sortino ratios for all the combinations of strike prices and the amount of protection in the Cash-Call strategy. “M” represent the multiplier for the strike price and “B” represent the number of options (call options) used on the portfolio in the protection strategy. The in-sample period starts in January 1990 and last until the end of December 2003. The performance is presented as annualized Sortino ratios.

M \ B	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25
0.90	0.71	0.70	0.70	0.69	0.68	0.68	0.67	0.67	0.66	0.66	0.65
0.95	0.51	0.48	0.46	0.44	0.41	0.39	0.37	0.34	0.32	0.29	0.26
1.00	-0.09	-0.19	-0.29	-0.39	-0.49	-0.60	-0.70	-0.80	-0.90	-0.99	-1.08
1.05	-0.74	-1.05	-1.41	-1.77	-2.06	-2.21	-2.23	-2.19	-2.12	-2.05	-1.99
1.10	-0.26	-0.68	-1.36	-1.99	-1.90	-1.66	-1.49	-1.38	-1.30	-1.24	-1.20
	Sortino ratio BH = 0.78										

Table 9: Presents the Sortino ratios for all the combinations of strike prices and the amount of protection in the Protective-Put strategy. “M” represent the multiplier for the strike price and “B” represent the number of options (put options) used on the portfolio in the protection strategy. The in-sample period starts in January 1990 and last until the end of December 2003. The performance is presented as annualized Sortino ratios.

The CPPI strategy performs best in-sample when the initial guaranteed amount is 85% of

the wealth at the beginning of the period and when the multiplier is equal to 3. The strategy underperforms compared to the BH strategy and yields a Sortino ratio of 0.58. Table 10 presents the results.

CPPI strategy in-sample (1990-2003)											
M \ F	0.85	0.86	0.87	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.95
3	0.58	0.58	0.57	0.57	0.57	0.57	0.56	0.56	0.55	0.54	0.54
4	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.56	0.56	0.54
5	0.54	0.54	0.55	0.55	0.55	0.55	0.55	0.56	0.56	0.56	0.55
6	0.48	0.49	0.49	0.49	0.50	0.50	0.51	0.52	0.52	0.53	0.54
Sortino ratio BH = 0.78											

Table 10: Presents the Sortino ratios for all the combinations of the initial guaranteed amounts (floor values) and multipliers. “M” represent the multiplier and “F” represent the floor value, at which is the minimum acceptable wealth of the initial wealth. The in-sample period starts in January 1990 and last until the end of December 2003. The performance is presented as annualized Sortino ratios.

SL strategy in-sample (1990-2003)	
$v$	Performance
0	0.60
1	0.84
2	0.41
3	0.72
4	0.73
5	0.97
6	0.99
7	1.02
8	0.98
9	1.00
10	0.94
11	1.30
12	1.30
13	1.30
14	1.10
15	1.04
Sortino ratio BH	
	0.78

Table 11: Presents the Sortino ratios for the best SL strategies in-sample.  $v$  represent the percentage change of price, in which buy and sell signals are generated. The in-sample period starts in January 1990 and last until the end of December 2003. The performance is presented as annualized Sortino ratios.

The Stop Loss strategy outperforms the Buy-and-Hold strategy and the optimal drawdown and drawup percentage for when buy- and sell-signals are triggered is 11%. When a sell-signal is generated, we sell the stock and invest in the risk-free asset until a buy-signal is generated. We optimise the strategy by testing for percentages between 0% to 15%. One can read from

Table 11 that the performance of the strategy is highest when buy- and sell-signals (drawup and drawdown) are generated by a relatively high percentage.

Table 12 reports that the Moving Average strategies which perform the best in-sample, are the SMAC (2,11) and the SMAC (2,12). These MA strategies generate Sortino ratios of 1,19 while the Buy-and-Hold strategy shows a Sortino ratio of 0,78. For this strategy, we optimise the parameters and test for short window sizes between 1 and 5, and long window sizes between 2 and 15. Since there are two combinations of parameters that generate identical performance, we choose to test the SMAC (2,11) out-of-sample.

MA strategy in-sample (1990-2003)		
Rank	Rule	Performance
1	SMAC(2,11)	1.19
2	SMAC(2,12)	1.19
3	P-SMA(15)	1.17
4	P-SMA(12)	1.13
5	SMAC(2,15)	1.13
6	SMAC(2,10)	1.13
7	SMAC(2,14)	1.12
8	SMAC(4,15)	1.11
9	P-SMA(13)	1.10
10	SMAC(3,15)	1.08
11	SMAC(5,13)	1.06
12	SMAC(5,14)	1.04
13	SMAC(5,15)	1.04
14	SMAC(2,13)	1.04
15	SMAC(3,14)	1.03
16	SMAC(4,14)	1.02
17	SMAC(4,13)	1.02
18	SMAC(5,12)	0.99
19	SMAC(4,10)	0.98
20	SMAC(4,11)	0.98
	B&H	0.78

Table 12: Presents the Sortino ratios for the best MA strategies in-sample. SMAC(S,L) is the simple moving average crossover with a short and a long window size. P-SMA is equivalent to SMAC(1,L). "S" denotes the short window size (which we have set to go from 1 to 5) and L denotes the long window size (which we have set to go from 2 to 15). The best performing strategy, SMAC(2,11) has e.g. a short window size of 2 and a long of 11, which means that a sell or buy signal is generated when the 2 month moving average crosses the 11 month moving average. The in-sample period starts in January 1990 and last until the end of December 2003. The performance is presented as annualized Sortino ratios.

A summary of descriptive statistics for all five strategies can one read in Table 13 and we conclude from the in-sample period that the Stop Loss and the Moving Average strategies



perform best. The MA and SL strategies outperform the BH strategy, while the CPPI, the PP, and the CC strategy underperform if we compare them to the BH strategy. We draw no conclusions on the real-life performance from the in-sample period, but we use the best performing parameters in the out-of-sample test.

Summary of all strategies in-sample (1990-2003)						
Statistics	BH	Cash-Call	Protective-Put	CPPI	MA	SL
Min return	-14.460	-7.800	-11.150	-12.570	-14.461	-14.461
Average return	12.110	9.257	11.186	6.298	12.671	13.665
Max return	11.430	8.614	11.368	7.693	11.430	11.430
Std. dev	14.886	10.842	14.521	5.706	11.358	11.996
Skewness	-0.483	-0.370	-0.394	-0.529	-0.386	-0.383
Kurtosis	0.554	0.020	0.101	4.392	2.998	2.154
Lower Std.dev	2.887	2.112	2.827	1.143	2.036	2.131
Average drawdown	0.057	0.040	0.058	0.020	0.041	0.043
Maximum drawdown	0.447	0.364	0.466	0.120	0.154	0.169
Sortino ratio	0.784	0.681	0.706	0.512	1.191	1.299
P-value		0.931	0.930	0.914	0.136	0.166

Table 13: A summary of statistics for all strategies (Cash-Call, Protective-Put, CPPI, MA and SL) including BH, in-sample (1990-2003). The in-sample period starts in January 1990 and last until the end of December 2003.

## 6.2 Out-of-Sample 2004-2016

Figure 7 presents the cumulative returns of all the strategies for this period and Table 14 reports the results. In this test, we use the parameters for each strategy, which have the highest performance in the in-sample period. We find that the option-based strategies underperform against the BH strategy. This is not surprising as the option-buying strategies also underperform in the in-sample test. The relative performance of the strategies tends to decrease in the out-of-sample test compared to the in-sample test. The CPPI strategy also underperforms in relation to the BH strategy. The risk is lower, but so is the average excess return. The reduced risk does not compensate the lower return, judging by the Sortino ratio. The SL and MA strategies outperform the BH strategy in the out-of-sample test. We find the Sortino ratios to be economically significantly higher, but the p-values are above conventional levels.

In addition, to look at the performance, measured by the Sortino ratio, it is interesting to look at the drawdowns the strategies generate. The CPPI strategy average half as large

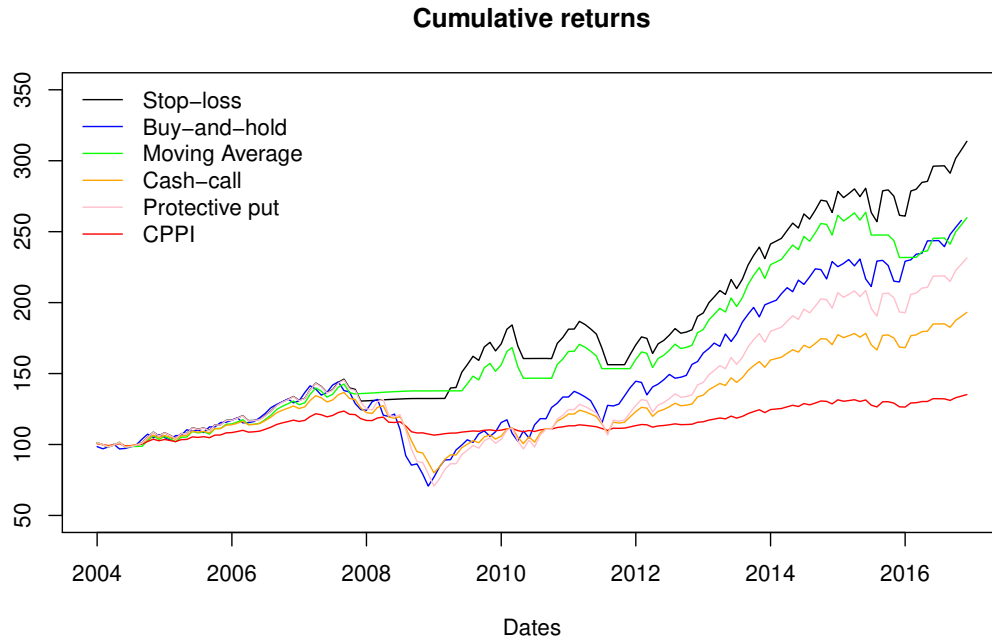


Figure 7: Present the cumulative returns for all of the active protection strategies (Protective-Put, Cash-Call, Moving Average, Stop Loss and Constant Proportion Portfolio Insurance) and the passive buy-and-hold strategy out-of-sample (2004-2016). The out-of-sample period starts in January 2004 and last to January 2017.

drawdowns as the next ranking strategy, which is the Moving Average strategy. Figure 8, 9, 10, 11 and 12 illustrates the drawdowns for all five strategies. We can observe from the graphs for the option-based strategies that they allow for the portfolio to decrease more in value. The maximum drawdown for these strategies is much deeper than for all the other ones. An interesting observation to take notice of is that they, however, reach a new all-time-high faster than the CPPI strategy after the Global Financial Crisis of 2007-2008. This is despite from the fact that the CPPI strategy manages to provide better protection during the crisis. The drawdowns for the SL and the MA strategy, which Figures 12 and 11 illustrates, show quite similar behaviour. The protection from the MA and SL strategy are better than the option-based strategies, and they reach a new peak earlier.

