

On break-away forces in actuated motion systems with nonlinear friction

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Abstract

The phenomenon of so-called break-away forces, as maximal actuation forces at which a sticking system begins to slide and thus passes over to a steady (macro) motion, is well known from the engineering practice but still less understood in its cause-effect relationship. This note analyzes the break-away behavior of systems with nonlinear friction, which is well-describable analytically by combining the Coulomb friction law with the rate-independent presliding transitions and, when necessary, Stribeck effect of the velocity-weakening steady-state curve. The break-away conditions are brought into an analytic form of the system description and shown to be in accord with a relationship between the varying break-away force and actuation force rate – that is observable in experiments reported in various independently published works.

Keywords: Friction, break-away force, nonlinearity, presliding, modeling, stiction force, hysteresis

1. Introduction

The break-away force and related break-away conditions belong to significant and well-known, but yet still not fully studied, aspects of nonlinear dynamic friction in the actuated motion systems. The break-away as phenomenon can be seen as a brief, but not discrete, transition between the presliding and gross sliding, when an idle system with friction is subject to a continuously increasing actuation (input) force. Due to the lack of direct measurements and complexity of nonlinear friction transitions, the break-away instant and force are particularly challenging for detecting accurately and describing in a closed analytic form. Some researchers even noted that "quantitative prediction of the break-away friction level seems not yet possible" [1].

The early detailed studies of presliding frictional characteristics and transitions to the gross sliding may be credited to the works of Dahl, e.g. [2, 3]. Later, in the well-celebrated survey on the friction modeling and control [4] the authors also addressed the break-away friction while noting that the break-away is not instantaneous and the corresponding modeling should account for a translational distance. In further works on dynamic friction modeling [5, 6, 7], the authors have paid attention to, and extracted from the numerical simulations, a dependency of the break-away force on the actuation force rate. In favor to that observation, quite similar relationships have been demonstrated in various experimental setups reported in [8, 6, 9]. Also the system sticking, and the related stick-slip behavior, have been analyzed and compared for various existing dynamic friction models, and that from a theoretical perspective in [10] and experimentally in [11, 12].

Further aspects of measuring the presliding friction transitions, continuous sliding, static friction, and dynamic friction effects the interested reader can find in [13, 14, 15, 16].

Despite the break-away phenomenon, as observation, is well-known from the engineering practice and has been addressed, or at least mentioned, in several studies on the kinetic friction, its modeling and control, less works have been dedicated to formulating the straightforward analytic conditions and deriving the expressions for break-away. Note that such developments should be in line with the corresponding system modeling. It seems that an explicit analysis and mathematical notation of the break-away states has been solely provided in [17], while the break-away force has been considered rather as a function of dwell time, and the given deviations appear cumbersome for a direct practical use.

With this note we address the relationship between the break-away friction force and actuation force rate in a possibly simple and, at the same time, coherent way, based on the established modeling assumptions and results published in several independent works. The following analysis and presentation should contribute to a better understanding of frictional break-away behavior and help in predicting and controlling the actuated presliding transitions.

2. Sliding and presliding friction

The tangential friction force, acting in opposite direction to the relative motion in x coordinates, is the generalized nonlinear function

$$F = f(\dot{x}, z, t). \quad (1)$$

The velocity argument can be seen as capturing the steady-state friction behavior including the amplitude-

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constant and $\text{sign}(\dot{x})$ -dependent Coulomb friction, the viscous velocity-dependent friction, and Stribeck velocity-weakening curves as well. All three can be described by the well-known steady-state characteristic curve

$$F_{ss}(\dot{x}) = \text{sign}(\dot{x}) \left(F_c + (F_s - F_c) \exp(-|\dot{x}|^\delta V^{-\delta}) \right) + \sigma \dot{x}, \quad (2)$$

often referred to as a static Stribeck friction model. The free parameters are the Coulomb friction coefficient $F_c > 0$, Stribeck (or stiction) friction level $F_s > F_c$, linear viscous friction coefficient $\sigma \geq 0$, and two shape factors of the velocity-weakening curve $V > 0$ and $\delta \neq 0$. For more details on the Stribeck effect and steady-state characteristic friction curve (2) we refer to [18, 4].

The time-dependency of friction in (1) summarizes the weakly known and often non-deterministic fluctuations in the frictional behavior due to e.g. wear, adhesion effects, contact surface irregularities, lubrication conditions, dust and others. Such effects may cause some non-systematic parameter drifting in the friction modeling and, in what follows, are excluded from an explicit consideration.

