

Research Article

\mathcal{H}_∞ Model Reduction for Discrete-Time Markovian Jump Systems with Deficient Mode Information

Yanling Wei,¹ Mao Wang,¹ Hamid Reza Karimi,² Nan Wang,¹ and Jianbin Qiu¹

¹ Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, China

² Department of Engineering, Faculty of Engineering and Science, University of Agder, 4898 Grimstad, Norway

Correspondence should be addressed to Jianbin Qiu; jbqiu@hit.edu.cn

Received 4 April 2013; Accepted 3 June 2013

Academic Editor: Xiaojie Su

Copyright © 2013 Yanling Wei et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper investigates the problem of \mathcal{H}_∞ model reduction for a class of discrete-time Markovian jump linear systems (MJLSs) with deficient mode information, which simultaneously involves the exactly known, partially unknown, and uncertain transition probabilities. By fully utilizing the properties of the transition probability matrices, together with the convexification of uncertain domains, a new \mathcal{H}_∞ performance analysis criterion for the underlying MJLSs is first derived, and then two approaches, namely, the convex linearisation approach and iterative approach, for the \mathcal{H}_∞ model reduction synthesis are proposed. Finally, a simulation example is provided to illustrate the effectiveness of the proposed design methods.

1. Introduction

Markovian jump linear systems (MJLSs), initially proposed by Krasovskii and Lidskii [1], have been attracting increasing attention over the past decades. The appeal for the study of this class of hybrid systems is that MJLSs are powerful to model plants whose structures are subject to random abrupt changes, for example, the fault-prone manufacturing systems [2], power systems [3], economic systems [4], networked control systems (NCSs) [5–10], and so on. As a critical factor, the transition probabilities (TPs) in the Markov chain determine the jump rules of different modes and are assumed to be completely known in most of the existing literature [11–14].

However, it is noted that, in many practical engineering systems, not all the mode transition information can be obtained exactly. Recently, there have appeared some results on the analysis and synthesis of MJLSs with uncertain TPs [15–18] or partially unknown TPs [19–21]. To mention a few, the authors in [15] considered the robust stochastic stability analysis problem for a class of MJLSs with norm-bounded uncertainties in the TPs; the author in [16] dealt with the robust stability analysis and stabilization problems for MJLSs with polytopic-type uncertain TPs; the authors in [17] studied the robust stability analysis for a class of discrete-time

Markovian jumping neural networks with defective statistics of mode transitions; the authors in [19] investigated the \mathcal{H}_∞ filtering design for MJLSs with partially unknown TPs. In particular, it is noted that the introduction of deficient mode information, which simultaneously contains the exactly known, partially unknown, and uncertain TPs, is very important and useful in both theoretical advances and practical applications of MJLSs.

On the other hand, in many engineering applications, high- or even infinite-order complex mathematical models are frequently used to describe physical systems and processes [22–24]. These high-order models bring serious difficulties to analysis and synthesis of the underlying systems. Therefore, model simplification/reduction with respect to certain criteria has received considerable attention in recent years. Correspondingly, many important results on model reduction have been obtained, involving various efficient approaches such as the Hankel norm approximation method [25], the \mathcal{H}_2 approach [26, 27], the \mathcal{H}_∞ approach [28–30], and the \mathcal{L}_2 - \mathcal{L}_∞ approach [31]. More recently, the linear matrix inequality (LMI) technique has been exploited to solve the model reduction problem for different classes of systems, such as bilinear systems [27], time-delay systems [28], and T-S fuzzy systems [32–36]. Specifically, the authors in [4, 20]

considered the \mathcal{H}_∞ model reduction problem for a class of MJLSs with completely known TPs and partially unknown TPs, respectively. However, to the authors' best knowledge, few results have been reported on \mathcal{H}_∞ model reduction for MJLSs with deficient mode information, which simultaneously consists of the exactly known, partially unknown, and uncertain TPs. This motivates us for the present study.

According to the issues mentioned previously, in this paper, we will make an attempt to tackle the \mathcal{H}_∞ model reduction problem for a class of discrete-time MJLSs with deficient mode information. Such deficient mode information simultaneously contains the exactly known, partially unknown, and uncertain TPs, which is a more practical scenario. By fully utilizing the properties of the transition probability matrices, together with the convexification of uncertain domains, a new \mathcal{H}_∞ performance analysis criterion for MJLSs with deficient mode information will be firstly derived. To solve the model reduction problem, two distinctly different approaches will be then proposed. The first approach is based on a linearisation procedure, which casts the model reduction into a convex optimization problem. The second one, which is based on the cone complementarity linearisation (CCL) idea [37, 38], casts the model reduction into a sequential minimization problem subject to LMI constraints. The effectiveness and superiority of the proposed approaches will be illustrated by a simulation example.

Notations. The notations used throughout the paper are standard. \mathbf{R}^n and $\mathbf{R}^{m \times n}$ denote, respectively, the n -dimensional Euclidean space and the set of all $m \times n$ real matrices; \mathbb{N}^+ represents the set of positive integers; the notation $P > 0$ means that P is real symmetric and positive definite; \mathbf{I} and $\mathbf{0}$ represent the identity matrix and a zero matrix, respectively; $\|\cdot\|$ refers to the Euclidean vector norm; $l_2[0, \infty)$ is the space of square summable infinite sequence and, for $w = \{w(k)\} \in l_2[0, \infty)$, its norm is given by $\|w\|_2 = \sqrt{\sum_{k=0}^{\infty} \|w(k)\|^2}$; $(\mathcal{S}, \mathcal{F}, \mathcal{P})$ denotes a complete probability space, in which \mathcal{S} is the sample space, \mathcal{F} is the σ algebra of subsets of the sample space, and \mathcal{P} is the probability measure on \mathcal{F} ; $\mathcal{E}[\cdot]$ stands for the mathematical expectation and, for the sequence $z = \{z(k)\} \in l_2((\mathcal{S}, \mathcal{F}, \mathcal{P}), [0, \infty))$, its norm is given by $\|z\|_{\mathcal{E}_2} = \sqrt{\mathcal{E}[\sum_{k=0}^{\infty} \|z(k)\|^2]}$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation and Preliminaries

A discrete-time Markovian jump linear system (MJLS) on a complete probability space $(\mathcal{S}, \mathcal{F}, \mathcal{P})$ can be described as

$$\begin{aligned} (\Sigma): \quad & x(k+1) = A(r(k))x(k) + B(r(k))u(k), \\ & z(k) = C(r(k))x(k) + D(r(k))u(k), \end{aligned} \quad (1)$$

where $x(k) \in \mathbf{R}^{n_x}$ is the state vector; $u(k) \in \mathbf{R}^{n_u}$ is the control input vector which belongs to $l_2[0, \infty)$; $z(k) \in \mathbf{R}^{n_z}$ is the output vector. The process $\{r(k), k \geq 0\}$ is described by a discrete-time homogeneous Markov chain, which takes

values in a finite set $\mathcal{S} := \{1, \dots, N\}$ with mode transition probabilities (TPs) $\Pr(r(k+1) = j \mid r(k) = i) = \pi_{ij}$, where $\pi_{ij} \geq 0$, for all $i, j \in \mathcal{S}$, and $\sum_{j=1}^N \pi_{ij} = 1$. For $r(k) = i, i \in \mathcal{S}$, the system matrices of the i th mode are denoted by (A_i, B_i, C_i, D_i) , which are real and known. Throughout the paper, we assume that system (Σ) is stochastically stable, which is a prerequisite for model reduction design.

