

# Full- and Reduced-order Model of Hydraulic Cylinder for Motion Control

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**Abstract**—This paper describes the full- and reduced-order models of an actuated hydraulic cylinder suitable for system dynamics analysis and motion control design. The full-order model incorporates the valve spool dynamics with combined dead-zone and saturation nonlinearities – inherent for the orifice flow. It includes the continuity equations of hydraulic circuits coupled with the dynamics of mechanical part of cylinder drive. The resulted model is the fifth-order and nonlinear in states. The reduced model neglects the fast valve spool dynamics, simplifies both the orifice and continuity equations through an aggregation, and considers the cylinder rod velocity as output of interest. The reduced model is second-order that facilitates studying the system behavior and allows for direct phase plane analysis. Dynamics properties are addressed in details, for both models, with focus on the frequency response, system damping, and state trajectories related to the load pressure and relative velocity.

## I. INTRODUCING REMARK

Hydraulic cylinders [1] are the first choice in numerous applications which require high forces and robust operation in outside, harsh, and hard-accessible environments. Motion control [2], force control [3], and hybrid of both [4] represent steady challenging tasks for the operation of hydraulic cylinders. Due to a complex nonlinear behavior, their reliable modeling, identification, and control [5] are crucial for subsequent exploitation. This note addresses the full- and reduced-order modeling of hydraulic cylinders seen from the system plant point of view required for the analysis and control design.

## II. FULL-ORDER MODEL

We first consider the full-order model of hydraulic cylinder controlled by a directional control valve (DCV). Note that the following modeling relies on the basics of hydraulic servo systems, provided e.g. in [1], and is close to developments which can also be founded in the previously published works on hydraulic servo systems, e.g. [5], [4]. However, whenever differences appear we will highlight these with the corresponding references. The DCV allows for a volumetric flow  $Q_n$  of hydraulic medium from, correspondingly to, both chambers (A and B) of hydraulic cylinder, see Fig. 1. Note that here and later on the subscript  $n = \{A, B, S, T\}$  refers to the cylinder chambers A and B, hydraulic pressure supply (for instance pump), and the tank correspondingly. The pressure difference between both chambers generates the hydraulic actuation force which moves the cylinder piston with a rod in direction of the pressure gradient. The external mechanical loads, inertial force,

and cylinder friction counteract the induced pressure-gradient force, thus resulting in the motion dynamics of cylinder drive.

In the standard configuration with a constant supply pressure  $P_S$ , the single available control input is that of the DCV denoted by  $u$ . Since the DCV spool is actuated by a proportional solenoid, the choice of the  $u$  quantity strictly depends on the way how the solenoid, and correspondingly DCV, is low-level controlled. Most simple case,  $u$  is the coil voltage applied through an amplifier so as to energize the electro-magnetic circuits of the solenoid. However, the more advanced and common case, which we also assume in the following, is when the control input  $u$  is directly proportional to the relative spool position. That means the latter is low-level controlled by an embedded DCV electronics, which includes an internal coil current loop and external spool position loop. Note that the spool relative displacement  $\nu$  is provided by either a single or by a pair of electro-magnetic solenoids. In the first case, a pre-stressed returning spring ensures the bidirectional spool actuation, cf. with Fig. 1. In the second case, two synchronized solenoids are arranged in antagonistic way, thus making spool pushing and pulling fully identical.

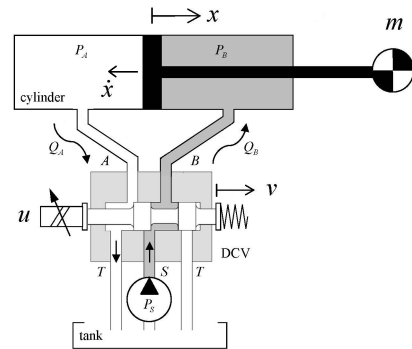


Fig. 1. Principal structure of hydraulic cylinder controlled by DCV.

