UNIVERSITY OF AGDER

Improving Understanding Of Logarithms By Using The Approach Of Repeated Division

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ABSTRACT

Empirical evidence from research (Chua & Wood, 2005) and my own experience as a teacher point to the difficulties in teaching logarithms in schools. Students find logarithm related questions difficult. They see 'log' as a common factor in expression $\log x + \log y$ and write it as $\log(x + y)$. Students have difficulty in recognizing logarithm as a number (Berezovski & Zazkis, 2006).

This research aims to develop an approach that enhances students' performance, conceptual understanding and retention of logarithms. It is hoped that the approach could be used to reduce the problems faced by the students.

In this research, I used a mixed method approach and developed teaching and learning materials to replace the entire topic (logarithms) in the students' text book. Pre-test, post-test and audio recordings of interviews with students and a teacher were used to collect data. The same test items were applied in administering both pre and post-test. The participating students were in Form1 and Form 2 (grade 10 and 11, age 15-17). The Form 2 students had been taught the topic before while the Form 1 students had no pre-knowledge. The data collected during the study were used to compare the performance of the students in Forms 1 & 2.

The study investigates the following questions: (a) Does the Repeated Division approach of learning logarithm improve the performance of students? (b) Does the Repeated Division approach of learning logarithm improve the conceptual understanding of students in logarithm? (c) Does the Repeated Division approach sustain the interest of students in learning logarithms?

The sample size of the consented participants who completed pre & post – test were: (*Form 1, n = 103 and Form 2, n = 39*). The retention test experienced dwindling of the sample size (from *n = 103, to n = 81*), thus 22 Form 1 students who took part in pre & post – test were absent. The quantitative analysis done could not give clear evidence to support hypothesis set for students' performance in the study. But, it was revealing in the qualitative analysis results that: the conceptual understanding and interest of the students were enhanced through the use of Repeated Division approach. The recommendations for future research and popularizing the approach are suggested.

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I am also legitimately proud and appreciative of my lectures who took me through research method course. My next thanks goes to my brother John P. Ansah (Assistant Professor) for his support, Mr. Benjamin Nana Mensa Dadzie, Mr. Kofi Ampah (class tutor) and Mr. Baffour (head of mathematics department) of Mfantsipim School for their wonderful assistance during my field work.

Many are those whose names I could not mention here, for wants of space, but for whom I owe a lot of gratitude for their moral and spiritual support in my educational endeavor, I would want to share the success of this study with them.

DEDICATION

I dedicate this work to my family and loved ones.

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CHAPTER ONE

1.1 Introduction/Background

Mathematics places a very vital role which cuts across all aspects of human life: social, cultural interpersonal, economic and physical lives. Algebra is one of the most critical aspects of mathematics (Carraher et al, 2006). Students of algebra are often introduced to powerful reasoning tool with applications in many different fields. Logarithm which is an aspect of algebra is no exception to the benefits algebra offers. It finds application in areas such as medicine, engineering, etc. It can be used to serve the purpose of comparison, measuring, forecasting, explaining, illustrating and interpreting. In economics, for example, it can be used to measure the growth of money at a fixed interest rate (Kalid, 2012). In medicine, nurses and doctors make use of logarithm to plot the intensity of various sounds that we hear, in a graphical scale. Doctors use logarithm in obstetrics- determine when pregnancy occurred and predict growth of the fetus (Kinsella, 2007). Aaron (1997), identified some applications of logarithms in graphing or measuring. He suggested that logarithm scale can be applied in calibrating axis of a graph which sought to measure different objects that cover a huge range of sizes on the same scale. Examples were given to support his point of comparing different measurement on the same scale: (a) the time one takes to get home, (b) the time one takes to drive across the entire country, (c) the time earth goes round the sun and (d) the time it takes light to go through sun to another galaxy.

Engineers and scientists use logarithms in solving complex problems. Aaron (1997) applied logarithm in his field as an engineer to design a special nozzle to measure the amount of rain that affects the wings of an airplane, vibration isolation system to measure the acceleration of a gadget in a space shuttle. Richter scale was developed to measure the pressure of sound our ears can accommodate and the magnitude of an earthquake by the use of logarithm (Kalid, 2012). According to Vural (2012), the logarithm is applied in studying the dynamics involve in the areas such as population growth, radioactive decay, and compound interest.

Notwithstanding the benefits associated with logarithms, students do have difficulties in grasping the concept. Many students struggle with logarithms and become disgruntled as they continue to encounter difficulties.

Empirical evidence from research (Chua & Wood, 2005) and my own experience as a teacher confirm that students—at the secondary school level--lack the basic understanding of logarithms. According to (Chua & Wood, 2005) some of the plausible causes of errors in learning logarithms are: (a) the wrong use of already known concepts in explaining new ideas, (b) students considering ‘log’ as a variable, but not an operation, (c) statements made by teachers which are not clear to students, (d) students’ own thinking or ideas and (e) wrong interpretation and explanation coupled with an incomplete statement given by their peers. Berezovski & Zazkis (2006), in their studies, realized that students over dependence on the algorithm and inefficient use of the digital tool in the algorithmic approach of logarithms contribute to the problem. The situation propelled them to think about: what actually necessitates the students’ choice of a particular method, the level of students’ understanding of the logarithms and how that facilitate reasoning rather than just the use of digital tool in solving logarithms. Berezovski & Zazkis realised that students’ ability to deal with logarithmic expression does not imply they understand their operational meaning. Weber (2002b), also emphasized on the inadequacy of students understanding and the difficulties they encounter in learning the concept. Weber designed activities and tasks for the students to improve their understanding (Weber,2002a & weber,2002b).

To address this apparent problem, mathematics educators should adopt new and convenient approaches in teaching logarithms to improve conceptual understanding and sustain the interest of students in learning logarithm, hence improve performance. This research focuses on one of such new and effective approach—repeated division—which has been found to improve students’ conceptual understanding of logarithms significantly.

1.2 Limitations of the Conventional Method

The current approach used in logarithm has many limitations. It has many laws and properties that students ought to be conversant with before applying them successfully. Due to that, students in most cases misquote the following laws and properties:

$\log 1=0$	as:	$\log 0=0$
$\log a + \log b = \log(ab)$	as:	$\log a + \log b = \log(a+b)$ (Matz, 1980).
	as:	$\log a \cdot \log b = \log(a \cdot b)$
$\log 10 = 1$	as:	$\log 1=1$
$\log a^x = x \log a$	as:	$\log a^x = (\log a)^x$

log a – log b= $\log(\frac{a}{b})$ as: log a – log b = $\frac{\log a}{\log b}$ (Lee&Heyworth, 1999).

as: $\frac{\log a}{\log b}$ = $\log(\frac{a}{b})$ (Kaur & Sharon, 1994).

Any misquotations of these properties (laws) will lead to the wrong result. This research is based on the premise that, the conventional approach to learning logarithm is inefficient. Consequently, new approaches to learning logarithm devoid of the limitations of the current approach, which can effectively be taught in secondary schools would improve outcome. One of such approaches, developed by Pauline Vos, a mathematics professor at the University of Agder, Kristiansand-Norway, is the Repeated Division (RD). The RD approach, described in the next section, have some advantage over the conventional approach (CA), as evident by the findings from a study conducted by (Espedal, 2015). The RD approach is based on prerequisite skills and concept familiar to students—these concepts are taught in the elementary schools—hence the potential of the RD approach for effective learning of logarithm.

1.4 Statement of the Problem

A closer look at the mathematics syllabus reveals that the topic logarithm is delayed until second year in secondary school (Form 2)—equivalent to 11 years in school—before it is introduced (core mathematics syllabus, 2010). The reason for the delay is due in part to the level of difficulty of the conventional approach to teaching logarithm. The quantum of relevant previous knowledge (RPK) required for grasping the concept is a contributory factor. My personal experience as a mathematics teacher in a senior high school confirms this observation. Empirical evidence from the research conducted by Chua and Wood (2005) further confirms the difficulties in teaching logarithm in schools. The examination report of the West Africa Examination Council (WAEC), the body in charge of external examinations in the West Africa sub-region, which includes Ghana, the site for this research confirms poor performance in mathematics, and in particular students' performance in answering logarithm questions during external examination (Chief examiners' report of WASSCE, 2014).

1.5 Objectives of the Study

According to Silver (1997), the significance of algebra in all levels of education indicate its importance. The main objective of this study is to improve the performance and conceptual understanding of students in learning logarithm. To achieve this, the study aims:

1. To introduce the RD approach of learning logarithm to students.
2. To compare the relative effectiveness of learning logarithm with the conventional approach to that of the RD approach.
3. To elicit students' preference of learning logarithm between the conventional approach and RD approach.

1.6 Research Questions

The poor performance and indifferent attitude of most students to learning logarithm are the reason for this research. The questions this research seeks to answer are:

- Does the Repeated Division approach of learning logarithm improve the performance of students?

  Hypothesis:

  Null hypothesis: There is no significant improvement in performance between the two approaches.

  Alternative hypothesis: There is a significant improvement in performance between the two approaches.

- Does the Repeated Division approach of learning logarithm improve the conceptual understanding of students in logarithm?

  Hypothesis:

  Null hypothesis: There is no difference in conceptual understanding of learning logarithm between the two approaches.

  Alternative hypothesis: There is the difference in the conceptual understanding of learning logarithm between the two approaches.

- Does the Repeated Division approach sustain the interest of students in learning logarithms?

  Hypothesis:

  Null hypothesis: There is no difference in the interest of students in learning logarithm.

Alternative hypothesis: There is the difference in the interest of students in learning logarithm.

1.8 The Significance of the Study

The research study could provide an alternate approach to help students learn logarithm effectively. It will help students avoid applying the laws and properties of the conventional approach without actually understanding them. It is hypothesized that the RD approach will help students to confidently demonstrate the reason for applying the approach in finding the logarithm of a number. The nature of the RD approach and how it is applied will improve students' reasoning ability. The process oriented RD approach makes it reasonable and practical, even for students who are not good in mathematics (Vos & Espedal, 2016). Improvement in dealing with logarithms and mathematics, in general, will "to give students the ability and skills most likely to be useful in pursuits of their future endeavour" (CRDD SSS core mathematics syllabus, 1989).

Teachers can use the approach to teaching logarithm to students with different abilities—both average and high achievers.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter focuses on a review of the literature on the evolution of logarithm and the development of the RD approach. In addition, the reason why logarithm is considered a part of algebra, as well as, the role of the logarithm in mathematics education in Ghana is discussed.

2.2 Development of Earliest Solution in Logarithm

In 1614, John Napier, Scottish mathematician discovered logarithm in his quest to reduce errors and save time in his astronomical calculations. Napier was very much interested in astronomy and carried out a lot of studies which involved lengthy calculations – time consuming and many errors. After many years of his work endeavor, he developed a logarithm concept (Smith, 2000) which helped astronomers to reduce error and save time (Katz, 2004). The definition of logarithm given by Napier was quite different currently perceived. In that era, astronomers often used trigonometric functions-particularly sine - thus, the definition was in consonant with trigonometry (Vz'llarrieal-Caldieron, 2012).

Later, he worked together with the English mathematician Henry Briggs and set logarithm of 1 equal to 0 thus (log 1=0) and the logarithm of 10 equal to 1 (log 10 =1).

Logarithm from the way it was perceived changed over time and Leonhard Euler in the late 1700s came out with notations to represent logarithms. He used the notation to define $\log_x y = z$ to be true when $x^z = y$ (Vz'llarrieal-Caldieron, 2012). In Napier's discovery of logarithm, he divided unite into 10,000,000 (10^7) portions, then subtracted from the unit its 10,000,000th portion (1/10,000,000), thus (1-1/10,000,000) (Cairns, 1928). He did that to get a number that can be approximate to 1, so that the successive terms of the geometric sequence will increase gradually or by a small margin (Maor,1994; Pierce,1977). He considered $N = (1-1/10{,}000{,}000)^L$, where L is the Napier's logarithm of the number N. For him not to deal with decimals, he multiplied each power by ten thousand (10,000,000)- $10^{7(}1- 1/10{,}000{,}000)^L$ and generated the following sequence of numbers: $10^7 = 10{,}000{,}000$, $10^7(1- 1/10{,}000{,}000) = 9{,}999{,}999$, $10^7(1- 1/10{,}000{,}000)^2 = 9{,}999{,}998$, $10^7(1- 1/10{,}000{,}000)^3 = 9{,}999{,}997$… $10^7(1- 1/10{,}000{,}000)^{100} = 9{,}999{,}900$. These are the numbers generated for the first table.

Napier thought of dividing the last term by the first term of the numbers generated (9,999,900/10,000,000 = 0.99999) from the first table to develop his second table. He started the second table with 10^7 and multiplied each power by $10^{7\cdot}$ The proportion of the first and last term which resulted in 0.99999 was converted to fraction 1/10,000 and used in generating the following sequence: 10^7= 10,000,000, $10^7(1\text{-}1/100,000)$ = 9,999,900, $10^7(1\text{-}1/10,000)^2$ = 9,999,800 , …$10^7(1\text{-}1/100,000)^{50}$ =9,995,001.

Again, he considered dividing the first and the last terms of the second table – (9995001/10000000= 0.9995001) to generate the following: 10^7=10,000,000, $10^7(1\text{-}1/10000)$ = 9,995,000, $10^7(1\text{-}1/10000)^2$=9990002, …$10^7(1\text{-}1/10000)^{20}$= 9900473.5. He continued by generating additional numbers by using the fraction of the last entry to the first entry of the successive tables developed.

In 1622, a slide rule was brought to bare by an English mathematician William Oughtred in solving logarithm problems. The slide rule was generally used by scientists and engineers (Stoll, 2006). Joost Biirgi, a court clockmaker by profession was once faced with issues of computation. The situation motivated him to find ways of dealing with it by developing logarithm table to assist him in computation. He developed a single table for multiplication, a single table for arithmetic and another for geometry. His ambition was to develop a single table to perform all the arithmetic operations (Clark & Montelle, 2010).

2.3 Logarithm As Part of Algebra

Algebra is very important and contributes to the mastery of logarithms. According to Carraher et al (2007), from the research conducted on the longitudinal study of early algebra, they described algebra by considering the relationship that exists among the numbers, the symbolism used and the study structure. Usiskin (1995) stated that “Algebra is the language of relationships between quantities. The description of algebra given by the above researchers and many others compelled Espedal (2015) to draw an analogy that logarithm can be considered as part of algebra if it is perceived from the following angle:

(a) Algebra as a generalized arithmetic
(b) Algebra as a study process/ procedure
(c) Algebra as a relationship between size and study structures**.**

In his quest to clarify his assertion, he demonstrated it with examples. Firstly, he looked at algebra as a generalized arithmetic and stated that, if 2+5 = 5+2 and 10+3 = 3+10, then we can generalized that a+b = b+a. Similarly, if log 2+ log 3 = log (2x3) = log 6, and log 3 + log 5 = log (3x5) = log 15, then one can emphatically generalized that log x + log y = log (xy) or log (xy) = log x + log y.

Furthermore, he delved into algebra as a process and stated that, if log (y+2) = 3, students will arrive at the equation $y + 2 = 10^3$, which is now recognized as a simple linear equation. The simplified equation can be solved procedurally if the concept is well grasped. The RD approach with its cover-up strategy could be applied to this situation.

Finally, Espedal indicated that, questions like log x + log (x-1) = log (3x + 12), does not demand generalization nor follows a pattern. One ought to manipulate and establish a relationship among the expressions. These are the perspectives for which logarithm is seen as part of algebra.

