UNIVERSITY OF AGDER

Performance of Technical Trading Rules in the US and Swedish Stock Markets

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This master's thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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Abstract

In this thesis I am testing the performance of two of the most popular market-timing strategies, the simple-moving average and time-series momentum, compared to the performance of the passive buy-and-hold strategy using in-sample testing. The strategies are implemented on two sets of data, one being the S&P500 for the period 1926 to 2013, and the second the Swedish stock market for the period 1919 to 2006. I replicate the results of previous studies on the S&P500, and also contribute to the ongoing research by extending the testing of the performance of the market-timing strategies to the Swedish stock market. The results of the in-sample test show that the market-timing strategies have better performance than the passive buy-and-hold strategy. However there are too many uncertain factors to conclude that the market-timing strategies are superior to the passive buy-and-hold strategy without conducting further extensive research.

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Innhold

1. Introduction

Technical analysis dates back to the 1800's and is a security analysis methodology that is used to forecast price direction through studying historical market data. It is a study of the markets based only on price actions as opposed to the fundamental analysis, which looks at the economic fundamentals of the underlying security. The users of technical analysis believe that all information which is relevant to forecasting the movement of a security is already included in the price, and that by analyzing the price action in the recent past and again in the more faraway past, one can then apply their knowledge to identify future trading opportunities.

There is a lot of discussion between academics and users of the method about whether or not technical analysis can improve portfolio return when applied to the markets, compared to the buy-and-hold strategy. The buy-and-hold strategy is a passive strategy in which investors purchase stocks and hold on to these for many years, in contrast to active trading in which ones goal is to actively trade in and out of a stock to try and maximize the gains and minimize the losses. In more recent times the efficient market hypothesis, which I will discuss later in this thesis, has been losing some of its support. Contributing to this are the economic crises such as those in 2000 and 2008, in which the latter one turned into a global crisis leading to the bankruptcy of countries like Iceland. As a result of the wish of investors to minimize their risk of losses, while at the same time benefitting from the booms in the market, technical trading strategies have been gaining popularity.

With technical analysis one is not so much attempting to forecast the direction of the market, but rather one is trying to identify the lower-risk high probability opportunities that a new emerging trend will continue and in the case that it does not do so, you exit the market without question and move on to the next opportunity. There are many trading strategies included in technical analysis, but I will focus on two of the widely used strategies in the market, which are the simple-moving average and time-series momentum strategies.

For the simple-moving average strategy the current price is compared against the mean of the last "k" observations. The price is trending upward if the current price proves to be higher

than the mean, and in this case a buy signal is generated and the asset should be bought and held. If the price is below the mean, a sell signal is generated and the asset should be sold. The length of the moving-average varies, but the 10-month mean on monthly data and 200-day mean on daily data are commonly used. When it comes to time-series momentum we only have one market index and we a buy signal is generated when the price increases over a certain period of time, and similarly a sell signal is generated when the price decreases over a certain period of time.

There has been published numerous research papers based on the study of the simple moving average strategy and time-series momentum. Many of these research papers have gotten favorable results for the market timing strategies, and praised their ability to outperform the market by generating notably higher risk-adjusted returns than the passive buy-and-hold strategy. Such studies include those by Brock, Lekonishok and LeBaron (1992), who found that technical strategies could generate higher return than the passive strategy, Bassembinder and Chan (1998) applied moving average rules on a group of Asian stock markets and found that they were useful when forecasting index returns. Further Faber (2007) concluded in his study that simple average model could reduce losses in the bear market, and by reducing losses generate a higher return. Siegel (2002) also found that market timing strategies generated higher risk-adjusted returns than the passive strategy. While in the study of time-series momentum, Markowitz, Ooi and Pedersen (2012) concluded that the strategy had consistent performance in many diverse asset classes, and performed well even in extreme periods. These results are contradictory to the claims of the efficient market hypothesis that the market is unbeatable because the current stock prices reflect and include all relevant information. And as such, one should not be able to predict future stock prices by looking for trends in the market.

The goal of this thesis is to replicate the results of the previous studies on S&P500, and to further extend the research by applying the strategies to the Swedish stock market. The performance of market-timing strategies has never been tested on the Swedish stock market previously, and as such I will be the first to do so. The purpose of this is to contribute to the ongoing in-sample testing of the performance of the market-timing strategies.

In this thesis I will work through literature that is relevant to technical analysis and the established benefits of the simple moving average and time series momentum strategy based on published research. Further I will test the performance of simple moving average, time series momentum and the passive buy-and-hold strategy, to see which of these strategies shows best performance. These strategies will be applied to the data from two different stock markets. The first one is US stock market index, Standard & Poor's 500. The Standard & Poor's 500 is regarded as the foremost indicator of the US stock market, this is mainly due to its size including 500 US stocks from 500 major companies selected by analysts and economists at Standard & Poor's. It is the most watched index in the US market, and nearly all American investors compare their results to those of the Standard & Poor's 500. The second data is from the Swedish stock market which consists of market returns for the Swedish stock market for the past 150 years. It operates under the name Nasdaq Stockholm, and one of its notable indexes is the OMX Stockholm 30, which is a market-value weighted index that follows 30 of the most traded stocks. However I will be looking at the Swedish stock market as a whole.

This thesis is organized as follows: to begin with it will have an overview of the relevant literature; thereafter the market portfolio theory and the market efficient hypothesis will be presented. Further I will briefly talk about the data that is used, and thereafter the methodology on relevant aspects will follow, and the empirical results of the in-sample testing will be presented. Nearing the end I will have a discussion, and lastly present a conclusion.

2. Literature Review

 Technical analysis is one of two types of analyses that are used in the financial markets, the second being fundamental analysis. In a fundamental analysis one looks at the economic factors known as fundamentals. Technical analysis is a methodology for forecasting the price movements of financial securities by using historical price and volume data, it attempts to forecast future price movements by studying past prices. (Kirkpatrick and Dalhquist, 2006). The roots of technical analysis are attributed to Charles Dow and date back to the 1800's, and since then it has been utilized. It is believed that technical analysis was the original form of investment analysis.

There have been many critics of technical analysis who have questioned its validity and discipline. Much of the criticism has roots in academic theory, especially because it breaks with the efficient market hypothesis (EHM). Only recently has technical analysis begun to gain recognition and credibility in the financial market. This change of viewpoint is mainly because many studies on technical analysis have shown to be more profitable than the buy-and-hold strategy. In 1992 a paper was released by Brock, Lekonishok and LeBaron in which they tested a number of trading rules, including the moving average (MA) and trading-range breaks. In their test they utilitized a series of data from 1897 to 1986 on the Dow Jones Industrial Average index (DJIA). They used both the t-test and bootstrapping, which are widely used in scientific studies, and found evidence that a majority of the trading rules gained abnormal returns over this period. Brock, Lekonishok and LeBaron were able to show that the technical trading rules could generate higher returns than the buy-and-hold strategy, and their results provided support for the technical strategies which considered useless by many earlier. However it is worth mentioning that their paper did not incorporate transaction costs, and therefore we do not know of if the results would be as favorable if they had been included.

Bassembinder and Chan (1998) testes whether the technical rules used by Brock, Lekonishok, and LeBaron in their research were useful for predicting index returns for a group of Asian stock markets. When testing the technical trading rules, Bassembinder and Chan (1998) used observations on stock price indices for Malaysia, Thailand, Taiwan, South Korea, Hong Kong and Japan. The first three countries were emerging markets that got better results in

predicting stock price movements. The results for averages in all of the countries and evaluated trading rules, showed that the average percentage changes in stock indices on days that have buy signals exceeded the averages of days have sell signals by a margin of 26,8 % on an annual basis. Although they were able to show that technical trading rules were useful in predicting Asian stock markets, they found that when transaction costs of 1.57 % were inclusive, the gains from using the trading strategies were eliminated.

Fifield et al. (2005) analyzed the predictability power of trading strategies by examining data for 11 stock markets, in a study that lasted over 10 years from January 1991 to December 2000. She found that the results varied greatly, as emerging markets displayed a certain degree of predictability in share return but not for the developed markets. The results for the different markets proved to be quite inconsistent, and since the trading rules seemed to be efficient only for specific markets at certain times, it showed that previous attempts by analytics to indicate that countries could be grouped together by their geographical location when testing and formulation the trading strategies might be inaccurate.

Simple moving average (SMA) is one of the most commonly used strategies by traders, it is simply an average of a series of data points over a given period of time. A well-known newer paper which showcases the efficiency of the SMA strategy is the Faber (2007) paper "A Quantitative Approach to Tactical Asset Allocation". Although this paper has been released since 2007, Faber continues to update results of the paper throughout recent years, it has last been updated in 2013. In a study from 1973, he tested the SMA strategy on monthly observations over a period of 10 months. In this study he applied the SMA strategy to multiple equity markets in addition to other publicly traded assets like the commodity market. He applied the strategy to the stock market from 1900 to 2008, and from 1970-2009 for the others. Through his observations he found that the risk adjusted returns were almost always improved. The simulated strategy showed increased compounded returns from 9.21% to 10.45%, over the period of 1900-2008, and reduced the volatility by over 500 bp (5%). In addition the maximum drawdown was reduced from 83.66% to 50.31%. Faber concluded that using a SMA model reduced losses in bear market, and he stated: "Avoiding these massive losses would have resulted in equity like returns with bond-like volatility and drawdown." These results were similar to those of Brock et al (1992) and Bassembinder and Chan (1998)

Another test on the SMA strategy was done by Siegel (2002) in his book "Stocks for the Long Run". He investigates the use of the 200-day SMA strategy in timing the Dow Jones Industrial Average (DJIA) from 1886-2006, and discovers that market timing increases the absolute and risk-adjusted returns over a simple buy-and-hold strategy. Similarly, when adjusted to include all transaction costs such as taxes, bid-ask spread and commissions, the risk-adjusted returns were still higher when market timing, even though the timing falls short on a total return measure. When using the signaling structure where the sell signal is produced if the index is under 1 % of the moving average, and a buy signal if over 1 %, he produce 4 % higher returns than the buy-and-hold strategy, while having 25 % lower volatility.