The low-level controlled DCV has the closed-loop transfer characteristics of the second-order system described by

$$G(s) = \frac{\nu(s)}{u(s)} = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}. \quad (1)$$

The closed-loop parameters are the eigen-frequency  $\omega_0$ , damping ratio  $\xi$  and, ideal-case, unity gain which ensures the steady-state accuracy of the spool position control loop. Depending on the input amplitude  $|u|$  and working pressure in hydraulic circuits, the transfer characteristics  $G(s)$  can be varying in parameters and, therefore, bear some minor uncertainties in tran-

sient behavior of the controlled spool position response. Some typical  $|u|$ -dependent variations in the nominal (measured) frequency response characteristics of the controlled DCV are in vicinity to corner frequencies, and therefore DCV control bandwidth. This can be found in several manufacturer data sheets of the commercially available DCVs with embedded electronics that implements a low-level spool position control.

When the DCV assembly is equipped with a closed center spool, which is one of the most common configurations aimed for reducing the valve leakage and sensitivity to the small input signals, the spool-controlled flow characteristics are inherently subject to the dead-zone nonlinearity. That is a relative spool displacement below certain magnitude, in both directions across (zero) center position, does not lead to an orifice and therefore hydraulic flow. Furthermore, the orifice, which is governed by the spool displacement, is subject to saturations due to the maximally possible opening given by the inner structure of the valve body. Therefore, an internal flow-governing control variable  $z$  can be introduced as being related to the controlled spool position through two static nonlinearities, dead-zone and saturation, connected in series. Here we note that several previous works either neglect or only partially account for the above nonlinearities, while these can have an impact on the overall system dynamics in view of cylinder feedback control in the applications. So e.g. in [5], [3], [2], [6], [4] both nonlinearities are neglected. In [7] both nonlinearities are taken into account while disregarding the second-order DCV closed-loop dynamics and approximating the spool position response as a first-order system. A more detailed and sophisticated model of proportional valves with internal flow dynamics [8] explicitly analyzed this type of nonlinearities. However the spool travel  $\nu$  has been assumed as an available external input, therefore without considering its controlled behavior and associated (internal) dynamics.

Combining the DCV closed-loop dynamics (1) with the overall static nonlinearity described by

$$h(\nu) = \begin{cases} \alpha \operatorname{sign}(\nu), & \text{if } |\nu| \geq \alpha + \beta, \\ 0, & \text{if } |\nu| < \beta, \\ \nu - \beta \operatorname{sign}(\nu), & \text{otherwise,} \end{cases} \quad (2)$$

results in the mechanical sub-model of DCV

$$\ddot{\nu} + 2\xi\omega_0 \dot{\nu} + \omega^2\nu = \omega^2 u, \quad (3)$$

$$z = h(\nu), \quad (4)$$

where the output  $z$  is the orifice state which governs the valve flow. The parameters  $\alpha$  and  $\beta$  denote the saturation level and dead-zone size correspondingly. For the sake of simplicity, we assume the same parameter values for both directions of the spool displacement, hence a symmetric DCV, while some more specific DCV assemblies can have different direction-dependent  $\alpha, \beta$  values, see e.g. [8] for details. Further we note that while the saturation level can be considered as constant and known a-priori, or at least after system identification, the dead-zone size can be subject to state-dependent uncertainties and altering (wear-related) effects. The state-dependent variations lead back to e.g. viscoelastic properties and clarity of the hydraulic medium, working pressure, temperature, and others.

The volumetric flow of hydraulic medium associated with both outlet ports of DCV, and thus with both chambers of the

connected hydraulic cylinder, is governed by the corresponding pressure differences and given by the orifice equations

$$Q_A(z) = \begin{cases} zK\sqrt{P_S - P_A}, & \text{for } z > 0, \\ zK\sqrt{P_A - P_T}, & \text{for } z < 0, \\ 0, & \text{otherwise;} \end{cases} \quad (5)$$

$$Q_B(z) = \begin{cases} -zK\sqrt{P_B - P_T}, & \text{for } z > 0, \\ -zK\sqrt{P_S - P_B}, & \text{for } z < 0. \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

For the sake of clarity, we note that the configuration depicted in Fig. 1 implies  $z < 0$ , so that the (in) flow through the port B has the positive sign, and the (out) flow through the port A correspondingly negative. The valve flow coefficient

$$K = C_d w \sqrt{\frac{2}{\rho}}, \quad (7)$$

denoted also as a valve gain, depends of the oil density  $\rho$ , discharge coefficient of the orifice  $C_d$ , and wight of rectangular orifice area  $w$ . Last two are the structural parameters of the DCV assembly at hand, cf. with e.g. [9]. Also we note that while the flow direction in (5), (6) is determined by the sign of the orifice state  $z$ , including dead-zone, several works decide the flow direction depending on the sign of control input [5], spool position [3], [2], or pressure drop in the chambers [7].