Statistics for all of the strategies out-of-sample (2004-2017)

Statistics	BH	Cash-Call	Protective-Put	CPPI	MA	SL
Min return	-16.736	-8.408	-12.178	-3.631	-8.043	-8.043
Average return	8.376	5.572	7.370	2.391	7.746	9.316
Max return	10.958	7.223	9.853	2.880	6.690	8.483
Std. dev	13.796	9.976	13.344	3.720	8.717	9.935
Skewness	-0.751	-0.665	-0.648	-0.636	-0.407	-0.283
Kurtosis	2.215	0.892	0.989	0.930	0.742	0.585
Lower Std.dev	2.762	2.028	2.695	0.774	1.577	1.756
Average drawdown	0.054	0.041	0.056	0.021	0.039	0.043
Maximum drawdown	0.510	0.415	0.514	0.137	0.129	0.164
Sortino ratio	0.751	0.624	0.662	0.448	1.201	1.336
P-value		0.952	0.939	0.914	0.277	0.195

Table 14: A summary of statistics for all strategies (Cash-Call, Protective-Put, CPPI, MA and SL) including BH, out-of-sample (2004-2017). The out-of-sample period starts in January 2004 and last to January 2017.

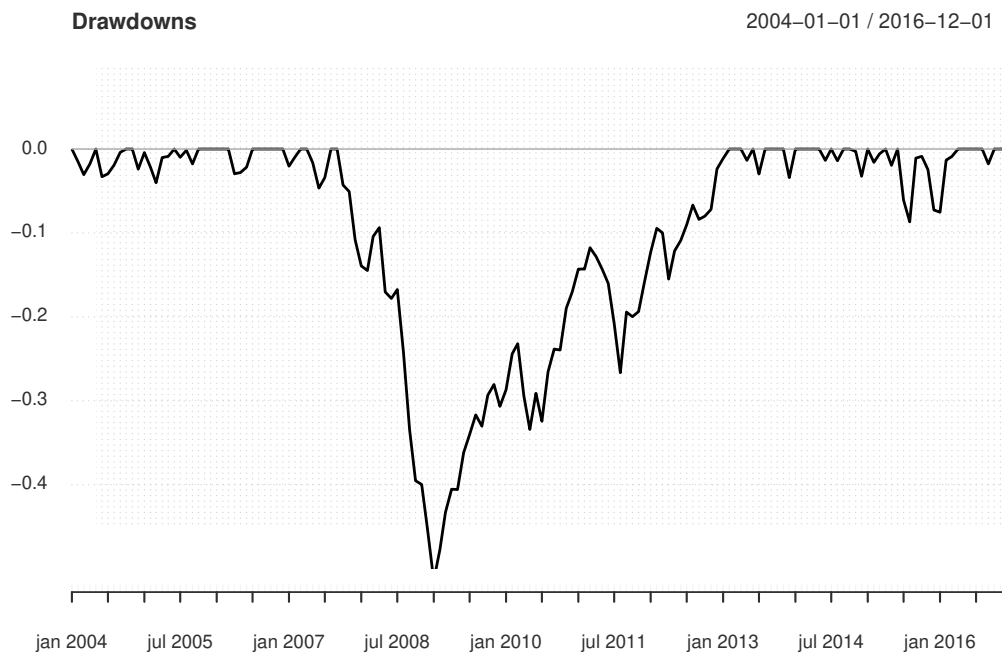


Figure 8: Shows the drawdown for the Protective-Put strategy out-of-sample (2004-2016). The drawdown as presented in this figure refers to the peak-to-trough for the S&P 500 index, and show the length and depth of decreasing prices in the index. Every time the graph reaches 0.0, we observe an all-time high for the index.

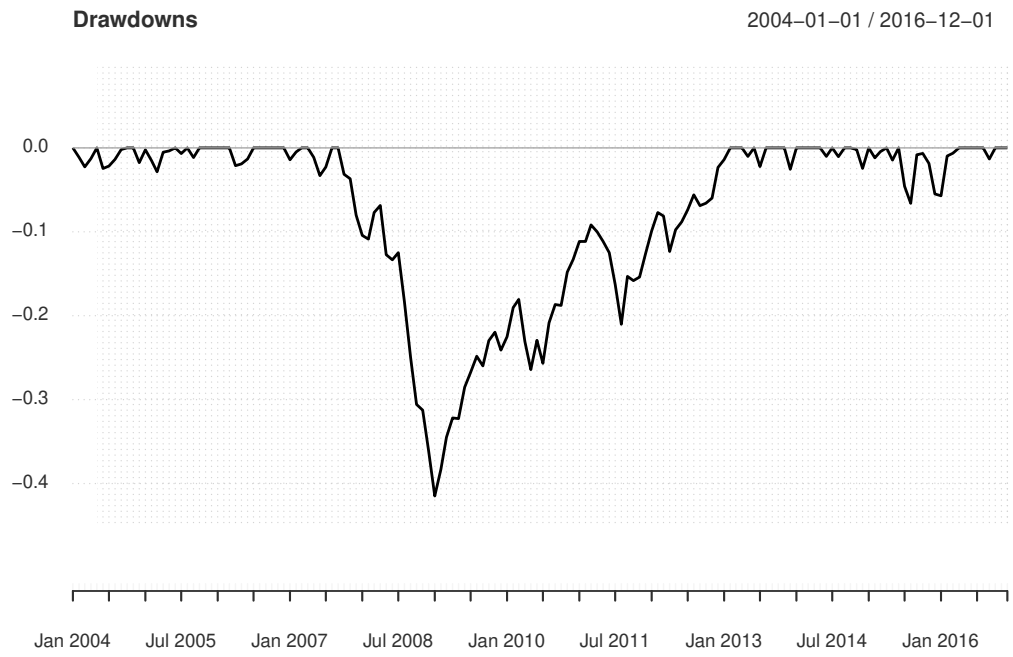


Figure 9: Shows the drawdown for the Cash-Call strategy out-of-sample (2004-2016). The drawdown as presented in this figure refers to the peak-to-trough for the S&P 500 index, and show the length and depth of decreasing prices in the index. Every time the graph reaches 0.0, we observe an all-time high for the index.



Figure 10: Shows the drawdown for the Constant Proportion Portfolio Insurance strategy out-of-sample (2004-2016). The drawdown as presented in this figure refers to the peak-to-trough for the S&P 500 index, and show the length and depth of decreasing prices in the index. Every time the graph reaches 0.0, we observe an all-time high for the index.

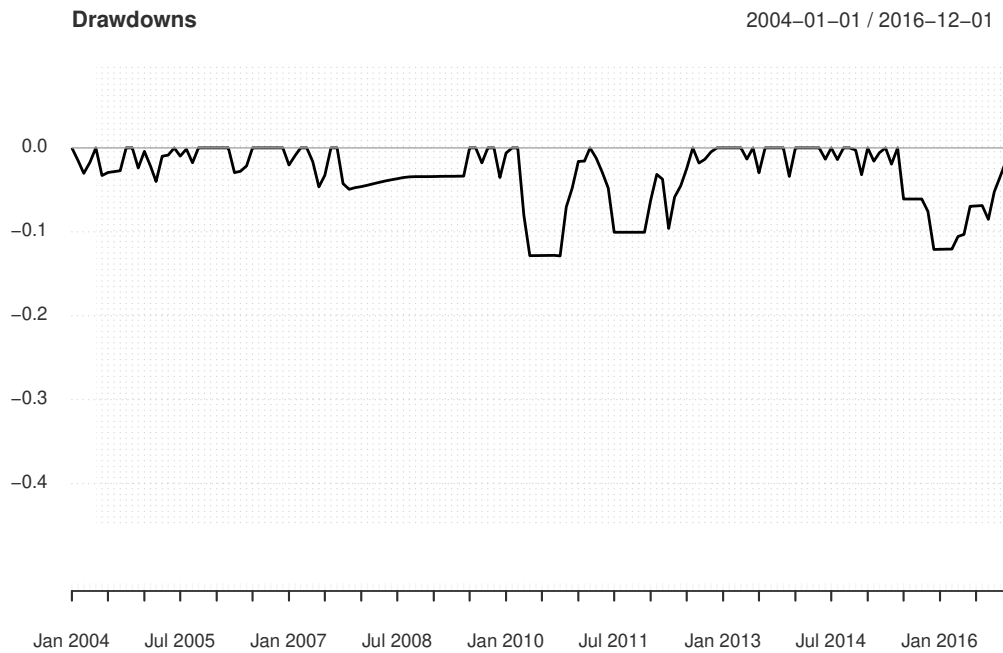


Figure 11: Shows the drawdown for the Moving Average strategy out-of-sample (2004-2016). The drawdown as presented in this figure refers to the peak-to-trough for the S&P 500 index, and show the length and depth of decreasing prices in the index. Every time the graph reaches 0.0, we observe an all-time high for the index.