In addition there is another strategy that has long been in existence and used by traders, but has only recently been presented in a study. The strategy was first written a study on by Moskowitz, Ooi, and Pedersen (2012), and they gave it an anomaly called "time-series momentum" (MOM). Time-series momentum focuses on that a security's *own* past return predicts its future returns. Moskowitz, Ooi, and Pedersen (2012) tested MOM on a security's past returns for nearly five dozen diverse future and forward contracts, which include country equity indices, commodities, currencies and sovereign bonds for more than 25 years of data. Over a period of 12-months they found that there was a positive correlation when it comes to the securities past returns and future returns. The profits by using MOM were positive, not only on an average across the assets, but for every asset contract that they examined which were 58 in total. They concluded that MOM exhibits a strong and consistent performance in many diverse asset classes, and that it has small loading on standard risk factors, in addition to performing well in extreme periods. All of these which challenged the random walk hypothesis and the standard rational pricing models.

In a study done by Zakamulin (2014), he tested SMA and MOM strategies by simulating the market timing strategies out-of-sample from 1926 to 2012. For this time period he used different fixed k (months) lengths and then found the k with best performance. The results proved the trading strategies to be less risky, which in return generated a lower return and thereby lower equity growth. He concludes that the market timing strategies seem to be greatly overrated, and that previous optimistic studies of their performance are likely the results of ignored market friction and data-mining. In addition it is concluded that the periods

of superior performance with the market timing strategies is short, and in the long run they are not likely to outperform the passive strategy.

In these studies of technical analysis human behavior is not included, I have discussed many aspects of these trading strategy rules which educate us about the different positive and negative aspects of these strategies. However the ones to put these strategies into practice are humans which are emotional beings. The choices conducted by human emotion can cause marginal errors when these strategies are put into practice.

3. Theory

3.1. Modern Portfolio Theory (MPT)

Modern Portfolio Theory (MPT) is one of the most important and influential theories within economics, it was first introduced in 1952 by Harry Markowitz in his paper "Portfolio Selection". The theory is based on attempting to find the balance between maximizing the portfolio expected return and minimizing the portfolio risk, which is the standard deviation for the portfolio. This is done by selecting different investments in a way that diversifies your risks whilst not reducing your expected return. In other words it is about choosing the right combination of stocks to reap the benefits of diversification.

The thought behind MPT is that within an investment portfolio, assets should not simply be selected based on their own merits, but instead based on how each of these asset could change in price relative to how the other asset could change in price. The general assumption within economics is that higher risk generates a higher return. The MPT is dependent on how much the investor is willing to risk, because the theory shows how to combine a portfolio with the highest expected return, or how to combine a portfolio with the lowest possible risk for a given return.

Expected return: the expected return is the amount one believes to receive after an investment. Expected return is not guaranteed. The formula for expected return for the portfolio:

$$E[R_p] = \sum_i X_i E[R_i]$$

X_i is the weighting of component assets i, Rp is the return on the portfolio, Ri is the return on the asset i.

Portfolio standard deviation: Standard deviation is an equipment to measure the risk by quantifying the variation of data. Portfolio standard deviation is the variance measure of expected returns of a portfolio of investments. The main reason for diversification is to reduce the risk of the portfolios. In order to benefit of the diversification, portfolio standard deviation of investments should not be higher than the weighted average of the individual investments standard deviation.

The formula for portfolio standard deviation is:

$$\sigma[R_p] = \sum_i X_i \sigma(R_i) Corr(R_i R_p)$$

In which X_i is the weighting of component assets i, R_i is the return on the asset i, R_p is the return on the portfolio, and $Corr(R_i, R_p)$ stands for correlation between the return on assets and the return for the portfolio.

Correlation coefficient (ρ): is a coefficient that shows how two or more random variables move in relation to each other. The correlation coefficient is:

$$Corr(R_i R_j) = \frac{Cov(R_i R_j)}{\sigma(R_i)\sigma(R_j)}$$

Where $cov(R_i R_j)$ is the covariance for R_i and R_j, while $\sigma(R_i)$ stands for standard deviation for R_i and $\sigma(R_j)$ stands for standard deviation for R_j.

When the prices of two variables move in similar direction the variables usually are considered to be correlated. The amount of correlation varies from$-1 \leq p \leq 1$, where 0 indicated no correlation, and 1 indicates perfect positive correlation. When the variable has perfect positive correlation, it means that the correlation is positive 100 % of the time. On the other hand, if the prices of the variables move in opposite directions they are negatively correlated, where 0 is no correlation, and -1 is perfect negative correlation, meaning the relationship between the variables is opposite 100 % of the time. No correlation variables move independently from each other, perfect positive correlated variables move in perfect symmetry, while perfect negatively correlated variables move perfectly opposite from each other.

Covariance is used to measure the linear relation between two variables.

Figure 1, zero covariance

If the covariance is zero, there will not be any relation between the variables, and they will be independent of each other.
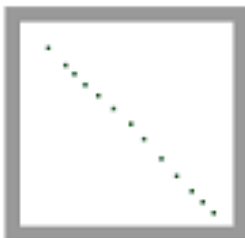

Figure 2, strong negative covariance

If the covariance is strong negative, then the variables will go together in a negative linear direction.

Figure 3, strong positive covariance

If the covariance is strong positive, the variables will go positively together in a linear direction.

The covariance formula is:

$$cov(X, Y) = E((X - E[x])(Y - E[y]))$$

Where X and Y are random variables, E[x] is expected value of the random variable X, E[y] is expected value of random variable Y.

If an investor chooses a combination of assets that are not perfectly positively correlated, the investor can reduce the portfolio risk. No correlation is important for portfolio diversification, when some assets in a portfolio lose money, other non-correlated assets are likely to still be gaining or not moving at all, which will reduce the losses or even gain for investors.

In his theory, Markowitz treats volatility and risk as the same thing. The risk is used as a measure of the possibility of whether an investment will go up and down in value, in addition to how much and how often. In the MPT it is assumed that the investor is rational, and as such the investor will prefer to minimize the risk. When given the option between two portfolios of the same return, the rational investors will choose the portfolio with the lowest level of risk. Should the investor take additional risk, they expect that there will be higher return.

By solving the following formula we can find the minimum variance for a targeted portfolio return, μ^*:

$$\min \frac{1}{2} \sigma^2 [R_p], subject \ to \ E[R_p] = \mu^* \ and \ \sum_{i=1}^{N} w_i = 1$$

Where $E[R_p]$ is the expected portfolio return, μ^* is targeted portfolio return, $\sigma^2[R_p]$ is the portfolio variance, w_i is portfolio weight and N is the number of different existing assets.
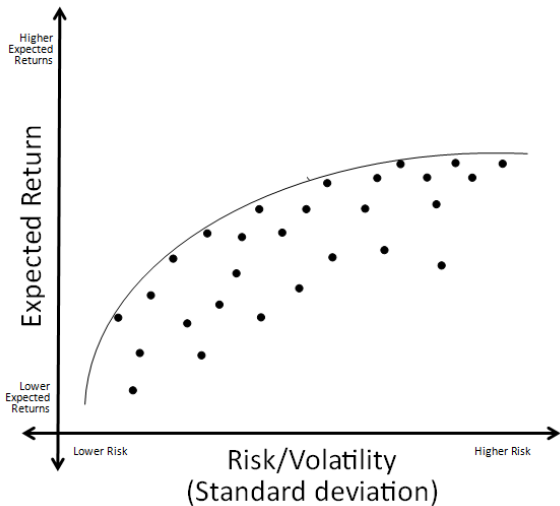
Figure 4, frontier with no risk free rate

When we resolve this quadratic problem the answer will be what combination of assets will give us a given level of return for the lowest risk. The combination that we get is called efficient and it will lie on the efficient frontier. For each achievable expected return there will be another asset set that will give us the same return with minimal risk. We can find the whole efficient return by varying portfolio expected return.

Investments usually involve a tradeoff between risk and return, the efficient frontier is a portfolio tool which demonstrates the maximum expected return one can get from their portfolio with the level of risk they are willing to take.

In Figure 4, each dot is representative of a portfolio. The curve is the efficient frontier which shows the optimal combination of risk and return. It is expected that the dots which are placed closest to the efficient frontier curve show the optimal expected return with lowest risk.