In addition, the TPs of the jumping process are considered to be polytopic-type uncertain and partially accessed; that is, the transition probability matrix (TPM) $\Pi = [\pi_{ij}]_{N \times N}$ is assumed to belong to a given polytope $P_\Pi := \{\Pi \mid \Pi = \sum_{s=1}^M \alpha_s \Pi_s; \alpha_s \geq 0, \sum_{s=1}^M \alpha_s = 1\}$, where vertices $\Pi_s = [\pi_{ij}]_{N \times N}$, $i, j \in \mathcal{S}, s = 1, 2, \dots, M$ are given TPMs containing unknown elements still. For instance, for system (Σ) with four operation modes, the TPM may be as

$$\begin{bmatrix} \pi_{11} & \tilde{\pi}_{12} & \hat{\pi}_{13} & \pi_{14} \\ \tilde{\pi}_{21} & \pi_{22} & \tilde{\pi}_{23} & \pi_{24} \\ \pi_{31} & \tilde{\pi}_{32} & \pi_{33} & \tilde{\pi}_{34} \\ \pi_{41} & \tilde{\pi}_{42} & \tilde{\pi}_{43} & \tilde{\pi}_{44} \end{bmatrix}, \quad (2)$$

where the elements labeled with “ \sim ” and “ $\hat{\cdot}$ ” represent the unknown information and polytopic uncertainties on TPs, respectively, and the others are known TPs. For notational clarity, for all $i \in \mathcal{S}$, we denote $\mathcal{S} = \mathcal{S}_{\mathcal{K}}^{(i)} \cup \mathcal{S}_{\mathcal{U}\mathcal{E}}^{(i)} \cup \mathcal{S}_{\mathcal{U}\mathcal{X}}^{(i)}$ as follows:

$$\begin{aligned} \mathcal{S}_{\mathcal{K}}^{(i)} &:= \{j : \pi_{ij} \text{ is known}\}, \\ \mathcal{S}_{\mathcal{U}\mathcal{E}}^{(i)} &:= \{j : \tilde{\pi}_{ij} \text{ is uncertain}\}, \\ \mathcal{S}_{\mathcal{U}\mathcal{X}}^{(i)} &:= \{j : \hat{\pi}_{ij} \text{ is unknown}\}. \end{aligned} \quad (3)$$

Moreover, if $\mathcal{S}_{\mathcal{K}}^{(i)} \neq \emptyset$ and $\mathcal{S}_{\mathcal{U}\mathcal{E}}^{(i)} \neq \emptyset$, it is further described as

$$\begin{aligned} \mathcal{S}_{\mathcal{K}}^{(i)} &:= \{\mathcal{K}_1^{(i)}, \dots, \mathcal{K}_{t_i}^{(i)}\}, \quad \forall 1 \leq t_i \leq N-2, \\ \mathcal{S}_{\mathcal{U}\mathcal{E}}^{(i)} &:= \{\mathcal{U}_1^{(i)}, \dots, \mathcal{U}_{v_i}^{(i)}\}, \quad \forall 1 \leq v_i \leq N, \end{aligned} \quad (4)$$

where $\mathcal{K}_{t_i}^{(i)} \in \mathbb{N}^+$ represents the t_i th known element with the index $\mathcal{K}_{t_i}^{(i)}$ in the i th row of TPM and $\mathcal{U}_{v_i}^{(i)} \in \mathbb{N}^+$ represents the v_i th uncertain element with the index $\mathcal{U}_{v_i}^{(i)}$ in the i th row of TPM, respectively. Obviously, $1 \leq t_i + v_i \leq N$. Also, we denote

$$\pi_{\mathcal{U}\mathcal{X}}^{(is)} := \sum_{j \in \mathcal{S}_{\mathcal{U}\mathcal{X}}^{(i)}} \hat{\pi}_{ij} = 1 - \sum_{j \in \mathcal{S}_{\mathcal{K}}^{(i)}} \pi_{ij} - \sum_{j \in \mathcal{S}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)}, \quad (5)$$

where $\tilde{\pi}_{ij}^{(s)}$ represents an uncertain TP in the s th polytope, for all $s = 1, \dots, M$.

To approximate the original MJLS (1), in this paper, we are interested in designing the following mode-dependent reduced-order model:

$$\begin{aligned} (\bar{\Sigma}): \quad & \hat{x}(k+1) = A_{r_i} \hat{x}(k) + B_{r_i} u(k), \\ & \hat{z}(k) = C_{r_i} \hat{x}(k) + D_{r_i} u(k), \end{aligned} \quad (6)$$

where $\hat{x}(k) \in \mathbf{R}^{n_r}$ ($n_r < n_x$), $\hat{z}(k) \in \mathbf{R}^{n_z}$, and $A_{ri} \in \mathbf{R}^{n_r \times n_r}$, $B_{ri} \in \mathbf{R}^{n_r \times n_u}$, $C_{ri} \in \mathbf{R}^{n_z \times n_r}$, and $D_{ri} \in \mathbf{R}^{n_z \times n_u}$ are the gains of the reduced-order models to be determined.

Define $\bar{x}(k) := [x^T(k) \ \hat{x}^T(k)]^T$ and $\bar{z}(k) := z(k) - \hat{z}(k)$. Then, by augmenting (1) and (6), the model error dynamics can be described as

$$\begin{aligned} (\bar{\Sigma}): \quad \bar{x}(k+1) &= \bar{A}_i \bar{x}(k) + \bar{B}_i u(k), \\ \bar{z}(k) &= \bar{C}_i \bar{x}(k) + \bar{D}_i u(k), \end{aligned} \quad (7)$$

where

$$\begin{aligned} \bar{A}_i &:= \begin{bmatrix} A_i & \mathbf{0} \\ \mathbf{0} & A_{ri} \end{bmatrix}, & \bar{B}_i &:= \begin{bmatrix} B_i \\ B_{ri} \end{bmatrix}, \\ \bar{C}_i &:= [C_i \ -C_{ri}], & \bar{D}_i &:= D_i - D_{ri}. \end{aligned} \quad (8)$$

Therefore, the purpose of this paper is to design a mode-dependent reduced-order model in the form of (6), such that the model error system $(\bar{\Sigma})$ in (7) with deficient mode information is stochastically stable and the induced l_2 -norm of the operator from $u(k)$ to the model error $\bar{z}(k)$ is less than γ ; that is, $\|\bar{z}(k)\|_{\mathcal{L}_2} < \gamma \|u(k)\|_2$, under zero initial conditions for any nonzero $u(k) \in l_2[0, \infty)$.

Before ending the section, we give the following lemma on the \mathcal{H}_∞ performance analysis of system (7) with *completely known* TPs, which will be used in the proof of our main results.

Lemma 1 (see [4]). *For the MJLS (7) with completely known TPs and a given scalar $\gamma > 0$, if the coupled inequalities*

$$\begin{aligned} \left[\begin{array}{cc} \bar{A}_i^T \mathcal{P}_i \bar{A}_i - P_i + \bar{C}_i^T \bar{C}_i & \bar{A}_i^T \mathcal{P}_i \bar{B}_i + \bar{C}_i^T \bar{D}_i \\ * & -(\gamma^2 \mathbf{I} - \bar{B}_i^T \mathcal{P}_i \bar{B}_i - \bar{D}_i^T \bar{D}_i) \end{array} \right] < 0, \\ \forall i \in \mathcal{I}, \end{aligned} \quad (9)$$

where $\mathcal{P}_i := \sum_{j=1}^N \pi_{ij} P_j$, have a feasible solution $P = \{P_1, P_2, \dots, P_N\}$ with $P_i > 0$, then the MJLS (7) with completely known TPs is stochastically stable with an \mathcal{H}_∞ performance γ .

3. Main Results

In this section, we will first derive the \mathcal{H}_∞ performance analysis criterion for the model error system $(\bar{\Sigma})$ in (7) with deficient mode information. Then, two distinctly different approaches will be proposed to solve the \mathcal{H}_∞ model reduction problem formulated in the previous section.

3.1. \mathcal{H}_∞ Performance Analysis. In the following, by exploiting the properties of the transition probability matrices (TPMs), together with the convexification of uncertain domains, an \mathcal{H}_∞ performance analysis criterion for the model error system $(\bar{\Sigma})$ in (7) with deficient mode information is presented, which will play an instrumental role in solving the \mathcal{H}_∞ model reduction problem.

Proposition 2. *The MJLS (7) with deficient mode information is stochastically stable with a guaranteed \mathcal{H}_∞ performance γ if there exist positive-definite symmetric matrices $P_i \in \mathbf{R}^{(n_x+n_r) \times (n_x+n_r)}$, $i \in \mathcal{I}$, such that the following matrix inequalities hold:*

$$\begin{aligned} \Phi_{ij}^{(s)} &:= \begin{bmatrix} \bar{A}_i^T \mathcal{P}_j^{(is)} \bar{A}_i - P_i + \bar{C}_i^T \bar{C}_i & \bar{A}_i^T \mathcal{P}_j^{(is)} \bar{B}_i + \bar{C}_i^T \bar{D}_i \\ * & -(\gamma^2 \mathbf{I} - \bar{B}_i^T \mathcal{P}_j^{(is)} \bar{B}_i - \bar{D}_i^T \bar{D}_i) \end{bmatrix} \\ &< 0, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}, s = 1, \dots, M, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \mathcal{P}_j^{(is)} &:= \sum_{j \in \mathcal{J}_{\mathcal{X}}^{(i)}} \pi_{ij} P_j + \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)} P_j + \frac{\pi_{\mathcal{U}\mathcal{X}}^{(is)} P_j}{\pi_{\mathcal{U}\mathcal{X}}^{(i)}}, \\ \pi_{\mathcal{U}\mathcal{X}}^{(is)} &:= 1 - \sum_{j \in \mathcal{J}_{\mathcal{X}}^{(i)}} \pi_{ij} - \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)}. \end{aligned} \quad (11)$$