2.4 Mathematics Proficiency

The mathematics curriculum in Ghana has been categorized into core mathematics and elective mathematics. The core mathematics is offered to all the senior high school students while the elective mathematics is offered to selected students offering particular programs.

A Logarithm is a topic treated in both core and elective mathematics. The logarithm taught under elective mathematics covers a broader perspective and it is much detailed, compared to that of core mathematics (CRDD Teaching syllabus core mathematics, 2010; & CRDD Teaching Syllabus elective mathematics, 2010).

For the purpose of this research, logarithm taught to students taking core mathematics is considered, in order to cover more students with different mathematics abilities. It is believed among students that elective mathematics is for students with superior ability in mathematics (the very clever students). The core mathematics curriculum has been organized around eight broad areas/topics and two profile dimension. A dimension is a unit for describing a particular learning behavior. In other words, the measure of students' actions during learning and the use of content. More than one of such action description constitute a profile dimension (CRDD Teaching syllabus core mathematics, 2010). The students' ability to recall what has been taught and apply the knowledge acquired which is termed Knowledge and understanding (KU) and application of knowledge (AK) form the main components of the profile dimensions in the curriculum. The curriculum is structured in such a way that logarithm is treated in the second years (Form 2). It's also placed emphases on the indices as a pre-requisite knowledge to the logarithm. Indices give a solid algebraic notation for repeated multiplication (a^4= a x a x a x a) and the inverse of it leads to logarithm introduction. Mathematics proficiency is linked with the students' competence and achievement. Hence, the need for students to be proficient in their pursuit of learning mathematics. Kilpatrick et al, (2002); Mathematics Learning Study Committee (2001) & Findell (2002) stated that conceptual understanding, procedural fluency, logical reasoning, ability to formulate and represent mathematical problems enable students to become good mathematics learner.

2.4.1 Conceptual Understanding

Conceptual understanding has been described as the ability of one knowing the facts and the why of it (Frederick & Kirsch, 2011). Conceptual understanding goes beyond just response to the test items. The essence of it is to probe into students' result more than just the correct answer. Wiggins (1998) explained conceptual understanding as the acquisition of enough concepts and skills to reflect, reassess and reformulate the already acquired knowledge- Thus when knowledge acquired is linked up in a rightful way to already existing knowledge (Davis, 1984; Skemp, 1971; Van Engen, 1953). According to Hiebert (2013), student's ability to establish a relationship between pieces of information is an indication of attaining conceptual understanding. He further explained that conceptual understanding can be developed through the student's ability to establish a relationship between the old knowledge acquired and the new knowledge being acquiring. According to Bruner (1961), conceptual understanding is developed through discovery learning. Kilpatrick et al, (2002) explanation outlines and summaries what other researcher have described conceptual understanding to be. According to them, it's constitute (a) comprehension of mathematical concepts (b) operations or process and (c) relations. According to Skemp (1978), the likelihood of a concept becoming part of students with clear understanding is certain than those who memorized a procedure. In other words, developing conceptual understanding of a concept is better retained and applied than memorizing it. Conceptual understanding—the ability of the student to demonstrate a clear understanding of a concept—helps student's to demonstrate their understanding of logarithm as Object and logarithm as Process, as stated by Sfard (1991).

2.4.2 Performance:

According to Winggins (1998), performance is how a student did in the light of what he attempted in a test. Performance measures/ gauges what the student has absorbed as against the outcome and the real output of a student against the benchmark. In assessing the student's performance, there must be a line of distinction between an optional and what is mandatory in student's work.

2.4.3 Interest:

The empirical evidence from some mathematics research emphases that interest is one of the important element and prerequisites for students to develop conceptual understanding of a subject (Shabani, 2006). Shabani described interest as an incentive that instigates students' activity power. The interest developed by the students' enhances the understanding of the material learned and its application (Shabani, 2006 cited in Khayati, & Payan, 2014). A study conducted by Swarat et al (2012), on "Activity matters: Understanding student interest in school science" revealed that genuine interest is a vital component of scientific literacy. The results from the study indicated that practical activities arouse students' interest in learning. Students' interest developed is not only needed for a career but also an important constituent of scientific knowledge (Rutherford & Ahlgren, 1991).

2.5 Concept Learning Through Process or Object

According to Sfard (1991), numbers and functions do have two different perspectives to be viewed at:

(a) Structural conception – as objects: is one's ability to solve the problem completely or to recognize mathematics steps to the solution as a holistic entity. Tall et al (1999); Cotrill et al (1996), see object oriented thinking as one's ability to recognize mathematical procedure or process as an entity without performing the procedure. The object is mostly linked with the product of the process.

(b) Operationally – as processes oriented: the sequential actions that are maximized when solving mathematical problems.

Students considering operational steps as an object helps them to develop the structural concept. For example, in expressing $2\log_b 3 + 3\log_b 5$ as objects, one has to represent $2\log_b 3$ by $\log_b 3^2 = \log_b 9$ and $3\log_b 5$ by $\log_b 5^3 = \log_b 125$ before rephrasing it by the use of the property: $\log_b x + \log_b y = \log (xy)$ to give $\log_b(9 \text{ x}125)$ (Chua & Wood, 2005). The misconception of mathematical structural ideas (objects) contribute to some of the mistakes students make in mathematics. Yen (1999) cited in (Chua & Wood,2005) mentioned that some students perceive "In" as variable in equation like In(7x -12), thus "In" is a common factor where it can be expanded and become In7x – In12. Others when given logarithmic equation log y = log 8, they divide both sides of the logarithmic equation by "log" to get log y = log 100 => y =100 (Lopez-Real, 2002) as a result of misconception of object ideas. Mathematics as object has a great beneficial effect which helps in making abstract ideas clear and assign meaning to it.

The empirical evidence from researches conducted by many mathematicians verifies that in the midst of acquiring new mathematical concept, the object comes after the process (Sfard, 1991, Sfard & Linchevski, 1994). The process based thinking augment object based thinking – thus, the steps or sequential actions followed in solving problem support the authenticity of the final answer.

2.6 Development and Description of Repeated Division

Vos & Espedal (2016), recently worked on logarithm by using an alternative didactical approach- Repeated Division. According to them, Repeated Division approach as the name implies involves an application of division. Repeated-division-until-you-reach-1 is the unconventional approach used in logarithms which facilitate the students' reasoning ability (discontinues the current approach of learning logarithms by laws and properties) right from the start. The nature of the approach makes it reasonable and practical to both good and weak students. It helps students' in developing a natural tendency to discover logarithm as function. Repeated-division-until-you-reach-1 could be illustrated from the perspective; (a) object side concept and (b) process side concept.

Espedal (2015), showed how logarithm can be introduced by using Repeated Division. He defined logarithm as:

$$x = 10a \Leftrightarrow \log x = a.$$

$$x = 10^{lgx}$$

$$\frac{x}{10^{lgx}} = 1$$

The above description can be translated as; how many times can *x* be divided by 10 exponent log *x* until reaching 1.

Espedal justified his approach with examples and used the students' RPK on a division in the introduction of the concept: Repeated Division until you reach 1. For example, if one has a number 81 and divides it by 3 (81/3 = 27). Then you continue the division by dividing the answer 27 by 3 (27/3 = 9). Until you reach 1, you continue to divide the successive answers by 3. So, 9 will be divided by 3 (9/3 = 3) and (3/3 = 1). The ultimate target is 1, of which we have obtained. After which, the number of times 81 was divided by 3 until 1 was obtained will be counted. In this case, 81was divided by 3 four times until 1 was obtained.

Thus: 81

: 3

27

: 3

9

: 3

3

: 3

1 Number of steps: 4

Therefore, log_3 81= 4 (reads as "log base 3 of 81 equals 4")

In some cases, exactly 1 cannot be reached in the repeated divion by the base. In such cases, estimation is done by giving the answers within a range). For example:

600

: 10

60

: 10

6

: 10

< 1

Number of steps: more than 2 but less than 3.

Therefore, log_{10} 600 = 2< log_{10} 600 < 3.

The approach establishes the following identities:

log10 = 1 log1 = 0.

The identity log 10 = 1, reads, 'log' base 10 of 10 equals 1. The repeated division approach meaning of the above identity reads, how many times the number 10 can be divided by the base 10 until you reach 1. Thus

10

1 step : 10

1

1

: 10

No step is needed to reach 1.

Using the identities, we can say that, log 600 is one step more than log 60 and log 60 is one step more than log 6. Therefore, we can say log600 = 1 + log60 = 1+1+ log6 = 2 + log6.

In finding the logarithm of decimal numbers, multiplication is regarded as the opposite of the division. In that sense, if the division provides positive (+ve) result, the multiplication will give negative (- ve) answer. For example, finding the 'log 0.01. The target is to obtain 1, therefore, 0.01 will be multiplied repeatedly by the base until the target 1 is obtained. Thus:

1

Opposite movement x 10

Direction opposite to direction of division(/)

0.1 -2

x 10

0.01

Therefore, we write log 0.01 = -2. (Negative sign is used because of the upwards movement). The division sign (downwards movement) gives the positive (+ve) result while the multiplication sign (upwards movement) gives the negative (-ve) result to the logarithm.

2.6.1 Logarithm of Negative Numbers

In dealing with the logarithm of negative numbers, the researcher adopted a logarithm function $\log_b(x) = y$. Since the base *b* is positive (*b*>0), the number X divided by *b* must be positive, hence, successive answers repeatedly divided by b must be positive. So the number X must be positive (x>0). The real base *b* logarithm of a negative number is undefined, thus, $\log_b(x)$ is undefined for $x \leq 0$.

2.7 Advantages of Repeated Division

According to Vos & Espedal (2016), Repeated-division-until-you-reach-1 is an unconventional approach to solving logarithms. It facilitates the students' reasoning ability (discontinues the current approach of learning logarithms by laws and properties) right from the onset. It has a process oriented meaning which makes the approach reasonable and practical to all students. It helps students to develop a natural tendency to discover logarithm as a function. Repeated-division-until-you-reach-1 could be illustrated from the viewpoint as (a) an object side concept and (b) process side concept. Again, unlike the conventional approach, it is efficient and very effective. In contrast to the complexity of the conventional approach, the Repeated Division approach is simple and by virtue of its inherent advantages, it has the potential of being easier to use. In view of this, it is believed that it will be preferred to the conventional approach whose formidable structure, including many laws and properties, make it difficult for students to comprehend. There is no need to spend precious time to teach any specialized pre-requisite skill before teaching this approach. Time is saved which could be used in teaching some other topics.

2.8 Recent Research Studies on Repeated Division and their Recommendation

Espedal (2015) conducted a study in Norway for high school students on learning logarithm using the RD approach. The findings from the study provided further evidence that learning logarithm with RD approach was much easier for students as compared to the conventional approach. The finding was especially relevant for those described as weak students. The Espedal's study emphasized that the process -oriented aspect of the approach enables students to reconstruct log1=0 and log0=1. Students demonstrated better understanding and explained why log1=0 and lg10 = 1. In his study, the tasks were not enough to close the gap between the repeated division and the logarithmic rules.

2.9 Logarithm Aspects

Logarithm and its definition lead to certain numerals students find it difficult to comprehend. Many others label the relations as laws or properties of a logarithm. For this study, I classify those logarithmic functions/relations as aspects of a logarithm. These aspects of logarithm have been shown in diagram 2.1 below. The aspects are important components of a logarithm. The conceptual understanding of these aspects does influence the effective application of logarithm concept in both mathematics and other fields where logarithm is applicable.

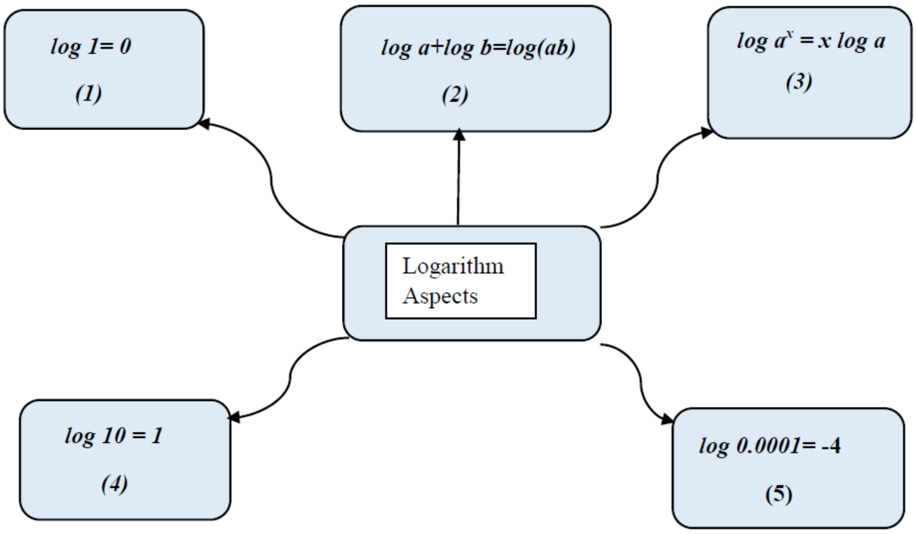


Diagram 2.1 Logarithm Aspects Diagram

The aspects of logarithm grants students' confidence and permits them in expressing logarithmic expressions in a variation of different ways (Mathcentre, 2009).

The aspects are implicitly defined and needed in solving every logarithmic problem. This study has categorized the logarithm aspects into five main parts. The aspects 1& 4 deal with the logarithm of one and ten. The aspect 1 reads: logarithm of one to base 10 is equal to zero while aspect 4 is: logarithm of ten to base 10 is equal to 1. The aspects 2 & 3 are characterized by many mathematics researchers as the "logarithm product rule" and "logarithm quotient rule" respectively. Aspect 2 is described as: "the logarithm of two or more positive factors to base 10 is equal to the sum of the logarithms of the factors to the same base". In a similar way, aspect 3 has also been described as: "the logarithm of the quotient of two factors to base 10 is equal to the difference of the logarithms of the factors to the same base" (Math-only-math.com, 2010 & Mathcentre, 2009). Aspect 5 of logarithm deals with the logarithm of decimal numbers (numbers with zeros preceding the positive integer). A typical example is what has been cited in diagram 2.1 above. Here, it is described as; logarithm of 0.0001 to base 10 is equal to -4 (thus number of times the decimal point moves until a whole number is attained). The direction at which the decimal point is moved gives the clue to the sign attached to the answer.

CHAPTER THREE

METHOD

3.0 Introduction

This chapter describes the methods applied in conducting the study. This includes the sample design used for the study and the procedure used in conducting the study. A detailed description of the experimental design and the analysis plan are also covered herein.

3.1Population and Sample

The students used for the study were the first and second-year students in their third term at Mfantsipim School, Cape Coast, in the central region of Ghana. The students were drawn from the general science program. Two classes were selected from Form 1 (i.e. 1S2 and 1S10) and one class from Form 2 (i.e. 2S3). The 1S2 & 1S10 classes constituted the experimental group while the 2S3 class was selected to be the control group. Students were selected from two different levels mainly for the purpose of comparing results from exposing students to the two approaches (the conventional approach and the RD approach). The students from the three classes were briefed about the research and they agreed to participate. Although they all agreed to partake, some students were unable to participate in the intervention instructional period, pre-test and the post-test. In view of these, the data was collected from the students who were able to fully participate in the whole process.