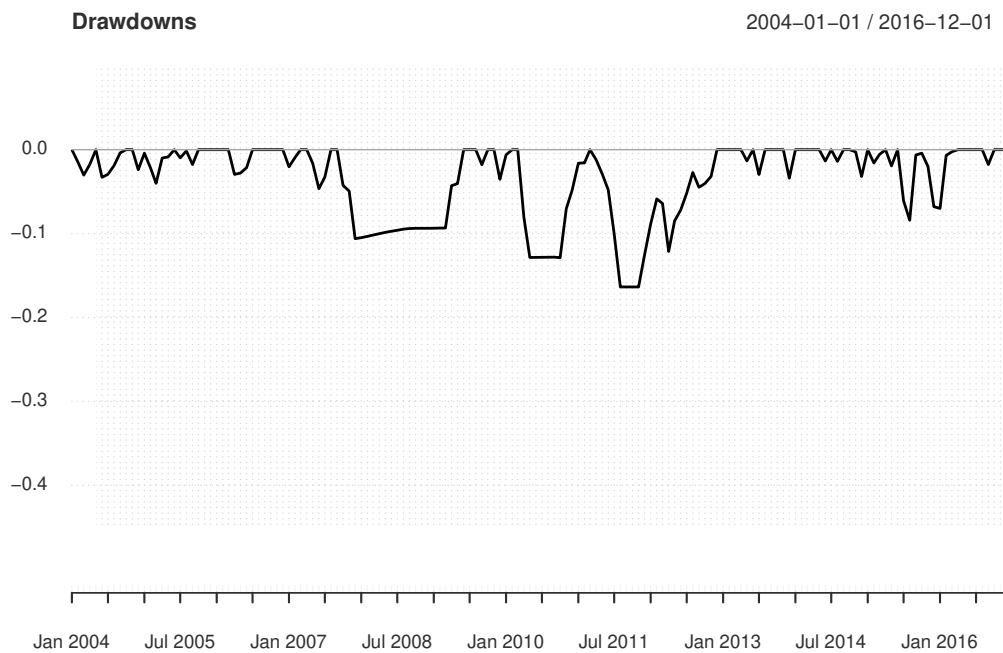


Figure 12: Shows the drawdown for the Stop Loss strategy out-of-sample (2004-2016). The drawdown as presented in this figure refers to the peak-to-trough for the S&P 500 index, and show the length and depth of decreasing prices in the index. Every time the graph reaches 0.0, we observe an all-time high for the index.

Finally, we present the Probability Density Functions (PDF) for the strategies. The PDF is an illustration of the probability for the variable (the return) to fall within a range of values. In Figure 13, we illustrate the PDFs for the PP and CC strategy, along with the PDF for the BH strategy. We see that the option-buying strategies have a similar PDF to the BH. In Figure 14, we show the PDFs for the MA, SL and CPPI strategy. We observe that the PDF's for these PI strategies are more concentrated around the centre than the PDF for the BH is. The tails of the BH strategy is longer than for the PI strategies, which indicates that the PI strategies can give protection against extreme losses.

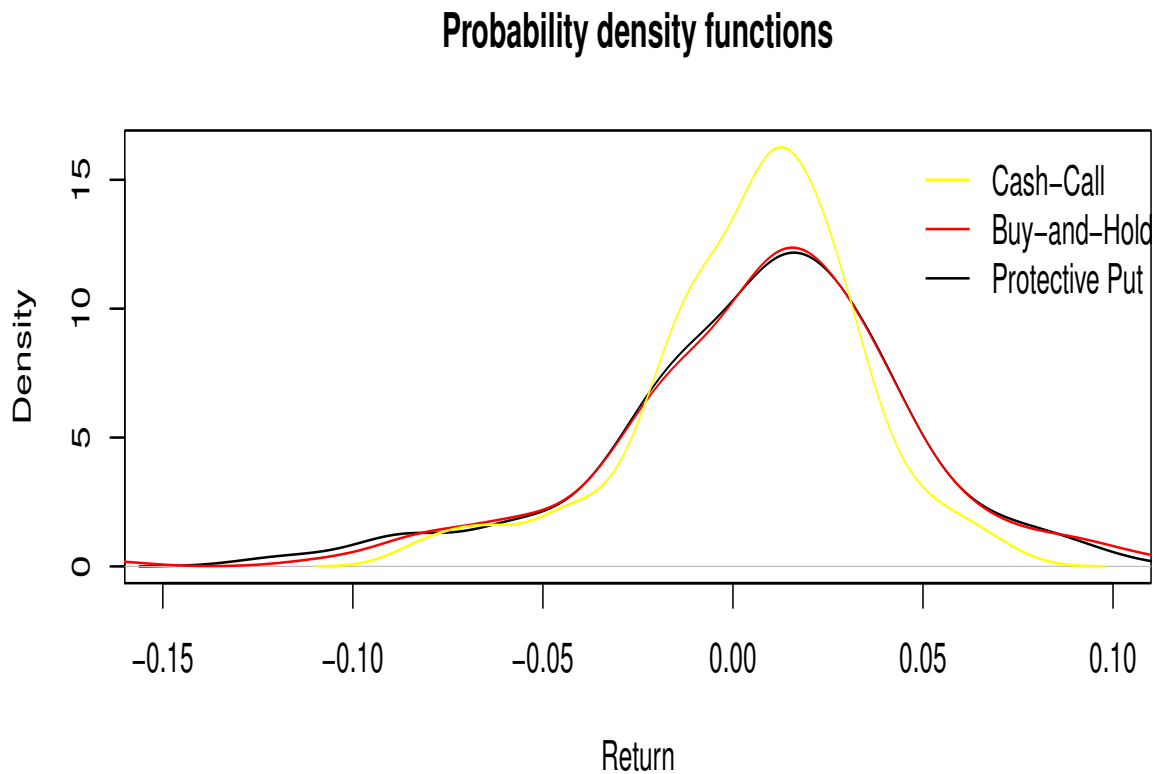


Figure 13: Presents the distributions of the option-based strategies and the Buy-and-Hold strategy, theoretically without dividends. Recall that CC is the Cash-Call strategy, PP is the Protective-Put strategy and BH is the passive Buy-and-Hold strategy.

## Probability density functions

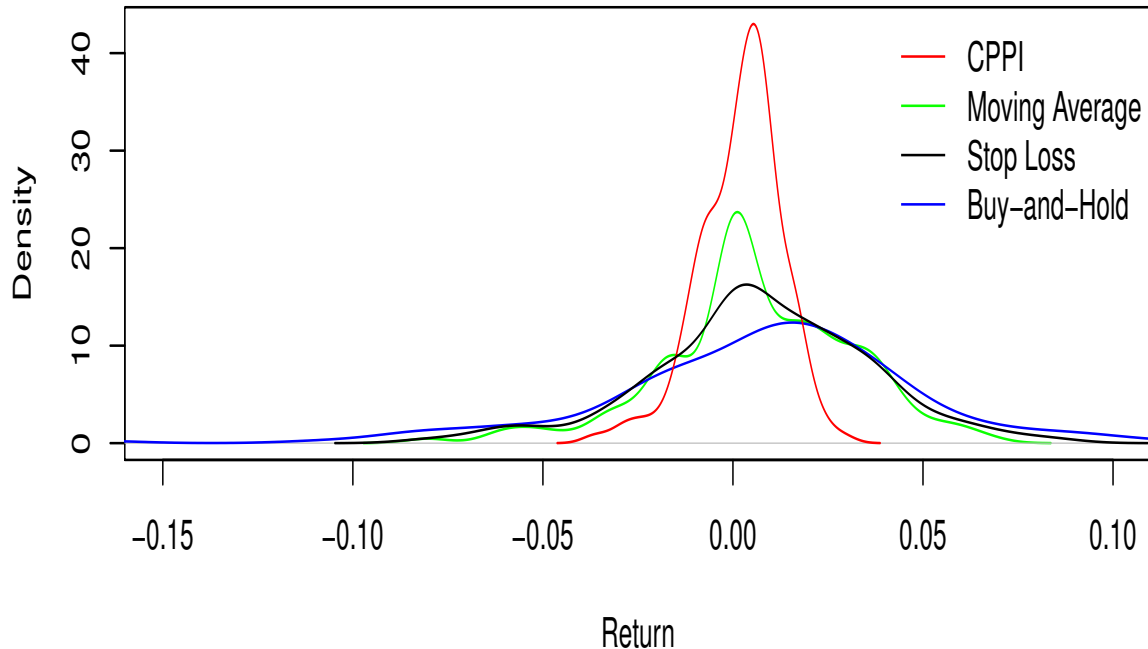


Figure 14: Presents the distributions of the option-based strategies and the Buy-and-Hold strategy, theoretically without dividends. Recall that CC is the Cash-Call strategy, PP is the Protective-Put strategy and BH is the passive Buy-and-Hold strategy.