Proof. Based on Lemma 1, it is known that system (7) with completely known transition probabilities (TPs) is stochastically stable with an \mathcal{H}_∞ performance γ if (9) holds. Now due to $\sum_{j=1}^N \pi_{ij} = 1$ and with deficient mode information, we rewrite the term $\sum_{j=1}^N \pi_{ij} P_j$ as

$$\sum_{j=1}^N \pi_{ij} P_j = \sum_{j \in \mathcal{J}_{\mathcal{X}}^{(i)}} \pi_{ij} P_j + \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{E}}^{(i)}} \left(\sum_{s=1}^M \alpha_s \tilde{\pi}_{ij}^{(s)} \right) P_j + \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}} \hat{\pi}_{ij} P_j. \quad (12)$$

Considering the fact that $0 \leq \alpha_s \leq 1$, $\sum_{s=1}^M \alpha_s = 1$, and $0 \leq \hat{\pi}_{ij} / \pi_{\mathcal{U}\mathcal{X}}^{(is)} \leq 1$, $\sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}} (\hat{\pi}_{ij} / \pi_{\mathcal{U}\mathcal{X}}^{(is)}) = 1$, (12) can be rewritten as

$$\begin{aligned} &\sum_{j=1}^N \pi_{ij} P_j \\ &= \sum_{s=1}^M \alpha_s \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}} \frac{\hat{\pi}_{ij}}{\pi_{\mathcal{U}\mathcal{X}}^{(is)}} \left(\sum_{j \in \mathcal{J}_{\mathcal{X}}^{(i)}} \pi_{ij} P_j + \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)} P_j + \pi_{\mathcal{U}\mathcal{X}}^{(is)} P_j \right). \end{aligned} \quad (13)$$

Thus, with deficient mode information, the left-hand side (LHS) of inequality (9) can be rewritten as

$$\text{LHS (9)} = \sum_{s=1}^M \alpha_s \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}} \frac{\hat{\pi}_{ij}}{\pi_{\mathcal{U}\mathcal{X}}^{(is)}} \Phi_{ij}^{(s)}, \quad (14)$$

where $\Phi_{ij}^{(s)}$ is defined in (10). Then (9) holds if and only if $\Phi_{ij}^{(s)} < 0$, which implies that, in the presence of unknown and uncertain TPs, inequality (9) is equivalent to (10). This completes the proof. \square

Remark 3. By fully exploiting the properties of the TPMs, together with the convexification of uncertain domains, a sufficient condition for the \mathcal{H}_∞ performance analysis has been derived for the model error system (Σ) with deficient mode information in Proposition 2. It is noted that there exist product terms between the Lyapunov matrices and system matrices in condition (10), which will bring some difficulties in the solutions of model reduction problem. By applying decoupling techniques, in the following, two distinctly different approaches to solve the \mathcal{H}_∞ model reduction problem will be proposed.

3.2. Model Reduction via Convex Linearisation Approach. In this subsection, based on a linearisation procedure, the model

$$\begin{bmatrix} \mathcal{P}_{j(1)}^{(is)} - G_{i(1)} - G_{i(1)}^T & \mathcal{P}_{j(2)}^{(is)} - HG_{i(2)} - G_{i(3)}^T & \mathbf{0} & G_{i(1)}A_i & H\bar{A}_{ri} & G_{i(1)}B_i + H\bar{B}_{ri} \\ * & \mathcal{P}_{j(3)}^{(is)} - G_{i(2)} - G_{i(2)}^T & \mathbf{0} & G_{i(3)}A_i & \bar{A}_{ri} & G_{i(3)}B_i + \bar{B}_{ri} \\ * & * & -\mathbf{I} & C_i & -C_{ri} & D_i - D_{ri} \\ * & * & * & -P_{i(1)} & -P_{i(2)} & \mathbf{0} \\ * & * & * & * & -P_{i(3)} & \mathbf{0} \\ * & * & * & * & * & -\gamma^2\mathbf{I} \end{bmatrix} < 0, \quad (15)$$

$$i \in \mathcal{I}, \quad j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}, \quad s = 1, \dots, M,$$

where

$$\begin{aligned} \mathcal{P}_{j(m)}^{(is)} &:= \sum_{j \in \mathcal{J}_{\mathcal{X}}^{(i)}} \pi_{ij} P_{j(m)} + \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)} P_{j(m)} + \underbrace{\pi_{\mathcal{U}\mathcal{X}}^{(is)} P_{j(m)}}_{j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}}, \\ m &= 1, 2, 3, \\ \pi_{\mathcal{U}\mathcal{X}}^{(is)} &:= 1 - \sum_{j \in \mathcal{J}_{\mathcal{X}}^{(i)}} \pi_{ij} - \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)}, \end{aligned} \quad (16)$$

$$H := [\mathbf{I}_{n_r \times n_r} \quad \mathbf{0}_{n_r \times (n_x - n_r)}]^T.$$

Moreover, if the above conditions have a set of feasible solutions $(P_{i(1)}, P_{i(2)}, P_{i(3)}, G_{i(1)}, G_{i(2)}, G_{i(3)}, \bar{A}_{ri}, \bar{B}_{ri}, C_{ri}, D_{ri})$, then an admissible n_r -order approximation model in the form of (6) can be constructed as

$$\begin{aligned} A_{ri} &= G_{i(2)}^{-1} \bar{A}_{ri}, & B_{ri} &= G_{i(2)}^{-1} \bar{B}_{ri}, \\ C_{ri} &= C_{ri}, & D_{ri} &= D_{ri}. \end{aligned} \quad (17)$$

Proof. It follows from Proposition 2 that if we can show (10), then the claimed results follow. By Schur complement, inequality (10) is equivalent to

$$\begin{bmatrix} (\mathcal{P}_j^{(is)})^{-1} & \mathbf{0} & \bar{A}_i & \bar{B}_i \\ * & -\mathbf{I} & \bar{C}_i & \bar{D}_i \\ * & * & -P_i & \mathbf{0} \\ * & * & * & -\gamma^2\mathbf{I} \end{bmatrix} < 0, \quad (18)$$

$$\forall i \in \mathcal{I}, \quad j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}, \quad s = 1, \dots, M,$$

reduction problem described in the previous section will be cast into a convex optimization problem. The following theorem presents a sufficient condition for the existence of admissible reduced-order models.

Theorem 4. Consider MJLS in (1) with deficient mode information and reduced-order model in the form of (6). The model error system in (7) is stochastically stable with an \mathcal{H}_∞ performance γ if there exist positive-definite symmetric matrices $P_i = \begin{bmatrix} P_{i(1)} & P_{i(2)} \\ * & P_{i(3)} \end{bmatrix} \in \mathbf{R}^{(n_x+n_r) \times (n_x+n_r)}$ and matrices $G_{i(1)} \in \mathbf{R}^{n_x \times n_x}$, $G_{i(2)} \in \mathbf{R}^{n_r \times n_r}$, $G_{i(3)} \in \mathbf{R}^{n_x \times n_r}$, $\bar{A}_{ri} \in \mathbf{R}^{n_x \times n_r}$, $\bar{B}_{ri} \in \mathbf{R}^{n_r \times n_u}$, $C_{ri} \in \mathbf{R}^{n_z \times n_r}$, and $D_{ri} \in \mathbf{R}^{n_z \times n_u}$, $i \in \mathcal{I}$, such that the following LMIs hold:

where

$$\begin{aligned} \mathcal{P}_j^{(is)} &:= \sum_{j \in \mathcal{J}_{\mathcal{X}}^{(i)}} \pi_{ij} P_j + \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)} P_j + \underbrace{\pi_{\mathcal{U}\mathcal{X}}^{(is)} P_j}_{j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}}, \\ \pi_{\mathcal{U}\mathcal{X}}^{(is)} &:= 1 - \sum_{j \in \mathcal{J}_{\mathcal{X}}^{(i)}} \pi_{ij} - \sum_{j \in \mathcal{J}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)}. \end{aligned} \quad (19)$$

Perform a congruent transformation to (18) by $\text{diag}\{G_i, \mathbf{I}, \mathbf{I}, \mathbf{I}\}$, and consider the following bounding inequality:

$$-G_i (\mathcal{P}_j^{(is)})^{-1} G_i^T \leq \mathcal{P}_j^{(is)} - G_i - G_i^T. \quad (20)$$