3.2 Task

Breen & O'Shea (2010), referred to task as homework problems and the classroom activities students try their hands on individually or in groups. In this study, the term task will refer to pre-test, post-test and retention test designed for the study. The series of tasks given to the students in the study were designed by the researcher with the help of the supervisor. According to Mason and Johnston-Wilder (2004), tasks are designed to provide an opportunity for practicing previous ideas, provide an opportunity to handle new ideas, act as revision, prompt reflections/ replication, prompt the connection and integration of various ideas. In regards to the characteristics of tasks given by Mason and Johnston-Wilder, the three different types of tasks prepared were not done to favor a particular approach but rather to meet the qualities a task should have. They were carefully prepared to exhibit those qualities mentioned above.

3.3 Preparation of Learning Materials

The new learning/intervention teaching material used for the study was prepared by the researcher under the guidance of the supervisor. The content of the new teaching material was examined and compared with the logarithm exercises in the already existing Mathematics textbook for Senior High Schools in Ghana. Cautions were taken in the preparation of the material in order to meet the standard of the education system in Ghana. The material underwent extensive vetting and supervision before it was finally approved to be used for the study.

The test items set for pre-test, post-test & retention test went through the process of vetting and modification with the help of the supervisor. The retention test items set initially by the researcher were rejected on the basis that, they were not equivalent to the pre & post- test, test items. This led to the setting of the new test items which were more suitable for all standard to test for the retention level of the students.

3.4 Intervention Design

The essence of the new learning material designed was to ensure:

1. The easiness of the learning challenges students do encounter in logarithm.
2. A strong basis for conceptual knowledge in logarithm among students.
3. Students' motivation to regulate their attitude and develop an interest in logarithm.

In view of the prime focus of the designed material, the experimental group was taken through series of the instructional period using an alternate approach – Repeated Division— which is not captured in the teaching syllabus. The intervention lessons were mainly activity based. Students were placed at the center of the intervention lessons to enable them to discover and develop conceptual understanding. The conventional approach captured in the teaching syllabus demand pre-requisite knowledge before a student can grasp the concept. In the RD approach, as the name depicts, the division which is well understood by every senior high school student forms the central core of the approach. RD was described as: 'the number of times a number x is divided until reaching 1'. Mathematically, we say logarithm of x and its notation is $\log x$.

Studies have been conducted to show many negative occurrences in the mathematics classroom. These negative occurrences are as a result of the absence of instructional materials and the nature of mathematical knowledge.

With regards to the students' misperception in logarithm, RD approach was introduced to bring to bare students' understanding in logarithm.

The intervention lessons covered three weeks (15 days) of the study period. Each week had three mathematics sessions and each session constituted two (2) or one (1) teaching period. In total, 15 days were used for the study: first and last days were used for pre-test and post-test respectively. According to West Africa Examination Council (WAEC) syllabuses and the school timetable, forty (40) minute per period has been allotted for mathematics in the SHS (Core mathematics syllabus, 2010, Elective mathematics syllabus, 2010). The lessons took place during the normal school session. During the intervention stage, students were made to perform a lot of activities in class (see Appendix C for the teaching material). The intervention set-up in table 3.1 below shows how the intervention was organized.

Table3.1:Intervention Set-Up

Days	Intervention	Experimental Group (Form 1)	Control Group (Form 2)
1	Pre – test	Yes	Yes
2 – 13	Lesson: Repeated Division	Yes	No
14	Revision	Yes	Yes
15	Lesson/Post – test	Yes	Yes

3.5 Instruments Used for Data collection

The instruments used for collecting the data were: the pre-test, post-test, retention-test, and interviews. All the test conducted consisted of four test items and forty (40) minute was allotted for each test. The same test items were used for both pre-test and post-test. The pre-test was conducted before the intervention teaching lessons to introduce RD to ascertain students' level of knowledge in logarithm. Since students conceptual understanding of logarithm was paramount to the study, considerable effort went into ensuring that tasks represented cognitive demand. All the test items were in general forms of the logarithm. However, item 3 was a type which could have been solved without logarithm yet, they were restricted to use logarithm. The post-test was administered after the significant lesson in RD approach to the logarithm. During the post-test, the Form 2 students were asked to use the conventional method while Form 1 students were asked to use the RD approach. Actually, the Form 1 students couldn't have used any other approach apart from RD because they hadn't studied logarithms using CA yet.

After four months, the retention test was administered to Form 1 the student who took part in the study following the same trend as post-test. These test items set were based on the pre & post-test items but in a varied form, (see Appendix A, iii for retention-test items). The test items contained inherent logarithm aspects the researcher needed for the analysis.

The study used descriptive statistics in the analysis to test if RD has any effect on students' performance, conceptual understanding and interest. The study was supplemented by interviews with the students and the class teacher. The students were selected randomly for the interview. The purpose of the interview was to solicit further evidence on their experience with the new approach. The Effort was made to investigate how easy it was for the students to use the RD approach and the impact the approach had on their understanding of logarithm. Recordings of the student's explanation during the intervention lesson were made and were analyzed to confirm students' test outcomes. Attempts were also made to investigate difficulties the students encountered in using the new approach-RD.

3.6 Data

3.6.1 Test**:** Students from the three classes selected for the study were tested at different levels. The first was the pre-test, which was organized at the very beginning of the study. In the next test was the post-test, organized after the intervention lessons. The last test was the retention test, organized after the post-test to measure students' retention of the concept. The test were 40minutes each. The test items were carefully designed based on the intervention teaching materials prepared by the researcher (see Appendix C). It covered all the aspects of logarithm the researcher needed for this study and cover-up strategy.

Pre-Test: The pre-test was conducted to find the difference in performance of students relative to the approach the students were exposed to, as well as their level of understanding of logarithm. The form 1 students' have not been taught logarithm yet, however, my interaction with them revealed that some of them are familiar with it.

Post-Test: The post-test was conducted after the intervention—exposure of Form 1 students to RD approach—hence, the Form 1 students were restricted to the use of RD approach to solving logarithm, while the Form 2 students used the conventional approach since they have only been exposed to the conventional approach as a control group.

Retention Test: Retention test was organized a week after the post- test, to understand how the RD approach lend itself to the possibility that students retain the knowledge acquired using the RD approach. After the researcher had a discourse with the supervisor, it was realized the use of calculator by the students during the three levels of the test conducted had affected the quality of the data needed for the analysis. I was asked by the supervisor to redesign similar test items (not the same as the pre& post-test items) to re-organize retention test in the month of November 2015. In view of that, the first retention test was discarded and second retention test (Appendix A, iii) was conducted in the third week of November 2015. Again, the use of calculator made the analysis of the pre & post-test took different dimension by drawing the line of distinction between the use of a calculator, avoiding/ skipping steps and others as described in chapter 4, section 4.1.3, analysis of the aspects – conceptual understanding.

3.6.2 Audio Recordings

The audio recordings made for the study were categorized into:

1. Interview with individual students
2. Interview with a group of six students (Focus group)
3. Interview with the class teacher
4. Class discussion

The interview with individual students: The interview questions centered on the views of students on the simplicity of the RD approach, the difficulties they encountered using the RD approach, the advantages offered by the approach and what they could recall after the intervention lessons.

The interview with a group of six students: A group of six student volunteers participated in the group interview conducted on a different day. This was done due to time constraint (i.e. the school had a pre-planned program for the students). The students respective views on the questions asked were recorded. The recorded interviews were transcribed and analyzed, as was the case for all the interviews.

Interview with the class teacher: The class teacher was interviewed after he had spent some number of days in the intervention lessons. His views regarding the RD approach and the conventional approach (CA) were solicited among other questions on how he could explain the concept to his colleague teacher. His preference in using any of the approaches after his experience with RD approach was sought (see Appendix E, i).

Classroom discussion: In the course of the intervention lessons, students performed a lot of activities in the class. Some of these activities took place in groups of which the recordings of student's conversation were made. In few occasions, some of the students were called to explain what they have done to the class. Those who were not clear with the explanation given by raised clarification questions.

3.6.3 Classroom Observation: Any time the students performed activities, the researcher went round to look at what the students were doing. Pictures of the students work during the intervention lessons were captured (see Appendix D). The data collection set-up in table 3.2 below shows how and where data were gathered.

Table 3.2:Data Collection Set-Up

Data Collected	Experimental Group (Form 1)	Control Group (Form 2)
Pre – test	Yes	Yes
Post – test	Yes	Yes
Retention test	Yes	No
Interviews	Yes	No
Class discussions	Yes	No
Class Observations	Yes	No

3.7 Data Analysis Method

The study used a mixed method research design (qualitative and quantitative) to analysis the aspects (see diagram 3.1). This is for the purpose of examining if RD approach to teaching logarithm compared to the conventional approach, improves students' performance and conceptual understanding of logarithm. Mixed method research, is defined as a research that systematically combines quantitative and qualitative methods (Brymen, 2012; Johnson, et al. 2007 & Brymen, 2015). Mixed method was preferred for this study due to the need to collect both numerical data as well as qualitative data on the opinion of students as well as teachers on the intervention—RD approach.

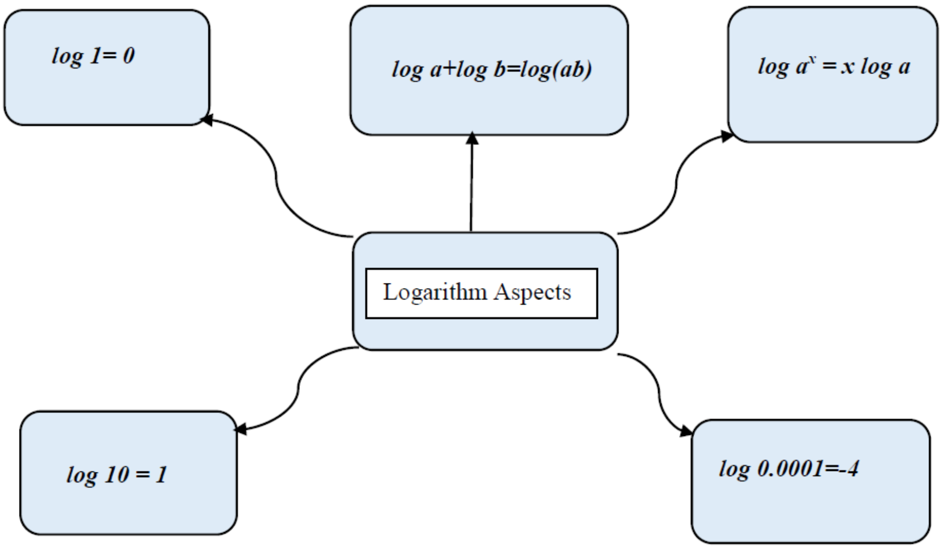


Diagram 3.1 Logarithm Aspects Diagram

CHAPTER FOUR

ANALYSIS AND RESULTS

4.0 Introduction

This chapter presents the analysis and results — students' performance, conceptual understanding and interest of logarithm — of the research. The analysis presents both quantitative and qualitative results.

4.1 Testing The Research Hypothesis

4.1.1 Analysis of Performance

Performance herein refers to the gain in scores by students in the post - test conducted or the scores attained by the students from the post - test conducted. Students were allocated 5 marks for each test item—there were four test items—summing up to 20 marks for the four test items that constituted the post - test. The focus was on getting the correct answer.

To compare the performance, students' scripts were marked, each script was coded in excel. Once the marking was done, marks were recorded against their names. The descriptive statistical analysis - group frequency distribution table was used. The table 4.1 below summaries the outcome of the results.

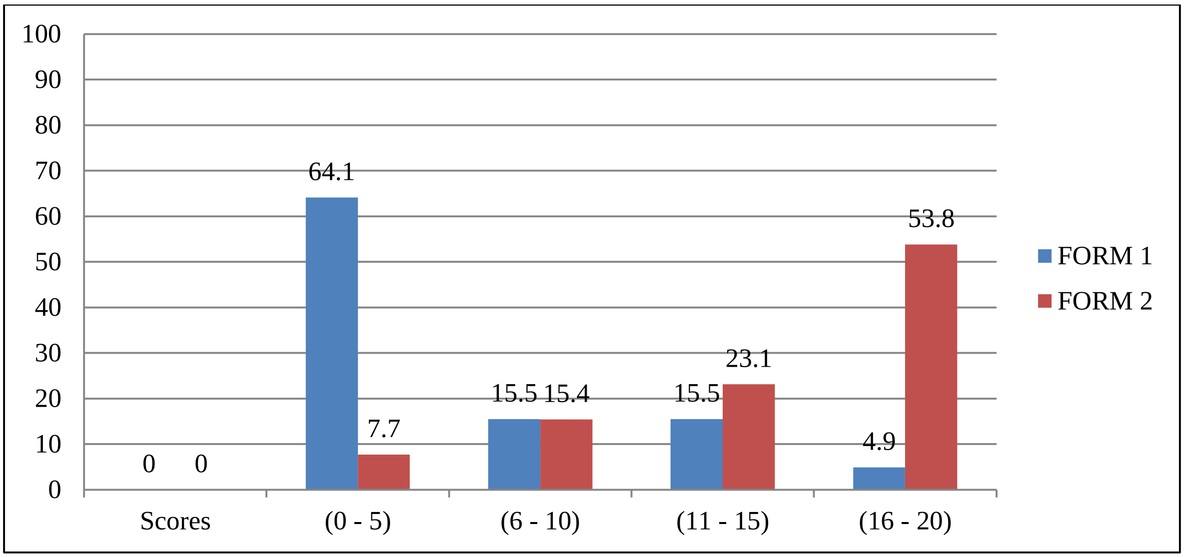


Figure 4.0 Performance analysis (pre-test)

Figure 4.0 shows the students' performance in the pre-test. The results show that more than 50% of the Form 2 students scored between (16 – 20) which represent the highest score. The highest score, in other words, is seen as the good performance of the students. The performance difference in the highest score between the two Forms was 48.9% (representing more than 9 fold increase). About 64% of the Form 1 students recorded the lowest mark – (0-5) whereas Form 2 students recorded about 7.7% in the pre – test.

Table 4.1:Performance Table (post-test)

Score	**Form 1 (Repeated Division)**			**Form 2 (Conventional Approach)**		
	Tally (t_1)	**Frequency (f_1)**	**Percentage (P_1)**	**Tally (t_2)**	**Frequency (f_2)**	**Percentage (P_2)**
0-5	*IIII*	4	3.9%			
6-10	~~IIII~~ ~~IIII~~ III	13	12.6%	I	1	2.6%
11-15	~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~	60	58.3%	~~IIII~~ I	6	15.4%
16-20	~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ I	26	25.2%	~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ II	32	82.0%

The table 4.1 above indicates that 25.2% of the Form 1 students had the quality pass (those who had the highest score) as against 82% of Form 2's. This represents about 56.8% variation of the quality performance in favor of the conventional approach. The greater number of Form 1's, about 58.3% scored between (11-15) marks. This percentage proportion is a little more than double fold their quality pass.

4.1.2 Categories of Data

The diagram below shows the aspects on logarithms that the researcher had included in the test. The aspects were incorporated into the various test items of the tests for the study as shown in the diagram below. They were not too obvious for the students to identify them so easily.