### 6.3 In-Sample 1975-1994

We conduct an additional empirical test for the CPPI, SL and MA strategies. The reason for this is to check if the strategies perform in a similar manner when a longer sample period is selected. Also, an increase in the sample-size should generate less biased performances. As mentioned earlier, we do not include the PP and CC strategies in this longer sample, due to the start of the VIX-index in 1990.

Table 15, 16, and 17 present the Sortino ratios for different combinations of parameters for the three strategies; CPPI, SL and MA. The optimal CPPI strategy in this in-sample period consists of a multiplier of 3 and a floor value of 95%. This combination yields a Sortino ratio of 0.28. The optimal SL strategy is when buy- and sell- signals are triggered by a 12% drawup/drawdown and the Sortino ratio then present a performance of 0.68. The best MA

strategy is the P-SMA(15), which implies a short window size equal to 1 and a long window size equal to 15. The strategy generates a Sortino ratio of 0.66.

CPPI strategy in-sample (1975-1994)											
M \ F	0.85	0.86	0.87	0.88	0.89	0.9	0.91	0.92	0.93	0.94	0.95
3	0.24	0.24	0.24	0.25	0.25	0.26	0.26	0.27	0.27	0.28	0.28
4	0.20	0.20	0.20	0.21	0.21	0.22	0.23	0.24	0.25	0.26	0.26
5	0.15	0.16	0.17	0.17	0.17	0.18	0.19	0.20	0.21	0.23	0.24
6	0.11	0.11	0.12	0.13	0.14	0.15	0.15	0.16	0.18	0.19	0.21
Sortino ratio BH = 0.73											

Table 15: Presents the Sortino ratios for all the combinations of the initial guaranteed amounts (floor values) and multipliers. “M” represent the multiplier and “F” represent the floor value, at which is the minimum acceptable wealth of the initial wealth. The in-sample period starts in January 1975 and last until the end of December 1994. The performance is presented as annualized Sortino ratios.

SL strategy in-sample (1975-1994)	
$v$	Performance
0	0.63
1	0.41
2	0.48
3	0.55
4	0.35
5	0.49
6	0.61
7	0.62
8	0.49
9	0.53
10	0.57
11	0.62
12	0.68
13	0.58
14	0.54
15	0.52
Sortino ratio BH	
	0.73

Table 16: Presents the Sortino ratios for the best SL strategies in-sample.  $v$  represent the percentage change of price, in which buy and sell signals are generated. The in-sample period starts in January 1975 and last until the end of December 1994. The performance is presented as annualized Sortino ratios.

Table 18 presents the results that the three strategies generate with the optimal parameters. The CPPI strategy underperforms compared to the BH strategy. This is consistent with what we find in the first in-sample period. The SL and MA strategies also underperform if we compare it to the passive strategy in this in-sample period. This is somewhat surprising since we expected that at least some variation of the strategies would outperform the BH strategy.



MA strategy in-sample (1975-1994)		
Rank	Rule	Performance
1	P-SMA(15)	0.66
2	SMAC(2,10)	0.65
3	SMAC(2,11)	0.65
4	SMAC(2,12)	0.65
5	P-SMA(2)	0.63
6	P-SMA(14)	0.61
7	P-SMA(13)	0.59
8	SMAC(2,15)	0.59
9	SMAC(2,14)	0.58
10	P-SMA(12)	0.57
11	SMAC(2,9)	0.57
12	SMAC(3,8)	0.57
13	SMAC(3,6)	0.56
14	SMAC(3,9)	0.54
15	SMAC(2,13)	0.52
16	SMAC(3,4)	0.52
17	SMAC(3,10)	0.52
18	SMAC(4,11)	0.51
19	SMAC(2,7)	0.50
20	SMAC(2,6)	0.50
	B&H	0.73

Table 17: Presents the Sortino ratios for the best MA strategies in-sample. SMAC(S,L) is the simple moving average crossover with a short and a long window size. P-SMA is equivalent to SMAC(1,L). “S” denotes the short window size (which we have set to go from 1 to 5) and L denotes the long window size (which we have set to go from 2 to 15). The best performing strategy, P-SMA(15) has e.g. a short window size of 1 and a long of 15, which means that a sell or buy signal is generated when the 1 month moving average crosses the 15 month moving average. The in-sample period starts in January 1975 and last until the end of December 1994. The performance is presented as annualized Sortino ratios.

Again, we do not conclude on the real-life performance of the strategies in the in-sample test.

We continue to test the best performing combinations of the strategies out-of-sample.

Summary of the strategies in-sample (1975-1994)				
Statistics	BH	CPPI	MA	SL
Min return	-21.539	-3.020	-21.539	-21.539
Average return	14.178	7.531	12.764	13.212
Max return	13.462	2.698	13.462	13.462
Std. dev	14.815	2.469	13.034	13.686
Skewness	-0.449	-0.211	-0.628	-0.547
Kurtosis	3.074	2.619	5.961	4.633
Lower Std.dev	2.829	0.489	2.517	2.633
Average drawdown	0.051	0.005	0.044	0.048
Maximum drawdown	0.296	0.034	0.233	0.245
Sortino ratio	0.727	0.283	0.655	0.675
P-value		1.000	0.685	0.662

Table 18: A summary of statistics for the strategies (CPPI, MA and SL) including BH, in-sample (1975-1994). The in-sample period starts in January 1975 and last until the end of December 1994.

## 6.4 Out-of-Sample 1995-2016

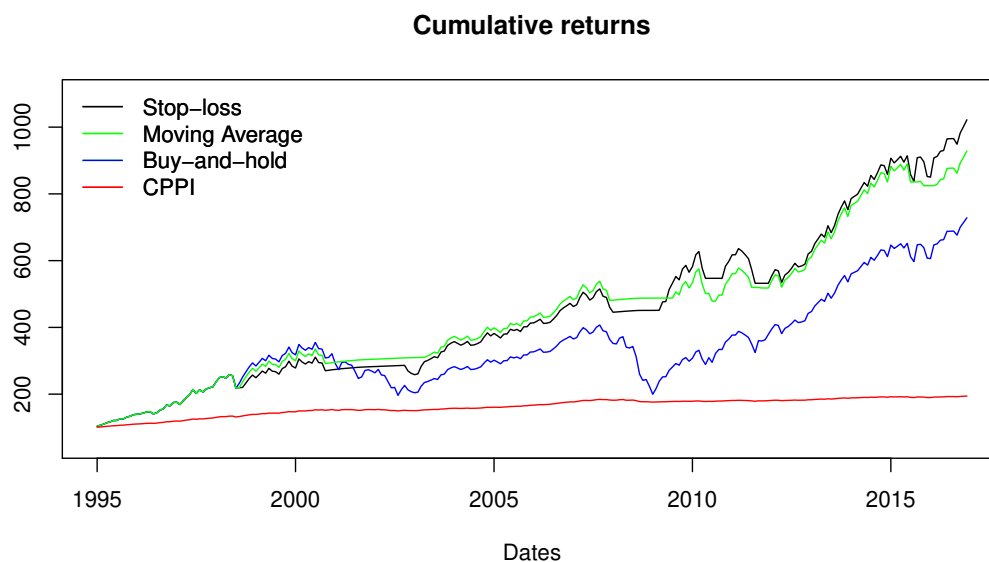


Figure 15: Present the cumulative returns for the active protection strategies (Moving Average, Stop Loss and Constant Proportion Portfolio Insurance) and the passive buy-and-hold strategy out-of-sample (1995-2016)

We use the best performing parameters for each strategy, to see how they perform out-of-sample. Figure 15 presents the cumulative returns for the strategies in this out-of-sample period and Table 19 shows the performance of the strategies. The CPPI strategy underperforms compared to the BH strategy, but relatively less than in the in-sample period. Both the SL and the MA outperform the BH strategy. The outperformance is statistically significant

at a 10% level. This is consistent to what we found in the out-of-sample period from 2004-2016 but does still somewhat surprise us. The reason is that the performance of the strategies tends to decrease out-of-sample compared to in-sample. The MA and SL strategies underperform against the BH strategy in-sample for all possible combinations, but out-of-sample they outperform the BH strategy.

Statistics for all of the strategies out-of-sample (1995-2017)				
Statistics	BH	CPPI	MA	SL
Min return	-16.736	-6.347	-14.461	-14.461
Average return	10.176	4.221	10.723	11.238
Max return	10.958	4.067	9.774	9.774
Std. dev	14.860	4.768	10.508	11.197
Skewness	-0.670	-0.480	-0.507	-0.442
Kurtosis	1.150	1.905	2.419	1.731
Lower Std.dev	2.959	0.957	1.905	2.017
Average drawdown	0.070	0.006	0.048	0.048
Maximum drawdown	0.510	0.044	0.168	0.169
Sortino ratio	0.761	0.575	1.265	1.268
P-value		0.890	0.087	0.086

Table 19: A summary of statistics for all strategies (CPPI, MA and SL) including BH, out-of-sample (1995-2016). The out-of-sample period starts in January 2005 and last to January 2017.

As for in the shorter data-sample period, the CPPI strategy generates the lowest average drawdown of the strategies simulated. The CPPI also generates the lowest maximum drawdown by a large margin. An interesting observation is that all the strategies, including the BH strategy, generates returns that are negatively skewed. Without conducting any statistical test of normality, we cannot say anything with certainty, but the descriptive statistics may indicate that the return distribution of the strategies is not normally distributed.

Similar to what we show in the first out-of-sample period, we also present illustrations of the drawdowns from the strategies here in Figures 16, 17 and 18. The drawdowns for the MA and the SL strategies do look somewhat similar in this sample period as in the shorter period. They have deeper drawdowns than the CPPI strategy but still generate superior Sortino ratios, due to higher excess returns.