The continuity equations of pressure the gradient [1] in both chambers are given by

$$\dot{P}_A = \frac{E}{V_A} (Q_A - A_A \dot{x} - C_L(P_A - P_B)), \quad (8)$$

$$\dot{P}_B = \frac{E}{V_B} (Q_B + A_B \dot{x} - C_L(P_B - P_A)), \quad (9)$$

where  $A_{A/B}$  are the corresponding effective piston areas. Note that for the single-rod cylinders  $A_A > A_B$ , cf. with Fig. 1, while for the double-rod cylinders  $A_A = A_B$  is mostly assumed. The bulk modulus  $E = -V(dP/dV)$  reflects the incompressibility of hydraulic medium in cylinder. While this is generally pressure-dependent, an effective bulk modulus is mostly assumed as a constant parameter for the rated operation pressure of hydraulic system at hand. The total volume in hydraulic circuits

$$V_A = V_A^0 + A_A \operatorname{sat}_h(x), \quad (10)$$

$$V_B = V_B^0 - A_B \operatorname{sat}_h(x) \quad (11)$$

depends each on the rod drive position, while  $V_{A/B}^0$  is the total chambers volume (including piping and fittings between the DCV and cylinder) when the rod drive is in the initial zero position  $x = 0$ . Note that the drive displacement is subject to saturation, captured by  $\operatorname{sat}_h$  operator in (10), (11), while  $h$  is the half-stroke of the symmetric cylinder.  $C_L$  is an internal leakage coefficient of cylinder which characterizes an additional pressure drop due to (minor) penetration of the hydraulic medium between both chambers. That is caused by a non-zero clearance between the barrel and piston of hydraulic cylinder. Note that in ideal case, the leakage coefficient can be assumed as zero, though it constitutes a rather uncertain factor related to the life cycle of hydraulic cylinder, temperature-dependent material expansion, oil viscosity and, if applicable, external radial stress affecting the piston rod during the operation.

The mechanical sub-model of cylinder drive can be directly formulated, for one translational degree-of-freedom (DOF), based on the Pascal's and Newton's second law as

$$m\ddot{x} = P_A A_A - P_B A_B - f - F_L. \quad (12)$$

The lumped moving mass of the cylinder piston with rod and, if applicable, external coupling (tool) is  $m$ , and all external counteracting forces are summarized by  $F_L$ . Note that the latter are the matter of application and, in the most simple case of stand-alone hydraulic cylinder, can be assumed to be zero. The counteracting nonlinear friction  $f$  mainly depends on the relative velocity of the piston drive and acts as an inherent damping of the relative motion. Note that in more complex modeling of the motion dynamics, the friction  $f$  can be also considered as depending on the normal load, relative displacement at motion reversals, working temperature, and other factors. Furthermore, the kinetic friction is well-known to exhibit a time-varying behavior so that its model parameters can be assumed as drifting or, at least, with an uncertainty range. For more details on varying kinetic friction and its impact on the motion control we refer to e.g. [10], [11].