The z -argument represents an internal presliding state, or the so-called relative presliding distance on the one-dimensional frictional interface. Most simple way, this is a relative displacement at each motion onset or motion reversal, and that until the dynamic friction force converges to the steady-state of gross sliding provided an unidirectional motion. Obviously, the presliding distance z is initialized (or reset) whenever the velocity sign changes and it explicitly maps the instantaneous state of presliding friction force transitions. Depending on the particular form of presliding friction $F = f(z)$, as a function of relative presliding distance, an additional scaling factor s is to be used, resulting in

$$z = s \int_{t_r}^t \dot{x} dt. \quad (3)$$

Note that here the scaling factor is strictly required due to the logarithmic map assumed further in (4). Therefore, the normalized through the scaling factor s presliding distance is defined on the interval $[-1, 1]$, while at the boundaries the friction force will saturate on the steady-state level. Obviously, t_r denotes the time instant of the last motion reversal so that $|z|$ always represents a scaled distance to position where the motion direction changed for the last time. That is the integral (3) is reset each time the motion direction changes at t_r . For the sake of simplicity, in what follows, we will assume $s = 1$. The friction-displacement curve at the motion reversals exhibit a hysteresis loop, see example shown in Figure 1. The shape of the hysteresis loops depends on multiple factors of the surface asperities, their elastic and plastic deformation and, as a consequence, on energy dissipated on the frictional interface during the motion cycles [19, 20, 21]. According to [22] the area of hysteresis loop increases in proportion to the n -th power of presliding distance. In particular,

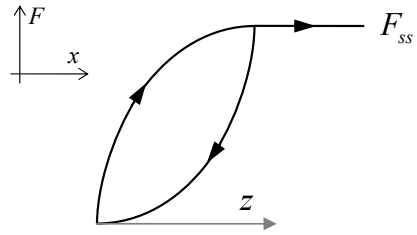


Figure 1: Friction-displacement curve with hysteresis loop.

it has been found and experimentally proved that this is quadratic relationship, i.e. $n = 2$. Therefore, the curvature of friction-displacement map during presliding can be described by

$$f(z) = z(1 - \ln(z)). \quad (4)$$

For details on deriving equation (4) from the above n -th, respectably second, power condition we refer to [22].

Assuming the hysteresis loop shape as in (4) and $s = 1$ it is obvious that for zero initial state $F_0 = 0$, i.e. at a motion onset, the presliding friction is given by

$$F(\dot{x}, z) = \text{sign}(\dot{x}) F_{ss}(\dot{x}) z(1 - \ln(z)). \quad (5)$$

Assuming the presliding transitions always converge to a steady-state F_{ss} and the instantaneous friction value at the last motion reversal is $F_r = F(t_r)$, the friction force during presliding is given by

$$F(\dot{x}, z) = |\text{sign}(\dot{x}) F_{ss}(\dot{x}) - F_r| z(1 - \ln(z)) + F_r. \quad (6)$$

Note that for zero initial friction state at motion reversal $F_r = 0$, which coincides with a case of motion onset, the general form (6) reduces to (5).

3. Break-away conditions

The problem of the break-away friction force can be seen as a problem of detection (alternatively prediction) of the minimal actuation force at which the motion system, being initially in the idle state, begins a continuous (macro) motion which is often denoted as gross sliding. This problem is closely related to the stiction and adhesion effects on the frictional surfaces and, at the same time, is of an empirical observation nature and practical relevance for engineering applications. The transitions from the system sticking into the gross sliding at an unidirectional motion have been observed in various actuated machines and mechanisms and reported in e.g. [13, 4, 15]. In the most of the previously published works, the varying break-away force (or torque) has been exposed in dependency of the actuation (input) force rate. That means the actuation (input) force has been linearly increased (starting from zero) i.e. $u = kt$, and the break-away transition has been observed and recorded as when a non-fluctuating quasi-constant acceleration occurs and the relative velocity grows continuously. Note that before this happens, the system is in the

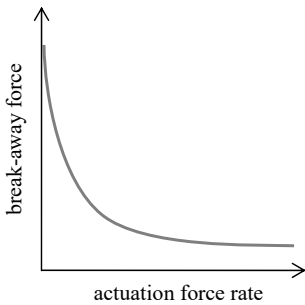


Figure 2: Break-away force as function of the actuation force rate.