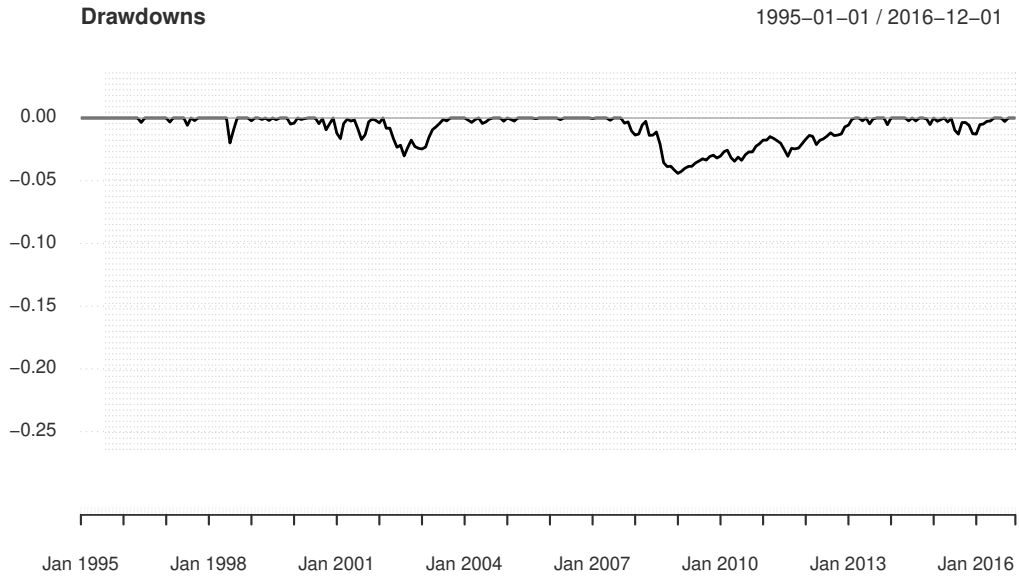


Figure 16: Shows the drawdown for the Constant Proportion Portfolio Insurance strategy out-of-sample (1995-2016). The drawdown as presented in this figure refers to the peak-to-trough for the S&P 500 index, and show the length and depth of decreasing prices in the index. Every time the graph reaches 0.0, we observe an all-time high for the index.

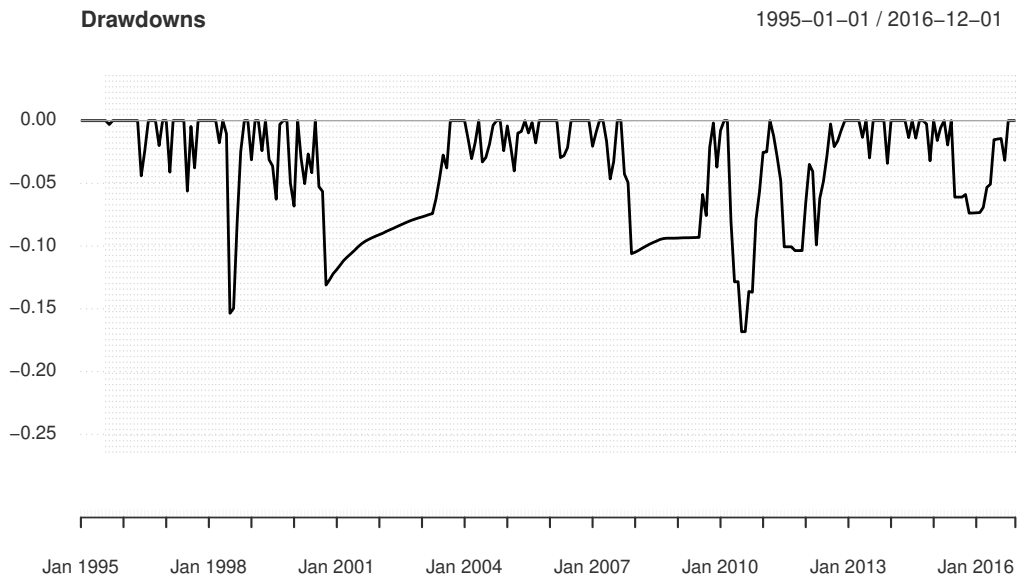


Figure 17: Shows the drawdown for the Moving Average strategy out-of-sample (1995-2016). The drawdown as presented in this figure refers to the peak-to-trough for the S&P 500 index, and show the length and depth of decreasing prices in the index. Every time the graph reaches 0.0, we observe an all-time high for the index.

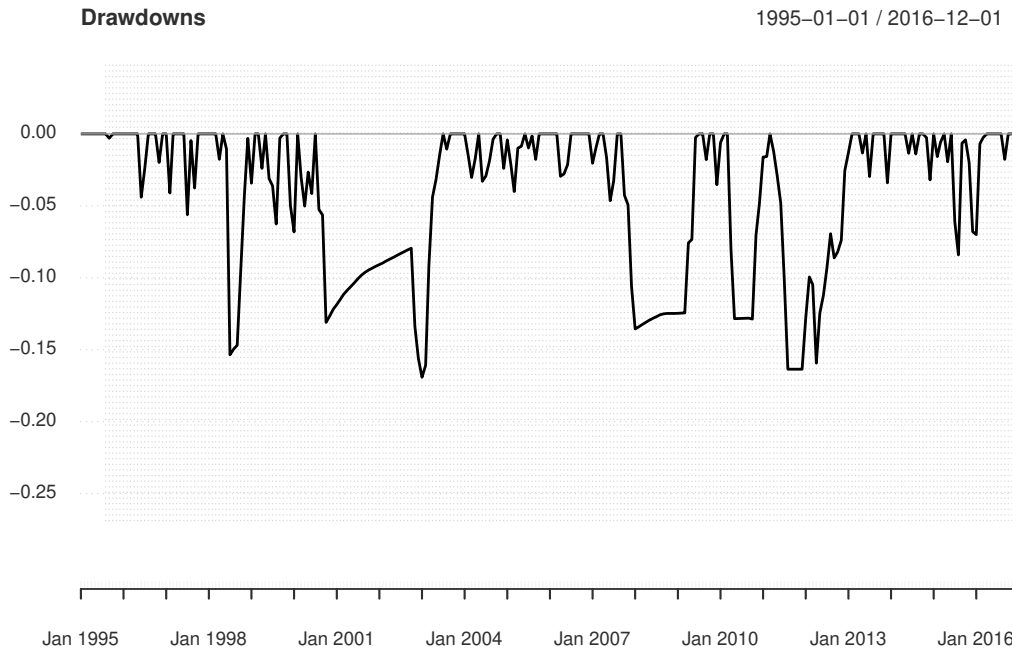


Figure 18: Shows the drawdown for the Stop Loss strategy out-of-sample (1995-2016). The drawdown as presented in this figure refers to the peak-to-trough for the S&P 500 index, and show the length and depth of decreasing prices in the index. Every time the graph reaches 0.0, we observe an all-time high for the index.

The probability density functions for the strategies are presented in Figure 19. We can also see in this illustration, that the CPPI strategy generates the lowest volatility. The PI strategies do generate smaller probability for losses than the BH strategy does.

## 7 Discussion

In the Monte Carlo simulation study, we demonstrate that the option-buying strategies outperform the Buy-and-Hold strategy. The return generating process for the stock is a Geometric Brownian Motion, with a positive drift. The mean return (10%) and standard deviation (20%) are realistic parameters observed on the S&P 500 index. In similar simulation studies for the CPPI, MA and SL strategies, the strategies generate weaker performances in bull and no-trending markets (Agić-Šabeta,2016).

Our findings from the empirical study generate conflicting results compared to the theoretical results. In the first out-of-sample test we find that the PP and CC strategy both underperform compared to the BH strategy. This is not expected based on the simulation study.

## Probability density functions

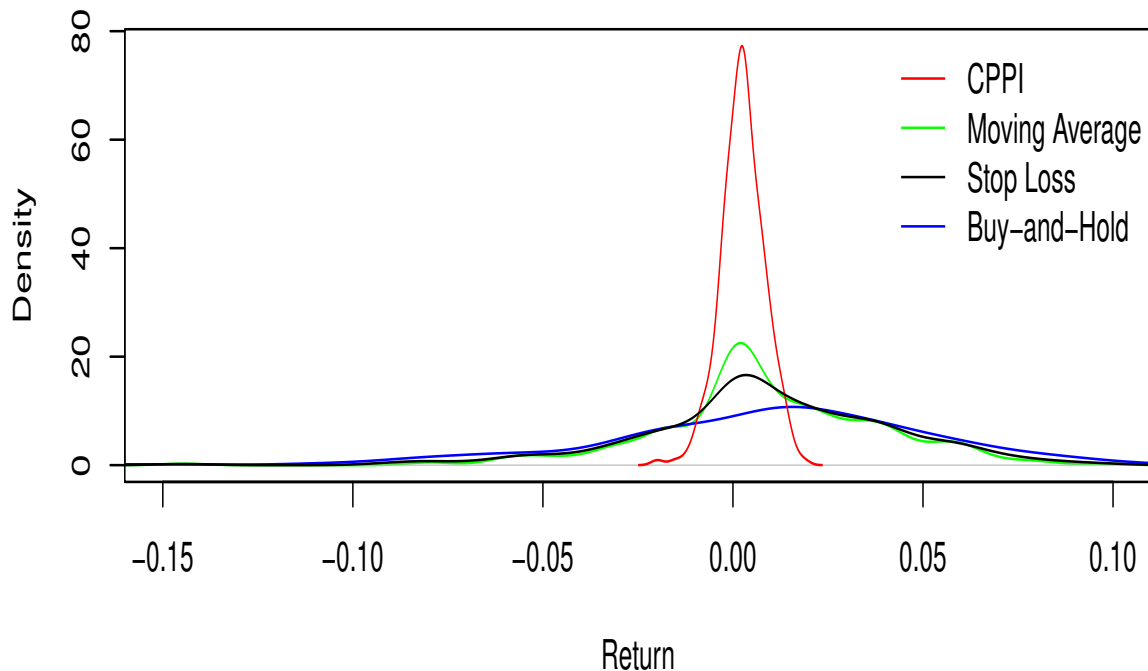


Figure 19: Presents the distributions of the option-based strategies and the Buy-and-Hold strategy, theoretically without dividends. Recall that CC is the Cash-Call strategy, PP is the Protective-Put strategy and BH is the passive Buy-and-Hold strategy.