It can be stressed that the friction in hydraulic cylinders is mainly due to the contacts between the rods and lip seals, and between the piston o-rings, seals, and cylinder [12]. Also the viscous effects of hydraulic fluid contribute to arising friction which counteracts the induced motion of the cylinder drive. However, the major source of friction remains due to the tight lip and piston seals that are required to prevent internal and external leaks in the system. In the following, we will assume the kinetic friction as function of solely the relative velocity  $\dot{x}$ , while some works [13], [14] made attempts to explicitly incorporate dependency on the pressure, correspondingly pressure difference, into the friction law. Further we note that more sophisticated dynamic friction models can also capture some transient and memory-driven effects of nonlinear friction, see [15] for survey, and pre-sliding rate-independent damping [16] as well. The steady-state friction can be described by the well-known Stribeck [17] characteristic curve

$$f(\dot{x}) = \text{sign}(\dot{x}) \left( F_c + (F_s - F_c) \exp(-|\dot{x}|^\delta \chi^{-\delta}) \right) + \sigma \dot{x}, \quad (13)$$

which takes into account the constant Coulomb and linear viscous friction, plus the velocity-weakening effects in a low velocity range around zero. The static model (13) is parameterized by the Coulomb friction coefficient  $F_c > 0$ , Stribeck (or stiction) friction coefficient  $F_s > F_c$ , linear viscous friction coefficient  $\sigma > 0$ , and two Stribeck shape factors  $\chi > 0$  and  $\delta \neq 0$ . In case the discontinuity at velocity zero crossing should be avoided in the model, to say without performing a more complex dynamic friction modeling, some heuristic approaches e.g. using tangens-hyperbolic functions instead of the sign-operator can be pursued, see for example [18]. Another reason for using the tangens-hyperbolic type smooth transitions, instead of the *sign* discontinuity, is that during fast transients the argument of square root in (5), (6) can yield temporary negative, thus making calculus improper. Several characteristic shapes of the steady-state friction determined in experiments on the hydraulic cylinder drives can be found in [19], [20], [3], [12], [6], [14]. It should be noted that the above friction parameters can be also subject to uncertainties dictated by the thermal and load conditions, life cycle and wear, dust, dwell-time, spatial properties of contacting surfaces and others.

### III. REDUCED MODEL

The reduced model takes advantages of the fast DCV spool dynamics so that the second-order closed-loop behavior of DCV can be neglected. Alternatively, this can be approximated by some constant time delay, if the corresponding phase lag is still to be taken into account when designing a feedback control of the hydraulic cylinder. Further, the cylinder piston velocity is assumed as the system output of interest, since this can be either directly derived from the available position measurement or observed, correspondingly estimated, using various robust velocity estimation techniques known from the literature, e.g. [21], [22]. In the following, we assume that the time-constants of hydraulic and mechanical sub-dynamics are significantly larger than that of the DCV control loop, to say by two to three orders of magnitude. Consequently, the input of the reduced model can be assumed as  $u^* = v$  which directly enters both nonlinearities according to (2). Therefore, the resulted reduced model is with static nonlinearities in the input channel.

Introducing the load-related pressure  $P_L = P_A - P_B$  and assuming a closed hydraulic circuit, i.e.  $|Q_A| = |Q_B|$ , the orifice equations (5), (6) can be aggregated into one

$$Q_L = zK \sqrt{\frac{1}{2} (P_S - \text{sign}(z)P_L)}, \quad (14)$$

while it is valid

$$P_A = \frac{P_S + P_L}{2}, \quad P_B = \frac{P_S - P_L}{2}. \quad (15)$$

Note that (14) represents an average load flow through the DCV and occurs as further nonlinearity in the input channel of the reduced model. This incorporates, however, a dynamic feedback of the load pressure  $P_L$  so that only (2) can be considered as a static input nonlinearity in forward. Furthermore it should be stressed that (14), (15) assume also zero pressure in the tank, cf. with (5), (6), which is however a reasonable simplification for various hydraulic cylinder drive systems.

Following the above way of aggregation, the hydraulic continuity equations (8), (9) can be transformed into one related to the load pressure gradient

$$\dot{P}_L = \frac{4E}{V_t} (Q_L - \bar{A}\dot{x} - C_L P_L). \quad (16)$$

Here the total hydraulic actuator volume is  $V_t = V_A^0 + V_B^0$  and the average effective piston area is  $\bar{A} = 0.5(A_A + A_B)$ . Note that the latter yields an exact value for double-rod cylinders, while for single rod cylinders it bears an averaging error of the half of the rod cross-section area.