presliding regime where a low relative displacement can be detected, while the measured relative velocity is mostly high-frequency oscillating around zero. For instance, such oscillating pattern with a relatively low average and increase has been shown in experiments in [23] (Fig. 8). The dependency of the observed break-away force on the actuation force rate $du/dt = k$ has been investigated and experimentally demonstrated in [8, 6, 9], and also shown for the numerically simulated dynamic friction in [5, 6, 7]. In all cases a typical inverse exponential map has been highlighted which is similar to that schematically shown in Figure 2 (cf. e.g. Fig. 4 in [5], Fig. 5 in [6], Fig. 13 in [7], Fig. 10 in [9]).

Now, we are in the position to analyze and to describe analytically the break-away conditions based on the modeling assumptions made in Section 2. We note that despite the break-away dependency on the actuation force rate has been known from experiments and confirmed by means of numerical simulations, no explicit analytic form has been proposed and validated so far, which would be in line with the modeled presliding friction behavior.

For the motion dynamics with a linearly increased actuation force we write

$$m\ddot{x} + f(\dot{x}, z) = kt. \quad (7)$$

During presliding, the macroscopic system inertia can be neglected due to a very low average acceleration so that the actuation force is mainly balanced by the counteracting friction, that yields $f(\dot{x}, z) \approx kt$. Taking the time derivative of (7) and neglecting the inertial dynamics one obtains

$$\frac{d}{dt}f(\dot{x}, z) = k, \quad (8)$$

while the full differential, as in [24], yields

$$\frac{d}{dt}f(\dot{x}, z) = \frac{\partial f}{\partial \dot{x}}\ddot{x} + \frac{\partial f}{\partial z}\dot{z}. \quad (9)$$

For the same reason as above and due to the fact that $\partial f/\partial \dot{x} = 0$ within presliding (this due to the rate-independency of presliding friction and therefore neglected viscous contribution), the first right-hand-side summand

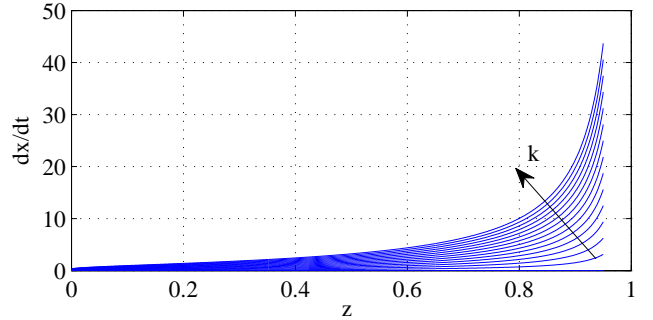


Figure 3: Relative velocity as a function of presliding distance in dependency of the rate of actuation force.

in (9) can be neglected and we obtain

$$\frac{\partial f}{\partial z}\dot{x} - k = 0. \quad (10)$$

Substituting the derivative of (4), with respect to z , into (10) results in

$$-F_{ss} \ln(z)\dot{x} = k. \quad (11)$$

It is obvious that the relative velocity during presliding

$$\dot{x} = -\frac{k}{F_{ss} \ln(z)} \quad (12)$$

can be estimated as a function of relative presliding distance and mainly depends on two factors k and F_{ss} . While k is fixed for the given slope of external actuation force, the steady-state friction value self depends on the instantaneous relative velocity. Nevertheless, from (2) we know that $F_c \leq |F_{ss}| \leq F_s$ so that either both boundary values, or an average

$$\hat{F}_{ss} = F_c + (F_s - F_c)/2,$$

can be considered when calculating (12).

The (z, \dot{x}) phase diagrams are shown in Figure 3 for the different actuation force rates k and a fixed \hat{F}_{ss} value. One can see that for all actuation force rates the relative velocity, starting from zero after a motion reversal ($z = 0$), increases exponentially when approaching the boundary of presliding distance ($z = 1$). The computed phase diagrams are for the actuation force rates $k \in [0.01, \dots, 30]$ with an increment equal to 2. Since a rapid (exponential) increase of the relative velocity (when $z \rightarrow 1$) is for all k , one can restrict the considered values by e.g. 95 % of presliding distance, further denoted by $z_{0.95}$. Here we should note that the transition from presliding to the gross sliding is not abrupt/stepwise at all, and the break-away conditions can be considered only for a certain, though well-specified interval, like for example $0.95 < z < 1$ we assumed. It is apparent that if increasing the left interval bound, like for example 0.99 or 0.999, a sharp asymptotic velocity increase (cf. Figure 3) will occur according to (12). However in this case the viscous damping and sliding friction ‘mechanisms’ (which are out of scope in the recent analysis) come into

