Jl. of Computers in Mathematics and Science Teaching (2008) **28**(2), 191-202

ICT for Inquiry in Mathematics: A Developmental Research Approach

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Computers and calculators are in general widely used in Norwegian schools, but with limited use in specific school subjects, as particularly in mathematics teaching. Various reports from surveys and research projects indicate that teachers' competence with ICT is a crucial point, and that teachers' lack of knowledge of how to utilise software for mathematics is a key challenge for further development. In the project ICT and mathematics learning (ICTML) at the University of Agder, the aim was to support the development of teachers' competence with ICT in order to improve mathematics teaching, learning, and problem solving with ICT.

The theoretical framework is a socio-cultural view of learning and the concept inquiry community. The project applies a developmental research methodology implementing an inquiry cycle in which teachers and didacticians collaborate in developing teaching and learning of mathematics with ICT and build understanding and competence with ICT for mathematics. In this article, I will present fundamental ideas of the project together with evidence from collaboration in inquiry communities with teachers and didacticians and characteristic features of activities in workshops and classrooms. The working model has proved to support and strengthen development, but development of teaching takes a long time.

INTRODUCTION AND BACKGROUND

In Norway, school authorities since 1984 through official documents and plan of actions (KUF, 1984; UFD, 2004), have promoted the use of Information and Communication Technology (ICT) as support for teaching and learning, and as a tool for work and problem solving. In this article, I mainly think of ICT-tools and digital tools as computers with software, although calculators and other digital equipment can be regarded as digital tools.

Computer studies or use of ICT was introduced in previous curriculum plans from 1987 and 1997. However, the use of computers has been scarce. In an evaluation of implementation of the previous curriculum plan in mathematics, hardly any activity with ICT was observed (Alseth, Breiteig, & Brekke, 2003). I can see several reasons for why implementation of ICT in mathematics teaching has been a slow process: Teachers lack knowledge of the tools and understanding of connections to mathematics, problems with equipment and software, organisation and leaders attitude, and not least the teachers' workload.

The situation has improved. Lack of equipment is no longer a problem. Computers are widely accessible in schools, with on average 4 to 7 pupils per machine in compulsory schools and more than 90% of the computers available for pupils have Internet access (Østerby, 2007). In general, computers are now used frequently and use of the word processor, the Internet, and Learning Management Systems (LMS) dominate, but with little use of ICT in specific school subjects. According to the ITU-report from 2007, no more than 17% of the pupils use ICT-tools in mathematics weekly, and 22% never in grades 7 and 9 in compulsory schools (Arnseth et al., 2007). It is reported that 46% to 55 % of the teachers' use of ICT is more than four hours per week, but mainly for preparation of their school work.

In the recently implemented curriculum plan from 2006, the Knowledge promotion (KD, 2006), use of digital tools is demanded in every school subject stating that "Being able to use digital tools" is one of five basic skills integrated in competence aims for all school subjects and with specific points given in the mathematics plan.

In my own experience, many teachers lack knowledge of how to utilise ICT-tools in mathematics teaching and often express the need to learn more about software and ideas for use in classes. The question is how can teachers develop their knowledge and competence to utilise digital tools in mathematics teaching? In the project *ICT and mathematics learning* (ICTML) at the University of Agder (UiA) the aim was to meet this challenge by inquiring into how digital tools can be utilised in mathematics, and in particular, how such tools can support inquiry approaches to teaching and learning. Inquiry is a key concept in the ICTML-project and in the project *Learning Communities in Mathematics* (LCM) with which ICTML collaborate closely.

DIGITAL COMPETENCE IN MATHEMATICS

What is digital competence in mathematics? – Basic skills with digital tools integrated in the competence aims. I would rather use the term digital competence, indicating a broader concept than just technical facilities. The curriculum plan states that basic skills involve using digital tools "for games, exploration, visualisation, and publication." Furthermore, it involves "learning how to use and assess digital aids for problem solving, simulation and modelling." And at last it states: "It is important to find information, analyse, process and present data with appropriate aids, and to be critical of sources, analyses, and results." (KD, 2006)

The plan for mathematics states some more specific requirements for use of digital tools, for example, use of spreadsheets and graph plotting. In many subtopics expressions like "with and without digital tools" indicate that digital tools can be used extensively for visualising mathematical concepts and as tools for problem solving.

I would suggest digital competence in mathematics is beyond the general knowledge, such as, to open programs and save files, know file systems and be able to use LMS systems or a word processor. Digital competence in mathematics deals with what is specific for the subject, like how mathematics can be represented and relations expressed in the software, and by that, provide opportunities for experimenting and exploring mathematical connections.