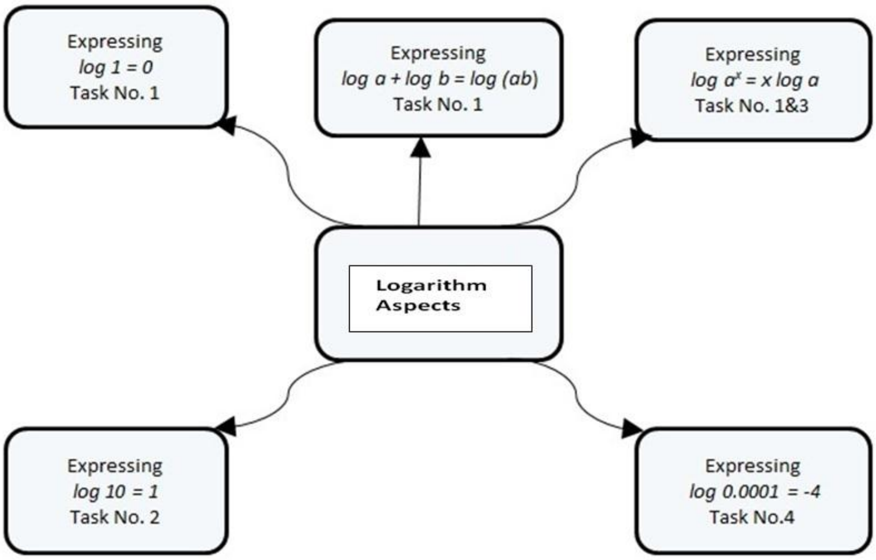


Diagram 4.1: Logarithm Aspects Diagram

4.1.3 Analysis of the Aspects - Conceptual Understanding

In a study conducted by Berezvoski & Zazkis (2006) on logarithm, they categorized students understanding into; logarithm as numbers, logarithm as operations and logarithm as functions. Here, I measured the changes in students understanding based on the approach they were exposed to. The aspects of the logarithm as depicted in chapter four, figure 4.1 were carefully chosen and used as a test standard to assess students conceptual understanding in logarithms. Again, the post –test was used in finding out whether there were a significant difference in the students conceptual understanding between the two approaches.

In this study, the test items answered by students in the post – test were analysed to find out if the following relationships of the logarithm aspects were clearly established: log 1=0, log a + log b = log (ab), log 10 = 1, log a^x = x log a, log 0.0001 = -4 and cover-up strategy. In the analysis of aspects, the names of the students were coded in the order the researcher collected the scripts. A detailed analysis was carried out by examining the participants' response to the test items one after the other. The following were used in coding: *"1"* was assigned to a student who demonstrated clear understanding, *"0"* was assigned to a student who demonstrated no understanding (wrong), *"2"* was assigned to a student who used calculator, *"3"* to a student who did not show vital steps or process to the answer and *"4*" was assigned to a student who was not consistent in establishing a particular relationship (at time he does it correctly/ at time he does it wrong). The table presented below gives the details of the aspects analysis.

Table 3.2:Aspects Analysis Table (post-test)

Aspects	**Number of correct responses. (1=correct)**		**Number of wrong responses. (0=wrong)**		**Number of calculator responses. (2=calculation)**		**Number of avoiding (not seen) responses. (3=avoiding)**		**Number of inconsistent responses. (4=inconsistency)**		**Percentages of correct responses (%)**	
	Form 1	**Form 2**	**Form 1**	**Form 2**	**Form 1**	**Form 2**	**Form 1**	**Form 2**	**Form 1**	**Form 2**	**Form 1**	**Form 2**
log 1=0	77	19	5	2	4	2	11	4	-	-	74.8	48.7
loga+logb= log (ab)	63	27	8	4	10	-	-	4	-	-	61.2	69.2
log 10=1	96	38	7	-	-	-	-	1	-	-	93.2	97.4
log a^x *= xlog a*	69	26	-	1	-	-	-	3	6	2	67.0	66.7
log 0.0001=-4	99	37	4	-	-	-	-	-	-	1	96.1	94.9
Cover up	89										86.4	

Log 1=0

From table 4.2, the number of Form 1 students who demonstrated clear understanding of *log 1=0* were more than Form 2's. The difference between Form 1 students to Form 2 students was 26.1% . The result suggests that 4.48% of the Form 1 students had no conceptual understanding -- *log1=0* —as compared to 5.1% of the Form 2 students. However, four out of 103 Form 1 students (3.9%) used a calculator to establish the relationship—*log1=0* —correctly. Approximately 10% of the students in each Form were unable to establish any relation. Students in both Forms did not demonstrate any inconsistency in their work.

Figures 4.1 to 4.4 depict some of the students' works that show how they exhibited their understanding in logarithm. The work illustrations in figure 4.1 & 4.2 shown below, clearly indicate the students' demonstration of their conceptual understanding of log 1.

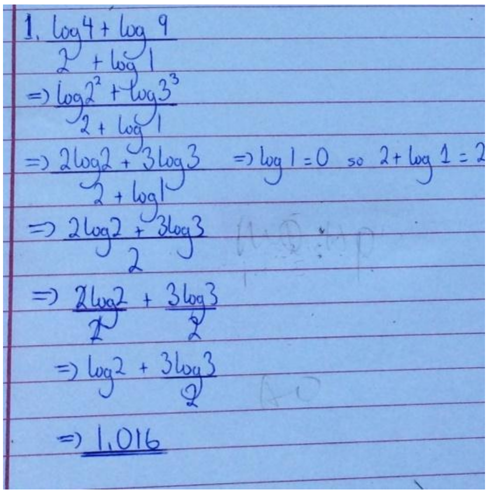


Figure 4.1

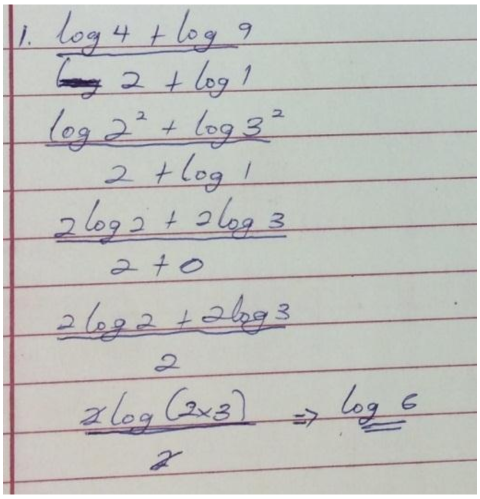
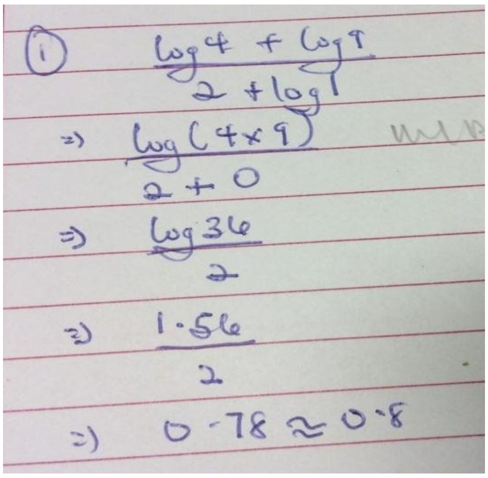


Figure 4.2



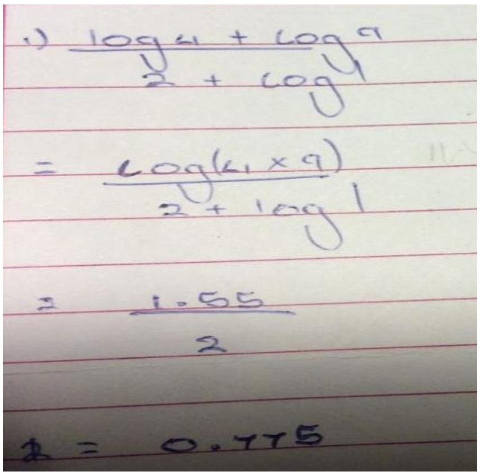
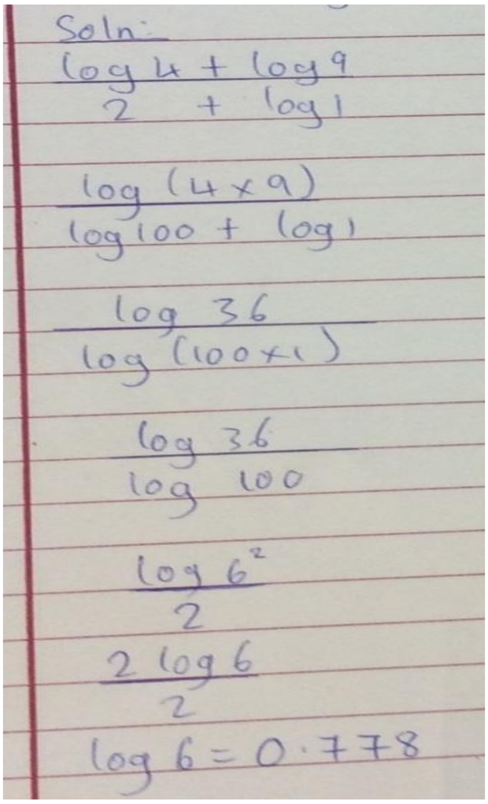


Figure 4.3

Figure 4.4

The exhibits in figure 4.3 and 4.4 indicate the students’ use of calculator and skipping an important step (avoiding).



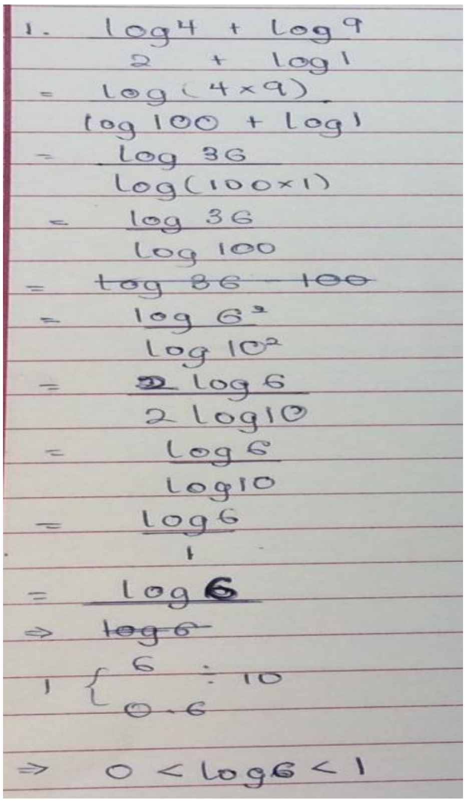


Figure 4.5

Figure 4.6

log a + log b = log (ab)

The tables 4.2 clearly indicates the percentage score by the students in the two Forms, demonstrating their conceptual understanding with respect to establishing the relationship between log a + log b and log (ab). The result shows that 61.2% of the Form 1 students were able to correctly establish a relationship while that of the Form 2 students was 69.2%.

Approximately 8% of the Form 1 students and 10% of the Form 2 students lack the conceptual understanding of the relationship between log a + log b and log (ab). With the use of a calculator, 9.7% of the Form 1 students were found to lack a conceptual understanding of the relationship between log a + log b and log (ab), while none was found for Form 2 students. Figure 4.5 and 4.6 are samples indicating students' demonstration of the relation.

Log 10 = 1

The analysis shows that the students were successful in answering logarithm Log 10 = 1, with the success rate of about 93.2% for Form 1 and 97.4% for the Form 2 students. However, a few number of the students were unsuccessful or lack the understanding and ignored or skipped a very vital step in arriving at establishing the relationship. About 6.8% of the Form 1 students were not successful and about 2.6% of the Form 2 students avoided a step to the final answer. An example of student's work in figure 4.11 demonstrating how he understood and used log 10 in the test items.

Log a^x = x log a

The success rate of log a^x = x log a was approximately 67% for both Forms. This suggests that most students demonstrated a reasonable understanding of solving this problem. About 2.7% of the Form 2 students were not successful, however, the Form 1 students had no wrong solution but about 5.8% were inconsistent in their solutions. Likewise, Form 2 students had about 5.1% inconsistencies in their solution. Figures 4.7 and 4.8 show the solutions from students.

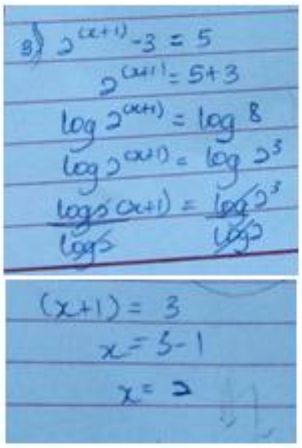


Figure 4.7

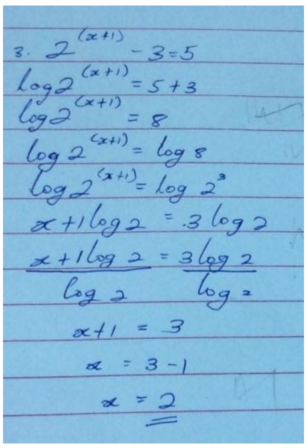


Figure 4.8

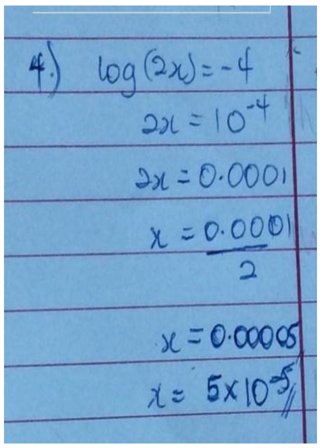


Figure 4.9

Log 0.0001 = -4

With log 0.0001 = -4, Form 1 had the highest percentage (96.1%) showing a clear understanding of the aspect, whiles that for Form 2 students was 94.9%. The percentage margin between the two Forms is 1.2%. Only 4 Form 1 students which represent approximately 3.9% exhibited a lack of understanding. However, 2.6% of the Form 2 students were inconsistent in establishing that log0.0001= -4. The first, second and third steps of the exhibit in Figure 4.9 is an example of how a student demonstrated his understanding.

Cover up

It is one of the strategies applied in RD approach for solving logarithmic equation. It is basically centered on the idea of solving the logarithmic equation in a backward direction as demonstrated in the *Logarithms, Senior High School, Core Mathematics* (handbook) prepared by the researcher (see appendix c page 7, hint to questions 4.2.3). About 86.4% of the Form 1 students who took part in the intervention lessons were successful in applying the strategy correctly.

Figure 4.10 below shows the understanding students developed in using the cover-up method.

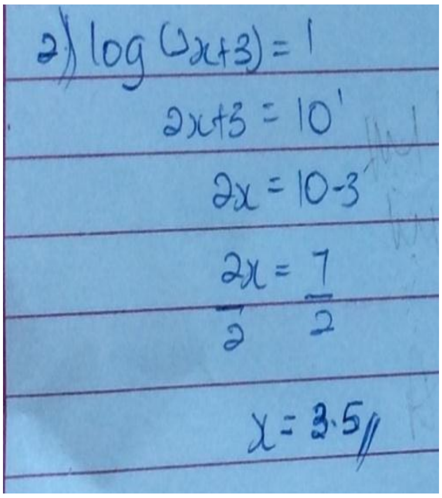


Figure 4.10

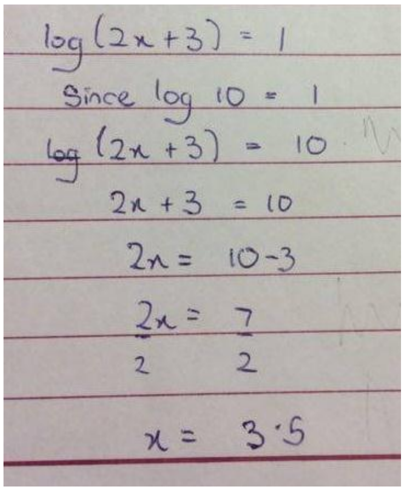


Figure 4.11

4.2 Interview Analysis

This section deals with the analysis of the results of the interview data from the students and the class teacher.