Dots that are placed below the curve show the sub-optimal portfolios. The return they give is not enough given the level of risk. Similarly dots which are placed to the right of the curve also sub-optimal, as they provide higher level of risk for area defined level of return. Whatever one's risk propensity is, one should be close to the efficient frontier. The degree if diversification is usually higher in optimal portfolios which compromise the efficient frontier, while the sub-optimal ones are usually less diversified.
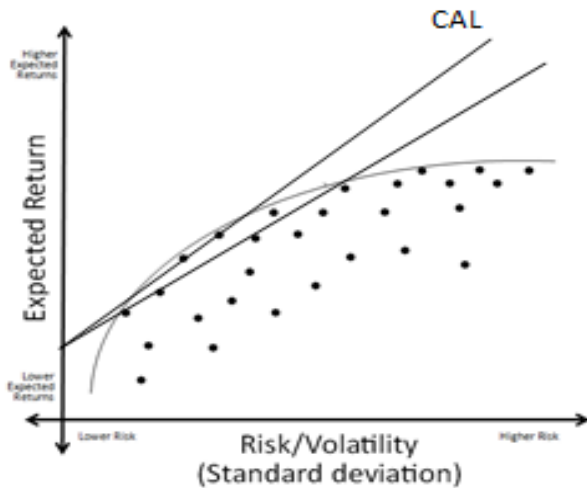
*Figure 5, frontier with risk free rat*e

When we include the risk free rate in our portfolio, the frontier theory looks like it does on Figure 5. The two frontier lines on the figure show us the result of diversification of portfolio and risk free rate. As mentioned in Figure 4, we choose the diversified portfolio with the best return after risk free rate is included. The optimal portfolio is chosen between the portfolios that are lying on the efficient frontier line. The efficient frontier line that is higher up the curve has higher expected return and lower risk. The investor's allocation consists in investing w in the optimal risky portfolio and 1-w in the risk-free asset. The return on the complete portfolio is:

$$R_c = wR_p^* + (1-w)R_f$$

R_c is the complete portfolio, R_p is the risky optimal portfolio return, R_f is risk free return, and w is the portfolio weight. There is zero covariance between risk free and risky asset, R_f must have volatility equal to zero in order to be risk free. The standard deviation for the portfolio is

$$\sigma_c = (1-w)\sigma_p$$

By combining the portfolio return and standard deviation we can calculate the best capital allocation line (CAL):

$$E[R_c] = R_f + \left(\frac{E[R_p^*] - R_f}{\sigma_P}\right)\sigma_c$$

In which R$_f$ is the risk free asset, w is the portfolio weight, E[R$_p$] is the expected return of optimal portfolio, σ_c is the total portfolio standard deviation, and σ_p is the risky portfolio standard deviation. The MPT theory claims that all risk-averse investors will adjust their portfolio along the CAL line somewhere between the best possible CAL.

The MPT theory is used by some investors, the method of diversifying a portfolio in order to minimize risk and increase return is regarded smart way of investment compared to investing all assets in one type of stocks.

3.2. Sharpe ratio

The Sharpe ratio was developed in 1966 by Nobel Laureate William F. Shape, and is a tool used to measure risk-adjusted portfolio performance. It has become a widely used standard for calculation in the industry. Sharpe ratio is the average return earned in excess of risk-free rate per unit of total risk or volatility. The Sharpe Ratio measures the value of investments by dividing the reward of the investments with the risk of investing in those assets. Basically it allows you to see how much additional return you are gaining for the added volatility of holding a risky asset, compared to a risk free asset.

The formula for the Shape ratio is:

$$SR_p = \frac{R_P - R_f}{\sigma_p}$$

Where SR_p stands for Sharpe ratio portfolio, R_p = mean portfolio return, σ_p is portfolio standard deviation and R_f is the risk free.

There is no need for an extensive financial mathematic background in order to be able to use the Sharpe Ratio measure. It can also be applied to nearly all types of assets, since one is not referring to an outside reference point as the standard for that particular investment.
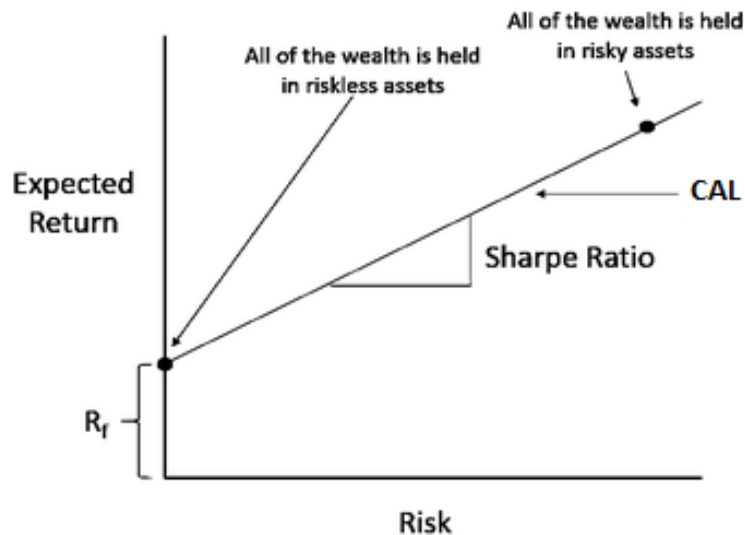
Figure 6: Sharpe Ratio

Figure 6 shows the risk free rate (R_f) and the slope CAL of the curve, which depicts the risk to expected return of a portfolio when there is a combination of different potions of risky and riskless assets. The steeper the slope is, the higher is the Sharpe ratio, and as such the expected return for taking on additional risk per unit will be higher. More risky assets are included if one moves rightwards up the slope to a point with higher risk and higher return, however it depends on how much risk the investor is willing to take on.

How profitable an investment is can be measured by the Sharpe ratio, the higher number the Sharpe ratio shows, the better the investment. If the Sharpe ratio is negative it would most likely be better to invest in risk-free platforms such as a bank, or treasury bonds. The Sharpe ratio can be inaccurate if applied to assets that do not have normal distribution of expected returns, as many assets might have negative skewness or a high degree of kurtosis.

3.3. Efficient Market Hypothesis (EHM)

Before technical analysis became more widely accepted, the efficient market hypothesis (EMH) was commonly used and accepted by academics. It is a theory that was partly introduced in the early 1960s by Eugene Fama through his article, "Efficient Capital Markets." The idea that all information is reflected in the stock market, and that new information is incorporated quickly into the prices of the securities without delay was the accepted view. The EMH theory is based on the belief that the current price of the market is always the correct one, meaning that the stocks always trade at fair value. Any of the past trading information is reflected in the price of the stock, and therefore it is impossible for investors to sell for inflated prices or purchase undervalued stocks. Based on this theory outperforming the overall market through expert stock selection or market timing should be impossible, and therefore purchasing riskier assets is the only way to obtain higher return.

The EMH hypothesis can be separated into three forms: 1) week form, 2) semi-strong form and 3) strong form:

1) The EMH in the weak form, otherwise known as soft-form, claims that the market reflects all public available information of the past is reflected in the present prices of the assets, and therefore the past prices do not have any predictability power of the future prices. This again means that no additional returns can be gained consistently in the long run by using strategies such as technical analysis, which are based on historical data. However some excess returns might be gained by using some forms of fundamental analysis. In the weak form EMH there must be a random walk to the prices, as it claims that the market users should be unable to systematically gain profit from market inefficiencies. The idea behind the random walk is that when information is absorbed and reflected in the prices, then the price changes of tomorrow will be reflective of the news of tomorrow only, independent of the prices today. Critics however have pointed out that there are a lot of studies that have demonstrated that there is a market tendency to the stock markets that can last for periods of weeks or more. In these market tendencies there is also positive correlation between degree of trending and the length of the time period. We can categorize the two weak form EHM test.

2) The EMH in the semi-strong form claims that the market adjusts quickly in order to reflect new public information in an instant and unbiased way. Investor will thereby not gain benefit over the market by trading on that new information, as one assumes that the current stock prices already reflect all new and available information and the stocks are purchased by investors after this information is out. Contrary to the weak-form efficiency, the semi-strong form claims neither technical nor fundamental analysis will be able to produce reliable excess returns. To be able to test the semi-strong efficiency the reaction to the new information must be immediate and of reasonable size.

3) The strong-form of EMH claims that the market reflects all information, both private and public. And so there would not be any investor that would profit above any other average investor even when given new information. In order to test strong-form efficiency the market has to be such that no investor can consistently gain excess return in the long run. The strong-form efficiency is impossible if there are legal legislations which hinder private information from being known publicly, such as when it comes to insider trading laws. The only exception that makes it possible even in such a situation is if the laws are being ignored universally.

In recent years the EHM theory has lost much of its foothold and is no longer as universally accepted as it was previously. Today many academics, financial economists and statisticians believe that stock prices could be somewhat predictable. Many studies done on strategies that test on historical data claim that investors can gain excess risk adjusted return. The EMH theory is based on the perception that all available information is exactly the same. With the many methods of valuing stocks the validity of EMH has been in question. Critics point out that investors might value the stocks differently, and so it is not possible to determine the worth of a stock under the efficient market. For instance if one investor deems the stock price to be overvalued, while another investor deems it undervalued, then the assessment of the fair value of the stock in the market will be different for these two.

Another point of the theory is that no investor should be able to gain higher profit than any other investor with the same total invested funds. Because they have the same information available to them, their return should also be identical. However, if it is true that no investor

has an advantage over other investors, then it should be unlikely that yearly returns in the mutual fund industry vary significantly between losses and profit. Critics state that this should be unlikely according to the EHM, as it says that the profit should be universal among the investors if one investor is profitable. Finally, in the EMH theory it should not be possible to beat the market over a long period of time. Yet there are many professional investors who consistently are able to beat the averages for years at a time, one example of this is Warren Buffett.

The efficient market theory is the opposite of the technical analysis, which we use to predict the future prices by analyzing historical data in this thesis. The claim of the EMH theory that the present market price is the correct price is something most people can agree on, however we cannot exclude that the traders evaluation of what the price should be might affect the price of today. The technical analysis is therefore seen by many as a tool that can assist in trying to predict the future prices, and the belief of being able to look at historical data in order to predict future prices affects what the trader evaluates the future price to be. This can be one of the effects of people not being rational like the EMH and MPT assume them to be.

4. Data

The data used in this thesis is gathered from two different sources. The monthly data on the Standard and Poor's Composite stock price index is gathered for a sample period from 1926 until 2013. The data was provide by Amit Goyal, and found on the website:
www.hec.unil.ch/agoyal.
The second collection of data used is on the Swedish stock market for the sample period from 1919-2006, it was provided by Daniel Waldenstrom and found on the website:
http://www.riksbank.se/sv/Riksbanken/Forskning/Historisk-monetar-statistik/Historical-Monetary-and-Financial-Statistics-for-Sweden-Exchange-Rates-Prices-and-Wages-1277-2008/.

For the data on the Swedish stock market the total return includes the dividend return in addition to the capital return. For the S&P500 the dividends were not accounted for on the price return index. Therefore in this thesis, the dividend return has been calculated and included in the total return together with capital return for the S&P500. As a result of this when talking about return further in this thesis, what is meant is the total return which includes capital return adjusted for dividend return. The risk free rate for the S&P500 is from Amit Goyale's database, and for the Swedish stock market from Daniel Waldenstrom.