It is possible to use a variety of computer software according to the statement in the plan. For games and exploration, small software presenting a specific task or problem can be used. General or generic software are flexible and have the potential to develop as a tool for the user and can be utilised in several topics and with a variety of problems. The user can decide what to do. In the ICTML-project we chose to use generic software like a spreadsheet, a graph plotter, and dynamic geometry software.

THEORETICAL BACKGROUND

The theoretical framework for the research and development project presented in this article is a socio-cultural view of learning where learning is seen as mediated by cultural tools. Key ideas are *learning communities* and *inquiry.* The concept learning community builds on Wengers' (1998) concept community of practice, with the modes of belonging: engagement, imagination, and alignment where the last is elaborated into critical alignment (Jaworski, 2006). The participants join the learning community, engage in discussions, and develop ideas for work by imagining ways of using tasks and digital tools, trying out and critically aligning themselves with the project by engaging in discussing and critically trying out ideas. The idea is that teachers and didacticians work together in developmental activities and discussions; teachers work together in school teams and with their pupils in schools.

Inquiry means to ask questions, investigate, acquire information, or search for knowledge. Other words like wondering, experiment, and explore might fill in a broader picture of what is a characteristic attitude that is aimed for in the project. A willingness to wonder, seek to understand, and collaborate with others implies being active in dialogic inquiry (Wells, 2001). This attitude can be characterised as "inquiry as a way of being" and has become an aim in both the ICTML- and the LCM-project.

The use of ICT can be characterised by the view of ICT-tools, as an amplifier or as a reorganiser (Pea, 1987; Dörfler, 1993). The metaphor amplifier implies doing the same as before, just more efficiently, but with no fundamental changes in the objects and tasks to work on. Seeing ICT-tools as reorganisers imply changes in the objects to work on and the way we work. According to Dörfler this implies more work on meta-level, from carrying out to the planning of what to do. Perhaps preparing a model on a spreadsheet that can replace several calculations. The calculations are left to the spreadsheet but the user has to plan the model and make the connections between cells. By using a graph plotter for the study of families of functions, the work can change and new objects are available for the study. Work can be done on experimenting with parameters for curves instead of just drawing the curve several times. In some software the function graph itself can be dragged and the representations in other windows change accordingly, for example, in a table of function values or displaying the formula for the curve. Dynamic geometry, with the possibility of dragging and deforming figures represent another new kind of objects to work on. In order to fully utilise the potential of ICT-tools this reorganising should be intended and encouraged according to Dörfler. Also Goos, Gailbrath, Renshaw & Geiger (2003) argue that ICT-tools are not just passive neutral objects, but can reshape interaction between teachers, students, and the technology itself.

ACTIVITIES IN THE ICTML-PROJECT

The ICTML-project is both a developmental and research project in which teachers in lower secondary schools work together with didacticians at the University of Agder to develop knowledge and understanding of how ICT can support mathematics teaching and learning. The main emphases have been on the development of teachers' competence and inquiring into ways of using ICT-tools for teaching mathematics. To avoid confusion, the term didacticians is used for teachers and researchers at the university including doctoral students.

Four schools on lower secondary level with pupils 13-to-16 years of age took part in the project. Three of the schools took part in the LCM-project which ran in parallel and teachers in the fourth school were invited to take part in LCM-workshops. The workshops in LCM provided examples and experience with an inquiry approach to teaching mathematics in general.

The activities in the ICTML-project in particular aimed to support implementation of ICT-tools in mathematics with emphasis on an inquiry approach to teaching and learning. Use of suitable pieces of computer software alone does not create inquiry, but the way they are used, kinds of tasks, and how they are presented can provide inquiry approaches to the work. By asking questions, investigating, and experimenting with mathematical concepts and relations using suitable computer software and mathematical problems, the learner (teachers, didacticians, or pupils) will develop knowledge and insight both in mathematics and how the software can be used. In ICTML and the LCM-project, the work involved to inquire into mathematics, into teaching mathematics and development of mathematics teaching and for ICTML, in particularly, the use of ICT-tools related to the various levels of work.

The key activities to support development in the project were workshops, work in school teams, and implementation of ICT in teaching. Didacticians held meetings to plan for workshops and discuss issues related to research and development and plan for workshop activities. Two didacticians and an experienced teacher were responsible for planning activities for the ICTML-workshops, but also colleagues involved in LCM were present in the ICTML-workshops and engaged in the discussions.