4.2.1 Interview – Responses from Students

The data from the interview were classified into three main domains—evidence of students' object understanding, evidence of students' process understanding and observation of students in class—to support the quantitative findings.

4.2.2 Evidence of Students' Object Understanding

It was identified from the interview data that; students were able to define/explain the concept of logarithm by the use of repeated division. They could accurately mention that, $log_{10}a$ is dividing a by 10, thus the base repeatedly. They could recognize a logarithmic expression such as $log_{10}100$ or $log_{10}10000$ as a number. Students were able to change forms of a logarithmic expression to their equivalent using the power, sum/product, and difference/quotient.

4.2.3 Evidence of Students' Process Understanding

Students were able to think of $\log_a b$ as a process by translating into verbal equation:

How many times can you divide b by a until you reach 1? In this case, a is the base. They were able to simplify expressions such as $\log_{10}100$ by employing the process. Example of responses given by students during the interview is:

> Interviewer: Can you mention some of the things you learnt and you still remember them?
>
> 'Student A*: Yes, I do remember. Eemm(0.5) I do remember that when you have a whole number and you want to find <u>the</u> logarithm of the certain whole number, what you have to do is just to ehh(0.3) continuously divide that particular number by the base until you get one. (.) And the number of times that you do divide till you reach one is the answer. So for instance if you have log 100, which is of course log (0.4)err 100 base ten, err the answer becomes (.) <u>2</u> because you would have to divide (.) <u>100</u> by ten two times in order to get one. Yes! so I have learnt that particular ( ).*
>
> Student A: ↑*And then to take another example,(0.3) errr say, log (.) 100000. Log 100000 too, you have to divide 100000 by ten (.) six times in order to get one, and so the six times of division is the answer'.*

From student A's second example given to demonstrate his understanding of log 100000 as a process, although the answer given was inaccurate, there was an indication from his response that shows his conceptual understanding of logarithm as a process.

Student could also establish pattern that could be used for certain category of numbers in logarithms. With whole numbers like 1000, 1000000, 100000000000 students could accurately give the answer for the logarithm of numbers in such category by counting the number of zeroes or the ending zeros.

A student gave some examples with answers without working out. He was asked if there was a pattern used of which he said:

> Interviewer: So do you see any pattern, because I can see you are able to tell the answer without working it out?
>
> Student: ↑*Yes, th : the pattern that I saw was that, if you have a number, a whole number that ehhh starts with one and the other numbers are zero, what happens is that, the number of zeros* £ *determines the answer. So the examples that I was using, 100 has two zeros (.) so the answer is two.* £ *100000 has six zeros and the answer is six (.)* <u>*Yes*</u>.

The student could use their knowledge from process definition to estimate values of logarithmic expressions which are not exactly divisible by 1. Although, it was a bit challenging to them in determining the two integers in which the logarithm of such numbers lie.

In dealing with logarithm of decimal figures, a student did explain how such task or problem could be solved. He gave a real example in his response by saying that:

> *Student B: yes, so so tha:t (.5)ehh this is the other thing that I also learnt(.2) If you have a decimal(.2) too,if you have a decimal number you would have to multiply the decimal number by the base until you get 1 and for th:that becomes also the answer (.) but this time it becomes the negative so for instance (.)th log (.2)ehh.1/100 is the same as log 0.(.)ehh 01. So I would have to multiply 0.01 (.)by 10 two times in other to get 1. So that two times, I will just(.) I negate the ( ) according to what you thought me,* £ *I have to negate the 2.*
>
> *Eh and also, the other thing is t<u>hat</u>, if you have emm (0.5) a number like (.2)100, 1/100,(.2) the logarithm of one divided by hundred (log1/100) becomes, ehh (0.5)th :the answer is the <u>negativ</u>e form of log hundred (log100). For instance, log1/100 is (.) -2 (.2) -2.You know it is because of log100 is <u>2</u> so log 1/100, the pattern is that,* £ *you just change the sign and it becomes negative.*

4.3 Observation of students in class – Responses from Students

During the intervention period- where repeated division approach was introduced to the students, they were given the opportunity to work out examples on the white board. A student led the class in solving one of the tasks in the logarithm handout prepared by the researcher. John could expressed log 0.126 as log(1.26×10^{-1}) which is equal to log1.26 + log 10^{-1} . John could apply what he learned in the intervention lessons of finding the logarithm of decimal numbers here. His target was to find the log 0.1 since the value of log 1.26 had been given in the question. John could use RD approach to deduce that, for him to reach 1 unless he multiply 0.1 which is the same as 10^{-1} by 10. He could reason through that if the use of division gives a positive answer to the logarithm of a particular number, then by applying multiplication will give him a negative answer. In other words, John recognized that multiplication is the opposite of a division – thus moving in opposite direction to a division. Below is John's explanation of question number (a) of exercise 4.2.15 which says, *given log 1.26 = 0.10, he should express log 0.126 in terms of log 1.26. (see Appendix B)*

John: *So we want to express this 0.126 in a form 1.26 and you multiply it by a certain number that will give you this 0.126 again. So let say 0.126 =( ).Now if you divide 1.26 ok, you have shifted the decimal place one so that means you multiply it by 10. So to get it back again, what do you have to multiply it by, 10 exponent negative 1 ($1.26 \times 10^{-1} = 0.126$). So you should come to ( ) 10 exponent negative 1. So this means that you've not done anything to it just that you have just expanded it. So we already know the log for this one (log1.26= 0.10) so it's becomes log 1.26 + log 0.1 or to this is already turn into -1 (log 0.1= -1). So here we have 0.10 + (-1) (.5) so we are going to get -0.9.*

Since John was explaining to the class, Peter who could not understand how John changed the multiplication sign into addition sign asked:

Peter: *why have you changed the multiplication sign into addition?*

John: *The last time we established here that if you have log x + log y is the same as log (xy) [logx + log y = log (xy)], so in the same way we have log (xy), is the same as log x + log y.'*

However, students were able to use their previous knowledge of continuously dividing a number by its base until getting one to establish the fact that, it is impossible to find a logarithm of a negative number.

A student demonstrated his understanding in cover-up strategy in answering a question in class. He did that by explaining to the entire class how the given question would be manipulated to become simple equation for him to deal with. In his response to how to go about solving a question *log (4x -2) = 1*, he said:

Emm (.0.5) in this case, amm we have from what we have been solving, we know that amm, from what we have been solving we know that log 10 is going to give us 1 (log 10 =1), that is the cover ( ) you cover what is here and then find out what value when put here (.0.2) will be equal to 1. And the value is this, so we equate this (4x – 2) to 10, I think ( ) so you equate what is here to 10 and then (.05) emm simple linear equation you send this (-2) to the other side, making it positive ( ). And 4x is equal to 10 + 2 thus (4x = 10 + 2), 10 + 2 = 12, divide both sides by 4, and ( ).

Another important area discovered from the interview response is their interest developed in logarithm. Student emphasizes that the nature of the RD approach makes it very simple for them to apply it in mathematics and even beyond. In investigating what would actually influence the students to apply the Repeated Division approach, the students responded that:

> Student A: *Yes, I think I would,(0.2)* £ *I think I would. I would still want to (0.5) I think it will be interesting to really apply this (0.2)err method of you know continuous multiplication and continuous division over there. Yes, I think I would love to do that if that (0.2)* £ *would be allowed (0.2)* £ *I think I would do it.*
>
> Student C*: I woul:d I woul:d love love to do that because errm I think it:s it is a method that I don't* <u>need</u> *much of a calculator and (.2) it offers practical hands on experience. Yes, it is not something that I really have to memorize a lot of things.(.) I just have to go to the exams hall or to what (.2) whatever I'm doing I just have continuously be dividing or multiplying. So.. yeah, I think because of this I would love to really apply it anywhere.*

4.4 Analysis of Class Teacher's Response

The analysis of the Form 1 mathematics teacher's interview revealed certain information that argument the response of the students. The teacher indicated that it is very challenging to him introducing logarithm to first-year students who have not been introduced to any form of indices or logarithm from basic school. The reason being that, indices is a pre-requisite knowledge for logarithm in secondary school. He reiterated that students use the conventional approach without any knowledge of explaining what went in the process to arrive at a particular result. In finding out how easy the two approaches are, he responded that; from the experienced gain in class with regards to the RD approach, students grasp the concept easily due to the fact that, the approach makes use of the students previous concepts in algebra-division and multiplication. Again, the nature of the RD approach will help students:

1. To better understand the concept
2. To develop interest in logarithm and appreciate the concept
3. To initiate self-reliance – they will do things on their own without relying on the solved question.

The teachers' responses revealed and confirmed some of the findings of the students. Moreover, the class teacher was willing to apply RD in teaching logarithm because the approach can easily be made student centered/ learner centered (thus, the focus of the instruction being shifted from the teacher to the students). When the teacher was asked the way he could explain the repeated division approach to his colleague mathematics teacher, he used log 10= 1, log 100= 2, etc in sequential form in explaining how the RD is applied. His reason for using examples is that it is easy for people to visualize and see the pattern.

4.5 Students' Retention

The number of students who participated in the retention exercise reduced from 103 to 81 due to absenteeism—the student had a prior engagement in other school activities. The results indicate that about 66.7% of both Forms were able to conceptualize and establish that $log\ a^x$ is equal to $x log\ a$ correctly. There was an increase from 61.2% in the post-test to 65.2% in the retention test (representing a margin of 4% increase) among the Form 1 on the problem $log\ a + log\ b = log\ (ab)$. The table below shows the summary of the students' retention.

Table 4.3:Retention Test Analysis table

Aspects	**Number of correct response. (1= correct)**	**Percentage of correct response %. (Retention test)**	**Percentage of correct response %. (Post-test)**
log1 =0	35	43.2	74.8
log a+log b= log (ab)	53	65.2	61.2
log 10 = 1	72	88.9	93.2
$log\ a^x = xlog\ a$	54	66.7	67.0
log 0.0001 = -4	63	77.8	96.1
Cover-up	56	69.1	

CHAPTER 5

DISCUSSION, CONCLUSION AND RECOMMENDAATION

5.1 Introduction

This chapter comprises of the discussion of the results and a summary of conclusions for the study. It also includes some recommendations based on the findings of the study.

5.2 Discussions and Conclusion

The purpose of the study was to investigate the question: does the RD approach of learning logarithms improve the performance of students. To answer the question, two Forms of secondary school students were selected into two groups: experimental group and control group. The experimental group (Form 1 students), while that of the control group (Form 2 students) were from the same school. The experimental group had not been taught logarithm since logarithm as a topic is taught only in the second year (Form 2). Hence, the experimental group was exposed to RD approach to learning logarithm. However, the control group had already been taught logarithm through the conventional approach and was used for the purpose comparing performance.

First, the results from the study suggest that learning logarithm with the CA improves performance compared to the RD approach of learning logarithms. The results failed to confirm the hypothesis: thus, the percentages of Form 2 students who scored higher marks were significantly more than the Form 1 students. Moreover, students who used RD approach in learning logarithm found the approach very simple and intuitive and very promising in helping alleviate anxiety associated with learning logarithm. There were exceptional situations where the Form 1 students outperformed the Form 2 students (for example in establishing log 1 = 0, the Form 1 students outperformed the Form 2 students by a significant margin of 26.1%). The finding led to the conclusion that, the RD approach makes it very simple for students to grasp the concept logarithms: i.e. of log 1= 0, by asking themselves, how many times they will divide 1 by 10 (which is the base) until they reach 1. They realized they were in 1 already, so they will need no division – hence their ability to conceptualize it.

The second hypothesis did measure the conceptual understanding developed by students through the use of the RD and CA in logarithm:

Does the Repeated Division approach of learning logarithm improve the conceptual understanding of students in logarithm?

Second, the results from the post-test and retention test could not give clear confirmation of the hypothesis. However, the results from the interviews, classroom discussions and observations confirm the hypothesis that: RD approach enhances students' conceptual understanding in logarithm. The Form 1 students developed object understanding in logarithm to explain and change forms of logarithmic expressions into their equivalents. Again, the development of process understanding of Form 1 students equipped them with the knowledge and ability to:

(a) Translate verbal equations into mathematical symbols.

(b) Develop a very easy pattern of finding logarithms of a certain category of numbers, for example 100, 100000, 10^{-6}, 10^{-10000}, etc.

(c) Apply the RD (RD explanation of a logarithm) in the approximating logarithm of numbers which are not divisible by the base.

(d) Use the RD to find the logarithm of decimal fractions.

(e) Develop internal representation of a logarithm and connect it to their RPK of division concept (Internalization of logarithm concept).

Third, the results of our study confirm the hypothesis that, RD approach improves students' interest in learning logarithm. The outcome of the analysis suggests that RD approach significantly aroused the interest of the students to the level that, they would want to apply the approach in other fields where logarithm is used. This may probably due to the nature of the approach with regards to its easiness/simplicity and practicality. The RD approach was found to promote activity-based teaching which is basically student centered. Students' participation in the activity tends to improve their social interaction which plays a major role in the learning process.

Lastly, the retention test and its organization met some challenges which the researcher presume might have affected the outcome. The low percentages recorded in the retention test may be attributed to the school's standardize exams planned coupled with other equally important school activities. The standardize exams formed part of the students continues assessment in the term. Students were preparing towards it so it was clear that, they did not put in an effort towards the preparation of retention test. The researcher's observation confirms the result of which the percentage scored in the post – test on log 1= 0 dwindled in the retention test (see table 4.3).

It was also deduced from the class teacher's responses that, it's not only the students who were willing to apply the approach. The teacher was willing as well, but their concern had to do with the external examination. Since the approach has not been prescribed in the mathematics teaching syllabus.

Another revealing aspect of the results is the students' demonstration of mathematical proficiency. A closer look at the students' response from the class observation reveals the conceptual understanding demonstrated by the students in applying what they had learned in a new situation. They were able to reason logically, applied the skills in carrying out procedure accurately and explained to justify the outcome.

The results support the assertions made by Carraher, (2007); Usiskin, (1995) and Espedal, (2015) that, logarithm is part of algebra. The results indicate that more than 60% of the Form 1 students could use process to establish relations and generalize all the aspects of logarithm correctly.

During the research, I played the role of a facilitator and guided the students where they had some difficulty. The initiative helped the students developed a self-reliance attitude towards learning. This experience suggests that, through supervision of class activities, a greater percentage of the students will show interest in learning logarithm.

5.3 Summary of Conclusion

The results of this study indicate that the Form 2 students who used CA outperformed Form 1 students who used RD approach. However, the Form 1 students demonstrated conceptual understanding and interest in logarithm through the use of RD. The meaningful meaning of logarithm through RD, its object and process orientation helped alleviate students fear associated with logarithm. It is hoped that with great awareness of RD approach through in-service training, teachers can plan teaching lessons and learning experiences that will improve conceptual understanding, improve the performance of students and sustain their interest in logarithm.

5.4 Recommendation

The following recommendations are put forward:

1. The study did not cover all the bases of logarithms. It's dealt with only base ten (10). So it can be extended to other bases, for example: base 4, 5, etc.