We find that the CPPI strategy underperforms the BH strategy. On the other hand, we find that both the MA strategy and the SL strategy economically significantly outperform the BH strategy. The statistical evidence of outperformance is weak in the out-of-sample period from 2004-2016.

We conduct a second out-of-sample test where we include a longer data-sample. In the second out-of-sample test, the period goes from the beginning of 1995 to the end of 2016. In this test, we do not include the option-buying strategies, due to the start of the VIX index in 1990. Again, we find that the CPPI strategy underperform the BH strategy, while the MA and SL strategies both economically outperform the BH. During this period the outperformance generated by the MA and SL strategies are statistically significant at a 10% level.

We conjecture that the weak performance of the option-buying strategies indicates that traded options are too expensive. The concept of volatility risk premium is described as the

positive difference between implied volatility and the actual realised volatility in the market (Bekaert, Hoerova, and Duca,2013). Volatility risk premium is well documented in the financial literature. The volatility risk premium in the option market can be interpreted as the premium the buyer is willing to pay the seller to get rid of uncertainty. Over a large sample size, the seller of an option will charge the buyer a premium and this makes options priced in the market more expensive than they would have been if the implied volatility reflected the realised volatility perfectly.

Latane and Rendleman (1976) point out that the Black and Scholes model consists of five variables and that they are all easily observed in the market, except the standard deviation on the underlying stock. According to Christensen and Prabhala (1998), implied volatility should be an efficient forecast for future volatility if option markets are efficient. The existence of volatility risk premium may indicate that the option market is inefficient. From Figure 20 we see that the implied volatility is systematically higher than the realised volatility on the S&P 500 index. The implied volatility is roughly 20% higher than realised volatility. This makes the options traded too expensive. In other words, we buy insurance for more extreme events than those who occur, and this makes the option-buying strategies perform poorly. We find that strategies that use options as insurance do not work. This is in line with conclusions from previous literature from Driessen and Maenhout (2007), Coval and Shumway (2001) and Danis (2007) amongst others.

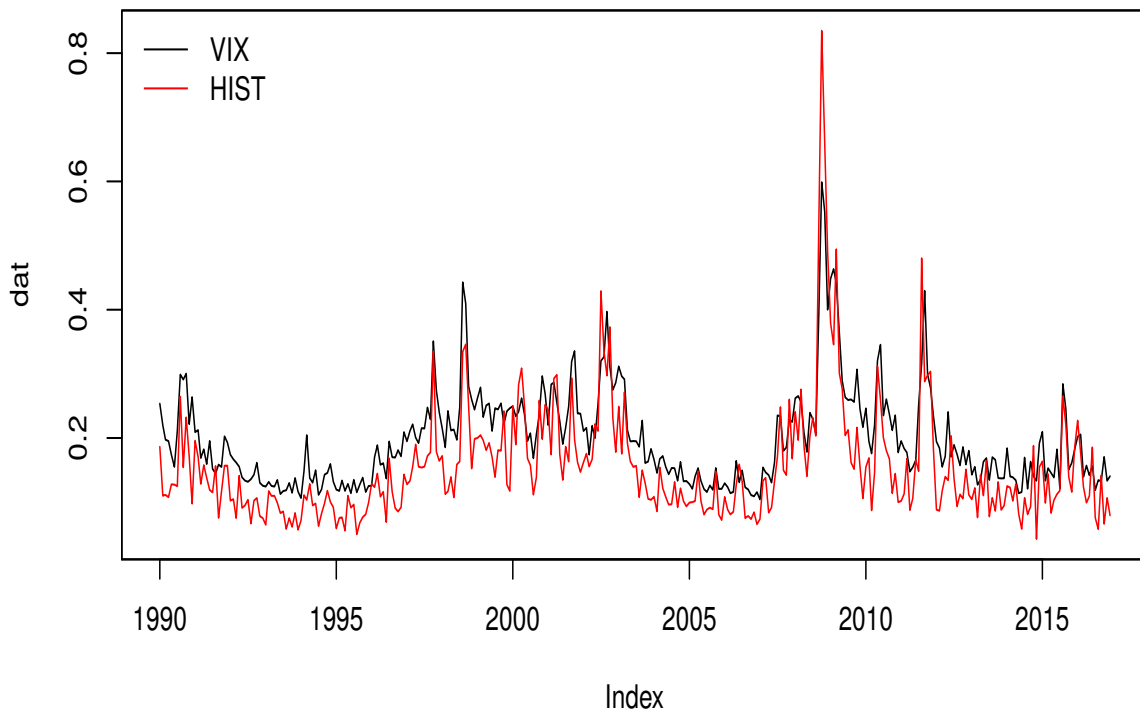


Figure 20: Illustrates the volatility risk premium as the difference between the actual observed volatility in the market (red graph) and the one-month implied volatility from the VIX index (black graph). The x-axis show the period of time and the y-axis present the volatility.

The CPPI strategy performs poorly in both out-of-sample tests. In the graphs that illustrate the cumulative returns for the strategies, we clearly see that the CPPI strategy generates very different returns than the rest of the strategies. The CPPI strategy presents low risk, but at the same time low returns. The lower risk (downside deviation) does not compensate for the lower excess returns, illustrated by the inferior Sortino ratio. We make an important change to the CPPI strategy when we adjust the floor value for each period. We increase the floor value by the return of the risk-free rate. Without the adjustment, the CPPI strategy will become identical to the BH strategy after the wealth increase enough, so that the exposure in the risky asset is 100%. It seems that the adjustment of the floor makes the CPPI strategy bear a low exposure in the risky asset, and thereby generating low returns. A possible improvement to the strategy would be to adjust the floor value in a different way. An example could be to adjust it after a certain amount of time, or after a certain wealth level is reached. This may increase



the performance. Our findings of the weak performance of the CPPI strategy are in contrast with the findings of Hakan and Hande (2009) and Ho et al. (2011). A difference between their findings of outperformance and our findings of underperformance is that they look at risk as standard deviation, while we consider the risk to be downside deviation.

The MA and SL strategies outperforms the BH strategy economically in both out-of-sample periods. The outperformance is statistically significant at a 10% level in the 1995-2016 period. These strategies are considered to be trend-following strategies, which means that when prices fall, the MA and SL strategies generate sell-signals (move capital from the risky asset to the risk-free asset) which protects the investor from additional losses. Similarly, the strategies generate buy-signals when it reverses to a positive trend (increasing prices), and thereby allows for the investor to take part in the subsequent bull period. The strong performance generated by the trend-following strategies in this thesis indicates that the S&P 500 index moves in trends. The average bull period is shorter over the period 1975-2016 than over the period 1990-2016, while the bear periods is of almost equal average length. We also see in Tables 1 and 2, that the bull-periods on average (average amplitude) is stronger in 1990-2016 compared to 1975-2016. Description of each bull/bear period is given in Table 21 in the appendix. It is well known that PI strategies perform relatively better compared to a BH strategy in bear markets. This is because such periods are when protection strategies provide the protection that the BH strategy does not. This may be some of the explanation of why the MA and SL generate better statistical evidence of outperformance in the longer out-of-sample test.

We observe that the downside deviation generated by the MA and SL strategies is drastically lower than the downside deviation generated by the BH strategy. This indicates that the MA and SL strategies do protect the investor from losses. We also see that the average return generated by the MA and SL strategies are as good as the BH strategy. Our results for the trend-following strategies are in line with the work by Kaminski and Lo (2014), Snorrason and Yusupov (2009) and Zakamulin (2017). Kaminski and Lo (2014) and Snorrason and Yusupov (2009) find that SL strategies perform well in the American futures market and the

Swedish stock market respectively. Zakamulin (2017) states that MA strategies tend to outperform the BH strategy post-1944, but the evidence is not statistically significant at conventional statistical levels.

A possible weakness of our thesis is that we do not include transaction cost. If we would have done that, the performance of the active strategies would decrease. The conclusion on the performance of the option-buying strategies and the CPPI strategy would not change. Another possible weakness of our thesis is that we use the VIX index as input for volatility. When we do this, we practically assume that the implied volatility is the same for all the different strike prices. In other words, we assume that the slope of the implied volatility smirk is flat. The implied volatility smirk (also known as implied volatility smile) is a phenomenon which occurs in the options market when the implied volatility for different strike prices, with the same expiration date, is plotted on a graph. The “smirk” is referring to the graph formation created, which resembles a smirk.