Correspondingly, the dynamics (12) of mechanical part transforms (for the reduced model) into

$$m \frac{d}{dt} \dot{x} = P_L \bar{A} - f(\dot{x}) - F_L, \quad (17)$$

with the cylinder drive velocity  $\dot{x}$  as system output and total friction force given by (13). Here we recall that the reduced model (13)-(17) with the input nonlinearity (2), (4) has the second-order dynamics, while the external load force  $F_L$  acts as an internal state disturbance interfering between the first and second integrators of the forward dynamics path.

#### IV. ANALYSIS OF SYSTEM DYNAMICS

In the following, we discuss dynamic properties of both models while making some qualitative comparisons between them and demonstrating the differences in terms of the measured frequency response functions and state trajectories. For the latter we consider the  $[P_L, \dot{x}]$  space, since both dynamic states are the most significant and characteristic for the response of hydraulic, correspondingly mechanic, parts of cylinder drives under an actuation. Before comparing the models, we provide a detailed analysis of the linearized reduced-order dynamics so as to disclose the principal system behavior with impact of linearization. For the accompanying

TABLE I. SYSTEM PARAMETERS OF NUMERICAL SIMULATION

Param.	Value	Units	Param.	Value	Units
$\alpha$	1e-3	m	$A_A$	5e-3	m <sup>2</sup>
$\beta$	3e-4	m	$A_B$	4.7e-3	m <sup>2</sup>
$\omega$	1200	rad/s	$V_A^0$	1.2e-3	m <sup>3</sup>
$\xi$	0.7	none	$V_B^0$	1.15e-3	m <sup>3</sup>
$C_d$	0.65	none	$m$	20	kg
$w$	0.02	m	$F_c$	600	N
$\rho$	850	kg/m <sup>3</sup>	$F_s$	900	N
$E$	1e8	Pa	$\sigma$	2000	kg/s
$P_S$	1e7	Pa	$\chi$	0.02	m/s
$P_T$	0	Pa	$\delta$	0.8	none
$C_L$	0	1/s	$h$	0.2	m

numerical simulations, the system parameters listed in Table I are assumed. All units are given in SI. The numerical values are assigned artificially but, at the same time, lie in the range of realistic values for the DVC controlled hydraulic cylinder actuators, cf. with parameters provided e.g. in [12], [6].

##### A. Linearized reduced-order dynamics

The purpose of a DCV is to supply hydraulic cylinders with the controlled volumetric flow which energizes the hydraulic circuits and mechanical drive according to (8), (9), (12), correspondingly (16), (17). Therefore, to obtain first an insight into dynamic behavior of the system, i.e. in frequency domain, the reduced-order model can be directly linearized between the load flow and rod velocity as

$$G(s) = \frac{\dot{x}(s)}{Q_L(s)} = \frac{\bar{A}^{-1}}{\omega_c^{-2} s^2 + 2\zeta\omega_c^{-1} s + 1}, \quad (18)$$

with the cylinder eigenfrequency and damping given by

$$\omega_c = 2\bar{A}\sqrt{\frac{E}{V_t m}}, \quad (19)$$

$$\zeta = \frac{\sigma}{4\bar{A}}\sqrt{\frac{V_t}{Em}}. \quad (20)$$

Note that the single step required to obtain the linearized dynamics (18) is that of assuming zero leakage coefficient and neglecting the nonlinear part of the system friction. The latter implies that a relative motion of cylinder rod becomes solely damped by the viscous friction with a linear damping coefficient  $\sigma$ . This represents an artificial case of the weakest mechanical damping, while a real dynamics of hydraulic cylinder is subject to additional rate-independent damping due to the Coulomb and presliding friction, correspondingly stiction.