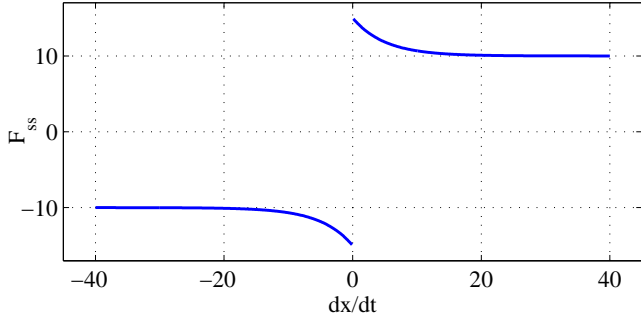


Figure 4: Steady-state (Stribeck) characteristic friction curve.

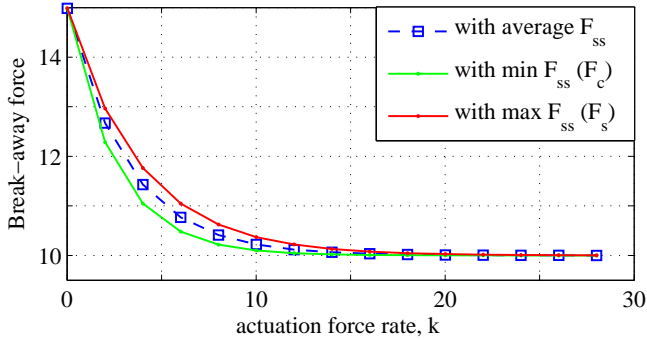


Figure 5: Break-away force as function of the actuation force rate computed by (2), (12), and (13) for minimal (F_c), maximal (F_s), and average (\hat{F}_{ss}) steady-state friction values.

play, so that a real velocity increase will remain bounded. This is also in accord with the experimental and numerical observations reported so far, while the break-away detection is mostly realized empirically “at the time where a sharp increase in the velocity could be observed” [5].

For the assumed presliding boundary the break-away force can be computed, based on (2) and (12), as

$$F_{ba} = F_{ss}(\dot{x}(z_{0.95})). \quad (13)$$

Now one can calculate the break-away force as a function of actuation force rate k , provided solely the friction model (2)-(4) is given. The assumed steady-state (Stribeck) characteristic curve is depicted in Figure 4. Recall that the linear viscous friction coefficient $\sigma = 0$ is assumed, for the sake of simplicity, and a relatively high difference $F_s = 1.5F_c$ between the minimal and maximal steady-state friction values is chosen, at the same time. The computed break-away force as a function of actuation force rate is depicted in Figure 5. In order to reveal the impact of F_{ss} value assumed for (12) computation, the minimal (F_c), maximal (F_s), and averaged (\hat{F}_{ss}) steady-state values are demonstrated opposite to each other. One can see that the functional dependency of the break-away force from k is similar for all three steady-state values. In particular, at lower (near to zero) and higher actuation force rates the curves are nearly coinciding with each other. On the opposite, the visible differences occur mostly during the

well-pronounced exponential decrease.

4. Conclusions

This note aimed to analyze and describe analytically the break-away conditions for which, at certain level of the external actuation force and its rate, the presliding friction behavior transits into the gross sliding and a continuous (macro) motion sets on. Using the straightforward formulation of presliding and steady-state friction force it has been explicitly shown how the relative velocity progresses with the relative presliding distance, starting from zero idle state. A rapid exponential increase of the relative velocity, that is characteristic for a break-away phenomenon, has been exposed. We have derived an analytic expression for computing the break-away force as a function of actuation force rate. The computed and demonstrated characteristic behavior is in accord with both, the previously published experimental observations and those obtained from the numerical simulations for which, however, an analytic expression and analysis have been missed.