By working together in the workshops and inquiring into the software, teachers and didacticians developed a learning community and developed their own knowledge about computer software for mathematics. Furthermore, suggested solutions were discussed, and solutions to examples from work in classrooms were presented and further ideas for development were discussed. Typically the workshops had sessions with introductions to possible software facilities with mathematical activities, a session of work on computers, and a summing up of experiences from the computer work and discussion of further development.

At each school it was intended that the teachers, at least three from each school, formed school teams for their collaboration in the project and set their own goals for the work. It was suggested to have regular meetings concerning their own work for discussing ideas for teaching, designing lessons, and developing materials for teaching. The didacticians took part in some of the school meetings. Work in the school team was expected to provide support for the teachers' implementation of ICT use in their own classes. The teachers were encouraged to, if possible, visit each others classes to observe and later discuss and reflect on what happened. Didacticians took part in the schools activities, observing work in classes, reflections after class visits, and further discussion of the work.

The design cycle can be seen as a guideline for the development work, with the main points: plan, act, observe, reflect, and feedback. The planning could start in a workshop or in a school team meeting when an idea for teaching was discussed. The planning was followed up by action or implementation of the plan in the classes and with observation by a didactician or other teachers. The reflection could follow just after the lesson or in a later school meeting with feedback and further discussion and perhaps start planning for a new cycle. As the inquiry approach developed, the cycle was seen as an inquiry cycle for the work.

RESEARCH METHODOLOGY

The methodology for research in the ICTML-project was closely connected to the developmental work. The development cycle informs and provides feedback to the research cycle, and also the research findings can inform the further work on development (Gravemeijer, 1994; Goodchild, 2008). The research dealt with all layers in the project and involved teachers, pupils, and didacticians in various roles: pupils work in class, teachers planning and work in class, workshops, didacticians planning and reflection meetings. For this reason all meetings with didacticians were tape-recorded. Workshops and classroom visits were audio- or video-recorded for observation and field notes were taken. Teachers were regarded as researchers on their own arena, and their discussions and reflections provided valuable insight. Teachers were also encouraged to take video- or audio-recordings of their work and could on some occasions provide material for further discussion and reflection on the work.

EXAMPLES FROM WORKSHOPS

A workshop for ICTML was planned in January 2006, a few weeks after an LCM workshop on algebra, and so the didacticians found it valuable to follow up with some ideas from the LCM workshop. The close relationship between the projects made this possible. The themes dealt with were functions and number patterns, and how to express connections. For the ICTML-workshop the plan was to use a spreadsheet and implement function relationships, number triangles and hidden formula to make investigations possible. At the start, the instructor gave an introduction to some of the facilities of the spreadsheet to express connections, some formatting issues and how to hide and protect formulae.

Various examples of number patterns and number triangles were made, some fairly similar to what the instructor had presented. Others utilised the features of hiding and protecting formulae making it possible to experiment with numbers and guess what the formula was.

Formler for geometriske figurer. Kan du finne hvilken?

x	Formel 1	Formel 2	Formel 3
1	6	0,5	3,14
2	12	2	12,56
3	18	4,5	28,26
4	24	8	50,24
5	30	12,5	78,5
6	36	18	113,04
7	42	24,5	153,86
8	48	32	200,96
9	54	40,5	254,34
10	60	50	314
11	66	60,5	379,94
12	72	72	452,16
13	78	84,5	530,66
14	84	98	615,44
15	90	112,5	706,5
16	96	128	803,84

Ark1 Ark2 Ark3

Figure 1. Geometrical figures – what formula?

The spreadsheet in Figure 1 was presented. In the columns C, D, and E calculations for three different geometrical figures were presented, with sizes related to x in the B column, with the question: "Formula for geometrical figures. Can you find which one?" The task is a variation of guess the formula. Reactions to the table in the discussion were: "It is an area of a rectangle …" The next comment was: "No, it is a perimeter …" The participants argued for their suggestions. Then the next question arouse: "Could it be both?" The discussion went on, could it be possible that the number calculated for perimeter and area of a rectangle is the same? Could the same be true for other figures? Which ones? The discussion provoked further questions. The inquiry approach led to further investigation and new ideas to discuss.

In the next workshop, the topic was to use more than one ICT-tool to solve a problem and compare and inquire into the solutions and connections between them. One of the suggested tasks was about building a sports arena within the area limited by the three roads. The area formed a right-angled triangle with smaller sides 30 and 40 metre.