Despite the above simplification, the linearized dynamics (18)-(20) offers a creditable estimation of the second-order system behavior. This is, above all, in terms of the system gain, eigenfrequency and therefore system stiffness, and damping which reveals the system as stronger or less oscillatory and has a direct impact on the closed-loop behavior when designing a feedback control. It is evident that the gain between the flow and cylinder velocity is inverse to the effective piston area, so that the cylinders with larger  $\bar{A}$  can achieve higher velocities. Also the eigenfrequency and therefore achievable control bandwidth increase with  $\bar{A}$ , according to (19). At the same time, one should keep in mind that increasing  $\bar{A}$  proportionally reduces the system damping, see (20), and therefore evokes additional issues of system stability. Furthermore from (19), one can see that both the moving mass and total hydraulic volume constitute the inertial terms which slow down the system eigenfrequency. From the energy transfer point of view it appears as logically consistent, since a higher hydraulic volume is more inertial, correspondingly with higher amount of kinetic energy for the same displacement rate. The bulk modulus  $E$  appears as an equivalent stiffness of hydraulic medium so that larger  $E$  increases the system eigenfrequency, according to (19). However, unlike the mechanical stiffness, the bulk modulus influences the system damping as well, cf. with (20). One can sum up that the parameters pair  $E/V_t$ , in addition to the effective piston area, is decisive for the system eigenfrequency and damping, as related to hydraulic circuits. This is crucial for specifying and dimensioning of hydraulic cylinder actuators during the design. Also it is obvious that the friction is the single factor influencing the overall system damping without having inverse impact on its eigenfrequency.

The associated closed-loop dynamics for system (18) can be analyzed when examining the root locus of the open-loop  $kG(s)s^{-1}$ . Here the position of cylinder rod is taken as output of interest (used for a feedback control), and  $k$  is a system feedback gain. The root locus diagram, depicted in Fig. 2 for

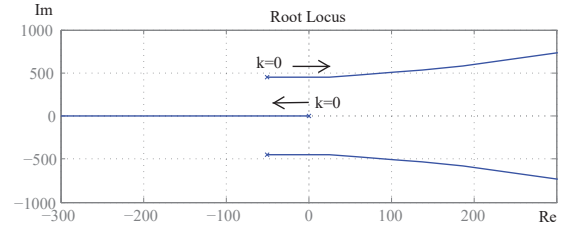


Fig. 2. Root locus for the open-loop  $kG(s)s^{-1}$ .

the parameters from Table I, discloses that the system remains oscillating due to the conjugate-complex pole pair and become unstable starting from certain feedback gain value.

Next the attention is to be paid to the impact of the load pressure feedback on the load flow according to (14). Keeping constant  $z$  value as a DCV operation point, the flow-pressure characteristics can be directly computed while  $\max P_L = P_S$ . The static flow/pressure characteristic curves for different  $z$ -values, starting from the neutral (closed) orifice state  $z = 0$  and going until its maximal value  $z = \alpha$ , are depicted in Fig. 3 for the assumed system parameters from Table I. Note that for negative load values the depicted characteristic curves are symmetrical with respect to both axes and lie in the third

quadrant for the negative  $(P_L, Q_L)$  pairs. Considering the working points  $\hat{z}$  and  $\hat{P}_L$  and positive DCV operation range, this without loss of generality, one can show that

$$\left. \frac{\partial Q_L}{\partial z} \right|_{\hat{P}_L} = K\sqrt{P_S - \hat{P}_L} =: C_q, \quad (21)$$

$$\left. \frac{\partial Q_L}{\partial P_L} \right|_{\hat{z}} = -\frac{K\hat{z}}{2\sqrt{P_S - \hat{P}_L}} =: -C_{qp}. \quad (22)$$

Both partial derivatives allow to introduce the so called flow-gain coefficient  $C_q$  and flow-pressure coefficient  $C_{qp}$ . Then, for linearizing the orifice equation (14), one can write

$$\hat{Q}_L = C_q z - C_{qp} P_L. \quad (23)$$

It is evident that the pressure-dependent coefficient  $C_q$  amplifies the input orifice state  $z$  and, therefore, introduces solely an additional gain to the transfer function (18). On the

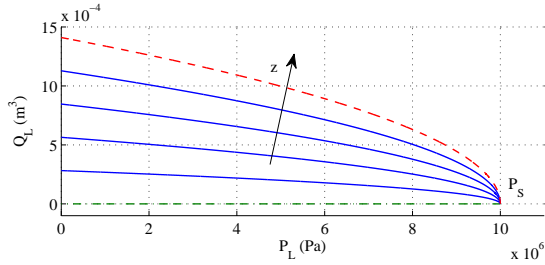


Fig. 3. Static load flow-pressure characteristic curves.