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References

- [1] F. Altpeter, Friction modeling, identification and compensation, PhD Thesis, EPFL (1999).
- [2] P. R. Dahl, A solid friction model, TOR 158(3107-18), The Aerospace Corporation, El Segundo (1968).
- [3] P. R. Dahl, Solid friction damping of mechanical vibrations, AIAA Journal 14 (12) (1976) 1675–1682.
- [4] B. Armstrong-Hélouvy, P. Dupont, C. C. De Wit, A survey of models, analysis tools and compensation methods for the control of machines with friction, Automatica 30 (7) (1994) 1083–1138.
- [5] C. C. De Wit, H. Olsson, K. J. Astrom, P. Lischinsky, A new model for control of systems with friction, IEEE Transactions on automatic control 40 (3) (1995) 419–425.
- [6] M. Iwasaki, T. Shibata, N. Matsui, Disturbance-observer-based nonlinear friction compensation in table drive system, IEEE/ASME transactions on mechatronics 4 (1) (1999) 3–8.
- [7] J. Swevers, F. Al-Bender, C. G. Ganseman, T. Projogo, An integrated friction model structure with improved presliding behavior for accurate friction compensation, IEEE Transactions on automatic control 45 (4) (2000) 675–686.
- [8] V. Johannes, M. Green, C. Brockley, The role of the rate of application of the tangential force in determining the static friction coefficient, Wear 24 (3) (1973) 381–385.
- [9] V. Lampaert, F. Al-Bender, J. Swevers, Experimental characterization of dry friction at low velocities on a developed tribometer setup for macroscopic measurements, Tribology Letters 16 (1-2) (2004) 95–105.
- [10] B. S. Armstrong, Q. Chen, The Z-properties chart, IEEE Control Systems Magazine 28 (5) (2008) 79–89.
- [11] R.-F. Fung, C.-F. Han, J.-R. Chang, Dynamic modeling of a high-precision self-moving stage with various frictional models, Applied Mathematical Modelling 32 (9) (2008) 1769–1780.

- [12] Y. Liu, J. Li, Z. Zhang, X. Hu, W. Zhang, Experimental comparison of five friction models on the same test-bed of the micro stick-slip motion system, *Mechanical Sciences* 6 (1) (2015) 15.
- [13] F. Heslot, T. Baumberger, B. Perrin, B. Caroli, C. Caroli, Creep, stick-slip, and dry-friction dynamics: Experiments and a heuristic model, *Physical review E* 49 (6) (1994) 4973.
- [14] A. Socoliuc, R. Bennewitz, E. Gnecco, E. Meyer, Transition from stick-slip to continuous sliding in atomic friction: entering a new regime of ultralow friction, *Physical review letters* 92 (13) (2004) 134301.
- [15] C.-H. Lee, A. A. Polycarpou, Static friction experiments and verification of an improved elastic-plastic model including roughness effects, *Journal of Tribology* 129 (4) (2007) 754–760.
- [16] A. Harnoy, B. Friedland, S. Cohn, Modeling and measuring friction effects, *IEEE Control Systems* 28 (6) (2008) 82–91.
- [17] B. Armstrong-Helouvry, Stick slip and control in low-speed motion, *IEEE Transactions on Automatic Control* 38 (10) (1993) 1483–1496.
- [18] R. Stribeck, Die wesentlichen Eigenschaften der Gleit- und Rollenlager, *VDI-Zeitschrift (in German)* 46 (36–38) (1902) 1341–1348,1432–1438,1463–1470.
- [19] M. Ruderman, M. Iwasaki, On damping characteristics of frictional hysteresis in pre-sliding range, *Journal of Physics: Conference Series* 727 (1) (2016) 012014.
- [20] M. Ruderman, Presliding hysteresis damping of LuGre and Maxwell-slip friction models, *Mechatronics* 30 (2015) 225–230.
- [21] M. Ruderman, D. Rachinskii, Use of prandtl-ishlinskii hysteresis operators for coulomb friction modeling with presliding, in: *Journal of Physics: Conference Series*, Vol. 811, 2017, p. 012013.
- [22] T. Koizumi, H. Shibazaki, A study of the relationships governing starting rolling friction, *Wear* 93 (3) (1984) 281–290.
- [23] M. Ruderman, T. Bertram, Friction model with elasto-plasticity for advanced control applications, in: *IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM2010)*, 2010, pp. 914–919.
- [24] M. Ruderman, M. Iwasaki, Analysis of linear feedback position control in presence of presliding friction, *IEEJ Journal of Industry Applications* 5 (2) (2016) 61–68.