It is thus easy to see that the following inequality implies (18),

$$\begin{bmatrix} \mathcal{P}_j^{(is)} - G_i - G_i^T & \mathbf{0} & G_i \bar{A}_i & G_i \bar{B}_i \\ * & -\mathbf{I} & \bar{C}_i & \bar{D}_i \\ * & * & -P_i & \mathbf{0} \\ * & * & * & -\gamma^2\mathbf{I} \end{bmatrix} < 0, \quad (21)$$

$$\forall i \in \mathcal{I}, \quad j \in \mathcal{J}_{\mathcal{U}\mathcal{X}}^{(i)}, \quad s = 1, \dots, M.$$

Furthermore, for model reduction synthesis purpose, we choose the slack matrix G_i as

$$G_i := \begin{bmatrix} G_{i(1)} & HG_{i(2)} \\ G_{i(3)} & G_{i(4)} \end{bmatrix}, \quad (22)$$

where $H := [\mathbf{I}_{n_r} \ \mathbf{0}_{n_r \times (n_x - n_r)}]^T$, $G_{i(1)} \in \mathbf{R}^{n_x \times n_x}$, $G_{i(3)} \in \mathbf{R}^{n_r \times n_r}$, $G_{(2)} \in \mathbf{R}^{n_r \times n_r}$, and $G_{(4)} \in \mathbf{R}^{n_r \times n_r}$. Then, similar to [22], performing a congruent transformation to

$$\begin{bmatrix} G_{i(1)} + G_{i(1)}^T & HG_{(2)} + G_{i(3)}^T \\ * & G_{(4)} + G_{(4)}^T \end{bmatrix} \quad (23)$$

by $\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & G_{(2)}G_{(4)}^{-1} \end{bmatrix}$ yields

$$\begin{aligned} & \begin{bmatrix} G_{i(1)} + G_{i(1)}^T & HG_{(2)}G_{(4)}^{-T}G_{(2)}^T + G_{i(3)}^T G_{(4)}^{-T}G_{(2)}^T \\ * & G_{(2)}G_{(4)}^{-T}G_{(2)}^T + G_{(2)}G_{(4)}^{-1}G_{(2)}^T \end{bmatrix} \\ & := \begin{bmatrix} G_{i(1)} + G_{i(1)}^T & H\bar{G}_{(2)} + \bar{G}_{i(3)}^T \\ * & \bar{G}_{(2)} + \bar{G}_{(2)}^T \end{bmatrix}. \end{aligned} \quad (24)$$

Thus, instead of (22), one can directly specify matrix G_i of the following form *without loss of generality*:

$$G_i = \begin{bmatrix} G_{i(1)} & HG_{i(2)} \\ G_{i(3)} & G_{i(2)} \end{bmatrix}, \quad i \in \mathcal{I}. \quad (25)$$

It is noted that in this way the matrix variable $G_{i(2)}$ can be absorbed by the reduced-order model gain variables A_{ri} and B_{ri} by introducing

$$\bar{A}_{ri} := G_{i(2)}A_{ri}, \quad \bar{B}_{ri} := G_{i(2)}B_{ri}. \quad (26)$$

This feature enables one to make no congruent transformation to the original matrix inequality, and all the slack variables can be set as Markovian switching.

Then, substituting matrices G_i given in (25) into (10), together with the consideration of the matrices defined in (26), leads to (15) exactly.

On the other hand, the conditions in (15) imply that $-G_{i(2)} - G_{i(2)}^T < 0$, which means that $G_{i(2)}$ is nonsingular. Thus, the reduced-order model gains can be constructed by (17). The proof is completed. \square

Remark 5. Theorem 4 provides a sufficient condition for the solvability of \mathcal{H}_∞ model reduction synthesis problem for the MJLS (1) with deficient mode information. A desired reduced-order model can be determined by solving the following convex optimization problem.

Problem MRLA (Model Reduction via Linearisation Approach). Minimize γ subject to (15) for $P_{i(1)}$, $P_{i(2)}$, $P_{i(3)}$, $G_{i(1)}$, $G_{i(2)}$, $G_{i(3)}$, \bar{A}_{ri} , \bar{B}_{ri} , C_{ri} , D_{ri} , $i \in \mathcal{I}$.

From the proof of Theorem 4, it is easy to see that the matrix inequality linearisation procedure is based on the bounding inequality in (20) with slack variables of a structural constraint in (25). This inevitably brings some degree of design conservatism. Another avenue to solve the nonlinear matrix inequality problem in (10) is by utilizing the cone complementarity linearisation (CCL) technique [37]. To this end, we will resort to an iterative approach to solve the model reduction problem.

3.3. Model Reduction via Iterative Approach. In this subsection, we will solve the model reduction problem by using the CCL technique. The following theorem gives another sufficient condition for the existence of admissible reduced-order models.

Theorem 6. Consider MJLS in (1) with deficient mode information and reduced-order model in the form of (6). The model error system in (7) is stochastically stable with an \mathcal{H}_∞ performance γ if there exist positive-definite symmetric matrices $\{P_i, X_i\} \in \mathbf{R}^{(n_x+n_r) \times (n_x+n_r)}$ and matrices $A_{ri} \in \mathbf{R}^{n_r \times n_r}$, $B_{ri} \in \mathbf{R}^{n_r \times n_u}$, $C_{ri} \in \mathbf{R}^{n_z \times n_r}$, and $D_{ri} \in \mathbf{R}^{n_z \times n_u}$, $i \in \mathcal{I}$, such that

$$\begin{bmatrix} \mathcal{X}_j^{(i)} & \mathbf{0} & F_i^{(s)} (\mathcal{A}_i + E\mathcal{E}_{ri}R) & F_i^{(s)} (\mathcal{B}_i + E\mathcal{E}_{ri}S) \\ * & -\mathbf{I} & \mathcal{C}_i + F\mathcal{E}_{ri}R & \mathcal{D}_i + F\mathcal{E}_{ri}S \\ * & * & -P_i & \mathbf{0} \\ * & * & * & -\gamma^2 \mathbf{I} \end{bmatrix} < 0,$$

$$i \in +F\mathcal{E}_{ri}R\mathcal{I}, \quad j \in \mathcal{I}_{\mathcal{U}\mathcal{X}}^{(i)}, \quad s = 1, \dots, M, \quad (27)$$

$$P_i X_i = \mathbf{I}, \quad i \in \mathcal{I}, \quad (28)$$

where

$$\mathcal{X}_j^{(i)} := \text{diag} \left\{ X_{\mathcal{X}_1^{(i)}}, \dots, X_{\mathcal{X}_i^{(i)}}, X_{\mathcal{U}_1^{(i)}}, \dots, X_{\mathcal{U}_v^{(i)}}, X_j \right\},$$

$$F_i^{(s)}$$

$$:= \left[\sqrt{\pi_{i\mathcal{X}_1^{(i)}}} \mathbf{I}, \dots, \sqrt{\pi_{i\mathcal{X}_i^{(i)}}} \mathbf{I}, \sqrt{\tilde{\pi}_{i\mathcal{U}_1^{(i)}}^{(s)}} \mathbf{I}, \dots, \sqrt{\tilde{\pi}_{i\mathcal{U}_v^{(i)}}^{(s)}} \mathbf{I}, \sqrt{\pi_{i\mathcal{U}\mathcal{X}}^{(is)}} \mathbf{I} \right]^T,$$

$$\mathcal{E}_{ri} := \begin{bmatrix} A_{ri} & B_{ri} \\ C_{ri} & D_{ri} \end{bmatrix}, \quad \mathcal{A}_i := \begin{bmatrix} A_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathcal{B}_i := \begin{bmatrix} B_i \\ \mathbf{0} \end{bmatrix},$$

$$\mathcal{C}_i := [C_i \ \mathbf{0}], \quad \mathcal{D}_i := D_i, \quad E := \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix},$$

$$F := [\mathbf{0} \ -\mathbf{I}], \quad R := \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad S := \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix},$$

$$\pi_{\mathcal{U}\mathcal{X}}^{(is)} := 1 - \sum_{j \in \mathcal{I}_{\mathcal{X}}^{(i)}} \pi_{ij} - \sum_{j \in \mathcal{I}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)}. \quad (29)$$

Proof. From Proposition 2, we know that for $i \in \mathcal{I}$ there exists a reduced-order model in the form of (6) such that the model error system (7) is stochastically stable with a guaranteed \mathcal{H}_∞ error performance γ if there exist matrices P_i satisfying (10). First, rewrite the matrices defined in (8) in the following form:

$$\begin{aligned} \bar{A}_i &:= \mathcal{A}_i + E\mathcal{E}_{ri}R, & \bar{B}_i &:= \mathcal{B}_i + E\mathcal{E}_{ri}S, \\ \bar{C}_i &:= \mathcal{C}_i + F\mathcal{E}_{ri}R, & \bar{D}_i &:= \mathcal{D}_i + F\mathcal{E}_{ri}S, \end{aligned} \quad (30)$$

where

$$\begin{aligned} \mathcal{G}_{ri} &:= \begin{bmatrix} A_{ri} & B_{ri} \\ C_{ri} & D_{ri} \end{bmatrix}, & \mathcal{A}_i &:= \begin{bmatrix} A_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, & \mathcal{B}_i &:= \begin{bmatrix} B_i \\ \mathbf{0} \end{bmatrix}, \\ \mathcal{C}_i &:= [C_i \ \mathbf{0}], & \mathcal{D}_i &:= D_i, & E &:= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}, \\ F &:= [\mathbf{0} \ -\mathbf{I}], & R &:= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, & S &:= \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}. \end{aligned} \quad (31)$$