The data return used for the S&P500 includes 1053 monthly data observations starting from Mars 1926 and ends in December 2013. While for the return data for the Swedish stock market is for 1052 monthly observations from April 1919 until December 2006. The SMA and MOM strategies will use the monthly return with a variable lookback period of 1-15 months (k), have done this by including the new monthly return and removing the oldest monthly return. This was done in order to find the k period with the best performance.

4.1 Standard & Poor's 500 (S&P500)

The Standard & Poor's 500, also called the S&P500, is a stock index for the American stock market which includes 500 large companies. It was first made in 1957 by the company Standard & Poor's, in order to assess the overall American stock market. The companies are selected by Standard & Poor's S&P Index Committee, which consists of economists and analysts. As the companies are representing the U.S. economy, they must achieve these liquidity based requirements to be included in the index:

1. The market capitalization has to be greater or equal to US $5.3 billion
2. Annual dollar value traded to float-adjusted market capitalization is greater than 1.0
3. For every one of the six months before the evaluation date, there must be a minimum of monthly trading volume of 250 000 shares.

The S&P500 is a market value weighted index, meaning that the stocks are weighted according to their market value. When looking at the average of stock performance of the 500 large companies which are from diverse industries, we can get an idea of how the U.S. stock

market performed last year. Since both value stock and growth stock is included in the index, it is seen as a good measure of the general stock prices level. As mentioned earlier, the S&P500 is a good measure of the U.S. stock market performance and that is why I use it in this thesis.

4.2 Swedish Stock market

The Swedish stock market appeared in the second half of the 19th century. It consists of the Swedish market returns for the past 150 years. The Stockholm Stock exchange was founded in 1863 and is considered to be the main securities market of Sweden. It merged with OM (Optionsmarknaden) in 1998 and in 2003 when OM merged with Helsinki Stock Exchange it formed the OMX. Since 2013 it has become a part of the NASDAQ groups and today goes by the legal brand name Nasdaq Stockholm. One of the significant indexes of the Swedish Stockholm Stock exchange is the market-value weighted OMX Stockholm 30, which follows 30 of the most traded stocks.
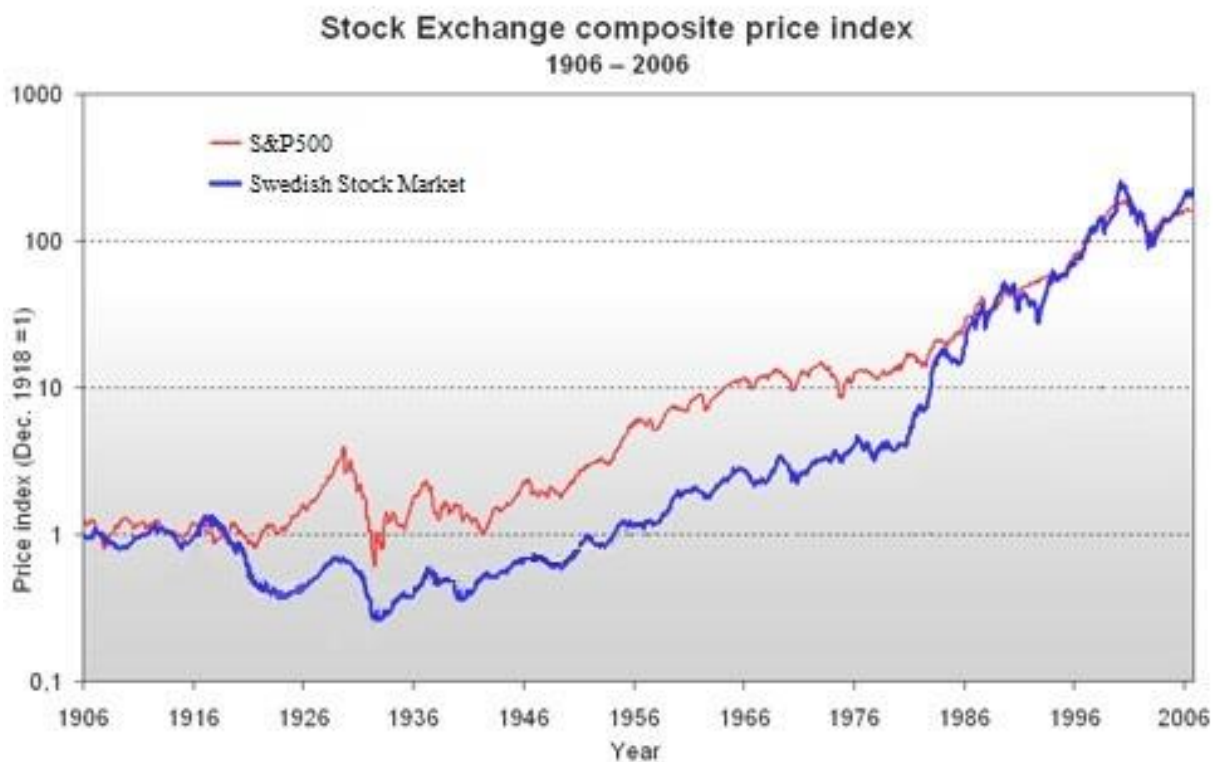
Figure 7: Shows the stock exchange composite price index for S&P500 and
Swedish stock market from 1906-2006

Table 1 and Table 2 below provide some basic information gathered on the two index markets that have been tested in this thesis. It shows total market return, capital appreciation return and risk free rate.

Table 1 – Descriptive statistics of the S&P500 data from 1926-2013

S&P500	MKT	CAP	RF
Mean return	0,95 %	0,62 %	0,30 %
Min retutn	-29,85 %	-29,94 %	0,00 %
Max return	43,50 %	42,22 %	1,36 %
Median	1,23 %	0,93 %	0,26 %
Stand.div	5,63 %	5,49 %	0,26 %
Kurtosis	13,08	9,321	1,29
Skewness	1,46	0,292	1,05

Table 1:Statistics of data that has been used in the thesis. Table shows MKT (the total market return), RF (risk free rate of return) and CAP (the capital appreciation return). Standard deviation, skewness, kurtosis, mean, median, min and max are given in percent.

Table 2 – Descriptive statistics of the Swedish stock market data for 1919-2006

Swedish stock market:	MKT	CAP	RF
Mean return	1,30 %	0,96 %	0,44 %
Min return	-27,00 %	-27,12 %	0,12 %
Max return	28,19 %	27,58 %	2,85 %
Median	1,31 %	0,96 %	0,37 %
Stand div.	5,26 %	5,16 %	0,25 %
Kurtosis	2,69	3,249	8,80
Skewness	-0,25	-0,122	1,90

Table 2:Statistics of data that has been used in the thesis. Table shows MKT (the total market return), RF (risk free rate of return) and CAP (the capital appreciation return). Standard deviation, skewness, kurtosis, mean, median, min and max are given in percent.

For Table 1 (S&P500) the variance is higher than in Table 2 (Swedish stock market), I calculated it by taking the difference between min and max of the MKT. This is also reflected in the standard deviation as it is higher for Table 1. The mean return is higher for Table 2, which proves by historical data that it has been more profitable to invest in. The skewness for total market return is higher in Table 1. Based on the results from the statistical data we can conclude that it riskier to invest in S&P500 than in the Swedish stock market, something which the standard deviation verifies as it is higher in Table 1.

5. Methodology

5.1 Computation of returns

In this thesis I will test the indices return for the Swedish stock market and Standard & Poor's 500 (S&P500) by comparing the simple moving average (SMA) and time-series momentum (MOM) to the passive strategy. This is done in order to find out which strategy performs the best. To begin with the SMA and MOM strategies which are active strategies will be compared to the passive strategy, and then the SMA and MOM strategies will be compared to each other. To measure the strategies performance we have found the monthly price return for indices.

My data are the stock prices and dividends. First I compute the capital appreciation return using the stock prices, then I compute the dividend yield using the stock prices and dividends, finally I compute the total return using the capital appreciation return and the dividend yield. In the subsequent sections I explain in details how I perform these computations.

5.1.1 Capital Appreciation Return (CAP)

The capital appreciation (CAP) return is used to measure the rise in the value of an asset in the market. Dividends and interest income are excluded in order to generate a signal which more effectively estimate the market movements, and also to avoid the noise that different dividend policies generate. Different companies have varying dividend policies which can give misleading results, for instance a company can issues dividend payouts although the true value of the company is dropping to give the impression that things are going well to their shareholders. We might get an inflated stock price, and therefore by removing dividends and other interest income it will give us a clearer signal for our trading rules.

The capital appreciation is given in percentage and the formula is:

$$CAP_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where P_t is the initial stock price and P_{t-1} is the stock price after 1st period

5.1.2 Dividend

A dividend is the periodical distribution of a portion a company's profit to its shareholders. The distribution of dividends can be issued in cash, as shares of stock or other property. A dividend yield is the total annual dividend payments of a company divided by the market capitalization of the company. Investors use it as a measure that shows how much cash flow they are earning for every dollar that is invested in a dividend paying stock. In times of financial difficulty a company can choose to reduce or eliminate the dividend, as the dividends are not guaranteed. The dividend yield is usually given in percentage, and the formula is:

$$Dividend\ Yield_t = \frac{D_t}{P_{t-1}}$$

In which D_t is the dividends for the period and P_{t-1} is the stock price after 1st period

5.1.3 Market Total Return (MTK)

Market total return (MKT) is a performance measurement on an investment over an evaluation period. It consists of both capital appreciation and income. Income can be dividends, distributions or other type of payouts to investors. The MKT is viewed as a good measure of the return of an investment and better than using dividends or interest alone. One can compare MKT to CAP in order to get a better overview of an investment performance, as the CAP shows the growth of value of an asset while MKT also includes the extra payout that have been paid out to investors.