The solution needed different approaches according to the software chosen for the task. When using a graph plotter, it was necessary to express functions for the area of the rectangle within the triangle for two possible positions. With a spreadsheet, a step-by-step solution was more accessible and with dynamic geometry, the geometric construction was the starting point and the various sizes could be investigated using the dragging feature.

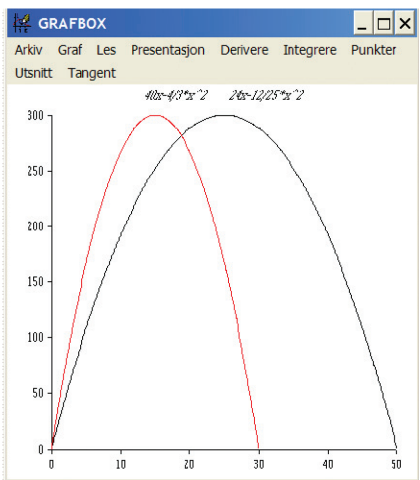

Figure 2. Graph plotter for sports arena.

Some of the participants expressed surprise about the result and had to use other tools to check. It seemed that the maximum was the same. Two didacticians used derivation to confirm their findings. Other solutions implied paper folding.

New questions came up – what if the question was not area, but perimeter? And will the questions give the same solution? What if the triangle is not right angled, is it possible to generalise the finding? Further questions are left for the reader to think about and inquire into.

INQUIRY CYCLE IN ACTION

In this section, I will present a brief narrative account of a case where a teacher developed some applications for inquiry using a spreadsheet. The tasks dealt with comparing decimal numbers, fractions, and percent for grade eight pupils. The story reveals how the development progressed and was tested and revised during two weeks. Two of the teachers had some experience with Excel before but were not experts according to their own judgement. A third colleague also took part in the meetings and observations in the class. He had strong experience with ICT. The teachers had set themselves the goal to develop their own library of spreadsheet tasks for investigations of mathematics. As this case started I had been with the school team in meetings and observed their classes a few times before.

One teacher wanted to develop some tasks for investigating relations between fractions, decimal numbers, and percentages. The first lesson on this started in the regular classroom where the teacher gave an introduction to the tasks, how to find and load the computer file, and how to proceed through the various sheets with tasks. The Excel document with tasks had been prepared the evening before. After the introduction, the pupils moved over to the computer lab and worked in pairs or alone on investigating the tasks, finding relationships between numbers, discussing and writing their comments. The instructor encouraged the pupils to write their comments directly into a text box in the Excel sheet with the task. I observed the work and video recorded some of the students' work. Two of the colleagues also came to the class, observed, and discussed a little with the students.

Just after the lesson, we had a meeting in the school team to discuss observations about the students work on computers. Simplifications to some tasks and improvements were discussed, together with new ideas for further development. Some constraints and possible ways to get around these were discussed together with other features of Excel to implement a new idea for investigating equal valued fractions. From the discussion, I noticed we inquired into various areas of the work, pupils' investigations and writing of explanations, improvements to the tasks and features of the software, constraints and affordances. Two days later, the teacher produced a new Excel file with tasks. The specific problem we had discussed was solved by some extra help from another colleague. The teacher expressed excitement and joy over the work, and was inspired to move on with even more tasks the coming weeks.

The cyclic pattern of plan, act, observe, reflect, and feedback is visible in the work which progressed over three cycles on this occasion. The support, joint reflection, and feedback from colleagues, and me as didactician helped the teachers to move on in their work. Although one teacher did most of the work on Excel this time, the help from the team and another colleague was also crucial for the success.

LEARNING FROM WORKING TOGETHER

Working together with teachers on development and inquiry into use of the ICT-tools played a key role in the ICTML-project and helped to build the learning community, or more specific inquiry community. Workshops were key activities for this. As teachers and didacticians worked together inquiring into mathematics and how ICT-tools might be utilised for inquiry, we developed mutual understanding and supportive relationship. Some didacticians and teachers both revealed a lack of understanding of some software. This balanced the roles and it became acceptable not to know the answer. Within the project community we had some expertise on ICT and could provide support and suggest further ways of exploring the software. Although the teachers had basic competence of spreadsheets at the start, many lacked knowledge of graph plotting and dynamic geometry.