Then, by Schur complement to (10), and with the notations $\mathcal{F}_{\mathcal{X}}^{(i)} := \{\mathcal{X}_1^{(i)}, \dots, \mathcal{X}_i^{(i)}\}$, $\mathcal{F}_{\mathcal{U}\mathcal{E}}^{(i)} := \{\mathcal{U}_1^{(i)}, \dots, \mathcal{U}_{v_i}^{(i)}\}$, we have that (10) is equivalent to

$$\begin{bmatrix} \overline{\mathcal{P}}_j^{(i)} & \mathbf{0} & F_i^{(s)} (\mathcal{A}_i + E\mathcal{G}_{ri}R) & F_i^{(s)} (\mathcal{B}_i + E\mathcal{G}_{ri}S) \\ * & -\mathbf{I} & \mathcal{C}_i + F\mathcal{G}_{ri}R & \mathcal{D}_i + F\mathcal{G}_{ri}S \\ * & * & -P_i & \mathbf{0} \\ * & * & * & -\gamma^2 \mathbf{I} \end{bmatrix} < 0, \quad i \in \mathcal{I}, \quad j \in \mathcal{F}_{\mathcal{U}\mathcal{X}}^{(i)}, \quad s = 1, \dots, M, \quad (32)$$

where

$$\begin{aligned} \overline{\mathcal{P}}_j^{(i)} &:= \text{diag} \left\{ P_{\mathcal{X}_1^{(i)}}^{-1}, \dots, P_{\mathcal{X}_i^{(i)}}^{-1}, P_{\mathcal{U}_1^{(i)}}^{-1}, \dots, P_{\mathcal{U}_{v_i}^{(i)}}^{-1}, P_j^{-1} \right\}, \\ F_i^{(s)} &:= \left[\sqrt{\pi_{i\mathcal{X}_1^{(i)}}} \mathbf{I}, \dots, \sqrt{\pi_{i\mathcal{X}_i^{(i)}}} \mathbf{I}, \sqrt{\tilde{\pi}_{i\mathcal{U}_1^{(i)}}^{(s)}} \mathbf{I}, \dots, \sqrt{\tilde{\pi}_{i\mathcal{U}_{v_i}^{(i)}}^{(s)}} \mathbf{I}, \sqrt{\pi_{i\mathcal{U}\mathcal{X}}^{(is)}} \mathbf{I} \right]^T. \end{aligned} \quad (33)$$

Setting $X_i := P_i^{-1}$ and considering (33), it is easy to see that (32) is equivalent to (27) and (28). This completes the proof. \square

Remark 7. Theorem 6 provides another sufficient condition for testing the solvability of n_r -order \mathcal{H}_∞ model reduction synthesis for MJLS (1) with deficient mode information. It is noted that the conditions in Theorem 6 are not strict LMI-based conditions due to the matrix equality in (28). However, with the CCL algorithm in [37, 38], we can resolve this nonconvex feasibility problem by formulating it into a sequential optimization problem subject to LMI constraints.

Based on the previous discussions and using a CCL technique, the nonconvex feasibility problem given in (27) and (28) is converted into the following nonlinear minimization problem that involves LMI conditions.

Problem MR1A (Model Reduction via Iterative Approach). Minimize Trace $(\sum_{i=1}^N P_i X_i)$ subject to (27) and

$$\begin{bmatrix} P_i & \mathbf{I} \\ * & X_i \end{bmatrix} \geq 0, \quad \forall i \in \mathcal{I}. \quad (34)$$

Then, the suboptimal performance of γ can be found by the following algorithm. The convergence of this algorithm is guaranteed in terms of similar results in [37, 38].

Algorithm MR1A: Suboptimal Performance of γ

Step 1. Choose a sufficiently large initial $\gamma > 0$, such that there exists a feasible solution to (27) and (34). Set $\gamma_0 = \gamma$.

Step 2. Find a feasible set $(P_i^{(0)}, X_i^{(0)}, A_{ri}^{(0)}, B_{ri}^{(0)}, C_{ri}^{(0)}, D_{ri}^{(0)})$, for all $i \in \mathcal{I}$ that satisfies the conditions in (27) and (34). Set $q = 0$.

Step 3. Solve the following LMI problem over the variables $P_i, X_i, A_{ri}, B_{ri}, C_{ri}$, and D_{ri} :

$$\text{Minimize Trace} \left(\sum_{i=1}^N (P_i^{(q)} X_i + P_i X_i^{(q)}) \right) \quad (35)$$

subject to (27) and (34).

$$\text{Set } P_i^{(q+1)} = P_i \text{ and } X_i^{(q+1)} = X_i.$$

Step 4. Substituting the gains A_{ri}, B_{ri}, C_{ri} , and D_{ri} obtained in Step 3 into (10) and if the LMIs in (10) are feasible with respect to the variables P_i , then set $\gamma_0 = \gamma$ and return to Step 2 after decreasing γ to some extent. If (10) are infeasible within the maximum number of iterations allowed, then exit. Otherwise, set $q = q + 1$, and go to Step 3.

Remark 8. It is noted that the \mathcal{H}_∞ model reduction problem for discrete-time MJLSs has been considered in [20]. However, there are some remarkable differences between our results and those in [20]. Firstly, it is noted that in this paper the deficient mode information, which simultaneously contains the exactly known, partially unknown, and uncertain transition probabilities (TPs), is considered for MJLS (1), which is more general and practical for engineering applications, whereas the TPs considered in [20] are only partially unknown. Secondly, by fully utilizing the properties of the TPM, including the relation $\sum_{j \in \mathcal{F}_{\mathcal{U}\mathcal{X}}^{(i)}} \tilde{\pi}_{ij} = 1 - \sum_{j \in \mathcal{F}_{\mathcal{X}}^{(i)}} \pi_{ij} - \sum_{j \in \mathcal{F}_{\mathcal{U}\mathcal{E}}^{(i)}} \tilde{\pi}_{ij}^{(s)}$, Theorem 4 gives a unified formulation of \mathcal{H}_∞ model reduction design for MJLS (1), while in [20] the above relation is neglected, which may cause possible conservatism. In addition, two different approaches, namely, the convex linearisation approach and iterative approach, to solve the model reduction problem are proposed, while in [20] only the linearisation approach was presented. It will be shown in the simulation part that the previous points are crucial to reduce the conservatism of model reduction design for MJLS in (1).

Remark 9. It is worth mentioning that the conditions given in Theorem 4 are convex and thus can be readily solved with

commercially available software. The design conservatism of Theorem 4 mainly comes from the bounding inequality in (20) with the slack variable of a structural constraint in (25). In the iterative approach, the conditions given in (27) and (34) are equivalent to the corresponding performance analysis results. This is the main advantage of Theorem 6 over Theorem 4. However, the numerical computation cost involved in Algorithm MRJA is also much larger than that involved in Theorem 4 (MRLA), especially when the number of iterations increases.

4. An Illustrative Example

In this section, we will present an illustrative example to demonstrate the effectiveness and superiority of the proposed approaches.