The MKT is expressed in percentage and the formula is:

$$\text{Market Total Return}_t = \frac{D_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}}$$

D_t is dividend yield, P_{t-1} is the stock price after 1st period and P_t initial stock price.

5.2 The simple moving average (SMA)

The SMA strategy is one of the most used technical analysis tools by traders, mostly because of the simplicity of the signals it gives. The strategy is mostly used to identify market direction, but is also used to generate buy and sell signals. I will let (P_1, P_2, P_3.., P_t) be the closing monthly indices prices. The way these signals are used to predict the market direction, is that when the index price is above the SMA value you hold the indexes, and when the index is below the SMA you move to cash. Using longer time frames is a good option when analyzing indices. The strategy can be calculated by adding prices over a set number of periods, and dividing the sum by the number of periods. This means that every time we add a new monthly closing price of an index, the oldest price is removed.

When using the SMA strategy the security prices in the look back period are equally weighted, and therefore the SMA at time t is:

$$SMA_t(k) = \frac{1}{k}\sum_{j=0}^{k-1} P_{t-j}$$

Where k stands for months, t for time, and P_t for closing price. For P_{t-j}, $j \in [1, k]$ in this case k is varied from 1 to 15.

The signals for SMA generated for time t+1 are:

Buy:

$P_t > SMA_t(k)$

Sell:

$P_t < SMA_t(k)$

5.3 Time-series momentum rule (MOM)

Time series momentum (MOM) focuses primarily on the process of a security's own past. It is used to show that a security's past return is a predictor of its future return. In the MOM strategy a buy signal is created when a k-month momentum is positive, or else a sell signal is generated.

We can assume whether the price is positive or negative by using the formula:

$MOM_t(k) = P_t - P_{t-k}$

k stands for months, t for time and P for indices prices

The signals for MOM month t+1 are generated by the following rules:

Buy:

$MOM_t(k) > 0$

Sell:

$MOM_t(k) < 0$

5.4 Buy-and-hold strategy

The buy-and-hold strategy is a passive investment strategy in which investors purchase stocks and hold on to them for a long period of time, no matter the fluctuations in the market. The strategy has a tax benefit in that long-term investments are usually taxed at a lower rate than short-term investments. For unexperienced traders it is seen as a better option to buy and hold, rather than using market timing where they enter and bail out of the market depending on the highs and lows. Another benefit is that this is a less time consuming strategy, as one does not need to follow the market as actively as those who employ a market timing strategy. Although investors who use this strategy are usually in it for the long-run, there are a few situations in which investors can decide to exit the market. For instant if a company files for bankruptcy, if the company has committed unmoral or unlawful actions, and if the company does something that goes against your values or beliefs.

5.5 In-sample test

In-sample test is a test in which you collect the results from a trading rule by applying the trading rule to the same data which we used to optimize the trading rule. To obtain the optimal market strategy we simulate the returns for different k-months. The best strategy is that which produces highest SR. This is a good way to measure the strategy performance in the past, however we cannot know if the strategies will continue to perform well in the future. That is because the market is unpredictable and might not behave similar to how it did in the past. For the in-sample test I will compare passive and active strategy to see if they are

similar in their results. And if they are not similar, to find out which strategy simulates the best historical results. The data on S&P500 and Swedish stock market will be tested monthly for $k \in [1,15]$ months.

5.6 Mean/Average

Within statistics a mean is the average of a set of two or more values. In this thesis I will only the arithmetical mathematic mean. One generally goes about to find the mean by adding every value in a set, and dividing it by the number of values within the set. This is also known as the arithmetic mean and is the type of mean I will use in this.

The arithmetic mathematical formula is:

$$\mu = \frac{1}{n} * \sum_{i=1}^{n} X_i$$

In which μ is the average/mean, n is the number of values and X_i is the value of each individual item in the list values.

5.7 Variance

Variance is a measure of how the spread among numbers is in a set of data. Variance is used within investments in order to measure the volatility of an investment. When the variance of the investment is calculated we can compare it to other similar investments in order to find out which one is the most risky. The higher the variance is, the greater is the volatility of the investment. And so the greater becomes the risk.

We can calculate variance by using the following formula:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2$$

σ^2 is the variance, X_i is each value of dataset, the symbol μ is the mean and n is the number of total data points.

5.8 Standard deviation (σ)

Standard deviation is an equipment to measure the risk by quantifying the variation of data. When the variation is high, than the data points are spread wider which leads to higher risk for the investment. When the variation is low it brings the standard deviation closer to the expected value, which decreases the risk for the investment. The formula of standard deviation is:

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2}$$

σ is the standard deviation, X_i is each value of dataset, the symbol μ will indicate the population mean in this formula and n is the number of total data points.

5.9 Skewness

Skewness is a measure of asymmetry of probability distribution of a real-valued random variable about its mean, the value of skewness can either be positive or negative, and even undefined. If skewness is positive it indicated that more of the returns are positive, and if it is negative then it indicates that more of the returns are negative.

Consistent with Excel we used this formula to calculate the skewness of S:

$$S = \frac{n\sum_{i=1}^{n}(x_i - \mu)^3}{((n-1)(n-2))\sigma^3}$$

μ is the mean, σ is the standard deviation of x. The formula requires that n >2 in order to avoid division by zero.

If $S(x) = 0$ (mean = median), the distribution is symmetrical

If $S(x) < 0$ (mean < median), the skewness is negative, and distribution is skewed to the left

If $S(x) > 0$ (mean > median), the skewness is positive, the distribution is skewed to the right

5.10 Kurtosis

Kurtosis is used as a measure of the "peakedness" or flatness of the probability distribution of a real-valued random variable. If the kurtosis is negative it indicates a relatively flat distribution. And if kurtosis is positive it indicates a relatively peaked distribution.

Consistent with Excel we used this formula to calculate kurtosis:

$$K = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\frac{\sum_{i=1}^{n}(X_i - \mu)^4}{\sigma^4} - \frac{3(n-1)^2}{(n-2)(n-3)}$$

In which μ is the mean, and σ is the standard deviation of K. This formula requires that n >3 to avoid division by zero.

If K(x) is large, it means that large realizations (positive or negative) are more likely to occur:

If K(x) = 0, distribution has the same tails as normal

If K(x) < 0, distribution has thinner tails than normal

If K(x) > 0, distribution has thicker tails than normal

We can use the kurtosis to measure the concentration of returns in any part of the distribution. The KURT function in excel refers to the "excess kurtosis", where 0 is normal distribution. The normal distribution is at 3 for other formulas of calculating kurtosis other than "sample excess kurtosis". An asset with low to negative excess kurtosis is preferable as it indicates more predictable returns. An exception to this is if there is a combination of high excess kurtosis and high positive skewness.
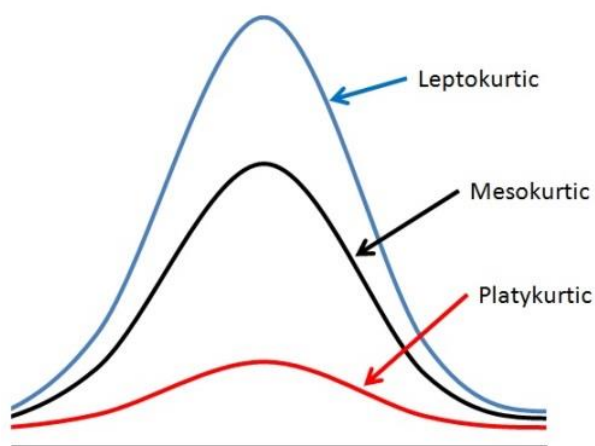
Figure 7: Kurtosis distribution

If the distribution of kurtosis is more than 3 it has positive excess kurtosis which is said to have heavy tails, the distribution puts more mass on the tails of its support than a normal distribution does. Such distribution contains more extreme values. If the kurtosis distribution is less than 3 it is platykurtic, while it is mesokurtic if the kurtosis distribution is similar or identical to the kurtosis of normal distribution.

5.11 Hypothesis

A hypothesis is an assumption that may be either true or false, and one might fail to reject it or may reject it on the basis of information. In statistical analysis a hypothesis is used on data that is subject to variability and uncertainty. Firstly in hypothesis testing one has to set up a null hypothesis, denoted by H_0. The null hypothesis refers to there being no relationship between two measured phenomena, or among groups. A null hypothesis is a hypothesis about one or more population parameter, which we hold to be true and do not reject unless there is enough statistical evidence to conclude otherwise. While the alternative hypothesis, denoted by H_1, is a contradiction to the null hypothesis that can be tested against it. If the null hypothesis is rejected or disregarded, then it is in favor of the alternative hypothesis.

In this thesis I have a significance level of 5 %, in order to claim that the results from the observations in the dataset lies with 95 % certainty inside the boundaries of a normal probability distribution. With this hypothesis we want to compare of if there is any difference between the passive strategy and an active strategy trading.

5.12 t-test

A t-test is a statistical method that is used to compare a sample mean of x and y to an accepted value, and it can be used to test whether there is a significant difference between the averages of two sets of data. I have used the two sample t-test for equality of two means.

The t-test formula is:

$$t = \frac{(\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

Where μ_x and μ_y are the sample means of x and y, σ_x^2 *and* σ_y^2 are the sample standard deviations of two sets of data of the size n_x *and* n_y.

And the following hypotheses have been used:

Null hypothesis H_0: $\mu_x = \mu_y$

Alternative hypothesis H_a: $\mu_x \neq \mu_y$

5.13 F-test

The F-test is a statistical method used to test if two variances are equal or not. The F-test simple random samples, we cannot make inferences about a population unless our sample is randomly selected can be either one or two-tailed. The difference between the two is that the significant level (α) for the two-tailed F-test needs to be divided by 2. I will be using a two-tailed test, also known as the Two-Sample hypothesis testing, to test if the variances of two samples are equal. So instead of $\alpha = 0.05$ we use $\alpha = 0.025$.