2. Since the use of calculator contaminated the data collected, the study can be conducted again without the use of a calculator. And also to extend the study period to may be two months.
3. Since students are educated to conceptualize what they are taught, it will be helpful if teachers and educators discover and apply different approaches of teaching logarithm that will make it easy for the students to understand.SF

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APPENDIX A – Test Items

Pre-test (Solving logarithmic equations)

Answer the following questions below:

1. Simplify to the lowest form

   $$\frac{log4 + log9}{2 + log1}$$

2. Find x

   log (2x +3) =1

3. Find x

   $2^{(x+1)} - 3 = 5$

4. Find x. (NB: leave your answer in a standard form).
   log(2x) = -4

Post-test (Solving logarithmic equations)

Answer the following questions below:

1. Simplify to the lowest form

   $$\frac{log4 + log9}{2 + log1}$$

2. Find x

   log (2x +3) =1

3. Find x

   $2^{(x+1)} - 3 = 5$

4. Find x. (NB: leave your answer in a standard form).
   log(2x) = -4

Retention test

***Note*: No calculator to be used!**

Answer the following questions below:

1. Simplify to the lowest form

   $$\frac{log8 + log27}{log1 + 3}$$

2. Find x

   log (x +5) =1

3. Find x

   $$5^{(x+1)} - 10 = 15$$

4. Find x. (NB: leave your answer in a standard form).

   log ($\frac{x}{2}$) = - 4

APPENDIX B – Transcription of Audio recording

Interview with the class teacher - Transcription

Class: Form 1

Gender: Male

Age: 35year

Interviewer: Can you say something about the easiness and the difficulties of teaching logarithms?

Class Teacher: Ehmm with regards to what I have been using (0.5) ehmm it has been quite difficult teaching it using th:e old theory (0.2) where you just have to follow the strict rules that are there, the laws to teach the students.Now in most cases, most at times the students wants to commit the laws into memory and use them in solving the questions (0.4) in those cases its quite difficult.

Interviewer: *What approach do you normally use in teaching the topic?*

Class Teacher: Ehmm (0.2) most at times w:e have always said that, ehh log and indices ( ) so when we teach indices, I think indices has those powers that are used and so moving from indices straight into logarithms, ehmm there is a lean way through that and that helps us to quickly do ( ).

Interviewer: Does it mean that you use the rules that you are talking about it?

Class Teacher: Yes, in most cases ehmm after deriving the rules from the indices, i:t is just like (.) an inverse reflection sort of, so we go ahead the same way to develop the(.) the log rules and we use them.

Interviewer: *Do you find your approach very challenging in your teaching?*

Class Teacher: Infact, very challenging where you have ehmm students who have come From the:e the basic schools who didn't appreciate any little form of indices or logarithms at the basic school, it makes it a little bit problematic trying to give them ( ) ehh going through the laws of log.

Interviewer: Can you mention some of the difficulties you face in using your approach in teaching logarithm? What are the advantages?

Class Teacher: Ehmm (0.2) the advantages rare are that £ you just put down the laws and explain to them that this is what the laws are meant for and so you can go back ( ) and they are supposed to take that. That one immediately you do that and you put question there is easier to get ( ) ehh if you get good students who. Follow there its interesting because they quickly finish. That is the only thing, but the other side is when most of them do not understand. You will be solving the questions, they seems to be following but at the end when you give them exercise you see that they haven't appreciated what exactly it is. They know how to solve the question but they don't know how (0.2) it came about to do those things.

Interviewer: So what are some of the advantages?

Class Teacher: For the old one in

Interviewer: It's your method that I'm talking about.

Class Teacher: Yes, in in, using my method, ehmm if the students are good, it's easier to finish. If they are able to commit those laws into memory, its easier to finish the topic ehh as quickly as possible. But if you get as I said ehh weak students, you need to work more examples to gear them to appreciate it.

Interviewer: *What do you see different from the repeated division approach as compare to your approach. Do you think it's an appropriate approach for teaching and learning logarithms?*

Class teacher: Ehmm I think that the the repeated division method ehh using their basic arithmetic rules of division and multiplication. In that case they identify with it easily. And so when they identify these divisions which they have done from their basic school, when you teach them ( ) when you are using division, they appreciate and when using multiplication it is easier to appreciate as well. You don't need to talk too much to get them to understand ( ).

Interviewer: So do you think it's an appropriate approach for teaching and learning logarithms?

Class Teacher: Yes, from what I have seen, its very appropriate ( ) it will help the students to understand and appreciate and then to do things on their own rather than always relaying on questions that have always been solved and they follow.

Interviewer: *Understand in what sense, they will be able to explain how they got the answer or just getting the final answer correct?*

Class Teacher: No. This time round they are doing division, they are doing multiplication. They know what multiplication is, they know what division is, so going through it is not difficult for them. Because they know that I have to divide and divide until, for example the one you have get the 1. You have to divide until you get one. Division is ( ) no problem for the students and they can divide and get the 1, even they are able to explain to others in going through that.

Interviewer: Would you like to apply repeated division approach in your teaching hence forth?

*Class Teacher:*Of course, when I went through it I marveled so I think it will be an interesting way of teaching logarithm.

Interviewer: In your own estimation, is the approach easy or difficult to use in teaching and learning logarithms?

Class Teacher: Emm, as compared to what we were using, I think that this emm is pretty much easier and you get the students involved.

Interviewer: How would you explain the approach to your colleague mathematics teacher?

Class Teacher: Emmm (0.5) basically emm the approach is explained on our normal multiplication and division, so for me emm (0.3) emm (.) ↑I will just have to tell him that base, for example if I take emm # log 100, I will be able to, infact, I will even start from log 10 so that I can serialize for the teacher to know that if I move from log of 10 and I'm able to establish that it is emm 1 and I'm also able to establish log of 100 is equal to 2, I am able to get a sequence and that make the teacher understands th:e the sequence that is growing and have to follow it.

Interviewer: *Does it mean that your examples and explanations will follow certain pattern that will help the teach easily identify the answer immediately the question is put down?*

Class Teacher: Exactly, exactly exactly when yeah, in most cases when you having sequences or patterns, it is easier for people to follow.

A Student's view on Repeated Division

Class: Form 1 Gender: Boy Age: 16years

Interviewer: *Did you benefit from the repeated division instructional teaching?*

Student: Yes I:d did benefit, (.) It's kind of a very new experience and ehh I think I have learnt something from it.

Interviewer: *Can you mention some of the things you learnt and you still remember them?*

Student A*:* Yes, I do remember. Eemm(0.5) I do remember that when you have a whole number and you want to find <u>the</u> logarithm of the certain whole number, what you have to do is just to ehh(0.3) continuously divide that particular number by the base until you get one. (.) And the number of times that you do divide till you reach one is the answer. So for instance if you have log 100, which is of course log (0.4)err 100 base ten, err the answer becomes (.) <u>2</u> because you would have to divide (.) <u>100</u> by ten two times in order to get one. Yes! so I have learnt that particular ( ).

Student A: ↑And then to take another example,(0.3) errr say, log (.) 100000. Log 100000 too, you have to divide 100000 by ten (.) six times in order to get one, and so the six times of division is the answer'.

Interviewer: *So do you see certain pattern because I can see that you are able to just* *mention the answer without working it out?*

Student: Yes, th : the pattern that I saw was that, if you have a number, a whole number that ehhh starts with one and the other numbers are zero, what happens is that, the number of zeros £ determines the answer. So the examples that I was using, 100 has two zeros (.) so the answer is two. £ 100000 has six zeros and the answer is six (.) Yes.

Student: Eh and also, the other thing is that, if you have emm (0.5) a number like (.2)100, 1/100,(.2) the logarithm of one divided by hundred (log1/100) becomes, (.5)ehh (.)th :the answer is the negative form of log hundred (log100). For instance, log1/100 is (.) -2 (.2) -2.You know it is because of log100 is 2 so log 1/100, the pattern is that, £ you just change the sign and it becomes negative.

Interviewer: *I know when you divide 1 by 100 which in decimal, it is 0.01, when you divide it by the base ten, you will never get to one. So how do you get negative one?*

Student B: yes, so so tha:t (.5)ehh this is the other thing that I also learnt(.2) If you have a decimal (.2) too, if you have a decimal number you would have to multiply the decimal number by the base until you get 1 and for th:that becomes also the answer (.) but this time it becomes the negative so for instance (.)th log (.2)ehh.1/100 is the same as log 0.(.)ehh 01. So I would have to multiply 0.01 (.)by 10 two times in other to get 1. So that two times, I will just(.) I negate the ( ) according to what you thought me, £ I have to negate the 2.

Interviewer: why do you have to negate the two?

Student: It's hehe difficult err the, the opposite of multiplication is division.

Interviewer: The opposite of multiplication is division. So which…. that it means that multiplication is positive and division is negative or what?

Student: Yes emmm (.) that is what I see, such that anytime if I have to emrm if I have to multiply till I get one, the answer is negative and if I have to divide a number till I get one, then the answer is positive.

Interviewer: Can you mention any more benefit or anything you learnt and you still remember?

Student: Emm (.9) yes Ok it is about also another trend and ehh the decimals.

Interviewer: What is it about?

Student: And that one, sometimes you don't really get one per sey, but you get a number that is close to one or yea, so that one as you taught me, you get a range. The answer may fall between maybe one and two yes!

Interviewer: Is it only the decimals or other numbers as well?

Student: Well a:as at now, yes that's what I see with it ( ) the decimals.

Interviewer: What do you think are some of the benefits of using repeated division?

Student: W:ll it it's about with multiplication you multiply £ till you get 1 or £ you divide (h) till you get 1 and this is what we have been studying since primary school. At least when it comes to multiplication and division, yes, we have always been doing it in Maths. So if we can solve logarithm by multiplication and division, then I think it's something that is so good and yeah it's offers a great benefit.

Interviewer: *So with this method, do you see logarithm as difficult or otherwise?*

Student: No, uh except except there are other difficult (h) things ahead that I have not yet been expose to but for now, with this multiplication and division thing, I think logarithm is not that (.) very difficult.

Interviewer: Do you want to apply repeated division in solving logarithm questions in other subject area, for example in physics?

Student: Yes, I think I would, I think I would. I would still want to….I think it will be interesting to really apply this ehh method of you know continuous multiplication and continuous division over there. Yes, I think I would love to do that if that £ would be very allowed £, I think I would do it.

Interviewer: why would you do that (apply the method in other subject area)?

Student: I would I would… I would love, love to do that because emmm I think it is a method that I don't need much of a calculator and (.) it offers practical hands on experience. Yes, it is not something that I really have to memorize a lot of things. I just have to go to the exams hall or to what…. whatever I'm doing I just have continuously be dividing or multiplying. So.. yeah, I think because of this I would love to really apply it anywhere.

Transcription of Students explanation

'John: *So we want to express this 0.126 in a form 1.26 and you multiply it by a certain* *number that will give you this 0.126 again. So let say 0.126 =( ).Now if you divide 1.26 ok, you have shifted the decimal place one so that means you multiply it by 10. So to get it back again, what do you have to multiply it by, 10 exponent negative 1 (1.26 x 10^{-1} = 0.126). So you should come to ( ) 10 exponent negative 1. So this means that you've not done anything to it just that you have just expanded it. So we already know the log for this one (log1.26= 0.10) so it's becomes log 1.26 + log 0.1 or to this is already turn into -1 (log 0.1= -1). So here we have 0.10 + (-1) (.5) so we are going to get -0.9.*

Peter: why have you changed the multiplication sign into addition?

John: Last time we established that log x + log y = log xy, so in the same way if we have log xy, we can write it as log x + log y.

Transcription of Students Interview last

Interviewer: What is repeated division?

Student A: Repeated division is a method for solving logarithm questions where you follow steps such that if you've given a question like, find the log of 1000, you are suppose to divide it by 10, where the number of times the steps involves that will be your answer. So for example, log 1000 divided by 10 three times and you find out that you took 3 steps so the answer will be 3.

Interviewer: Do you divide the number by 10 until you reach certain point or you divide it indefinitely?

Student B: You divide it until you get to 1, the number 1.

Interviewer: can you say something about the easiness and the difficulty of using repeated division?

Student A: Oh.. it wasn't difficult, it wasn't difficult eem when we were dividing, it wasn't difficult dividing repeatedly by 10.

Interviewer: What is the easiness in using repeated division in general?

Student A: Eh I believe its quite easy because its just a matter of dividing by 10 untill you get to one and £ every one can divide a number by 10 so its easy.

Interviewer: What about the difficulty in using repeated division? Did you encounter any difficulty in using repeated division?

Student A: The difficulty involve is sometimes when the number is not completely divisible by 10, where you have to use the range.

Student B: where you have to use the range.

Interviewer: Is it always the case that we divide a number by 10 or we can also divide it by different number?

Student A; you can only divide by 10.

Interviewer: comment- ok because I taught you only base 10, but you can equally divide a number by its base.

Interviewer: do you find the approach very challenging?

Student C: no sir, its quite easy so we don't find it challenging.

Interviewer: …. Challenging in a sense that, if the RD gives you new insight or approach of solving logarithm?

Student C: It was the first time we met the approach when you introduced it in a class.

Interviewer: what are some of the advantages of using RD?

Student A: Emm I believe it's easy to remember, you don't have to follow too many steps, it's just a matter of dividing by 10 if you are dealing with a number to the base of ten or ( ) if it's in any other base, you divide it by that same number and also you don't need to use the calculator before you can solve the question.

Interviewer: do you think RD is an appropriate approach for teaching logarithm?

Student C: Since the repeated division can also be use to obtain the same answers as the other approaches, I think emmm, its ok, it's can be used emm to treat logarithm. And me, I find it difficult using the other approaches but emm repeated division is quite easy for me.

Interviewer: would you like to apply RD in solving logarithm related problems in other fields?

Student A: yes ehh as long as its divisible by 10 or the base number, we will surely use it.

Interviewer: Will you encourage other students who have not used RD before to use the approach in solving logarithm?

Student B: Ehh once they get the understanding on emm how to apply the repeated division, I think they will also find it very easy so they will (0.3) they will easily use it, yes eem I will encourage our friends to use it.

Interviewer: Are you too confident to explain RD to a friend who doesn't know anything *about it?*

Student B: Yes, yes yes please

Interviewer: ….and how will explain it?

Student A: as I said early, I use an example like log 1000 to the base of ten. So its simple, you just have to take the number 1000 and then divide it by 10, that is one step, you will get 100. You divide it by 10 again, you will get 10, that is 2nd step, when you divide by 10 again, you are going to get 1 and that is the 3rd step. So for that one, the answer to log 1000 will be 3. So its that simple to explain to any one.

Interviewer: it means you can easily remember how to apply it any time you want to use it?

Student B: yes, yes

Student C: yes, we will not forget it. It's something that can easily £ be remembered. Its something we can't easily forget. Its something we can easily remember.

Interviewer: were you able to use the approach effectively after you have been taught in the test?

Student B: yes I was able to use it.

Student C: yes, we could easily apply it but eem when eem we were working on it in class,

because we are also taught the different method, I don't know if we can start using this method but eeem that's the only thing. Like, if we are allowed to use it then we can easily use it (.) to solve any question. If we are giving the go ahead to use it in class then ( ).

Interviewer: Does it mean that when you compare repeated division to other method, you prefer repeated division to other methods?

Student B: yes, sir yes, we believe when our friends colleagues also hear of the RD and understand the concept eem comparing it to the other method of solving logarithm, I believe emm they will also chose to use the repeated division..