contrary, the pressure feedback coefficient  $C_{qp}$  reshapes the  $G(s)$  dynamics so that the frequency response characteristics changes depending on the operation point. Already from the characteristic curves shown in Fig. 3, one can see that the feedback gain  $C_{qp}$ , which is a slope of the tangent to the  $(Q_L, P_L)$  curve for each  $(\hat{z}, \hat{P}_L)$  working point, is rather low for small amplitudes. For zero load flows, i.e. at neutral DCV state with zero orifice, the feedback gain remains zero even for higher load pressures. This explain why the hydraulic cylinders under pressure often exhibit a steady-state vibration noise, even when no apparent motion of cylinder, i.e. no load flow, occur.

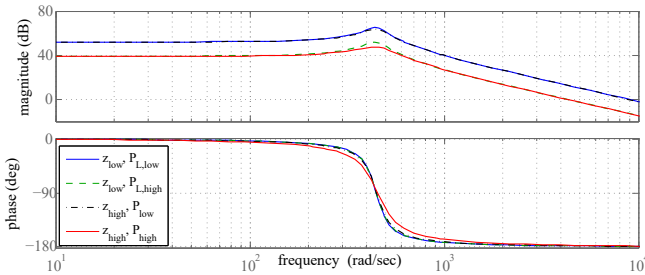


Fig. 4. FRFs of linearized system (24) at different  $(\hat{z}, \hat{P}_L)$  operation points.

For better exposition of the above analysis consider the linearized transfer function between the orifice state  $z$  and rod velocity  $\dot{x}$ , which can be obtained from (18)-(20) and (23) as

$$\hat{G}(s) = \frac{\dot{x}(s)}{z(s)} = \frac{C_q G(s)}{1 + C_{qp}(ms + \sigma)A^{-1}G(s)}. \quad (24)$$

Obviously, for zero flow-pressure coefficient, i.e. for no load pressure feedback, the (24) transforms to the previously considered dynamics (18) amplified by the flow gain  $C_q$ . To provide a quantitative comparison between the linearized dynamics (24) at different operation points assume four pairs by combining the relatively low and high values  $\hat{z}_{low,high} = \{0.05\alpha, 0.95\alpha\}$  and  $\hat{P}_{L,low,high} = \{0.05P_S, 0.95P_S\}$ . Recall that  $\alpha$  and  $P_S$  constitute the upper bounds for  $z$  and  $P_L$  correspondingly. The frequency response functions (FRFs) for all four  $(\hat{z}, \hat{P}_L)$  combinations are shown opposite to each other in Fig. 4. One can see that the low and high load pressure values change significantly the system gain, while the principal shape of the frequency response, with corresponding eigenfrequency and damping, remains quite similar. It can be noted that an increased system damping is solely in case of the maximal orifice and load pressure (red solide line).

### B. Magnitude-dependent frequency response

In what follows, the numerical simulations of the full-order and reduced models, according to Sections II, III, have been used while assuming the parameters from Table I and Forward-Euler integration with a fixed step (0.0001) solver of MathWorks Simulink<sup>®</sup> software. The sign( $\dot{x}$ ) in (13) has been replaced by  $\tanh(400\dot{x})$  so as to capture the negative pressure differences under square root in the orifice equations.

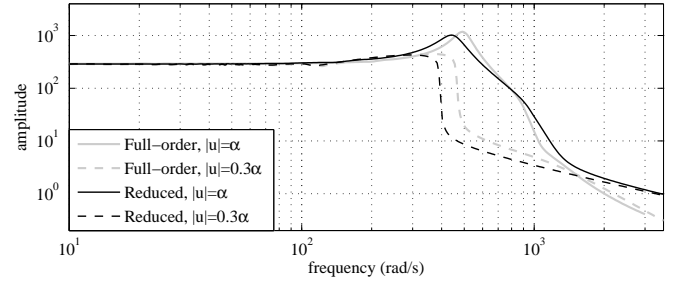


Fig. 5. FRFs of full-order and reduced models at different input amplitudes.