For the Two-Sample F-test for variances the following criteria have to be fulfilled for fairness:

- The data sets have to be simple random samples, because we cannot simply make inferences about a population unless the sample is randomly selected
- The data sets have to be independent, as the individuals cannot be measured twice
- The data sets have to be chosen from normally distributed populations, regardless of the size

The following formula is used to calculate the F-test statistics:

$$F = \frac{\sigma_x^2}{\sigma_y^2}$$

F stands for ratio of how big deviation we can have in our test, and σ_x^2 is the highest variance of one sample, and σ_y^2 is the lowest variance of the other sample.

And the following hypothesis has been used:

Null hypothesis: H_0: $\sigma_x^2 - \sigma_y^2 = 0$

Alternative hypothesis: H_a: $\sigma_x^2 - \sigma_y^2 \neq 0$

For the F-test the null hypothesis is always that the two variances are equal.

5.14 Sharpe ratio test

To test the statistical significant of changes which we found in the Sharpe ratios, I performed the SR test by Jobson and Korkie (1981), with the Mammel (2003) correction. Given two portfolios x and y, which in this thesis x stands for active strategy, and y stands for passive strategy. With SR_x, SR_y, the correlation coefficient ρ estimates their Sharpe ratios over a sample of size T.

The test of the null hypothesis:

H_0: $SR_x - SR_y = 0$

H_a: $SR_x - SR_y \neq 0$

This is obtained via the z-test statistic:

$$z = \frac{SR_x - SR_y}{\sqrt{\frac{1}{T}\left[2(1 - \rho^2) + \frac{1}{2}\left(SR_x^2 + SR_y^2 - 2SR_x SR_y \rho^2\right)\right]}}$$

Which is asymptotically distributed as a standard normal distribution. The z stands for z-test statistics, T is for number of observations, ρ is the correlation coefficient, SR_x is the Sharpe ratio of x and SR_y is the Sharpe ratio of y.

After using the z-test statistic, I found probability by using the formula $P = N(-|z|)$ for the Sharpe test. P stands for probability and N is the cumulative normal distribution, If the p-value is below the predetermined significant level, which is usually 0.05, one rejects the null hypothesis.

6 Empirical Results

I will analyze the performance of the SMA and MOM strategies. The SMA strategy is tested using a variable lookback period of 1-15 months (k) by using the in-sample test. The same will be done with the MOM strategy. The performance of the active strategies will be compared to the performance of the passive buy-and-hold strategy. Thereafter I will compare the performance of these two active strategies, in order to see which strategy delivers better performance based on the historical data. SMA and MOM signals are generated using prices not adjusted for dividends. Since we want to find the best performance, I calculate CAP and thereafter include dividends, which gives us the total return. When a sell signal is generated, the capital is invested in risk free assets, and when a buy signal is generated the stocks are held active. In total it will all show how the active strategy performs. The comparison will be based on which of the strategies generates the highest Sharpe ratio. I also added the p-value in order to test the null hypothesis, that the Sharpe ratio given by the active strategies is equal to the Sharpe ratio given by the passive strategy. If the p-value is lower than 0.05 the null hypothesis will be rejected and I will conclude that the performance of the active strategies are unequal to the performance of the passive strategy, which means that it can be better or worse than that of the passive strategy.

6.1 S&P500 (1926-2013)

The performance of the different SMA (k) lengths which were tested in the period 1926-2013 and MOM (k) lengths which were tested in the period 1926-2013 are shown below in Table 3,4,5 and 6.

Simple moving average

Table 3: Performance of SMA (k) lengths for S&P500 for the period 1926-2013

SMA (K)	Passive	1	2	3	4	5
Mean return	0,65 %	0,48 %	0,42 %	0,34 %	0,48 %	0,57 %
Min return	-29,46 %	-29,46 %	-24,48 %	-24,48 %	-24,48 %	-22,05 %
Max return	42,87 %	42,87 %	38,36 %	38,36 %	38,36 %	38,36 %
Stand.div	5,51 %	3,80 %	3,61 %	3,77 %	3,62 %	3,56 %
Kurtosis	9,61	30,07	17,58	16,36	17,06	16,22
Skewness	0,41	1,44	0,94	0,00	0,37	0,67
Sharpe ratio	0,12	0,13	0,12	0,09	0,13	0,16

SMA (K)		6	7	8	9	10
Mean return		0,51 %	0,55 %	0,58 %	0,60 %	0,60 %
Min return		-22,05 %	-22,05 %	-22,05 %	-22,05 %	-23,52 %
Max return		16,32 %	16,32 %	16,32 %	16,32 %	16,32 %
Stand.div		3,40 %	3,47 %	3,49 %	3,44 %	3,56 %
Kurtosis		5,54	5,77	5,67	5,50	6,65
Skewness		-0,61	-0,66	-0,64	-0,54	-0,75
Sharpe ratio		0,15	0,16	0,17	0,18	0,17

SMA (K)		11	12	13	14	15
Mean return		0,60 %	0,59 %	0,58 %	0,61 %	0,61 %
Min return		-23,52 %	-23,52 %	-23,52 %	-23,52 %	-23,52 %
Max return		16,32 %	16,32 %	16,32 %	16,32 %	16,32 %
Stand.div		3,56 %	3,57 %	3,57 %	3,53 %	3,55 %
Kurtosis		6,63	6,57	6,57	6,21	6,08
Skewness		-0,72	-0,72	-0,73	-0,58	-0,57
Sharpe ratio		0,17	0,17	0,16	0,17	0,17

Looking at Table 3 we can see that the best performance for S&P500 is given by the SMA (9) months. The mean return for this period is 0.60%. Although it is not the highest mean return compared to the other SMA (k) lengths, we still consider it to be the best performance length because it has the highest SR at 0.18. There are SMA (k) lengths which generate equal or higher mean return than the SMA (9). The reason why they do not have an equal or better SR than the SMA (9) is because they also have a standard deviation which is higher than that of SMA (9) caused by the variance using all returns. The higher standard deviation causes these SMA (k) periods to have lower SR than the SMA (9). Overall the average monthly returns for the SMA (k) lengths range from 0.34% to 0.61%. The passive strategy has higher mean return at 0.65 %, a total of 0.04% higher than the best mean return for the SMA strategy. However it has the highest standard deviation at 5.51%, and a SR of only 0.12. Overall SMA (3) computes the lowest SR at 0.09 and lowest mean return at 0.34 %.

Table 4: Statistical test performance of SMA (k) lengths for S&P500 in period from 1926-2013

SMA (K)	1	2	3	4	5
t-test	0,38	0,25	0,13	0,39	0,70
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,41	0,48	0,19	0,33	0,10

SMA (K)	6	7	8	9	10
t-test	0,49	0,61	0,72	0,79	0,78
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,17	0,12	0,08	0,05	0,07

SMA (K)	11	12	13	14	15
t-test	0,80	0,75	0,74	0,83	0,83
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,07	0,08	0,09	0,06	0,06

For table 4 the t-test has a significant level of $\alpha = 0.05$, but since we are conducting a two-tailed t-test we divide the alpha by 2 and operate with $\alpha = 0.025$. We can see from the table that none of the p-values for the SMA (k) lengths are lower than the significant level, and therefore we keep the null hypothesis and conclude with a 95% confidence that there is no difference between the active and passive strategy. We also have a two tailed F-test and therefore we use $\alpha = 0.025$. None of the p-values of the F-statistics for the SMA (k) lengths are higher than the significant level, and therefore we reject the null hypothesis and conclude that the passive and active strategy have unequal variances. For the Sharpe test, if the p-value is below the significant level of $\alpha = 0.05$, we reject the null hypothesis. We only reject the null hypothesis for SMA (9) because it is right on the verge of the significant level of 0.05, however if the significant level had been for instance 0.10, more of the p-values for the SMA would have rejected the null hypothesis. Based on the results from the t-test we do not reject the null hypothesis for any of the SMA (k) lengths. While for the Sharpe test we reject the null hypothesis for SMA (9), and for F-test we reject the null hypothesis for all of the SMA (k) lengths.

Time series momentum

Table 5: Performance of MOM (k) lengths for S&P500 in period from 1926-2013

MOM (K)	Passive	1	2	3	4	5
Mean return	0,65 %	0,47 %	0,34 %	0,43 %	0,51 %	0,62 %
Min return	-29,46 %	-29,46 %	-24,48 %	-29,46 %	-22,05 %	-22,05 %
Max return	42,87 %	42,87 %	38,36 %	38,36 %	16,86 %	16,86 %
Stand.div	5,51 %	3,80 %	3,88 %	3,83 %	3,40 %	3,48 %
Kurtosis	9,61	30,05	16,16	18,29	5,77	5,56
Skewness	0,41	1,45	0,29	-0,30	-0,41	-0,38
Sharpe ratio	0,12	0,12	0,09	0,11	0,15	0,18

MOM (K)		6	7	8	9	10
Mean return		0,54 %	0,58 %	0,54 %	0,61 %	0,65 %
Min return		-22,05 %	-22,05 %	-23,52 %	-23,52 %	-23,52 %
Max return		16,32 %	16,32 %	13,57 %	42,87 %	42,87 %
Stand.div		3,47 %	3,46 %	3,52 %	3,80 %	3,77 %
Kurtosis		5,67	5,52	6,65	19,21	19,33
Skewness		-0,68	-0,61	-0,87	0,67	0,84
Sharpe ratio		0,16	0,17	0,15	0,16	0,17

MOM (K)		11	12	13	14	15
Mean return		0,57 %	0,56 %	0,50 %	0,49 %	0,49 %
Min return		-23,52 %	-23,52 %	-23,52 %	-22,05 %	-22,05 %
Max return		42,87 %	16,32 %	16,32 %	16,32 %	16,32 %
Stand.div		3,80 %	3,63 %	3,67 %	3,60 %	3,60 %
Kurtosis		18,92	5,86	5,62	4,38	4,37
Skewness		0,78	-0,62	-0,60	-0,36	-0,36
Sharpe ratio		0,15	0,15	0,14	0,14	0,14

The results from Table 5 show that the best performing MOM (k) is 5 months. It gives us the highest SR out of all the MOM (k) lengths with a SR of 0.18. MOM (5) does not generate a high mean return, but the reason why it is the best performing is because the standard deviation in this situation is lower than in the other MOM (k) lengths where the mean is equal or higher than for MOM (5). For MOM (k) lengths we have a mean return that varies from 0.34% to 0.65%. The MOM (2) length gives the lowest SR of 0.09, and it also has the lowest

mean return of 0.34%. The second best performing MOM (k) lengths are MOM (7) and MOM (10) which have an equal SR of 0.17. MOM (10) has a mean return of 0.65 %, which is the highest mean return out of all the MOM (k) lengths. However it has a standard deviation at 3.77 %, which is higher than the 3.48 % of MOM (5), and thus it is not the best performing. The passive strategy has a SR of 0.12, only MOM (2) and MOM (3) have lower SR at 0.09 and 0.011 respectively. MOM (1) has equal SR to the passive strategy.