Interviewer: What do you see so special about the repeated division?

Student B: It's so easy emm in other approaches, there are so many laws and properties you have to memories to be able to solve some questions but over here you just adding only two rules ( ) then you have your answer. Its very easier.

Student A: Comparing the RD to other method, the RD as I said is earlier, it's a method that is easy to apply so I will suggest that,.. it's a method that should be introduced to everyone. We should all be able to use it in …….. , emmm we should all be able to use it in solving questions if giving the go ahead, we can use it in solving questions and it will be easier for us so I do recommend that.

APPENDIX C – Intervention Material Prepared

Logarithms

Senior High School

Core Mathematics

Isaac Ansah

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Contents

Objectives:

In the teaching syllabus for core mathematics in the senior high school, the objectives for logarithms are as follows:

By the end of the logarithm lesson; students will be able to:

- Write in exponent form the repeated factors of a number.
- Solve equations involving indices.
- Relate indices to logarithms in base ten.
- Deduce the rules of logarithms and apply them.
- Find the anti-logarithms of a given number.

An aspect of the general aims state that:

"To meet the demands expressed in the rationale, the SHS Core Mathematics syllabus is designed to help the student to:

- Develop the ability and willingness to perform investigations using various mathematical ideas and operations.
- Use the calculator and the computer for problem solving and investigations of real life situations.

4.2. Logarithms and Repeated Division

Objectives:

By the end of this section, students will be able to:

- Explain the logarithm of a number.
- Find the value of a logarithm of a simple number without the use of a calculator.
- Give an estimation of a logarithm when given a number.
- Deduce simple rules and explain why these work.

Using REPEATED DIVISION
(Base 10)

Example 1

How many times can 100 be divided by 10 until you reach 1?

Solution:

$$\begin{array}{l} 100 \\ \left.\begin{array}{r} \div 10 \\ 10 \\ \div 10 \end{array}\right\} \text{2 times} \\ 1 \end{array}$$

log100=

Number of times 100 can be divided by 10 until you reach 1?

100 can be divided by 10 two (2) times until you reach 1.

Therefore, we write:

$$\log_{10} 100 = 2$$

Example 2

How many times can 10,000 be divided by 10 until you reach 1?

Solution:

10000

÷ 10

1000

÷ 10

100

÷ 10

10

÷ 10

1

10,000 can be divided by 10 four (4) times until you reach 1.

Therefore, we write:

$$\log_{10} 10000 = 4$$

Example 3

How many times can 1,000,000 be divided by 10 until you reach 1?

Solution:

1000000

÷ 10

100000

÷ 10

10000

÷ 10

1000

÷ 10

100

÷ 10

10

÷ 10

1

1,000,000 can be divided by 10 six (6) times until you reach 1.

Therefore, we write:

$$\log_{10} 1000000 = 6$$

NOTE:

When we write $\log_{10} 10000$, the number 10 is the base. When 10 is the base we can write $\log_{10} 10000$, but we may leave out the base. So $\log_{10} 10000$ = log 10000.

Therefore, $\log_{10} 1000000$ = log 1000000 = 6

Exercise 4.2.1 **Without calculator**

By the use of repeated division, find:

a) log 100 000

b) log 1000

c) log 10

d) log 1 (*Hint: how many times you will divide 1 by 10 until you reach 1?*)

Exercise 4.2.2 **Without calculator**

Find the value of x in the following equations:

a) log x = 5
b) log x = 3
c) log x = 1
d) log x = 0

Exercise 4.2.3 **Without calculator**

Find the value of x in the following equations:

a) log (4x-2) = 1
b) log 10 = x+1
c) log 100 = $\frac{x}{2}$
d) log $\frac{x}{2}$ = 1

HINT TO QUESTIONS IN 4.2.3

Use the **cover-up method** in dealing with such questions.

For example:

Find the value of x in the following equation: log 2x = 2

Solution:

log $\boxed{}$ =2

So, you cover up 2x. Then you find what number should be under the cover.

log 100 = 2

So the 100 helps in equation log 2x = 2

so 2x=100.

This becomes a simple equation which you can solve for x.

Definition:

The representation $\log_a b$ can be interpreted or translated into the equation:

How many times can you divide *b* by *a* until you reach 1? In this case, *a* is the base.

We have already established that:

- $\log_{10} 10 = \log 10 = 1$ log $= 1$
- $\log_{10} 1 = \log 1 = 0$ log $1 = 0$

Summary: $\boldsymbol{\log_a a = 1}$
$\boldsymbol{\log_a 1 = 0}$

Exercise 4.2.4 **Without calculator**

Formulate a logarithmic equation of the following:

Example 4: How many times can you divide 10^2 by 10 until you reach 1?

Solution: $\log 10^2 = x$

a) How many times can 10^4 be you divide by 10 until you reach 1?

b) How many times can 10^{100} be divided by 10 until you reach 1?

c) How many times can 10^n be divided by 10 until you reach 1?

Exercise 4.2.5 **Without calculator**

Find the values of the following:

a) $\log 10^6$
b) $\log 10^{36}$
c) $\log 10^1$
d) $\log 10^0$
e) Study the pattern of answers to questions a) - d) of exercise 4.2.5 to establish a rule to find $\log 10^n$.

Using REPEATED DIVISION
(Giving answers within a range)

Example 5

Estimate the value of $\log 6000$.

Solution:

6000
: 10
600
: 10
60
: 10
6
: 10
< 1

Hints:

1. In between what integers lies the answer?
2. Which one of the two integers is the closest?
3. Check your answer with your calculator.

1. When you continue to divide the successive numbers by 10, you will never reach 1 exactly. You can see that 3 steps will give you 6 (which is more than 1) and 4 steps will give you 0.6 (which is less 1).
   Therefore, $\log_{10} 6000$ is a number between 3 and 4.
   $$\mathbf{3 \ < \log 600 < 4}$$
2. You can see that log 6000 must be closer to 4, because 0.6 is closer 1 than 6.
3. With a calculator you can check: log 6000 = 3.78

Exercise 4.2.6 **Without calculator**

In between which integers lie the following logarithms:

a)
   i) $\log 60$
   ii) $\log 353$
   iii) $\log 6214$
   iv) $\log 7$

b) Which of the integers is the closest?

c) Use a calculator to find the values of question (a). What does it yield?

Exercise 4.2.7 **Use a calculator**

a) Find the values of the following logarithms:
   (i) log 8
   (ii) log 80
   (iii) log 8000
   (iv) log 80 000
b) What is the relationship between the numbers in (a).
c) Give reason(s) for your answer given in question (b) by using repeated division and counting the number of times until you reach 1.

Exercise 4.2.8 **Without calculator**

By the use of repeated division, find the value of the following logarithms:

a) $\log(-20)$
b) $\log(-100)$
c) What do you notice in questions (a) – (b) and why?
d) Use a calculator to find the values of questions (a) – (b). What does it yield and what is your conclusion?

Exercise 4.2.9 **Without calculator**

By the use of repeated division, find the value of the following logarithm:

a) $\log 0$
b) What do you notice in question (a) and why?
c) Use a calculator to find the value of question (a). What does it yield and what is your conclusion?

Exercise 4.2.10

Find log 0.01.

Solution:

Opposite movement

1

X 10

0.1

X 10

0.01

-2

How many times was 0.01 multiplied by 10 until reaching 1?

Therefore, we write log 0.01 = -2

Example 7

How many times can 0.0001 be multiplied by 10 until reaching 1?

Solution

1

X 10

0.1

X 10

0.01

X 10

0.001

X 10

0.0001

log0.0001=

Number of times 0.0001 can be multiplied by 10 until reaching 1?

0.0001 can be multiplied by 10 four (4) times until reaching 1.

Therefore, log 0.0001 = -4 (*Negative sign is used because we went upwards).*

Exercise 4.2.11 **Without calculator**

Find:

a) $\log 0.00001$

b) $\log 0.1$

c) $\log 0.001$

d) Use your calculator to find the values in questions (a)-(c). What does it yield?

e) Considering questions (a)-(c), count the number of zeros after the decimal point of each question and compare with their respective answers.

f) What is the relationship between the number of zeros and the answer for each question?

g) Study the trend of answers to questions (a) – (c) to establish a rule to find $\log 10^{-n}$. (NB: see page 8 for positive n).

Exercise 4.2.10 **Without calculator**

In between which two integers lie the following logarithms?

a) $\log 0.08$

b) $\log 0.0005$

c) $\log 0.035$

d) Which of the integers is the closest?

e) Use your calculator to find the values in questions (a)-(c). What does it yield?

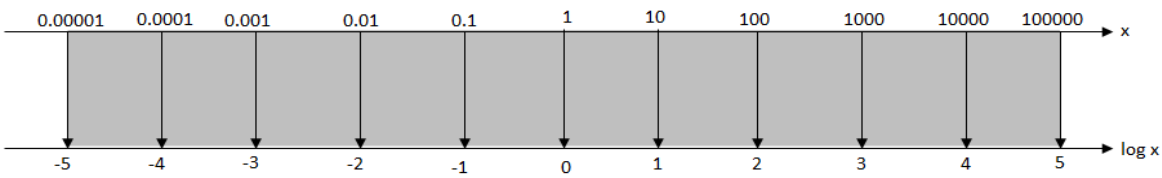
Note:

(a) $0.01=10^{-2}$ (b) $0.00001=10^{-5}$

Therefore, $\log 0.01=\log 10^{-2} = -2$ and $\log 0.00001=\log 10^{-5} = -5$

Exercise 4.2.11

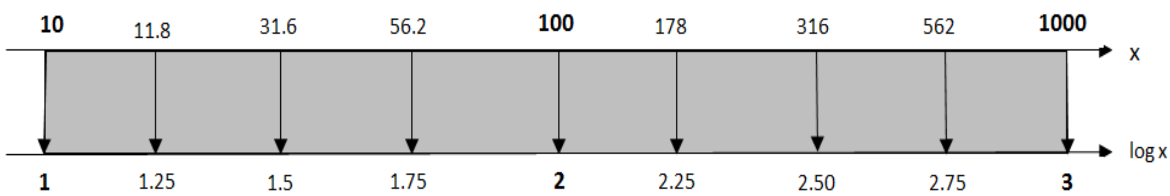
The figure below shows the values of x on a logarithmic scale.



a) Use the logarithmic scale to find the values of log 0.001 and log 0.1.
b) Find log 0.001 + log 0.1, using the values from the logarithmic scale and compare with log 0.0001.
c) Use the logarithmic scale to find the values for log10 and log1000.
d) Find log10 + log1000, using the values from the logarithmic scale and compare with log10000.
e) Find log 10 + log0.1, using the values from the logarithmic scale and compare with log 1.
f) Study the pattern of questions (a)-(e), and use the logarithmic scale to establish a rule.

Exercise 4.2.12

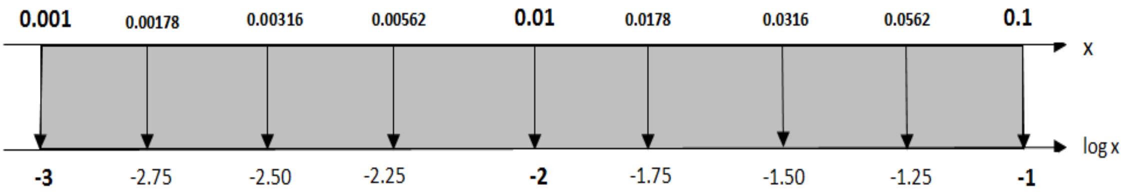
Here you enlarge a small part of the logarithmic scale in exercise 4.2.11:



a) Use the logarithmic scale above to find the values of log 17.8, log 31.6 and log 56.2.
b) Find log17.8 + log 56.2, using the logarithmic scale and compare with log 1000.
c) Find log 17.8 + log 31.6, using the logarithmic scale and compare with log 562.

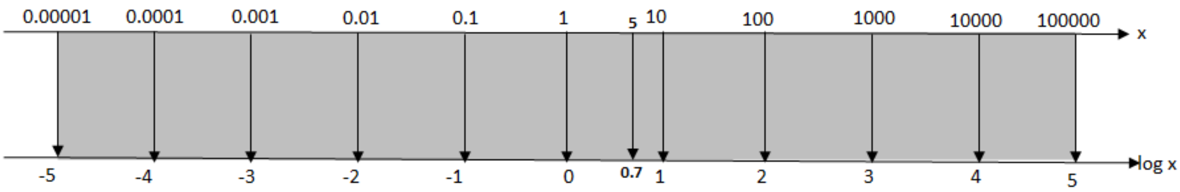
Exercise 4.2.13

Here you enlarge a small part of the logarithmic scale in exercise 4.2.11:



a) Use the logarithmic scale to find the values of log 0.0562, log0.0316 and log 0.1.
b) Find log 0.0562 + log 0.0316, using the logarithmic scale and compare with log 0.00178.
c) Find log 0.1 + log 0.0316, using the logarithmic scale and compare with log 0.00316.

Exercise 4.2.14



Use the logarithmic scale to find the values of:

a) log 0.05
b) log 0.5
c) log 500

(*Hint: the figure can be converted to standard form*)

Exercise 4.2.15 **Without calculator**

Given that log 1.26 = 0.10, write down the logarithms of the following numbers:

a) 0.126
b) 0.00126
c) 0.000126

Exercise 4.2.16

Given that log 6.3 = 0.8, write down the numbers whose logarithms are:

a) 2 + 0.8
b) 3 + 0.8
c) 3.2

Exercise 4.2.17

Given that log 6.3 = 0.8, write down the numbers whose logarithms are:

a) -2 + 0.8
b) -3 + 0.8
c) -3.2

Exercise 4.2.18

By the use of a calculator, find:

a)

i) log 40 and log $(\frac{1}{40})$
ii) log 1000 and log $(\frac{1}{1000})$
iii) log 5 and log $(\frac{1}{5})$
iv) log 0.01 and log $(\frac{1}{0.01})$
v) log 0.1 and log $(\frac{1}{0.1})$

b) Find three different pairs by following the pattern.

c) What is the relationship between the pairs?

d) By studying the pattern, establish a rule for log x and log $(\frac{1}{x})$.

Exercise 4.2.19

a) Copy and complete the following table.

x	10 000	1000	100	10	1	0.1	0.01	0.001
log x				1	0	-1		

b. From the table, find the range of values of log x for the following ranges of x.

i) 1000 < x < 10 000

ii) 100 < x < 1000

iii) 10 < x < 100

iv) 1 < x < 10

v) 0 < x < 1

4.3 Use of logarithmic Rules

From the previous section, we considered the number of times a number *a* can be divided by 10 until reaching 1. We were able to write its symbolic representation as logarithm *a*, thus (log *a*). We also estimated the values of certain logarithms which lie between two integers and used calculators to confirm the correctness of our answers. In this section, we shall use the concept learnt 'repeated division' in solving problems that we deem it fit.