The FRFs of the full-order and reduced models have been obtained from the numerical simulation at different excitation amplitudes,  $|u| = \{\alpha, 0.3\alpha\}$ , when applying a down-chirp 600–1 Hz and excluding the input nonlinearities (2). Estimated with a standard H1 correlation algorithm and smoothed, the FRFs are depicted opposite to each other in Fig. 5. One can see that for both models the low inputs do not excite the hydraulic-related resonance peak. The resonance peak of the reduced model is also shifted to the lower frequencies. At higher frequencies the reduced model has inherently lower,  $-40$  dB per decade, decrease due to the second order dynamics.

### C. State trajectories

The  $(P_L, \dot{x})$  state trajectories are analyzed, for both models, when applying a low sinusoidal amplitude  $0.4e-3$ , which is close to the dead-zone size  $\beta$ , and high sinusoidal amplitude  $1e-3$ , which corresponds to the saturated orifice state. Both input sinusoids are at 0.1 Hz frequency without phase shift. The velocity responses and phase plane portraits of both models are shown in Fig. 6, for the low (on the top) and high (on the bottom) excitation amplitudes. One can see that while the velocity patterns of both models coincide well with

each other, the load pressure trajectories are quite different in both, transient oscillations and steady-state locations. This is not surprising since at relatively low load pressures, the reduced model averaging of the orifice and continuity equations yields the hydraulic circuits as ideally symmetrical. On

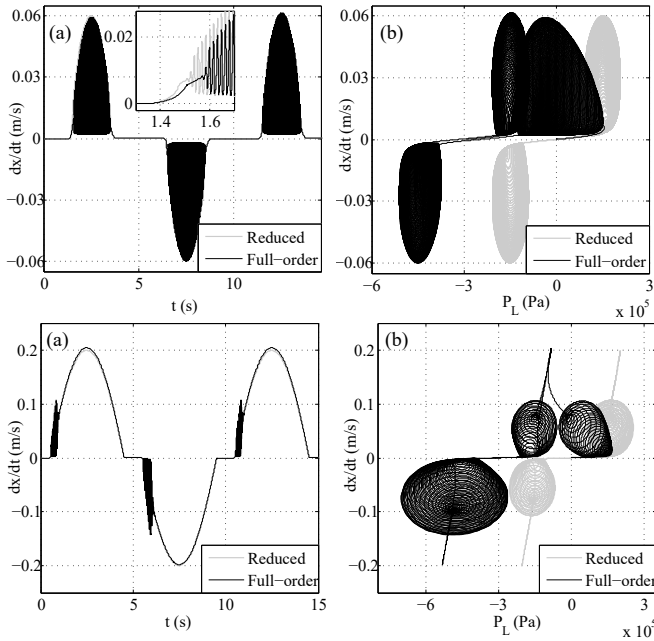


Fig. 6. Velocity trajectories (a) and phase plane portraits (b) for the sinusoidal inputs with low (upper) and high (below) amplitudes.

the contrary, the full-order model takes into account the one-side rod cylinder that reflects in the shifted (unbalanced) load pressure during a cyclic open-loop excitation. From both, velocity trajectories and phase plane portraits, one can see zero velocity regions corresponding to the DCV dead-zone. Further one can recognize the transient oscillations of both states in the anti-phase each time the relative motion restarts in an opposite direction. After the fast transients, the  $(P_L, \dot{x})$ -trajectory attains a non-oscillating pattern which corresponds to the steady-state motion in case of the higher excitation amplitude.

## V. CONCLUSIONS

The full- and reduced-order dynamic models for DCV controlled hydraulic cylinders have been described in details. A possible reduction from the 5th to the 2nd order dynamics has been provided while explaining the related assumptions and implications. The system behavior has been analyzed and the basic equations have been exposed in terms of their parametrization and coupling between each other. Numerical examples with parameters assumed close to the real hydraulic servo systems have been considered, with particular focus on the open-loop behavior in frequency domain and trajectories of the most relevant system states, load pressure and cylinder velocity. The given modeling and analysis should serve for better understanding the dynamics of hydraulic cylinder drives and associated identification and motion control design.

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