Table 6: Statistical test performance of MOM (k) lengths for S&P500 in period from 1926-2013

MOM (K)	1	2	3	4	5
t-test	0,38	0,13	0,27	0,46	0,86
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,44	0,35	0,43	0,19	0,04

MOM (K)	6	7	8	9	10
t-test	0,58	0,71	0,59	0,81	0,97
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,13	0,08	0,15	0,10	0,05

MOM (K)	11	12	13	14	15
t-test	0,68	0,63	0,47	0,43	0,43
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,17	0,15	0,29	0,30	0,30

The t-test significant level we operate with was $\alpha = 0.025$. From Table 6 we can see that all of the MOM (k) lengths p-values are higher than significant level, and so we keep the null hypothesis and conclude with a 95% confidence that there is no difference between the MOM (k) lengths and the passive strategy. The significant level for F-test is also $\alpha = 0.025$, and as we can see all of the p-values of the F-statistics for the MOM (k) lengths are lower than the significant level, and as such we reject the null hypothesis. For the Sharpe test we reject the null hypothesis if the p-value is below the significant level of $\alpha = 0.05$, There are no p-values for the S&P500 that are lower than $\alpha = 0.05$, except for MOM (5) which has a p-value of 0.04 and MOM (10) which has a p-value of 0.05 that is on the verge of the significant level. If the significant level was for instance 0.10, more p-values would have rejected the null hypothesis, and if it had been 0.01, none of the p-values would reject the null hypothesis. Therefore we only reject the null hypothesis for MOM (5) and MOM (10) lengths. This means that there is a significant difference between the two MOM (k) lengths and the passive strategy SR. Out of all the tests, only the F-test and the Sharpe test for the MOM (5) and MOM (10) rejects the null hypothesis.

6.2 Swedish stock market (1919-2006)

The performance results of the different SMA (k) lengths which were tested in the period 1919-2006 and MOM (k) lengths which were tested in the period 1919-2006 are shown below in Table 7,8,9 and 10.

Simple moving average

Table 7: Performance of the SMA (k) lengths for Swedish stock market in period 1919-2006

SMA (K)	Passive	1	2	3	4	5
Mean return	0,52 %	0,63 %	0,62 %	0,62 %	0,60 %	0,61 %
Min return	-27,53 %	-21,19 %	-21,19 %	-21,19 %	-21,19 %	-21,19 %
Max return	26,67 %	26,67 %	17,77 %	17,77 %	17,77 %	17,77 %
Stand.div	4,84 %	3,51 %	3,50 %	3,55 %	3,50 %	3,56 %
Kurtosis	3,23	7,61	5,12	4,59	4,85	4,57
Skewness	-0,19	0,69	0,20	0,17	0,14	0,12
Sharpe ratio	0,11	0,18	0,18	0,18	0,17	0,17

SMA (K)		6	7	8	9	10
Mean return		0,62 %	0,63 %	0,68 %	0,66 %	0,70 %
Min return		-21,19 %	-21,19 %	-21,19 %	-21,19 %	-21,19 %
Max return		17,77 %	17,77 %	17,77 %	17,77 %	17,77 %
Stand.div		3,50 %	3,50 %	3,44 %	3,45 %	3,42 %
Kurtosis		4,74	4,71	4,73	4,28	4,39
Skewness		0,13	0,13	0,23	0,09	0,11
Sharpe ratio		0,18	0,18	0,20	0,19	0,20

SMA (K)		11	12	13	14	15
Mean return		0,71 %	0,70 %	0,71 %	0,71 %	0,70 %
Min return		-21,19 %	-21,19 %	-21,19 %	-21,19 %	-21,19 %
Max return		17,77 %	17,77 %	17,77 %	17,77 %	17,77 %
Stand.div		3,42 %	3,44 %	3,45 %	3,43 %	3,46 %
Kurtosis		4,39	4,27	4,23	4,34	4,18
Skewness		0,11	0,11	0,15	0,15	0,13
Sharpe ratio		0,21	0,20	0,21	0,21	0,20

From Table 7 we can see that the best performance SMA is 11 months, when we rank according to SR and standard deviation. The 11, 13 and 14 months have equal SR ratio of 0.21. Both SMA (13) and SMA (14) perform very similarly on standard deviation at 3.45%

and 3.43% respectively, compared to 3.42% of SMA (11). The results are quite equal for the 15 different SMA lengths which are tested, with the monthly average returns ranging from 0.52% to 0.71%. When compared to the SMA strategy, the passive strategy performs very poorly, having the lowest SR of 0.11 and lowest mean return of 0.52% in the table. Among the SMA (k) lengths, the lowest SR performance is given by SMA (4) and SMA (5). Both SR of 0.17, which is still 0.06 higher than that of the passive strategy, and they have an average mean return of 0.60% and 0.61%, and a standard deviation of 3.50% and 3.56% respectively.

Table 8: Statistical test performance of SMA (k) lengths for Swedish stock market in period 1919-2006

SMA (K)	1	2	3	4	5
t-test	0,57	0,64	0,64	0,71	0,69
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,04	0,05	0,04	0,06	0,28

SMA (K)	6	7	8	9	10
t-test	0,65	0,62	0,46	0,55	0,43
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,05	0,04	0,02	0,03	0,02

SMA (K)	11	12	13	14	15
t -test	0,38	0,39	0,35	0,35	0,39
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,02	0,02	0,02	0,02	0,02

For Table 8 the t-test α is 0.025 as previously determined. The significant level is exceeded by all of the SMA (k) lengths for the Swedish stock market. Because of this, we keep the null hypothesis and conclude with a 95% confidence that there is no difference between the SMA (k) lengths and the passive strategy. The F-test α is also 0.025. All p-values of the F-statistics for the SMA (k) lengths that are lower than $\alpha = 0.025$, and therefore we reject the null hypothesis that the active and passive strategy are equal. For the Sharpe test the significant level is 0.05. The only p-values that are higher than $\alpha = 0.05$, are SMA (4) and SMA (5), for these k lengths we do not reject the null hypothesis. The p-value is lower or on the verge of the significant level for all the other SMA (k) lengths, and therefore the null hypothesis is rejected for those, meaning there is a significant difference between the passive strategy SR and the SR for those SMA (k) lengths.

Time series momentum

Table 9: Performance of MOM (k) lengths for Swedish stock market in period 1919-2006

MOM (K)	Passive	1	2	3	4	5
Mean return	0,52 %	0,63 %	0,56 %	0,63 %	0,58 %	0,60 %
Min return	-27,53 %	-21,19 %	-27,53 %	-21,19 %	-22,66 %	-22,66 %
Max return	26,67 %	26,67 %	17,77 %	17,77 %	17,77 %	17,77 %
Stand.div	4,84 %	3,51 %	3,56 %	3,47 %	3,55 %	3,51 %
Kurtosis	3,23	7,61	7,83	4,75	5,79	5,95
Skewness	-0,19	0,69	-0,37	0,08	-0,17	-0,09
Sharpe ratio	0,11	0,18	0,16	0,18	0,16	0,17

MOM (K)		6	7	8	9	10
Mean return		0,66 %	0,57 %	0,62 %	0,60 %	0,68 %
Min return		-22,66 %	-21,19 %	-21,19 %	-22,66 %	-21,19 %
Max return		17,77 %	17,77 %	17,77 %	17,77 %	17,77 %
Stand.div		3,55 %	3,52 %	3,46 %	3,61 %	3,56 %
Kurtosis		5,77	4,30	4,47	5,25	4,32
Skewness		-0,03	-0,08	-0,01	-0,22	0,11
Sharpe ratio		0,19	0,16	0,18	0,17	0,19

MOM (K)		11	12	13	14	15
Mean return		0,67 %	0,69 %	0,70 %	0,64 %	0,60 %
Min return		-21,19 %	-21,19 %	-21,19 %	-21,19 %	-22,66 %
Max return		17,77 %	17,77 %	17,77 %	17,77 %	17,77 %
Stand.div		3,53 %	3,61 %	3,59 %	3,59 %	3,65 %
Kurtosis		4,11	3,87	3,97	3,97	5,09
Skewness		0,11	0,06	0,04	0,03	-0,24
Sharpe ratio		0,19	0,19	0,19	0,18	0,16

The results for Table 9 shows that MOM (k) length with the best performance is MOM (13), it has an average mean return of 0.70 and standard deviation of 3.59%, which generates the highest SR at 0.1949 when we include more decimals. MOM (10) and MOM (12) have an equal SR at 0.19, but when we take a closer look at the numbers, they generate a SR at 0.1910 and 0.1911 respectively, which is still lower than the SR of MOM (13). The standard deviation for MOM (10) and MOM (12) months is 3.56% and 3.61% respectively, varying only a little from 3.59% of MOM (13) months. The results are similar for the 15 different MOM lengths which are tested, with the monthly average returns ranging from 0.52% to

0.70%.The passive strategy has the lowest SR of 0.11 and mean return of 0.52% when compared to the MOM strategy. Among the MOM (k) lengths MOM (2) has the lowest performance, with an average mean return of 0.56%, standard deviation at 3.56% and a SR of only 0.16, causing it to have lower performance than the other MOM (k) lengths that generate and equal SR.