Example 8

Approach 1

log (4 x 10)

Solution:

4 x 10
÷ 10 — 1 step
4
÷ 10 — log 4
0.4

1step + log4

Therefore, log (4 x 10) = 1 + log 4 = 1 + 0.60 = 1.60

log (a x 100)
=
Number of times 10 divides 100 until reaching 1 plus log

Approach 2

Given that log 4= 0.60

Then log 40 = 1.60 (see 4.2.7 and 4.2.16)
so
log 40 =1 + log 4
log 40 = log 10 + log 4
log (10 x 4) = log 10 +log 4

Example 9

Given that log 2= 0.30 and log 5 = 0.70

log 2 + log 5 = 0.30 + 0.70
= 1 = log (2 x 5)
Therefore, log(2 x 5) = 1 = log 2 + log 5

(See logarithmic scale in exercise 4.2.11)

Exercise 4.3.1 **Without calculator**

Given that log 531 = 2.73, find the following and write it as an addition of logarithms:

a) log(531 x 100)
b) log(531 x 10^3)
c) log(531 x 0.0001)
d) log(521 x 10^{-3})

Exercise 4.3.2

Given that log 7.055 = 0.85, write down the logarithms of the following numbers using an addition of logarithms:

a)

i) 70.55
ii) 0.7055
iii) 0.000 705 5

b)

i) 70 550

ii) 7 055 00

Exercise 4.3.3

a) Copy and complete the table below.

m	n	log m	log n	log(m x n)
10^{-3}	10^{2}			
0.01	0.001			
1000	100			
24	351			
35	1			

b) Explain why log (m x n) is not equal to log m x log n.
c) What is the relationship between log m, log n and log (m x n)?

Exercise 4.3.4 **Without calculator**

With the help of the result from exercise 4.3.3 (c), show that the following identities are correct using log *a* + log *b* = log (*a* x *b*):

a) log50 + log 20 = 3
b) log0.01 + log0.1 = -3
c) log5x + log2 = 1 + log x
d) log5x – log50 = -1 + log x
e) log9x – log 9 = log x

Exercise 4.3.5 **Without calculator**

Given that log 7= 0.85

Let x be an arbitrary number greater than 0.

a) How much greater is log (7x) than log x?
b) Find log (7x) – log x

Example 10

Given that log 243= 2.39

$\log(243^4) = \log (243 \times 243 \times 243 \times 243)$

$= \log 243 + \log 243 + \log 243 + \log 243$

$= 5 \times 2.39 = 11.95$

let log a=3.41

log (a^3)=

log (a x a x a)

you can divide each 'a' by the base 3.42 times until you reach 1. Therefore,

log(a^3) = log a +log a + log a

= 3 x 3.41

Exercise 4.3.6 **Without calculator**

Given that log 123 = 2.09

Find log (123^3)

Exercise 4.3.7

a) Complete the table below.

a	***n***	**log a^n**	***n*log *a***
5	2		
4	3		
2.3	1.7		

b) Expand the table with three different values for *a* and *n*, *a*>0.
c) Study the pattern of the answers to question (a) and establish a rule for it.
d) Explain the reason why the rule you formulated works.

Example 11. **Without calculator**

Given that log 523 = 2.72

$\log\left(\frac{523}{100}\right) = \log 523 - \log 100$

$= 2.72 - 2 = 0.72$

(see exercises 4.2.10, 4.2.11 and 4.2.15)

Exercise 4.3.8 **Without calculator**

Given log 873=2.94

Find:

a) $\log\left(\frac{873}{100}\right)$
b) $\log\left(\frac{873}{10^3}\right)$
c) Is the answer to questions (a) – (b) reasonable? Why?

Exercise 4.3.9 **Without calculator**

Given that log 6534 = 3.82 and log 601 = 2.78

Find $\log\left(\frac{6534}{601}\right)$

Exercise 4.3.10 **Without calculate**

Given that log 6 = 0.78

Find:

a) $\log 6 + \log\left(\frac{1}{6}\right)$
b) $\log 6x + \log\left(\frac{x}{6}\right)$

(see exercise 4.2.18)

Summary:

1. log(ab)=log(a x b) = log a + log b
2. $\log\left(\frac{a}{b}\right) = \log a - \log b$
3. $\log(a^b) = b \log a$

4.4 logarithmic Equation

In this section, we will deal with equations where the unknown appears as the exponent. Examples are given below.

Example 12

Find x, if $2^x = 64$

Solotion:

$2^x = 64$

Both sides of the equation can be divided equally by 10 until reaching 1. This means that

$\log(2^x) = \log 64$

$\log(2^x) = \log(2^4)$

$x\log 2 = 4\log 2$

$x = \frac{4\log 2}{\log 2}$

$x = 4$

Excercise 4.4.1

Find x:

a) $4^{(x+2)} = 14$
b) $10^x = \frac{12}{8}$
c) $2^x = 7$
d) $2 + 3^{(2x-3)} = 10$
e) $3(2^{x+4}) = 350$

Excercise 4.4.2

a) $8^{(x+3)} = 11^x$
b) $6^{(3x+1)} = 4^{(5x+2)}$

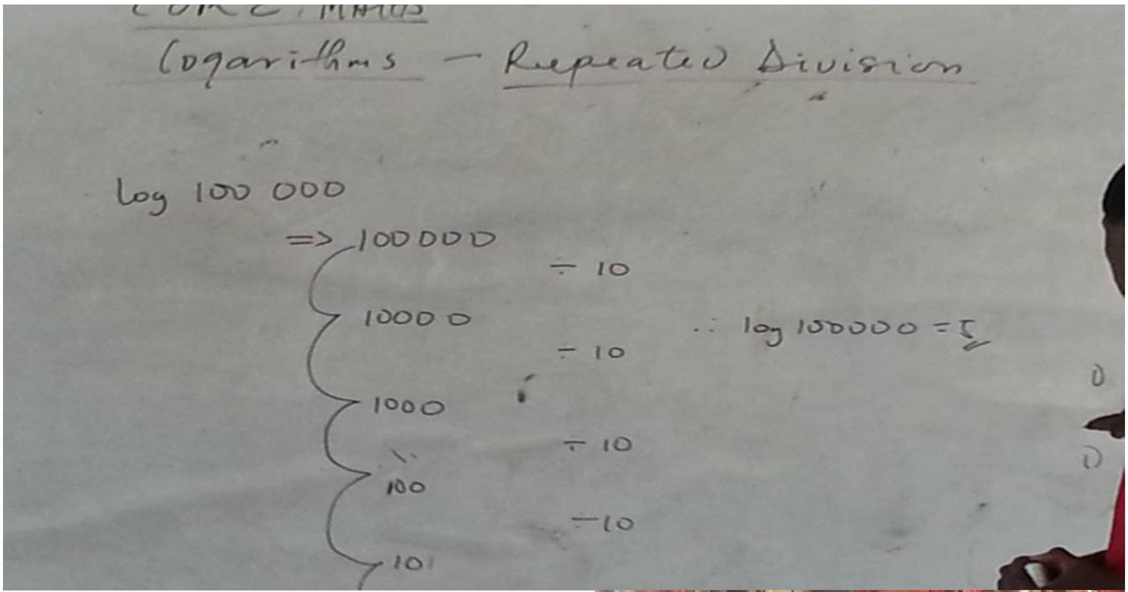
Excercise 4.4.3 Without calculator

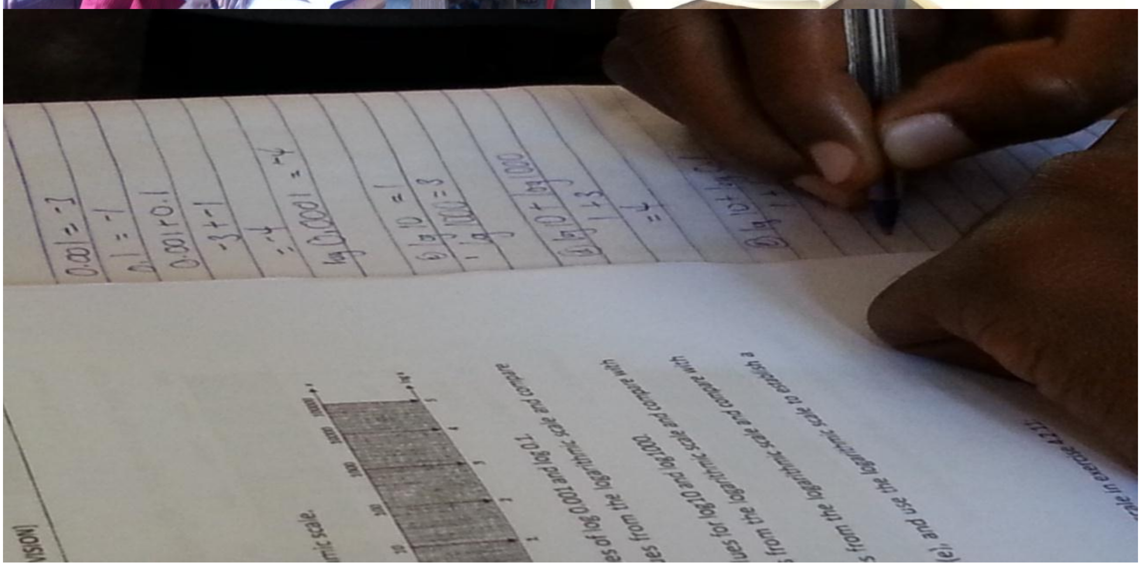
Given that log 45.6 = 1.66

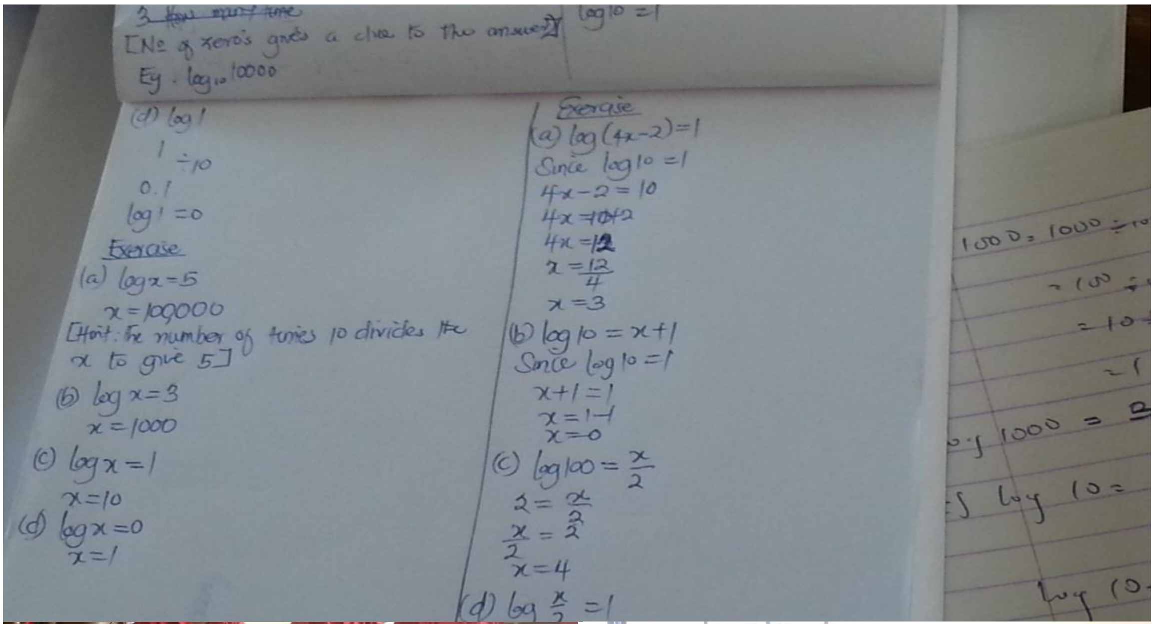
Find:

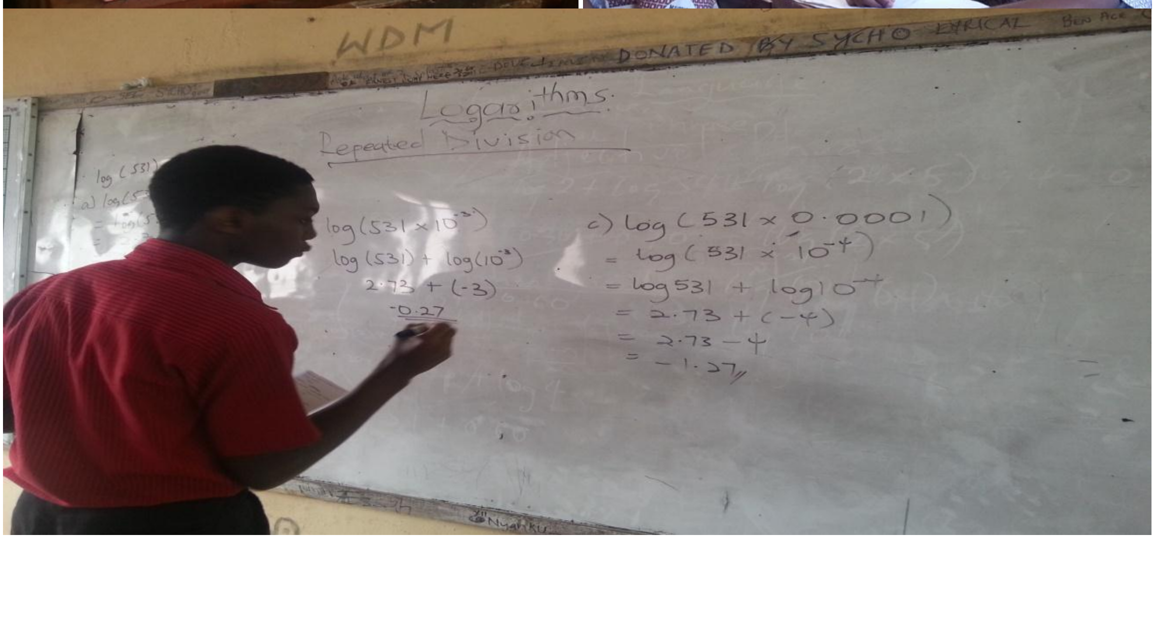
a) $10^x = 456$
b) $100^x = 4.56$

APPENDIX D – Exhibits of the intervention Lessons






[No of zero's gives a clue to the answer]
Eg. log10 10000
log 10 = 1
(d) log 1
1 ÷10
0.1
log 1 = 0
Exercise
(a) log x = 5
x = 100000
[Hint: The number of times 10 divides the x to give 5]
(b) log x = 3
x = 1000
(c) log x = 1
x = 10
(d) log x = 0
x = 1
Exercise
(a) log (4x−2) = 1
Since log 10 = 1
4x−2 = 10
4x = 12
x = 12/4
x = 3
(b) log 10 = x+1
Since log 10 = 1
x+1 = 1
x = 1−1
x = 0
(c) log 100 = x/2
2 = x/2
x = 4


WDM
DONATED BY SYCHO LYRICAL
Logarithms
Repeated Division
log(531 × 10⁻³)
log(531) + log(10⁻³)
2.73 + (-3)
-0.27
c) log(531 × 0.0001)
= log(531 × 10⁻⁴)
= log 531 + log 10⁻⁴
= 2.73 + (-4)
= 2.73 − 4
= −1.27

APPENDIX E – Interview Questions

Interview Questions (for Teacher)

1. Can you say something about the easiness and the difficulties of teaching logarithms?
2. What approach do you normally use in teaching the topic?
3. Do you find your approach very challenging in your teaching?
4. Can you mention some of the difficulties you face in using your approach in teaching logarithm? What are the advantages?
5. What do you see different from the repeated division approach as compare to your approach. Do you think it's an appropriate approach for teaching and learning logarithms?
6. Would you like to apply repeated division approach in your teaching hence forth?
7. In your own estimation, is the approach easy or difficult to use in teaching and learning logarithms?
8. How would you explain the approach to your colleague mathematics teacher?