The analysis of summary discussions in workshops looking at various solutions from the computer lab confirmed that inquiry took place and questions stimulated further inquiry. Unexpected results stimulated further inquiry, a closer examination, and justification. It stimulated reflections and sometimes use of other software or paper-and-pencil methods to confirm the result. Reflections discussed in plenary led to further explorations and investigations. To investigate various ways of solving a problem, using different pieces of software provided better insight and provoked inquiry into the connections and search for explanations. In some cases questions were raised concerning what the software could afford and their constraints. Limitations to the software tools appeared to challenge creativity to get around the constraints. This was seen when some teachers wanted to produce a symbolic algebra expressions in a triangular arrangement, like a number pattern.

The implementation of the project aims in schools varied. For the cases where teachers worked as a team they claim they benefited highly from the support of their colleagues and didacticians as they engaged in team meetings and observations in the class. The engagement and inspirations was observed both with teachers and didacticians, new ideas were created when experiences were discussed, and both parts inspired follow up with further work.

Features of the work that appeared to be successful with pupils were a clear introduction at the start of the lesson and summary of what has been learned in the end. It seemed clear that investigative work needs some structure in the organisation. It was observed that students struggle to write explanations.

CONCLUDING REMARKS

A key feature of the ICTML-project is the inquiry community to stimulate development of teachers' competence with ICT-tools and an inquiry approach to teaching mathematics. Observation data recorded during the project prove that collaborative inquiry can occur with teachers and didacticians when working on ICT. The experiences suggest that the close relationship, with mutual understanding and engagement in the work was important for the community building and provided opportunities for both development and research.

Development of teachers' competence with ICT-tools and use of inquiry approach to teaching takes time, and so far the project has only provided a start. The workshops and school team meetings proved to be a good way to build the community and provided stimulation for the work. Teachers need support to "dare to use ICT-tools" as expressed by some participants. Still teachers claim their practice has developed and they were becoming more conscious about their teaching.

Considering ICT-tools as reorganisers imply some reorganising of ways of working with mathematics using ICT and have further implications for tasks and problems to present. The ICTML-workshops provided substantial support for development of teachers' competence. However, three years of work with teachers is short time for such development and it is reasonable that further work is needed to improve teachers and pupils' understanding of the available tools and how mathematical problems might be solved.

The developmental model of the inquiry cycle of plan, act, observe, reflect, and feedback provide a helpful tool for developmental work and also support for the collection of data for research. The close relationship and mutual support of colleagues and didacticians with critical alignment, which implies critical discussion and inquiring into ICT-tools and their use, proves to be a way further for developing competence with ICT-tools.

References

Alseth, B., Breiteig, T., & Brekke, G. (2003). *Endringer og utvikling ved L97 som bakgrunn for videre planlegging og justering - matematikkfaget som kasus*. Notodden: Telemarksforsking.

Arnseth, H. K., Hatlevik, O., Kløvstad, V., Kristiansen, T., & Ottestad, G. (2007). *ITU monitor 2007 Skolens digitale tilstand 2007*. Oslo: Universitetsforlaget.

Dörfler, W. (1993). Computer use and the views of the mind. In C.Keitel & K. Ruthven (Eds.), *Learning from computers: Mathematics education and technology* (pp. 159-186). Springer.

Goodchild, S. (2008). A quest for good research. The mathematics teacher educator as practicioner researcher in a community of inquiry. In B.Jaworski & T. Wood (Eds.), *International Handbook on Mathematics Teacher Education: Vol. 4. The Mathematics Teacher Educator as a Developing Professional: Individuals, teams, communities and networks.* Rotterdam, Holland: Sense Publisher.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behaviour, 22,* 73-89.

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education. *Journal for Research in Mathematics Education, 25,* 443-471.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical Inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9,* 187-211.

KD (2006). Mathematics. In KD (Ed.), *Knowledge Promotion (http://www.udir.no)* Oslo: Kunnskapsdepartementet.

KUF (1984). *St.mld. nr 39 Datateknologi i skolen*. Oslo: Kirke og undervisningsdepartementet.

Østerby, S. (2007). *Utstyrs- og driftssituasjonen i grunnopplæringen 2006 - 2007 En gjentatt kartlegging og en sammenlikning med 2004 - 2005*. Utdanningsdirektoratet.

Pea, R. D. (1987). Cognitive technologies for mathematics education. In A.Schoenfeld (Ed.), *Cognitive Science and Mathematics Education* (pp. 89-122). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.

UFD (2004). *Program for digital kompetanse 2004-2008*. Oslo: Undervisnings- og forskningsdepartementet.

Wells, G. (2001). The development of a community of inquirers. In G.Wells (Ed.), *Action, talk, and text: Learning and teaching through inquiry.* (pp. 1-22). New York: Teachers College Press.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.