Table 10: Statistical test performance of MOM (k) lengths for Swedish stock market in period 1919-2006

MOM (K)	1	2	3	4	5
t-test	0,59	0,87	0,61	0,82	0,72
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,04	0,09	0,04	0,08	0,06
MOM (K)	6	7	8	9	10
t-test	0,48	0,86	0,66	0,78	0,50
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,03	0,08	0,04	0,06	0,02
MOM (K)	11	12	13	14	15
t-test	0,51	0,41	0,40	0,60	0,76
F-test	0,00	0,00	0,00	0,00	0,00
Sharpe test	0,02	0,02	0,02	0,03	0,06

In Table 10 the significant level of t-test was 0.025. All of the p-values for the MOM (k) lengths are higher than the significant level. Therefore the null hypothesis is true, and we can conclude with a 95% confidence that there is no difference between the MOM (k) lengths and the passive strategy. The significant level of the F-test is also $\alpha = 0.025$ and there are no p-values for the MOM (k) lengths that are higher than the significant level. That is why we reject the null hypothesis for the F-test. As for the Sharpe test, if the p-value is below the significant level of $\alpha = 0.05$, we reject the null hypothesis. The MOM (k) lengths with a higher p-value than the significant level are MOM (2), MOM (4), MOM (5), MOM (7), MOM (9) and MOM (15). For these values we do not reject the null hypothesis, but for all the other Sharpe test p-values which are lower than $\alpha = 0.05$ we reject the null hypothesis.

This means that there is a significant difference between all the other MOM (k) lengths and the passive strategy SR. If the significant level had been 0.01, there would not be any p-values that rejected the null hypothesis.

7 Discussion

From the results I can see that the active strategies, which included the SMA and MOM, perform better than the passive buy-and-hold strategy. This was the case for both the S&P500 and the Swedish stock market. In this section I will attempt to explain the differences in performance of the strategies.

The main indicator of the performance of the strategies is SR, as it is good measure of the returns generated per unit of volatility by a portfolio. The highest SR for both the MOM and SMA strategy for the S&P500 is 0.18, which is 0.06 higher than the SR of the passive strategy. While for the Swedish stock market the SMA with the highest SR is 0.21, which is 0.10 higher than the SR for the passive strategy. For MOM the highest SR is 0.19, that is 0.08 higher than for the passive strategy. Based on the SR generated from the performance of the different strategies, it is evident that the SMA and MOM strategies perform better than the passive strategy. This is due to standard deviation, which is much lower for the active strategies than the passive. The results might have been altered if transaction costs were included, as this would have decreased the SR generated by the active strategies.

One weakness to SR is that it sees risk as volatility, and volatility is bad. All the volatility is treated the same, meaning it does not differentiate between upside and downside volatility. Those who use upside volatility strategies do not think it should be viewed as a bad thing. However though some view this as a weakness, we cannot overlook that volatility that goes up or down, no matter the return, has an instability that is a risk which SR includes in its calculations.

For S&P500 the passive strategy generated highest mean return of 0.65%, which was higher than the mean return for all of the SMA (k) lengths. The only MOM (k) length that could

compare with the passive strategy was MOM (10), which had an equal mean return of 0.65%. While passive strategy generated the lowest return for the Swedish stock market compared to both the active strategies. Looking at the SR of the performance only might be misleading, as we can see the passive strategy also proves it can generate a high mean return compared to the active strategies. However the passive strategy has higher standard deviation than the active strategies both for S&P500 and the Swedish stock market, which confirms that traders who use a passive strategy also have a higher risk tied to their investment. This is also why SR is lower for the passive strategy than the active strategies. Even when the active strategies generate a lower mean return, they have reduced risk and therefore have a higher SR.

The general perception within economics is that the reduction in standard deviation means reduced risk, which ultimately generates a lower return. According to MPT we know that that traders can attempt to maximize the portfolio expected return and minimize the portfolio risk by choosing the right combining of stocks through diversification. This way one can try to reduce the risk while not reducing the expected return. From the results we can see that the active strategies have lower standard deviation while not having adverse impact on the mean return. As the return was higher for the active strategies than the passive strategy for the Swedish stock market despite having lower standard deviation, and for the S&P500 MOM (10) had similar return to the passive strategy even though it also had lower standard deviation. This shows that the active strategies have been able to reduce risk while not having to compensate with return for the most part in our results. The reason why standard deviation is lower for market-timing strategies is because investors who use them, invest their assets in risk free investment while they are out of the marked by selling their stock. Investors who use the passive strategy hold their assets regardless of the fluctuations in the market, and therefore they are more exposed to risk, which is why standard deviation is higher for the passive strategy.

When investors use the market timing strategies it can potentially go both ways, meaning that they can exit a market and thereby avert major losses, but also miss out on large profits. The SMA strategy works as a reactionary indicator. Longer term SMA becomes a lagged version of previous realization and that gives it the ability to exit a market in the case of a historical negative trend. However it might react too slowly and in the time it takes to switch back to a positive trend, investors might have already missed out on a big return. In practice investors

can potentially solve this problem by using a shorter time frame SMA when trading, because then it takes less time to react as one is calculating more recent data. So a negative thing for the users of the market timing strategies in practice is that they are at risk of having less returns than passive strategy users as a result of not being present in the market at the right time and therefore missing out on major returns.

When comparing the results for active strategies, I can see that for S&P500 the highest SR is 0.18 for both MOM (5) and SMA (9), while the lowest SR is 0.09 for both SMA(2) and MOM(3). Based on these results the strategies perform similarly and neither of the two strategies outperforms the other. For the Swedish stock market the highest SR is 0.21, generated by SMA(11), and 0.19 for MOM(13). Lowest SR for SMA is 0.17 generated by SMA(4) and SMA(5), and 0.16 for MOM(2). The performance of the SMA strategy seems to be slightly better than that of the MOM strategy; however the difference is neither big nor substantial enough to conclude that the SMA strategy is superior to the MOM strategy. The optimal "k" length for the US stock market and the Swedish stock market is different for both SMA and MOM. This indicates that there is no single optimal "k" length that can be used for all markets. For the Swedish stock market the p-values are lower and therefore I could reject the null hypothesis for more p-values in the Swedish stock market than for S&P500. In addition the difference in SR is higher between the SMA and MOM (k) length in the Swedish stock market. This tells us that in the Swedish stock market it is easier to implement the market timing strategies and they have better performance. The fact that the market timing strategies show better performance for the Swedish stock market means, according to the efficient market hypothesis that the Swedish stock market is less efficient than the S&P500.

When comparing the results on the performance of the market timing strategies tested in this thesis to existing research papers, we can see that the results are consistent with many of them. Some of these are Faber (2007) and Brock, Lekonishok and LeBaron (1992), who got favorable results for the active strategies in that they generated higher returns than the passive strategy, and reduced losses which contributed to increased returns. In addition to Markowitz, Ooi and Pedersen (2012) who found MOM to have a consistent good performance. Similarly in this thesis the market-timing strategies outperformed the passive strategy. However despite the fact that our in-sample testing results show that the active strategies are superior to the passive buy-and-hold strategy in their performance, there is not substantial enough evidence

to claim that the efficient market theory is incorrect in claiming that historical prices do not have any impact on the current prices. One of reasons why we cannot make such a claim is because there has not been performed an out-of-sample test for this thesis, and as such it could be affected by data mining. In addition I do not include the transaction costs, and we saw from previous studies done by Bassembinder and Chan (1998) that the advantage of using the technical trading rules were eliminated after the transaction costs were included. And so I do know if the results would have been as favorable for the active strategies if we had included transaction costs.

Based on the aspects discussed above there is not strong enough evidence to conclude which strategy is best to use. In order to be able to determine this, it is necessary to conduct a further extensive research as to which strategy outperforms the other and under which circumstances it does so.

8 Conclusion

In this thesis I have analyzed the performance of two of the most well-known technical strategies, the simple moving average and time-series momentum strategy. I used data for S&P500 and the Swedish stock market, for 1053 and 1052 months respectively, for a variable lookback period of 1-15 months (k) using in-sample testing. The performance of the active strategies was compared to the performance of the passive strategy in order to find out which strategy has the best performance.

I find that when using the SMA and MOM strategies with a variable lookback period of 1-15 (k) months and applying them to the two sets data by using in-sample testing, the SMA and MOM strategies were able to outperform the buy-and-hold strategy for many sets of "k" months. The active strategies had reduced standard deviation compared to the passive strategy, and were still for the most part able to generate equal or higher mean return than the passive strategy. This was despite the view that a lower risk should be followed by lower return, and opposite. The results of this thesis are consistent with, and reflective of the results

of others such as Faber (2007), Brock, Lekonishok and LeBaron (1992), and Markowitz, Ooi and Pedersen (2012).

Regardless of the favorable results on the performance of the active strategies, and in that especially the SMA strategy for the Swedish stock market, I did not have substantial enough evidence to conclude that the active strategies are in fact superior to the passive strategy. One of the reasons being that we have only conducted an in-sample test, which does not provide us accurate enough results to be able to come to such a conclusion. And secondly because the favorable performance of the market-timing strategies could have less favorable if transaction costs were included. I conclude that based on the results of this thesis the market-timing strategies outperform the passive buy-and-hold strategy, but in order for this to be a conclusive answer there needs to be conducted further extensive research.

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