Consider a Markovian jump linear system (MJLS) described by (Σ) with parameters as follows:

$$\begin{aligned} & \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array} \right] \\ &= \left[\begin{array}{cccc|c} 0.05 & -0.27 & 0.44 & 0.39 & 0.5 \\ 0.55 & 0.33 & 0.38 & 0.55 & 0.2 \\ 0.1 & 0.17 & 0.27 & 0.44 & 0.3 \\ 0.05 & 0.22 & 0.16 & 0.11 & 0.1 \\ \hline 1 & 0.1 & 0.2 & -0.3 & 0.5 \end{array} \right], \\ & \left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array} \right] \\ &= \left[\begin{array}{cccc|c} 0.11 & -0.17 & 0.27 & 0.44 & -0.8 \\ 0.55 & 0.06 & 0.22 & 0.55 & -0.2 \\ 0.05 & 0.17 & 0.28 & 0.44 & -0.1 \\ 0.17 & 0.05 & 0.06 & -0.11 & -1 \\ \hline 0.5 & -0.8 & 0.3 & 0.5 & 0.5 \end{array} \right], \\ & \left[\begin{array}{c|c} A_3 & B_3 \\ \hline C_3 & D_3 \end{array} \right] \\ &= \left[\begin{array}{cccc|c} 0.16 & 0.06 & -0.02 & 0.18 & 0.2 \\ 0.04 & -0.37 & 0.53 & -0.04 & -0.2 \\ -0.08 & -0.32 & -0.05 & -0.11 & -0.1 \\ -0.17 & 0.40 & 0.04 & 0.29 & 0.1 \\ \hline 1.4 & 0.7 & 0.2 & -0.8 & 0.5 \end{array} \right], \\ & \left[\begin{array}{c|c} A_4 & B_4 \\ \hline C_4 & D_4 \end{array} \right] \\ &= \left[\begin{array}{cccc|c} 0.23 & 0.01 & -0.55 & -0.38 & 0.9 \\ -0.33 & 0.36 & -0.48 & -0.10 & -1.1 \\ -0.20 & -0.45 & 0.1 & -0.19 & -0.7 \\ 0.23 & 0.16 & 0.5 & -0.30 & -1.2 \\ \hline -0.7 & 1.2 & 1.2 & -0.6 & 0.5 \end{array} \right]. \end{aligned} \quad (36)$$

Four different cases for the transition probability matrix (TPM) are given in Table 1, where the transition probabilities (TPs) labeled “~” and “~” represent the unknown and uncertain elements, respectively. Specifically, Case 1, Case

TABLE 1: Four different TPMs.

Case 1: completely known TPM	Case 2: deficient TPM1
$\begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ 0.3 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.3 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ \tilde{\pi}_{21} & \tilde{\pi}_{22} & 0.3 & 0.2 \\ \tilde{\pi}_{31} & \tilde{\pi}_{32} & \tilde{\pi}_{33} & \tilde{\pi}_{34} \\ 0.2 & \tilde{\pi}_{42} & \tilde{\pi}_{43} & \tilde{\pi}_{44} \end{bmatrix}$
Case 3: deficient TPM2	Case 4: completely unknown TPM
$\begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ \tilde{\pi}_{21} & \tilde{\pi}_{22} & 0.3 & 0.2 \\ \tilde{\pi}_{31} & \tilde{\pi}_{32} & \tilde{\pi}_{33} & \tilde{\pi}_{34} \\ 0.2 & \tilde{\pi}_{42} & \tilde{\pi}_{43} & \tilde{\pi}_{44} \end{bmatrix}$	$\begin{bmatrix} \tilde{\pi}_{11} & \tilde{\pi}_{12} & \tilde{\pi}_{13} & \tilde{\pi}_{14} \\ \tilde{\pi}_{21} & \tilde{\pi}_{22} & \tilde{\pi}_{23} & \tilde{\pi}_{24} \\ \tilde{\pi}_{31} & \tilde{\pi}_{32} & \tilde{\pi}_{33} & \tilde{\pi}_{34} \\ \tilde{\pi}_{41} & \tilde{\pi}_{42} & \tilde{\pi}_{43} & \tilde{\pi}_{44} \end{bmatrix}$

 TABLE 2: Comparison of minimum \mathcal{H}_∞ performance with deficient TPM2 in Case 3.

Methods	Three-order	Two-order	One-order
Theorem 4	1.2423	1.5231	2.5952
[20]	1.3815	1.7320	2.9094

2, Case 3, and Case 4 stand for the completely known TPs, deficient mode information (including known, partially unknown, and uncertain TPs), partially unknown TPs, and completely unknown TPs, respectively.

For Case 2 shown in Table 1, the uncertain TPs comprise two vertices Π_s , $s = 1, 2$, where the third rows $\Pi_{s(3)}$, $s = 1, 2$, are given by

$$\begin{aligned} \Pi_{1(3)} &= [\tilde{\pi}_{31} \quad 0.1 \quad \tilde{\pi}_{33} \quad 0.3], \\ \Pi_{2(3)} &= [\tilde{\pi}_{31} \quad 0.3 \quad \tilde{\pi}_{33} \quad 0.4], \end{aligned} \quad (37)$$

and the other rows in the two vertices are defined with the same elements; that is,

$$\begin{aligned} \Pi_{s(1)} &= [0.3 \quad 0.2 \quad 0.1 \quad 0.4], \\ \Pi_{s(2)} &= [\tilde{\pi}_{21} \quad \tilde{\pi}_{22} \quad 0.3 \quad 0.2], \\ \Pi_{s(4)} &= [0.2 \quad \tilde{\pi}_{42} \quad \tilde{\pi}_{43} \quad \tilde{\pi}_{44}], \quad s = 1, 2. \end{aligned} \quad (38)$$

The objective is to design an n_r -order model of the form (6) to approximate the above system such that the model error system (7) is stochastically stable with an \mathcal{H}_∞ performance index γ . Here, we first assume that the MJLS is subject to the partially unknown TPs in Case 3, and then a detailed comparison between the minimum \mathcal{H}_∞ performance indices γ_{\min} obtained based on Theorem 4 in this paper and the results in [20] is summarized in Table 2. It is observed from Table 2 that the design approach proposed in Theorem 4 is less conservative than [20].

In the following, we will further demonstrate the advantage and less conservatism of Theorem 6 over Theorem 4 proposed in this paper. By solving the problems MRLA and MRJA, another detailed comparison of the minimum \mathcal{H}_∞ performance indices γ_{\min} is listed in Table 3, where

TABLE 3: Comparison of minimum \mathcal{H}_∞ performance for different TPMs.

TPMs	Three-order		Two-order		One-order	
	Theorem 4 (MRLA)	Theorem 6 (MRIA)	Theorem 4 (MRLA)	Theorem 6 (MRIA)	Theorem 4 (MRLA)	Theorem 6 (MRIA)
Case 1	0.8798	0.4 (35)	0.9595	0.8 (139)	2.0057	1.5 (38)
Case 2	1.1687	0.6 (39)	1.4829	1.2 (145)	2.4748	1.9 (40)
Case 3	1.2423	0.7 (87)	1.5231	1.3 (157)	2.5952	2 (43)
Case 4	1.7332	1 (147)	2.1914	1.6 (165)	3.3299	2.4 (137)

the bold numbers in the brackets represent the number of iterations. By inspection of Tables 1 and 2, it is easy to see that the results in Theorem 6 are less conservative than those in Theorem 4. It is also shown that the more information on TPs is available, the better \mathcal{H}_∞ performance can be obtained, which is effective to reduce the conservatism of model reduction design. Therefore, the introduction of the polytopic-type uncertain TPs is significant.

Specifically, with $n_r = 3$, we obtain $\gamma_{\min} = 1.1687$ by Theorem 4 with deficient TPM1 shown in Table 1, and the three-order model parameters are given by

$$\begin{aligned}
 & \left[\begin{array}{c|c} A_{r1} & B_{r1} \\ \hline C_{r1} & D_{r1} \end{array} \right] \\
 &= \left[\begin{array}{ccc|c} -0.0540 & 0.2529 & 0.2176 & -0.8490 \\ 0.3274 & 0.6285 & 0.4862 & 0.1883 \\ -0.1543 & 0.3250 & 0.2913 & -0.8224 \\ \hline -0.9425 & 0.1661 & -0.0302 & 0.6282 \end{array} \right], \\
 & \left[\begin{array}{c|c} A_{r2} & B_{r2} \\ \hline C_{r2} & D_{r2} \end{array} \right] \\
 &= \left[\begin{array}{ccc|c} 0.1724 & 0.1095 & 0.3894 & 0.8742 \\ 0.3654 & 0.4674 & 0.2785 & 0.3566 \\ -0.0484 & 0.3457 & 0.2416 & 0.1962 \\ \hline -0.6496 & 0.6678 & -0.7150 & 0.6001 \end{array} \right], \\
 & \left[\begin{array}{c|c} A_{r3} & B_{r3} \\ \hline C_{r3} & D_{r3} \end{array} \right] \\
 &= \left[\begin{array}{ccc|c} 0.2655 & 0.2048 & -0.1861 & -0.2537 \\ 0.1033 & -0.1618 & 0.2405 & 0.1205 \\ -0.1747 & -0.3936 & 0.1018 & 0.3127 \\ \hline -1.2638 & -0.0046 & -0.0662 & 0.4385 \end{array} \right], \\
 & \left[\begin{array}{c|c} A_{r4} & B_{r4} \\ \hline C_{r4} & D_{r4} \end{array} \right] \\
 &= \left[\begin{array}{ccc|c} 0.1162 & -0.5423 & -0.5004 & -0.7796 \\ -0.3767 & 0.0416 & -0.3589 & 1.1859 \\ 0.0724 & -0.3765 & 0.0533 & 0.6505 \\ \hline 0.5008 & -0.9345 & -1.1617 & 0.3712 \end{array} \right].
 \end{aligned} \tag{39}$$