The Black-Scholes-model predicts that the implied volatility slope is flat. In other words, the Black-Scholes-model assumes that the volatility is the same for all options which expire at the same time. The implications of differing implied volatility levels are that the further away from ATM the options are, the more expensive the options are relative, based on the Black-Scholes pricing model. There are several possible explanations for the appearance of the implied volatility smile. Ederington and Guan (2013) list the hedging pressure hypothesis, transaction cost hypothesis, skewness hypothesis and jump risk as possible explanations, but they state that the most popular theory is that the volatility smile occurs due to the stochastic nature of volatility and its tendency to be negatively correlated with recent stock market returns. Another possible reason may be that log-returns are not normally distributed. Empirical distributions have negative skewness and high kurtosis. We will not go deeper into the reasons for the existence of the volatility smile, but we do acknowledge that it exists, and it is well documented in the literature.

The existence of the smirk does, however, impact our study in the way that our estimated stock prices may be inaccurate compared to the actual option prices. We use the VIX index

as input for volatility in the Black-Scholes option-pricing formula. The VIX index is, as explained earlier, based on multiple strike prices on options expiring 30-days out. The effect is that both In-The-Money call options and Out-of-The-Money put options are more expensive in reality, if we compare them to our estimations. This could lead to a higher performance of both the CC strategy buying options ITM, and the PP strategy buying OTM put options, due to the “discount” in option premiums. It may also be that the estimated option prices in the strategies which buy options At-The-Money are too high and consequently decrease the performance of the strategies. This issue is unfortunately beyond the scope of this study, but it may be interesting for future studies. An improvement to future studies could be to use actual option prices.

## **8 Summary and Conclusions**

We conduct this study with the objective to test whether PI strategies could outperform the simple Buy-and-Hold strategy. We include several PI strategies in the same study, as this is a contribution to the literature and it helps investors to find a conclusion on which PI strategies that are superior. Our formal research question is; “Are there any portfolio insuring strategies that outperform a Buy-and-Hold strategy on the S&P 500 index?”. We include strategies based on Moving Averages and Stop Loss in the study, to see how trend-following strategies perform compared to more traditional insurance strategies. Using the VIX index as an estimate for volatility on the option-based strategies is also an implementation that very few researchers use. We try to overcome weaknesses of previous studies by implementing out-of-sample testing and we use a valid statistical test for outperformance. A non-parametric stationary block bootstrap is used to test for significant outperformance. In addition, we use the Sortino ratio as the measure of performance, where many other studies use the Sharpe ratio.

We conduct a Monte Carlo simulation study, using Black-Scholes option prices, that illustrate that the option-buying strategies; the Protective-put and the Cash-Call, perform better than the Buy-and-Hold strategy theoretically. On the other hand, we know that the CPPI and

the trend-following strategies (MA and SL) will underperform against a BH strategy in a similar simulation study. We use the theoretical simulations as motivation to test the strategies empirically. In the empirical study, we calculate the performance of the option buying strategies by the use of an estimate of the option prices. Option-buying strategies are implemented instead of option replicating strategies. They are more relevant today, due to the increase of liquidity in the option market.

We evaluate the performance of the strategies using two different time periods. First, we study the performance of all the strategies; Protective-Put, Cash-Call, Constant Proportion Portfolio Insurance, Stop Loss and Moving Average in the period from 1990 to 2016. The out-of-sample period is from the beginning of 2004 to the end of 2016. We find that the MA and SL strategies perform better than the BH strategy, but the statistical evidence of outperformance is weak. The option-buying strategies and the CPPI strategy all underperform compared to the BH strategy. The economical outperformance by the MA and SL strategies and the underperformance by the option-buying strategies are in contrast with the results we find in theoretical simulations. In the second out-of-sample test from 1995 to 2016, we also find that the MA and SL strategies economically significantly outperform the BH strategy. The outperformance for this period is statistically significant at a 10% level. We observe from this thesis that option-buying strategies perform weak compared to the BH strategy. This makes us believe that such strategies do not work in practice, and the same conclusion can be drawn for the CPPI strategy. We do, however, find evidence that the MA strategy and the SL strategy manage to outperform the BH strategy, yet they do not fully provide protection from losses which investors should be aware of. For future studies, we think it would be interesting to use realised option prices instead of estimating the prices using the VIX index, to confirm the weak performance generated.

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## 9 Appendix

M \ B	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25
0.90	0.68145	0.68107	0.68074	0.68044	0.68018	0.67994	0.67973	0.67953	0.67935	0.67919	0.67903
0.95	0.39770	0.39732	0.39699	0.39670	0.39643	0.39619	0.39598	0.39578	0.39560	0.39544	0.39529
1.00	-0.54228	-0.54277	-0.54320	-0.54358	-0.54392	-0.54423	-0.54450	-0.54475	-0.54498	-0.54519	-0.54538
1.05	-2.03451	-2.03549	-2.03632	-2.03703	-2.03765	-2.03818	-2.03865	-2.03907	-2.03944	-2.03977	-2.04007
1.10	-2.04888	-2.05150	-2.05364	-2.05543	-2.05693	-2.05820	-2.05928	-2.06021	-2.06100	-2.06169	-2.06229

Table 20: Presents the Sortino ratios for all the combinations of strike prices and the amount of protection in the Cash-Call strategy. “M” represent the multiplier for the strike price and “B” represent the number of options (call options) used on the portfolio in the protection strategy. The in-sample period starts in January 1990 and last until the end of December 2003. The performance is presented as annualized Sortino ratios.

Bull periods			Bear periods		
Dates	Duration	Amplitude	Dates	Duration	Amplitude
Jan 1975 to Dec 1976	24.00	40.00	Jan 1977 to Feb 1978	14.00	-15.00
Mar 1978 to Nov 1980	33.00	58.00	Dec 1980 to Jul 1982	20.00	-21.00
Aug 1982 to Jun 1983	11.00	40.00	Jul 1983 to May 1984	11.00	-7.00
Jun 1984 to Aug 1987	39.00	115.00	Sep 1987 to Nov 1987	3.00	-28.00
Dec 1987 to May 1990	30.00	46.00	Jun 1990 to Oct 1990	5.00	-15.00
Nov 1990 to Jan 1994	39.00	49.00	Feb 1994 to Jun 1994	5.00	-5.00
Jul 1994 to Aug 2000	74.00	231.00	Sep 2000 to Sep 2002	25.00	-43.00
Oct 2002 to Oct 2007	61.00	75.00	Nov 2007 to Feb 2009	16.00	-50.00
Mar 2009 to Apr 2011	26.00	71.00	May 2011 to Sep 2011	5.00	-16.00
Oct 2011 to May 2015	44.00	68.00	Jun 2015 to Sep 2015	4.00	-7.00
Oct 2015 to Jan 2017	16.00	10.00			

Table 21: Presents an overview of the periods between 1975 and 2016 the S&P 500 index is considered to be bull and bear periods.

### 9.1 Reflection Note, Ruben T. Jensen

In our master thesis my co-writer and I investigate the real-life performance of actively managed portfolio insuring (PI) strategies, using out-of-sample testing. The goal of a PI strategy is to limit the risk of losses, but still maintain the potential for gains. We compare the performance of the PI strategies against the performance of a passive Buy-and-Hold (BH) strategy on the American S&P500 stock index. The strategies we study in our thesis is the Protective-Put (PP), Cash-Call (CC), Constant Proportion Portfolio Insurance (CPPI), Stop Loss (SL) and Moving Average (MA). We expand the literature in the field when we include all the strategies mentioned in the same study. We also implement valid statistical test for outperformance. We use a non-parametric test called stationary block bootstrap. PI strategies are believed to be attractive for investors who are loss-averse. We evaluate the performance of the strategies

using the performance measure Sortino ratio. We argue that this performance measure is more appropriate for loss-averse investors, than the Sharpe ratio. We find in our study that the MA and SL strategy manage to outperform the BH strategy out-of-sample. The outperformance is statistically significant at a 10% significance level in the out-of-sample period starting in 1995 and ending in 2016. On the other hand, we find that the PP, CC and CPPI strategy underperform compared to the BH strategy. The underperformance the PP and CC strategy generates empirically contrast with what we find in the theoretical Monte Carlo study we conduct. The outperformance the MA and SL strategy generates are also in contrast to what we find in similar theoretical studies.

Since the invention of the internet the trading volume on the worlds stock and currency exchanges has exploded. Trading is a lot easier and cheaper over the internet, than it was earlier, when investors had to call a broker. A lot of the volume today is what can be considered short-term trading. Short-term trading means that the investor does not intend to own the stock or the currency (or any other security or asset) for more than a year. In fact, a lot of the short-term trading volume is today trading by investors who is popularly classified as day-traders. They often own the security for less than a day or even less than an hour. Day-traders try to take advantage of small and short-term movements in securities. Often, they rely on trading techniques that use past information, for example previous price levels. In addition to day-traders, alongside the development of stronger computers, robot traders have become increasingly popular over the recent years. Robot traders are trading programs that trades on the worlds financial exchanges based on various trading rules. The trading rules are programmed in advance, and then the trading robot automatically executes their trades according to the trading program/algorithm. In some ways, a trading robot can be viewed as a non-human day-trader. One of the advantages trading robots have over human traders is that they do not have human emotions. In other words, they can trade strictly according to the trading rules, which can be very difficult for humans. Another advantage is the speed of which they can execute a trade. A lot of the biggest investment banks pay huge amounts of money to increase the speed of their trading robots. Even though trading robots is not cheap to develop

and operate, they can be cheaper than to have a large team of analysts and/or traders managing the capital.