With $n_r = 3$, we obtain $\gamma = 0.6$ by solving the Algorithm MR1A after 39 iterations with deficient TPM1 shown in Table 1, and the three-order model parameters are given by

$$\begin{aligned}
 & \left[\begin{array}{c|c} A_{r1} & B_{r1} \\ \hline C_{r1} & D_{r1} \end{array} \right] \\
 &= \left[\begin{array}{ccc|c} 0.2800 & 0.0772 & 0.6328 & -0.4352 \\ 0.5261 & 0.2307 & 0.7350 & -0.1521 \\ 0.0462 & 0.2468 & 0.3092 & -0.0833 \\ \hline -1.5239 & 0.2064 & 0.4439 & 0.4937 \end{array} \right], \\
 & \left[\begin{array}{c|c} A_{r2} & B_{r2} \\ \hline C_{r2} & D_{r2} \end{array} \right] \\
 &= \left[\begin{array}{ccc|c} 0.3132 & -0.0048 & 0.7263 & 0.5510 \\ 0.4900 & 0.0264 & 0.7604 & -0.2455 \\ 0.0826 & 0.1022 & -0.0038 & 0.5862 \\ \hline -0.7215 & 0.8753 & -0.3862 & 0.4679 \end{array} \right], \\
 & \left[\begin{array}{c|c} A_{r3} & B_{r3} \\ \hline C_{r3} & D_{r3} \end{array} \right] \\
 &= \left[\begin{array}{ccc|c} 0.1946 & -0.0921 & 0.0870 & -0.0826 \\ 0.1133 & -0.2445 & -0.5027 & 0.2327 \\ -0.1452 & 0.1980 & 0.3471 & -0.0378 \\ \hline -1.9205 & -0.3685 & 0.8553 & 0.4941 \end{array} \right], \\
 & \left[\begin{array}{c|c} A_{r4} & B_{r4} \\ \hline C_{r4} & D_{r4} \end{array} \right] \\
 &= \left[\begin{array}{ccc|c} -0.0592 & -0.2981 & -0.6252 & -0.3605 \\ -0.5600 & -0.0061 & 0.0120 & 0.5109 \\ 0.1490 & -0.0012 & -0.3114 & 0.8336 \\ \hline 0.5634 & -2.4550 & -0.7422 & 0.4789 \end{array} \right].
 \end{aligned} \tag{40}$$

The feasible solutions for the other cases are omitted for brevity.

In order to further illustrate the effectiveness of the designed approximation models, simulations have been carried out. Specifically, choose the zero initial condition and input $u(k)$ as

$$u(k) = \begin{cases} e^{(-0.2k+1)} \sin(0.2k), & 10 \leq k \leq 30, \\ 0.3 \cos(0.25k), & 50 \leq k \leq 90, \\ 0, & \text{otherwise.} \end{cases} \tag{41}$$

With the above obtained approximation models under Case 2 given in Table 1, and one of the possible realizations of

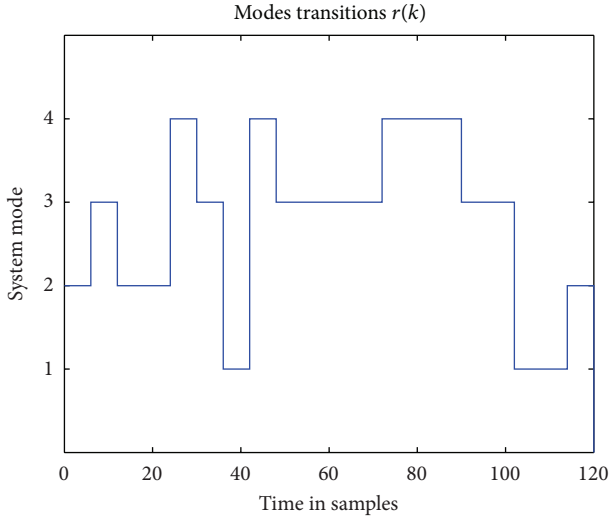


FIGURE 1: One possible system mode evolution.

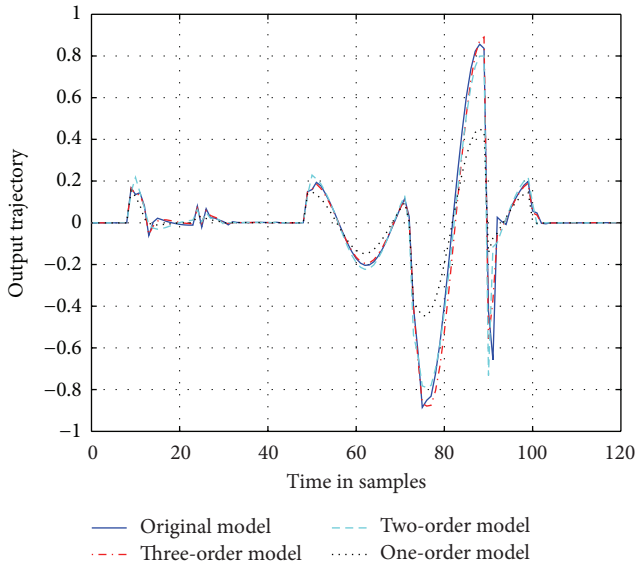


FIGURE 2: Output trajectories of the original system and reduced-order models with deficient TPM1 based on Theorem 4 (MRLA).

the Markovian jumping mode shown in Figure 1, the output trajectories of the original system and approximation models obtained based on Theorem 4 (MRLA) and Theorem 6 (MRIA) are shown in Figures 2 and 3, respectively. It can be clearly observed from the simulation curves that, despite the deficient TPs, the designed reduced-order models can approximate the original system very well.

5. Conclusions

This paper has studied the problem of robust \mathcal{H}_∞ model reduction for a class of discrete-time Markovian jump linear systems (MJLSs) with deficient mode information, which simultaneously consists of the exactly known, partially

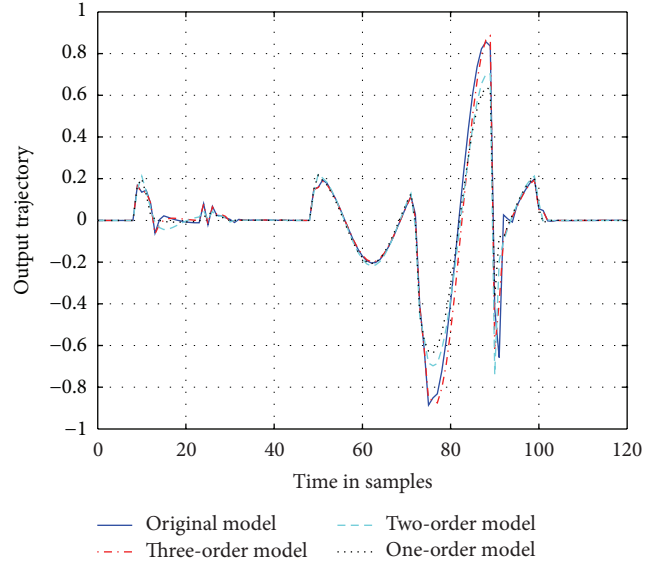


FIGURE 3: Output trajectories of the original system and reduced-order models with deficient TPM1 based on Theorem 6 (MRIA).

unknown, and polytopic-type uncertain transition probabilities and is thus more general and practical. By fully utilizing the properties of the transition probability matrices, together with the convexification of uncertain domains, a new \mathcal{H}_∞ performance analysis criterion has been developed. Two distinctly different approaches, namely, the convex linearisation approach and iterative approach, have been proposed to solve the model reduction problem. A simulation example has been given to illustrate the less conservatism and effectiveness of the proposed approaches.

Acknowledgments

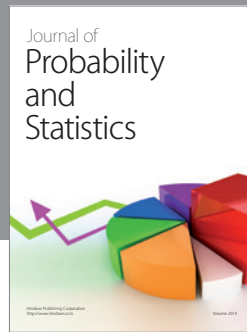
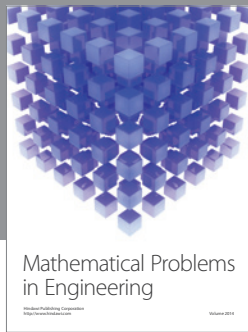
The authors are grateful to the guest editors and anonymous reviewers for their constructive comments based on which the presentation of this paper has been greatly improved. This work was supported in part by the National Natural Science Foundation of China (61004038), in part by the National 973 Project of China (2009CB320600), in part by the Program for New Century Excellent Talents in University, in part by the Fundamental Research Funds for the Central Universities (HIT.BRETIII.201214), and in part by the Alexander von Humboldt Foundation of Germany.