The trend of using more and more automated trading strategies will likely continue in the coming years and decades. When the operators of the trading robots program the trading rules the robots are to follow, it is important to test that the trading rules works. In our thesis we study the real-life performance of portfolio insurance strategies, using out-of-sample testing. We also implement valid a statistical test. The use of out-of-sample testing mimics the behaviour of an investor who apply trading strategies based on in-sample testing. To find the true performance of strategies is important. The trend in finance is that bigger and bigger funds employ automated trading machines. This can be funds managed by large investment banks or large hedge-funds. If multibillion dollar funds implement strategies that actually do not perform well, even though they think they do because of a invalid testing regime, the fund can suffer massive losses. The trading robot (the program) can work effectively, but if the true performance of the strategy (trading rules) is bad, the performance of the fund may suffer. Our thesis seek to find true performance of trading strategies, and our thesis can therefore be linked to testing of trading programs as described above.

During the 1970s the liquidity in the options market was low. Consequently, alternative PI strategies was developed, such as the synthetic put strategy and the CPPI strategy. Today the options market is highly liquid, and there is no problem to use options to insure stock portfolios. The problem is that the options traded are too expensive, judging by option-pricing theory, like the Black-Scholes model. We illustrate this in our thesis when we compare the realised volatility on the S&P500 against the VIX index. In other words, the premium an option buyer pays to the option writer is on average to high. The option buyer pays for protection against more extreme events than on average occur. Consequently, even though it is possible to protect portfolios against losses using options, the performance of the portfolios suffers consequently. We show in our thesis that the strategies based on options perform poorly. Exactly why the options are too expensive is a field of research itself. It may very well be that we do not really understand options enough. The lacking understanding of options may be the

reason for why there exist so many theories about why they are traded with high premiums. In my opinion the innovation needed is more research and knowledge on options and the utility they provide for option buyers. More research will increase our understanding of how options should be priced.

In the pre-internet era most of the investors on in the stock market was professionals with formal economic education or a lot of financial knowledge. Today a lot of the investors in the stock markets have limited formal education in finance, because it is easier for people to access the stock market over the internet. Many educate themselves by reading books, articles, blogs etc. written by experts/financial advisors. The experts providing the advices and knowledge to the hobby investors have an ethical responsibility, especially when the experts make money by giving the advice. Of course, all the advice includes disclaimers so that investors using the advice given cannot legally sue the advisors, but is that enough? The industry of giving financial advice is worth billions, if not trillions, each year. There are a lot of examples of experts giving advice that simply do not work. In some cases, the experts believe in their own advice, but the problem is when they do not give their audience an objective picture of the risk involved. For example, let us say an expert write about a trading strategy that he has used himself with success in the past, he has an ethical responsibility to inform his audience that the strategy may not work in the future under different conditions. Experts advertising certain trading strategies etc are often very bad at giving objective information, and it is not easy for people with no education within finance to see that the information given is not objective and correct. It is not clear what action could be taken to deal with this problem. Experts have the right to formulate their advice in what way they want to, if they include a disclaimer stating that the author is not responsible for any loss. On the other side, in order for experts to get credibility and respect, they should have an incentive to give as good advice as possible. The problem is often that advice/strategies that promise investors huge profits are more attractive than advice/strategies that give realistic predictions.

## 9.2 Reflection Note, Håvard B. Sæstad

In our master thesis, we test different active Portfolio Insurance strategies and compare them to a passive investment strategy. We test five different strategies; Protective-Put, Cash-Call, Constant Proportion Portfolio Insurance, Stop Loss and a Moving Average strategy, and see if any of them manage to outperform a passive Buy-and-Hold strategy. We aim to improve previous research about portfolio insurance when we compare a range of different strategies in the same study. We also use the VIX index as an estimate for volatility in the two first strategies which use options. This thesis implements out-of-sample tests which deal with the problems regarding “data-mining-bias which is an improvement from many other studies conducted on Portfolio Insurance. We also use a different performance measure than some other papers, the Sortino ratio which we believe is an improvement and make the results more reliable.

In addition to the empirical results this thesis presents, we run a so-called Monte Carlo simulation which tests the strategies based on options in a theoretical environment. We find that the Protective-Put strategy and the Cash-Call strategy manage to outperform the Buy-and-Hold strategy theoretically. Previous studies test the CPPI, Stop Loss and strategies based on Moving Average and show that these strategies do not outperform a passive Buy-and-Hold strategy in similar theoretical simulations. Some of the results that base on empirical research conflict with those results we find in a theoretical environment. We find that the Protective-Put, the Cash-Call and the CPPI strategy generate lower performance than the Buy-and-Hold strategy when we use the Sortino ratio as the performance measure. The Stop Loss and the strategy that bases on Moving Average do however manage to outperform the passive Buy-and-Hold strategy empirically and these results are statistically significant. The CPPI strategy is the only strategy where we find results that coincide with theoretical and empirical tests.

At the end, we present conclusions of why the Protective-Put and the Cash-Call strategy generate a poor performance which conflict with those results we find in the Monte Carlo simulation. Our results indicate that the options that sell on the S&P 500 index are overpriced and therefore yield these results. The volatility risk premium is a phenomenon of the difference between the implied and the actual realised volatility in the market. When we calculate option

priced with implied volatility, higher volatility increases the prices of the options. We also point at the implied volatility smirk as a possible explanation of our results.

Writing this thesis has both helped me use what I have learned on this master course, but also to acquire new knowledge. The subject on what we write our thesis about builds on several of the subjects we have had, especially from the finance courses on the master level. In addition, to learn about the subject of portfolio insurance, I have learned both some programming in “R” and how to use a typesetting program called “Latex”. We chose to use these programs to gain information from large data samples and to present the thesis in a more professional manner. Improvements on the master course for the future could be to implement more programming and use of data programs, such as Excel. This would have helped us to easier implement programming in the thesis, in addition to the fact that computer programs continue to get more important and relevant as the world gets more computerised.

This master thesis relates to international trends as Portfolio Insurance become more and more popular worldwide. Portfolio insurance was introduced by Hayne Leland in the 1970s and the concept of Portfolio Insurance has later developed to include a wide range of products and different methods to achieve protection on portfolios of risky assets. In this thesis, we include five different strategies, some of them which date back in time and some which not have been possible to use in practice until the latest years. Especially including option-based strategies are in line with international trends. On many risky assets and indices, the use of option-based strategies has become increasingly popular as options have been more frequently traded. This makes them cheaper to implement and the use of them has protection as been possible in practice.

Hayne Leland introduced the concept of Portfolio Insurance after seeing that many pension funds had withdrawn their money from risky assets after experiencing significant losses and failed to invest them to take advantage of the subsequent period with increasing prices. As the financial markets have experienced several so-called bear periods with declining prices after this, Portfolio Insurance has become more and more popular. The stock markets in several countries experience all-time-highs and this may frighten investors that a new long period of

declining prices is around the corner. Regardless if this is the main reason, investors through pension funds and hedge funds place an ever-increasing amount of money in strategies that are actively traded to secure them against losses. Our master thesis reflects this trend when we aim to find the optimal strategy for investors who seek protection.

The field of research about Portfolio Insurance is ever increasing but there are still areas that have not been thoroughly investigated. When we test the performance of option-based strategies on the S&P 500 index, we use the implied volatility in the market to calculate option prices. The VIX index is the estimate for volatility that we have used and can be interpreted as the one-month implied volatility in the market. Our results suggest that the implied volatility is too high if we compare it to the realised volatility, which makes options about 20% too expensive. Because of these findings, the performance from the Cash-Call and the Protective-Put strategies yield an inferior performance in relation to the Buy-and-Hold strategy. For further research, it had been interesting to see if implied volatilities on other indices present the same results. In this, I mean to include a larger number of stock indices which again have indices that reflect the one-month implied volatility on them. You would then investigate whether the implied volatility is consistently higher in financial markets and if option-based strategies present similar results as we have observed.

A different line of innovation on the field of Portfolio Insurance would be to use actual realised volatility instead of implied volatility when we test the option-based strategies. If we use realised volatility to calculate option prices in the Black-Scholes formula, the strategies would most likely perform better than what we found. This is because the sellers of options require a “volatility risk premium” from the buyers. It would have been interesting to know if the protection strategies still underperform compared to the Buy-and-Hold strategy if we adjust for this premium or if the performance of those strategies would conflict with ours with this adjustment.

Most of the studies and the papers about Portfolio Insurance include only one or a few protection strategies. To the best of my knowledge, the very few studies that implement many Portfolio Insurance strategies only test these in a theoretical environment or in a Monte Carlo



simulation. We do however include five strategies and test them empirically. Further research where more strategies are included, e.g. synthetic option strategies could improve the knowledge that investors have. When Previous papers only conduct testing of one or few strategies, it is often difficult to compare them. Including an even larger number of strategies and test them empirically could be a useful innovation in the field of Portfolio Insurance.

As mentioned above, an increasing amount of money is invested in active Portfolio Insurance strategies for private investors through pension funds and hedge funds. The values in the pension and hedge funds are placed from professional traders. When private investors with limited knowledge about investments pay professional to invest for them, the professionals have a huge responsibility to take care these values. If there is a limited amount of research about active Portfolio Insurance strategies, professional institutions may offer a product which is either too expensive or a product which do not protect as is promised when prices decline. This could both be due to limited knowledge from the professionals or because they can exploit the limited knowledge of other. More literature and research within the field of Portfolio Insurance will make it more difficult for professional institutions to offers active protection strategies that do not reflect what is best for the investors with limited knowledge. A suggestion on how to make sure that professional investors act in the best interest of small investors with limited knowledge of investments is difficult to come up with. One proposal could, however, be to make the institutions that offer active protection strategies to tell their costumers about what conclusions the research within the relevant field yields.