References

- [1] N. N. Krasovskii and È. A. Lidskii, “Analytical design of controllers in systems with random attributes. I. Statement of the problem, method of solving,” *Automation and Remote Control*, vol. 22, pp. 1021–1025, 1961.
- [2] E.-K. Boukas, *Stochastic Switching Systems: Analysis and Design*, Birkhäuser, Boston, Mass, USA, 2006.
- [3] Y. Liu, Z. Wang, and W. Wang, “Reliable \mathcal{H}_∞ filtering for discrete time-delay Markovian jump systems with partly unknown

- transition probabilities," *International Journal of Adaptive Control and Signal Processing*, vol. 25, no. 6, pp. 554–570, 2011.
- [4] L. Zhang, B. Huang, and J. Lam, " \mathcal{H}_∞ model reduction of Markovian jump linear systems," *Systems & Control Letters*, vol. 50, no. 2, pp. 103–118, 2003.
- [5] R. Yang, P. Shi, G.-P. Liu, and H. Gao, "Network-based feedback control for systems with mixed delays based on quantization and dropout compensation," *Automatica*, vol. 47, no. 12, pp. 2805–2809, 2011.
- [6] R. Yang, P. Shi, and G.-P. Liu, "Filtering for discrete-time networked nonlinear systems with mixed random delays and packet dropouts," *IEEE Transactions on Automatic Control*, vol. 56, no. 11, pp. 2655–2660, 2011.
- [7] J. Qiu, G. Feng, and J. Yang, "A new design of delay-dependent robust \mathcal{H}_∞ filtering for discrete-time T-S fuzzy systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 5, pp. 1044–1058, 2009.
- [8] J. Qiu, G. Feng, and H. Gao, "Fuzzy-model-based piecewise \mathcal{H}_∞ static-output-feedback controller design for networked nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 5, pp. 919–934, 2010.
- [9] J. Qiu, G. Feng, and H. Gao, "Nonsynchronized-State estimation of multichannel networked nonlinear systems with multiple packet dropouts Via TS Fuzzy-Affine dynamic models," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 1, pp. 75–90, 2011.
- [10] L. Huang and J. Chen, "Fuzzy PD control of networked control systems based on CMAC neural network," *Mathematical Problems in Engineering*, vol. 2012, Article ID 347217, 10 pages, 2012.
- [11] P. Shi, E.-K. Boukas, and R. K. Agarwal, "Robust control for Markovian jumping discrete-time systems," *International Journal of Systems Science*, vol. 30, no. 8, pp. 787–797, 1999.
- [12] H. R. Karimi, "Robust delay-dependent \mathcal{H}_∞ control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters," *IEEE Transactions on Circuits and Systems. I. Regular Papers*, vol. 58, no. 8, pp. 1910–1923, 2011.
- [13] G. He, J.-A. Fang, and X. Wu, "Robust stability of Markovian jumping genetic regulatory networks with mode-dependent delays," *Mathematical Problems in Engineering*, Article ID 504378, 18 pages, 2012.
- [14] J. Liu, Z. Gu, and S. Hu, " \mathcal{H}_∞ filtering for Markovian jump systems with time-varying delays," *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 3, pp. 1299–1310, 2011.
- [15] M. Karan, P. Shi, and C. Y. Kaya, "Transition probability bounds for the stochastic stability robustness of continuous- and discrete-time Markovian jump linear systems," *Automatica*, vol. 42, no. 12, pp. 2159–2168, 2006.
- [16] C. E. de Souza, "Robust stability and stabilization of uncertain discrete-time Markovian jump linear systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 5, pp. 836–841, 2006.
- [17] Y. Zhao, L. Zhang, S. Shen, and H. Gao, "Robust stability criterion for discrete-time uncertain markovian jumping neural networks with defective statistics of modes transitions," *IEEE Transactions on Neural Networks*, vol. 22, no. 1, pp. 164–170, 2011.
- [18] Y. Wei, M. Wang, and J. Qiu, "A new approach to delay-dependent \mathcal{H}_∞ filtering for discrete-time Markovian jump systems with time-varying delay and incomplete transition descriptions," *IET Control Theory and Applications*, vol. 7, no. 5, pp. 684–696, 2013.
- [19] L. Zhang and E.-K. Boukas, "Mode-dependent \mathcal{H}_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 6, pp. 1462–1467, 2009.
- [20] L. Zhang, E.-K. Boukas, and P. Shi, " \mathcal{H}_∞ model reduction for discrete-time Markov jump linear systems with partially known transition probabilities," *International Journal of Control*, vol. 82, no. 2, pp. 343–351, 2009.
- [21] Y. Wei, J. Qiu, H. R. Karimi, and M. Wang, "A new design of \mathcal{H}_∞ filtering for continuous-time Markovian jump systems with time-varying delay and partially accessible mode information," *Signal Processing*, vol. 93, no. 9, pp. 2392–2407, 2013.
- [22] H. Gao, J. Lam, and C. Wang, "Model simplification for switched hybrid systems," *Systems & Control Letters*, vol. 55, no. 12, pp. 1015–1021, 2006.
- [23] Q. Liu, W. Wang, and D. Wang, "New results on model reduction for discrete-time switched systems with time delay," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 5, pp. 3431–3440, 2012.
- [24] H. R. Shaker and R. Wisniewski, "Model reduction of switched systems based on switching generalized gramians," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 7, pp. 5025–5044, 2012.
- [25] K. Zhou, "Frequency-weighted \mathcal{L}_∞ norm and optimal Hankel norm model reduction," *IEEE Transactions on Automatic Control*, vol. 40, no. 10, pp. 1687–1699, 1995.
- [26] W.-Y. Yan and J. Lam, "An approximate approach to \mathcal{H}_2 optimal model reduction," *IEEE Transactions on Automatic Control*, vol. 44, no. 7, pp. 1341–1358, 1999.
- [27] L. Zhang and J. Lam, "On \mathcal{H}_2 model reduction of bilinear systems," *Automatica*, vol. 38, no. 2, pp. 205–216, 2002.
- [28] H. Gao, J. Lam, C. Wang, and S. Xu, " \mathcal{H}_∞ model reduction for discrete time-delay systems: delay-independent and dependent approaches," *International Journal of Control*, vol. 77, no. 4, pp. 321–335, 2004.
- [29] L. Wu and W. X. Zheng, "Weighted \mathcal{H}_∞ model reduction for linear switched systems with time-varying delay," *Automatica*, vol. 45, no. 1, pp. 186–193, 2009.
- [30] S. Xu, J. Lam, S. Huang, and C. Yang, " \mathcal{H}_∞ model reduction for linear time-delay systems: continuous-time case," *International Journal of Control*, vol. 74, no. 11, pp. 1062–1074, 2001.
- [31] J. Lam, H. Gao, S. Xu, and C. Wang, " \mathcal{H}_∞ and $\mathcal{L}_2/\mathcal{L}_\infty$ model reduction for system input with sector nonlinearities," *Journal of Optimization Theory and Applications*, vol. 125, no. 1, pp. 137–155, 2005.
- [32] L. Wu, X. Su, P. Shi, and J. Qiu, "Model approximation for discrete-time state-delay systems in the TS fuzzy framework," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 2, pp. 366–378, 2011.
- [33] X. Su, L. Wu, P. Shi, and Y. Song, " \mathcal{H}_∞ model reduction of Takagi-Sugeno fuzzy stochastic systems," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 42, no. 6, pp. 1574–1585, 2012.
- [34] X. Su, P. Shi, L. Wu, and Y. Song, "A novel approach to filter design for T-S fuzzy discrete-time systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 6, pp. 1114–1129, 2012.
- [35] X. Su, P. Shi, L. Wu, and Y. Song, "A novel control design on discrete-time Takagi-Sugeno fuzzy systems with time-varying delays," *IEEE Transactions on Fuzzy Systems*, no. 99, 2012.

- [36] X. Su, L. Wu, and P. Shi, "Sensor networks with random link failures: distributed filtering for T-S fuzzy systems," *IEEE Transactions on Industrial Informatics*, 2012.
- [37] L. El Ghaoui, F. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 1171–1176, 1997.
- [38] J. Qiu, G. Feng, and H. Gao, "Asynchronous output-feedback control of networked nonlinear systems with multiple packet dropouts: T-S fuzzy affine model-based approach," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 6, pp. 1014–1030, 2011.



Hindawi

Submit your manuscripts at
<http://www.hindawi.com>

